A radiative transfer framework for non-exponential media

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Magnus Wrenninge [4] Jan Novák [3] Steve Marschner [5] Wojciech Jarosz [1]

[1] Dartmouth College [2] ETH Zurich [3] Disney Research [4] Pixar Animation Studios [5] Cornell University

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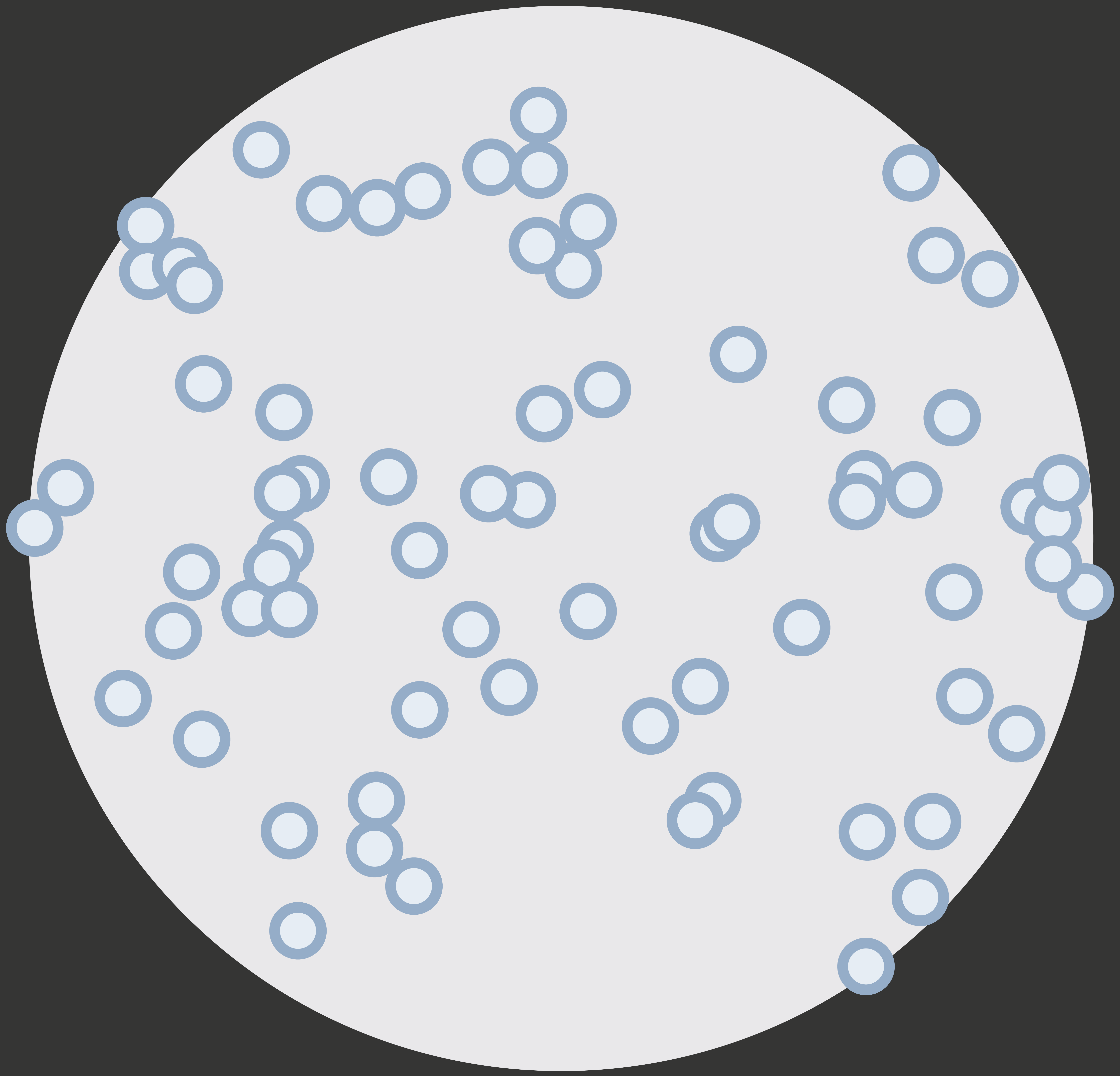
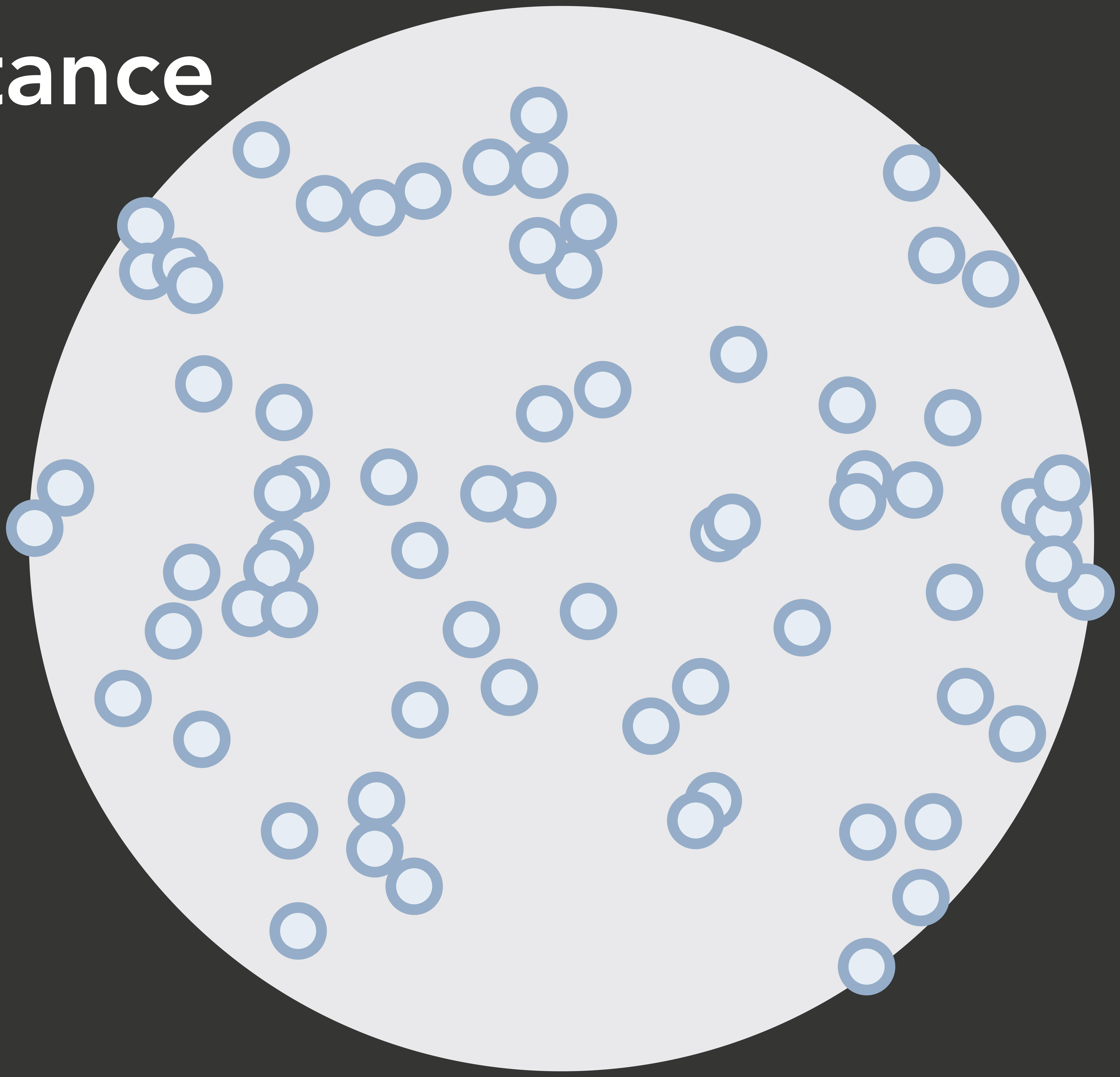
Ross Burgener


Jan Novak


NASA



Transmittance



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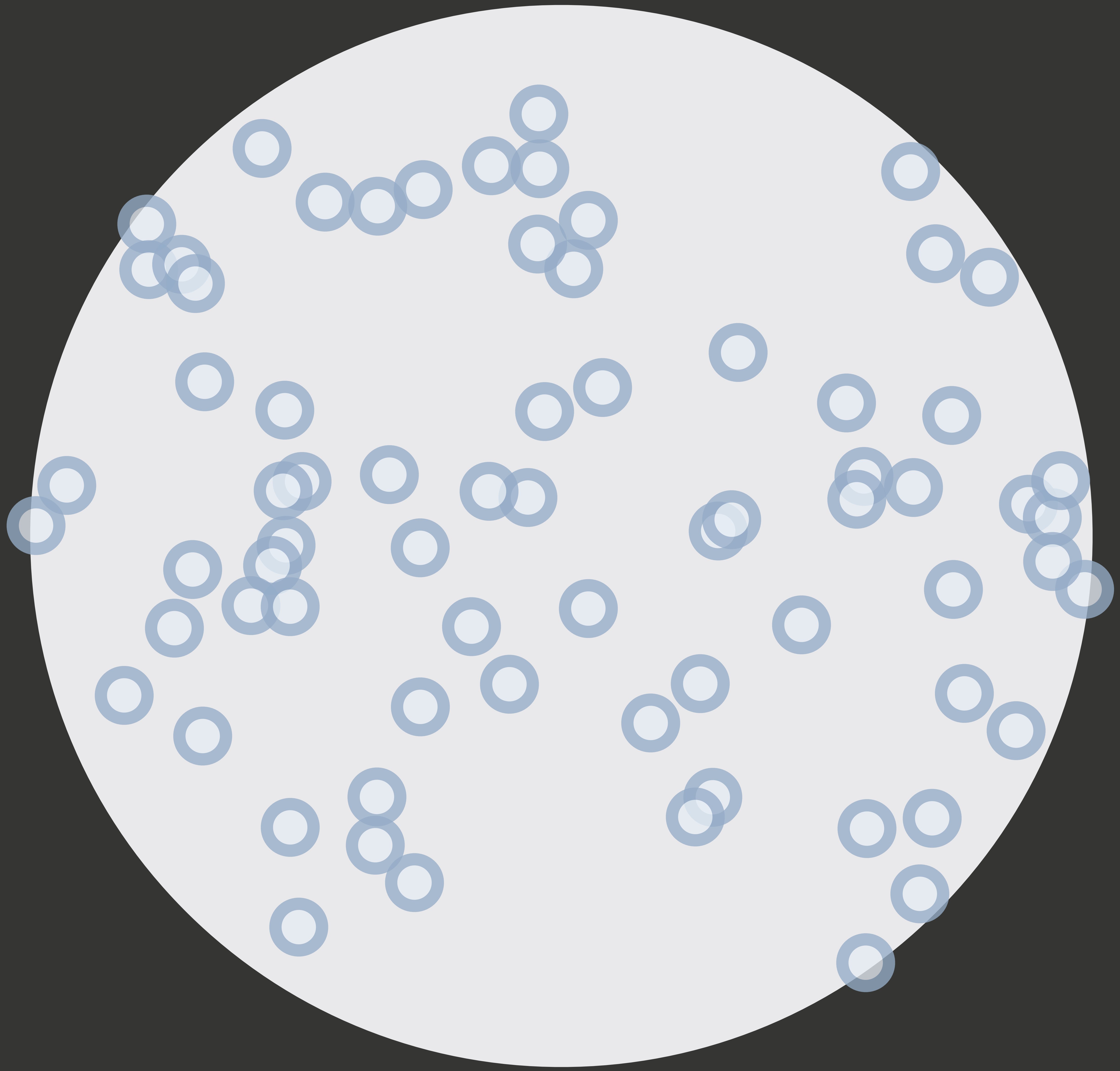
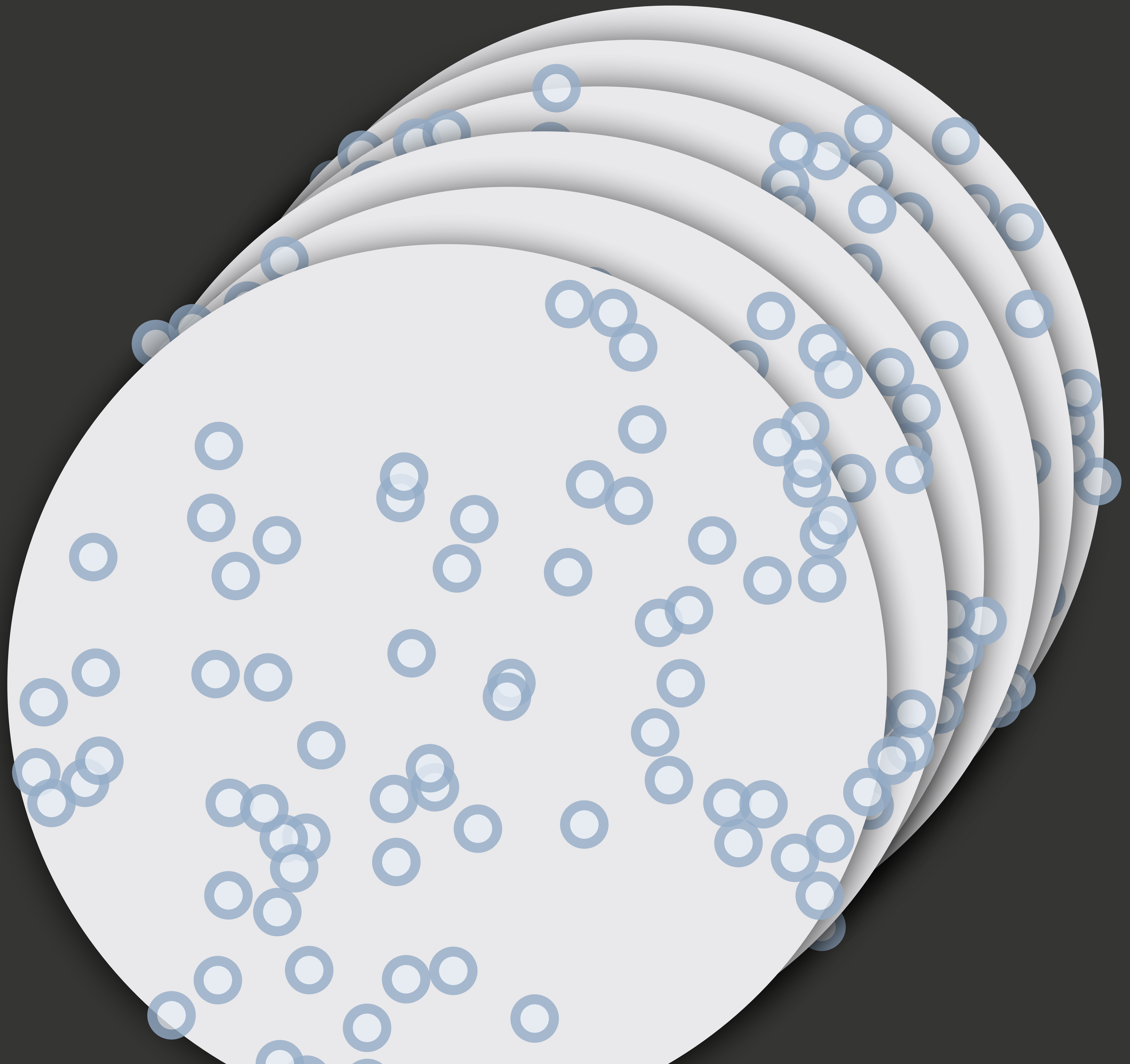
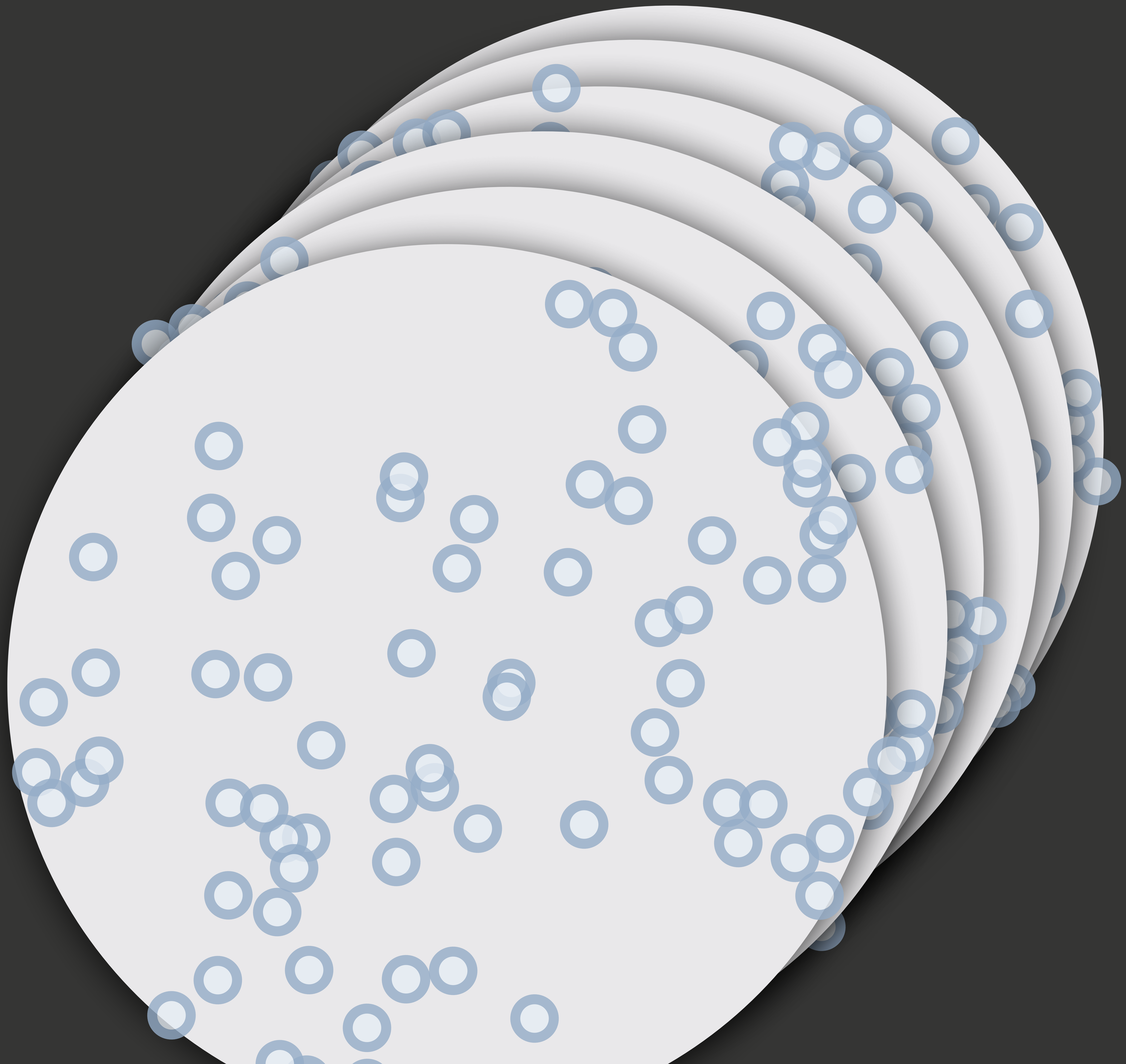




$\mathbb{E}\Big[$  $\Big]$

Transmittance

$\mathbb{E}[\quad]$

Radiative Transfer Theory

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- **Radiative Transfer,** *Chandrasekhar,* **1960**

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- Assumes that particle positions are *independent*

Transmittance

$\mathbb{E}[\quad]$

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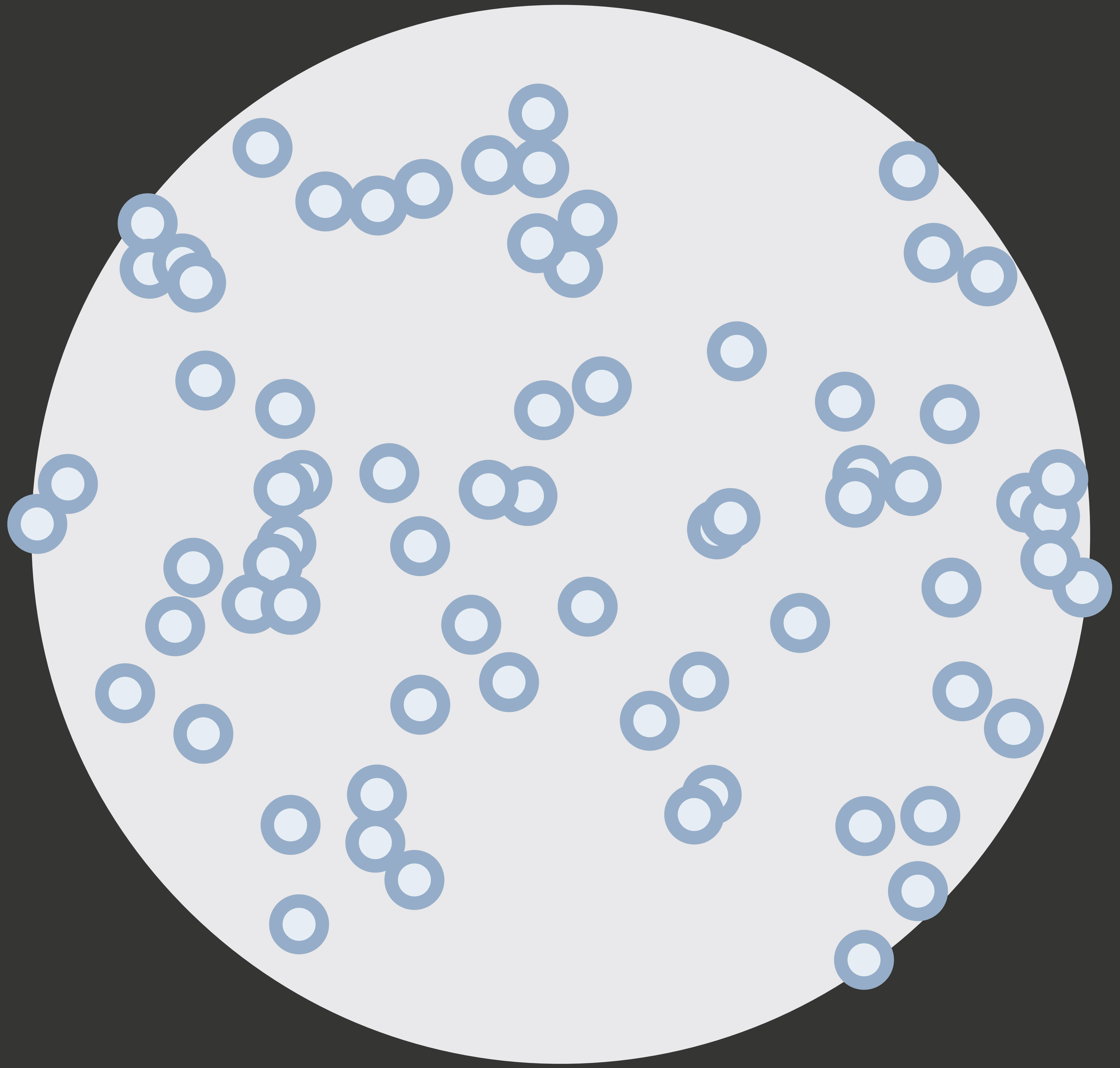
Particle Correlations

Inter-particle Forces



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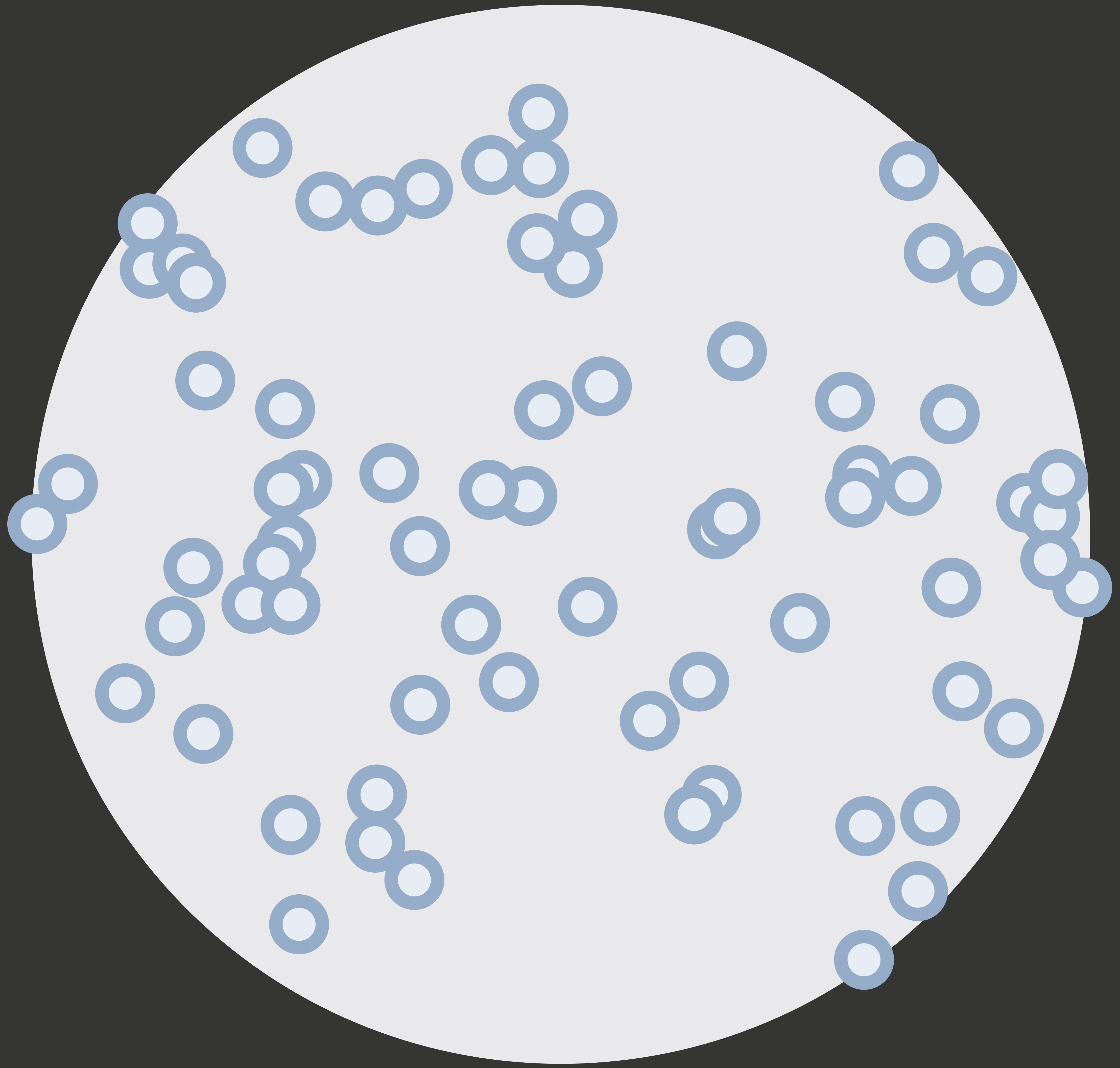

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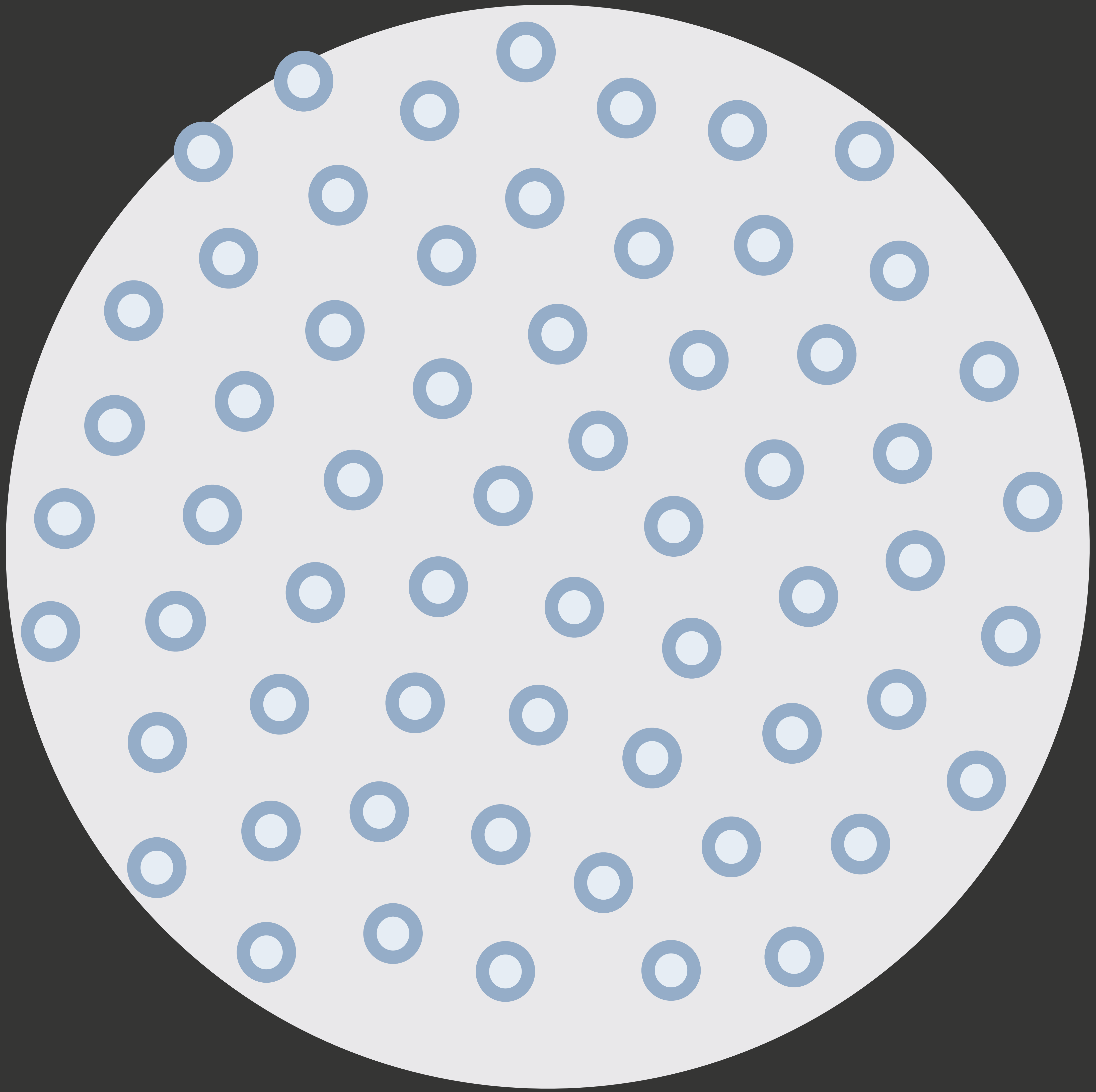

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Clouds

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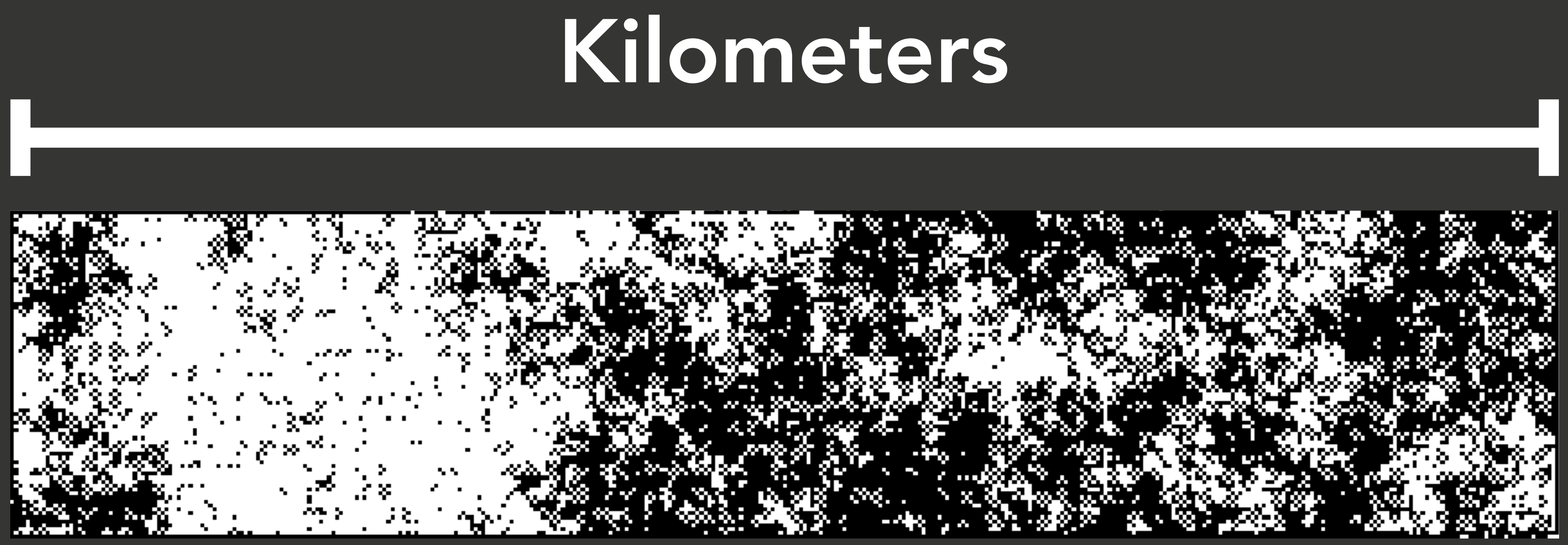

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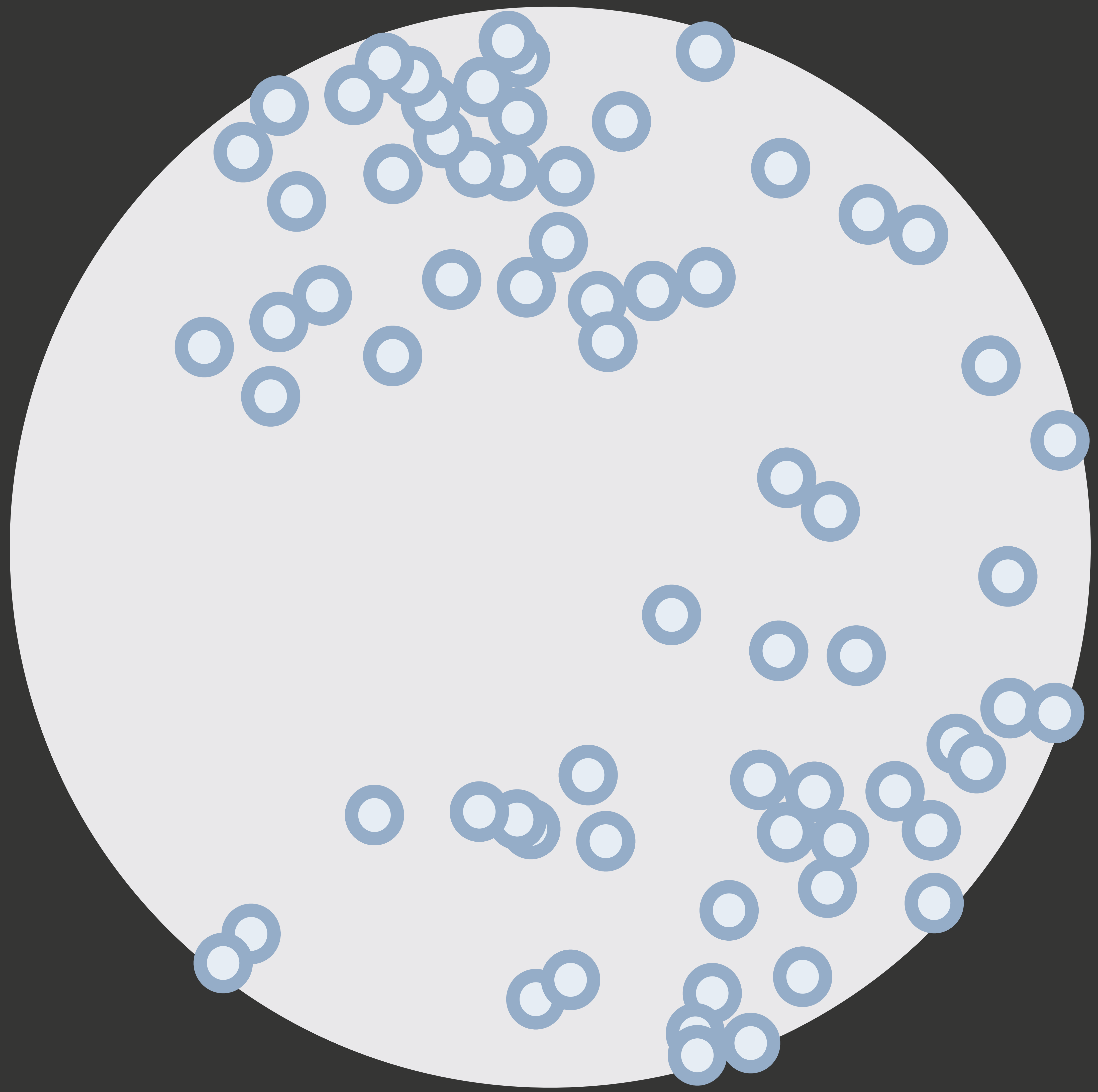

