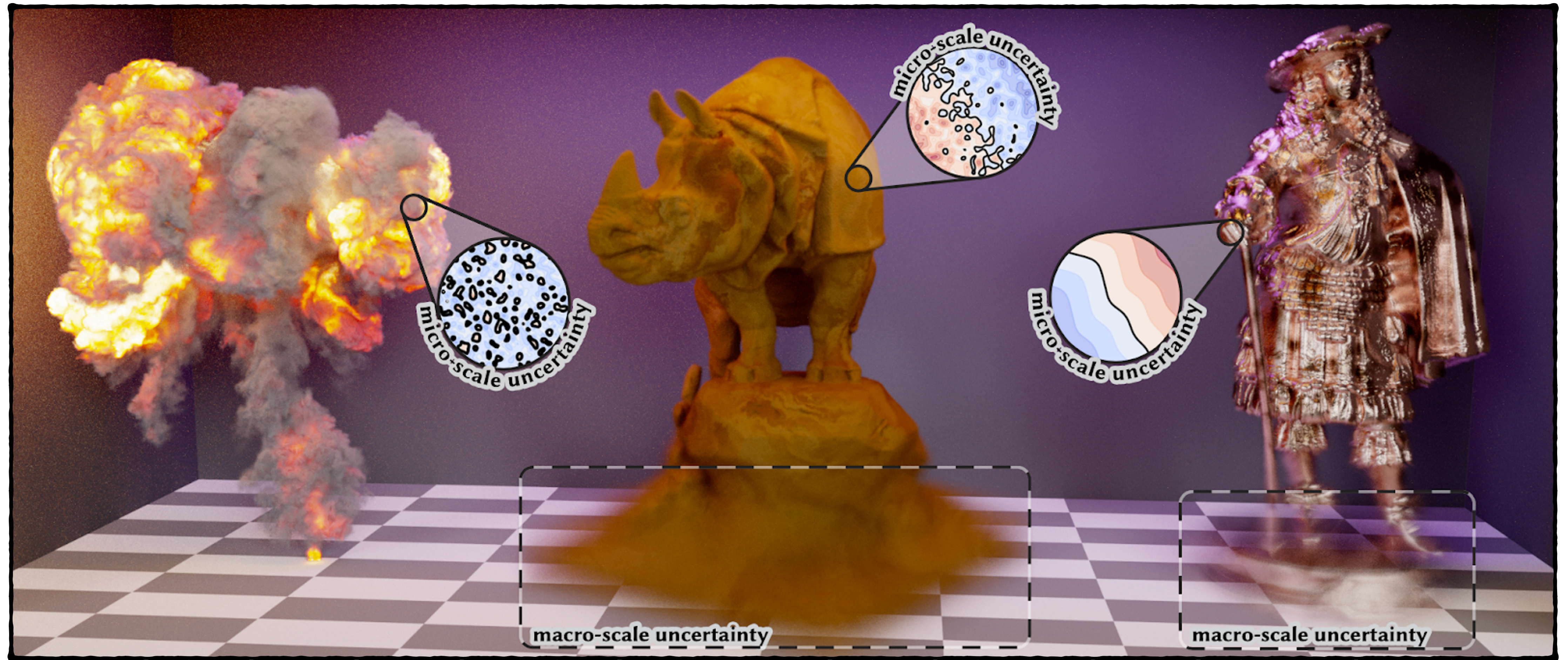


A UNIFIED TRANSPORT MODEL FOR CORRELATED DISORDER

microfacets, volumes, porous materials, and beyond



Wojciech Jarosz

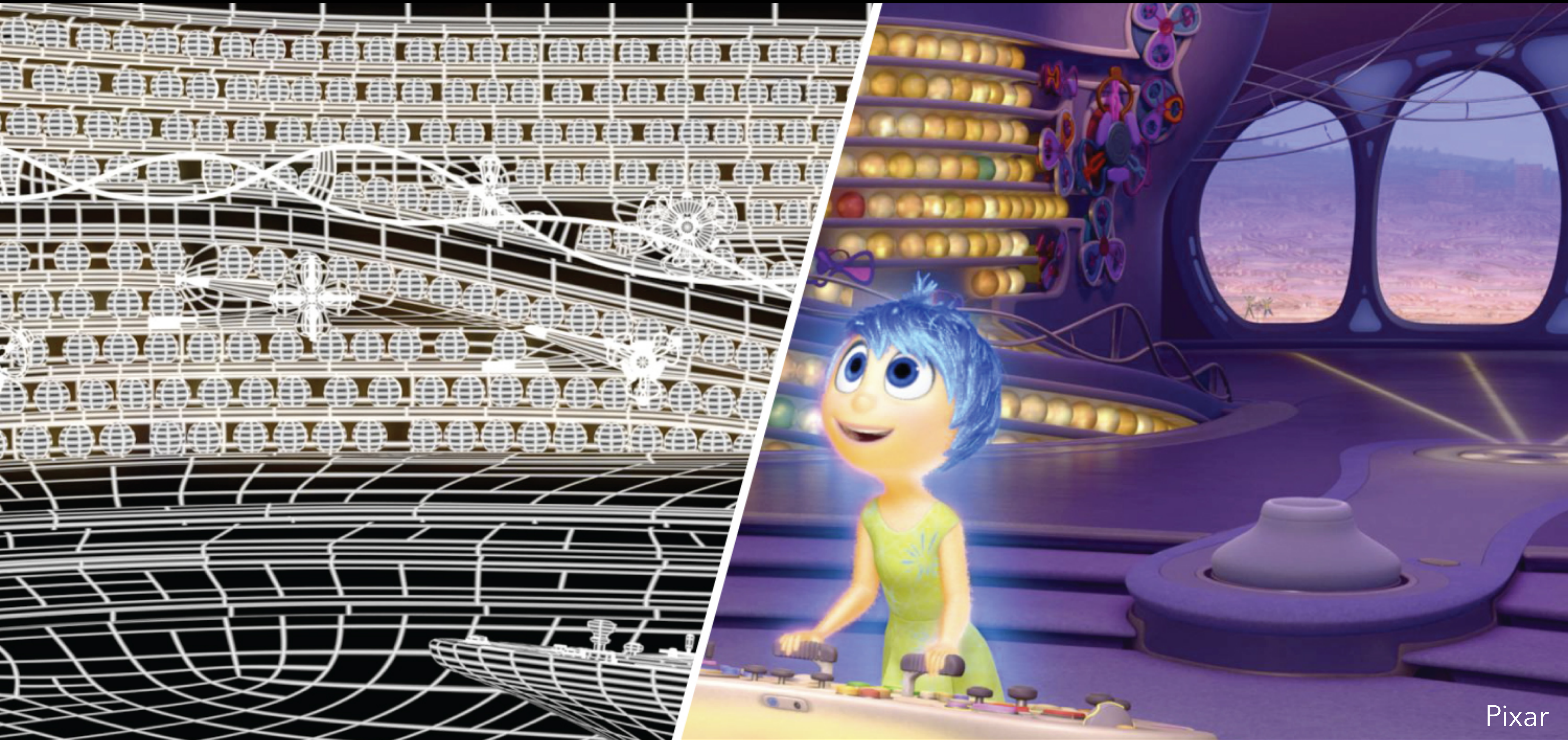
Dario Seyb

Kehan Xu

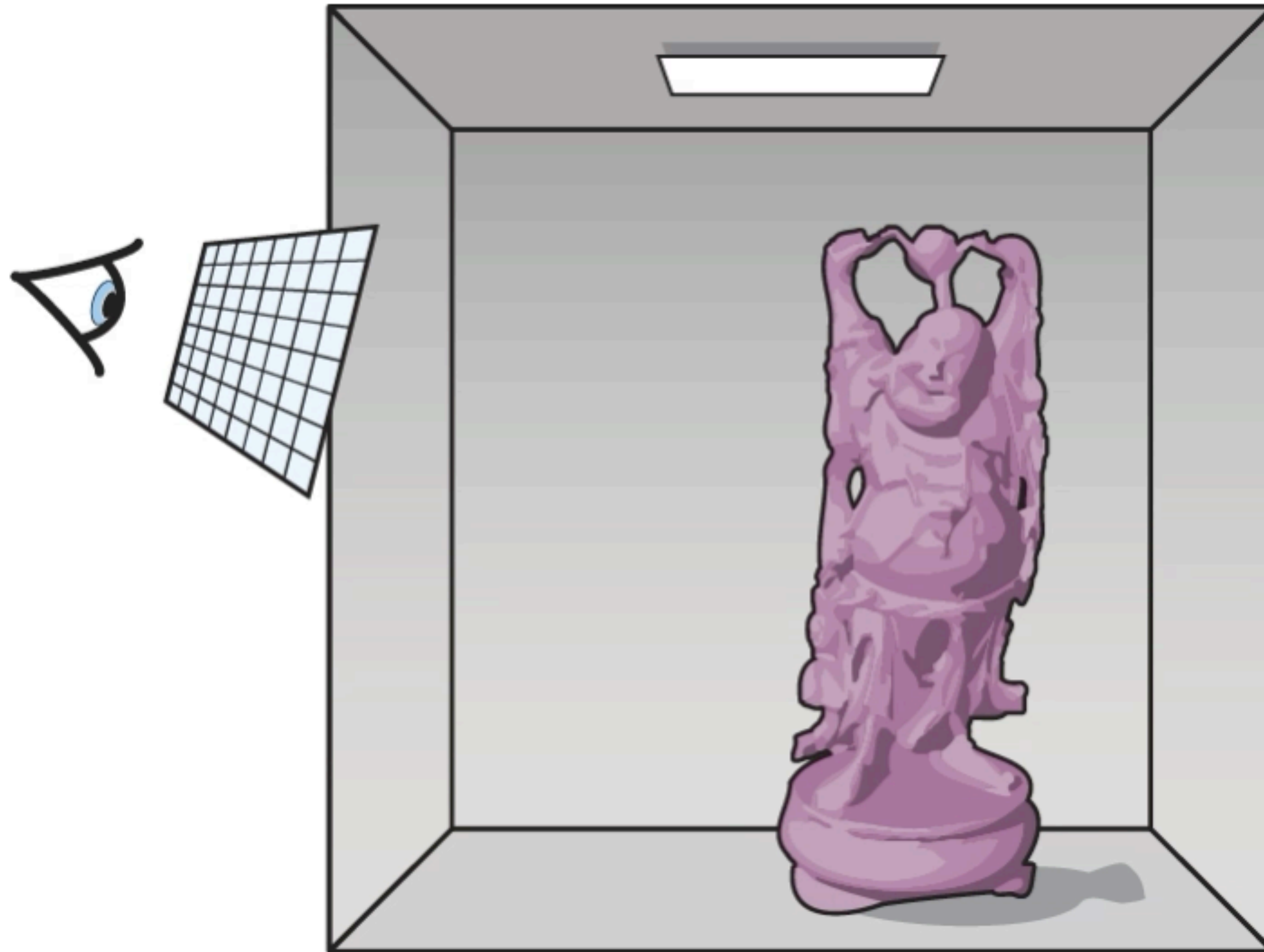
Eugene d'Eon

Benedikt Bitterli

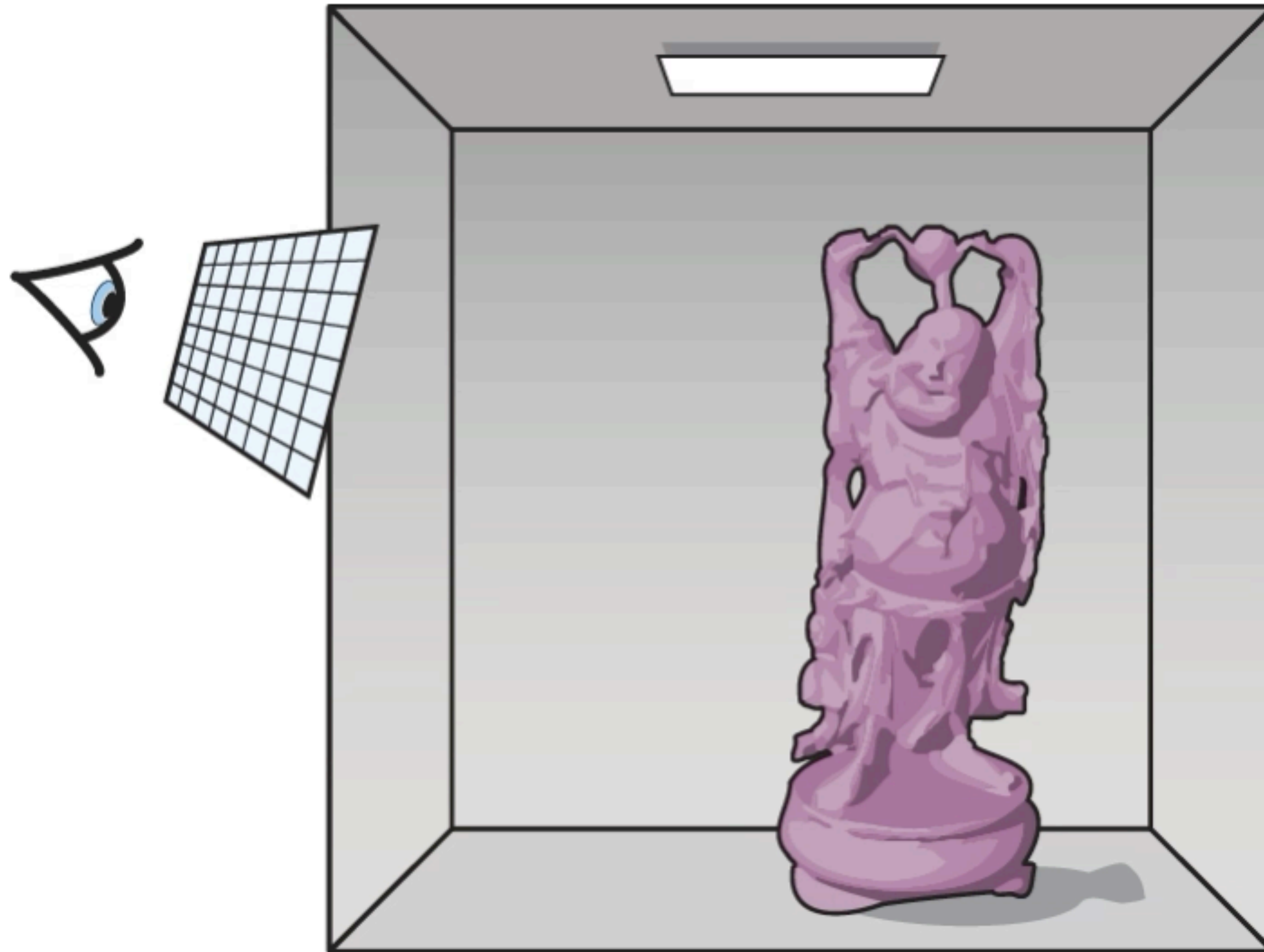
Rendering a virtual scene



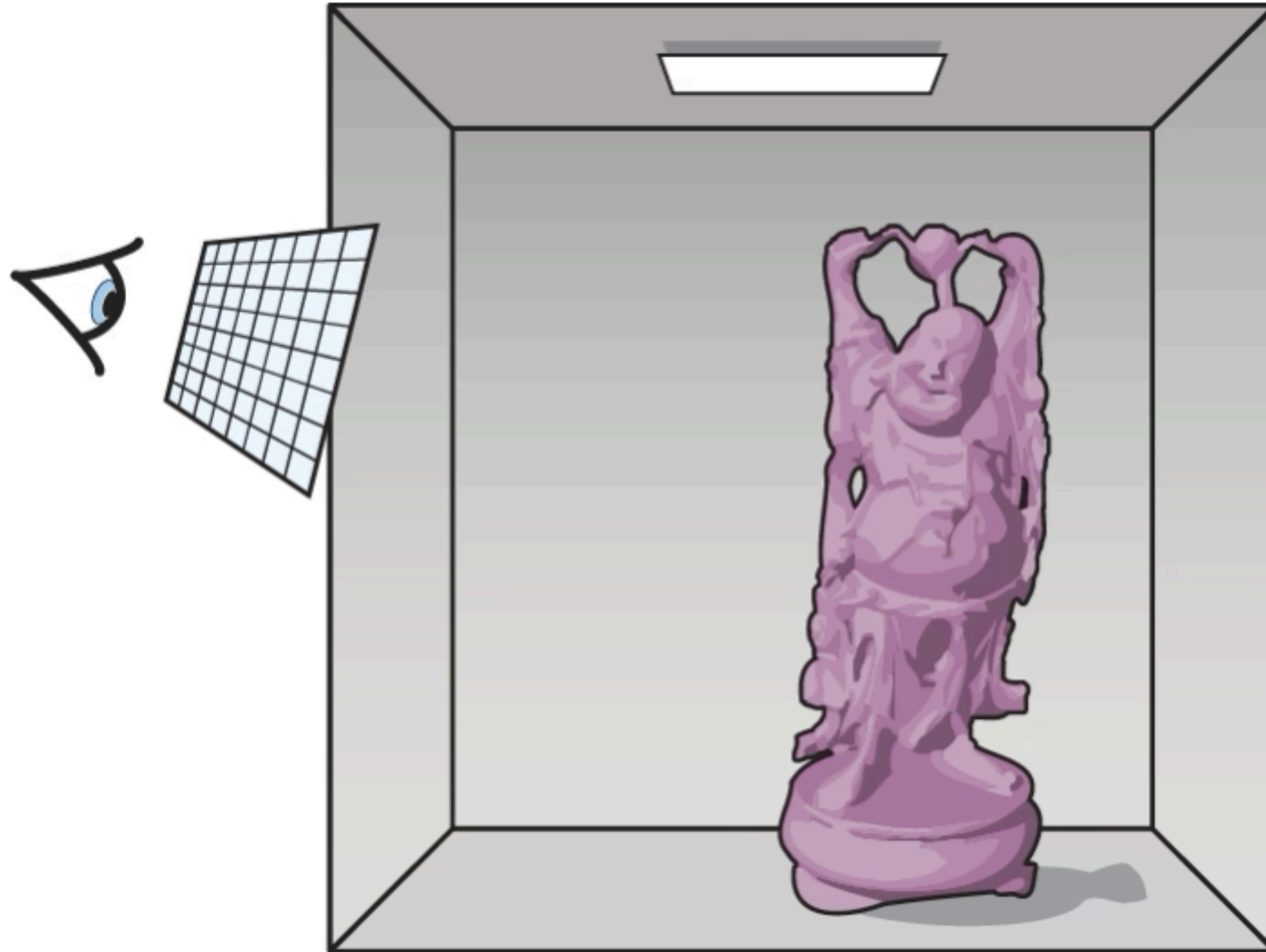
Monte Carlo light transport simulation



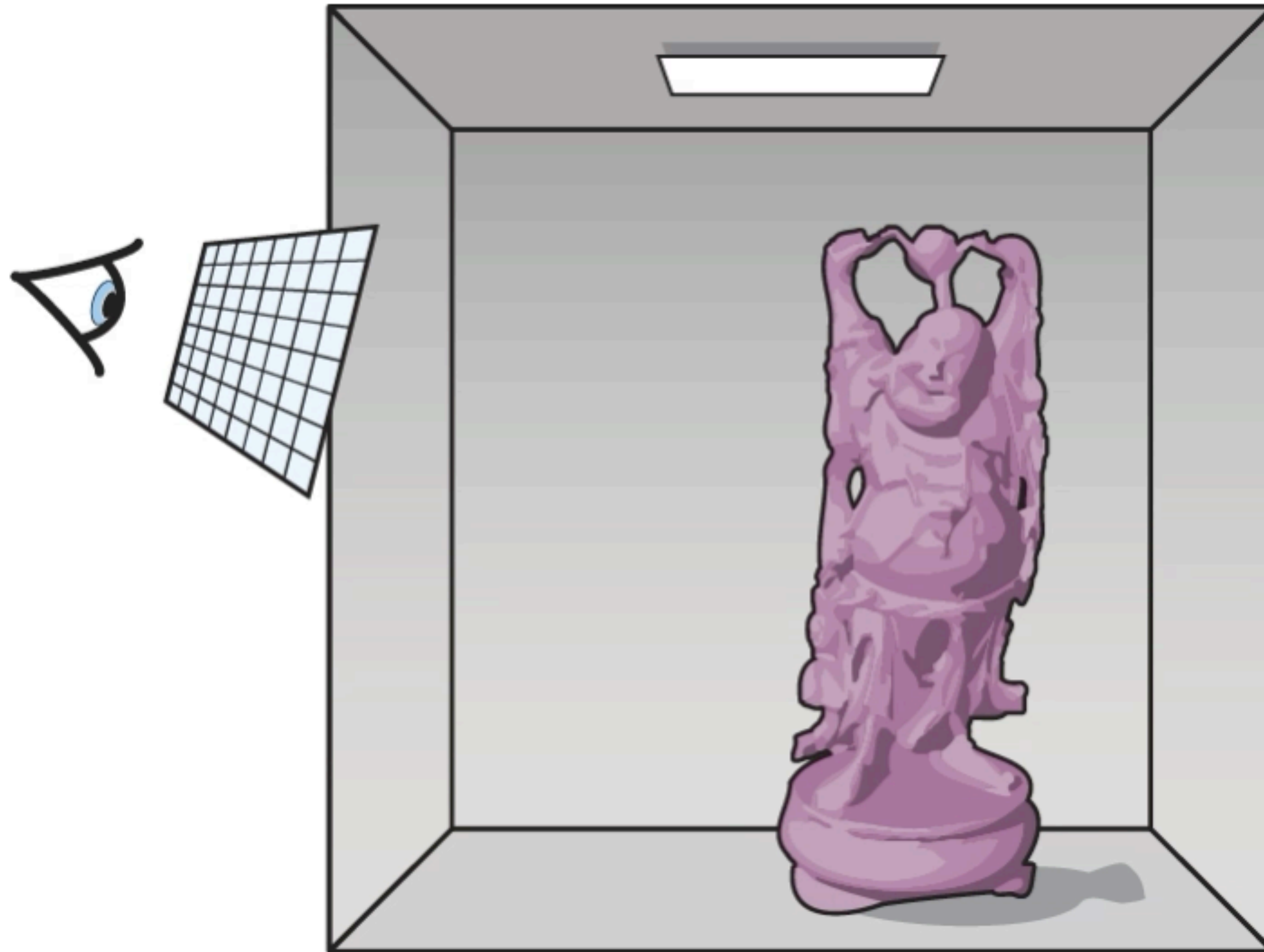
Monte Carlo light transport simulation



Monte Carlo path tracing (adjoint)

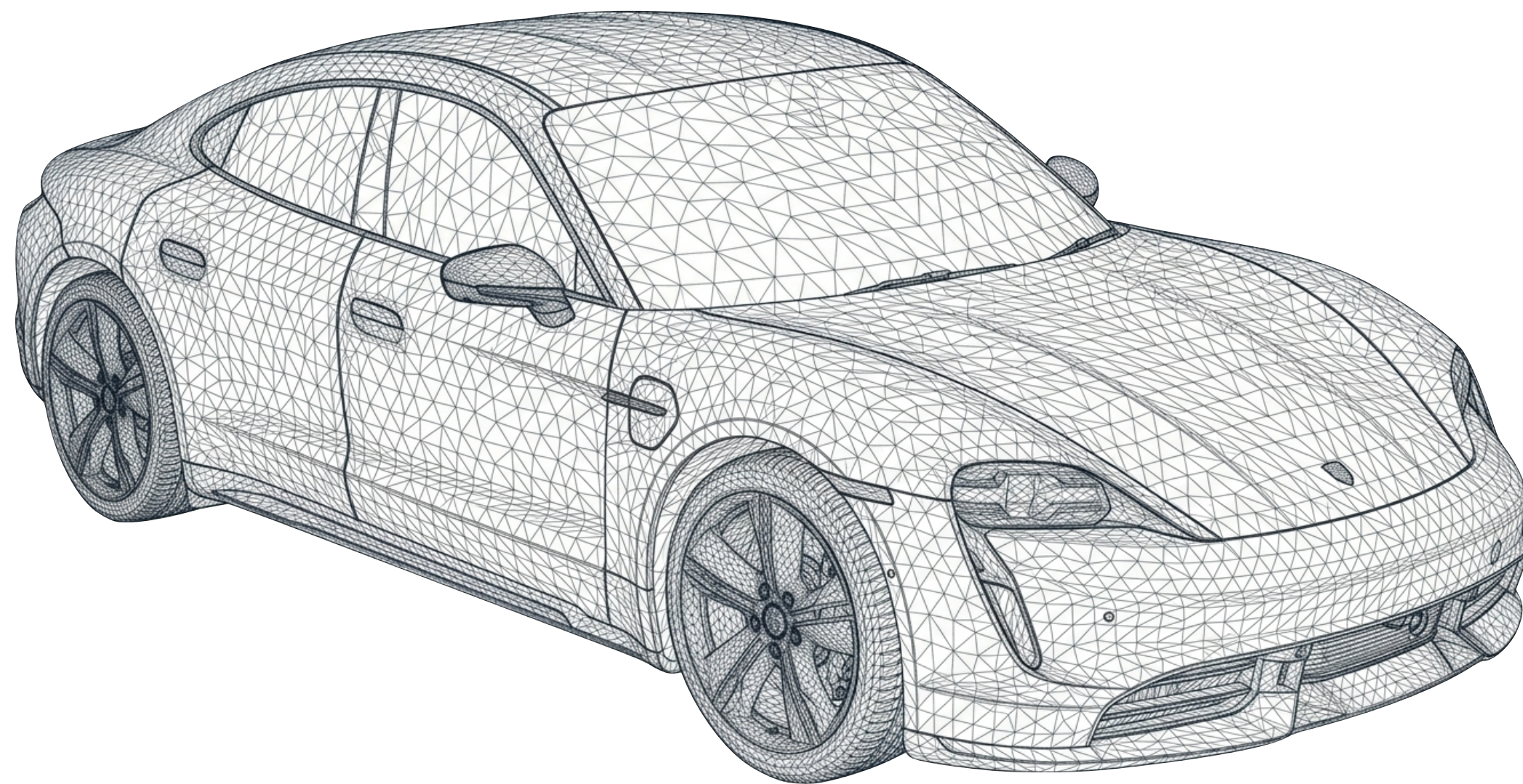


Monte Carlo path tracing (adjoint)



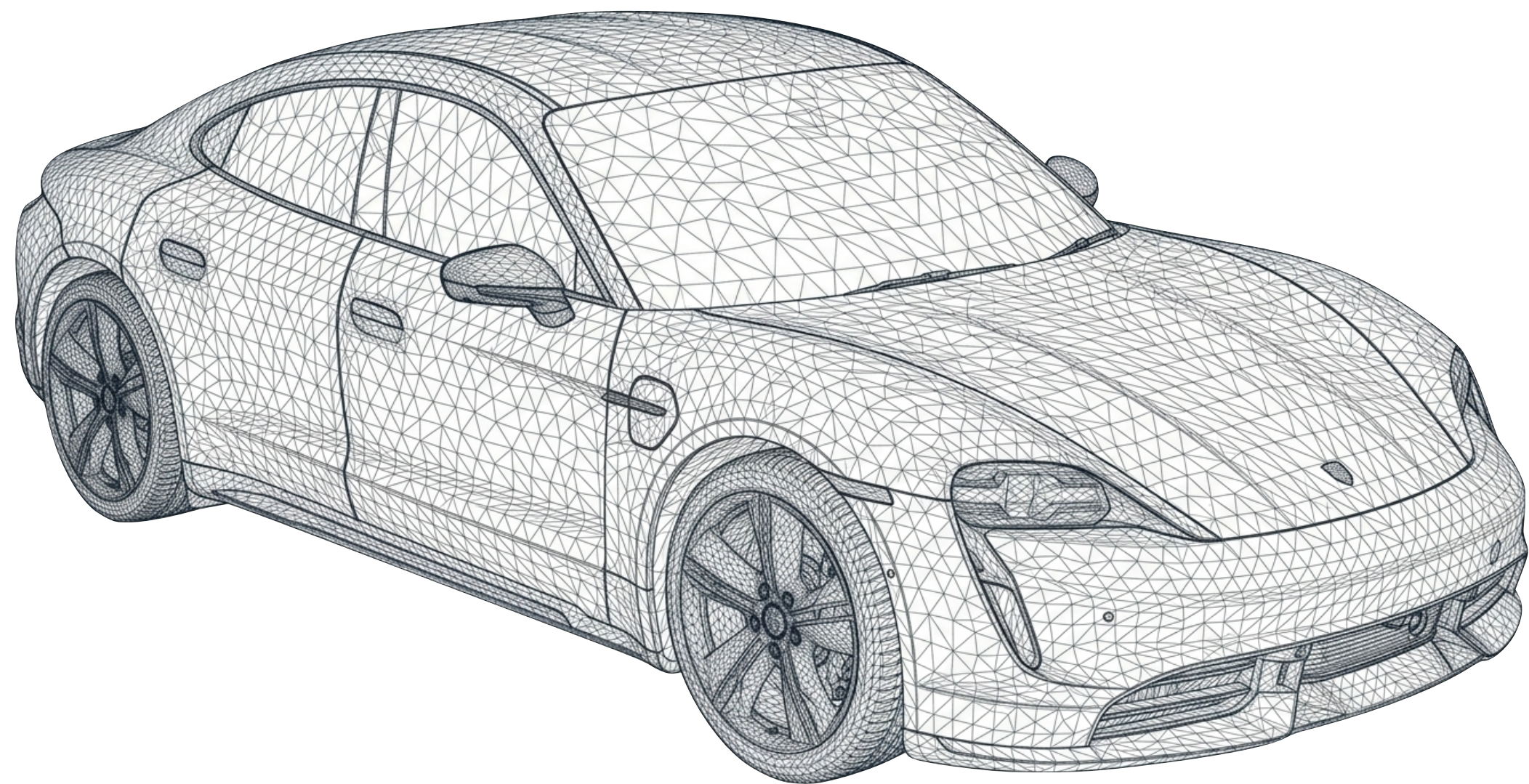
Define the macroscopic shape

Surfaces: Triangle Mesh

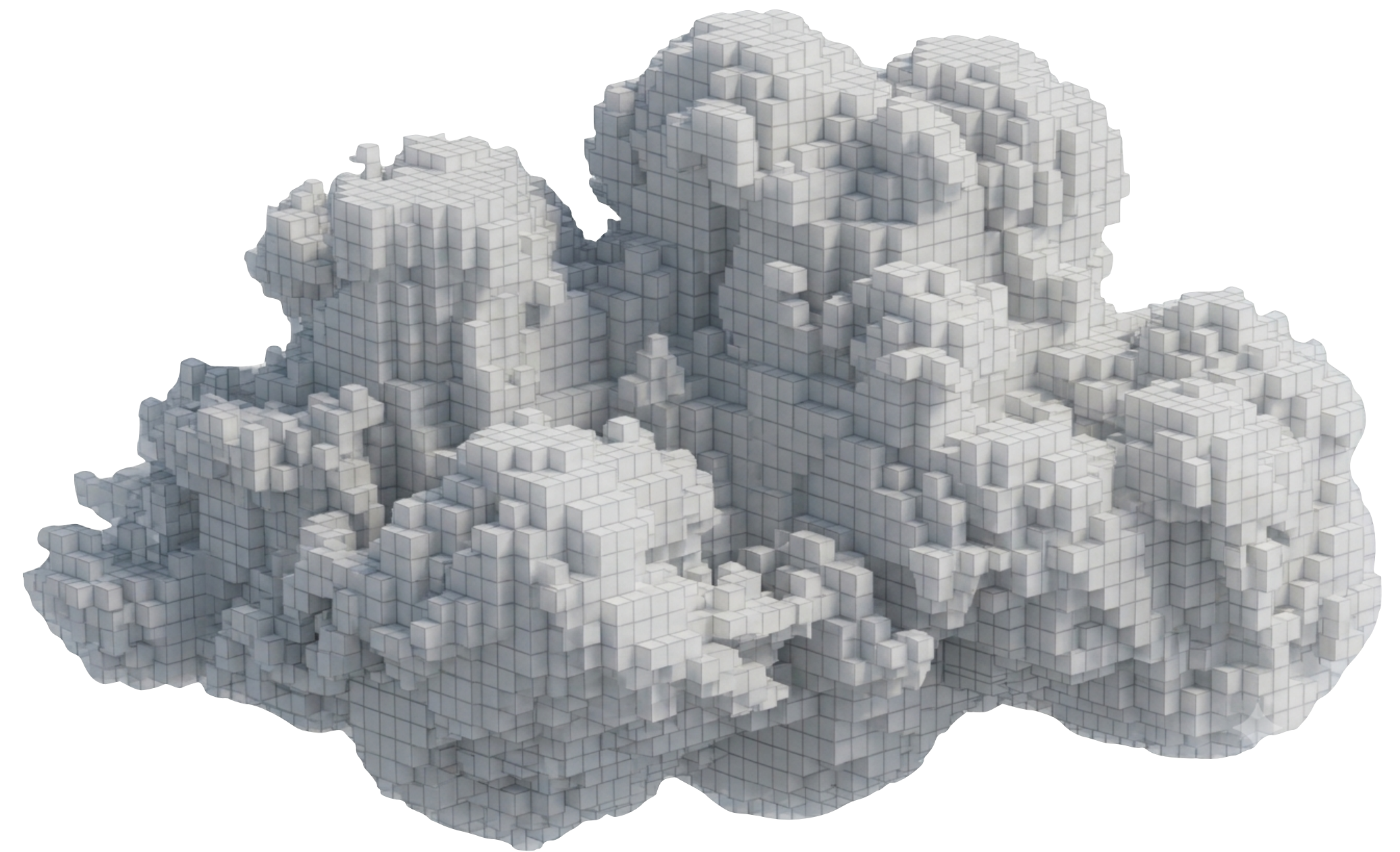


Define the macroscopic shape

Surfaces: Triangle Mesh



Volumes: Voxel Density Grid



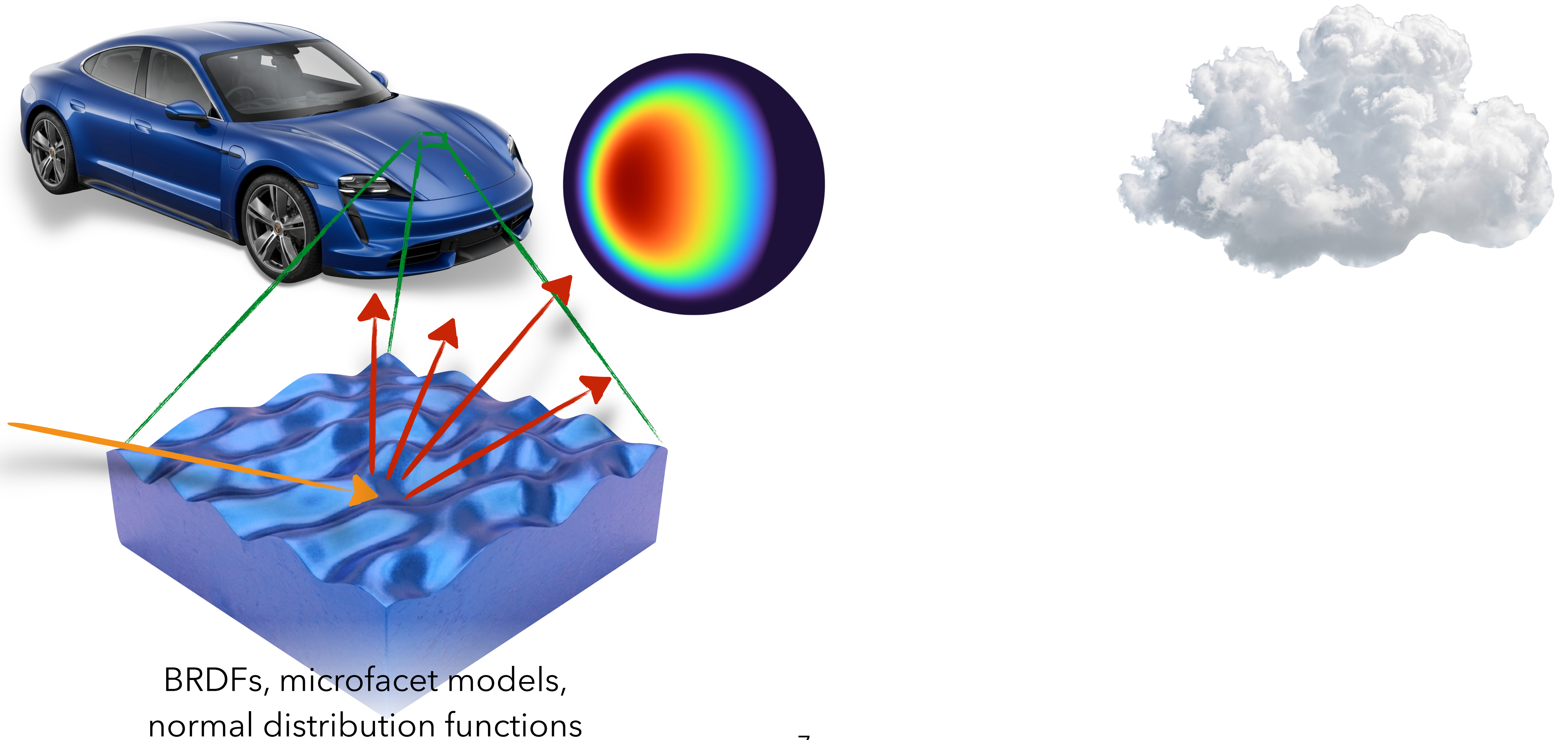
Define appearance



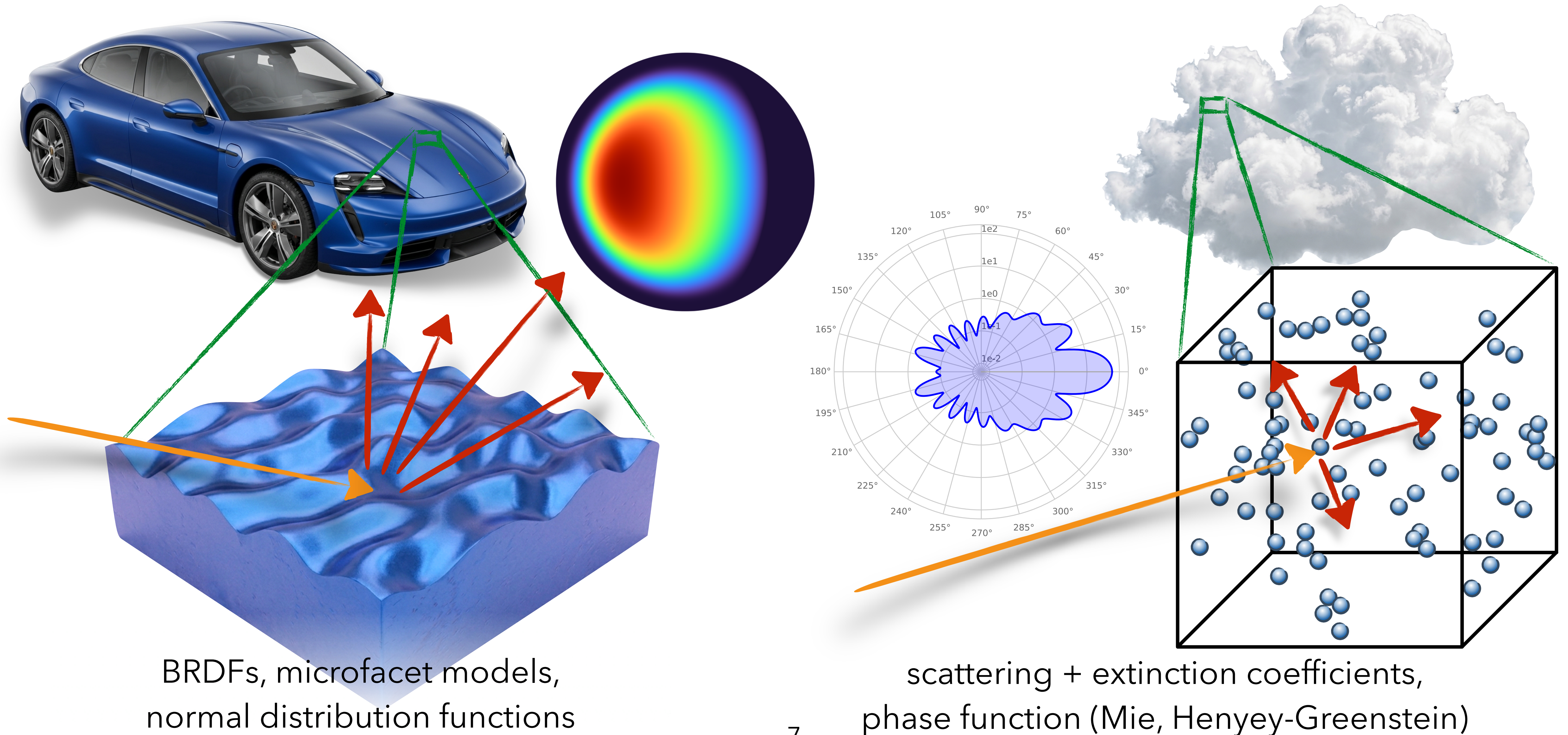
Statistics at microscopic scale



Statistics at microscopic scale



Statistics at microscopic scale



Two modeling scales

Surfaces



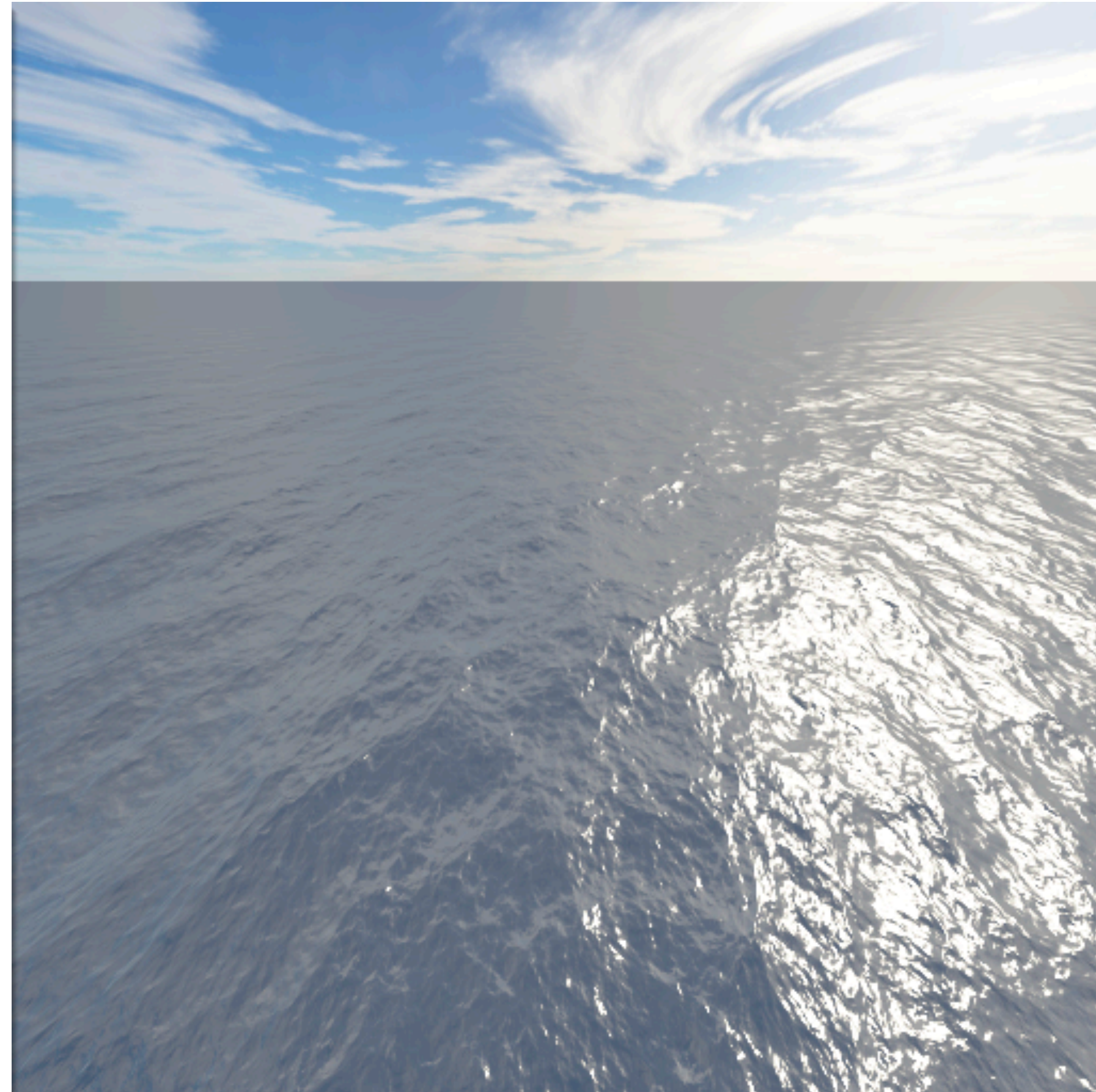
Volumes



Shape	e.g. triangles	e.g. voxel grids
Appearance	microfacet BRDFs	coefficients, phase functions

Transitioning between scales?

Surfaces



[Dupuy et al. 2013]

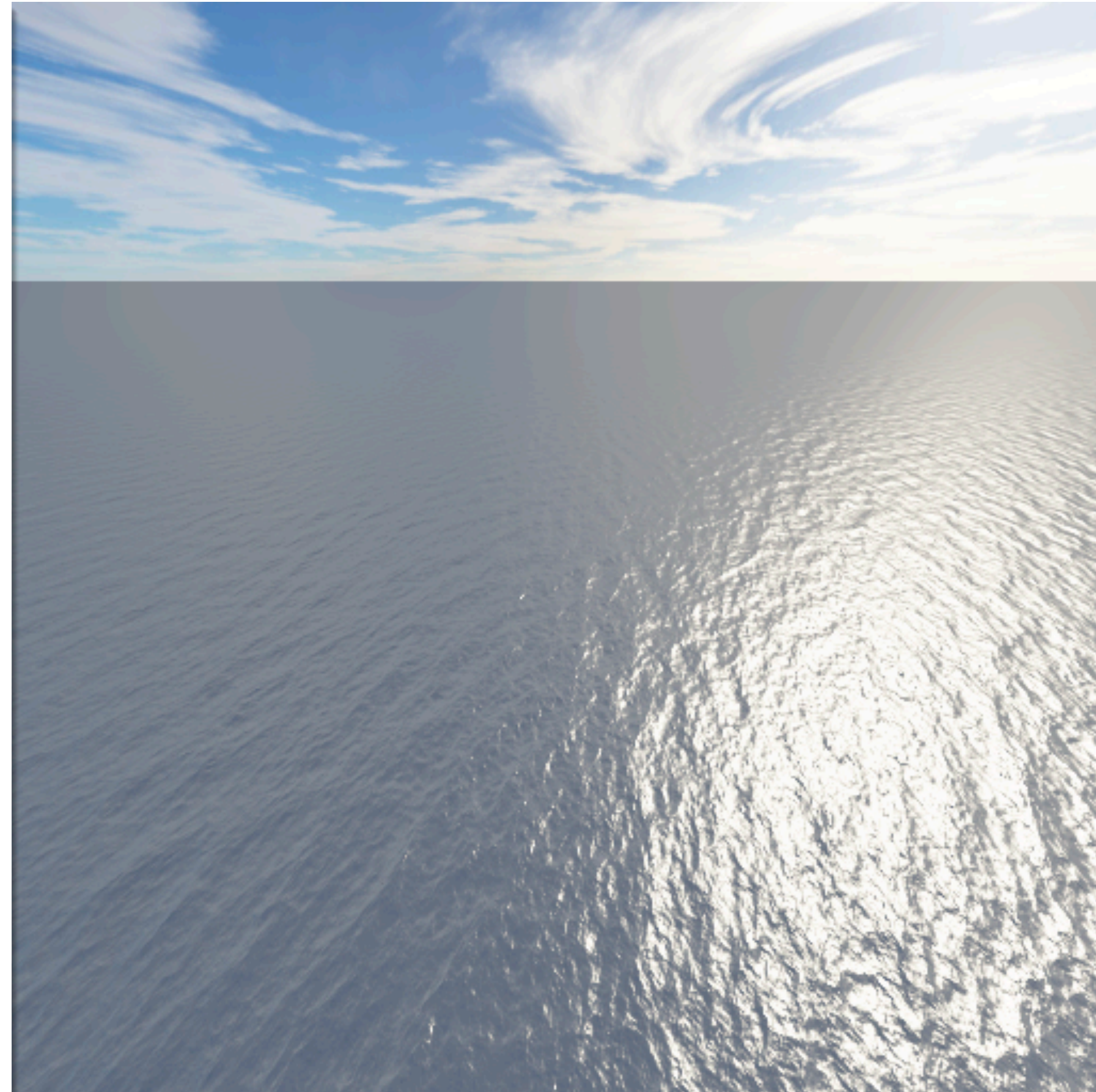
Volumes



Shape	e.g. triangles	e.g. voxel grids
Appearance	microfacet BRDFs	coefficients, phase functions

Transitioning between scales?

Surfaces



[Dupuy et al. 2013]

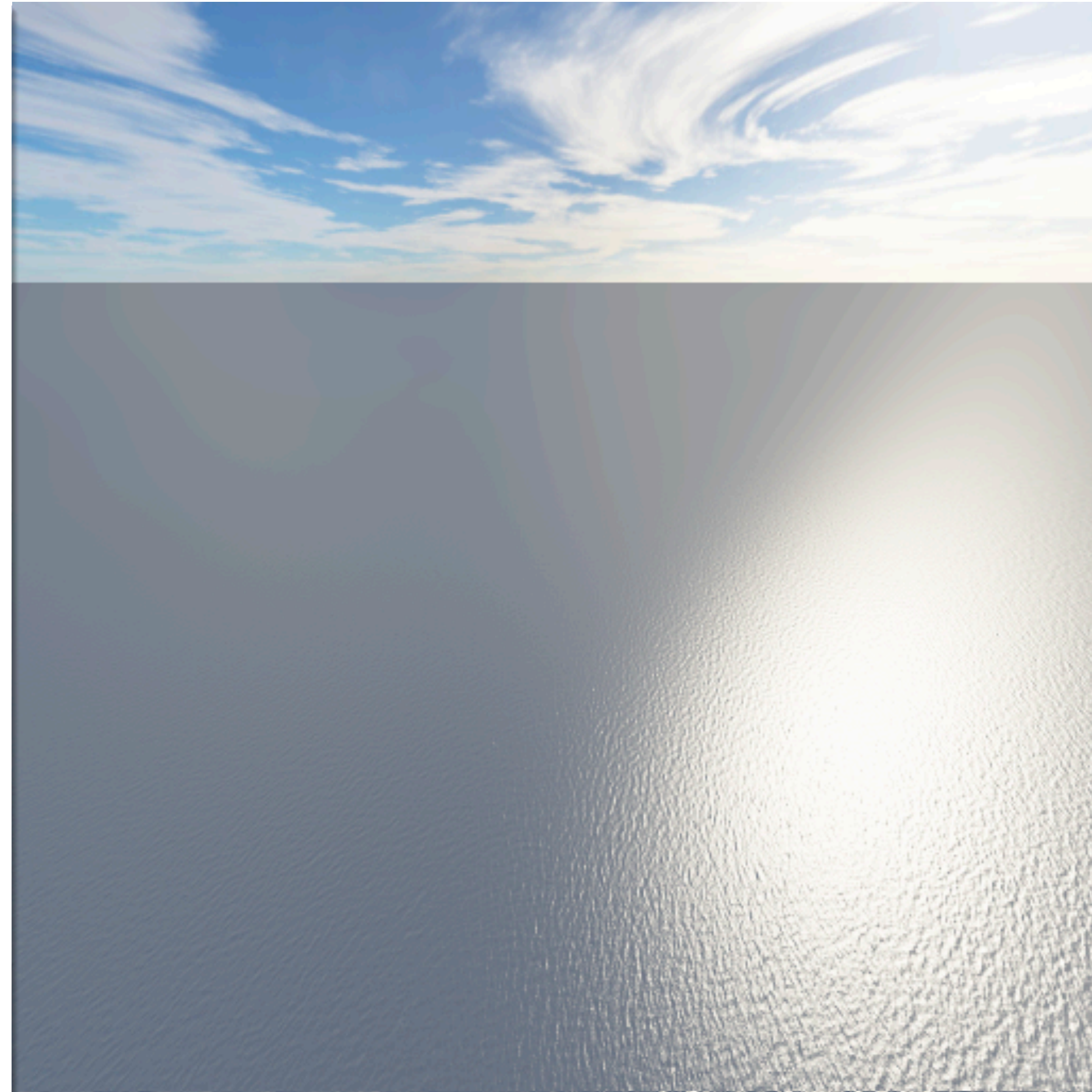
Volumes



Shape	e.g. triangles	e.g. voxel grids
Appearance	microfacet BRDFs	coefficients, phase functions

Transitioning between scales?

Surfaces



[Dupuy et al. 2013]

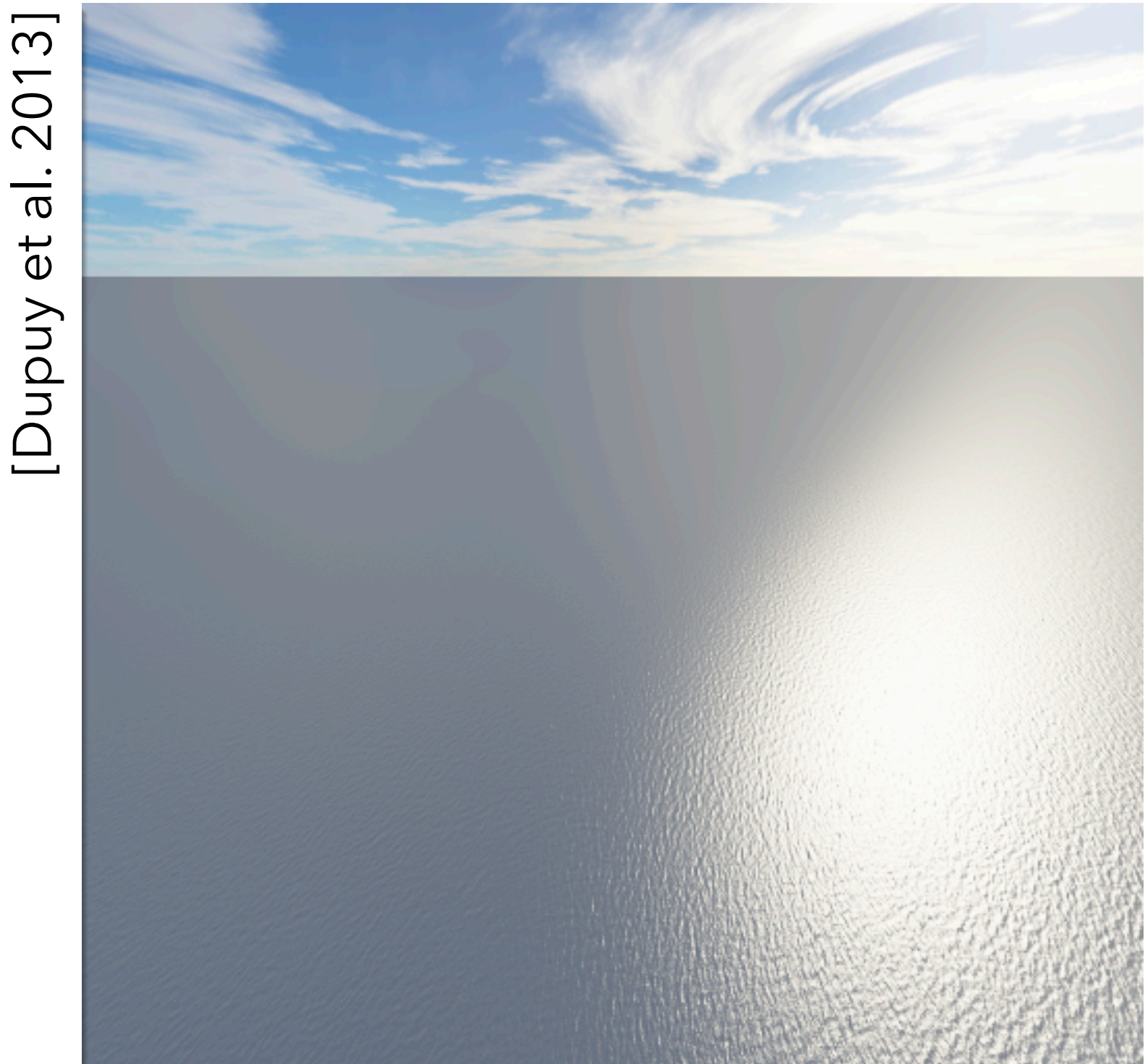
Volumes



Shape	e.g. triangles
Appearance	microfacet BRDFs

Different representations & equations

Surfaces



Volumes



Transport eqn.	$L = L_{\text{emi}} + \int_{H^2} p_{\text{sca}} \cdot L_{\text{inc}} \, \text{d}\Omega$	$\neq$	$(\nabla \cdot \Omega) L = \Sigma_{\text{sca}} L_{\text{sca}} + \Sigma_{\text{abs}} L_{\text{emi}} - \Sigma_{\text{ext}} L$
Shape	e.g. triangles	$\neq$	e.g. voxel grids
Appearance	microfacet BRDFs	$\neq$	coefficients, phase functions



Surface or volume?



Surface or volume?



Can we reason about
everything with one
representation &
equation?

A way forward?

A way forward?



Benedikt Bitterli
Darmouth → NVIDIA



Dario Seyb
Dartmouth → Valve

From microfacets to participating media: A unified theory of light transport with stochastic geometry

DARIO SEYB, Dartmouth College, USA
EUGENE D'EON, NVIDIA, New Zealand
BENEDIKT BITTERLI, NVIDIA, USA
WOJCIECH JAROSZ, Dartmouth College, USA

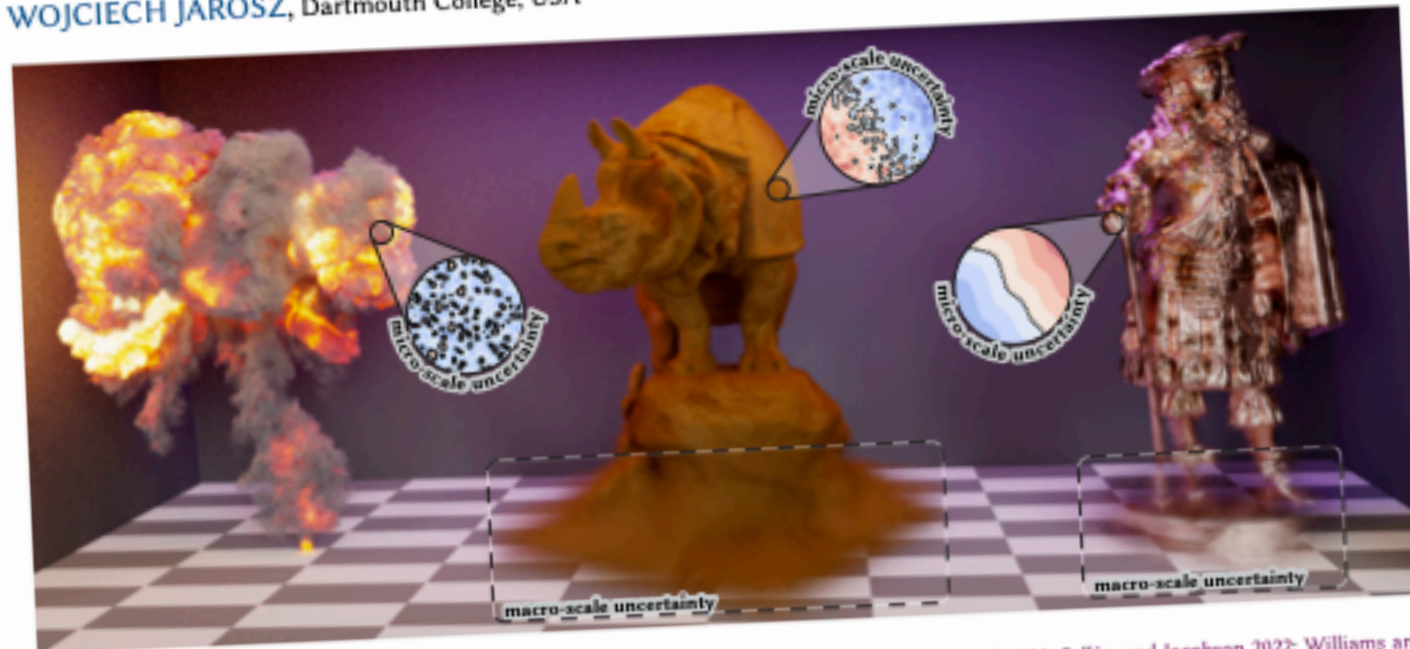


Fig. 1. We develop a theory of light transport for scenes with stochastic implicit surfaces [Dragiev et al. 2011; Sellán and Jacobson 2022; Williams and Fitzgibbon 2006] and show that this results in an expressive range of appearance behaviors that covers microfacet surfaces, classical participating media, and a novel continuum in between. Each object shown above is completely described by a 3D volume that encodes the mean and covariance kernel of a non-stationary Gaussian process and is rendered using the same rendering algorithm, agnostic of its “appearance type”. The insets show example realizations of the process at the highlighted points in the scene. Note how the appearance of the stochastic geometry transitions from volumetric (left) to hard-surface (right) as the correlations in the process are strengthened. Increasing the variance of the process allows us to visualize uncertainty at the macro-scale (bottom).

Stochastic geometry models have enjoyed immense success in graphics for modeling interactions of light with complex phenomena such as participating media, rough surfaces, fibers, and more. Although each of these models operates on the same principle of replacing intricate geometry by a random process and deriving the average light transport across all instances thereof, they are each tailored to one specific application and are fundamentally distinct. Each type of stochastic geometry present in the scene is firmly encapsulated in its own appearance model, with its own statistics and light transport average, and no cross-talk between different models or deterministic and stochastic geometry is possible.

In this paper, we derive a theory of light transport on stochastic implicit surfaces, a geometry model capable of expressing deterministic geometry, microfacet surfaces, participating media, and an exciting new continuum in between containing aggregate appearance, non-classical media, and more. Our model naturally supports spatial correlations, missing from most existing stochastic models.

Our theory paves the way for tractable rendering of scenes in which all geometry is described by the same stochastic model, while leaving ample future work for developing efficient sampling and rendering algorithms.

CCS Concepts: • Computing methodologies → Volumetric models.

