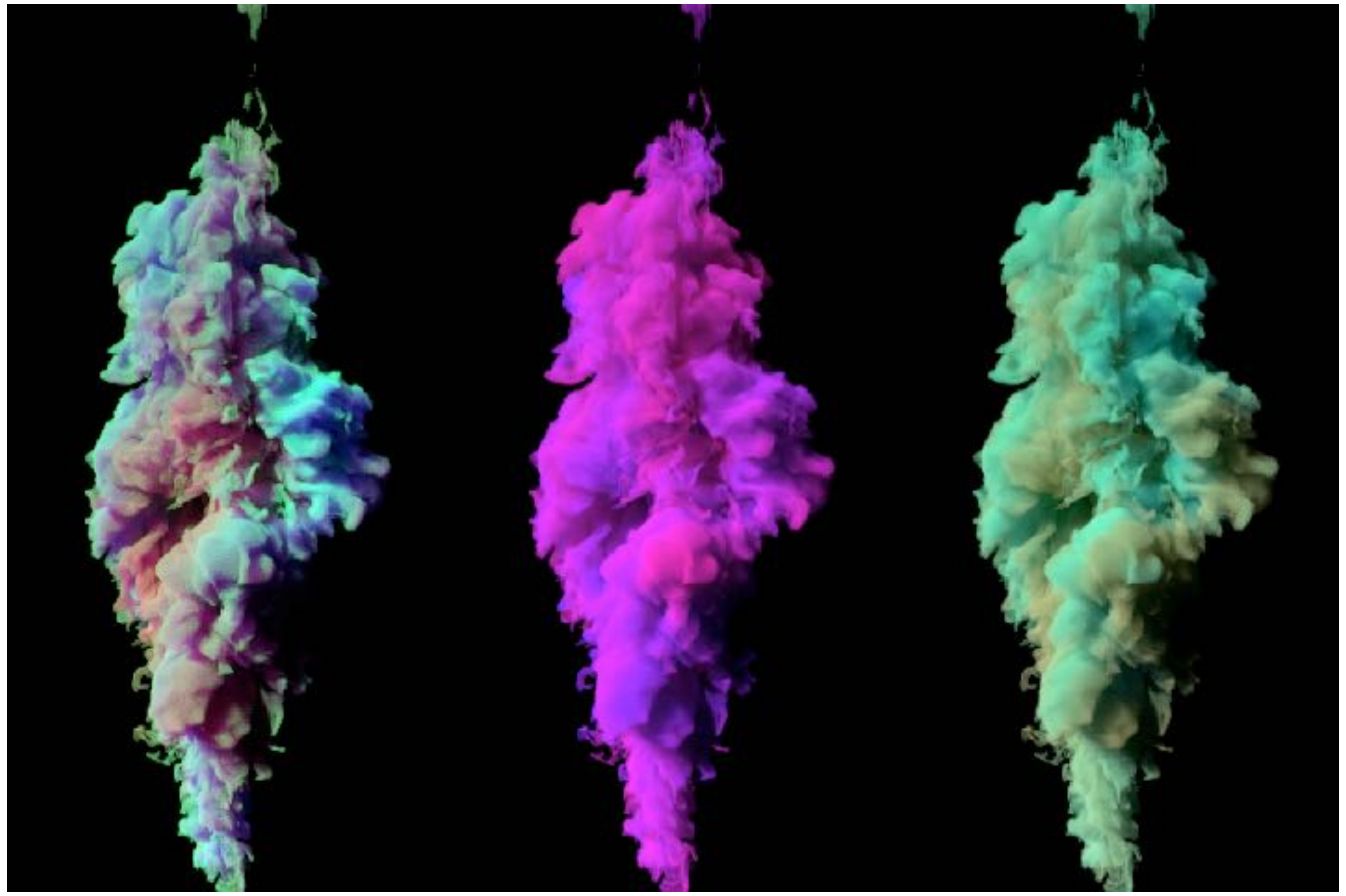
A NULL SCATTERING PATH INTEGRAL FORMULATION OF LIGHT TRANSPORT



Bailey Miller[1*] Iliyan Georgiev[2*] Wojciech Jarosz[1]
Dartmouth College[1] Autodesk[2]
authors with equal contribution*

DARTMOUTH
VISUAL COMPUTING LAB

AUTODESK

thrive
SIGGRAPH2019
LOS ANGELES • 28 JULY – 1 AUGUST

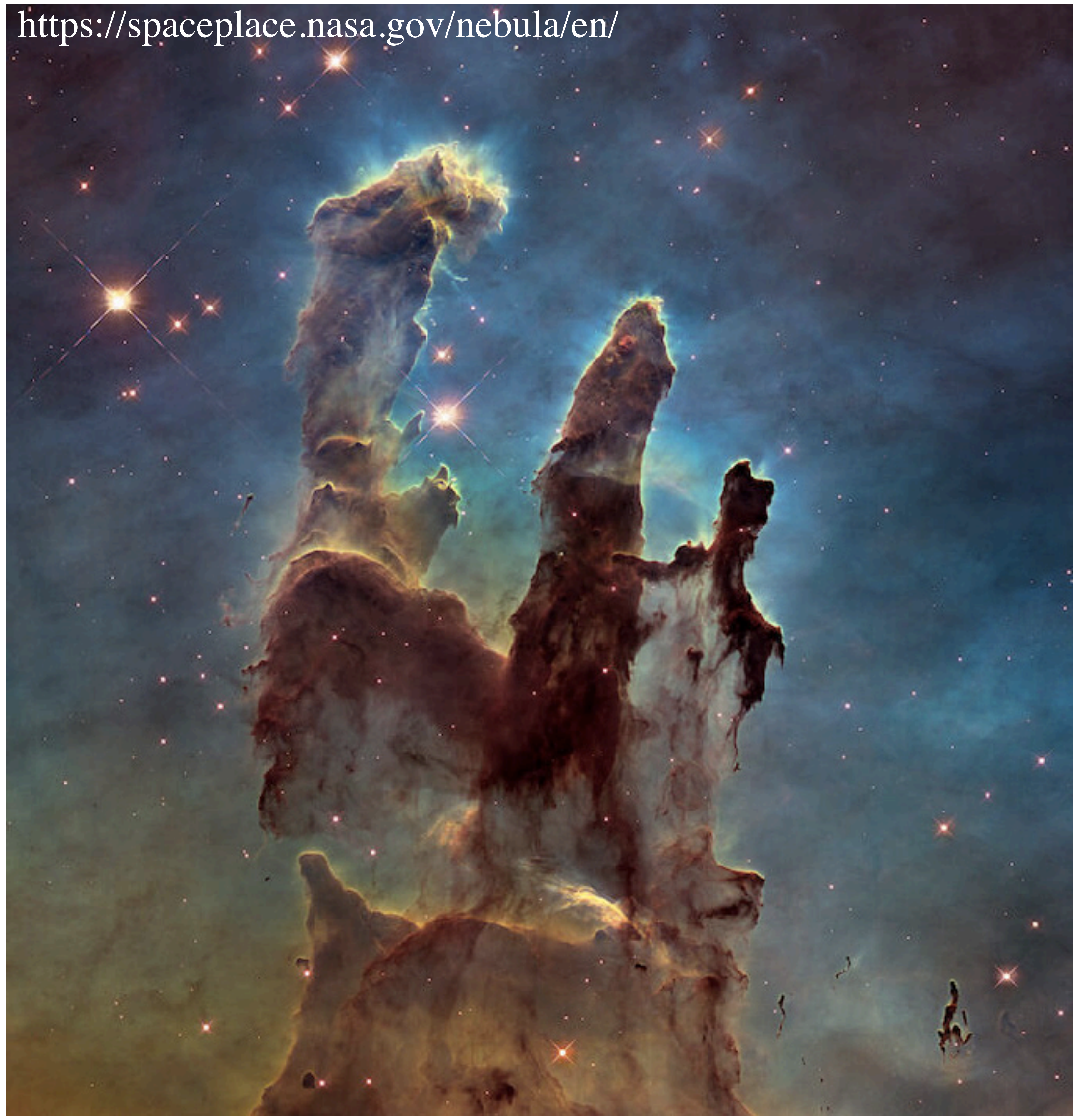
Motivation

https://www.tripadvisor.com/LocationPhotoDirectLink-g189180-d10486874-i314214380-Porto_Bridge_Climb-Porto_Porto_District_Northern_Portugal.html

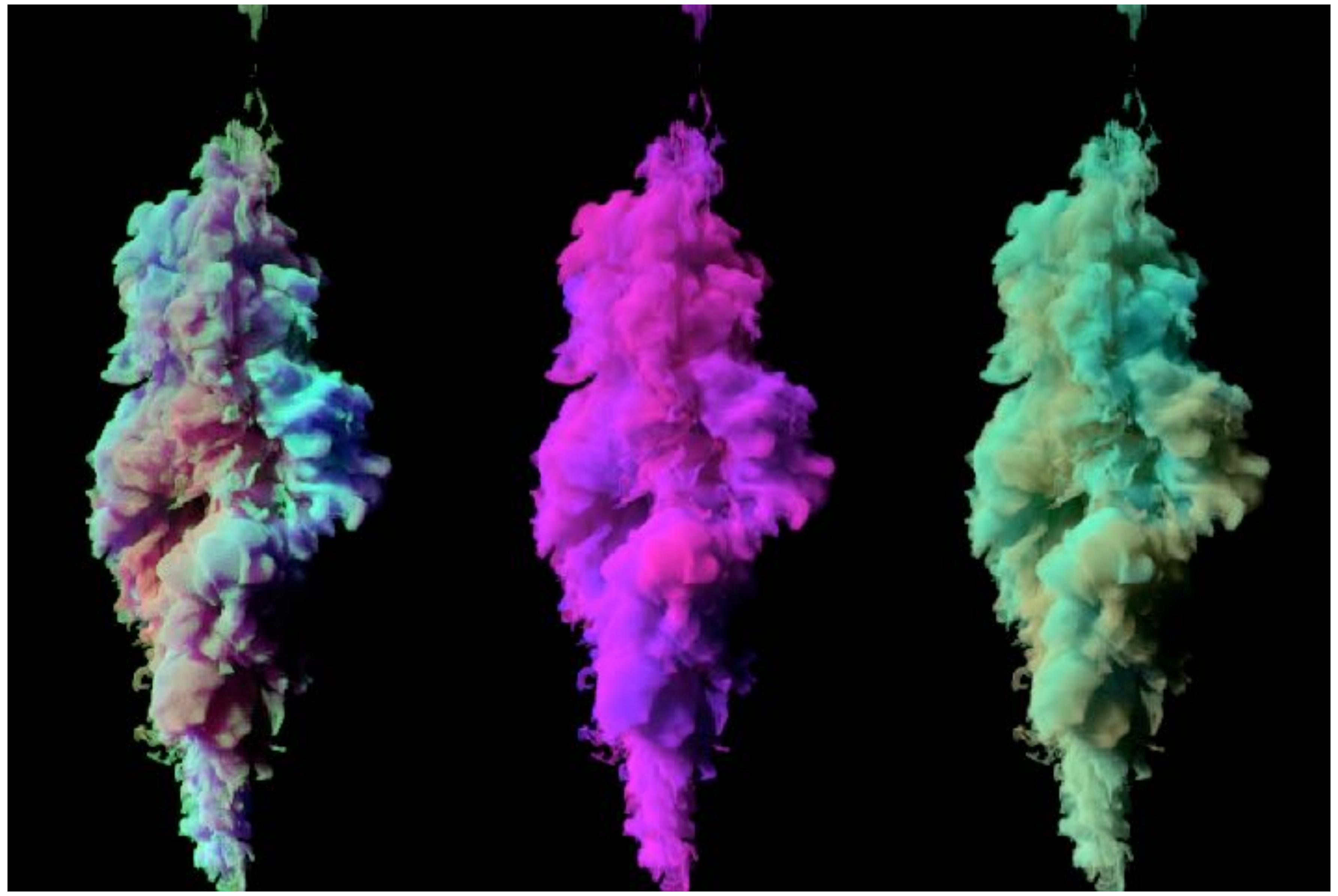
https://hiveminer.com/Tags/bored%2Cboring

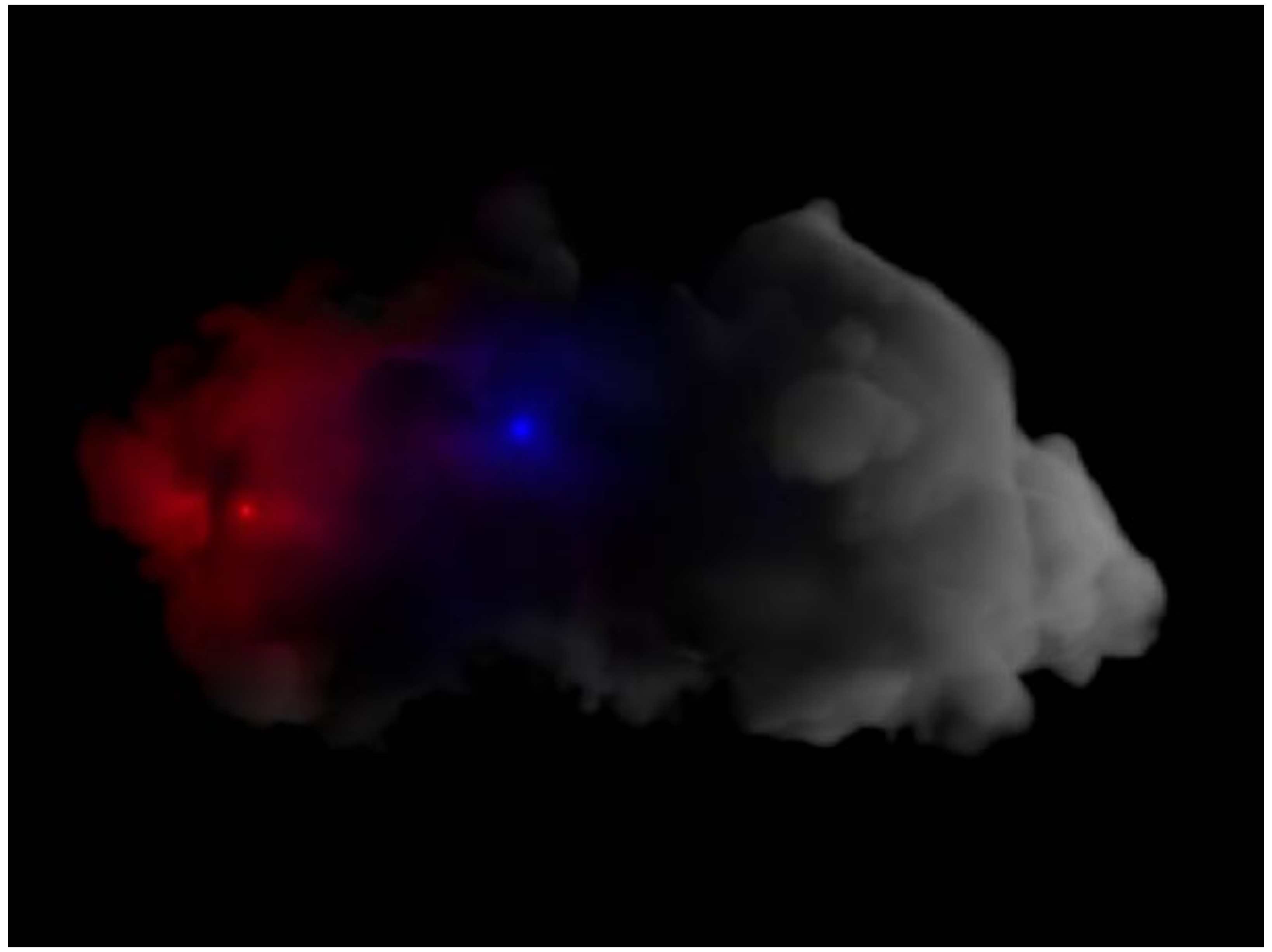
Motivation


https://www.pinterest.com/pin/437694275598495929/


https://spaceplace.nasa.gov/nebula/en/

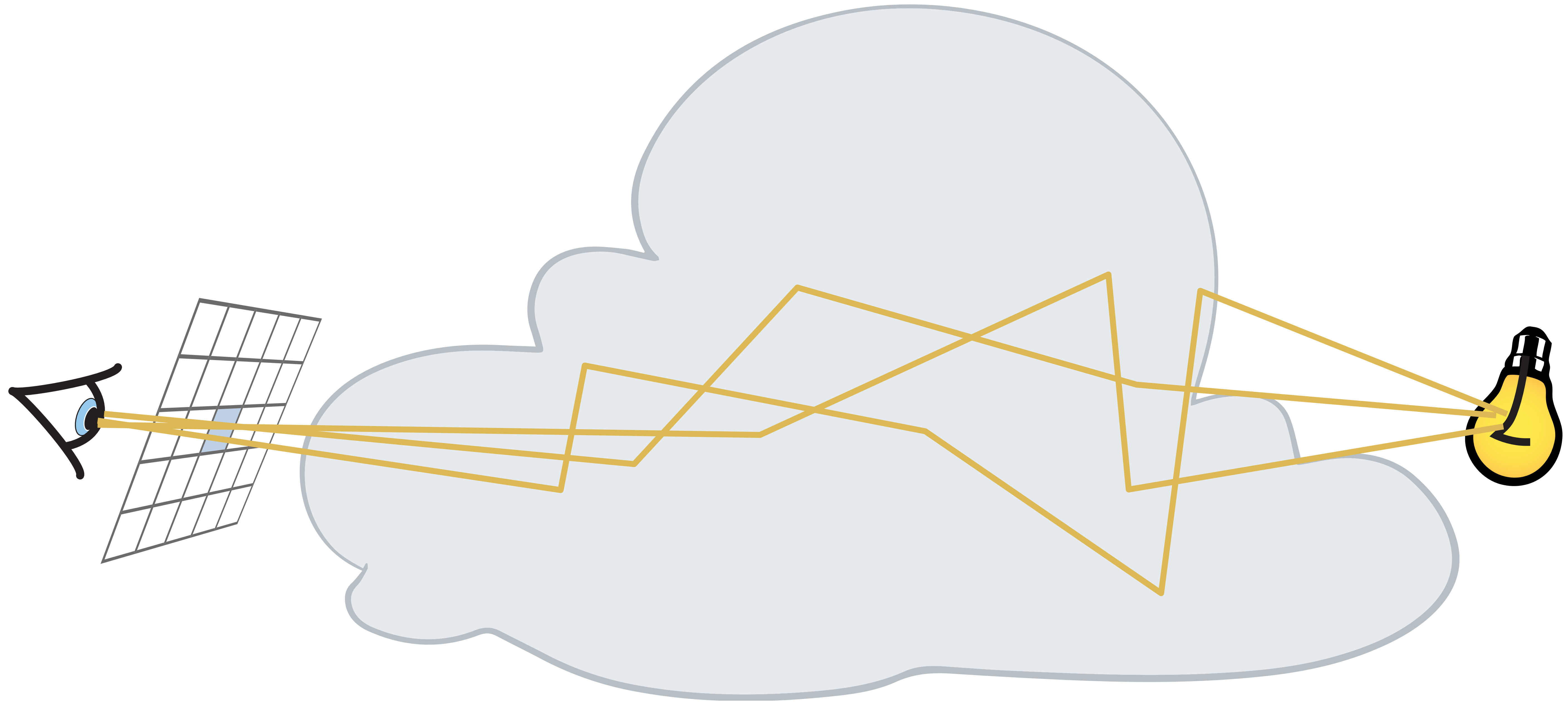
Motivation



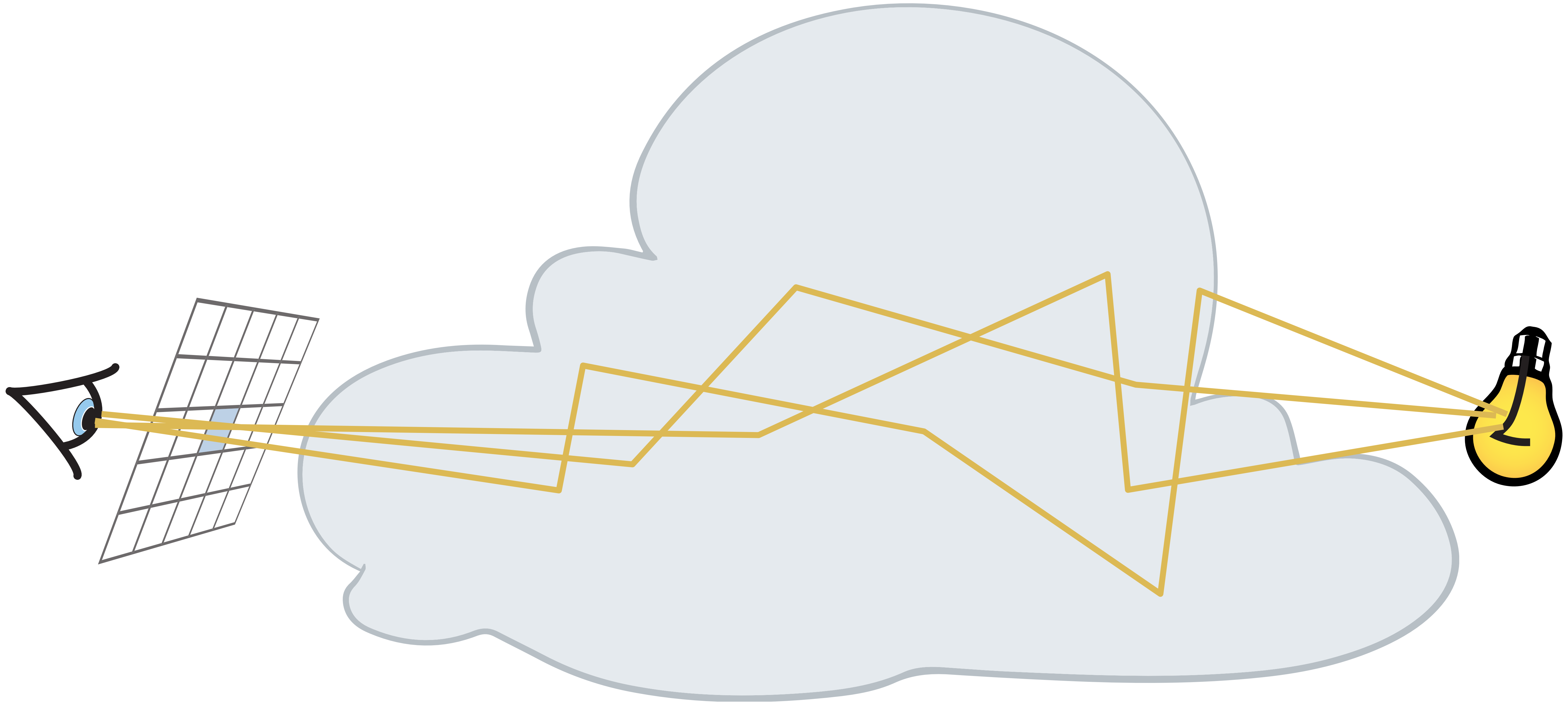


Background: Path Tracing

Background: Path Tracing



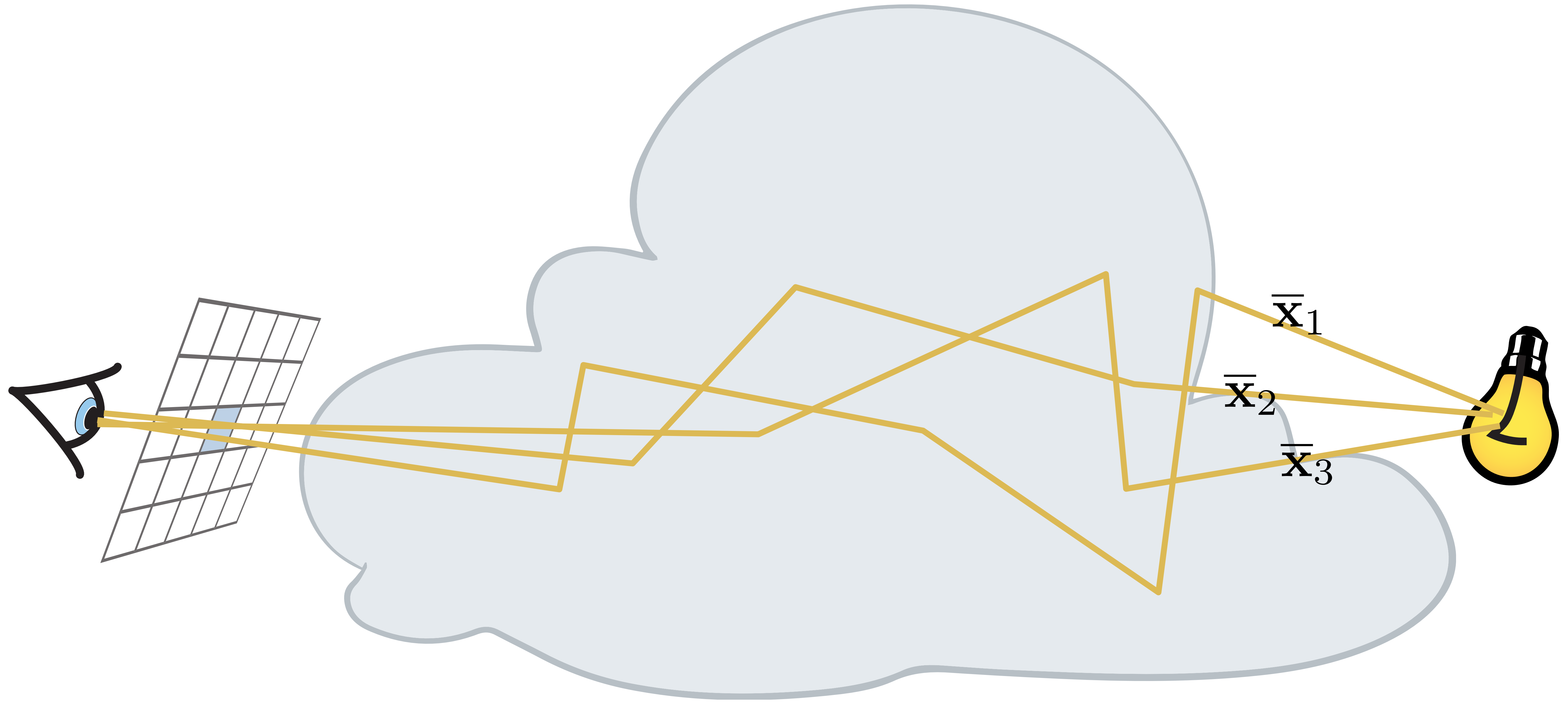
Background: Path Tracing



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\overline{\mathbf{x}}) d\overline{\mathbf{x}}$$

Background: Path Tracing



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\overline{\mathbf{x}}) d\overline{\mathbf{x}}$$

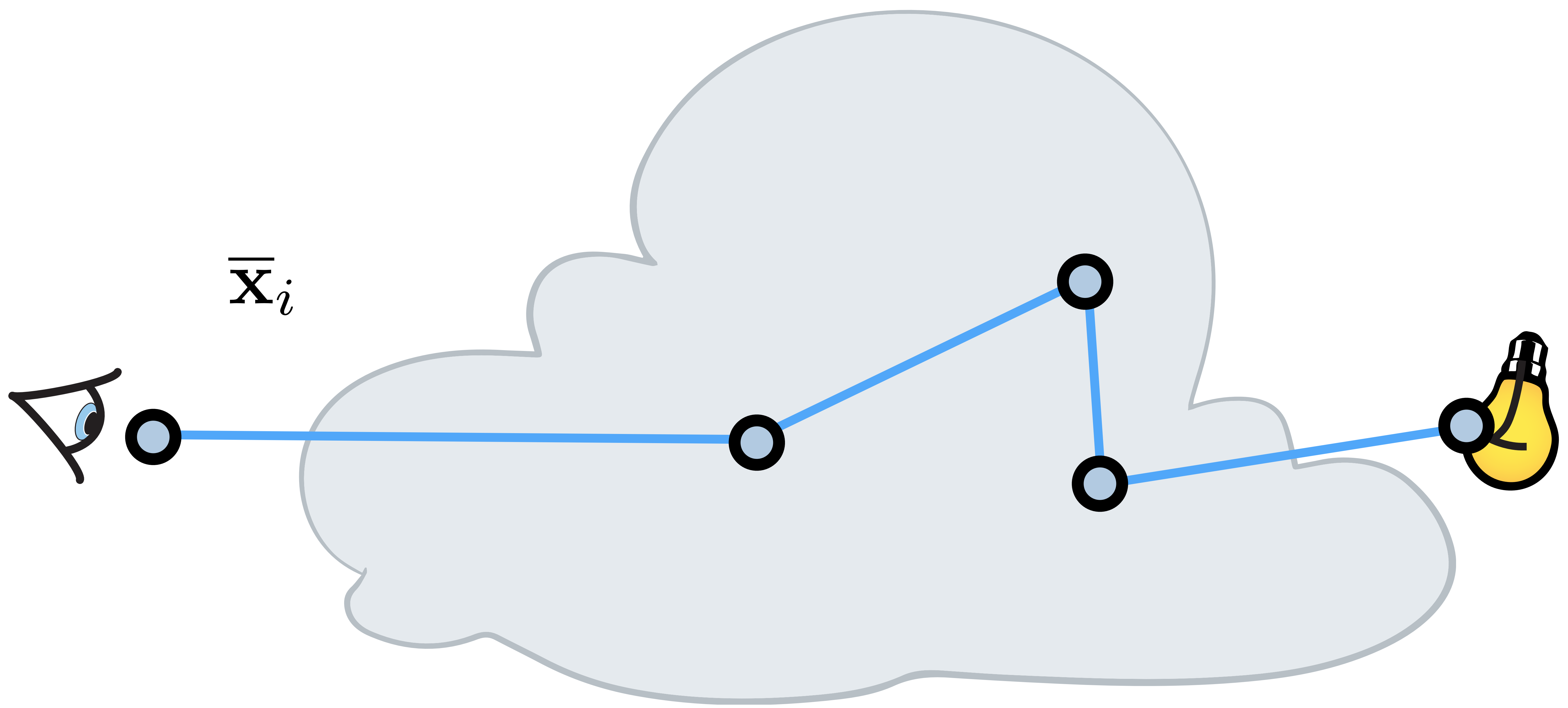
Pixel estimator

$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(\overline{\mathbf{x}}_i)}{p(\overline{\mathbf{x}}_i)}$$

path contribution

path pdf

Background: Path Tracing



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\overline{\mathbf{x}}) d\overline{\mathbf{x}}$$

Pixel estimator

$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(\overline{\mathbf{x}}_i)}{p_k(\overline{\mathbf{x}}_i)}$$

path contribution

path pdf for strategy k

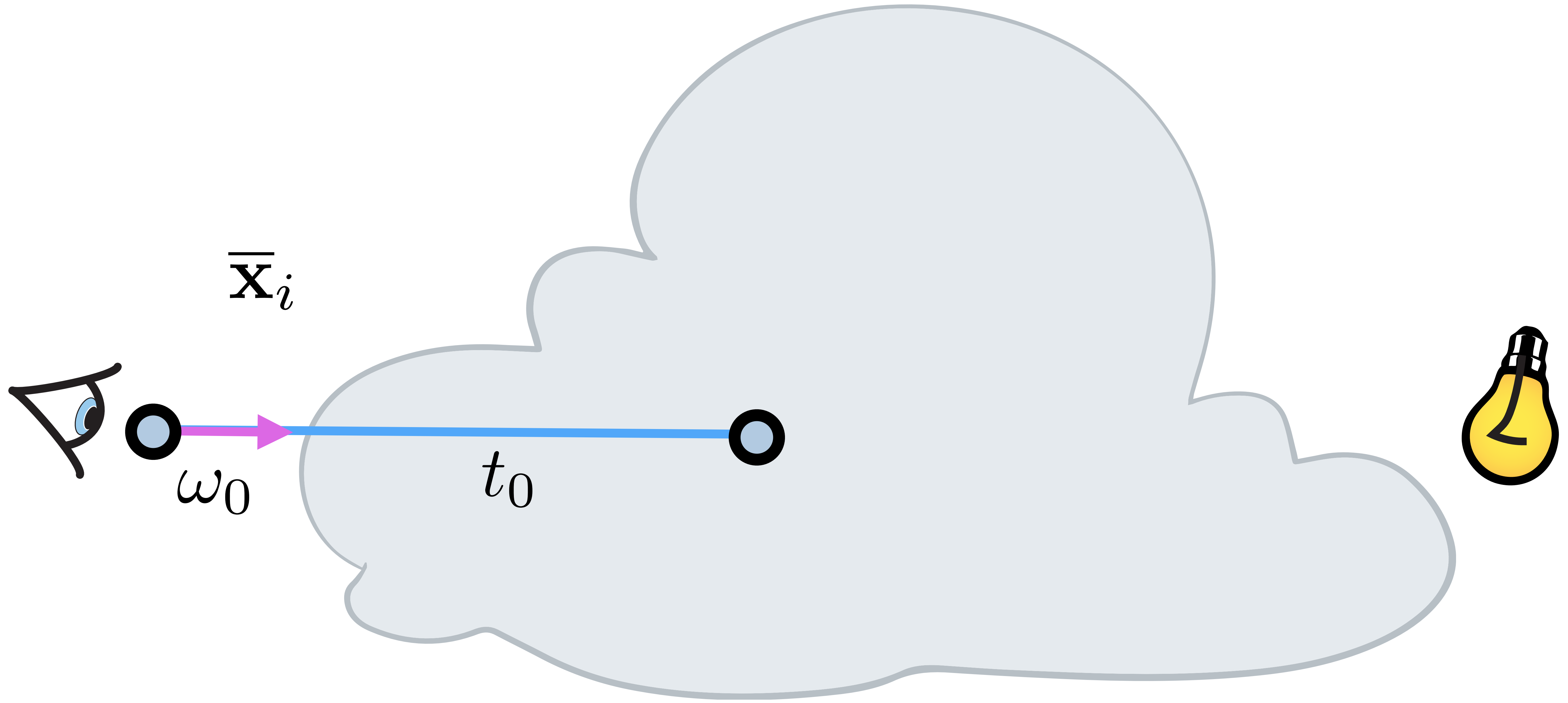
Background: Path Tracing

$$p_1(\overline{\mathbf{x}}_i) =$$

Background: Path Tracing

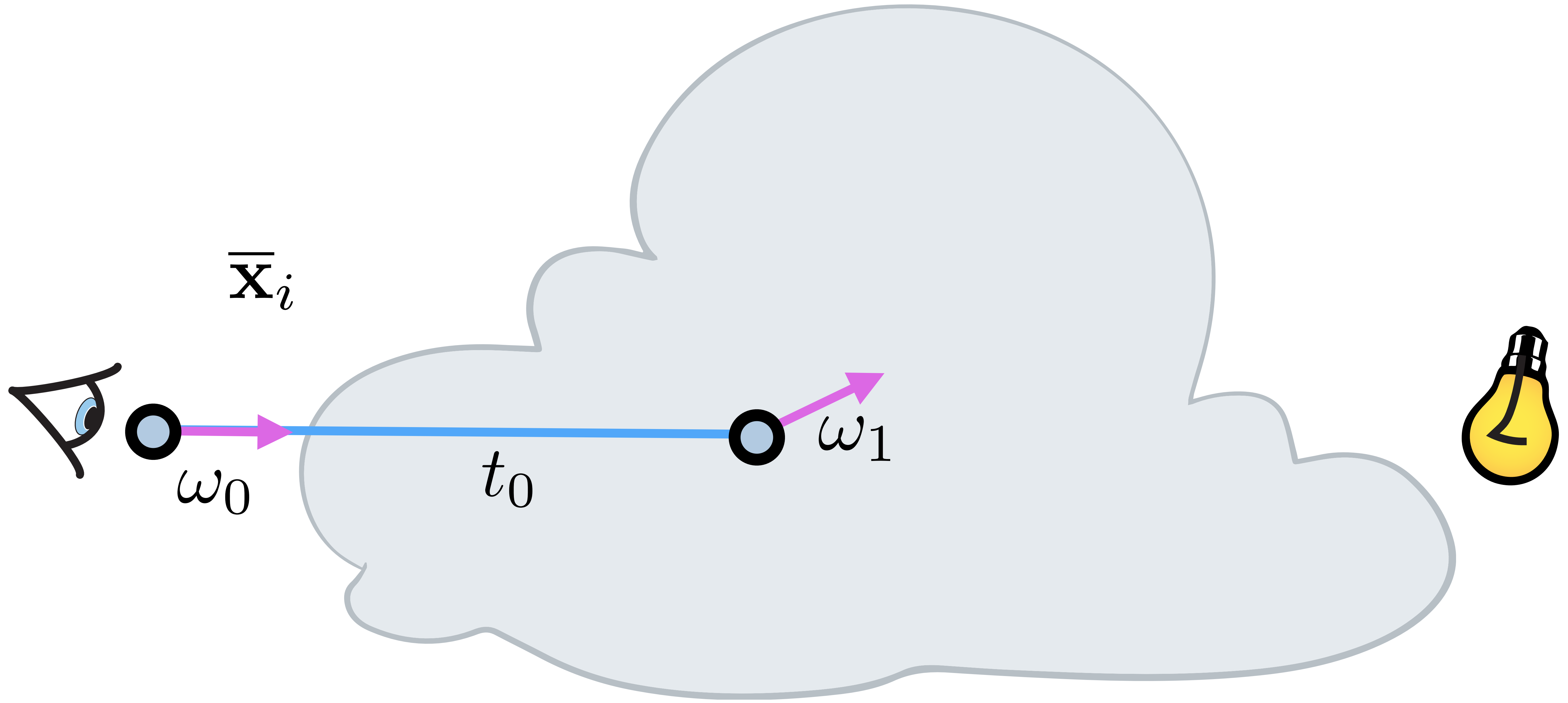
$$p_1(\overline{\mathbf{x}}_i) = p(\omega_0)$$

Background: Path Tracing



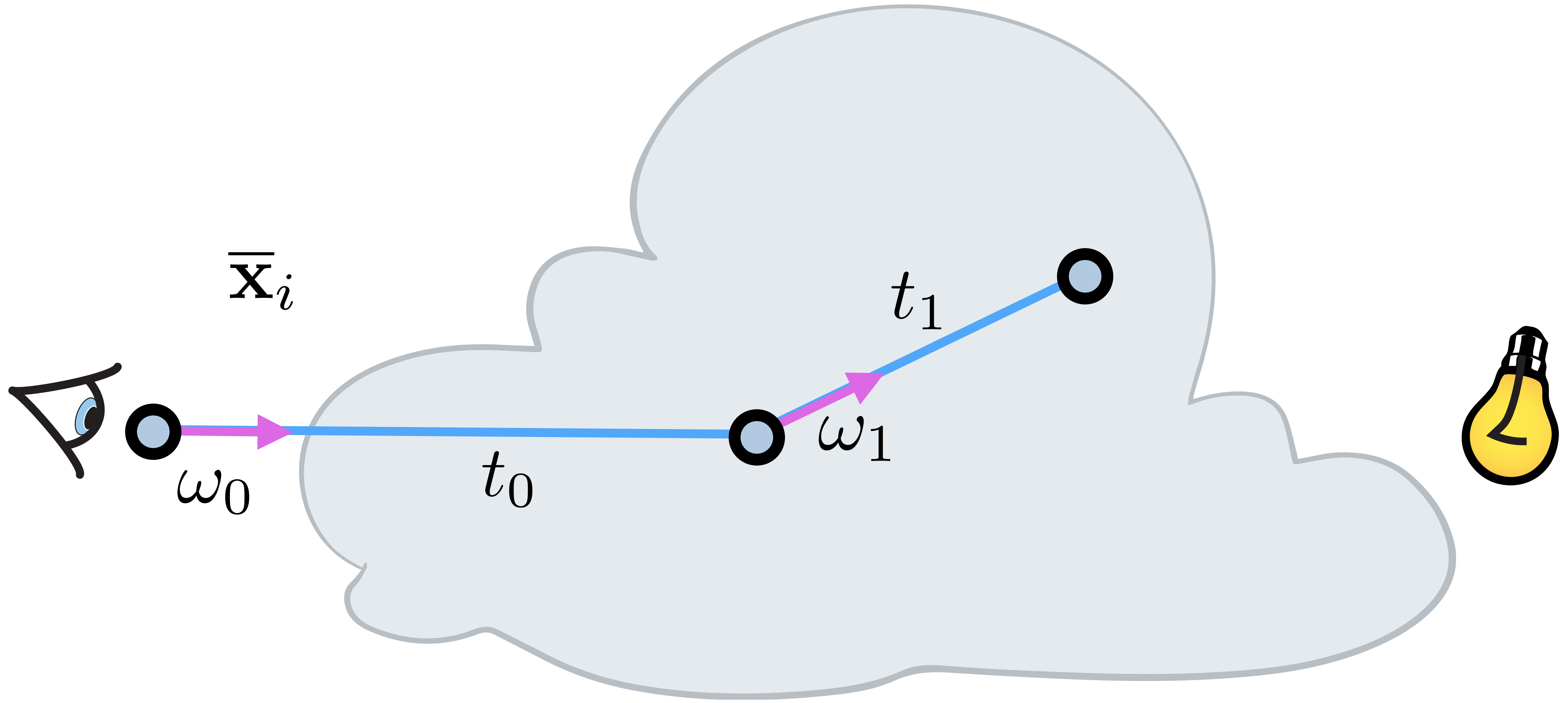
$$p_1(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)$$

Background: Path Tracing



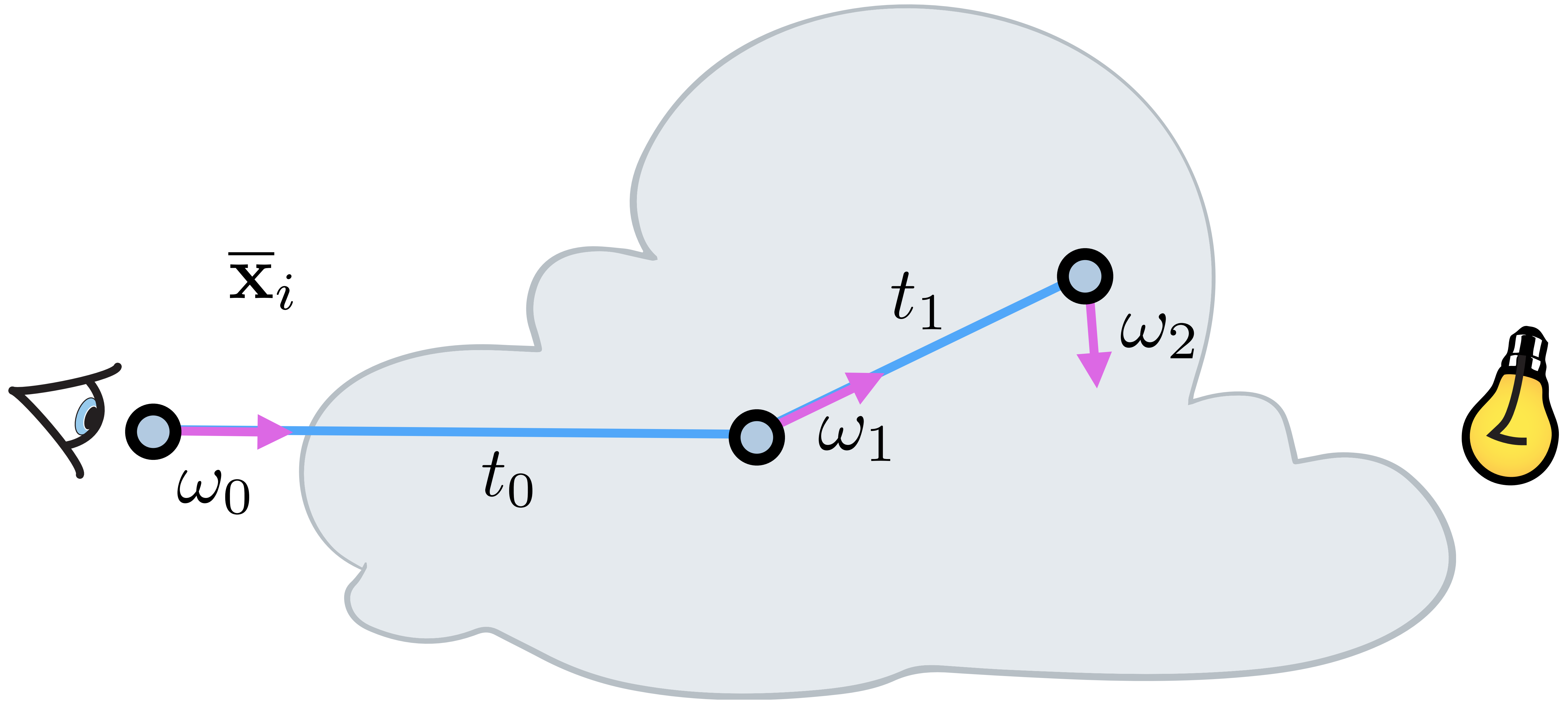
$$p_1(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(\omega_1)$$

Background: Path Tracing



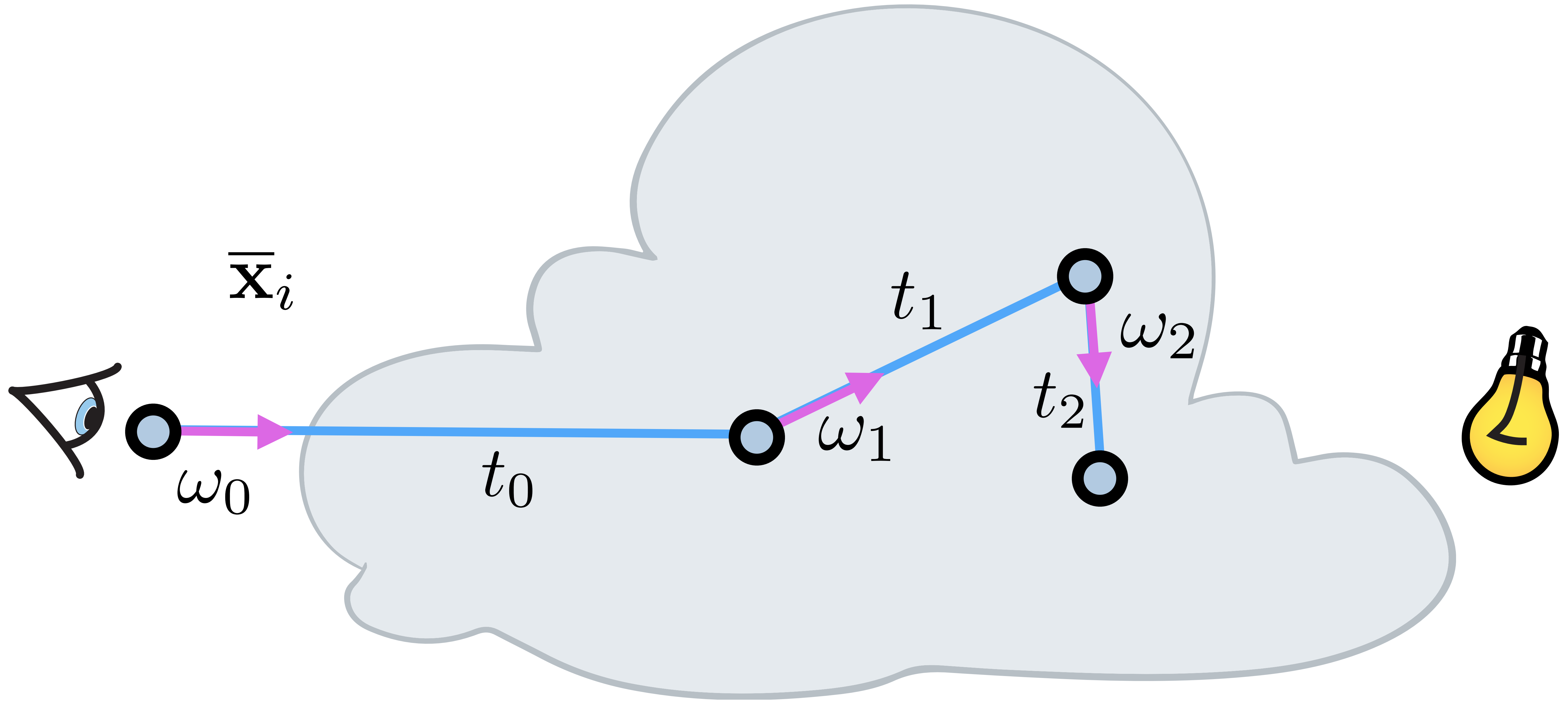
$$p_1(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(\omega_1)p(t_1)$$

Background: Path Tracing



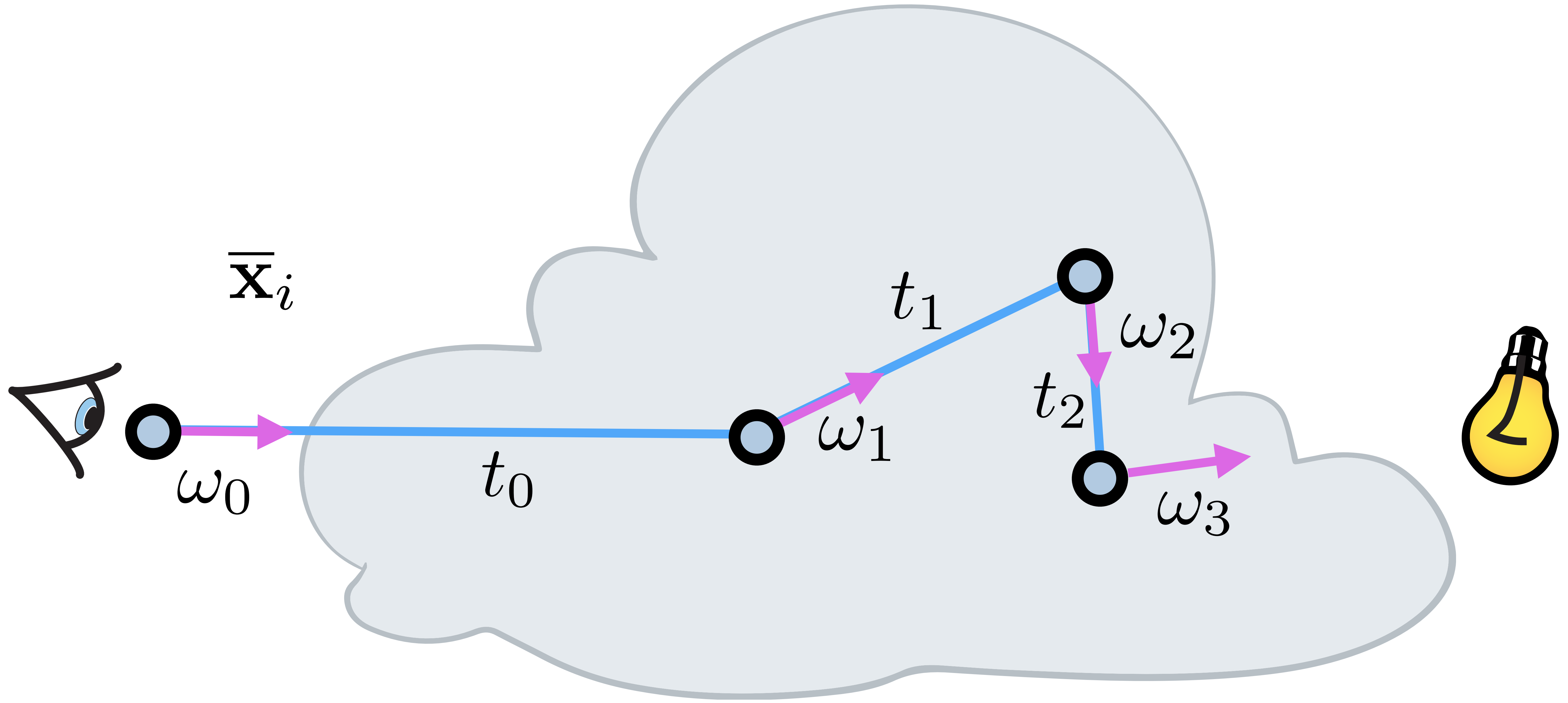
$$p_1(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(\omega_1)p(t_1)p(\omega_2)$$

Background: Path Tracing



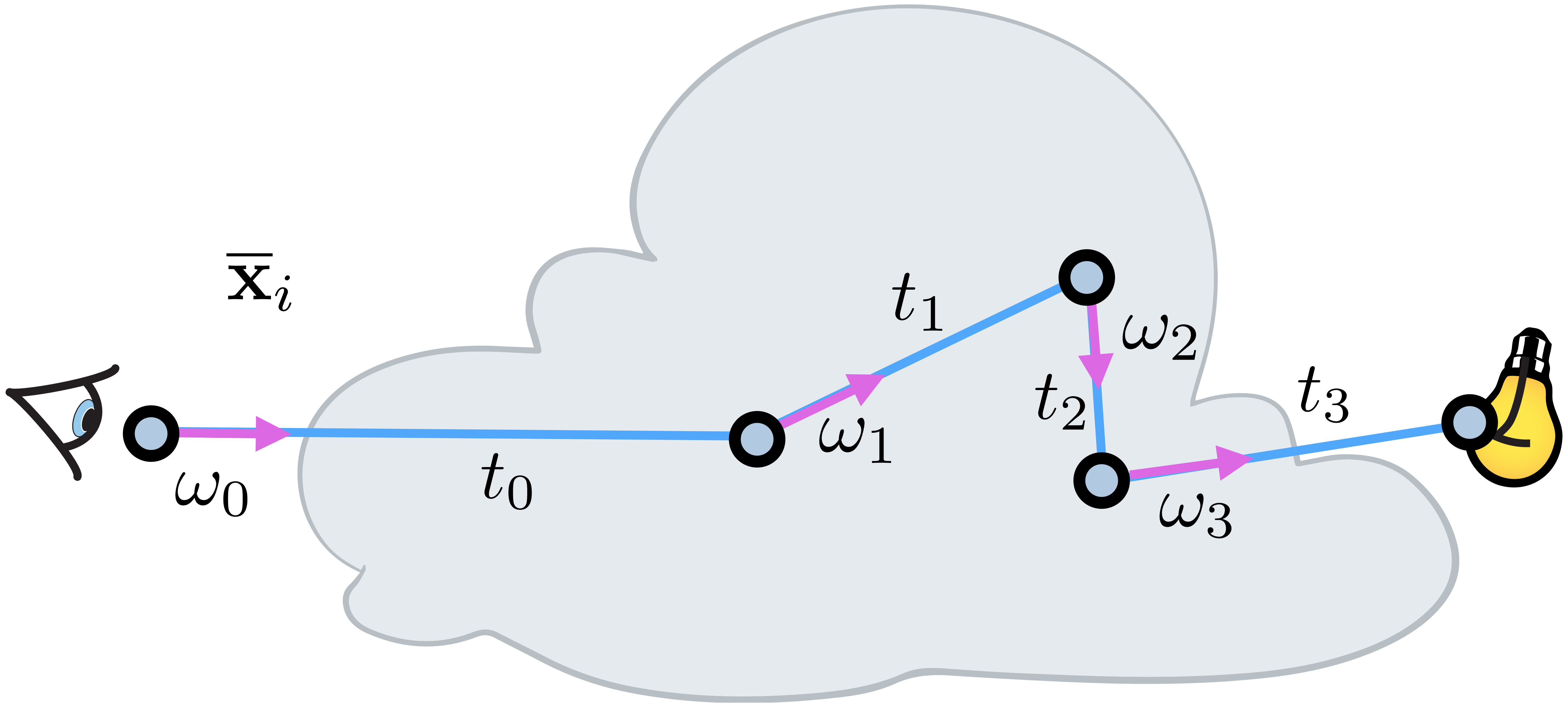
$$p_1(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(\omega_1)p(t_1)p(\omega_2)p(t_2)$$