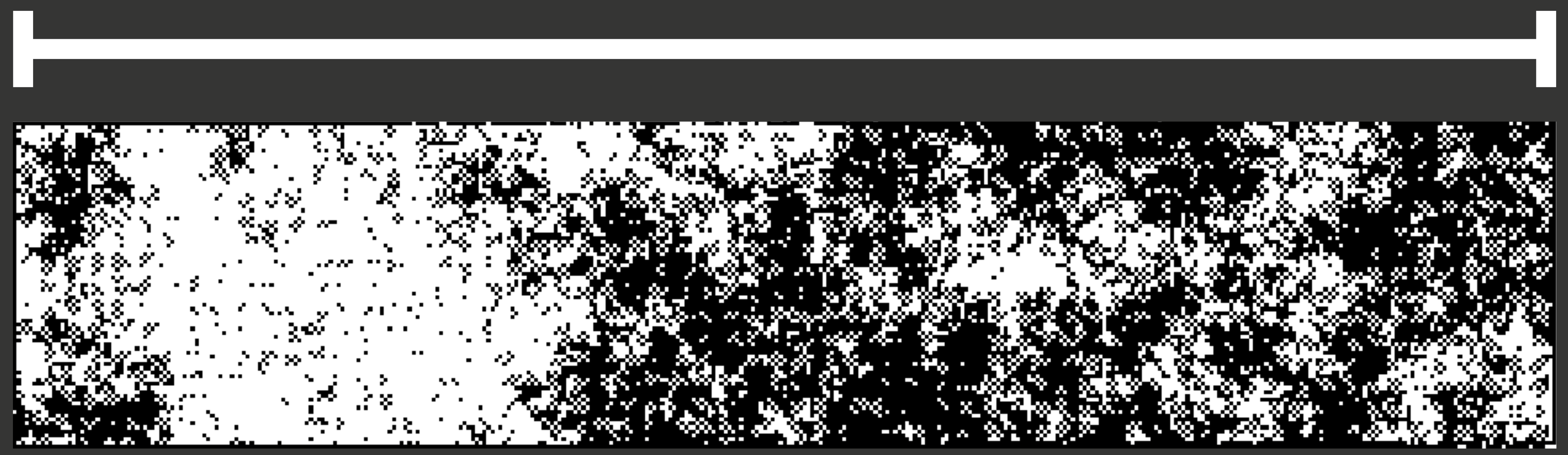

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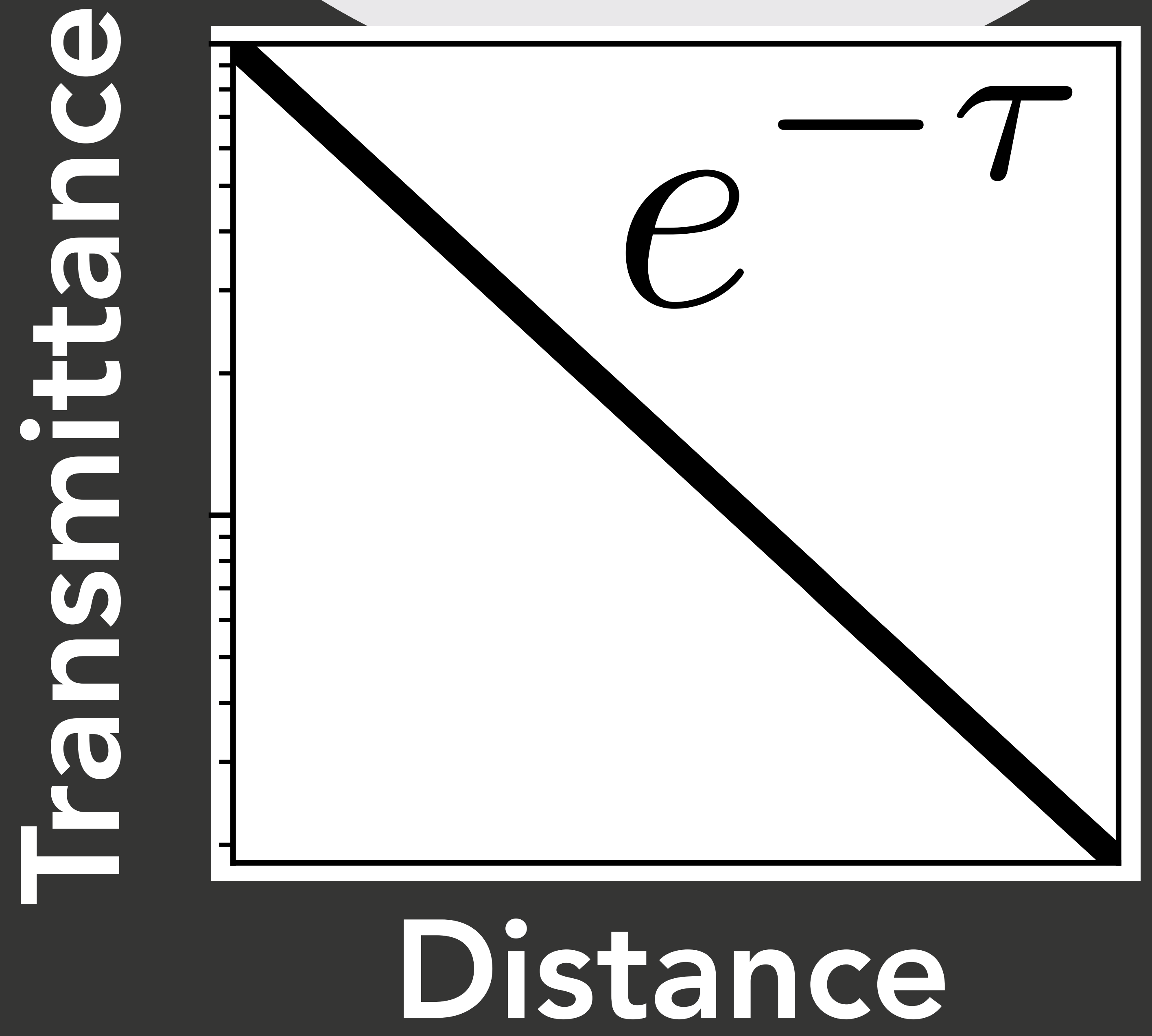

$e^{-\tau}$
Transmittance
Distance


Transmittance
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The Rendering Equation

$$L_i(\mathbf{x}, \omega) = \mathrm{Tr}(\mathbf{x}, \mathbf{x}_s) L_o(\mathbf{x}_s, \omega) + \int_0^s \mathrm{Tr}(\mathbf{x}, \mathbf{x}_t) \sigma_s(\mathbf{x}_t) L_s(\mathbf{x}_t, \omega) \mathrm{d}t$$

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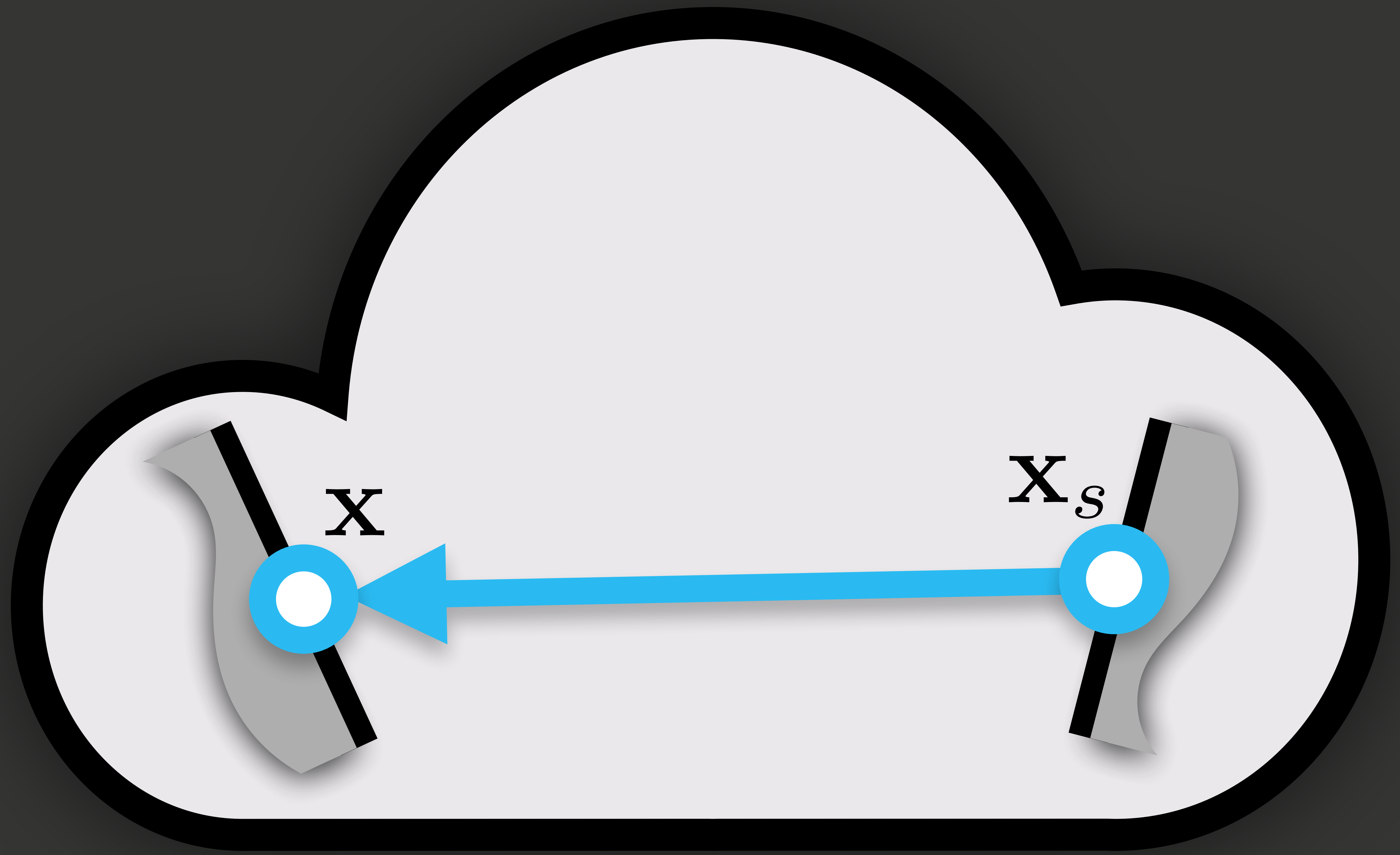

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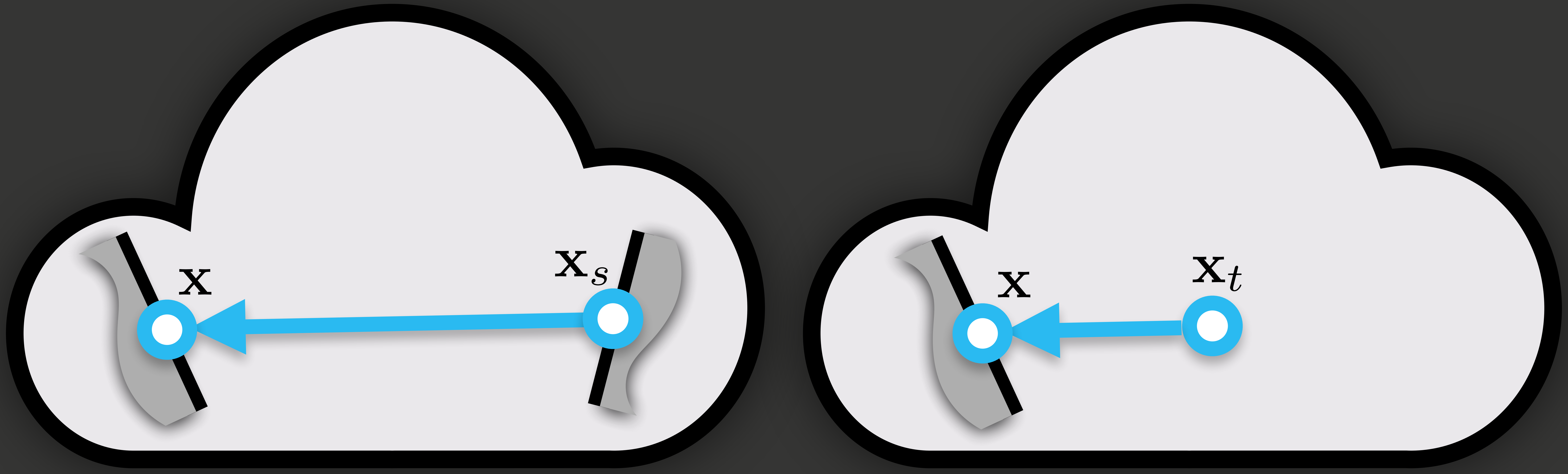

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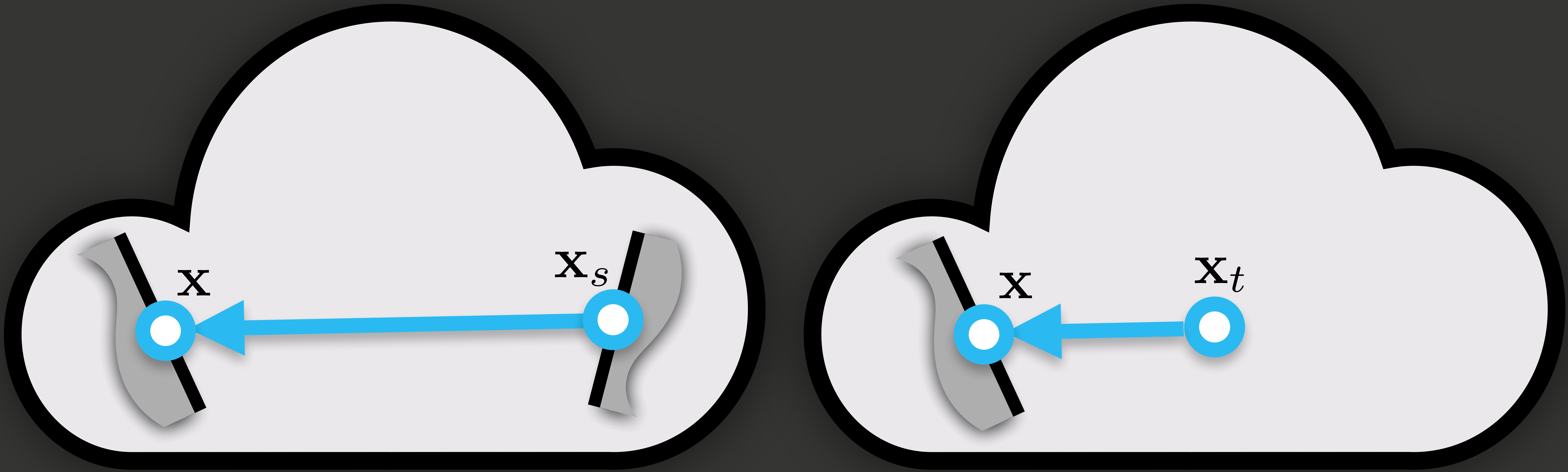

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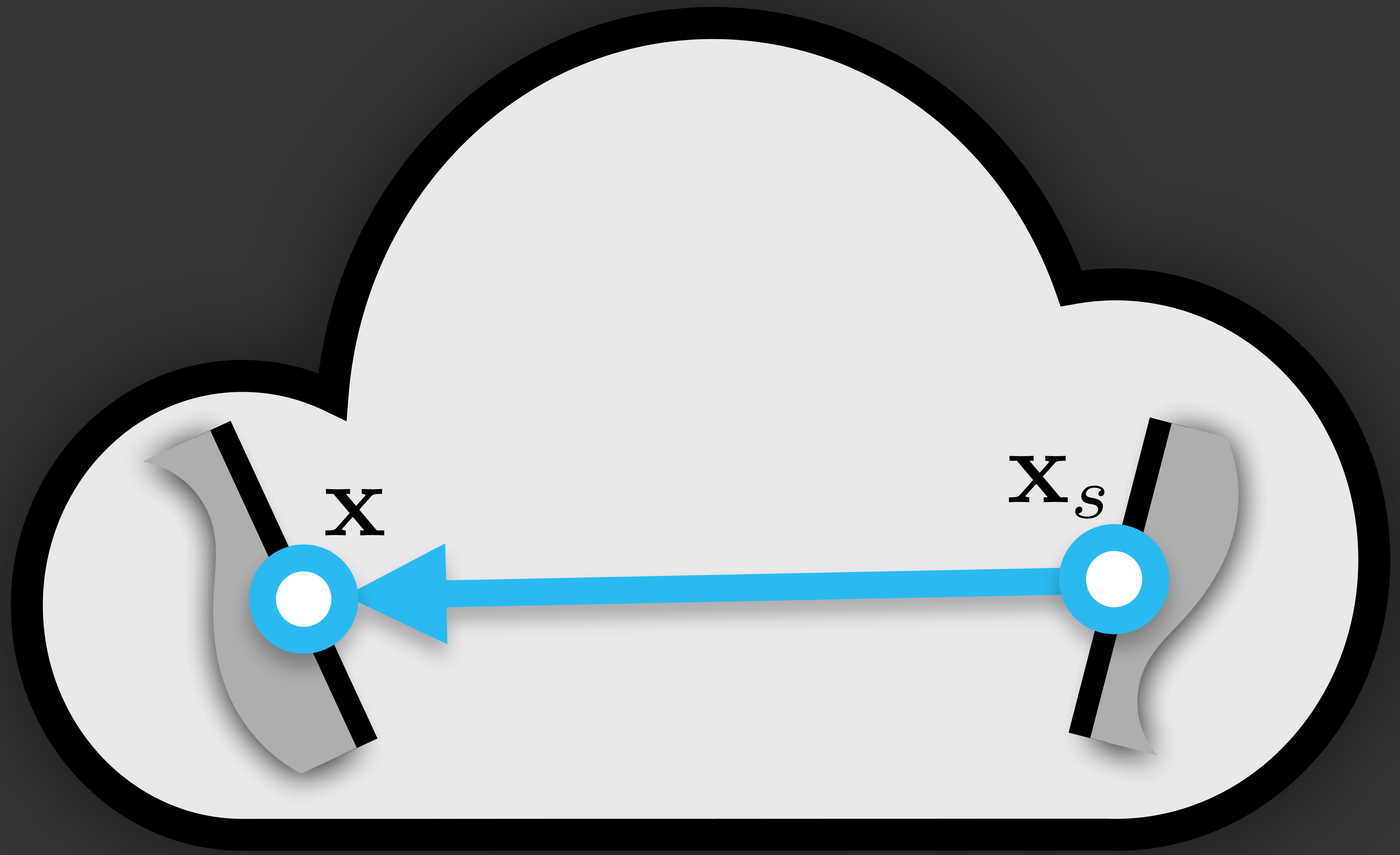

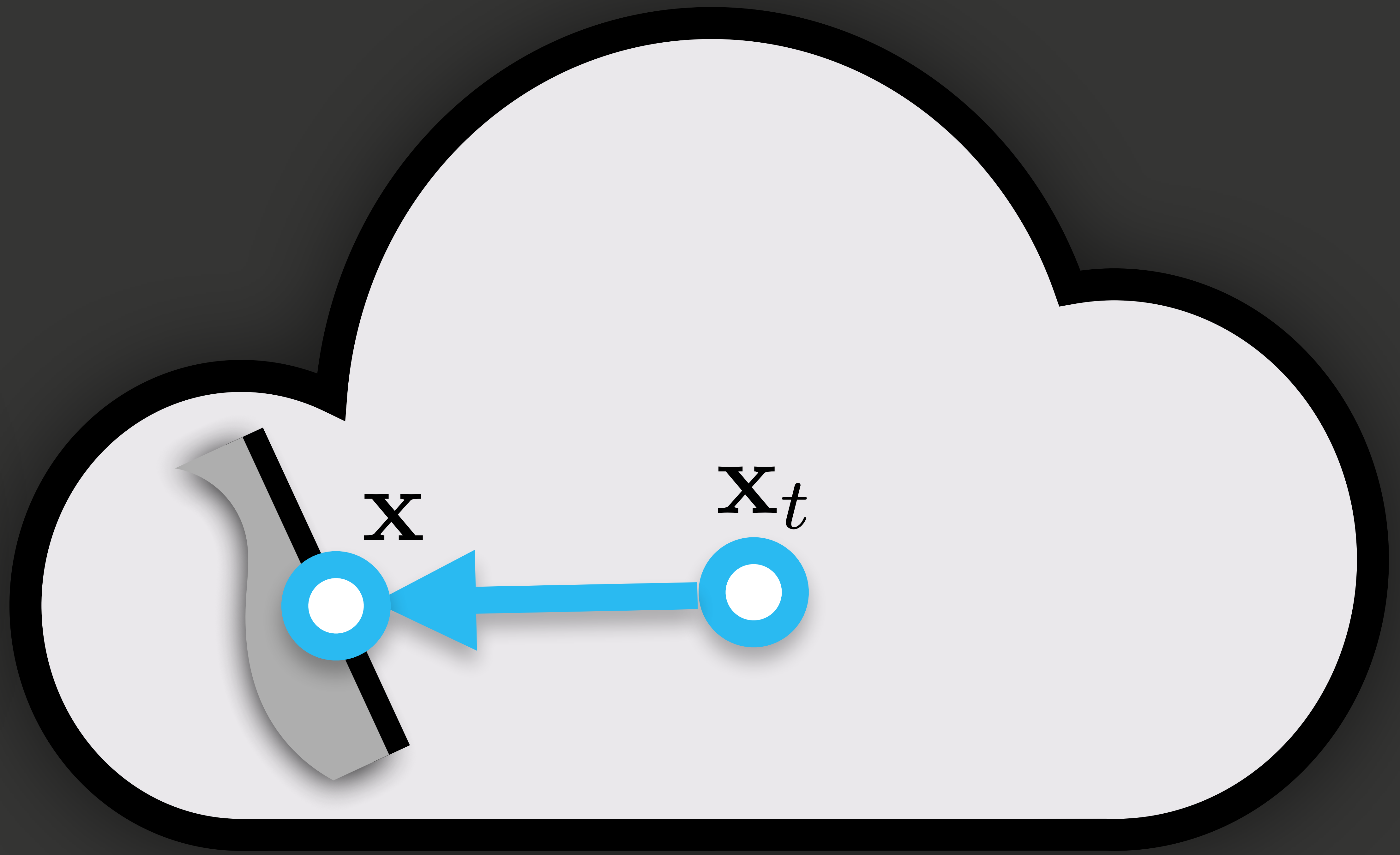

$$L_i(\mathbf{x},\omega) = e^{-\tau(\mathbf{x},\mathbf{x}_t)} L_o(\mathbf{x}_s,\omega) + \int_0^s e^{-\tau(\mathbf{x},\mathbf{x}_t)} \sigma_s(\mathbf{x}_t) L_s(\mathbf{x}_t,\omega)\mathrm{d}t$$

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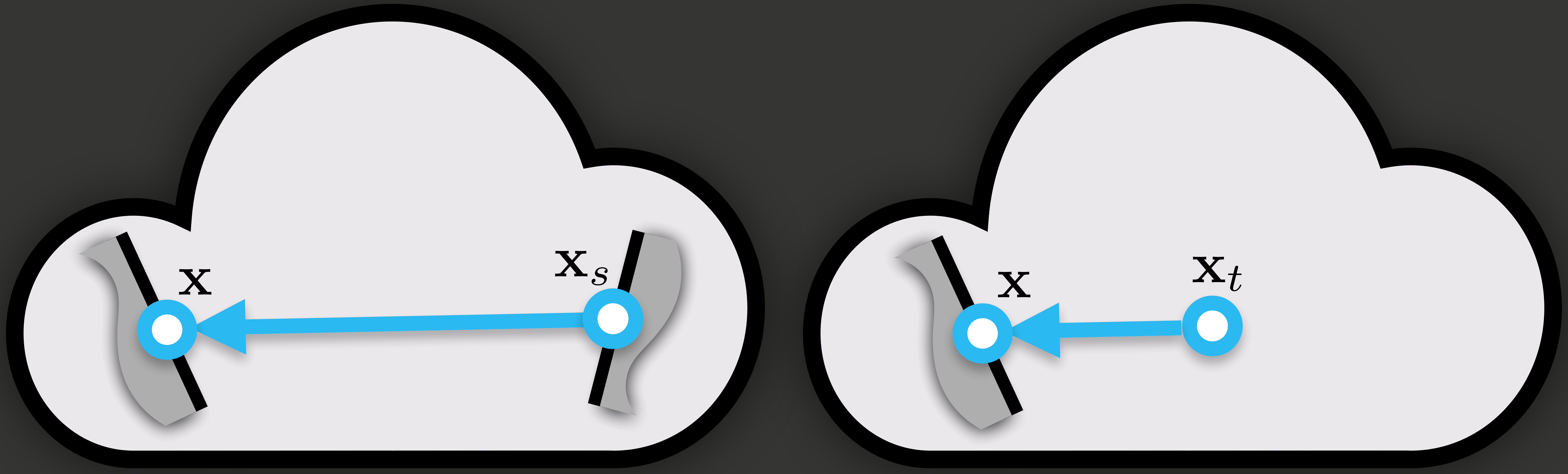

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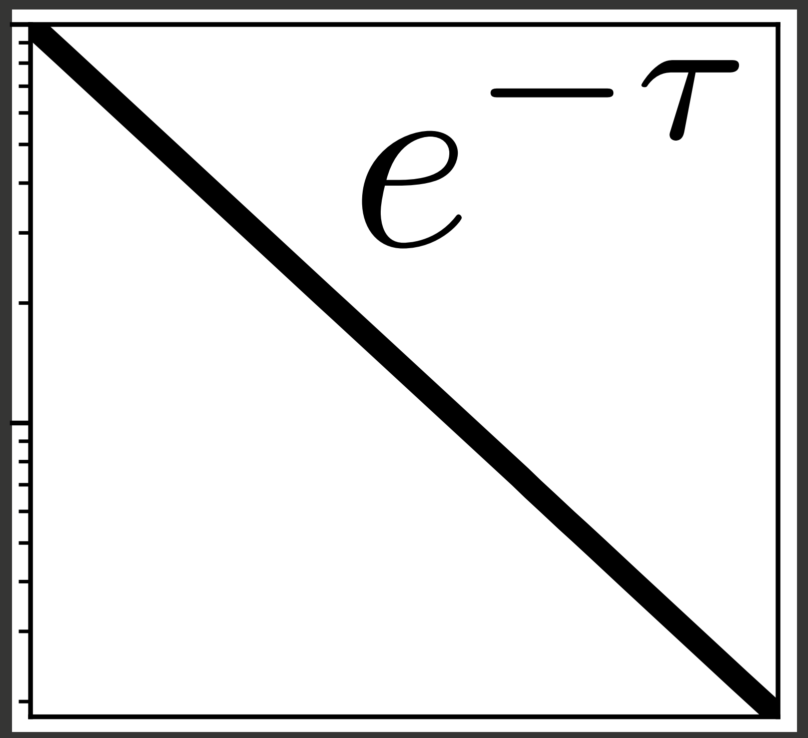
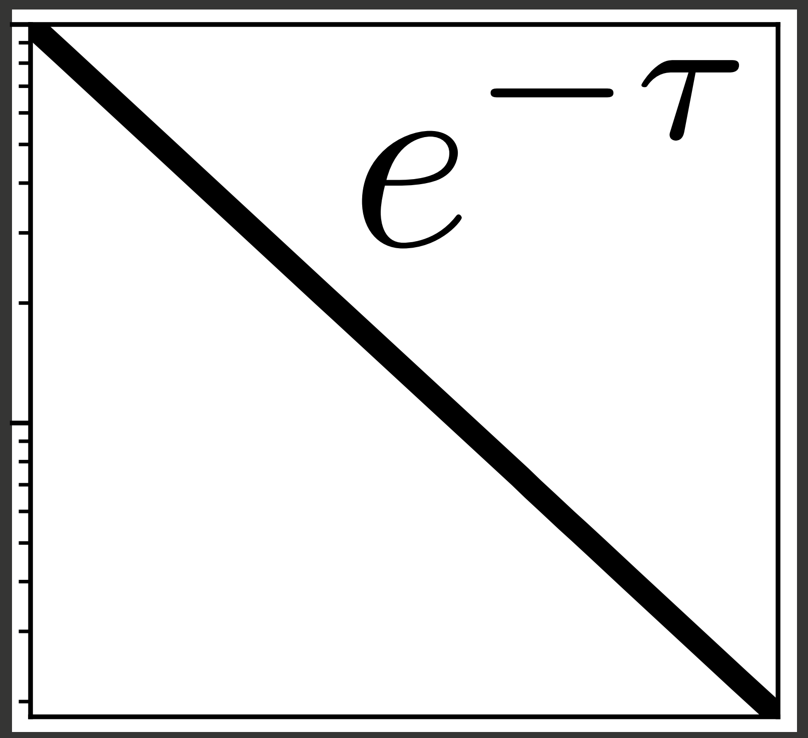
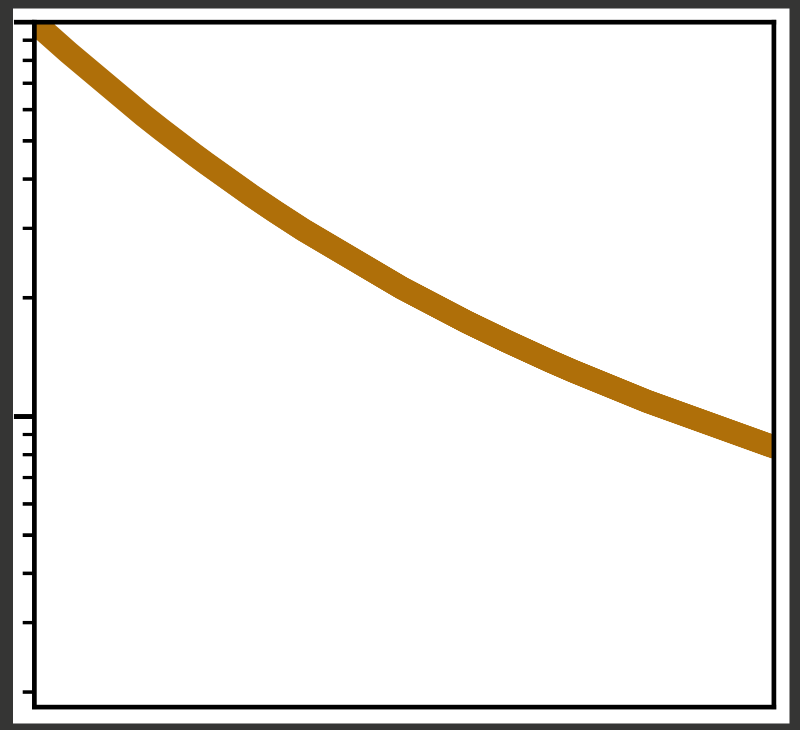
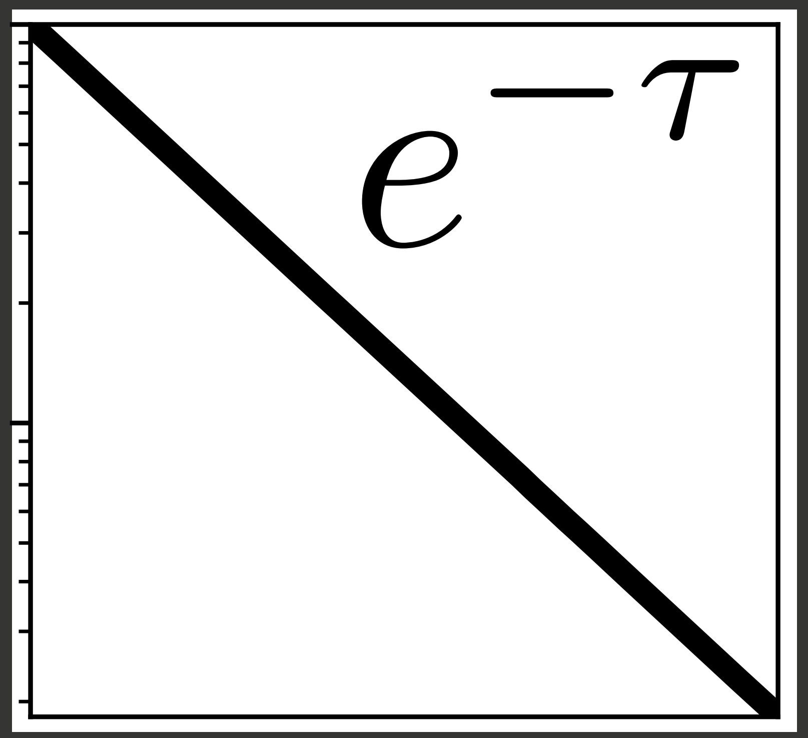
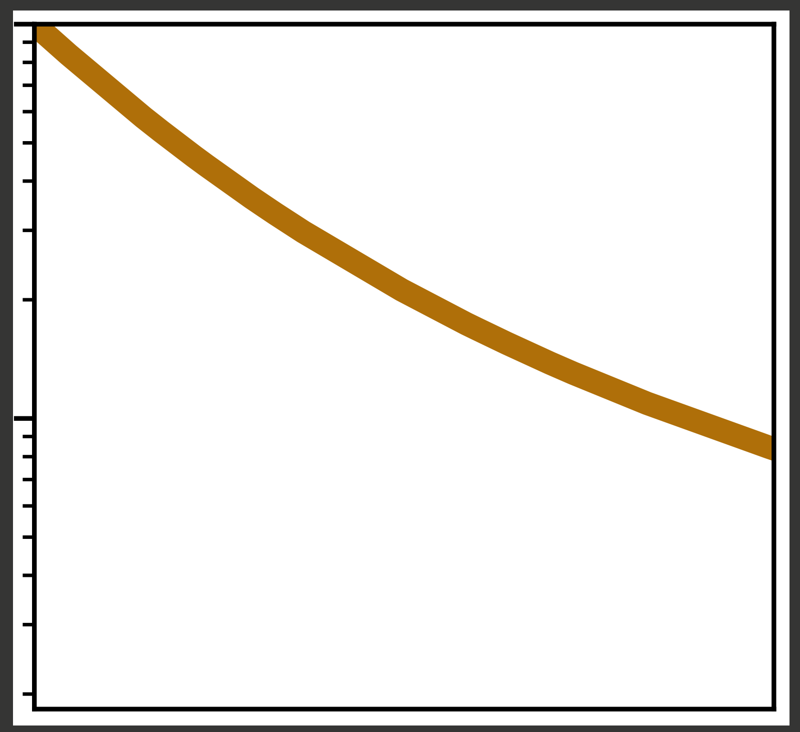
$$\mathbf{Tr} =$$