Additional Key Words and Phrases: volumetric light transport, stochastic processes, implicit surfaces

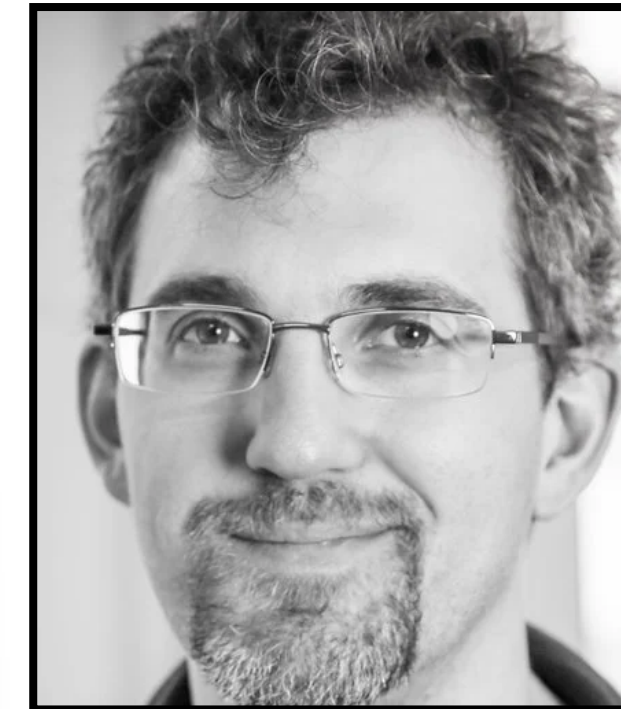
ACM Reference Format:

Dario Seyb, Eugene d'Eon, Benedikt Bitterli, and Wojciech Jarosz. 2024. From microfacets to participating media: A unified theory of light transport with stochastic geometry. ACM Trans. Graph. 43, 4, Article 112 (July 2024), 17 pages. <https://doi.org/10.1145/3658121>

1 INTRODUCTION

Computer graphics has seen significant progress over the past few decades, with advances in light transport, material fidelity, and

ACM Trans. Graph., Vol. 43, No. 4, Article 112. Publication date: July 2024.



Eugene d'Eon
NVIDIA

A way forward?



Benedikt Bitterli
Dartmouth → NVIDIA



Dario Seyb
Dartmouth → Valve

From microfacets to participating media: A unified theory of light transport with stochastic geometry

DARIO SEYB, Dartmouth College, USA
EUGENE D'EON, NVIDIA, New Zealand
BENEDIKT BITTERLI, NVIDIA, USA
WOJCIECH JAROSZ, Dartmouth College, USA

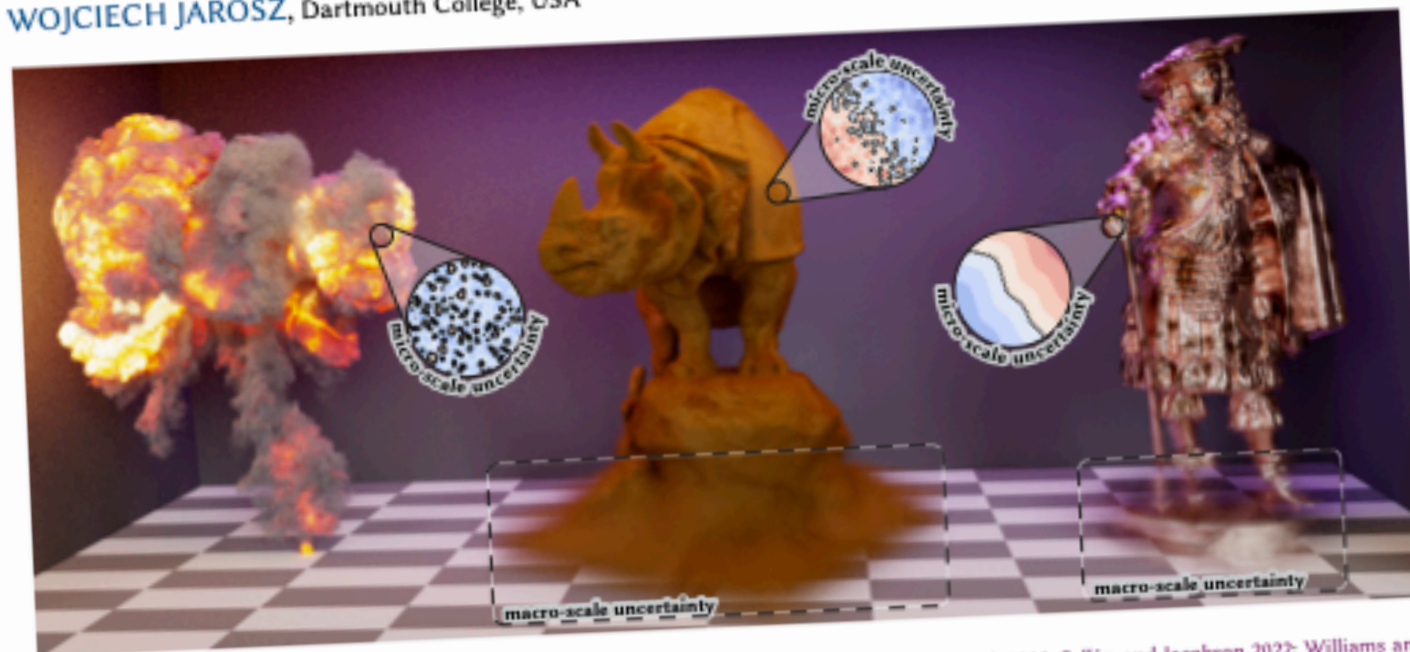


Fig. 1. We develop a theory of light transport for scenes with stochastic implicit surfaces [Dragiev et al. 2011; Sellán and Jacobson 2022; Williams and Fitzgibbon 2006] and show that this results in an expressive range of appearance behaviors that covers microfacet surfaces, classical participating media, and a novel continuum in between. Each object shown above is completely described by a 3D volume that encodes the mean and covariance kernel of a non-stationary Gaussian process and is rendered using the same rendering algorithm, agnostic of its “appearance type”. The insets show example realizations of the process at the highlighted points in the scene. Note how the appearance of the stochastic geometry transitions from volumetric (left) to hard-surface (right) as the correlations in the process are strengthened. Increasing the variance of the process allows us to visualize uncertainty at the macro-scale (bottom).

Stochastic geometry models have enjoyed immense success in graphics for modeling interactions of light with complex phenomena such as participating media, rough surfaces, fibers, and more. Although each of these models operates on the same principle of replacing intricate geometry by a random process and deriving the average light transport across all instances thereof, they are each tailored to one specific application and are fundamentally distinct. Each type of stochastic geometry present in the scene is firmly encapsulated in its own appearance model, with its own statistics and light transport average, and no cross-talk between different models or deterministic and stochastic geometry is possible.

In this paper, we derive a theory of light transport on stochastic implicit surfaces, a geometry model capable of expressing deterministic geometry, microfacet surfaces, participating media, and an exciting new continuum in between containing aggregate appearance, non-classical media, and more. Our model naturally supports spatial correlations, missing from most existing stochastic models.

Our theory paves the way for tractable rendering of scenes in which all geometry is described by the same stochastic model, while leaving ample future work for developing efficient sampling and rendering algorithms.

CCS Concepts: • Computing methodologies → Volumetric models.

Additional Key Words and Phrases: volumetric light transport, stochastic processes, implicit surfaces

ACM Reference Format:
Dario Seyb, Eugene d'Eon, Benedikt Bitterli, and Wojciech Jarosz. 2024. From microfacets to participating media: A unified theory of light transport with stochastic geometry. *ACM Trans. Graph.* 43, 4, Article 112 (July 2024). 17 pages. <https://doi.org/10.1145/3658121>

1 INTRODUCTION

Computer graphics has seen significant progress over the past few decades, with advances in light transport, material fidelity, and

ACM Trans. Graph., Vol. 43, No. 4, Article 112. Publication date: July 2024.



Eugene d'Eon
NVIDIA



Kehan Xu
Dartmouth

Practical Gaussian Process Implicit Surfaces with Sparse Convolutions

KEHAN XU, Dartmouth College, USA
BENEDIKT BITTERLI, NVIDIA, USA
EUGENE D'EON, NVIDIA, Switzerland
WOJCIECH JAROSZ, Dartmouth College, USA



Fig. 1. Our novel sparse convolution representation for Gaussian Process Implicit Surfaces (GPISes) enables, for the first time, the visualization of single non-stationary realizations (split screen, middle right) alongside their ensemble-averaged light transport (right). The single realization allows examining the underlying structures of the stochastic geometry that lead to the aggregate rough surface or volumetric appearance. Furthermore, our algorithm (middle left) significantly boosts rendering efficiency for ensemble light transport compared to previous work by Seyb et al. [2024] (left) at equal time (40 min). The rightmost column presents magnified insets comparing the two methods, with speedup quantified by the ratio of mean squared error (MSE). This improvement stems from a combination of our efficient sparse convolution representation and the ability to perform next-event estimation even on specular micro-surfaces.

A fundamental challenge in rendering has been the dichotomy between surface and volume models. Gaussian Process Implicit Surfaces (GPISes) recently provided a unified approach for surfaces, volumes, and the spectrum in between. However, this representation remains impractical due to its high computational cost and mathematical complexity. We address these limitations by reformulating GPISes as procedural noise, eliminating expensive linear system solves while maintaining control over spatial correlations. Our method enables efficient sampling of stochastic realizations and supports flexible conditioning of values and derivatives through pathwise updates. To further enable practical rendering, we derive analytic distributions for surface normals, allowing for variance-reduced light transport via next-event estimation and multiple importance sampling. Our framework achieves efficient, high-quality rendering of stochastic surfaces and volumes with significantly simplified implementations on both CPU and GPU, while preserving the generality of the original GPIS representation.

CCS Concepts: • Computing methodologies → Rendering; Ray tracing.

Authors' addresses: Kehan Xu, Dartmouth College, USA, kehan.xu.gr@dartmouth.edu; Benedikt Bitterli, NVIDIA, USA, bbitterli@nvidia.com; Eugene d'Eon, NVIDIA, Switzerland, edeon@nvidia.com; Wojciech Jarosz, Dartmouth College, USA, wjarosz@dartmouth.edu.

© 2025 Copyright held by the owner/author(s). This is the author's version of the work. It is posted here for your personal use. Not for redistribution. The definitive Version of Record was published in *ACM Transactions on Graphics*, <https://doi.org/10.1145/3763329>.

Additional Key Words and Phrases: stochastic processes, implicit surfaces

ACM Reference Format:

Kehan Xu, Benedikt Bitterli, Eugene d'Eon, and Wojciech Jarosz. 2025. Practical Gaussian Process Implicit Surfaces with Sparse Convolutions. *ACM Trans. Graph.* 44, 6, Article 188 (December 2025). 18 pages. <https://doi.org/10.1145/3763329>

1 INTRODUCTION

Traditional light transport simulation categorizes scene components as either solid surfaces or participating media, with each governed by a distinct rendering equation. This binary distinction is inadequate for capturing the diverse appearances found in the real world, particularly when considering level-of-detail techniques that might require turning hard surfaces into volumetric effects with surface-like correlation.

Seyb et al. [2024] demonstrated a unified framework using ensemble-averaged light transport through stochastic geometry, covering surfaces, volumes, and the spectrum in between. A key contribution was establishing a rigorous mathematical formalism for ensemble-averaged transport. Despite its potential, this method remains impractical, with rendering speeds at least two orders of magnitude slower than conventional path tracing. This performance gap arises

ACM Trans. Graph., Vol. 44, No. 6, Article 188. Publication date: December 2025.

This talk: intuition + summary



Benedikt Bitterli
Dartmouth → NVIDIA



Dario Seyb
Dartmouth → Valve

From microfacets to participating media: A unified theory of light transport with stochastic geometry

DARIO SEYB, Dartmouth College, USA
EUGENE D'EON, NVIDIA, New Zealand
BENEDIKT BITTERLI, NVIDIA, USA
WOJCIECH JAROSZ, Dartmouth College, USA

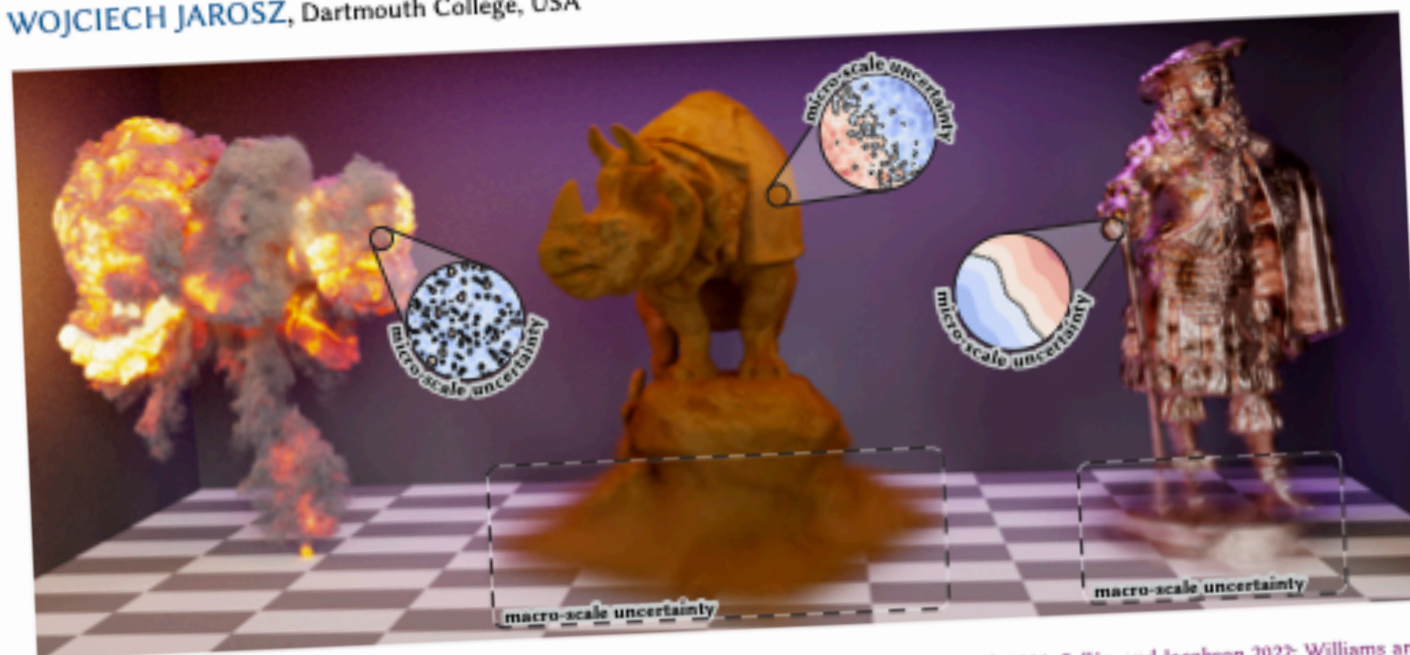


Fig. 1. We develop a theory of light transport for scenes with stochastic implicit surfaces [Dragiev et al. 2011; Sellán and Jacobson 2022; Williams and Fitzgibbon 2006] and show that this results in an expressive range of appearance behaviors that covers microfacet surfaces, classical participating media, and a novel continuum in between. Each object shown above is completely described by a 3D volume that encodes the mean and covariance kernel of a non-stationary Gaussian process and is rendered using the same rendering algorithm, agnostic of its “appearance type”. The insets show example realizations of the process at the highlighted points in the scene. Note how the appearance of the stochastic geometry transitions from volumetric (left) to hard-surface (right) as the correlations in the process are strengthened. Increasing the variance of the process allows us to visualize uncertainty at the macro-scale (bottom).

Stochastic geometry models have enjoyed immense success in graphics for modeling interactions of light with complex phenomena such as participating media, rough surfaces, fibers, and more. Although each of these models operates on the same principle of replacing intricate geometry by a random process and deriving the average light transport across all instances thereof, they are each tailored to one specific application and are fundamentally distinct. Each type of stochastic geometry present in the scene is firmly encapsulated in its own appearance model, with its own statistics and light transport average, and no cross-talk between different models or deterministic and stochastic geometry is possible.

In this paper, we derive a theory of light transport on stochastic implicit surfaces, a geometry model capable of expressing deterministic geometry, microfacet surfaces, participating media, and an exciting new continuum in between containing aggregate appearance, non-classical media, and more. Our model naturally supports spatial correlations, missing from most existing stochastic models.

Our theory paves the way for tractable rendering of scenes in which all geometry is described by the same stochastic model, while leaving ample future work for developing efficient sampling and rendering algorithms.

CCS Concepts: • Computing methodologies → Volumetric models.

Additional Key Words and Phrases: volumetric light transport, stochastic processes, implicit surfaces

ACM Reference Format:
Dario Seyb, Eugene d'Eon, Benedikt Bitterli, and Wojciech Jarosz. 2024. From microfacets to participating media: A unified theory of light transport with stochastic geometry. *ACM Trans. Graph.* 43, 4, Article 112 (July 2024). 17 pages. <https://doi.org/10.1145/3658121>

1 INTRODUCTION

Computer graphics has seen significant progress over the past few decades, with advances in light transport, material fidelity, and

ACM Trans. Graph., Vol. 43, No. 4, Article 112. Publication date: July 2024.



Eugene d'Eon
NVIDIA



Kehan Xu
Dartmouth

Practical Gaussian Process Implicit Surfaces with Sparse Convolutions

KEHAN XU, Dartmouth College, USA
BENEDIKT BITTERLI, NVIDIA, USA
EUGENE D'EON, NVIDIA, Switzerland
WOJCIECH JAROSZ, Dartmouth College, USA

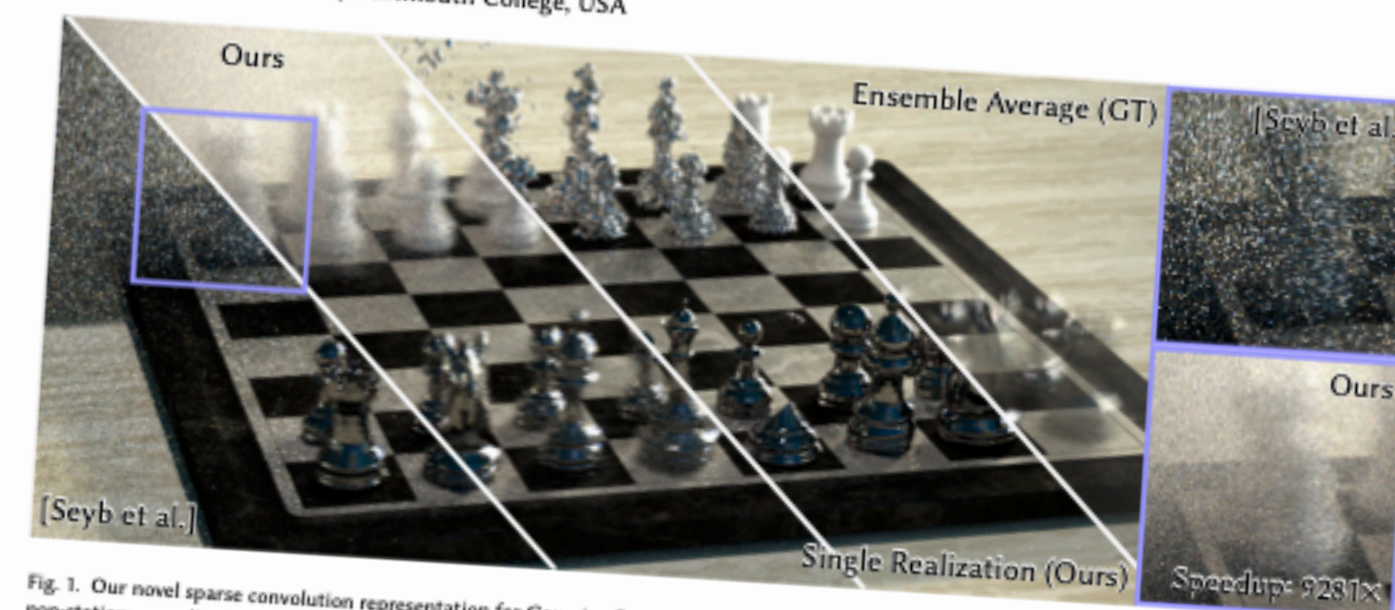


Fig. 1. Our novel sparse convolution representation for Gaussian Process Implicit Surfaces (GPISes) enables, for the first time, the visualization of single non-stationary realizations (split screen, middle right) alongside their ensemble-averaged light transport (right). The single realization allows examining the underlying structures of the stochastic geometry that lead to the aggregate rough surface or volumetric appearance. Furthermore, our algorithm (middle left) significantly boosts rendering efficiency for ensemble light transport compared to previous work by Seyb et al. [2024] (left) at equal time (40 min). The rightmost column presents magnified insets comparing the two methods, with speedup quantified by the ratio of mean squared error (MSE). This improvement stems from a combination of our efficient sparse convolution representation and the ability to perform next-event estimation even on specular micro-surfaces.

A fundamental challenge in rendering has been the dichotomy between surface and volume models. Gaussian Process Implicit Surfaces (GPISes) recently provided a unified approach for surfaces, volumes, and the spectrum in between. However, this representation remains impractical due to its high computational cost and mathematical complexity. We address these limitations by reformulating GPISes as procedural noise, eliminating expensive linear system solves while maintaining control over spatial correlations. Our method enables efficient sampling of stochastic realizations and supports flexible conditioning of values and derivatives through pathwise updates. To further enable practical rendering, we derive analytic distributions for surface normals, allowing for variance-reduced light transport via next-event estimation and multiple importance sampling. Our framework achieves efficient, high-quality rendering of stochastic surfaces and volumes with significantly simplified implementations on both CPU and GPU, while preserving the generality of the original GPIS representation.

CCS Concepts: • Computing methodologies → Rendering; Ray tracing.

Authors' addresses: Kehan Xu, Dartmouth College, USA, kehanxu.gr@dartmouth.edu; Benedikt Bitterli, NVIDIA, USA, bbitterli@nvidia.com; Eugene d'Eon, NVIDIA, Switzerland, edeon@nvidia.com; Wojciech Jarosz, Dartmouth College, USA, wjarosz@dartmouth.edu.

© 2025 Copyright held by the owner/author(s). This is the author's version of the work. It is posted here for your personal use. Not for redistribution. The definitive Version of Record was published in *ACM Transactions on Graphics*, <https://doi.org/10.1145/3763329>.

Additional Key Words and Phrases: stochastic processes, implicit surfaces

ACM Reference Format:

Kehan Xu, Benedikt Bitterli, Eugene d'Eon, and Wojciech Jarosz. 2025. Practical Gaussian Process Implicit Surfaces with Sparse Convolutions. *ACM Trans. Graph.* 44, 6, Article 188 (December 2025). 18 pages. <https://doi.org/10.1145/3763329>

1 INTRODUCTION

Traditional light transport simulation categorizes scene components as either solid surfaces or participating media, with each governed by a distinct rendering equation. This binary distinction is inadequate for capturing the diverse appearances found in the real world, particularly when considering level-of-detail techniques that might require turning hard surfaces into volumetric effects with surface-like correlation.

Seyb et al. [2024] demonstrated a unified framework using ensemble-averaged light transport through stochastic geometry, covering surfaces, volumes, and the spectrum in between. A key contribution was establishing a rigorous mathematical formalism for ensemble-averaged transport. Despite its potential, this method remains impractical, with rendering speeds at least two orders of magnitude slower than conventional path tracing. This performance gap arises

ACM Trans. Graph., Vol. 44, No. 6, Article 188. Publication date: December 2025.

For math: see papers & chat with me



Benedikt Bitterli
Dartmouth → NVIDIA



Dario Seyb
Dartmouth → Valve

From microfacets to participating media: A unified theory of light transport with stochastic geometry

DARIO SEYB, Dartmouth College, USA
EUGENE D'EON, NVIDIA, New Zealand
BENEDIKT BITTERLI, NVIDIA, USA
WOJCIECH JAROSZ, Dartmouth College, USA

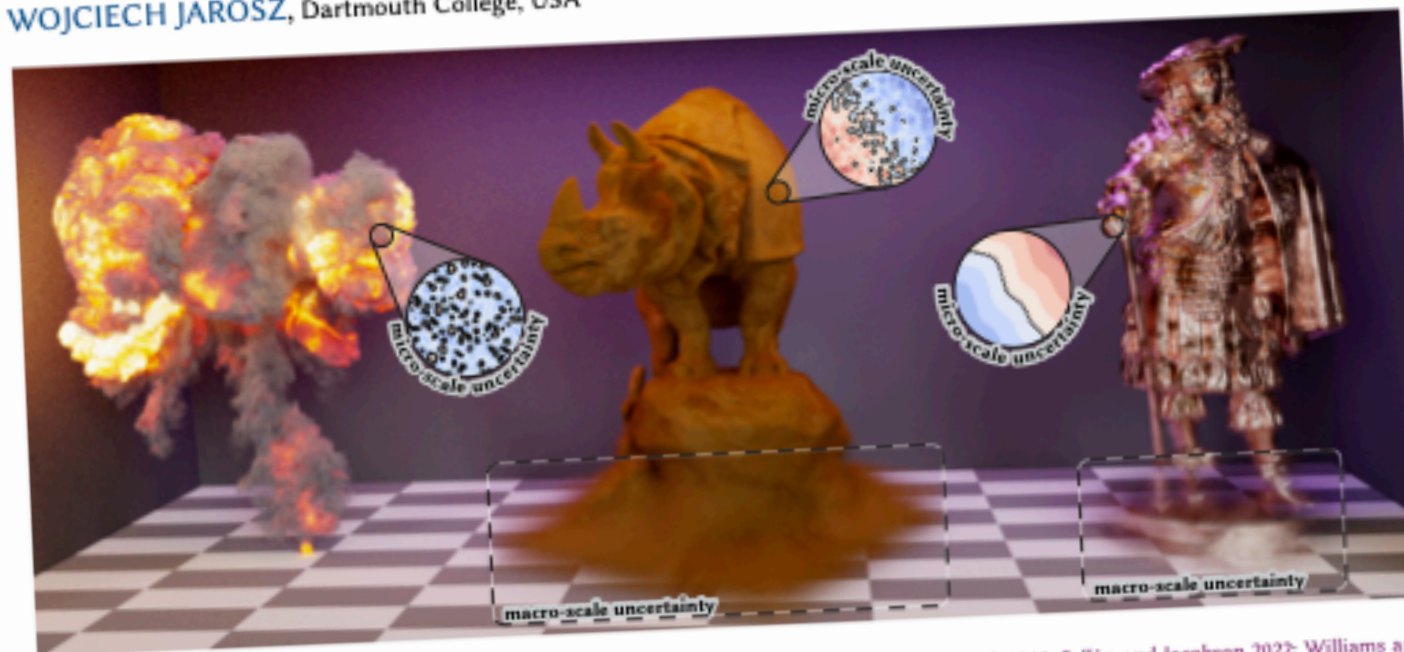


Fig. 1. We develop a theory of light transport for scenes with stochastic implicit surfaces [Dragiev et al. 2011; Sellán and Jacobson 2022; Williams and Fitzgibbon 2006] and show that this results in an expressive range of appearance behaviors that covers microfacet surfaces, classical participating media, and a novel continuum in between. Each object shown above is completely described by a 3D volume that encodes the mean and covariance kernel of a non-stationary Gaussian process and is rendered using the same rendering algorithm, agnostic of its “appearance type”. The insets show example realizations of the process at the highlighted points in the scene. Note how the appearance of the stochastic geometry transitions from volumetric (left) to hard-surface (right) as the correlations in the process are strengthened. Increasing the variance of the process allows us to visualize uncertainty at the macro-scale (bottom).

Stochastic geometry models have enjoyed immense success in graphics for modeling interactions of light with complex phenomena such as participating media, rough surfaces, fibers, and more. Although each of these models operates on the same principle of replacing intricate geometry by a random process and deriving the average light transport across all instances thereof, they are each tailored to one specific application and are fundamentally distinct. Each type of stochastic geometry present in the scene is firmly encapsulated in its own appearance model, with its own statistics and light transport average, and no cross-talk between different models or deterministic and stochastic geometry is possible.

In this paper, we derive a theory of light transport on stochastic implicit surfaces, a geometry model capable of expressing deterministic geometry, microfacet surfaces, participating media, and an exciting new continuum in between containing aggregate appearance, non-classical media, and more. Our model naturally supports spatial correlations, missing from most existing stochastic models.

Our theory paves the way for tractable rendering of scenes in which all geometry is described by the same stochastic model, while leaving ample future work for developing efficient sampling and rendering algorithms.

CCS Concepts: • Computing methodologies → Volumetric models.

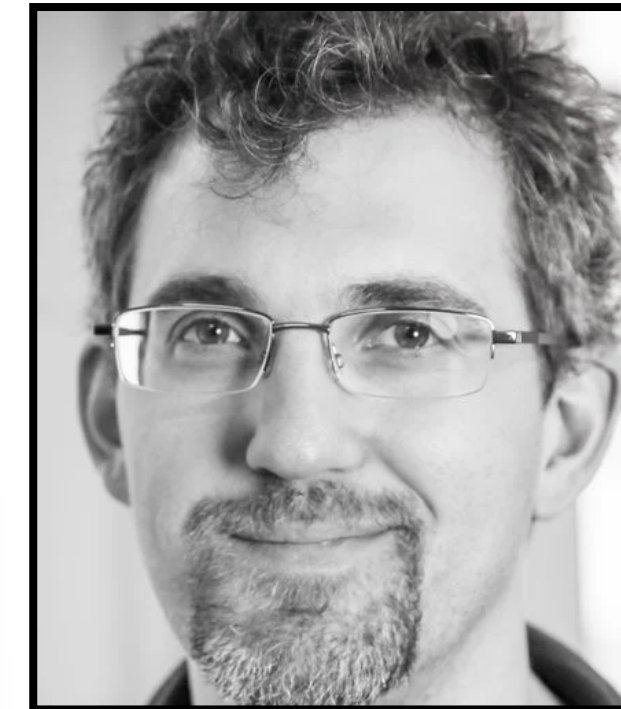
Additional Key Words and Phrases: volumetric light transport, stochastic processes, implicit surfaces

ACM Reference Format:
Dario Seyb, Eugene d'Eon, Benedikt Bitterli, and Wojciech Jarosz. 2024. From microfacets to participating media: A unified theory of light transport with stochastic geometry. *ACM Trans. Graph.* 43, 4, Article 112 (July 2024). 17 pages. <https://doi.org/10.1145/3658121>

1 INTRODUCTION

Computer graphics has seen significant progress over the past few decades, with advances in light transport, material fidelity, and

ACM Trans. Graph., Vol. 43, No. 4, Article 112. Publication date: July 2024.



Eugene d'Eon
NVIDIA



Kehan Xu
Dartmouth

Practical Gaussian Process Implicit Surfaces with Sparse Convolutions

KEHAN XU, Dartmouth College, USA
BENEDIKT BITTERLI, NVIDIA, USA
EUGENE D'EON, NVIDIA, Switzerland
WOJCIECH JAROSZ, Dartmouth College, USA



Fig. 1. Our novel sparse convolution representation for Gaussian Process Implicit Surfaces (GPISes) enables, for the first time, the visualization of single non-stationary realizations (split screen, middle right) alongside their ensemble-averaged light transport (right). The single realization allows examining the underlying structures of the stochastic geometry that lead to the aggregate rough surface or volumetric appearance. Furthermore, our algorithm (middle left) significantly boosts rendering efficiency for ensemble light transport compared to previous work by Seyb et al. [2024] (left) at equal time (40 min). The rightmost column presents magnified insets comparing the two methods, with speedup quantified by the ratio of mean squared error (MSE). This improvement stems from a combination of our efficient sparse convolution representation and the ability to perform next-event estimation even on specular micro-surfaces.

A fundamental challenge in rendering has been the dichotomy between surface and volume models. Gaussian Process Implicit Surfaces (GPISes) recently provided a unified approach for surfaces, volumes, and the spectrum in between. However, this representation remains impractical due to its high computational cost and mathematical complexity. We address these limitations by reformulating GPISes as procedural noise, eliminating expensive linear system solves while maintaining control over spatial correlations. Our method enables efficient sampling of stochastic realizations and supports flexible conditioning of values and derivatives through pathwise updates. To further enable practical rendering, we derive analytic distributions for surface normals, allowing for variance-reduced light transport via next-event estimation and multiple importance sampling. Our framework achieves efficient, high-quality rendering of stochastic surfaces and volumes with significantly simplified implementations on both CPU and GPU, while preserving the generality of the original GPIS representation.

CCS Concepts: • Computing methodologies → Rendering; Ray tracing.

Authors' addresses: Kehan Xu, Dartmouth College, USA, kehan.xu@dartmouth.edu; Benedikt Bitterli, NVIDIA, USA, bbitterli@nvidia.com; Eugene d'Eon, NVIDIA, Switzerland, edeon@nvidia.com; Wojciech Jarosz, Dartmouth College, USA, kjarosz@dartmouth.edu.

© 2025 Copyright held by the owner/author(s). This is the author's version of the work. It is posted here for your personal use. Not for redistribution. The definitive Version of Record was published in *ACM Transactions on Graphics*, <https://doi.org/10.1145/3763329>.

Additional Key Words and Phrases: stochastic processes, implicit surfaces

ACM Reference Format:

Kehan Xu, Benedikt Bitterli, Eugene d'Eon, and Wojciech Jarosz. 2025. Practical Gaussian Process Implicit Surfaces with Sparse Convolutions. *ACM Trans. Graph.* 44, 6, Article 188 (December 2025). 18 pages. <https://doi.org/10.1145/3763329>

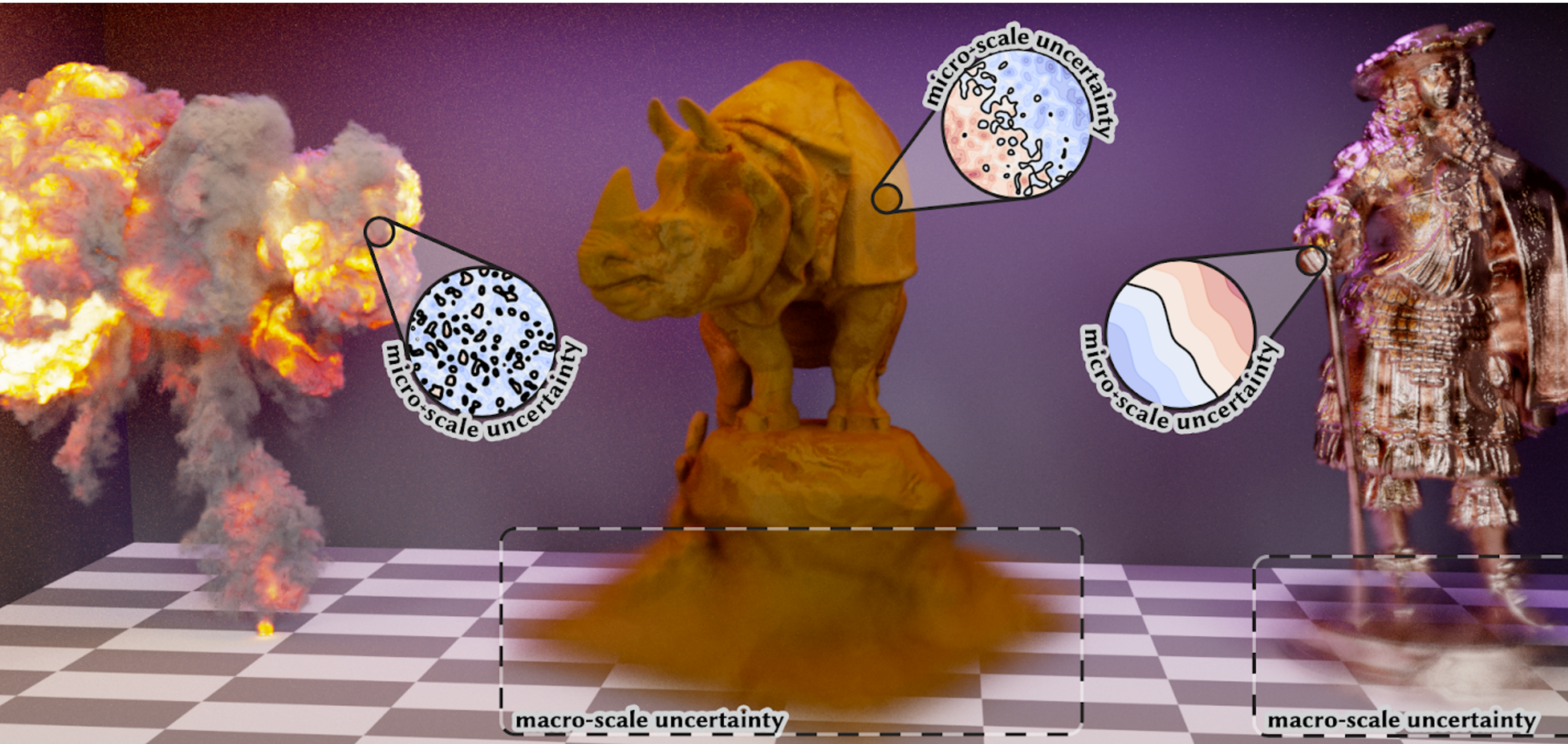
1 INTRODUCTION

Traditional light transport simulation categorizes scene components as either solid surfaces or participating media, with each governed by a distinct rendering equation. This binary distinction is inadequate for capturing the diverse appearances found in the real world, particularly when considering level-of-detail techniques that might require turning hard surfaces into volumetric effects with surface-like correlation.

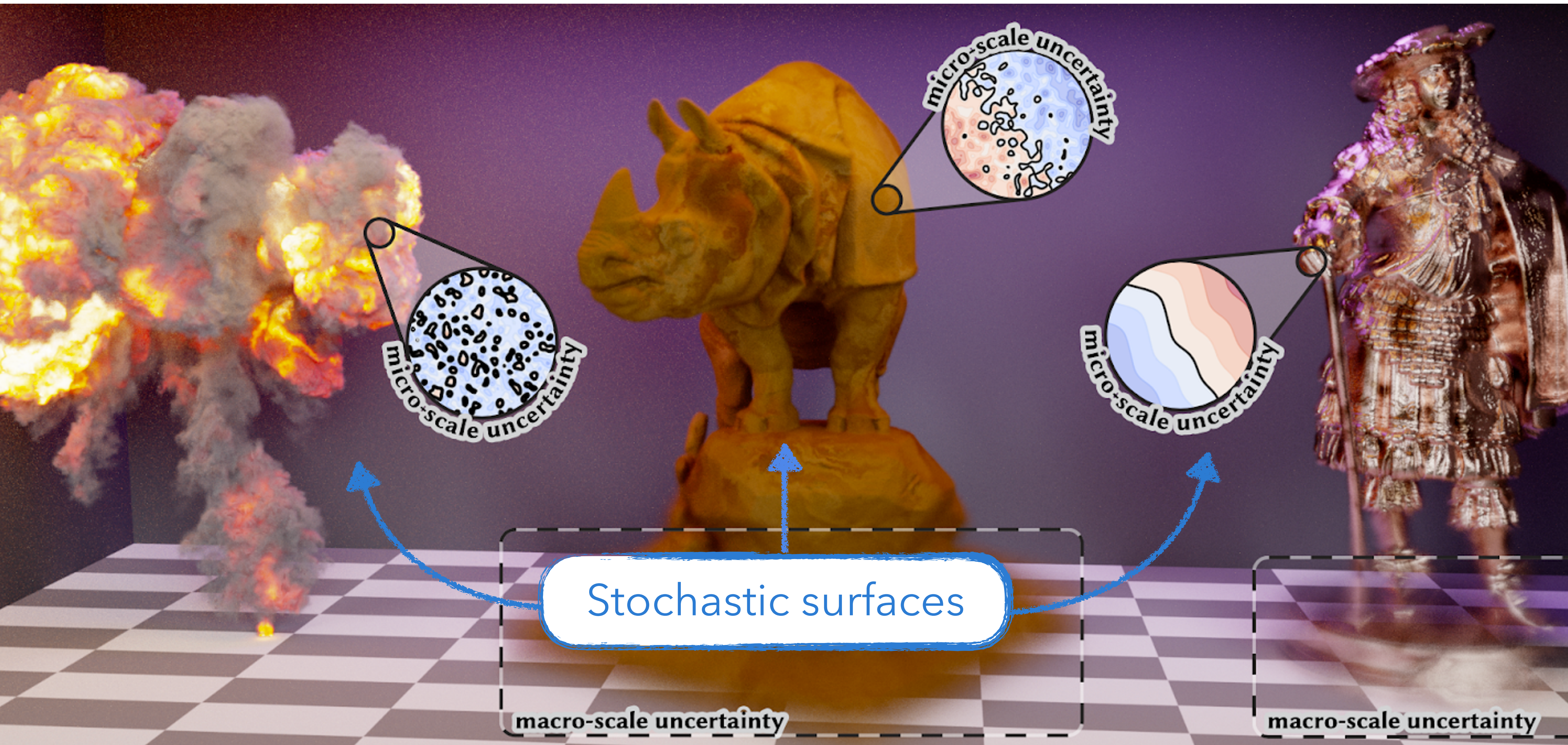
Seyb et al. [2024] demonstrated a unified framework using ensemble-averaged light transport through stochastic geometry, covering surfaces, volumes, and the spectrum in between. A key contribution was establishing a rigorous mathematical formalism for ensemble-averaged transport. Despite its potential, this method remains impractical, with rendering speeds at least two orders of magnitude slower than conventional path tracing. This performance gap arises

ACM Trans. Graph., Vol. 44, No. 6, Article 188. Publication date: December 2025.

A Unified Theory for Volumes, Surfaces and In-between



A Unified Theory for Volumes, Surfaces and In-between

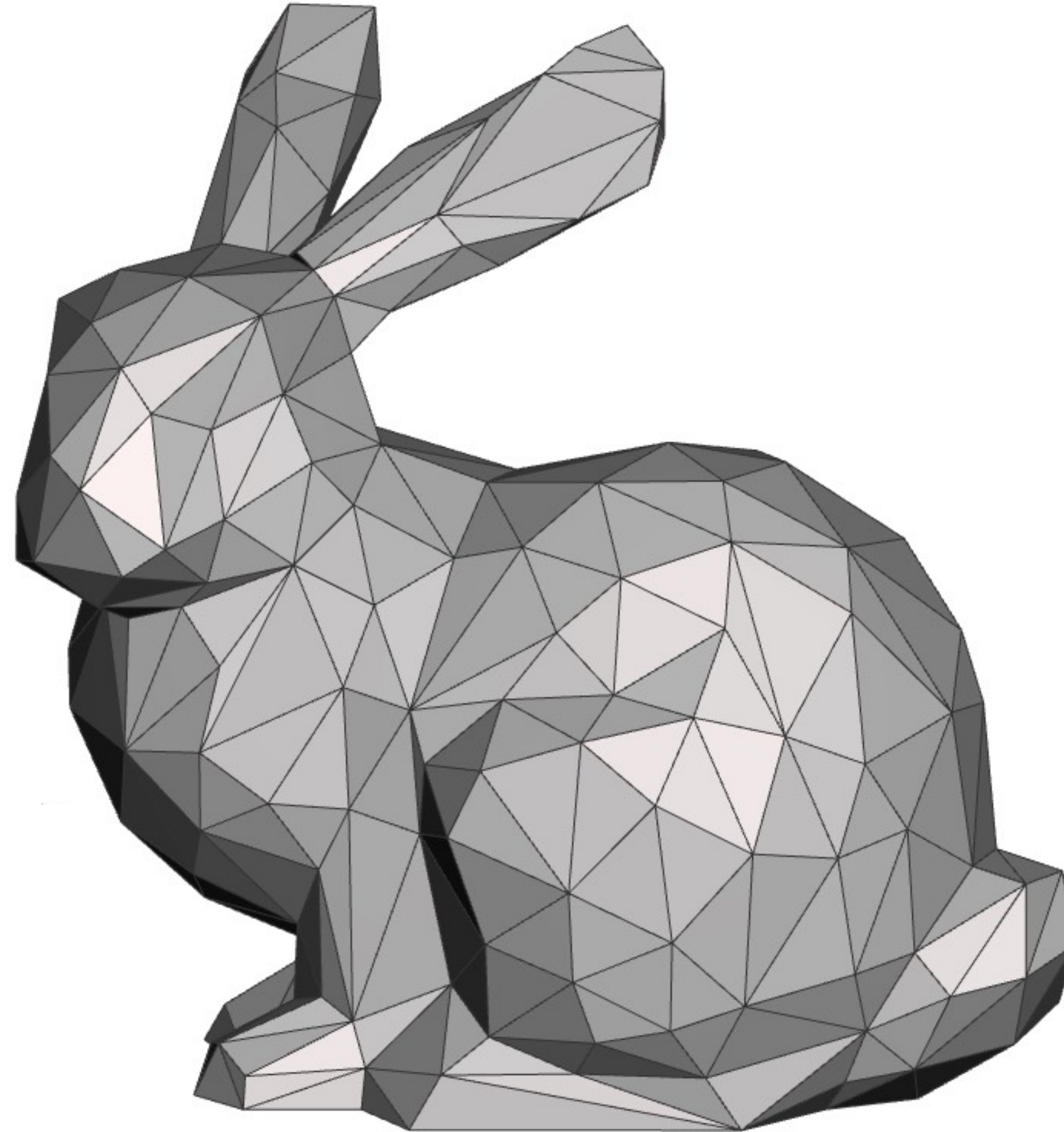


Gaussian Process Implicit Surfaces

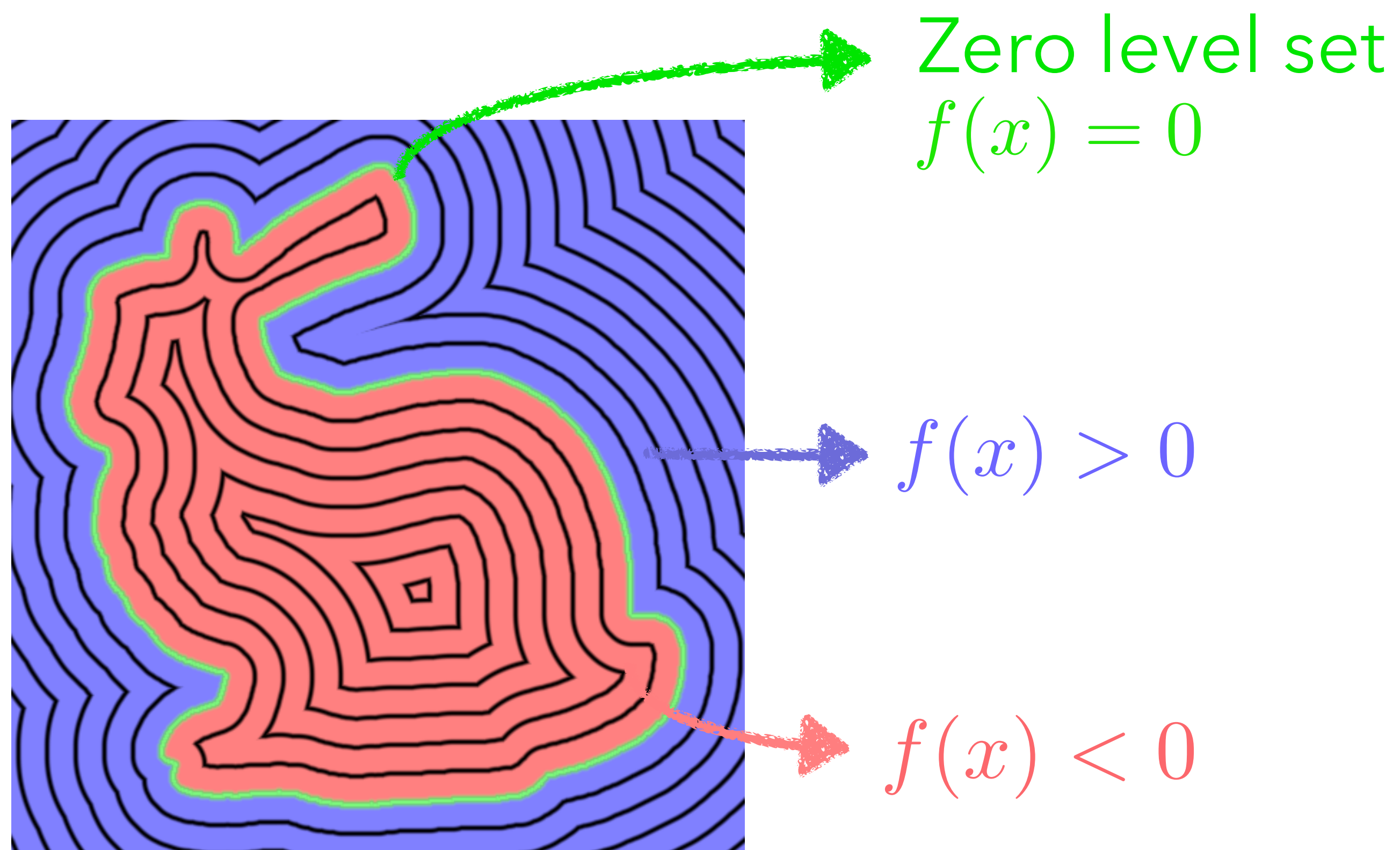
Gaussian Process

Implicit Surfaces

A triangle mesh



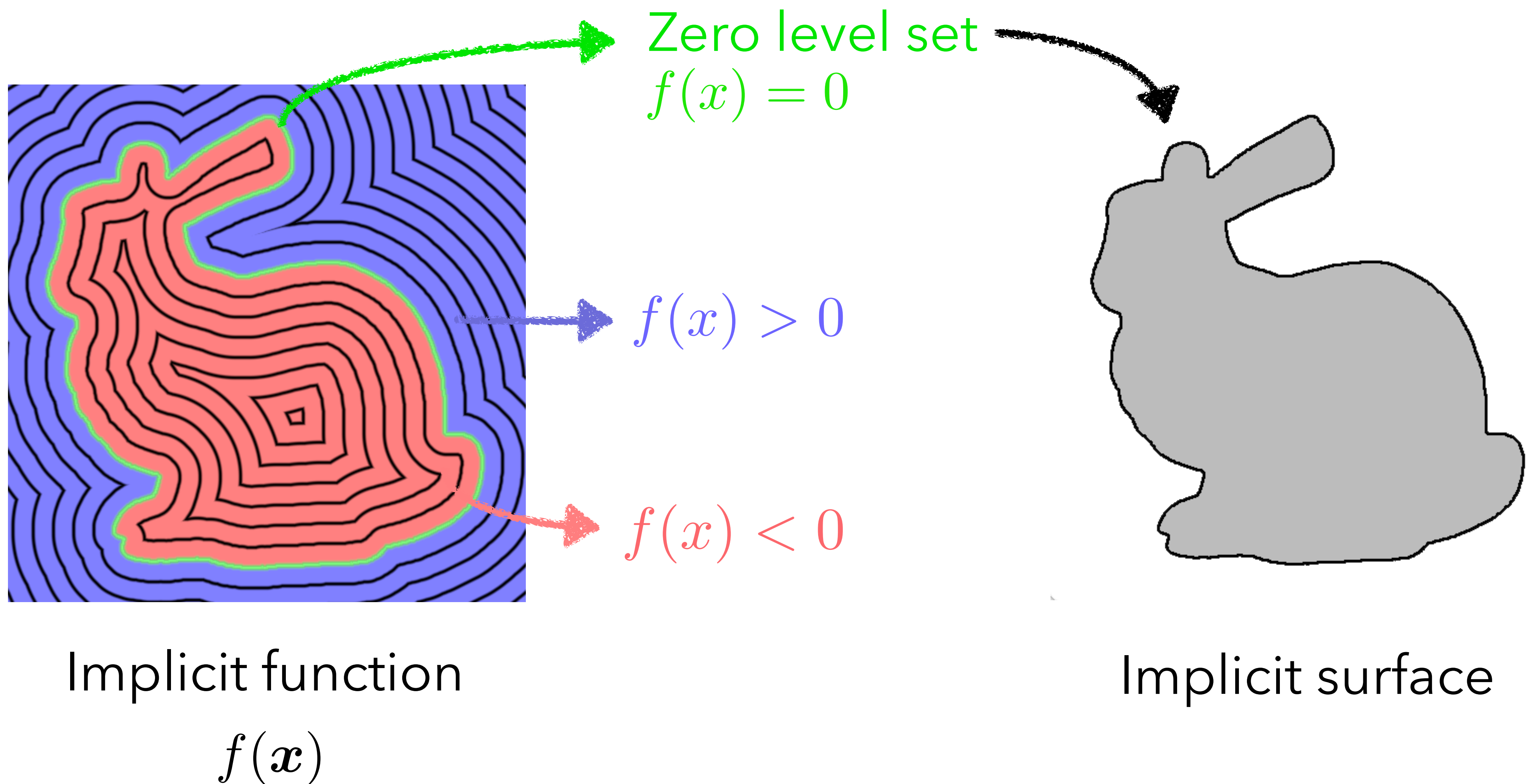
An Implicit Surface



Implicit function

$$f(x)$$

An Implicit Surface



Gaussian Process

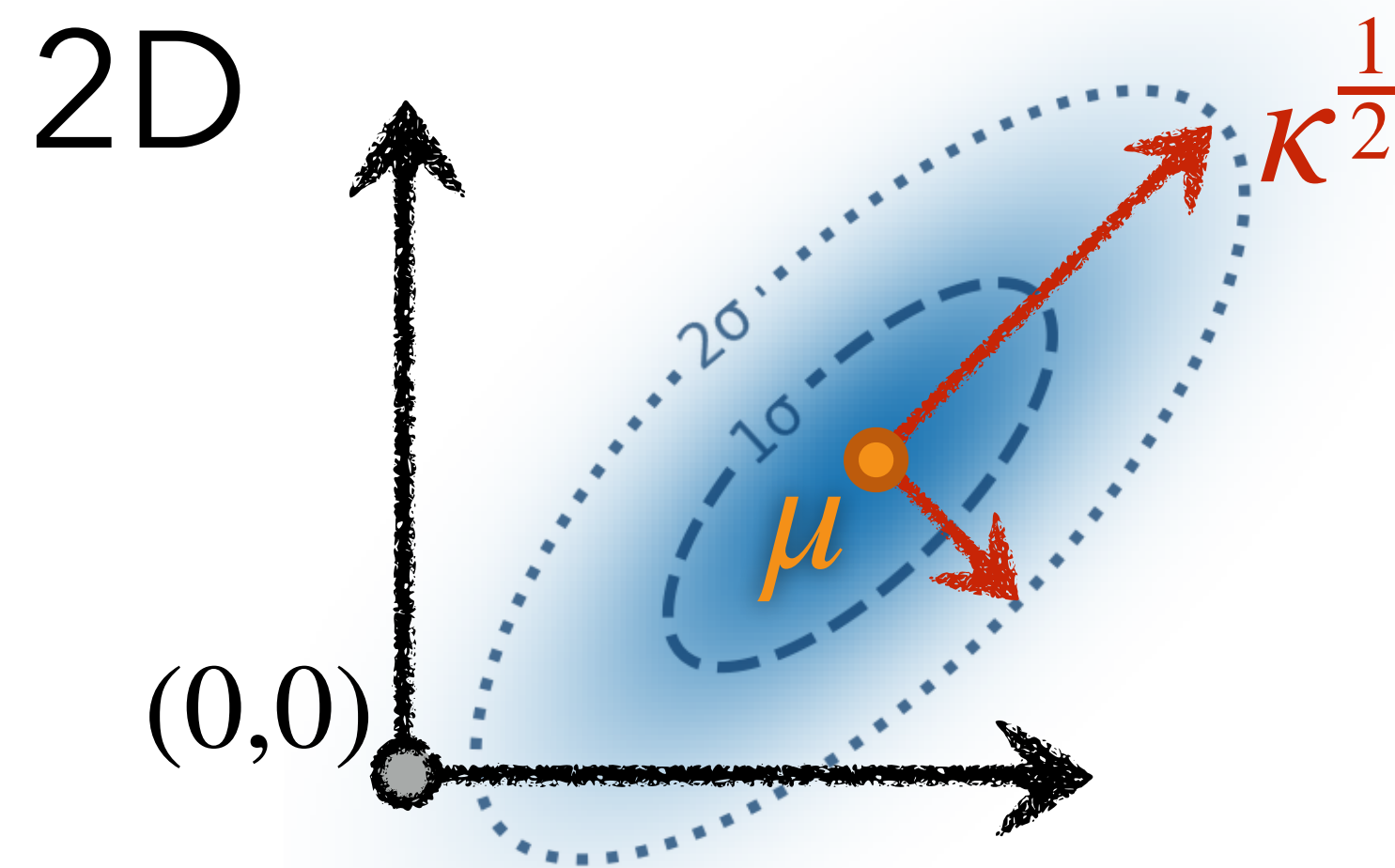
Implicit Surfaces

Gaussian Distribution

$$f \sim \mathcal{N}(\mu, \kappa)$$

Mean

Covariance

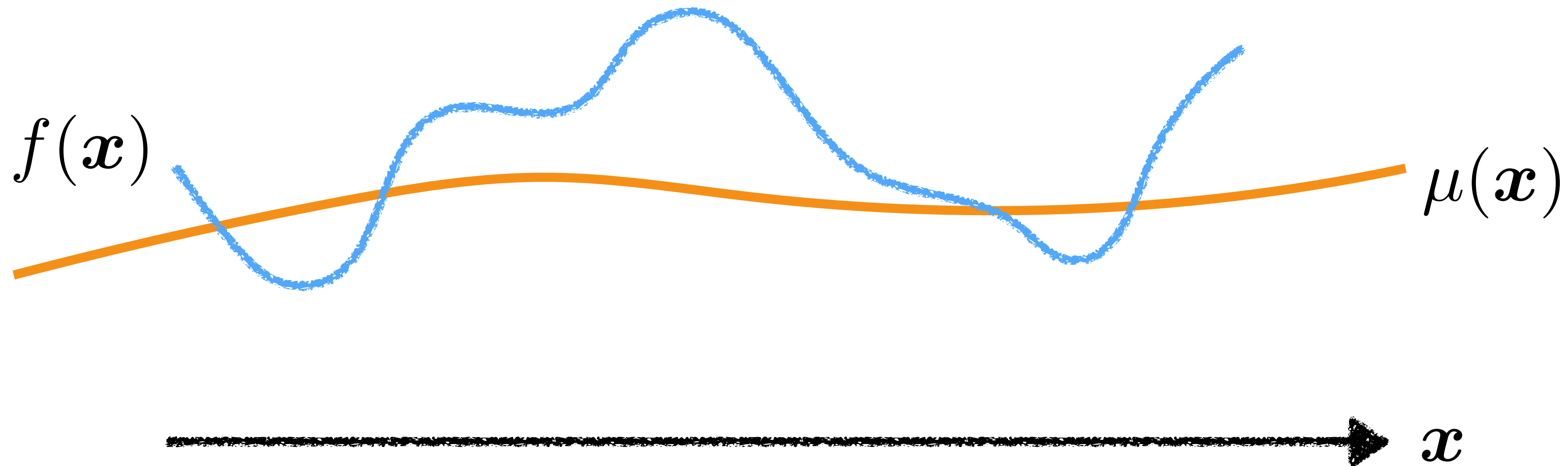


Gaussian Distribution → Gaussian Process

$$f(x) \sim \mathcal{N}(\mu(x), \kappa(x, x'))$$

Mean

Covariance

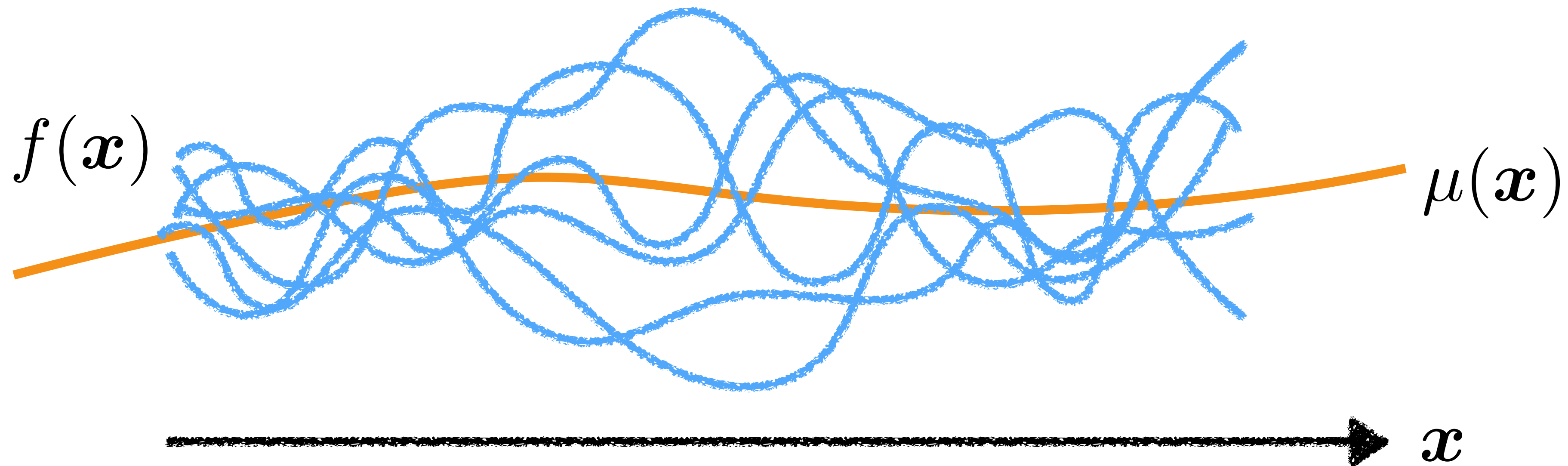


Gaussian Distribution → Gaussian Process

$$f(x) \sim \mathcal{N}(\mu(x), \kappa(x, x'))$$

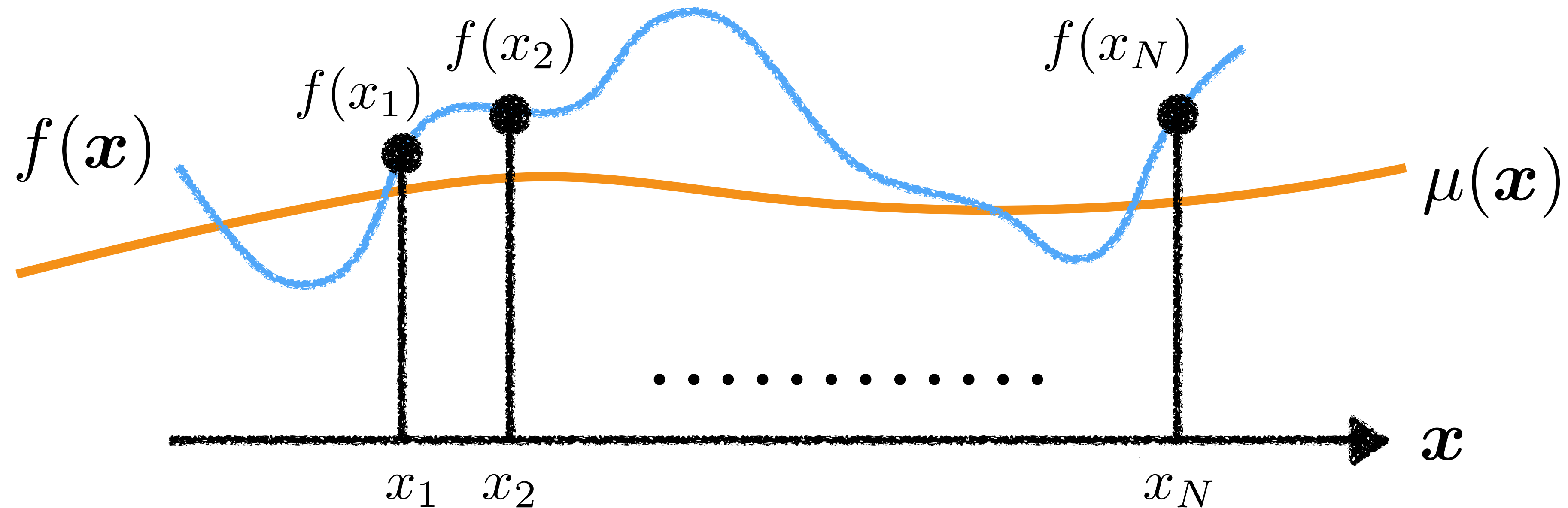
Mean

Covariance



Any N points jointly distributed as N-D Gaussian

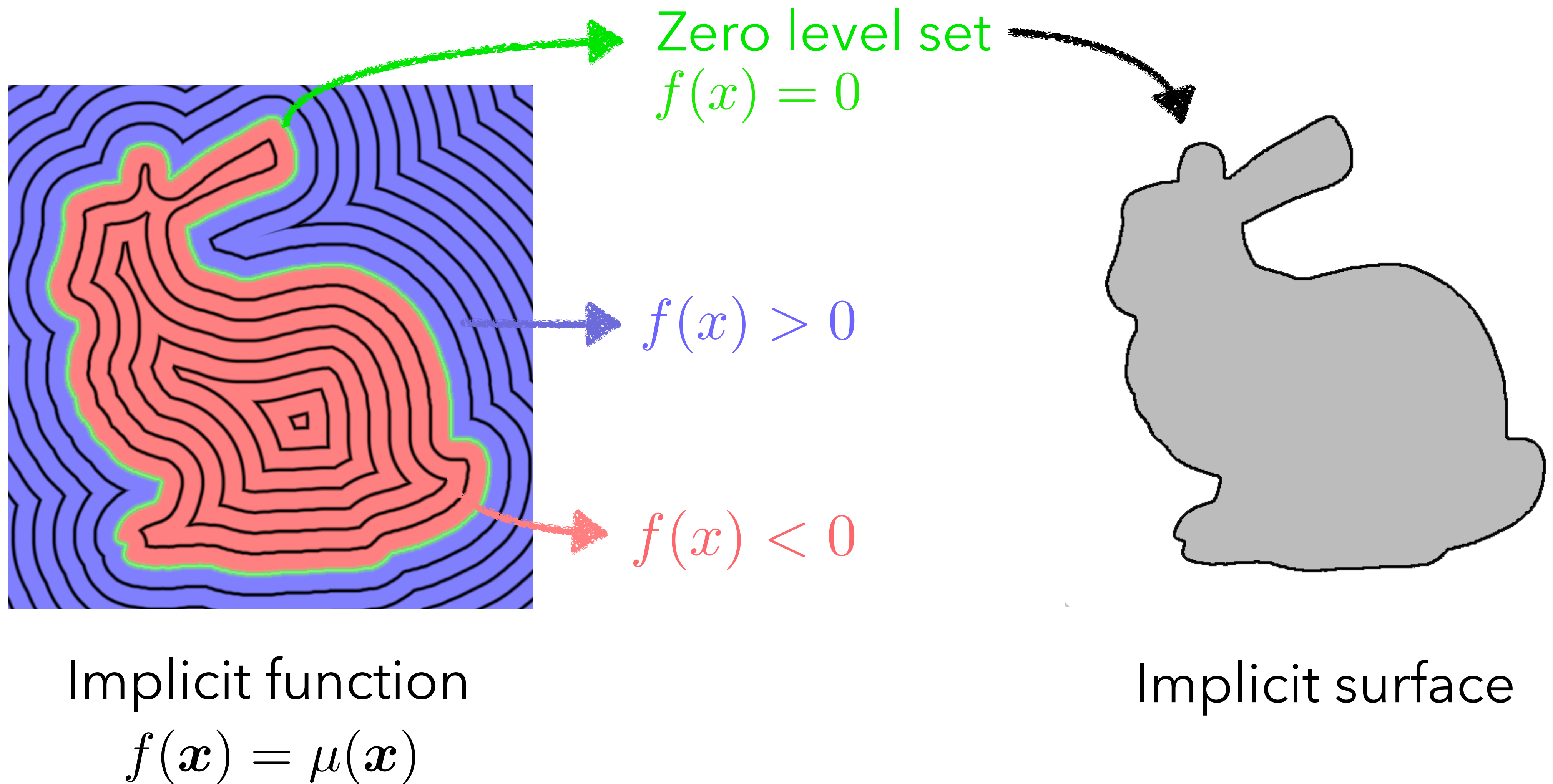
$$f\left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}\right) \sim \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{bmatrix}, \begin{bmatrix} \kappa_{11} & \kappa_{12} & \dots & \kappa_{1N} \\ \kappa_{21} & \kappa_{22} & \dots & \kappa_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_{N1} & \kappa_{N2} & \dots & \kappa_{NN} \end{bmatrix}\right)$$



