# Santa Fe Institute
# 2005 Complex Systems Summer School

## Week I: Introduction to Nonlinear Dynamics

**Instructor:**

Liz Bradley
Department of Computer Science
University of Colorado
Boulder CO 80309-0430 USA
lizb@cs.colorado.edu
www.cs.colorado.edu/∼lizb

**Syllabus:**

1. Introduction; Dynamics of Maps — chs 1 & 10 of [49]
   - a brief tour of nonlinear dynamics — [31] *(in [17])*
   - an extended example: the logistic map
     - how to plot its behavior
     - initial conditions, transients, and fixed points
     - bifurcations and attractors
     - chaos: sensitive dependence on initial conditions, $\lambda$, and all that
     - pitchforks, Feigenbaum, and universality — [22] *(in [17])*
     - the connection between chaos and fractals — [23], ch 11 of [49]
     - period-3, chaos, and the u-sequence — [30, 33] *(latter is in [17])*
     - *maybe:* unstable periodic orbits — [2, 25, 48]
2. Dynamics of Flows — [49], sections 2.0-2.3, 2.8, 5, and 6 (except 6.6 and 6.8)
   - maps vs. flows
     - time: discrete vs. continuous
     - axes: state/phase space — [9]
   - an example: the simple harmonic oscillator
     - some math & physics review — [8]
     - portraying & visualizing the dynamics — [9]
   - trajectories, attractors, basins, and boundaries — [9]
   - dissipation and attractors — [41]
   - bifurcations

- how sensitive dependence and the Lyapunov exponent manifest in flows
- anatomy of a chaotic attractor: [23]
  - stretching/folding and the un/stable manifolds
  - fractal structure and the fractal dimension ch 11 of [49]
  - unstable periodic orbits [2, 25, 48]
  - shadowing
  - *maybe:* symbol dynamics [26] *(in [13])*; [28]

3. Tools [1, 9, 36, 39]

- ODE solvers and their dynamics [8, 32, 34, 43]
- PDE solvers [8, 43]
- Poincaré sections [27]
- stability, eigenstuff, and un/stable manifolds (plus a bit of control theory)
- embedology [29, 38, 45, 46, 44, 51] *([38] is in [36] and [44] is in [52];)*
- *maybe:* calculating Lyapunov exponents and fractal dimensions [1, 9, 36, 39]

4. Applications [13, 36, 37]

- prediction [3, 4, 5, 14, 15, 52]
- filtering [20, 21, 24]
- control [7, 6, 11, 35, 47] *([35] is in [36])*
- communication [16, 40]
- classical mechanics [10, 42, 50, 53, 54]
- music, dance, and image [12, 18, 19]

# References


[1] H. Abarbanel. *Analysis of Observed Chaotic Data*. Springer, 1995.

[2] D. Auerbach, P. Cvitanovic, J.-P. Eckmann, G. Gunaratne, and I. Procaccia. Exploring chaotic motion through periodic orbits. *Physical Review Letters*, 58:2387–2389, 1987.

[3] T. Bass. *The Eudaemonic Pie*. Penguin, New York, 1992.

[4] T. Bass. *The Predictors*. Owl Books, 2000.

[5] D. Berreby. Chaos hits Wall Street. *Discover*, 14:76–84, March 1993.

[6] E. Bollt and J. Meiss. Targeting chaotic orbits to the moon. *Physics Letters A*, 204:373–378, 1995.

[7] E. Bradley. Using chaos to broaden the capture range of a phase-locked loop. *IEEE Transactions on Circuits and Systems*, 40:808–818, 1993.

[8] E. Bradley. Numerical solution of differential equations. Research Report on Curricula and Teaching CT003-98, University of Colorado, Department of Computer Science, 1998. `www.cs.colorado.edu/~lizb/na/ode-notes.ps`.

[9] E. Bradley. Analysis of time series. In *Intelligent Data Analysis: An Introduction*, pages 199–226. Springer-Verlag, 2 edition, 2000. M. Berthold and D. Hand, eds.

[10] E. Bradley. Classical mechanics. Research Report on Curricula and Teaching CT007-00, University of Colorado, Department of Computer Science, 2000. `www.cs.colorado.edu/~lizb/chaos/classmech.ps`.

[11] E. Bradley and D. Straub. Using chaos to broaden the capture range of a phase-locked loop: Experimental verification. *IEEE Transactions on Circuits and Systems*, 43:914–822, 1996.

[12] E. Bradley and J. Stuart. Using chaos to generate variations on movement sequences. *Chaos*, 8:800–807, 1998.

[13] D. Campbell, R. Ecke, and J. Hyman. *Nonlinear Science: The Next Decade*. M.I.T. Press, 1992.

[14] M. Casdagli. Nonlinear prediction of chaotic time series. *Physica D*, 35:335–356, 1989.

[15] M. Casdagli and S. Eubank, editors. *Nonlinear Modeling and Forecasting*. Addison Wesley, 1992.

[16] K. M. Cuomo and A. V. Oppenheim. Circuit implementation of synchronized chaos with applications to communications. *Physical Review Letters*, 71:65–68, 1993.

[17] P. Cvitanovic. Introduction. In *Universality in Chaos*. Adam Hilger, Bristol U.K., 1984.

[18] D. Dabby. Musical variations from a chaotic mapping. *Chaos*, 6:95–107, 1996.

[19] D. Dabby. A chaotic mapping for musical and image variation. In *Proceedings of the Fourth Experimental Chaos Conference*, 1997.

