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Transient Photon Beams

Julio Marco[1] Wojciech Jarosz[2] Diego Gutierrez[1] Adrian Jarabo[1]

[1] Universidad de Zaragoza, I3A [2] Dartmouth College

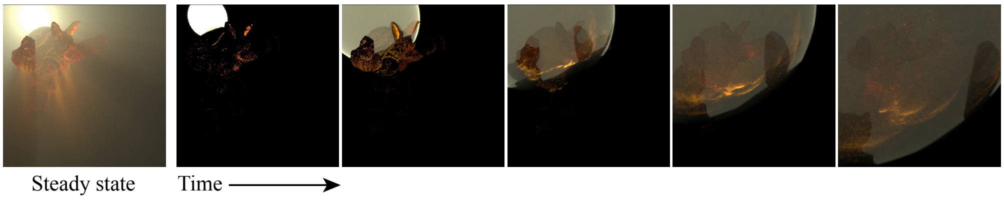


Figure 1: We present a method for efficient transient rendering of participating media based on the time-resolved radiative transfer equation and photon beams techniques. In the left image we can observe a classic steady-state render of a glass armadillo within a participating media. On the right we show the frame sequence of the time-resolved response under a delta pulse of light. We leverage density estimation properties of photon-based methods for mitigating aggravated variance of Monte Carlo sampling in the light temporal domain. Our method allows to efficiently render complex media effects such as caustics and multiple scattering in transient state.

Abstract

Recent advances on transient imaging and their applications have opened the necessity of forward models that allow precise generation and analysis of time-resolved light transport data. However, traditional steady-state rendering techniques are not suitable for computing transient light transport due to the aggravation of the inherent Monte Carlo variance over time. These issues are specially problematic in participating media, which demand high number of samples to achieve noise-free solutions. We address this problem by presenting the first photon-based method for transient rendering of participating media that performs density estimations on time-resolved precomputed photon maps. We first introduce the transient integral form of the radiative transfer equation into the computer graphics community, including transient delays on the scattering events. Based on this formulation we leverage the high density and parameterized continuity provided by photon beams algorithms to present a new transient method that allows to significantly mitigate variance and efficiently render participating media effects in transient state.

CCS Concepts

•***Computer Graphics*** → *Three-dimensional graphics and realism; Raytracing; Transient rendering;*

1. Introduction

The recent technological advances on transient imaging have led to the emergence of a wide number of techniques that leverage information on the temporal domain of light transport for applications in computer graphics and vision [JMMG17]. As a consequence, accurate time-resolved light transport information is key to provide insights and analysis of transient imaging techniques. In that sense, forward rendering models are a powerful tool to generate this kind of data under controlled synthetic setups. The recent work by Jarabo and colleagues [JMMn*14] formally introduced a generalized transient path integral formulation for surfaces and media. They demonstrated how traditional steady-state methods fail when rendering light in transient state, specially due to radiance-aimed importance sampling techniques that create uneven distributions of variance growing over time. They address these issues by proposing new time-based importance sampling methods and progressive approaches in a time-resolved bidirectional path tracer. Still, their method remains very sensitive to variance due to the underlying nature of path tracing methods. Other existing solutions [SSD08, Jar12, ABW14, Bit16] have addressed transient light transport, but they either are too narrowly scoped or generate sub-optimal straightforward solutions.

Steady-state methods based in photon tracing have proved successful in reducing variance of Monte Carlo solutions for participating media rendering. These methods trade variance for bias by performing density estimations on stored light paths across the scene. In particular, techniques based on *photon beams* [JNSJ11, JNT*11, KGH*14] leverage the information of light tracing by densely populating media with full photon trajectories, which significantly increases efficiency during the rendering process. One of the major drawbacks in transient rendering lies on the requirement of much higher sampling rates to fill up the extended temporal domain, where usual steady-state samples are sparsely and unevenly distributed along time. We make the key observation that continuity of full photon trajectories allows to render media at arbitrary temporal resolutions thanks to closed-form parametrized radiance estimations between camera rays and photon beams. Along with increased media sampling density provided by photon beams, these features make this kind of algorithm very suitable for transient rendering.

Based on these principles, in this paper we introduce a new method for efficiently computing transient light transport in participating media. While original radiative transfer theory [Cha60] accounts for light time of flight, for practical issues its integral form used in classic computer graphics is time-agnostic due to the assumption of infinite speed of light. Therefore, in this paper we introduce the time-dependent integral form of the radiative transfer equation into the computer graphics community, including temporal delays in the scattering events not accounted in the original formulation. This naturally allows modeling transient light transport in participating media. We build upon this formulation to present a new method based on photon beams [JNSJ11] for efficiently rendering participating media in transient state. We finally demonstrate how our method is capable of producing noise-free time-resolved renders in a variety of scenarios, including indirect illumination, multiple scattering and complex caustics.