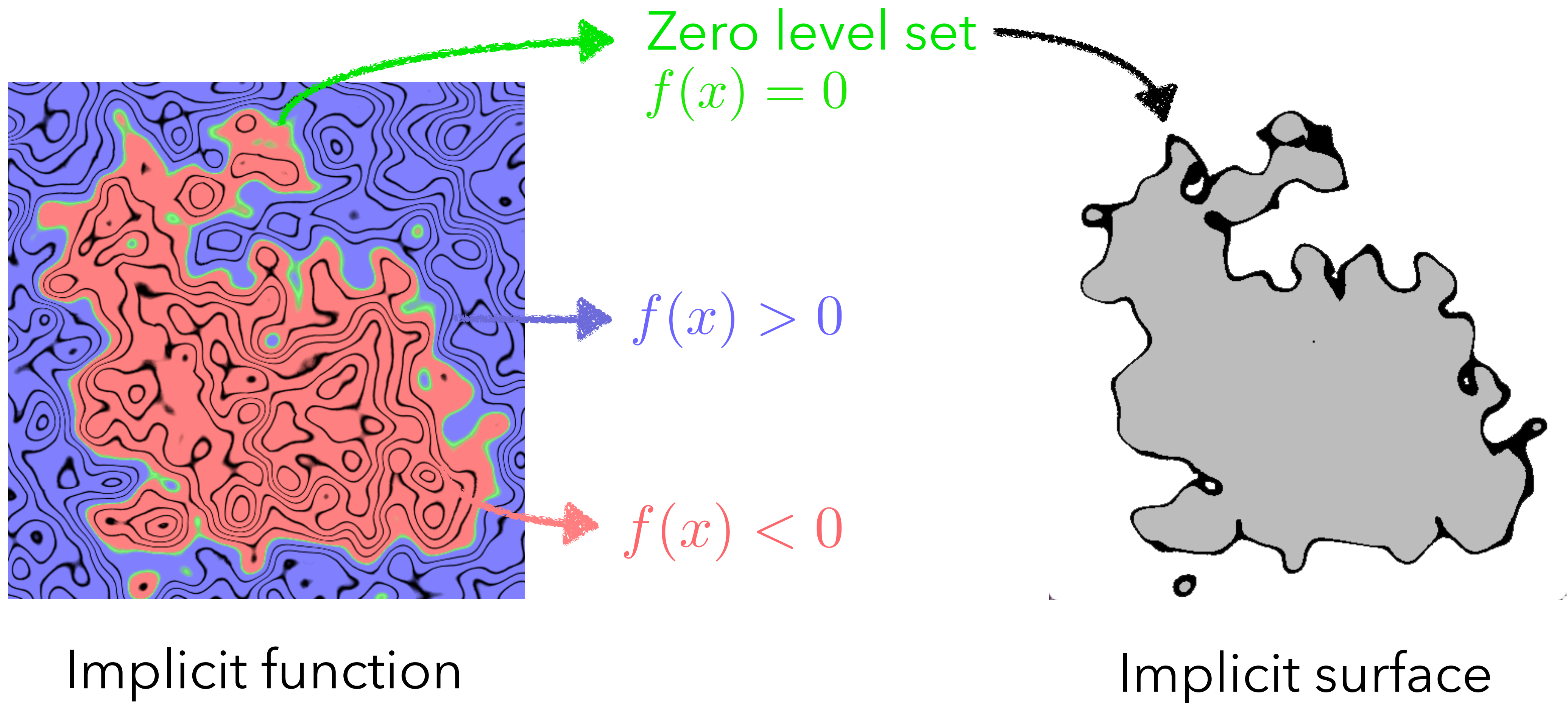
Gaussian Process

Implicit Surfaces

Implicit Surface

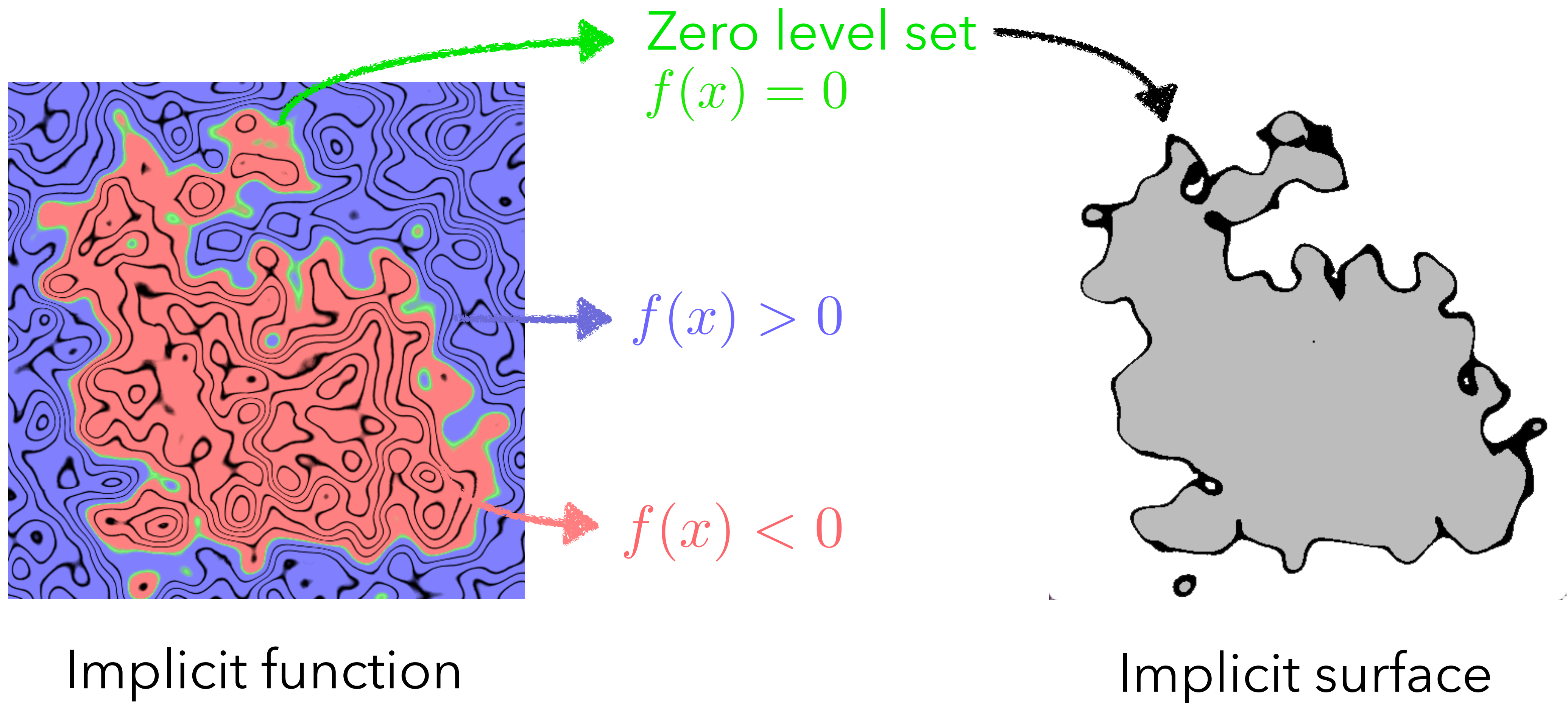


Gaussian Process Implicit Surface (GPIS)



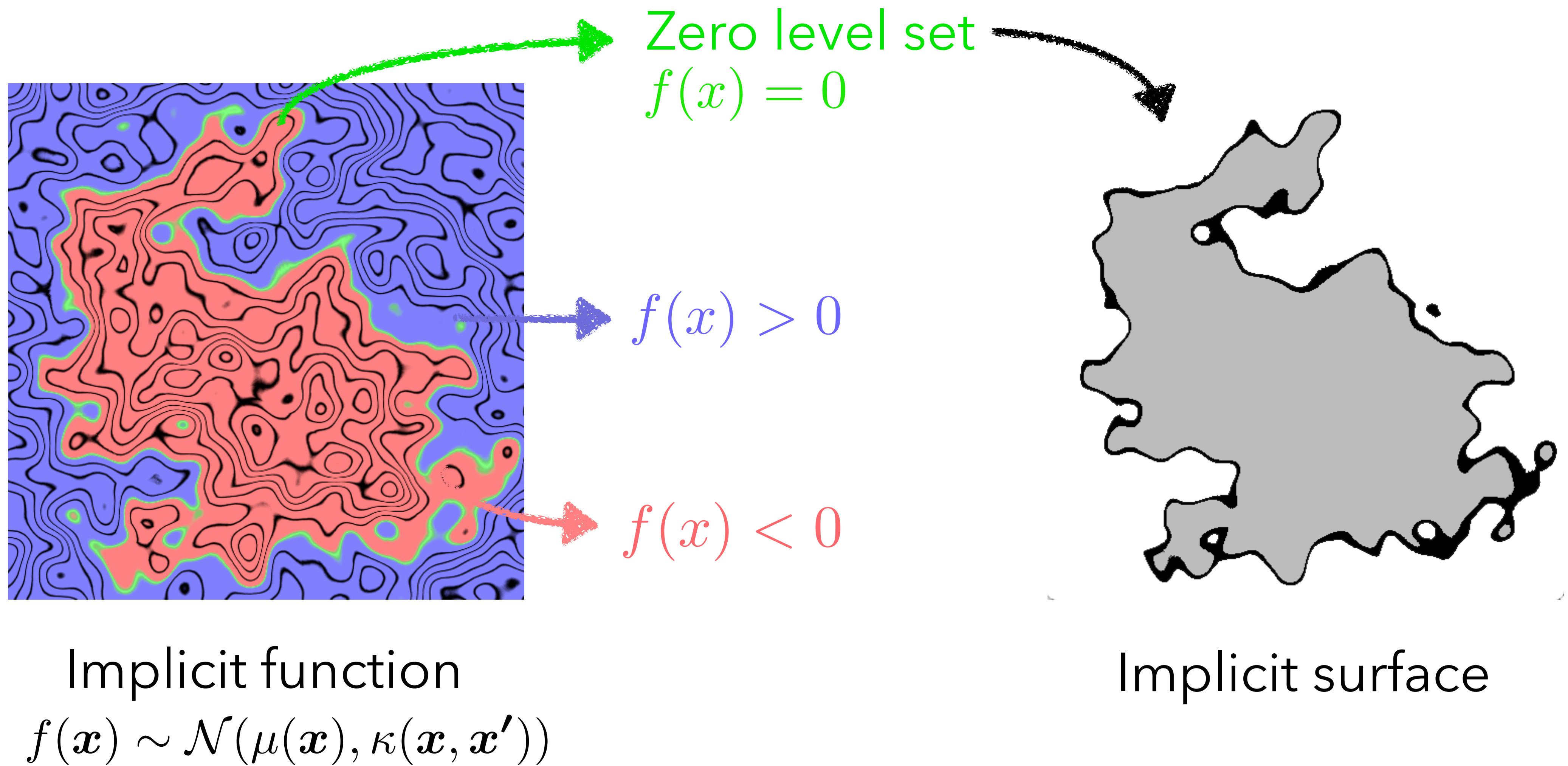
$$f(\mathbf{x}) \sim \mathcal{N}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}'))$$

Gaussian Process Implicit Surface (GPIS)

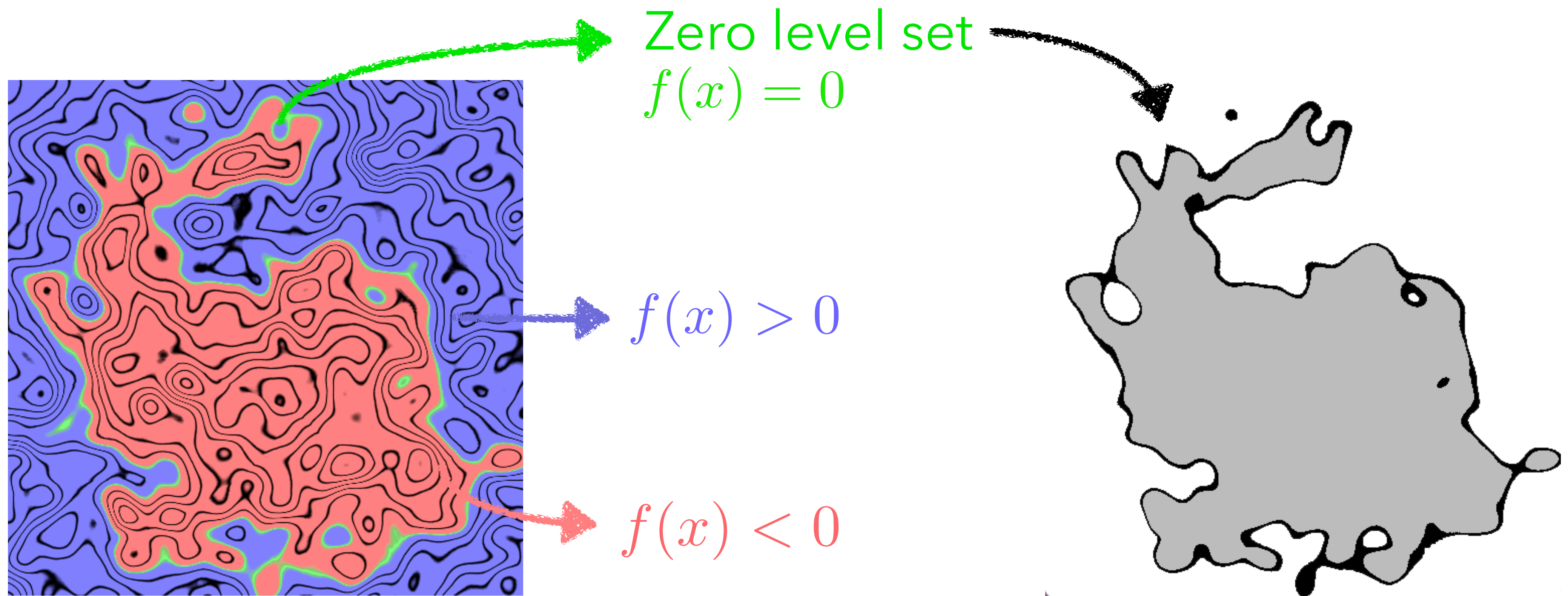


$$f(\mathbf{x}) \sim \mathcal{N}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}'))$$

Gaussian Process Implicit Surface (GPIS)

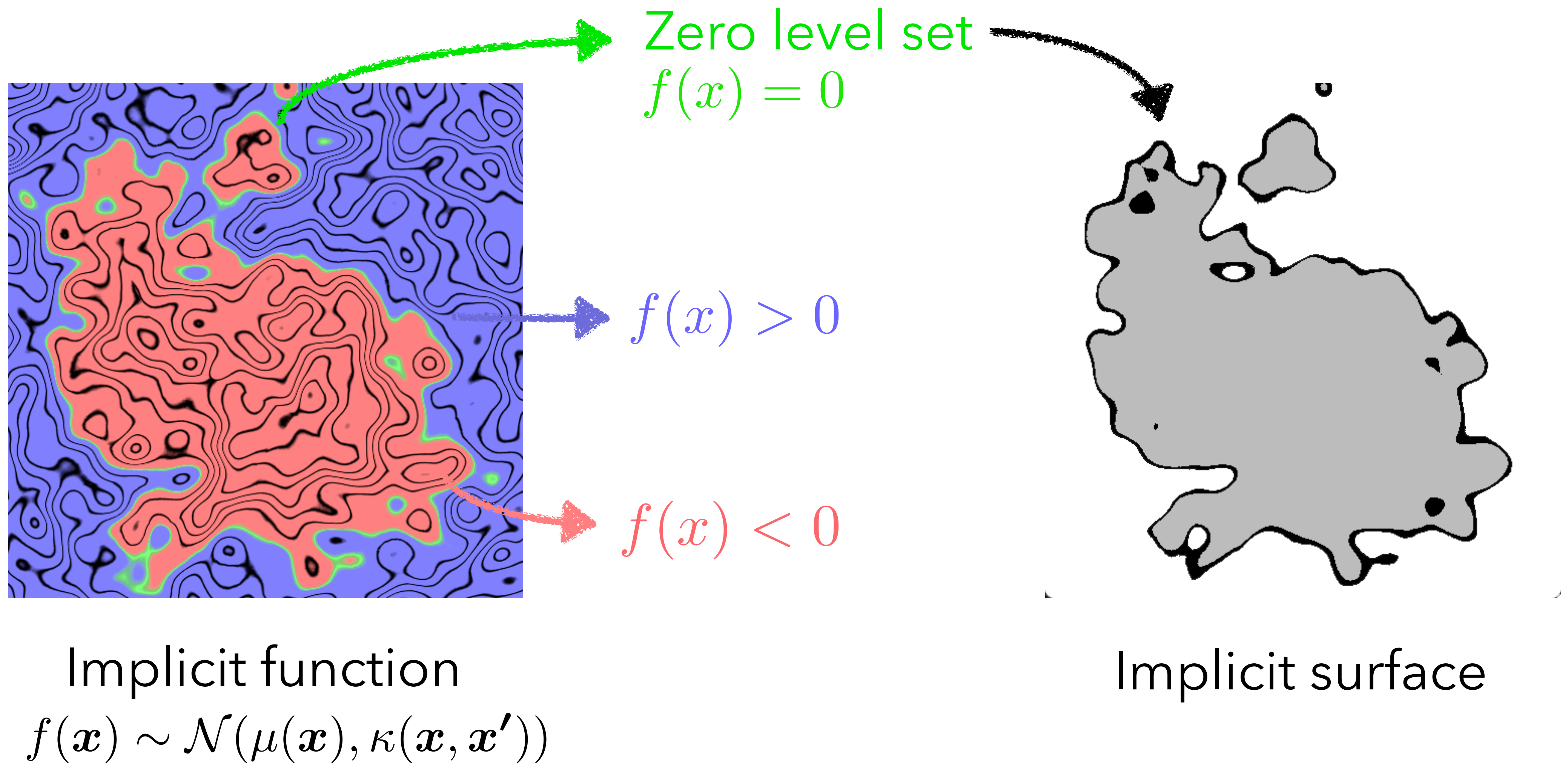


Gaussian Process Implicit Surface (GPIS)

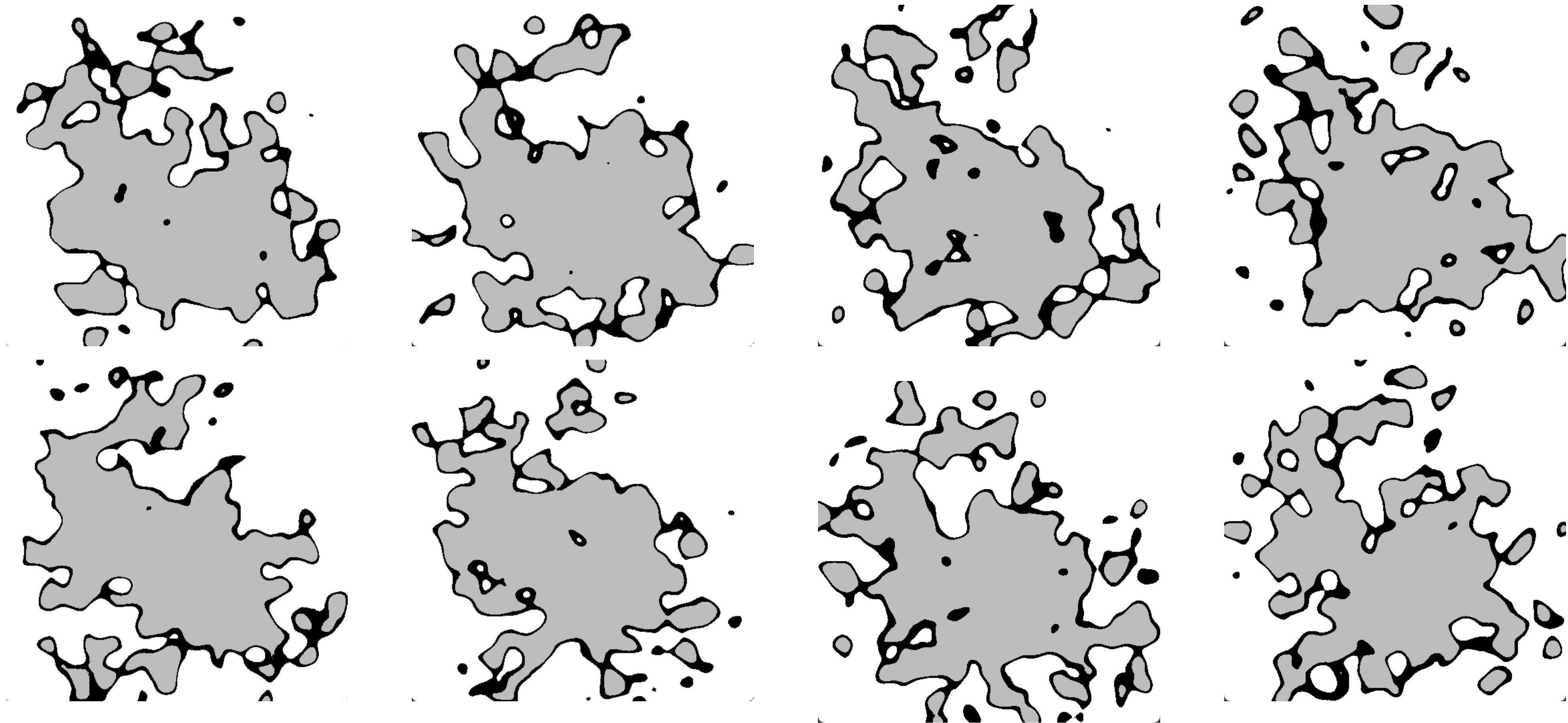


Implicit function
 $f(\mathbf{x}) \sim \mathcal{N}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}'))$

Gaussian Process Implicit Surface (GPIS)

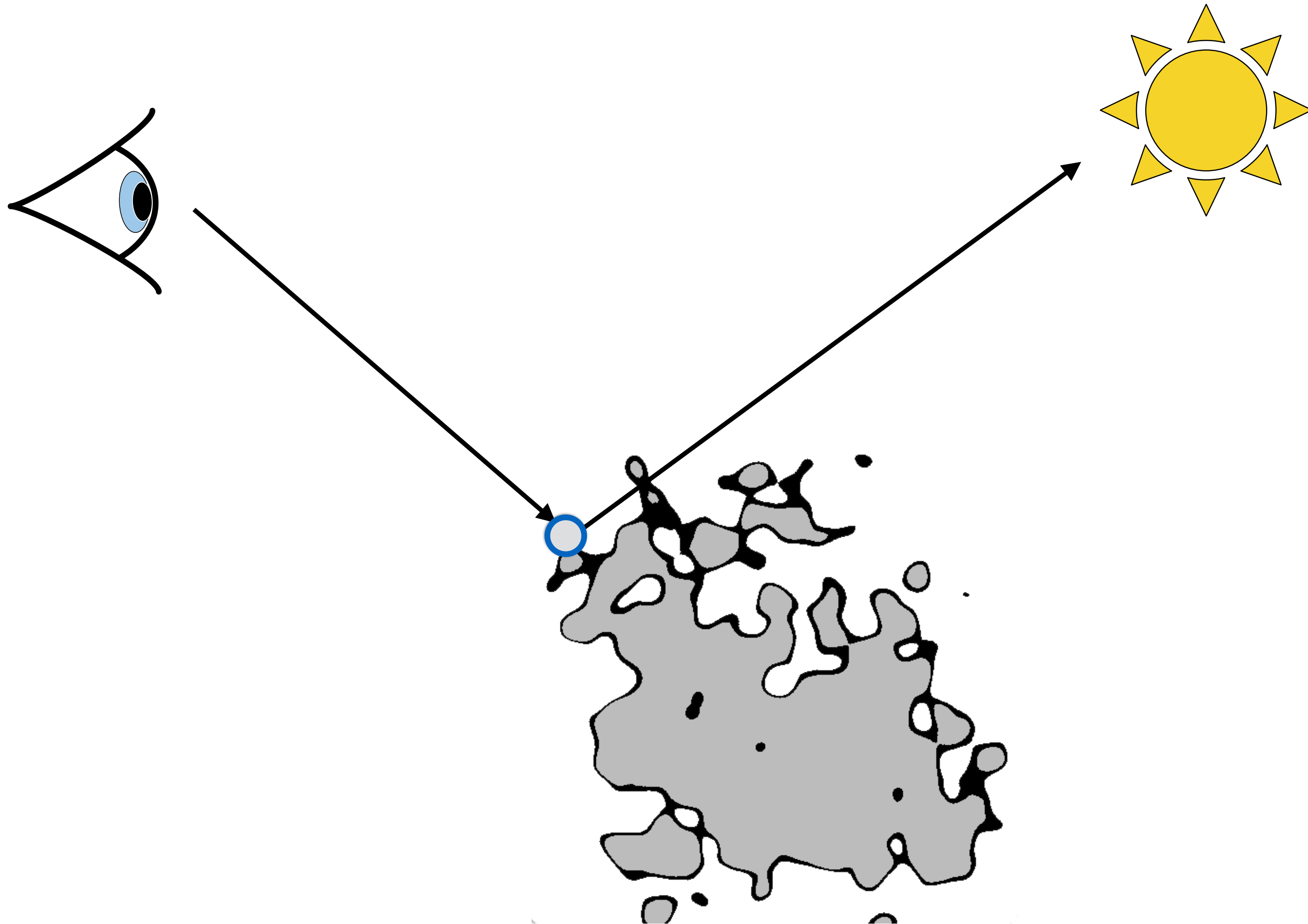


A distribution of random surfaces

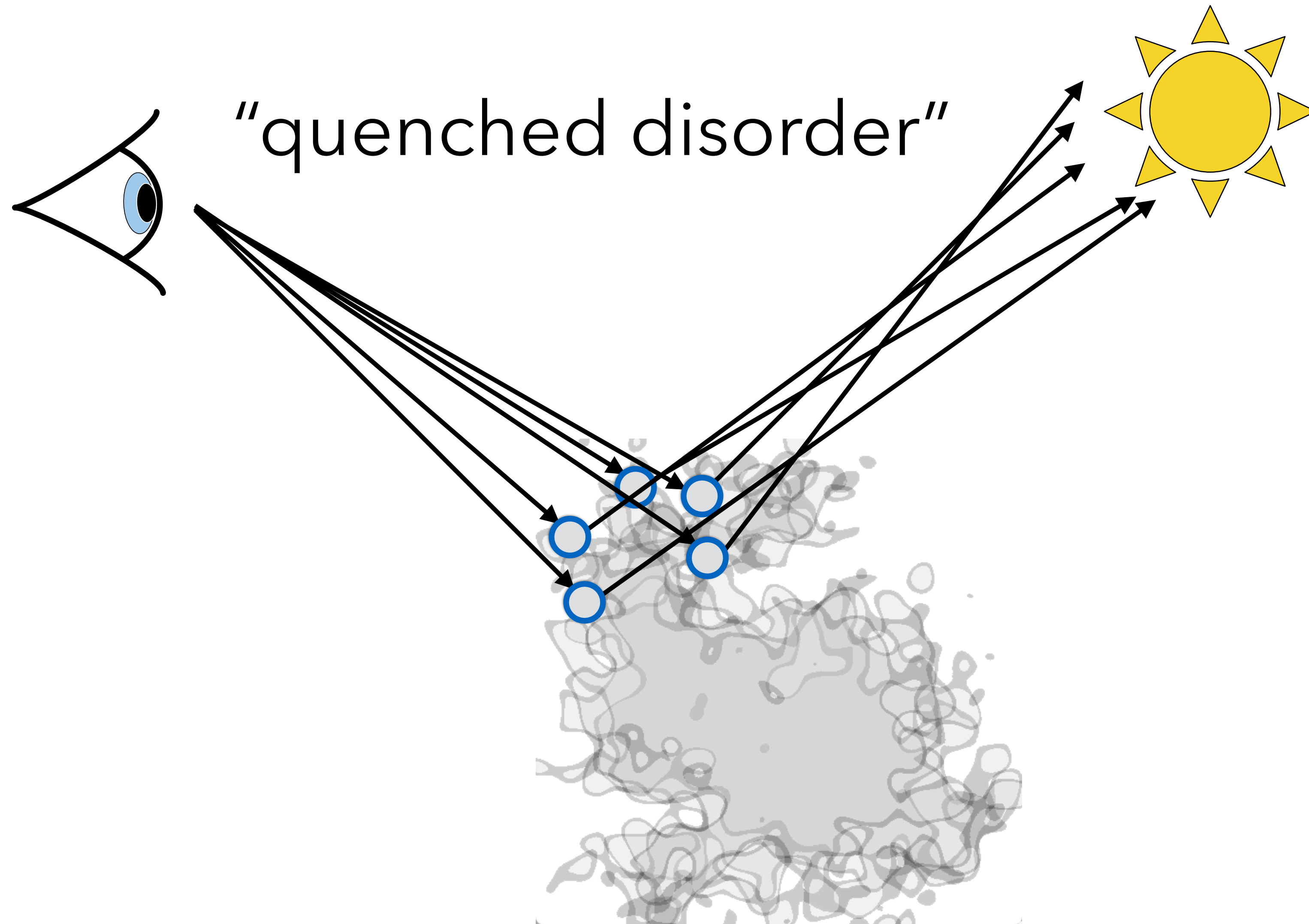


$$f(x) \sim \mathcal{N}(\mu(x), \kappa(x, x'))$$

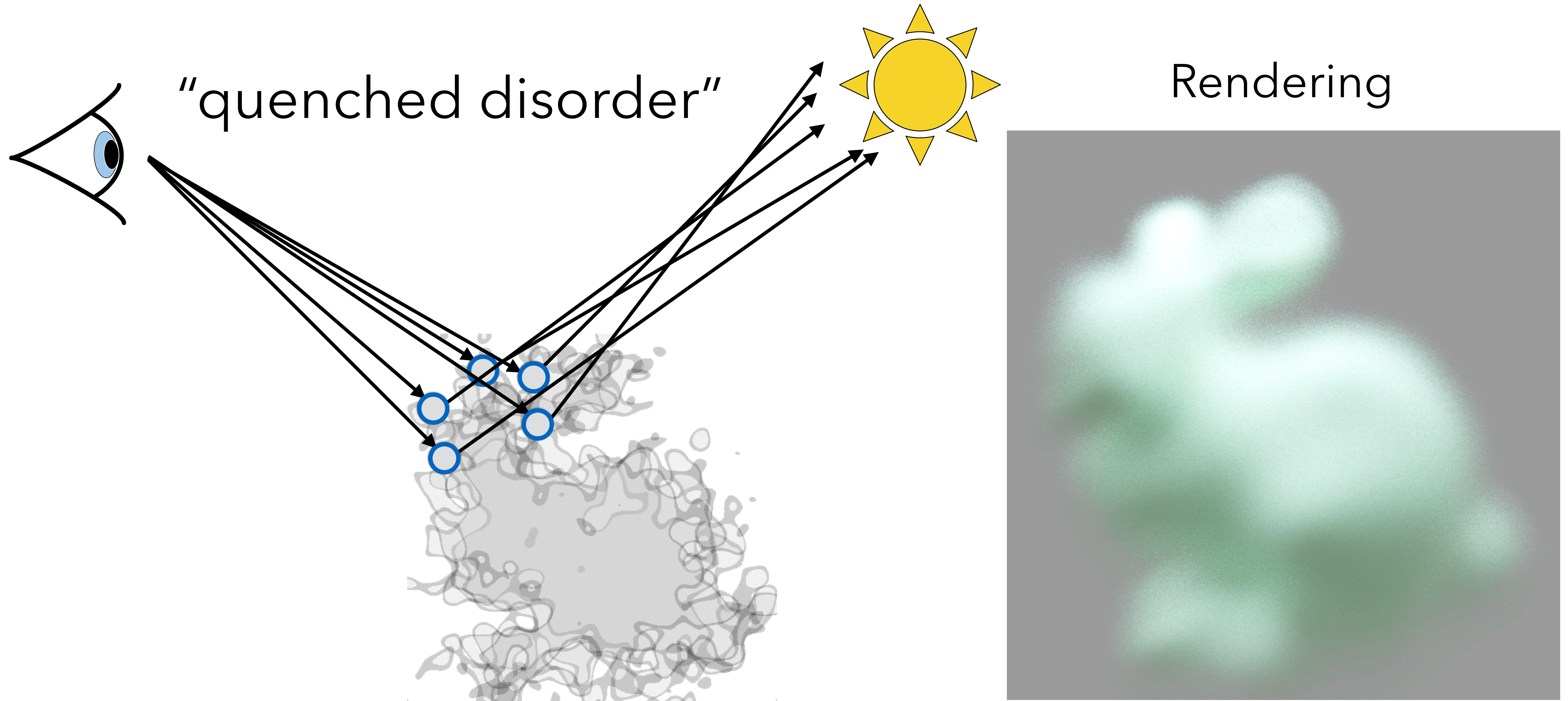
Light transport with one realization



Ensemble-averaged light transport



Ensemble-averaged light transport





Appearance Space of GPISes

GPISeS with different parameters

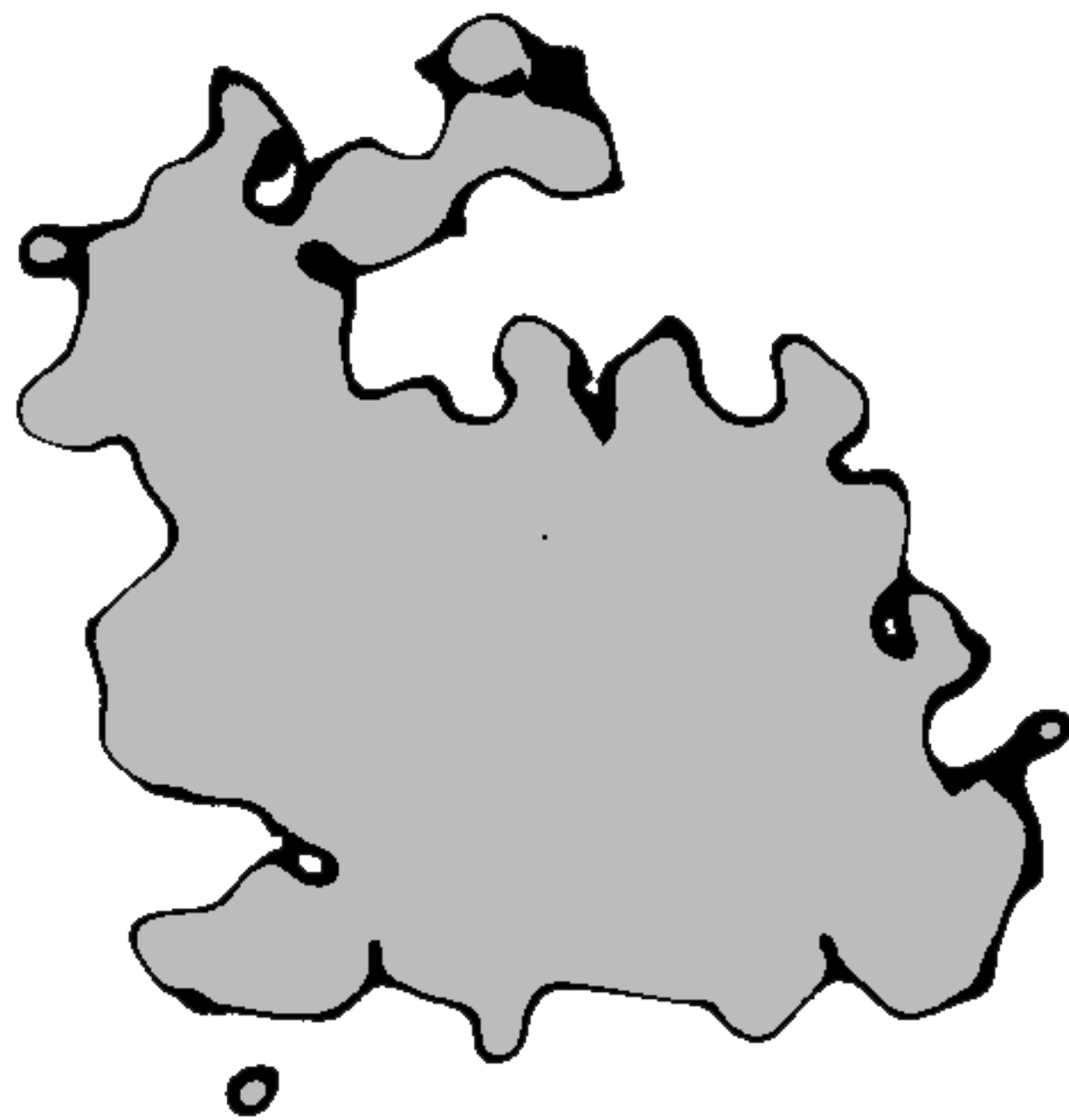
Mean **Covariance**

$$f(\mathbf{x}) \sim \mathcal{N}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}'))$$

GPISeS with different parameters

Mean Covariance

$$f(\mathbf{x}) \sim \mathcal{N}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}'))$$

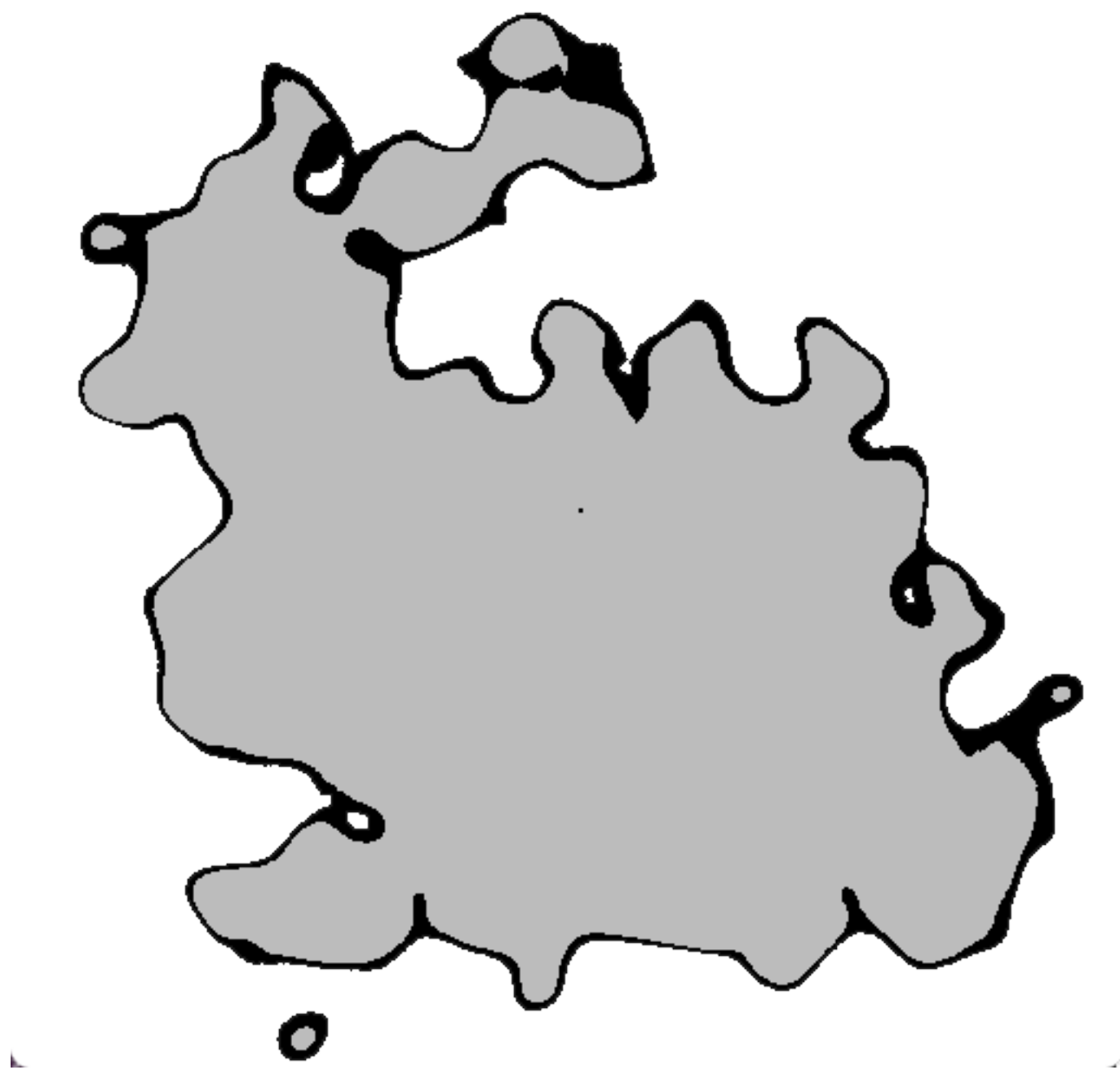


Surface-like

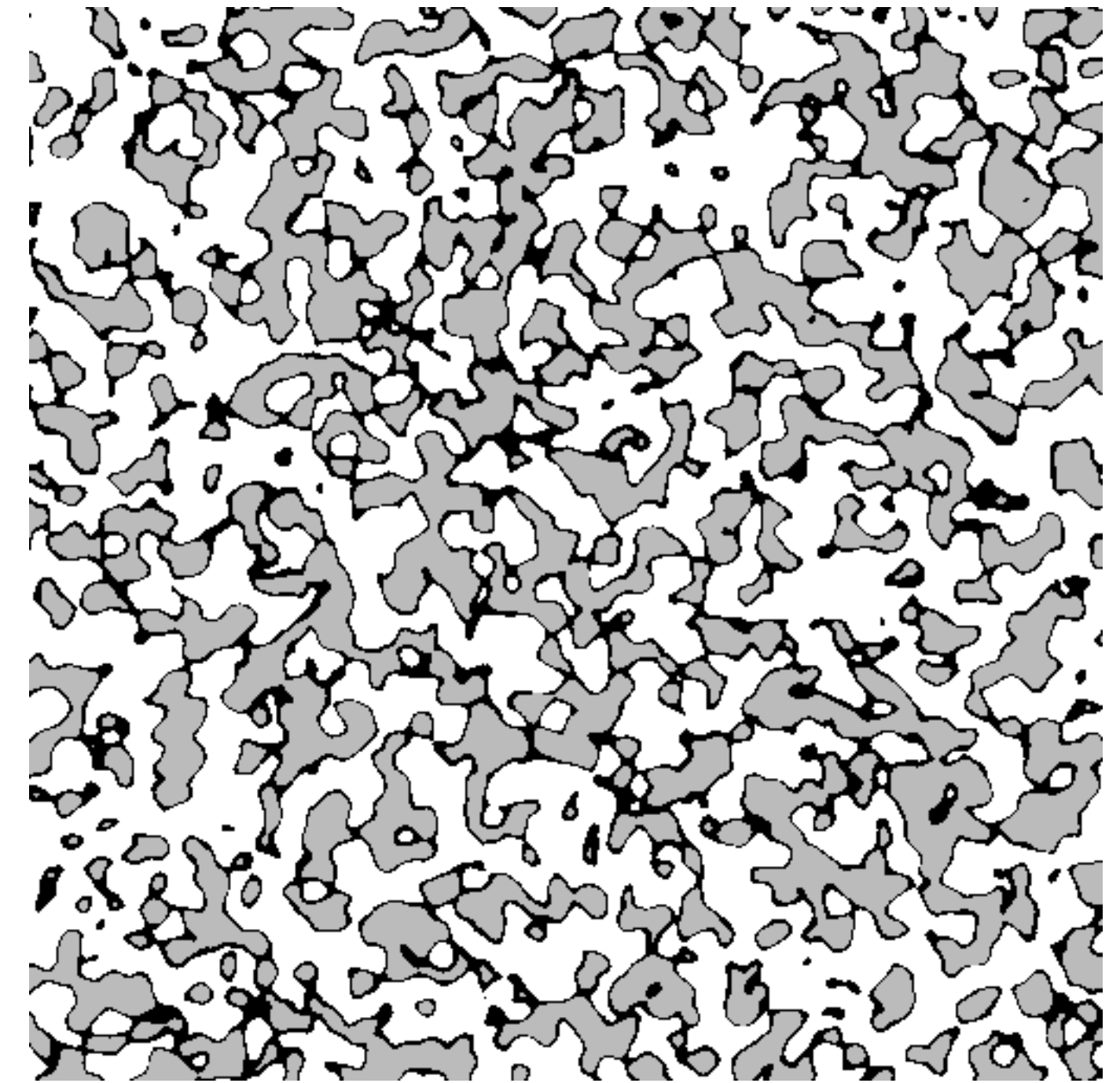
GPISeS with different parameters

Mean Covariance

$$f(\mathbf{x}) \sim \mathcal{N}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}'))$$



Surface-like

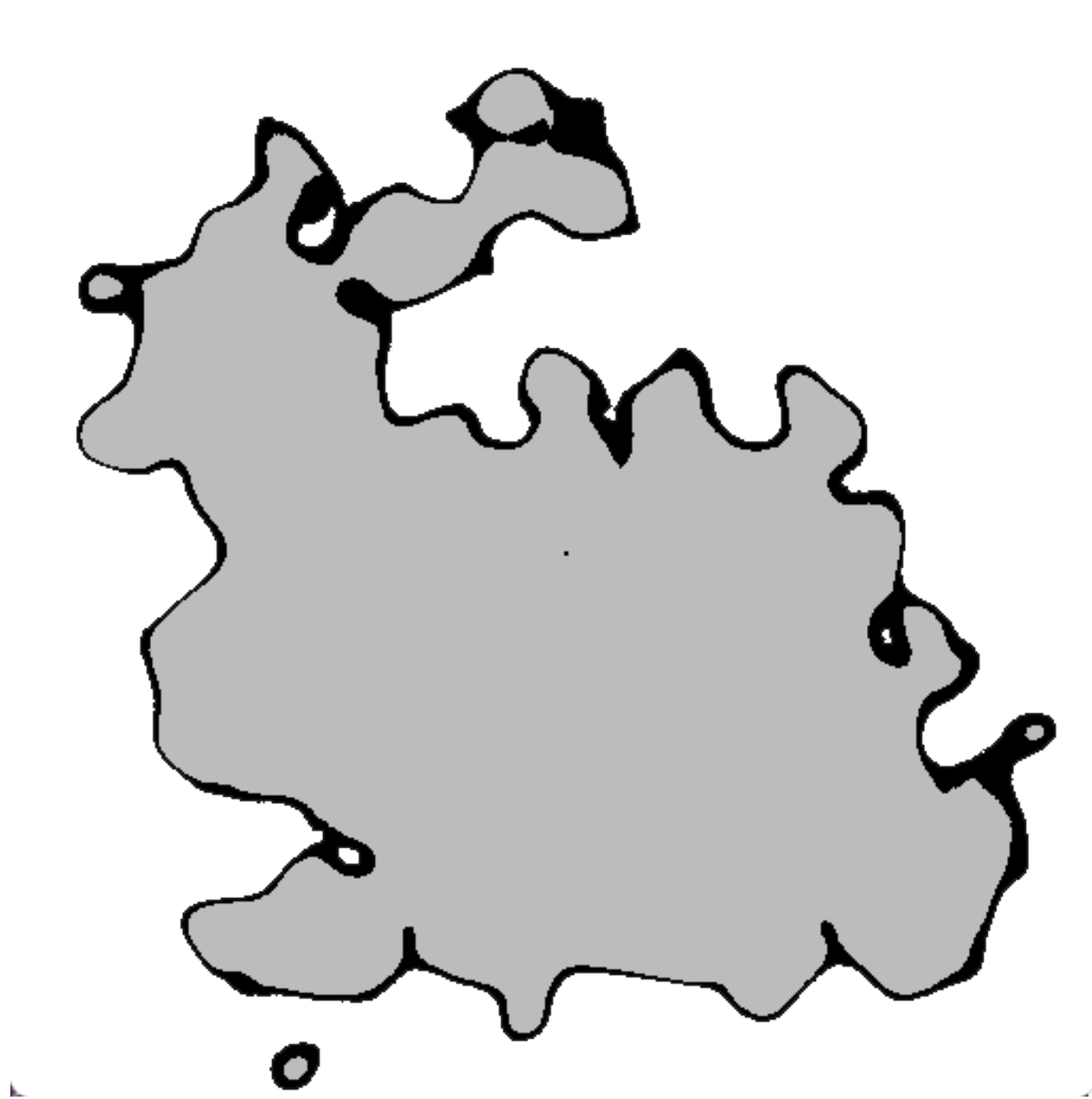


Volume-like

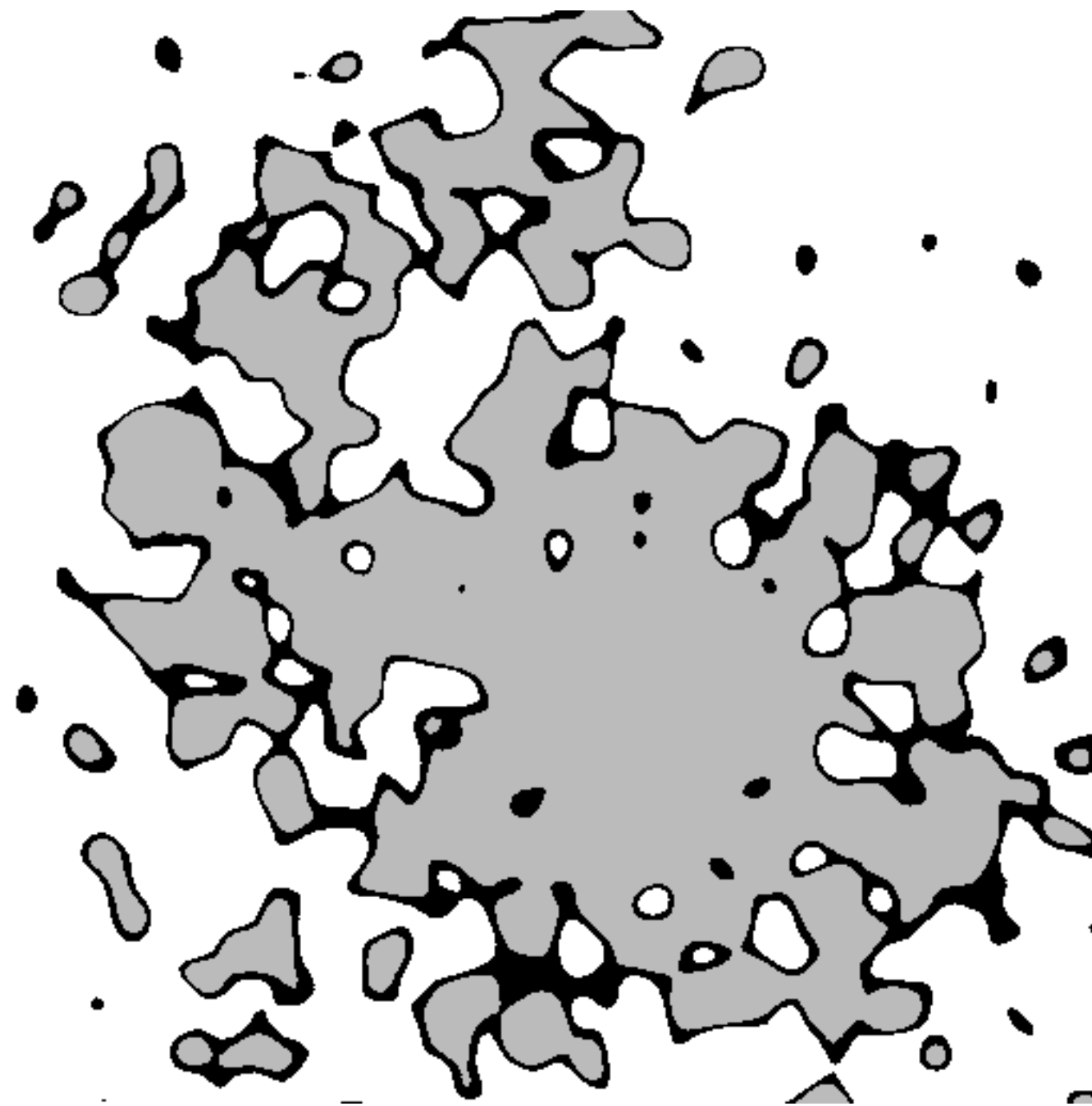
GPISeS with different parameters

Mean Covariance

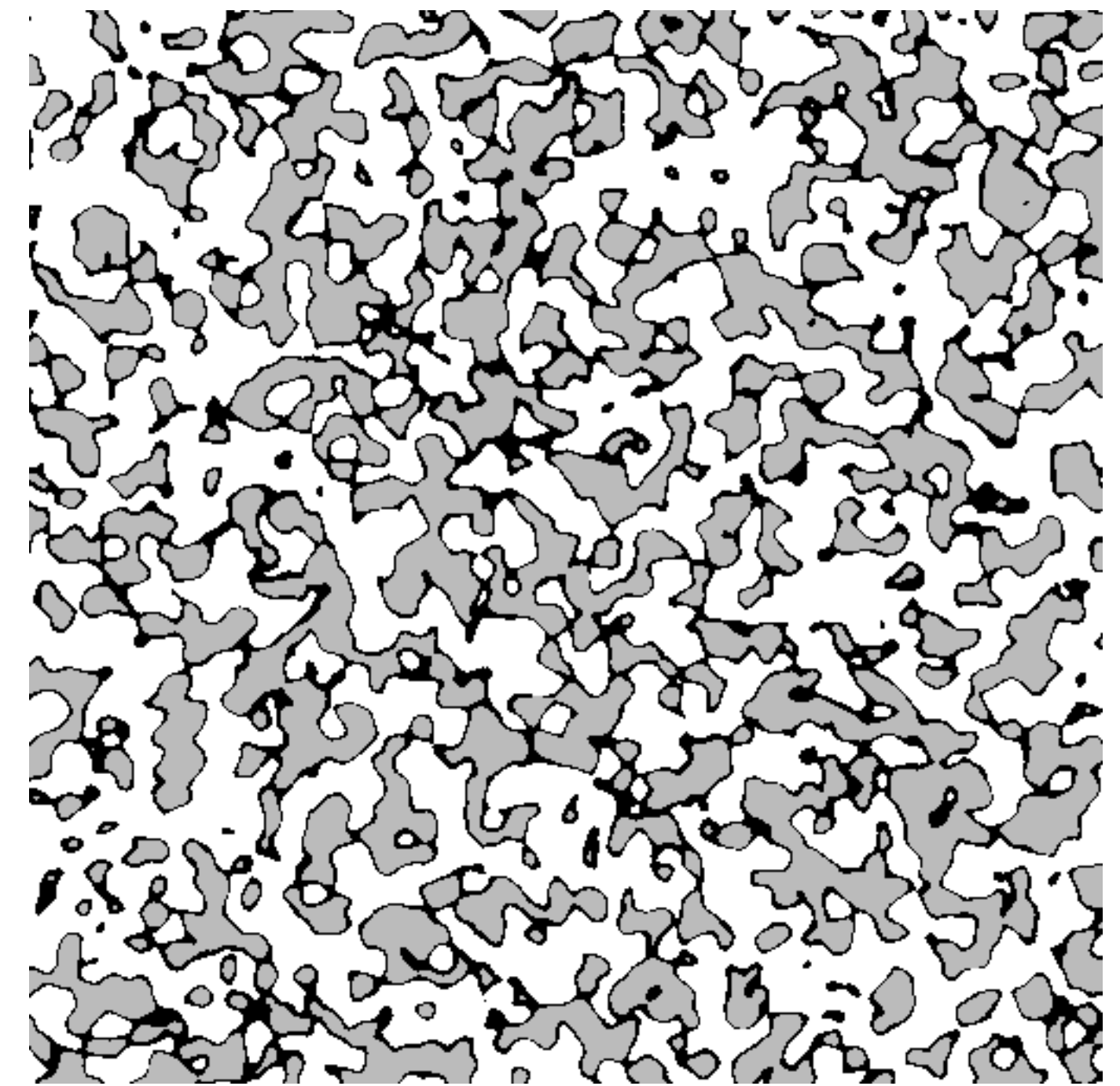
$$f(\mathbf{x}) \sim \mathcal{N}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}'))$$