Background: Path Tracing



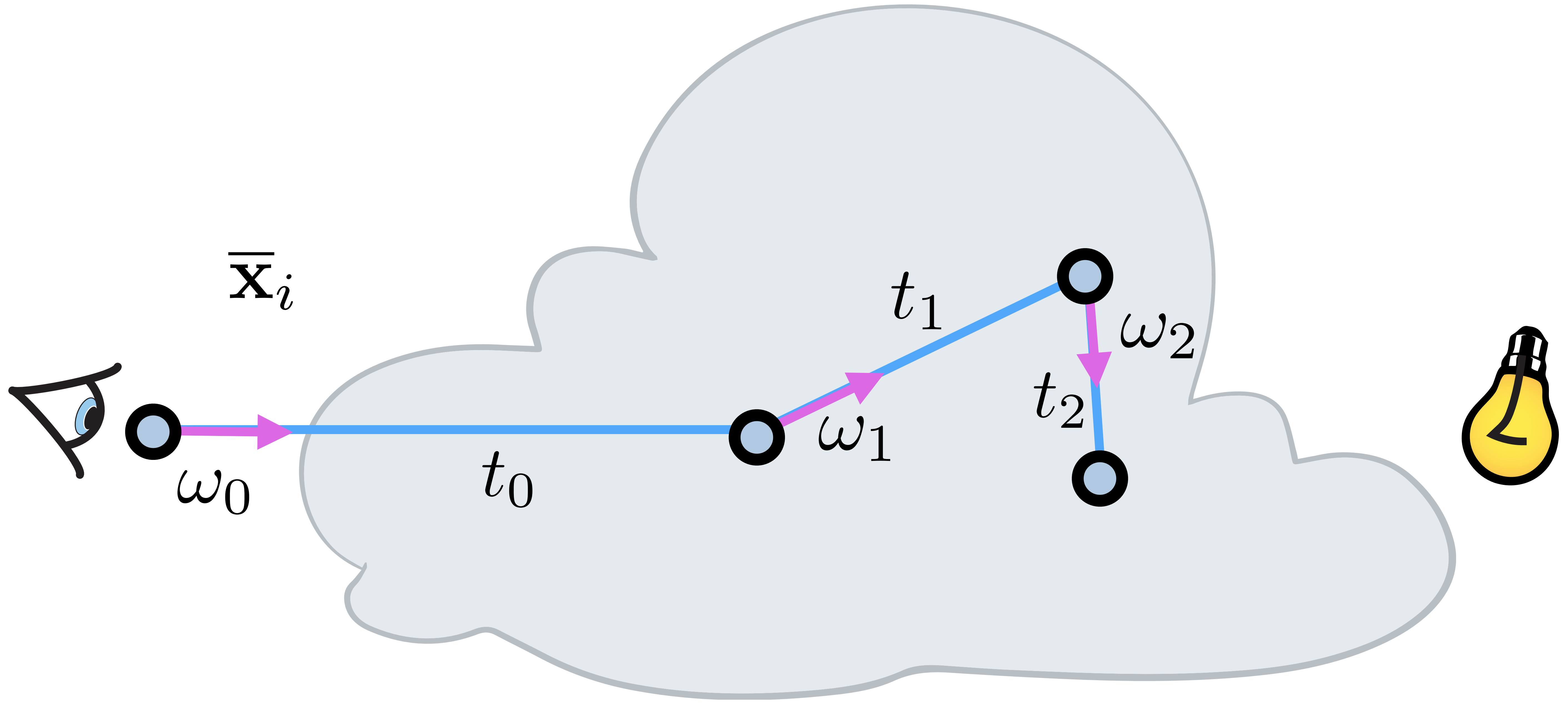
$$p_1(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(\omega_1)p(t_1)p(\omega_2)p(t_2)p(\omega_3)$$

Background: Path Tracing



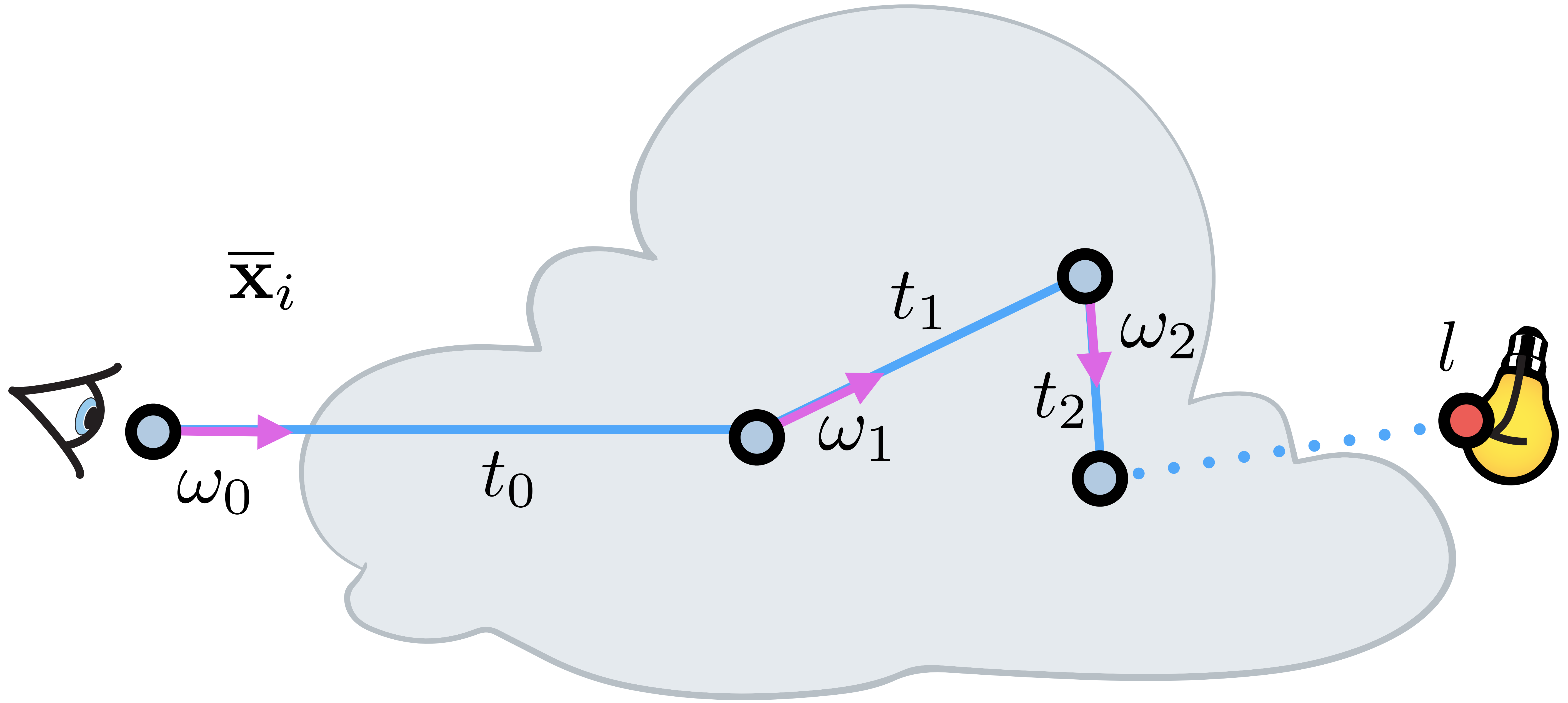
$$p_1(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(\omega_1)p(t_1)p(\omega_2)p(t_2)p(\omega_3)p(t_3)$$

Background: Path Tracing



$$p_2(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(\omega_1)p(t_1)p(\omega_2)p(t_2)$$

Background: Path Tracing

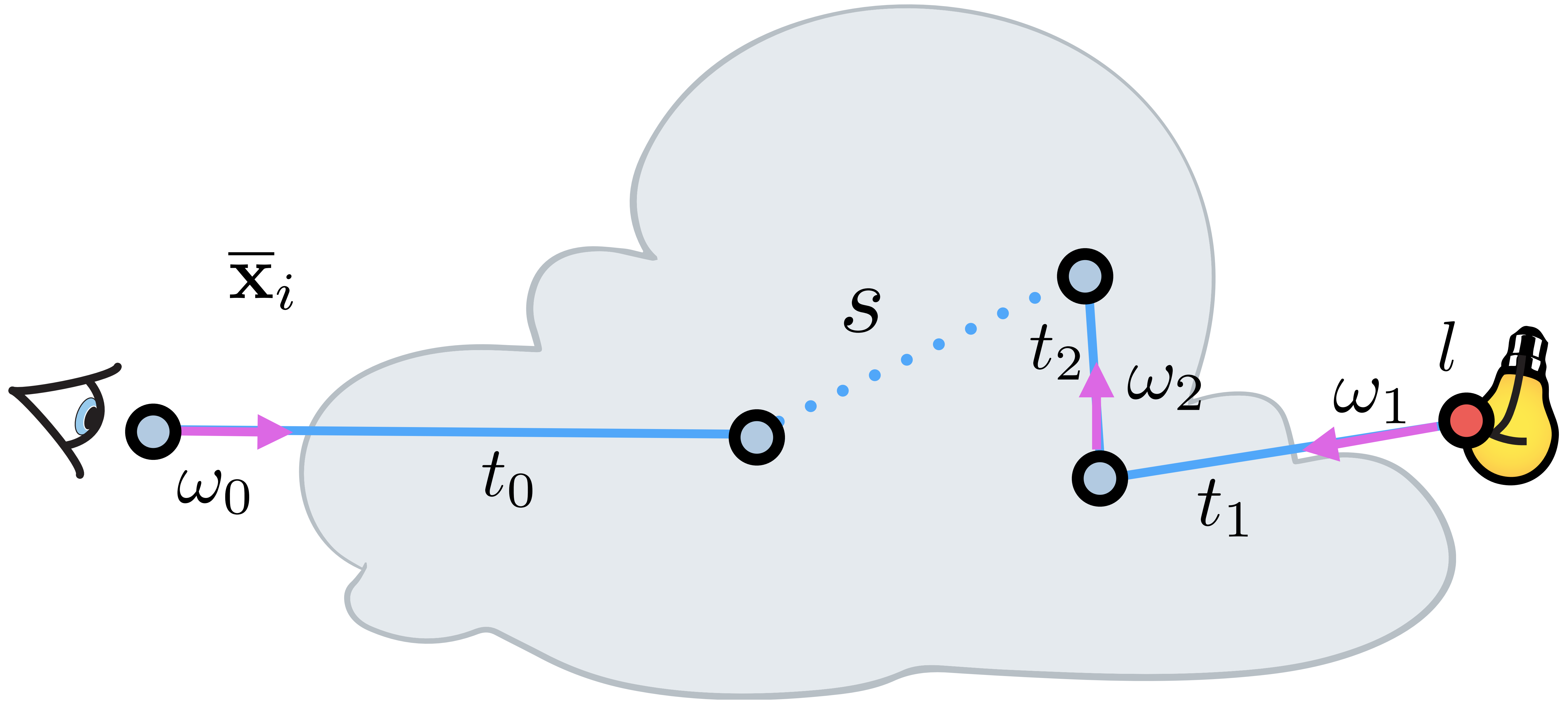


$$p_2(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(\omega_1)p(t_1)p(\omega_2)p(t_2)p(l)$$

Background: Path Tracing

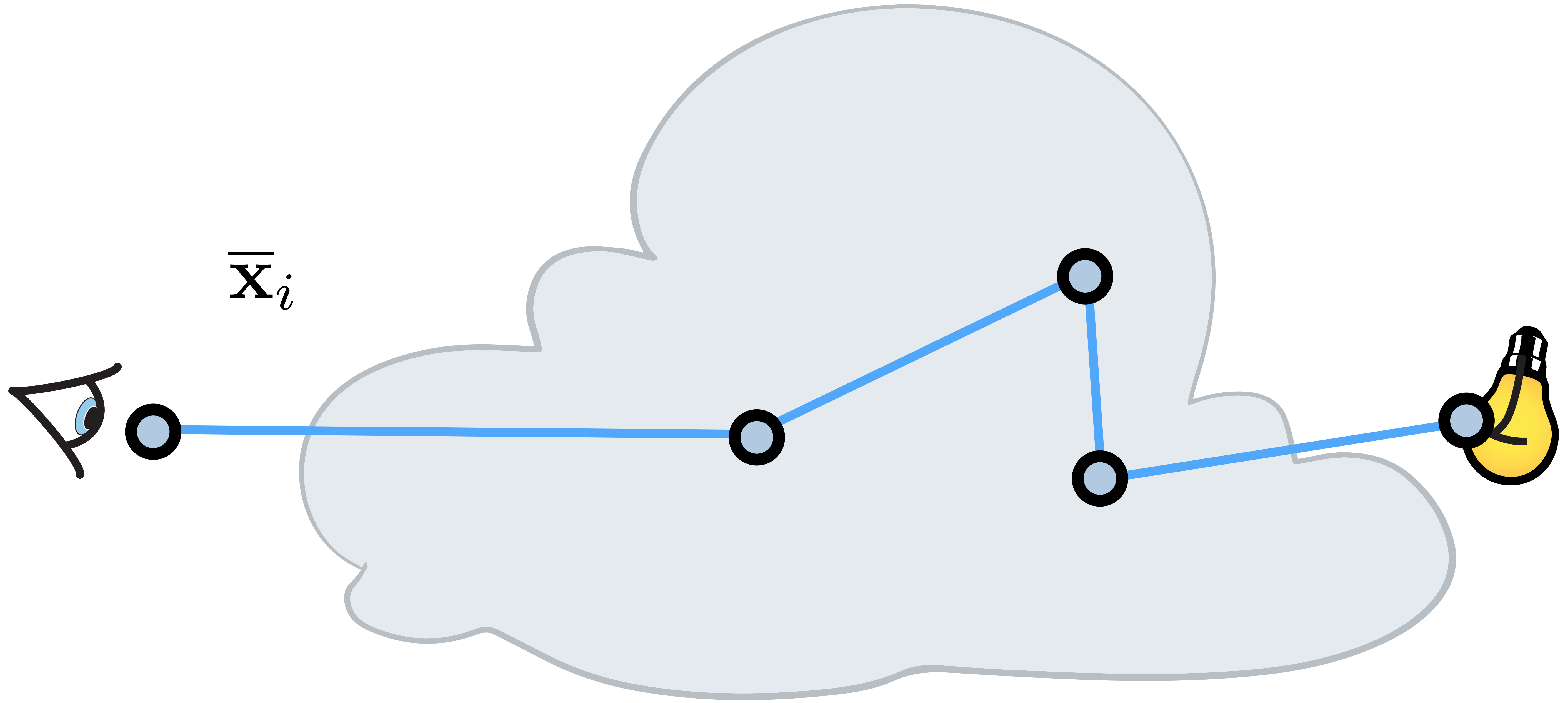
$$p_3(\overline{\mathbf{x}}_i) =$$

Background: Path Tracing



$$p_3(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(l)p(\omega_1)p(t_1)p(\omega_2)p(t_2)p(s)$$

Background: Importance Sampling



Pixel estimator

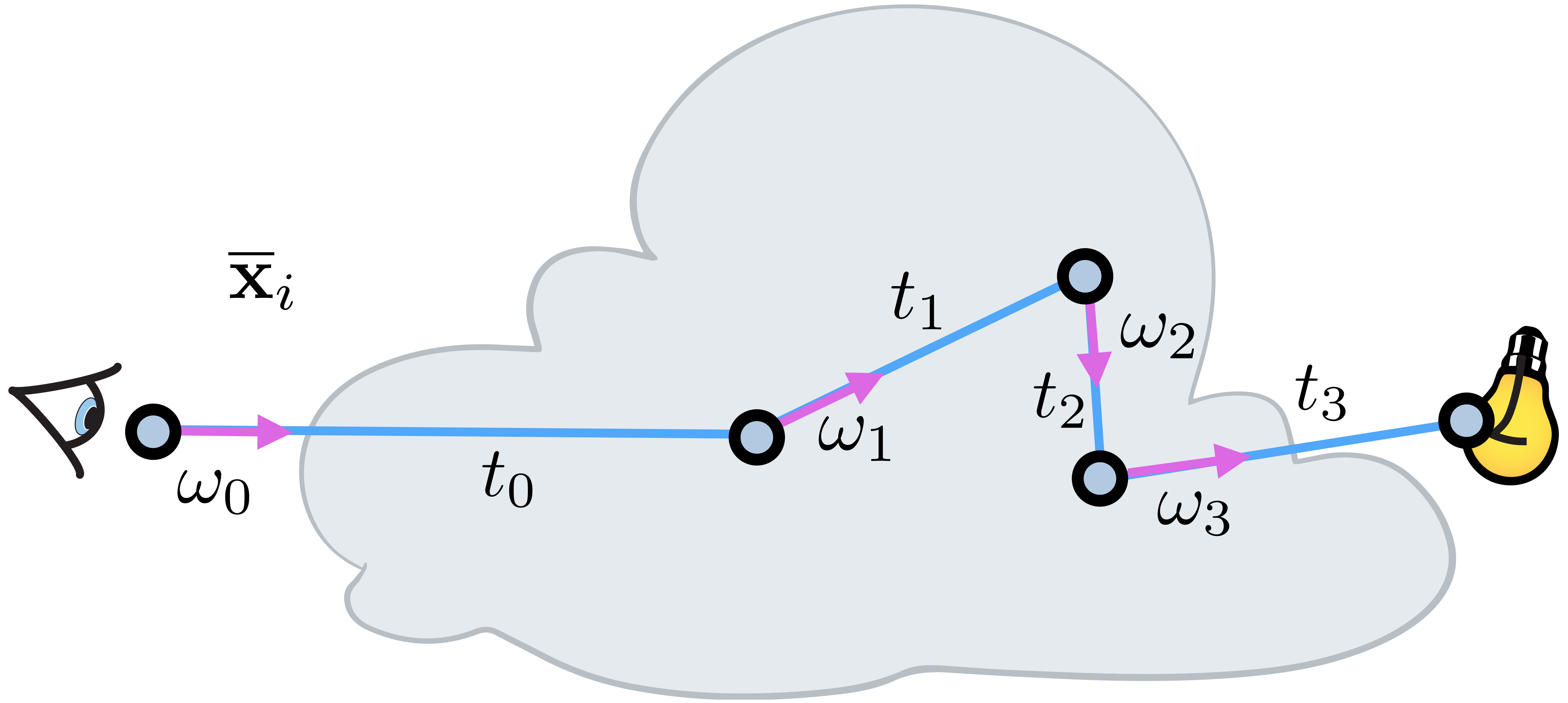
$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(\overline{\mathbf{x}}_i)}{p_k(\overline{\mathbf{x}}_i)}$$

path contribution

path pdf for strategy k

$\propto$

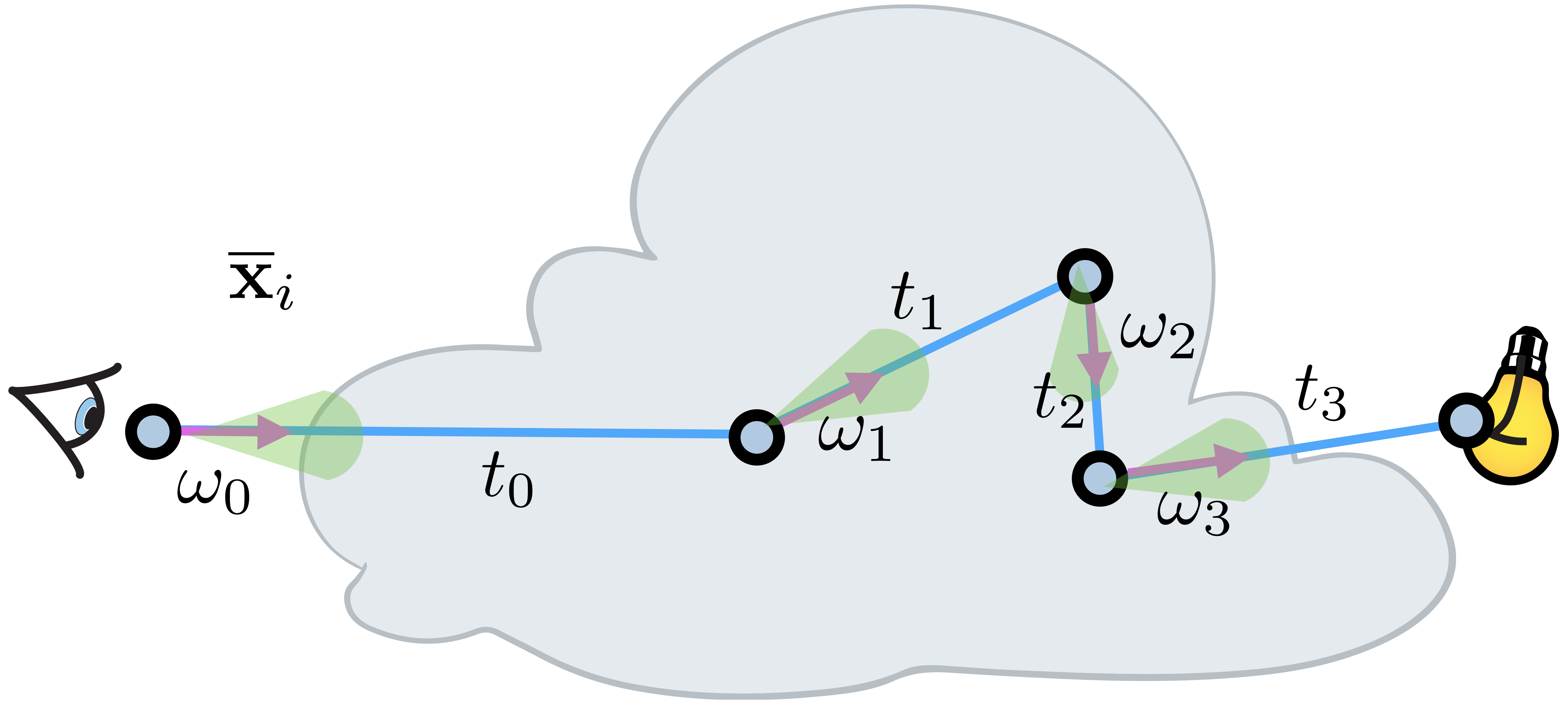
Background: Importance Sampling



Scattering transmittance

$$f(\overline{\mathbf{x}}_i) = \ldots f_s(\omega_j)T(t_j)\ldots$$

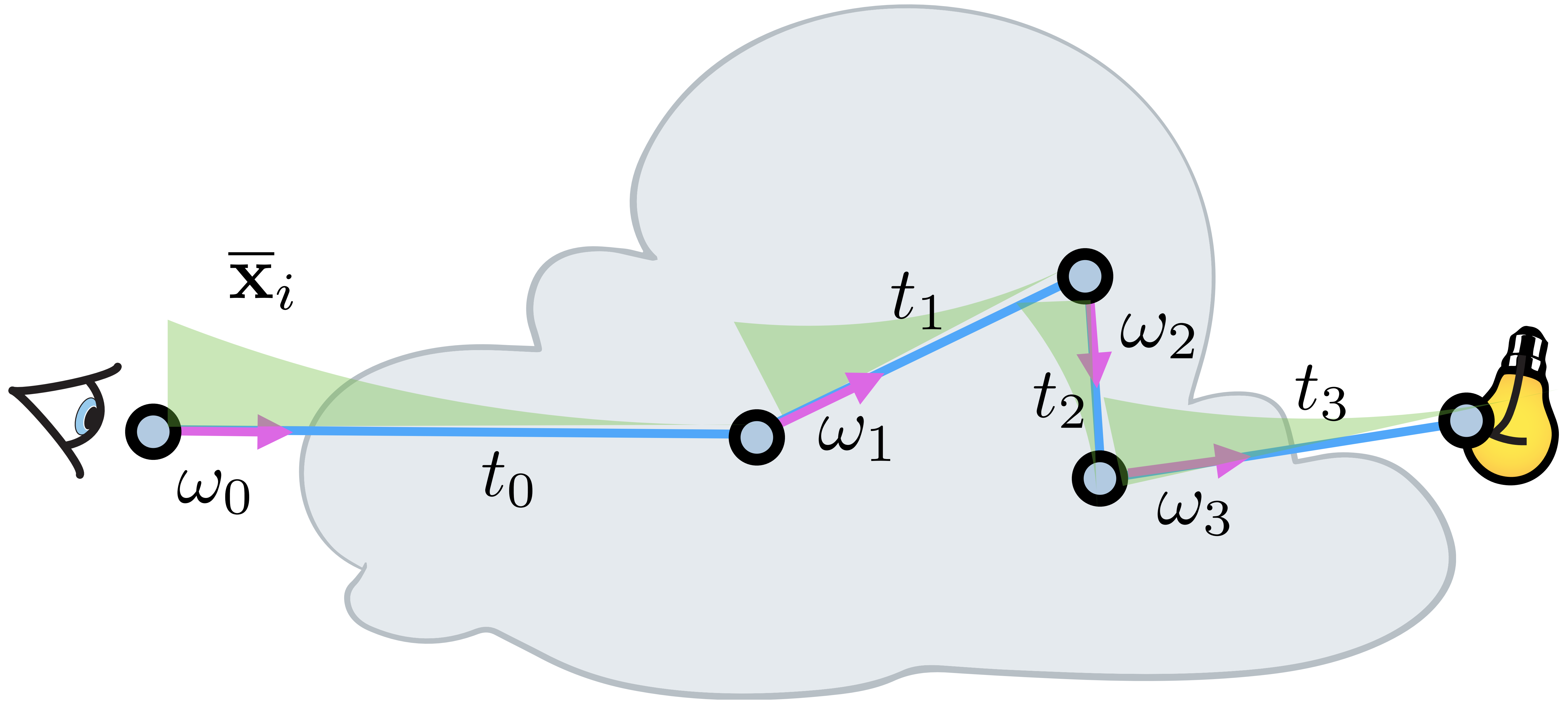
Background: Importance Sampling



Scattering transmittance

$$f(\overline{\mathbf{x}}_i) = \ldots f_s(\omega_j) T(t_j) \ldots$$

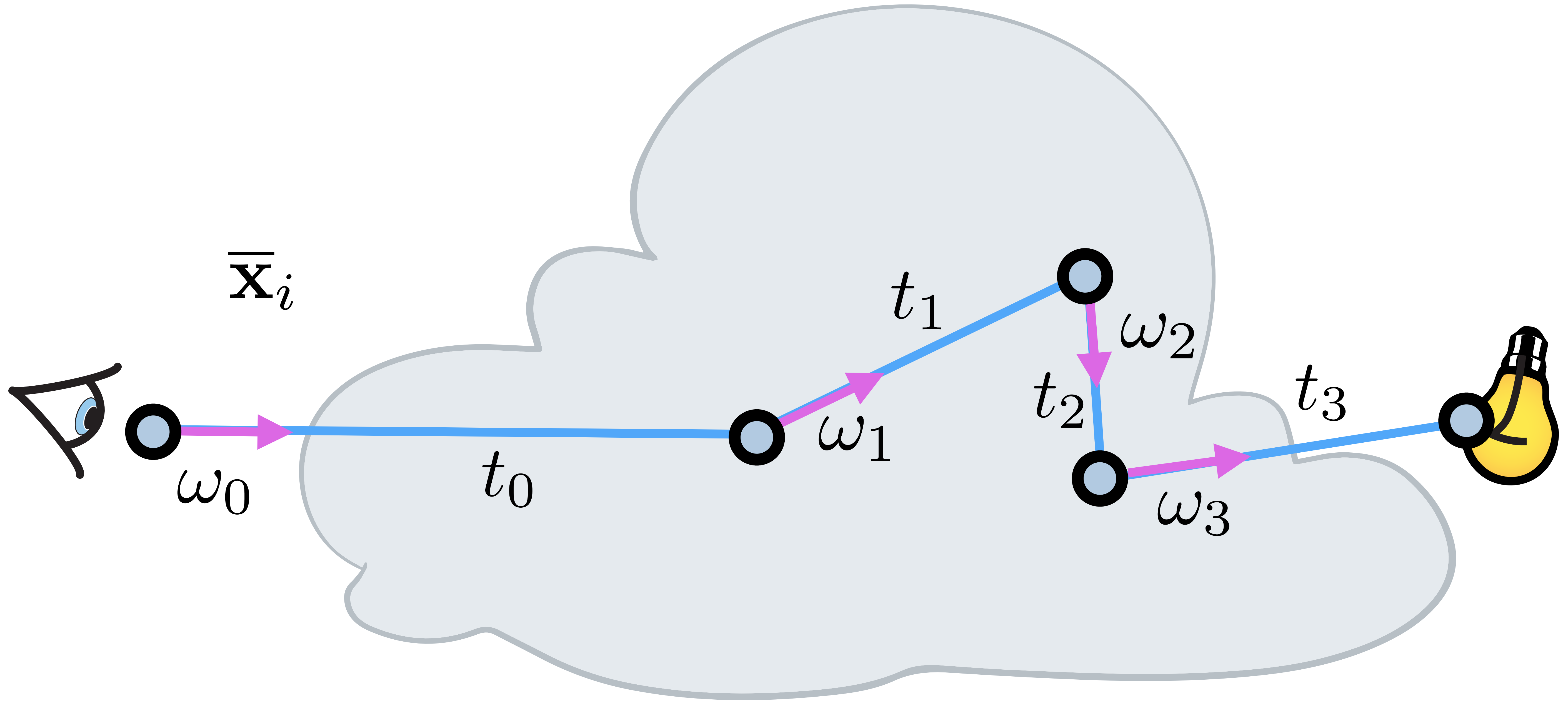
Background: Importance Sampling



Scattering transmittance

$$f(\overline{\mathbf{x}}_i) = \ldots f_s(\omega_j) T(t_j) \ldots$$

Background: Importance Sampling



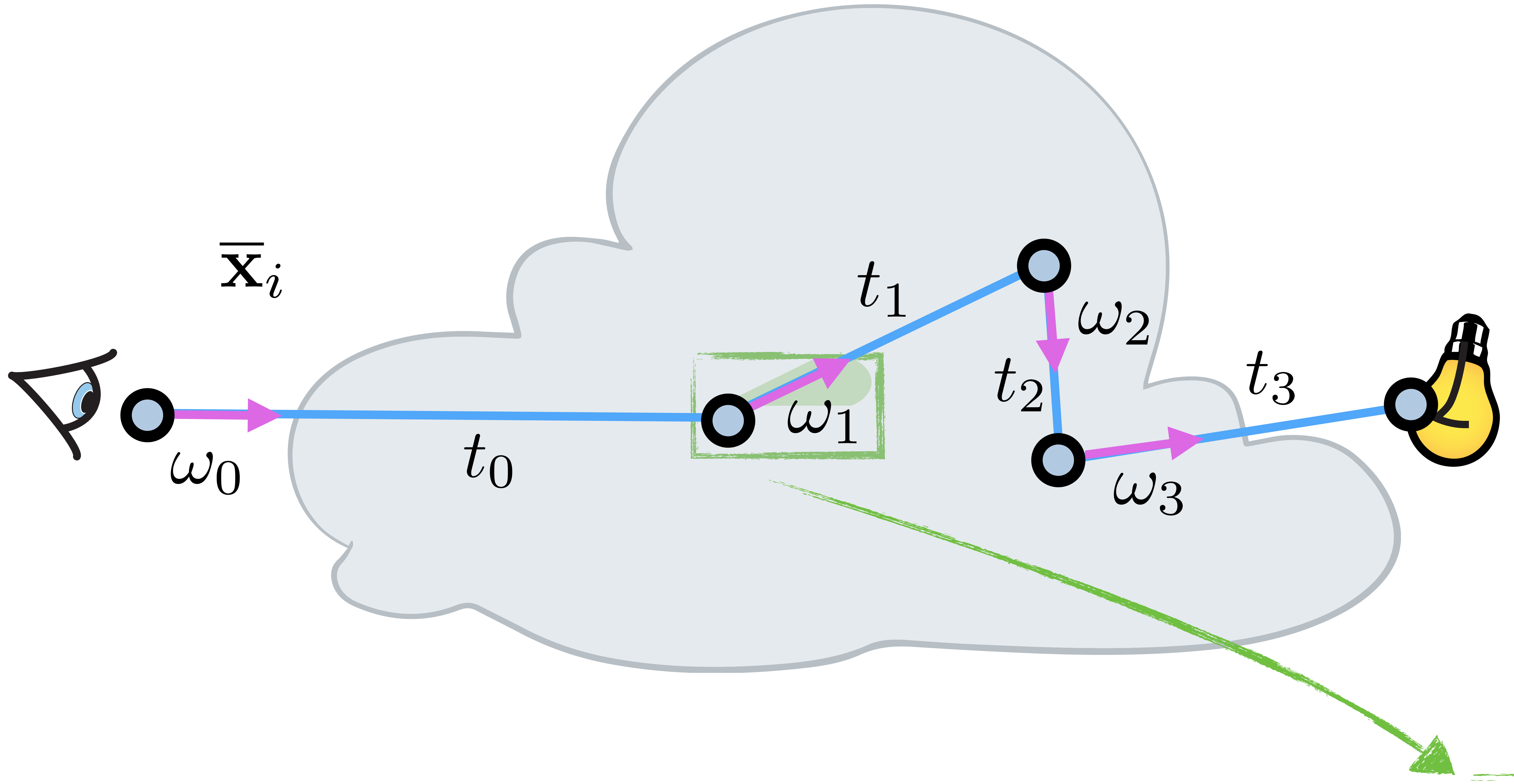
Scattering transmittance

$$\frac{f(\overline{\mathbf{x}}_i)}{p_k(\overline{\mathbf{x}}_i)} = \frac{\dots f_s(\omega_j)T(t_j)\dots}{\dots p(\omega_j)p(t_j)\dots}$$

Ideally

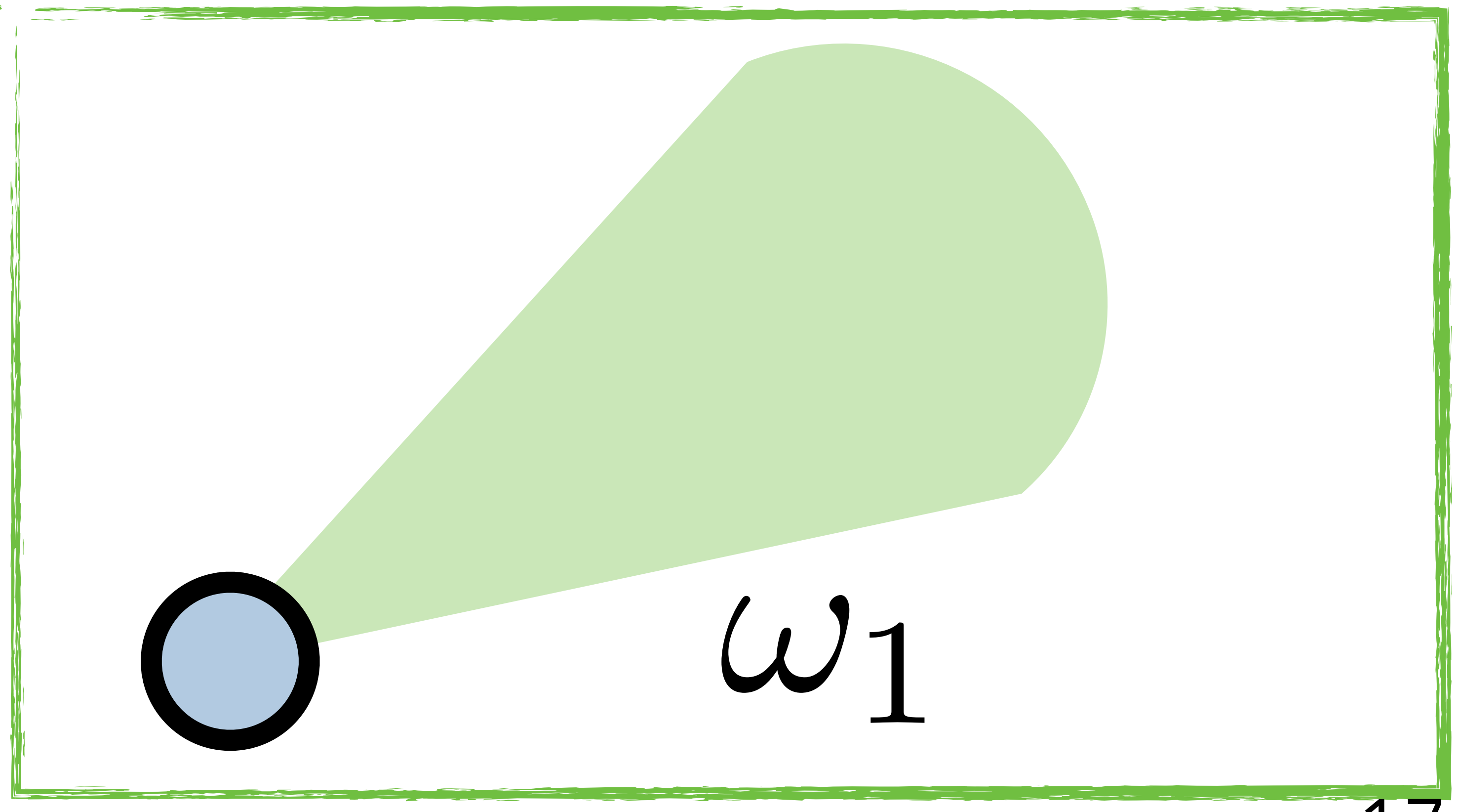
$$p(w_j) \propto f_s(w_j)$$
$$p(t_j) \propto T(t_j)$$

Background: Importance Sampling

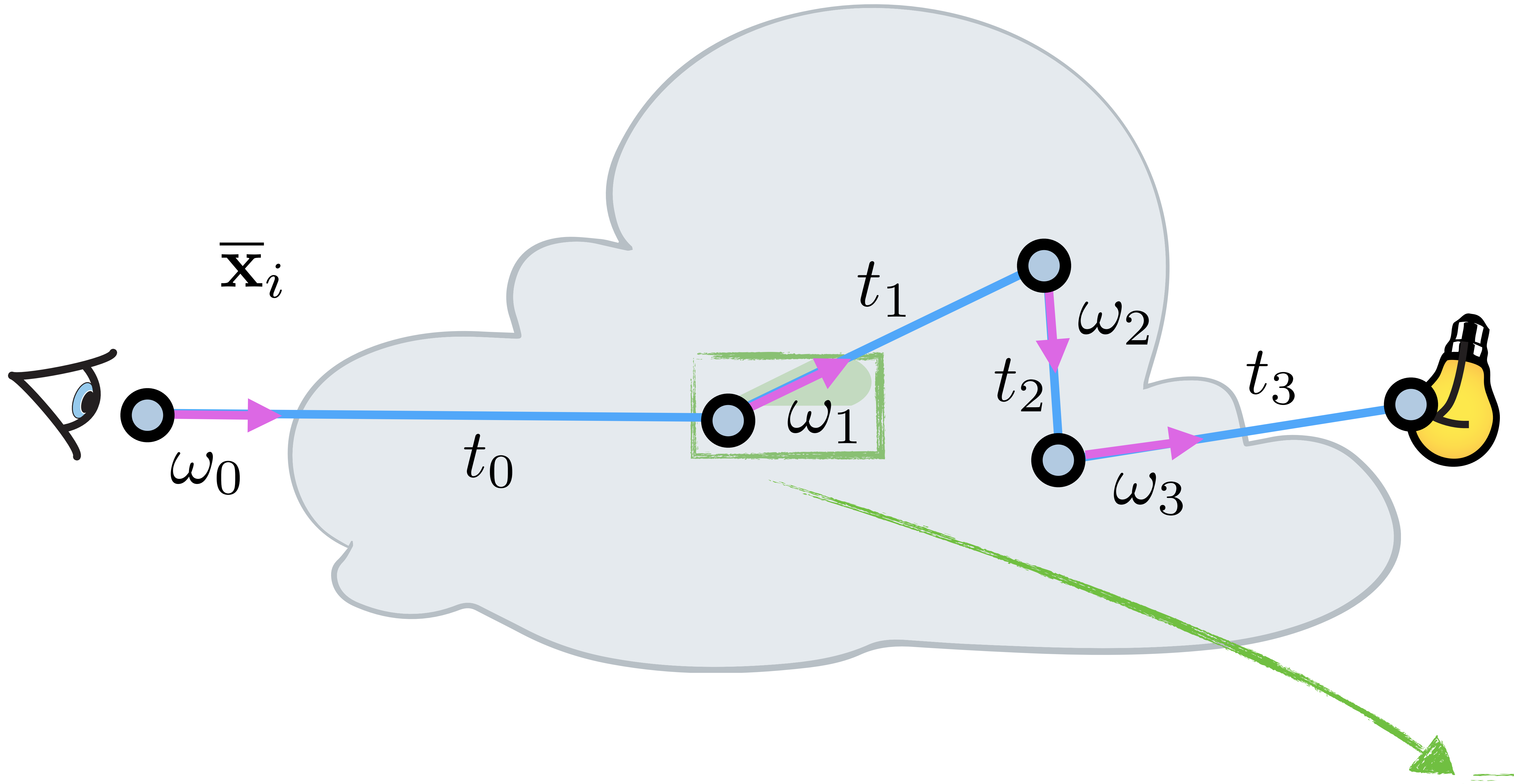


Scattering transmittance

$$\frac{f(\overline{\mathbf{x}}_i)}{p_k(\overline{\mathbf{x}}_i)} = \frac{\dots f_s(\omega_j) T(t_j) \dots}{\dots p(\omega_j) p(t_j) \dots}$$

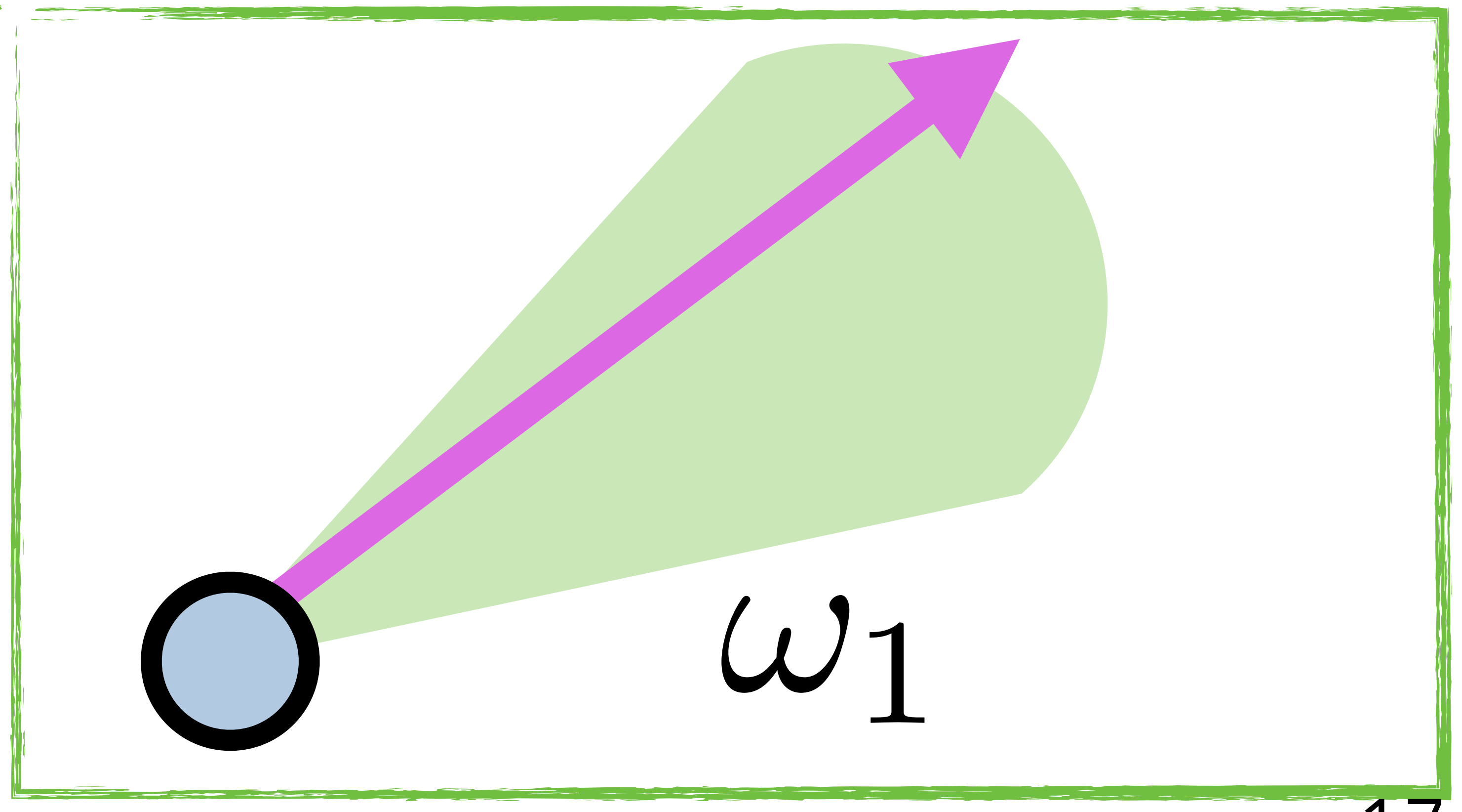


Background: Importance Sampling

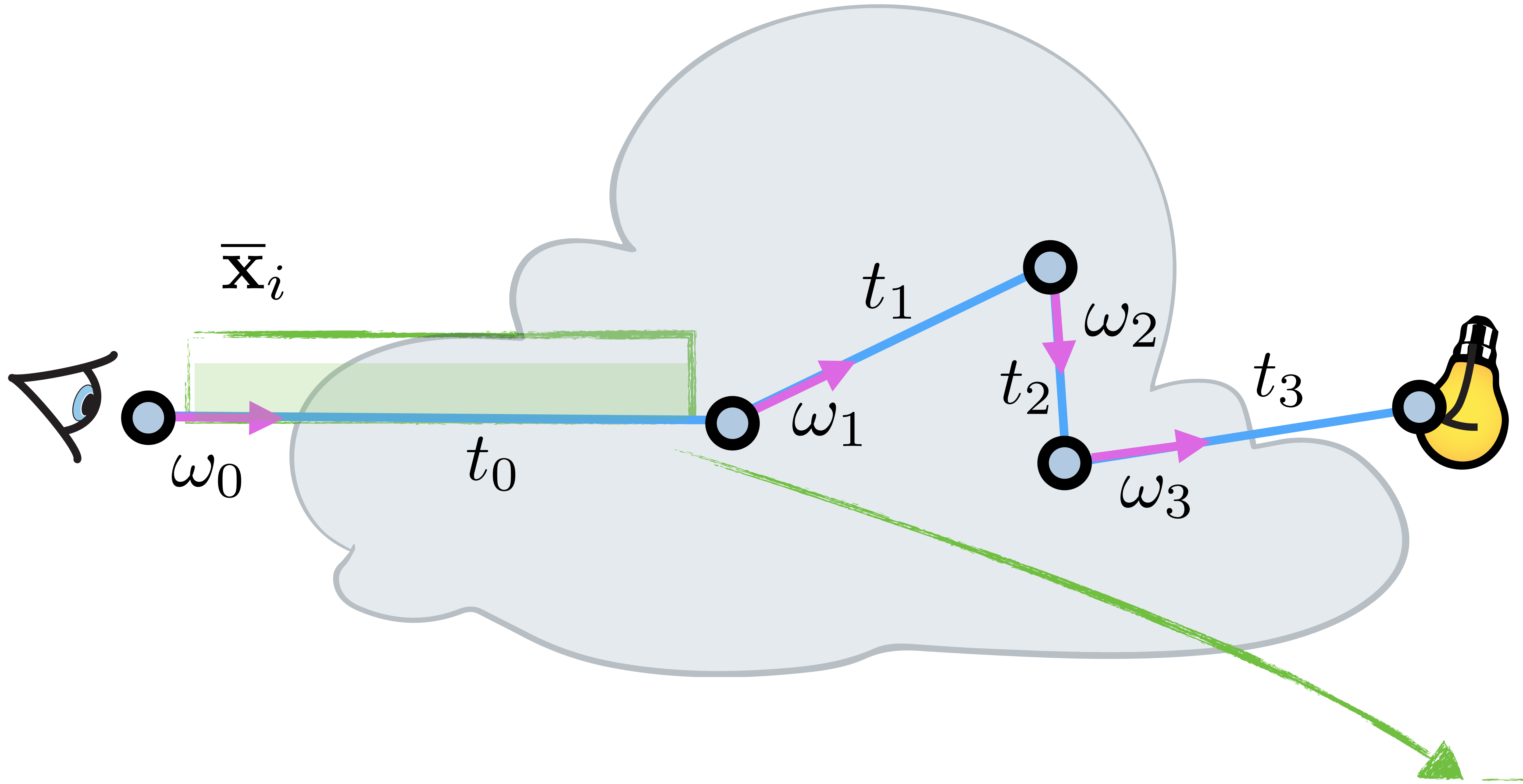


Scattering transmittance

$$\frac{f(\overline{\mathbf{x}}_i)}{p_k(\overline{\mathbf{x}}_i)} = \frac{\ldots f_s(\omega_j) T(t_j) \ldots}{\ldots p(\omega_j) p(t_j) \ldots}$$

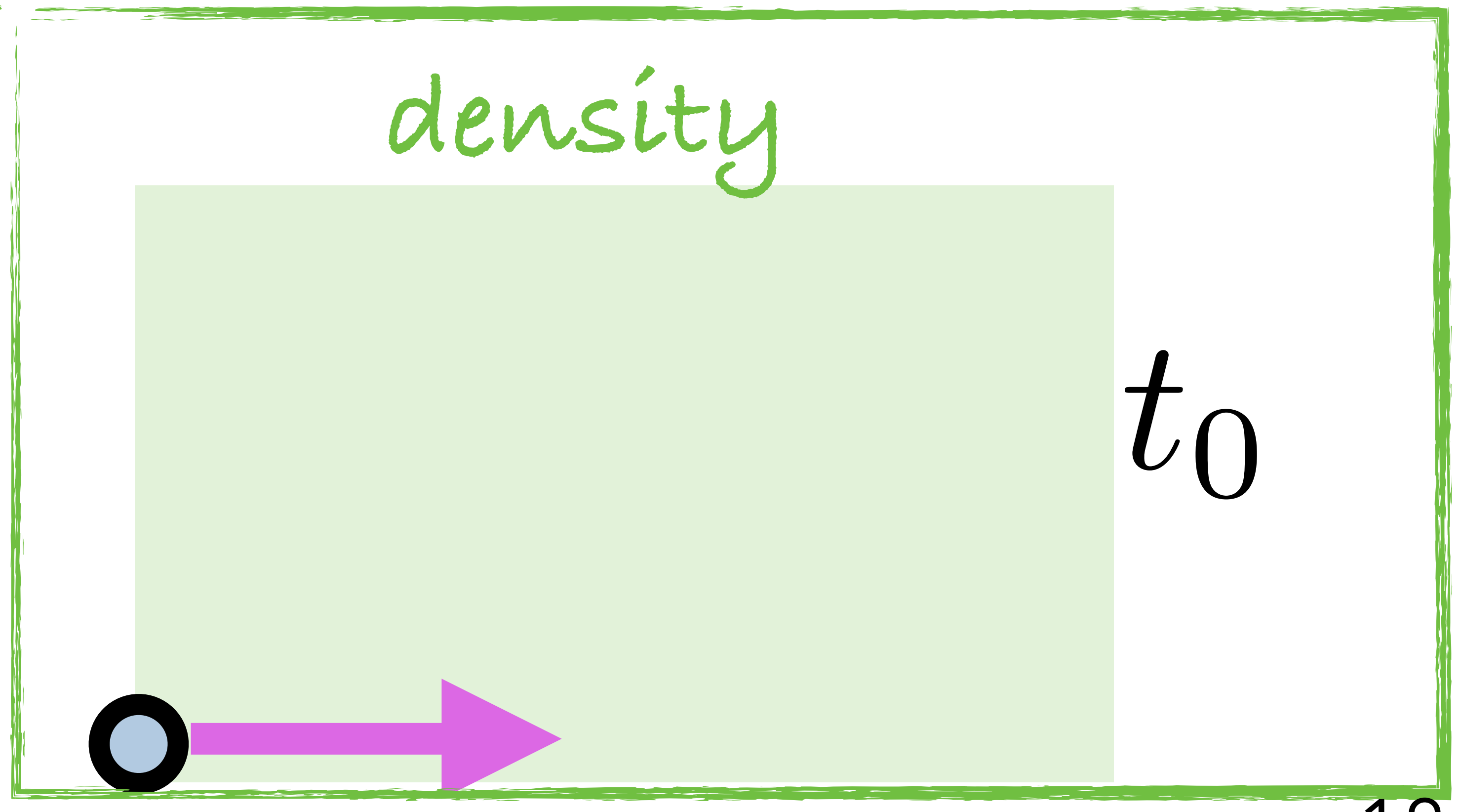


Background: Importance Sampling

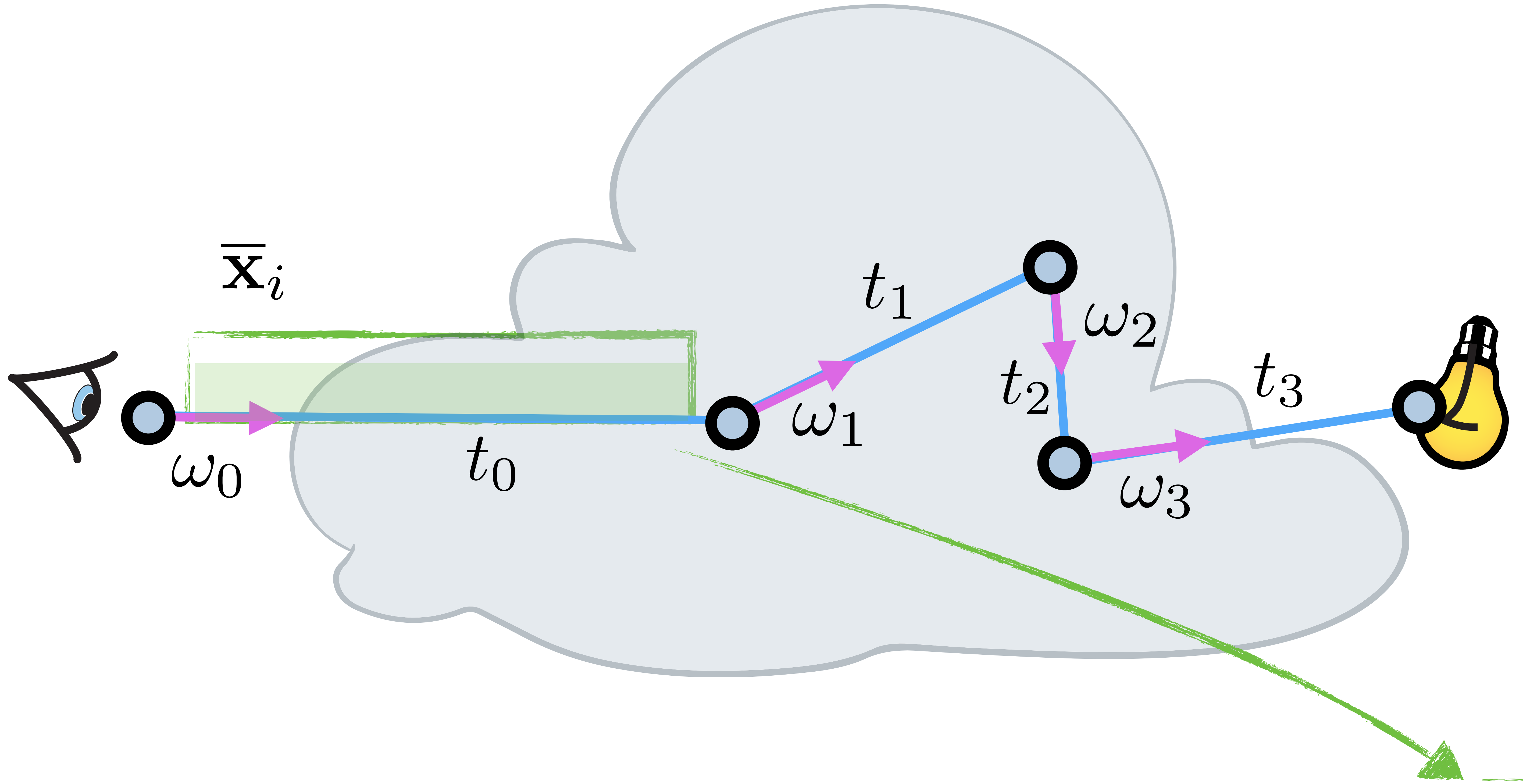


Scattering transmittance

$$\frac{f(\overline{\mathbf{x}}_i)}{p_k(\overline{\mathbf{x}}_i)} = \frac{\dots f_s(\omega_j)T(t_j)\dots}{\dots p(\omega_j)p(t_j)\dots}$$