2. Related Work

Transient rendering While light transport equations often used in computer graphics [BW02, Cha60] are originally defined in a time-resolved manner, steady-state rendering has usually assumed infinite speed of light by dropping any time dependence on these models. First introduction of light time of flight into the rendering equation [Kaj86] was presented by Smith et al. [SSD08]. Later Jarabo and colleagues [JMMn*14] presented a generalized time-resolved formulation for light propagation based on the path integral [Vea97], which allowed them to synthesize videos of light in motion including scattering and propagation delays on a bidirectional path tracer. Other works have addressed time-resolved light transport for more direct applications in transient imaging [NML*13, OHX*14, ADY*16], and as a forward model for inverse problems [KOKP07, KK09, FH08, Fuc10, JPMP12, JPMP14, Hul14, KPM*16]. Closer to our work, Meister and colleagues [MNJK13] addressed transient rendering also using photon-based techniques [MNK13], but scoped to time-of-flight imaging problems for diffuse surfaces propagation. Different to these works, we provide a transient method for rendering participating media by using the time-resolved form of the radiative transfer equation [Cha60].

Photon-based density estimation Classic steady-state rendering has heavily benefited from photon-based techniques for variance reduction of global illumination computation. Ever since the appearance of photon mapping [Jen01], subsequent works presented extensions for dynamic scenes [CJ02], progressive approaches [HOJ08, HJJ10, KZ11, KD13] or hybrid methods [GKDS12, HPJ12]. Jarosz and colleagues significantly improved efficiency in volumetric photon mapping by introducing the beam radiance estimate [JZJ08]. Generalization of beams to the tracing process by storing full photon trajectories (photon beams) [JNSJ11] led to a dramatic increase of density of photon maps at very little computational cost. Benefits provided by photon beams led to their counterpart progressive and hybrid techniques [JNT*11, HCJ13, KGH*14]. All these works are, however, restricted to steady-state renders where light is assumed to have infinite speed. In our work we introduce light propagation time into the photon beams technique and leverage beams continuity and spatial density estimations to mitigate variance in the temporal domain.

3. Transient Radiative Transfer

A beam of light reaching any region of the space different from vacuum will interact with matter that may alter its behavior in different ways. In the so-called *participating media*, light transport occurs all over the volume where both scattering and absorption effects play a significant role. The *radiative transfer equation* [Cha60] models the behavior of light traveling through a medium. While original formulation is time-resolved, its integral form used in traditional rendering drops this temporal dependence, and computes the radiance reaching any point $\mathbf{x}$ from direction $\vec{\omega}$ as

$$L(\mathbf{x},\vec{\omega}) = T_r(\mathbf{x},\mathbf{x}_s)L_s(\mathbf{x}_s,\vec{\omega}) + \int_0^s \mu_s(\mathbf{x}_r)T_r(\mathbf{x},\mathbf{x}_r)L_o(\mathbf{x}_r,\vec{\omega})\mathrm{d}r. \quad (1)$$

where L_s is the radiance from the closest surface point $\mathbf{x}_s$ at distance s along the ray; μ_s is the scattering coefficient at a media point $\mathbf{x}_r$; T_r indicates the transmittance between two points; the second summand integrates radiance across all media points $\mathbf{x}_r$ from direction $\vec{\omega}$; and $L_o(\mathbf{x}_r,\vec{\omega})$ models the in-scattered radiance at $\mathbf{x}_t$ exiting towards direction $\vec{\omega}$,

$$L_o(\mathbf{x},\vec{\omega}) = \int_\Omega \rho(\mathbf{x},\vec{\omega}_i,\vec{\omega})L_i(\mathbf{x},\vec{\omega}_i)\,\mathrm{d}\vec{\omega}_i, \quad (2)$$

where ρ represents the phase function and in general is dependent on the location $\mathbf{x}$ and the incoming and outgoing directions, $\vec{\omega}_i$ and $\vec{\omega}$ respectively.

Equations 1 and 2 assume that the speed of light is infinite, which is reasonable as long as we want to represent a scene as seen by a standard camera. However, if we want to solve the RTE at time scales comparable to the speed of light we need to include light travel time into the equations, and therefore provide new numerical solutions for these equations. Recently, Jarabo and colleagues [JMMn*14] introduced a transient version of the path integral formulation [Vea97] that inherently models transient light transport in participating media. In the following we summarize the main practical considerations for accounting time into the integral form of the RTE. We refer to the reader to the work by Jarabo and colleagues for a generalized formulation of transient light transport for both surfaces and media.