Surface-like

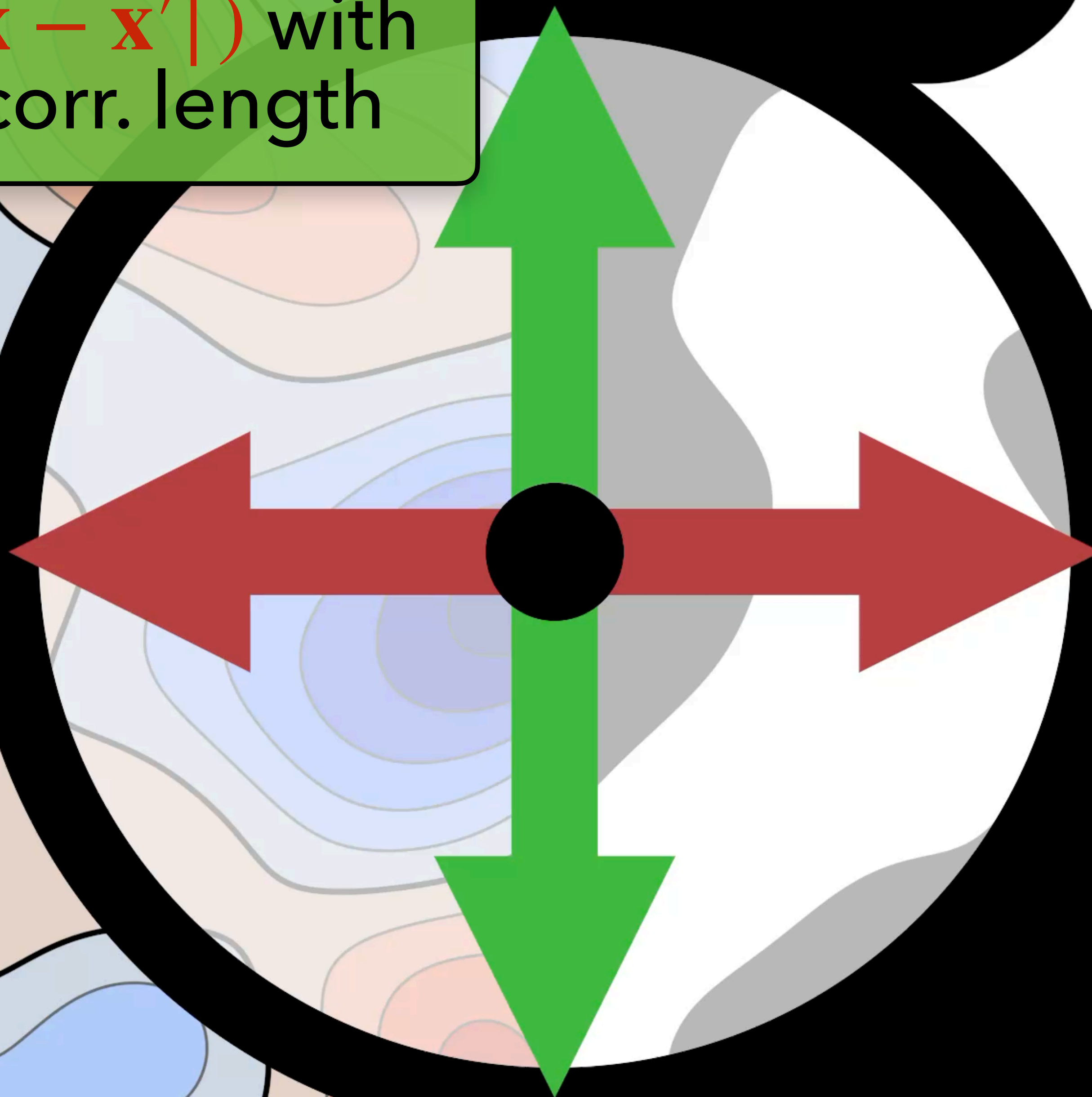


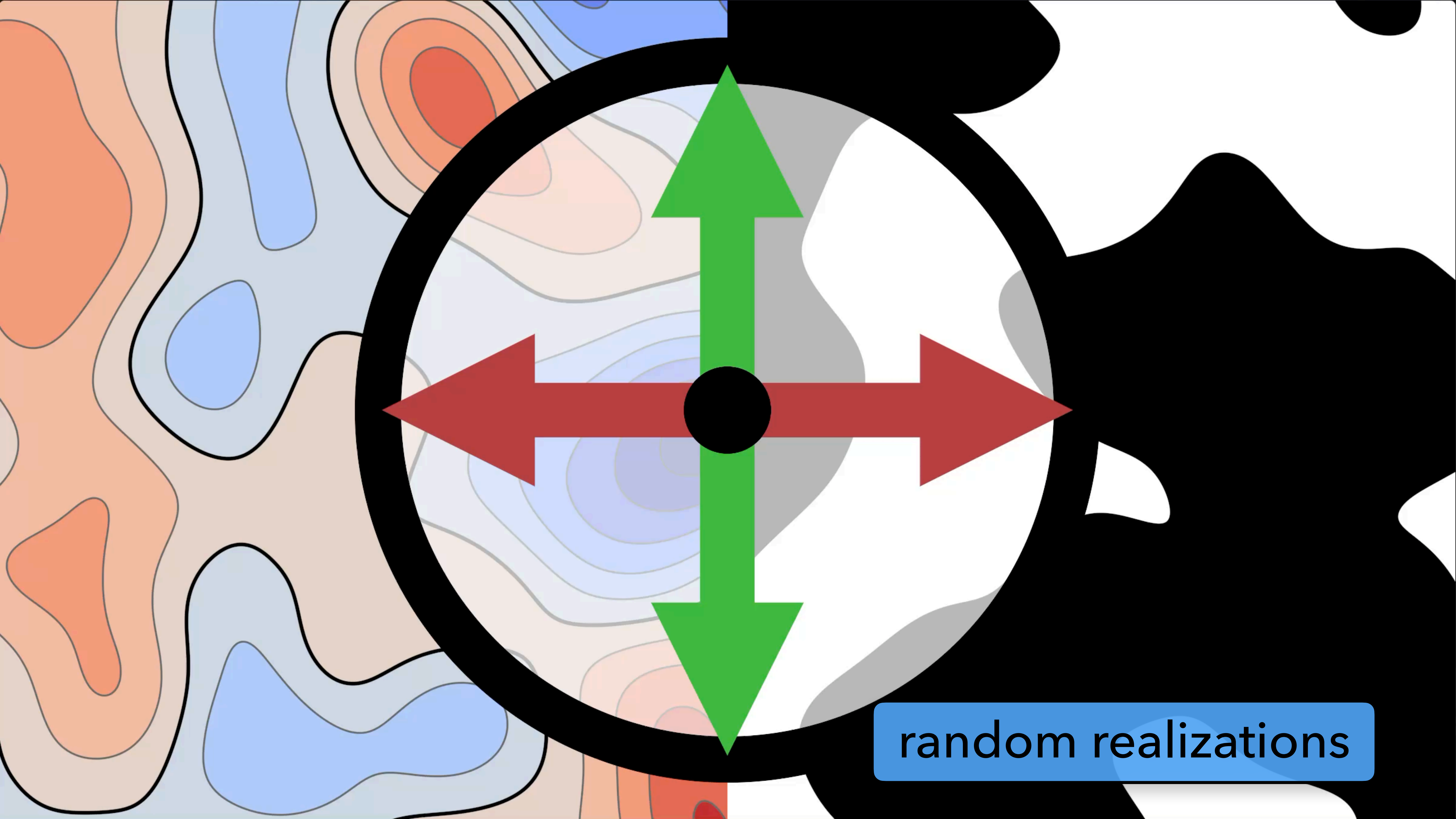
In-between



Volume-like

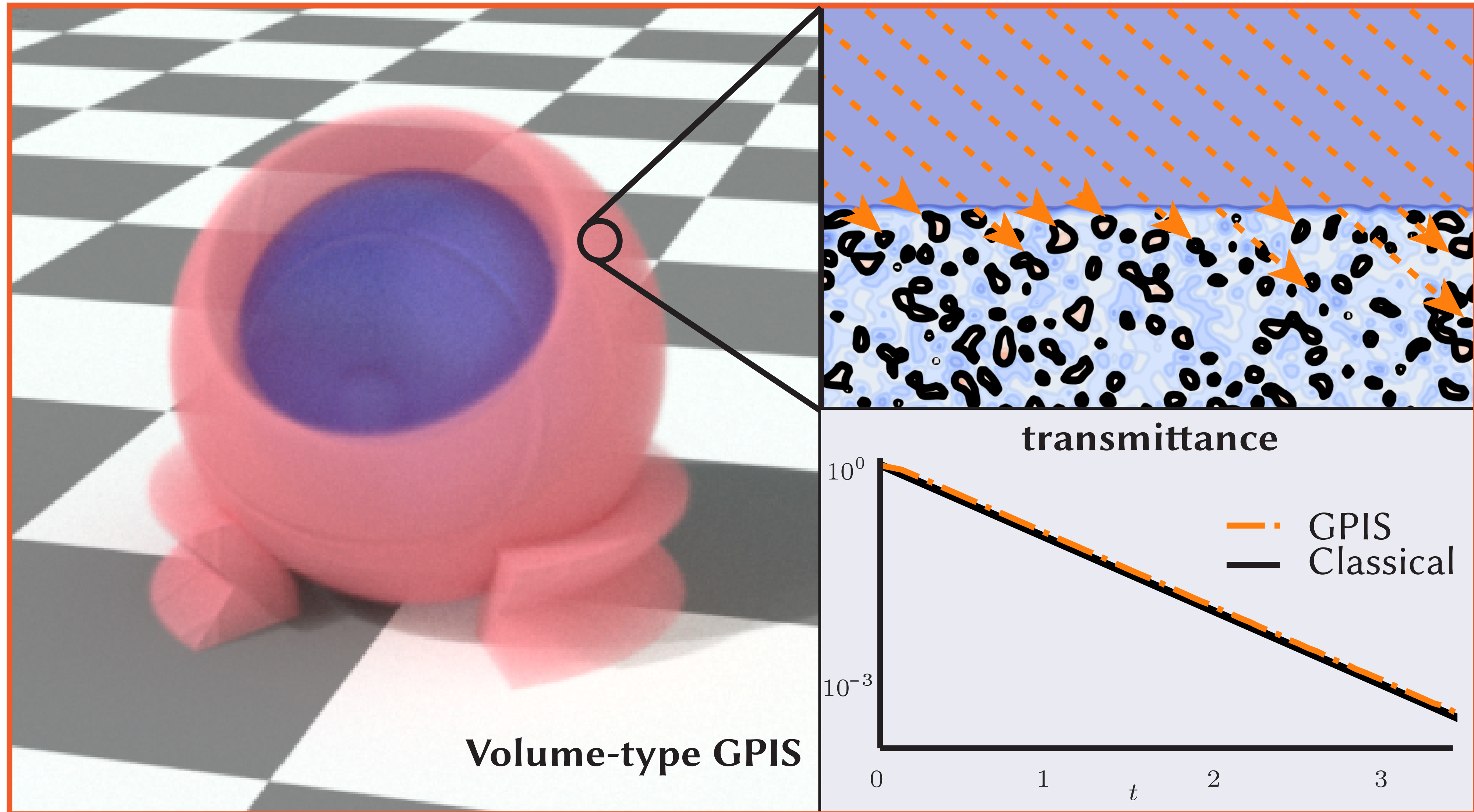
isotropic $\kappa(|\mathbf{x} - \mathbf{x}'|)$ with
decreasing corr. length



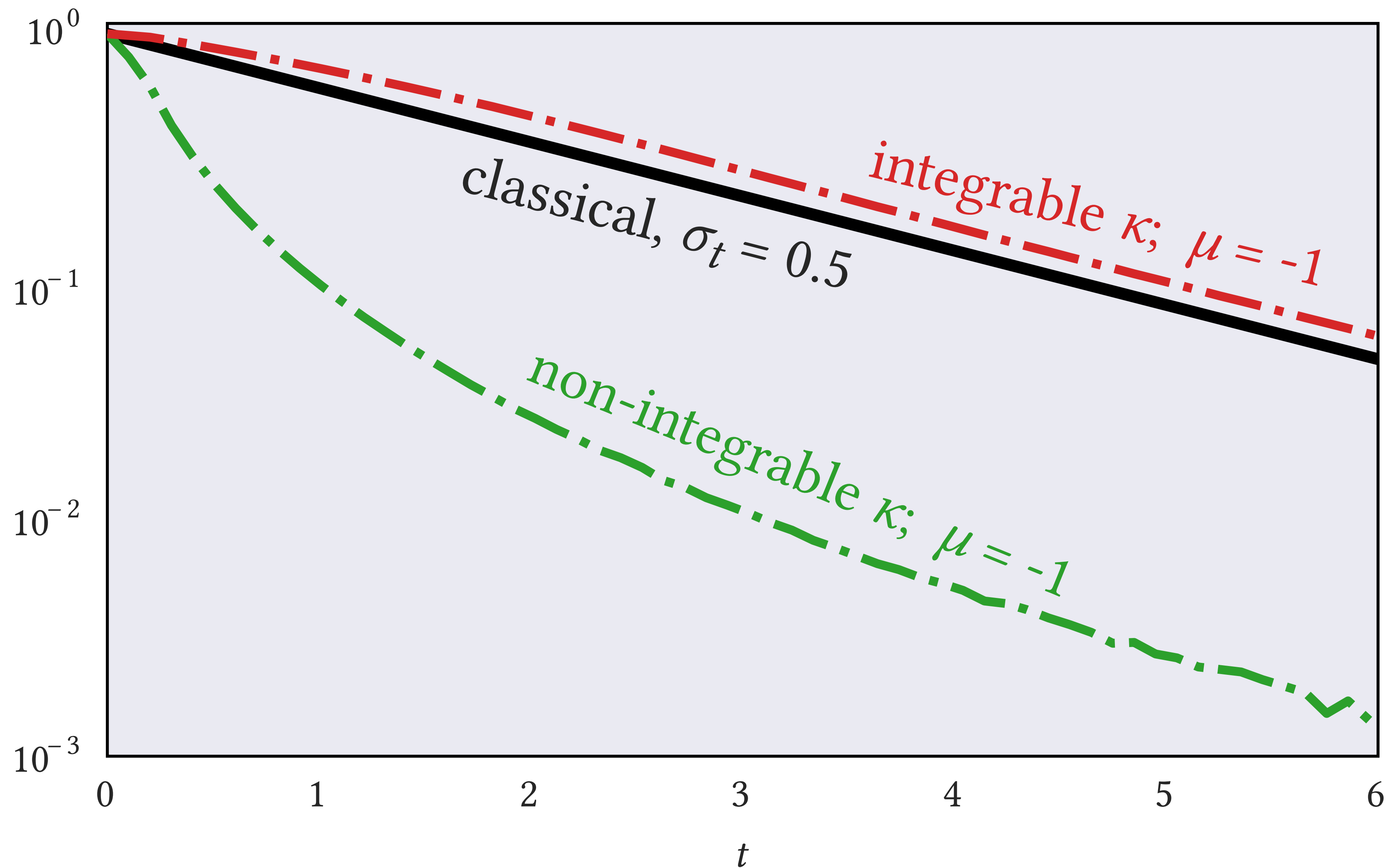


random realizations

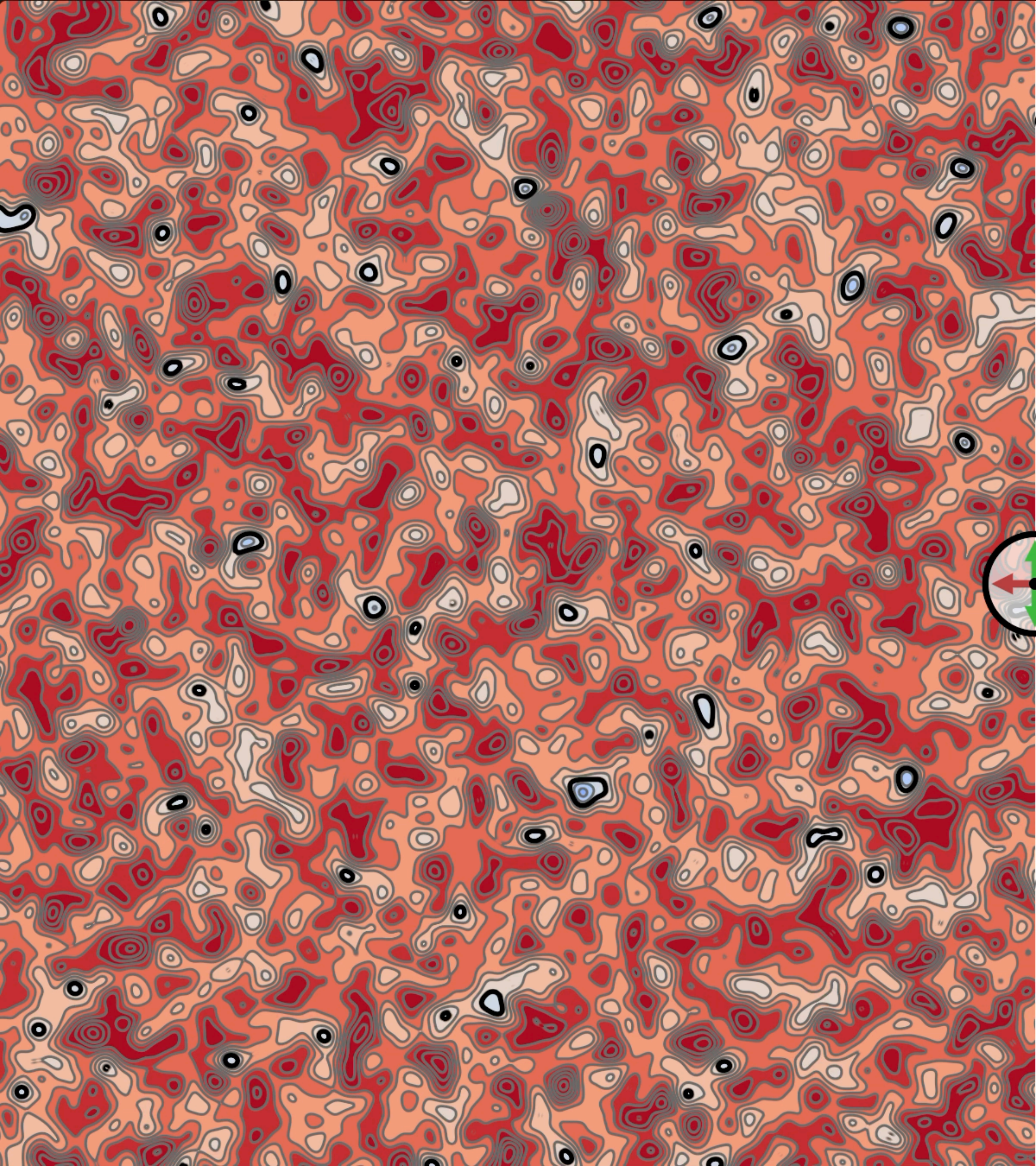
GPISeS can approximate classical media



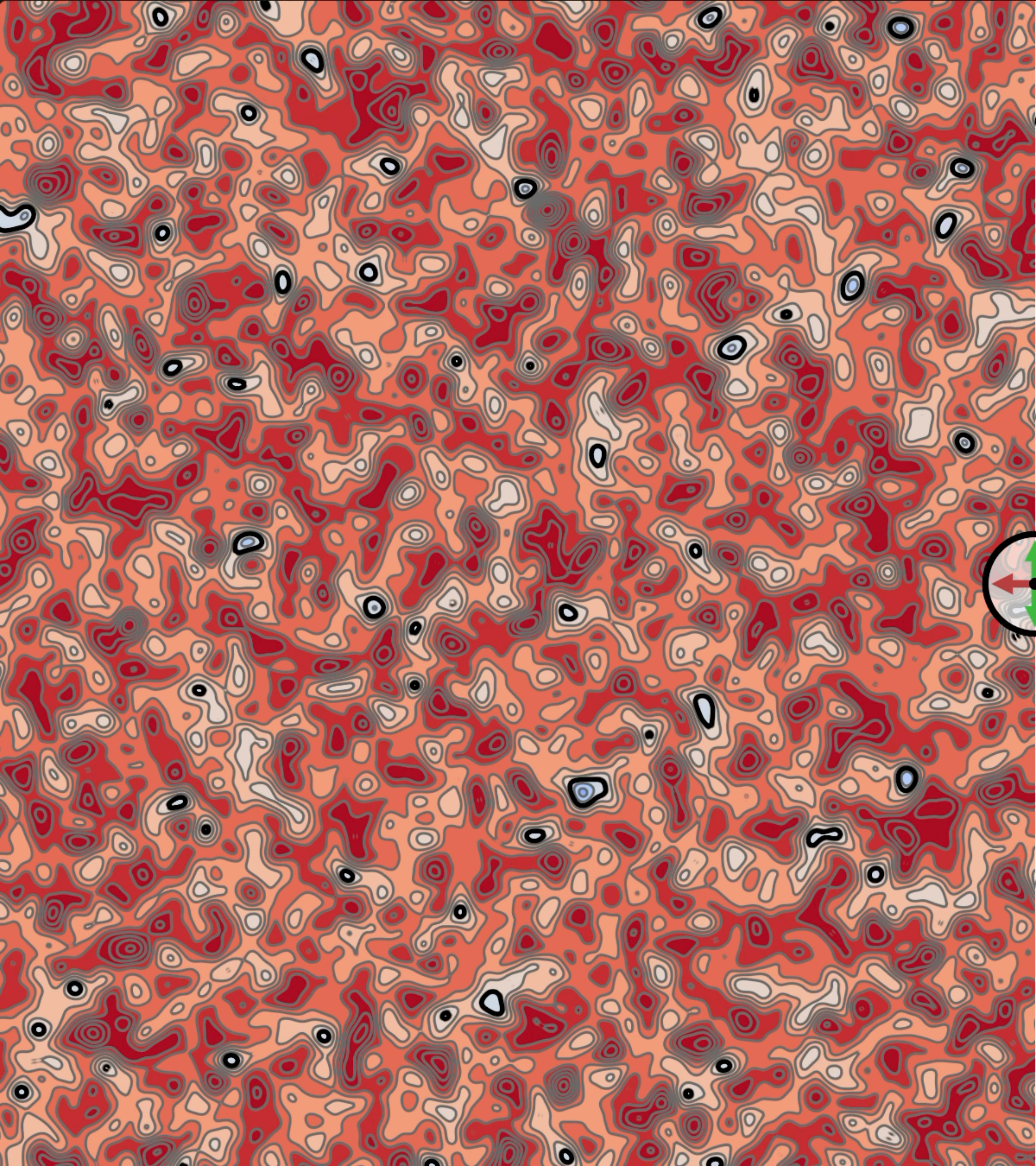
and include non-classical transport



non-stationary, linearly
increasing $\mu(y)$

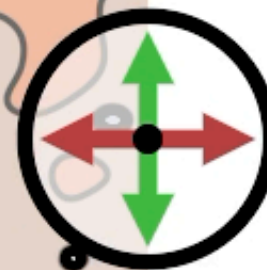


non-stationary, linearly
increasing $\mu(y)$

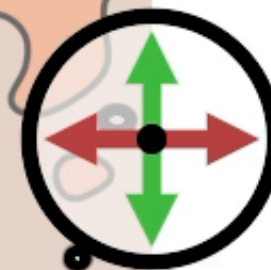


random porous media

stretching correlation
length along y

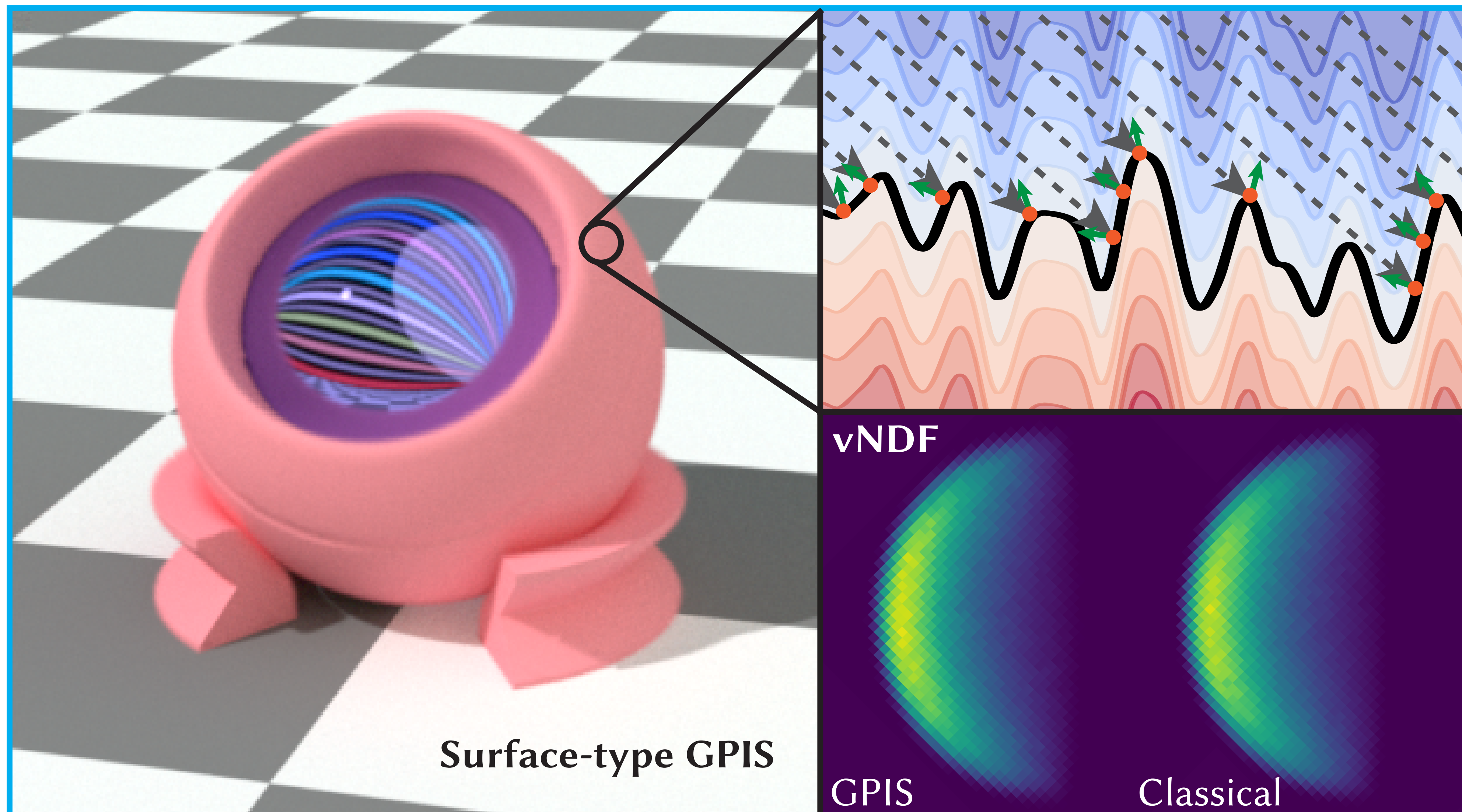


stretching correlation
length along y



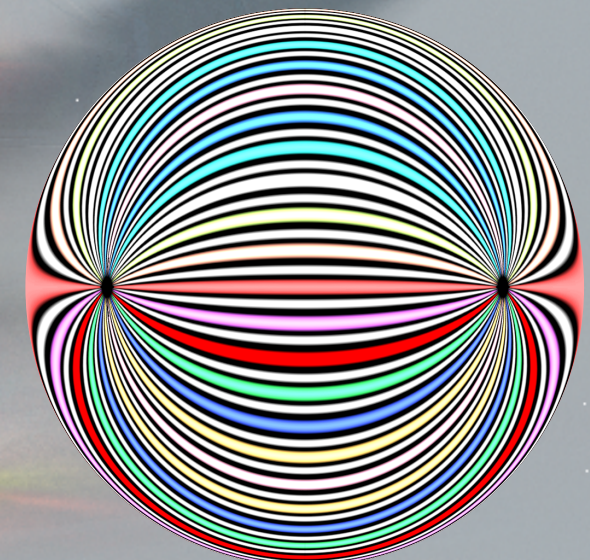
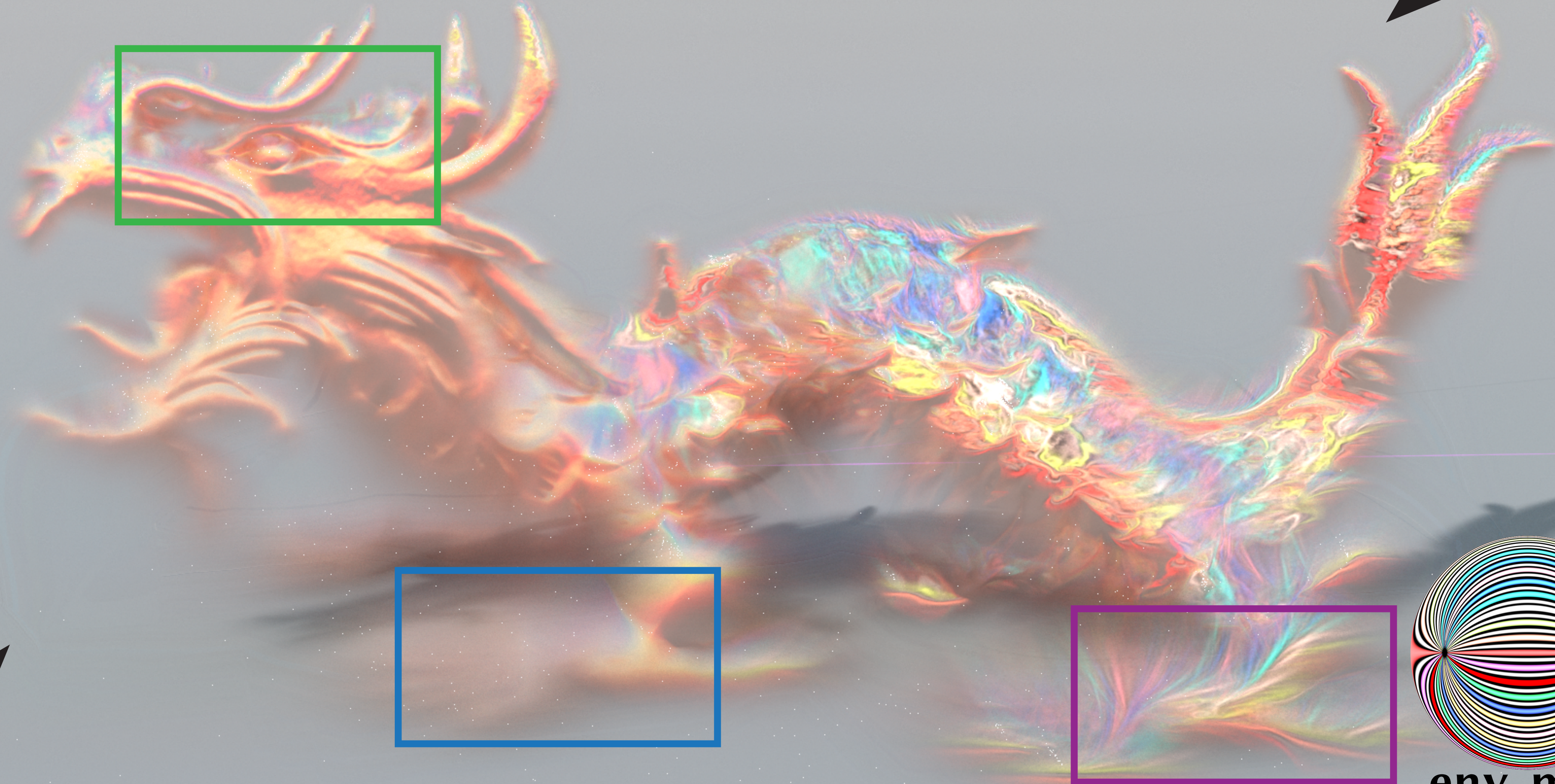
random heightfields

GPISeS include microfacet surfaces



increasing correlation

increasing variance



env. map

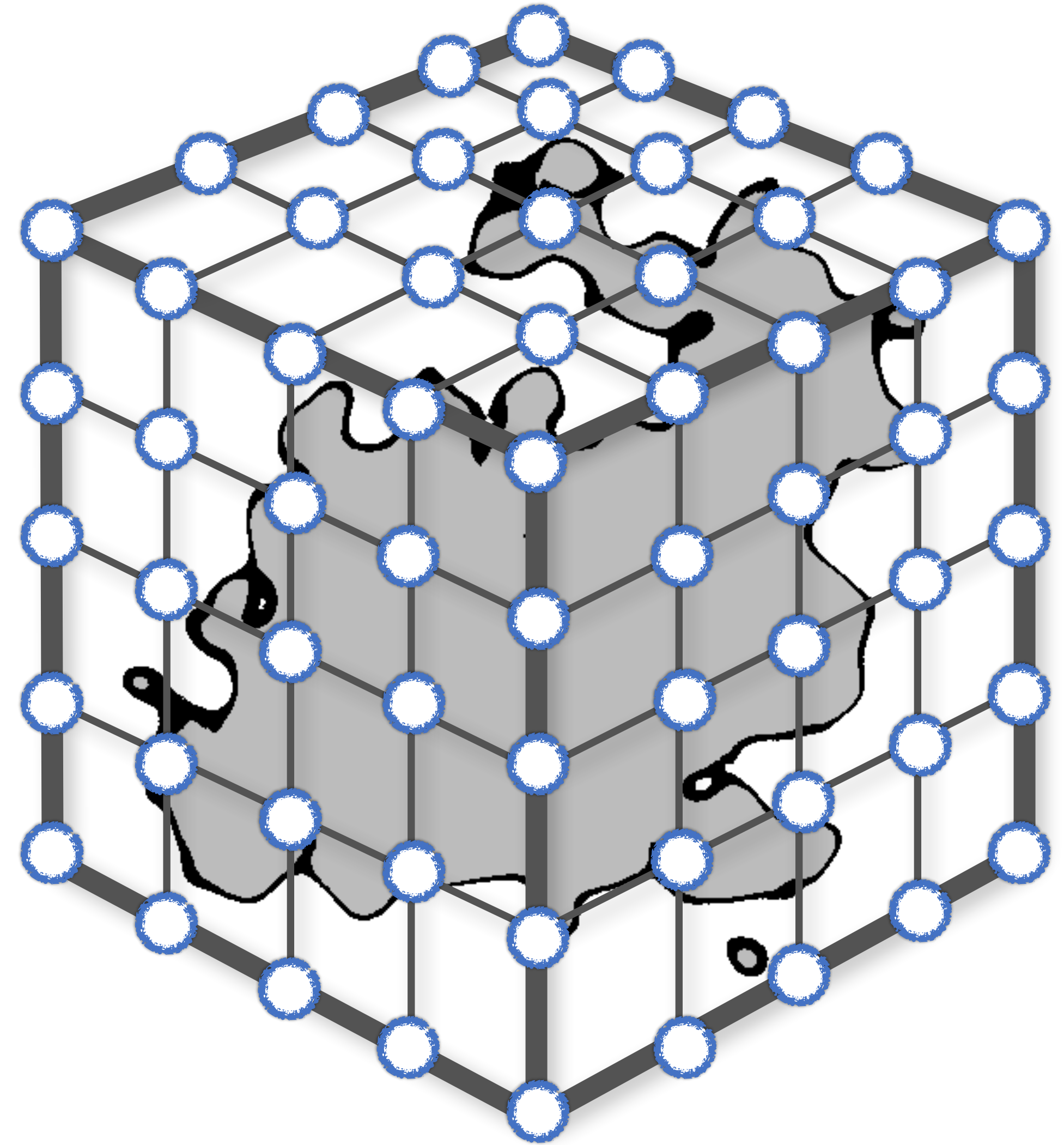




Rendering GPISes

How can we render GPISes?

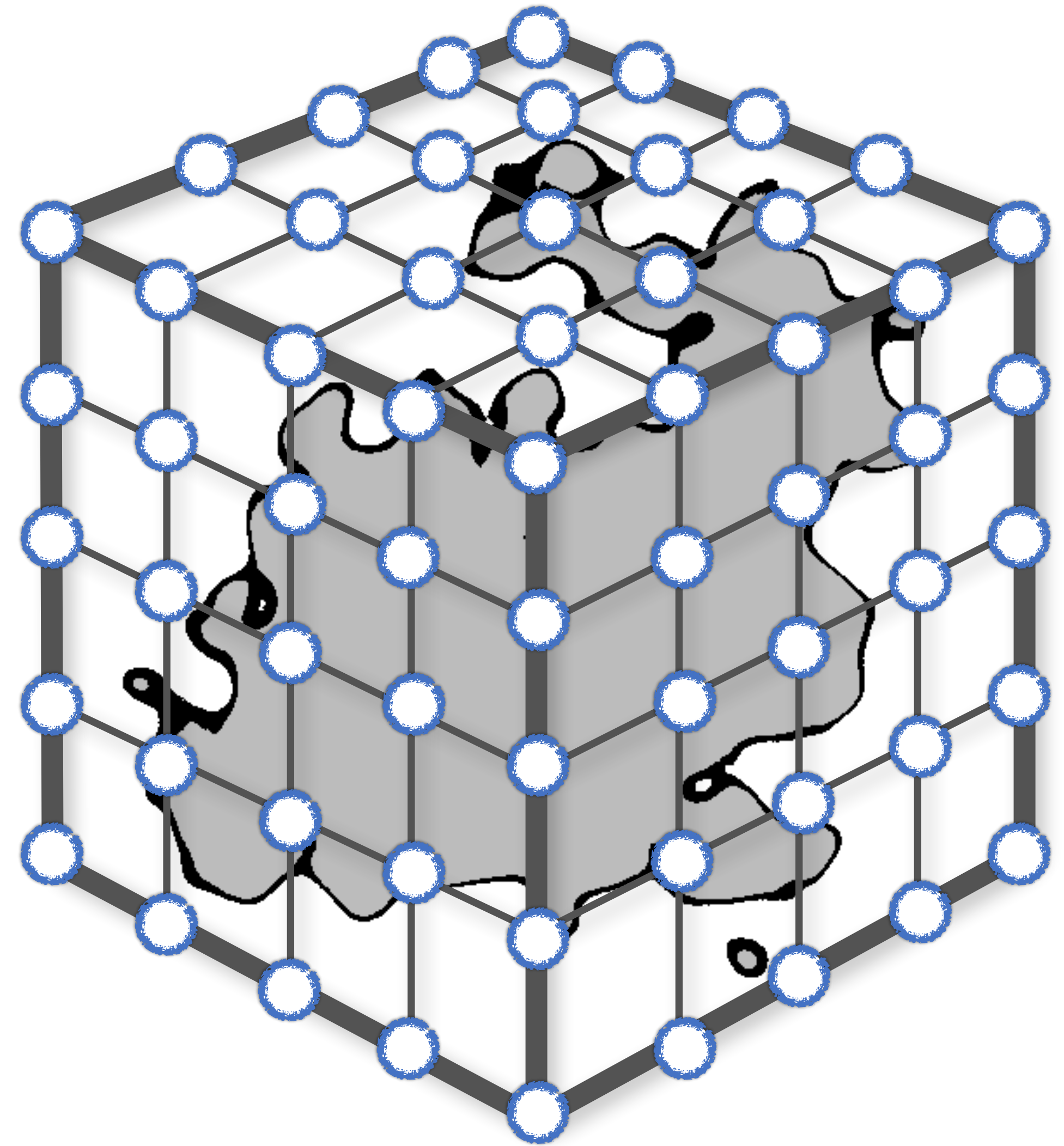
Correlated “function-space”
sampling of $f(\mathbf{x}) \sim \mathcal{N}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}'))$
at N locations $\mathbf{x}_1 \dots \mathbf{x}_N$ is **expensive**:
 $\mathcal{O}(N^3)$



How can we render GPISes?

Correlated “function-space”
sampling of $f(\mathbf{x}) \sim \mathcal{N}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}'))$
at N locations $\mathbf{x}_1 \dots \mathbf{x}_N$ is **expensive**:
 $\mathcal{O}(N^3)$

Also: hard to design valid (PSD)
non-stationary covariance function κ

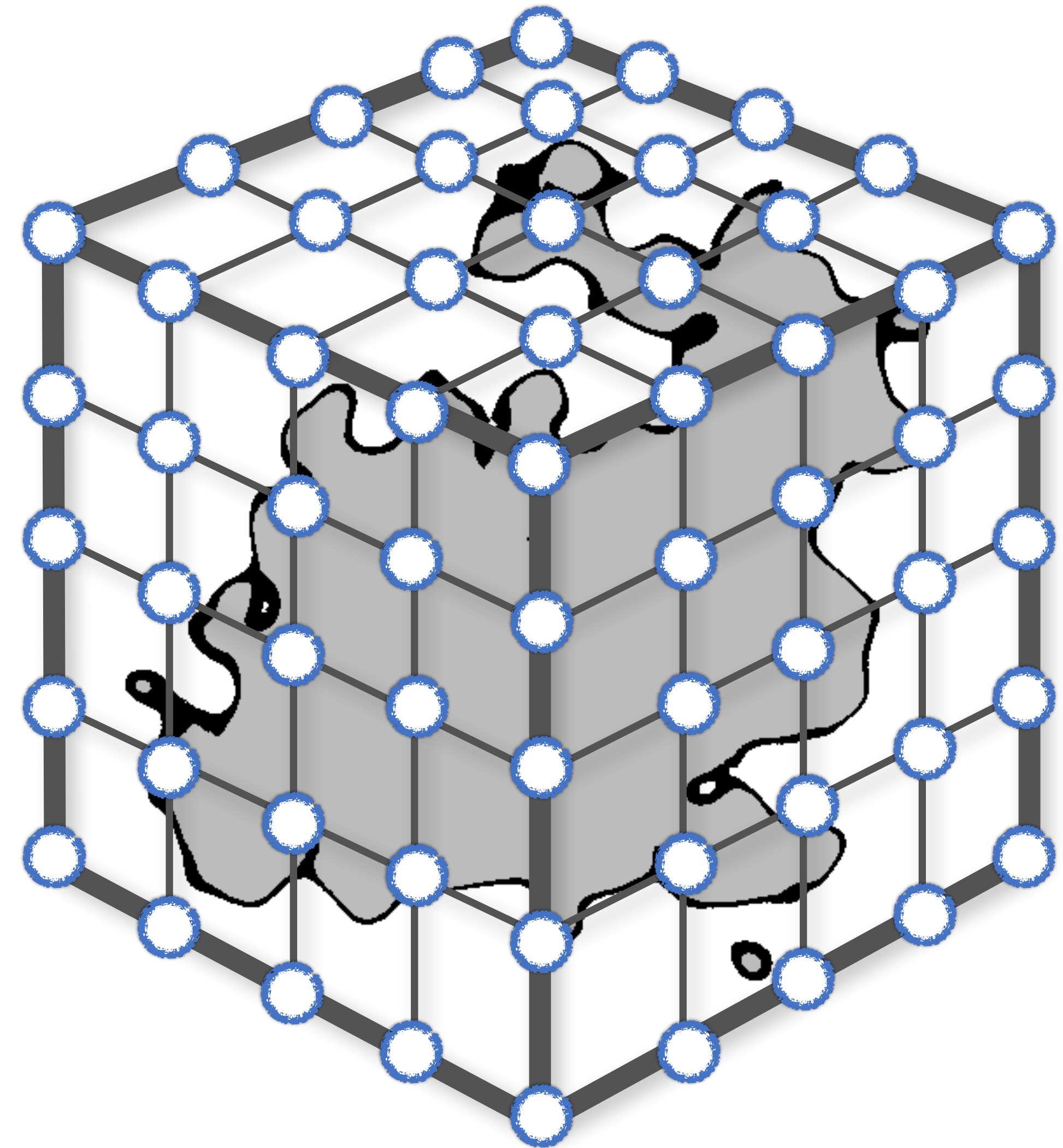


How can we render GPISes?

Correlated “function-space”
sampling of $f(\mathbf{x}) \sim \mathcal{N}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}'))$
at N locations $\mathbf{x}_1 \dots \mathbf{x}_N$ is **expensive**:
 $\mathcal{O}(N^3)$

Also: hard to design valid (PSD)
non-stationary covariance function κ

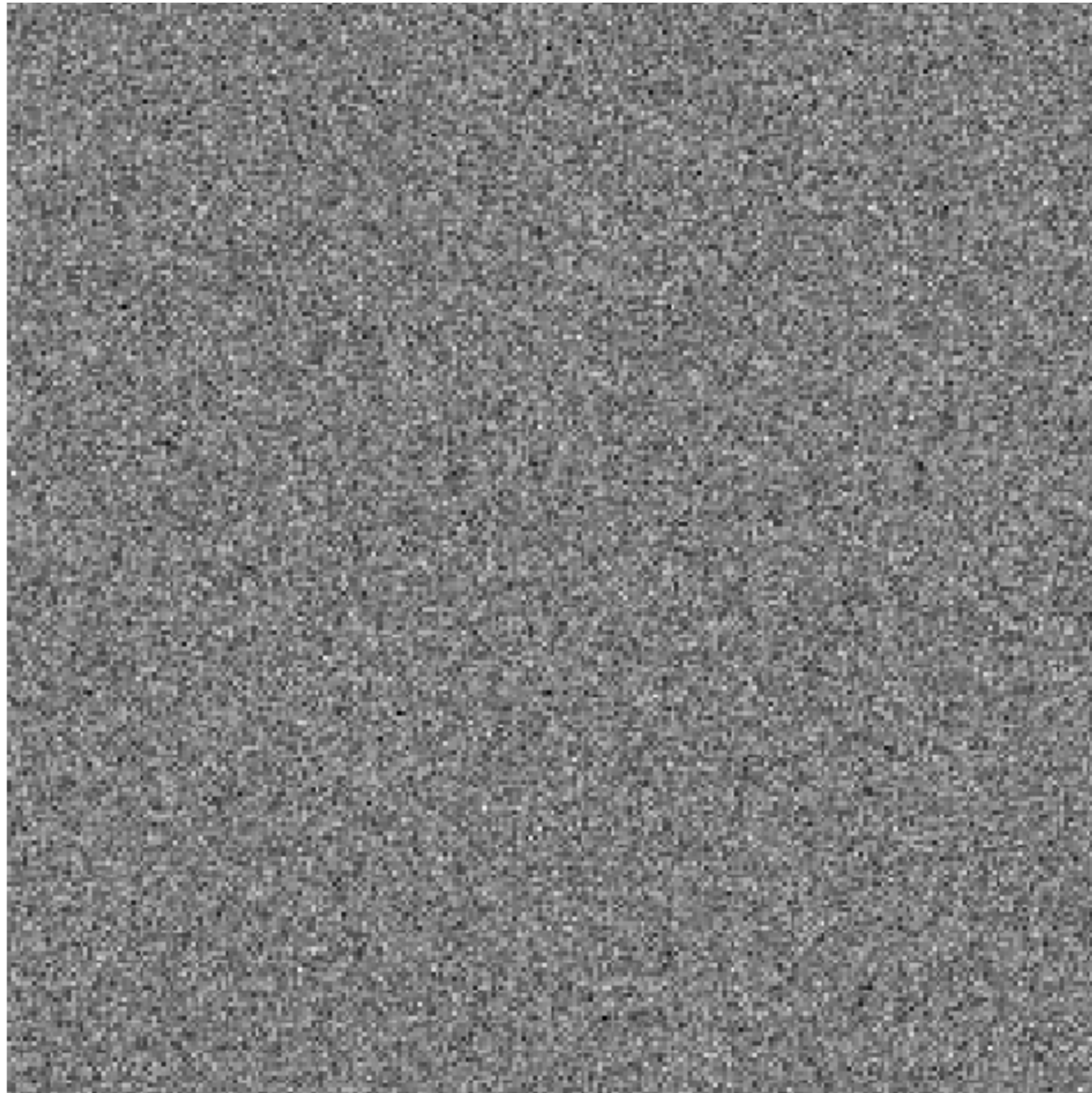
And that’s just for one realization!



Convolution view of Gaussian processes

Convolution view of Gaussian processes

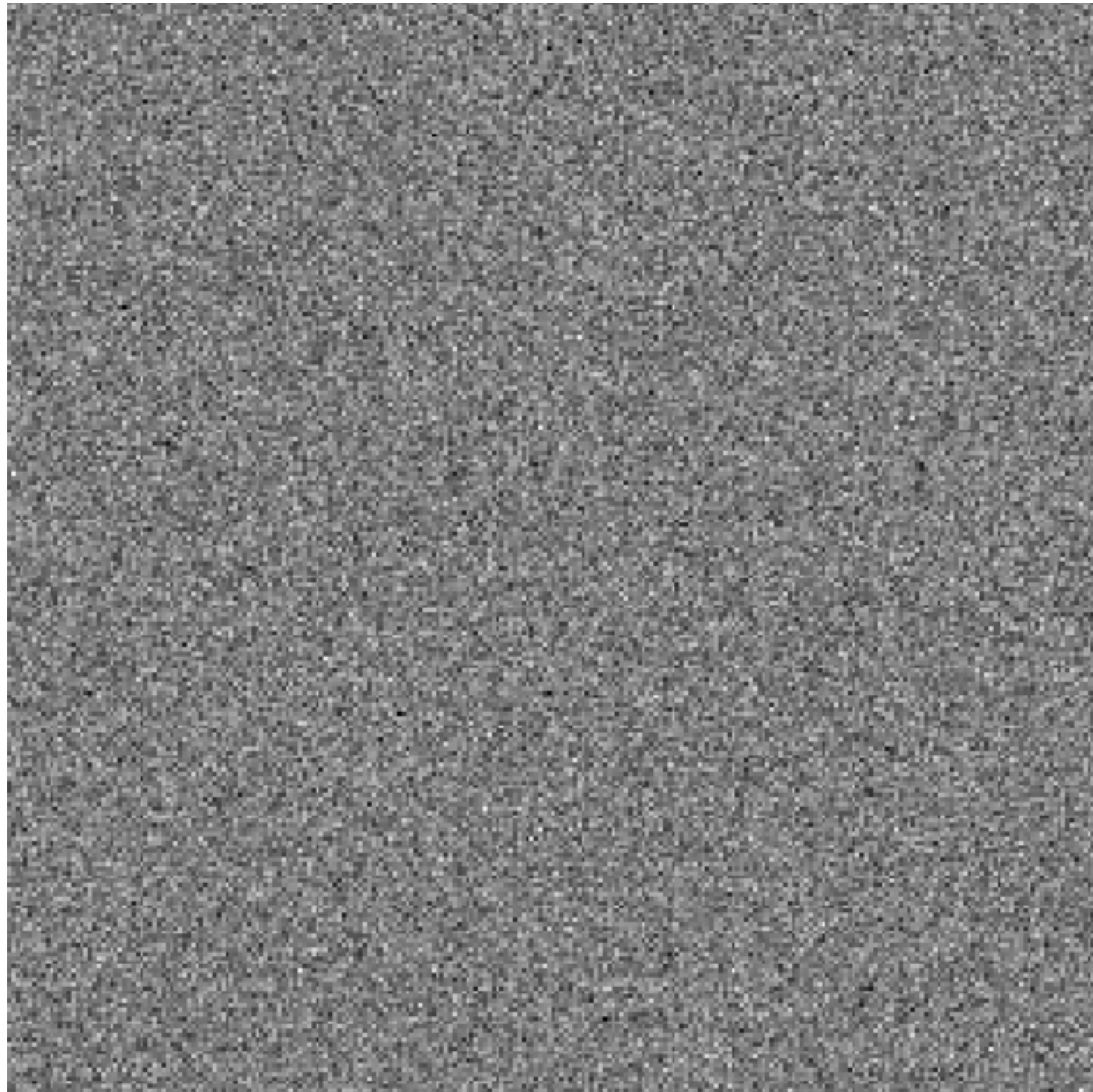
White noise



$$\mathcal{N}(0, \mathbb{I})$$

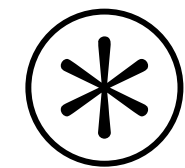
Convolution view of Gaussian processes

White noise



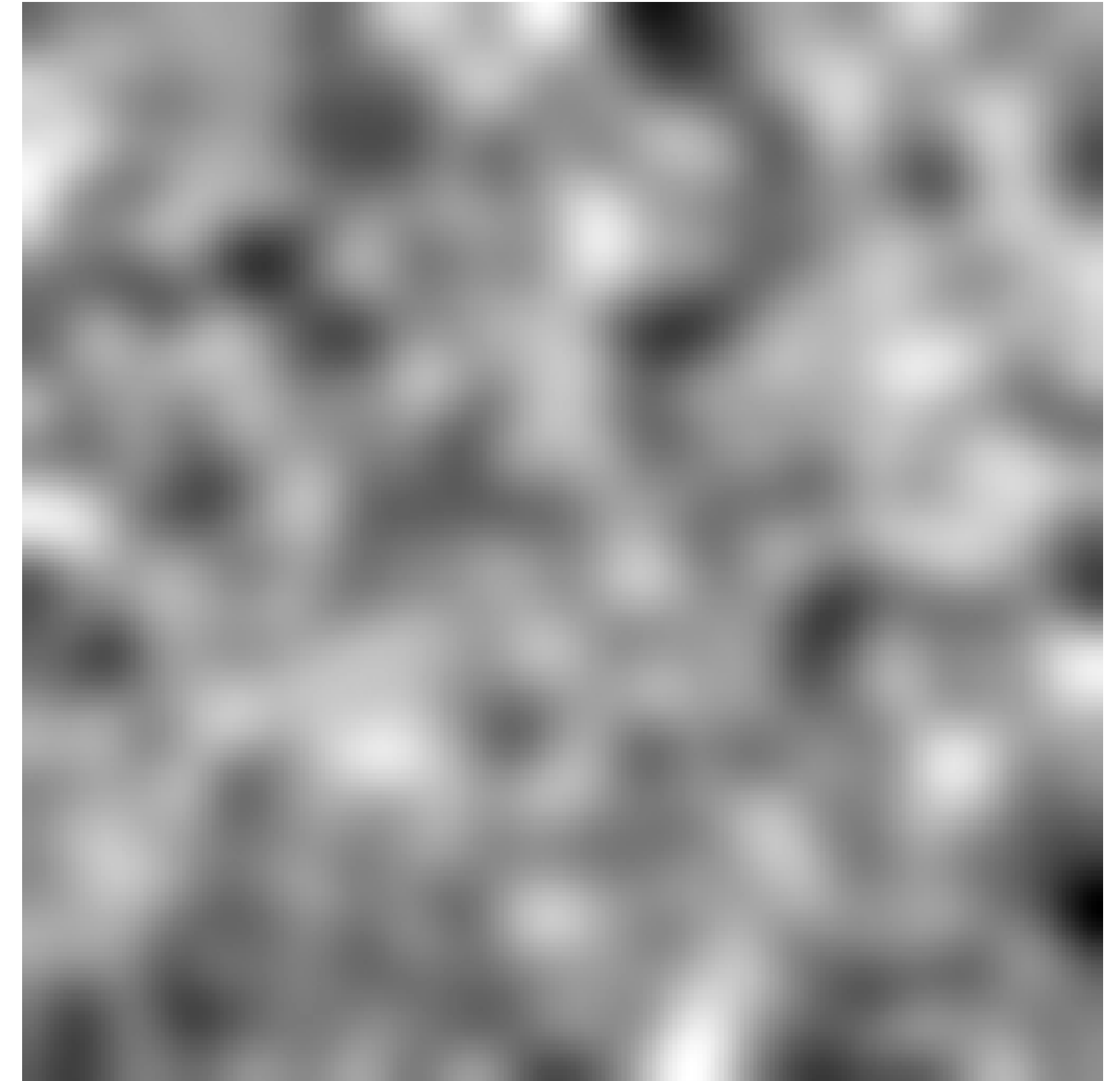
$$\mathcal{N}(0, \mathbb{I})$$

Kernel h



=

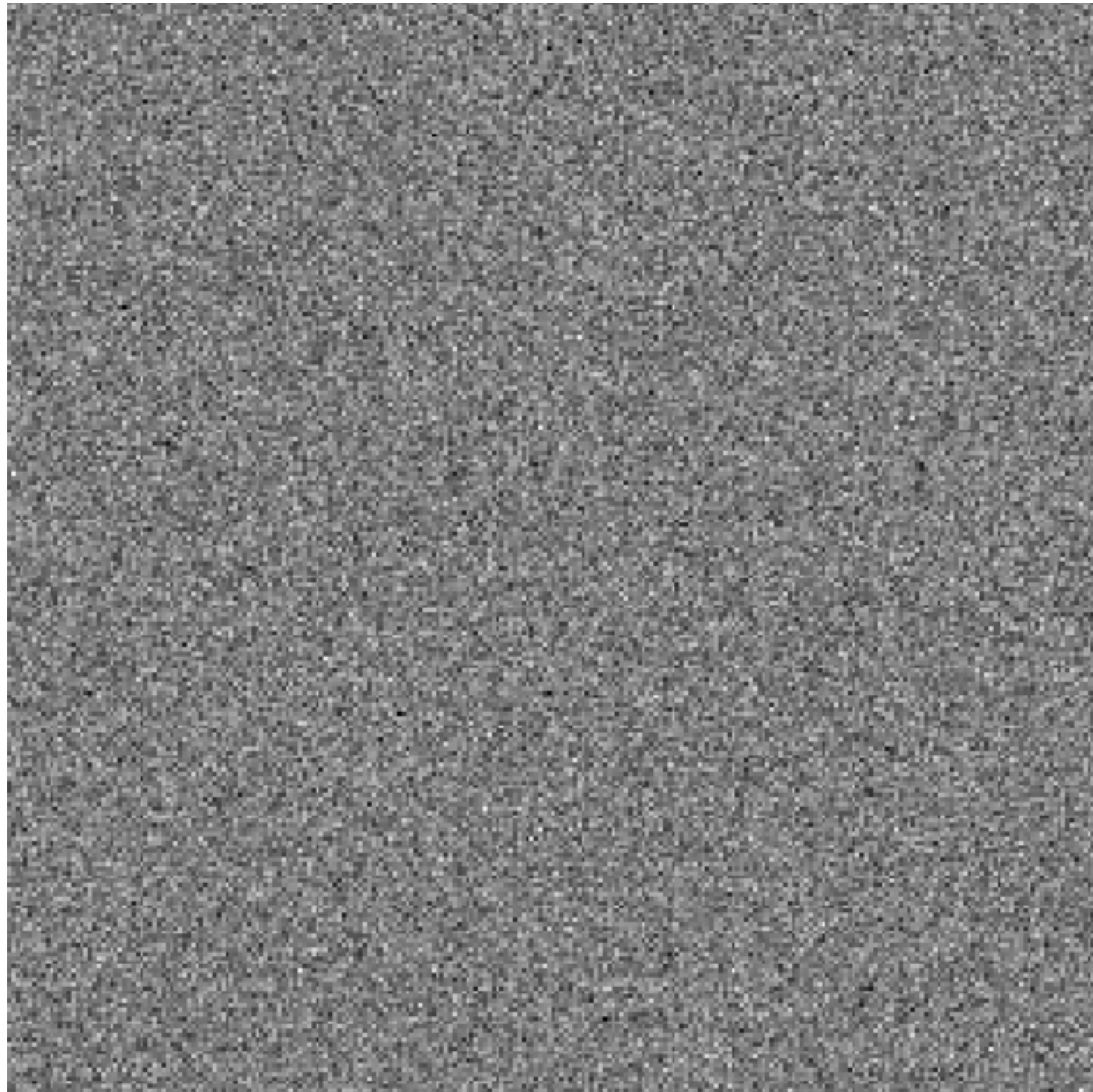
Convolved process



$$\mathcal{N}(0, h \circledast h(x, x'))$$

Convolution view of Gaussian processes

White noise

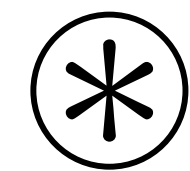


$$\mathcal{N}(0, \mathbb{I})$$

Convolved process



Kernel h



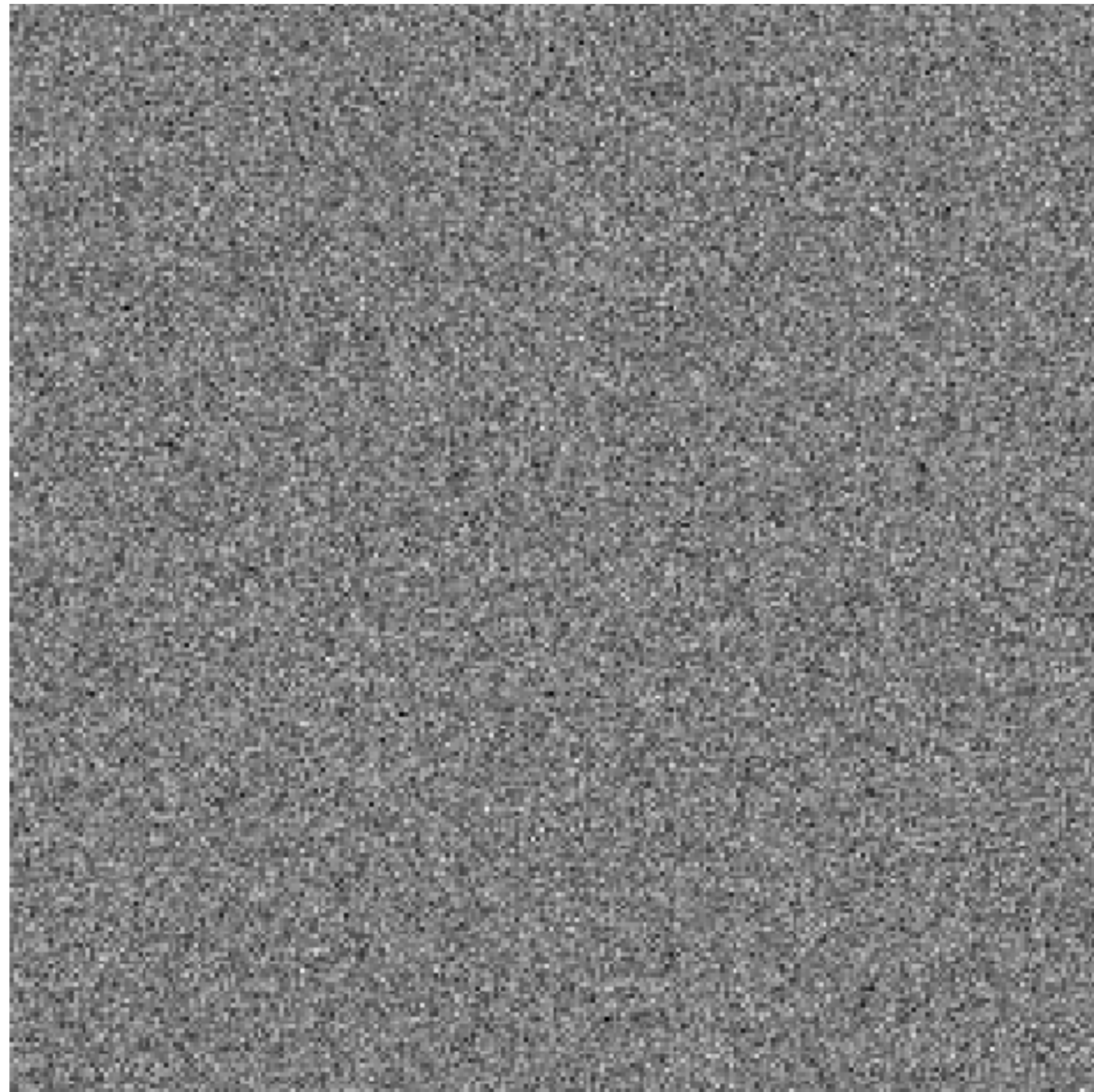
=

$$\kappa = h \circledast h$$

$$\mathcal{N}(0, \underbrace{h \circledast h}_{\kappa}(x, x'))$$

Convolution view of Gaussian processes

White noise

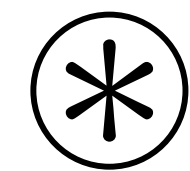


$$\mathcal{N}(0, \mathbb{I})$$

Convolved process



Kernel h



=

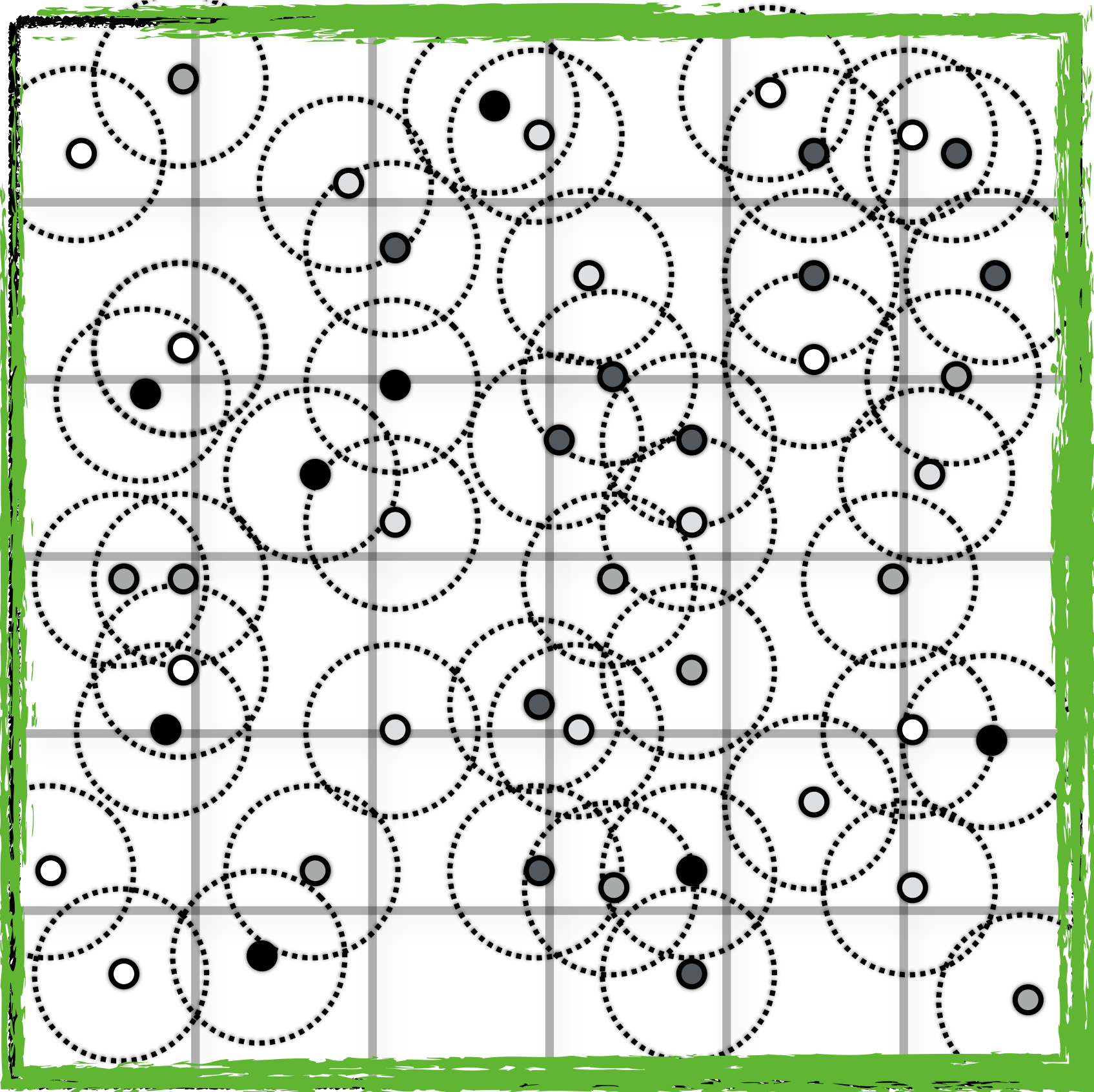
$$\kappa = h \circledast h$$

Positive semi-definite!