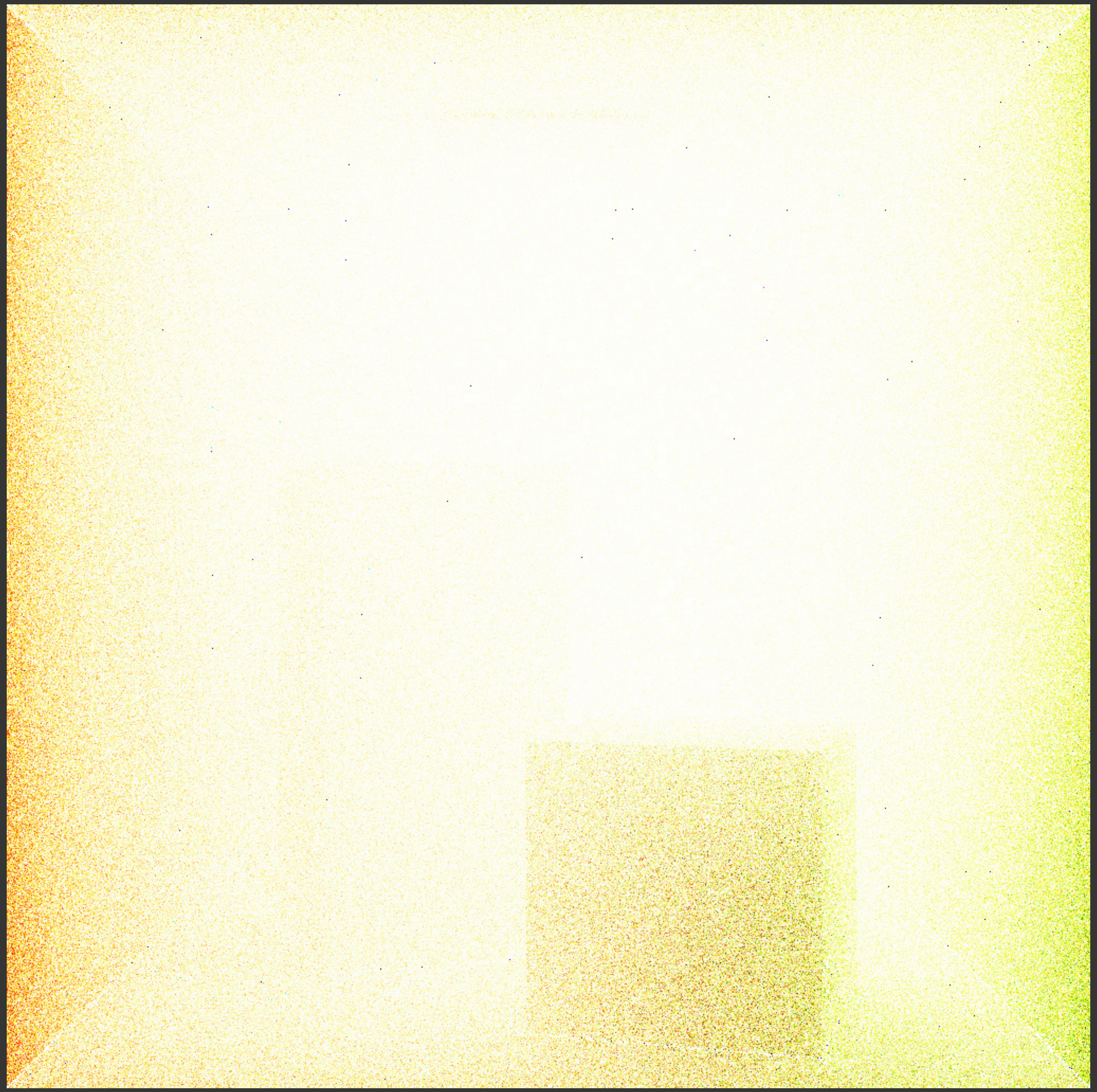


$\mathbf{Tr} =$ 

$\mathbf{Tr} =$ 

$\mathbf{Tr} =$ 

$\mathbf{Tr} =$ 



Summary

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- Correlated particles lead to non-exponential transmittance
- Non-exponential transmittance breaks classical transport
- We need a *new transport framework*

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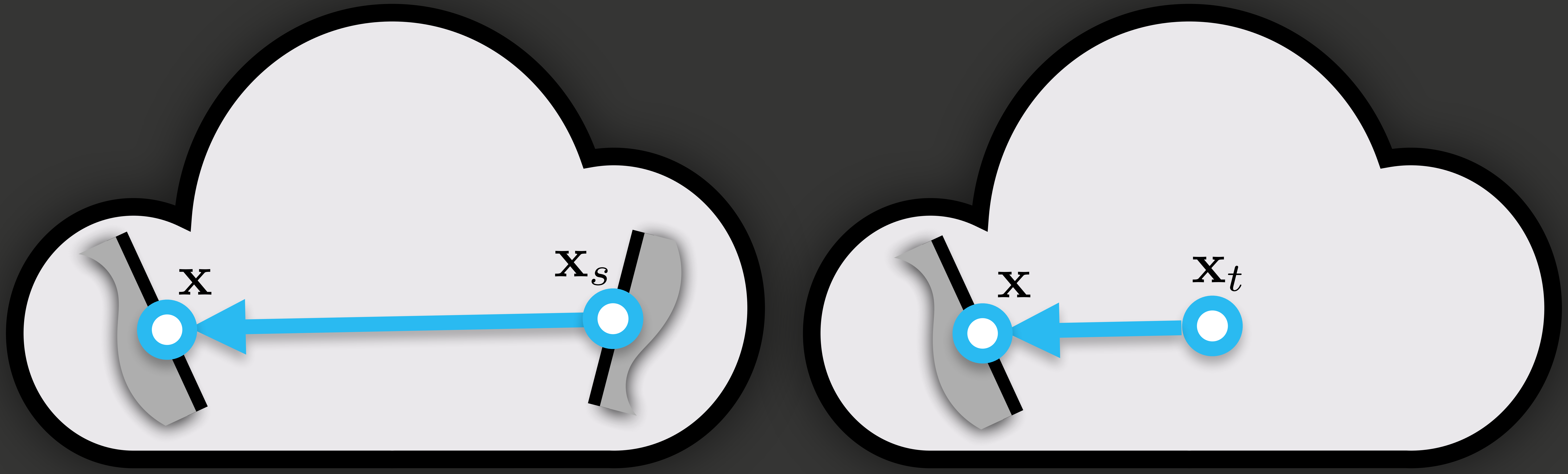

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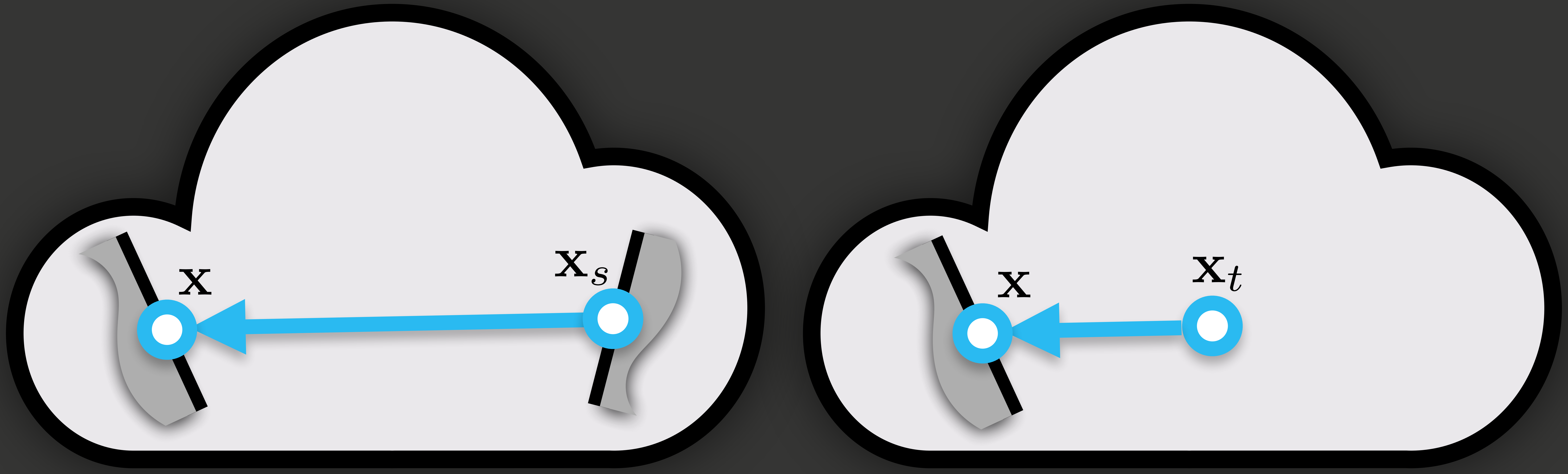

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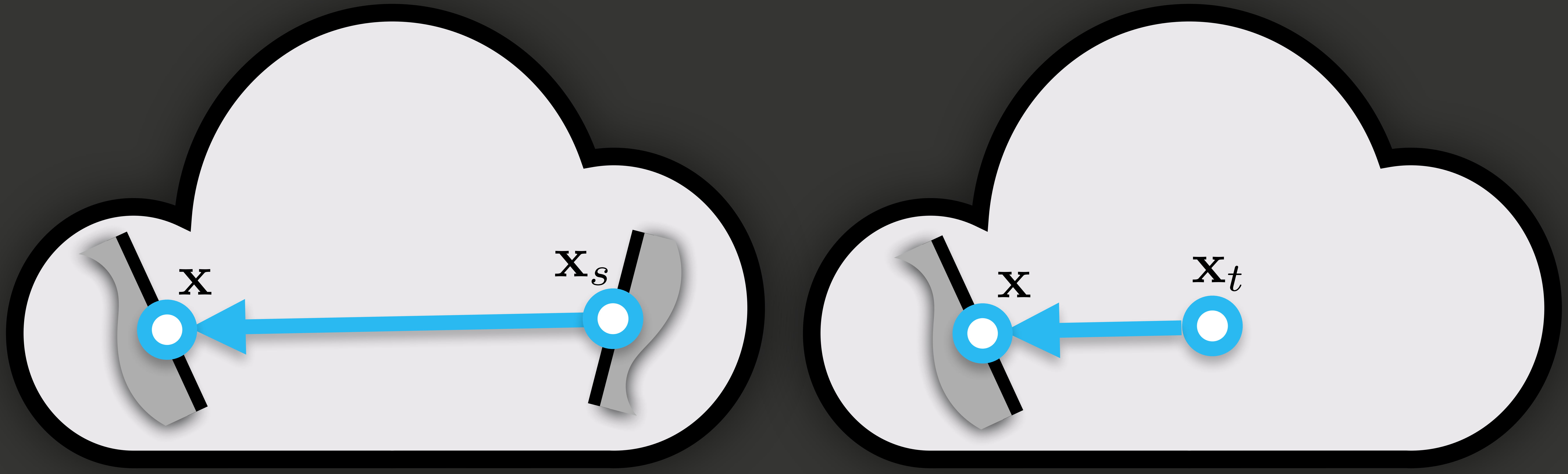

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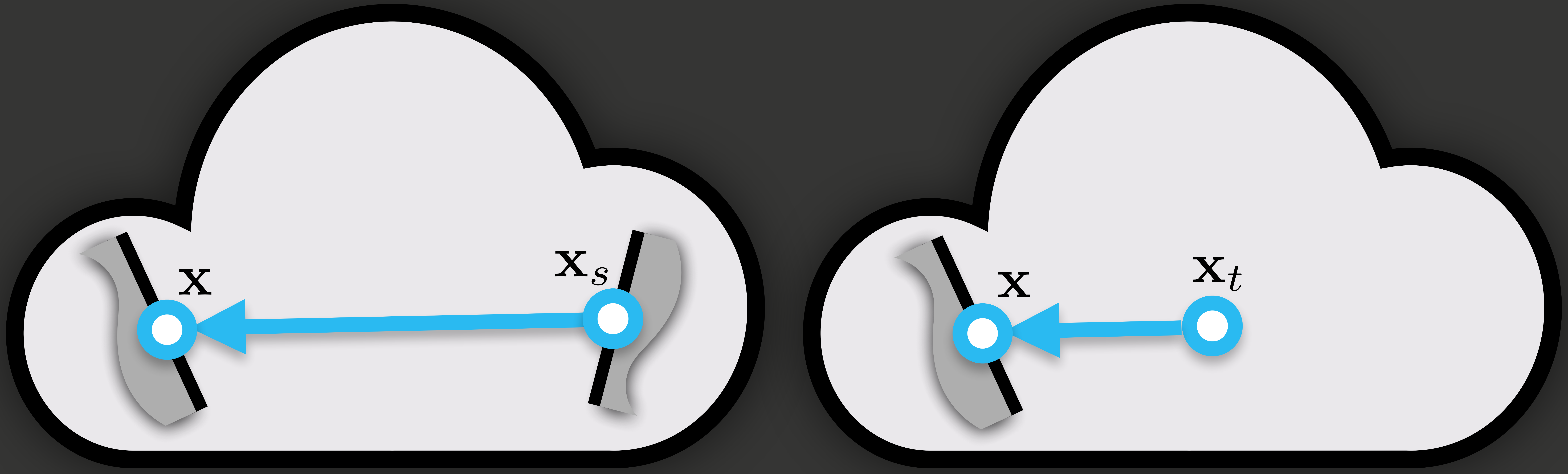

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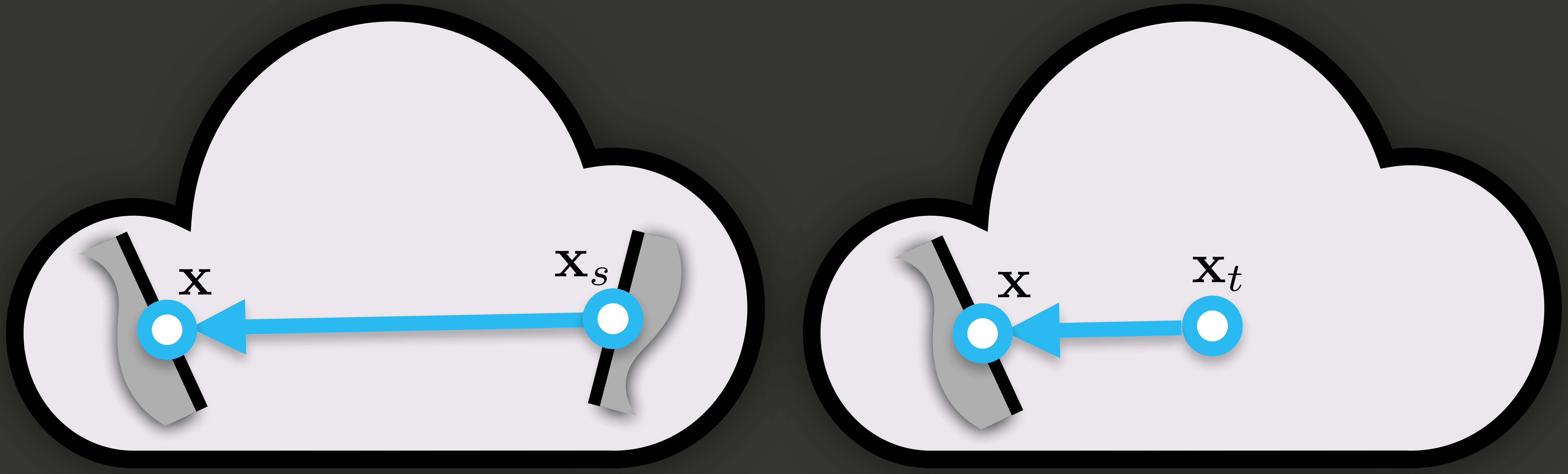

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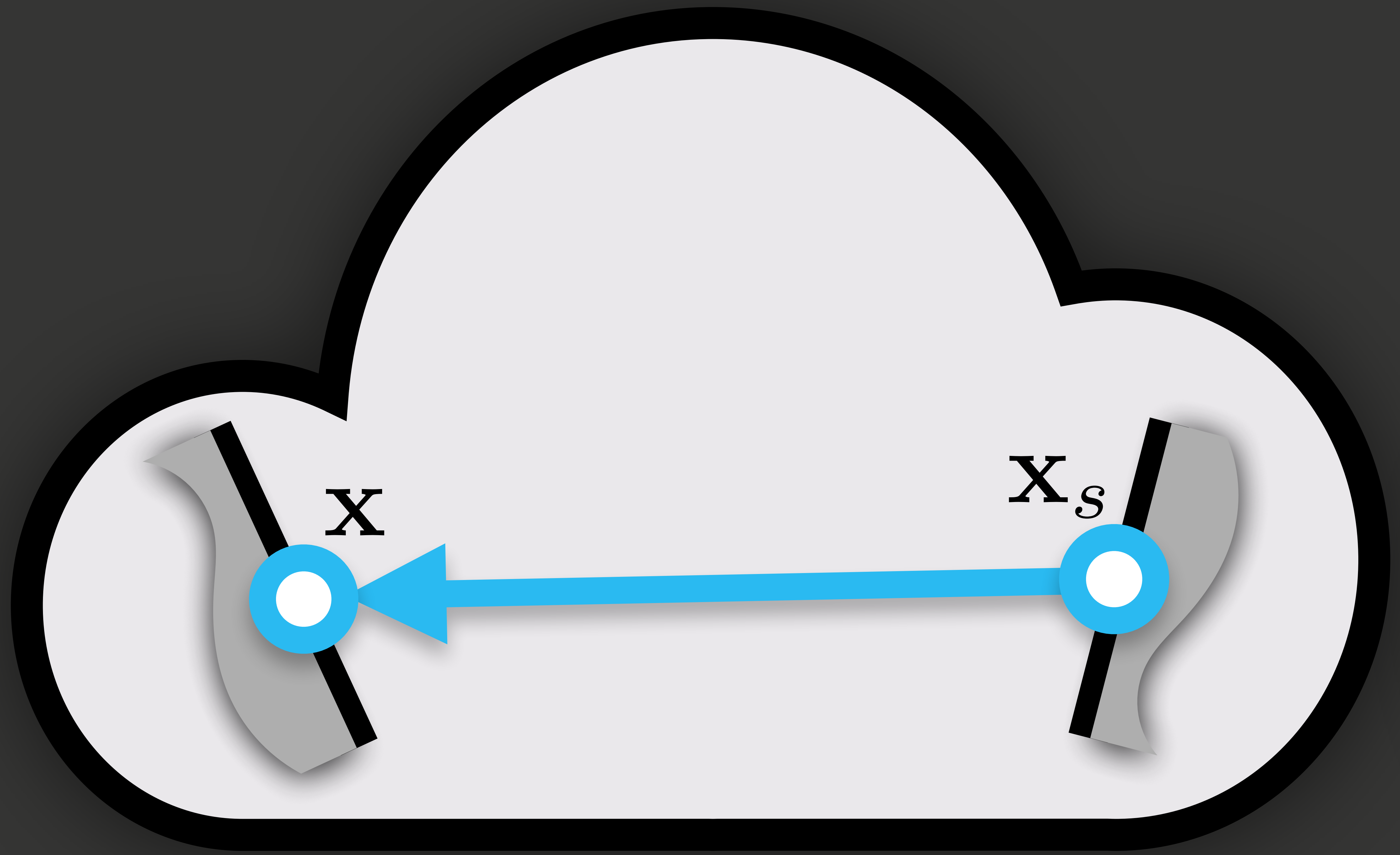

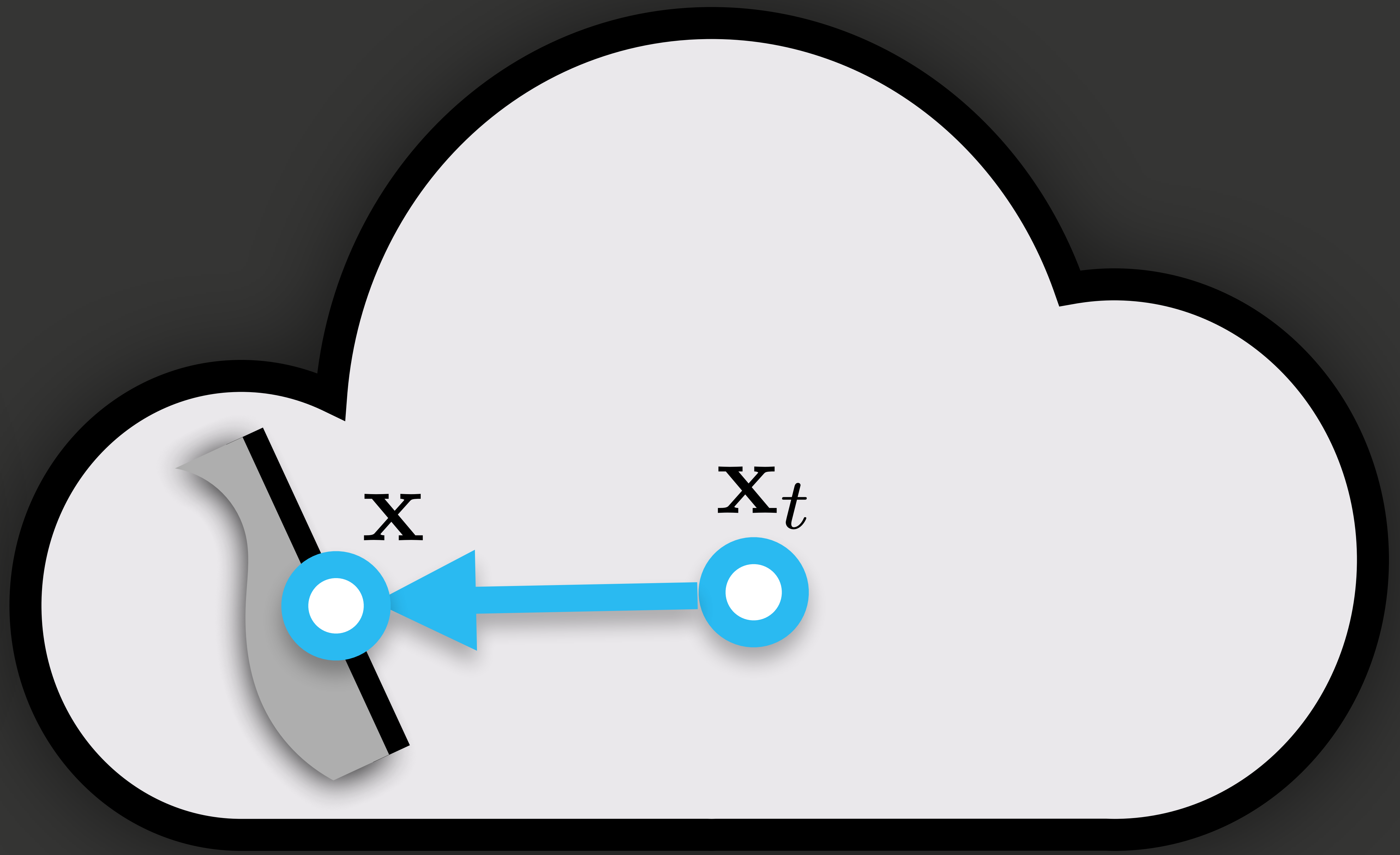

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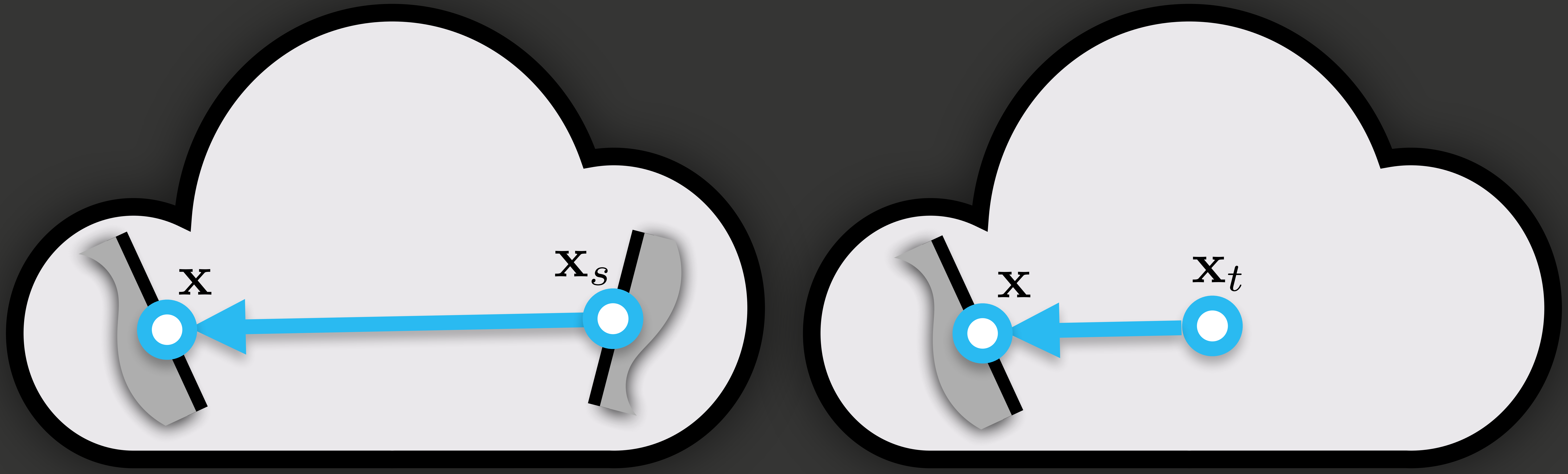

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Transport Functions

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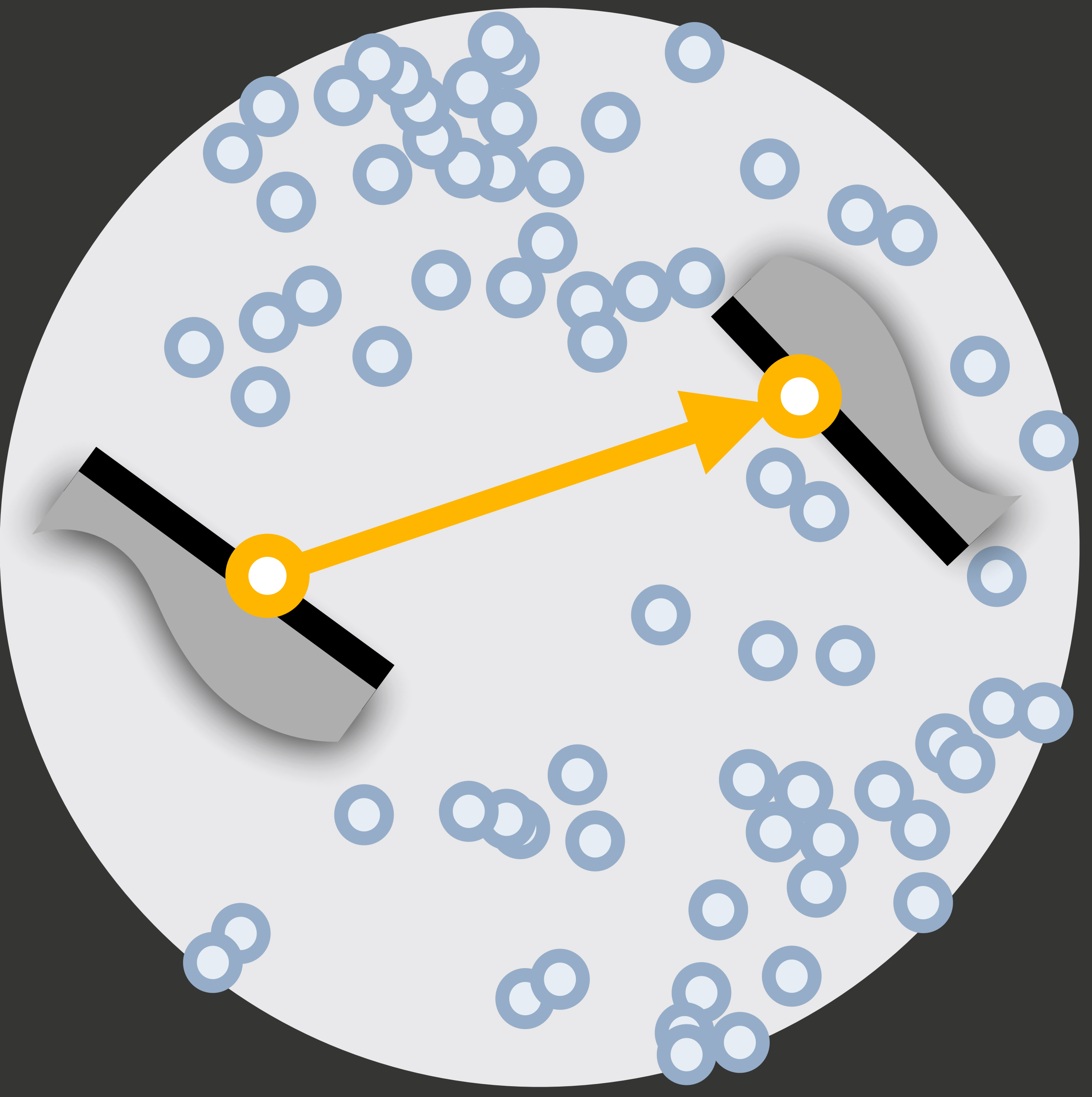
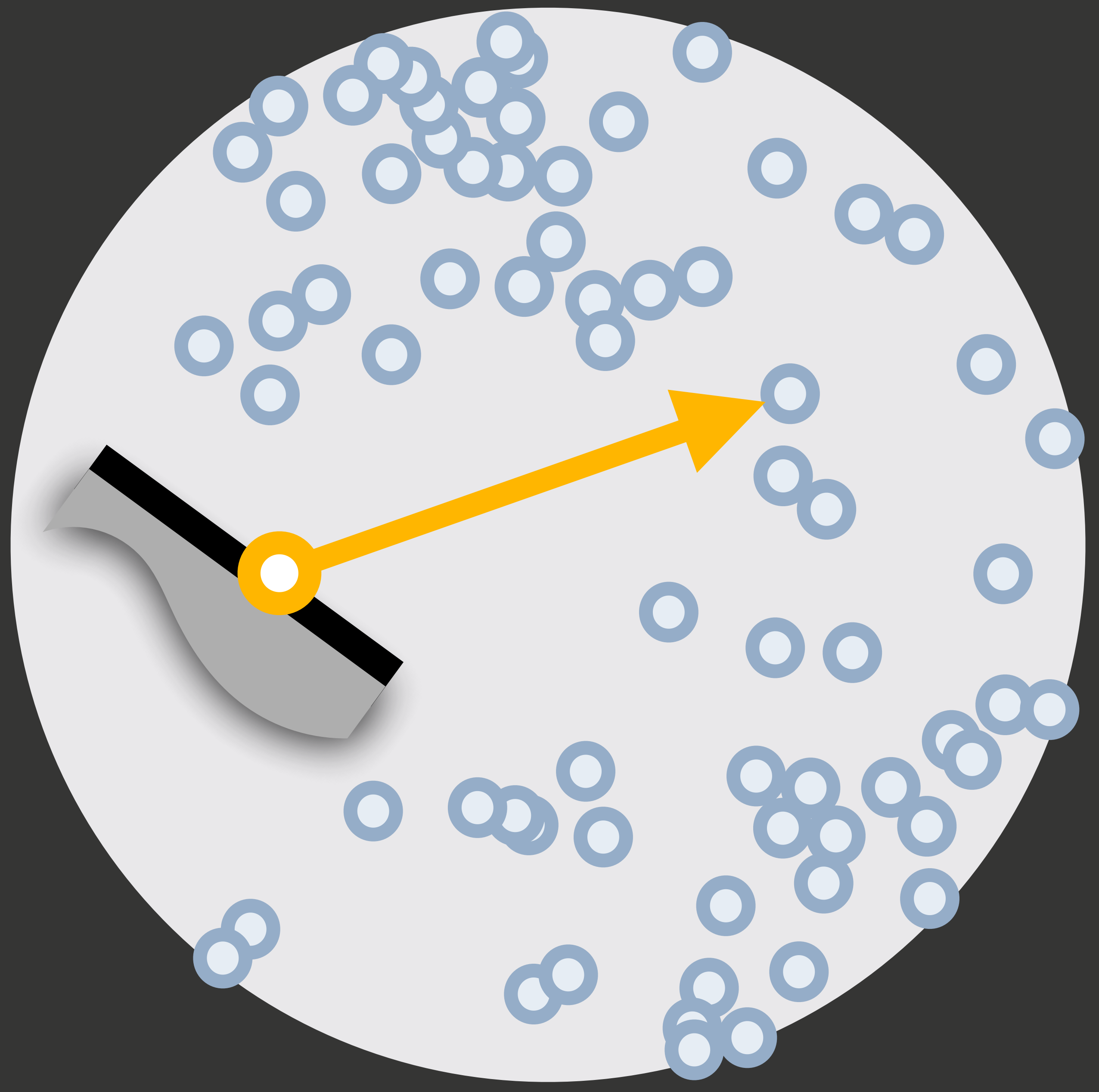


$\mathrm{Tr}(\mathbf{x}, \mathbf{x}_t)$



$\mathrm{pdf}(\mathbf{x}, \mathbf{x}_t)$

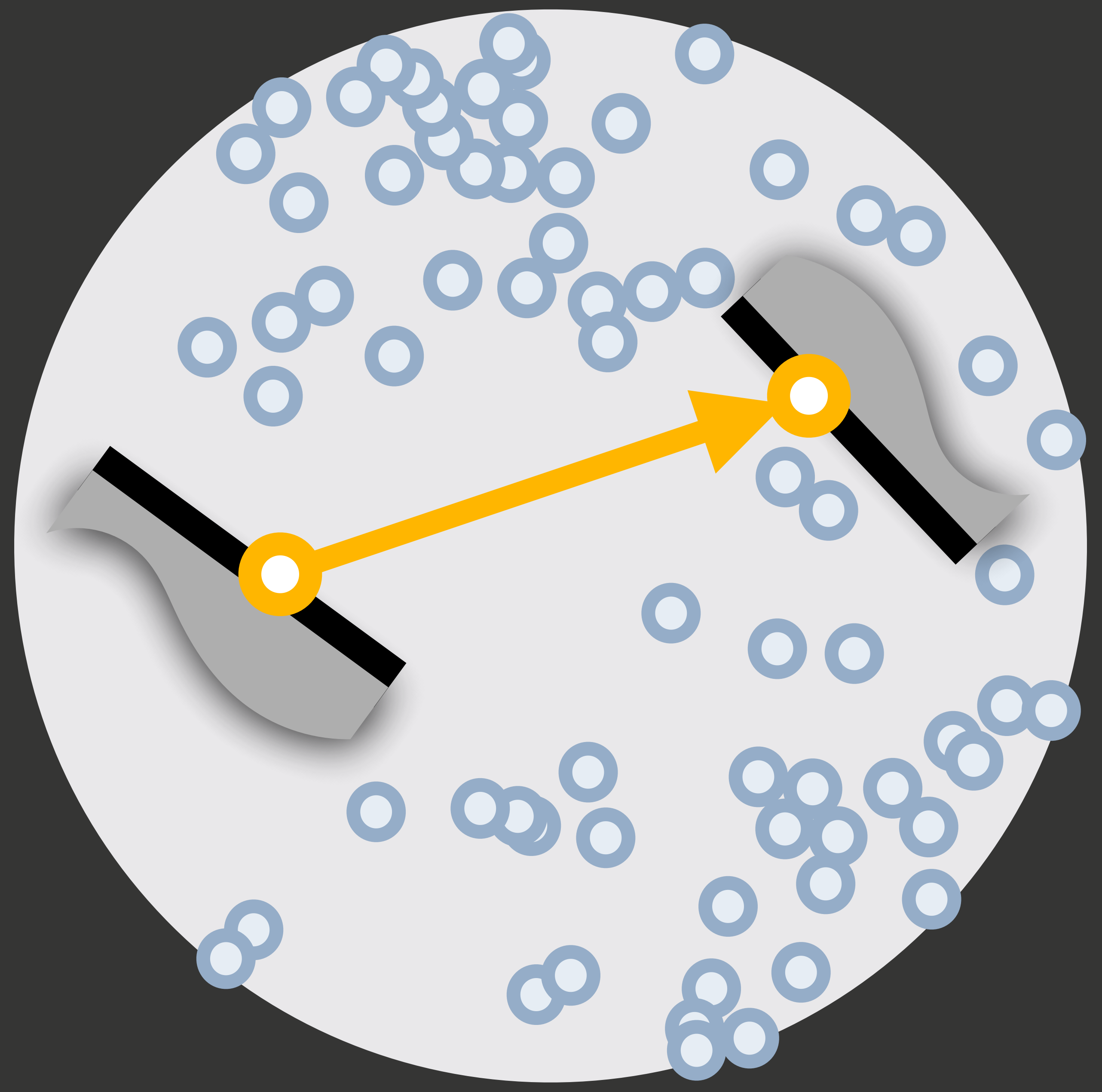
Transport Functions

$\mathrm{Tr}(\mathbf{x}, \mathbf{x}_t)$ $\mathrm{pdf}(\mathbf{x}, \mathbf{x}_t)$

Transport Functions

$\mathrm{ff}(\mathbf{x}, \mathbf{x}_t)$ $\mathrm{fp}(\mathbf{x}, \mathbf{x}_t)$ $\mathrm{pf}(\mathbf{x}, \mathbf{x}_t)$ $\mathrm{pp}(\mathbf{x}, \mathbf{x}_t)$

p: "particle" f: "free space"