Light takes a certain amount of time to propagate through space, and therefore light transport from a point $\mathbf{x}_0$ towards a point $\mathbf{x}_1$ does not occur immediately, having

$$L(\mathbf{x}_1, \vec{\omega}, t) = L(\mathbf{x}_0, \vec{\omega}, t - \Delta t), \quad (3)$$

where $\vec{\omega}$ is a direction outgoing from $\mathbf{x}_0$ towards $\mathbf{x}_1$, and Δt is the time it takes the light to go from $\mathbf{x}_0$ to $\mathbf{x}_1$. In turn, Δt is defined by

$$\Delta t(\mathbf{x}_0 \leftrightarrow \mathbf{x}_1) = \int_{\mathbf{x}_0}^{\mathbf{x}_1} \frac{\eta(\mathbf{x})}{c} \mathrm{d}\mathbf{x}, \quad (4)$$

where $\eta(\mathbf{x})$ is the index of refraction at a medium point $\mathbf{x}$ and c is the speed of light in vacuum. In a medium with a constant index of refraction $\eta(\mathbf{x}) = \eta_m$, between $\mathbf{x}_0$ and $\mathbf{x}_1$, Equation 4 can be expressed as

$$\Delta t(\mathbf{x}_0 \leftrightarrow \mathbf{x}_1) = \frac{\eta_m}{c} ||\mathbf{x}_1 - \mathbf{x}_0||. \quad (5)$$

In order to reformulate the RTE (Equations 1 and 2) in transient state we can introduce time dependence (Equation 3) as

$$L(\mathbf{x}, \vec{\omega}, t) = T_r(\mathbf{x}, \mathbf{x}_s, t) L_s(\mathbf{x}_s, \vec{\omega}, t - \Delta t_s) + \int_0^s T_r(\mathbf{x}, \mathbf{x}_r, t) L_o(\mathbf{x}_r, \vec{\omega}, t - \Delta t_r) \mathrm{d}r \quad (6)$$

$$L_o(\mathbf{x}, \vec{\omega}, t) = \int_\Omega \int_0^t \mu_s(\mathbf{x}_r, t - t') \rho(\mathbf{x}, \vec{\omega}_i, \vec{\omega}, t - t') L_i(\mathbf{x}, \vec{\omega}_i, t) \mathrm{d}t' \mathrm{d}\vec{\omega}_i \quad (7)$$

Observe that media scattering μ_s and absorption (implicit in transmittance T_r) can have variations at time scales comparable to the speed of light. While we include time-dependent absorption and scattering properties of the medium in this formulation, we assume that variations in the medium density due to particle size and concentration occur at much smaller time scales than the ones used in the remaining of this work. Consequently, since transmittance T_r is a function of absorption and scattering, we also assume it as time-independent.

The phase function ρ models angular light scattering at particle level. When light interacts with a micrometric particle (e.g. water droplets) it can follow paths within that particle before being redirected outside, which would result in a significant time delay within the phase function. For the sake of simplicity, in the remaining of this work we assume time-independent phase functions where light interactions within media particles occur instantaneously.

4. Transient Photon Beams

Equations 1 and 2, and their respective transient versions Equations 6 and 7 define recursive models which in general do not present analytical solutions for arbitrary scenes, therefore needing numerical computation to approximate them.

Photon beams Photon beams algorithm [JNSJ11] provides a numerical solution for rendering participating media in steady state by performing two passes. In the first pass, a series of paths are traced from the light sources by Monte Carlo sampling Equations 1 and 2. These paths represent packages of light (photons) traveling through the medium. Every interaction of a photon within the medium is stored on a map as a *beam* with a direction $\vec{\omega}_b$, position $\mathbf{x}_b$ and power Φ_b. In the second pass, rays are traced from the camera against the photon beams map. Every photon beam is considered to have certain radius r, and radiance seen by a camera ray is computed by performing a density estimation on every ray-beam intersection (see Figure 2a). Depending on the dimensionality of the density estimation, Jarosz and colleagues proposed three different estimators based on 3D, 2D and 1D kernels. For the sake of brevity we present our extension to transient state based on the 2D kernel within homogeneous media. Analogous concepts apply for extending 3D and 1D kernels to transient state.