$$\mathcal{N}(0, \underbrace{h \circledast h}_{\kappa}(x, x'))$$

Procedural noise $\approx$ Gaussian process

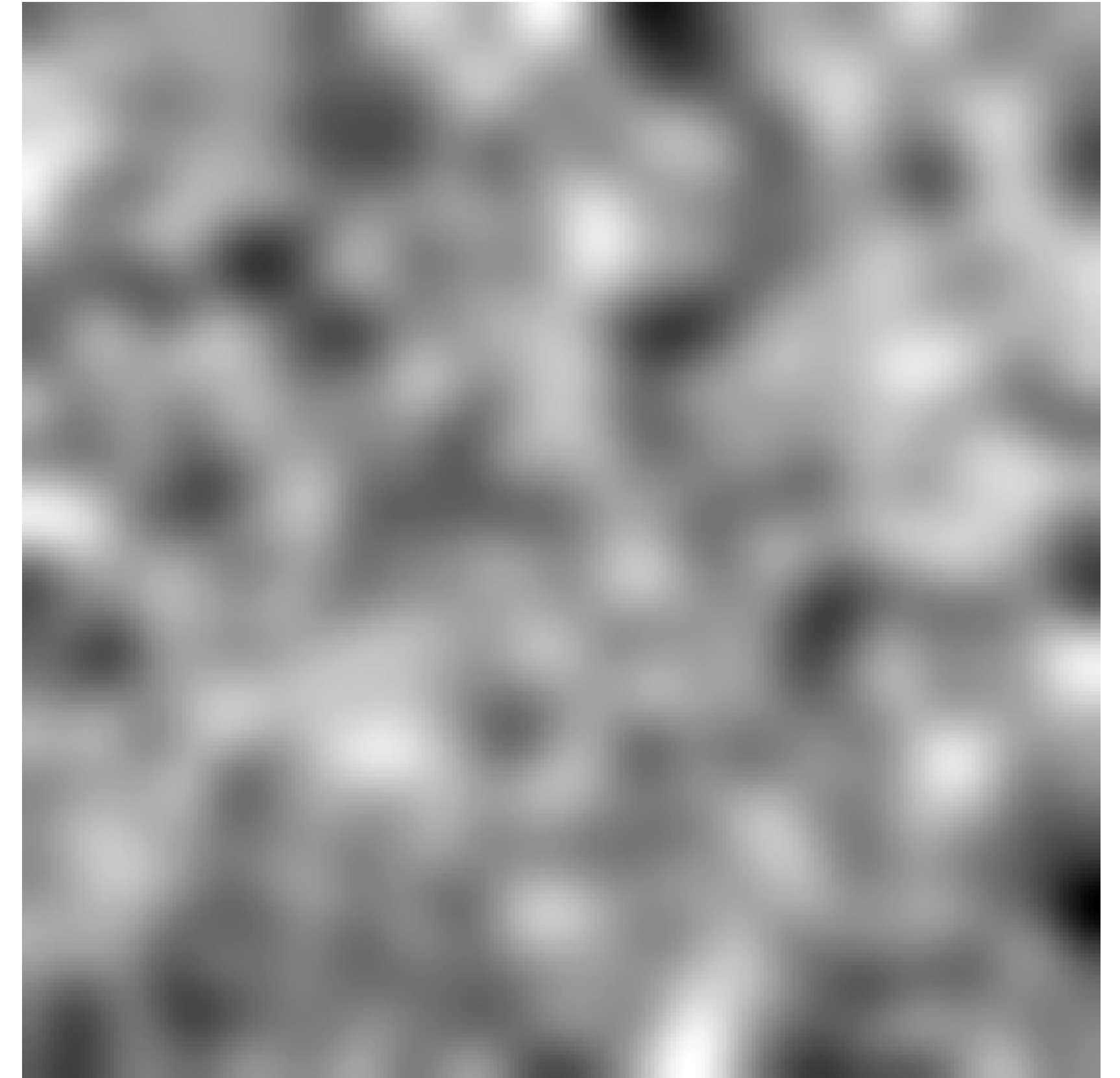
Sparse convolution noise



Lewis [1986, 1989]

$\approx$

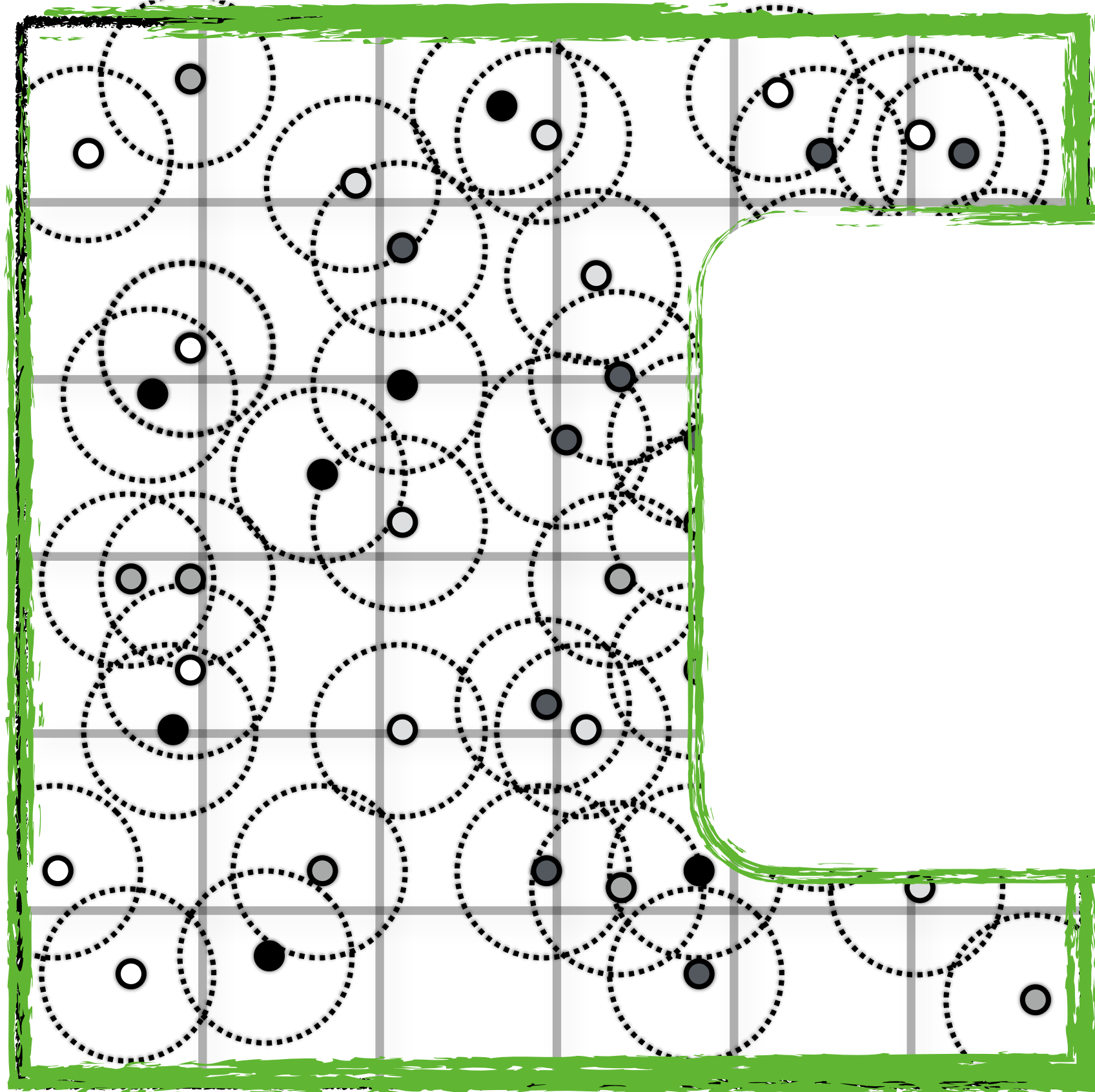
Convolved process



$$\mathcal{N}(0, h \circledast h(x, x'))$$

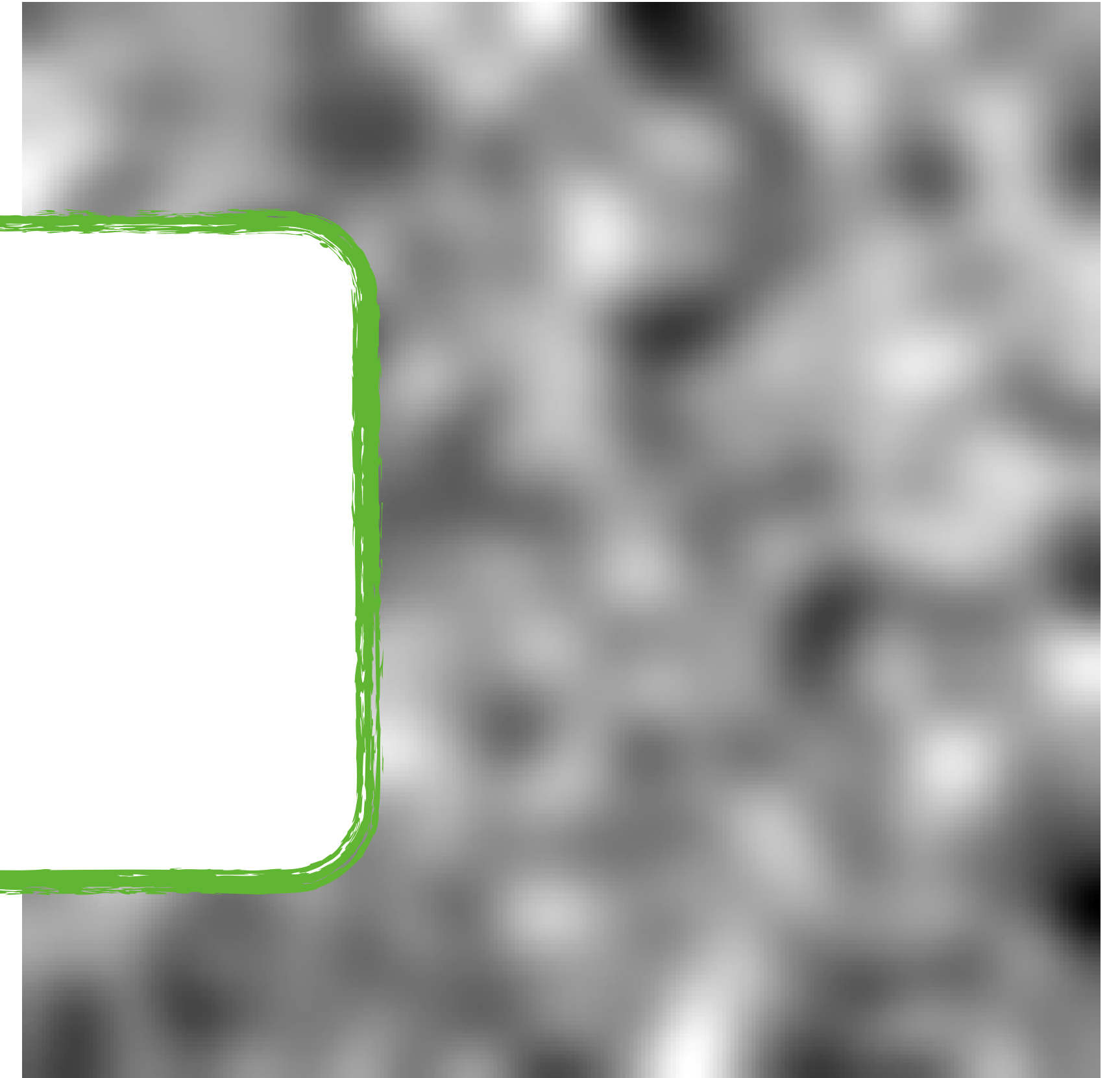
Procedural noise $\approx$ Gaussian process

Sparse convolution noise



Lewis [1986, 1989]

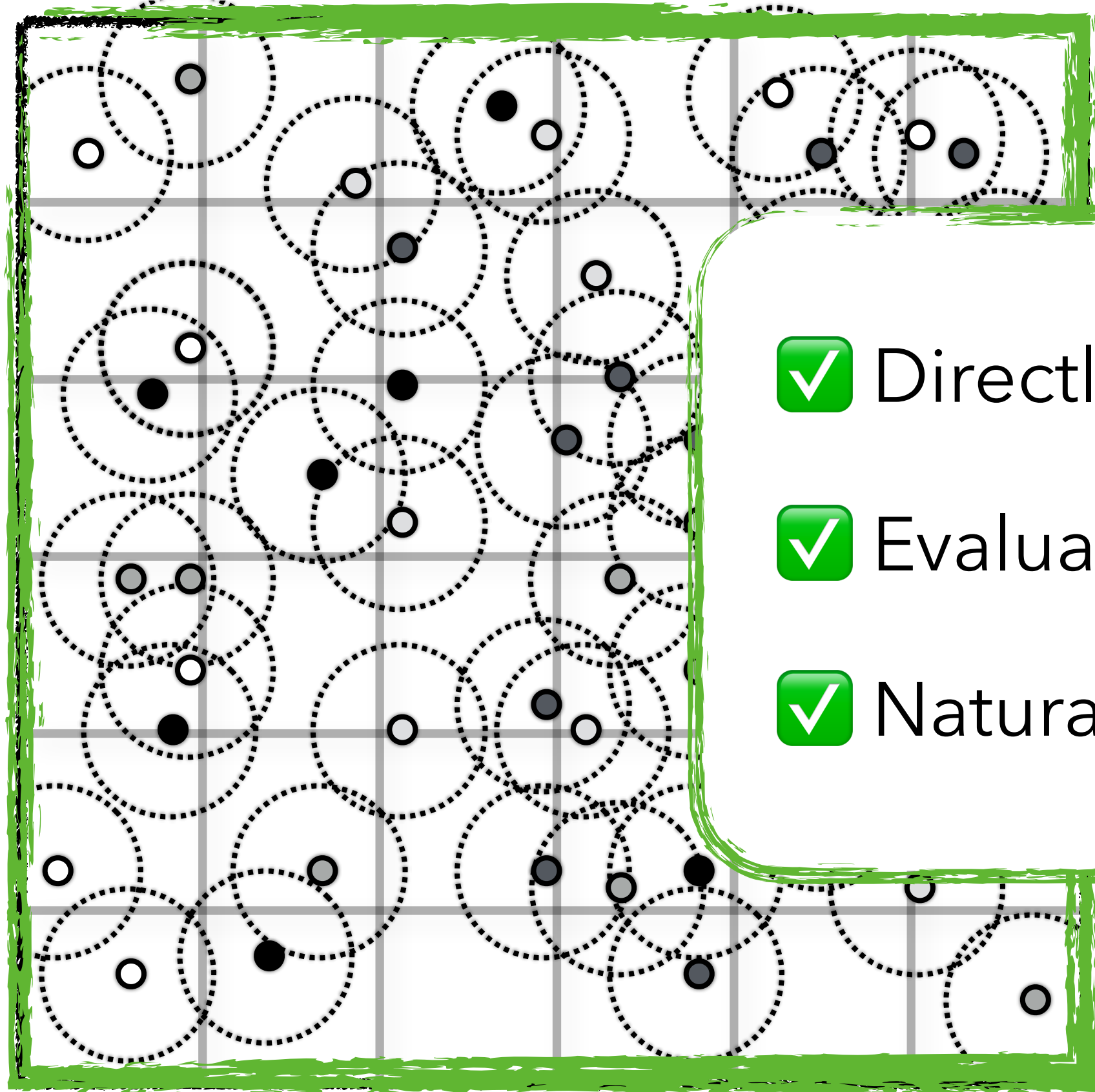
Convolved process



$$\mathcal{N}(0, h \circledast h(x, x'))$$

Procedural noise $\approx$ Gaussian process

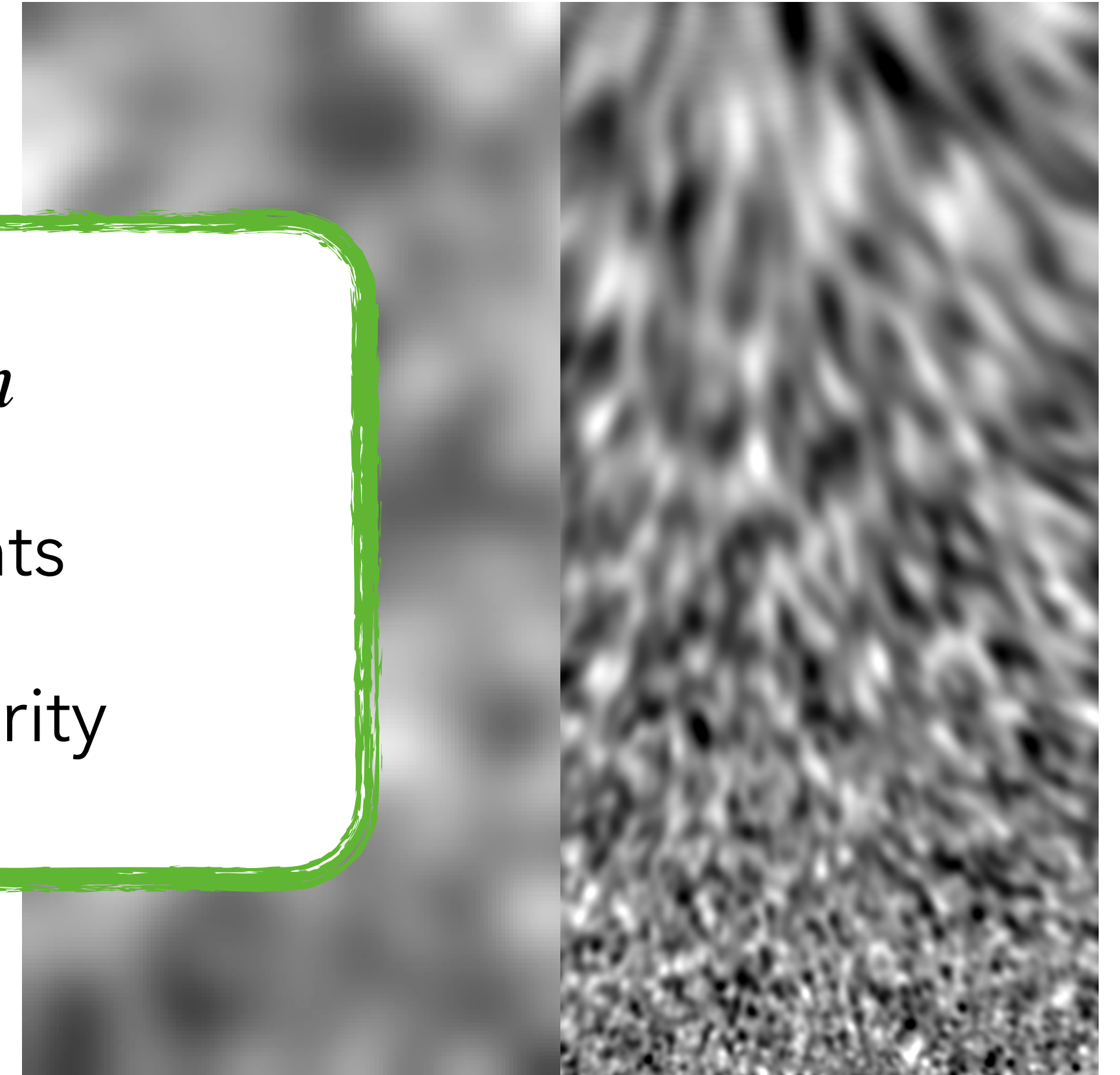
Sparse convolution noise



- ✓ Directly control covariance via h
- ✓ Evaluation time $\mathcal{O}(N)$ for N points
- ✓ Naturally supports non-stationarity

Lewis [1986, 1989]

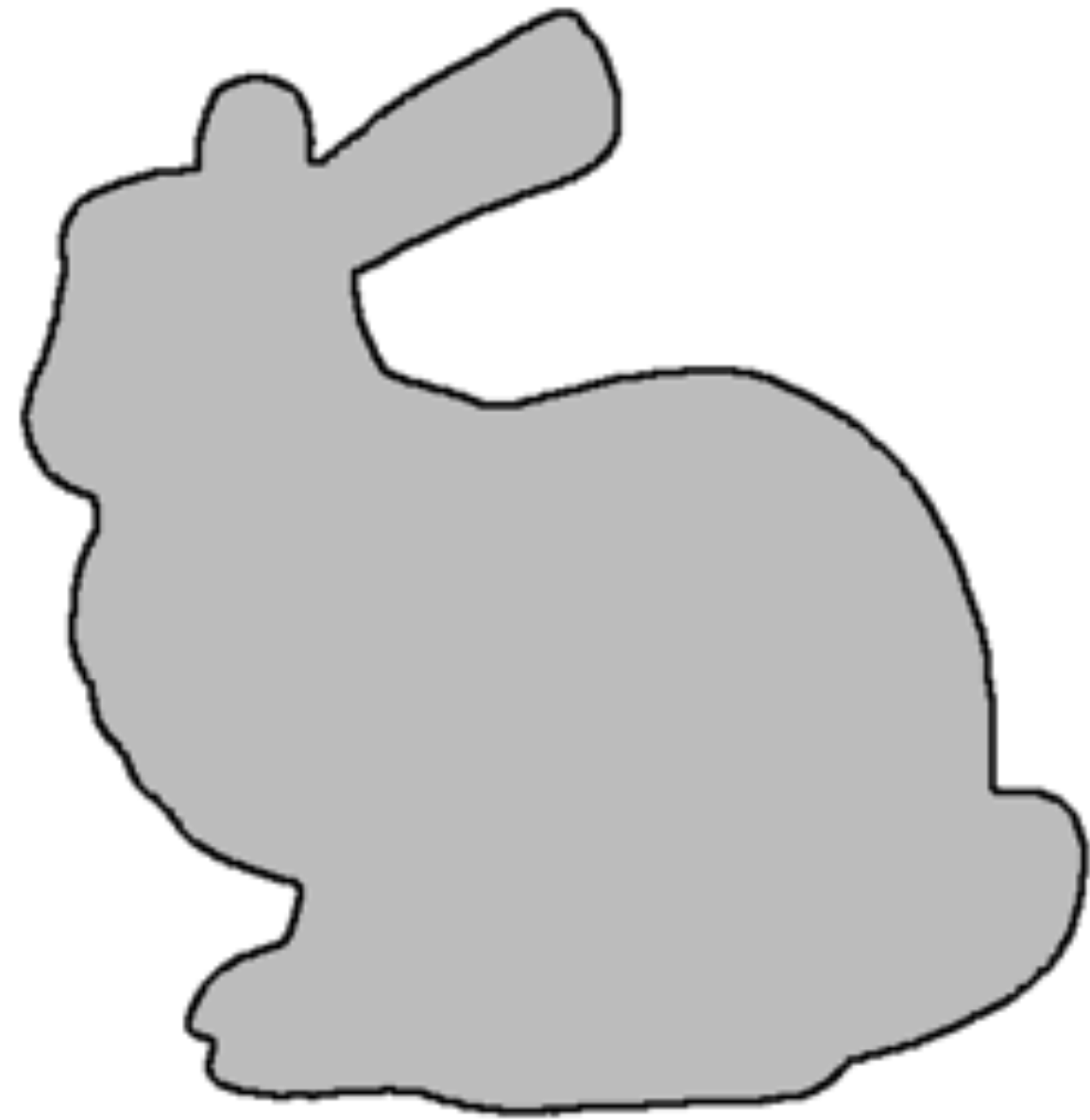
Convolved process



$$\mathcal{N}(0, h \circledast h(x, x'))$$

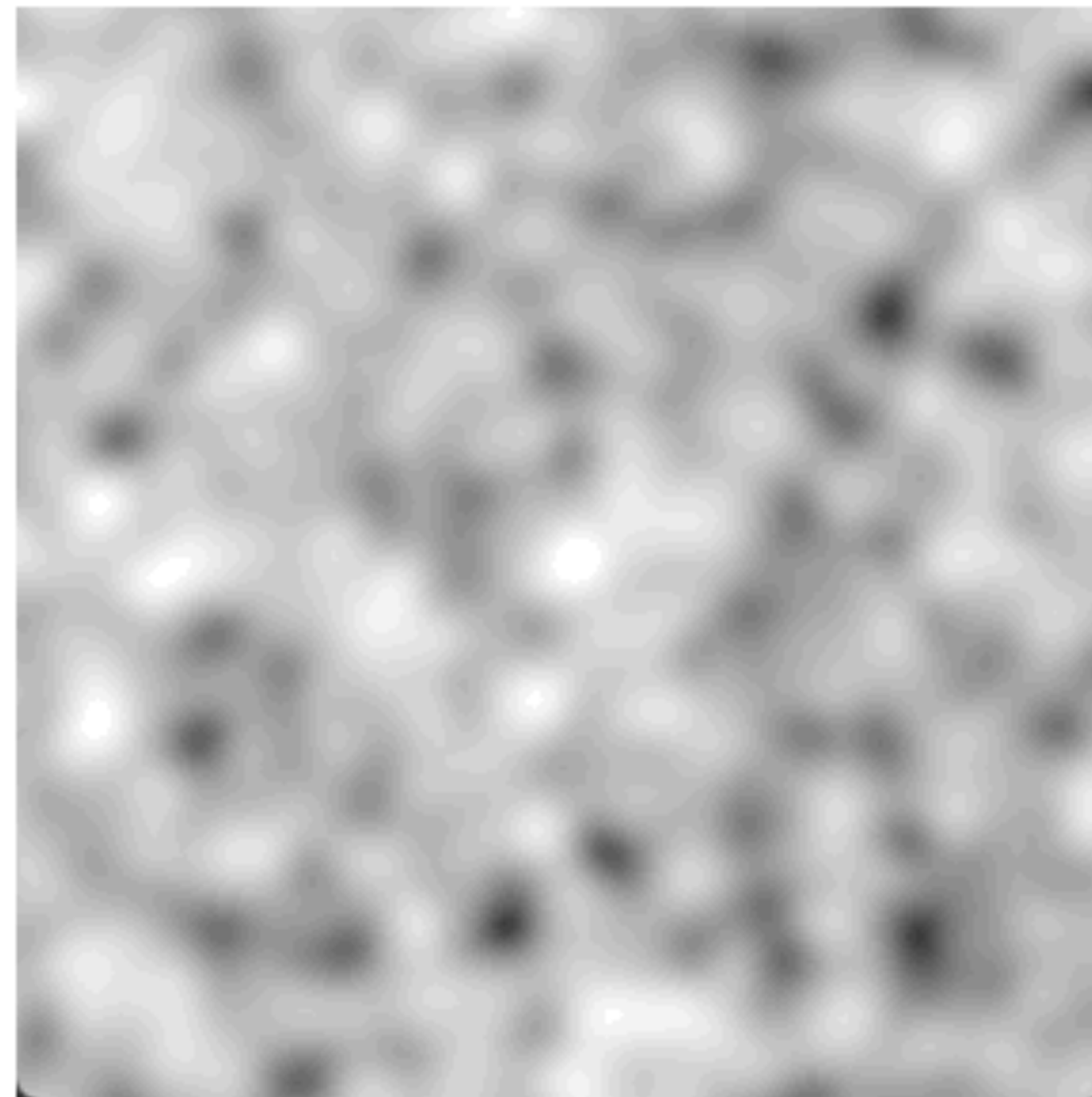
GPISeS via Procedural Noise

Mean



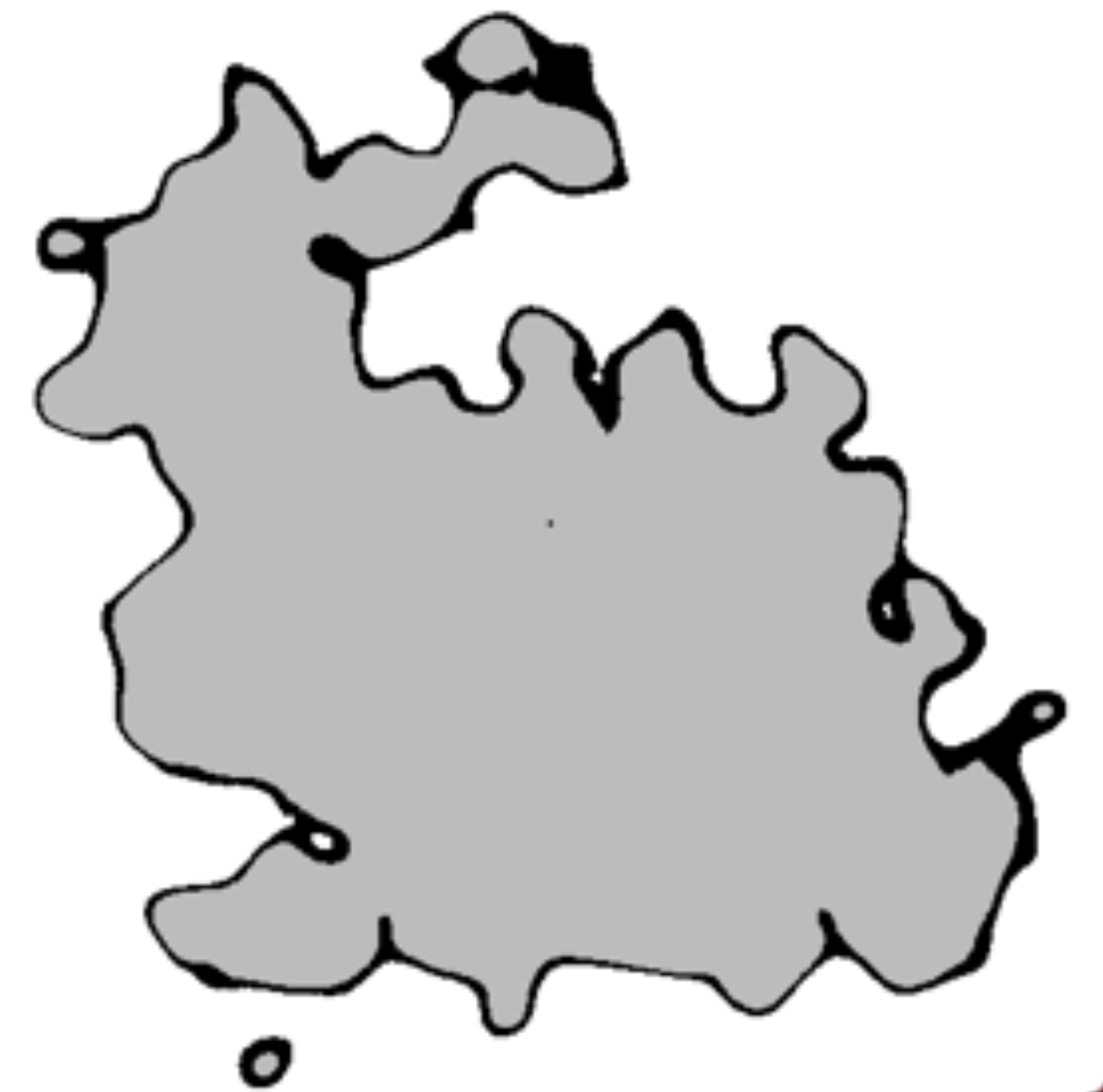
+

Procedural noise



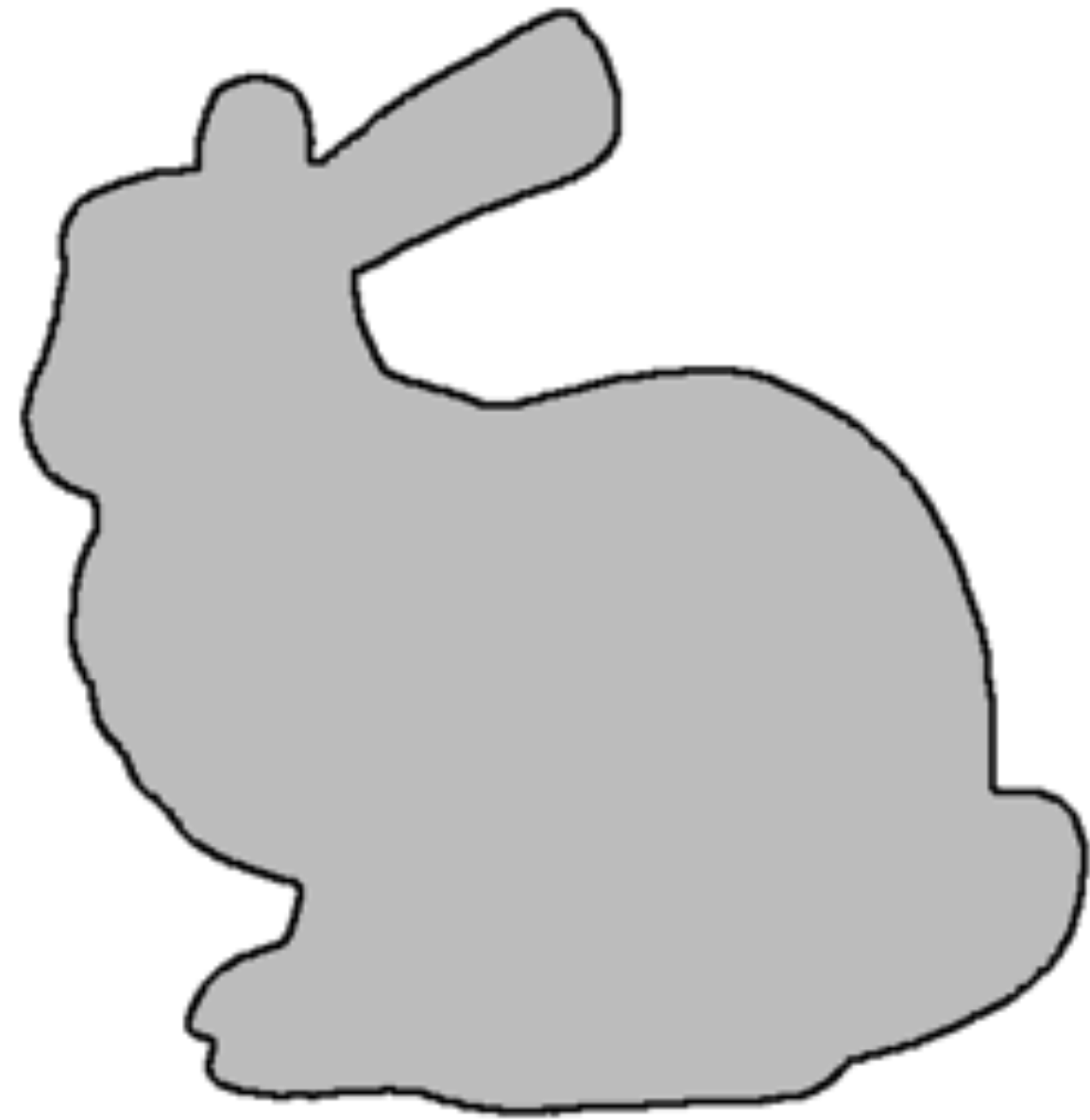
=

Single realization



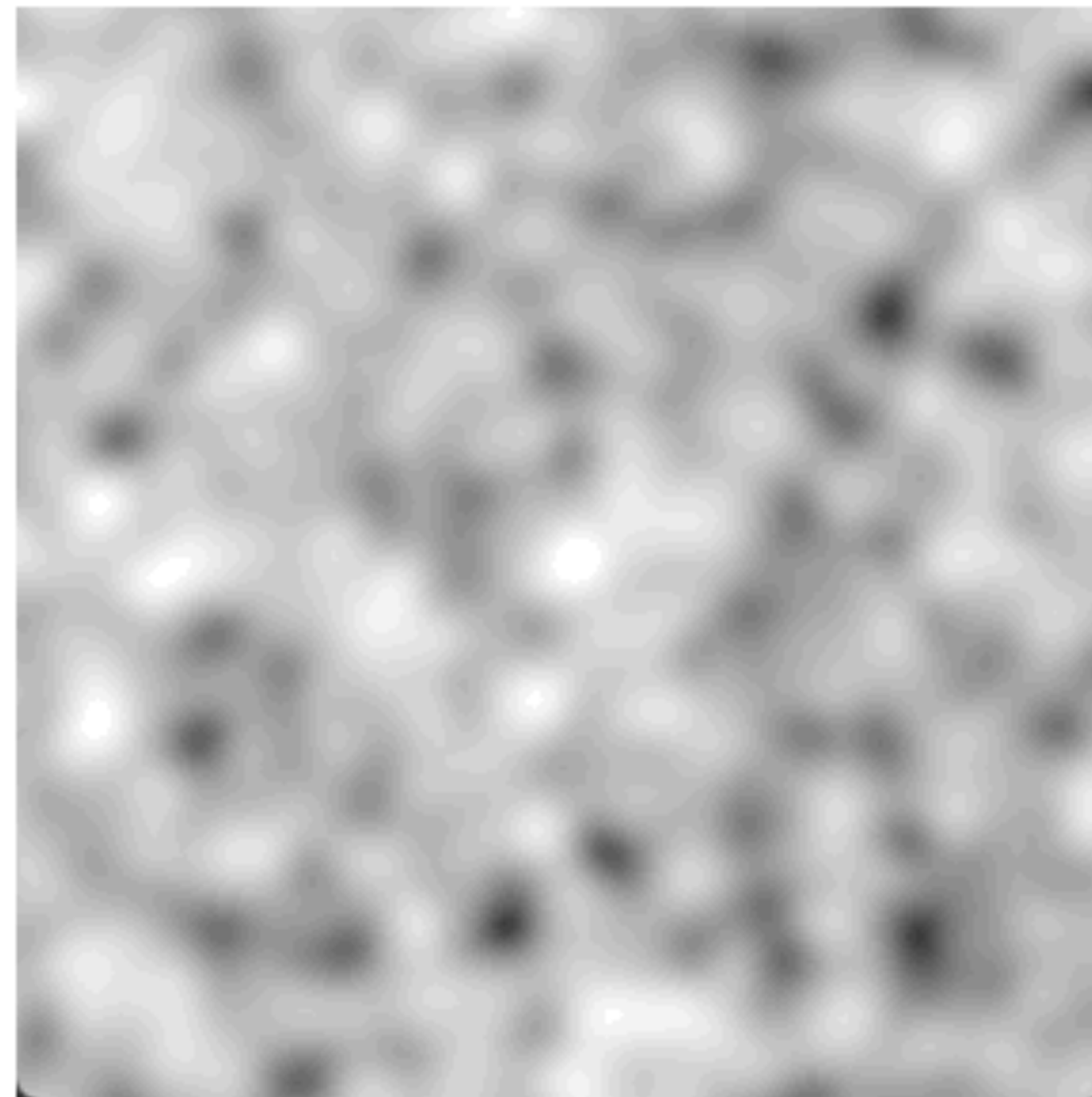
GPISeS via Procedural Noise

Mean



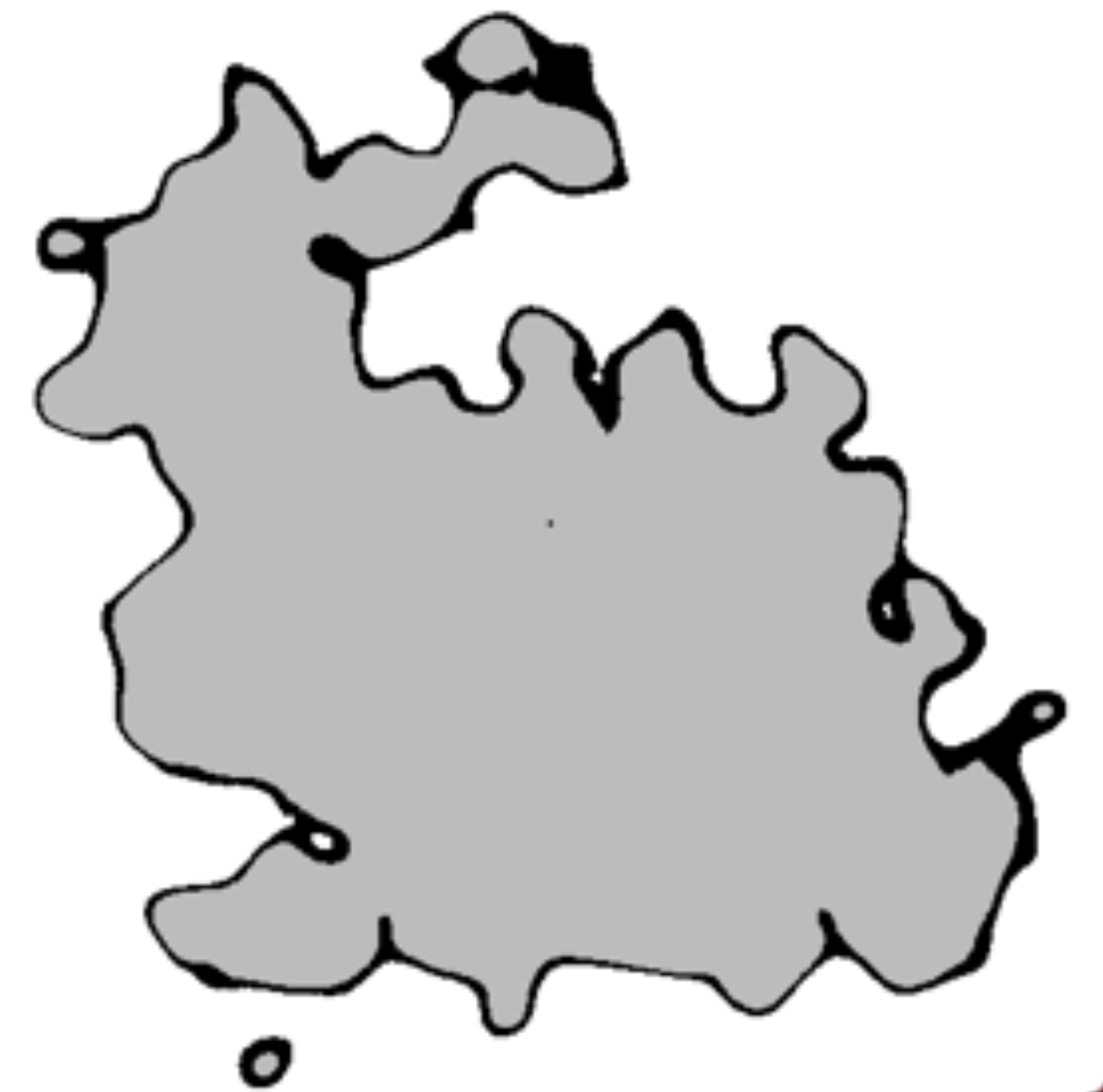
+

Procedural noise

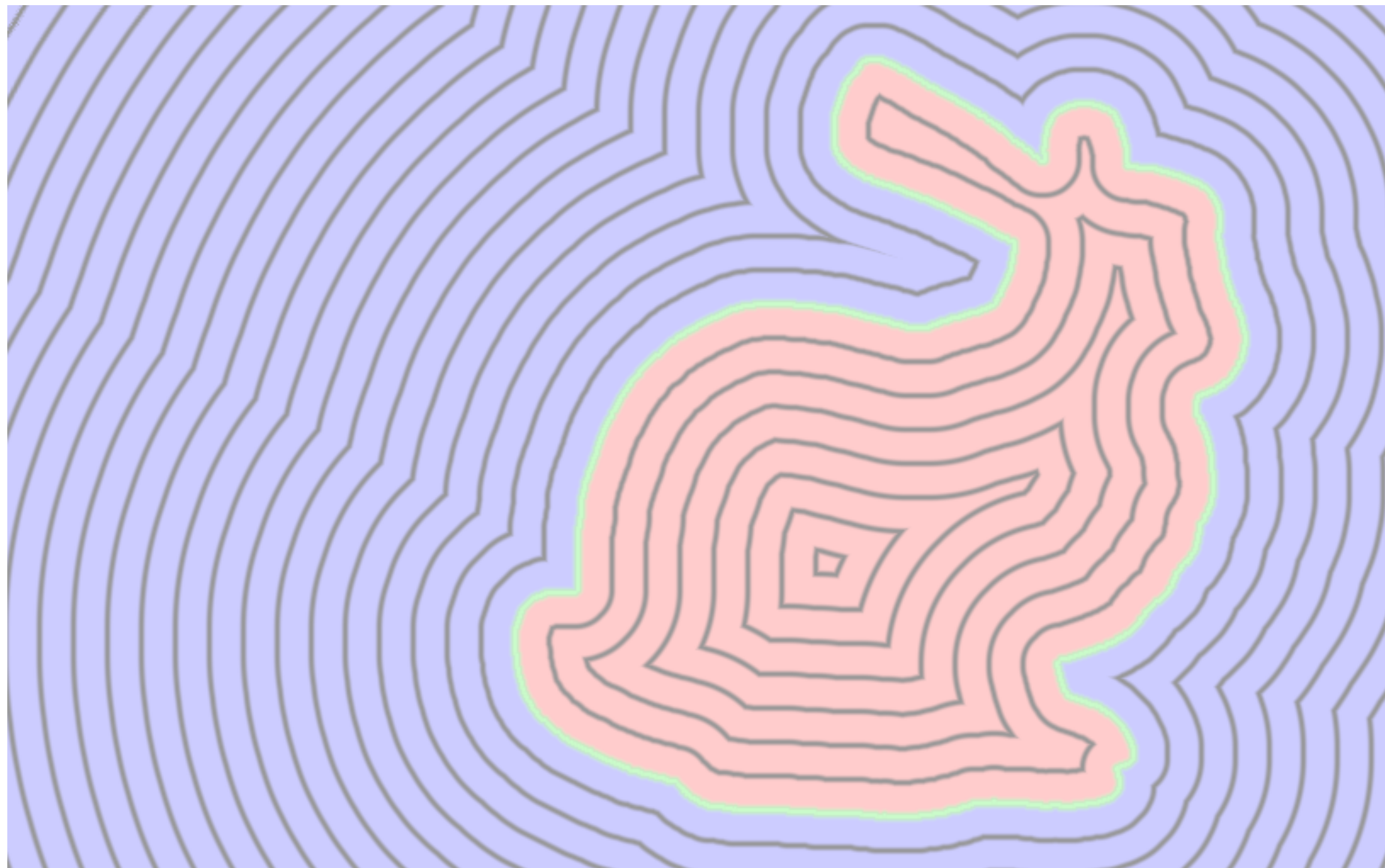
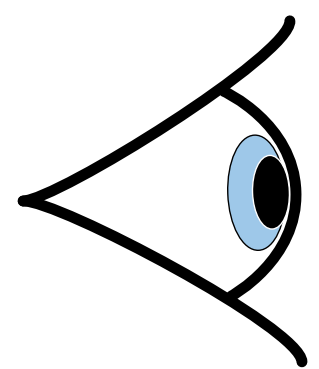


=

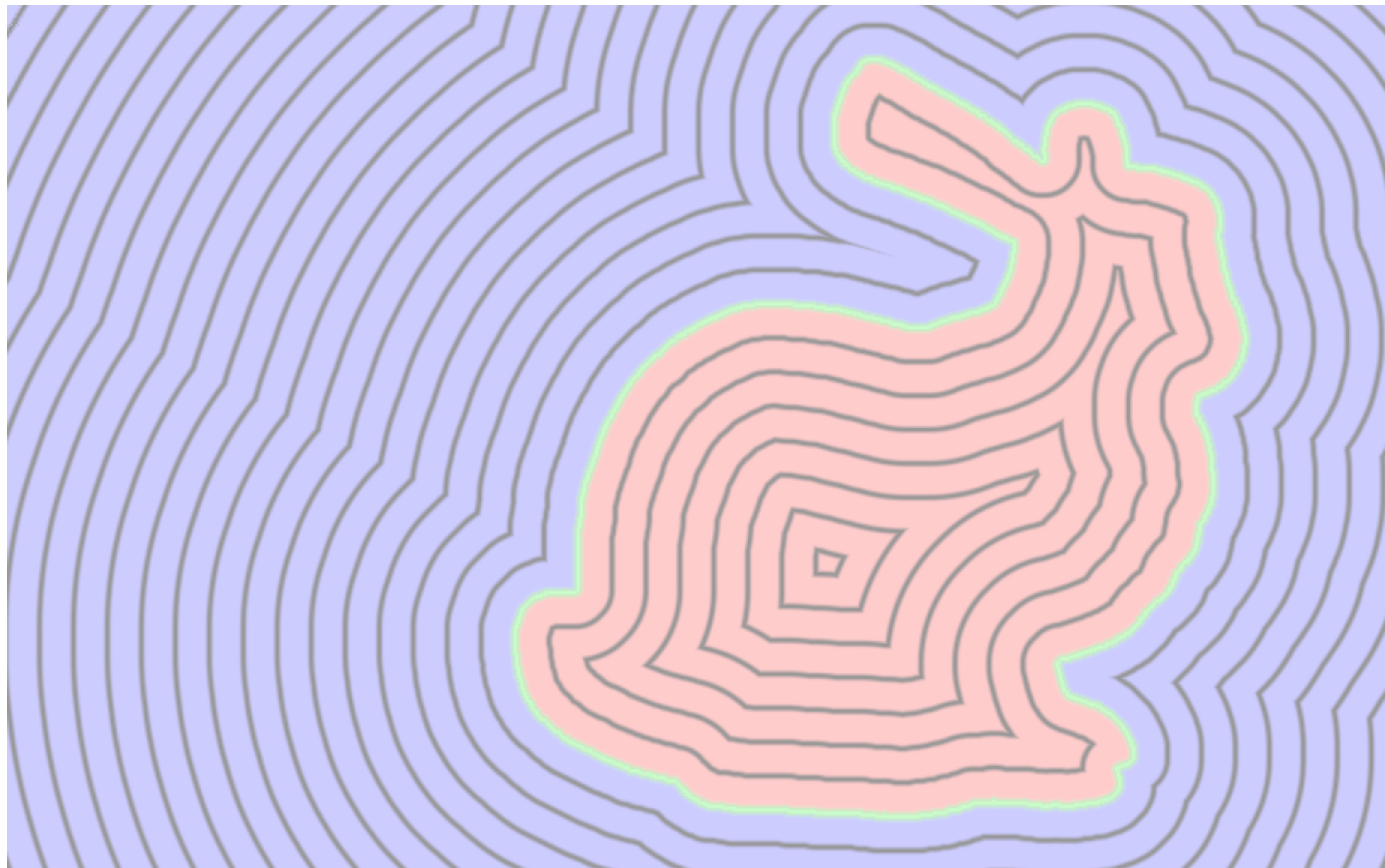
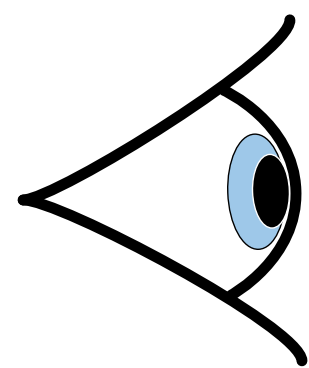
Single realization



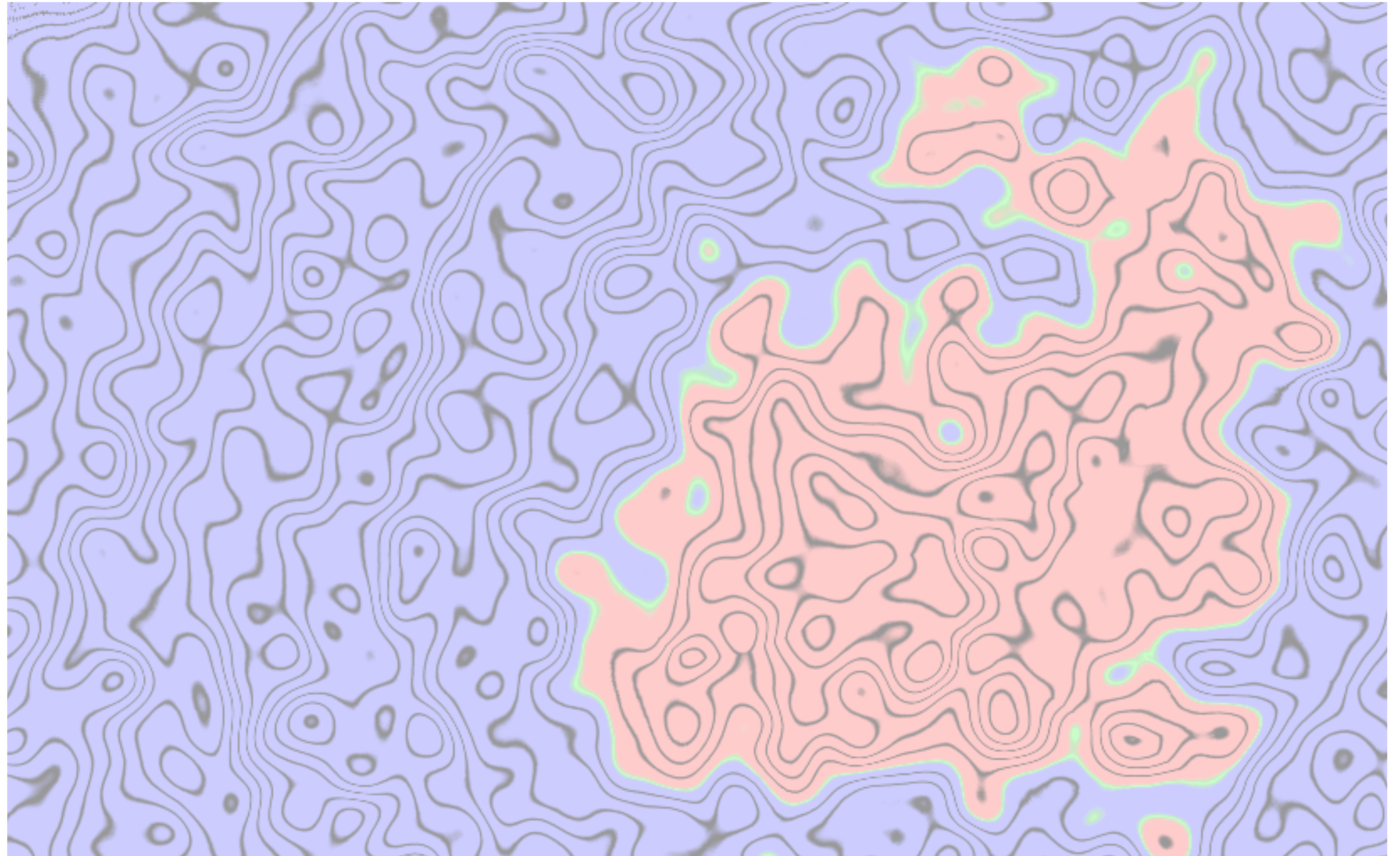
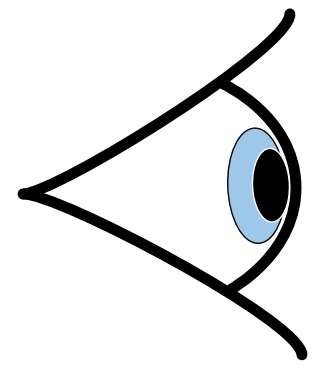
The Mean Implicit Function