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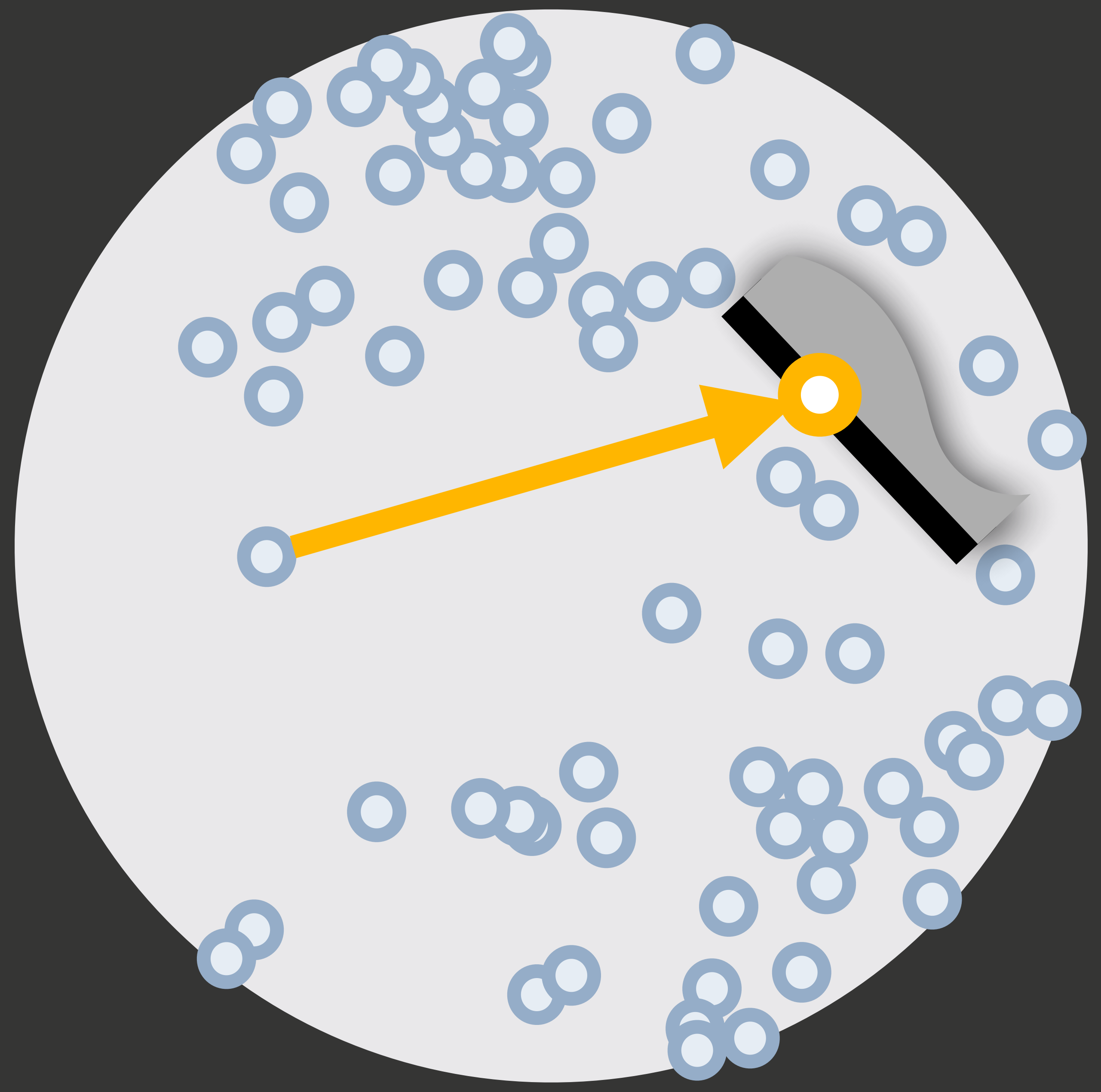

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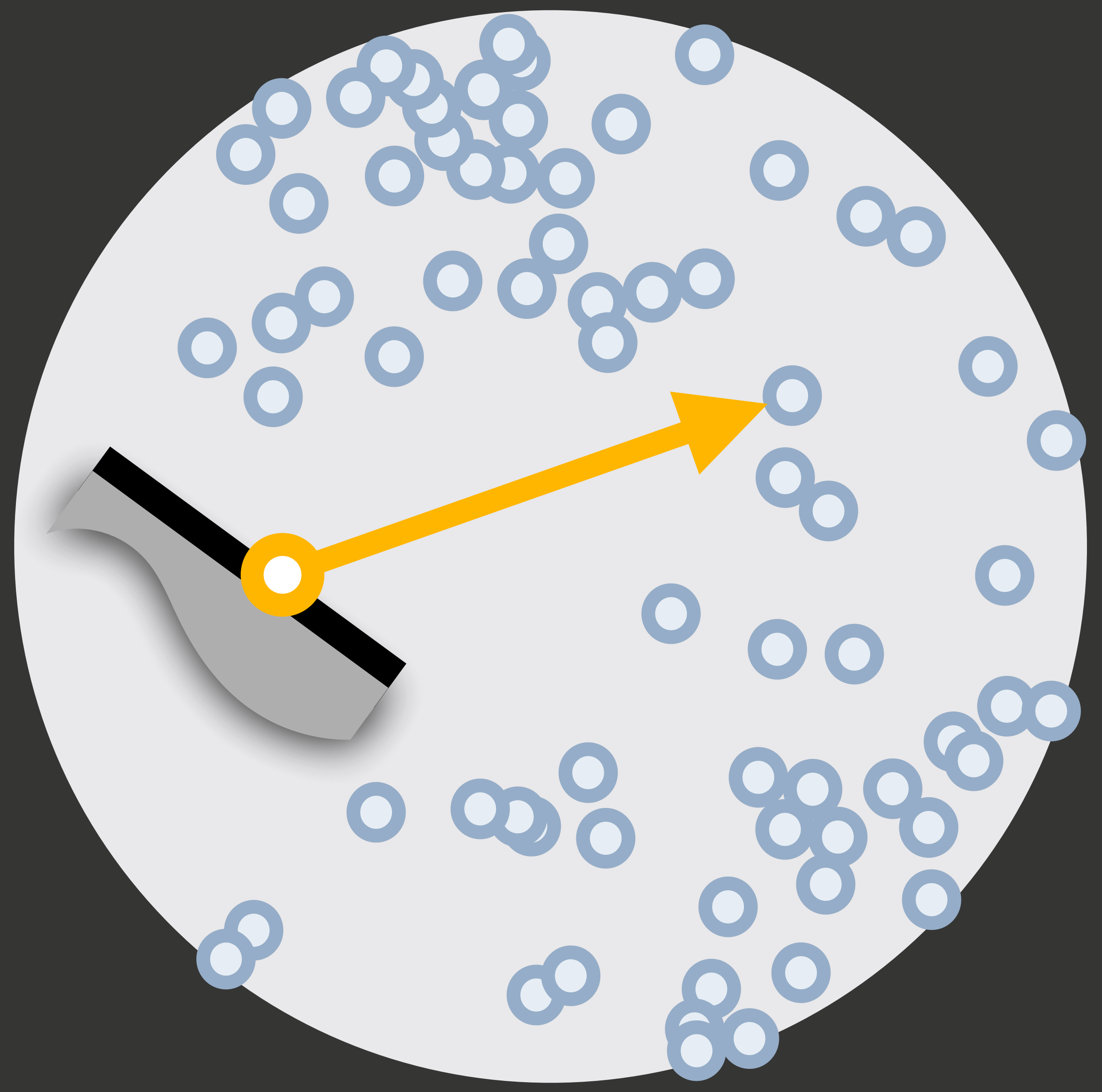

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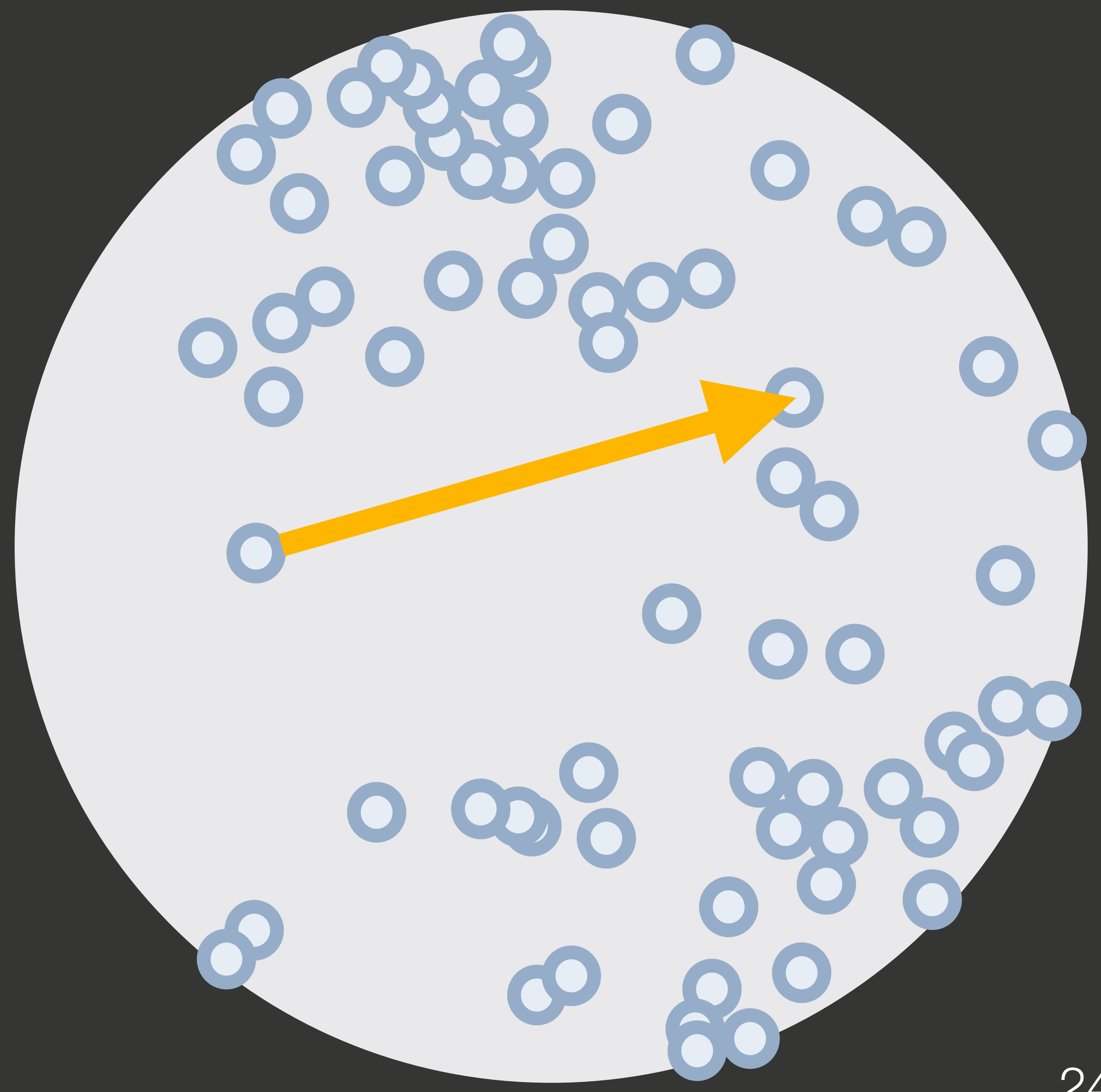
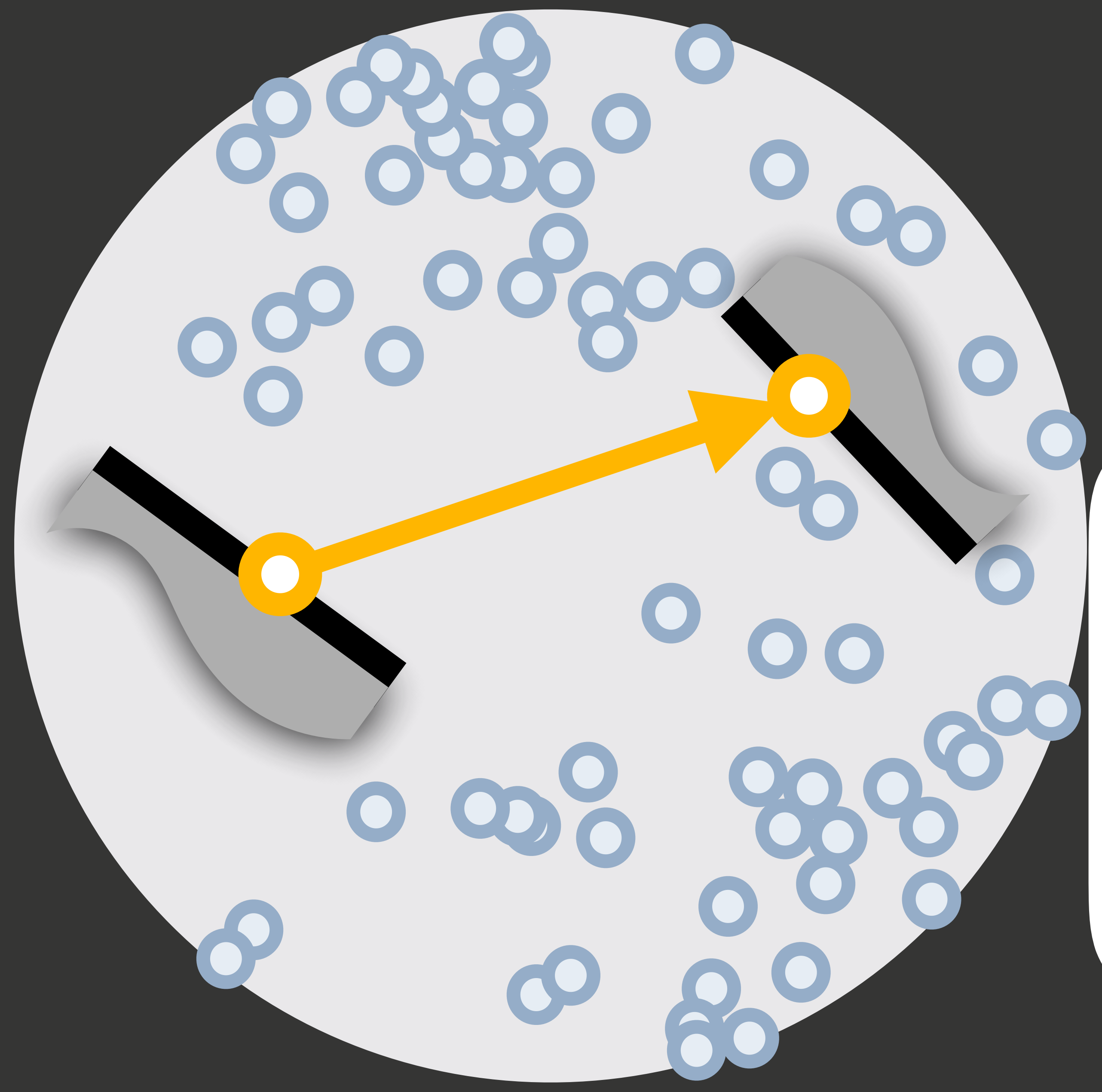


$\mathrm{pp}(\mathbf{x}, \mathbf{x}_t)$



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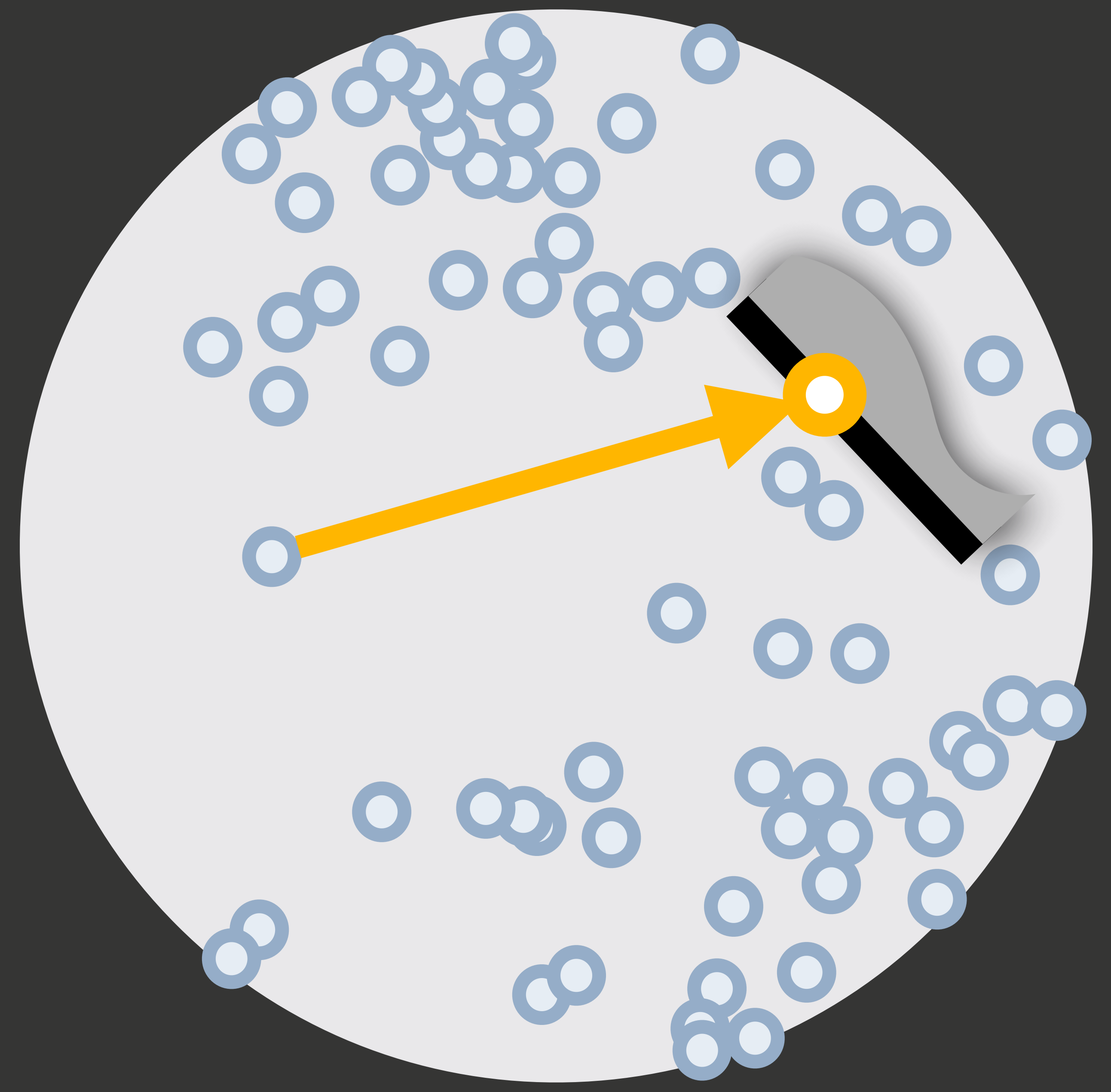
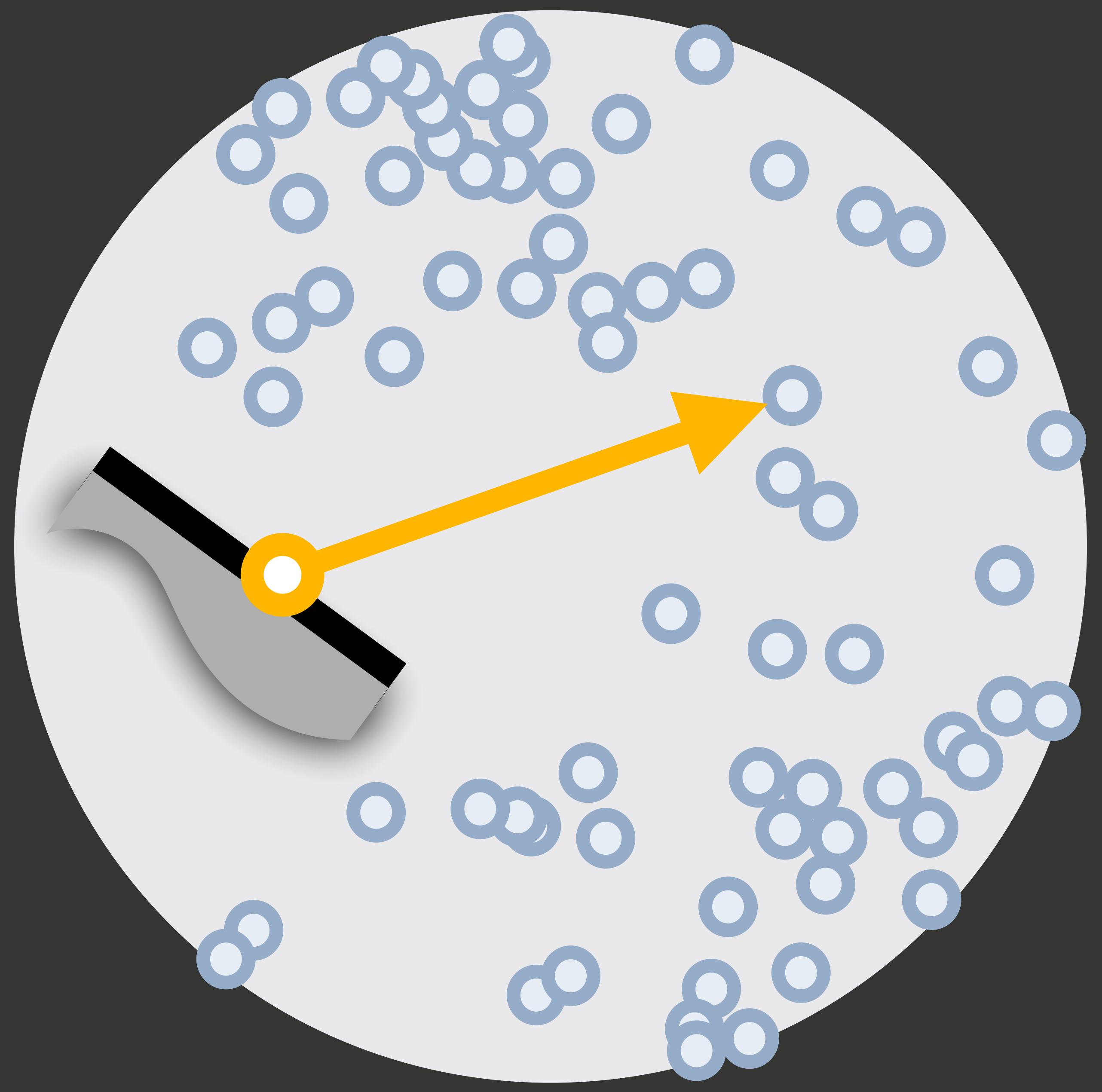


Transmittance

Probability: Starts at 1



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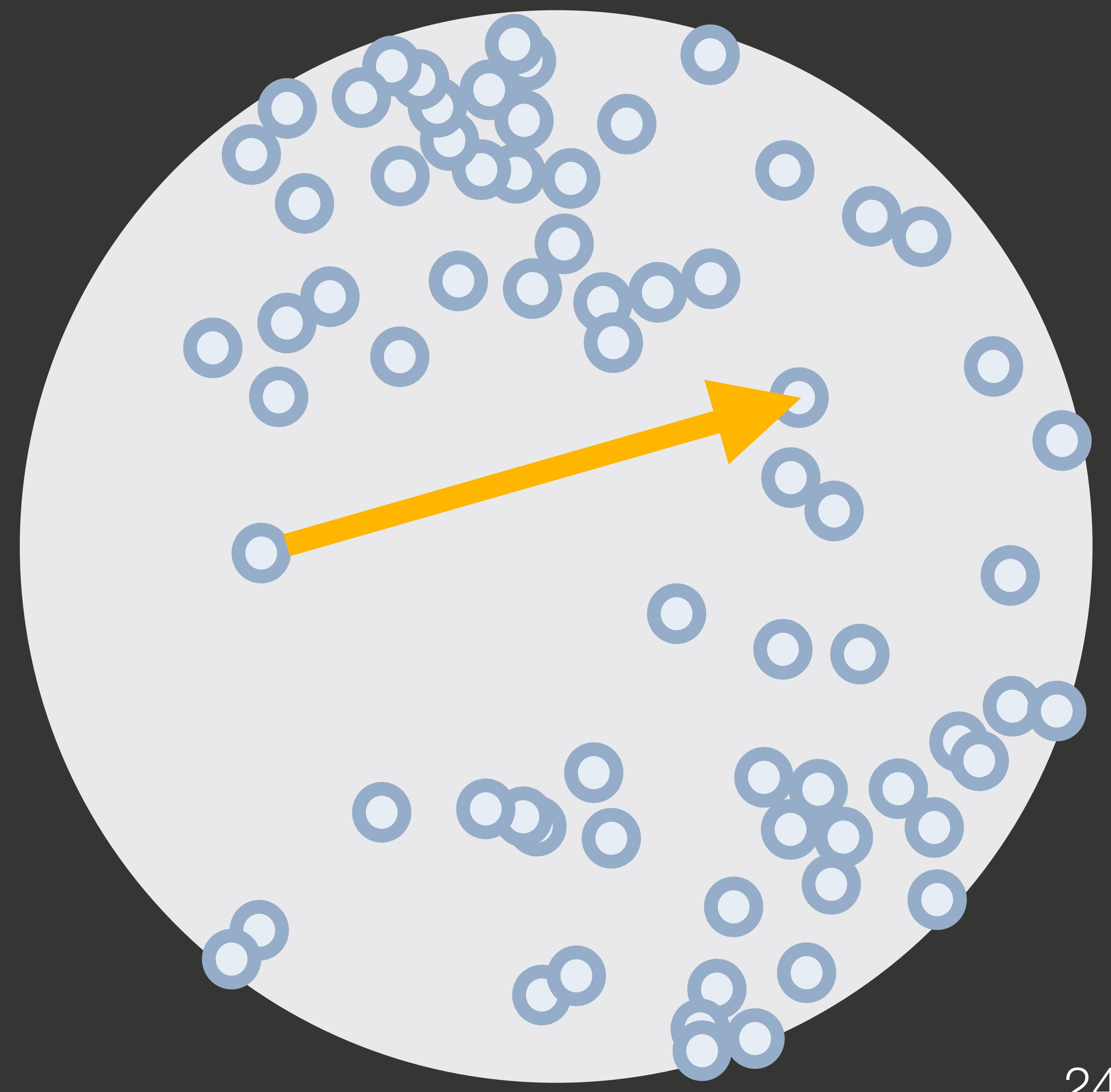