Given a camera ray defined by $\mathbf{x}_r + s_r \cdot \vec{\omega}_r$ and a photon beam b defined by $\mathbf{x}_b + s_b \cdot \vec{\omega}_b$ with energy Φ_b, the 2D density estimator for homogeneous media computes the radiance arriving at $\mathbf{x}_r$ analytically as

$$L_b(\mathbf{x}_r, \omega_c) = \frac{\mu_s}{\Omega_R(r^2)} \rho(\theta_b) \Phi_b \int_{s_r^-}^{s_r^+} T_r(s_r) T_r(s_b(s_r)) \, \mathrm{d}s_r \quad (8)$$

$$= \frac{e^{-\mu_t(s_b^- + s_r^-)} - e^{-\mu_t(s_b^- + \cos\theta_b (s_r^+ - s_r^-) + s_r^+)}}{\mu_t \cos\theta_b} \quad (9)$$

where $[s_r^-, s_r^+]$ are the limits of the ray-beam intersection (Figure 2c), θ_b is the angle between $\vec{\omega}_b$ and $\vec{\omega}_r$, and $\Omega_R(r^2)$ represents the 2D kernel of a beam of radius r. Finally, the total radiance at $\mathbf{x}_r$ is computed as the sum of all beam radiances along the camera ray

$$L(x_r, \vec{\omega}_r) \approx \sum_{b \in R_b} L_b(\mathbf{x}_r, \vec{\omega}_r). \quad (10)$$

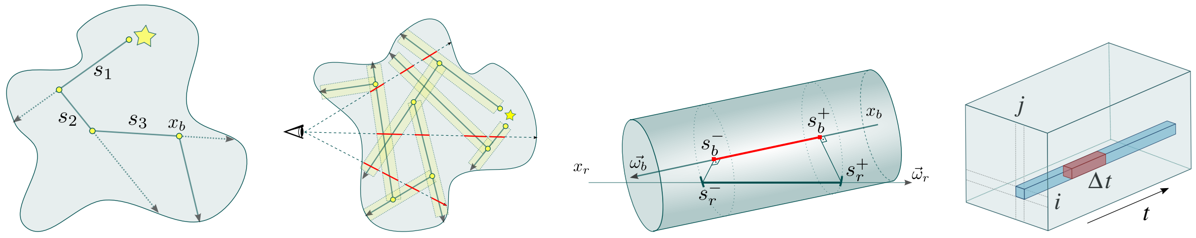
Our algorithm However, since we aim to compute time-resolved radiance, we have to account for photon timings along its way from the light sources to the camera. Photon time-of-flight is directly related to its optical path. While photon points only provide time information at discrete timings in the scene—which is a drawback when using small temporal resolutions—, a photon beam is by definition continuous since its trajectory is parameterized by $\mathbf{x}_b + s_b \cdot \vec{\omega}_b$. In the following we show how to account for time propagation in Equations 9 and 8, conveniently allowing us to span every ray-beam radiance estimation to any desired temporal resolution. This feature is very important since—as shown by Jarabo and colleagues [JMMn*14]—one of the main drawbacks in transient rendering is the increased sparsity and uneven distributions of radiance samples when increasing temporal resolution.

A photon located at $\mathbf{x}_b$ has taken a certain time to get there since it started traveling from the light source. Photon timing t_{b_0} at the origin of the photon beam s_{b_0} can be computed by Monte Carlo sampling our proposed transient version (Equations 6 and 7), keeping track of all the distances s_j traveled by that photon up to the beam starting point (see Figure 2a). The photon beam starting time t_{b_0} is therefore computed as

$$t_{b_0} = \sum_{s_j \in \Pi} \Delta t(s_j) = \sum_{s_j \in \Pi} \frac{\eta_{m_j}}{c} s_j \quad (11)$$

where $s_j \in \Pi$ represents the photon optical path from the light source to $\mathbf{x}_b$, and η_{m_j} represents the index of refraction of the different media crossed by the photon b.

Observe Figure 2c. For a point within the ray-beam blur region at distance s_{b_i} from the beam start $\mathbf{x}_b$ (i.e. $s_{b_0} = 0$, Figure 2c), the



(a) (b) (c) (d)

Figure 2: (a) A photon emitted from the light source will take a time $t_{b_0} = \frac{\eta_m}{c}(s_1 + s_2 + s_3)$ to get x_b. (b) Radiance estimation in the medium is done by intersecting every ray against the photon beam map, and performing density estimations at the ray-beam intersections (red). (c) Ray-beam intersection for density estimation using a 2D kernel, where the photon energy at every beam point $\mathbf{x}_b + s_b \cdot \vec{\omega}_b$ affects a single point within the ray $\mathbf{x}_r + s_r \cdot \vec{\omega}_r$ within a perpendicular 2D disk. Time delays within this spatial density estimation will depend on the ray-beam orientation and the blur region intersections, the speed of light and the index of refraction of the media. (d) Time-resolved volume for the rendered transient frames, showing the time interval of a ray-beam radiance at pixel ij.