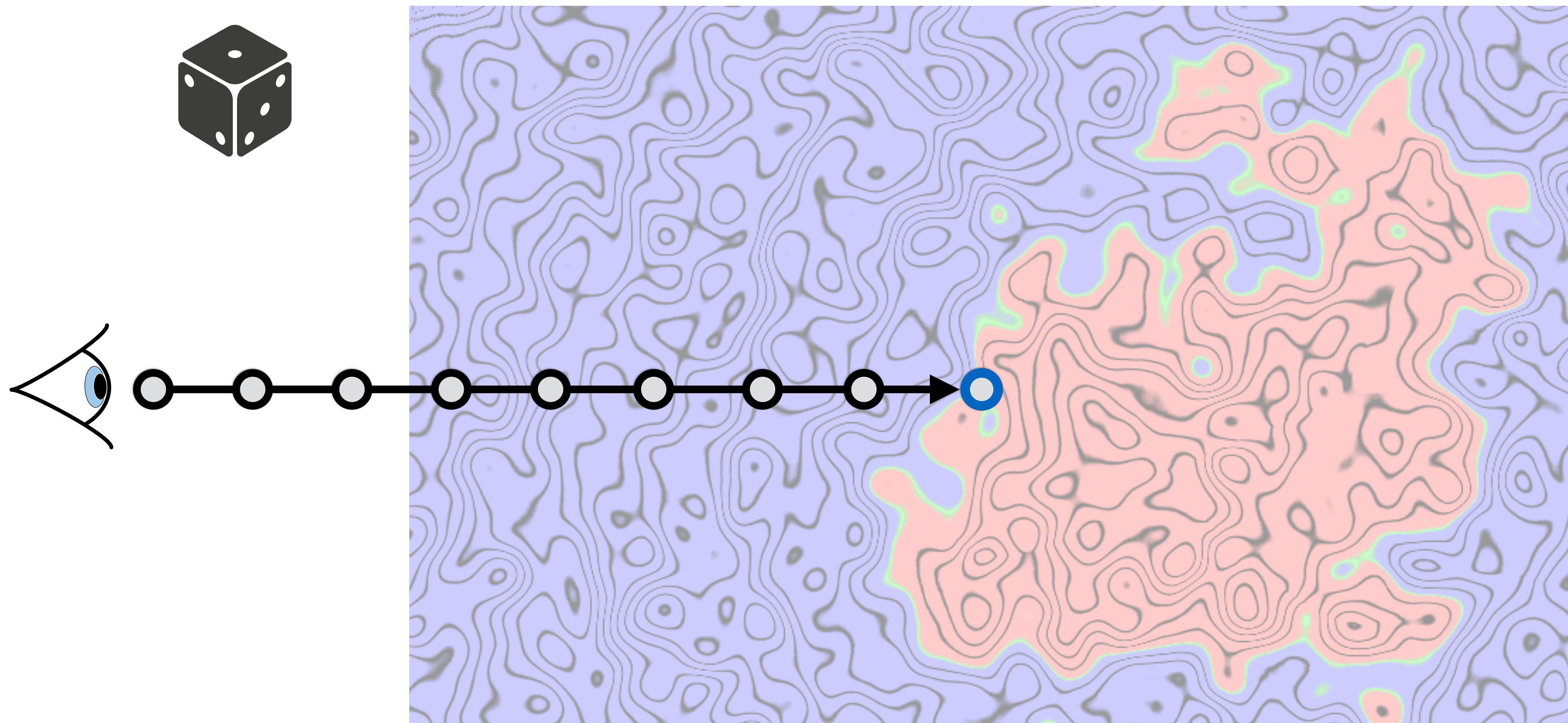
The Mean Implicit Function



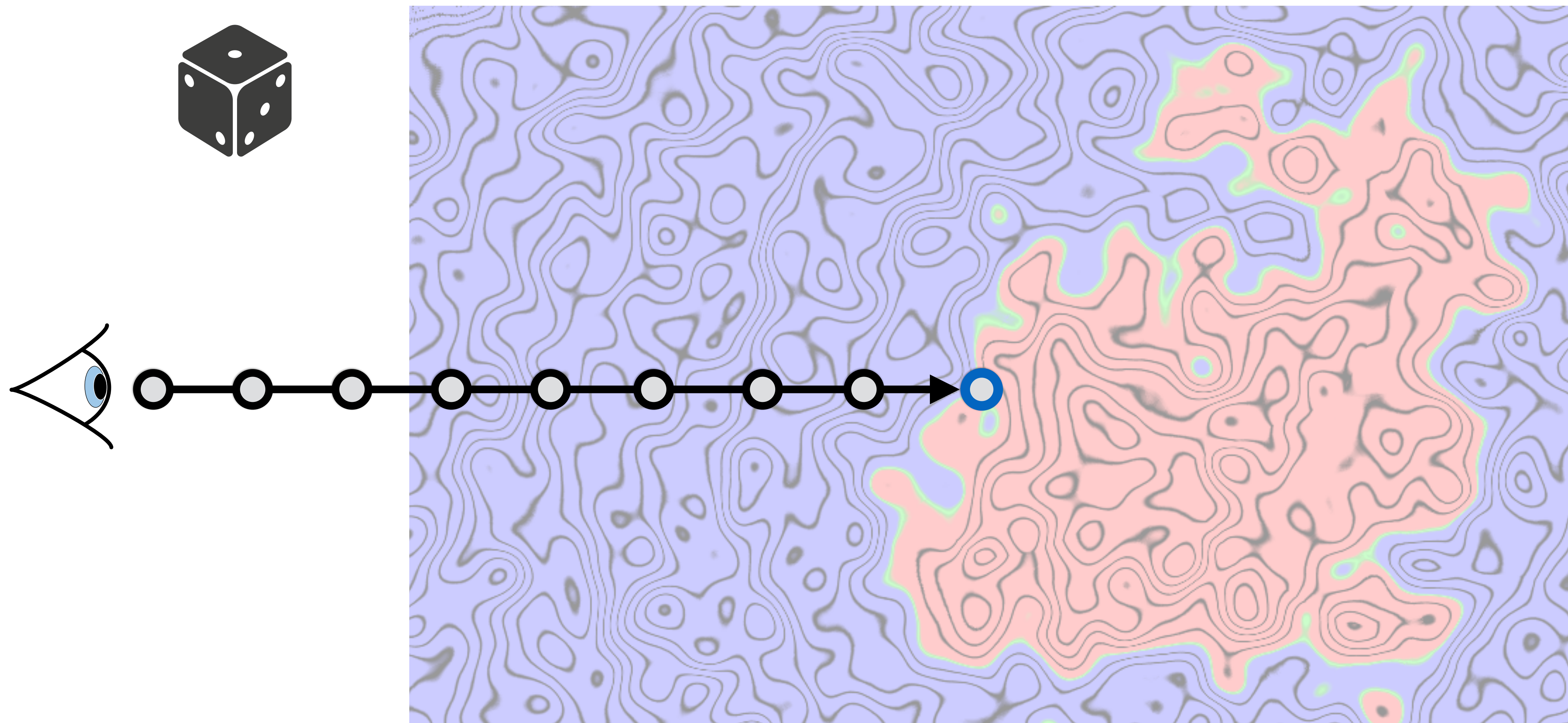
Ray March a GPIS



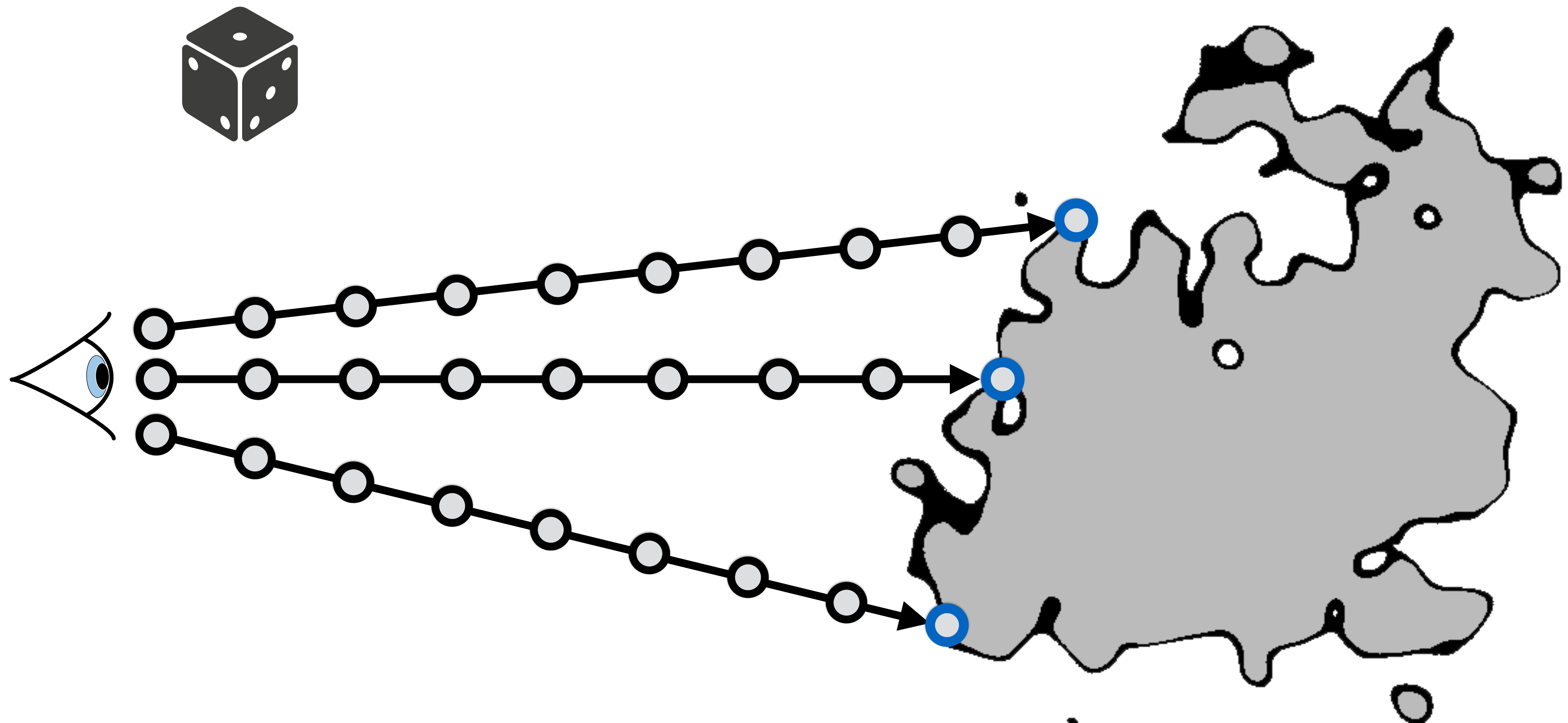
Ray March a GPIS



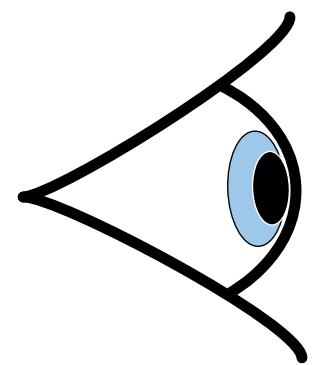
A Single Realization



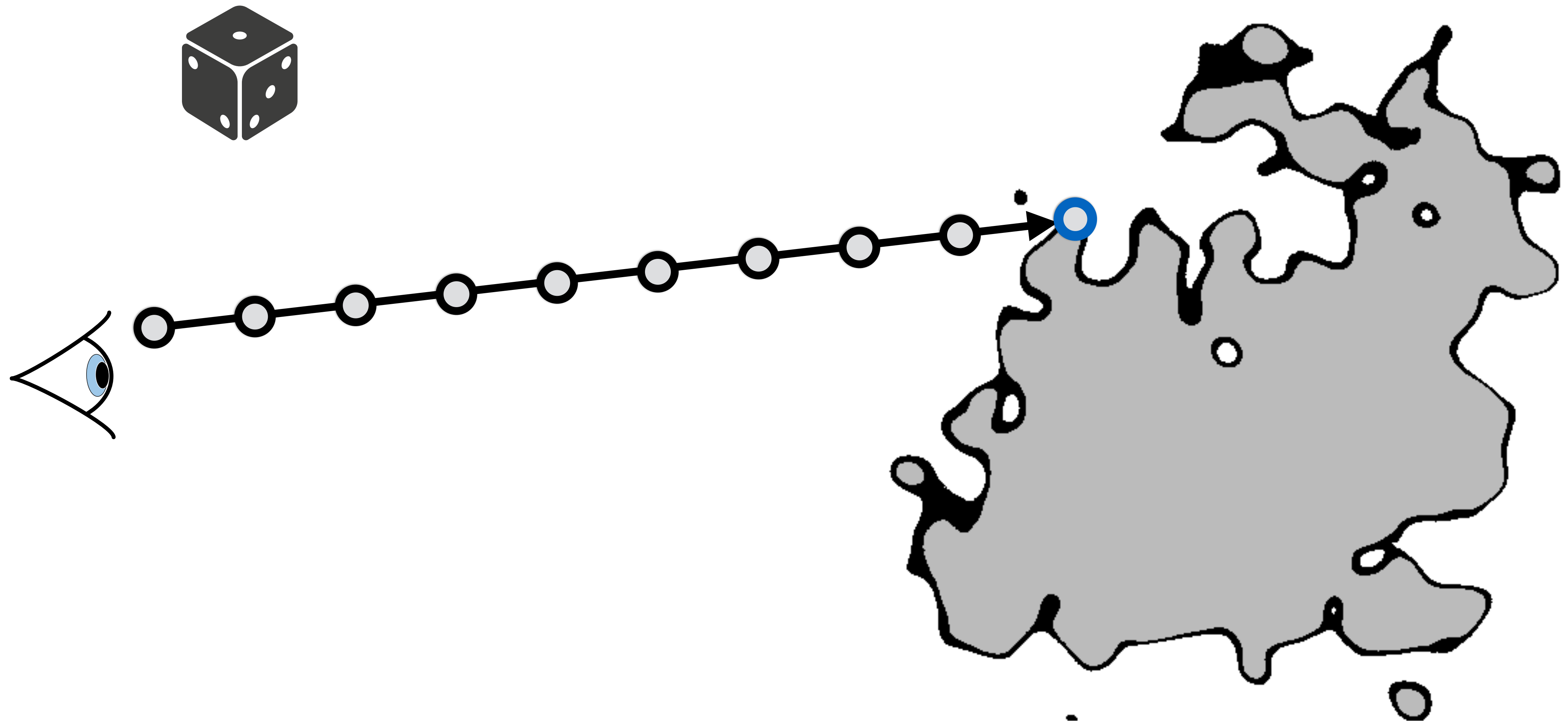
A Single Realization



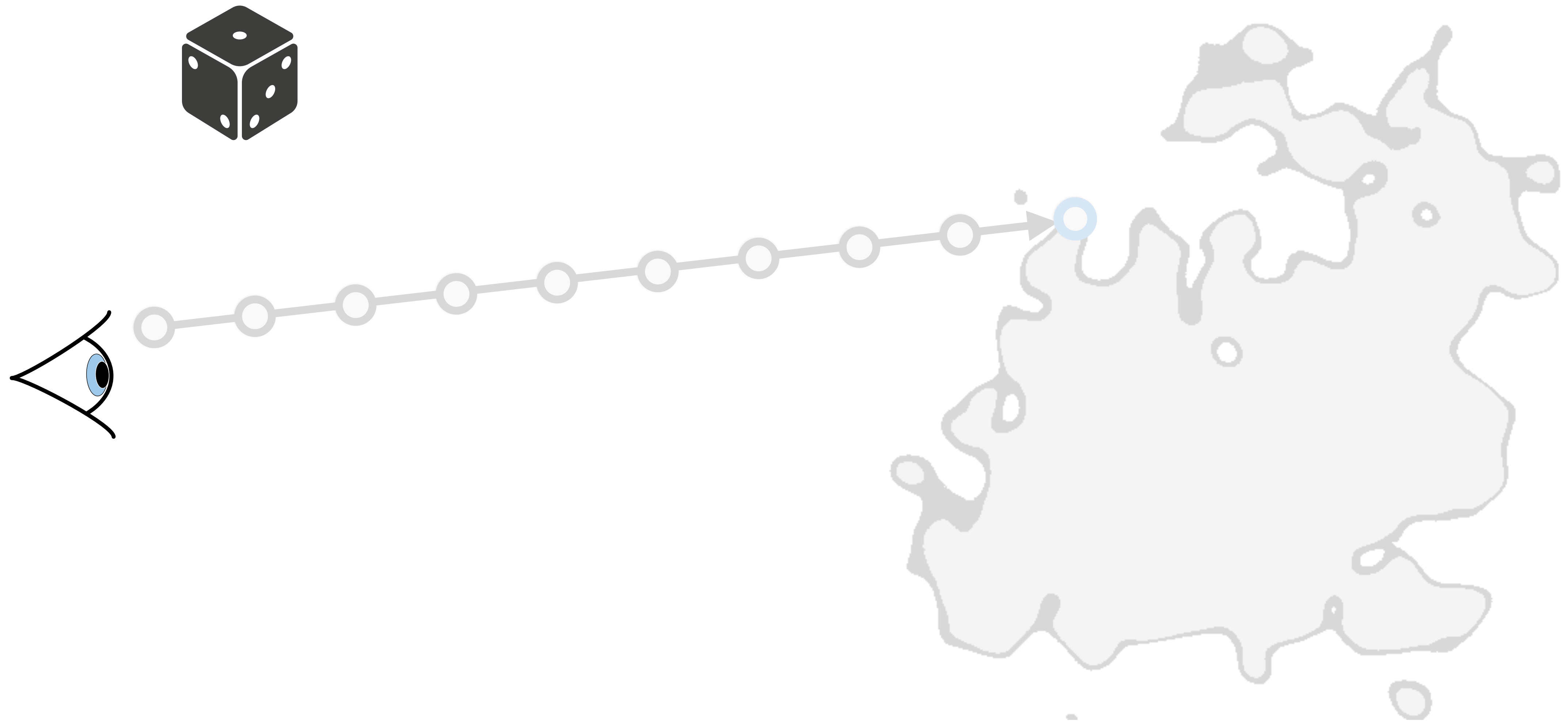
Ensemble-averaged Light Transport



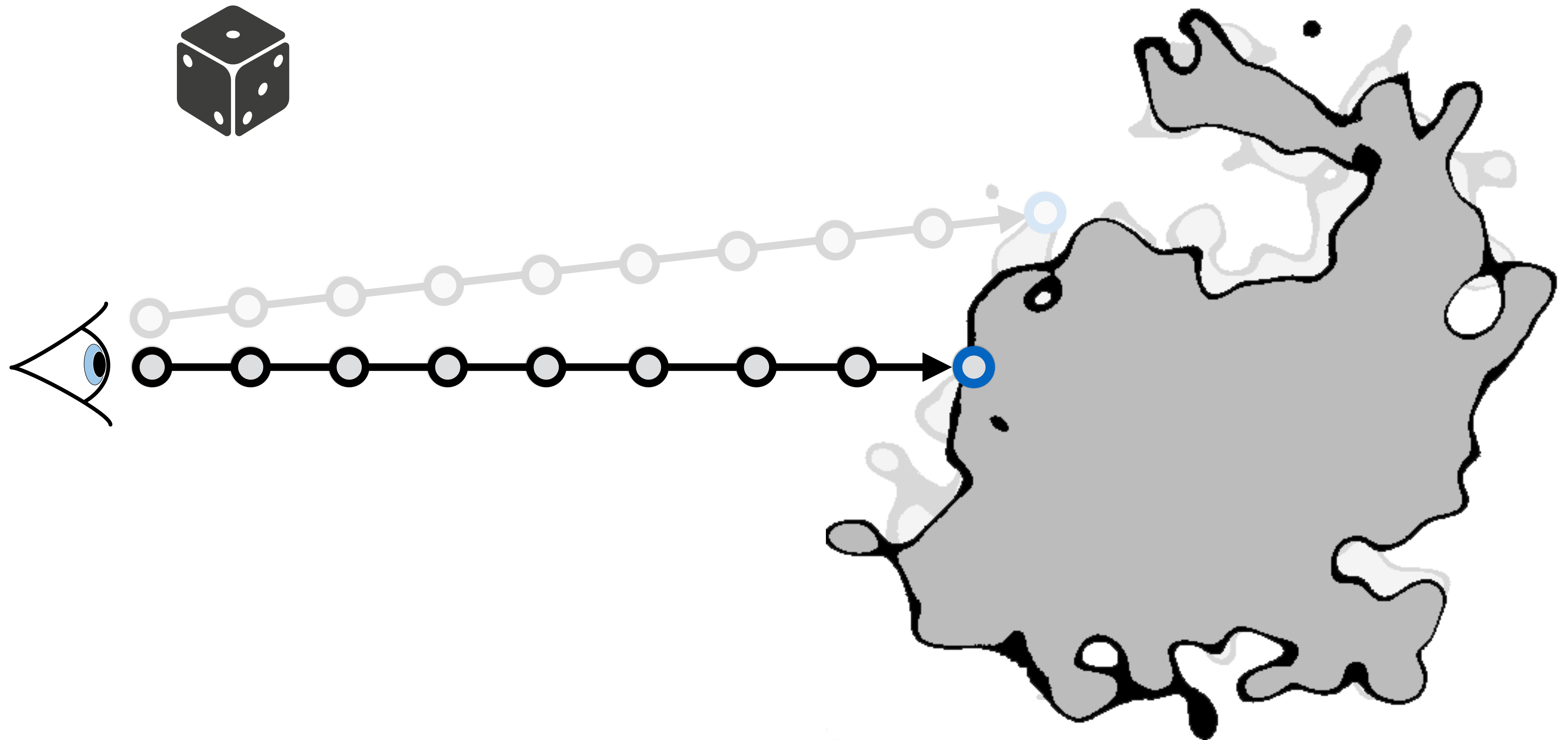
Ensemble-averaged Light Transport



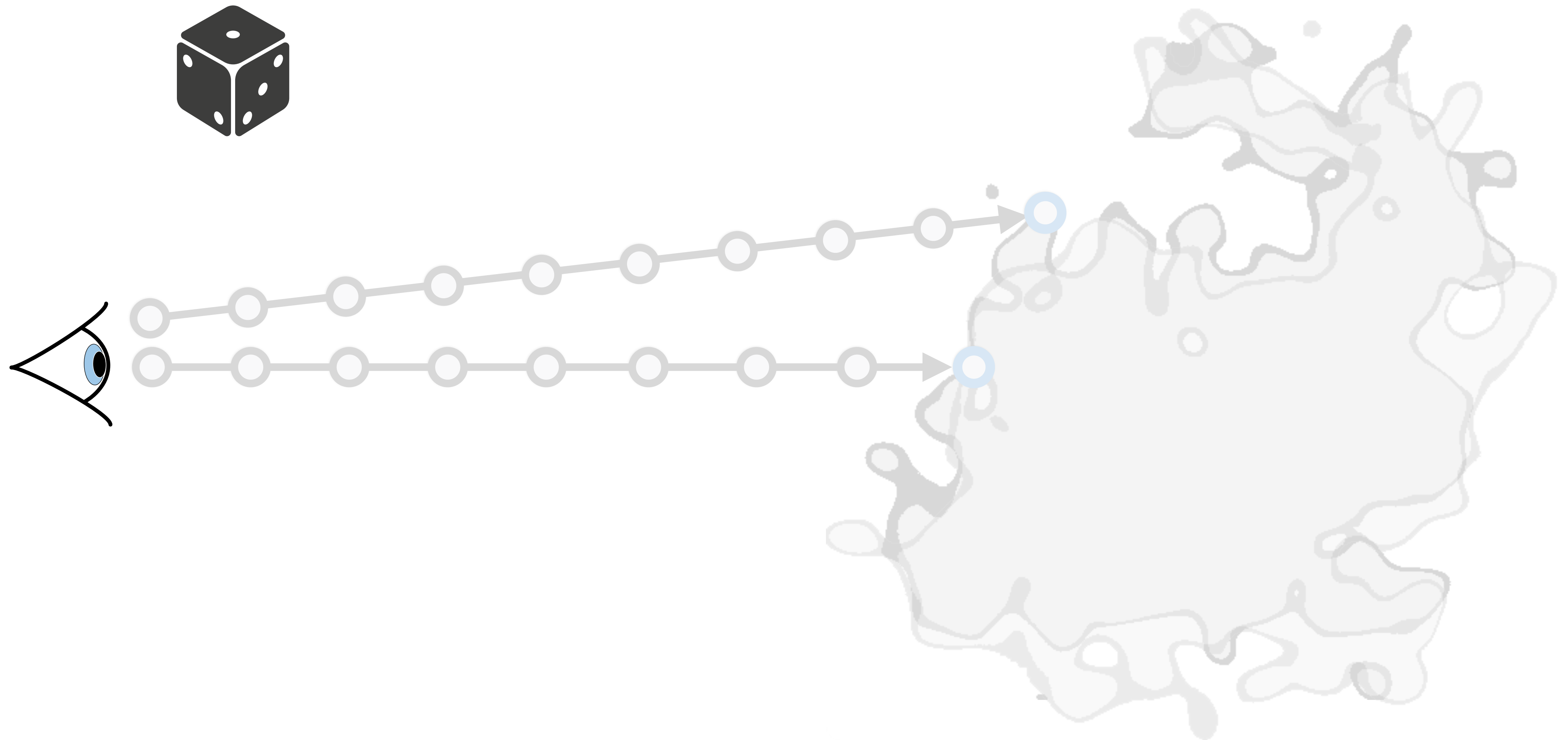
Ensemble-averaged Light Transport



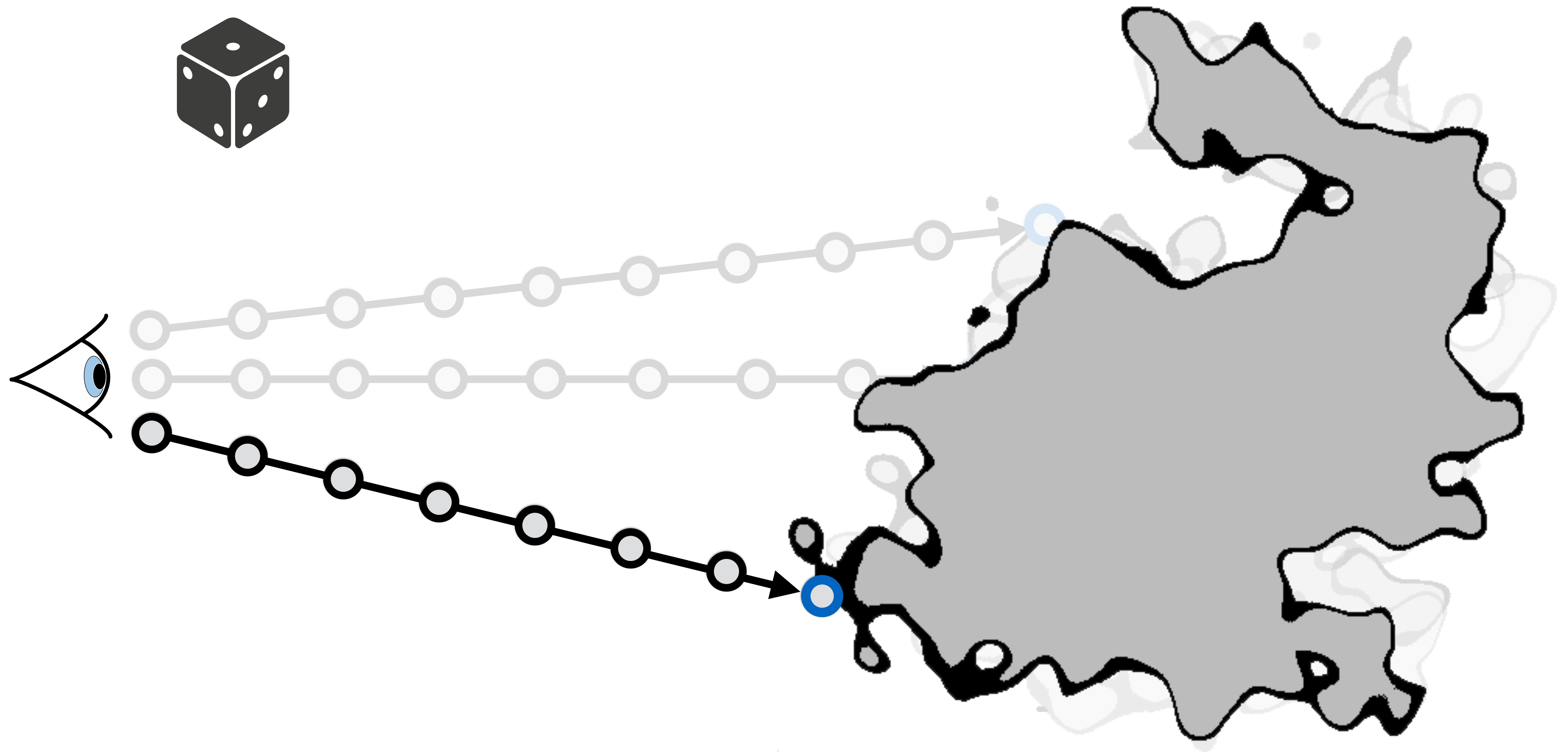
Ensemble-averaged Light Transport



Ensemble-averaged Light Transport



Ensemble-averaged Light Transport



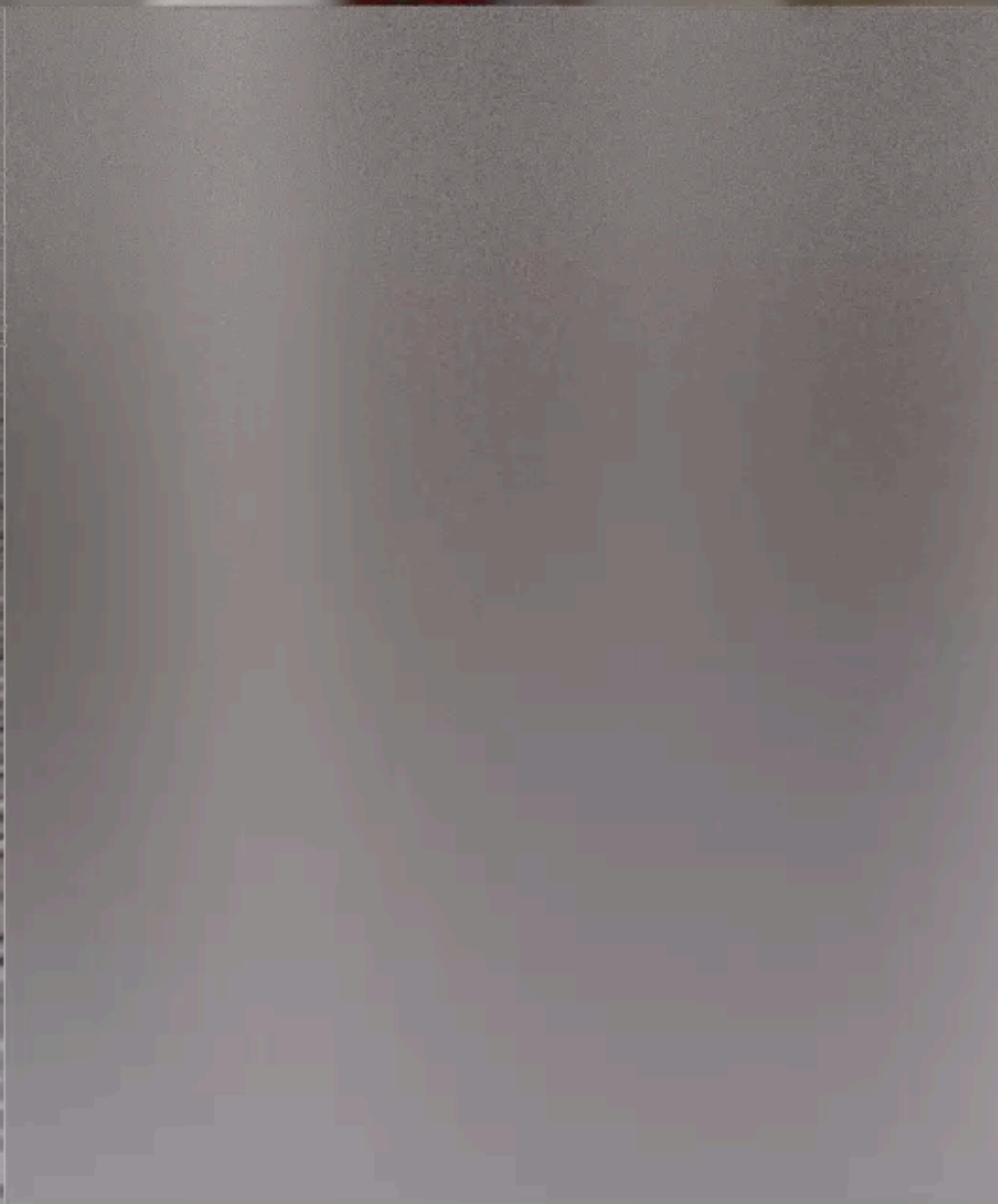
Ensemble-averaged Light Transport



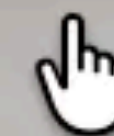
Single realization



Ensemble Average



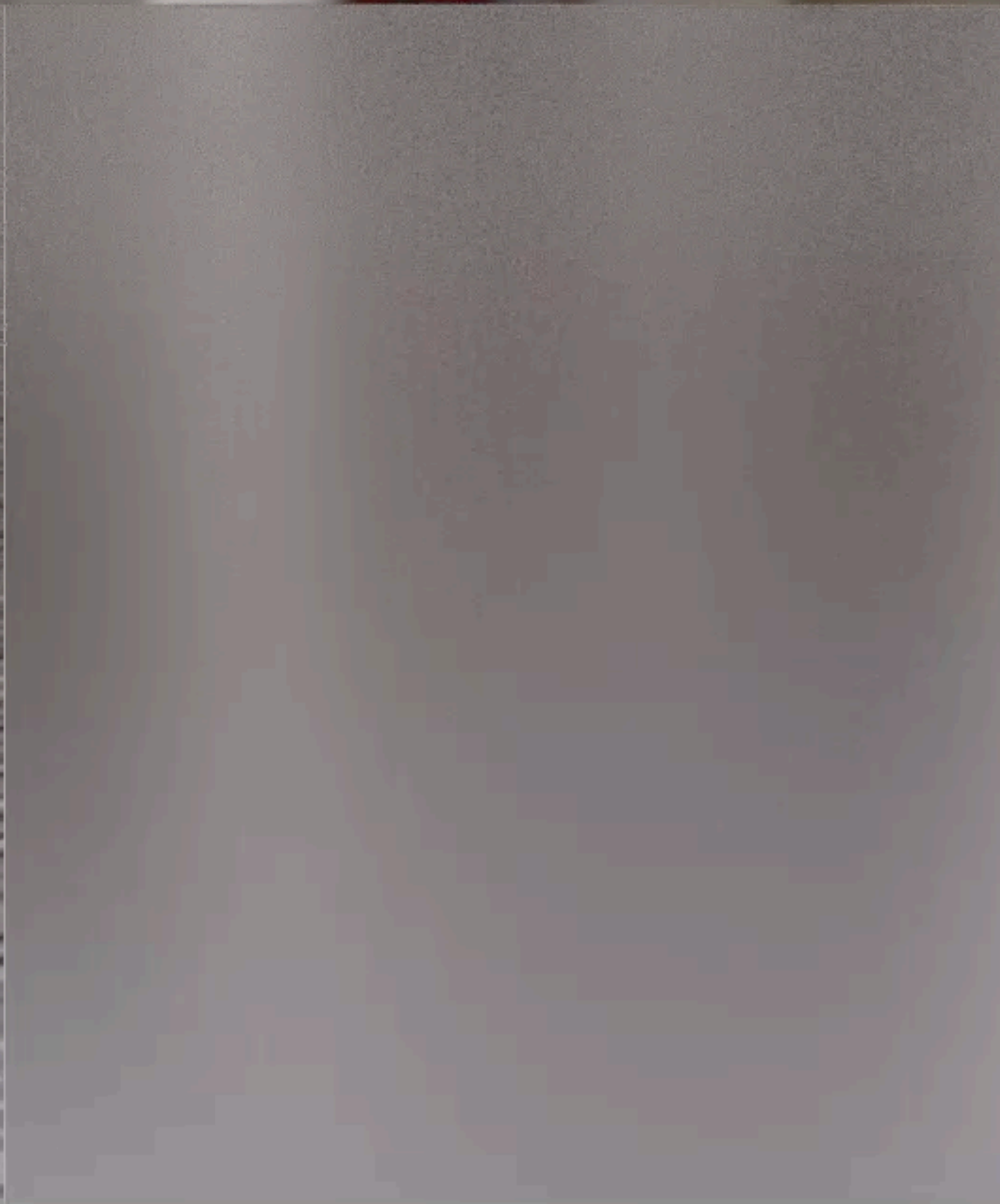
Microfacet model



Single realization



Ensemble Average

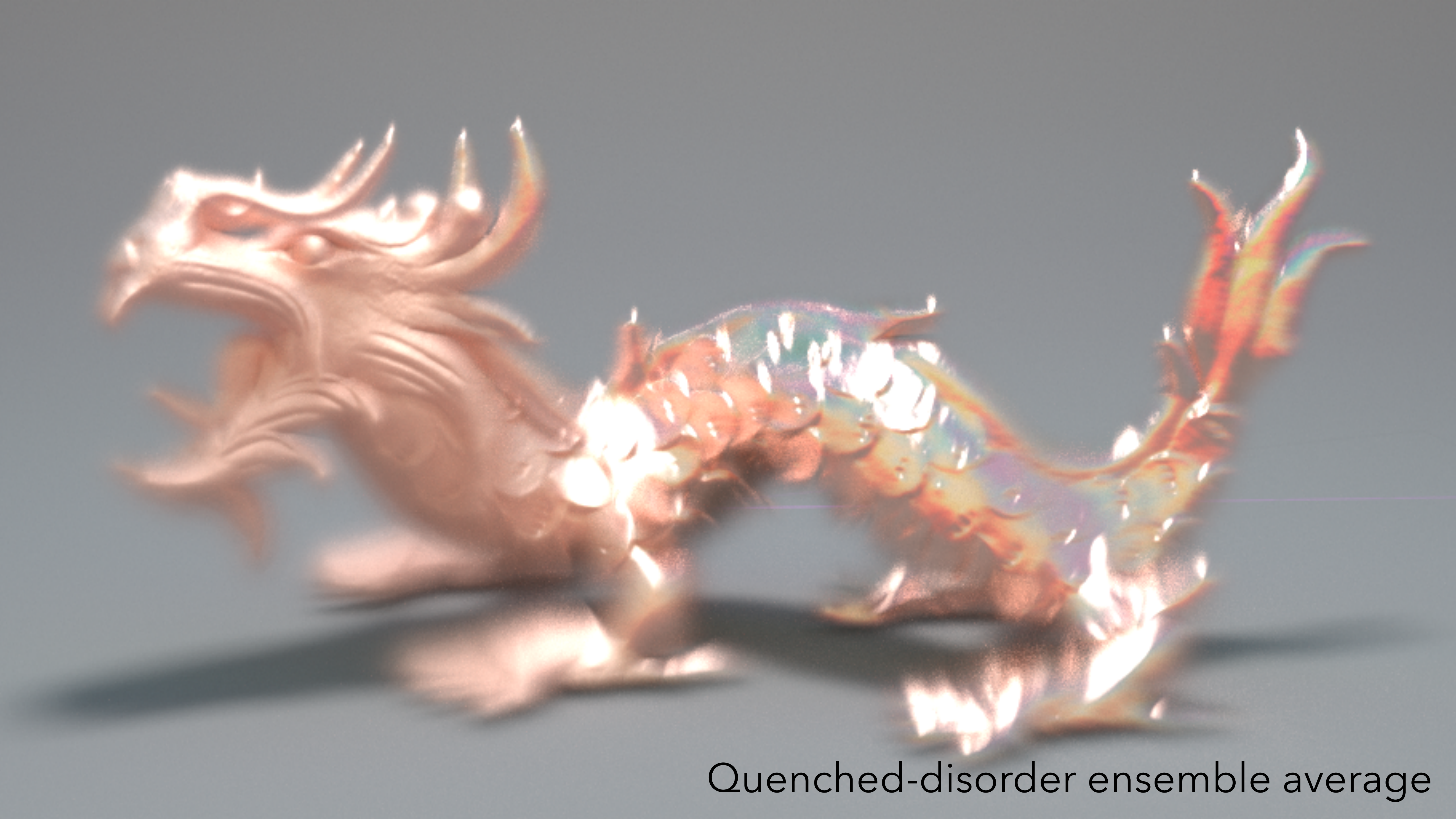


Microfacet model



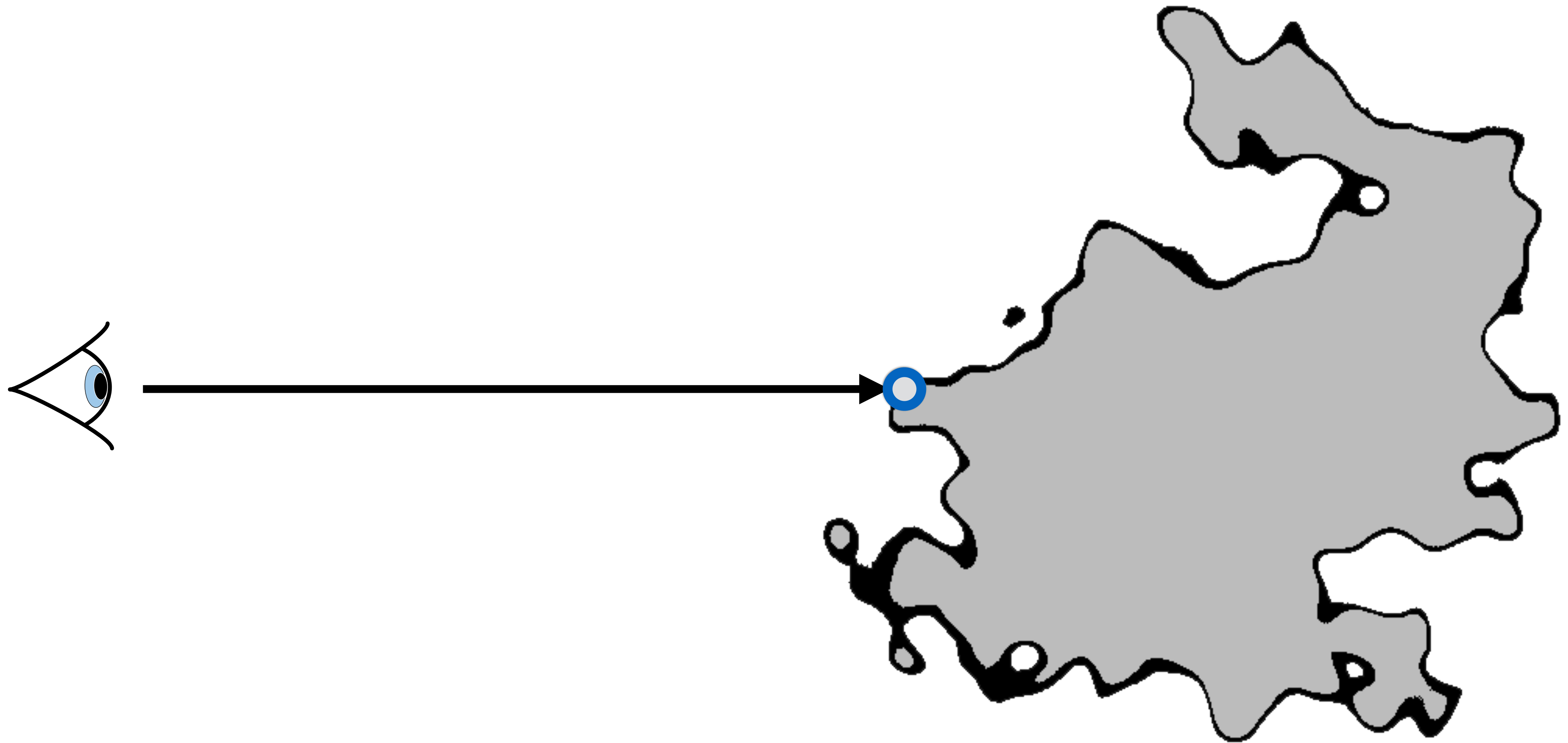


Single realization

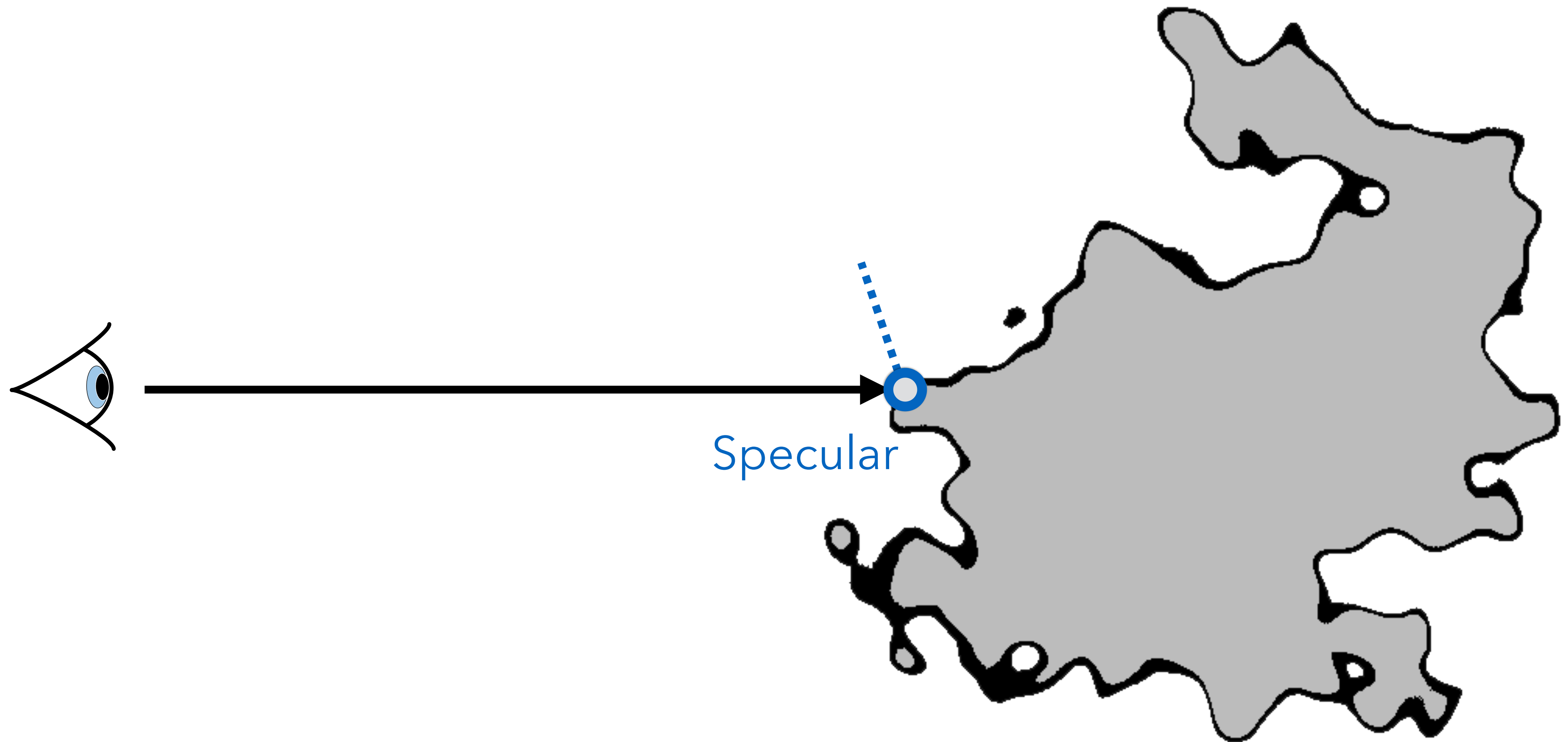


Quenched-disorder ensemble average

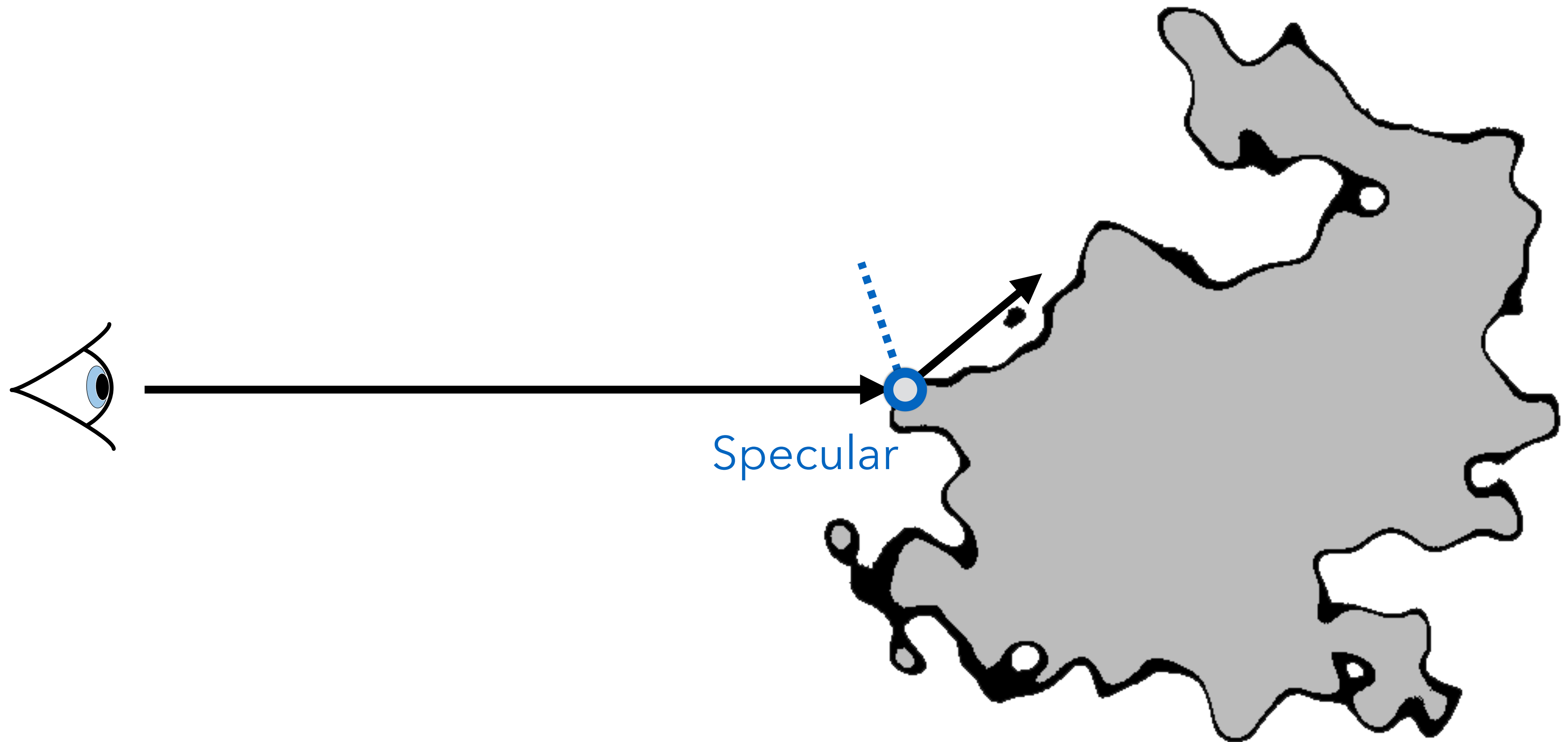
Full realizations lock everything down



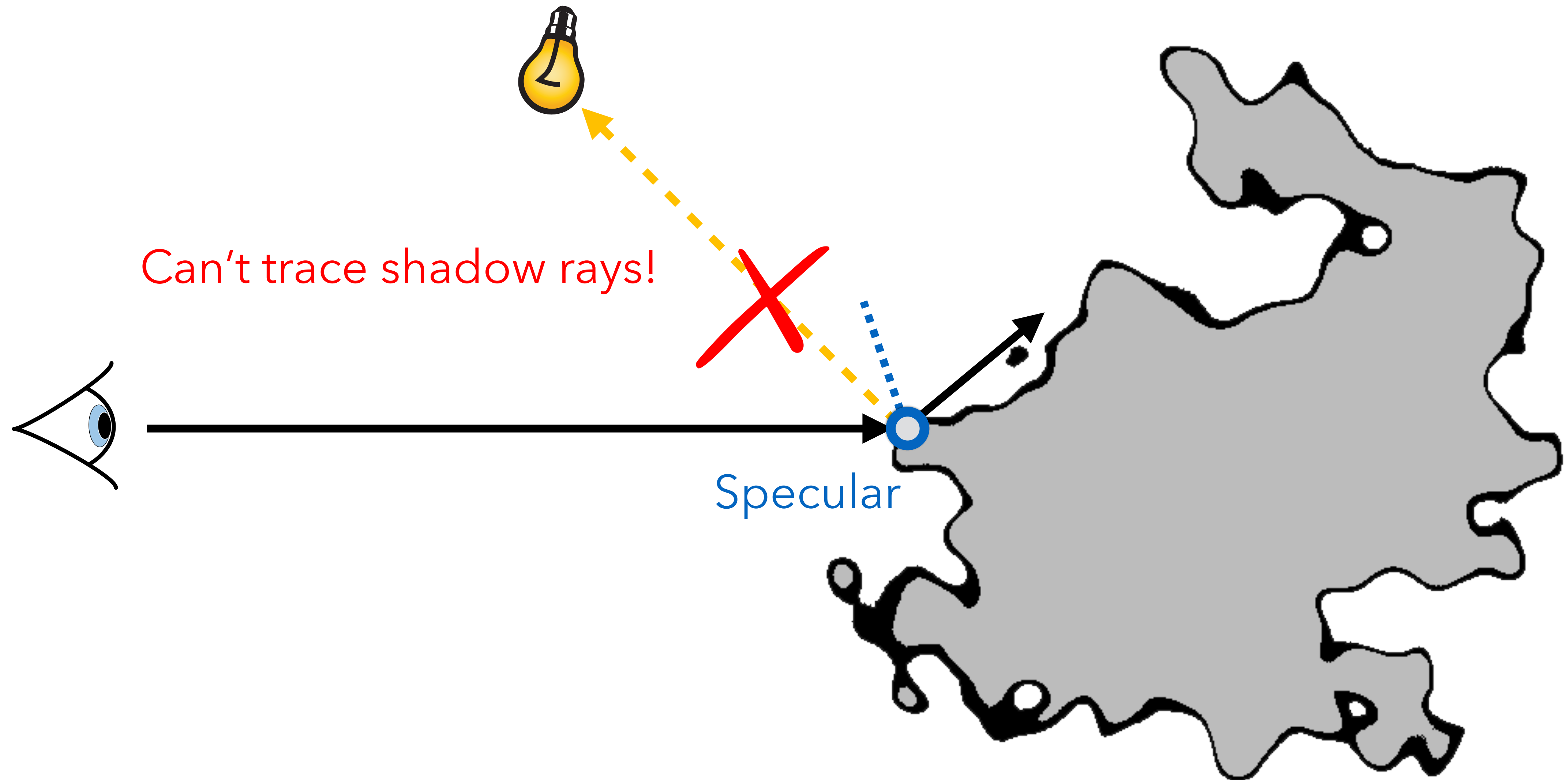
Full realizations lock everything down



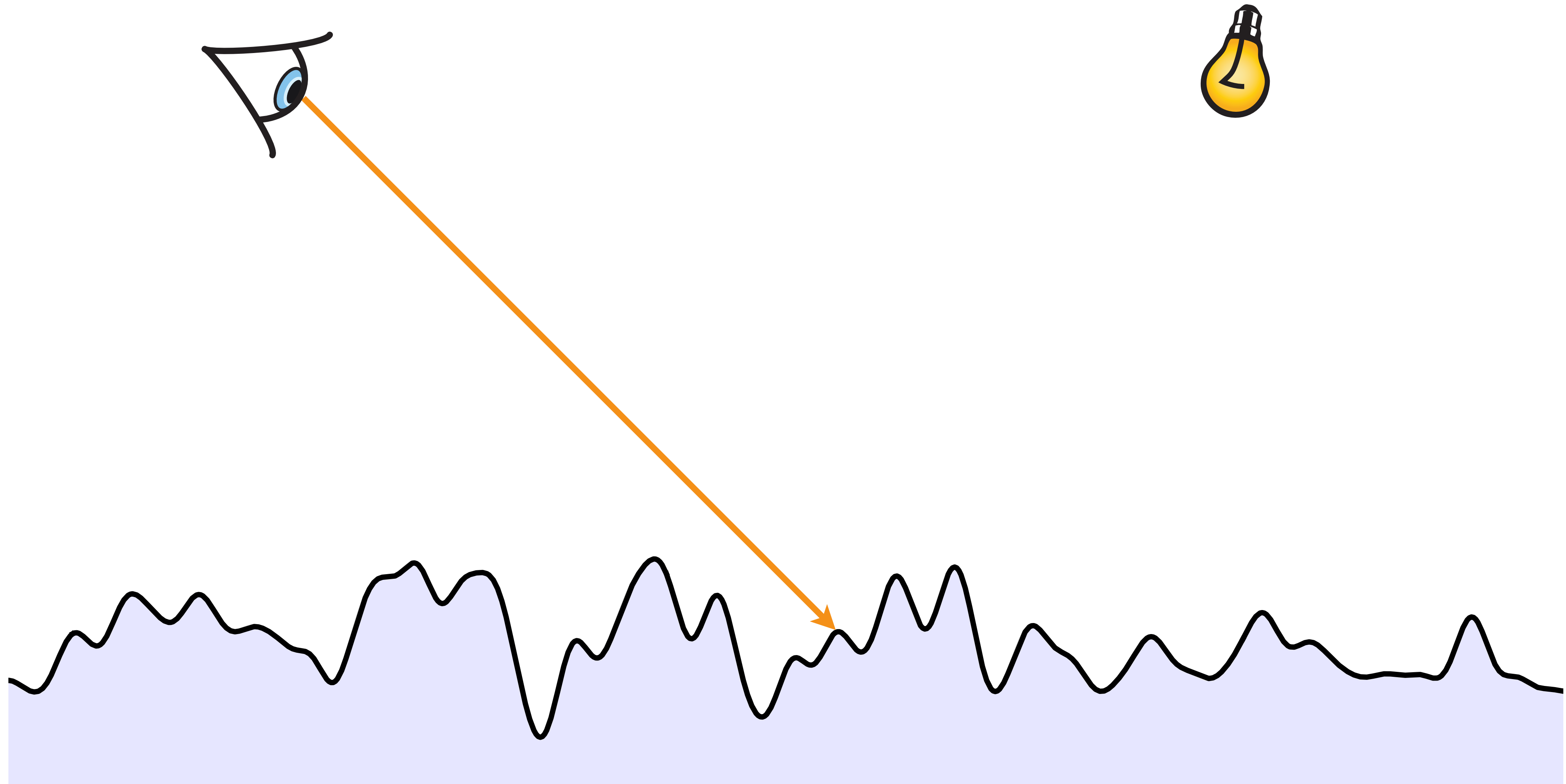
Full realizations lock everything down



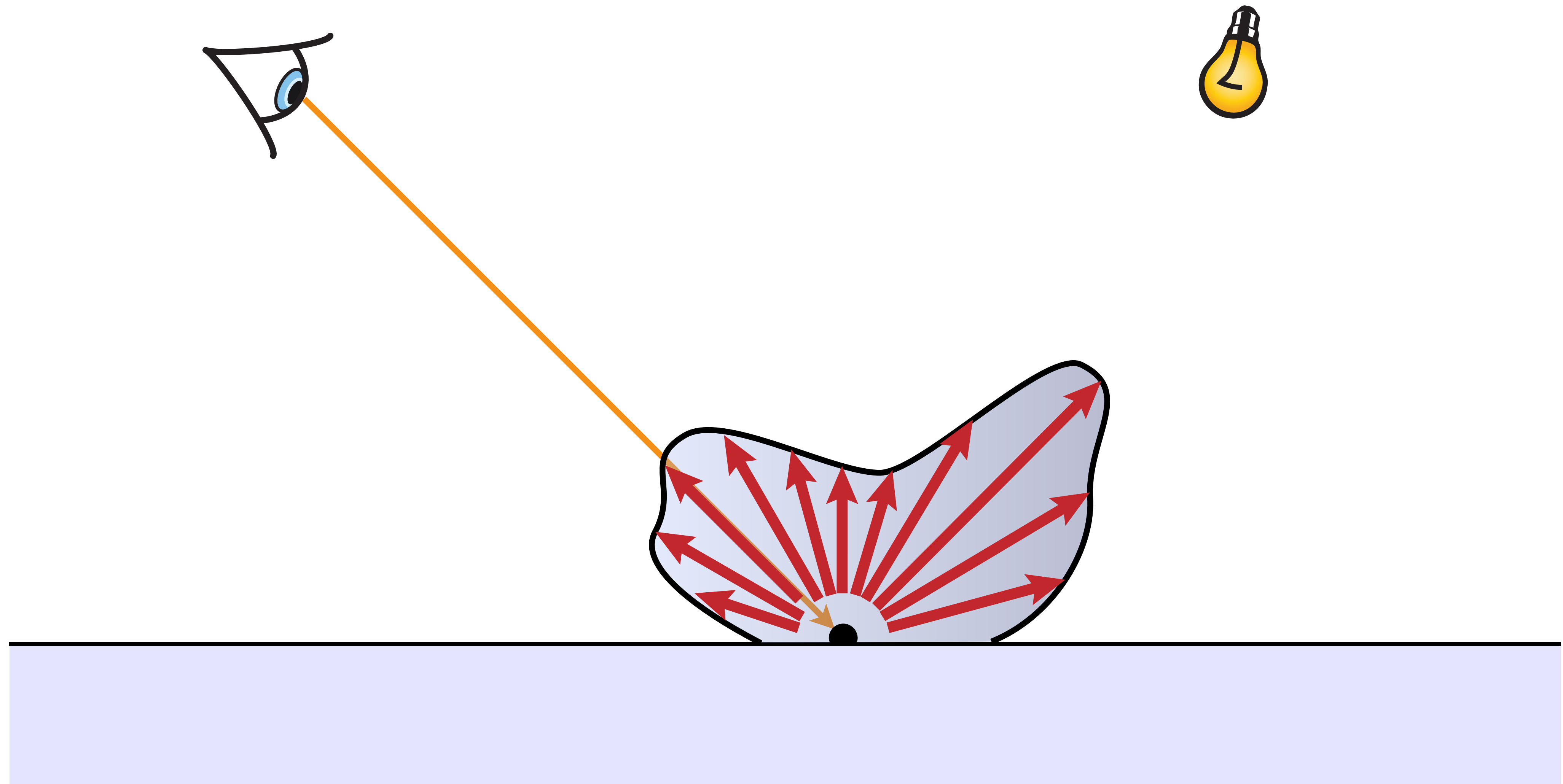
Full realizations lock everything down



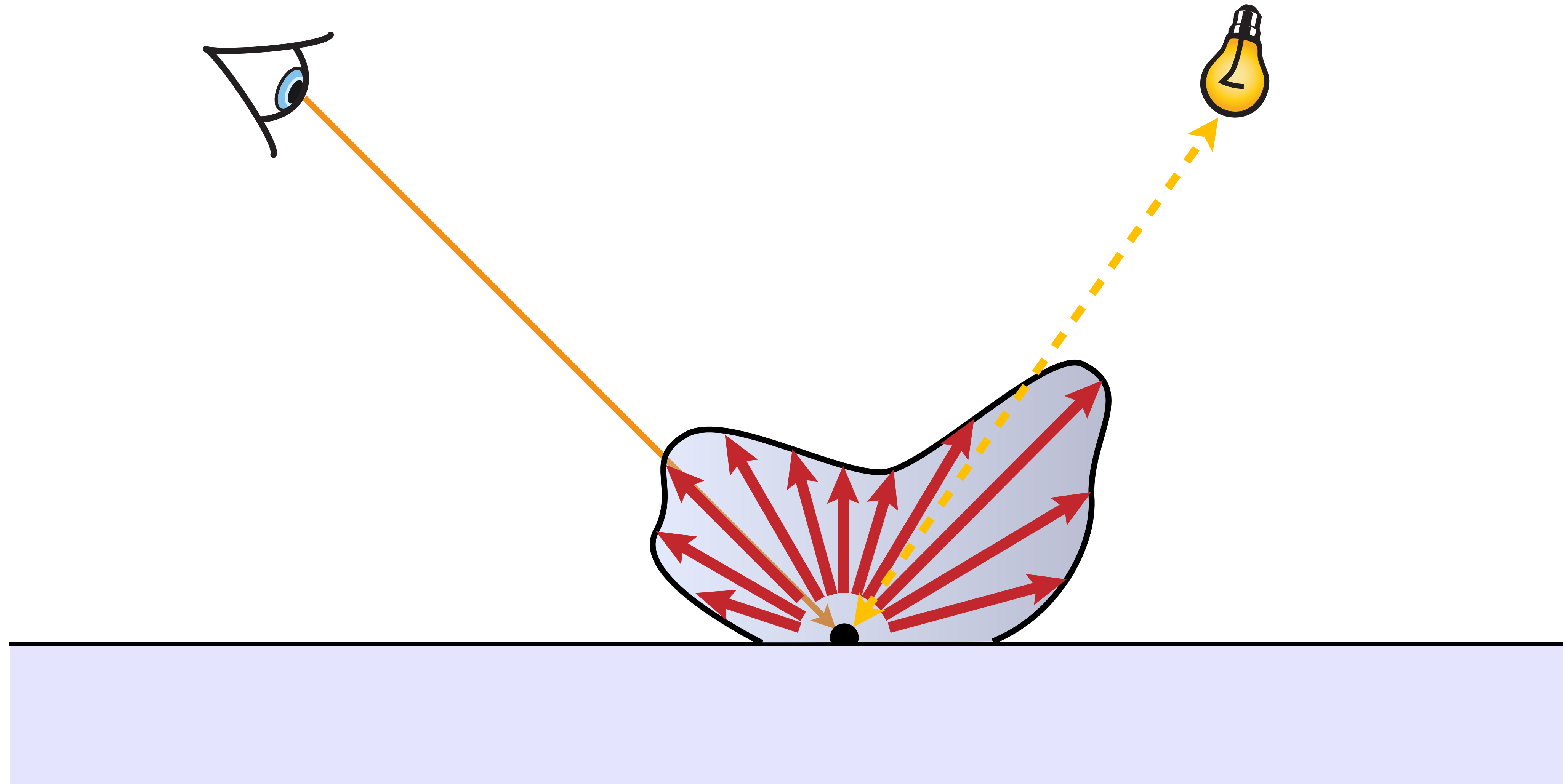
Compare to: specular microfacets



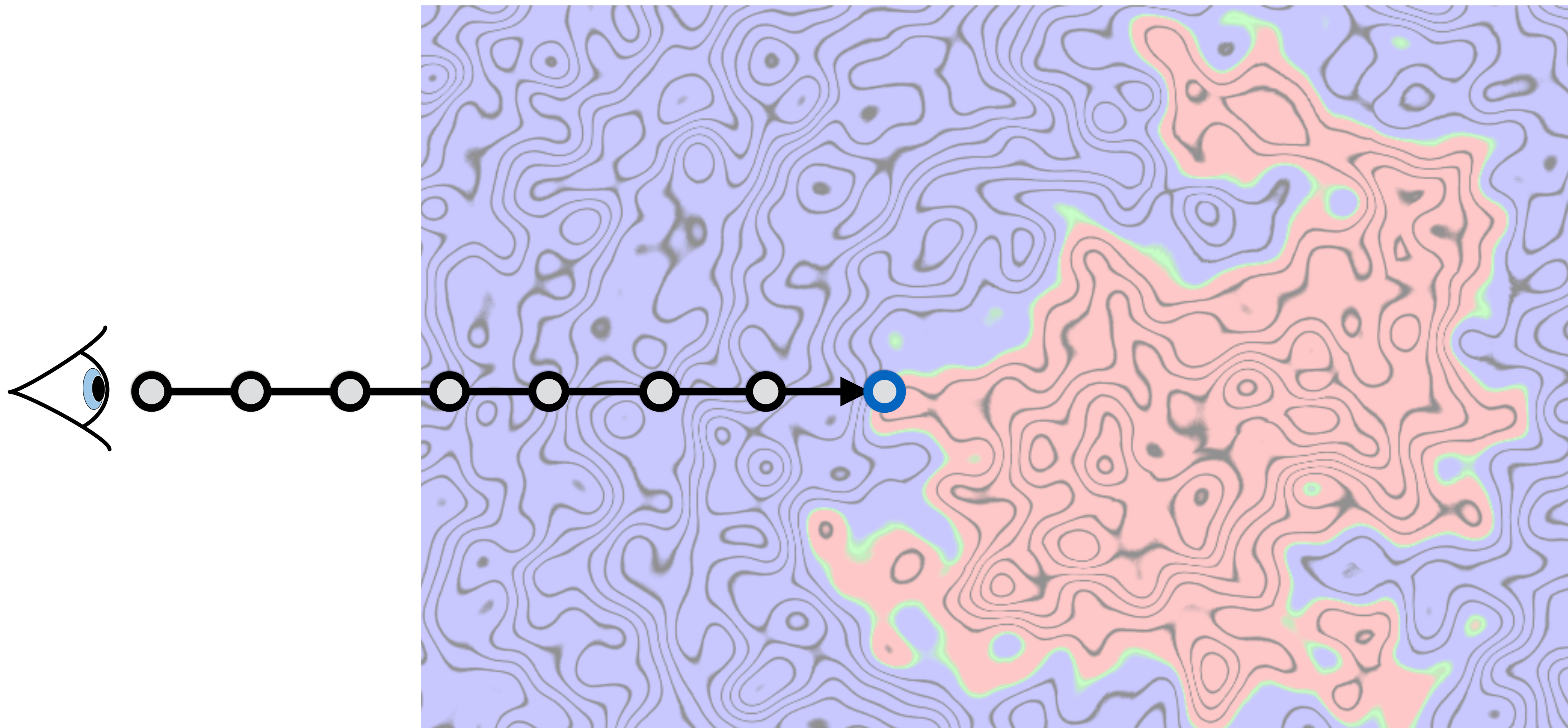
Compare to: specular microfacets



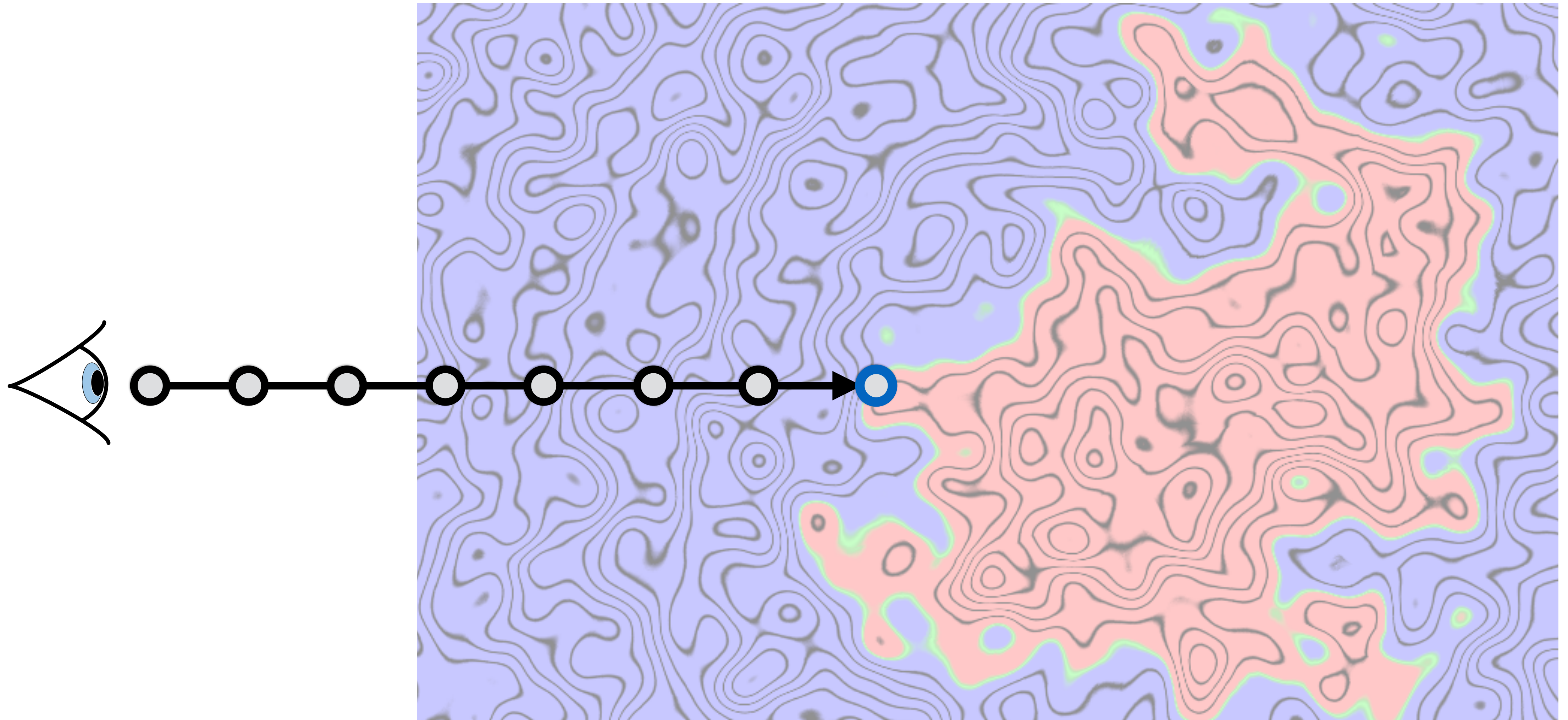
Compare to: specular microfacets



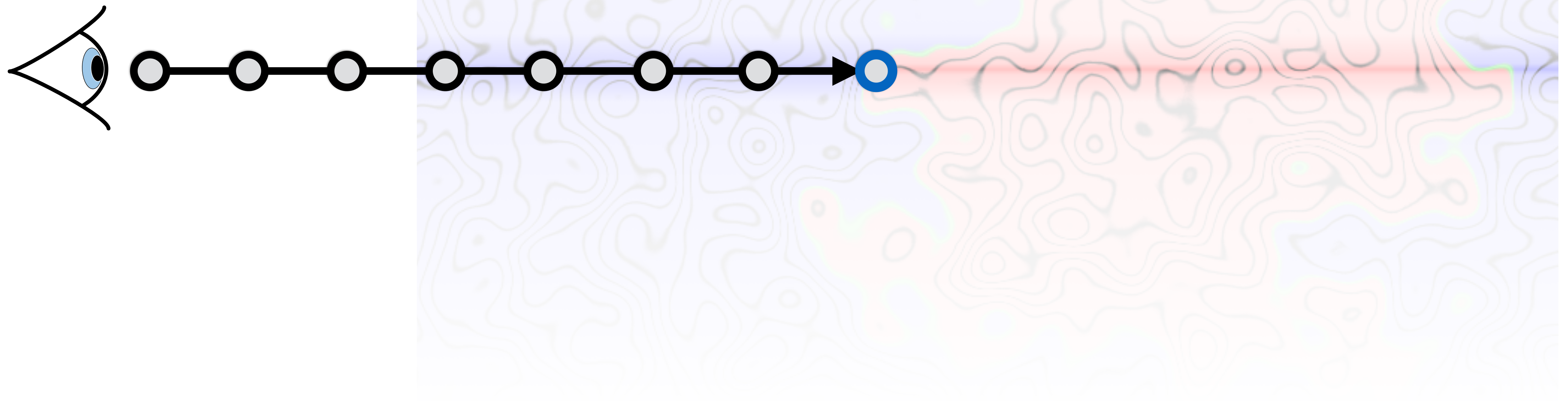
Only lock down along the ray



Only lock down along the ray



Only lock down along the ray

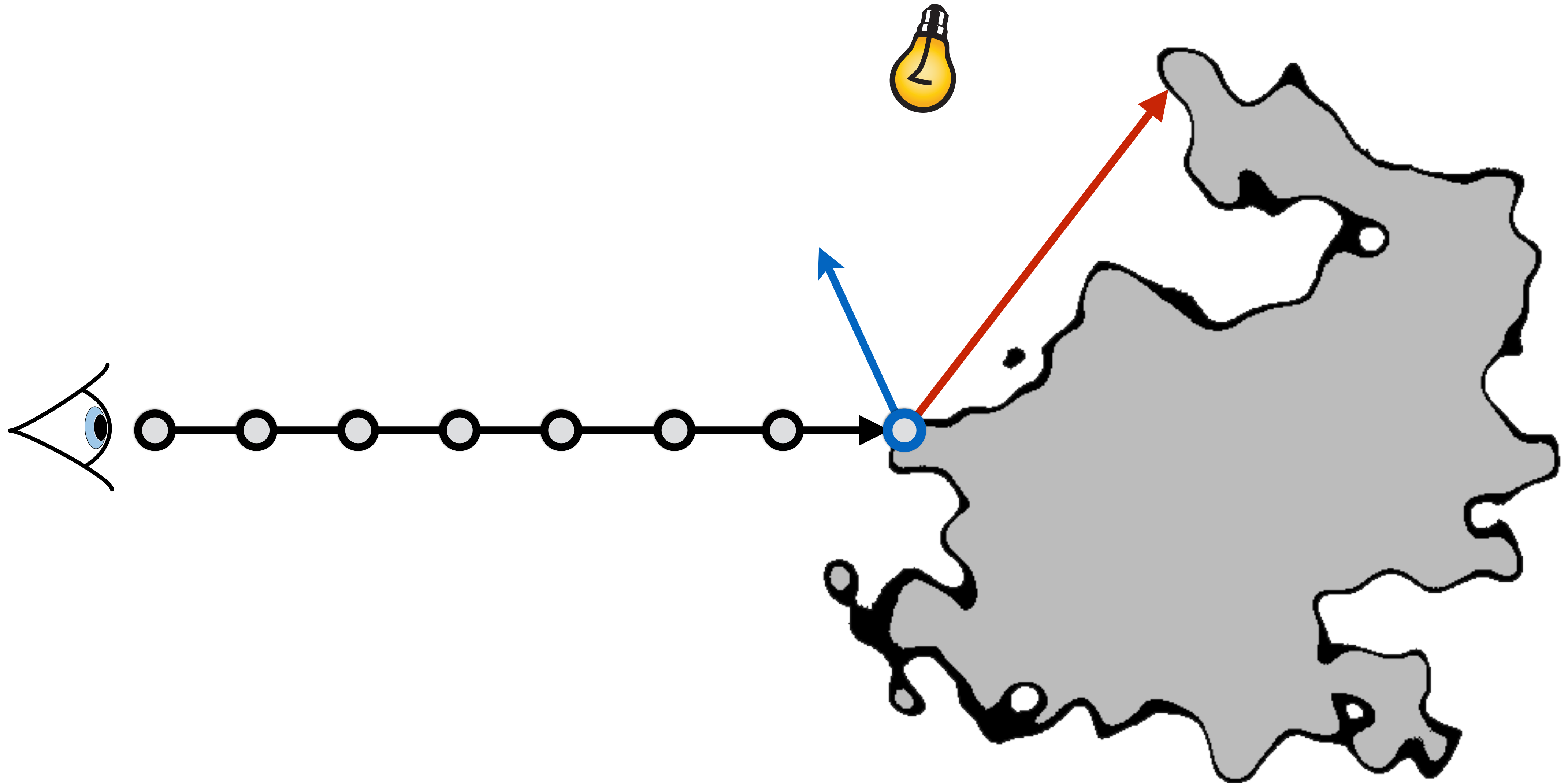


Only lock down along the ray

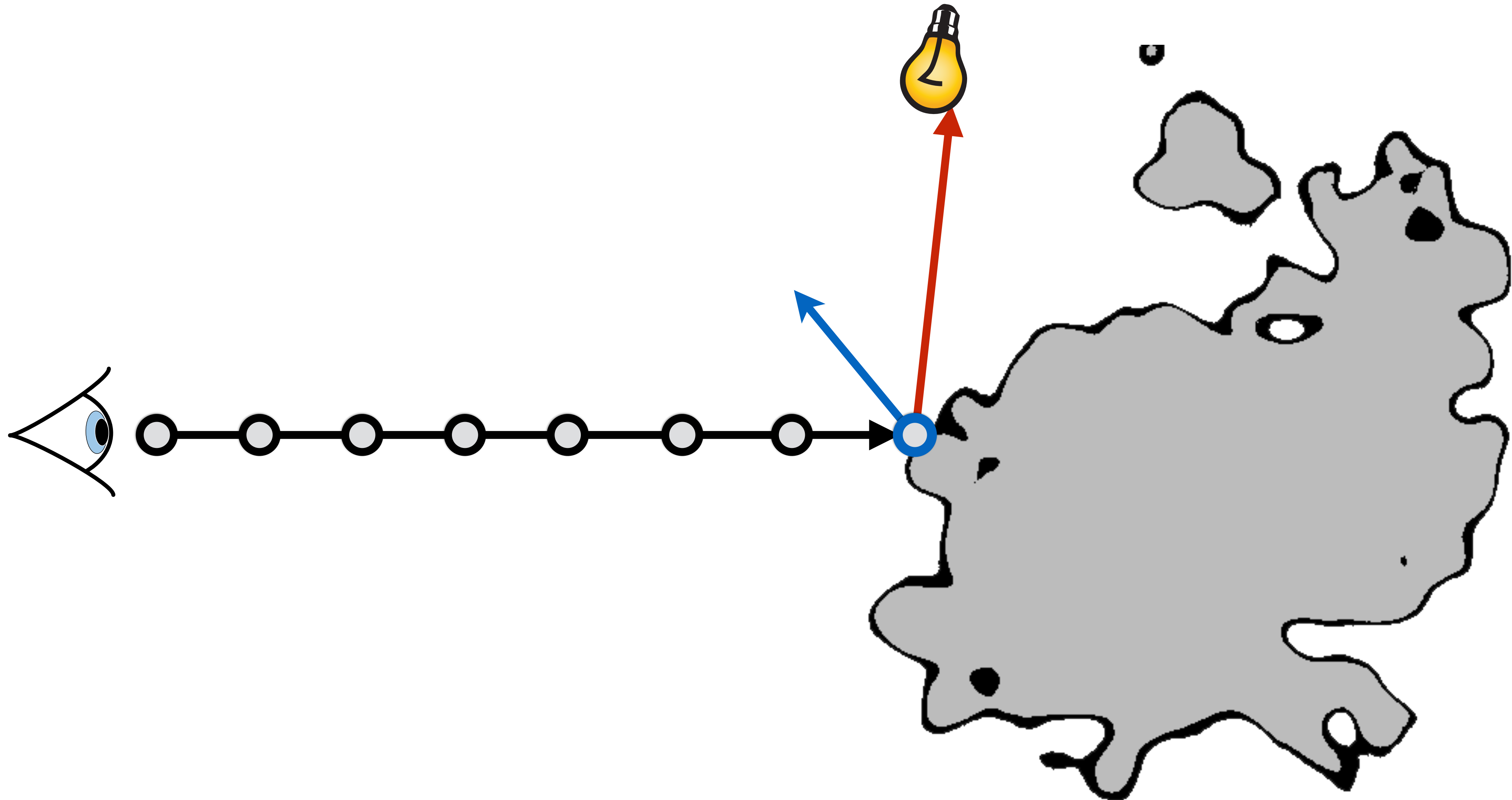
Only need 1D noise evaluations: **Faster!**