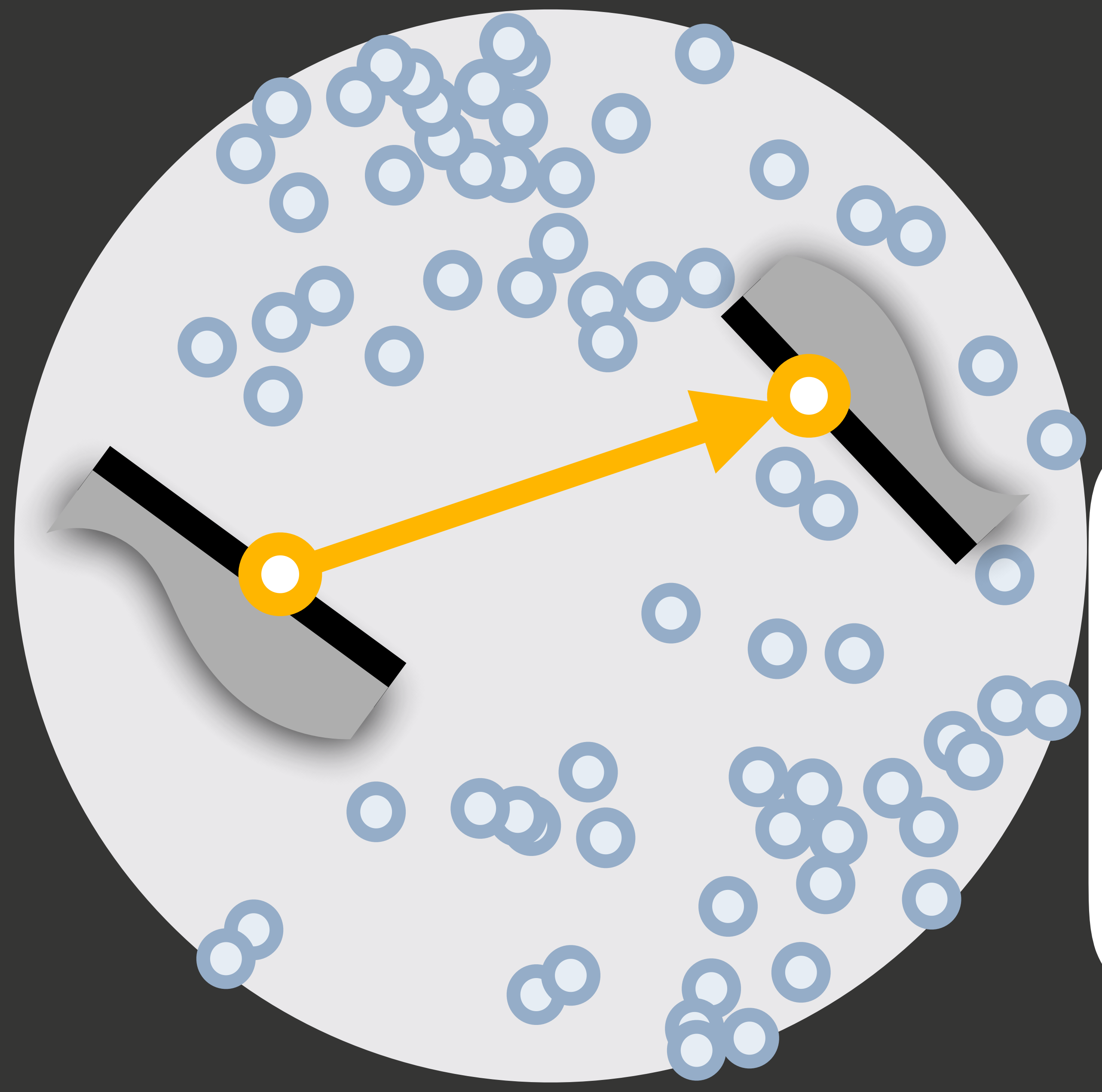

$\mathrm{ff}(\mathbf{x}, \mathbf{x}_t)$ $\mathrm{pf}(\mathbf{x}, \mathbf{x}_t)$

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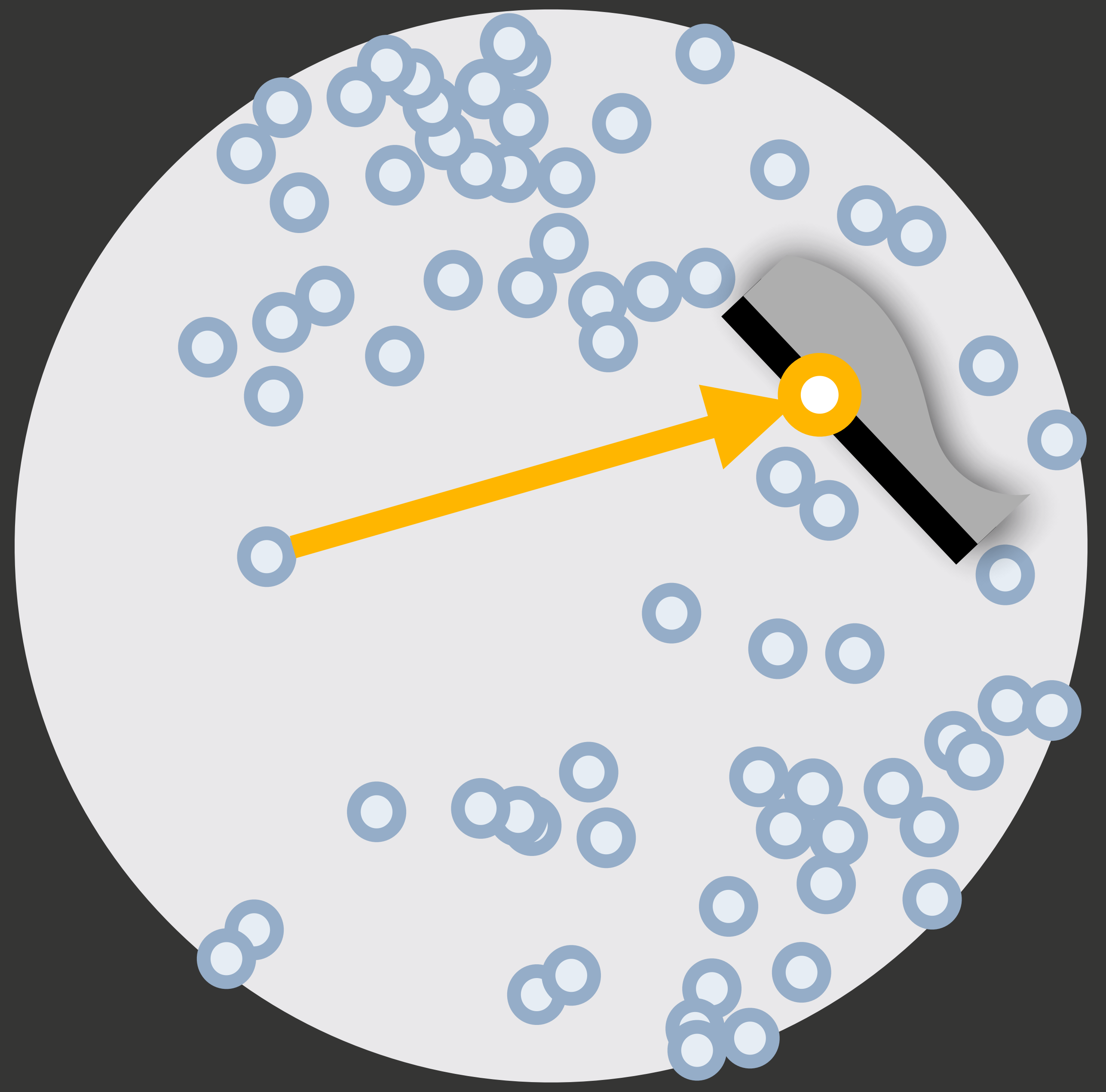
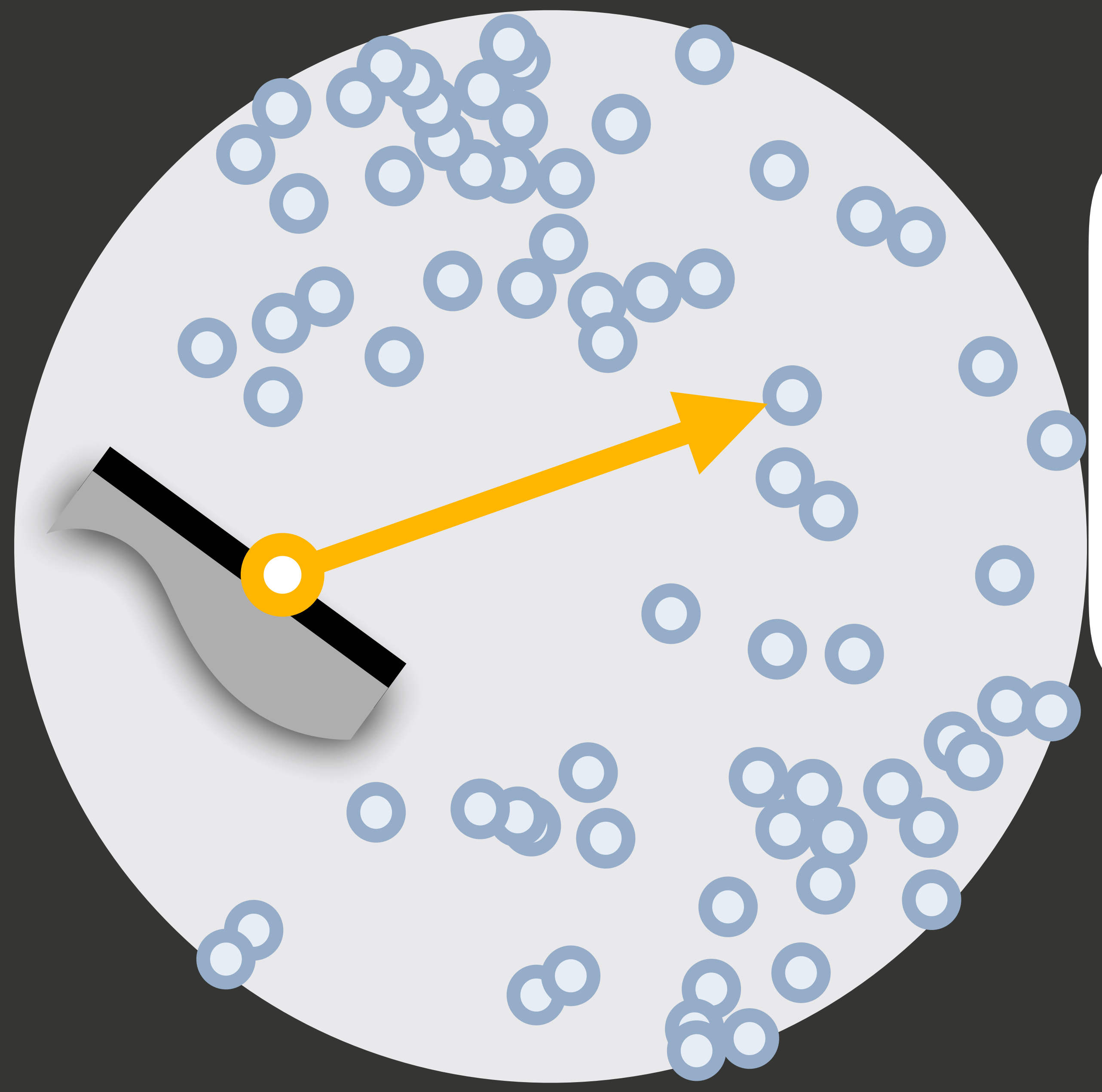
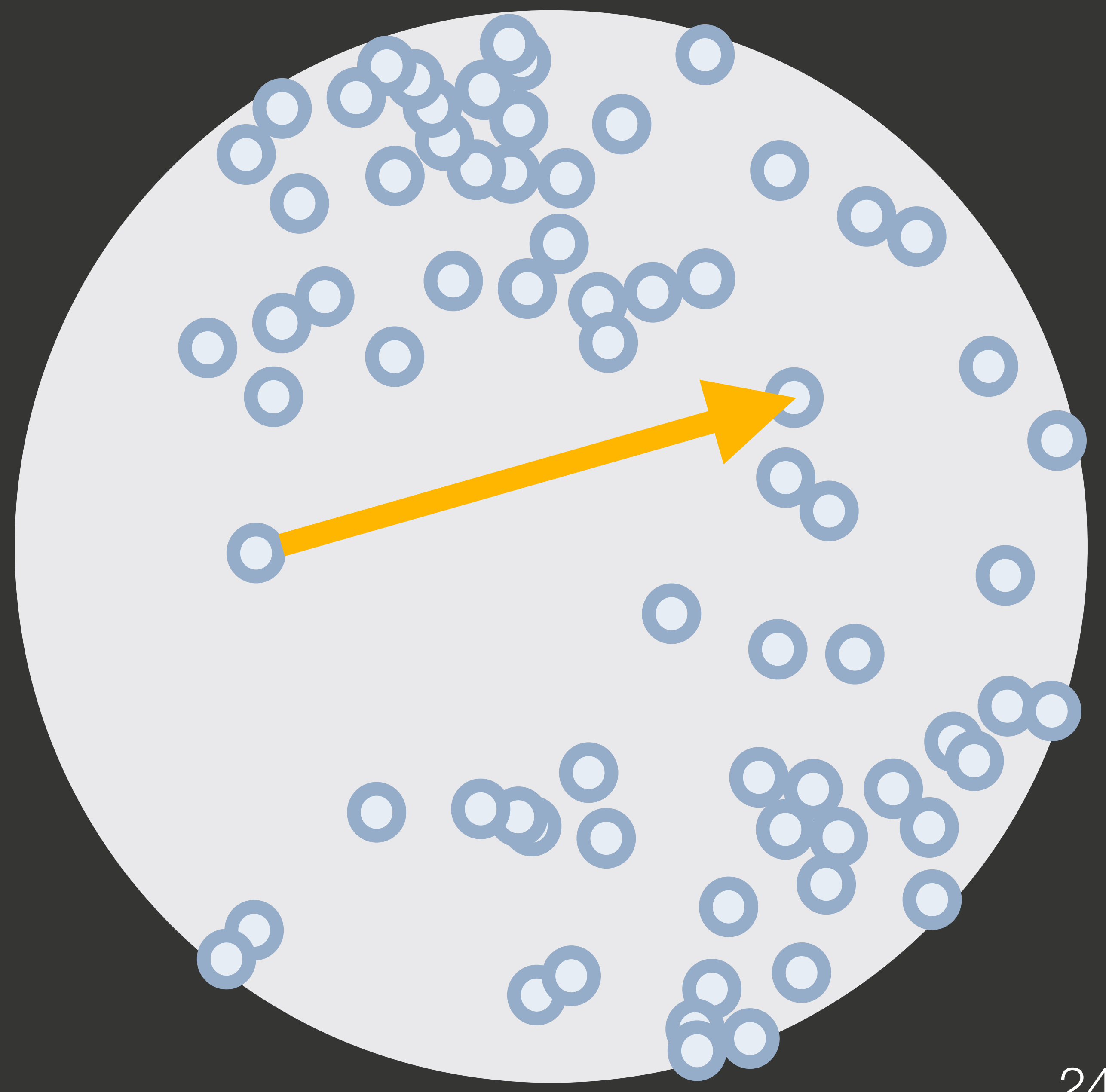


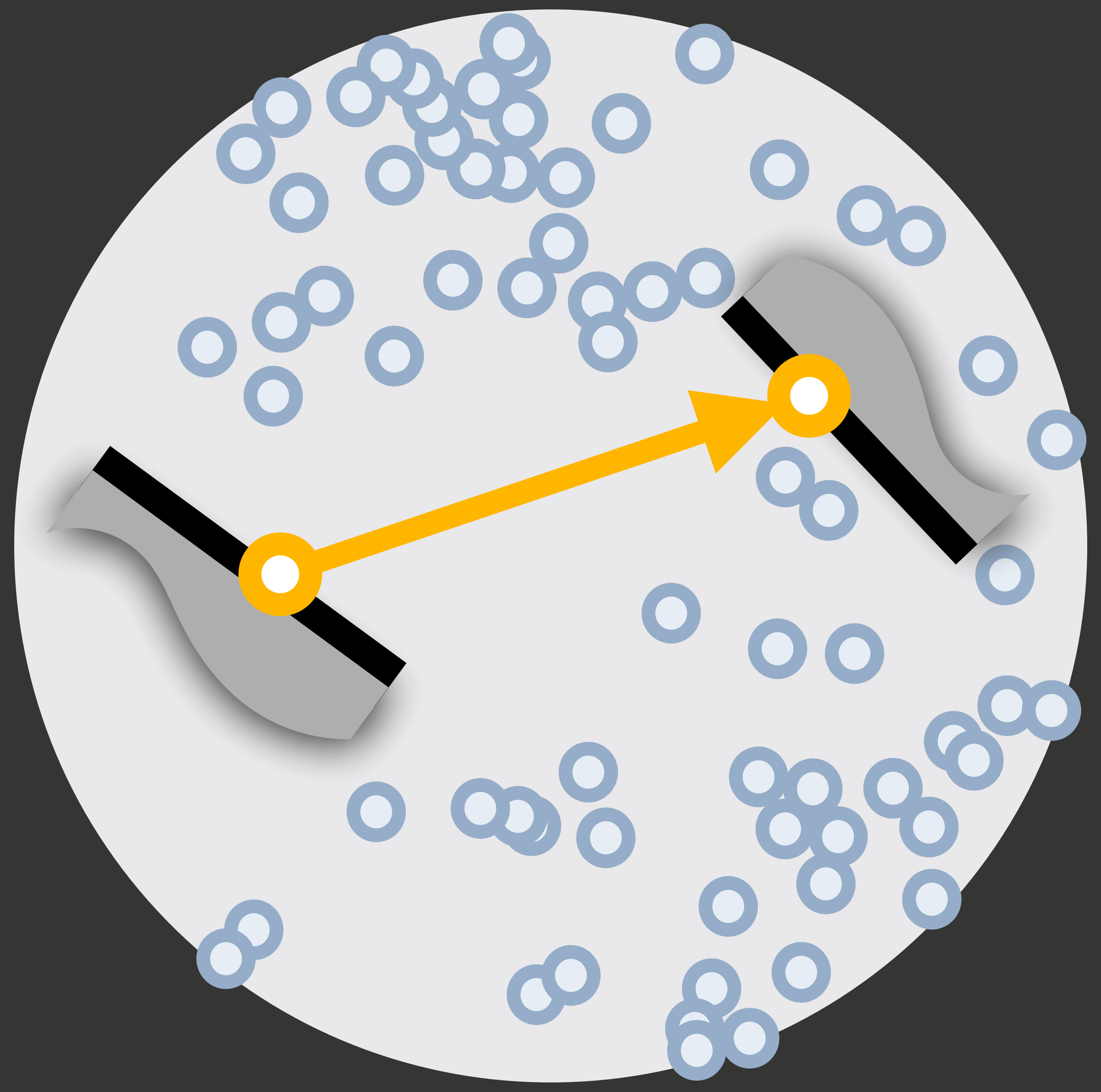


Free-flight PDF

Probability Density: Integrates to 1

$\mathrm{fp}(\mathbf{x}, \mathbf{x}_t)$ $\mathrm{pp}(\mathbf{x}, \mathbf{x}_t)$





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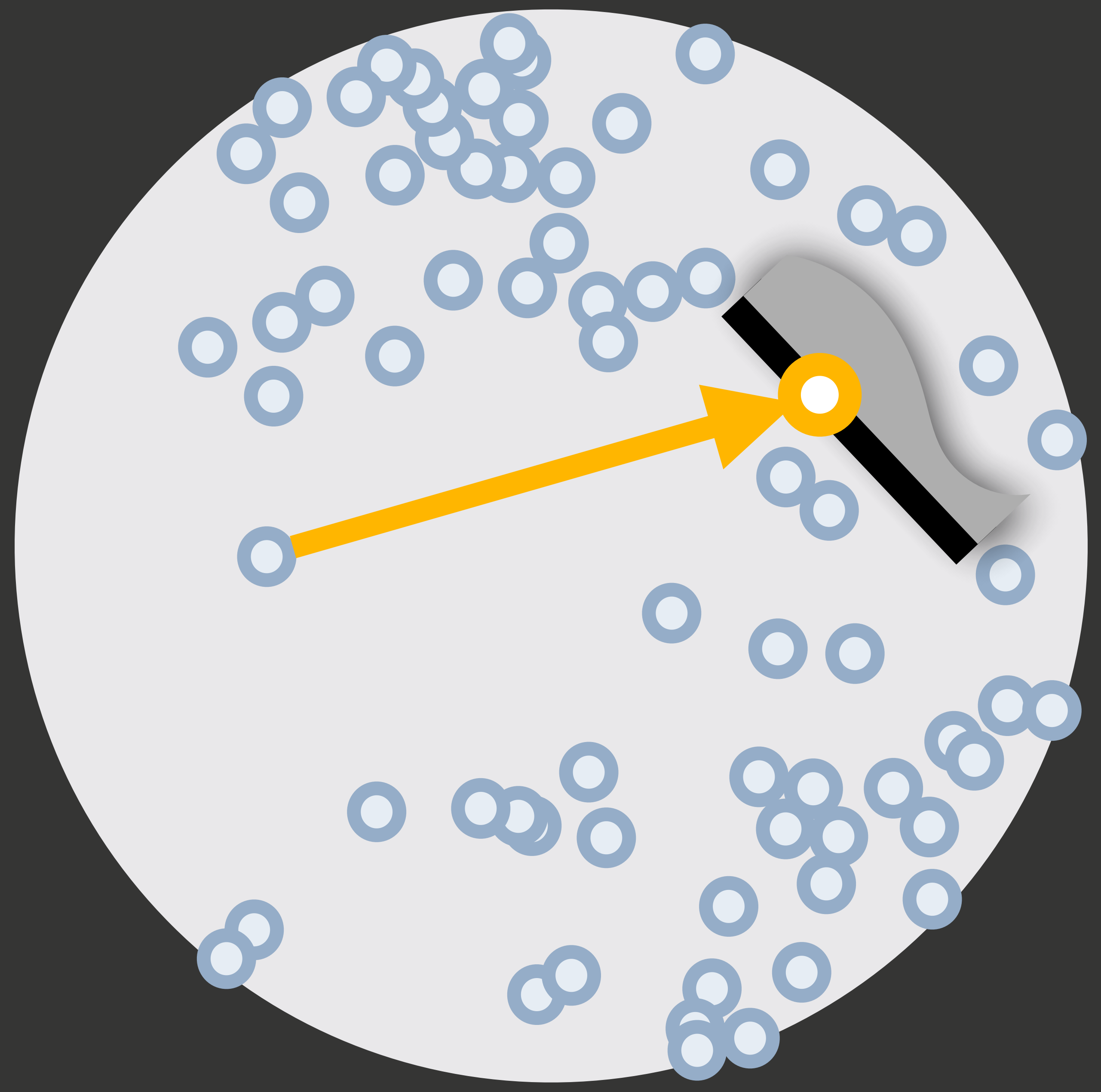

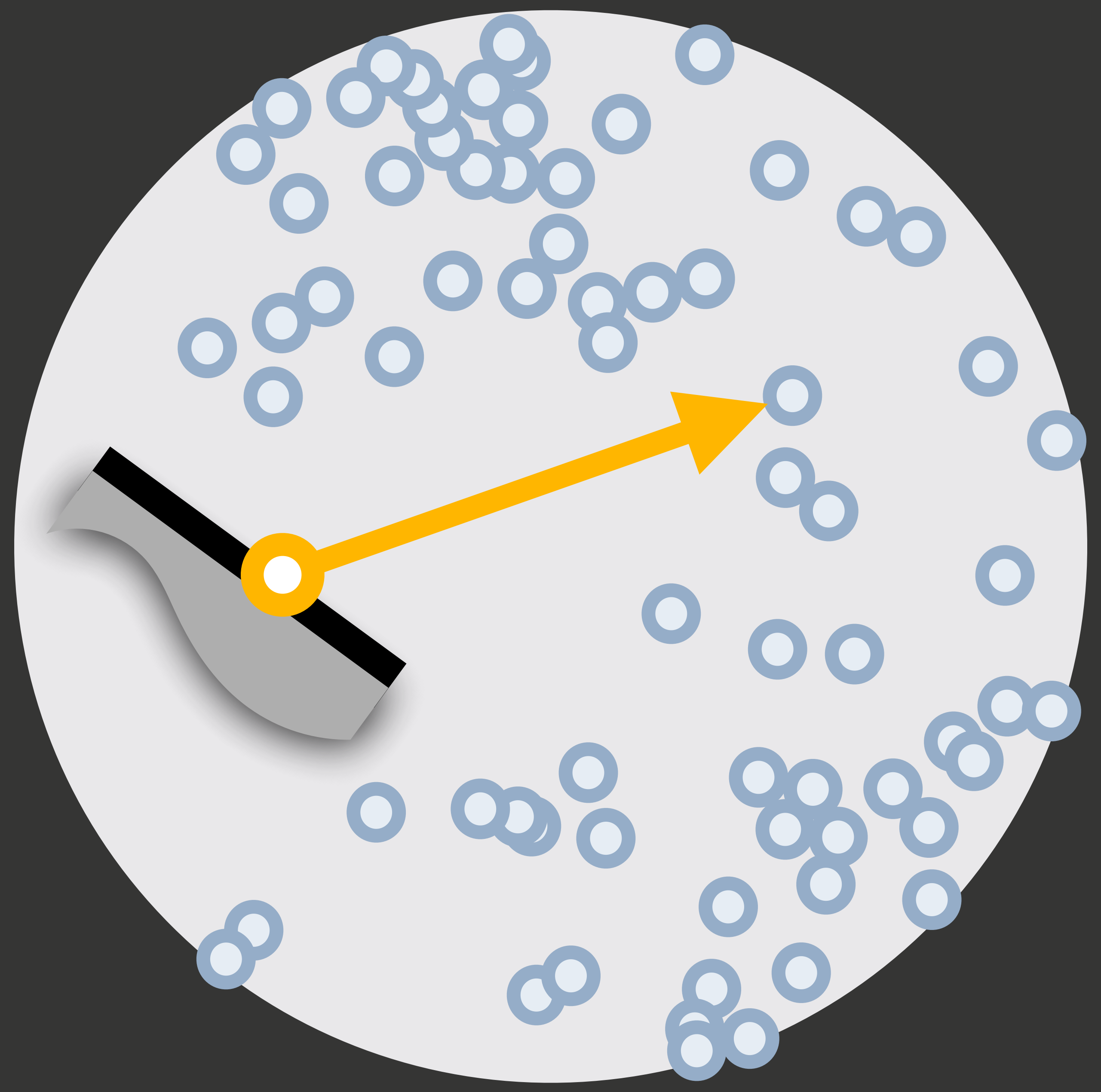

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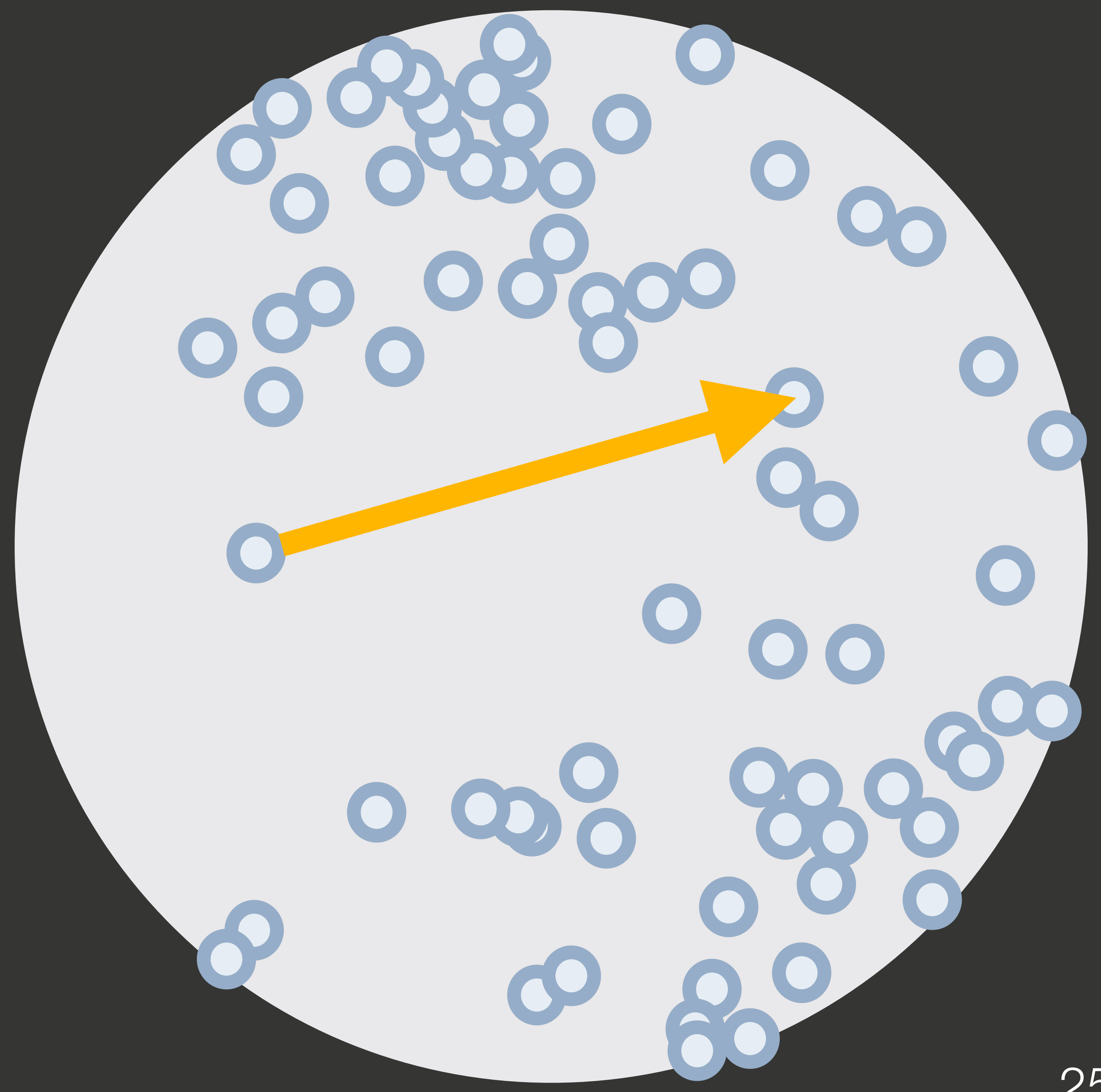
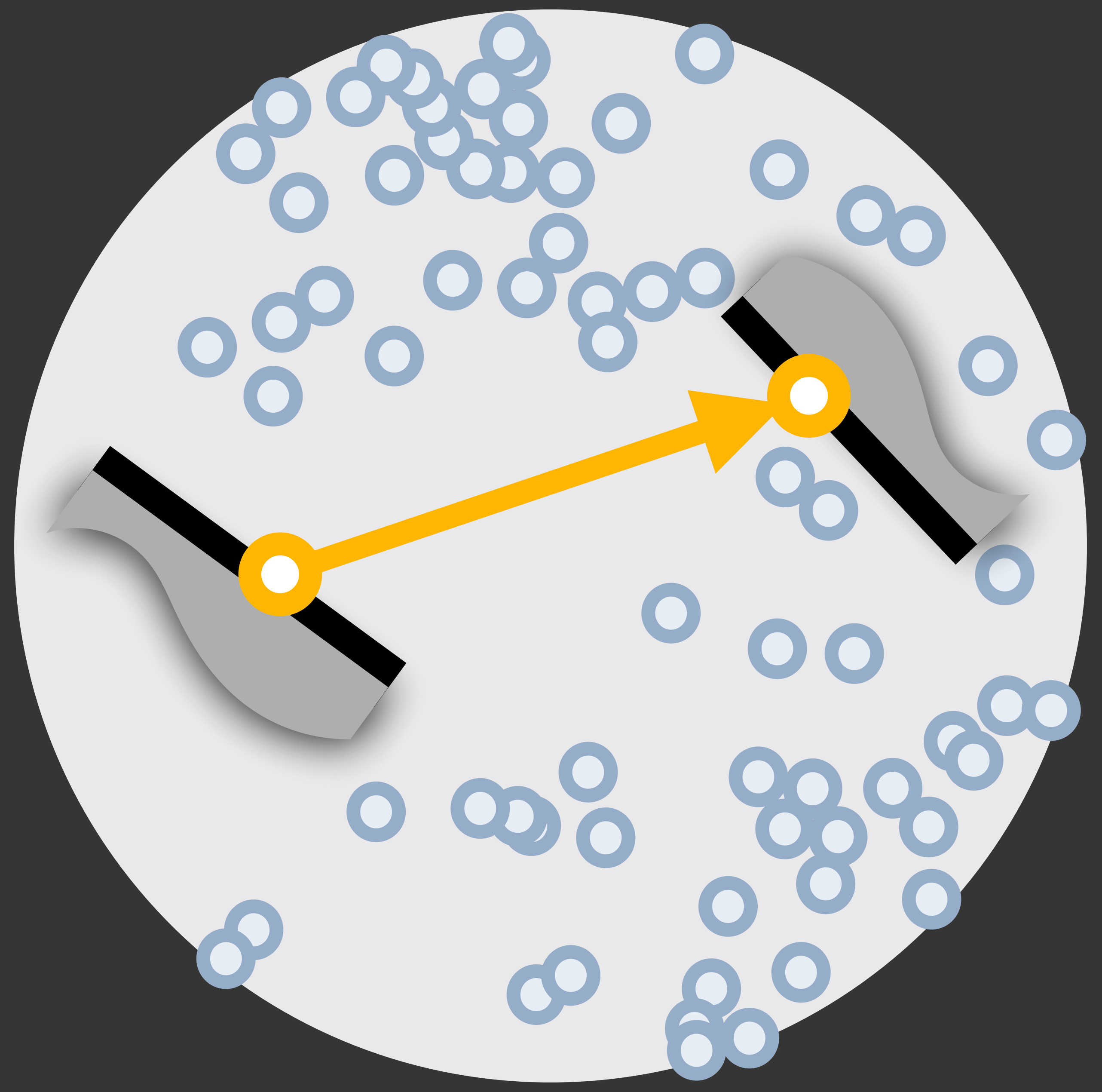


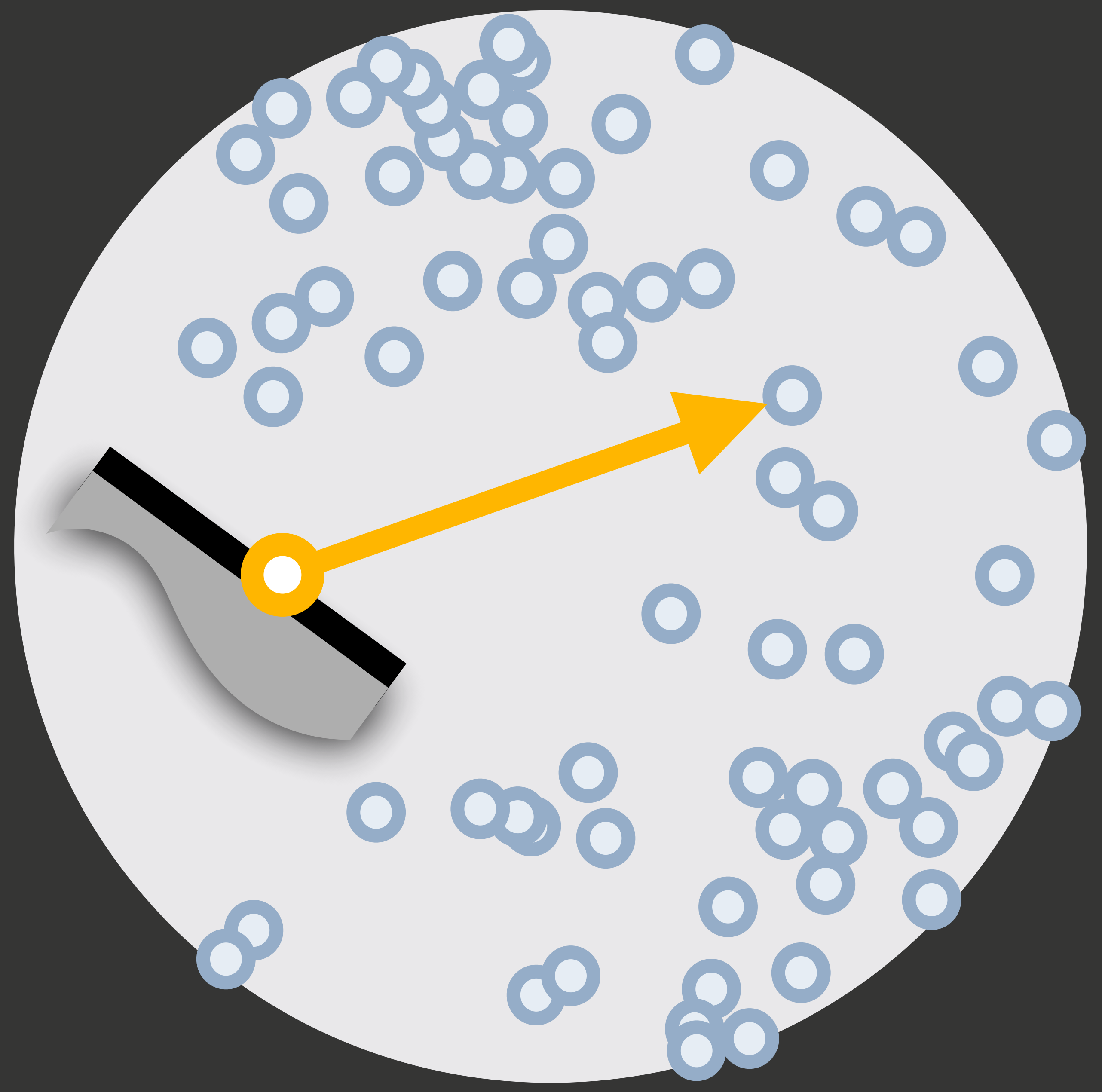

$\mathrm{ff}(\mathbf{x}, \mathbf{x}_t)$
$\int$
$\mathrm{fp}(\mathbf{x}, \mathbf{x}_t)$
$\mathrm{pf}(\mathbf{x}, \mathbf{x}_t)$
$\int$
$\mathrm{pp}(\mathbf{x}, \mathbf{x}_t)$

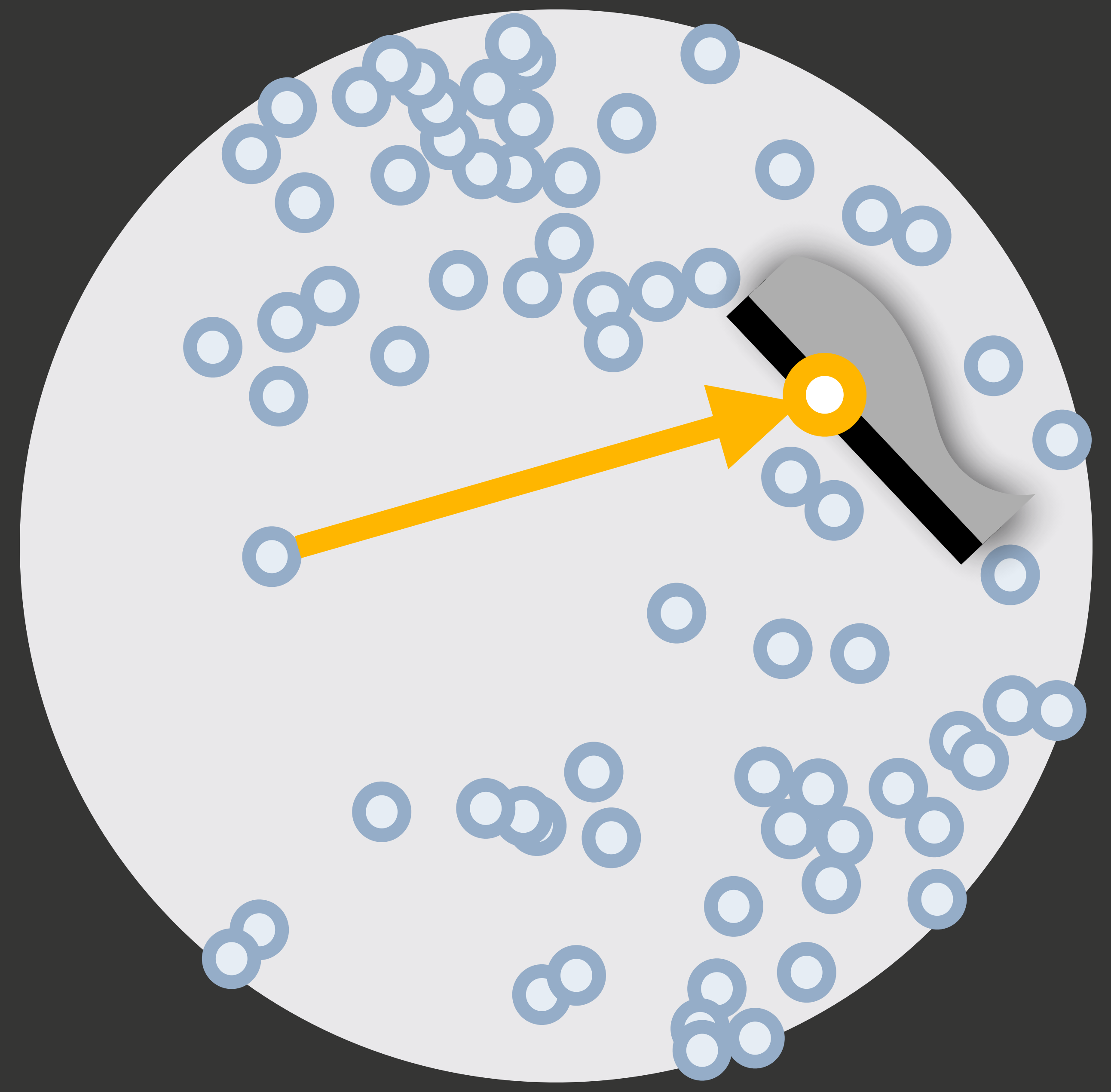


$\mathrm{ff}(\mathbf{x}, \mathbf{x}_t)$

$\frac{\mathrm{d}}{\mathrm{d}t} \downarrow \uparrow \int$

$\mathrm{fp}(\mathbf{x}, \mathbf{x}_t)$

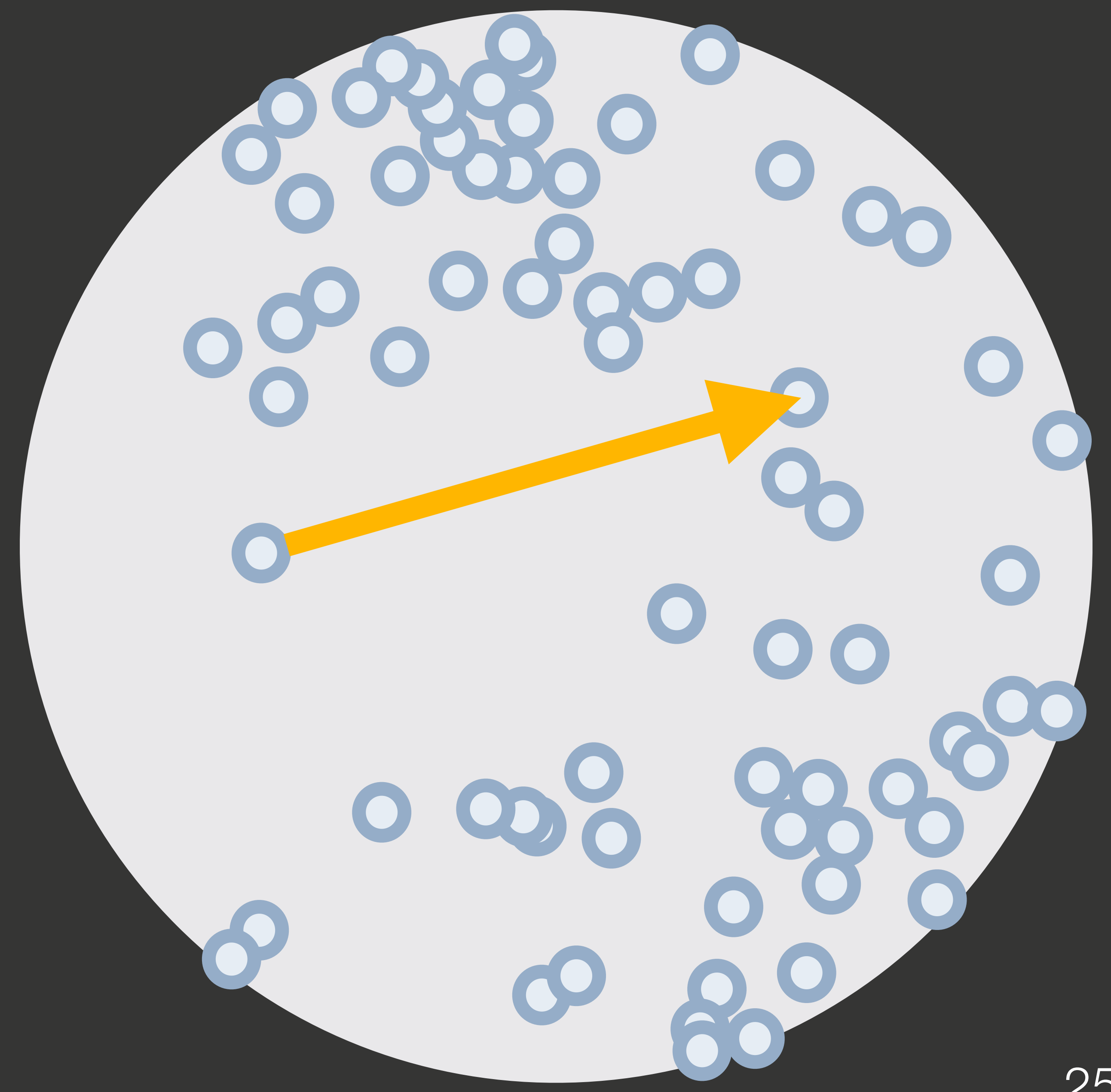




$\mathrm{pf}(\mathbf{x}, \mathbf{x}_t)$

$\frac{\mathrm{d}}{\mathrm{d}t} \downarrow \uparrow \int$

$\mathrm{pp}(\mathbf{x}, \mathbf{x}_t)$




$\mathrm{ff}(\mathbf{x}, \mathbf{x}_t)$
$\mathrm{pf}(\mathbf{x}, \mathbf{x}_t)$
$\frac{\mathrm{d}}{\mathrm{d}t}$
$\int$
$\bar{\sigma}$
$\frac{\mathrm{d}}{\mathrm{d}t}$
$\int$
$\mathrm{fp}(\mathbf{x}, \mathbf{x}_t)$
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$\int$
$\bar{\sigma}$
$\bar{\sigma}^{-1}$
$\frac{\mathrm{d}}{\mathrm{d}t}$
$\int$
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$\mathrm{pp}(\mathbf{x}, \mathbf{x}_t)$