photon takes

$$\Delta t([s_{b_0} = 0] \leftrightarrow s_{b_i}) = \frac{\eta_m}{c} s_{b_i} \quad (12)$$

time to get from s_{b_0} to s_{b_i} (Equation 5). Finally, that photon will take

$$\Delta t(s_{r_i} \leftrightarrow [s_{r_c} = 0]) = \frac{\eta_m}{c} s_{r_i} \quad (13)$$

to reach the camera $\mathbf{x}_r$ from the corresponding point at s_{r_i} within the 2D blur region. Therefore, the total time a photon takes to get from the light source to a position inside the ray-beam kernel at a distance s_{b_i} and then to the camera $\mathbf{x}_r$ can be computed as

$$t = t_{b_0} + \Delta t(s_{b_0} \leftrightarrow s_{b_i}) + \Delta t(s_{r_i} \leftrightarrow s_{r_c}). \quad (14)$$

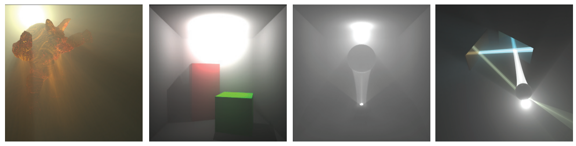
However, original closed-form of the density estimation (Equation 9) discards all light travel times within the 2D blur region, and directly computes the integrated radiance along the intervals defined by $[s_b^-, s_b^+]$ and $[s_r^-, s_r^+]$. While we could Monte Carlo sample the integral in Equation 8 to obtain discrete time–radiance samples, that would increase variance on our estimation, which is against one of the desired benefits from density estimation techniques. Additionally, this would introduce an unaffordable computational overhead since we would need to do this for every ray-beam intersection. Instead, we can evenly distribute the integrated ray-beam radiance L_b across the time interval $\Delta t(L_b) = [t^-, t^+]$ covered by the 2D blur region. In particular, this interval is defined by the light travel times corresponding to $[s_b^-, s_b^+]$ and $[s_r^-, s_r^+]$, yielding

$$t^- = t_r^- + t_b^-, \qquad t^+ = t_r^+ + t_b^+ \quad (15)$$

which can be computed from Equations 12 and 13.

Note that due to transmittance, the photon energy actually varies as it travels across the blur region. Evenly distributing the integrated radiance L_b across this interval introduces temporal bias, apart from the inherent spatial bias introduced by density estimation. However in our comparisons against path traced results (see Section 5) we observed this even distribution provides a good tradeoff between bias, variance, and computational overhead.

Implementation Unlike frame-to-frame steady-state rendering, to distribute transient radiance of a beam on a pixel ij we need to keep in memory the full temporal span of that pixel (see Figure 2d) up to some maximum time. During beam tracing, starting time t_{b_0} of each photon beam can be stored in the first pass of the algorithm along with its position $\mathbf{x}_b$, direction $\vec{\omega}_b$ and energy Φ_b in the photon map. In the second pass, we can keep track of the time interval $[t^-, t^+]$ determined by every ray-beam radiance L_b, and evenly distribute this radiance over the resulting pixels interval in the time domain.



Armadillo Cornell blocks Cornell spheres Mirrors

Figure 3: Steady-state renders for the scenes *Armadillo* (Figure 1), *Cornell blocks* (Figure 4), *Cornell spheres* (Figure 5) and *Mirrors* (Figure 6).

5. Results

In the following we illustrate the results of our proposed technique. All the results were taken on a desktop PC with Intel i7 and 4GB RAM. Unless stated otherwise, all observed media in the rendered scenes have index of refraction of vacuum (IOR = 1). We show results of our method in four scenes: *Armadillo*, *Cornell blocks*, *Cornell spheres* and *Mirrors*. Figure 3 shows the corresponding steady-state renders of these results. Please refer to the supplemental video for the full videos shown throughout this section.

Figure 4 shows an equal-time comparison of subsequent frames on the *Cornell blocks* scene rendered with a transient path tracer (bottom), and our transient photon beams implementation (top), both taking approximately 5 hours and 30 minutes. All surfaces in this scene present Lambertian reflection, and the light on the top emits a Dirac delta pulse of light. Indirect illumination through the media seen as color bleeding near the red and green blocks. We can observe the benefits of density estimation on variance reduction compared to the path traced solution, and how it holds over time due the continuity of beams.