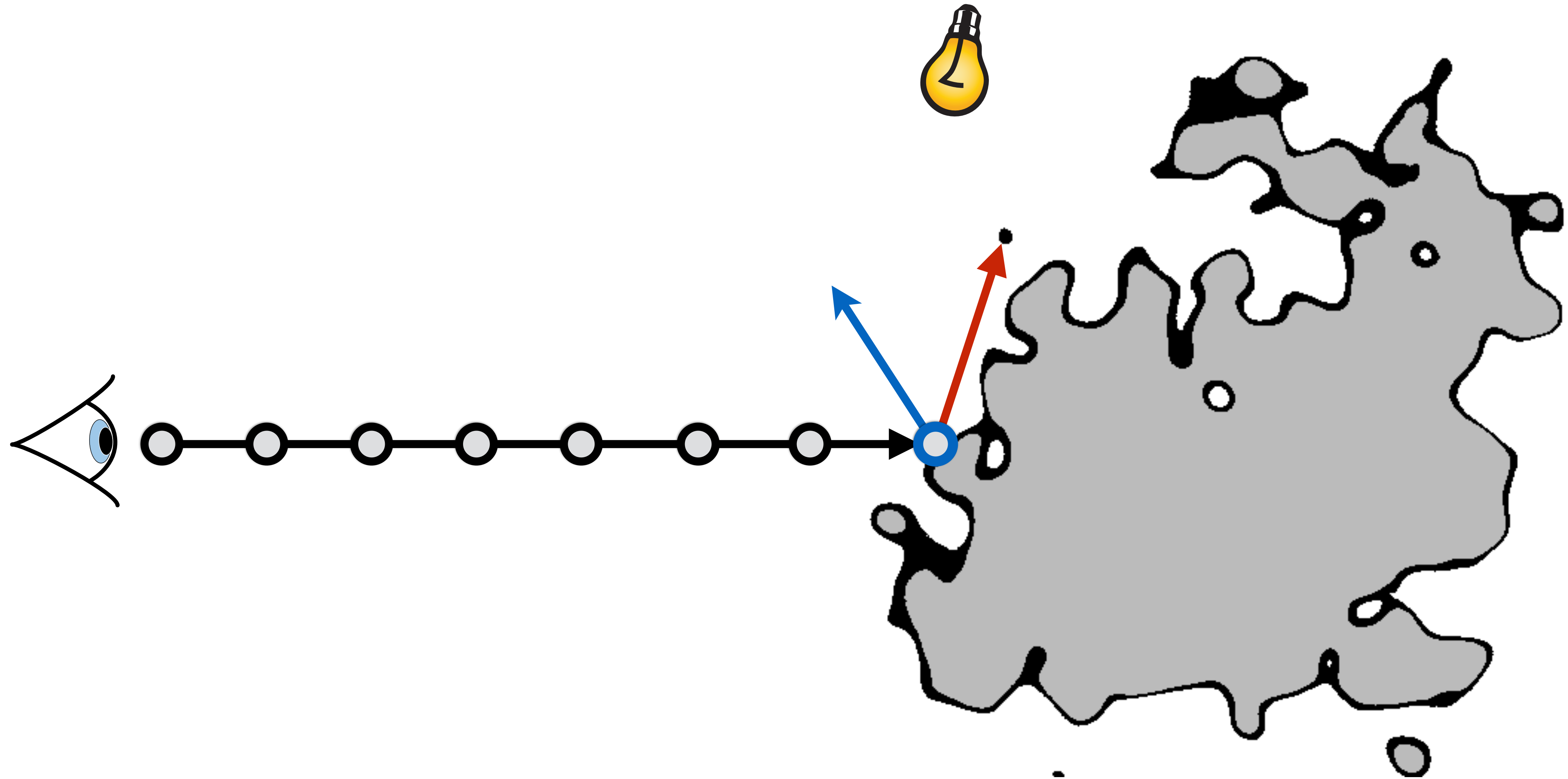
Still have a distribution of surfaces



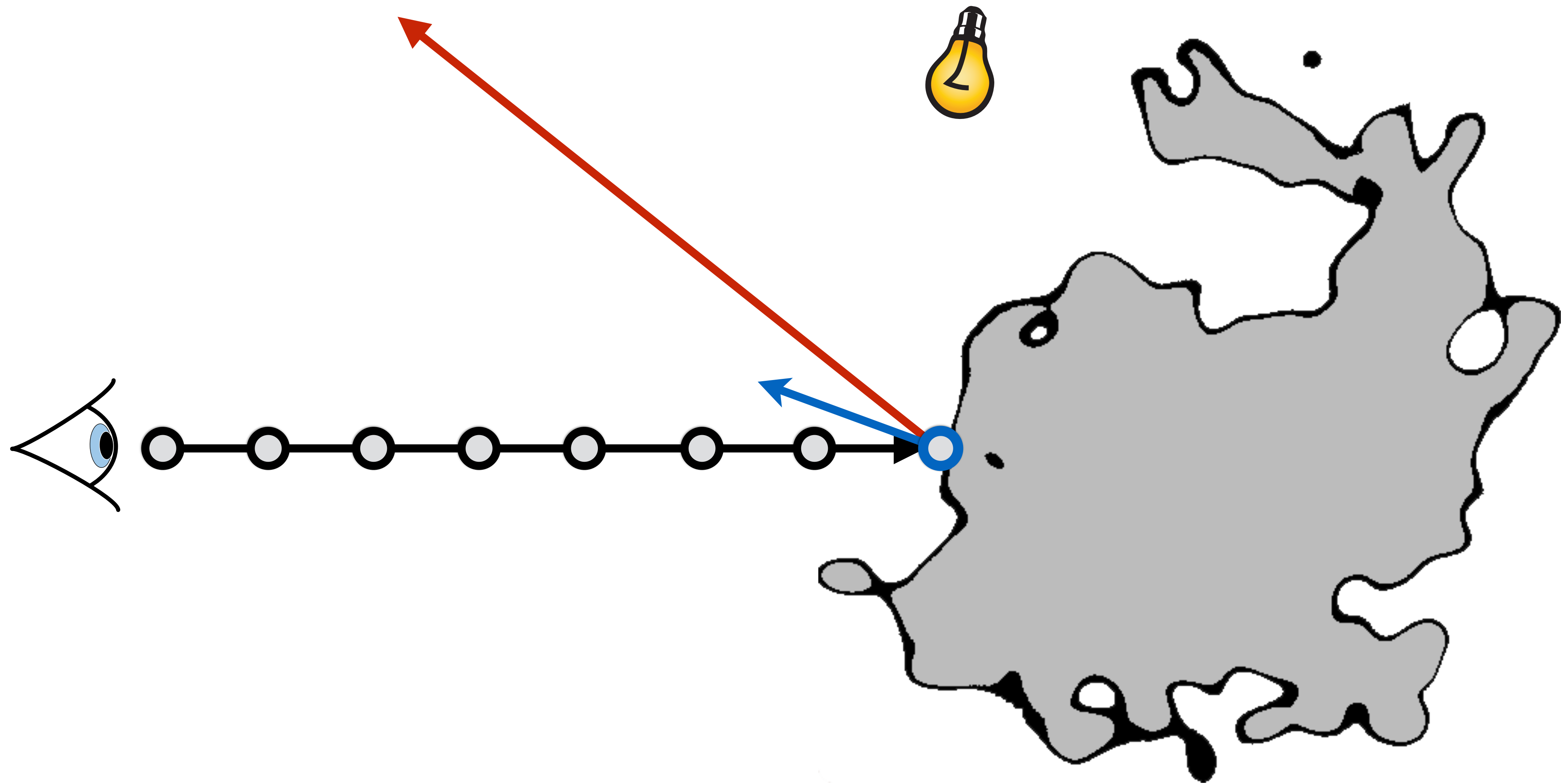
Still have a distribution of surfaces



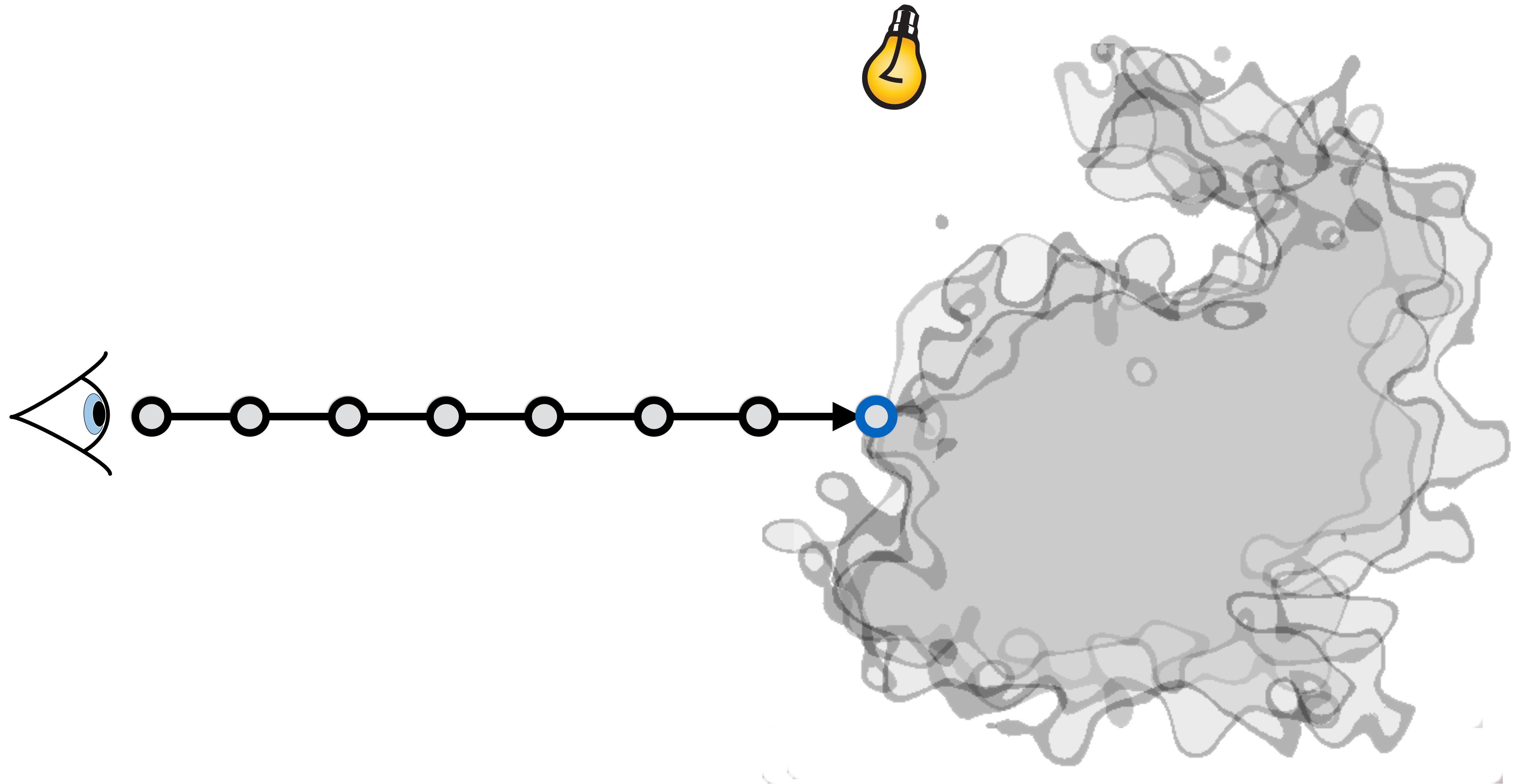
Still have a distribution of surfaces



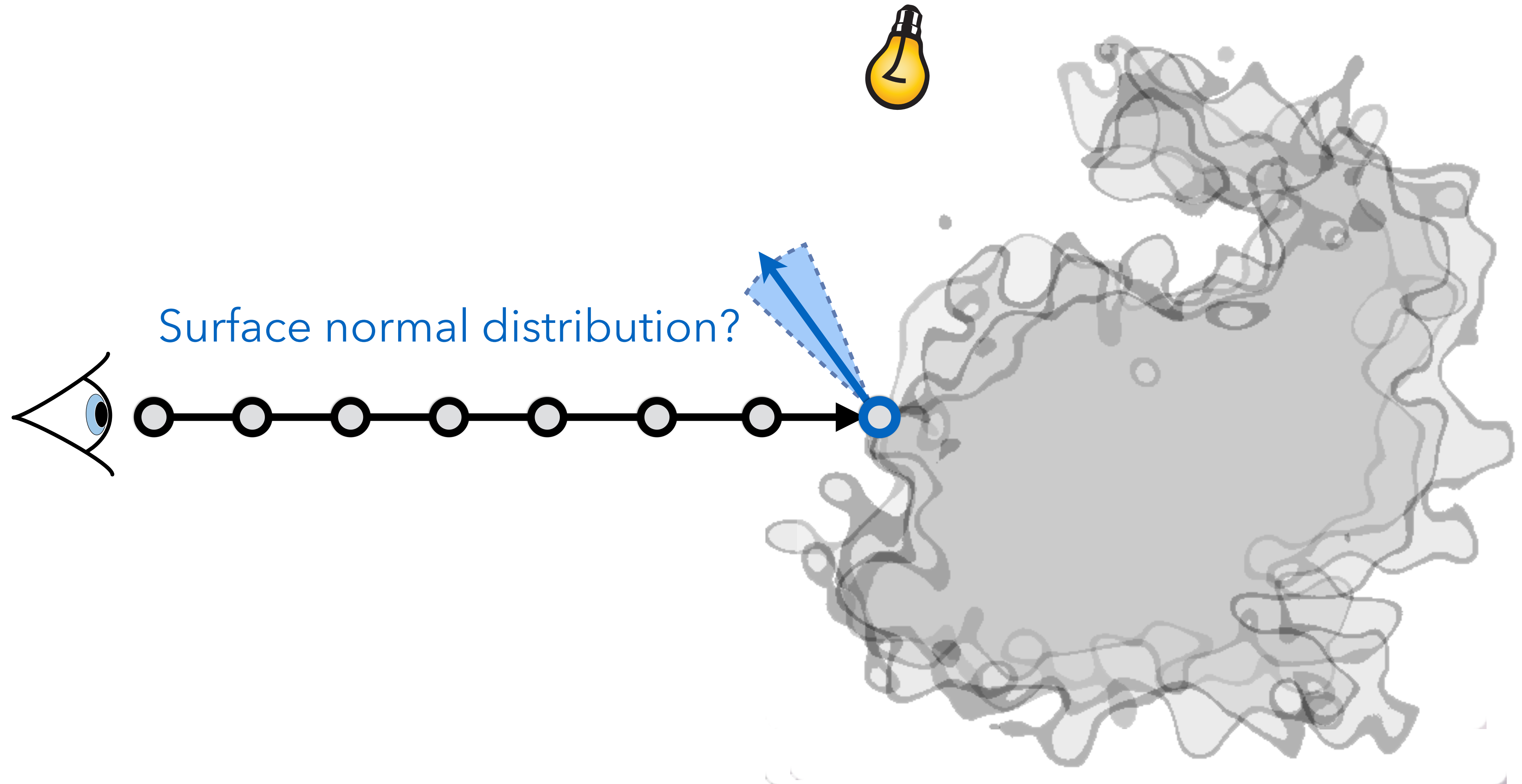
Still have a distribution of surfaces



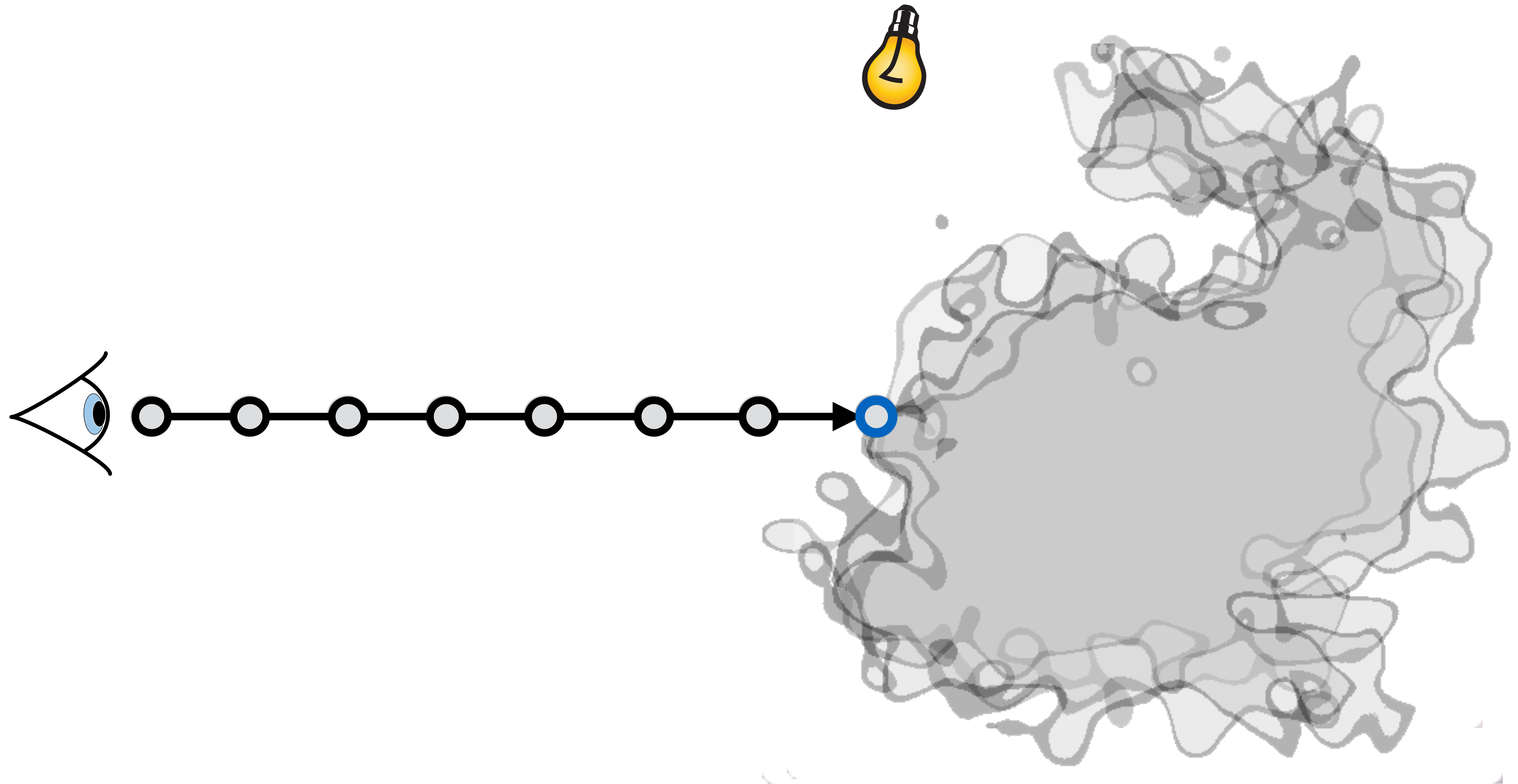
Still have a distribution of surfaces



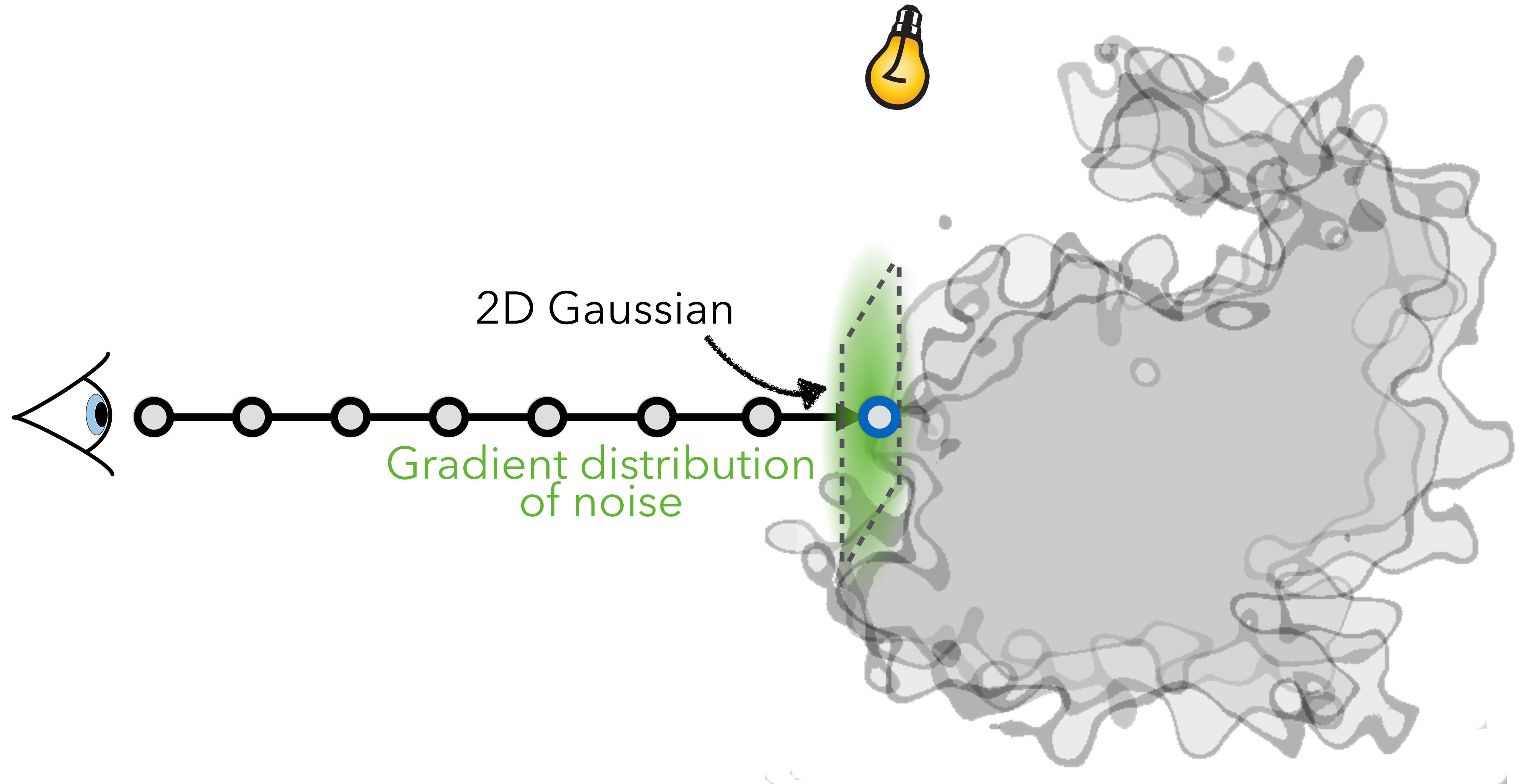
Still have a distribution of surfaces



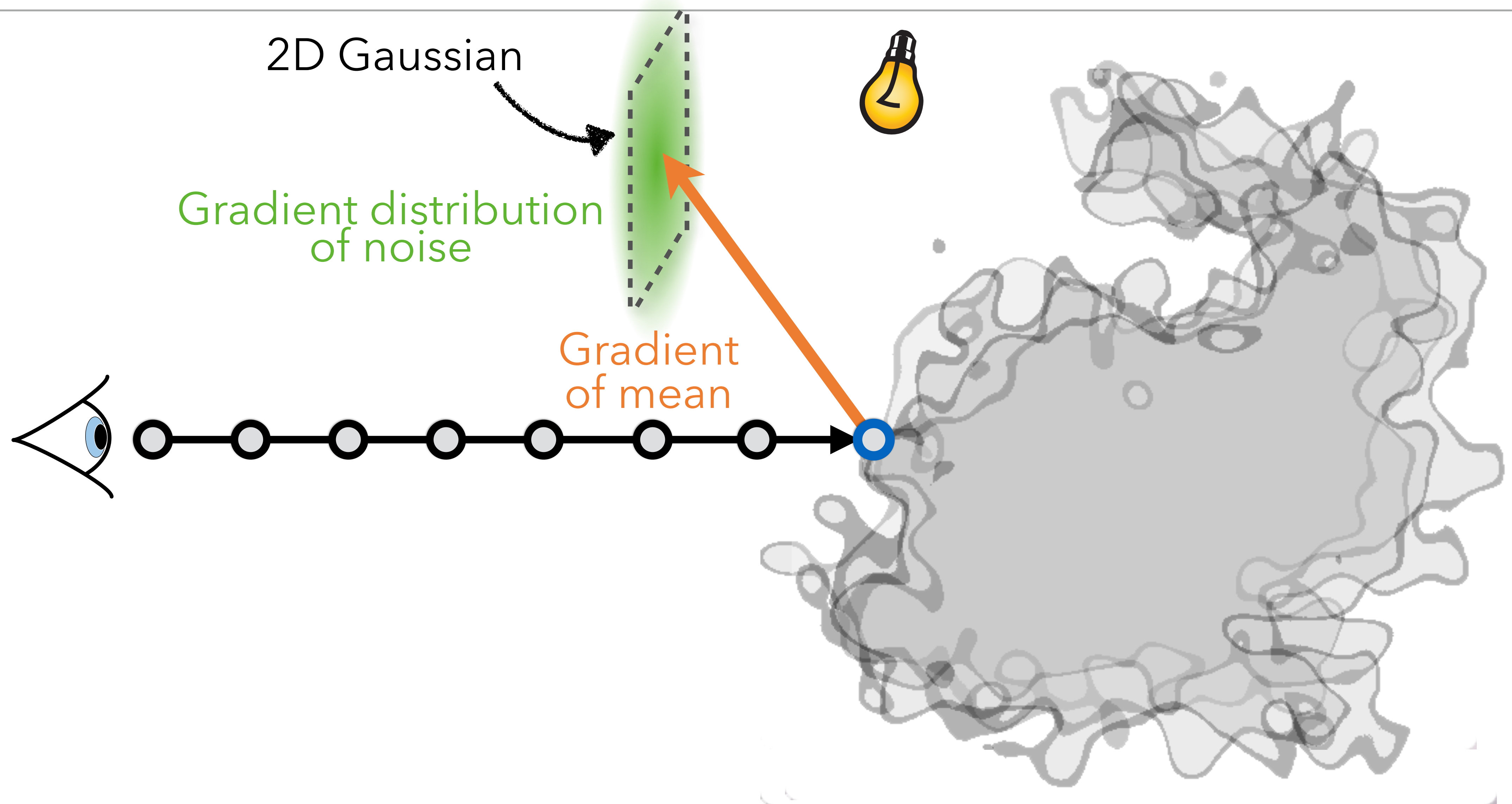
Distribution of normals



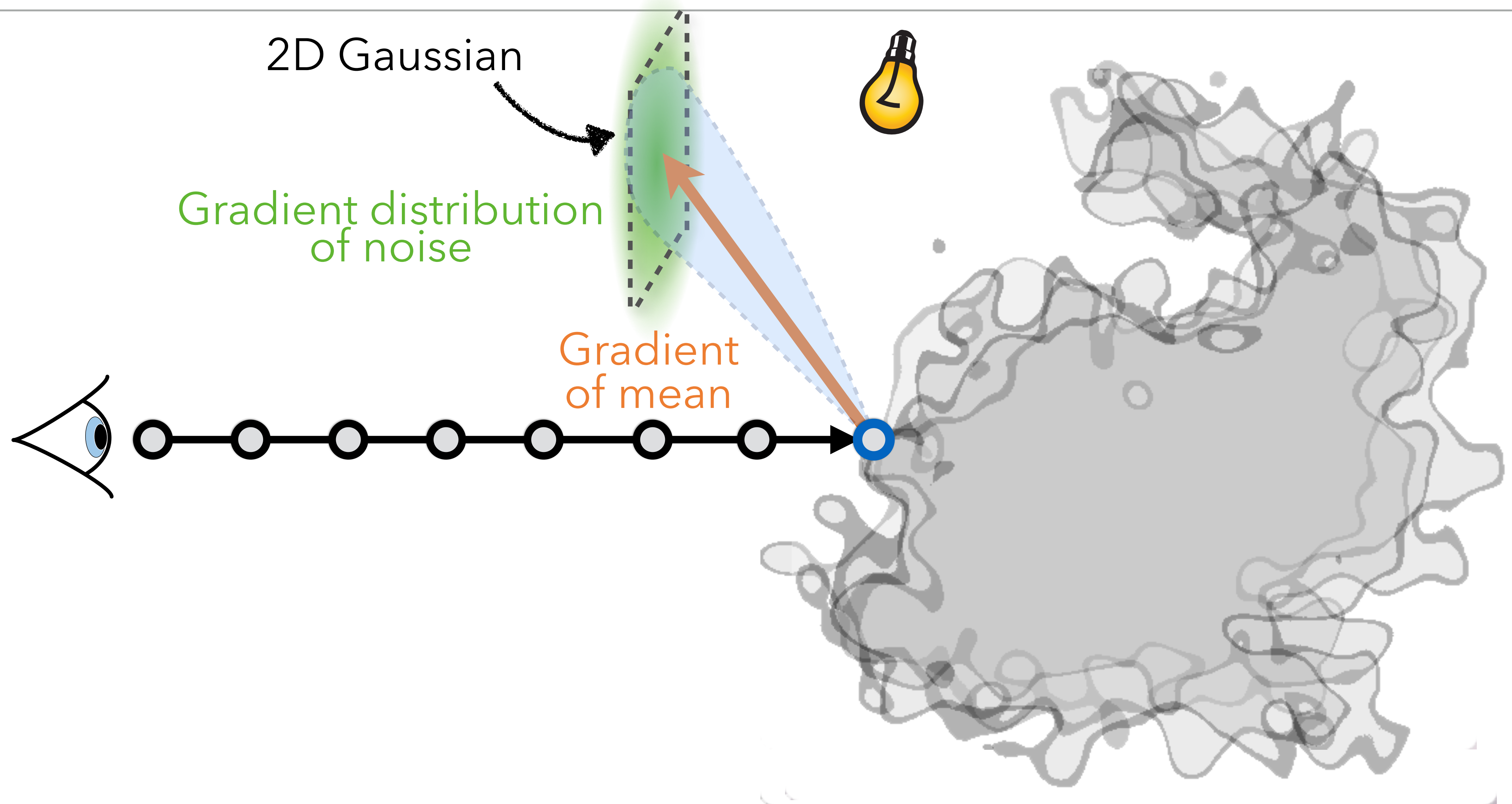
Distribution of normals



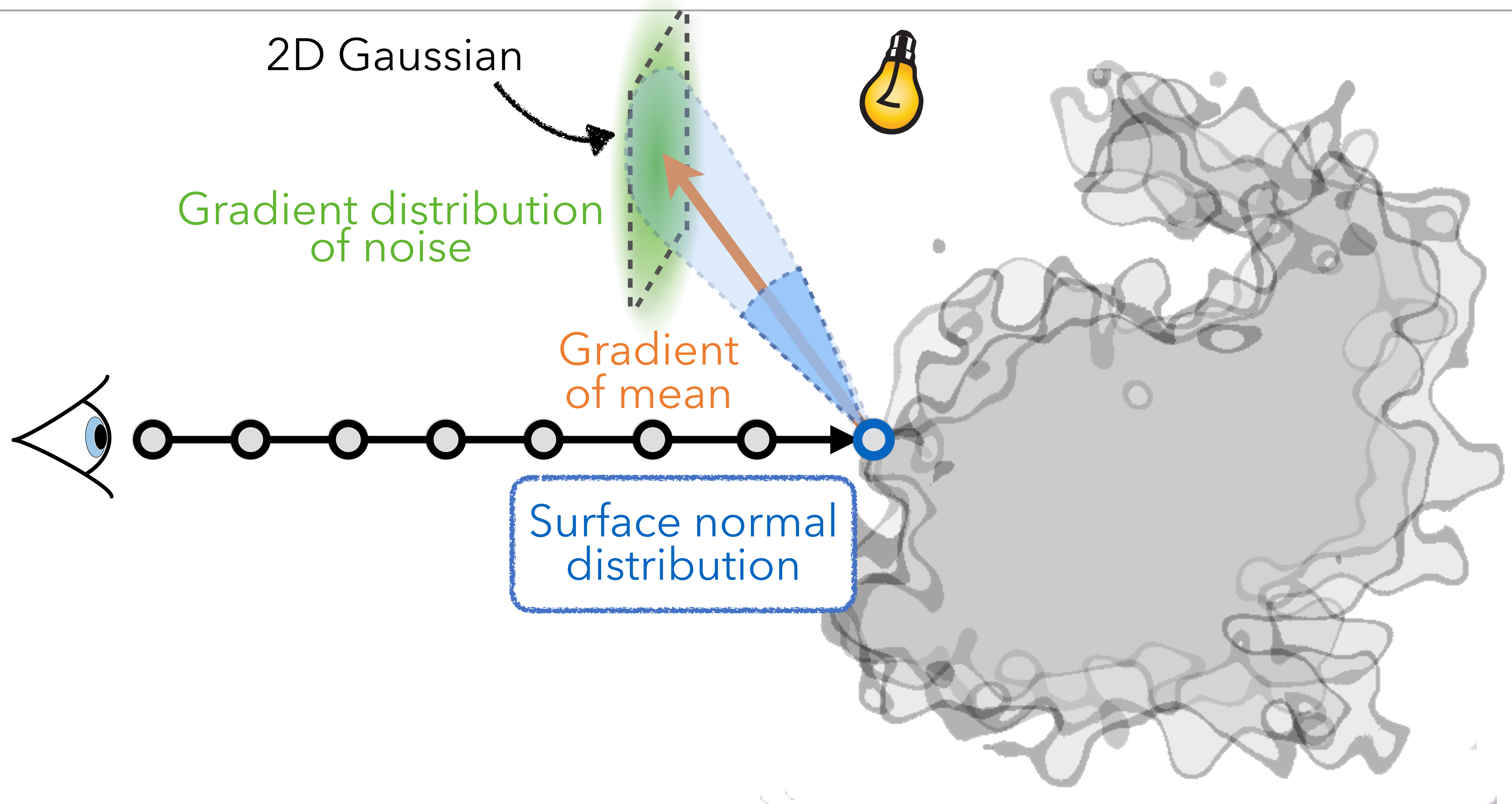
Distribution of normals



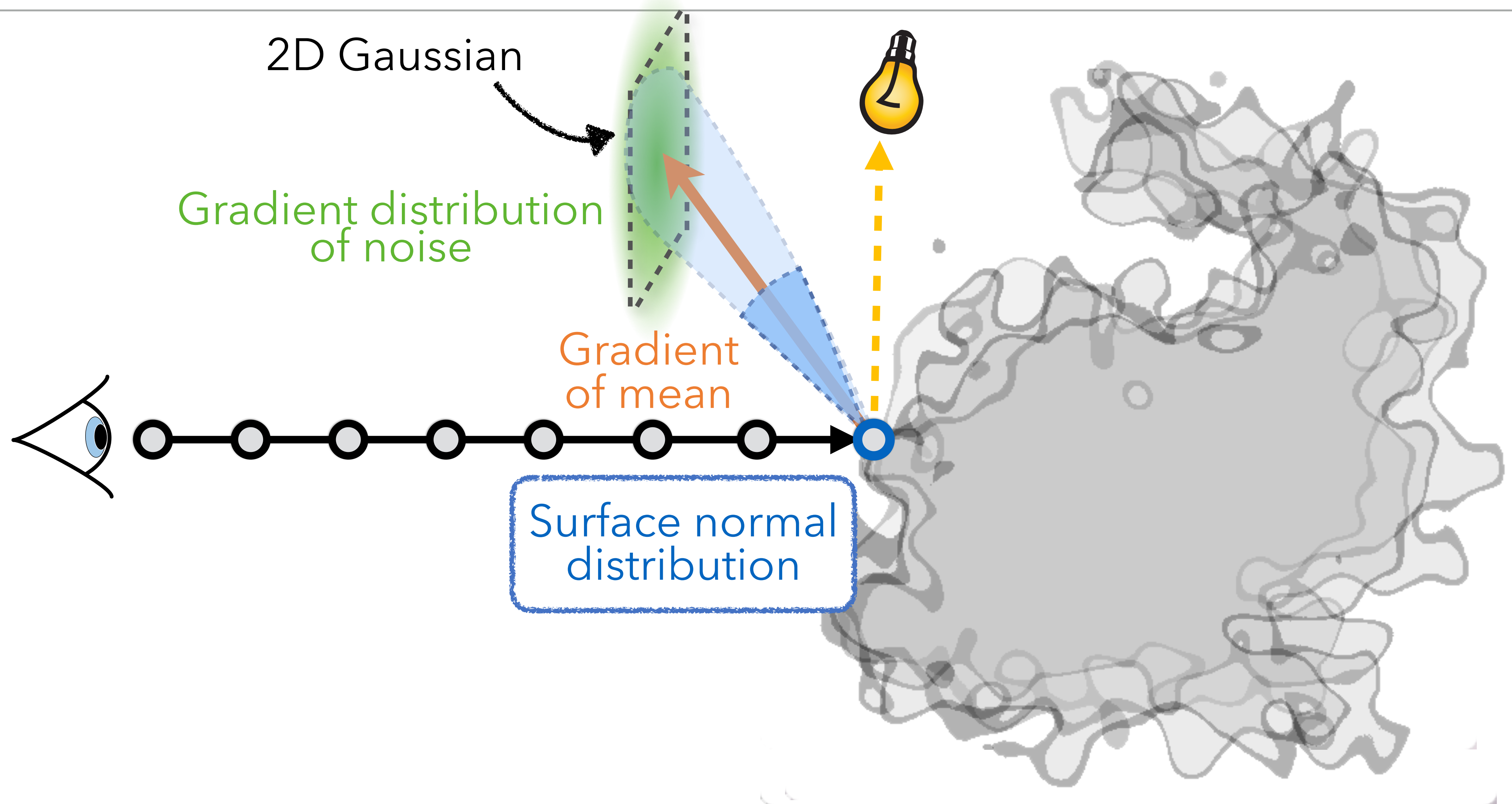
Distribution of normals



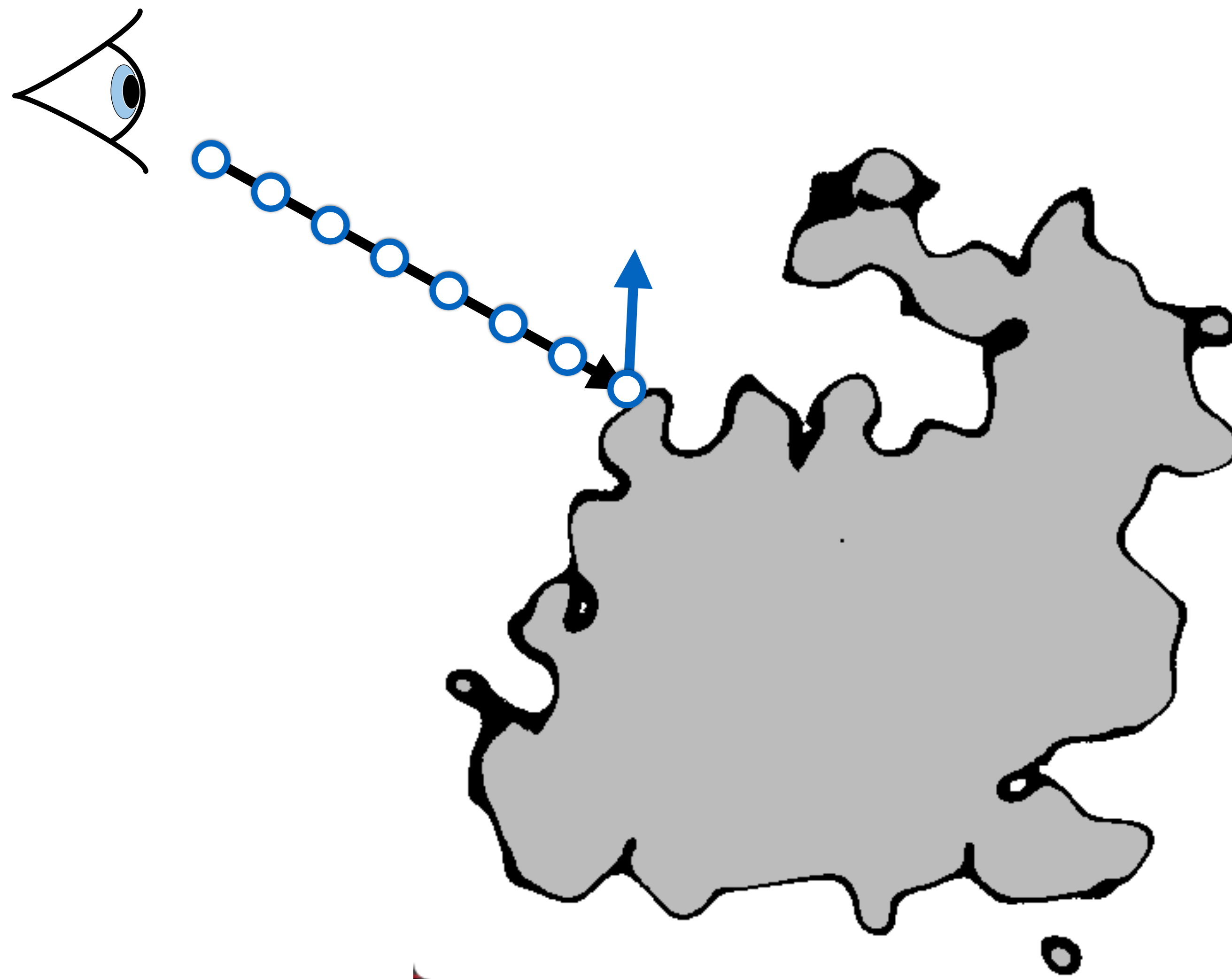
Distribution of normals



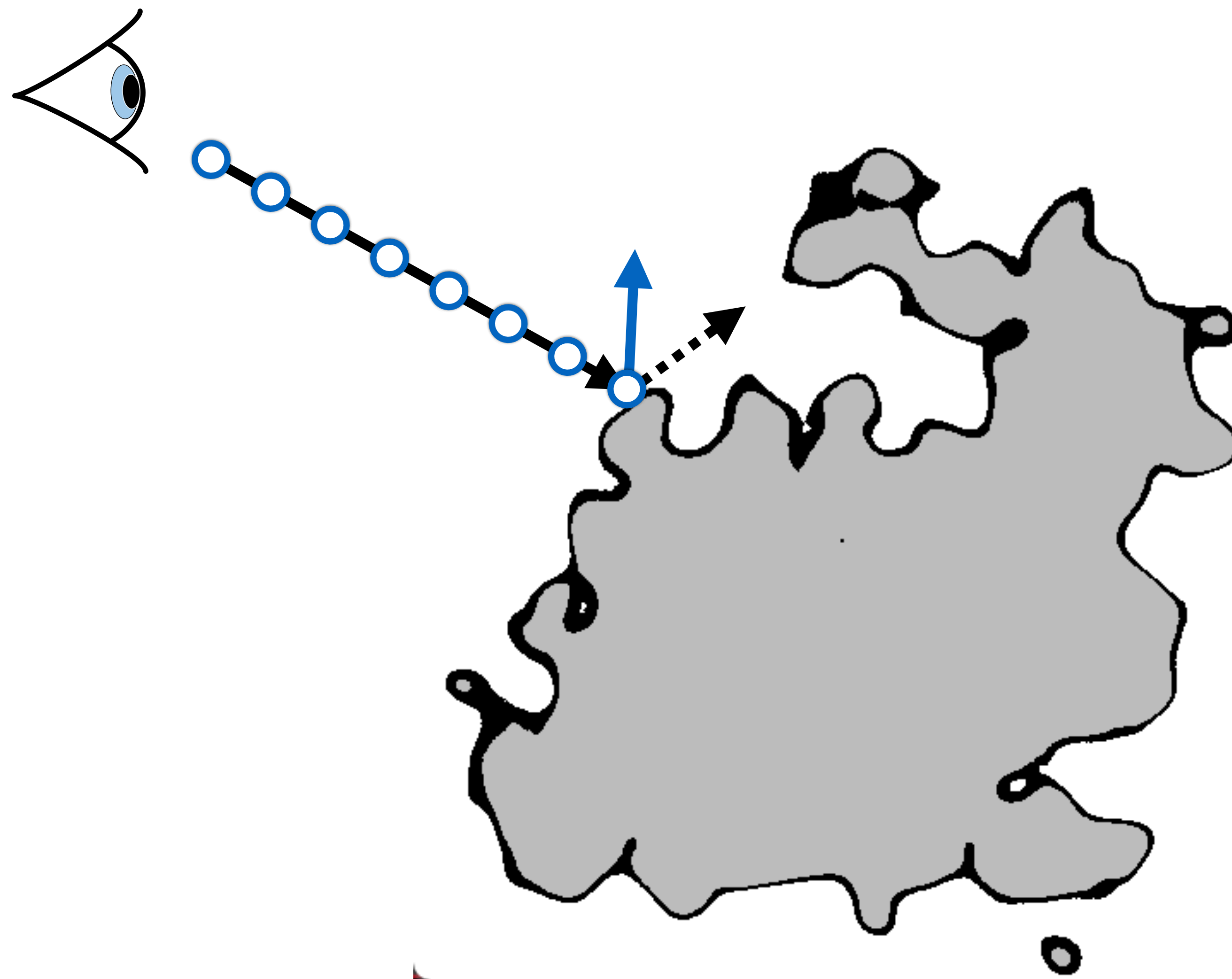
Distribution of normals



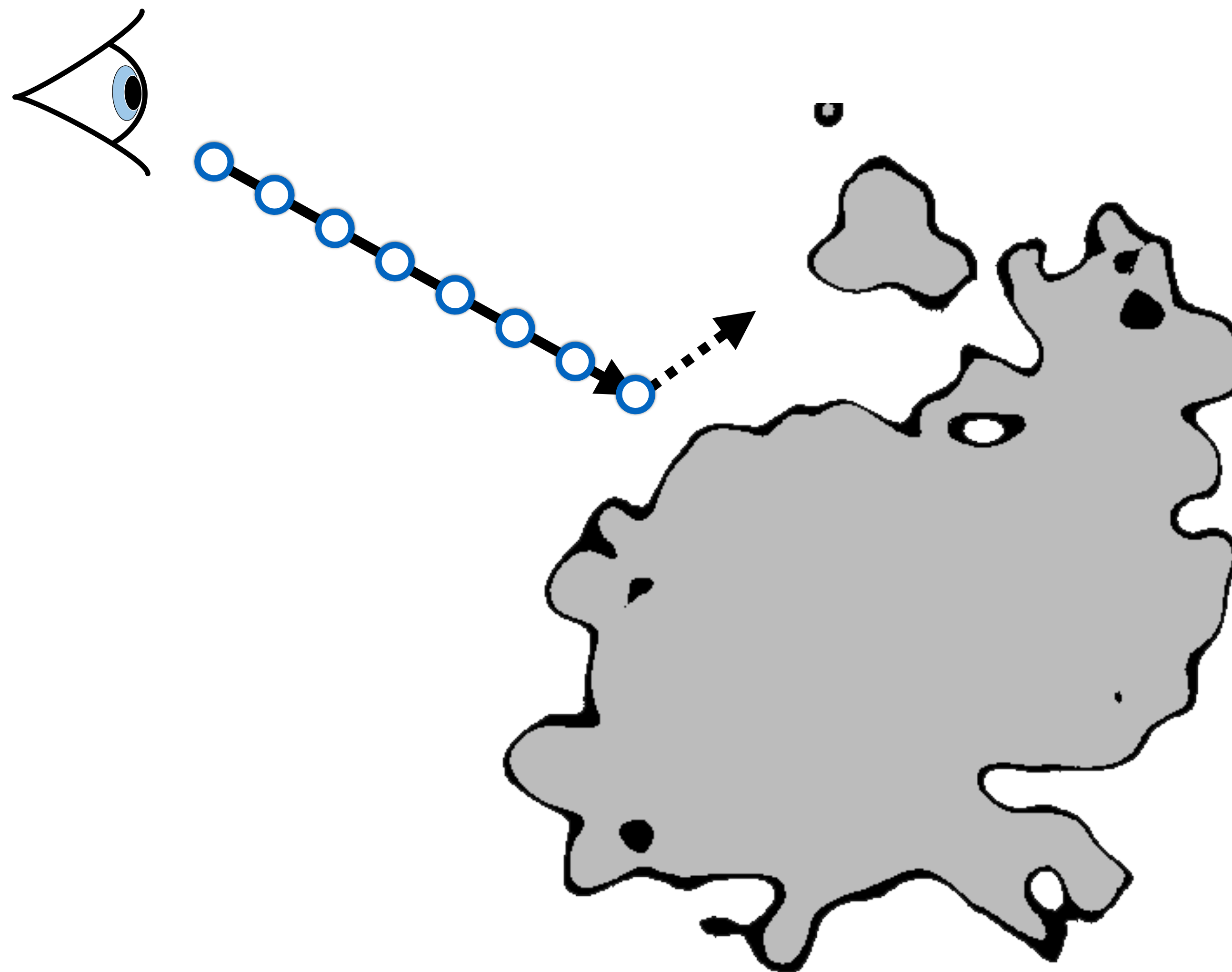
Tracing a Full Path



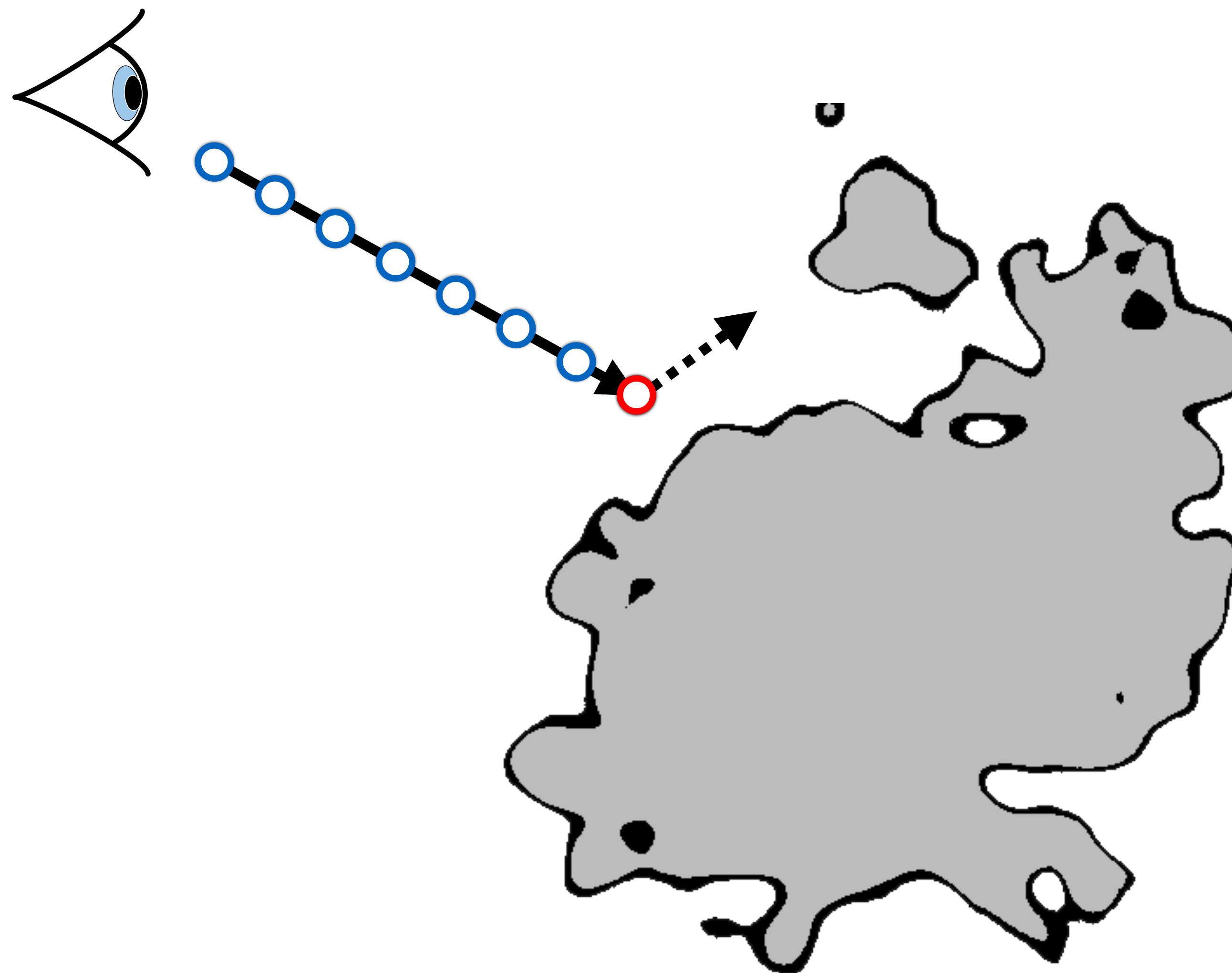
Tracing a Full Path



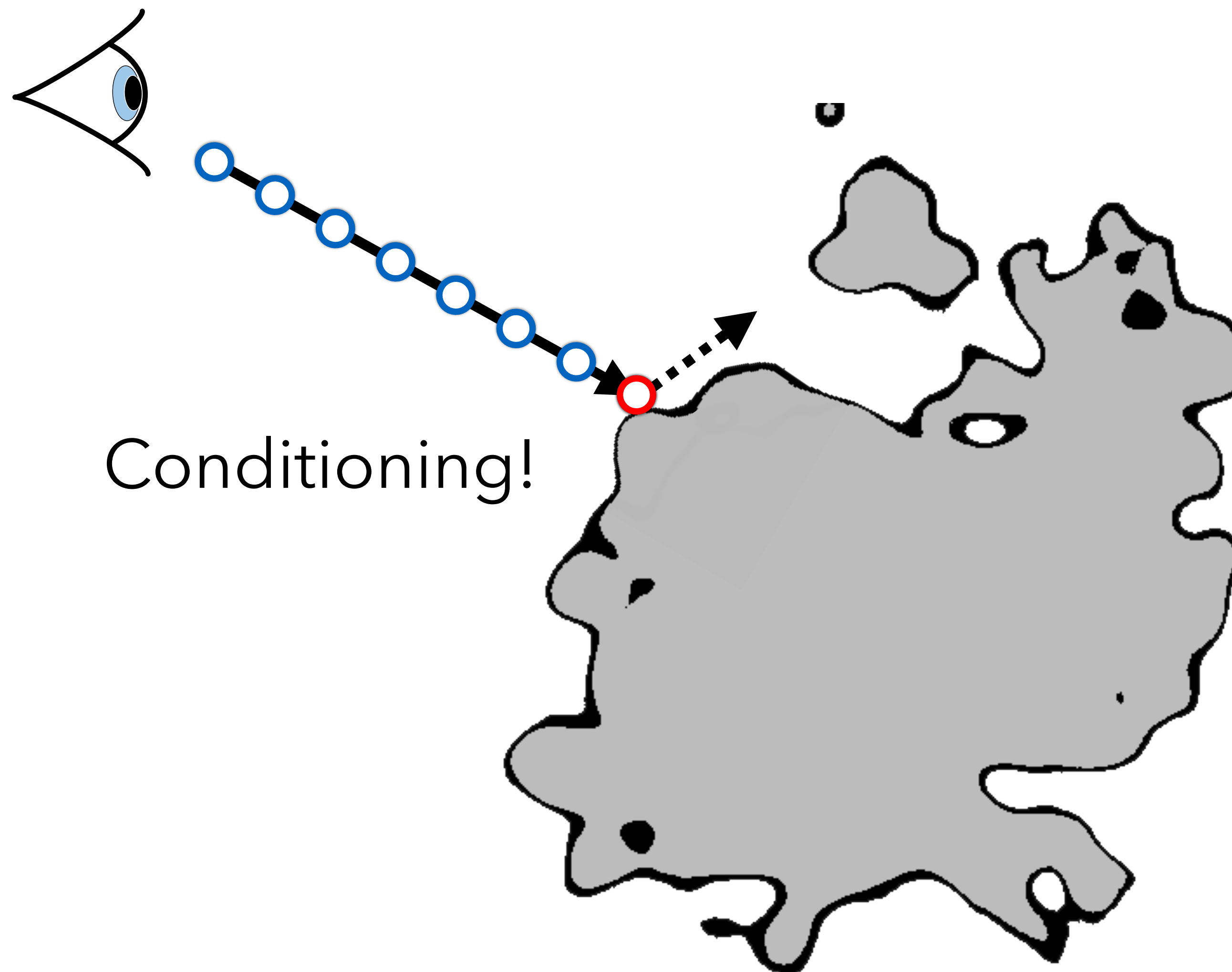
Tracing a Full Path



Tracing a Full Path

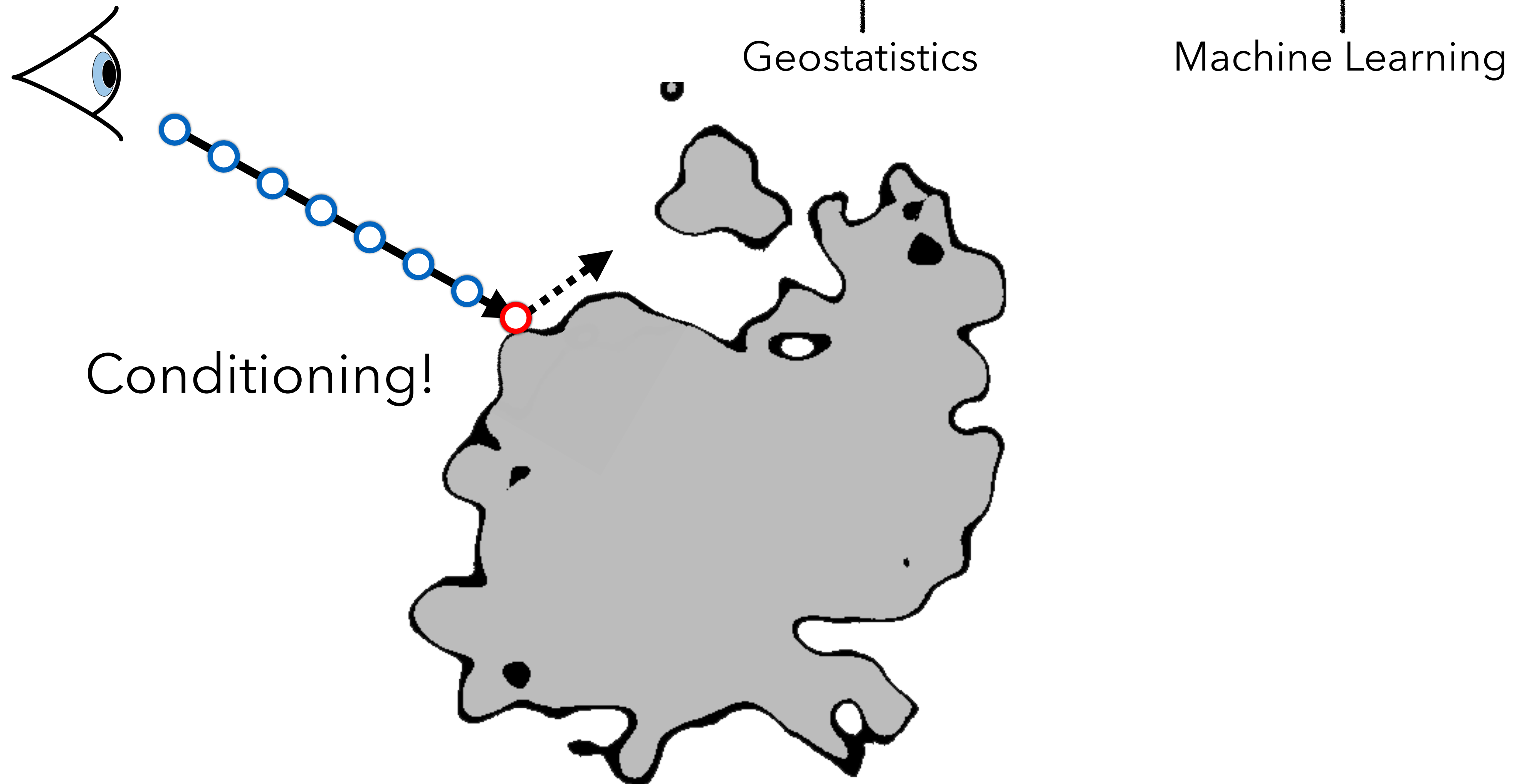


Tracing a Full Path



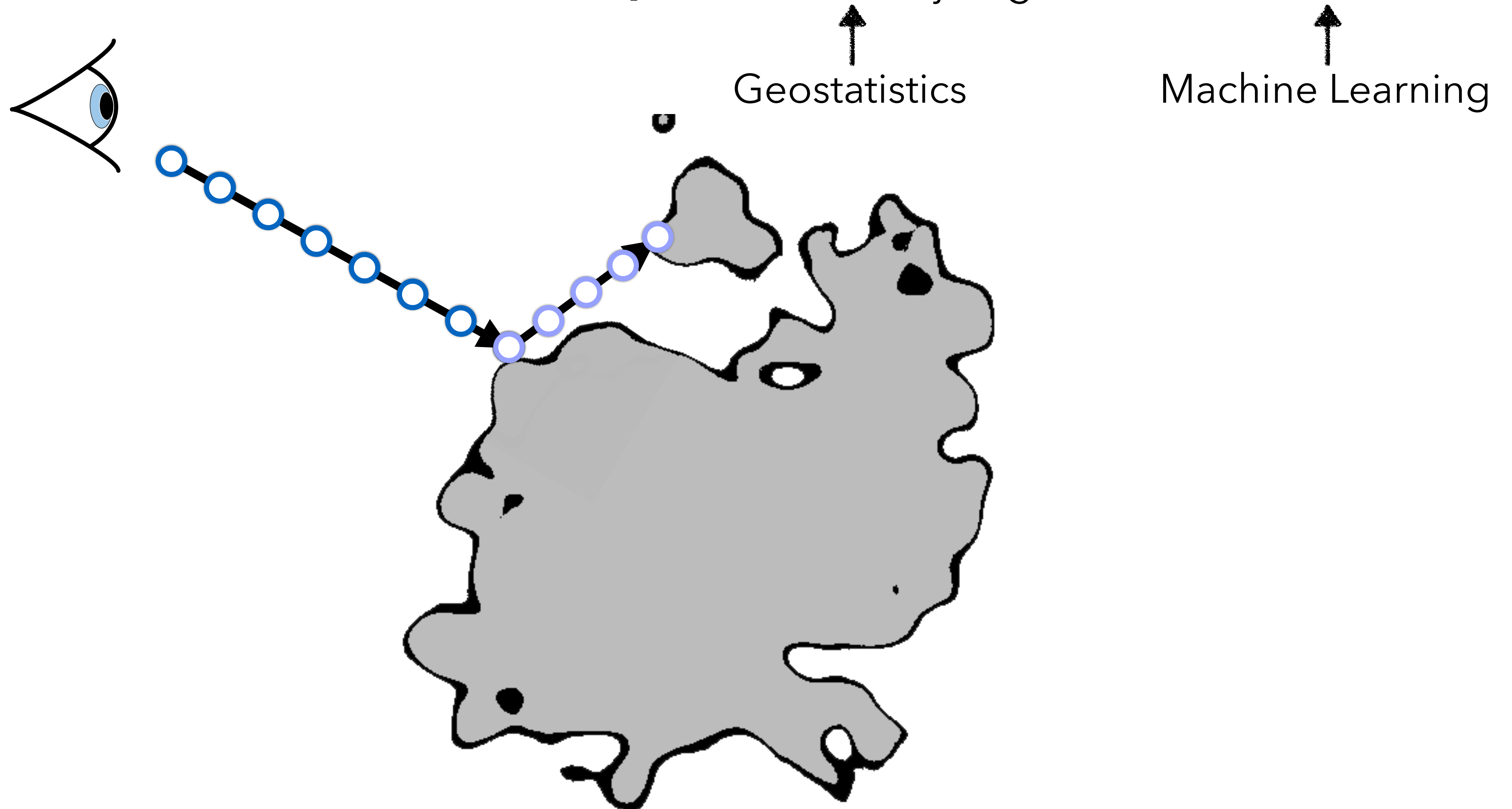
Tracing a Full Path

Conditioning via “pathwise updates”
[Journel and Huijbregts 1978, Wilson et al. 2020]



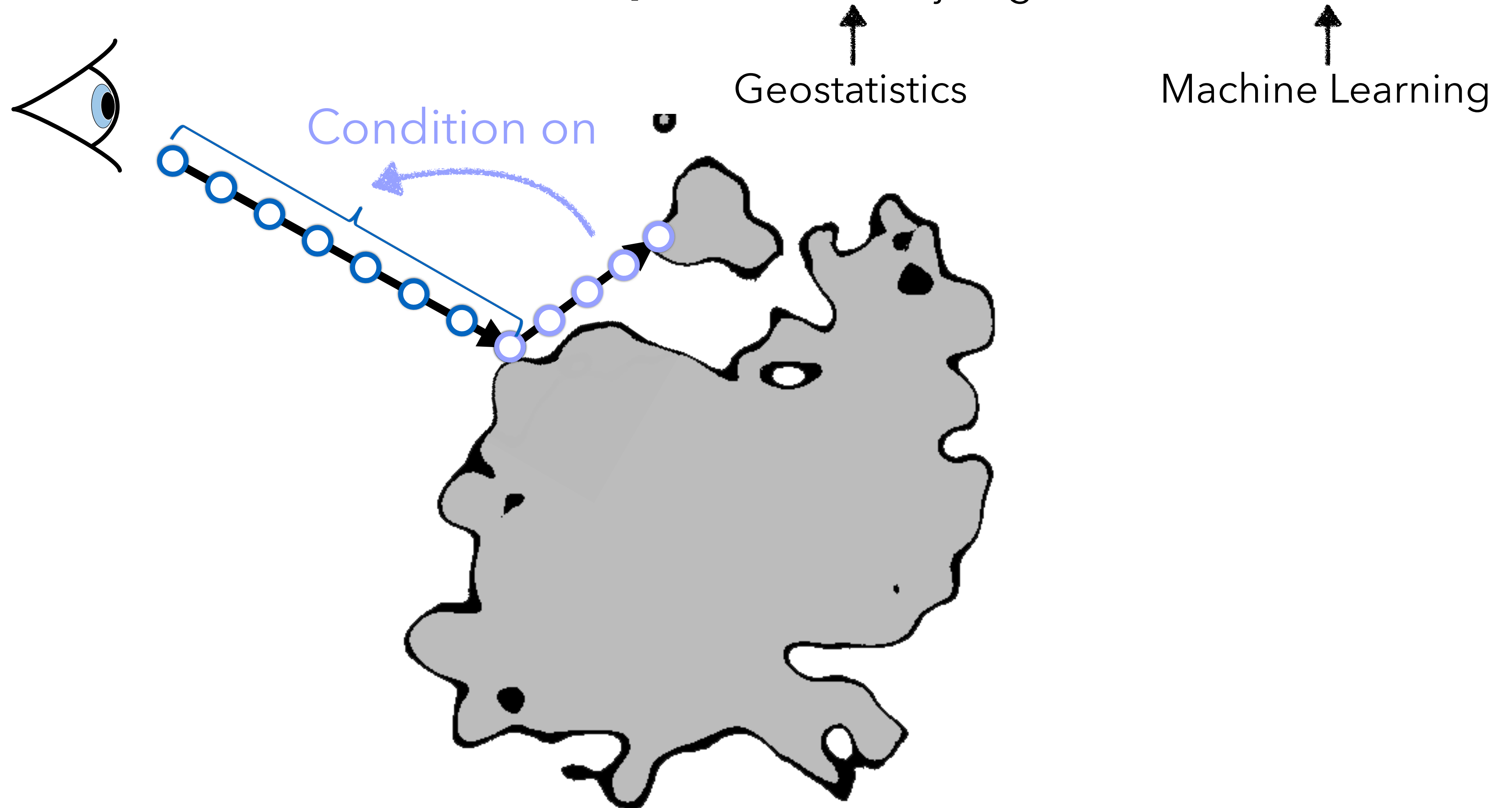
Tracing a Full Path

Conditioning via “pathwise updates”
[Journel and Huijbregts 1978, Wilson et al. 2020]



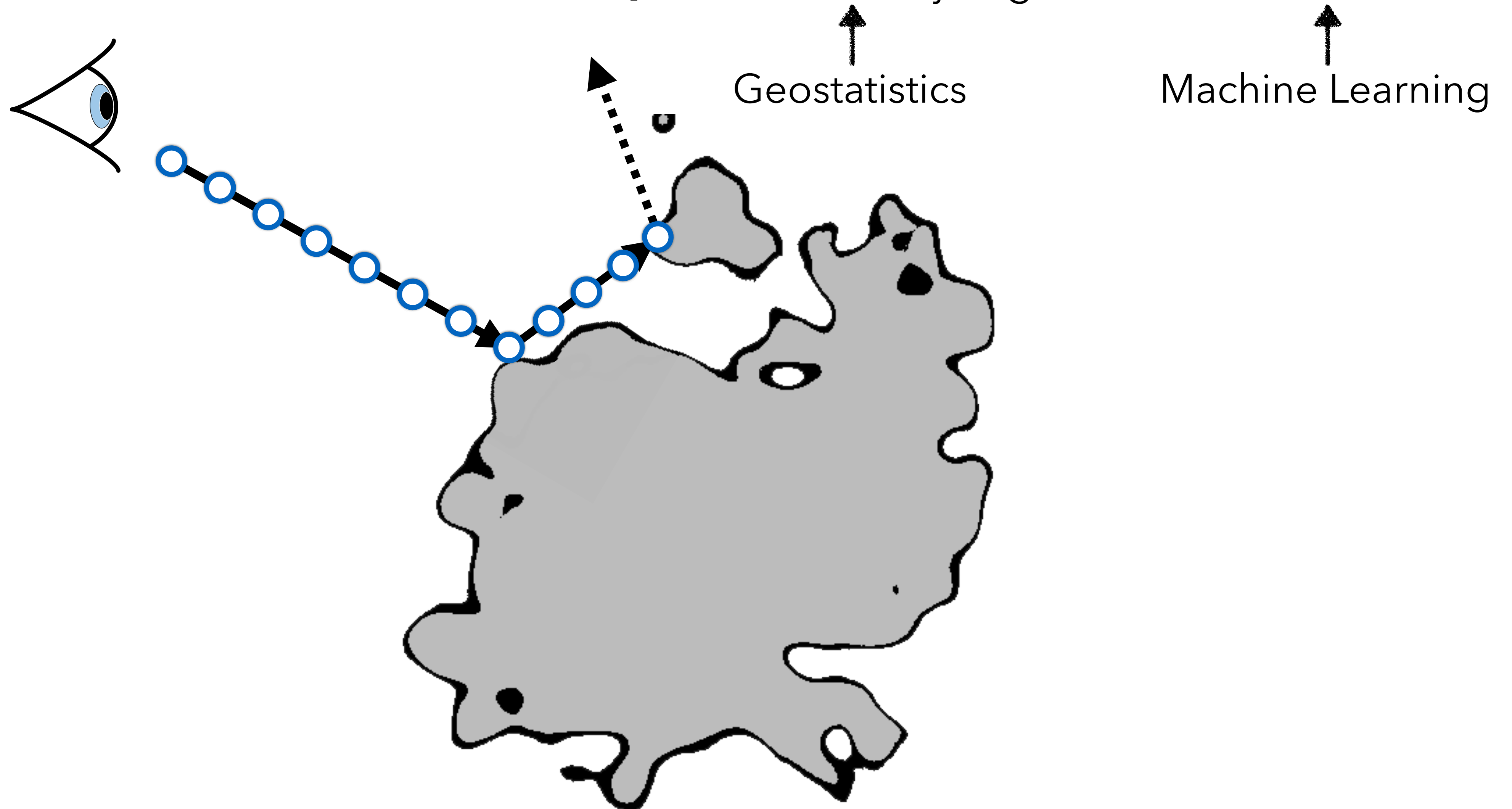
Tracing a Full Path

Conditioning via “pathwise updates”
[Journel and Huijbregts 1978, Wilson et al. 2020]



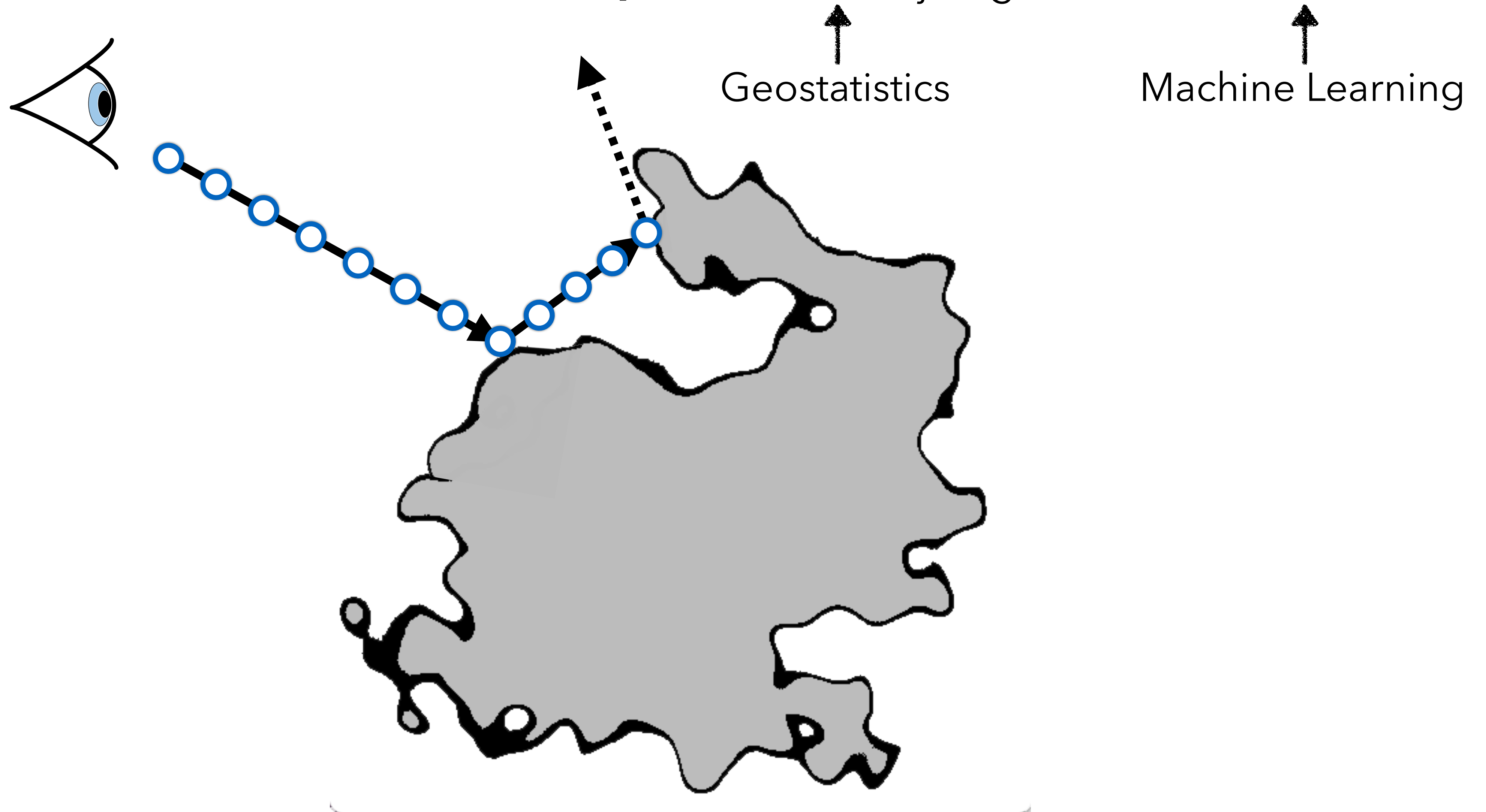
Tracing a Full Path

Conditioning via “pathwise updates”