Transport Kernel

$$T(\mathbf{x}, \mathbf{x}_t) = \begin{cases} \mathrm{pp}(\mathbf{x}, \mathbf{x}_t) & \text{if } \mathbf{x} \in p \text{ and } \mathbf{x}_t \in p \\ \mathrm{pf}(\mathbf{x}, \mathbf{x}_t) & \text{if } \mathbf{x} \in p \text{ and } \mathbf{x}_t \in f \\ \mathrm{fp}(\mathbf{x}, \mathbf{x}_t) & \text{if } \mathbf{x} \in f \text{ and } \mathbf{x}_t \in p \\ \mathrm{ff}(\mathbf{x}, \mathbf{x}_t) & \text{if } \mathbf{x} \in f \text{ and } \mathbf{x}_t \in f \end{cases}$$

A Non-Exponential Rendering Equation

$$L_i(\mathbf{x},\omega) = \boxed{\mathrm{Tr}(\mathbf{x},\mathbf{x}_s)} L_o(\mathbf{x}_s,\omega) + \int_0^s \boxed{\mathrm{Tr}(\mathbf{x},\mathbf{x}_t)\sigma_s(\mathbf{x}_t)} L_s(\mathbf{x}_t,\omega)\mathrm{d}t$$

A Non-Exponential Rendering Equation

$$L_i(\mathbf{x}, \omega) = \boxed{T(\mathbf{x}, \mathbf{x}_t)} L_o(\mathbf{x}_s, \omega) + \int_0^s \boxed{T(\mathbf{x}, \mathbf{x}_t)\alpha(\mathbf{x}_t)} L_s(\mathbf{x}_t, \omega)\mathrm{d}t$$

A Non-Exponential Rendering Equation

$$L_i(\mathbf{x},\omega) = T(\mathbf{x},\mathbf{x}_t) L_o(\mathbf{x}_s,\omega) + \int_0^s T(\mathbf{x},\mathbf{x}_t)\alpha(\mathbf{x}_t) L_s(\mathbf{x}_t,\omega)\mathrm{d}t$$

This Talk: Rendering Equation

A Non-Exponential Rendering Equation

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This Talk: Rendering Equation

In Paper: Path Integral

A Non-Exponential Rendering Equation

$$L_i(\mathbf{x}, \omega) = \boxed{T(\mathbf{x}, \mathbf{x}_t)} L_o(\mathbf{x}_s, \omega) + \int_0^s \boxed{T(\mathbf{x}, \mathbf{x}_t)\alpha(\mathbf{x}_t)} L_s(\mathbf{x}_t, \omega)\mathrm{d}t$$

This Talk: Rendering Equation

In Paper: Path Integral

Reciprocity, energy conservation, …

Summary

Summary

- In correlated media, transmittance becomes *four functions*

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- These represent different interactions at the end points

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- This talk: High level overview
- Paper: Rigorous derivation

Modelling Transmittance

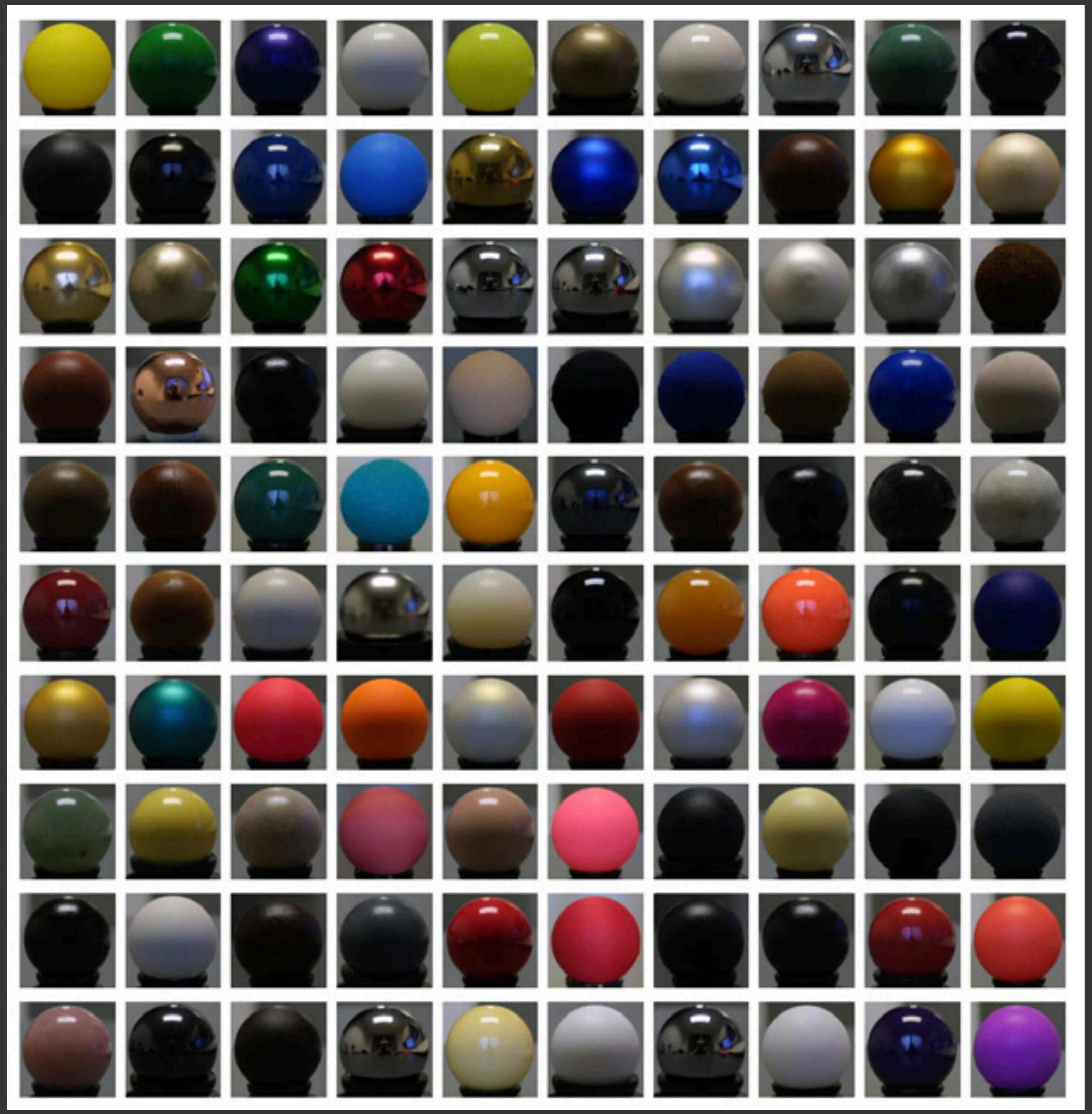
Modelling Transmittance

Data Driven

Modelling Transmittance

Data Driven

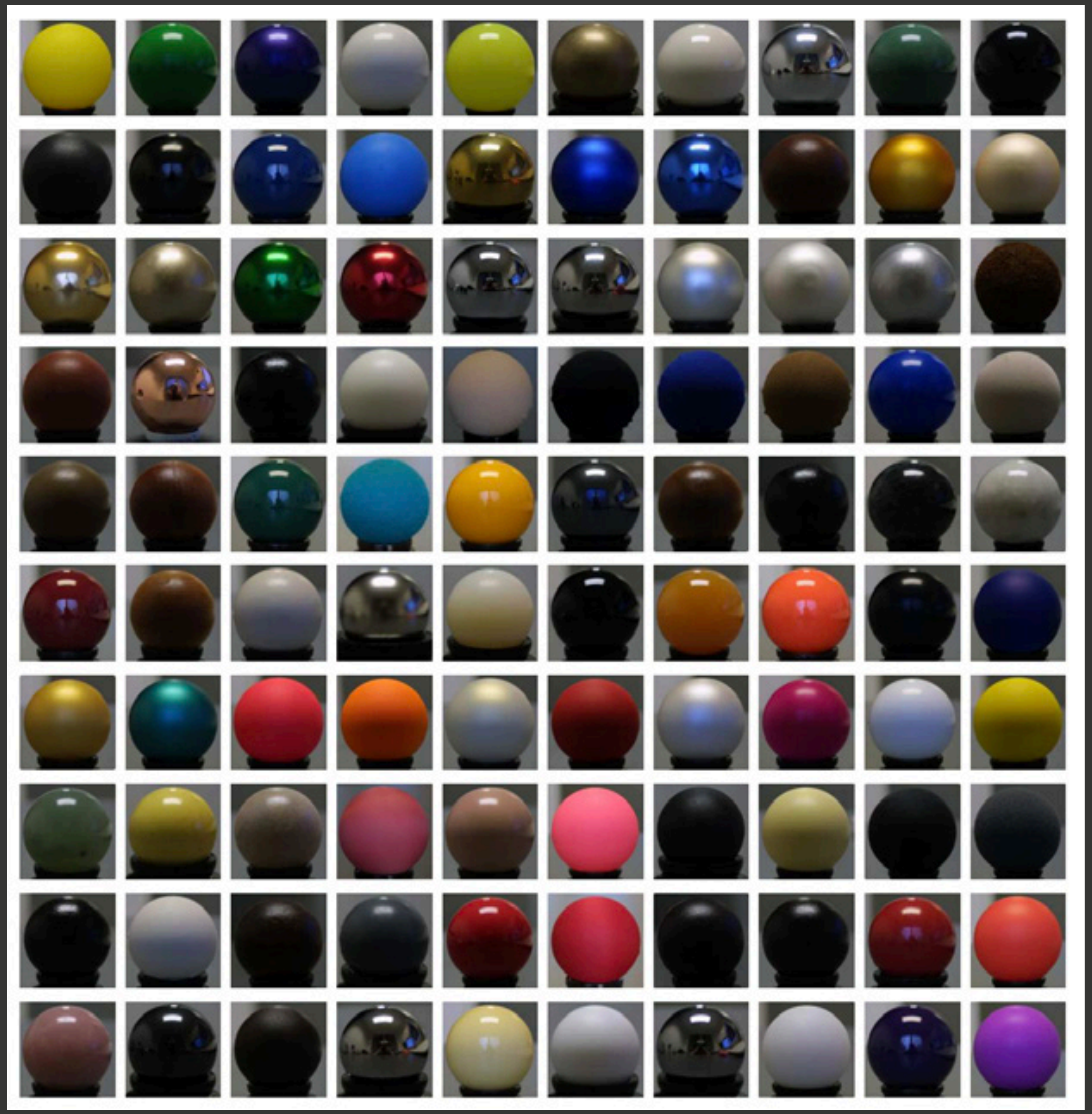
A Data-Driven Reflectance Model,
Matusik et al., **2003**



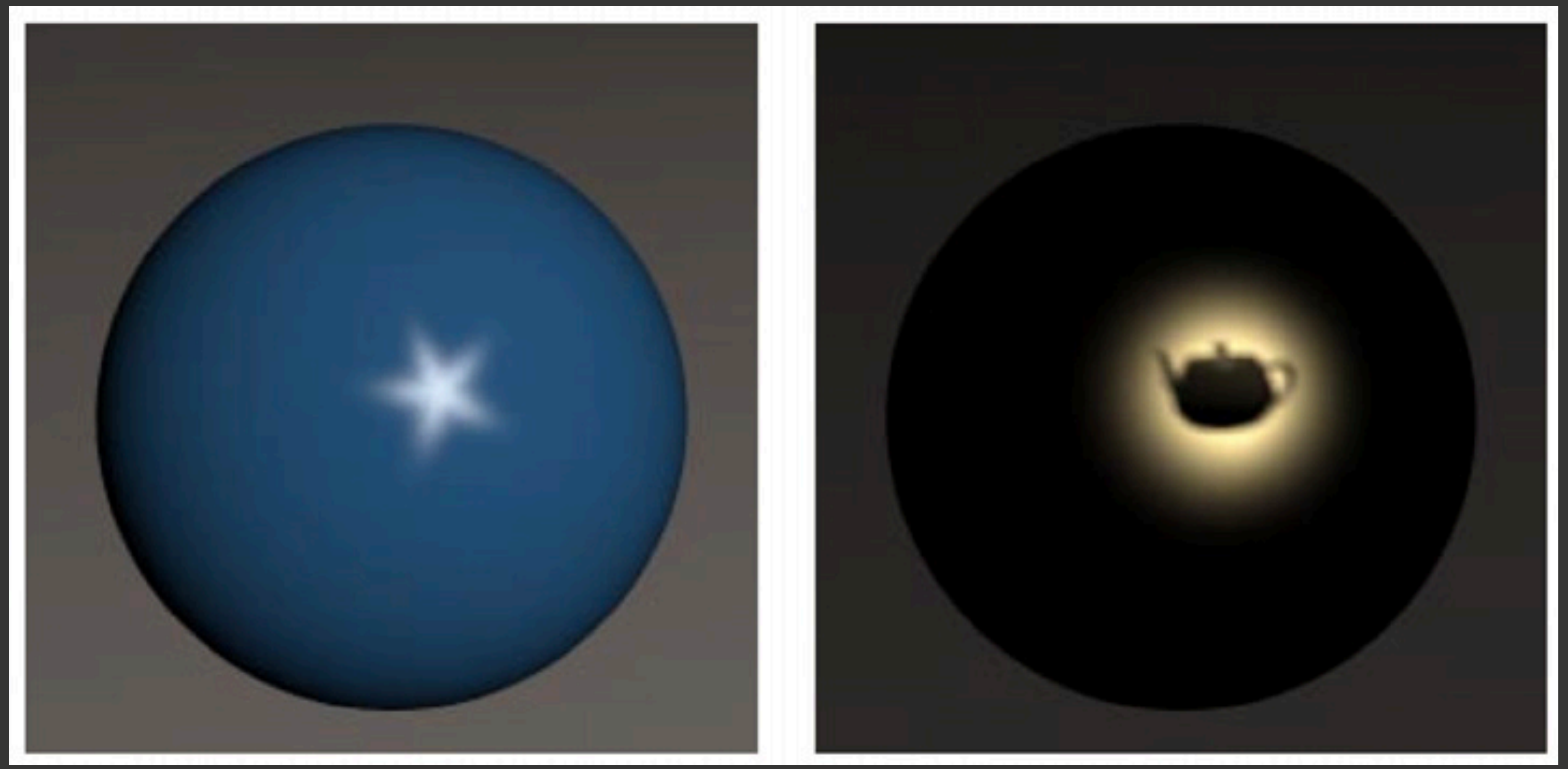
Modelling Transmittance

Data Driven

A Data-Driven Reflectance Model,
Matusik et al., **2003**



A Microfacet-based BRDF Generator,
Ashikhmin et al., **2000**



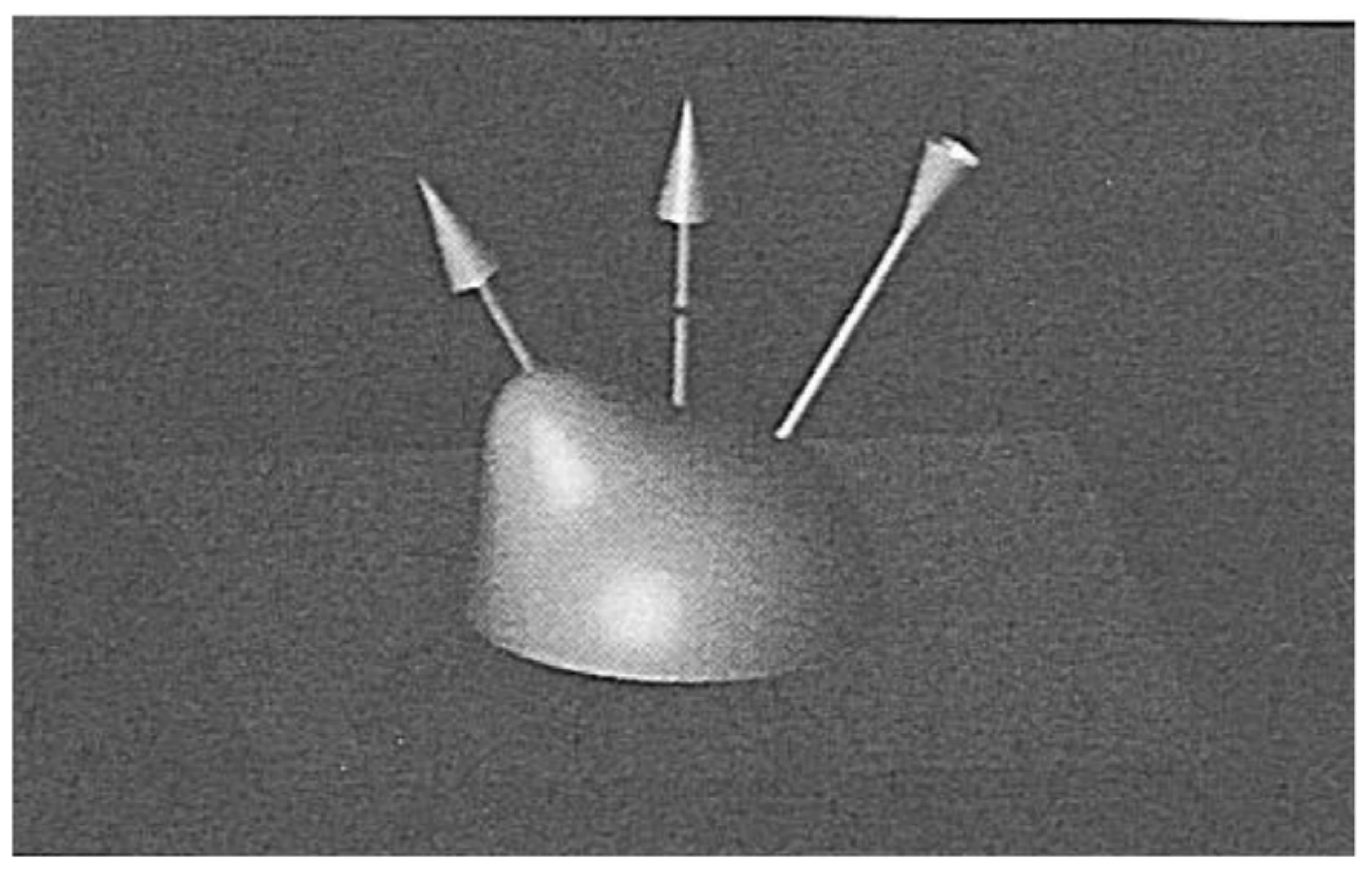

Transmittance
$e^{-\tau}$

Modelling Transmittance

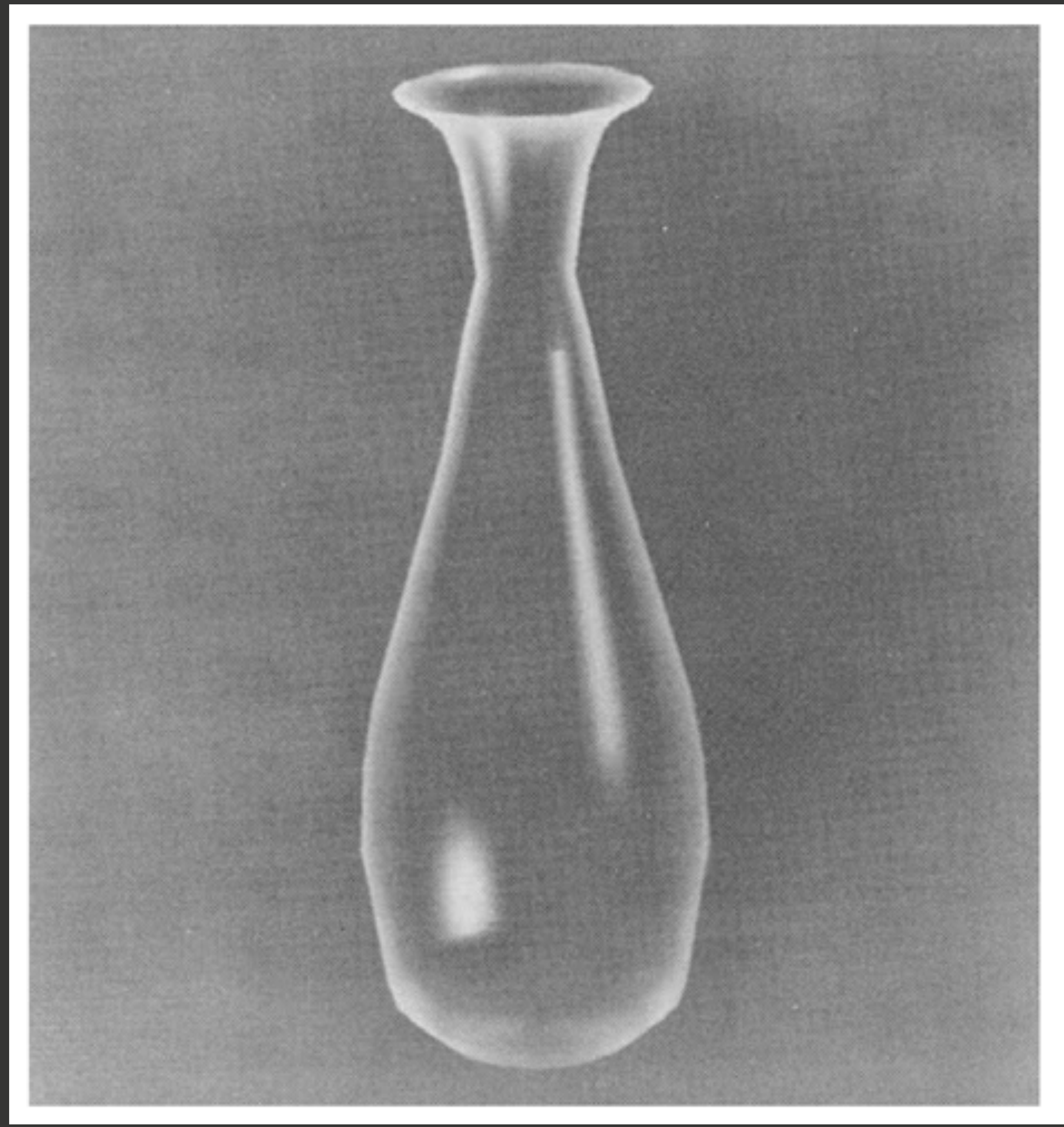
Phenomenological

Modelling Transmittance

Models of Light Reflection For Computer Synthesized Pictures
James F. Blinn, **1977**



Illumination for Computer Generated Pictures,
Bui Tuong Phong, **1975**



Phenomenological

Modelling Transmittance

Models of Light Reflection For Computer Synthesized Pictures
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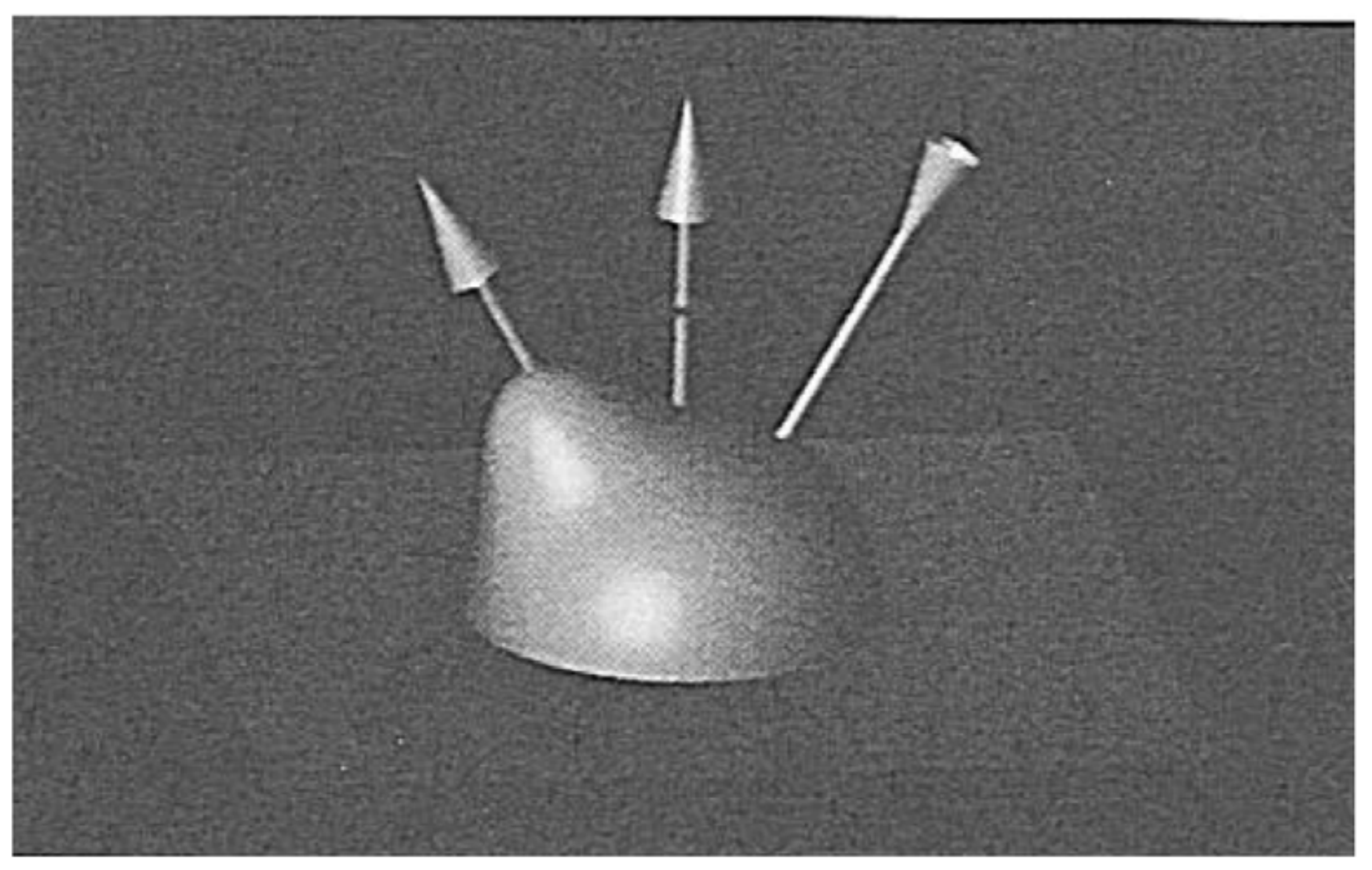

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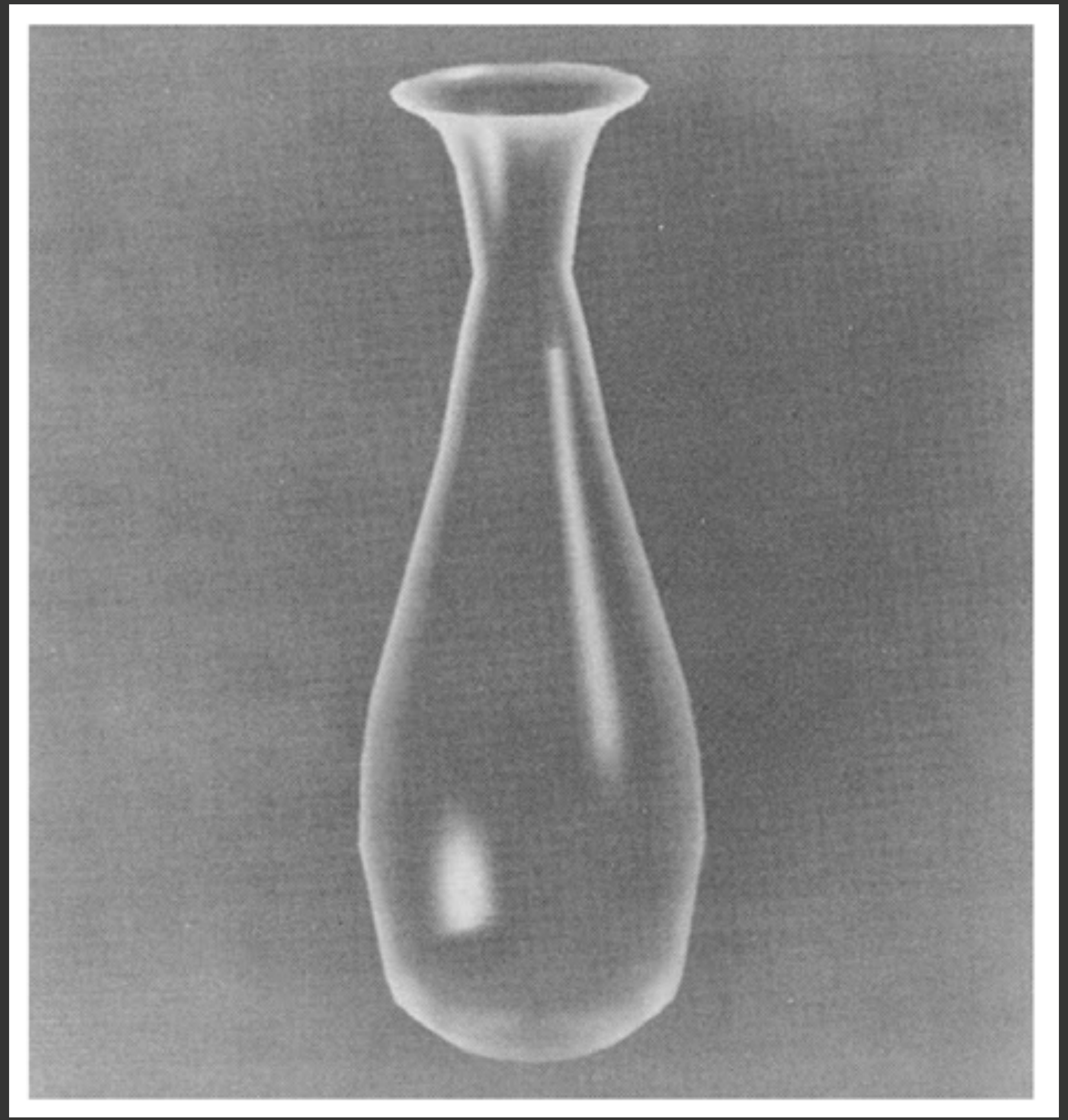
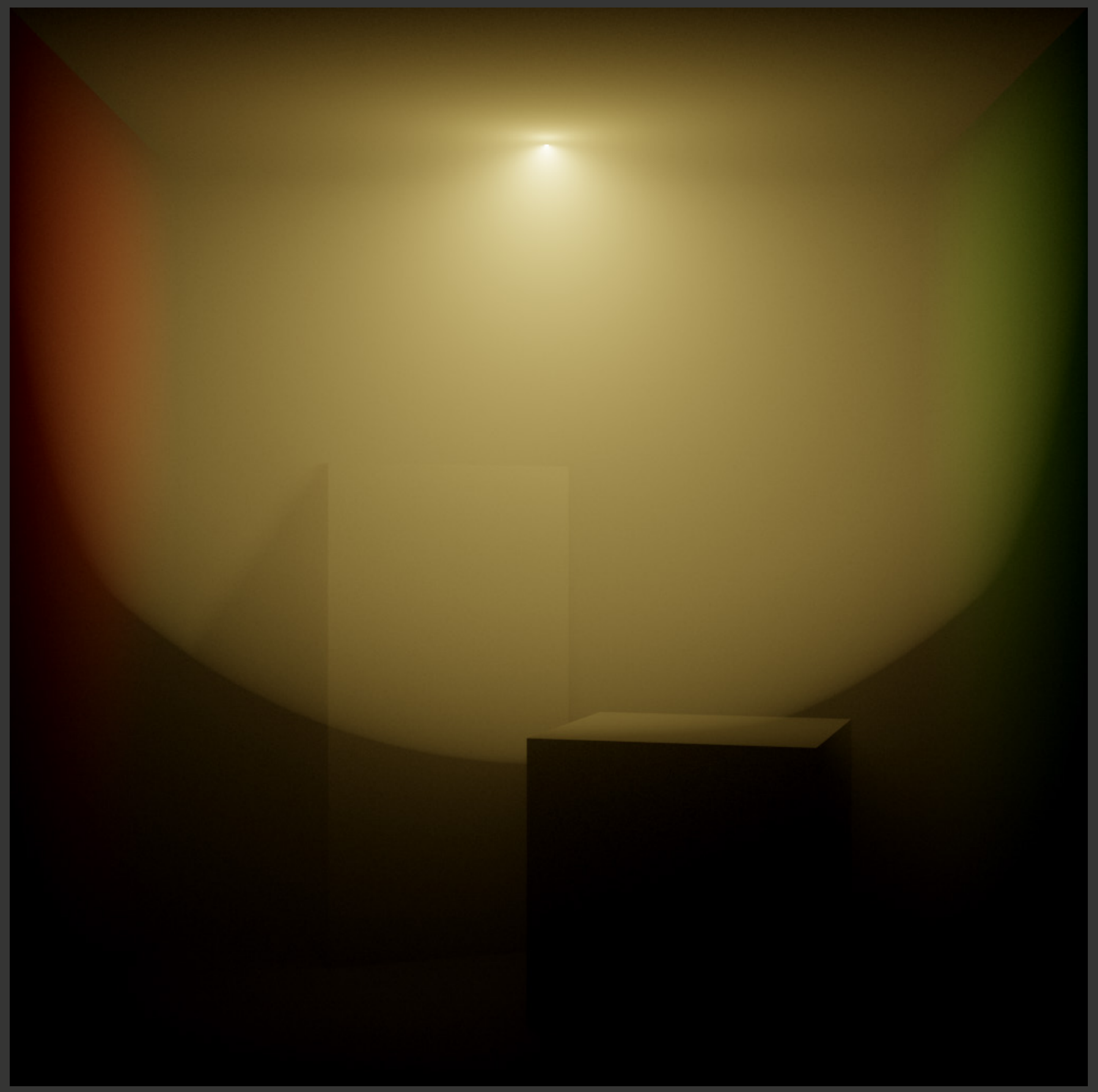
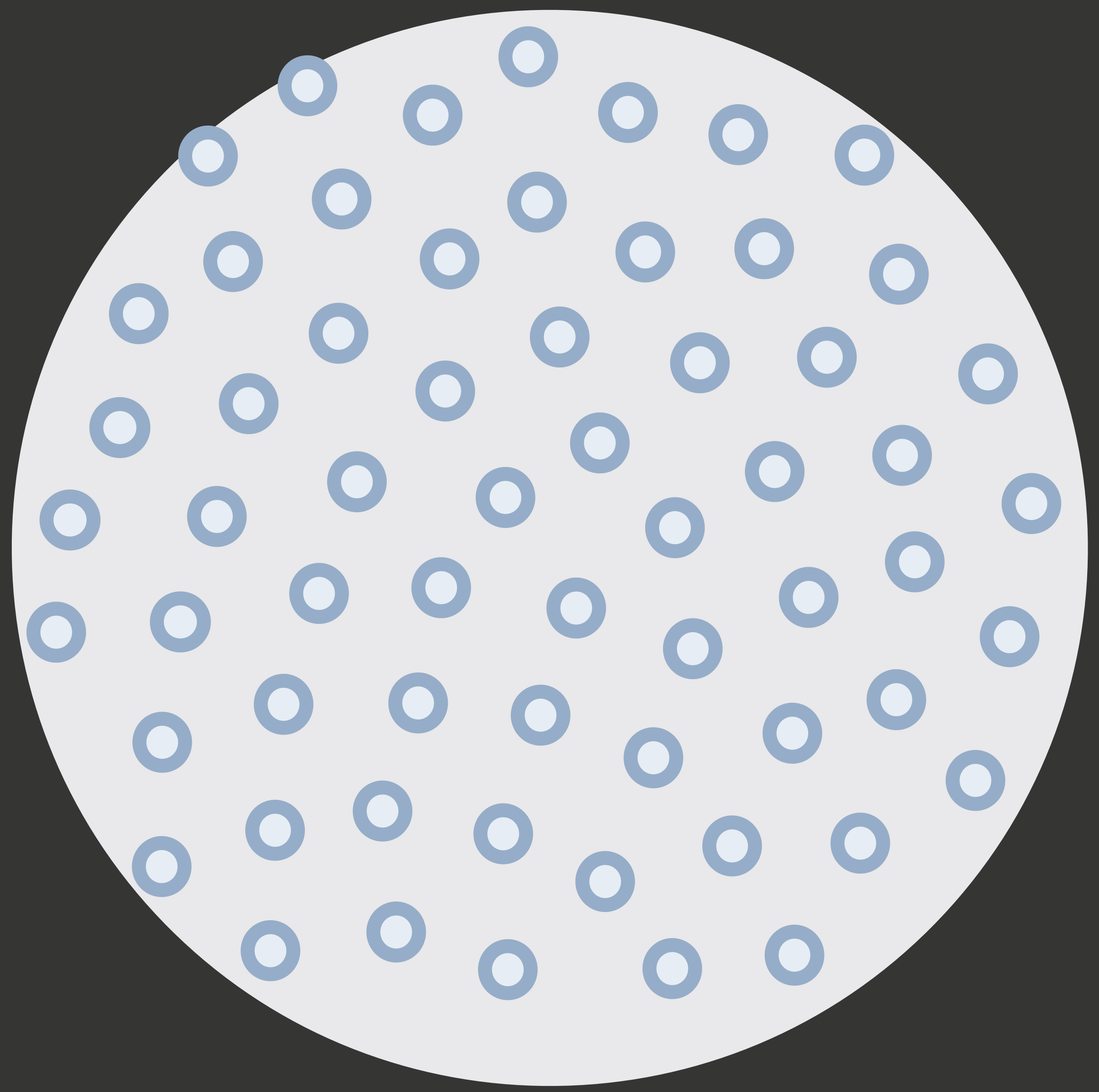