Camera unwarping [VWJ*13] is an intuitive way of visualizing

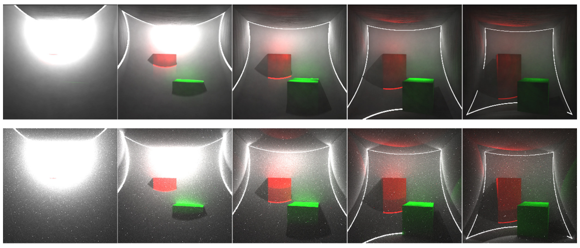


Figure 4: Frame sequence (from left to right) from the *Cornell blocks* scene rendered with a transient path tracer (bottom) and our proposed transient photon beams algorithm (top). Continuity of photon beams allows to keep the variance reduction provided by density estimation over time.

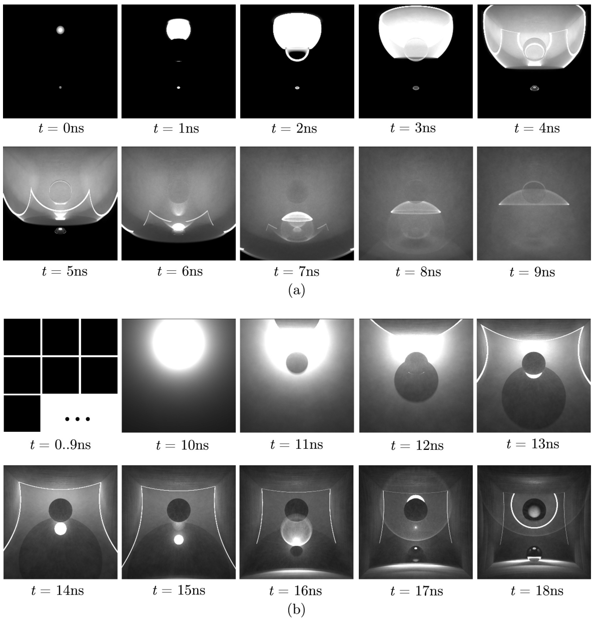


Figure 5: Comparison of *Cornell spheres* scene using (a) a *camera-unwarped* solution where we do not take into account the camera time, and (b) real propagation of light. In (b) the shape of the wavefront is altered by the camera time, as if we were scanning the scene from the viewpoint towards the furthest parts of the scene. This can be seen in the shape of the wavefront in the side walls. Camera unwarping (a) shows us more intuitively how light propagates *locally*.

how light propagates *locally* on the scene without accounting for the time light takes to reach the camera. In Figure 5 we compare the effects of accounting or not for this camera time. The scene consists of a diffuse Cornell box with a point light on the top, a glass refractive sphere (top, IOR = 1.5) and a mirror sphere (bottom). While Figure 5b shows the real propagation of light—including camera time—, Figure 5a depicts more intuitively how light comes out from the point light, travels through the refractive sphere, and the generated caustic bounces on the mirror sphere. Note how in the top sequence we can clearly see how light is slowed down through the glass sphere due to the higher index of refraction. We can also observe multiple scattered light (particularly noticeable in frames t=4ns to t=6ns) as a secondary wavefront.

Figure 6 compares visualizations of light propagation within the *Mirrors* scene under constant and Dirac delta light emission. The scene is composed by two colored mirrors and a glass sphere with IOR = 1.5, and was rendered using the previously mentioned camera unwarping. We can observe how delta emission generates wavefronts that go through the ball and bounce in the mirrors, creating wavefront holes where constant emission creates medium shadows. In the last frame of the top row Delta emission clearly depicts the slowed down caustic through the glass ball respect to the main wavefront.

Figure 1 shows an orange glass armadillo inside a yellow media and a point light emitting from the back. The light is emitted as a delta pulse and the scene is rendered using camera unwarping. We can see how light refracted through the glass comes out by the front of the armadillo showing propagation delays due to longer complex light paths within the object and the higher index of refraction of the glass.

6. Conclusions

In this paper we presented a novel method for efficient simulation of light transport participating media. We introduced the time-dependent integral-form of the radiative transfer equation into the computer graphics community, and imposed it to steady-state photon beams methods. As a result, we leveraged spatial density estimation techniques and high density of continuous photon trajectories to significantly mitigate variance of transient light transport simulation and render complex effects such as multiple scattering and caustics. Our contributions are of great importance in transient imaging, where continuously emerging techniques and hardware advances demand reliable transient data under controlled setups. For that purpose, our method can be used for efficiently obtaining this kind of data which could be used to obtain valuable insights on transient light transport.