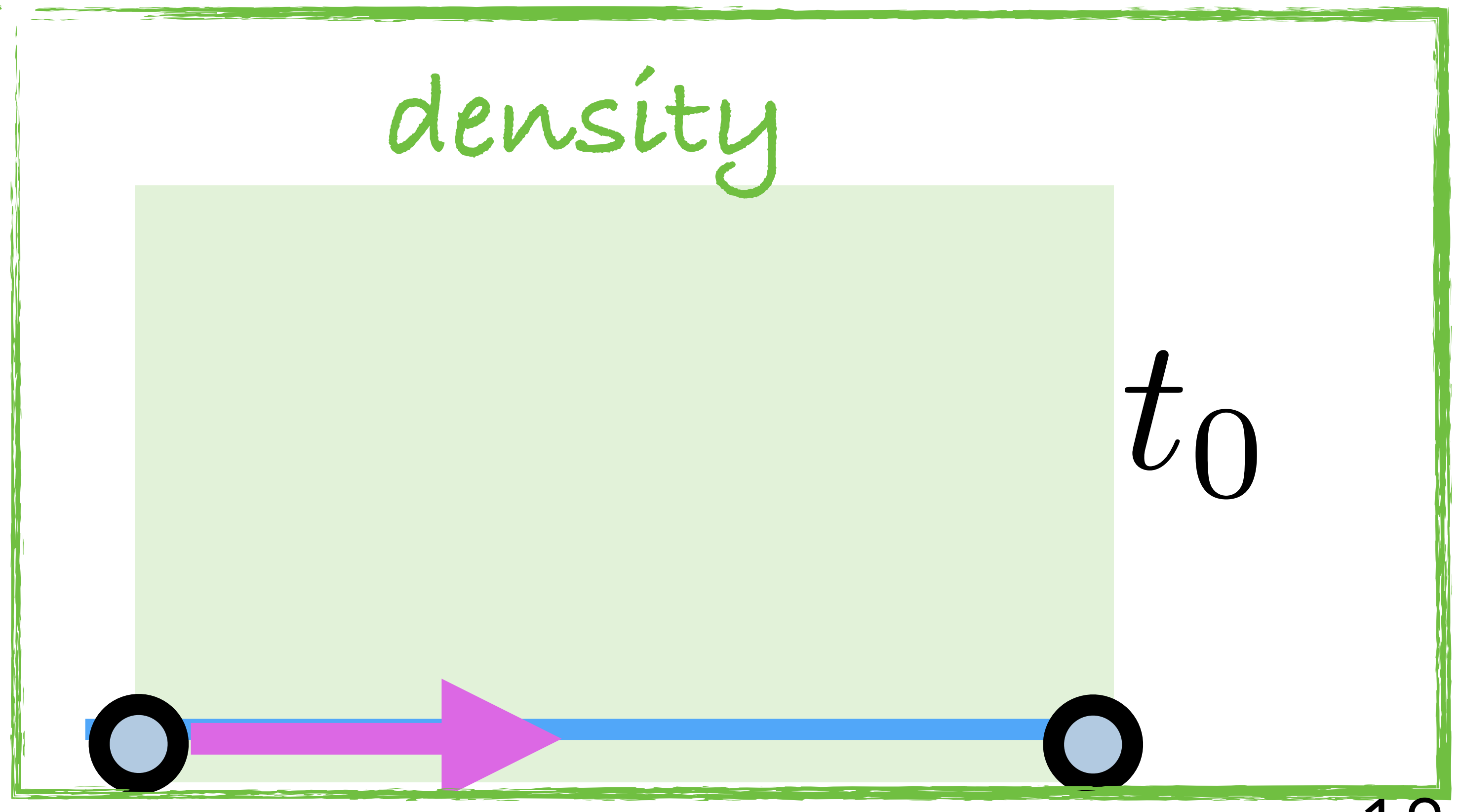


Background: Importance Sampling

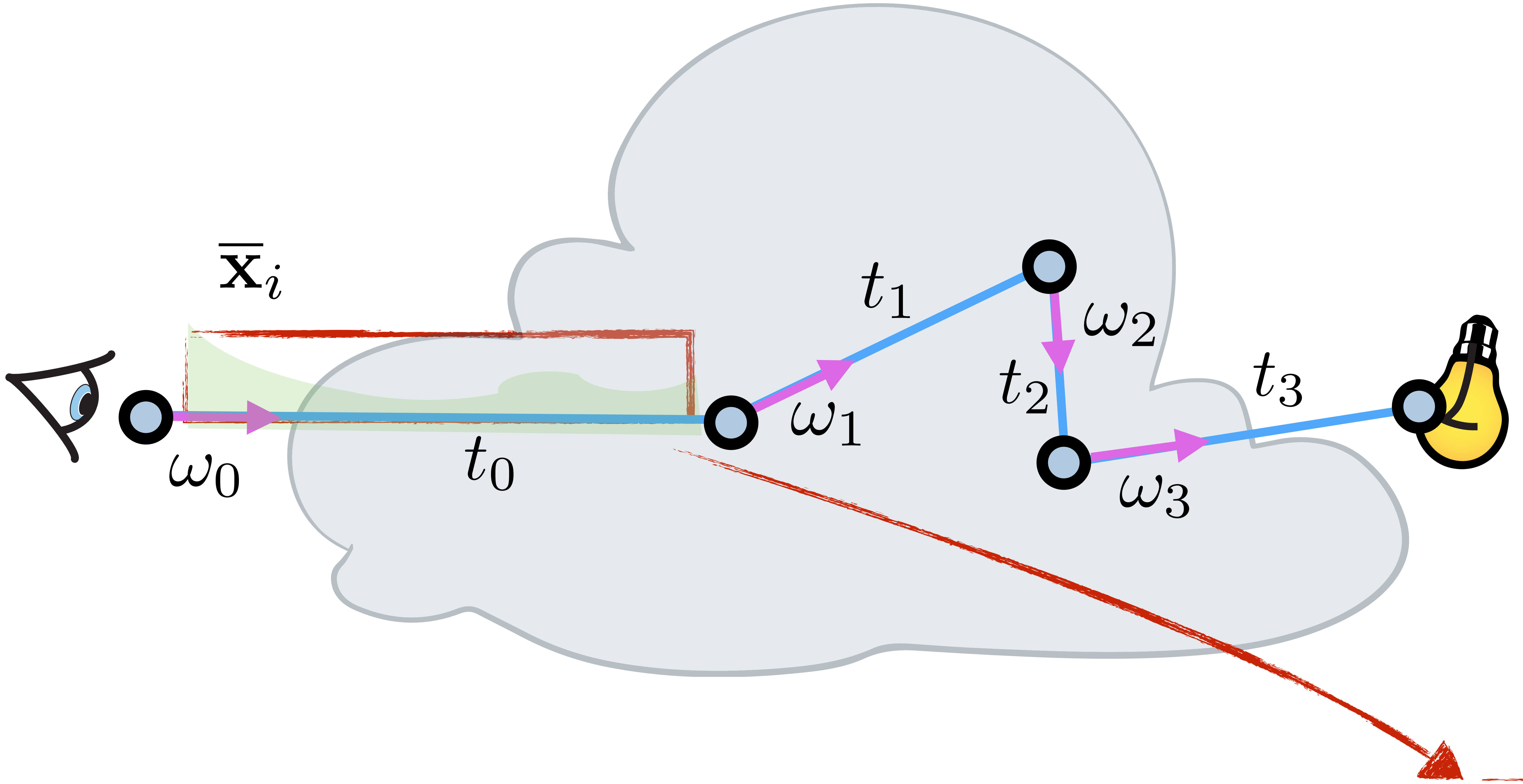


Scattering transmittance

$$\frac{f(\overline{\mathbf{x}}_i)}{p_k(\overline{\mathbf{x}}_i)} = \frac{\dots f_s(\omega_j) T(t_j) \dots}{\dots p(\omega_j) p(t_j) \dots}$$



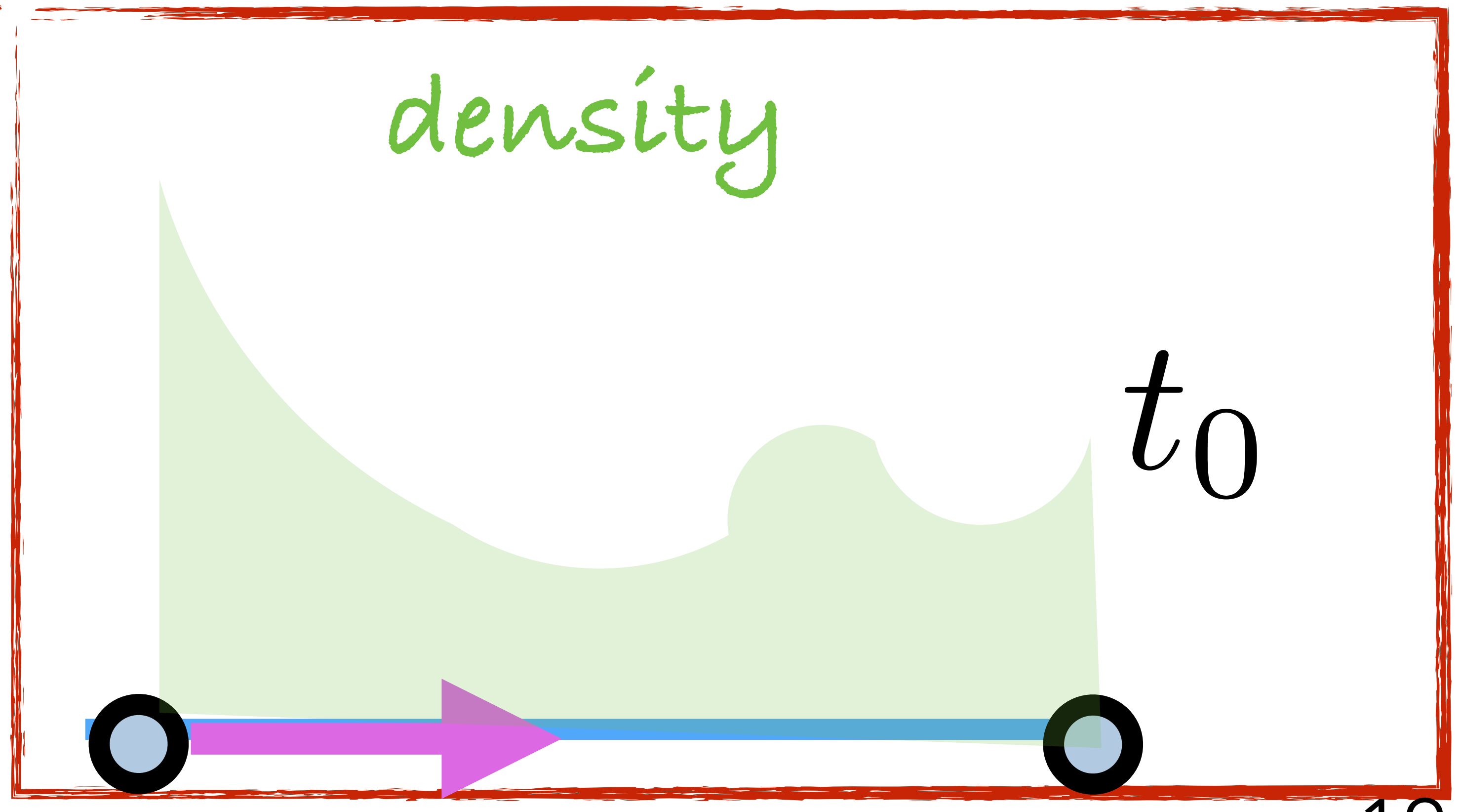
Background: Importance Sampling



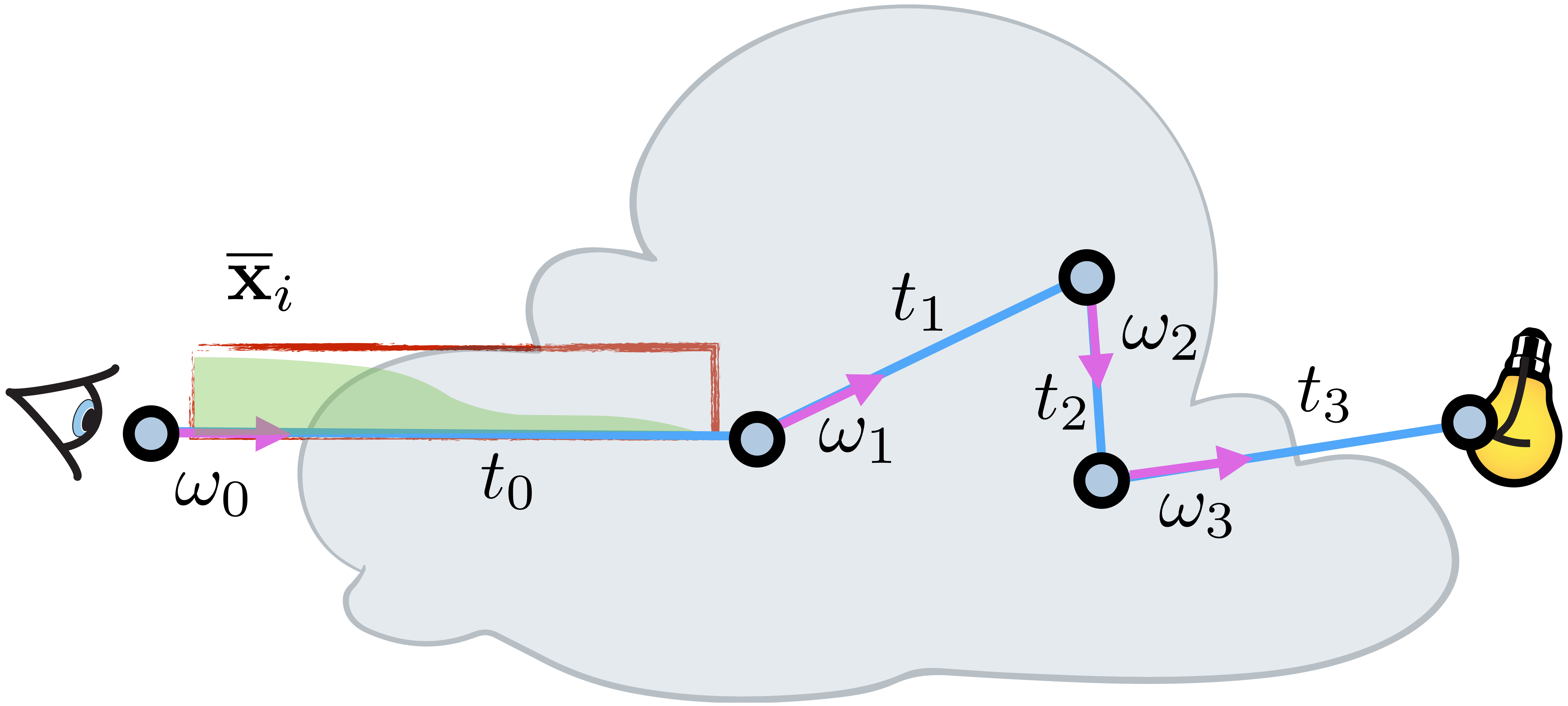
Scattering transmittance

$$\frac{f(\overline{\mathbf{x}}_i)}{p(\overline{\mathbf{x}}_i)} = \frac{\dots f_s(\omega_j)T(t_j)\dots}{\dots p(\omega_j)p(t_j)\dots}$$

??? ???



Background: Importance Sampling

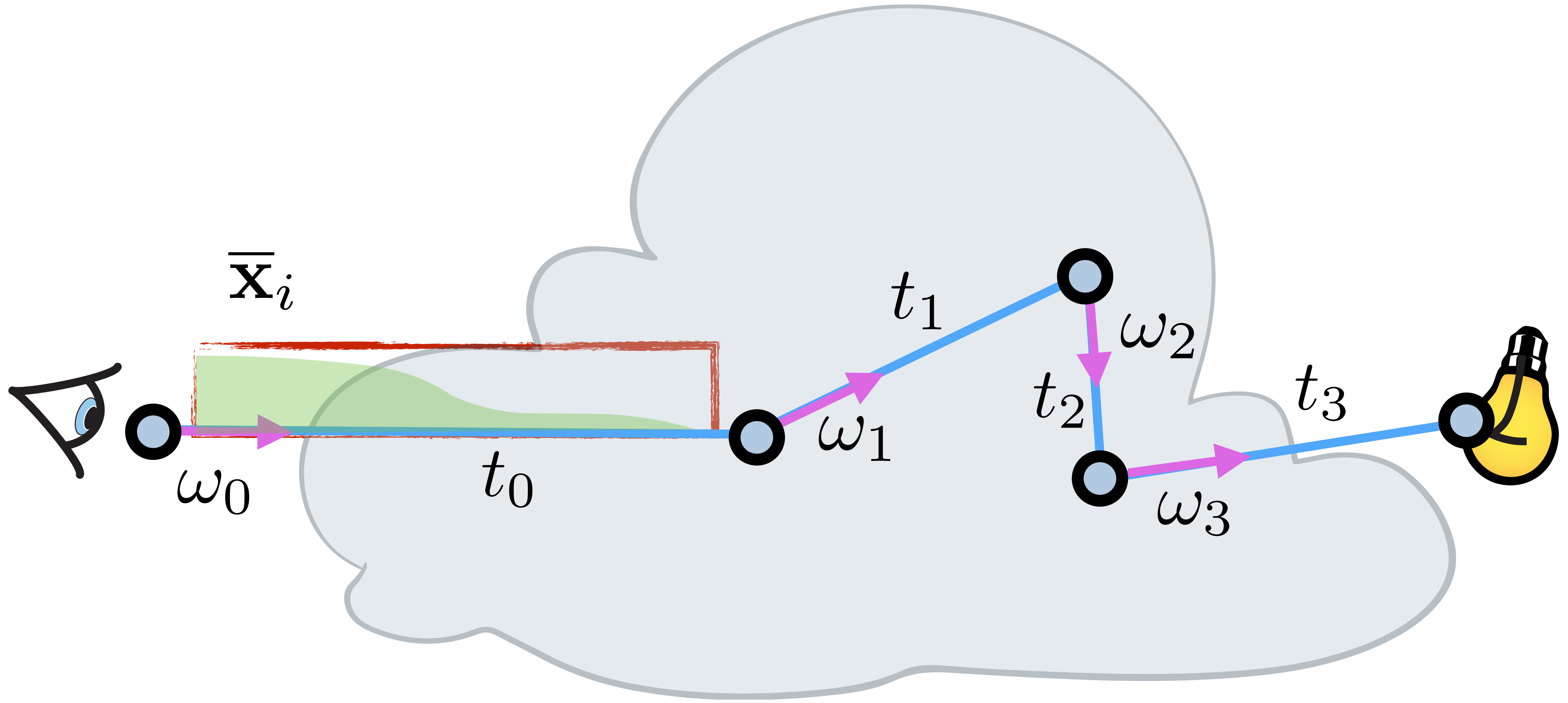


Scattering transmittance

$$\frac{f(\overline{\mathbf{x}}_i)}{p_k(\overline{\mathbf{x}}_i)} = \frac{\ldots f_s(\omega_j)T(t_j)\ldots}{\ldots p(\omega_j)cT(t_j)\ldots}$$

$$p(t_j) \propto T(t_j)$$

Background: Importance Sampling

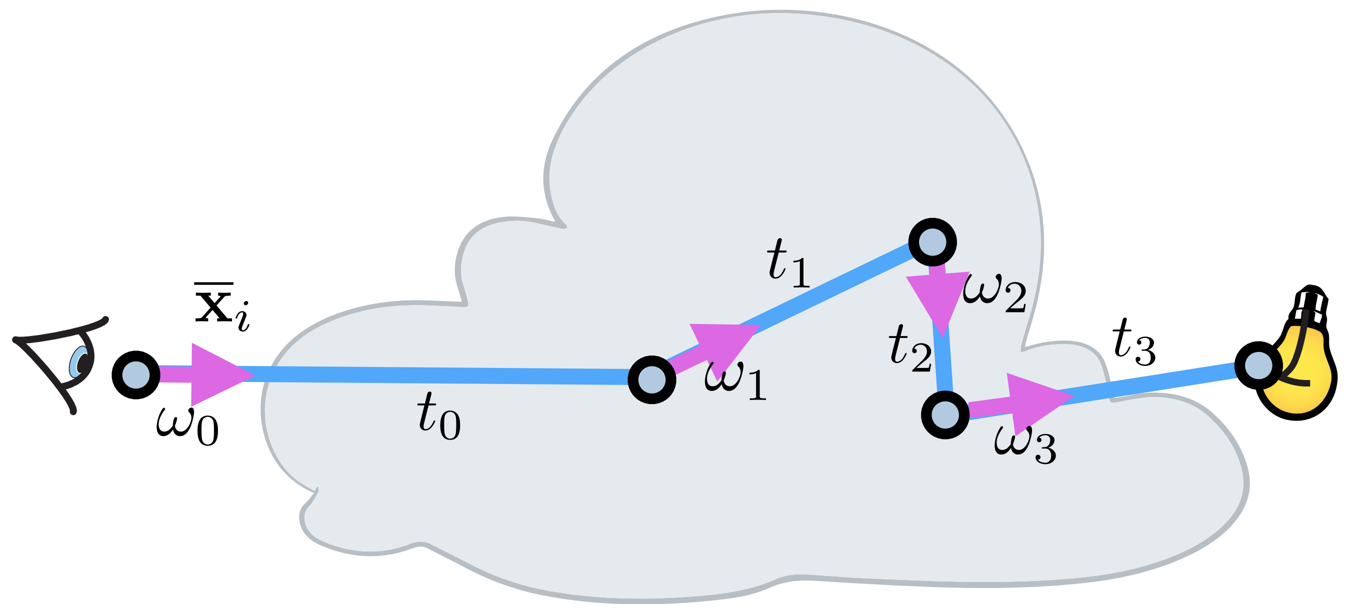


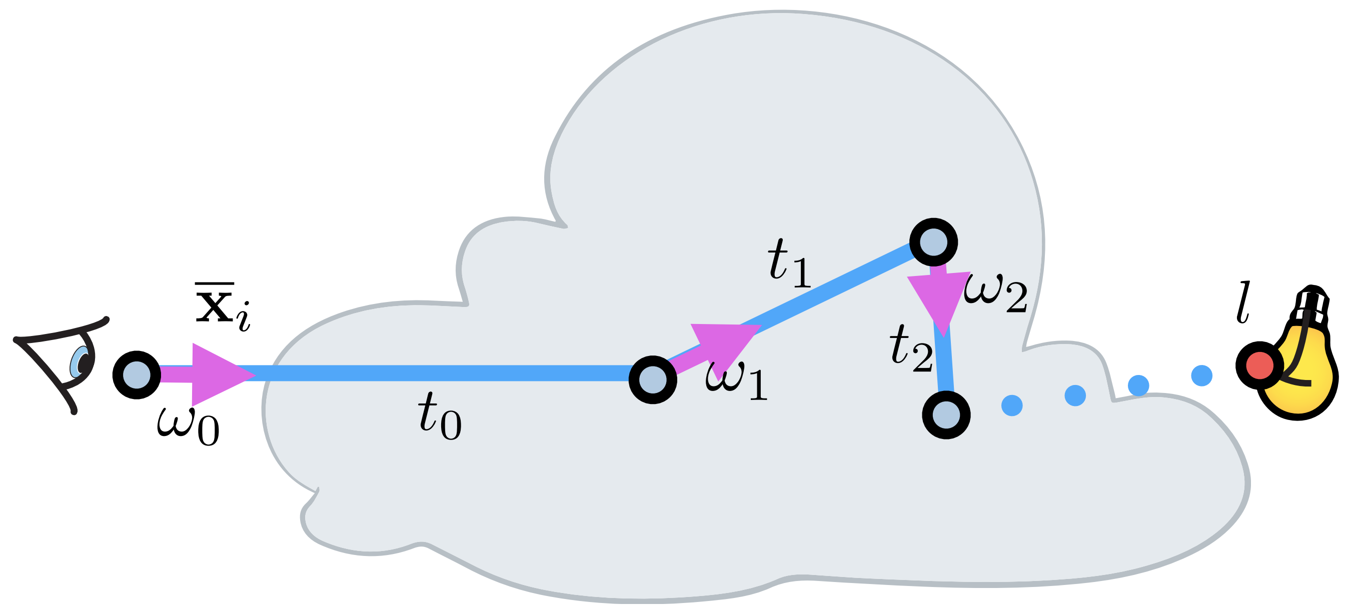
Scattering transmittance

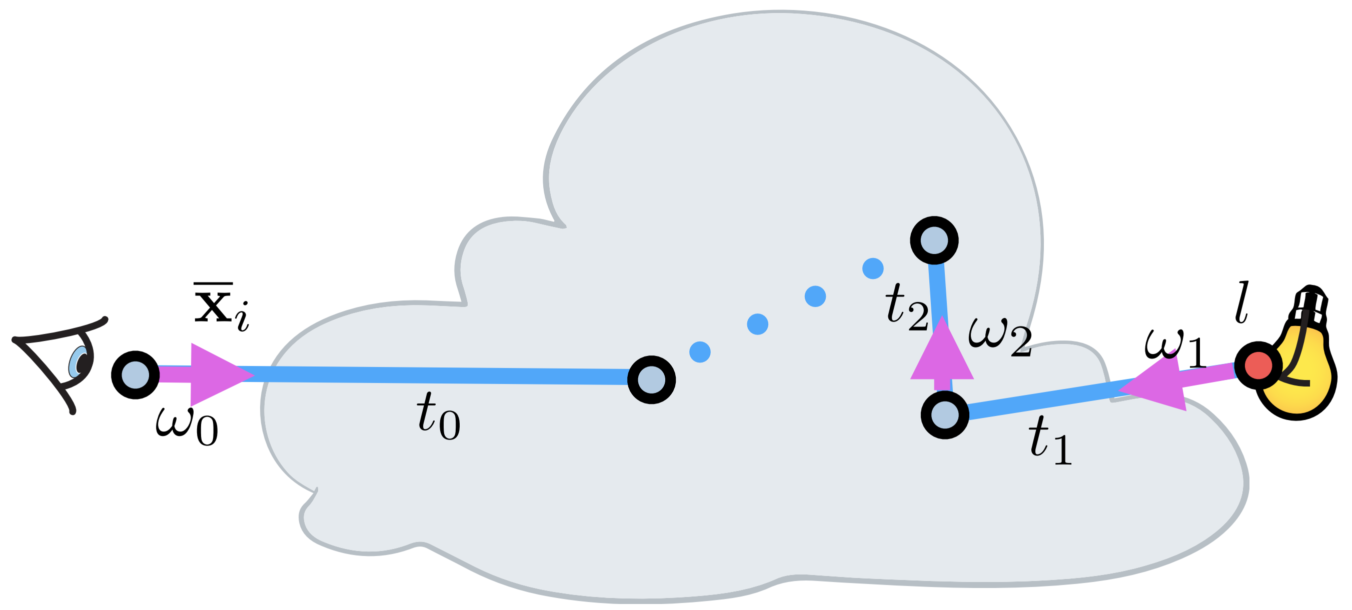
$$\frac{f(\overline{\mathbf{x}}_i)}{p_k(\overline{\mathbf{x}}_i)} = \frac{\ldots f_s(\omega_j)T(t_j)\ldots}{\ldots p(\omega_j)cT(t_j)\ldots}$$

cancellation trick

Background: Multiple Importance Sampling





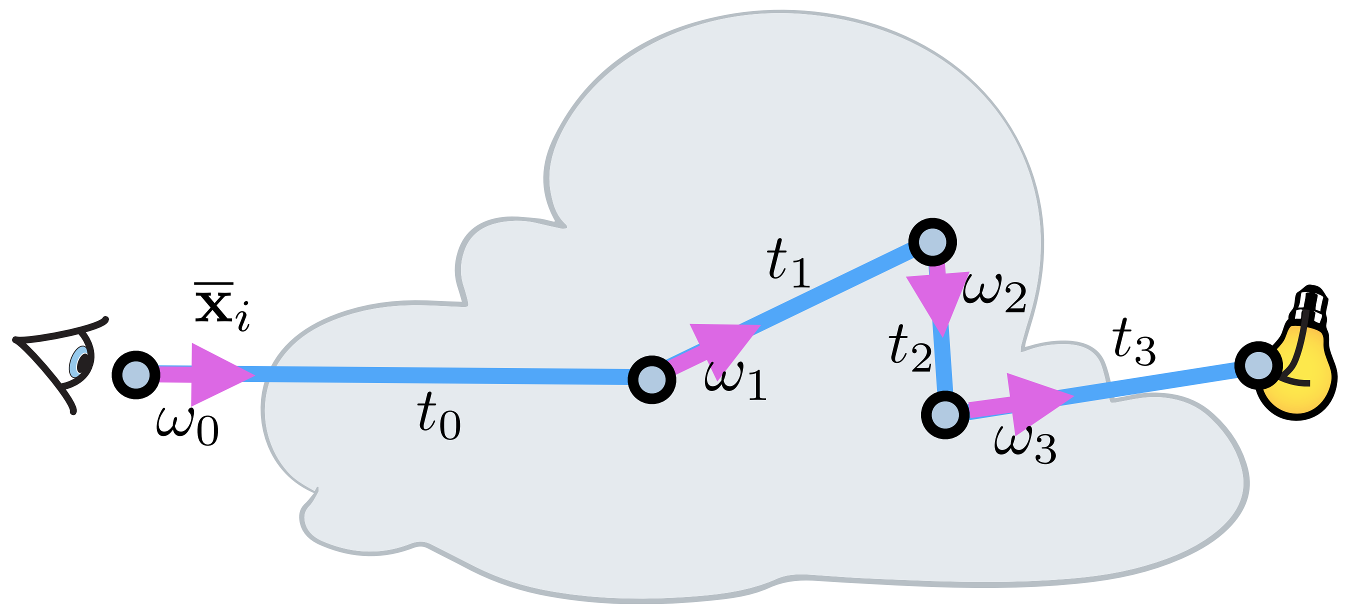


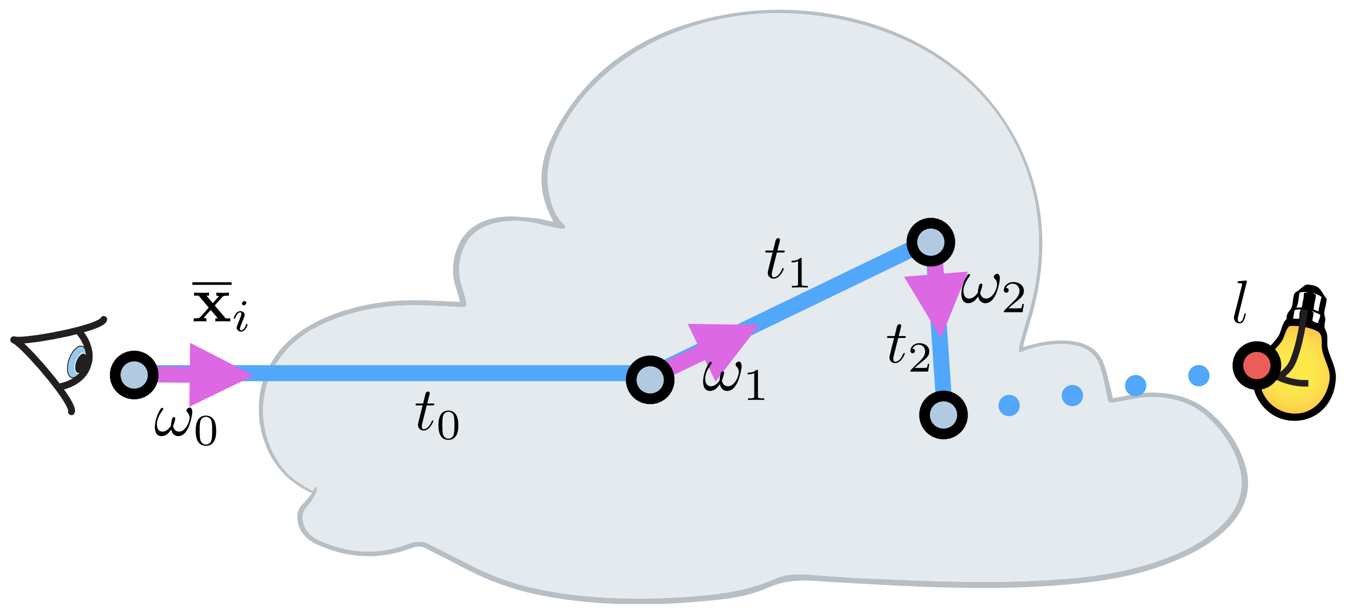
?

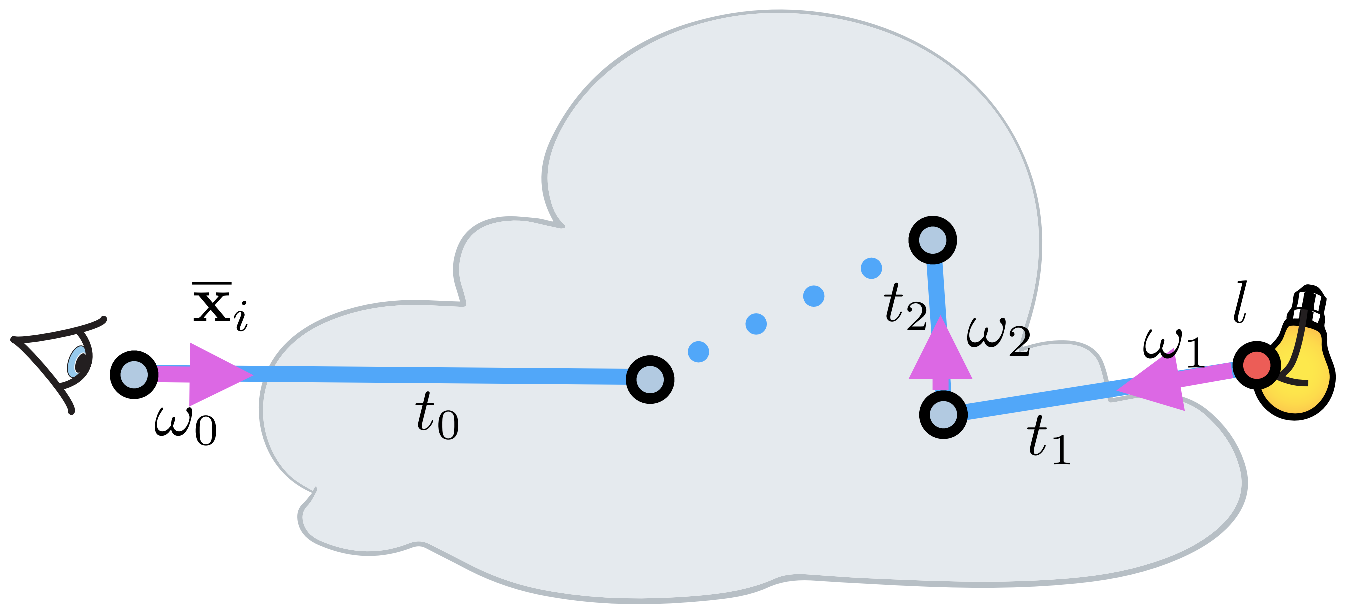
?

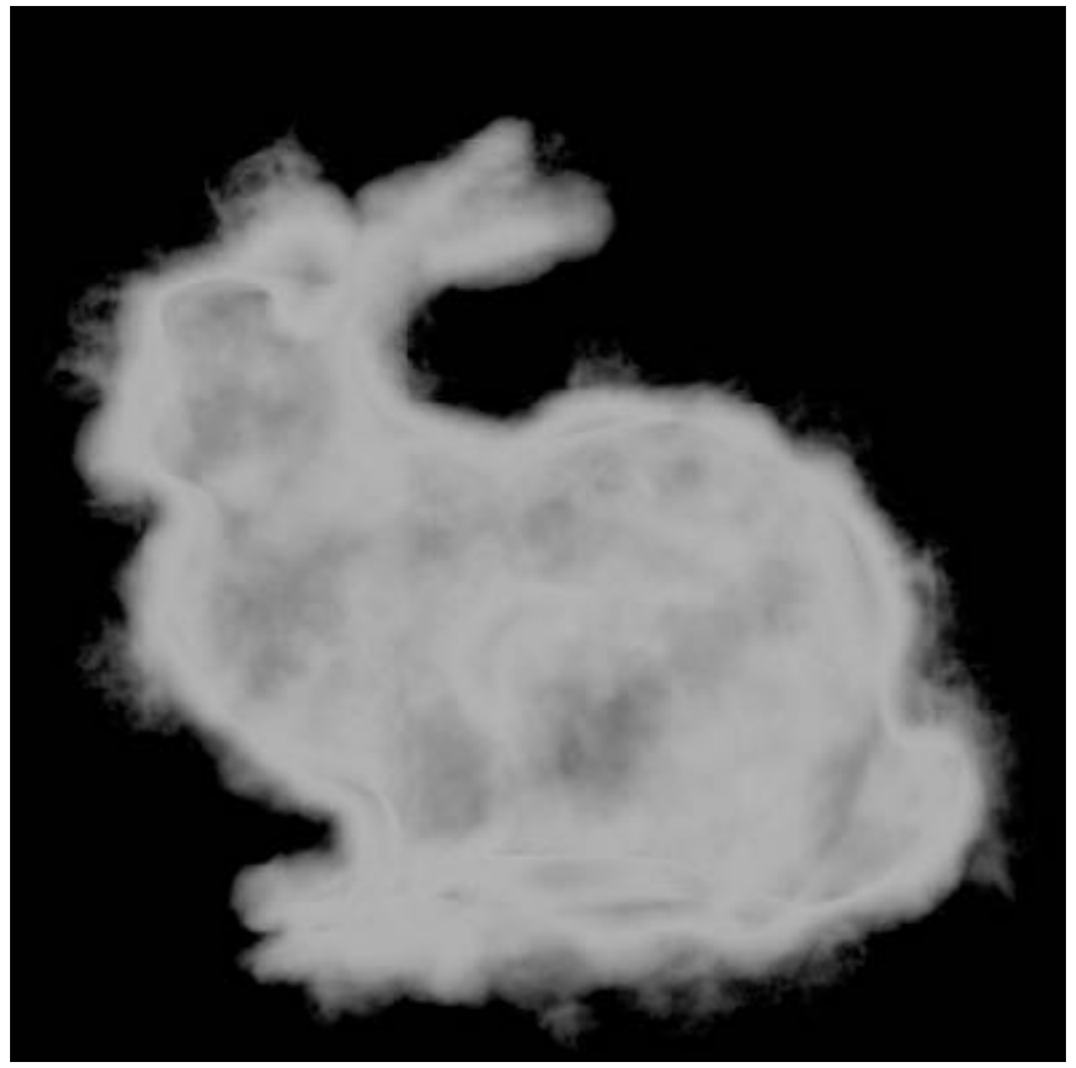
?

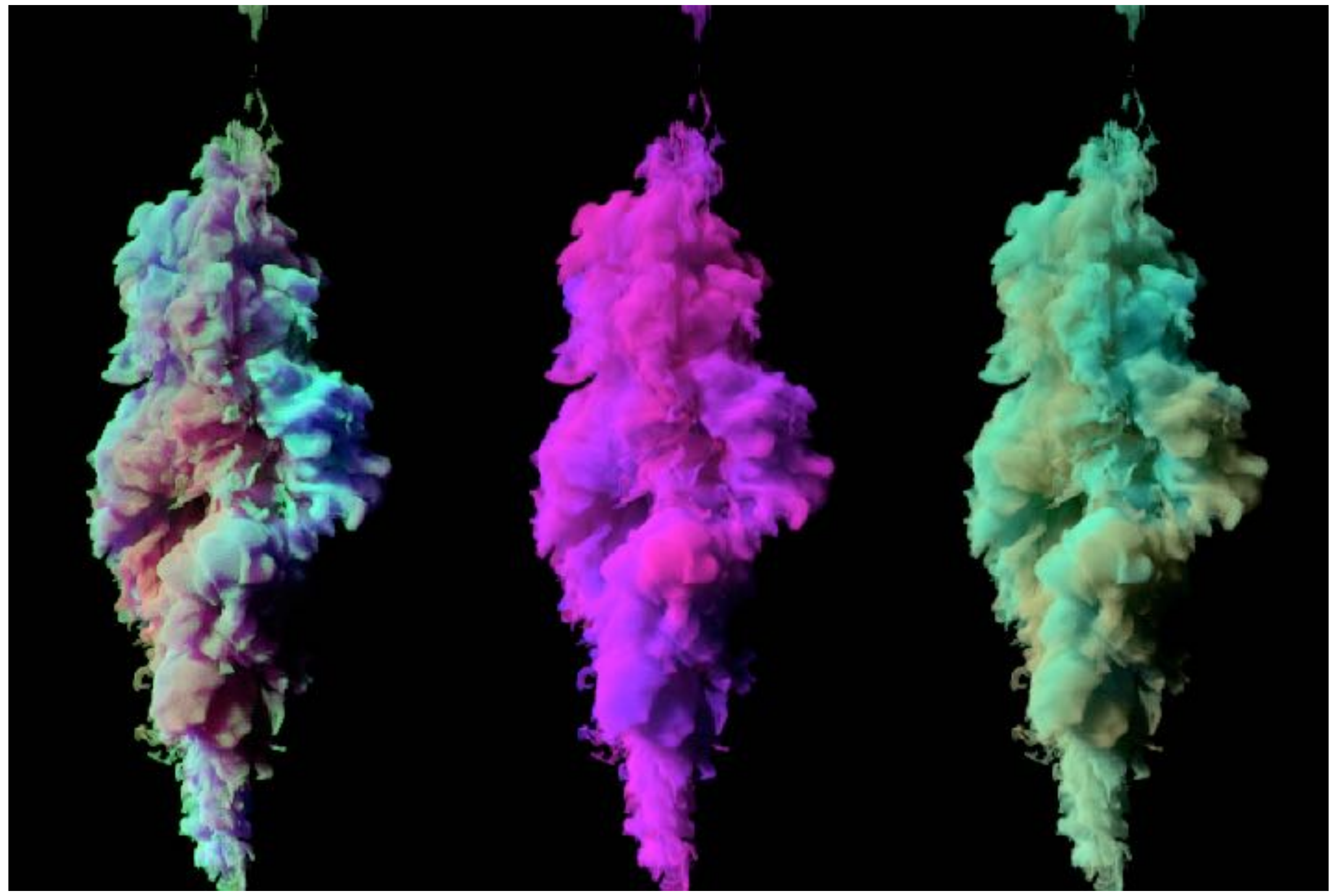
Background: Multiple Importance Sampling



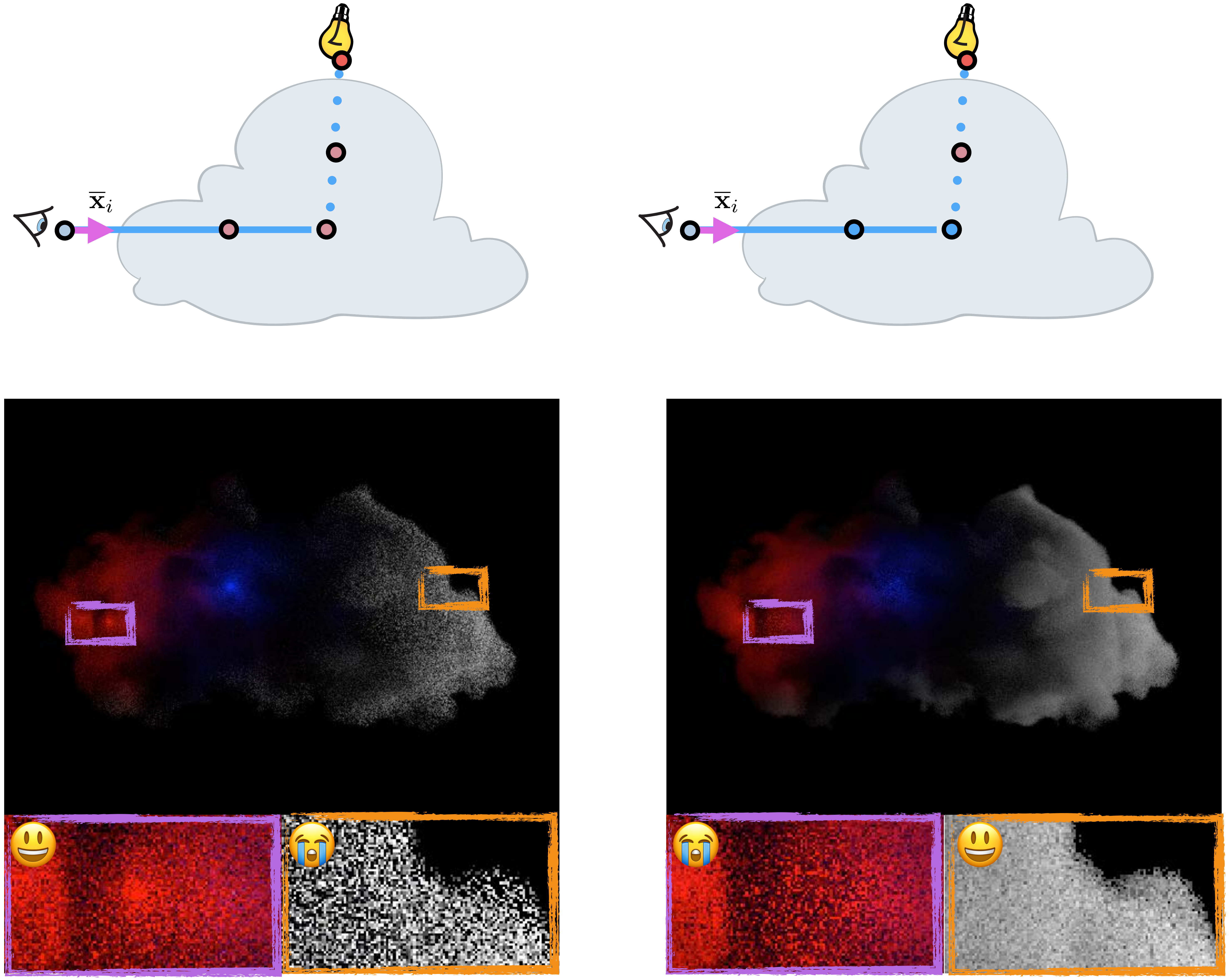




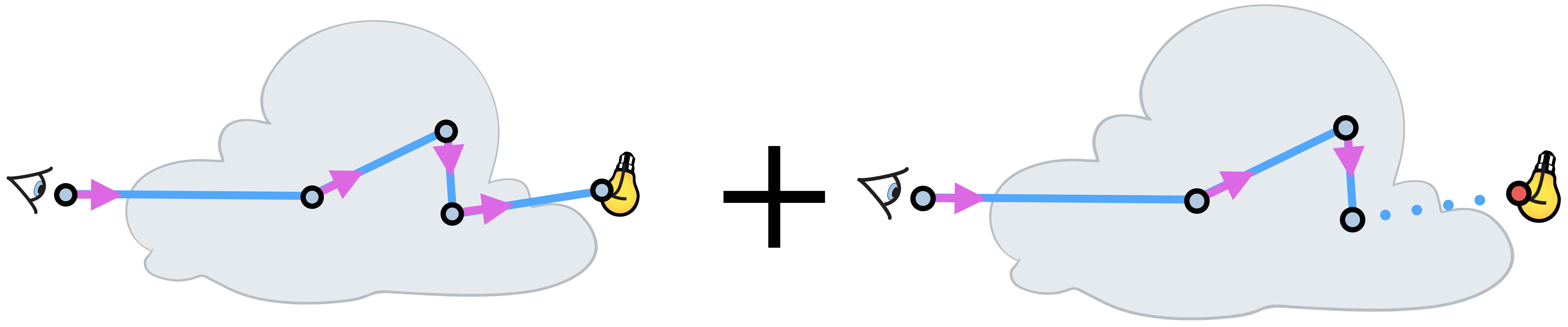




Background: Multiple Importance Sampling

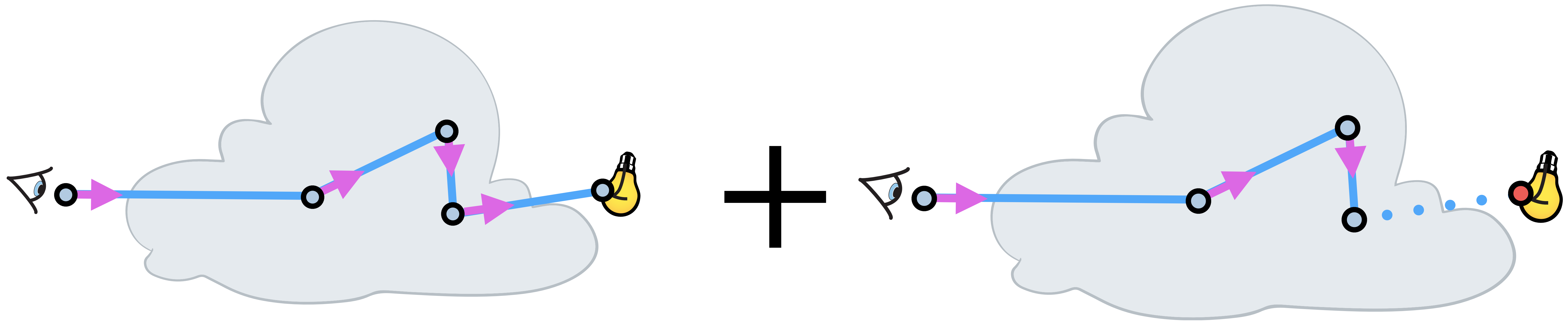


Background: Multiple Importance Sampling



$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(\overline{\mathbf{x}_i})}{\frac{1}{2}(p_1(\overline{\mathbf{x}_i}) + p_2(\overline{\mathbf{x}_i}))}$$

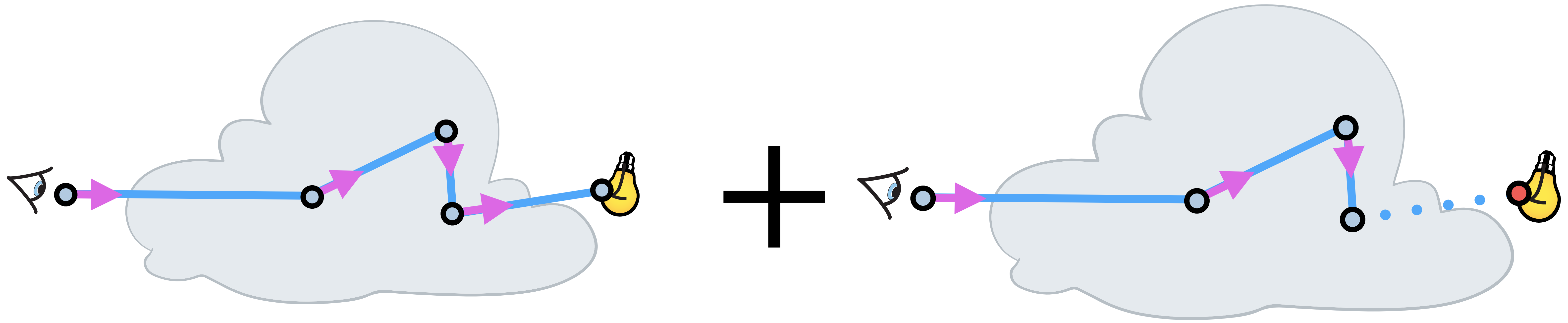
Background: Multiple Importance Sampling



$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(\overline{\mathbf{x}_i})}{\frac{1}{2}(p_1(\overline{\mathbf{x}_i}) + p_2(\overline{\mathbf{x}_i}))}$$

generally can't use cancellation trick

Background: Multiple Importance Sampling



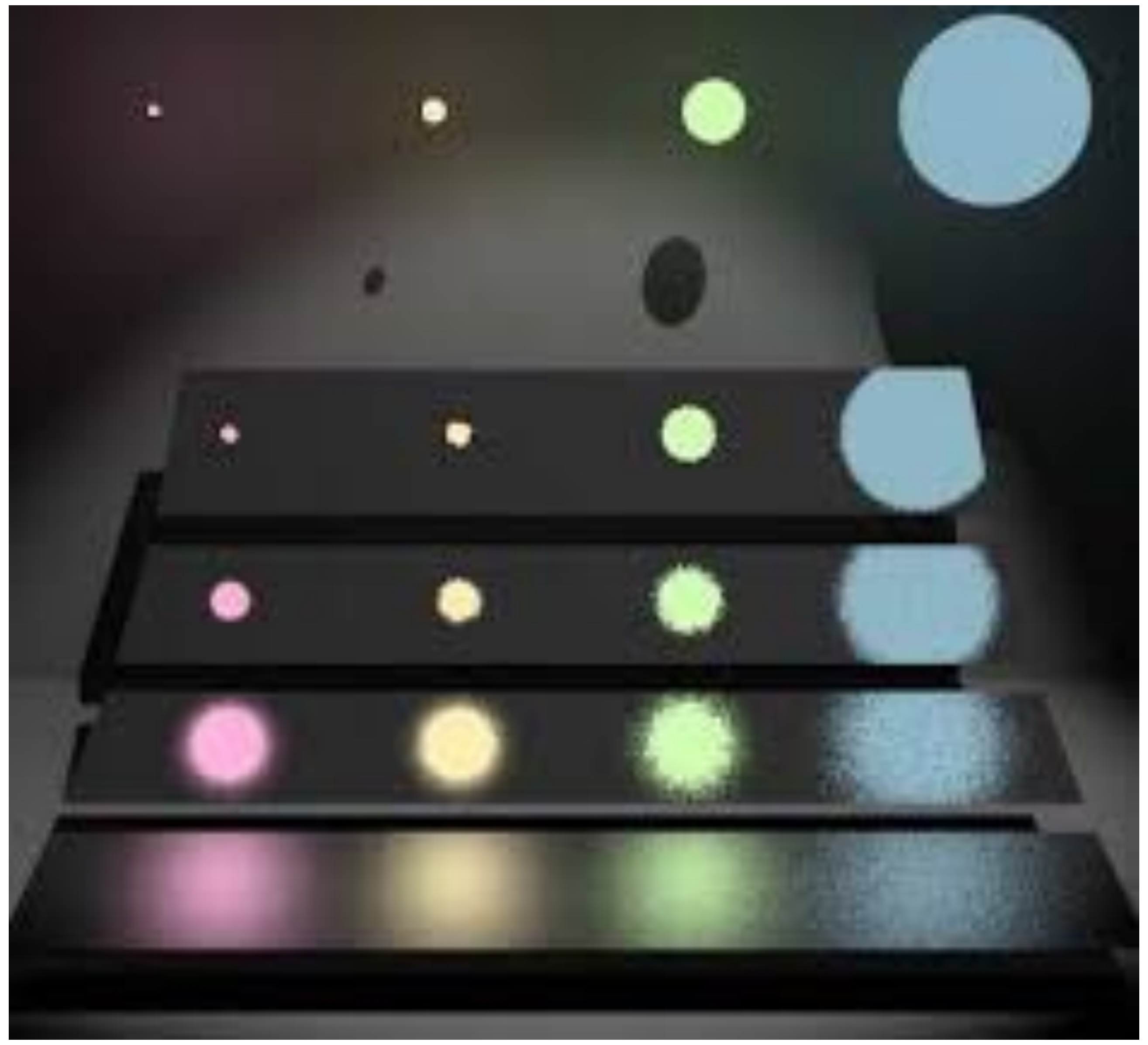
$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(\overline{\mathbf{x}}_i)}{\frac{1}{2}\left(\boxed{p_1(\overline{\mathbf{x}}_i)} + \boxed{p_2(\overline{\mathbf{x}}_i)}\right)}$$

generally can't use cancellation trick

Need the pdf!