Phenomenological

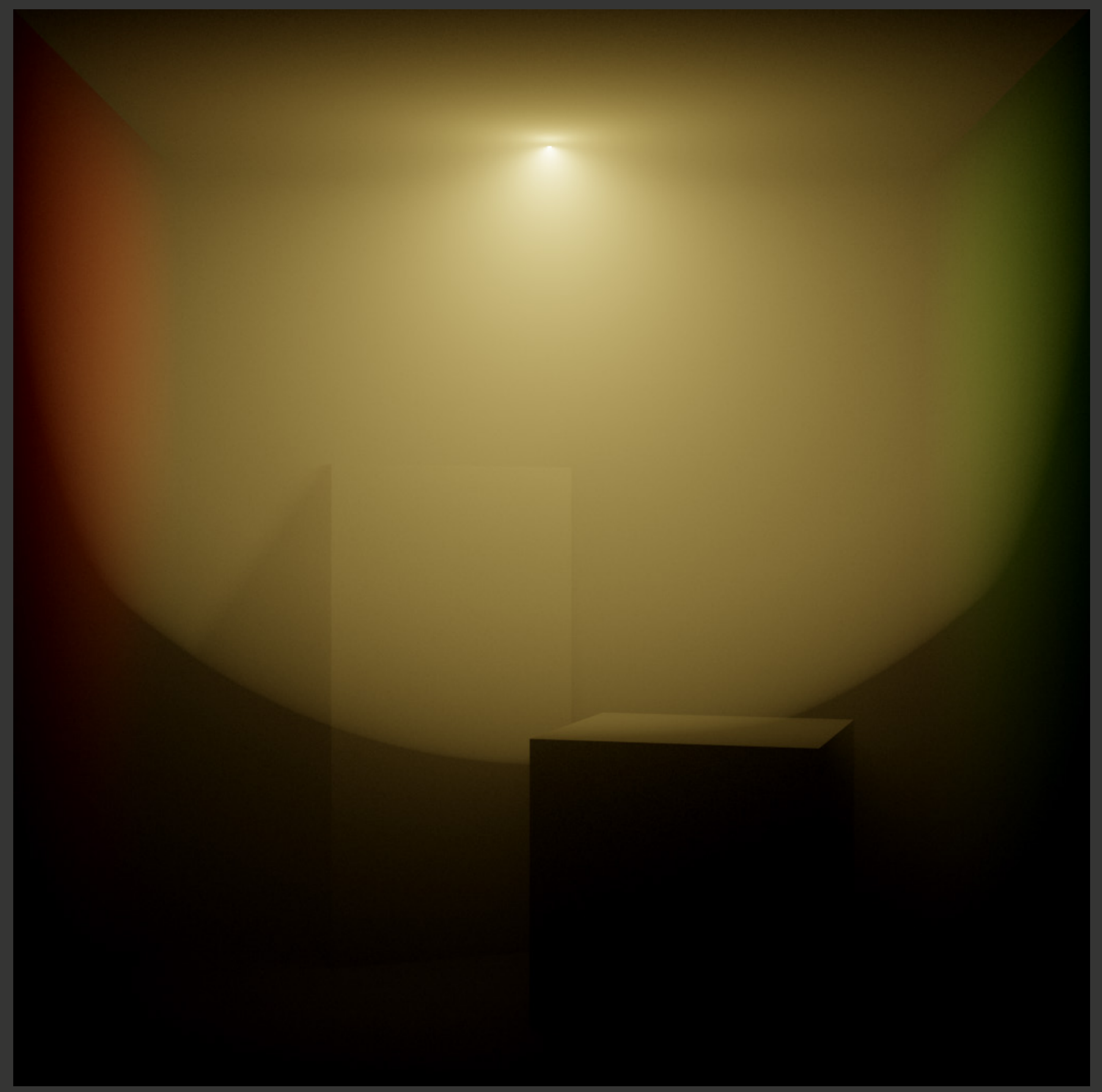


Modelling Transmittance

Data Driven



Phenomenological

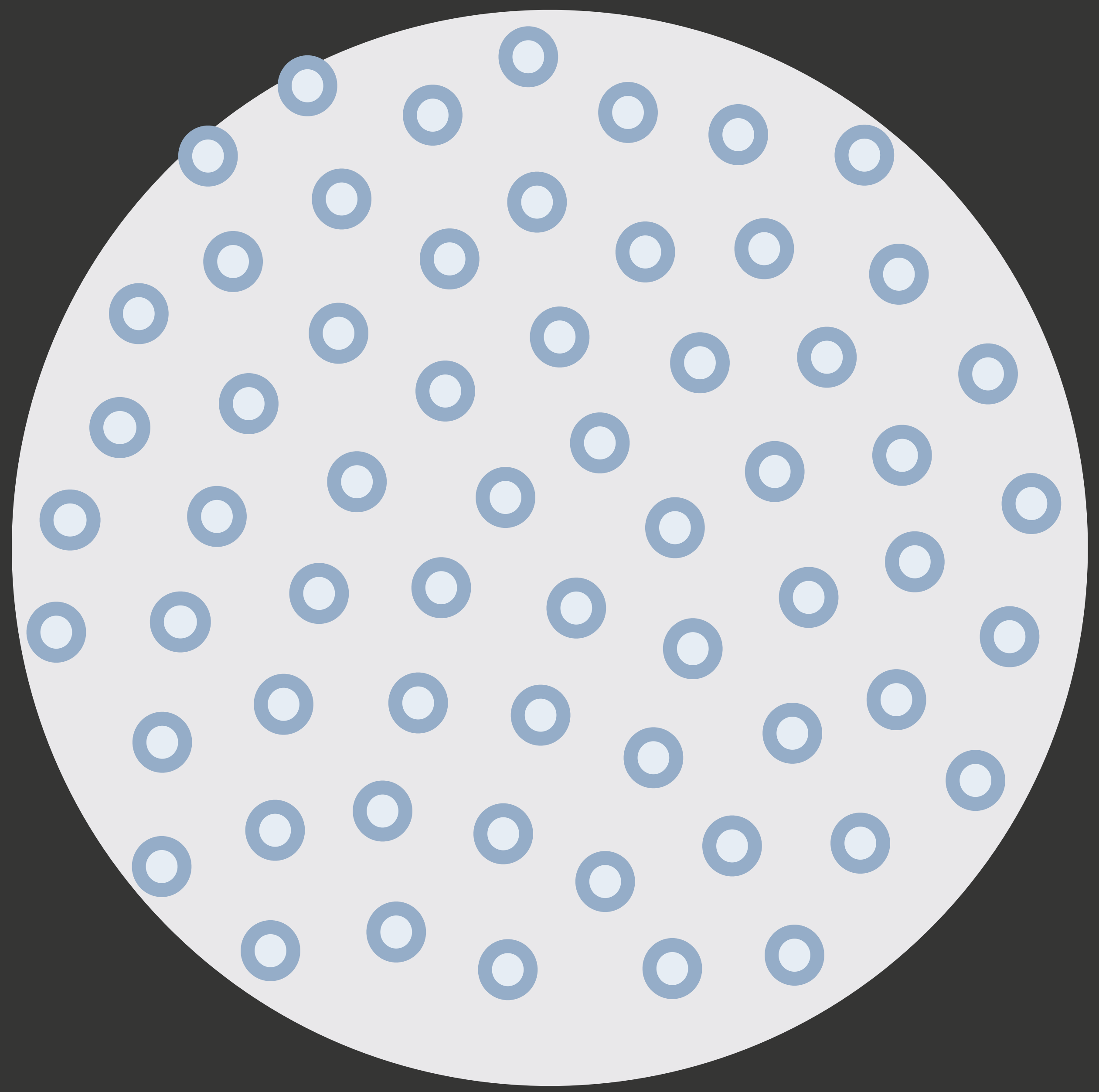


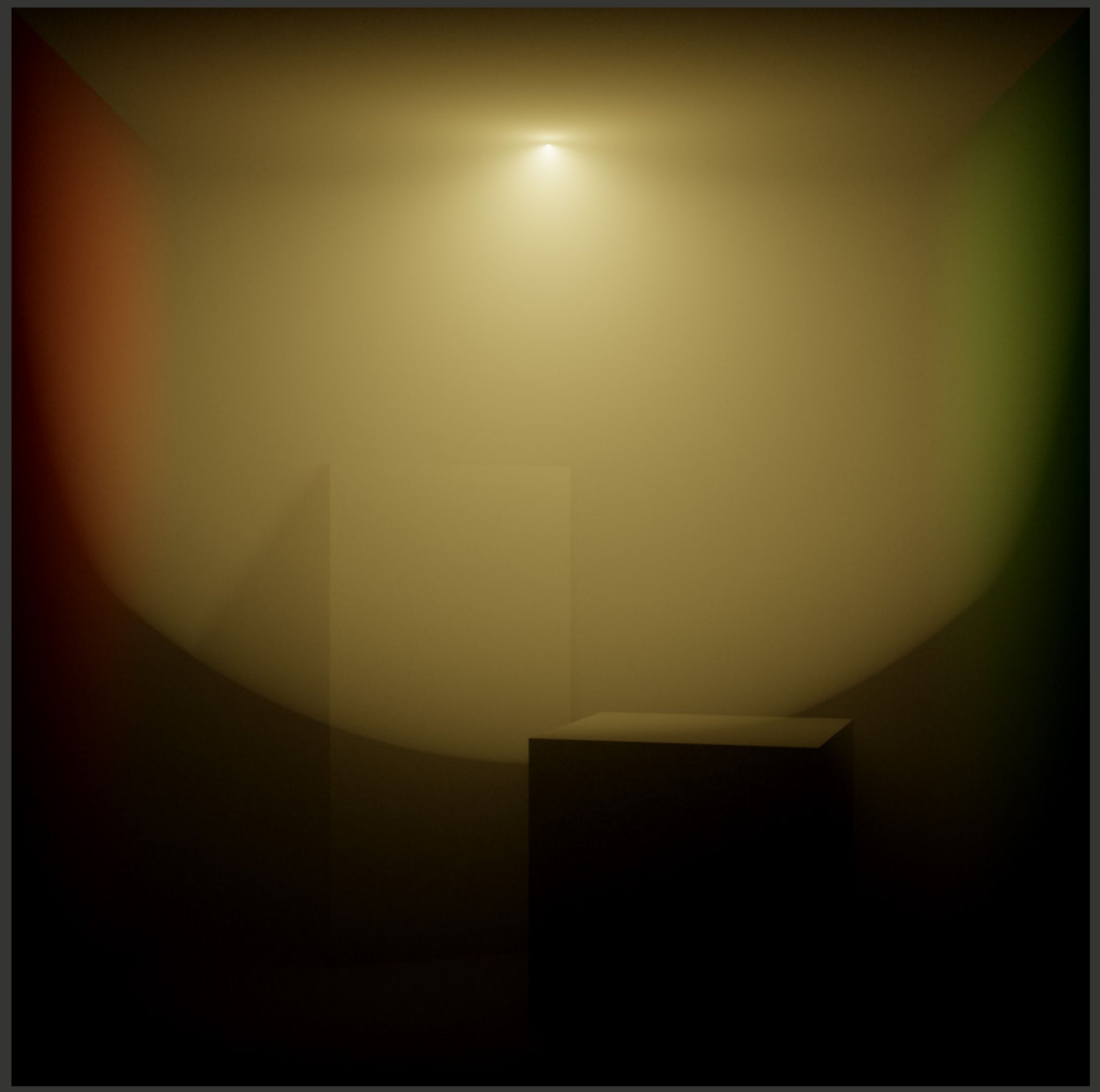
Modelling Transmittance

Data Driven

Statistical Models

Phenomenological



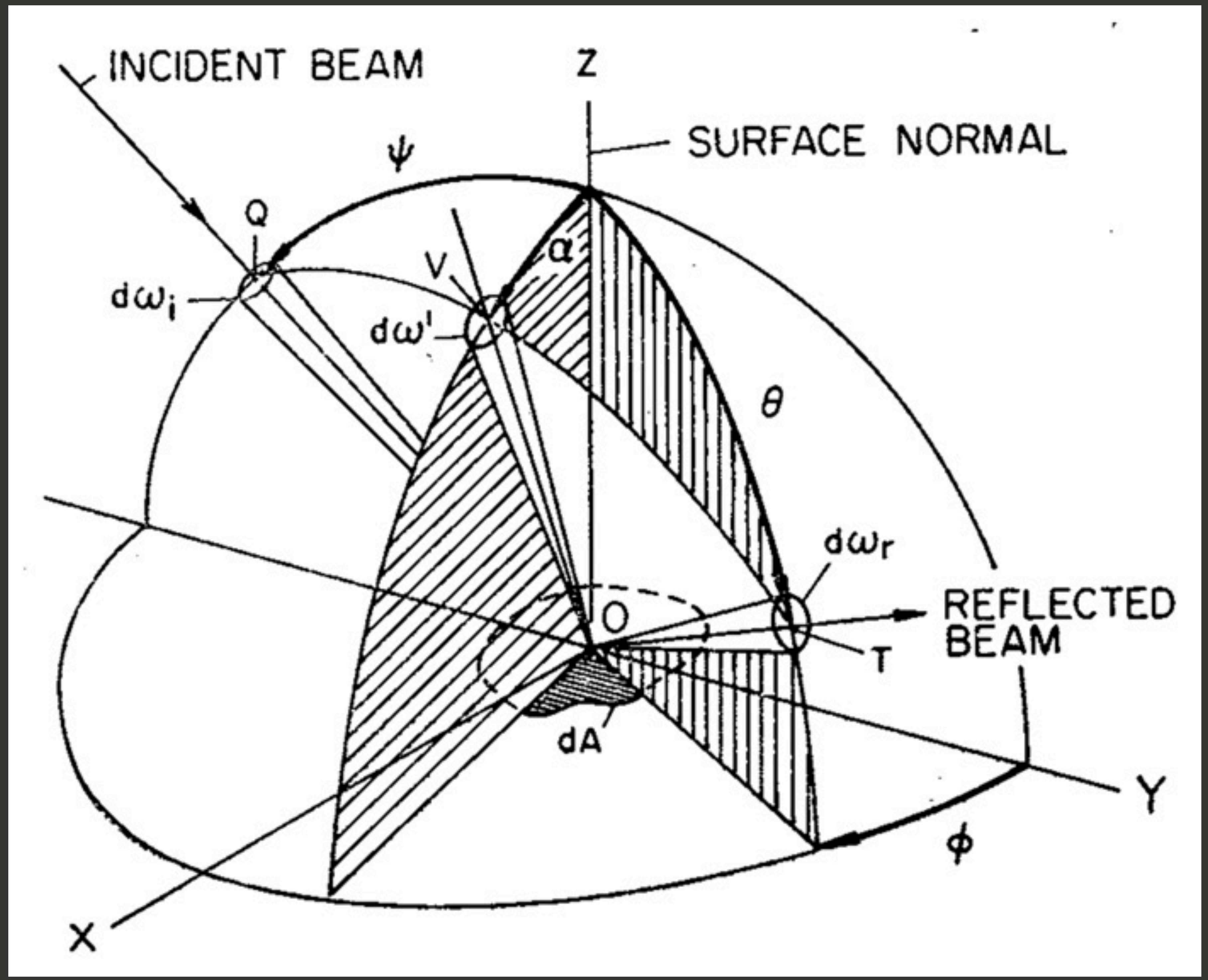


Modelling Transmittance

Microfacet Models for Refraction through Rough Surfaces,
Walter et al., **2007**

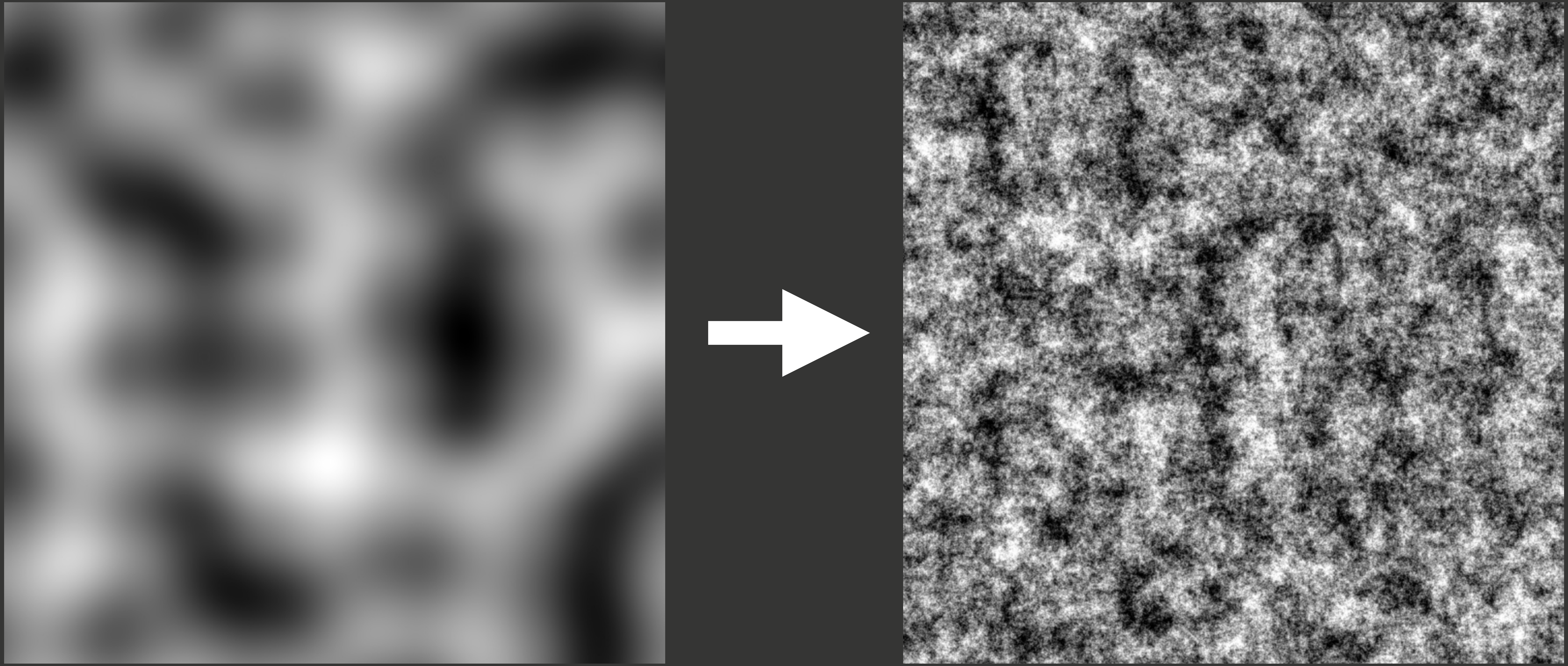
Statistical Models

Theory for Off-Specular Reflection From Roughened Surfaces,
Torrance and Sparrow, **1966**

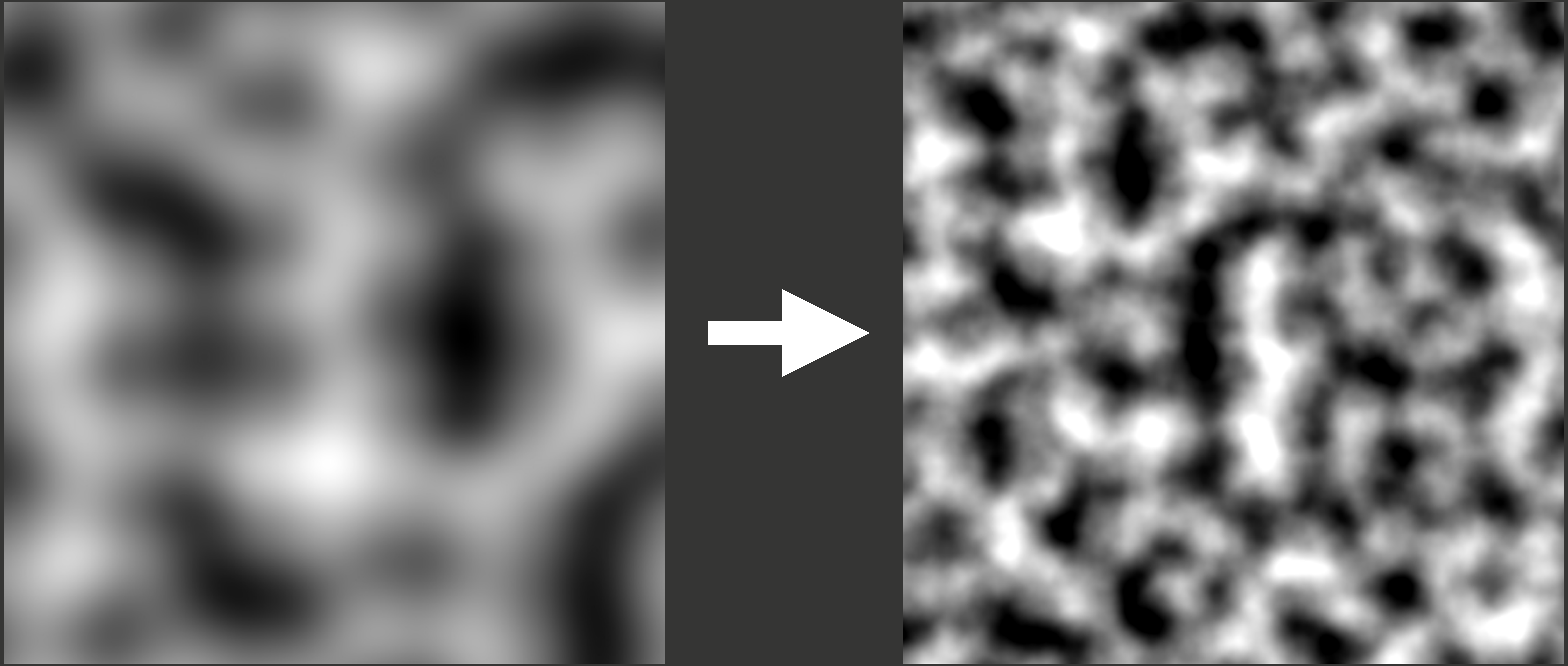


Statistical Transmittance

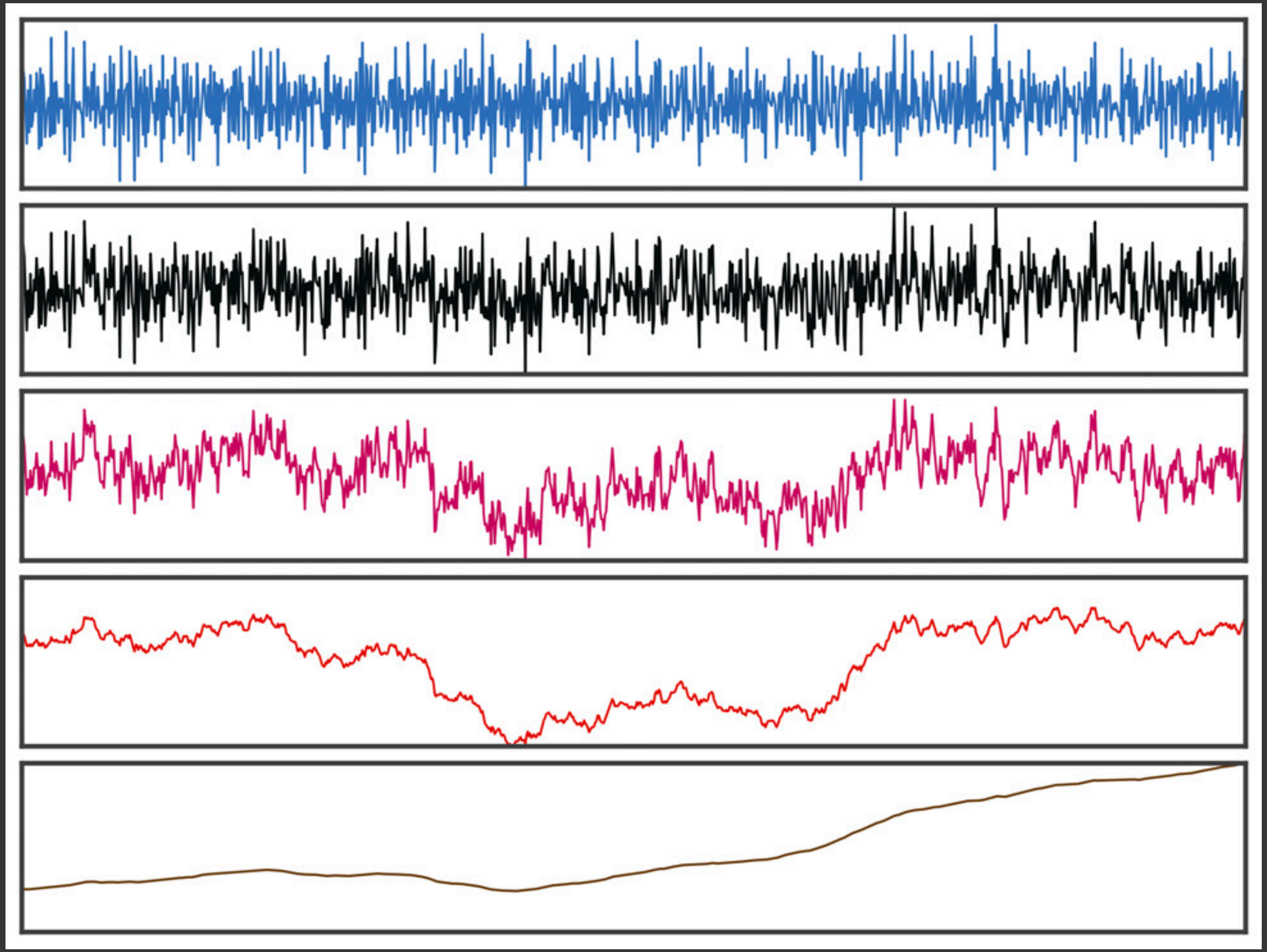
Statistical Transmittance



Statistical Transmittance



Statistical Transmittance



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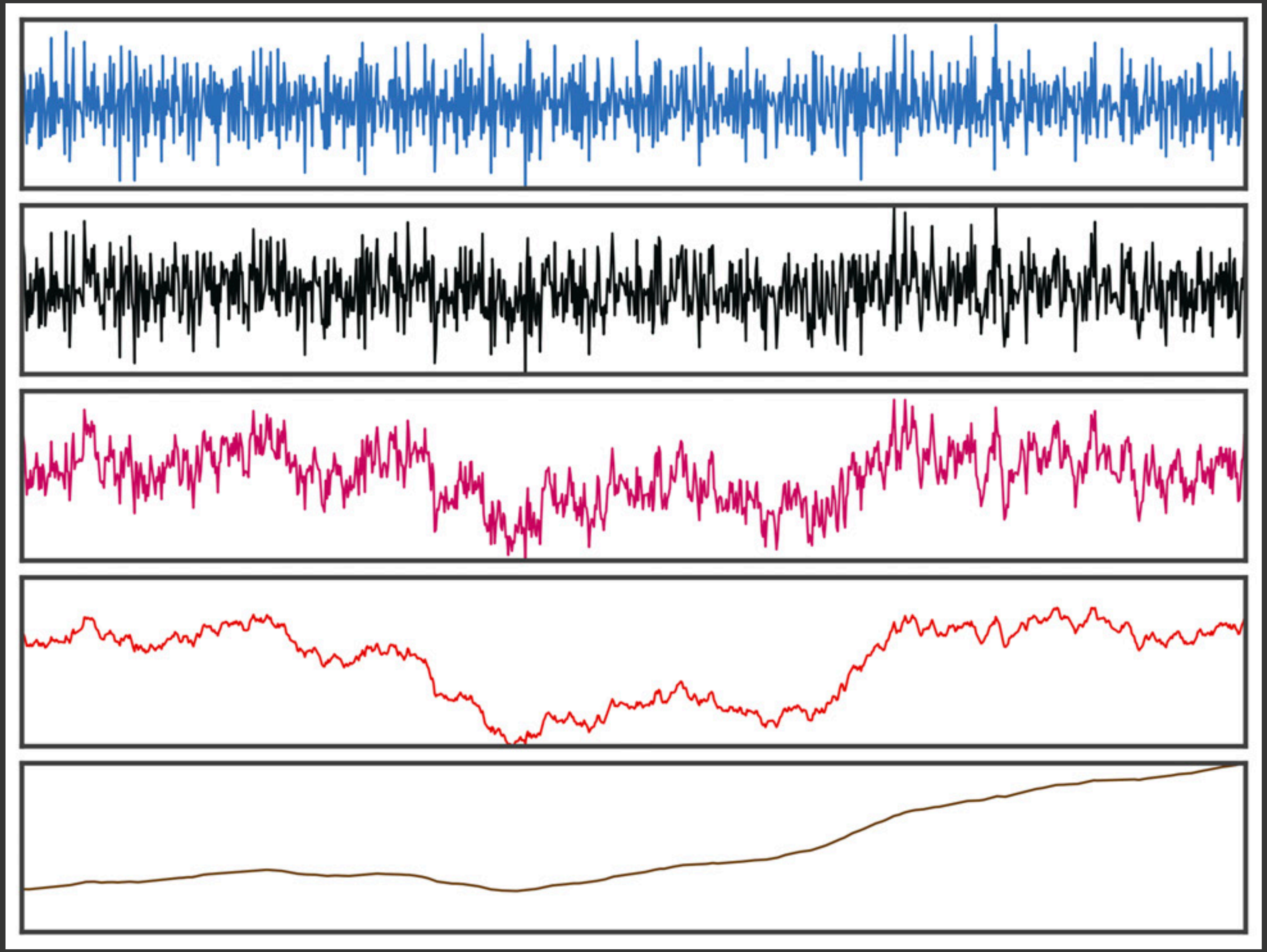
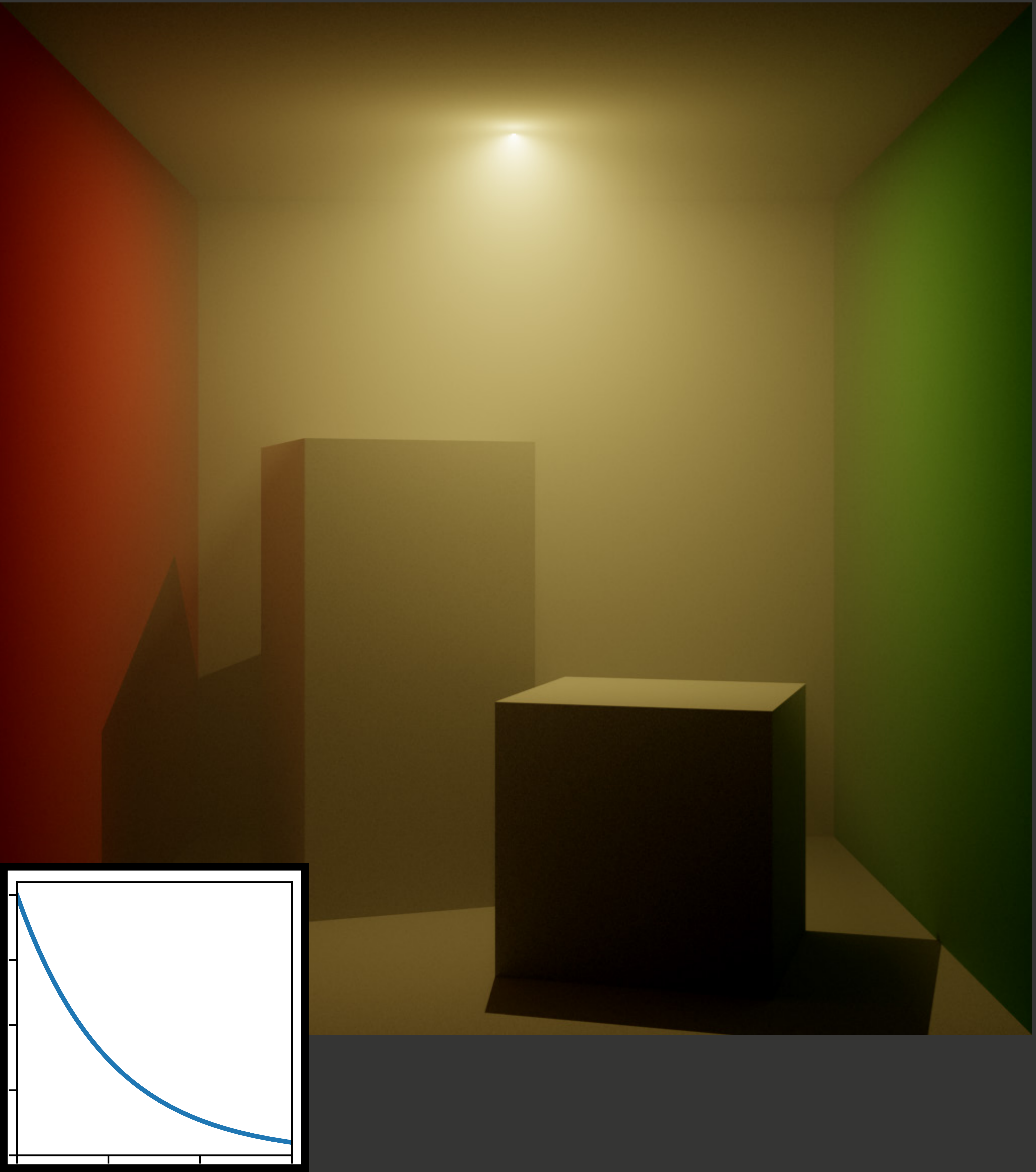