As future work we regard a more thorough analysis of variance reduction and bias impact in transient state under varying media characteristics, and how transient-state adaptations of different photon estimators and hybrid and progressive techniques [JNT*11, KGH*14] could improve performance of time-resolved light transport simulation in different media and geometry setups.

Acknowledgments

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References

[ABW14] AMENT M., BERGMANN C., WEISKOPF D.: Refractive radiative transfer equation. *ACM Trans. Graph. 33*, 2 (2014). 1

[ADY*16] ADAM A., DANN C., YAIR O., MAZOR S., NOWOZIN S.: Bayesian time-of-flight for realtime shape, illumination and albedo. *IEEE Trans. Pattern Analysis and Machine Intelligence* (2016). 2

[Bit16] BITTERLI B.: Virtual femto photography. https://benedikt-bitterli.me/femto.html, 2016. 1

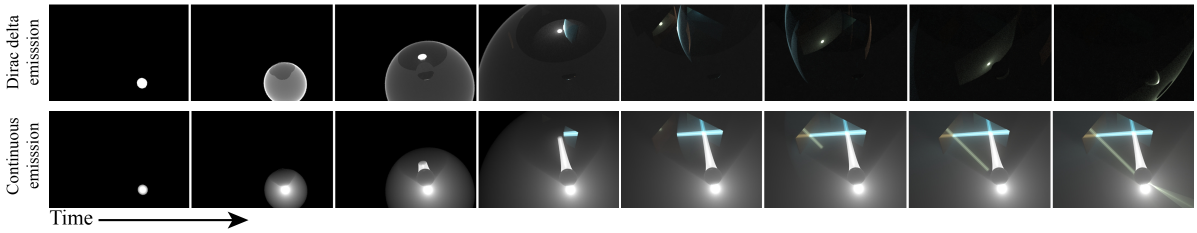

Figure 6: Comparison between (a) Delta Dirac and (b) continuous emission. Dirac delta emission lets us see how a pulse of light travels and scatters across the scene, depicting the light wavefronts bouncing on the mirrors and going through the glass ball. Continuous emission shows how light is emitted until it reaches every point in the scene, as if we were taking a picture with a camera at very slow-motion.

[BW02] Born M., Wolf E.: *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*. Cambridge University Press, 2002. 2

[Cha60] Chandrasekhar S.: *Radiative Transfer*. Dover, 1960. 2

[CJ02] Cammarano M., Jensen H. W.: Time dependent photon mapping. In *Eurographics Workshop on Rendering* (2002). 2

[FH08] Fuchs S., Hirzinger G.: Extrinsic and depth calibration of ToF-cameras. In *IEEE CVPR* (2008). 2

[Fuc10] Fuchs S.: Multipath interference compensation in time-of-flight camera images. In *IEEE International Conference on Pattern Recognition* (2010). 2

[GKDS12] Georgiev I., Křivánek J., Davidovič T., Slusallek P.: Light transport simulation with vertex connection and merging. *ACM Trans. Graph. 31*, 6 (2012). 2

[HCJ13] Habel R., Christensen P. H., Jarosz W.: Photon beam diffusion: A hybrid monte carlo method for subsurface scattering. In *Computer Graphics Forum* (2013), vol. 32, Wiley Online Library, pp. 27–37. 2

[HJJ10] Hachisuka T., Jarosz W., Jensen H. W.: A progressive error estimation framework for photon density estimation. In *ACM Trans. Graph. (TOG)* (2010), vol. 29, ACM, p. 144. 2

[HOJ08] Hachisuka T., Ogaki S., Jensen H. W.: Progressive photon mapping. *ACM Trans. Graph. (TOG) 27*, 5 (2008), 130. 2

[HPJ12] Hachisuka T., Pantaleoni J., Jensen H. W.: A path space extension for robust light transport simulation. *ACM Trans. Graph. (TOG) 31*, 6 (2012), 191. 2

[Hul14] Hullin M. B.: Computational imaging of light in flight. In *SPIE/COS Photonics Asia* (2014). 2

[Jar12] Jarabo A.: *Femto-photography: Visualizing light in motion*. Master's thesis, Universidad de Zaragoza, 2012. 1

[Jen01] Jensen H. W.: *Realistic Image Synthesis Using Photon Mapping*. AK Peters, 2001. 2