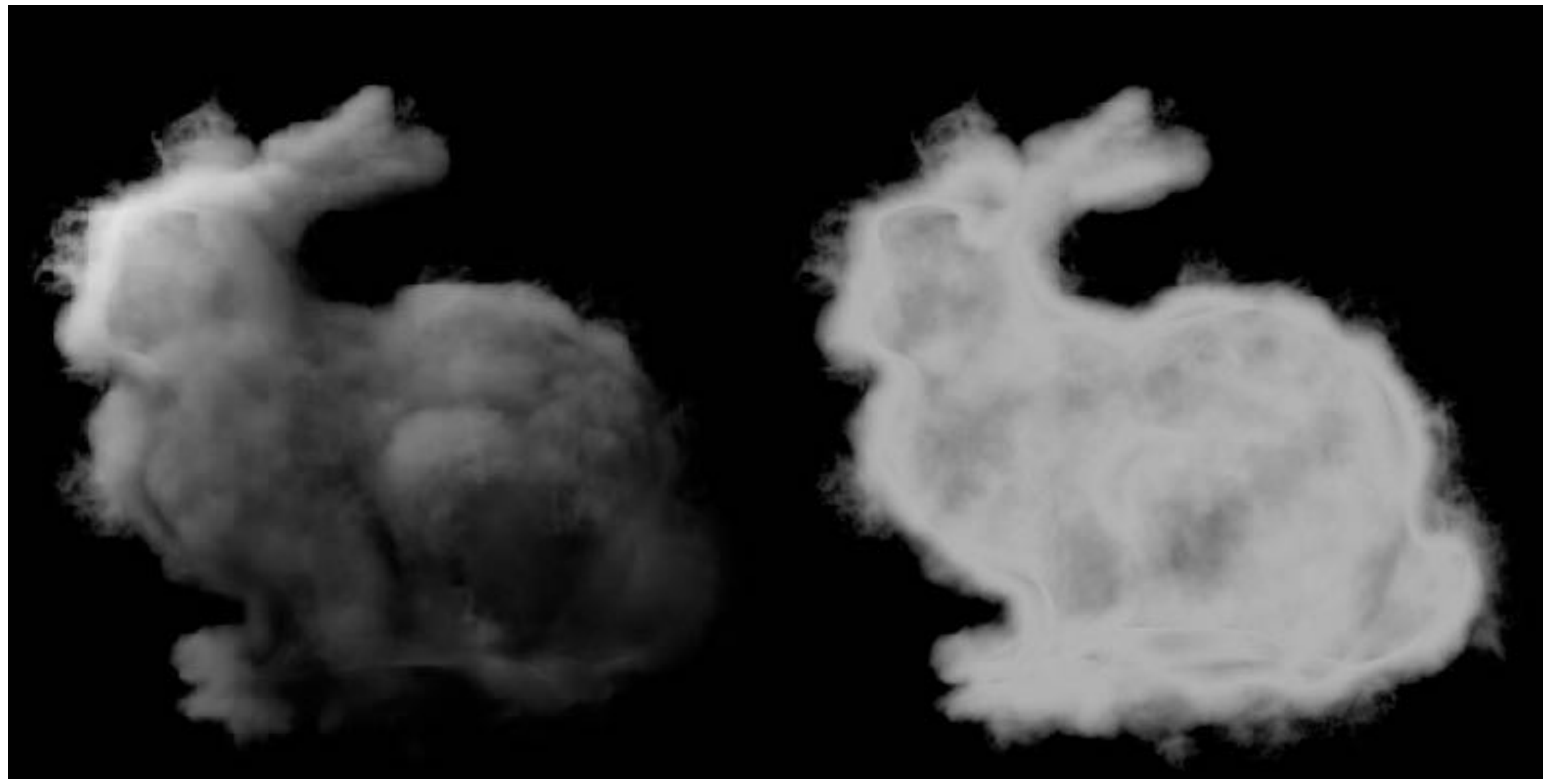
Goal

Surface MIS



[Veach and Guibas ’97]

Volume MIS



bring MIS to heterogeneous, volumetric media

Our Contributions

Our Contributions

- analytic path pdf for any media

Our Contributions

- analytic path pdf for any media
- MIS in any media

Our Contributions

- analytic path pdf for any media
- MIS in any media
- generalize and improve techniques including:
  - ratio tracking [Novak et al 2014]
  - hero wavelength sampling [Wilkie et al. 2014],
  - spectral tracking [Kutz et al. 2017]
  - equiangular sampling [Kulla and Fajardo 2012]
  - volumetric variants of bidirectional path tracing [Georgiev et al. 2013]

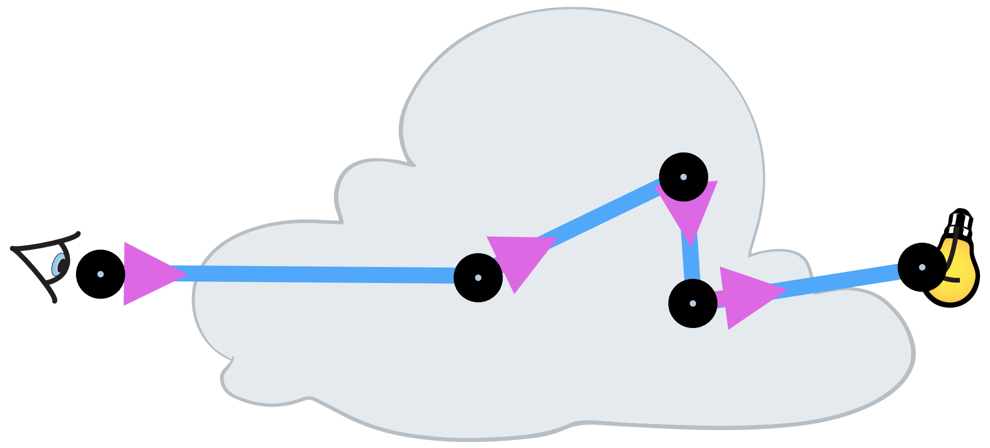
Previous Work: Overview

Class: **Techniques:**

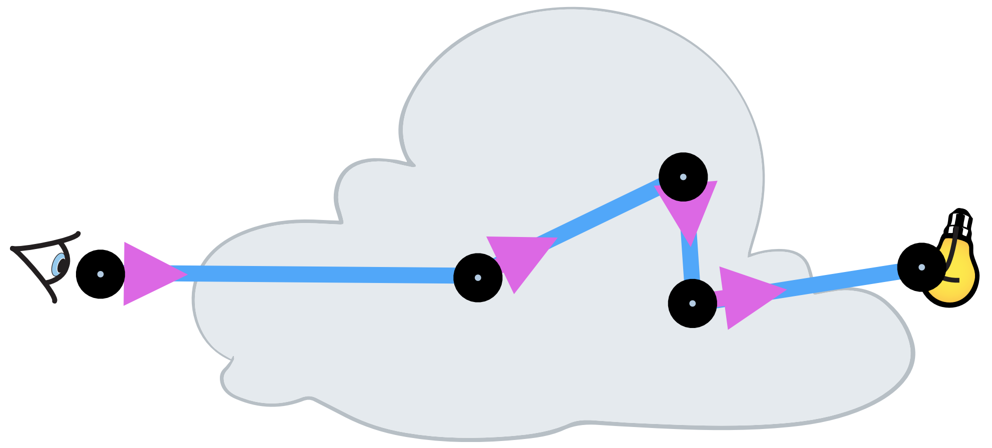
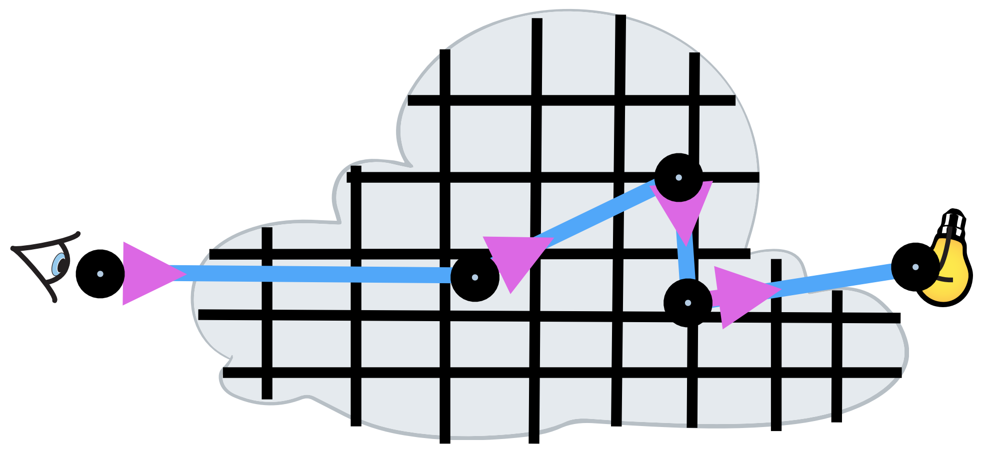
Previous Work: Overview

Class:	Techniques:	Any Media?	Unbiased?	PDF Known?

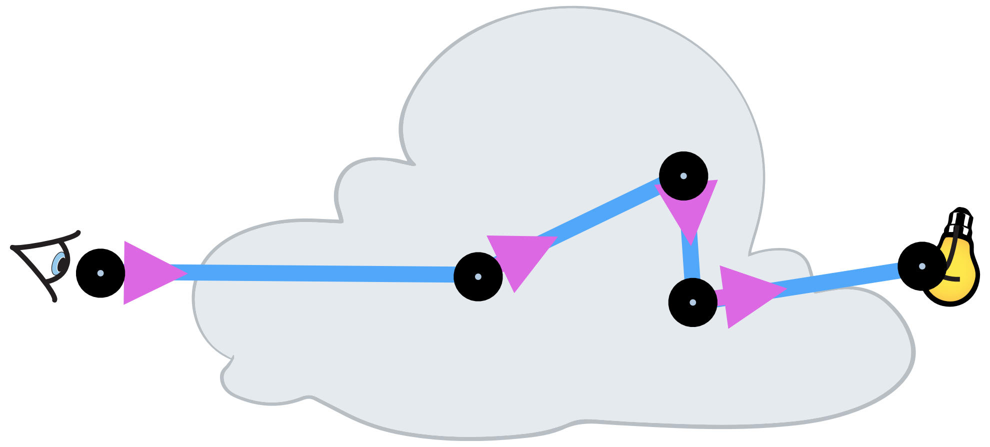
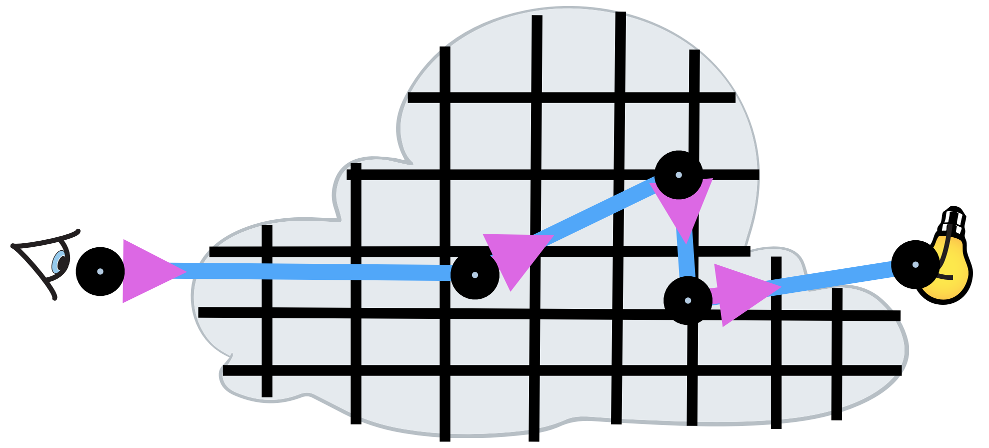
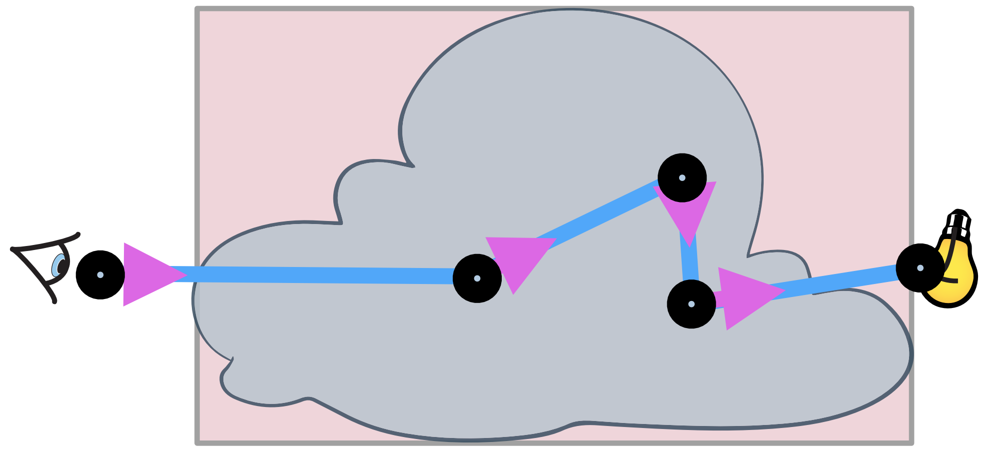
Previous Work: Overview

Class:	Techniques:	Any Media?	Unbiased?	PDF Known?
 analytic	Closed Form Tracking	✗	✓	✓

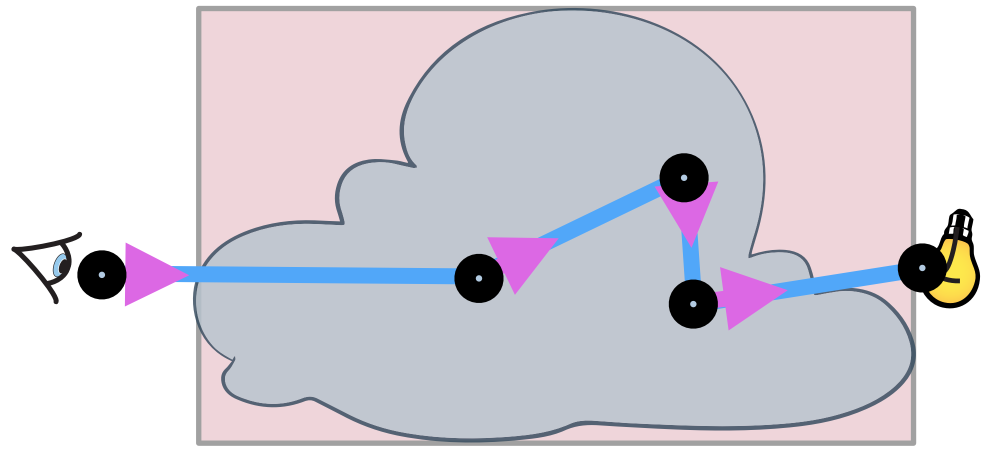
Previous Work: Overview

Class:	Techniques:	Any Media?	Unbiased?	PDF Known?
 analytic	Closed Form Tracking	✗	✓	✓
 approximations	Regular Tracking Ray Marching	✓	✗	✓

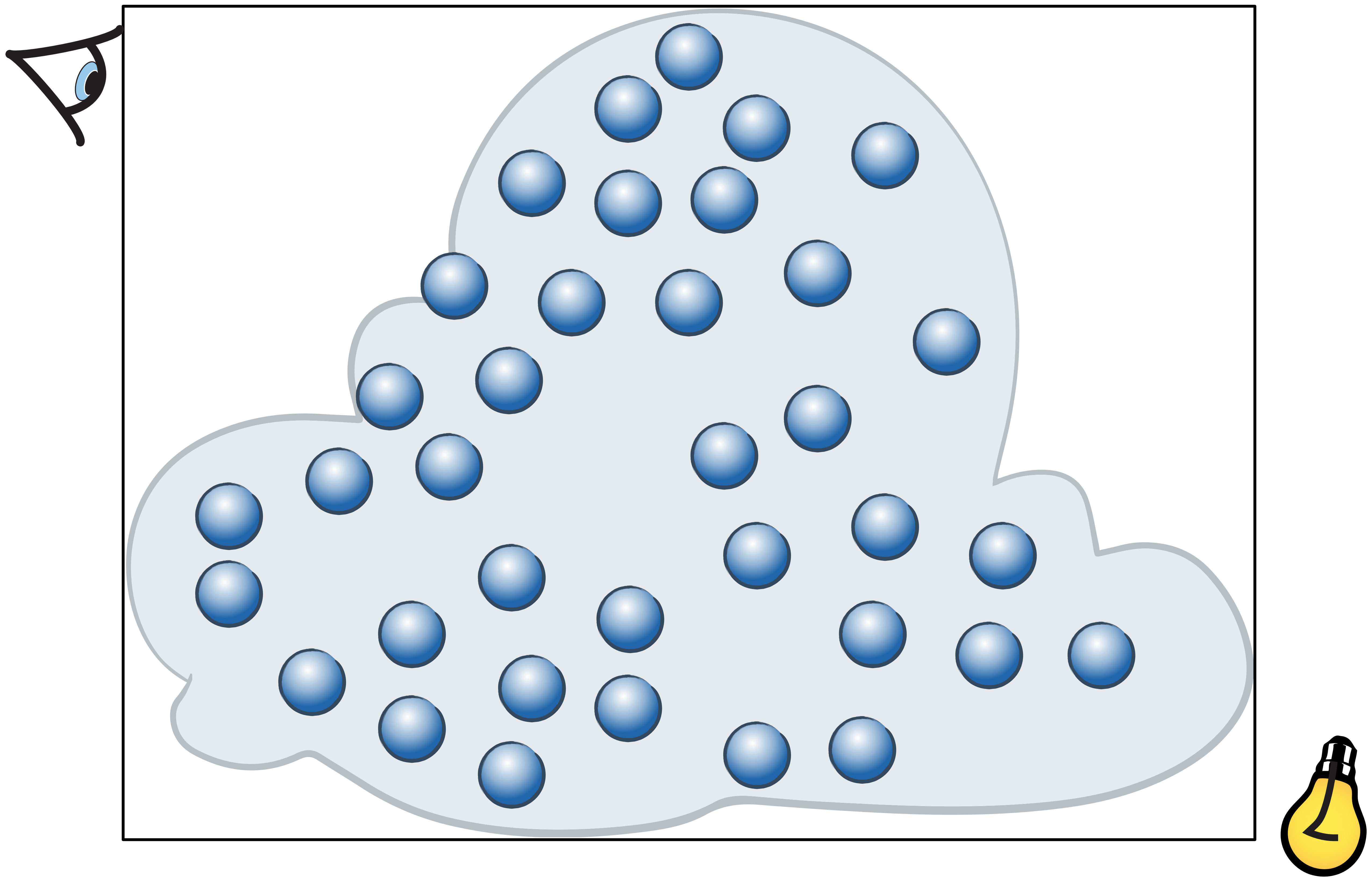
Previous Work: Overview

Class:	Techniques:	Any Media?	Unbiased?	PDF Known?
 analytic	Closed Form Tracking	✗	✓	✓
 approximations	Regular Tracking Ray Marching	✓	✗	✓
 null collisions	Delta Tracking [Woodcock et al '65] Spectral Tracking [Kutz et al '17]	✓	✓	✗

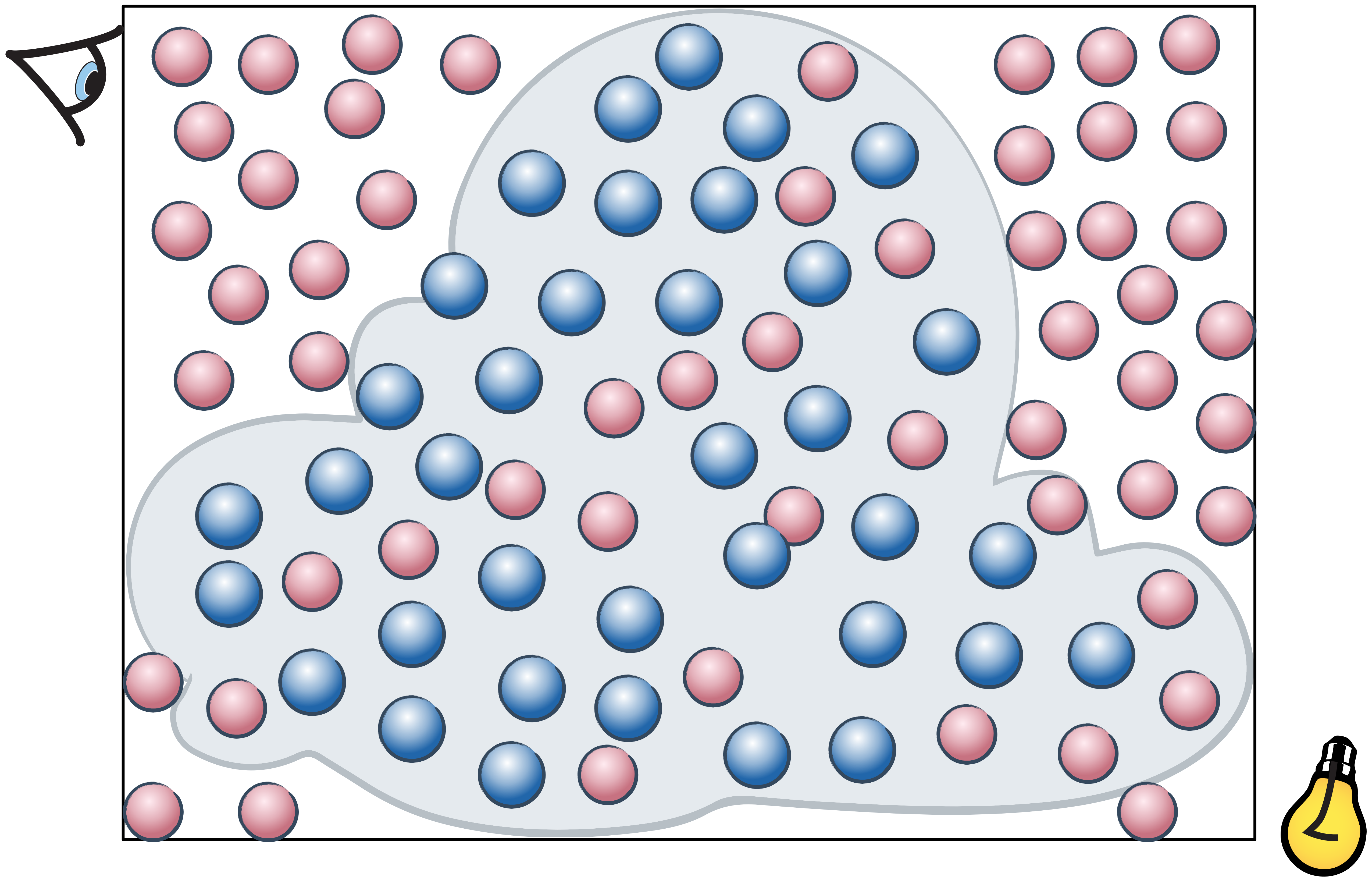
Previous Work: Overview

Class:	Techniques:	Any Media?	Unbiased?	PDF Known?
analytic	Closed Form Tracking	✗	✓	✓
approximations	Regular Tracking Ray Marching	✓	✗	✓
 null collisions	Delta Tracking [Woodcock et al '65] Spectral Tracking [Kutz et al '17]	✓	✓	✗