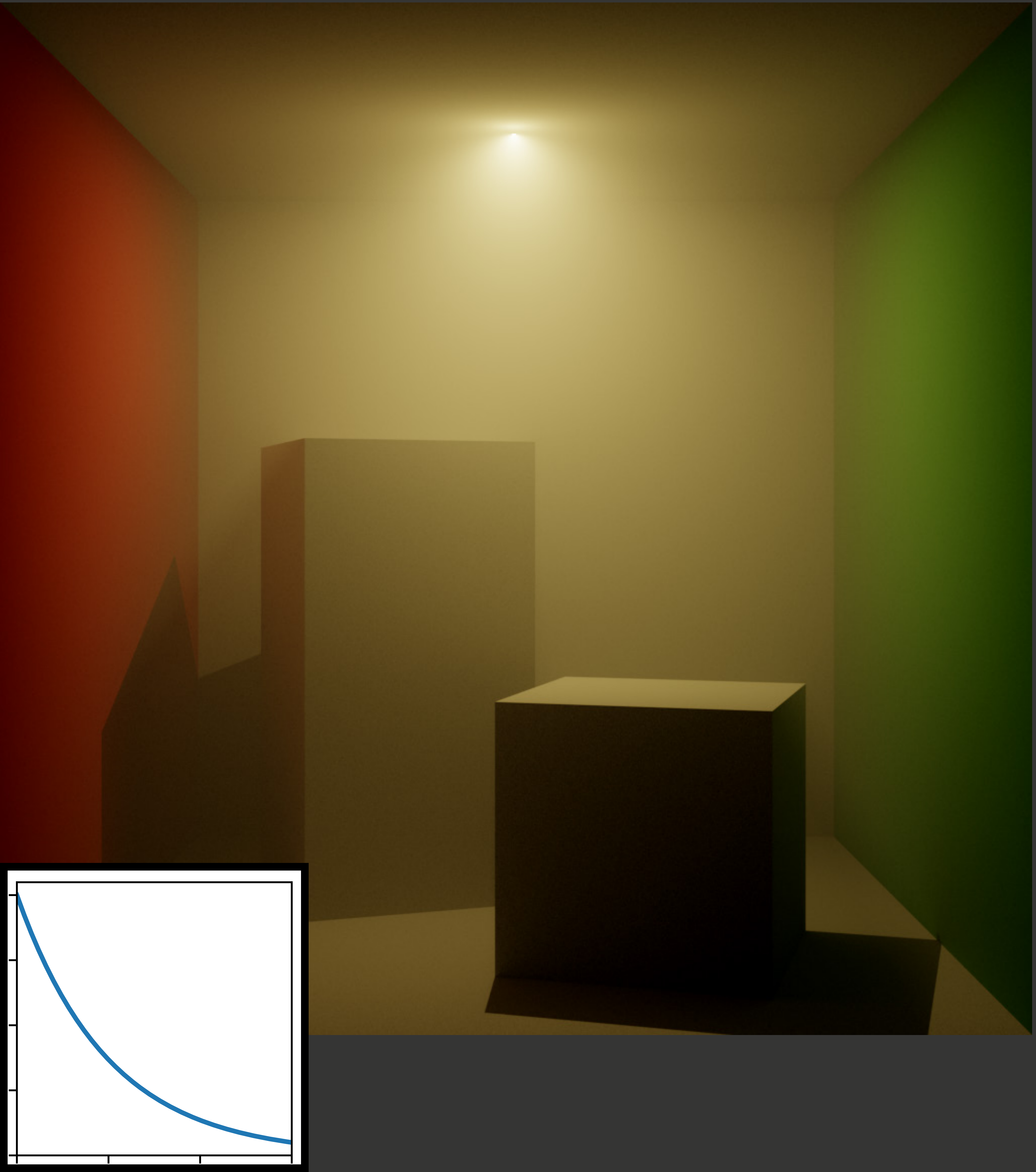
Davis and Mineev-Weinstein, **2011**

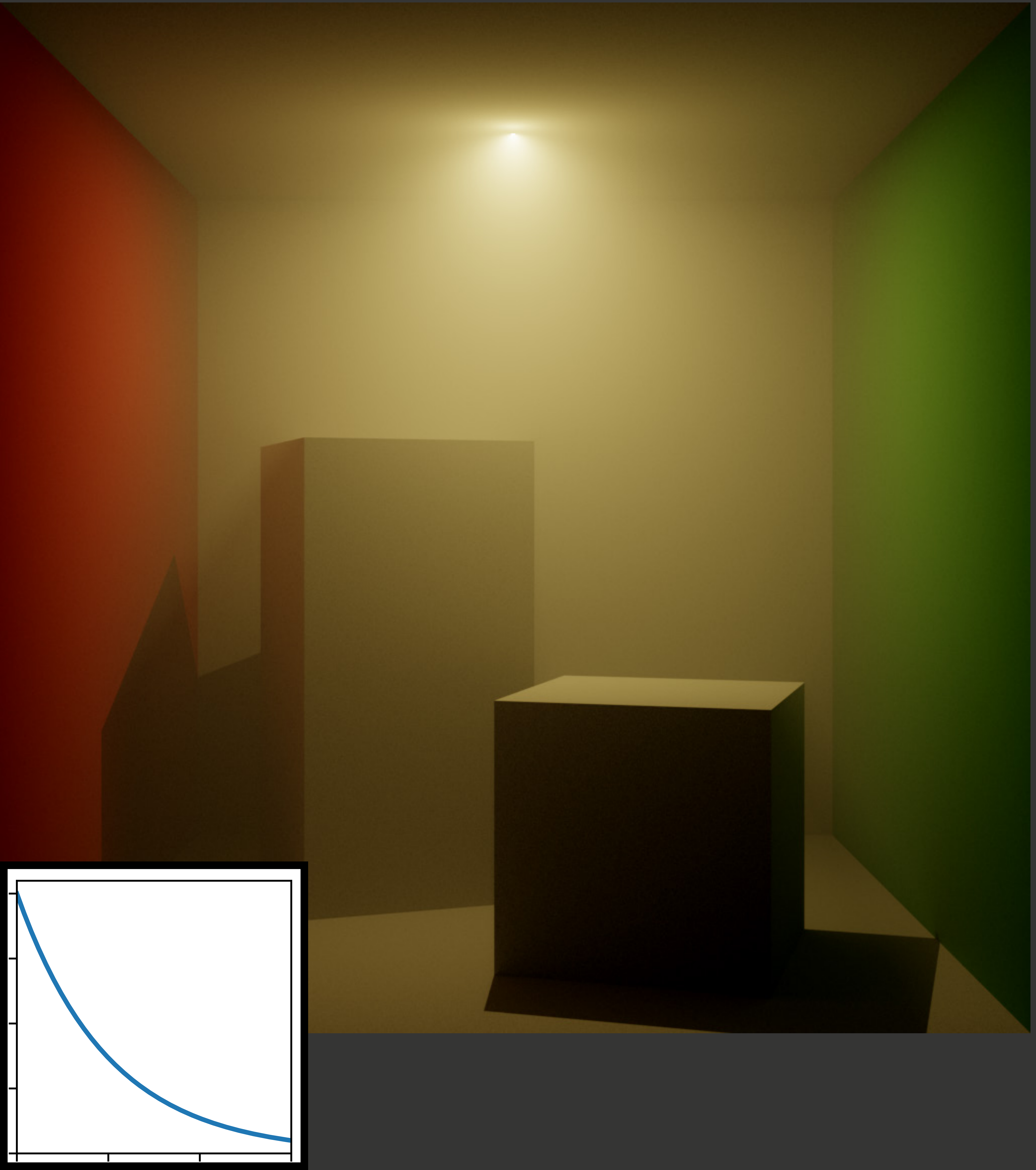
Results

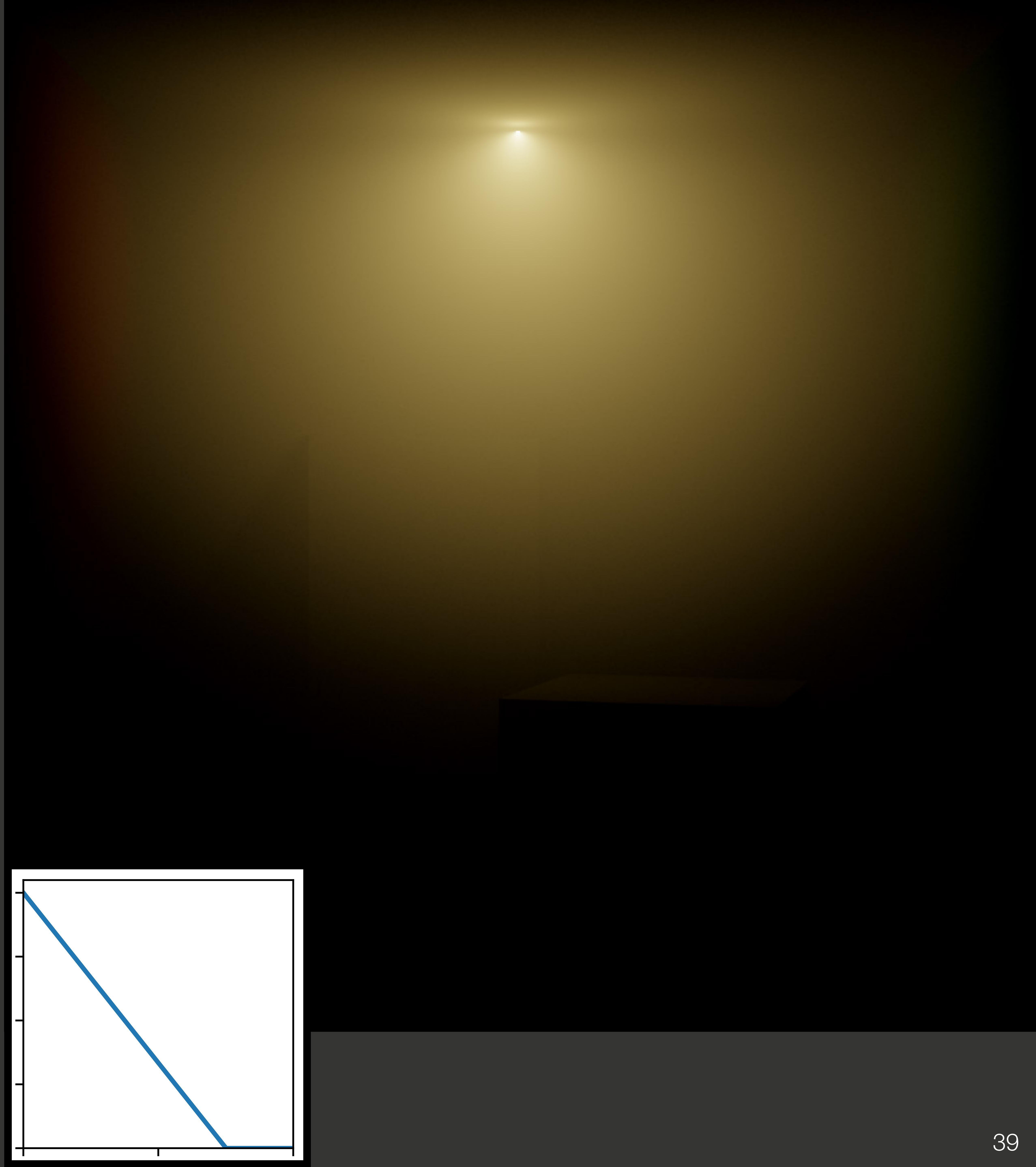
Phenomenological Transmittance

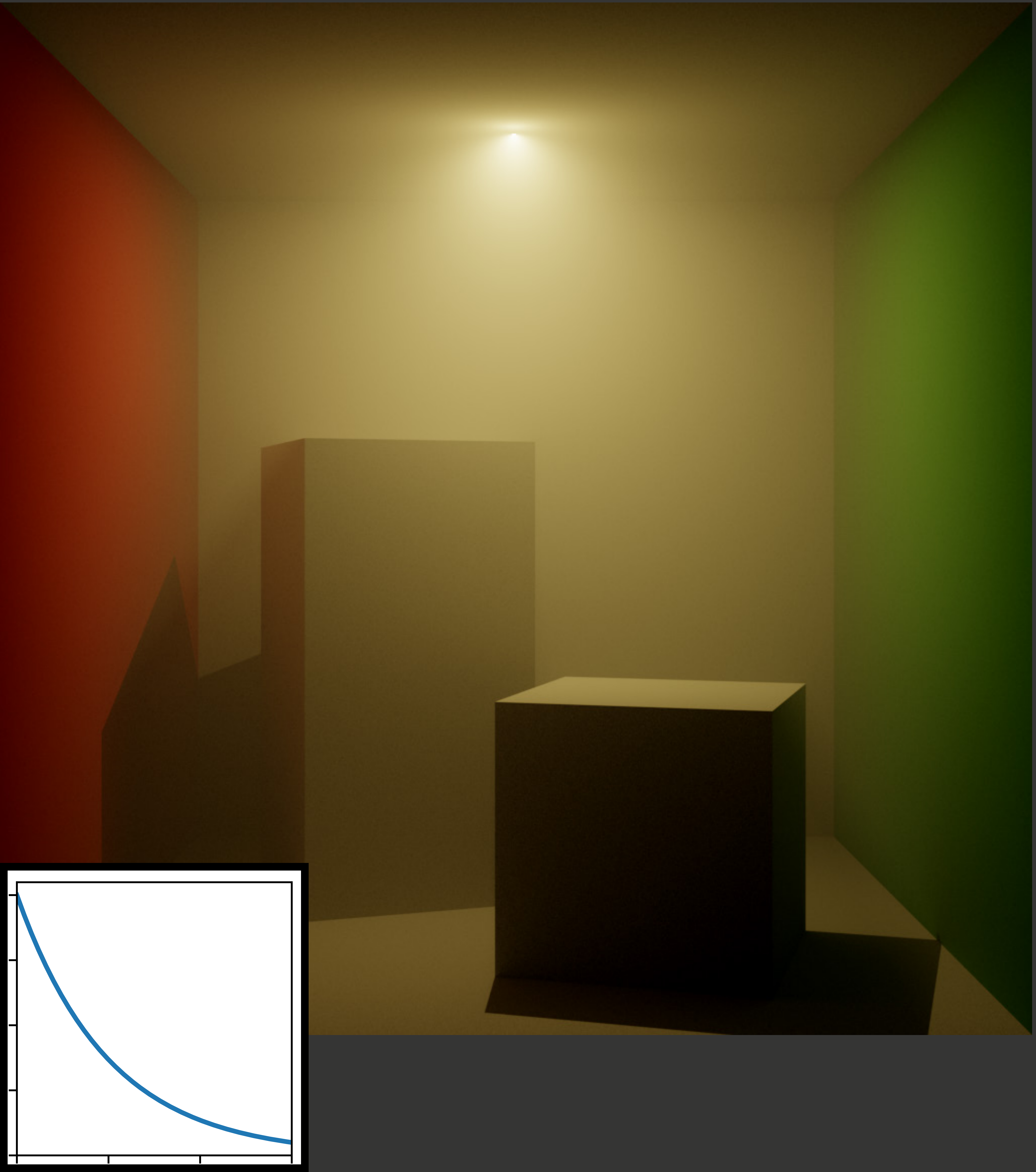
(non-physical)

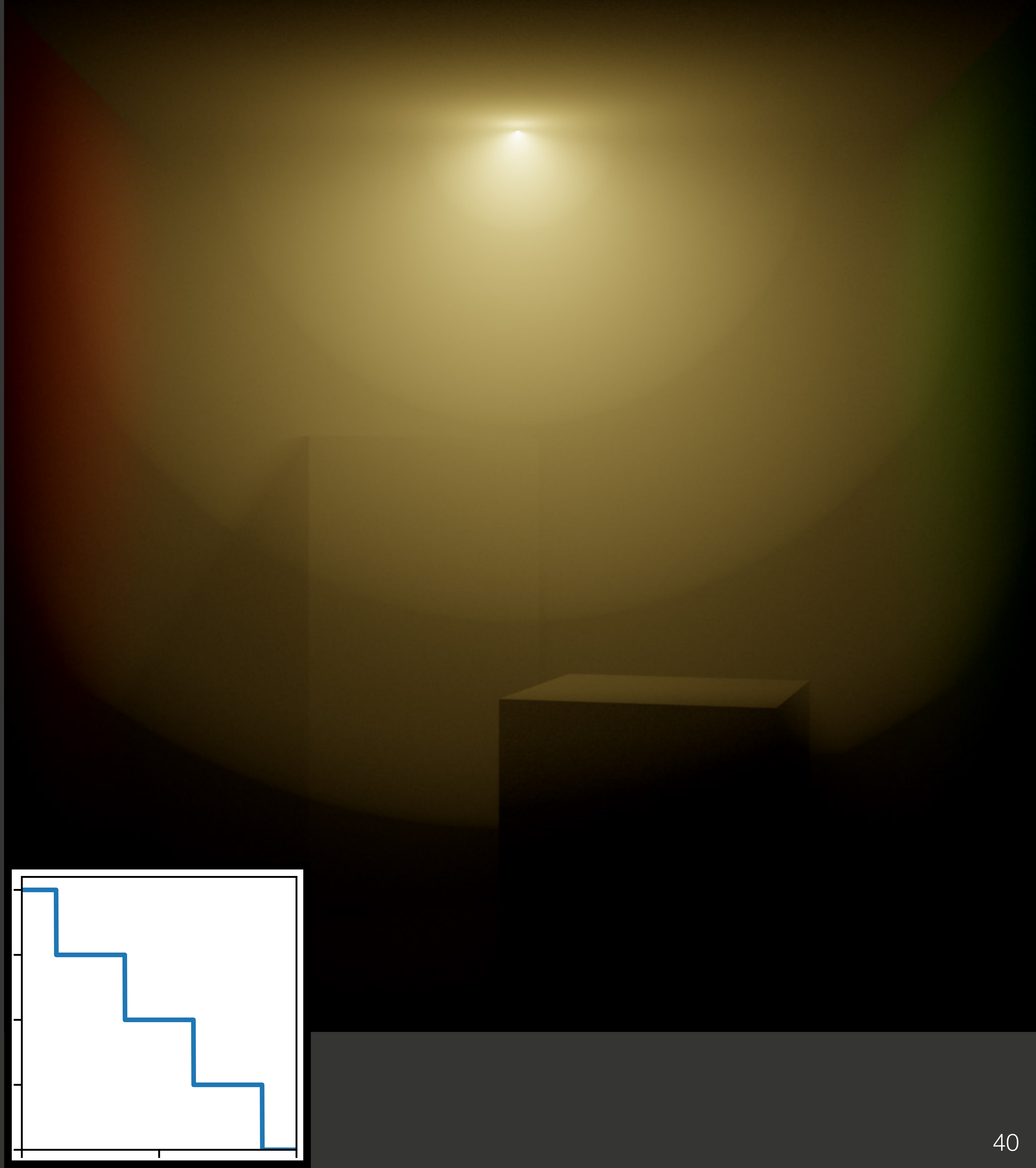


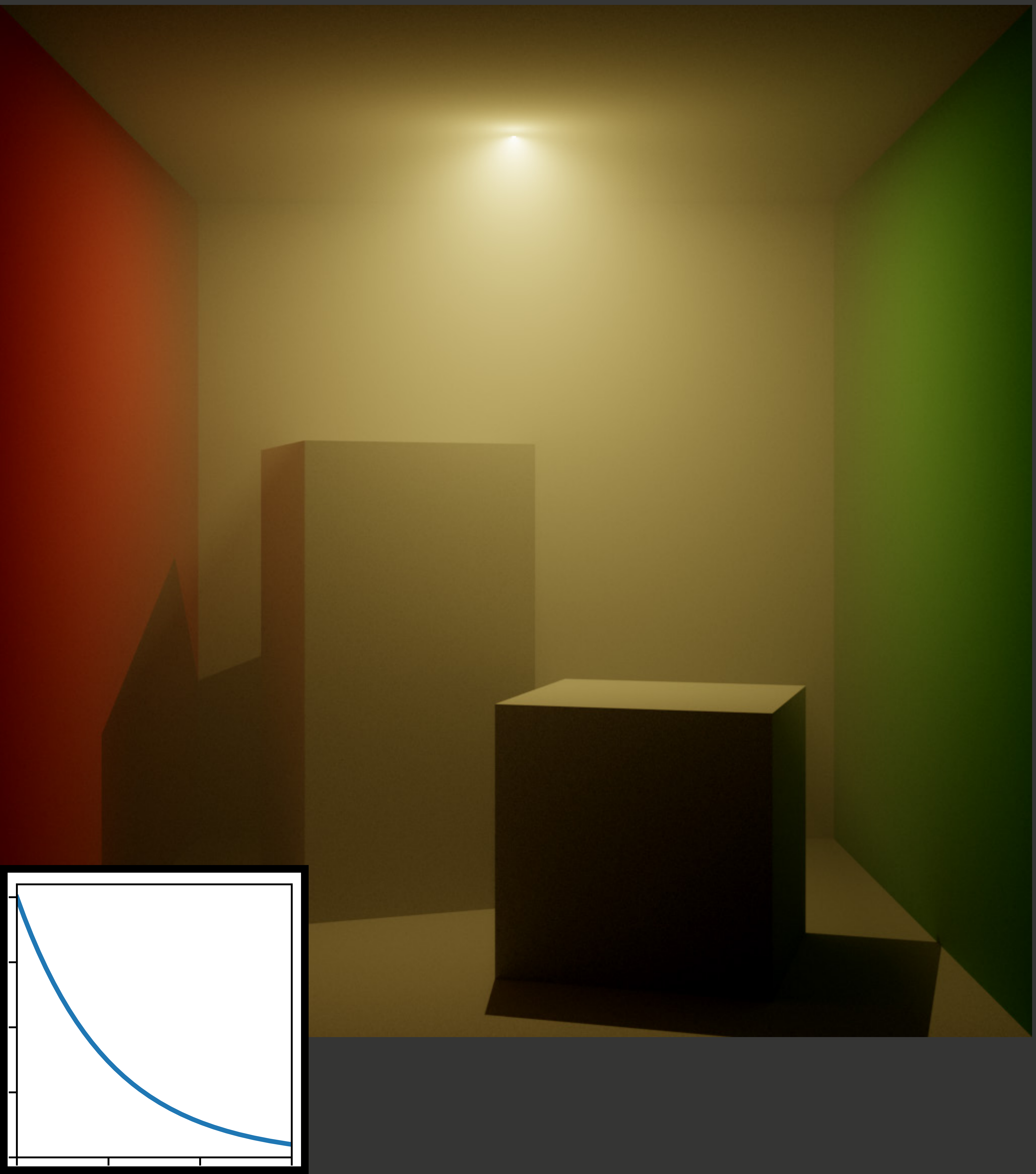
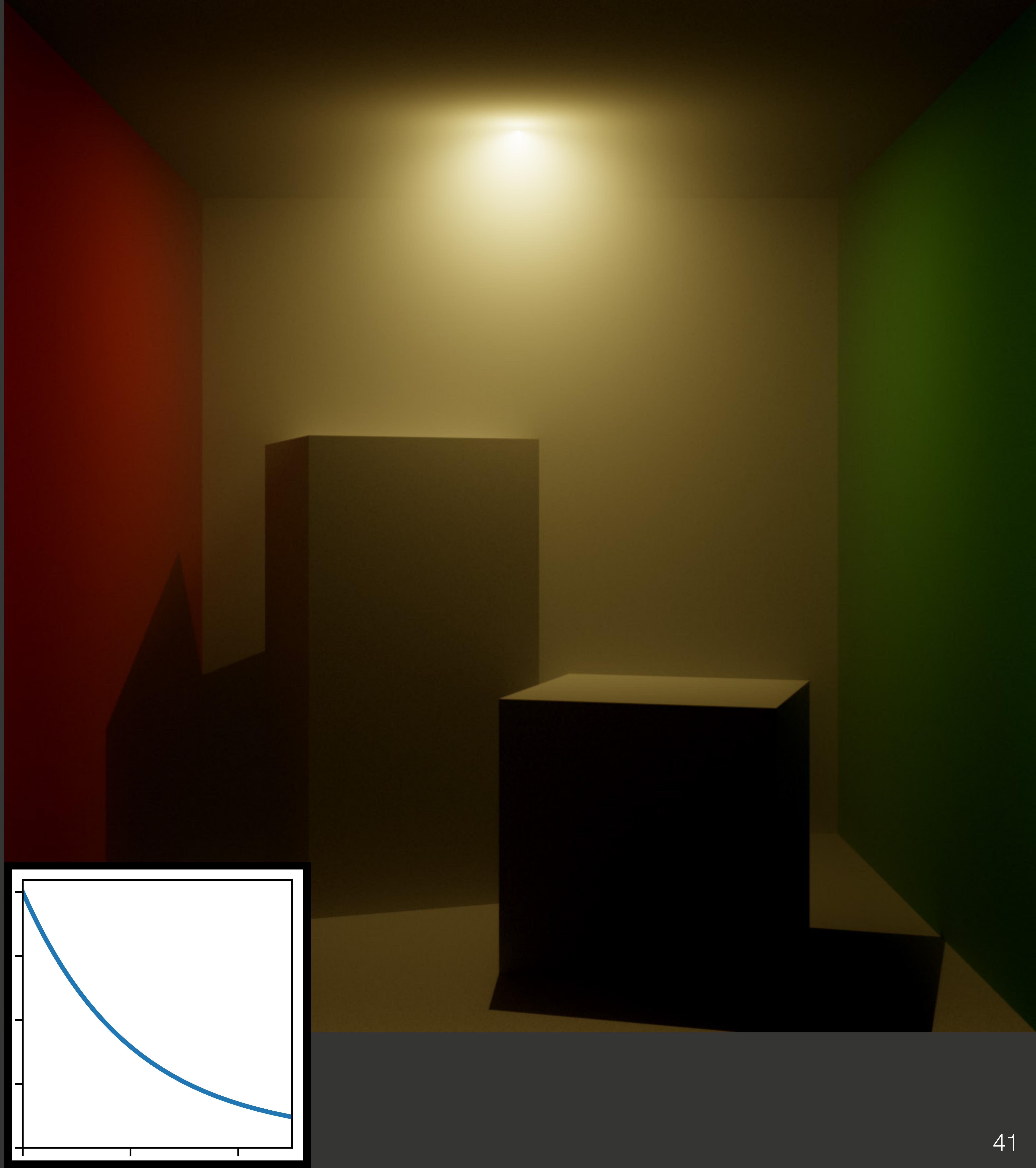











Transmittance (PF)


Transmittance (PF)

Davis-Mineev-Weinstein Model

(physically based)


β
C
Transmittance (PF)


β
C
Transmittance (PF)

Limitations

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- Same correlations everywhere

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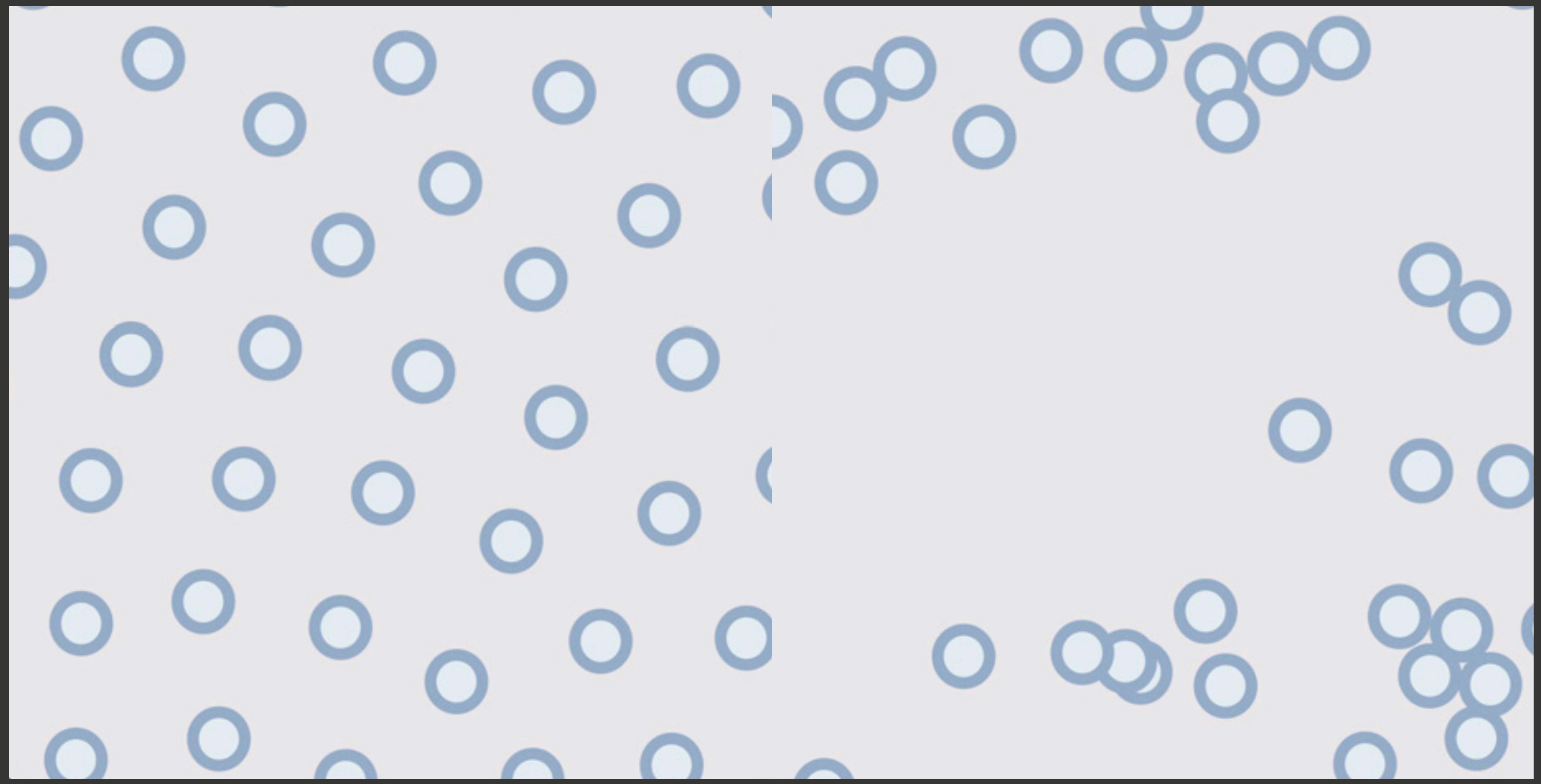

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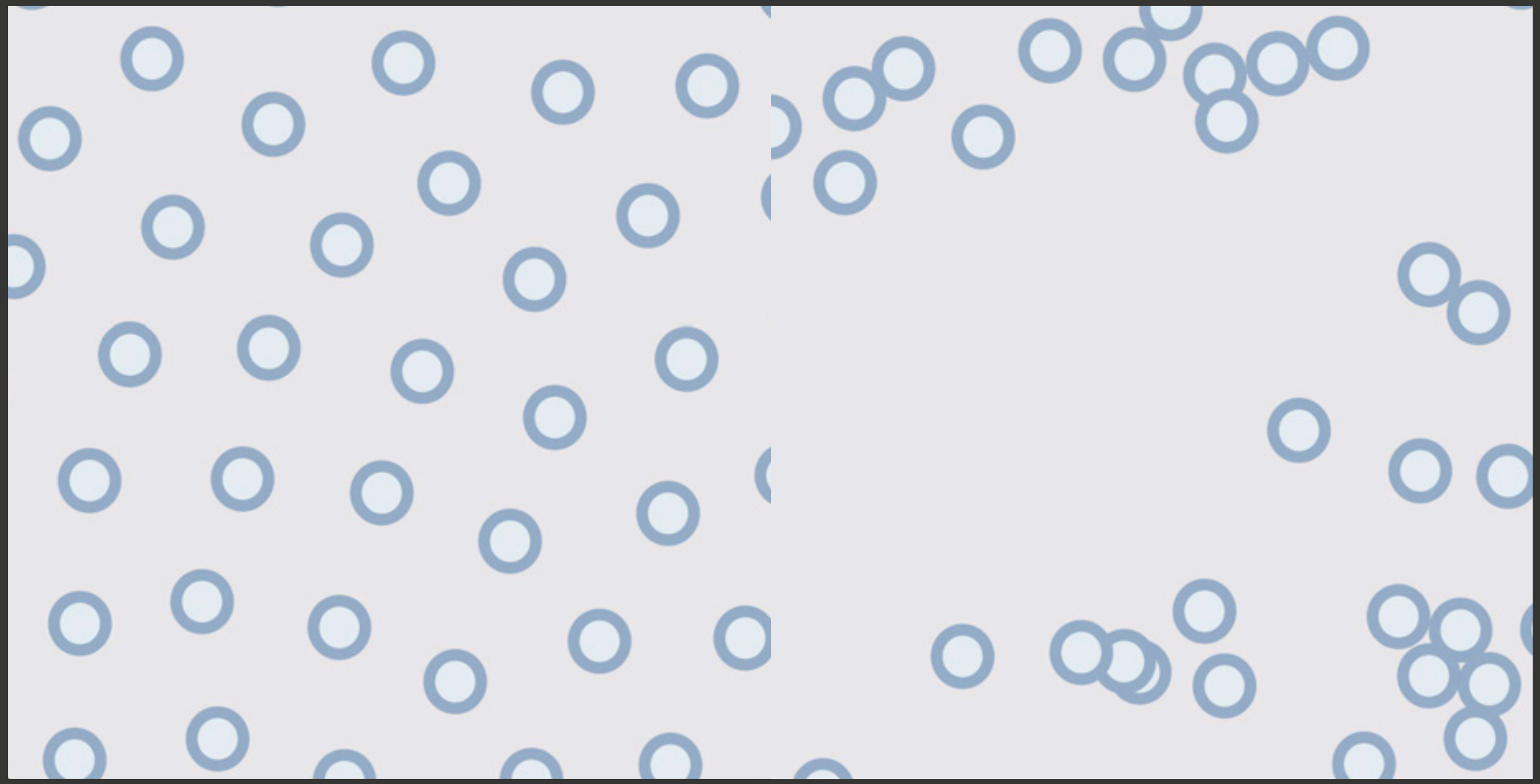

- Unbiased distance sampling in heterogeneous media only in special cases

Future Work

Future Work

- Non-exponentiality as a tool for…

Future Work

- Non-exponentiality as a tool for…
  - Multi-scattering approximation

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Oz: The Great and Volumetric,
Wrenninge et al. **2013**

Future Work

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Related Work

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- **Radiation propagation in random media: From positive to negative correlations in high-frequency fluctuations,** *Davis and Mineev-Weinstein*, **2011**

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- **A generalized linear Boltzmann equation for non-classical particle transport**, *Larsen and Vasquez*, **2011**

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- **A Radiative Transfer Framework for Spatially-Correlated Materials**, *Jarabo et al.* **2018**

Comparison to Related Work

Ours
Larsen and Vasquez, **2011**
Jarabo et al., **2018**

Comparison to Related Work

	Surfaces
Ours	✓
Larsen and Vasquez, **2011**	✗
Jarabo et al., **2018**	✓

Comparison to Related Work

	Surfaces	Heterogeneity
Ours	✓	✓
Larsen and Vasquez, **2011**	✗	✗
Jarabo et al., **2018**	✓	✗

Comparison to Related Work

	Surfaces	Heterogeneity	Reciprocity
Ours	✓	✓	✓
Larsen and Vasquez, **2011**	✗	✗	✓
Jarabo et al., **2018**	✓	✗	≈

Comparison to Related Work

	Surfaces	Heterogeneity	Reciprocity	Path Integral
Ours	✓	✓	✓	✓
Larsen and Vasquez, **2011**	✗	✗	✓	✗
Jarabo et al., **2018**	✓	✗	≈	✗


Thank you!
Pink Noise (ours)


Thank you!
Pink Noise (ours)