[JMMG17] Jarabo A., Masia B., Marco J., Gutierrez D.: Recent advances in transient imaging: A computer graphics and vision perspective. *Visual Informatics 1*, 1 (2017). 1

[JMMn*14] Jarabo A., Marco J., Muñoz A., Buisan R., Jarosz W., Gutierrez D.: A framework for transient rendering. *ACM Trans. Graph. 33*, 6 (2014). 1, 2, 3

[JNSJ11] Jarosz W., Nowrouzezahrai D., Sadeghi I., Jensen H. W.: A comprehensive theory of volumetric radiance estimation using photon points and beams. *ACM Trans. Graph. 30*, 1 (Feb. 2011), 5:1–5:19. 2, 3

[JNT*11] Jarosz W., Nowrouzezahrai D., Thomas R., Sloan P.-P., Zwicker M.: Progressive photon beams. *ACM Trans. Graph. (TOG) 30*, 6 (2011), 181. 2, 5

[JPMP12] Jiménez D., Pizarro D., Mazo M., Palazuelos S.: Modeling and correction of multipath interference in time of flight cameras. In *IEEE Computer Vision and Pattern Recognition* (2012). 2

[JPMP14] Jiménez D., Pizarro D., Mazo M., Palazuelos S.: Modeling and correction of multipath interference in time of flight cameras. *Image Vision Comput. 32*, 1 (Jan. 2014). 2

[JZJ08] Jarosz W., Zwicker M., Jensen H. W.: The beam radiance estimate for volumetric photon mapping. In *ACM SIGGRAPH 2008 classes* (2008), ACM, p. 3. 2

[Kaj86] Kajiya J. T.: The rendering equation. In *SIGGRAPH* (1986). 2

[KD13] Kaplanyan A. S., Dachsbacher C.: Adaptive progressive photon mapping. *ACM Trans. Graph. (TOG) 32*, 2 (2013), 16. 2

[KGH*14] Křivánek J., Georgiev I., Hachisuka T., Vévoda P., Šik M., Nowrouzezahrai D., Jarosz W.: Unifying points, beams, and paths in volumetric light transport simulation. *ACM Trans. Graph. (Proceedings of SIGGRAPH) 33*, 4 (July 2014). 2, 5

[KK09] Keller M., Kolb A.: Real-time simulation of time-of-flight sensors. *Simulation Modelling Practice and Theory 17*, 5 (2009). 2

[KOKP07] Keller M., Orthmann J., Kolb A., Peters V.: A simulation framework for time-of-flight sensors. In *International Symposium on Signals, Circuits and Systems 2007* (2007). 2

[KPM*16] Klein J., Peters C., Martín J., Laurenzis M., Hullin M. B.: Tracking objects outside the line of sight using 2D intensity images. *Scientific Reports 6* (2016). 2

[KZ11] Knaus C., Zwicker M.: Progressive photon mapping: A probabilistic approach. *ACM Trans. Graph. (TOG) 30*, 3 (2011), 25. 2

[MNJK13] Meister S., Nair R., Jähne B., Kondermann D.: *Photon Mapping based Simulation of Multi-Path Reflection Artifacts in Time-of-Flight Sensors*. Tech. rep., Heidelberg Collaboratory for Image Processing, 2013. 2

[MNK13] Meister S., Nair R., Kondermann D.: Simulation of time-of-flight sensors using global illumination. In *Vision, Modeling & Visualization* (2013). 2

[NML*13] Nair R., Meister S., Lambers M., Balda M., Hofmann H., Kolb A., Kondermann D., Jähne B.: Ground truth for evaluating time of flight imaging. In *Time-of-Flight and Depth Imaging. Sensors, Algorithms, and Applications*. 2013. 2

[OHX*14] O'Toole M., Heide F., Xiao L., Hullin M. B., Heidrich W., Kutulakos K. N.: Temporal frequency probing for 5D transient analysis of global light transport. *ACM Trans. Graph. 33*, 4 (2014). 2

[SSD08] Smith A., Skorupski J., Davis J.: *Transient Rendering*. Tech. Rep. UCSC-SOE-08-26, School of Engineering, University of California, Santa Cruz, 2008. 1, 2

[Vea97] Veach E.: *Robust Monte Carlo methods for light transport simulation*. PhD thesis, Stanford, 1997. 2

[VWJ*13] Velten A., Wu D., Jarabo A., Masia B., Barsi C., Joshi C., Lawson E., Bawendi M., Gutierrez D., Raskar R.: Femto-photography: Capturing and visualizing the propagation of light. *ACM Trans. Graph. 32*, 4 (2013). 5