[20] J. Farmer and J. Sidorowich. Predicting chaotic time series. *Physical Review Letters*, 59:845–848, 1987.

[21] J. Farmer and J. Sidorowich. Exploiting chaos to predict the future and reduce noise. In *Evolution, Learning and Cognition*. World Scientific, 1988.

[22] M. J. Feigenbaum. Universal behavior in nonlinear systems. *Los Alamos Science*, 1:4–27, 1980.

[23] C. Grebogi, E. Ott, and J. A. Yorke. Chaos, strange attractors and fractal basin boundaries in nonlinear dynamics. *Science*, 238:632–638, 1987.

[24] J. Guckenheimer. Noise in chaotic systems. *Nature*, 298:358–361, 1982.

[25] G. Gunaratne, P. Linsay, and M. Vinson. Chaos beyond onset: A comparison of theory and experiment. *Physical Review Letters*, 63:1, 1989.

[26] B.-L. Hao. Symbolic dynamics and characterization of complexity. *Physica D*, 51:161–176, 1991.

[27] M. Hénon. On the numerical computation of Poincaré maps. *Physica D*, 5:412–414, 1982.

[28] C. Hsu. *Cell-to-Cell Mapping*. Springer-Verlag, New York, 1987.

[29] J. Iwanski and E. Bradley. Recurrence plots of experimental data: To embed or not to embed? *Chaos*, 8:861–871, 1998.

[30] T.-Y. Li and J. A. Yorke. Period three implies chaos. *American Mathematical Monthly*, 82:985–992, 1975.

[31] E. N. Lorenz. Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*, 20:130–141, 1963.

[32] E. N. Lorenz. Computational chaos – A prelude to computational instability. *Physica D*, 35:229–317, 1989.

[33] N. Metropolis, M. Stein, and P. Stein. On finite limit sets for transformations of the unit interval. *J. Combinational Theory*, 15:25–44, 1973.

[34] R. Miller. A horror story about integration methods. *J. Computational Physics*, 93:469–476, 1991.

[35] E. Ott, C. Grebogi, and J. A. Yorke. Controlling chaos. *Physical Review Letters*, 64:1196–1199, 1990.

[36] E. Ott, T. Sauer, and J. Yorke. *Coping with chaos*. Wiley, 1994.

[37] J. M. Ottino. *The Kinematics of Mixing: Stretching, Chaos, and Transport*. Cambridge, Cambridge U.K., 1992.

[38] N. Packard, J. Crutchfield, J. Farmer, and R. Shaw. Geometry from a time series. *Physical Review Letters*, 45:712, 1980.

[39] T. S. Parker and L. O. Chua. *Practical Numerical Algorithms for Chaotic Systems*. Springer-Verlag, New York, 1989.

[40] L. M. Pecora and T. L. Carroll. Synchronization in chaotic systems. *Physical Review Letters*, 64:821–824, 1990.

[41] I. Percival. Chaos in Hamiltonian systems. *Proceedings of the Royal Society, London*, 413:131–144, 1987.

[42] I. Peterson. *Newton's Clock: Chaos in the Solar System*. W. H. Freeman, New York, 1993.

[43] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling. *Numerical Recipes: The Art of Scientific Computing*. Cambridge University Press, Cambridge U.K., 1988.

[44] T. Sauer. Time-series prediction by using delay-coordinate embedding. In *Time Series Prediction: Forecasting the Future and Understanding the Past*. Santa Fe Institute Studies in the Sciences of Complexity, Santa Fe, NM, 1993.

[45] T. Sauer. Interspike interval embedding of chaotic signals. *Chaos*, 5:127, 1995.

[46] T. Sauer, J. Yorke, and M. Casdagli. Embedology. *Journal of Statistical Physics*, 65:579–616, 1991.

[47] T. Shinbrot. Progress in the control of chaos. *Advances in Physics*, 44:73–111, 1995.

[48] P. So, E. Ott, S. Schiff, D. Kaplan, T. Sauer, and C. Grebogi. Detecting unstable periodic orbits in chaotic experimental data. *Physical Review Letters*, 76:4705–4708, 1996.

[49] S. Strogatz. *Nonlinear Dynamics and Chaos*. Addison-Wesley, Reading, MA, 1994.

[50] G. J. Sussman and J. Wisdom. Chaotic evolution of the solar system. *Science*, 257:56–62, 1992.

[51] F. Takens. Detecting strange attractors in fluid turbulence. In D. Rand and L.-S. Young, editors, *Dynamical Systems and Turbulence*, pages 366–381. Springer, Berlin, 1981.

[52] A. S. Weigend and N. S. Gershenfeld, editors. *Time Series Prediction: Forecasting the Future and Understanding the Past*. Santa Fe Institute Studies in the Sciences of Complexity, Santa Fe, NM, 1993.

[53] J. Wisdom. Chaotic behavior in the solar system. *Nuclear Physics B*, 2:391–414, 1987.

[54] J. Wisdom. Is the solar system stable? and Can we use chaos to make measurements? In D. K. Campbell, editor, *Chaos/XAOC : Soviet-American perspectives on nonlinear science*. American Institute of Physics, New York, 1990.

References [1, 3, 4, 13, 15, 17, 28, 36, 49, 52] are in the CSSS library.

**More Resources:** `http://www.cs.colorado.edu/~lizb`

`http://amath.colorado.edu/faculty/jdm/faq.html`