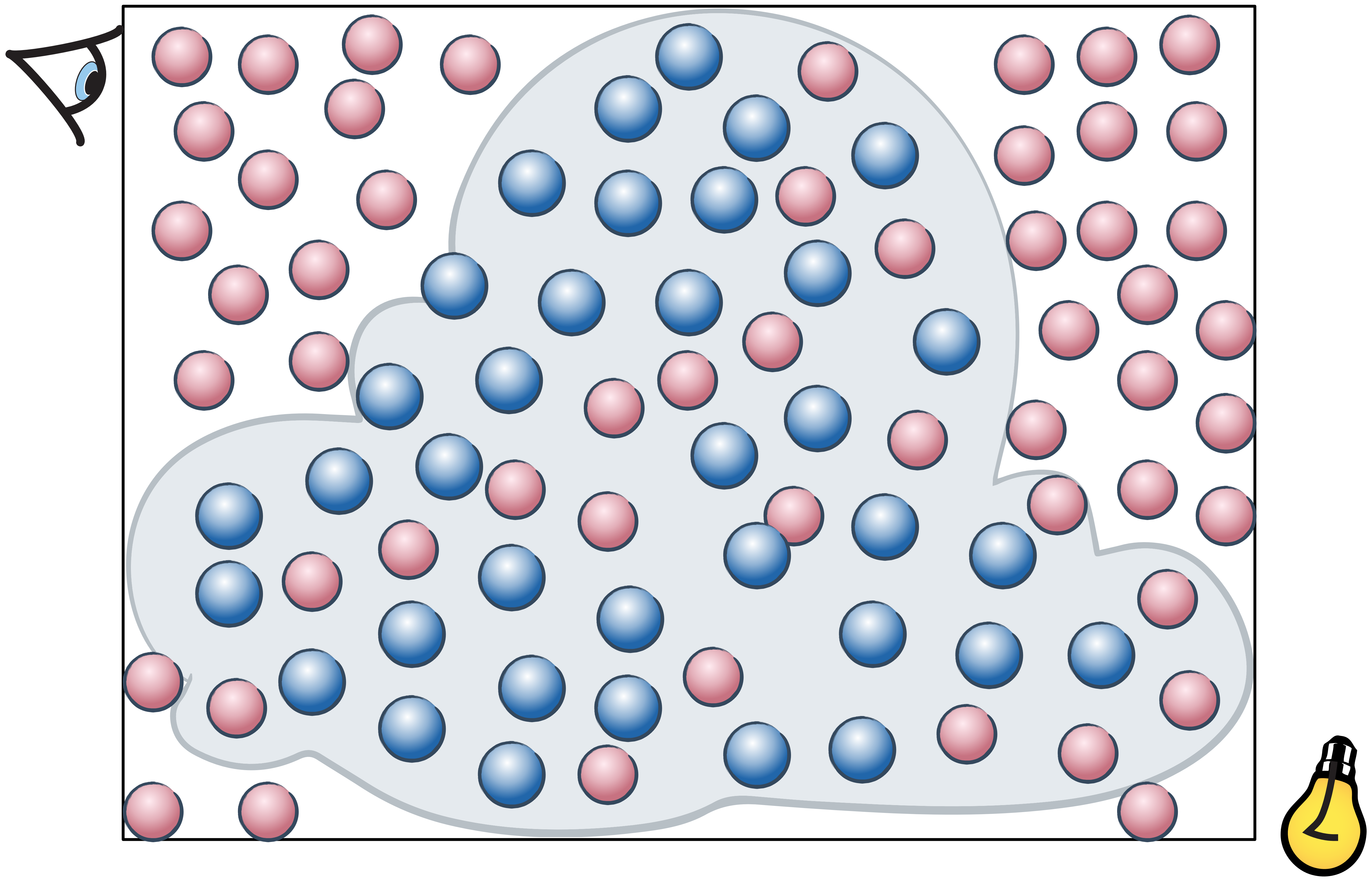
Background: Delta Tracking



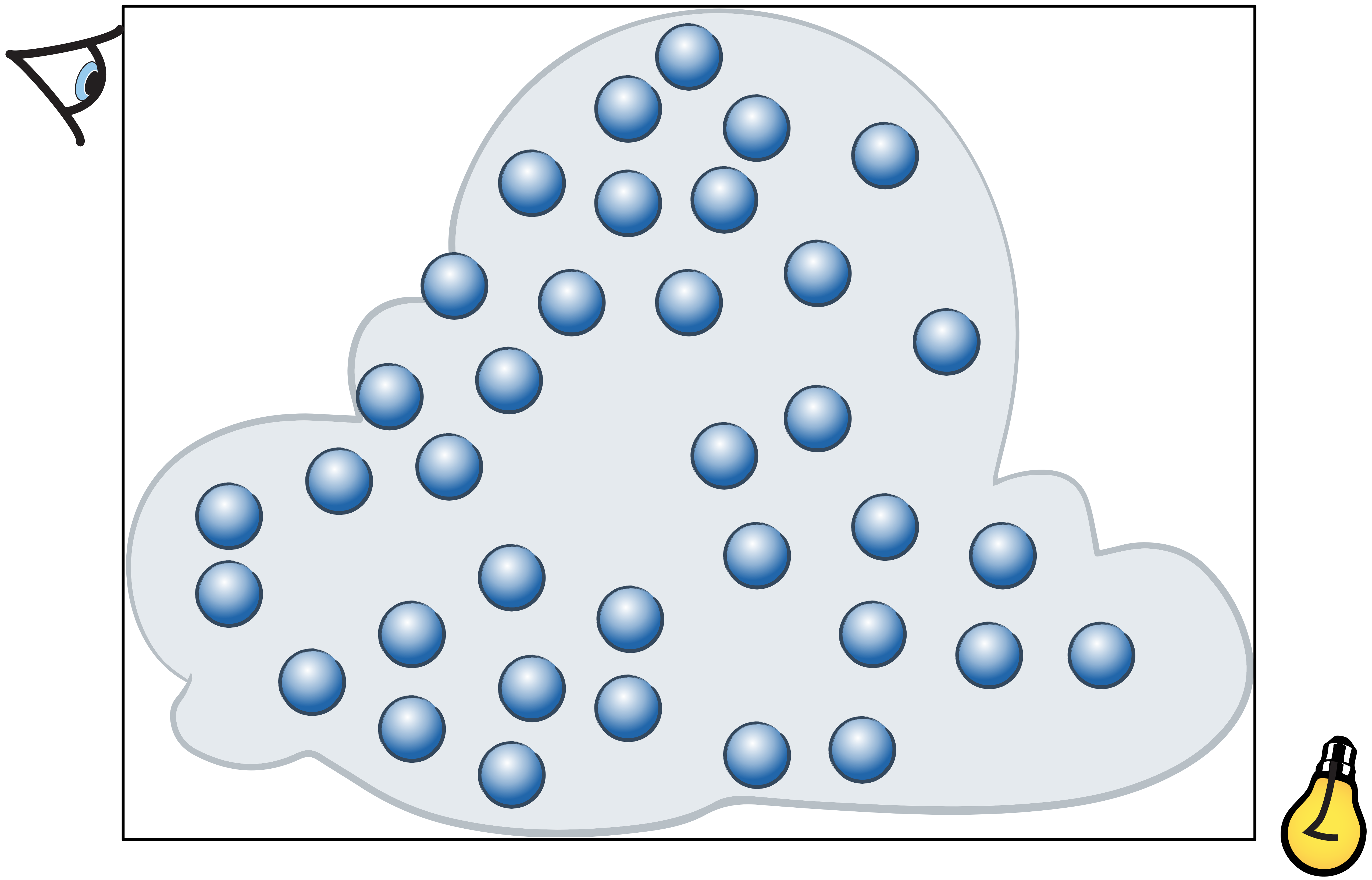
Background: Delta Tracking



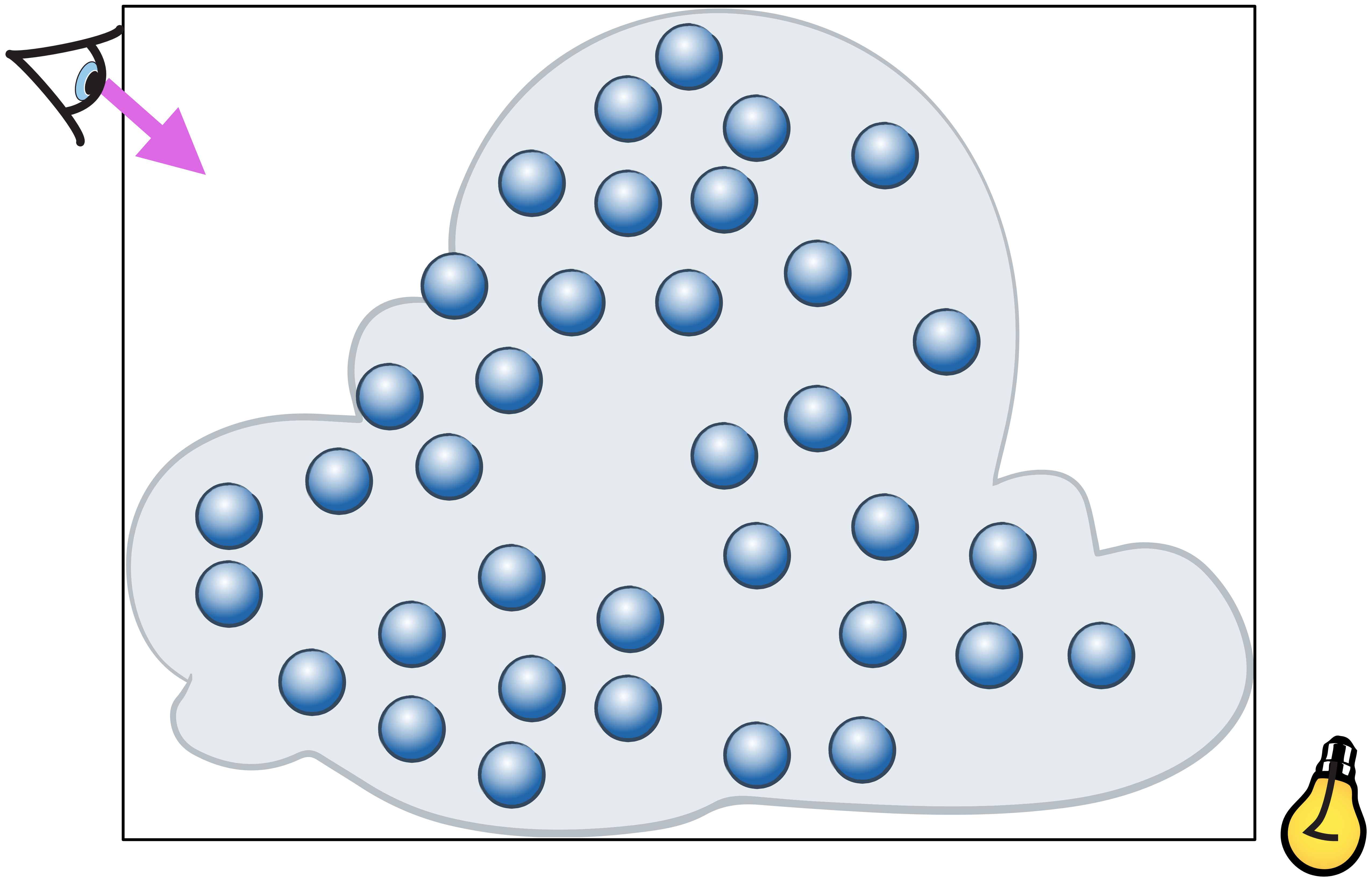
Background: Delta Tracking



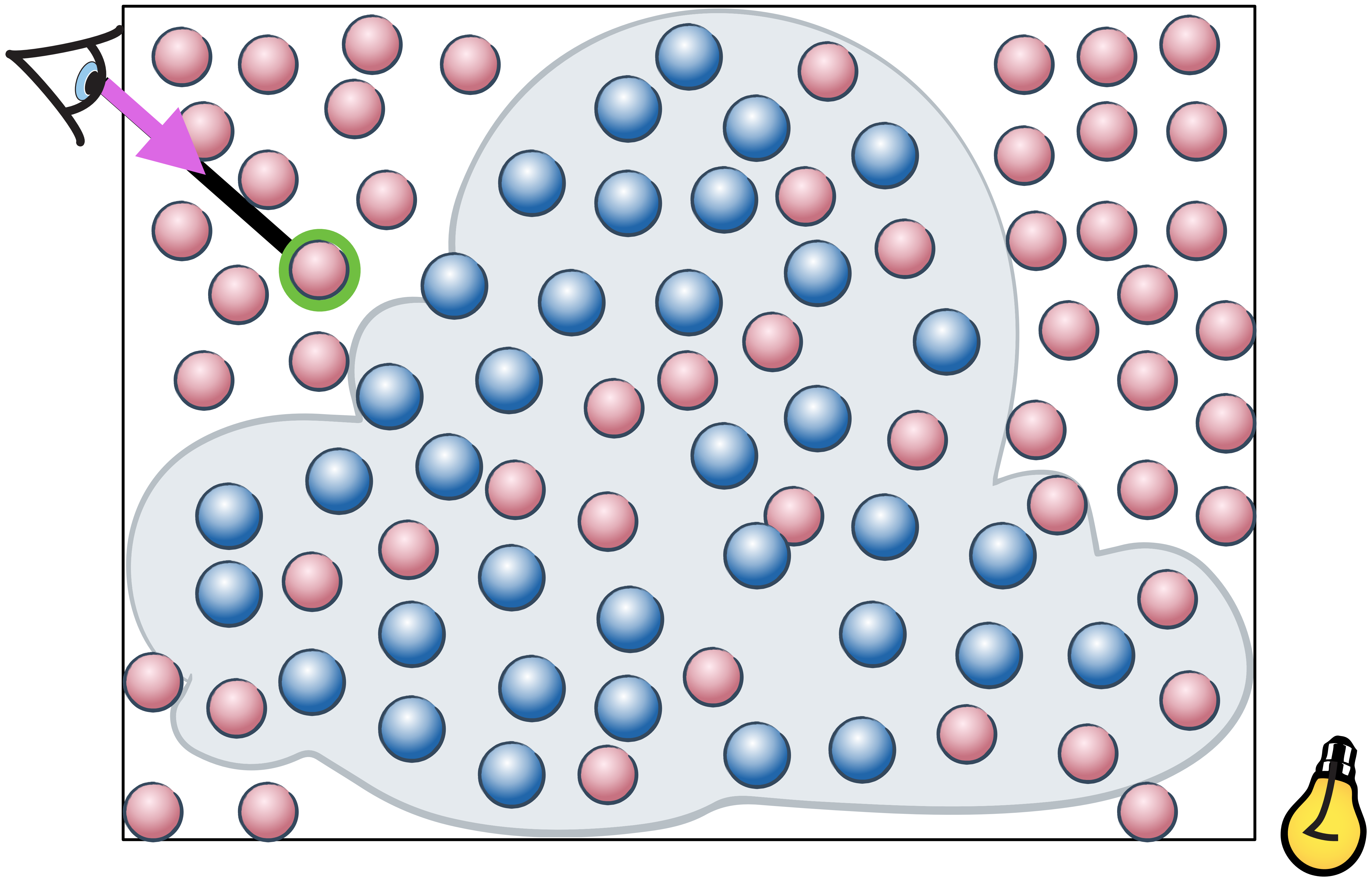
Background: Delta Tracking



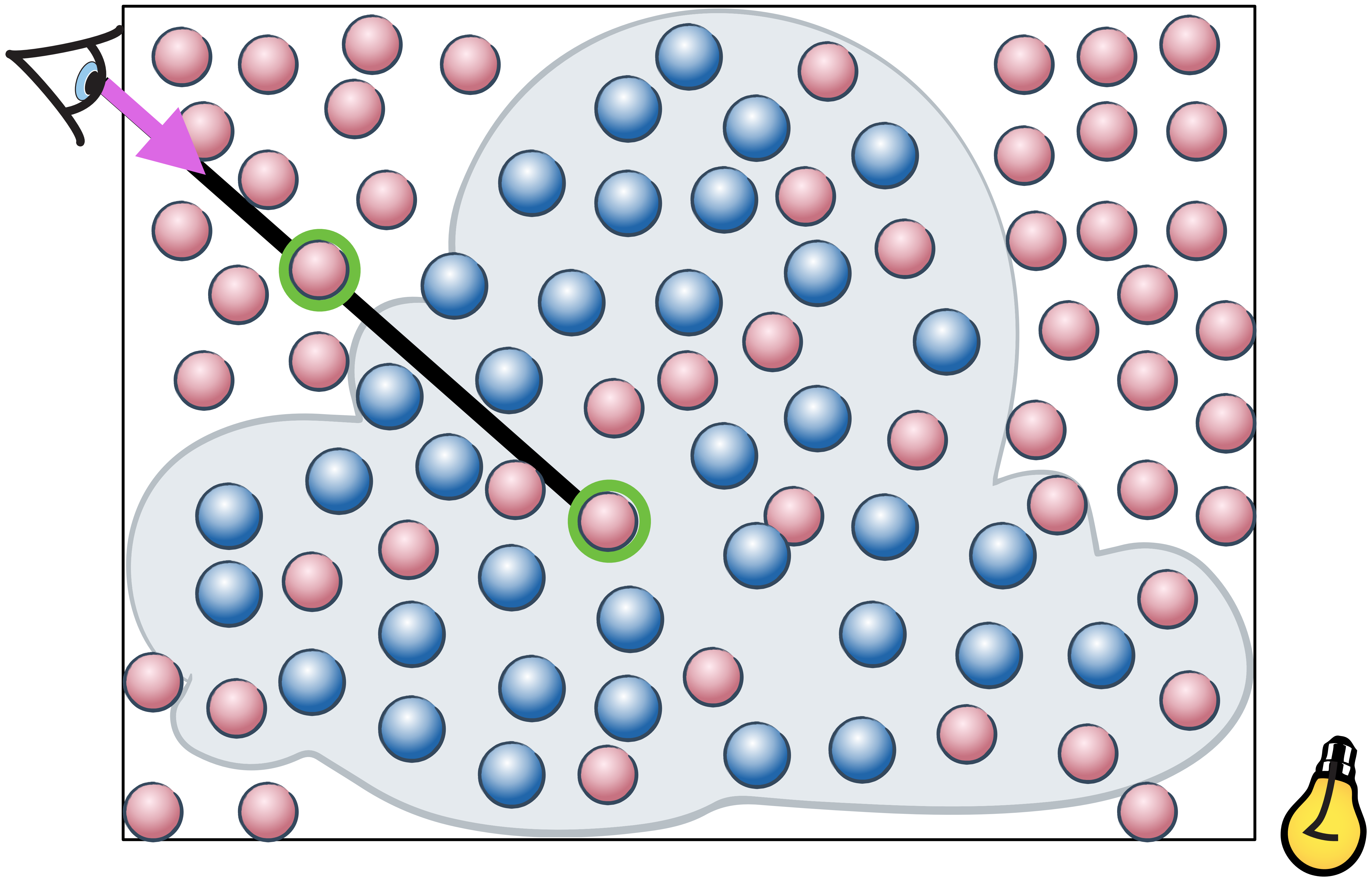
Background: Delta Tracking



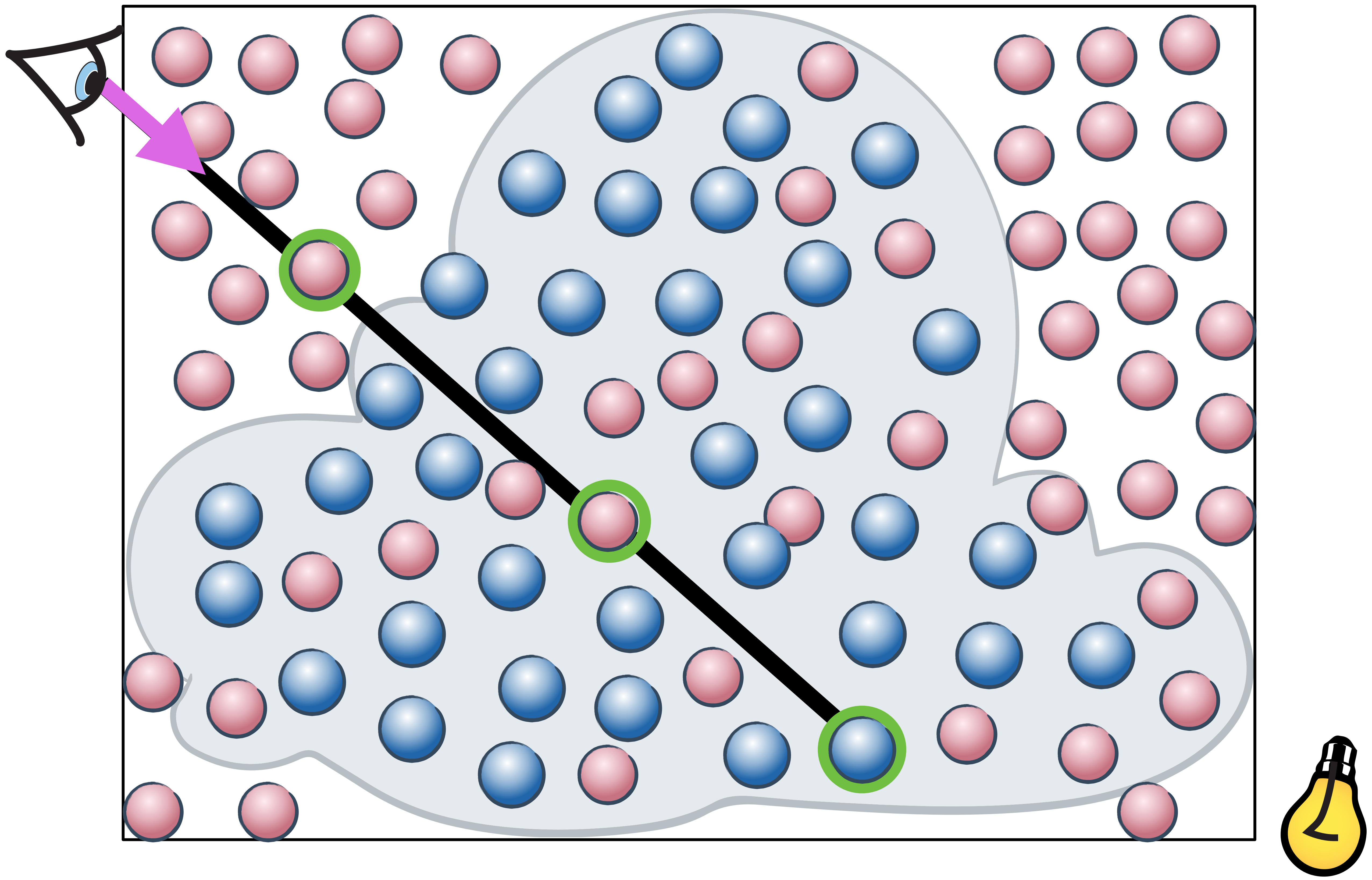
Background: Delta Tracking



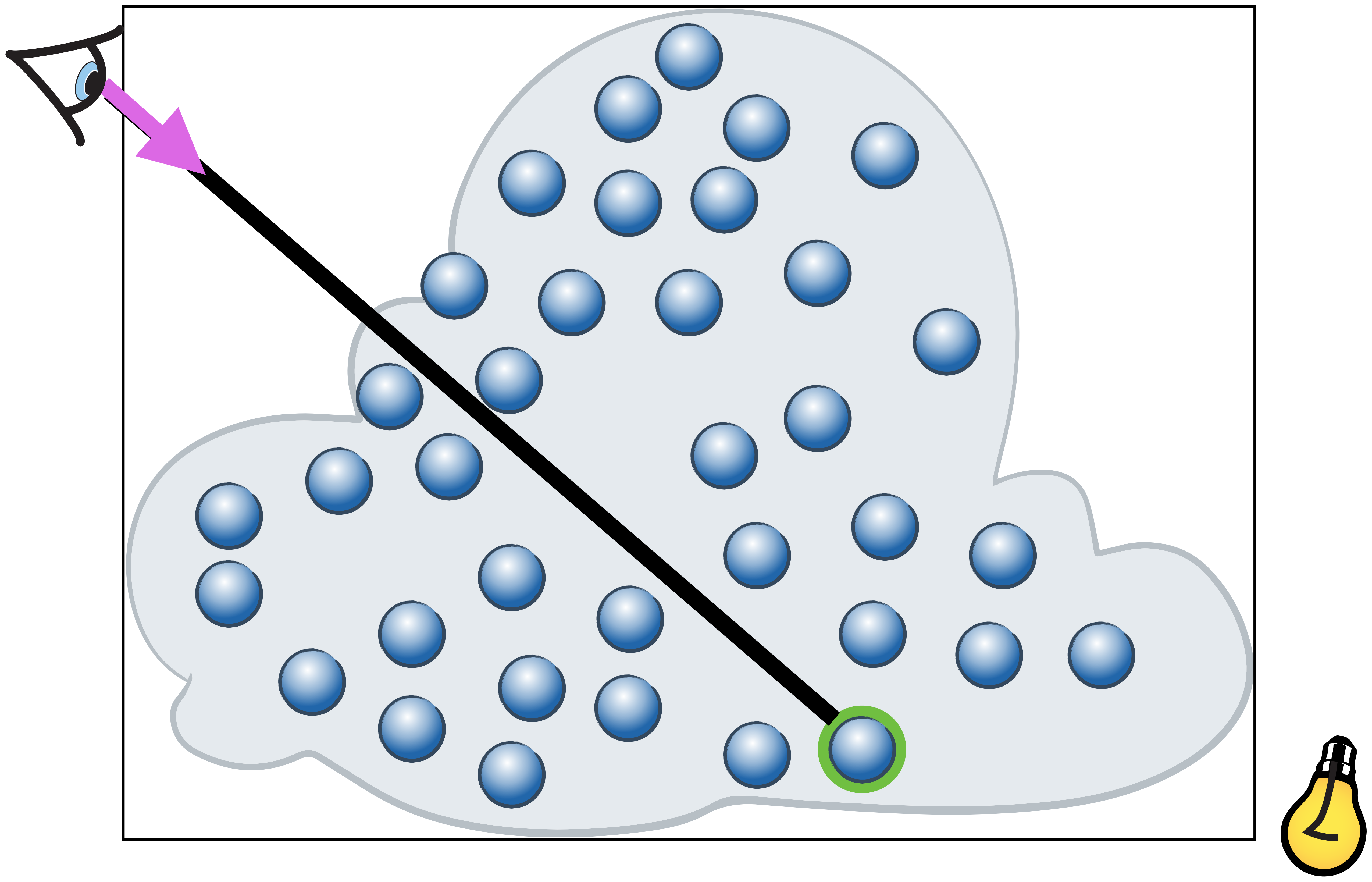
Background: Delta Tracking



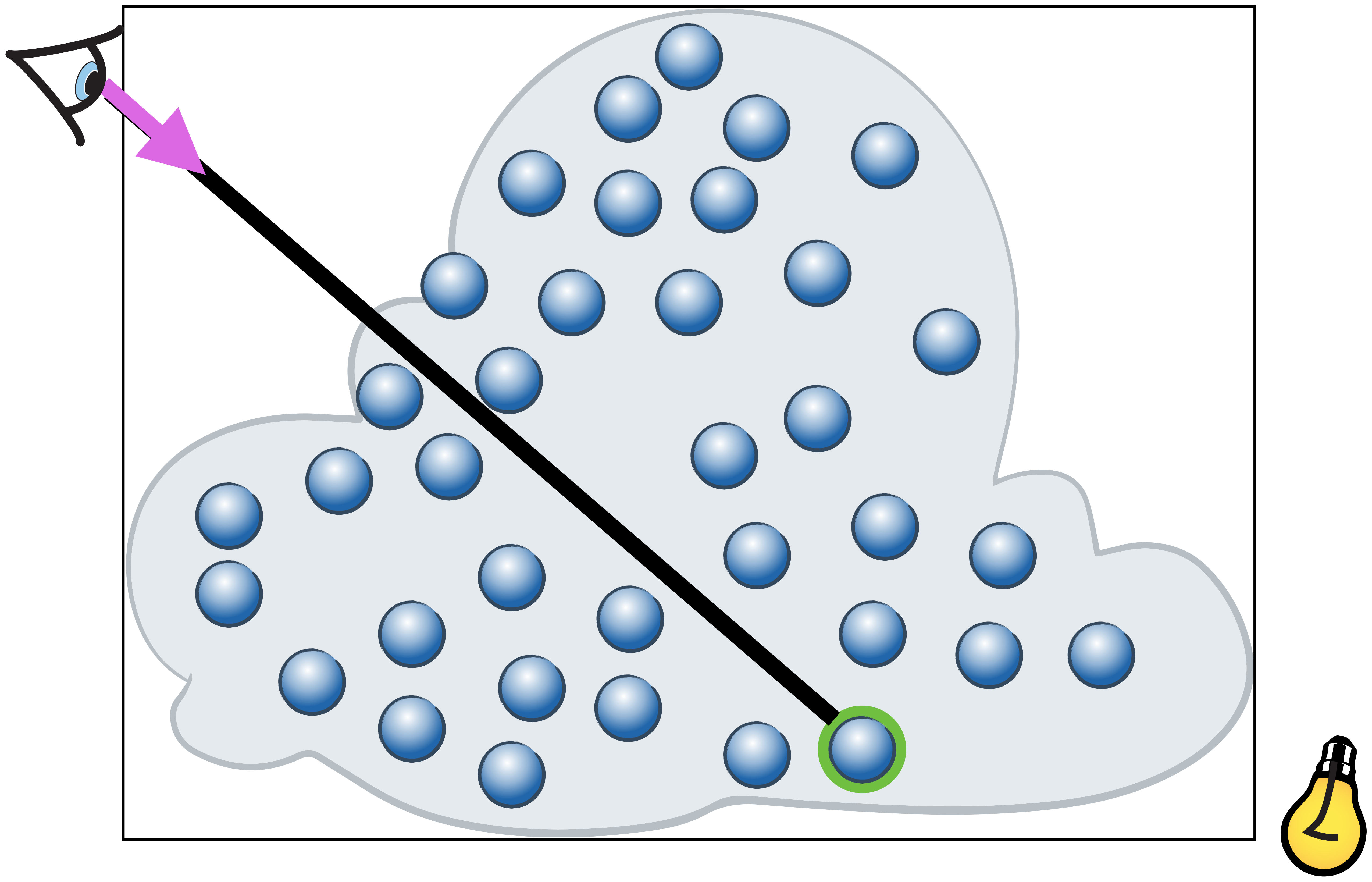
Background: Delta Tracking



Background: Delta Tracking

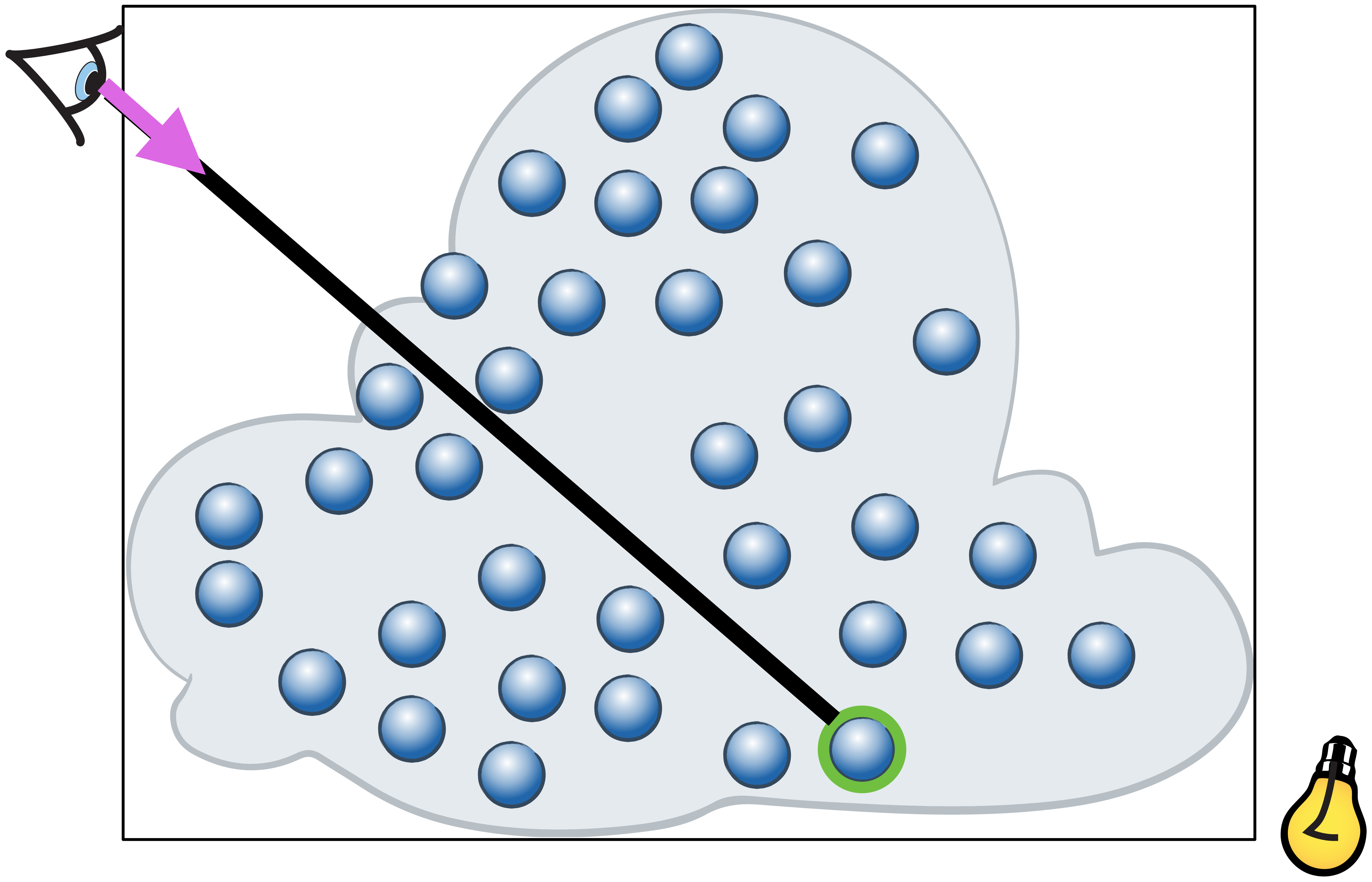


Background: Delta Tracking

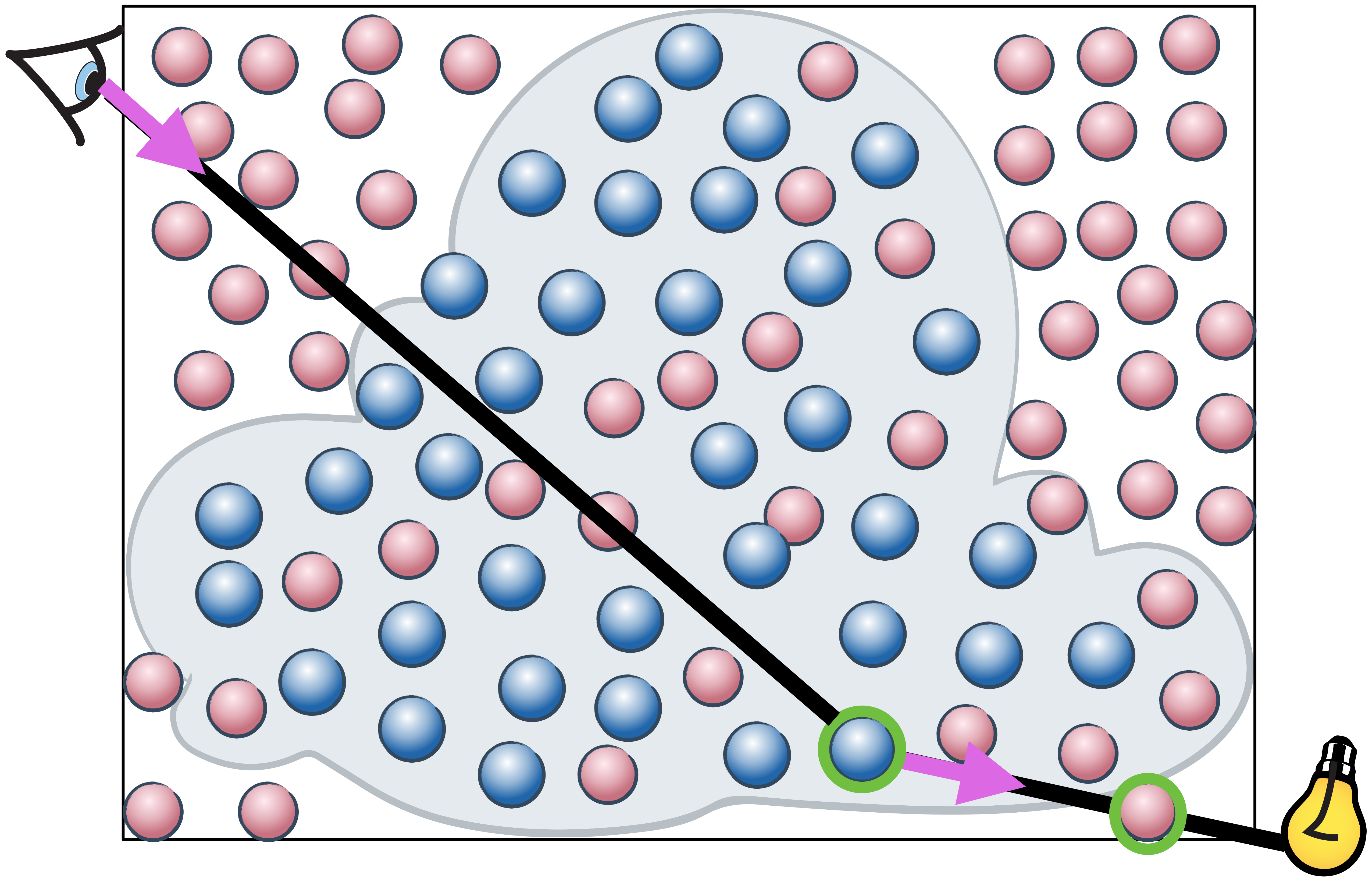


$$p(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0 + t_1 + t_2)$$

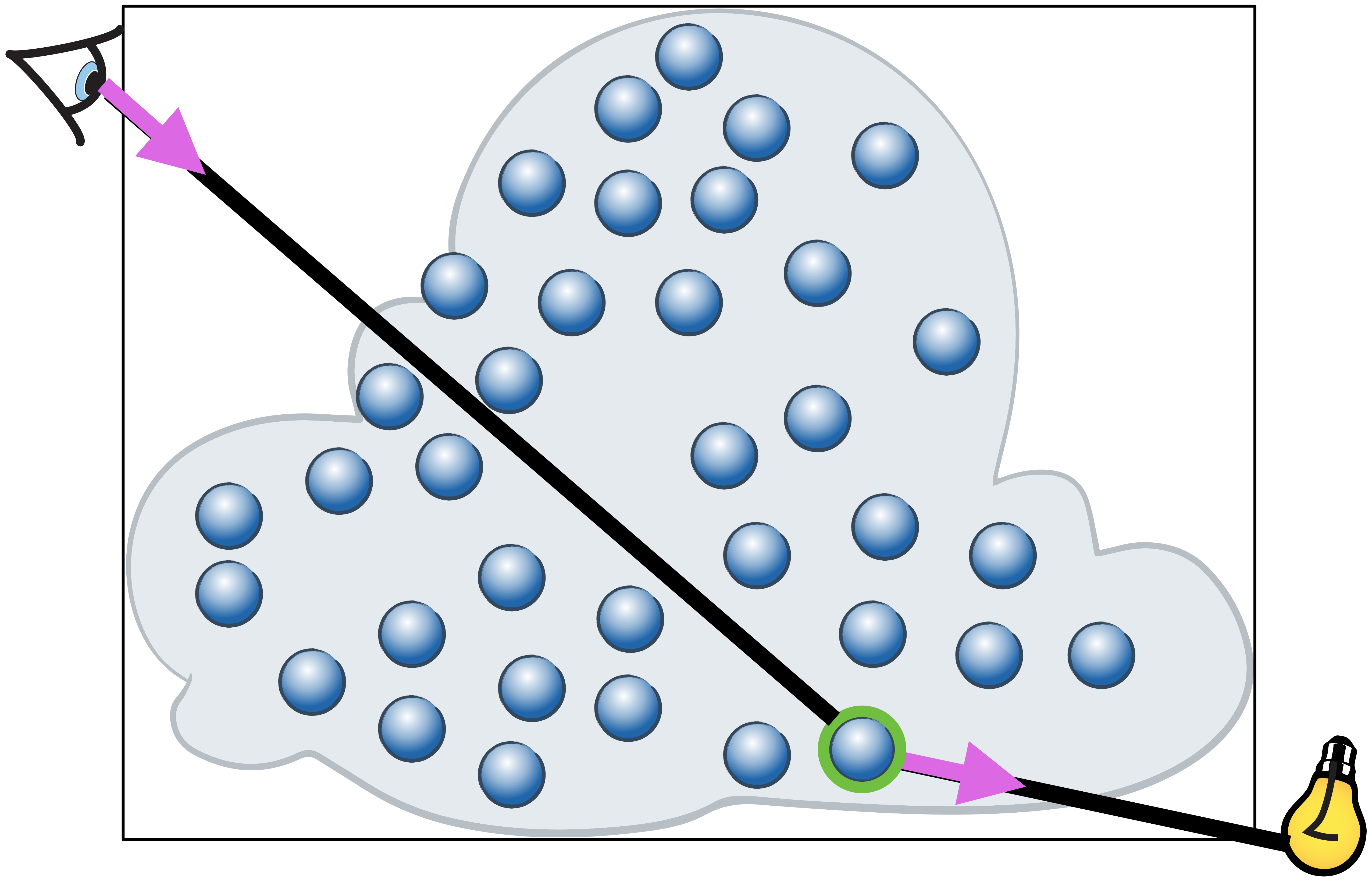
Background: Delta Tracking



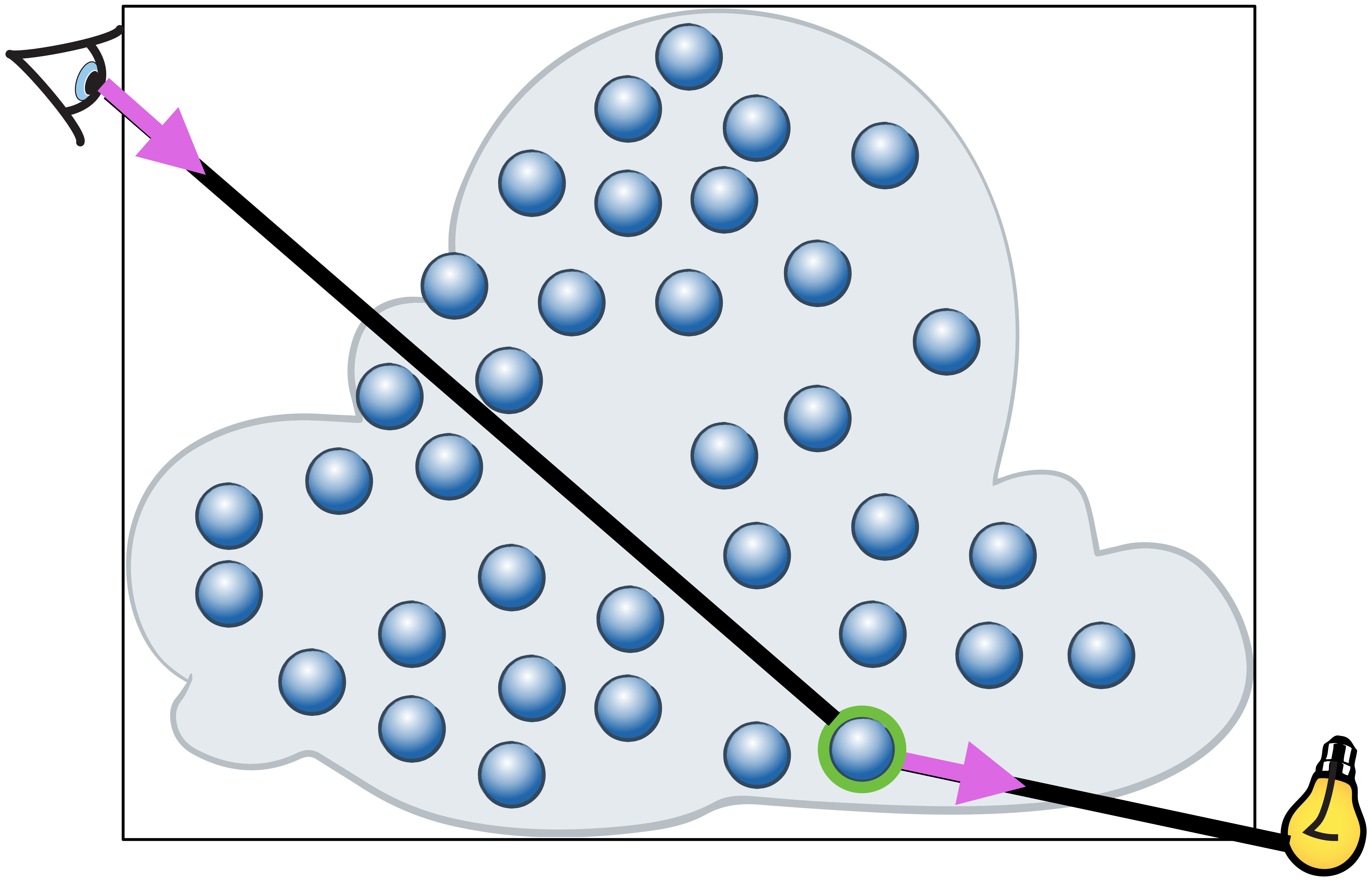
Background: Delta Tracking



Background: Delta Tracking

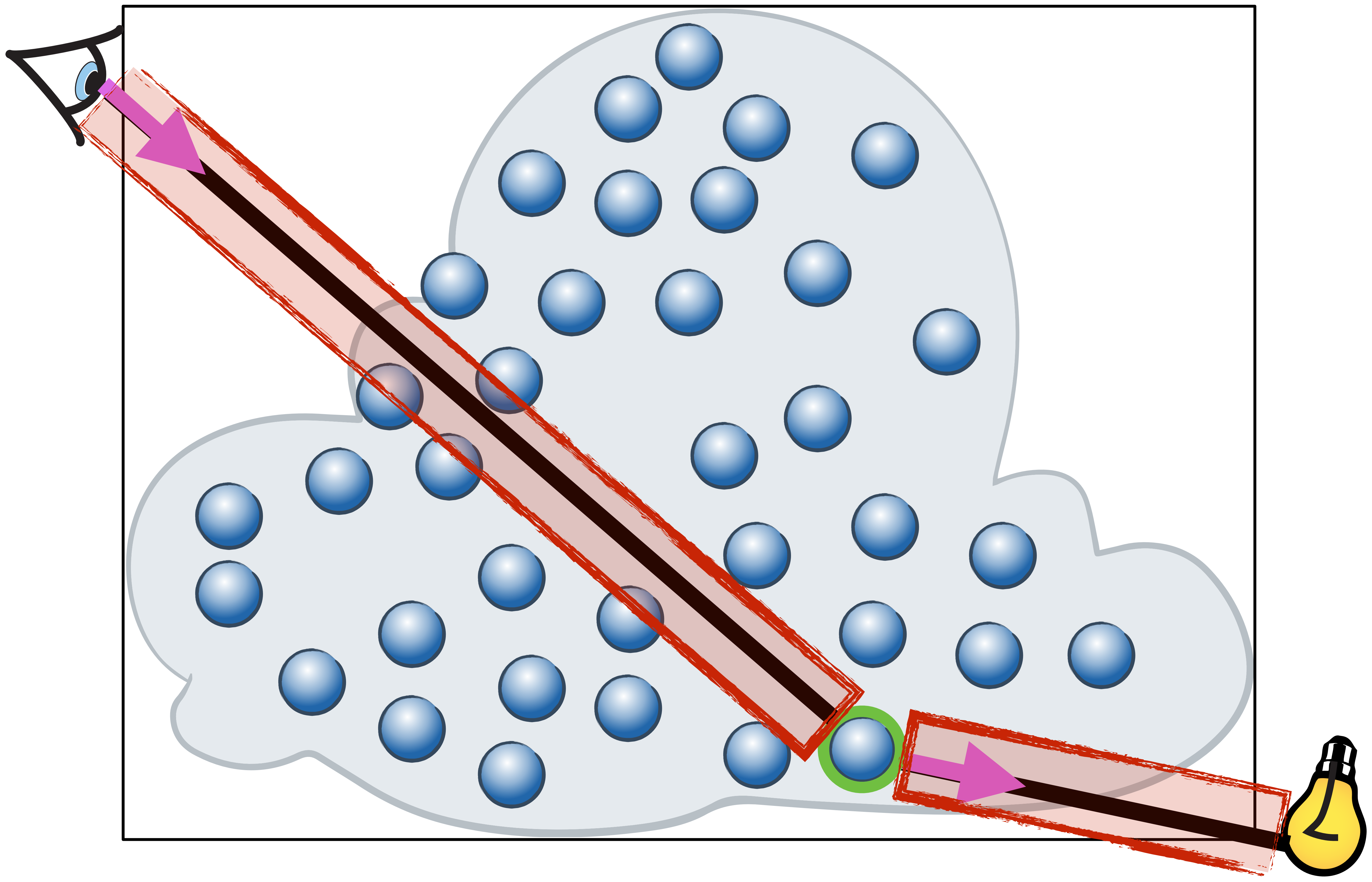


Background: Delta Tracking



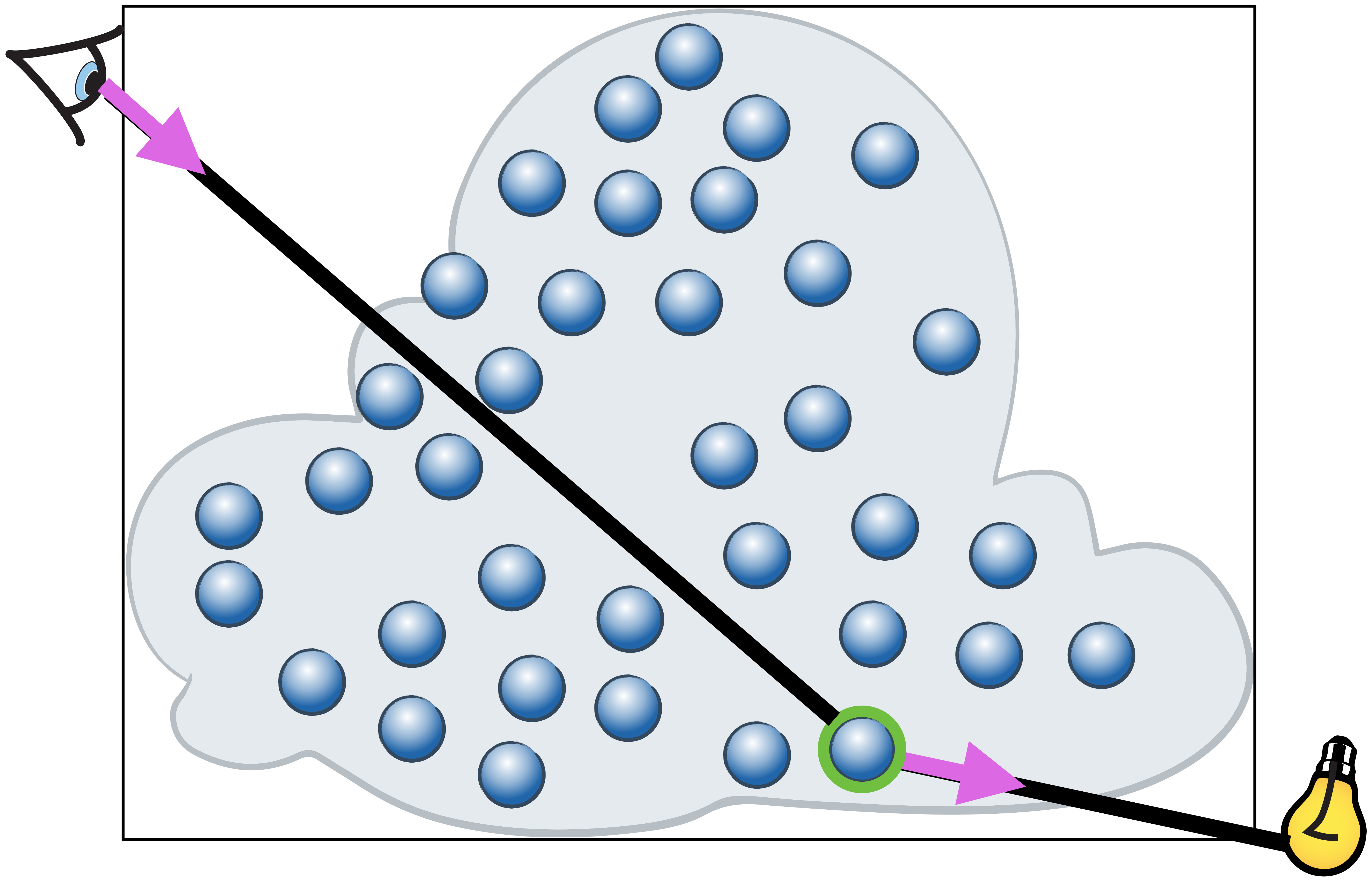
$$p(\overline{\mathbf{x}}_i) = p(\omega_0)\boxed{p(t_0 + t_1 + t_2)}p(\omega_3)\boxed{p(t_3 + t_4)}$$

Background: Delta Tracking



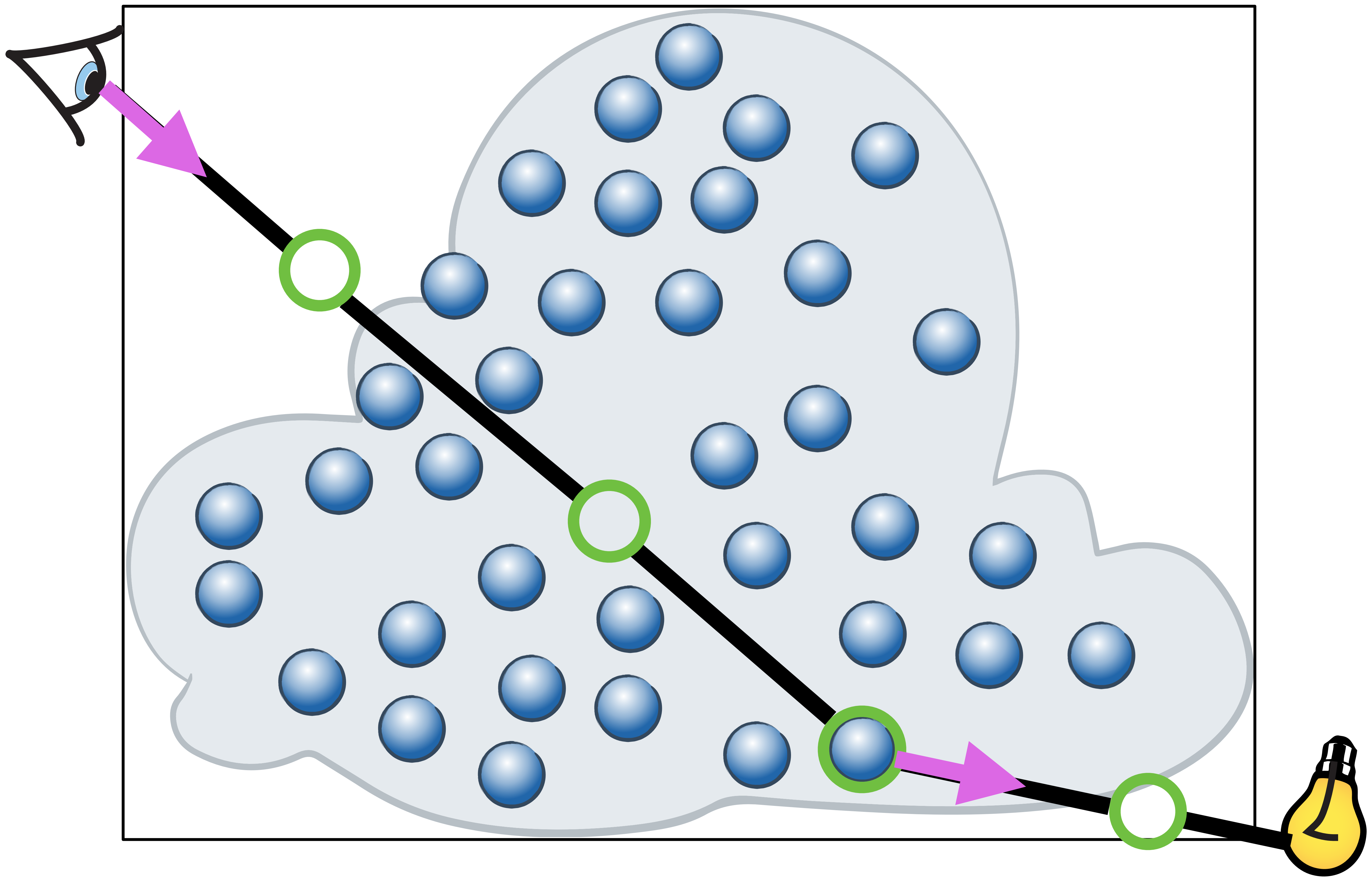
$$p(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0 + t_1 + t_2)p(\omega_3)p(t_3 + t_4)$$

Our Approach



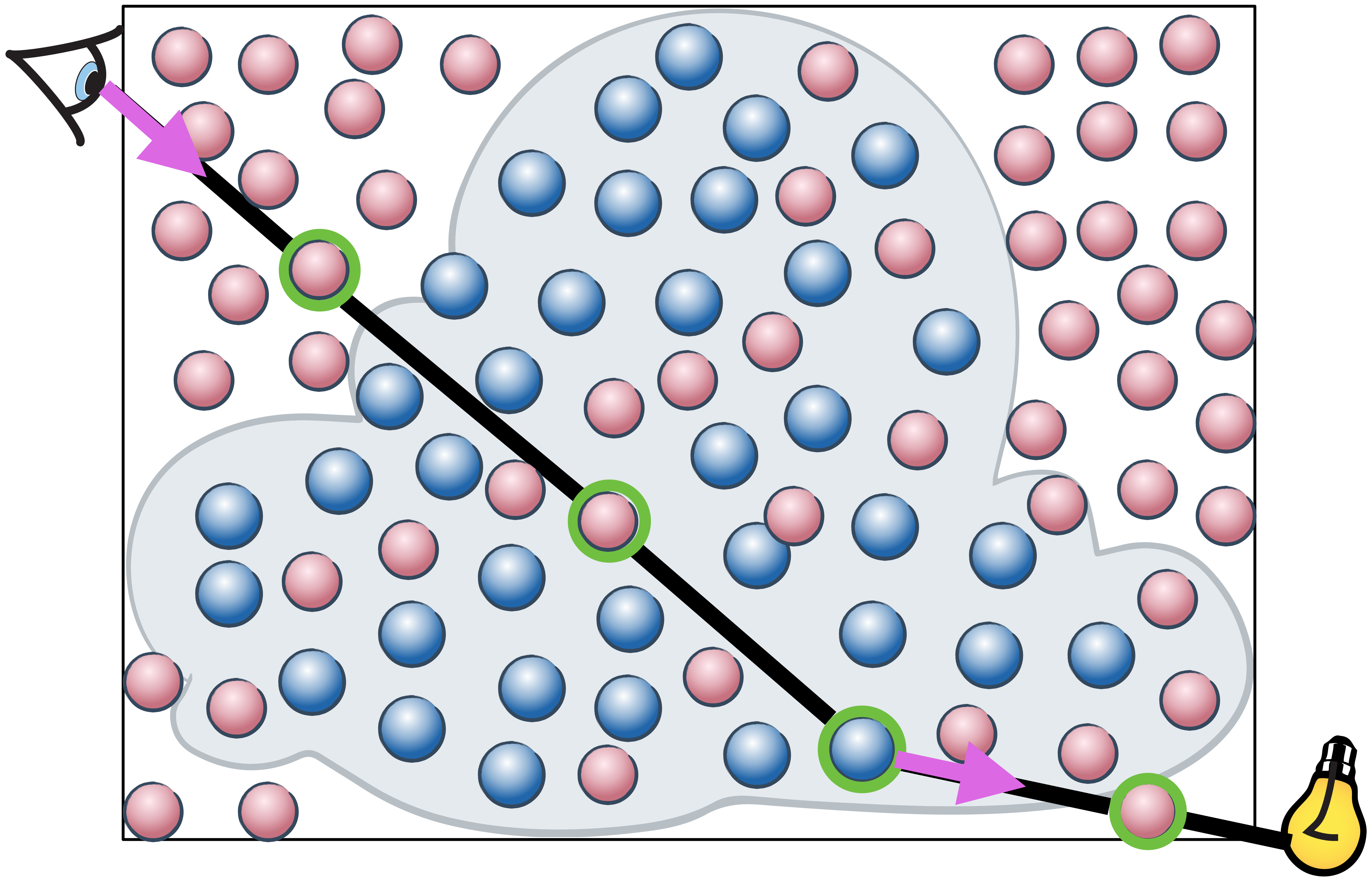
$$p(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0 + t_1 + t_2)p(\omega_3)p(t_3 + t_4)$$

Our Approach



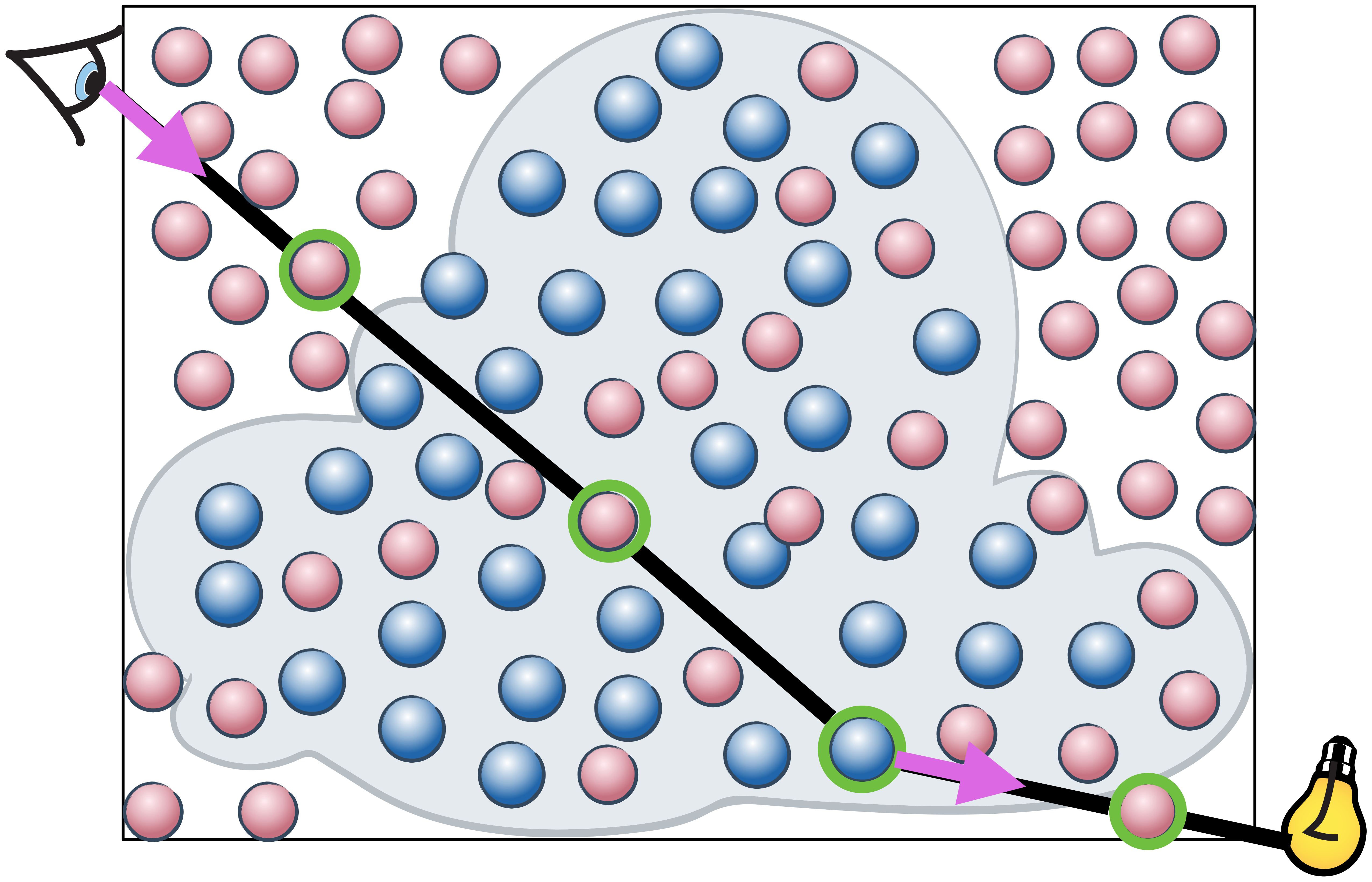
$$p(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0 + t_1 + t_2)p(\omega_3)p(t_3 + t_4)$$

Our Approach



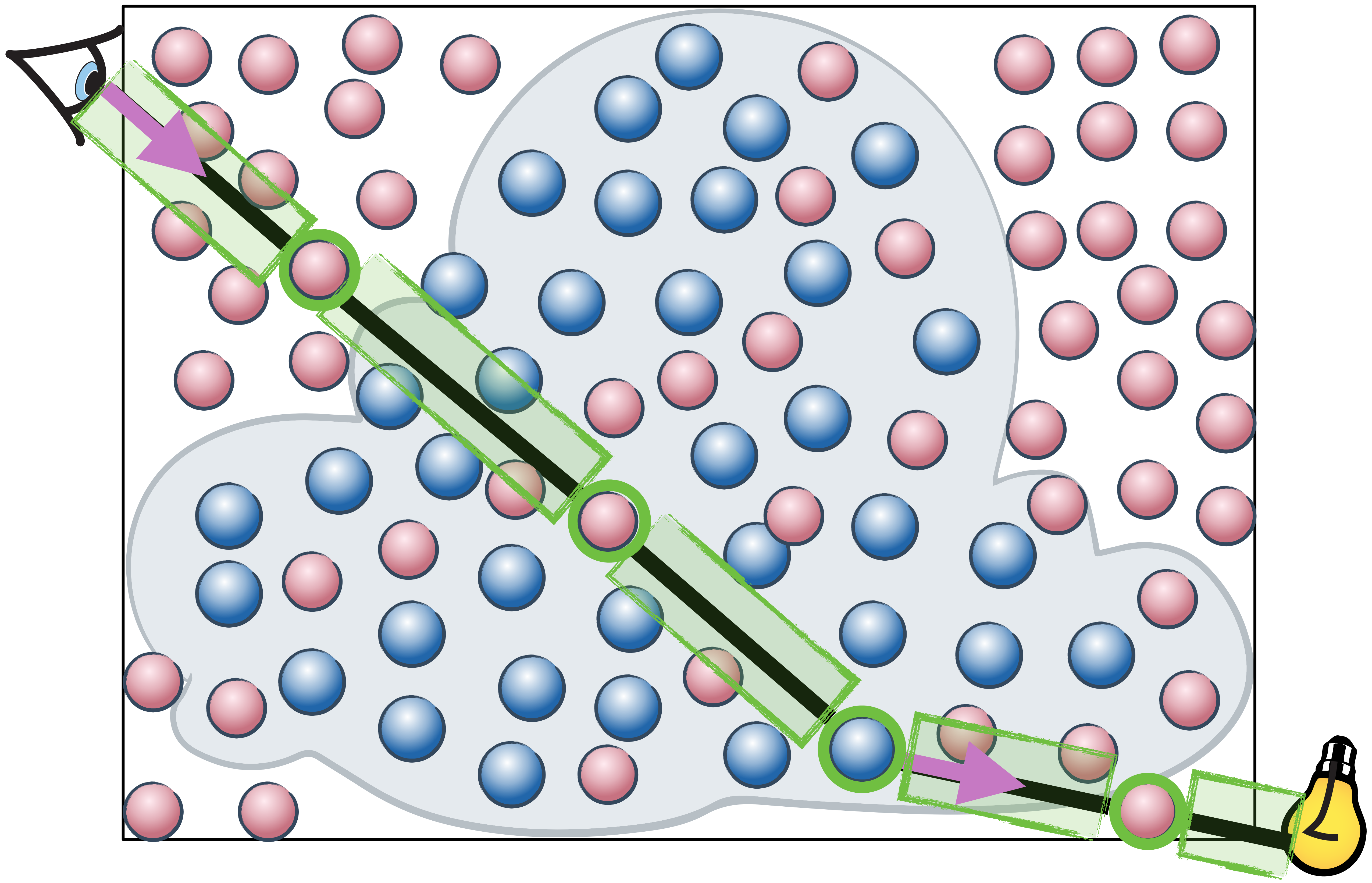
$$p(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0 + t_1 + t_2)p(\omega_3)p(t_3 + t_4)$$

Our Approach



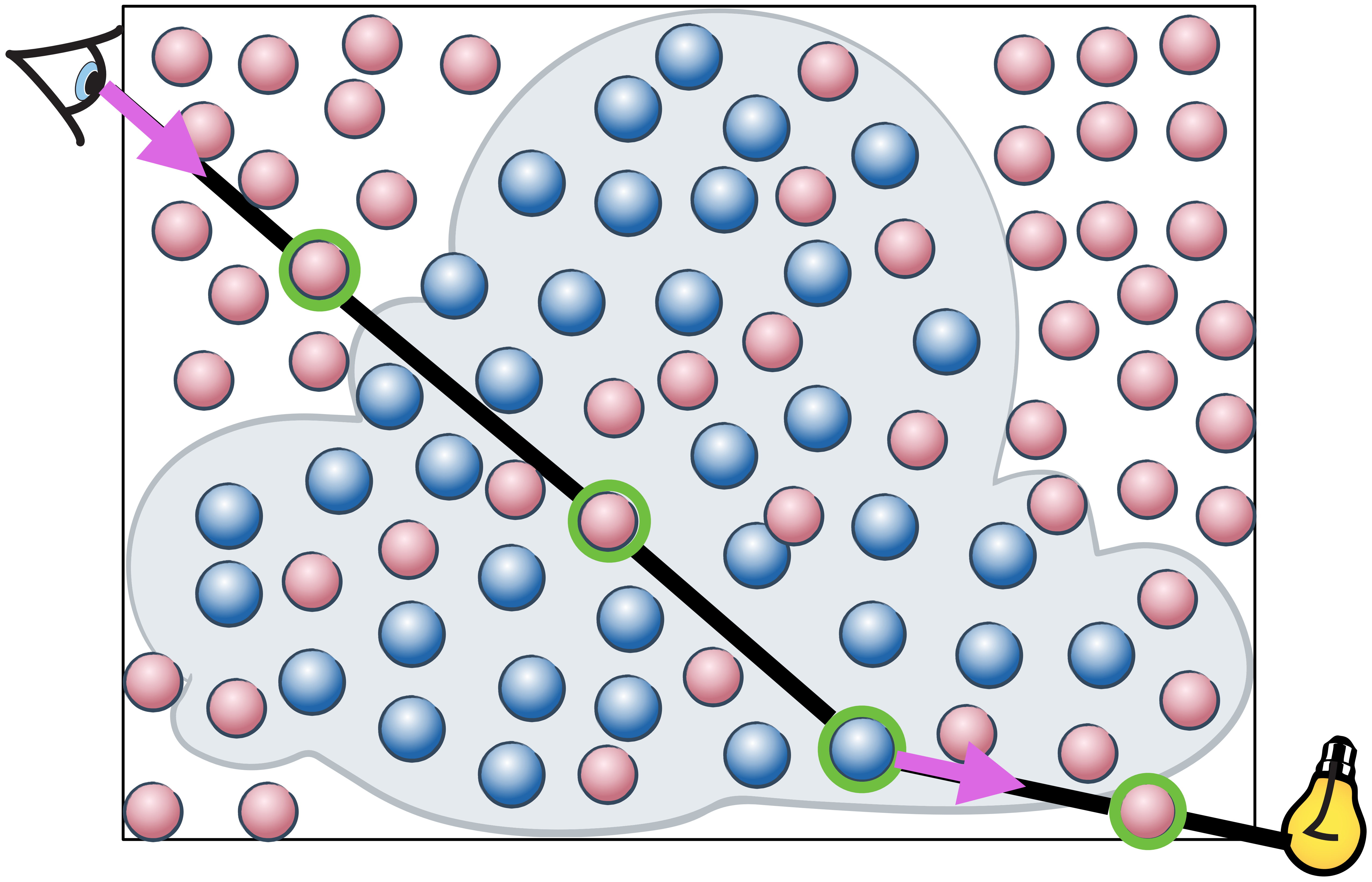
$$p(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(t_1)p(t_2)p(\omega_1)p(t_3)p(t_4)$$

Our Approach



$$p(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(t_1)p(t_2)p(\omega_1)p(t_3)p(t_4)$$

Our Approach

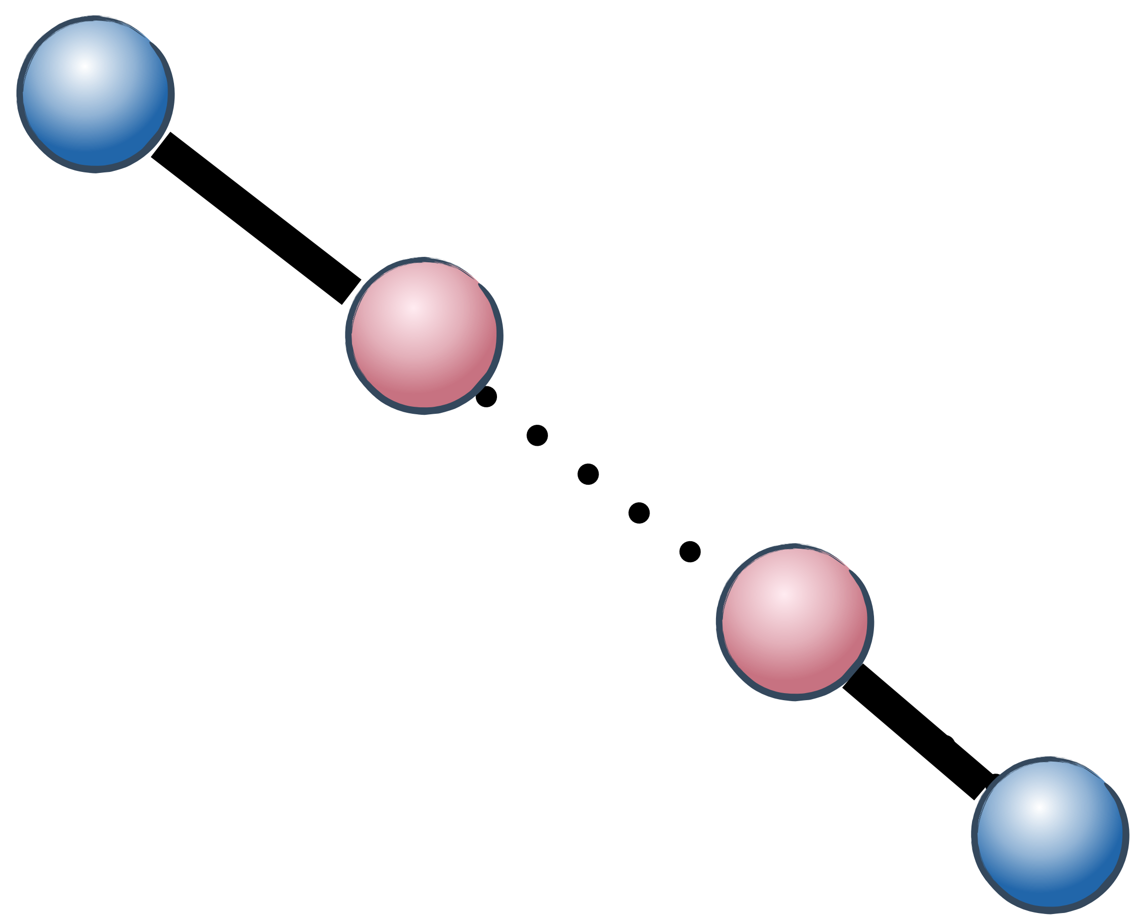


Our Approach

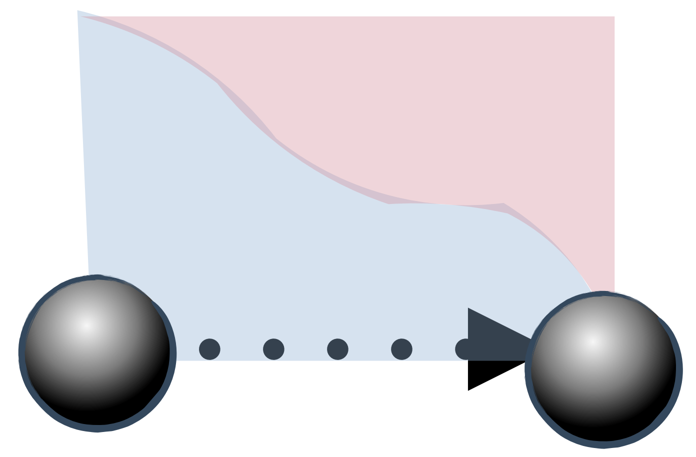
[Galtier et al. 2013]

Our Approach: Path Integral

Both Real and
Null Scatterers



Null Scattering
Always Forward



Homogenized
Transmittance

Our Approach: Path Integral

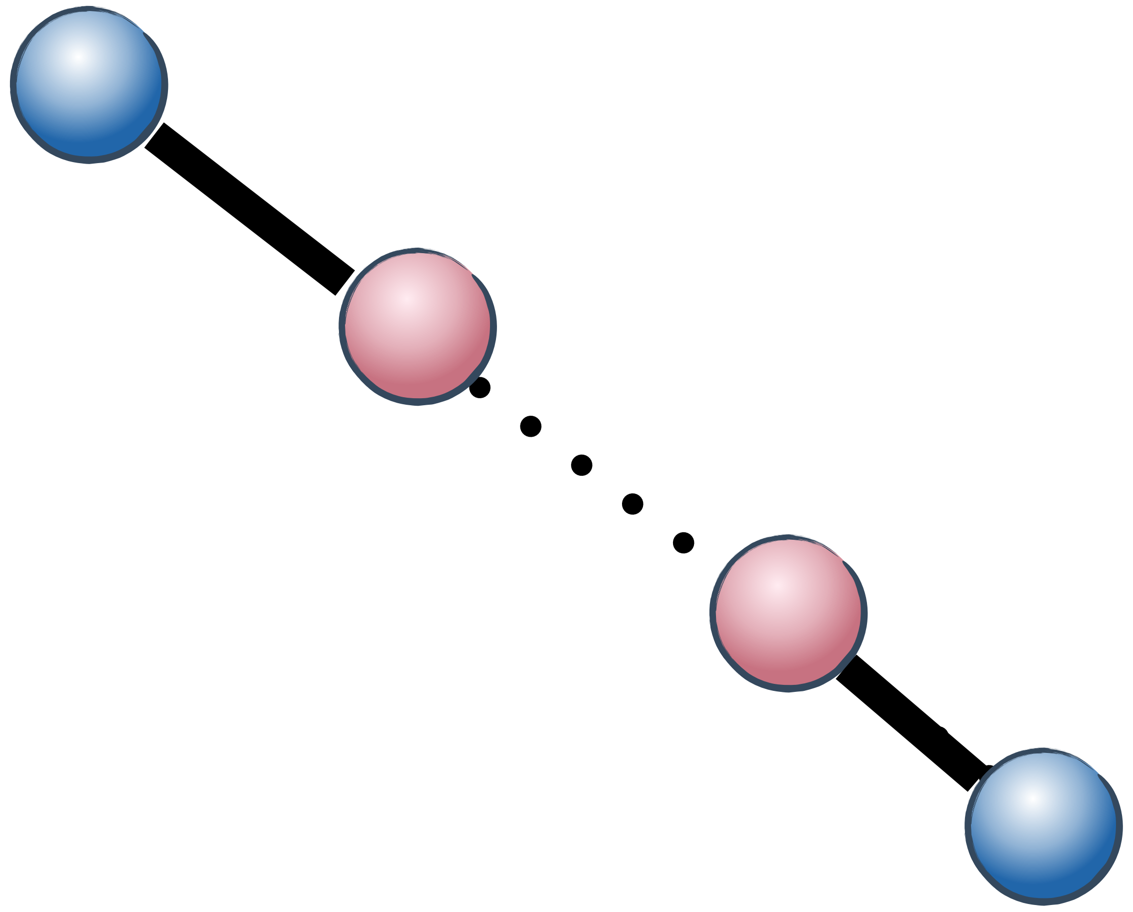
Both Real and
Null Scatterers

Null Scattering
Always Forward

Homogenized
Transmittance

Our Approach: Path Integral

Both Real and
Null Scatterers

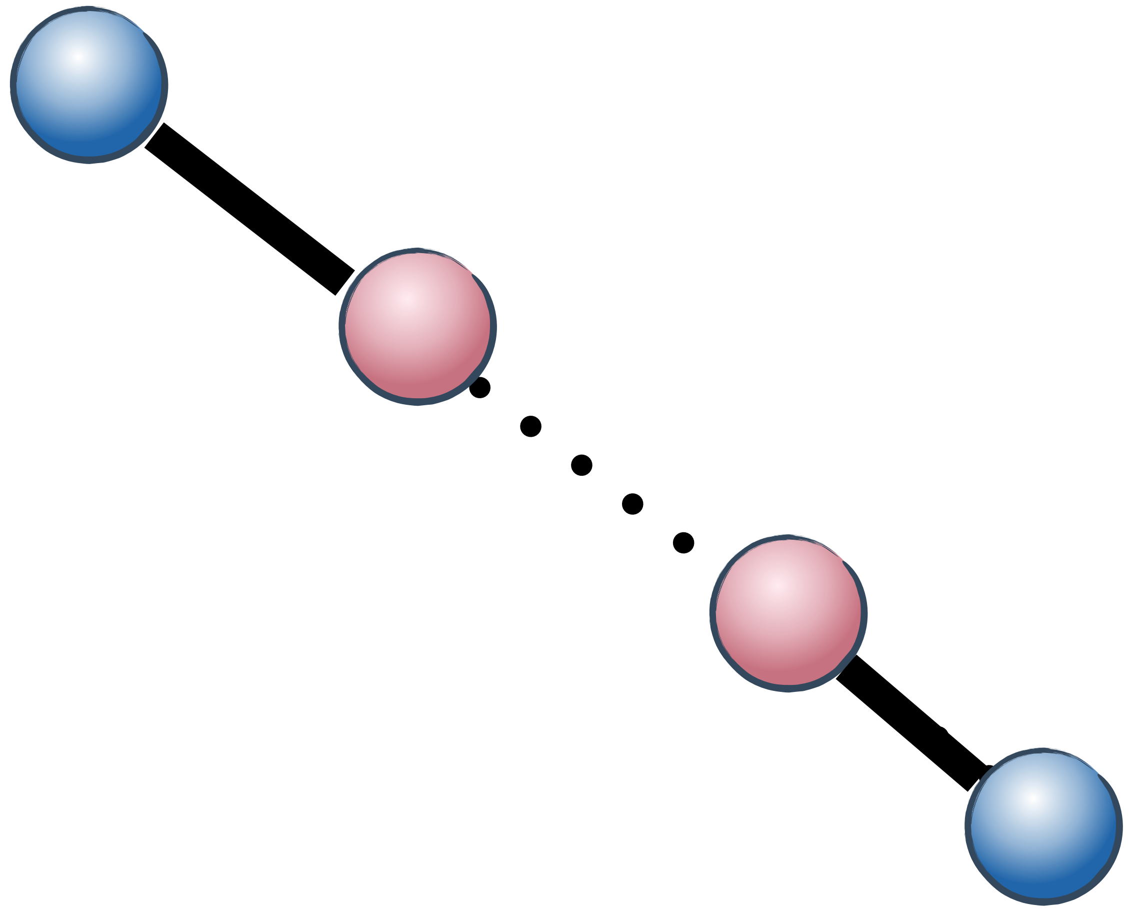


Null Scattering
Always Forward

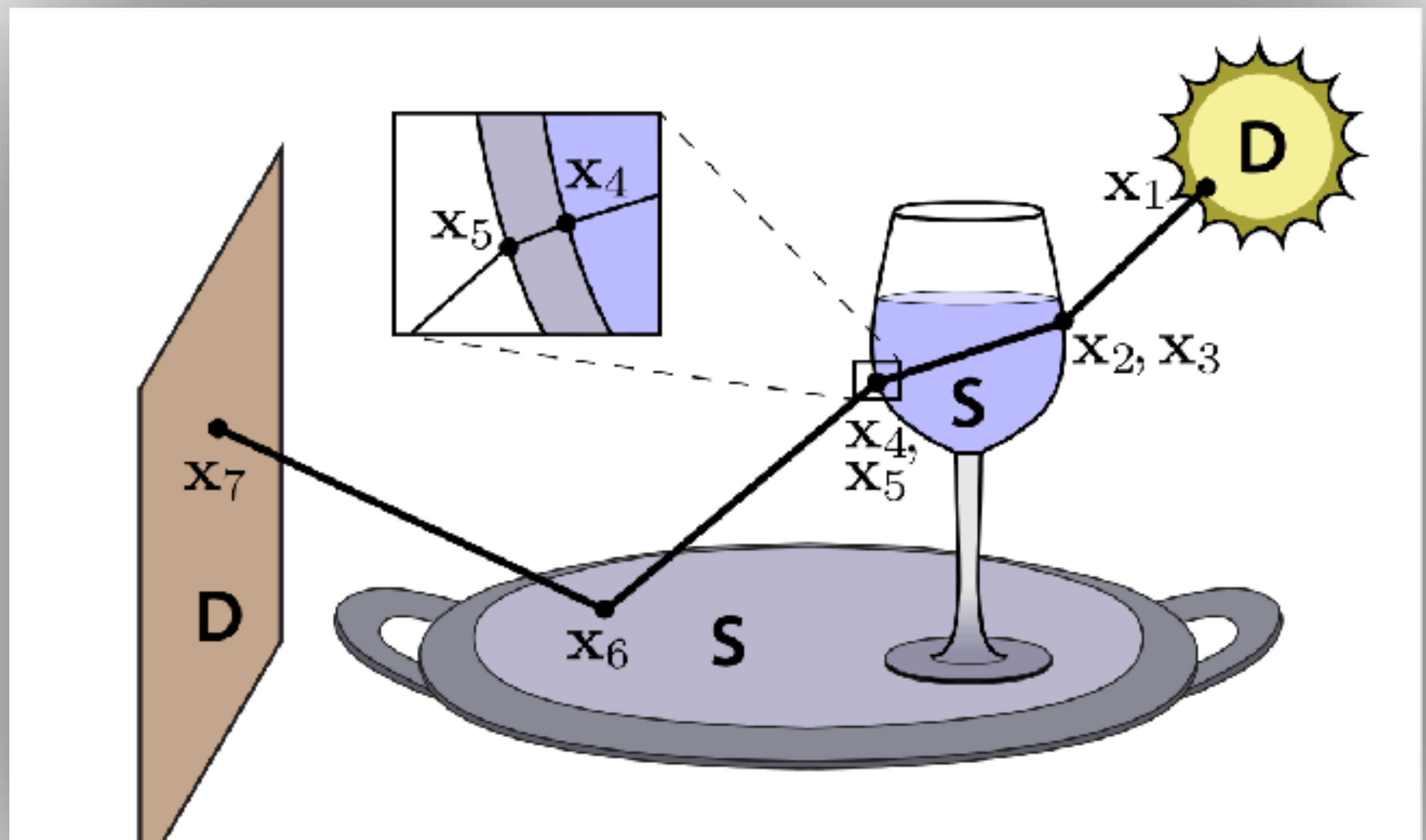
Homogenized
Transmittance

Our Approach: Path Integral

Both Real and
Null Scatterers



Null Scattering
Always Forward



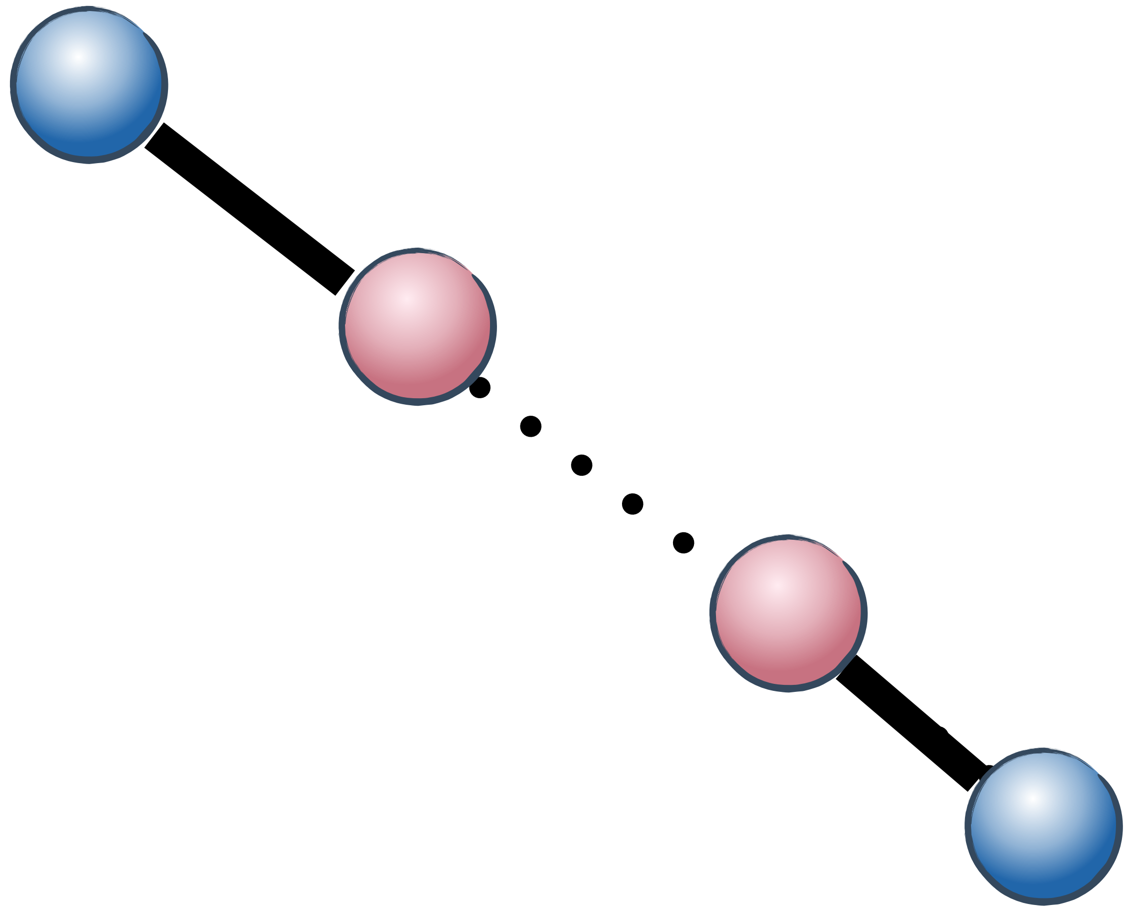
(a) An example path

Specular Manifolds

[Jakob 2013]

Our Approach: Path Integral

Both Real and
Null Scatterers



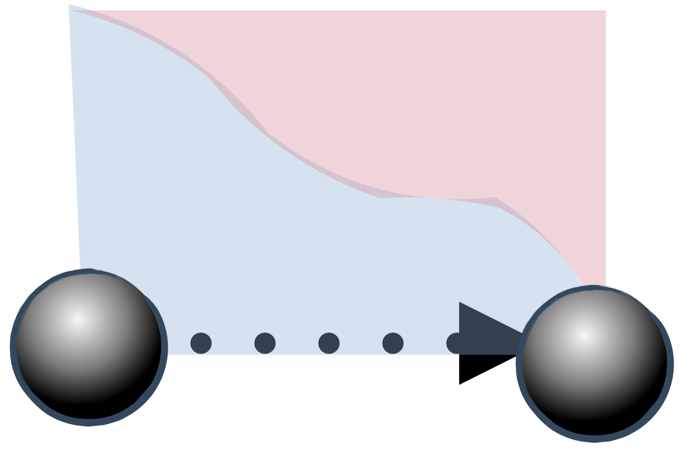
Null Scattering
Always Forward

Homogenized
Transmittance

Our Approach: Path Integral

Both Real and
Null Scatterers

Null Scattering
Always Forward



Homogenized
Transmittance

Our Approach: Path Integral

See paper for more details!

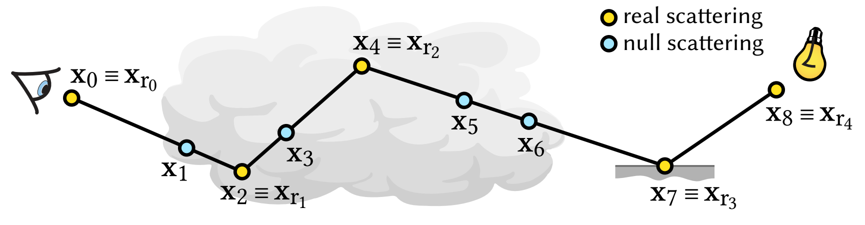


Fig. 3. Illustration of a light transport path in our integral formulation, which explicitly considers both real and null scattering at path vertices. Prior formulations consider real scattering only, where this length-8 path corresponds to the length-4 path $x_{r_0}x_{r_1}x_{r_2}x_{r_3}x_{r_4}$.

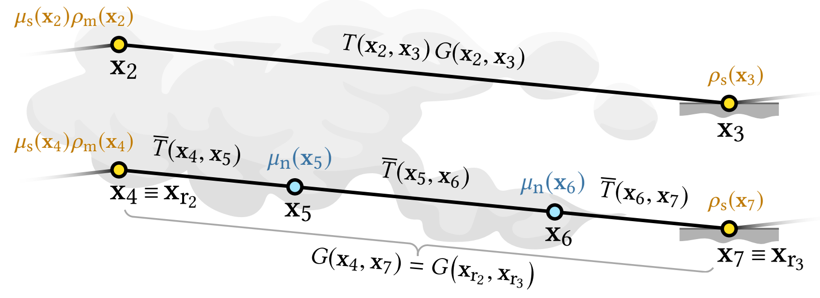


Fig. 4. The classical path integral formulation (top) considers only real scattering and thus has to evaluate the extinction transmittance T between such events. Our formulation (bottom) instead evaluates the combined transmittance $\overline{T}$ as it considers null-scattering events explicitly.

connecting every two consecutive vertices, which does not permit analytic evaluation or sampling in general heterogeneous media.

To address this inconvenience, we derive a path integral expression with the same form as Eq. (12) but starting from the null-scattering VRE (9). In contrast to the formulation of Pauly et al. [2000], our formulation considers both real and null scattering at path vertices, and replaces the extinction transmittance T by the analytically evaluable combined transmittance $\overline{T}$ (10).