[Journel and Huijbregts 1978, Wilson et al. 2020]

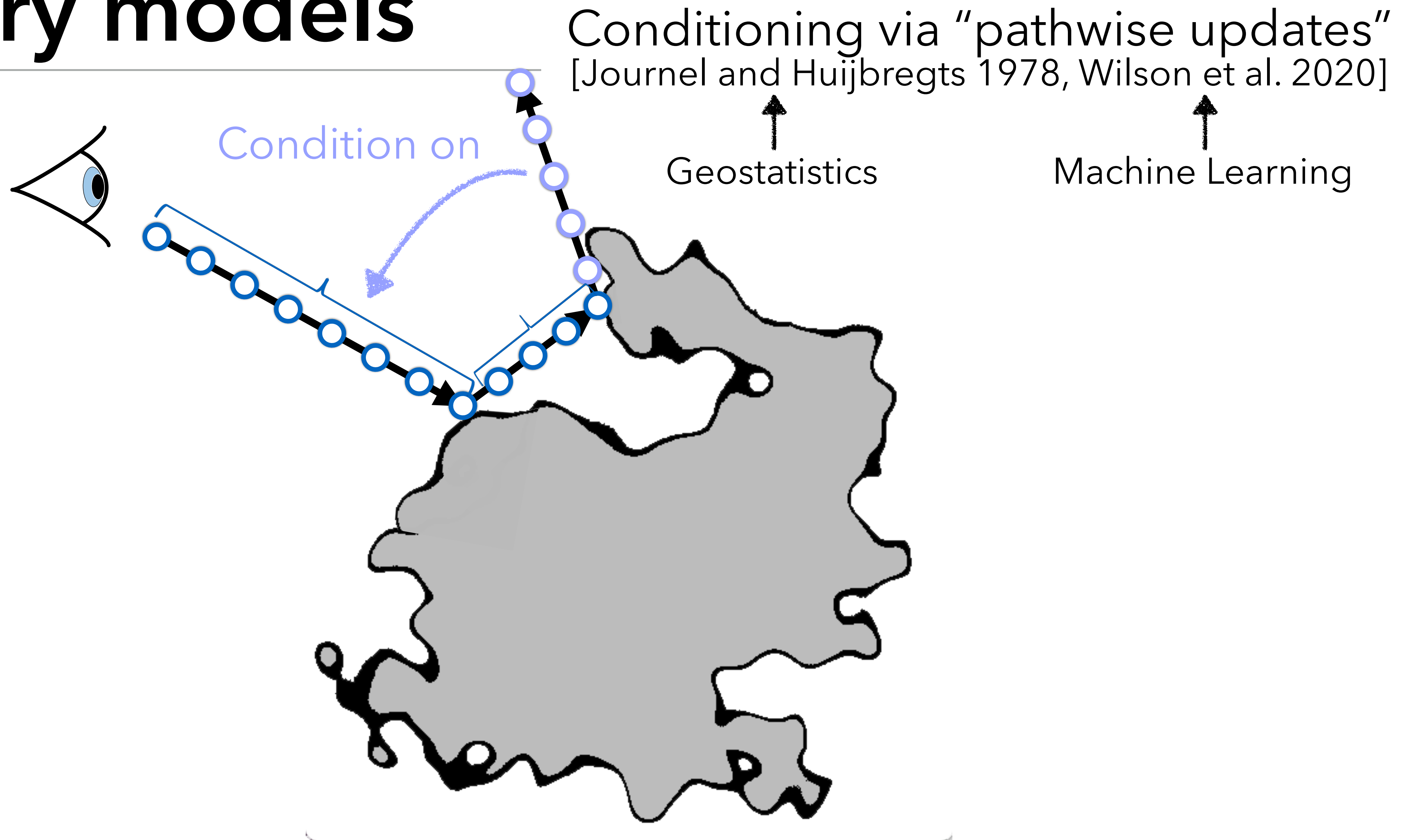


Tracing a Full Path

Conditioning via “pathwise updates”
[Journel and Huijbregts 1978, Wilson et al. 2020]



Memory models



Memory models

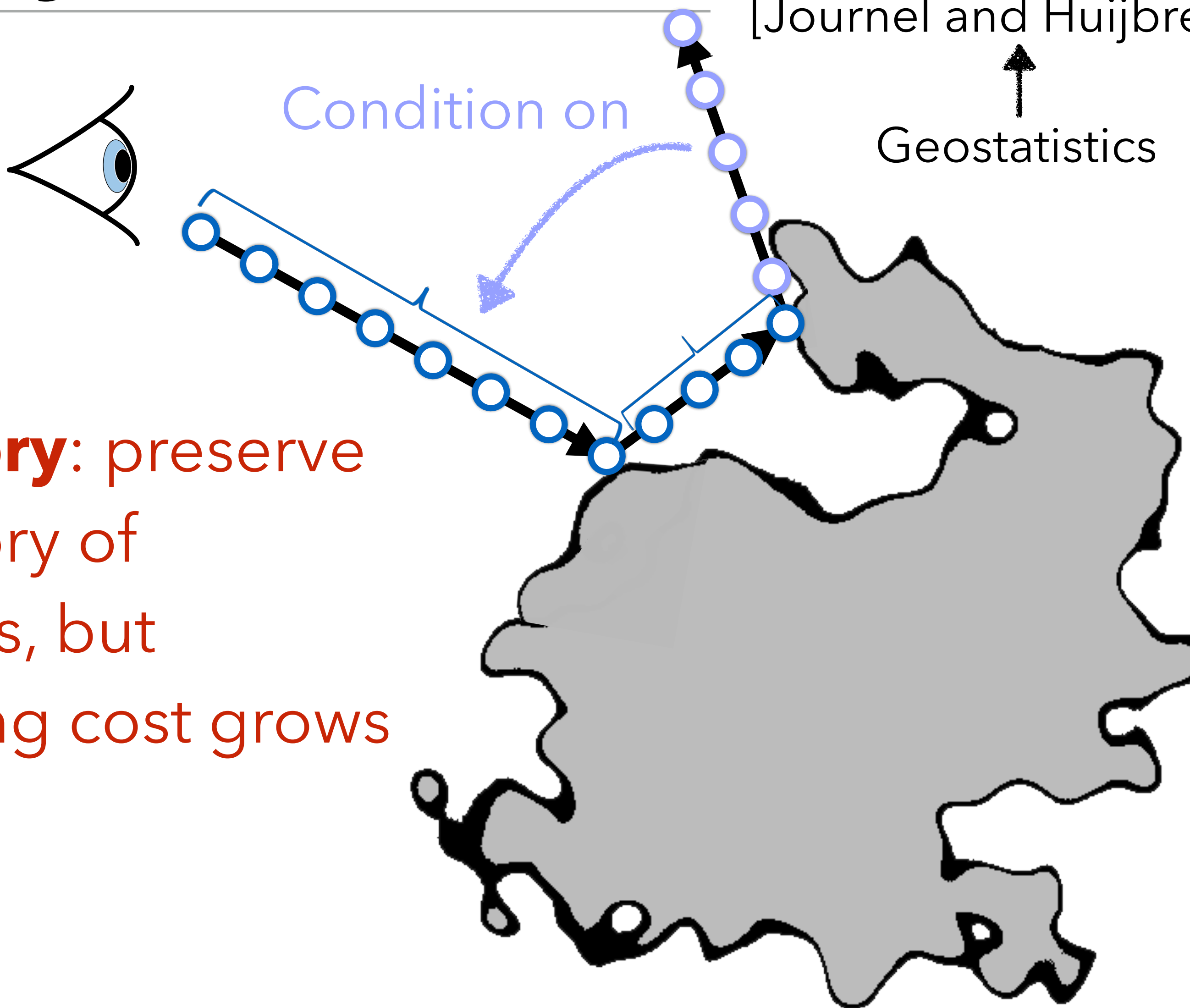
Conditioning via “pathwise updates”
[Journel and Huijbregts 1978, Wilson et al. 2020]

Geostatistics

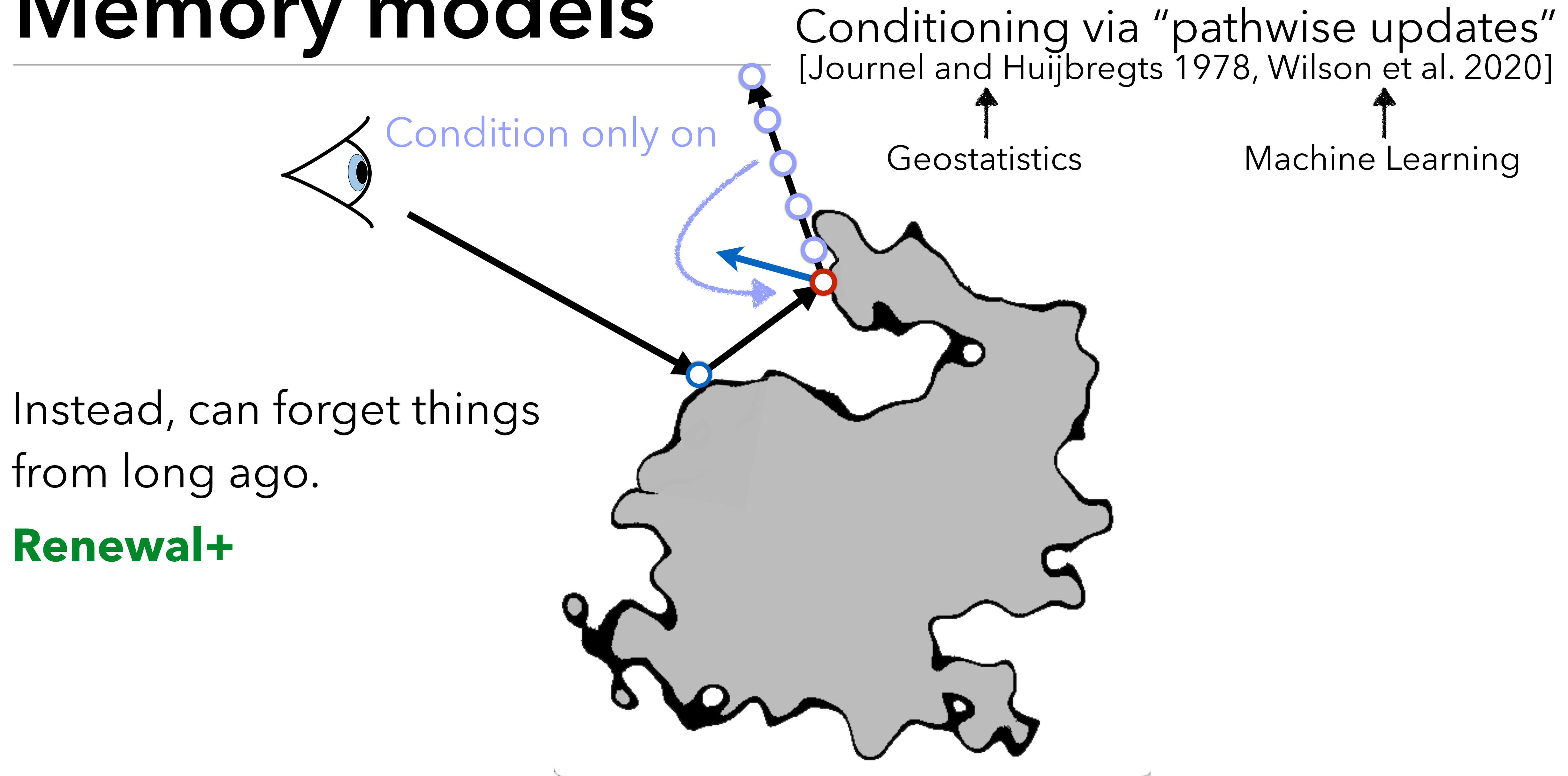
Machine Learning

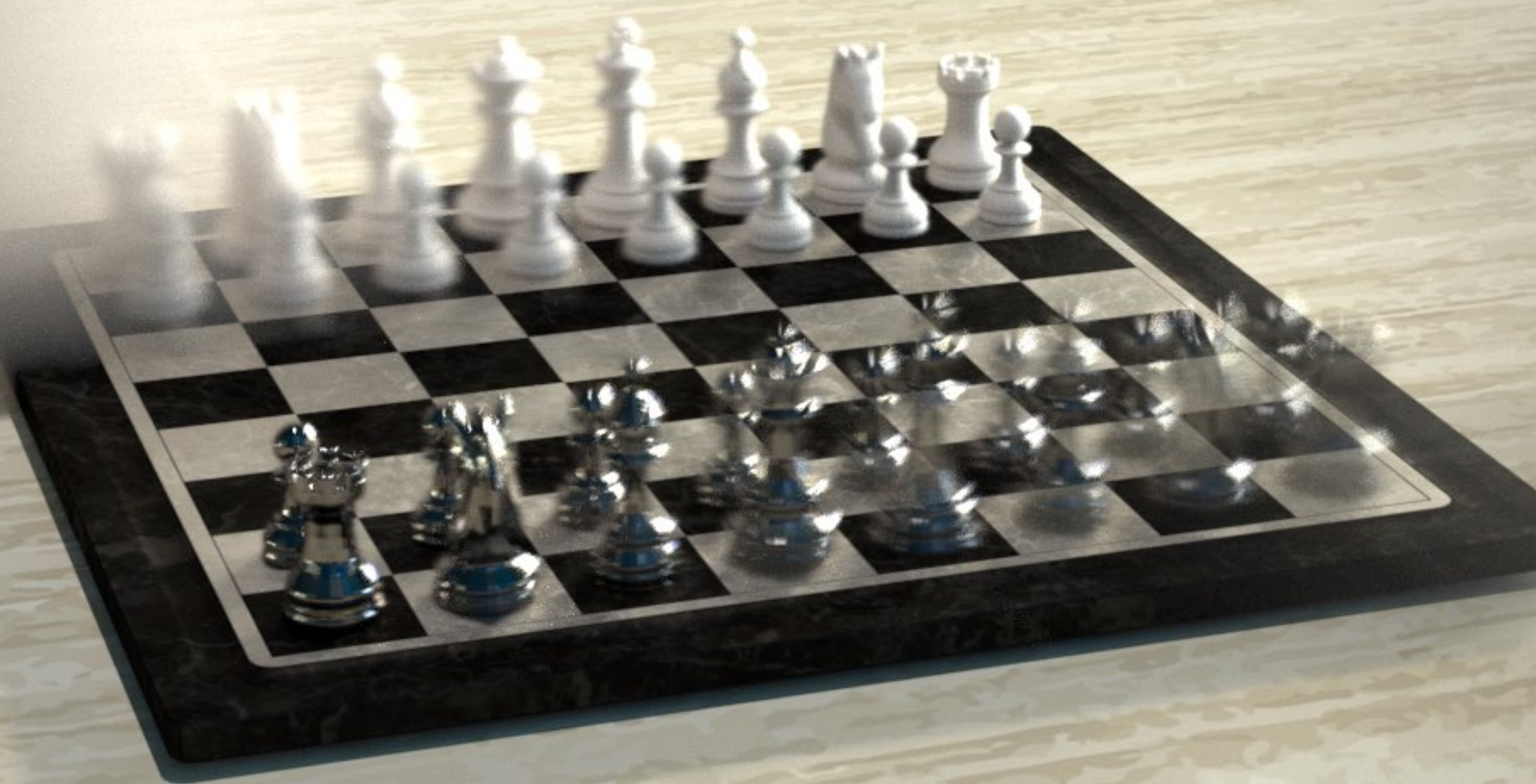
Condition on

Full memory: preserve
entire history of
correlations, but
conditioning cost grows



Memory models



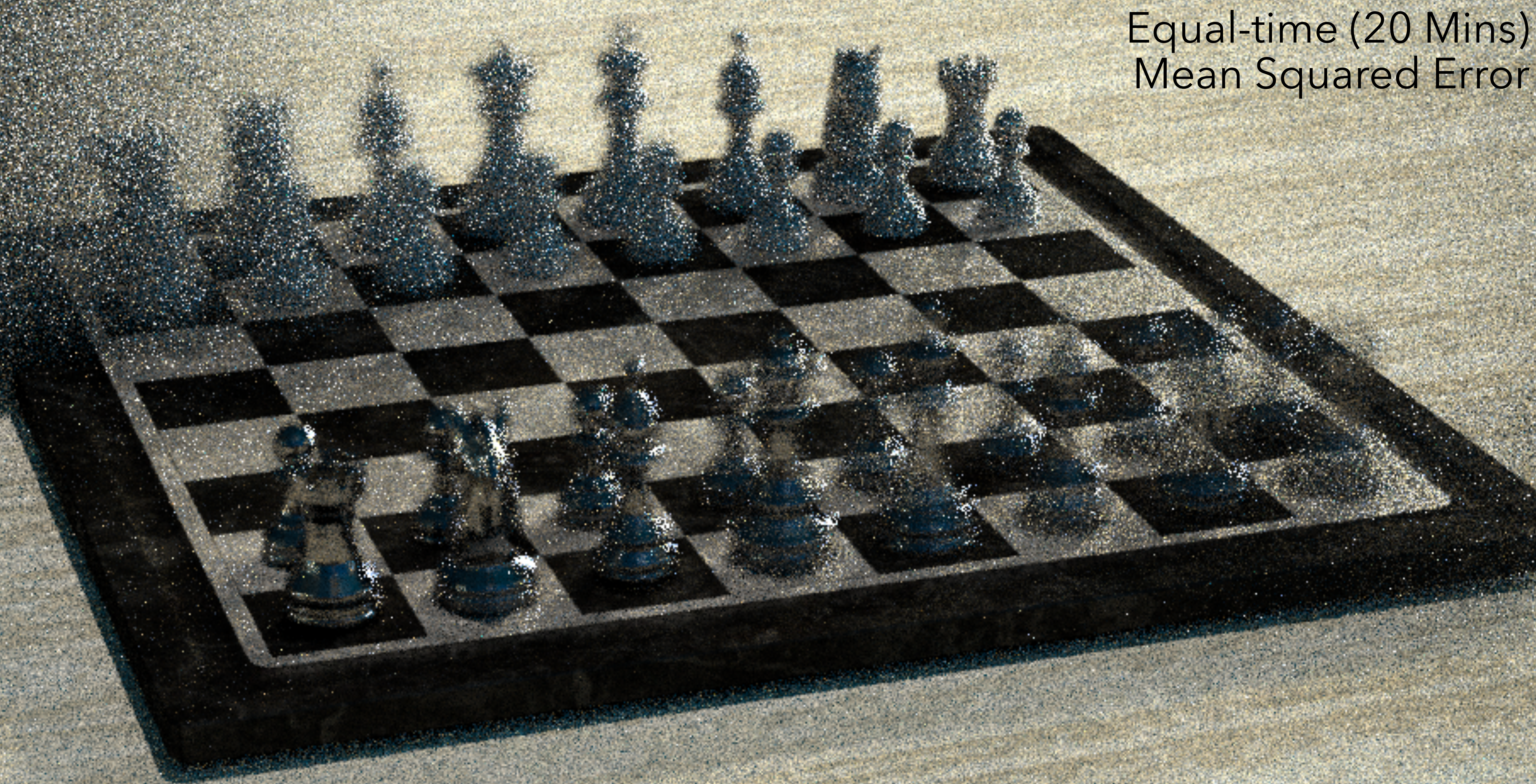


Ensemble average



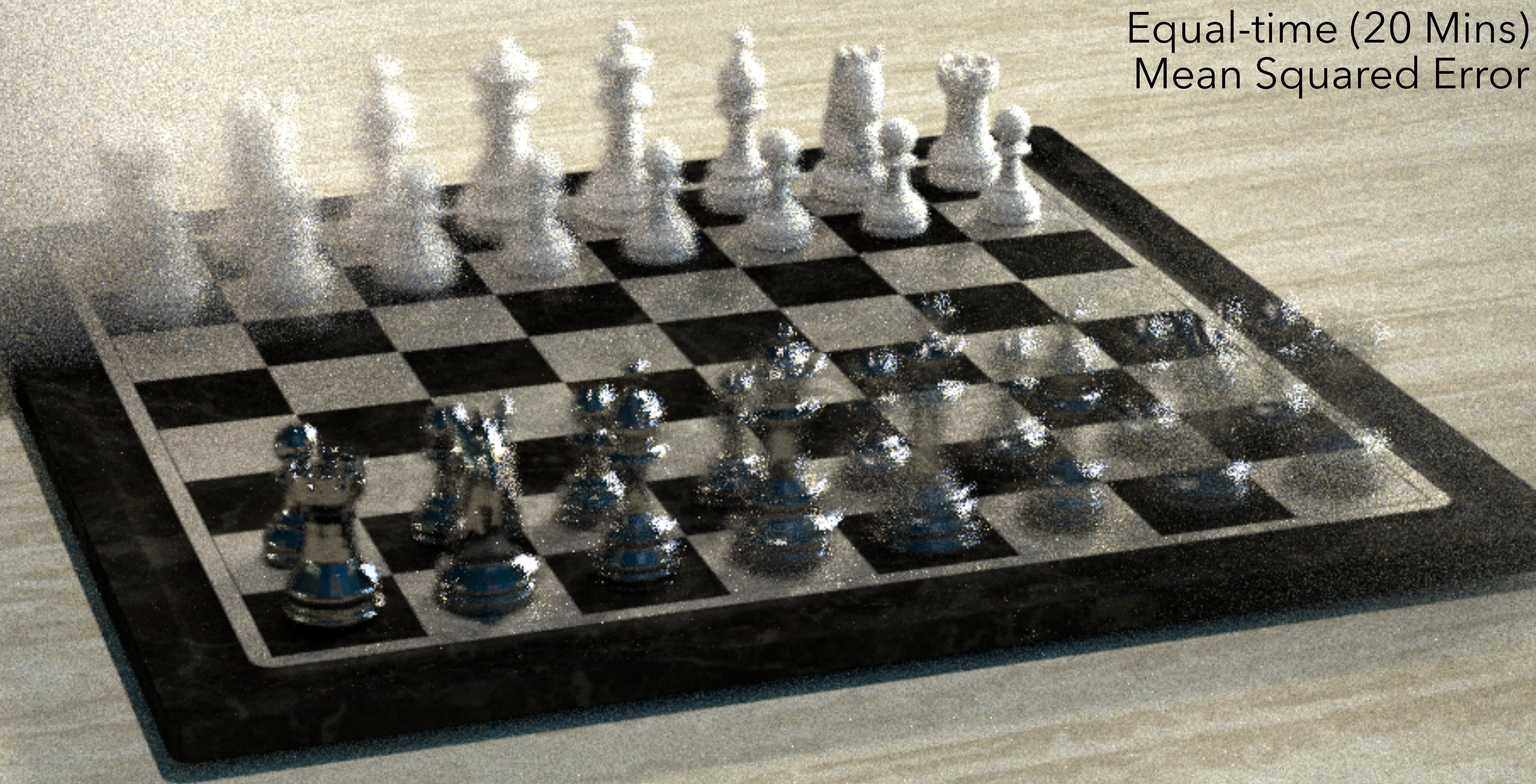
Single realization

Equal-time (20 Mins)
Mean Squared Error

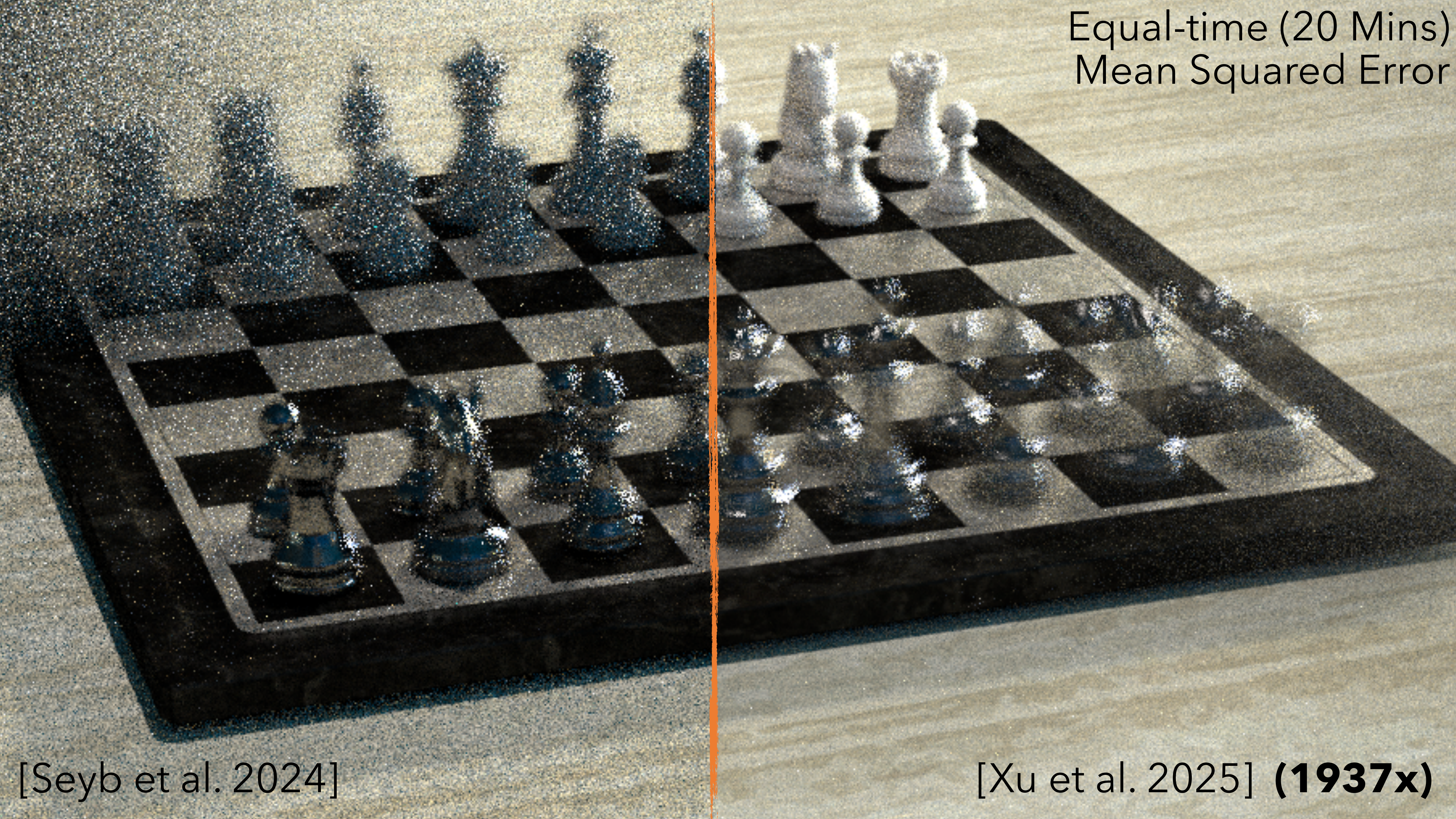


[Seyb et al. 2024]

Equal-time (20 Mins)
Mean Squared Error



[Xu et al. 2025] **(1937x)**



Equal-time (20 Mins)
Mean Squared Error

[Seyb et al. 2024]

[Xu et al. 2025] **(1937x)**



[Seyb et al. 2024]



[Xu et al. 2025]
(9281x)

What's next?

- Lots of interesting work remaining
 - further theory, speed, inverse problems, downstream tasks
- Applications in other domains?
 - glacier science
 - nuclear engineering, remote sensing?



Dario Seyb



Benedikt Bitterli



Kehan Xu



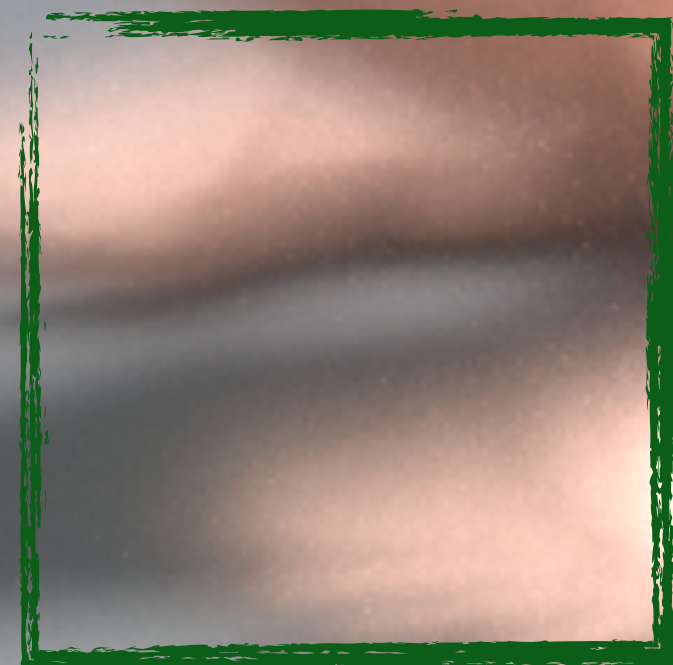
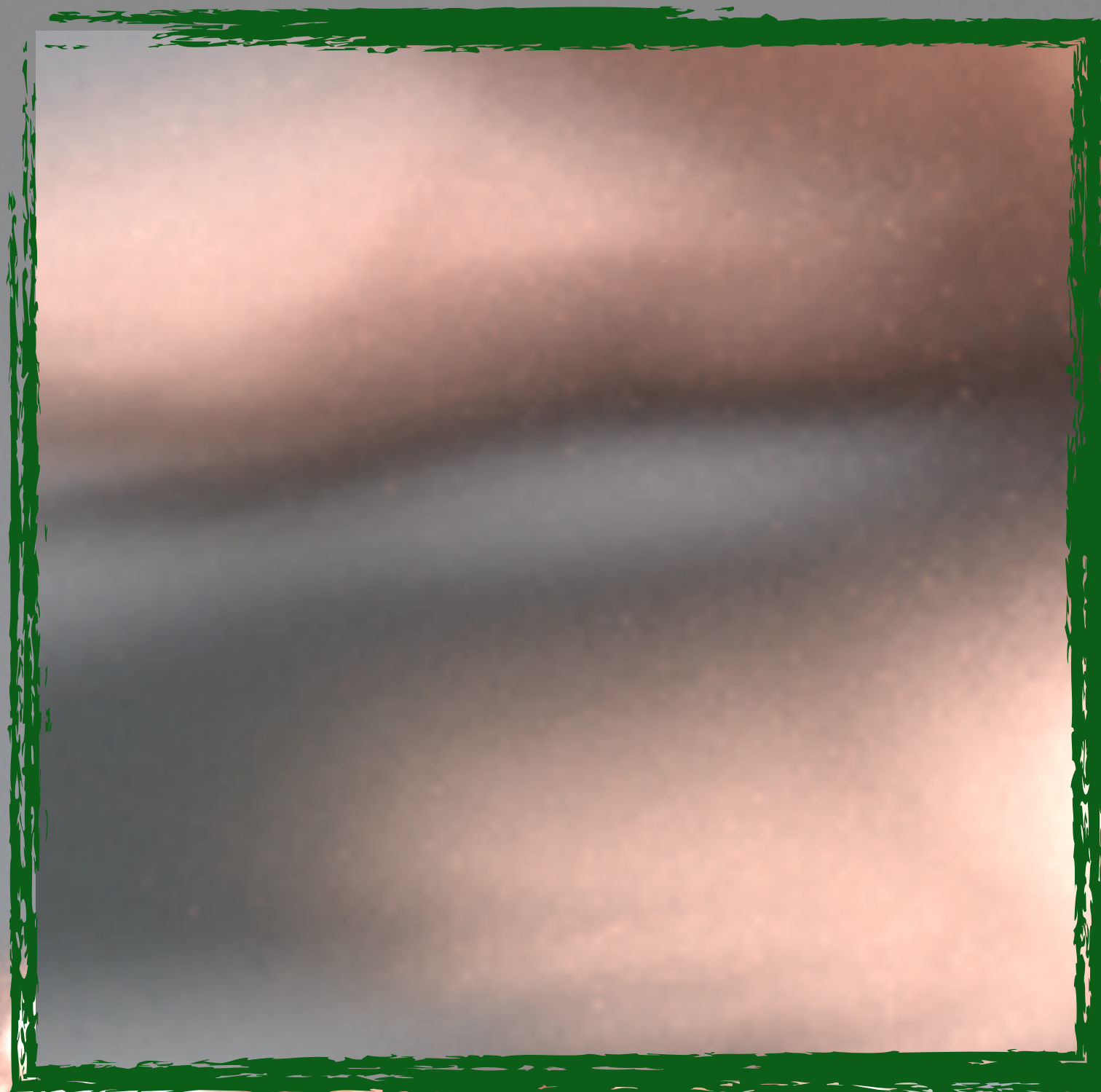
Eugene d'Eon

This work was supported by NSF awards 1844538 and 2440472.

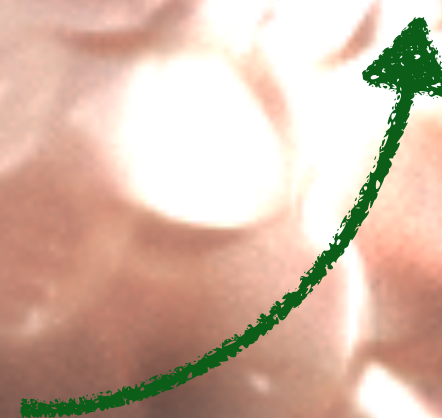
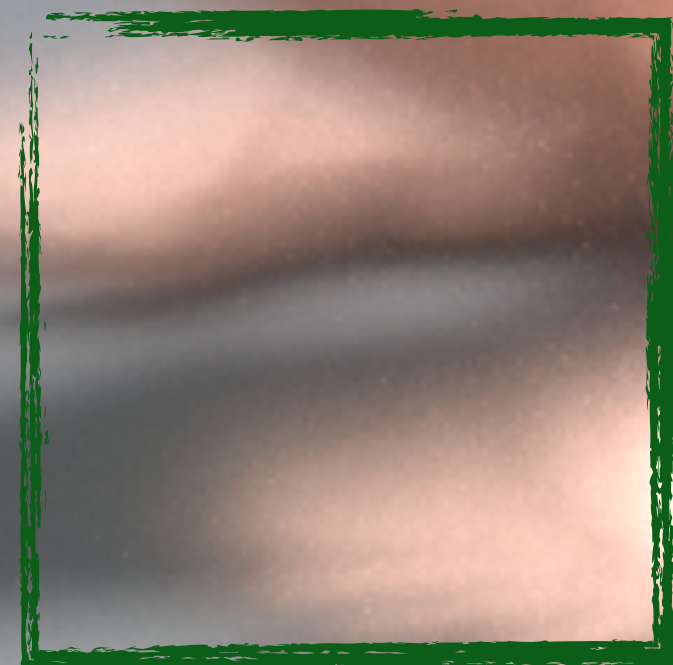
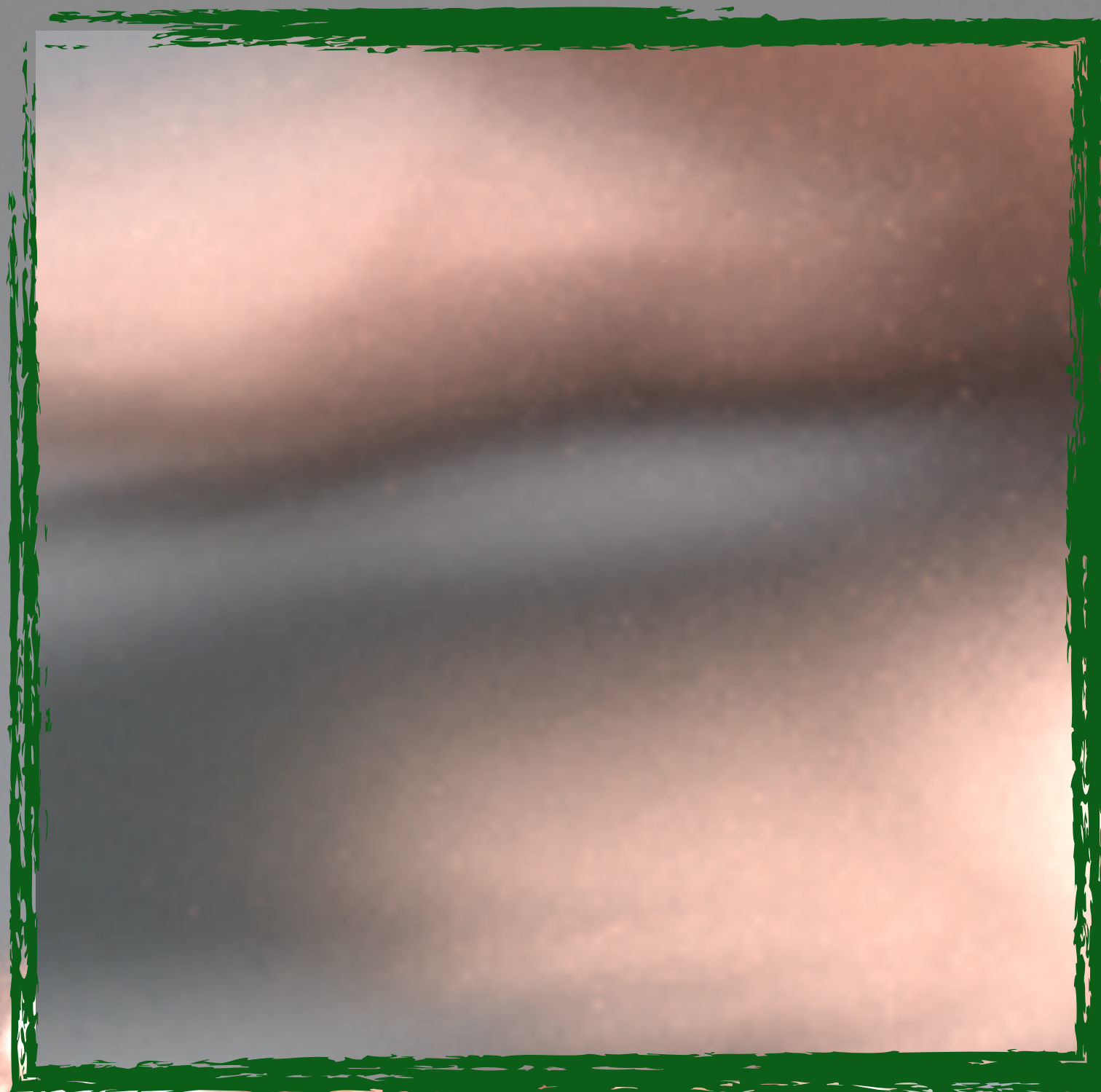


Interested? Let's chat & collaborate!

Thanks!



Ensemble average **with NEE** (4096 SPP)



Ensemble average **with NEE** (4096 SPP)