Path space and measure. To properly handle the geometry of null scattering, we isolate such events into a *null-scattering volume* $\mathcal{V}_\delta$ which is simply a copy of $\mathcal{V}$. This extends the traditional path space and its corresponding differential measure to

$$\mathcal{P} = \bigcup_{k=1}^{\infty} (\mathcal{A}\cup\mathcal{V}\cup\mathcal{V}_\delta)^{k+1}, \quad d\overline{x} = \prod_{i=0}^{k} dx_i, \quad dx_i = \begin{cases} dA(x_i), & \text{if } x_i \in \mathcal{A}, \\ dV(x_i), & \text{if } x_i \in \mathcal{V}, \\ dV_\delta(x_i), & \text{if } x_i \in \mathcal{V}_\delta. \end{cases} \quad (13)$$

Null-scattering vertices x_i are measured along the line connecting the preceding and succeeding real scattering vertices:

$$dV_\delta(x_i) = d\delta_{x_i^{r-} \leftrightarrow x_i^{r+}}(x_i), \quad (14)$$

where $\delta_{x_i^{r-} \leftrightarrow x_i^{r+}}(x_i)$ is a Dirac measure restricting the integration along the line segment connecting the preceding and succeeding real-scattering vertices x_i^{r-} and x_i^{r+}, respectively. The path length k is the number of segments between consecutive scattering events of any kind. Figure 3 illustrates a path of length 8 in the space $\mathcal{P}$.

Measurement contribution. For each path length k, the measurement

is given by Eq. (10), and

$$G(x, y) = \frac{D(x, \omega_{xy})V(x, y)D(y, \omega_{yx})}{\|x - y\|^2} \quad (16)$$

$$D(x, \omega) = \begin{cases} |n(x) \cdot \omega|, & \text{if } x \in \mathcal{A}, \\ 1, & \text{if } x \in \mathcal{V} \end{cases} \quad (17)$$

$$L_e(x, \omega) = \begin{cases} L_e(x, \omega_{xy}), & \text{if } x \in \mathcal{A}, \\ \mu_a(x)L_e(x, \omega_{xy}), & \text{if } x \in \mathcal{V} \end{cases} \quad (18)$$

$$\rho(\omega, x, \omega') = \begin{cases} \rho_s(\omega, x, -\omega'), & \text{if } y \in \mathcal{A}, \\ \mu_s(x)\rho_m(\omega, x, \omega'), & \text{if } x \in \mathcal{V}, \\ \mu_n(x)H(\omega \cdot \omega'), & \text{if } x \in \mathcal{V}_\delta, \end{cases} \quad (19)$$

where $V(x, y)$ is the binary visibility function between x and y. In contrast to prior definitions, our generalized scattering term ρ explicitly considers null scattering, where H is the heaviside function which enforces the ordering of the null vertices.[1]

Note that the geometry term $G(x, y)$ is evaluated only between real-scattering events $x, y \in \mathcal{A} \cup \mathcal{V}$. Null-scattering vertices are constrained to lie on the polyline connecting real-scattering events which is effectively a path-space manifold. This is similar to the manifolds studied by Jakob and Marschner [2012] and the changes in path density through chains of specular (i.e. delta) surface reflection and refraction between two scattering events. In our case, the geometry term through a null-scattering chain has a simple form.

3.2 Discussion

By explicitly accounting for null scattering, the problematic extinc-

4 NULL-SCATTERING PATH INTEGRAL DERIVATION

In this section we derive our path integral formulation (12) from the null-scattering VRE (9). We first expand the recursions in the VRE, followed by a change of variables in the resulting high-dimensional integrals, which we ultimately merge into one path-space integral. These are the same general steps done in derivations based on the real-scattering VRE (3) [Pauly et al. 2000; Jakob 2013]. However, in our case the added null-scattering recursion in Eq. (9) increases the complexity of the expansion, and the resulting null-scattering integrals require an appropriate change of variables. Readers not interested in these technical details may skip over to Section 5, where we discuss the practical applications of our formulation.

We begin by writing the null-scattering VRE in a compact form:

$$L(x, \omega) = \int_0^z \overline{T}(x, y)L_o(y, \omega)\,dy + \overline{T}(x, z)L_o(z, \omega) \quad (20)$$

$$L_o(x, \omega) = L_e(x,\omega) + \int_{S^2} \rho(\omega, x, \omega')L(x, \omega')D(x, \omega')\,d\omega' + \mu_n(x)L(x, \omega),$$

where $\overline{\mu}_t$ from Eqs. (8) and (9) cancels out and where we use the notation from Eqs. (17) to (19) to express the contributions from medium points y and surface points $z = x - z\omega$ in $L(x, \omega)$ using a common *outgoing radiance* term L_o. We do not consider null scattering (i.e. transparency) at surfaces, thus $\mu_n(x) = 0$ for $x \in \mathcal{A}$.

4.1 Operator formulation

As in prior formulations [Pauly et al. 2000; Jakob 2013], we will express the pixel measurement I as a path integral by recursively expanding the radiance L (20) in Eq. (11). To express this expansion succinctly, we make use of linear operators [Veach 1997; Arvo 1995]. Substituting L into L_o replaces the last two terms of L_o by four new terms, from which we extract four operators:

$$(R_m h)(x, \omega) = \int_{S^2}\int_0^z \rho(\omega, x, \omega')\overline{T}(x, y)D(x, \omega')h(y, \omega')\,dy\,d\omega' \quad (21)$$

$$(R_s h)(x, \omega) = \int_{S^2} \rho(\omega, x, \omega')\overline{T}(x, z)D(x, \omega')h(z, \omega')\,d\omega' \quad (22)$$

$$(N_m h)(x, \omega) = \mu_n(x) \int_0^z \overline{T}(x, y)h(y, \omega)\,dy \quad (23)$$

$$(N_s h)(x, \omega) = \mu_n(x)\overline{T}(x, z)h(z, \omega). \quad (24)$$

We then define the real- and null-scattering operators, respectively

Pixel measurement. Our next step is to write the pixel measurement (11) in non-recursive operator form, as a sum of nested integrals. To that end, including W_e temporarily in the definition of the scattering function ρ (19) allows us to treat the spherical integral in Eq. (11) as a real-scattering event, such that expanding L in Eq. (11) using Eq. (20) and then expressing L_o using Eq. (26) yields

$$I = \sum_{k=0}^{\infty} \sum_{S_k \in \{R,N\}^k} \int_{\mathcal{A}} (RS_k L_e)(x, \cdot)\,dx = \sum_{P \in \Omega_P} \int_{\mathcal{A}} (PL_e)(x, \cdot)\,dx, \quad (27)$$

where the operator under the integral is evaluated with a dummy direction. The set Ω_P includes all *path operators* of the form $P = RS_k$, where the "camera" real-scattering event is followed (in direction opposite of the light flow) by k scene scattering events of any type.

4.2 Scattering chain decomposition

To express the pixel measurement (27) as integration over a product (path) space of surface area and volume, we need to perform an appropriate change of variables in every path operator $P = RS_k \in \Omega_P$. For every k there are 2^k possible operators P, each corresponding to a different k-sequence of real- and null-scattering events. To handle this combinatorial explosion, we make the key observation that every such operator can be written as a sequence of scattering chains, each starting with a real-scattering event:

$$P = R\overbrace{N\cdots N}^{n_1 \text{ times}}R\overbrace{N\cdots N}^{n_2 \text{ times}}\cdots R\overbrace{N\cdots N}^{n_r \text{ times}} = RN^{n_1}RN^{n_2}\cdots RN^{n_r}, \quad (28)$$

where the number of real-scattering events is r and the total number of null-scattering events is $\sum_{i=1}^{r} n_i = k - r$, with $n_i \geqslant 0$. It thus suffices to find the change of variables for a general chain RN^n, which can then be applied to each chain in every path operator P.

In the absence of null scattering, i.e. when $n = 0$, the chain operator RN^n simplifies to R; we will address this special case in Section 4.3 below. When $n > 0$, assuming no null scattering at surfaces, we can expand RN^n using Eq. (25):

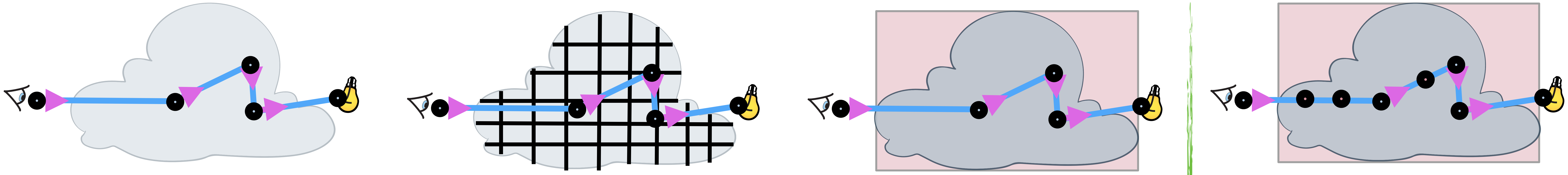
$$(RN^n h)(x, \omega) = (R_m N^n h)(x, \omega) + (R_s N^n h)(x, \omega) \quad (29)$$

$$= (R_m(N_m + N_s)^n h)(x, \omega) = (R_m(N_m + N_s)^{n-1}(N_m + N_s)h)(x, \omega)$$

$$= (R_m N_m^n h)(x, \omega) + (R_m N_m^{n-1} N_s h)(x, \omega). \quad (30)$$

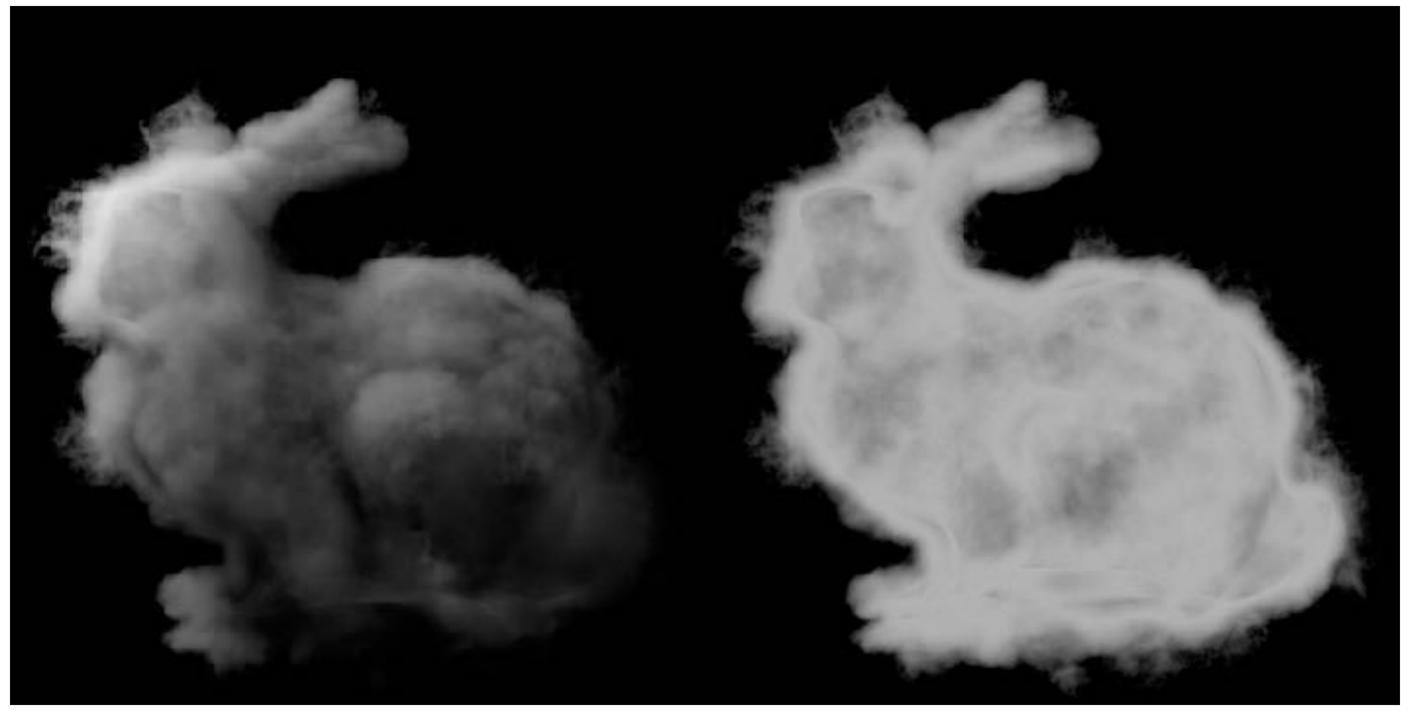
With no null scattering at surfaces, only the medium contribution

Our Approach

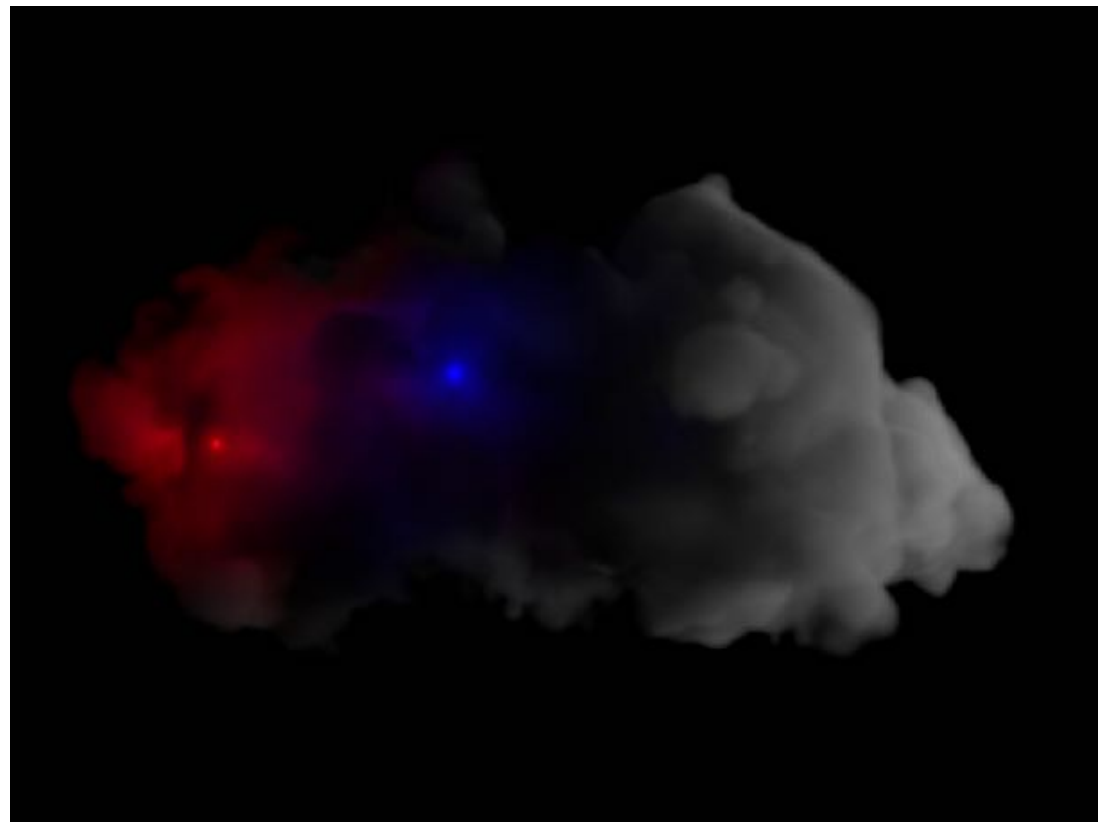


	analytic	approximations	null collision	ours
Any media?	✗	✓	✓	✓
Unbiased ?	✓	✗	✓	✓
Known PDF?	✓	✓	✗	✓

Results:

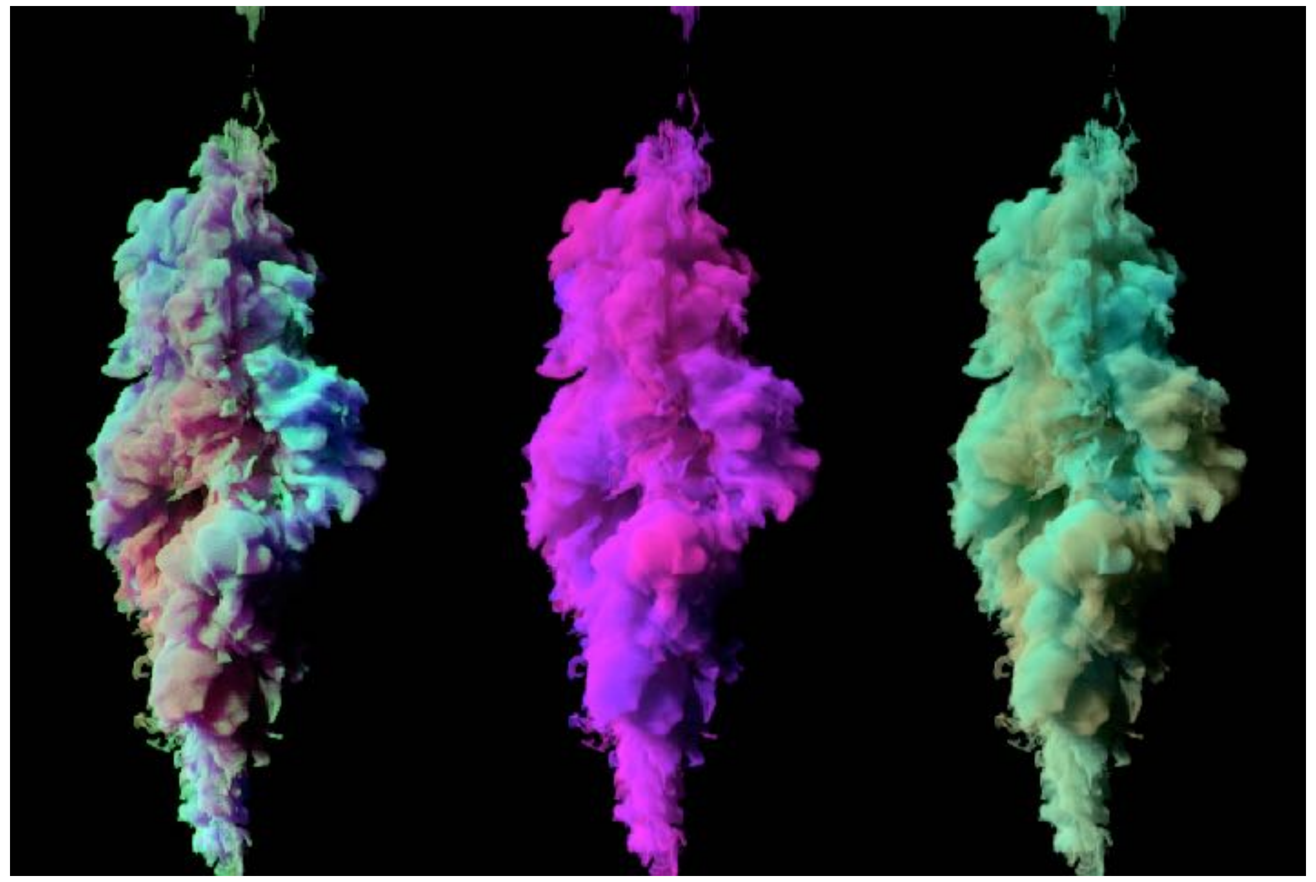


unidirectional + NEE



Equiangular + NEE

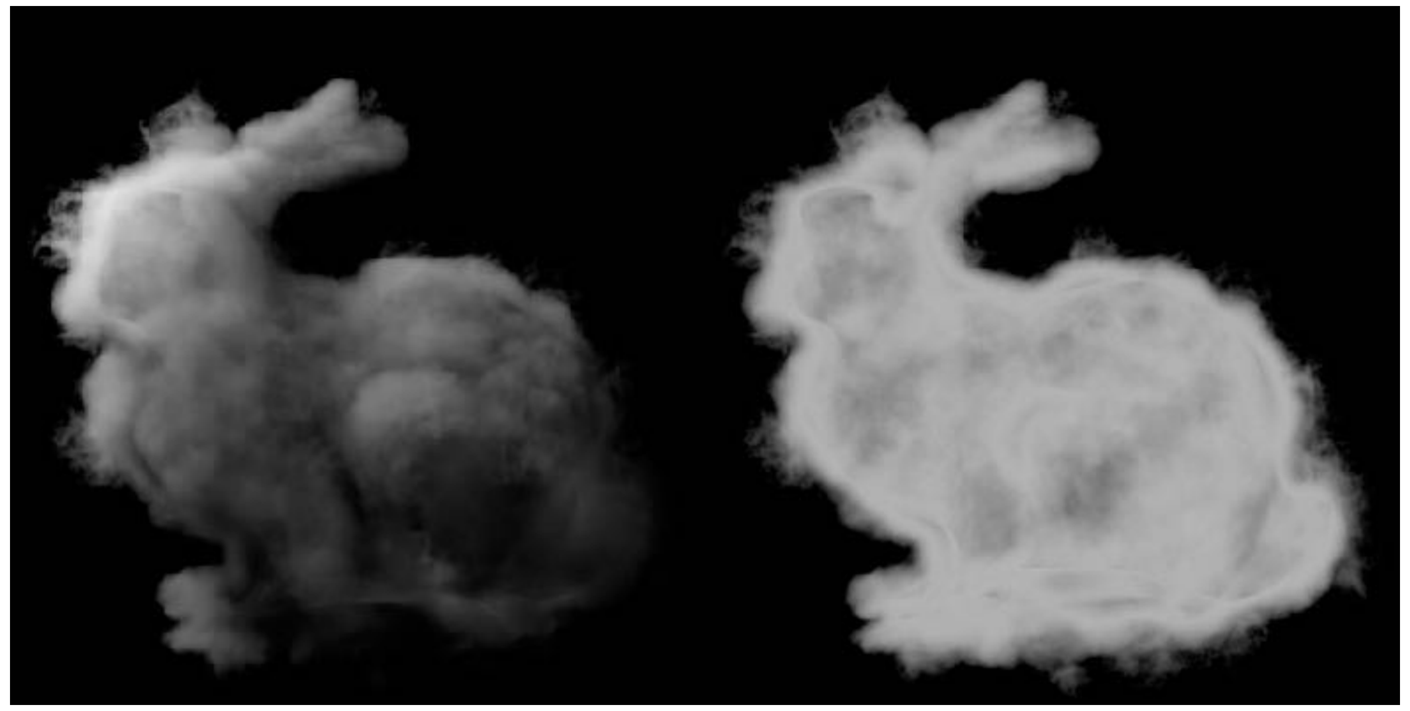
Spectral MIS



Spectral MIS

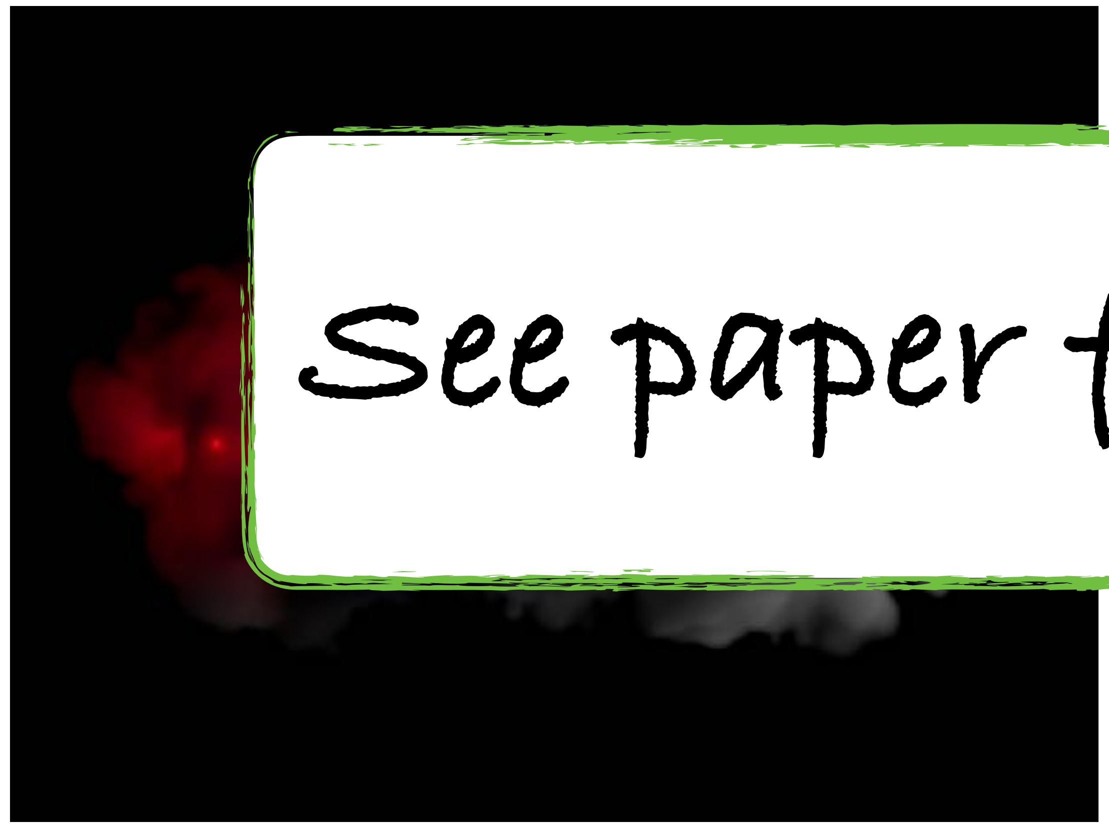
Bidirectional

Results:



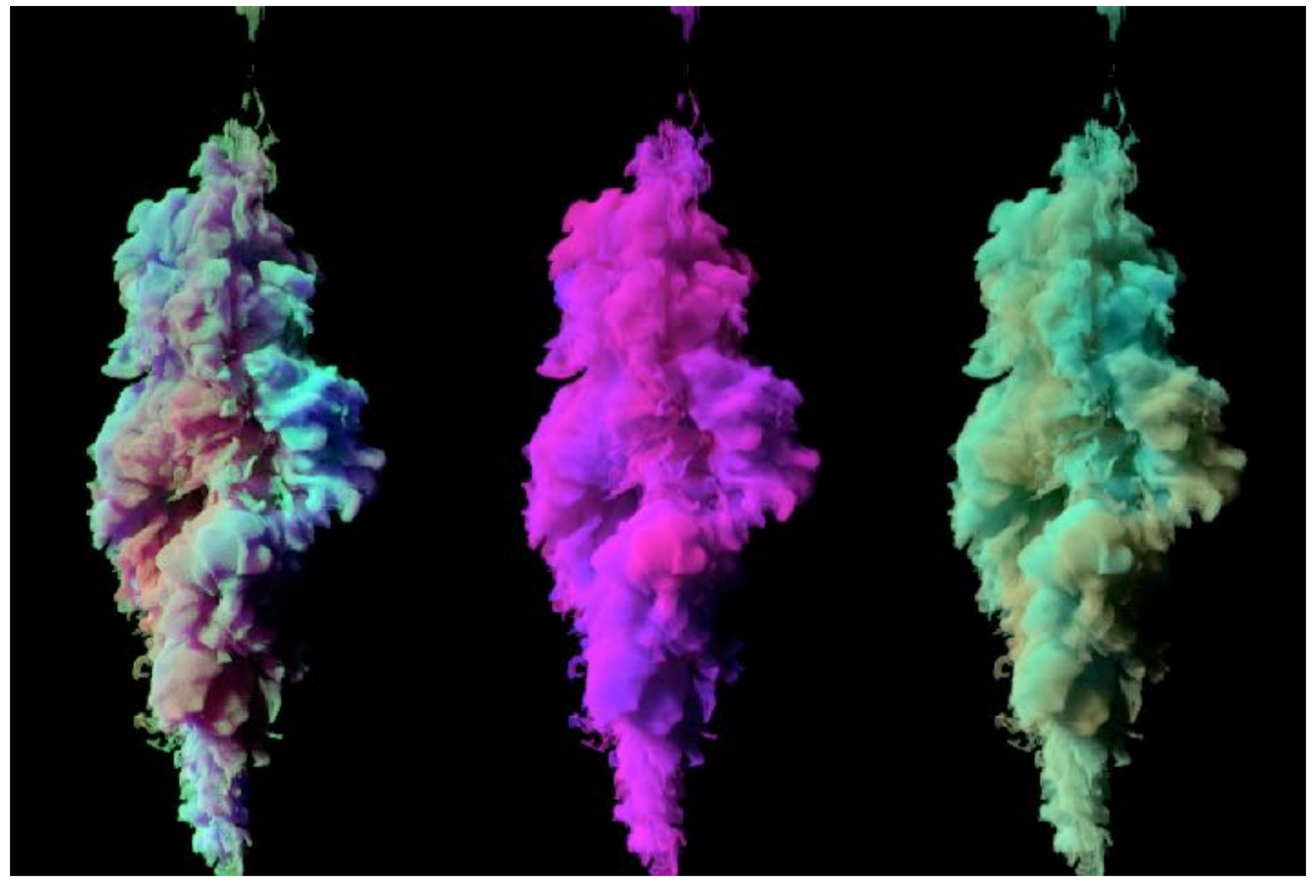
unidirectional + NEE

S

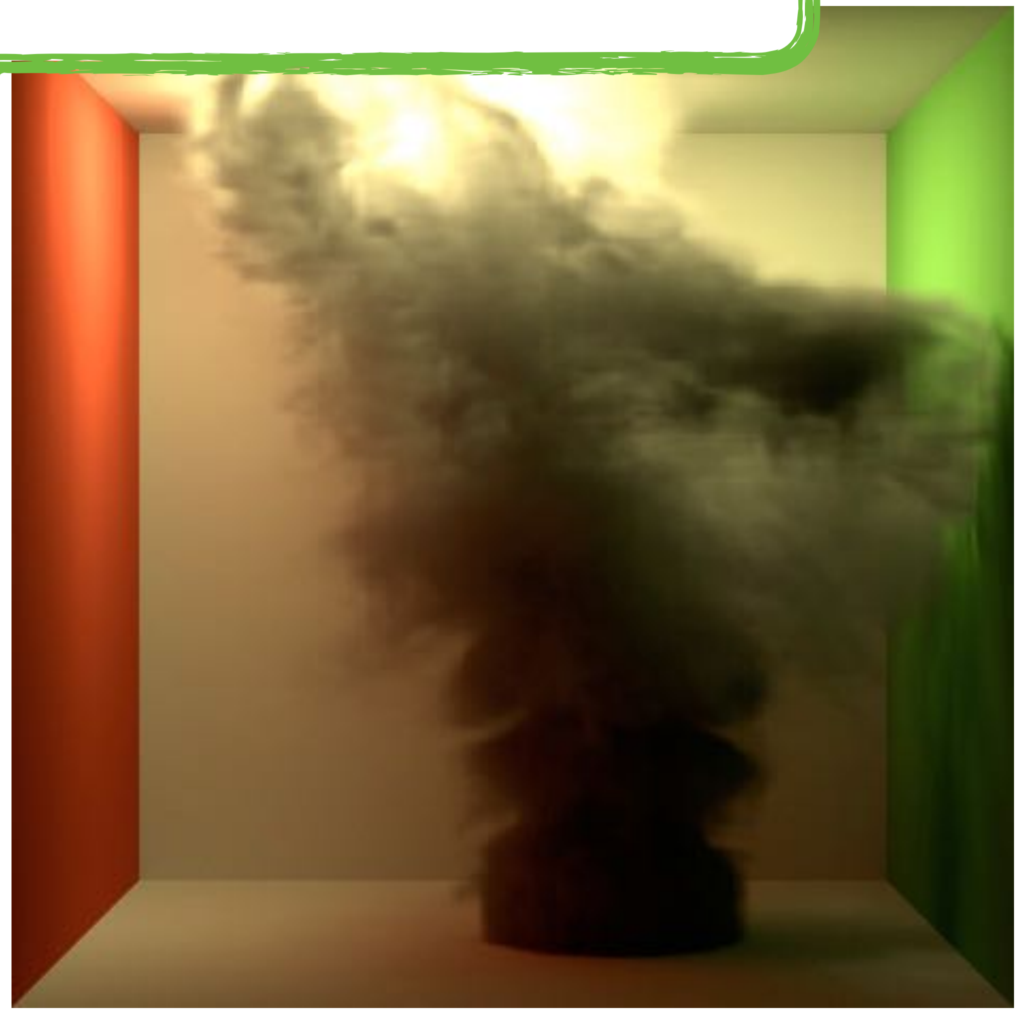


Equiangular + NEE

See paper for more results!



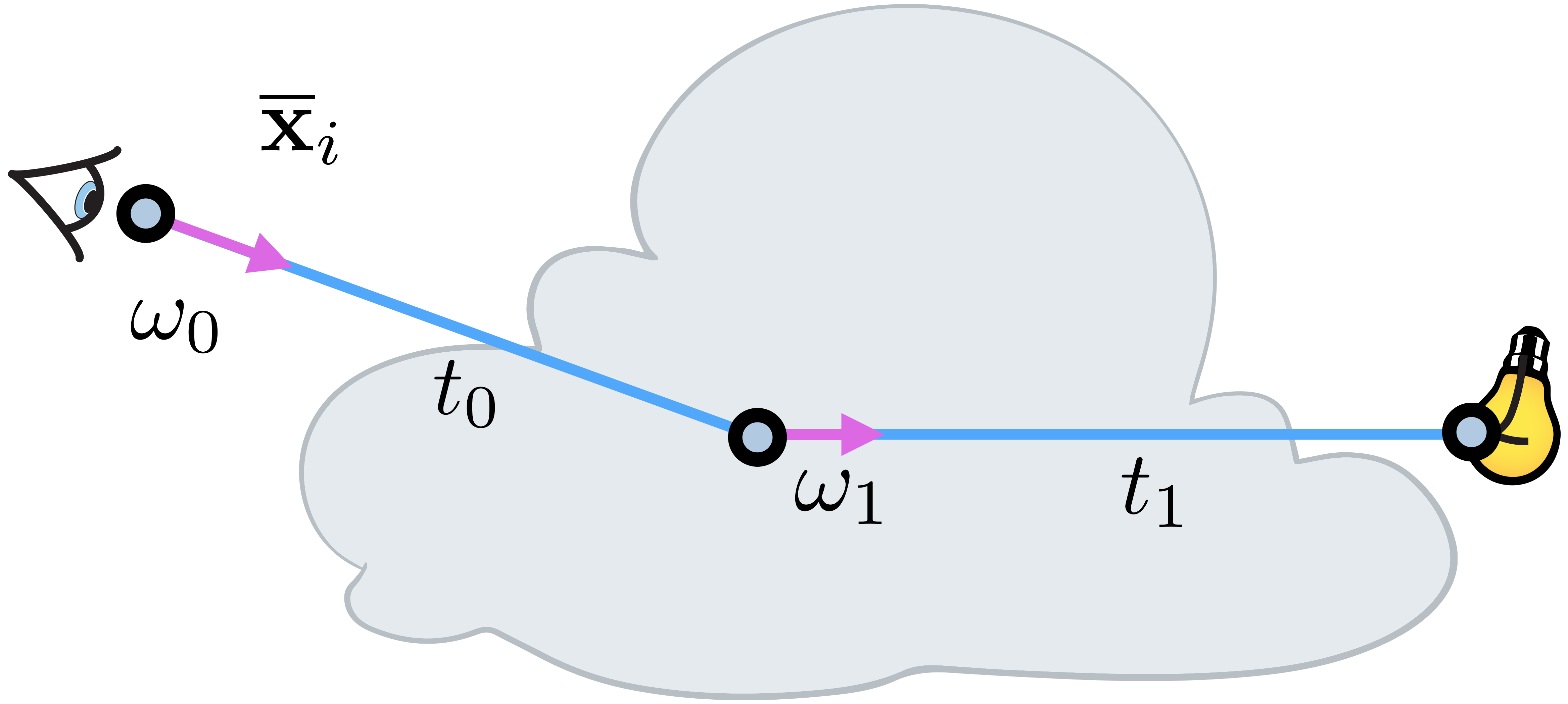
Spectral MIS



Bidirectional

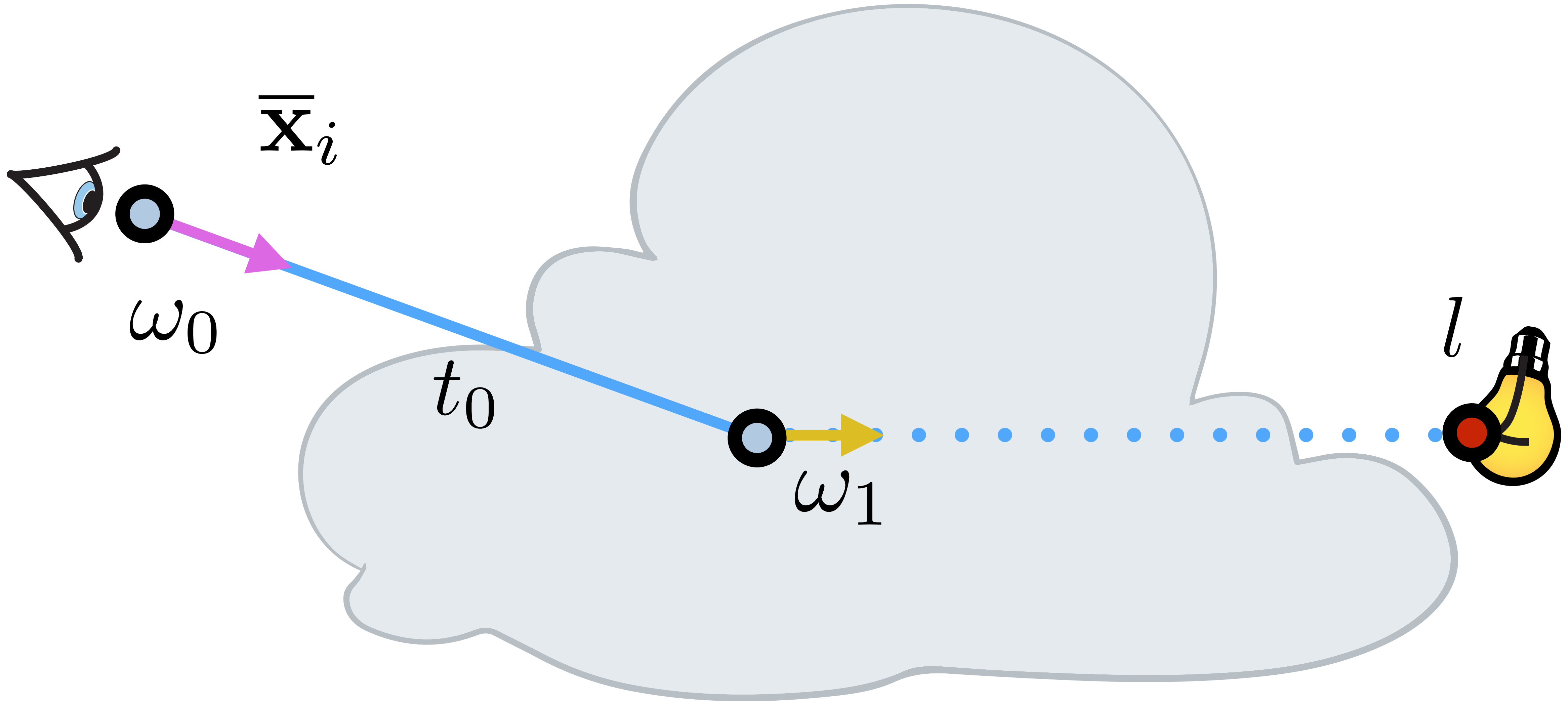
Spatially Varying Media

unidirectional



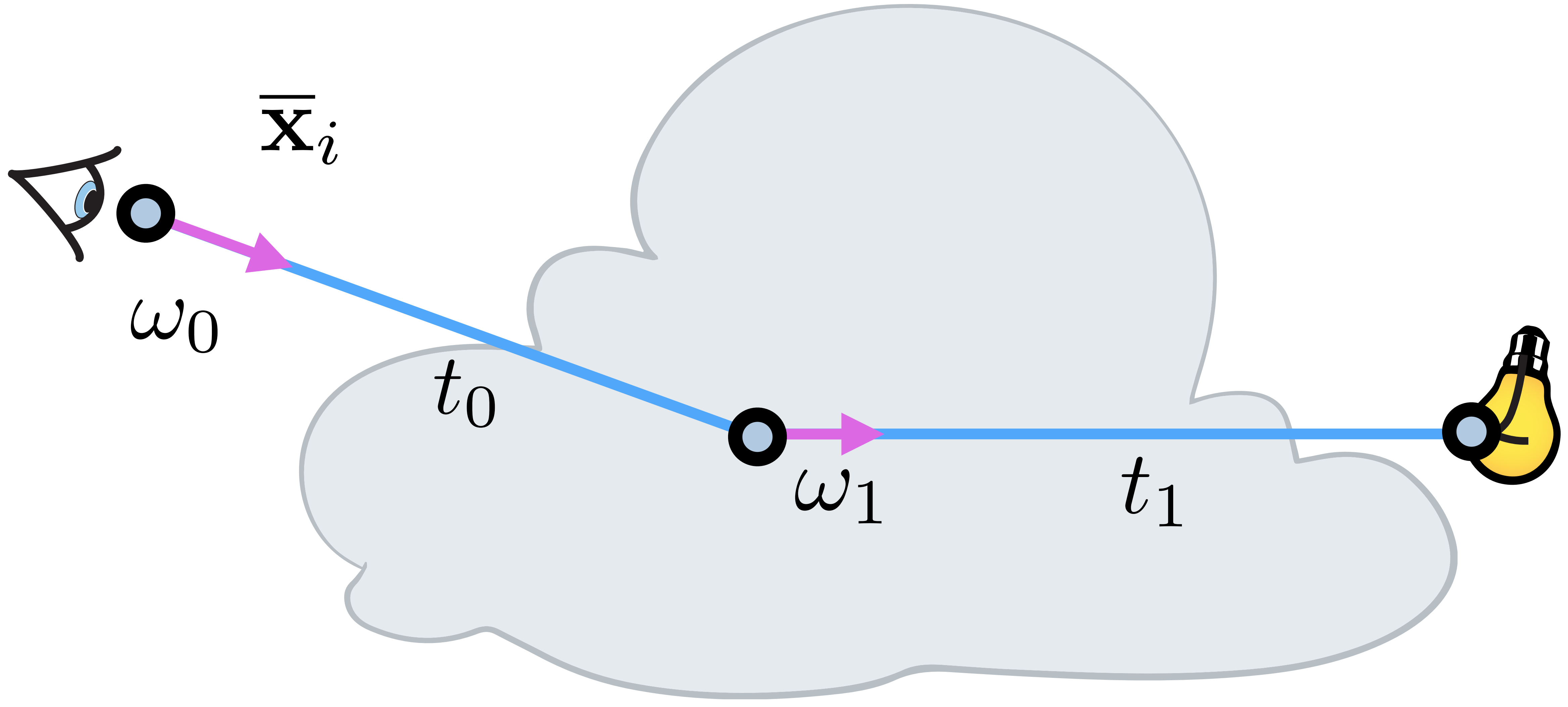
ratio next event (NEE)

[Novak et al. 2014]



Spatially Varying Media

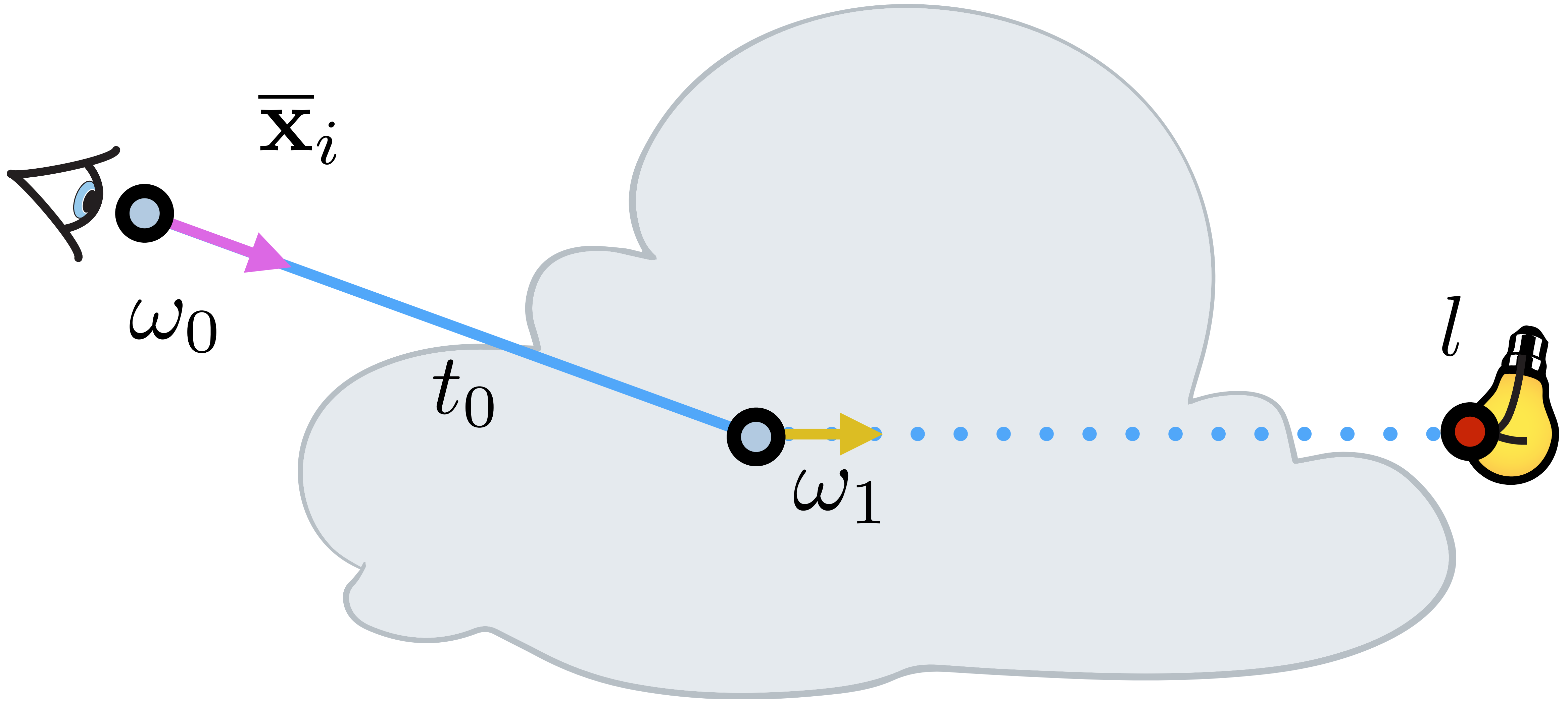
unidirectional



$$p(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(\omega_1)p(t_1)$$

ratio next event (NEE)

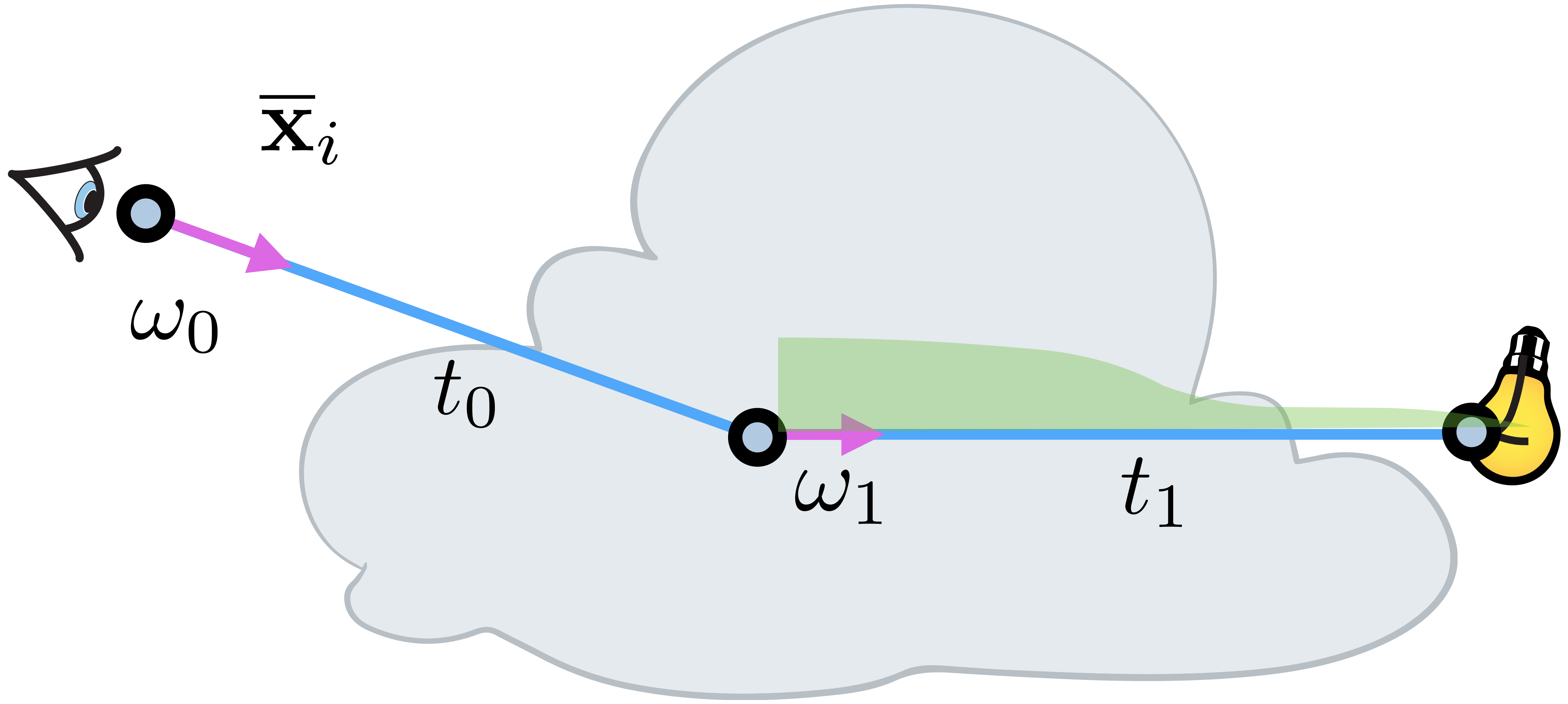
[Novak et al. 2014]



$$p(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(\omega_1)$$

Spatially Varying Media

unidirectional



$$p(\overline{\mathbf{x}}_i) = p(\omega_0)\boxed{p(t_0)}p(\omega_1)\boxed{p(t_1)}$$

ratio next event (NEE)

[Novak et al. 2014]

$$p(\overline{\mathbf{x}}_i) = p(\omega_0)\boxed{p(t_0)}p(\omega_1)$$

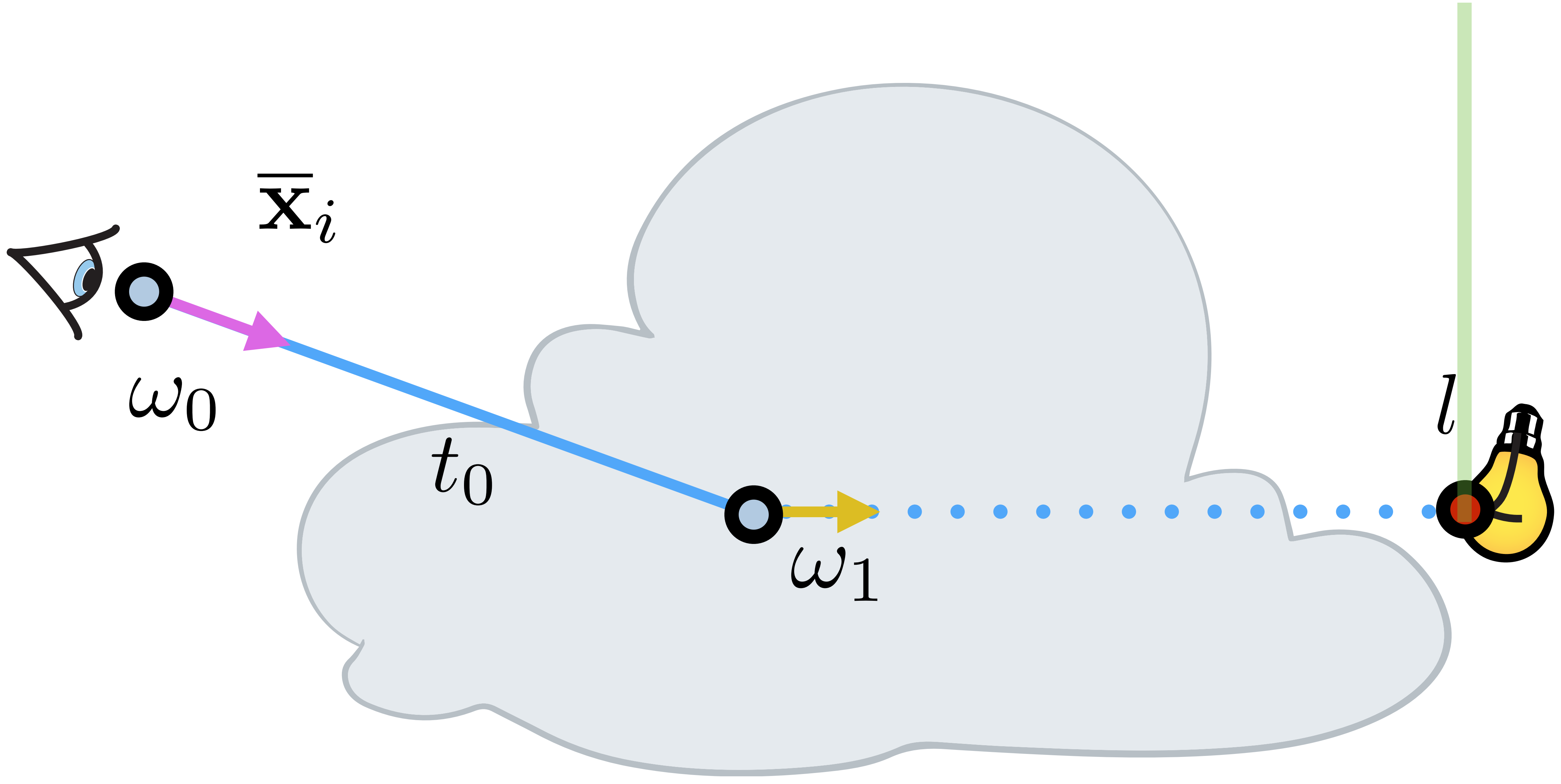
Spatially Varying Media

unidirectional

$$p(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(\omega_1)p(t_1)$$

ratio next event (NEE)

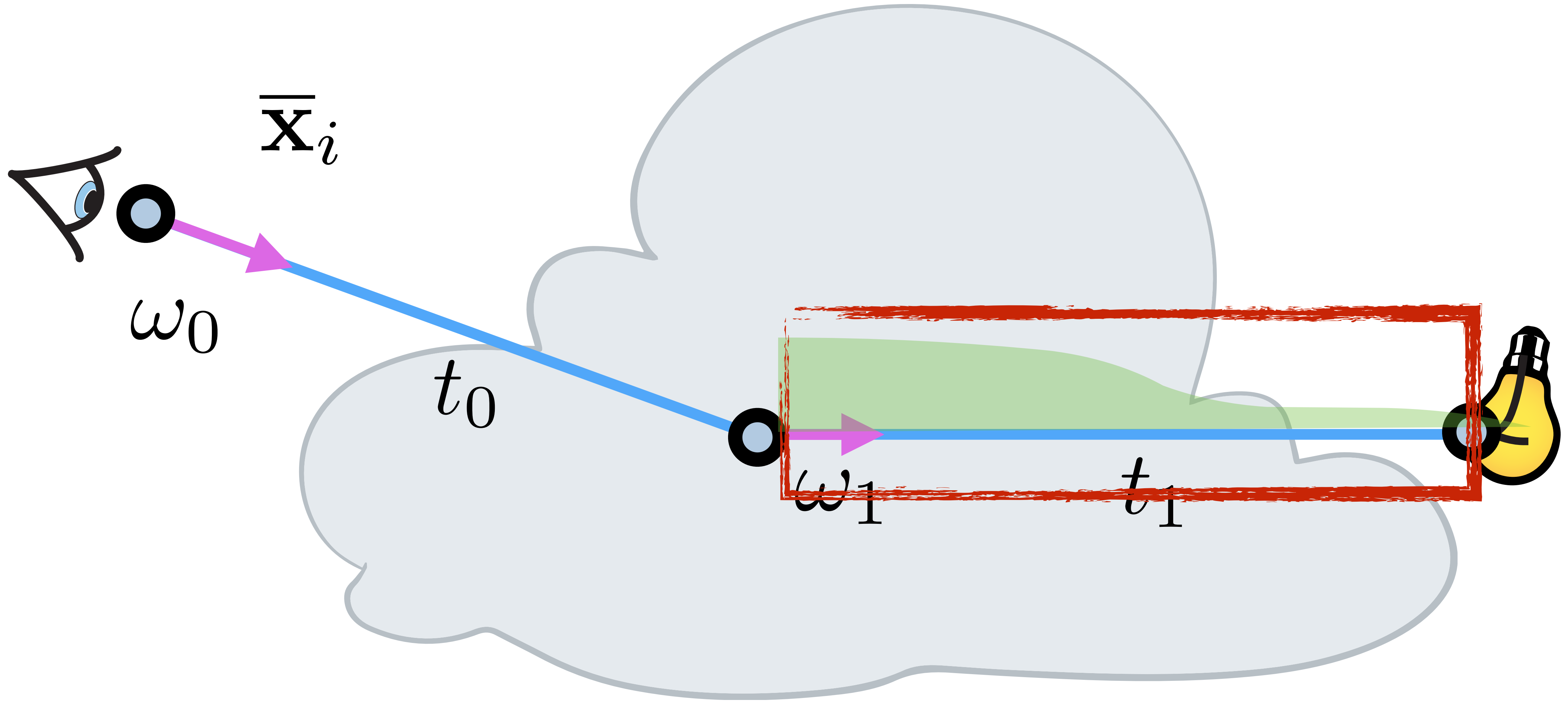
[Novak et al. 2014]



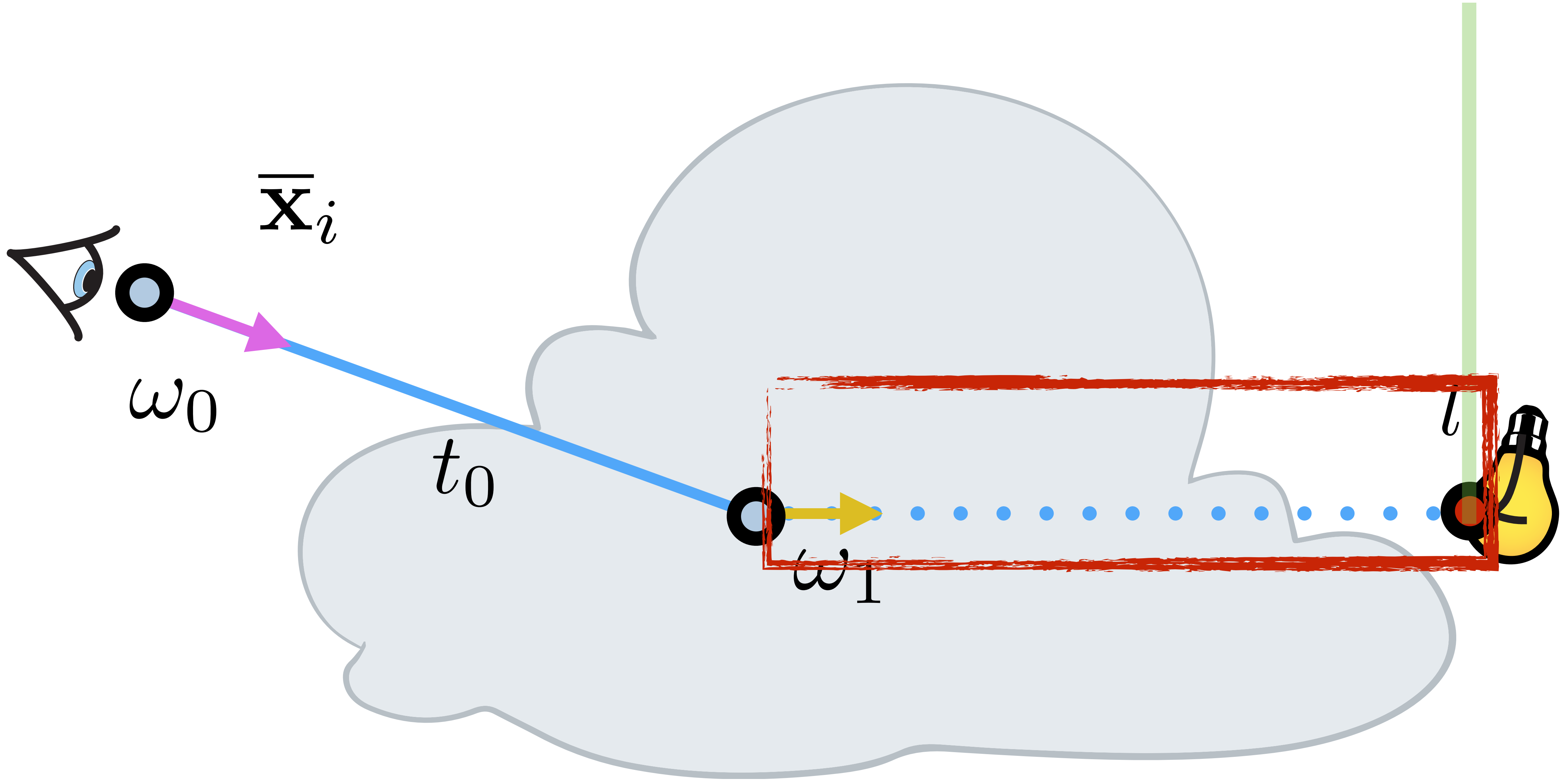
$$p(\overline{\mathbf{x}}_i) = p(\omega_0)p(t_0)p(\omega_1)$$

Spatially Varying Media

unidirectional

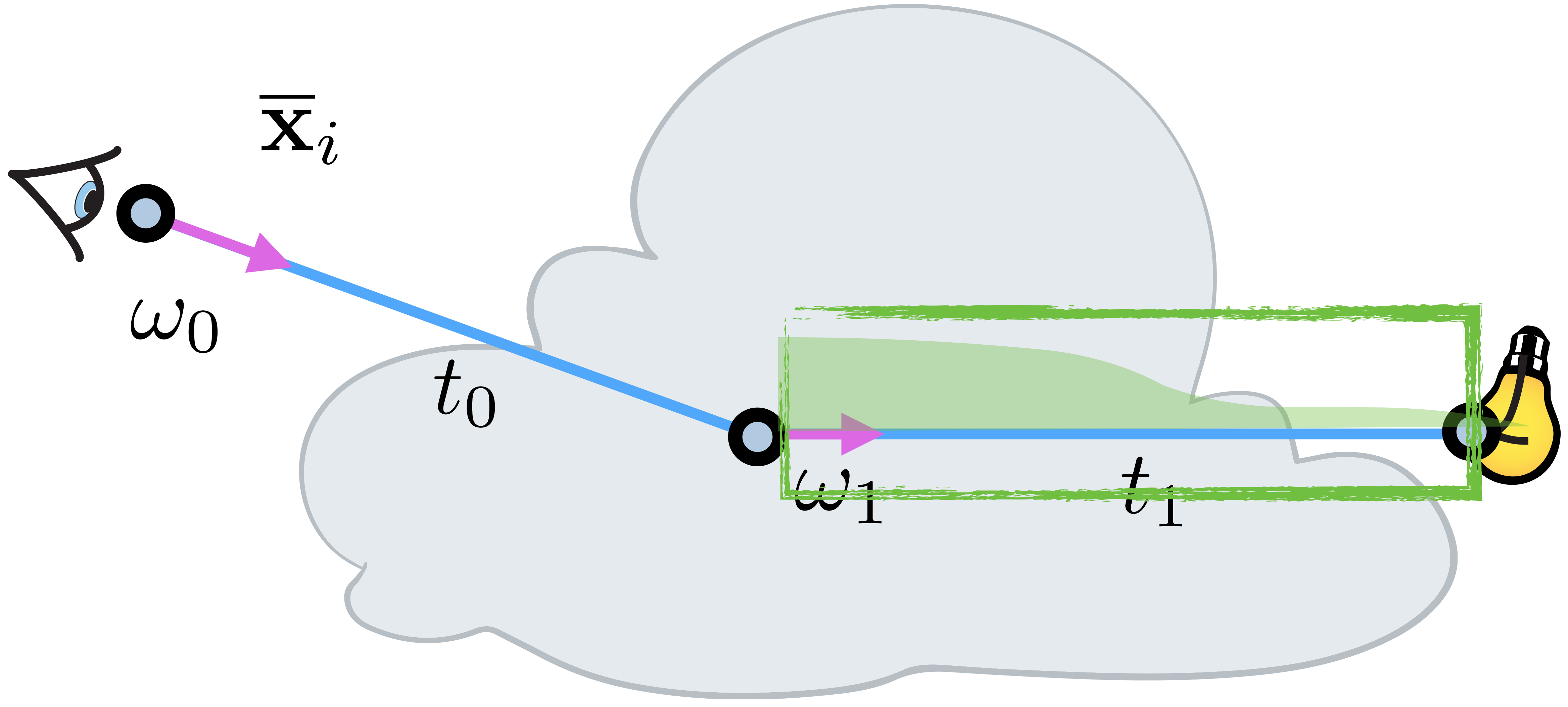


ratio next event (NEE)
[Novak et al. 2014]

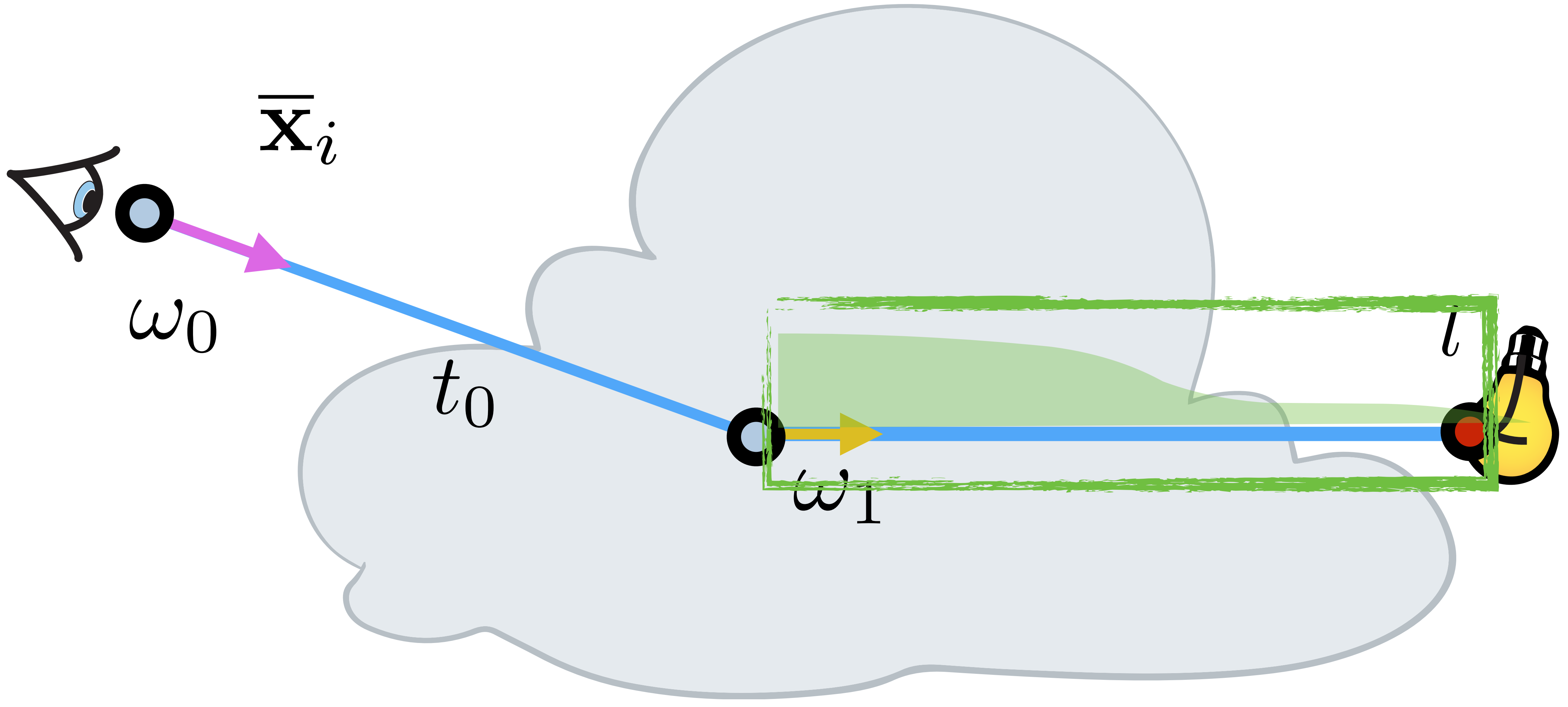


Spatially Varying Media

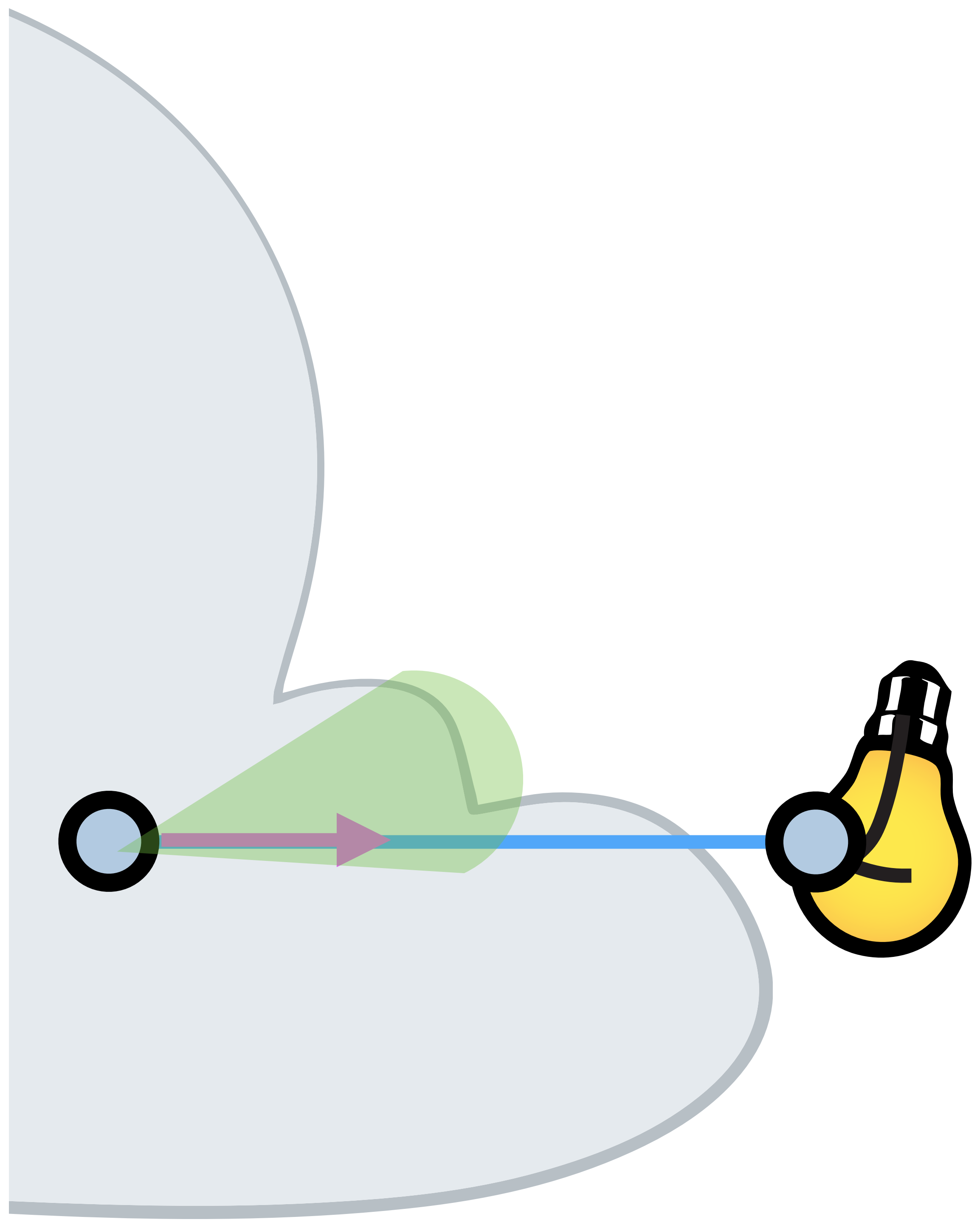
unidirectional



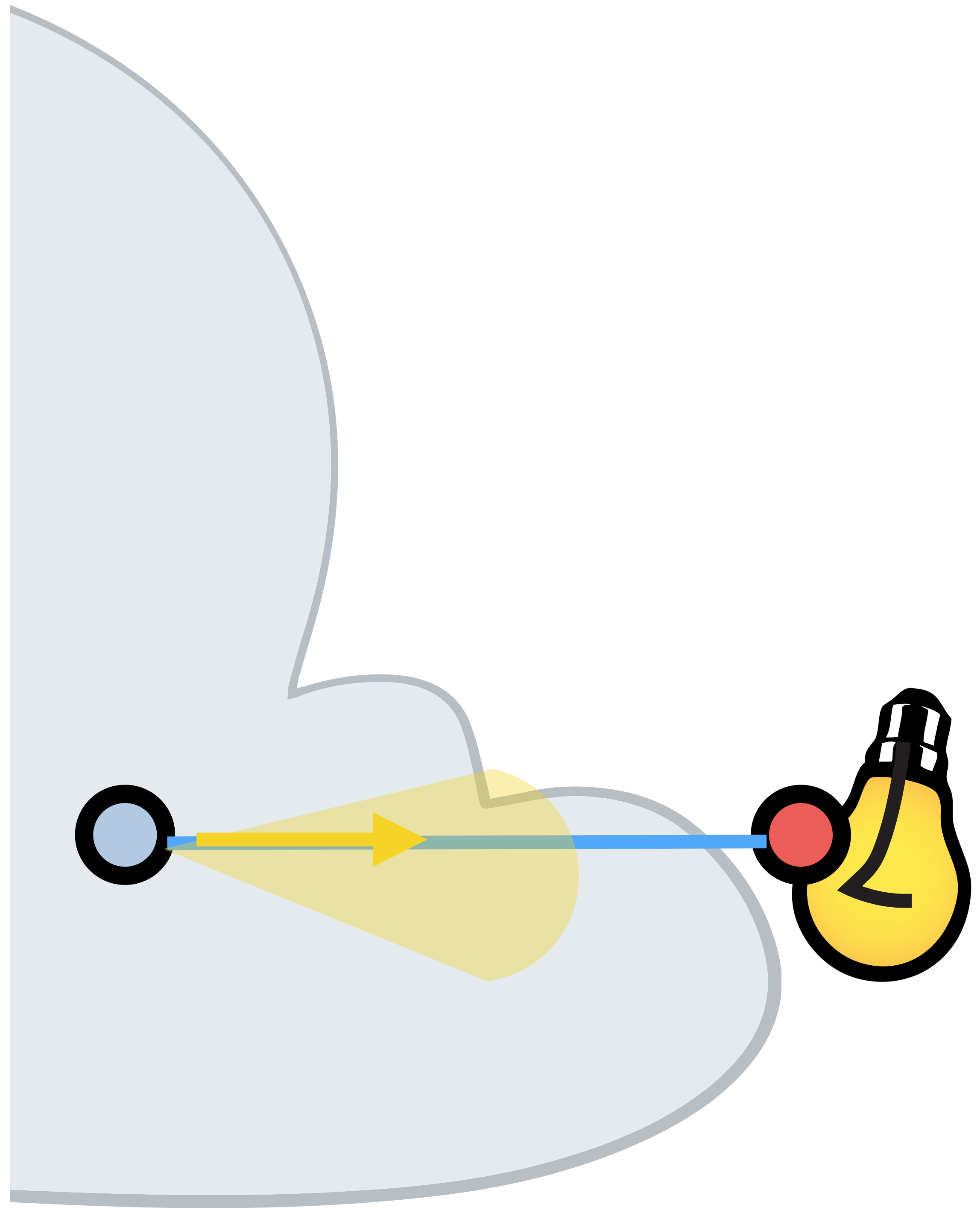
delta next event (NEE)
[Novak et al. 2014]



Directional MIS

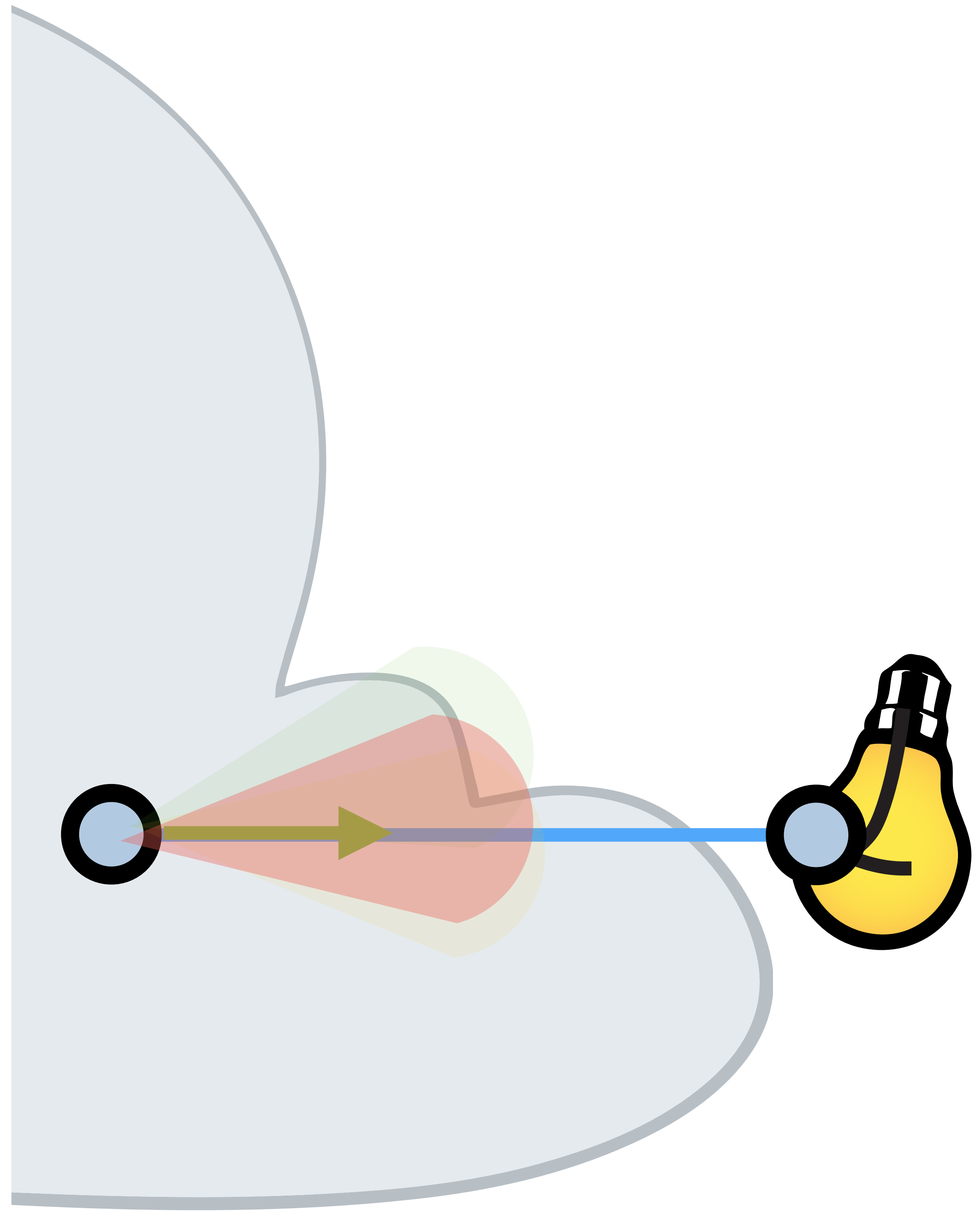


unidirectional



delta next Event (NEE)

[Novak et al. 2014]



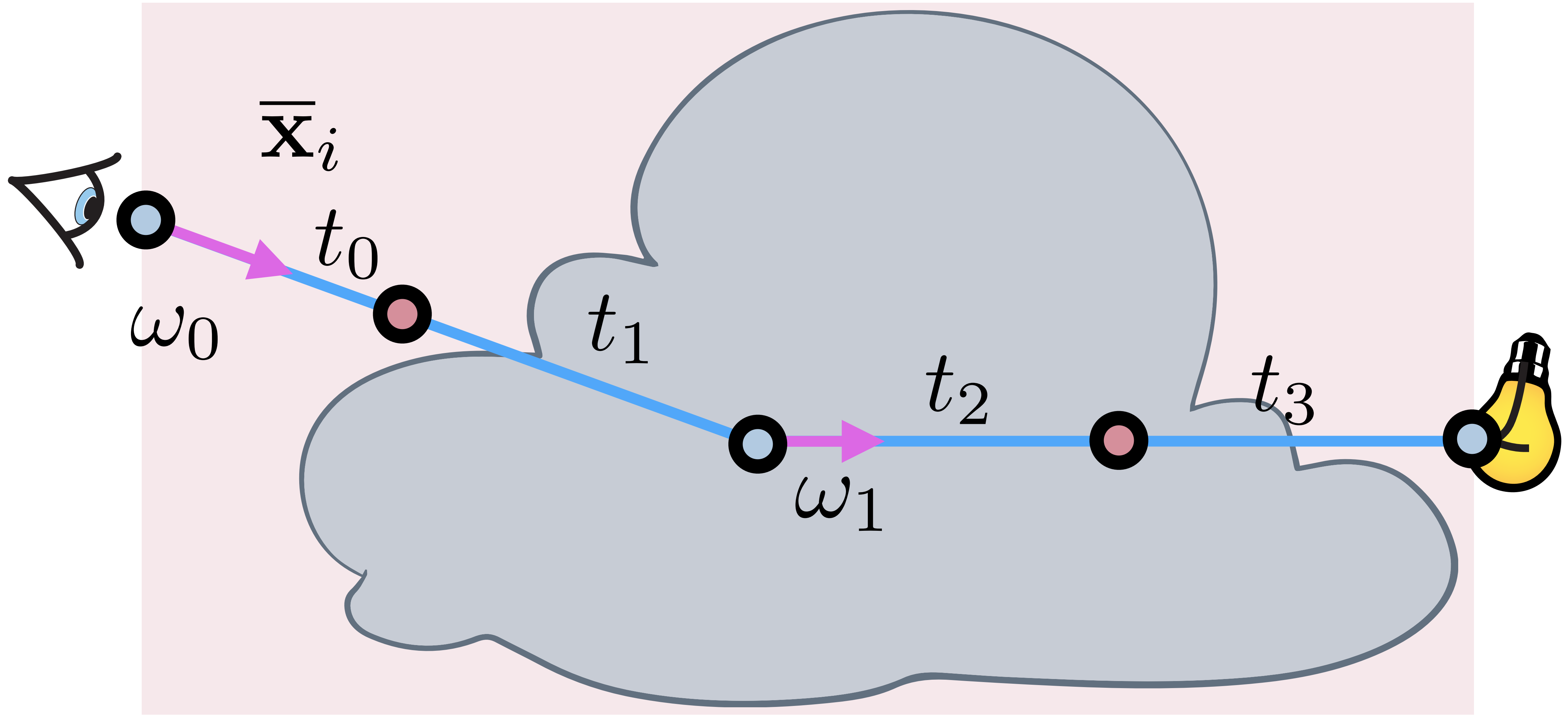
directional MIS

[Novak et al. 2014]

[Kutz et al. 2017]

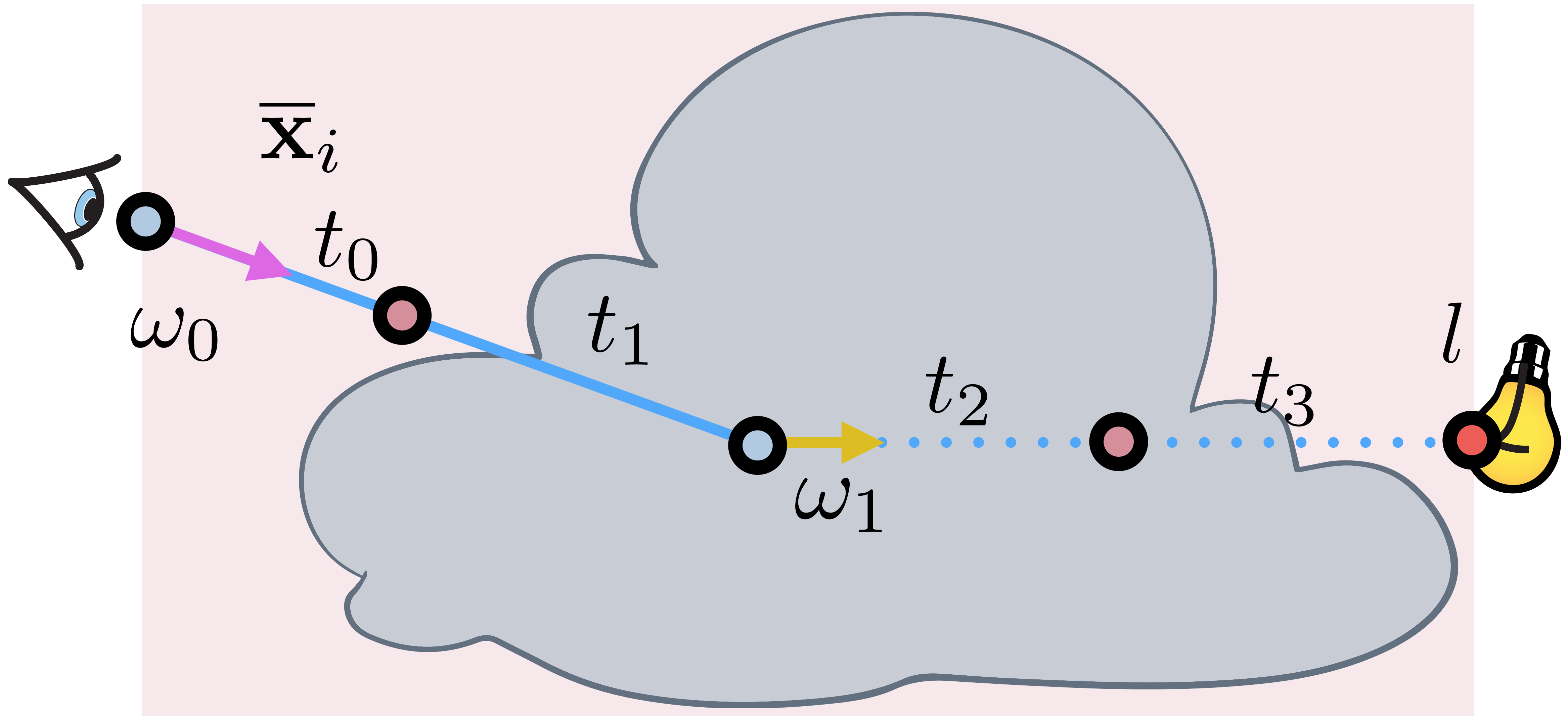
Spatially Varying Media

unidirectional



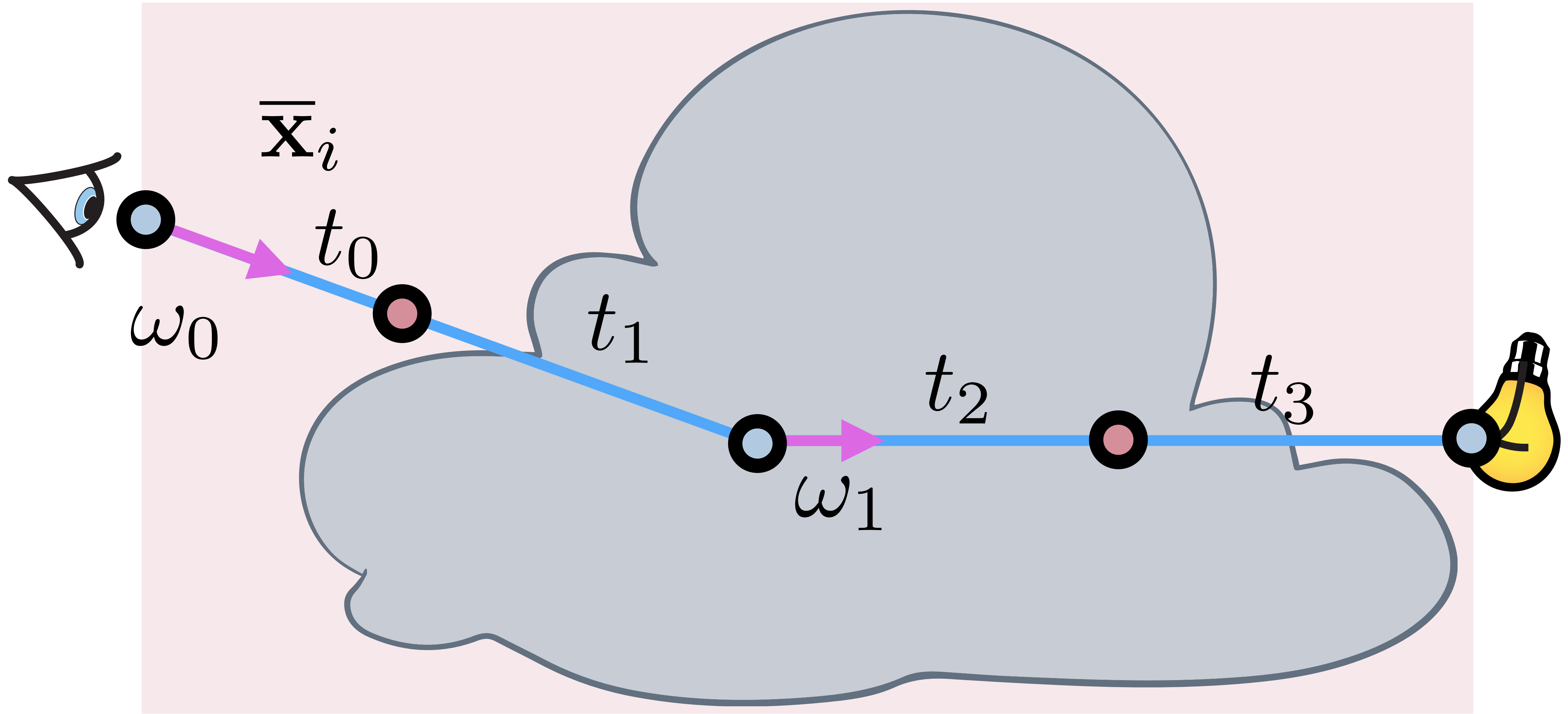
ratio next event (NEE)

[Novak et al. 2014]



Spatially Varying Media

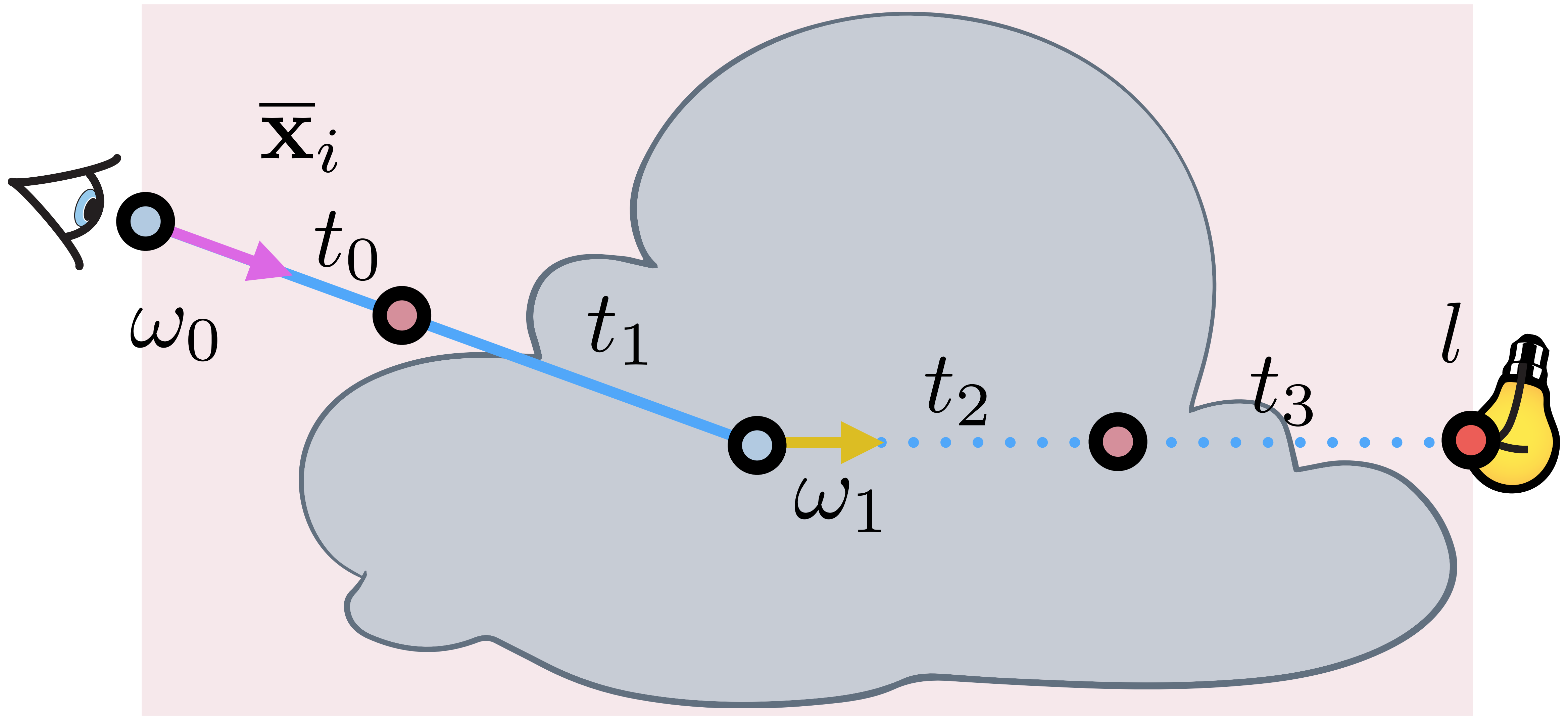
unidirectional



$$p(\overline{\mathbf{x}}) = p(\omega_0)p(t_0)p(t_1)p(\omega_1)p(t_2)p(t_3)$$

ratio next event (NEE)

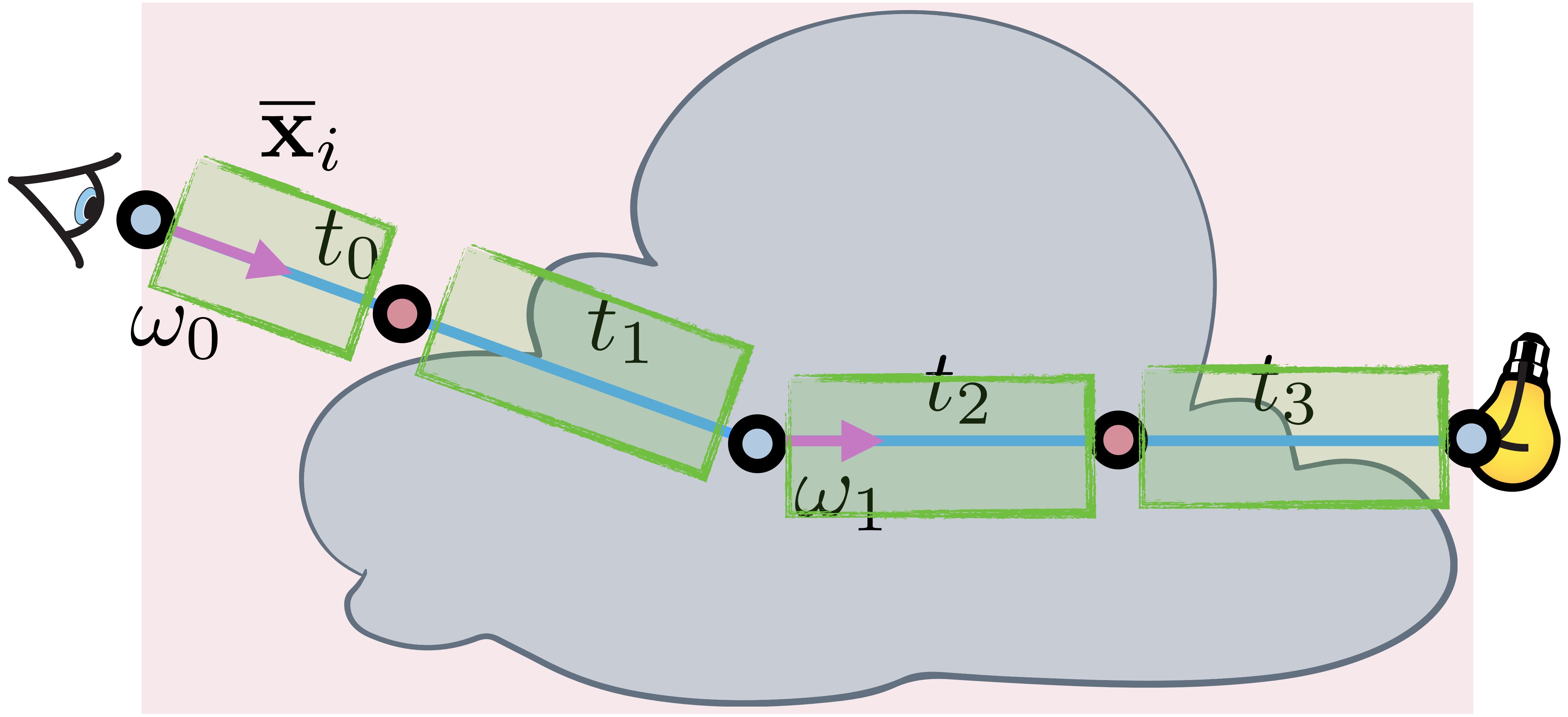
[Novak et al. 2014]



$$p(\overline{\mathbf{x}}) = p(\omega_0)p(t_0)p(t_1)p(\omega_1)p(t_2)p(t_3)$$

Spatially Varying Media

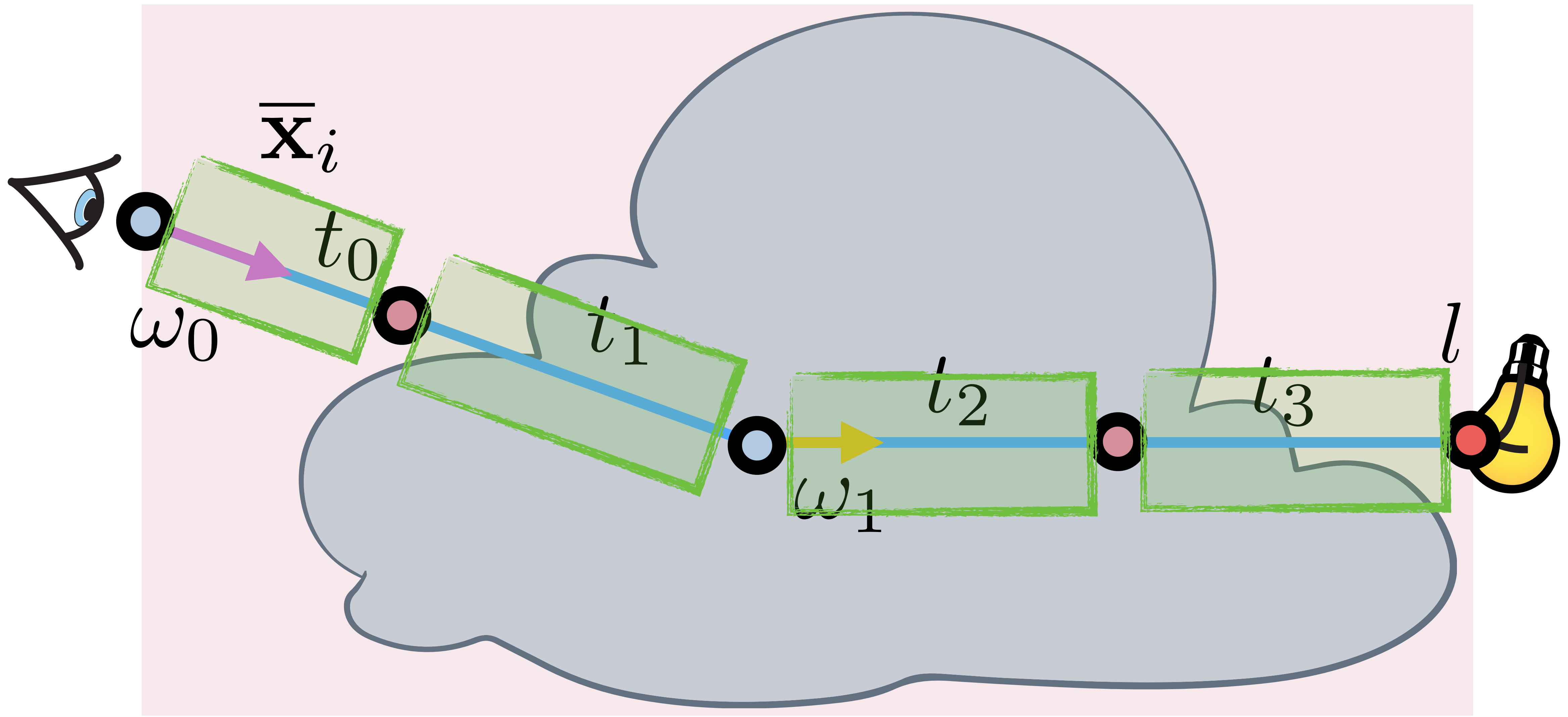
unidirectional



$$p(\overline{\mathbf{x}}) = p(\omega_0)p(t_0)p(t_1)p(\omega_1)p(t_2)p(t_3)$$

ratio next event (NEE)

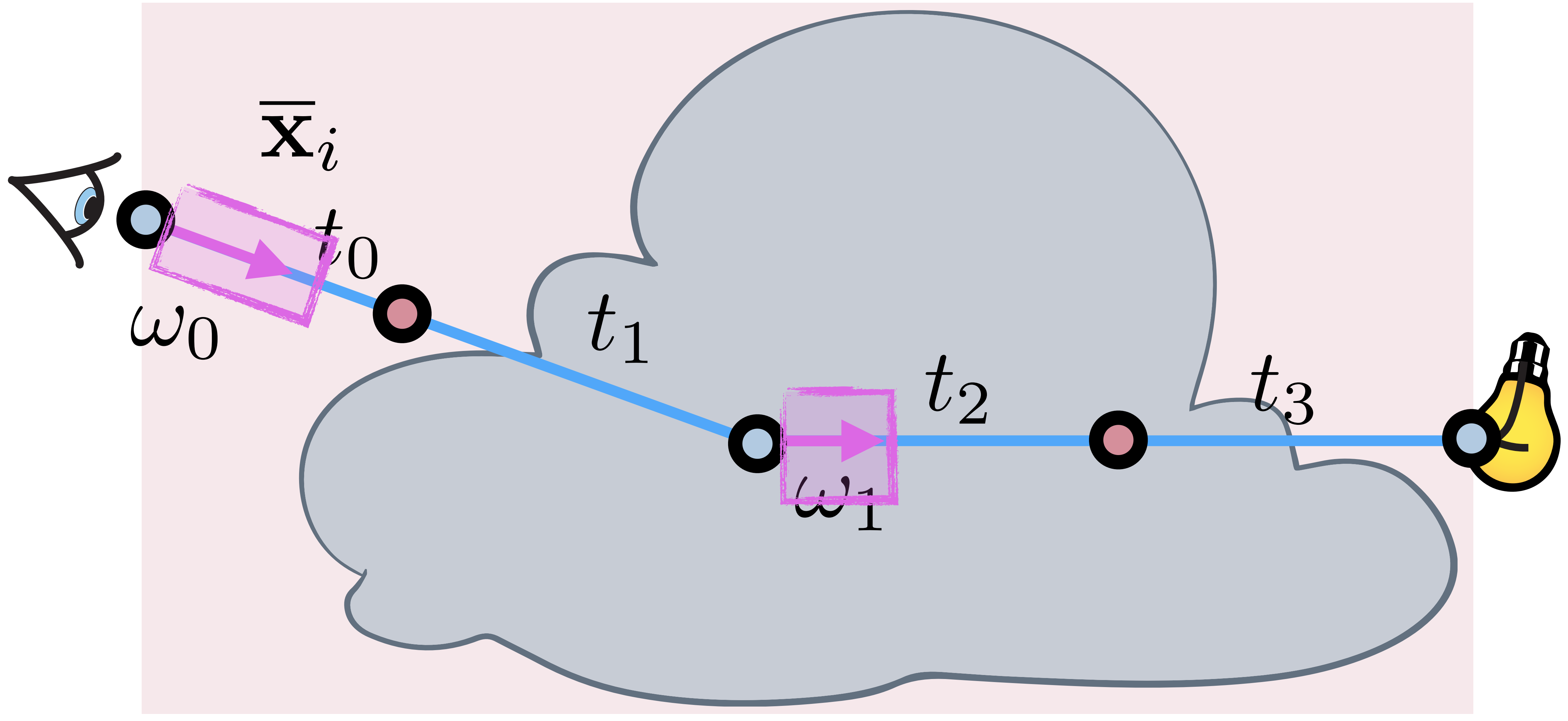
[Novak et al. 2014]



$$p(\overline{\mathbf{x}}) = p(\omega_0)p(t_0)p(t_1)p(\omega_1)p(t_2)p(t_3)$$

Spatially Varying Media

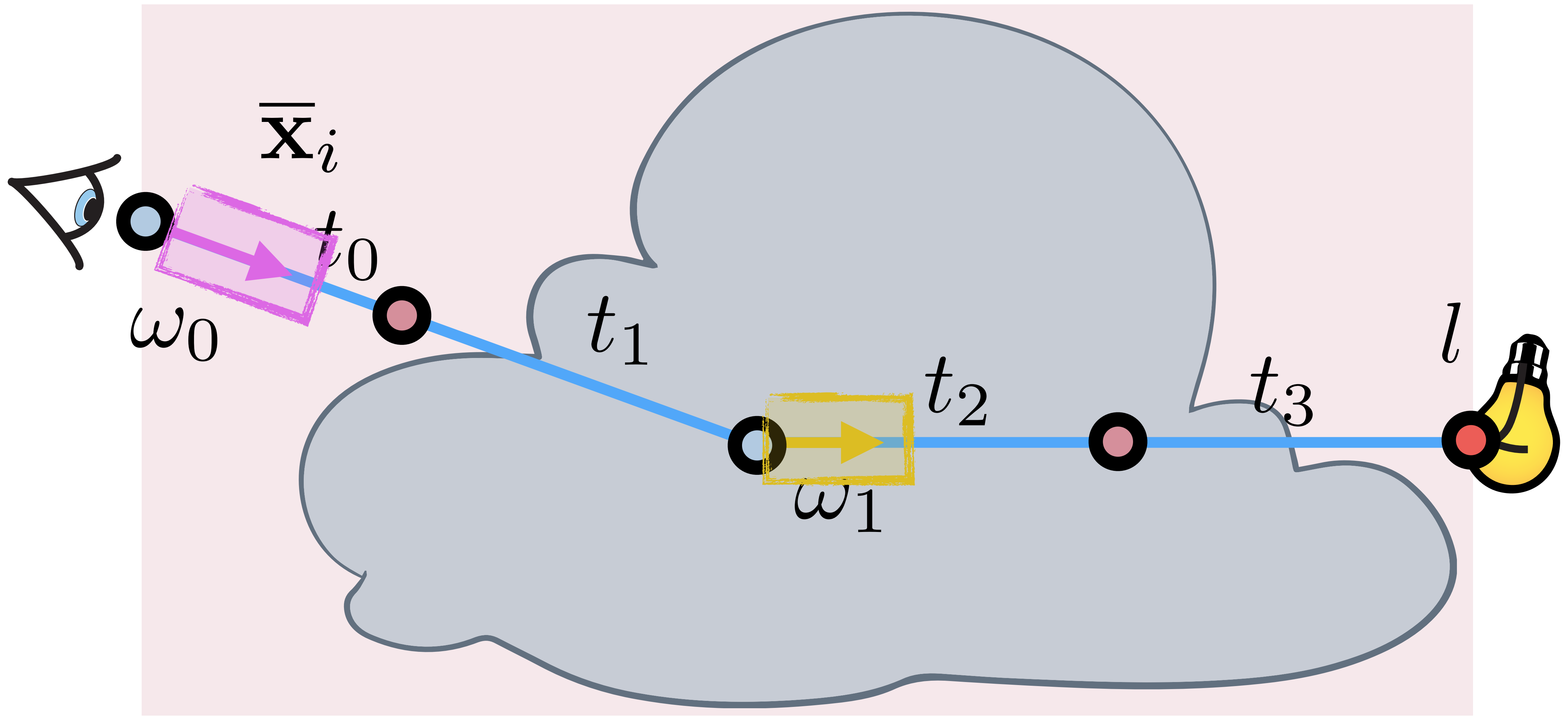
unidirectional



$$p(\overline{\mathbf{x}}) = p(\omega_0)p(t_0)p(t_1)p(\omega_1)p(t_2)p(t_3)$$

ratio next event (NEE)

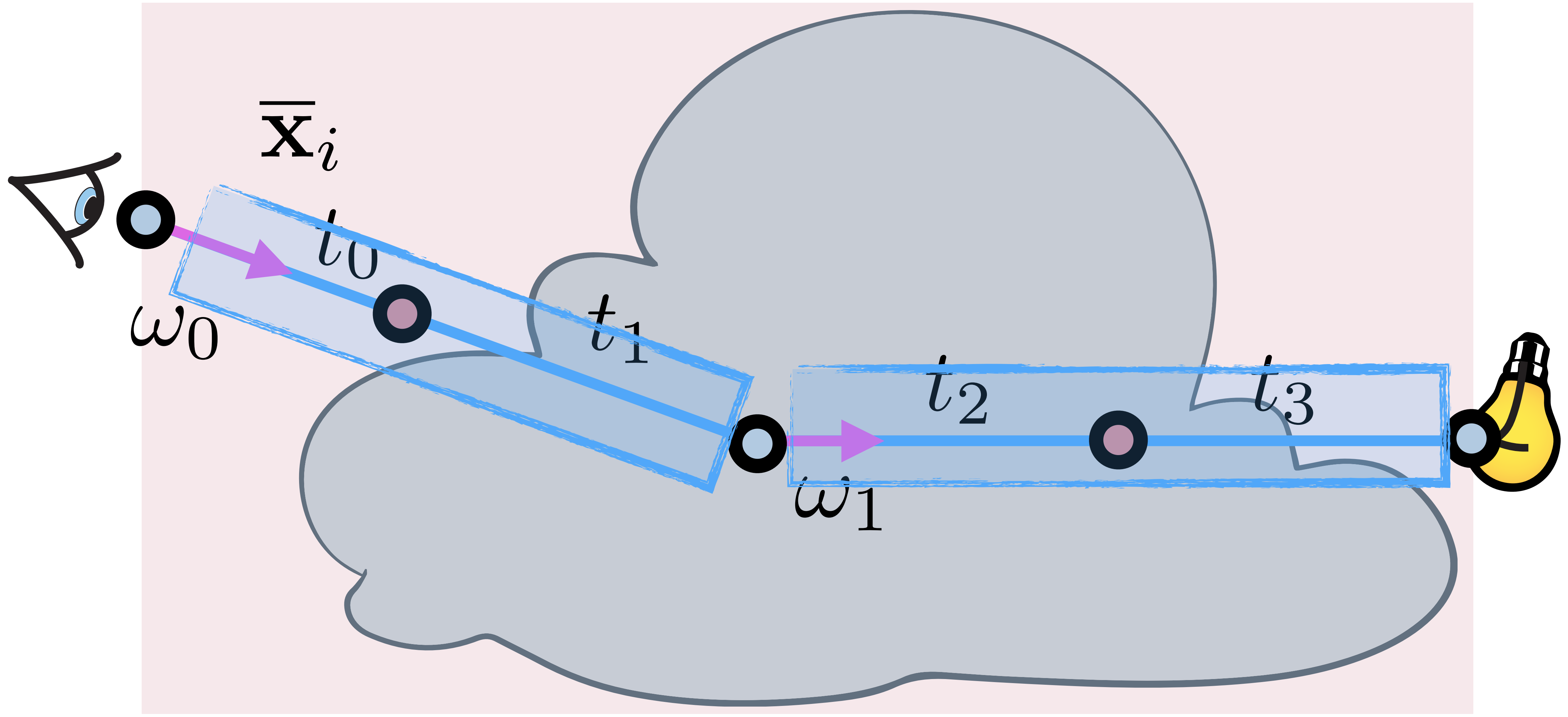
[Novak et al. 2014]



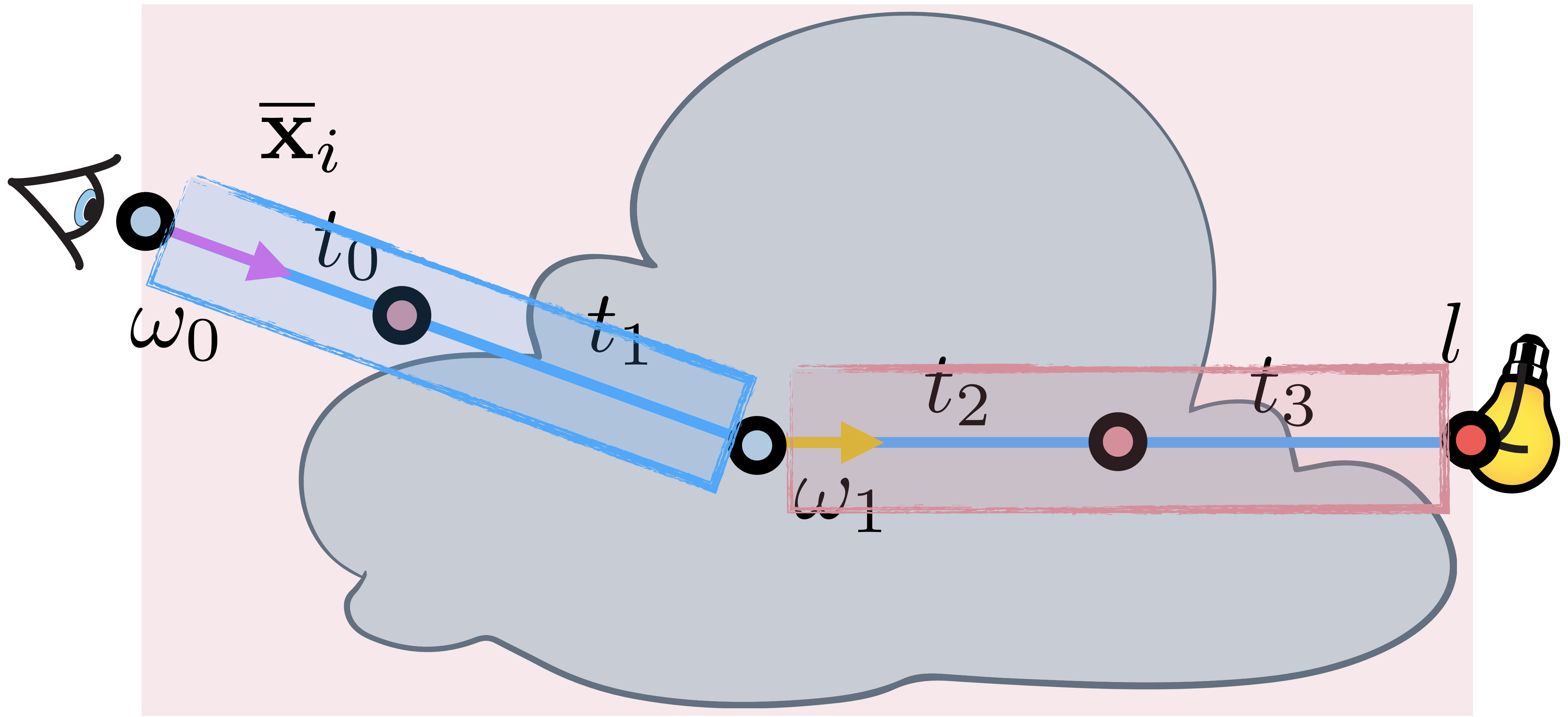
$$p(\overline{\mathbf{x}}) = p(\omega_0)p(t_0)p(t_1)p(\omega_1)p(t_2)p(t_3)$$

Spatially Varying Media

unidirectional



ratio next event (NEE)
[Novak et al. 2014]



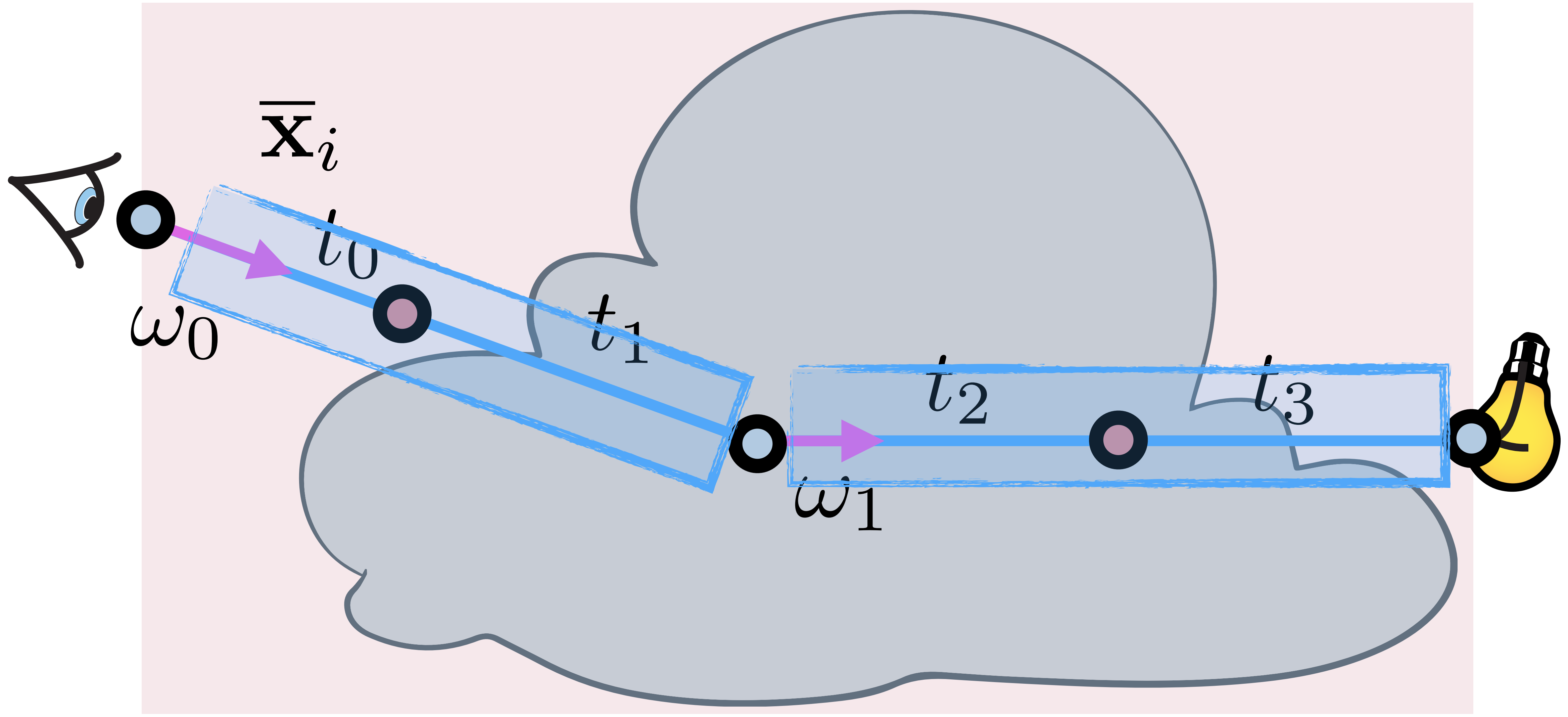
$$P(\textbf{real}) = \frac{\text{real density}}{\text{real + null density}} \qquad P(\textbf{null}) = \frac{\text{null density}}{\text{real + null density}}$$

$$P(\textbf{null}) = 1$$

Spatially Varying Media

unidirectional

ratio next event (NEE)
[Novak et al. 2014]

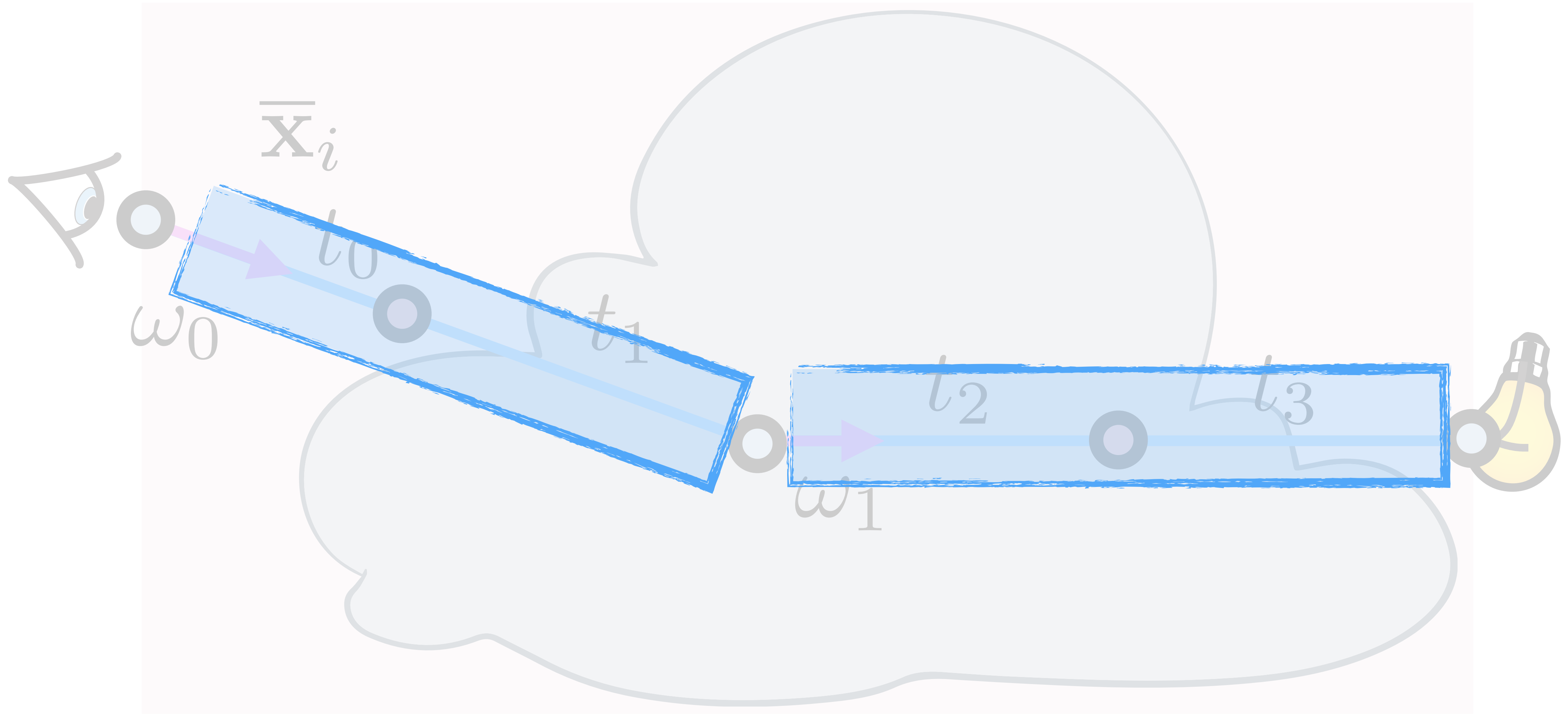


$$P(\textbf{real}) = \frac{\text{real density}}{\text{real + null density}} \qquad P(\textbf{null}) = \frac{\text{null density}}{\text{real + null density}}$$

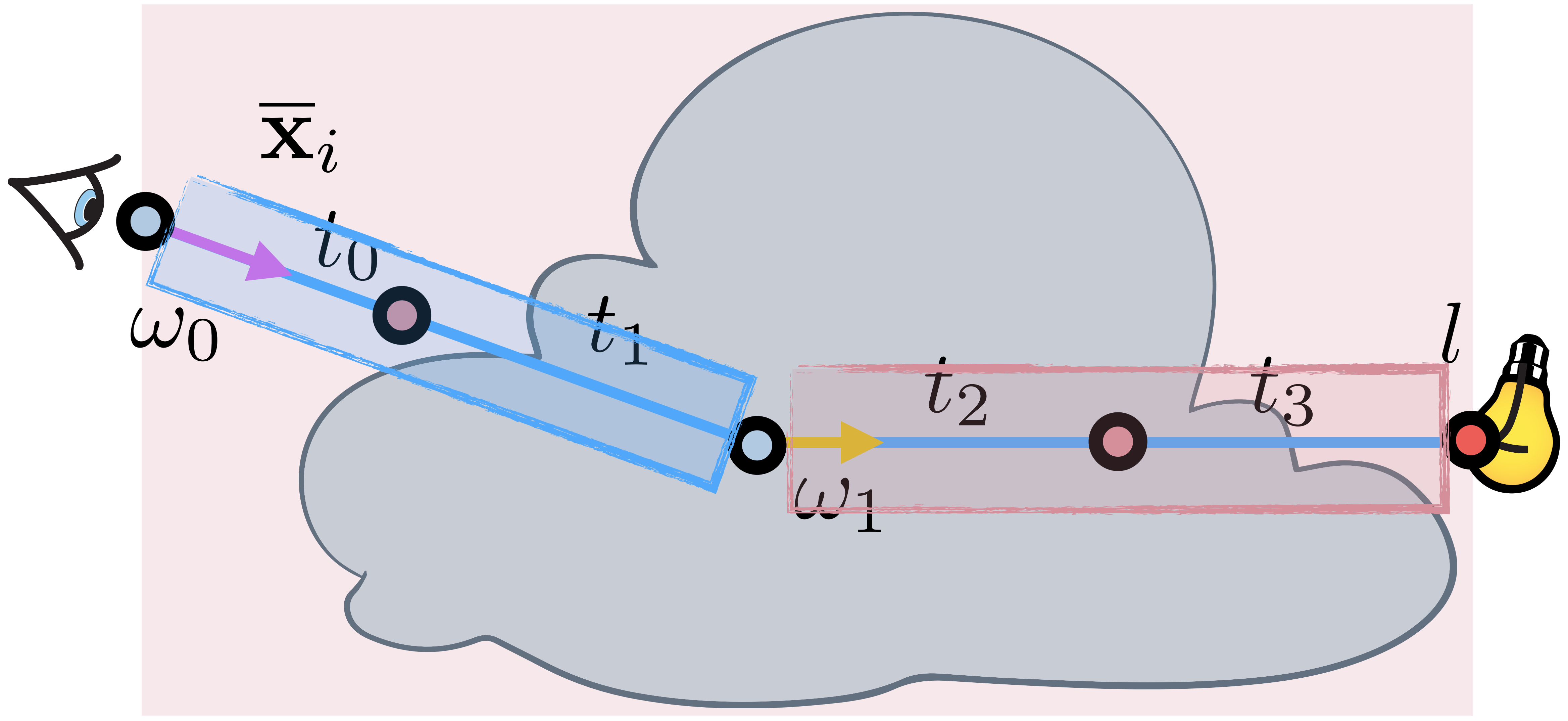
$$P(\textbf{null}) = 1$$

Spatially Varying Media

unidirectional



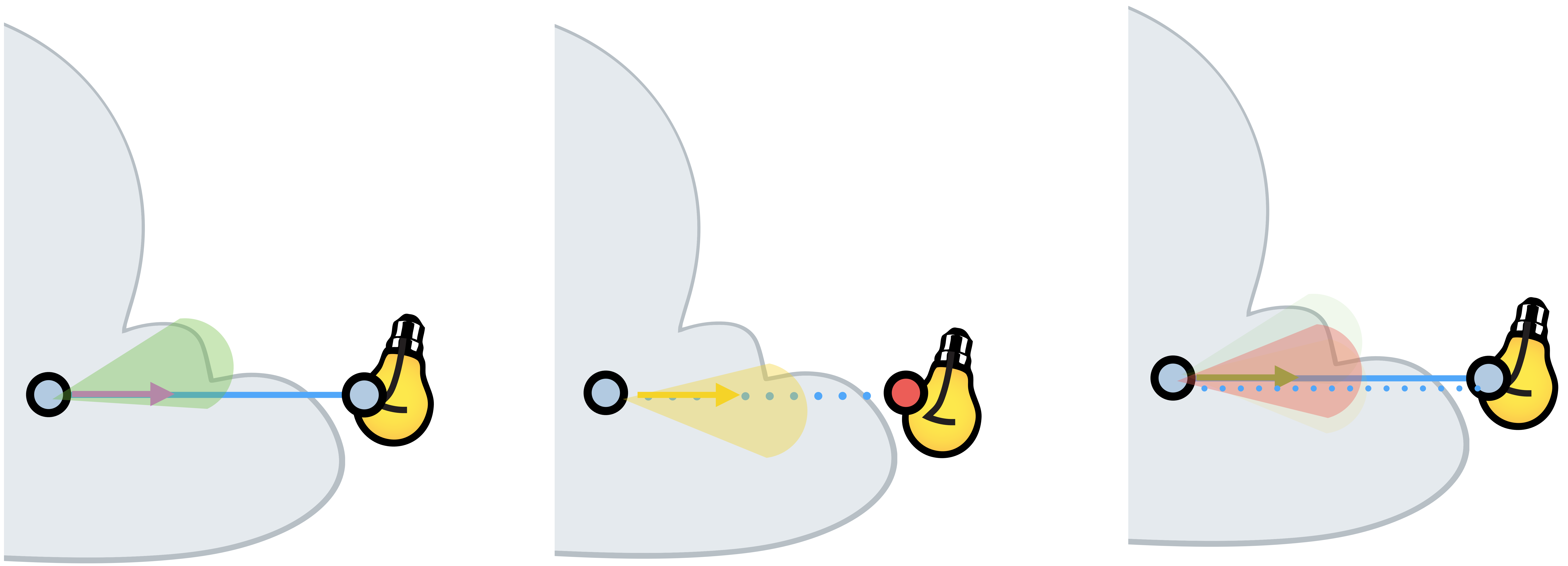
ratio next event (NEE)
[Novak et al. 2014]



$P(\textbf{real}) = \dfrac{\text{real density}}{\text{real + null density}}$ $\quad P(\textbf{null}) = \dfrac{\text{null density}}{\text{real + null density}}$

$P(\textbf{null}) = 1$

Unidirectional + NEE MIS

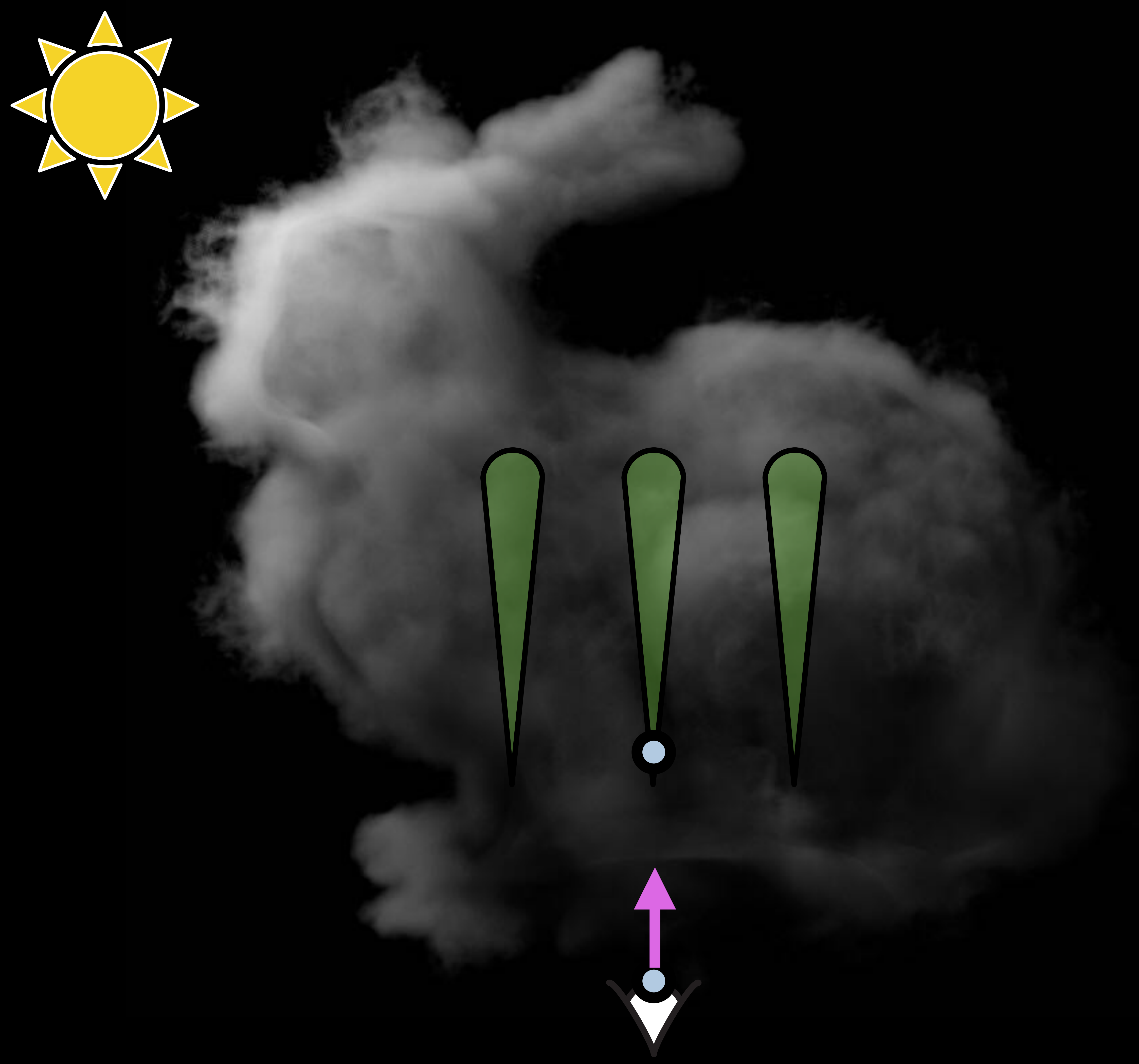


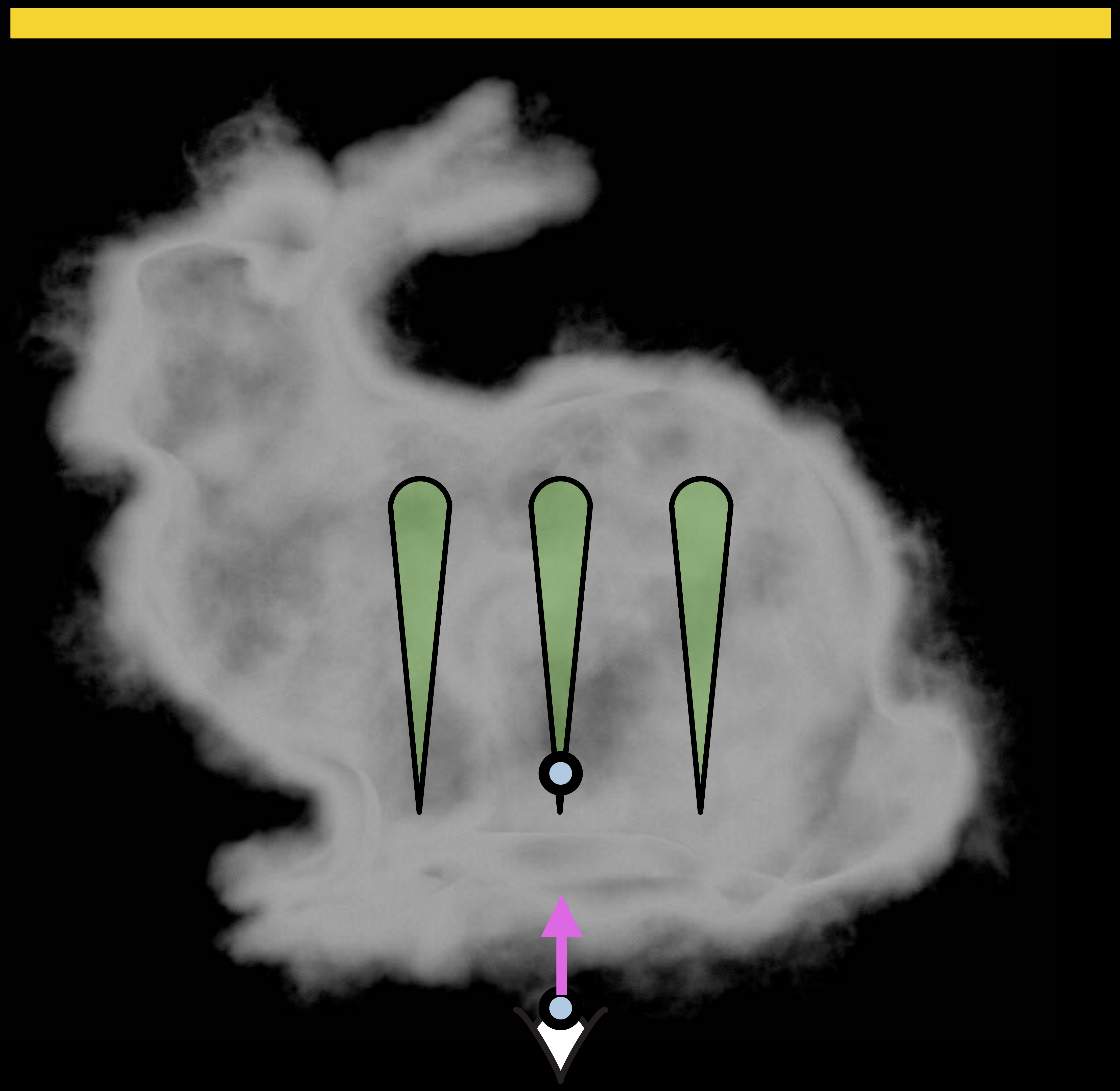
unidirectional | ratio next event (NEE) [Novak et al. 2014] | unidirectional + NEE MIS [Ours]

Results: Measuring Variance

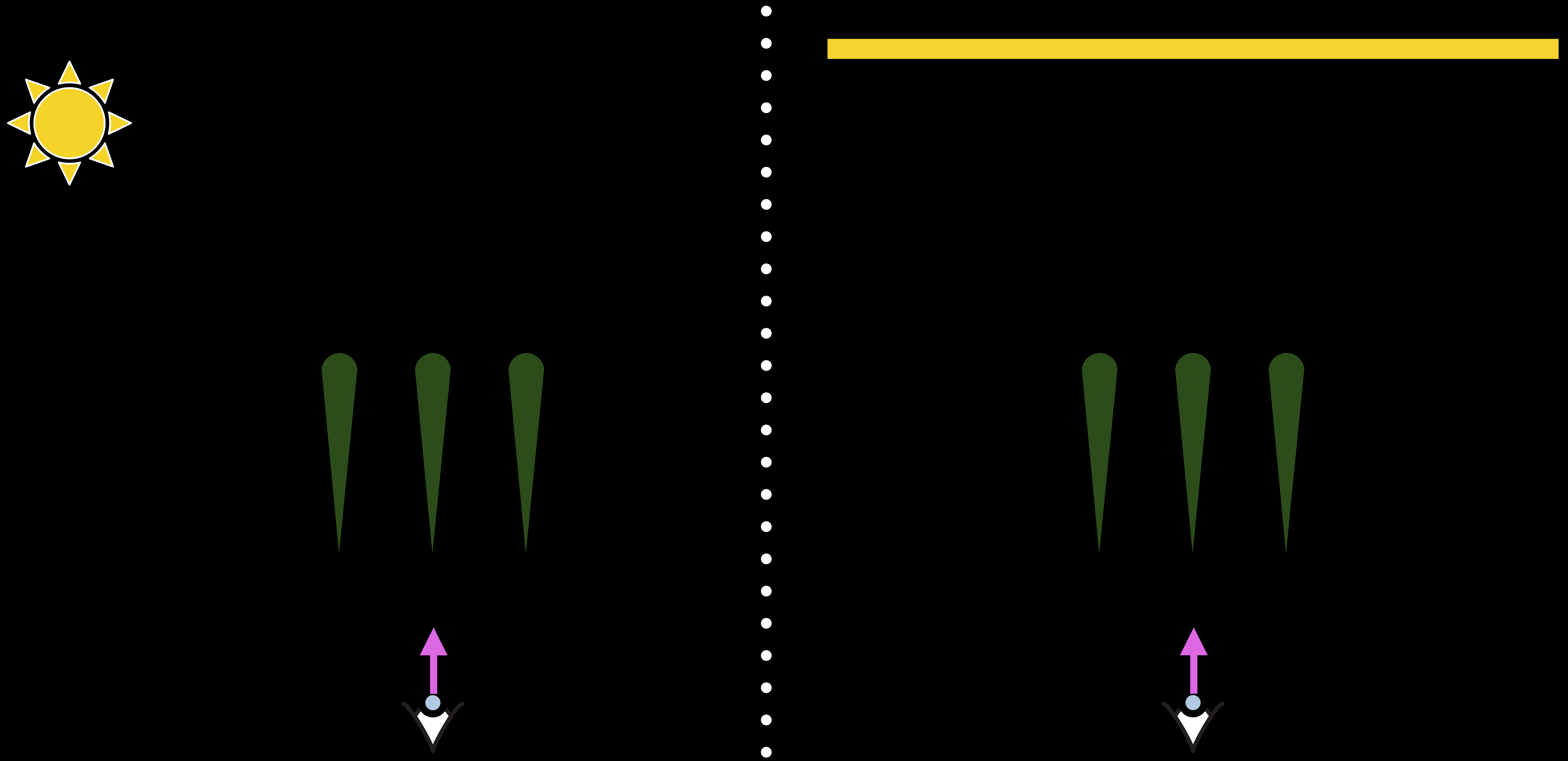
$$\textbf{LUTV} = \#\textbf{Lookups} \times \textbf{Unit Variance}$$

Bunny Scene

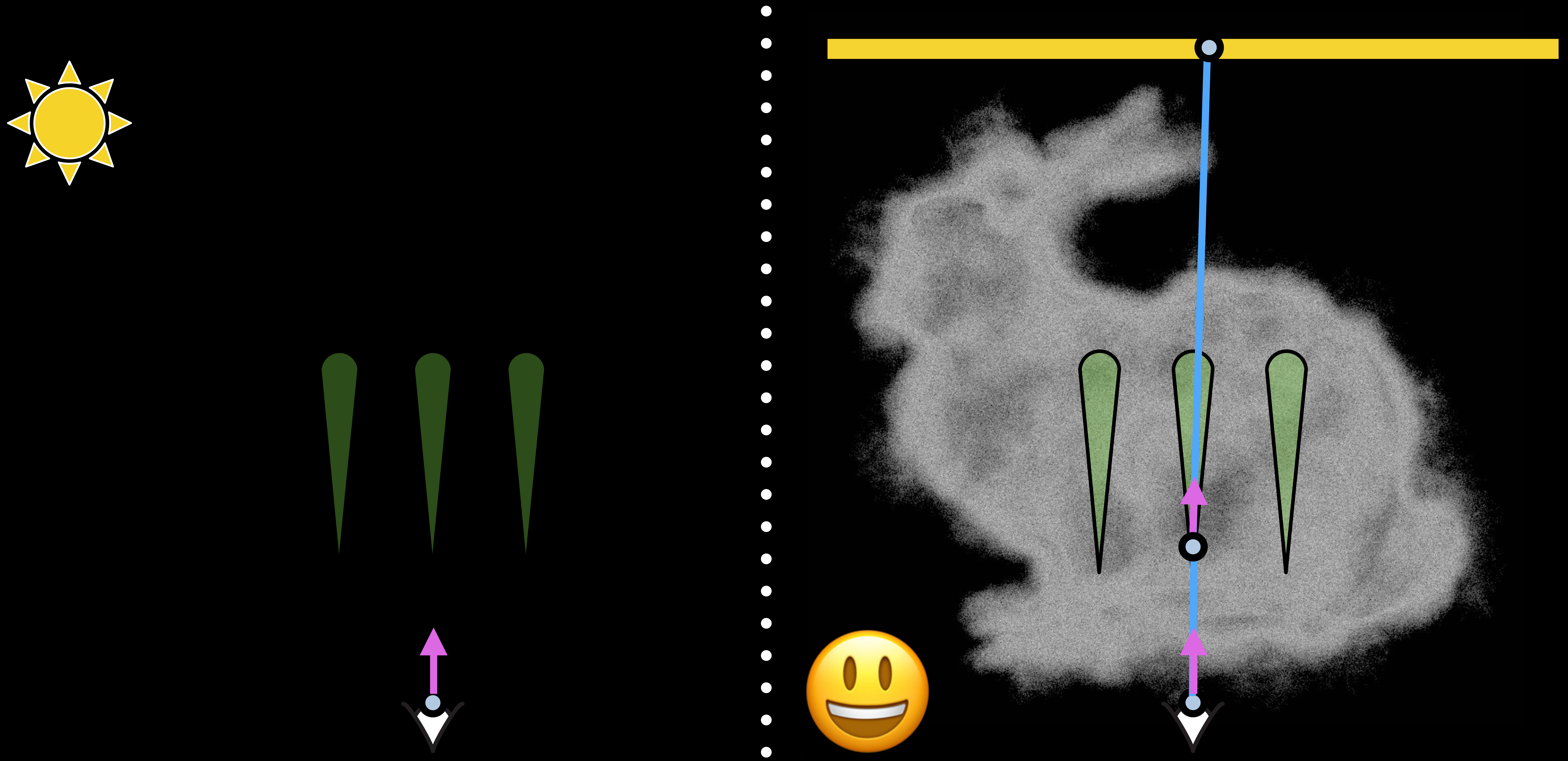




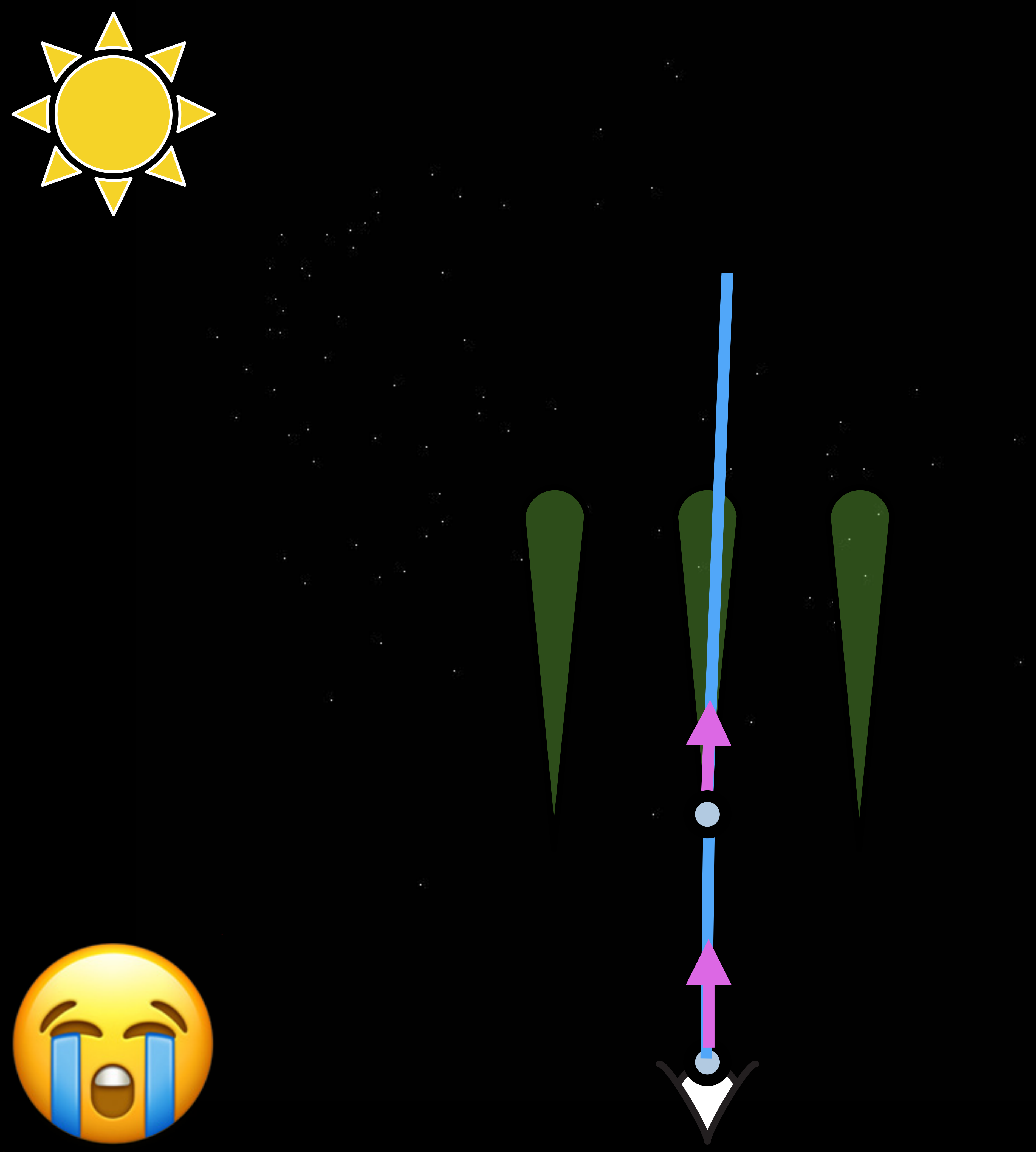
Bunny Scene: Unidirectional

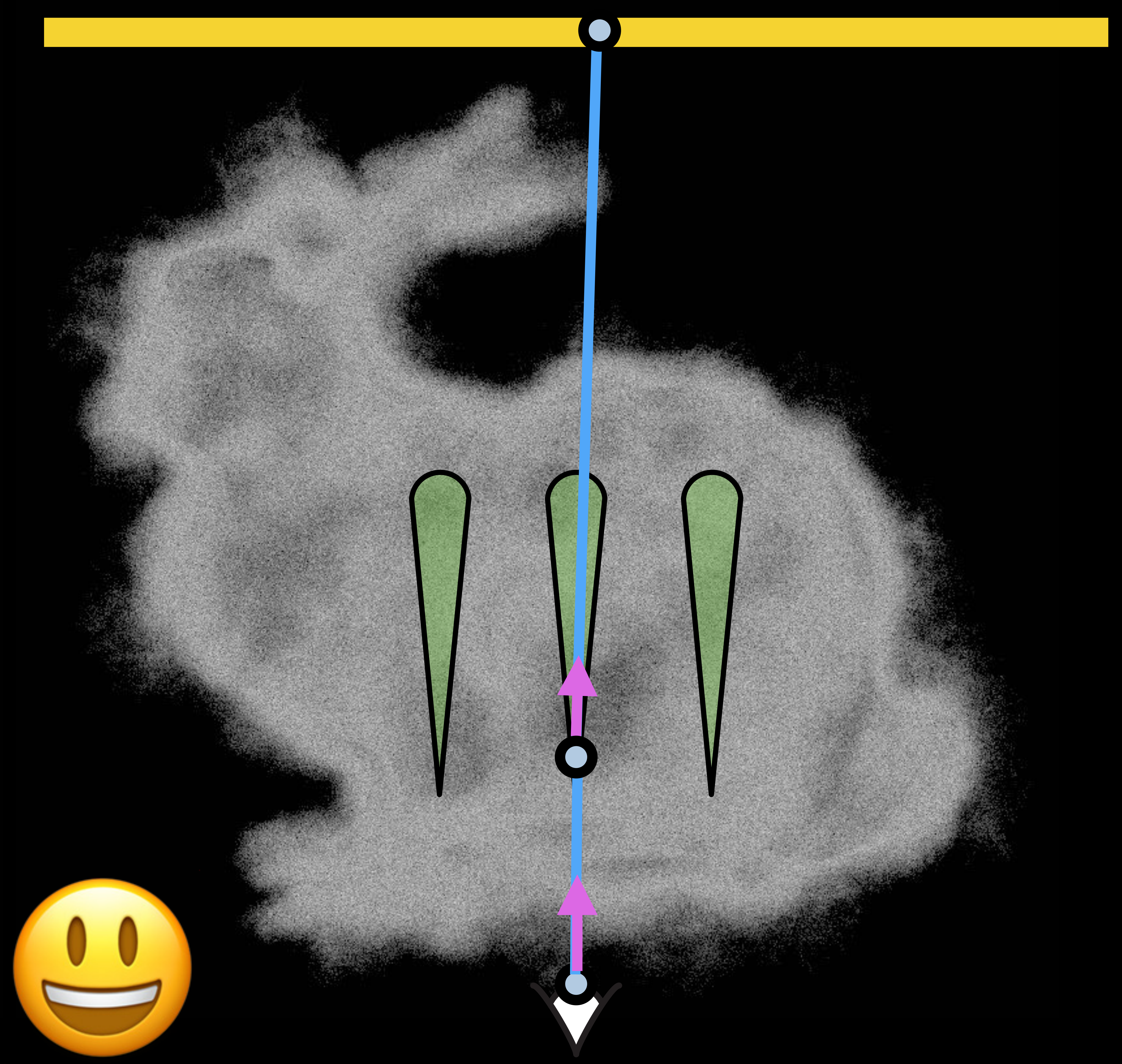


Bunny Scene: Unidirectional

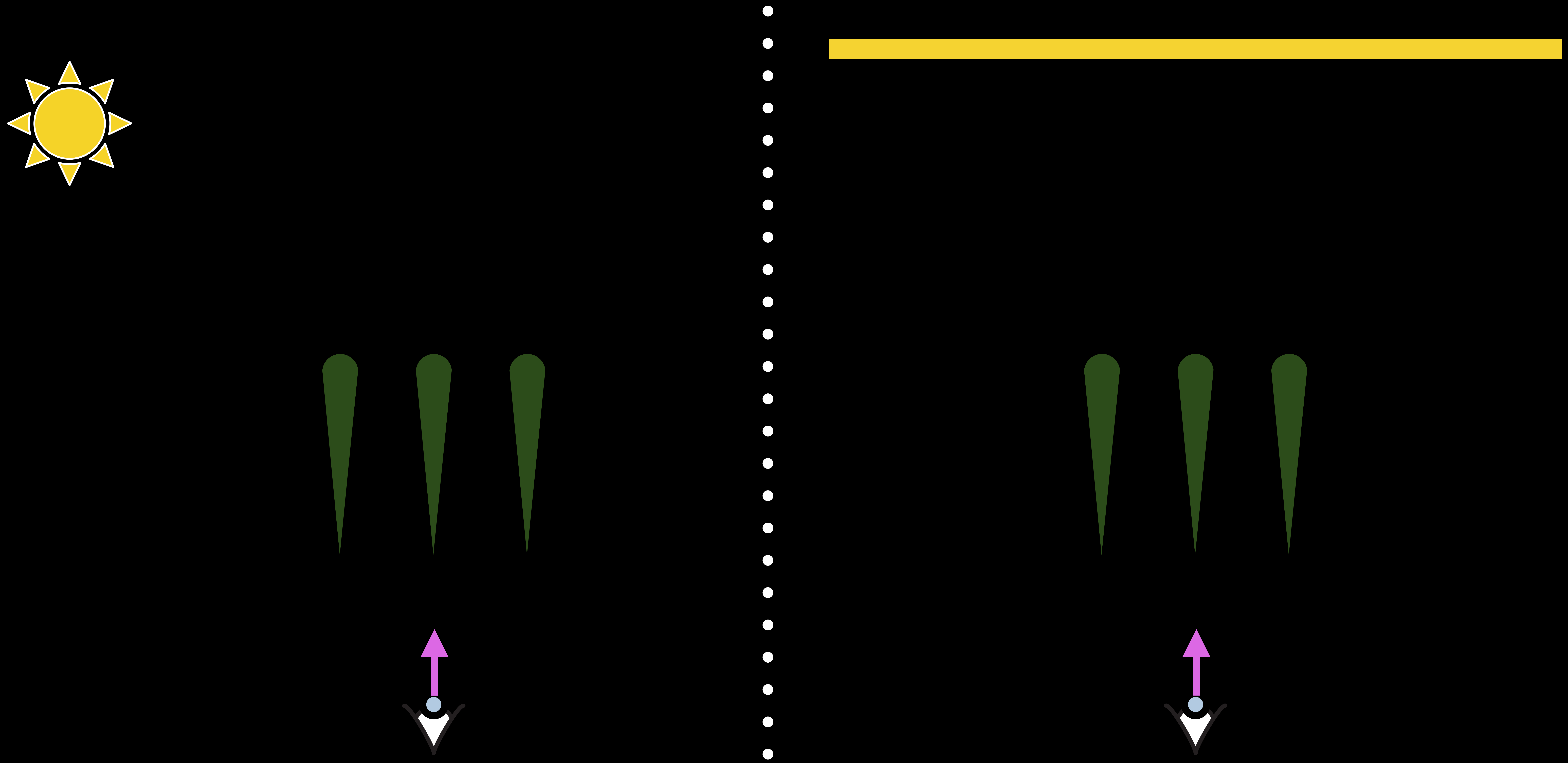


Bunny Scene: Unidirectional

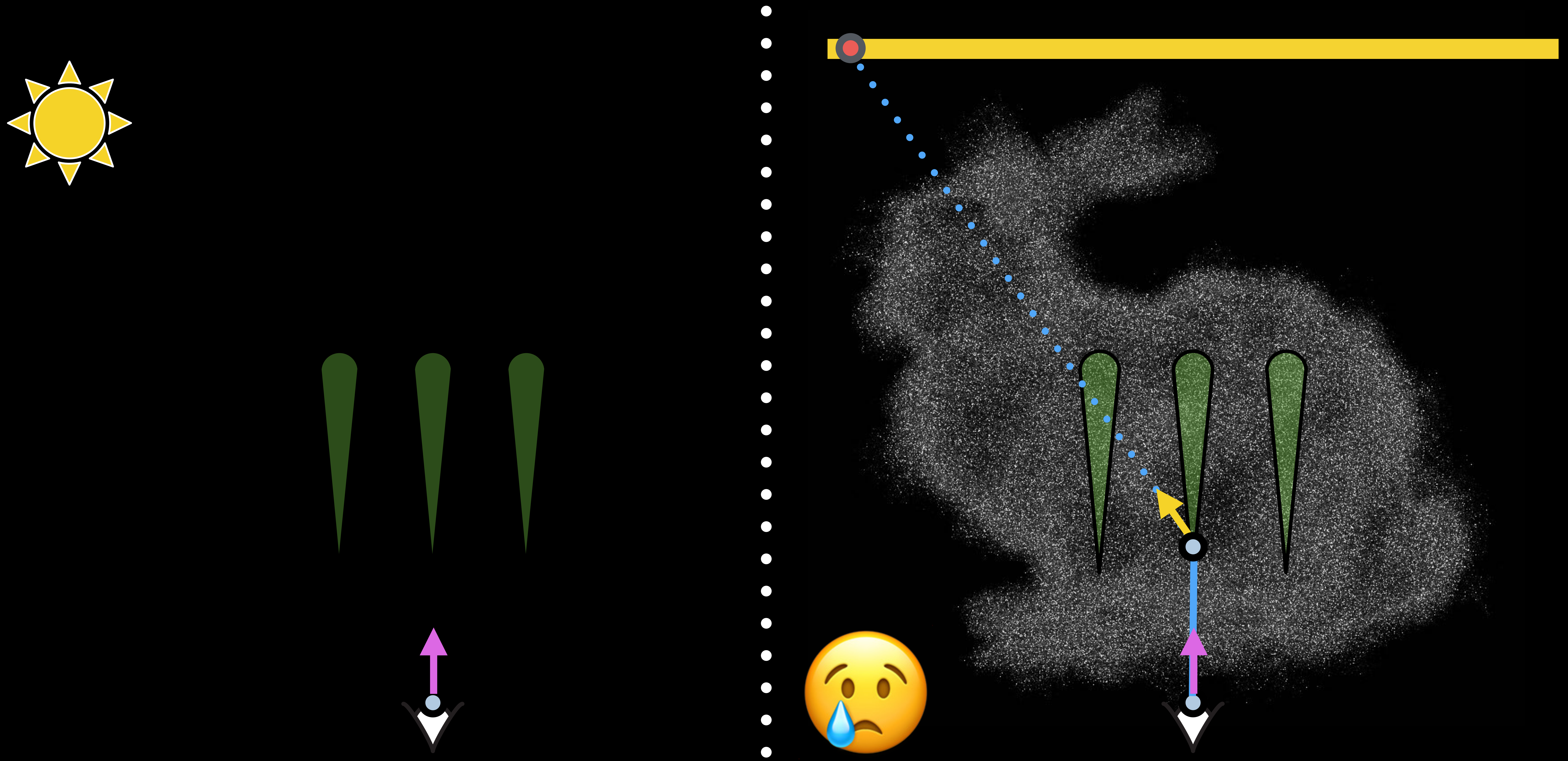




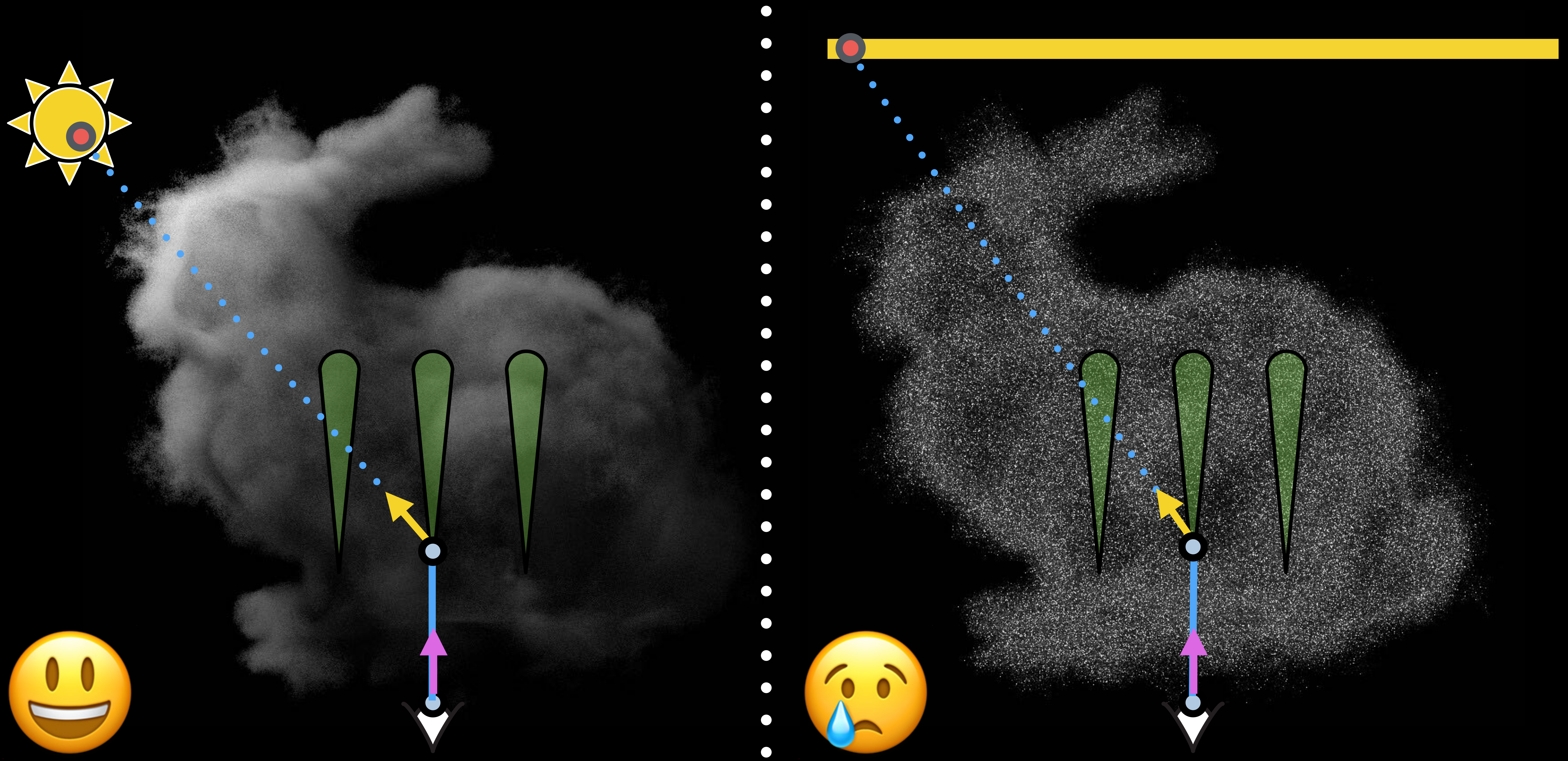
Bunny Scene: Ratio NEE [Novak et al 2014]



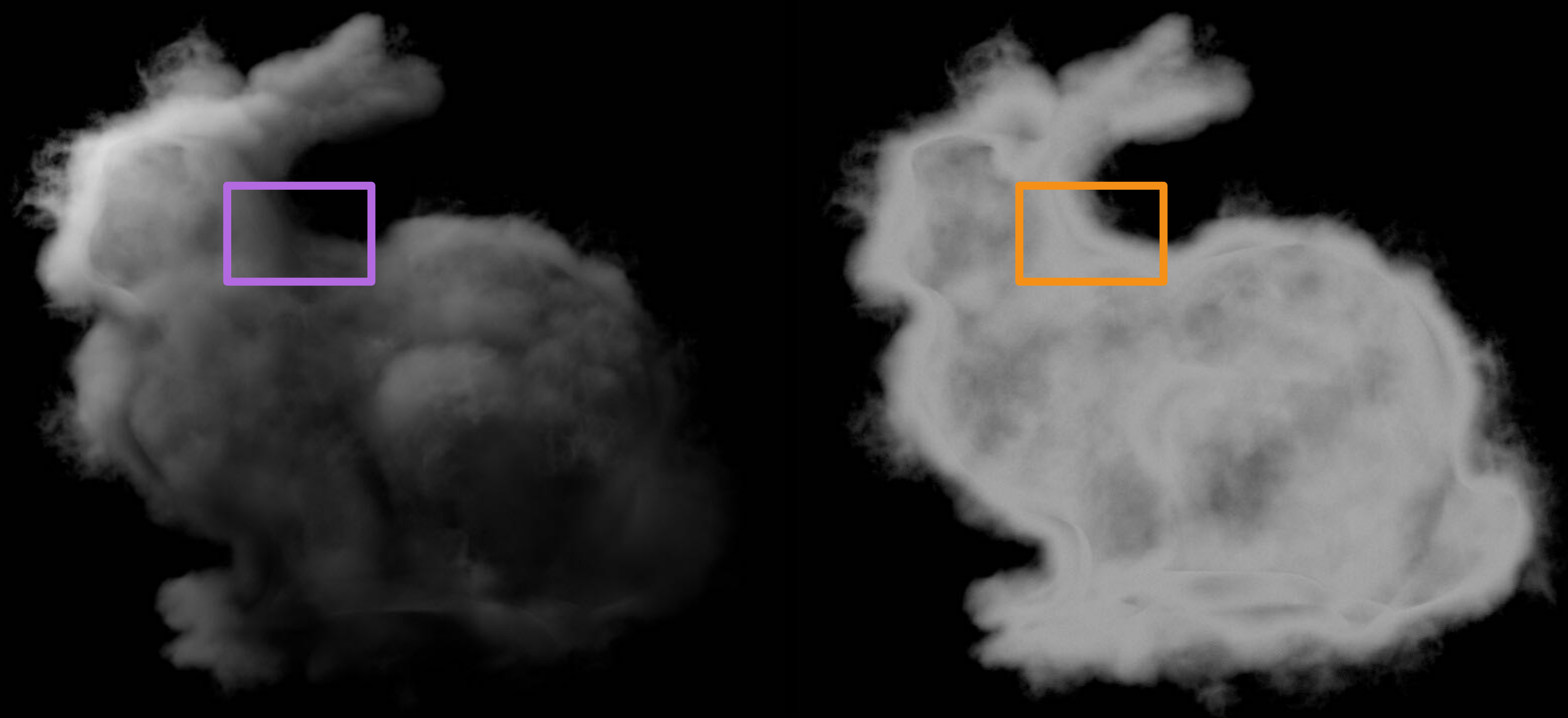
Bunny Scene: Ratio NEE [Novak et al 2014]



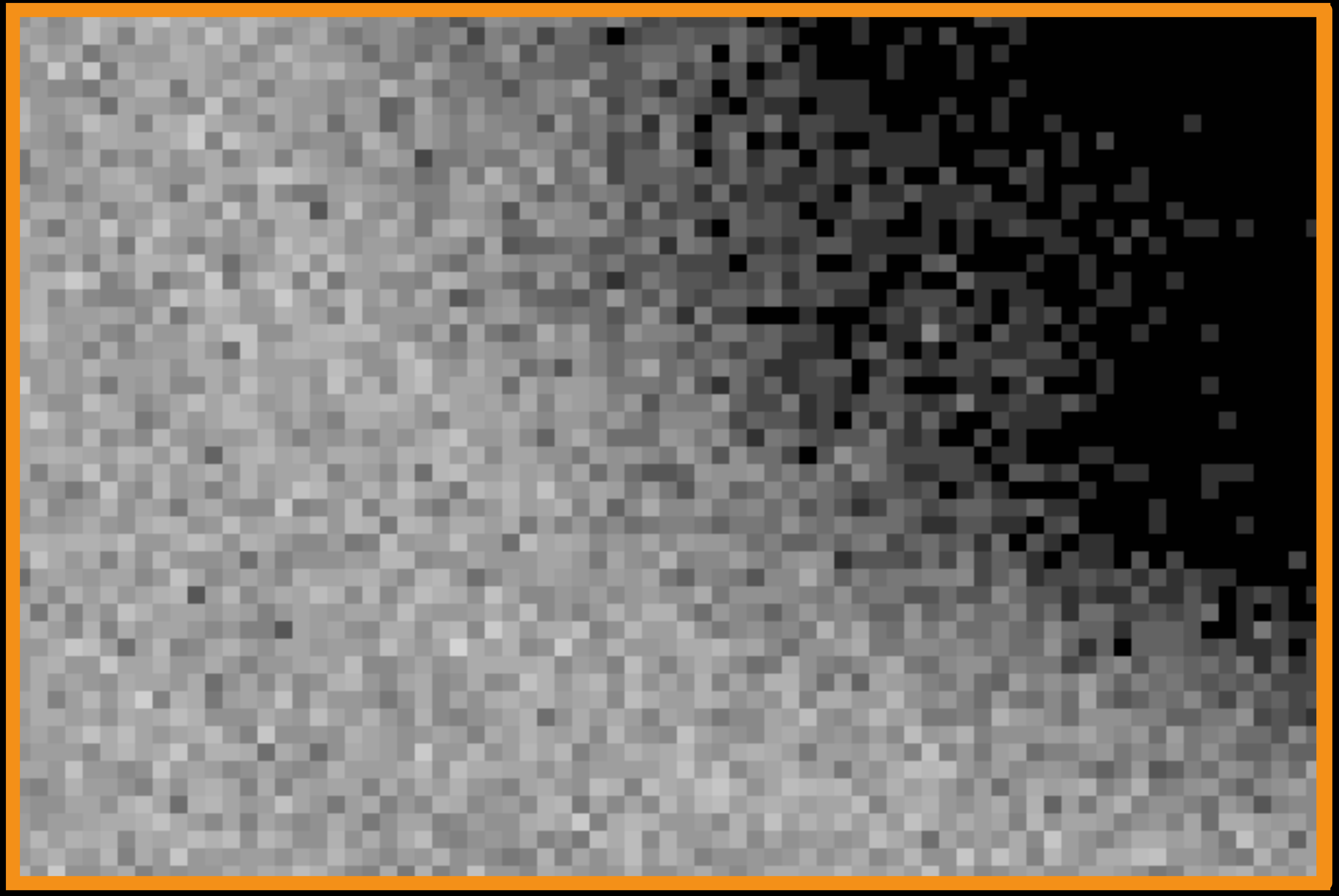
Bunny Scene: Ratio NEE [Novak et al 2014]

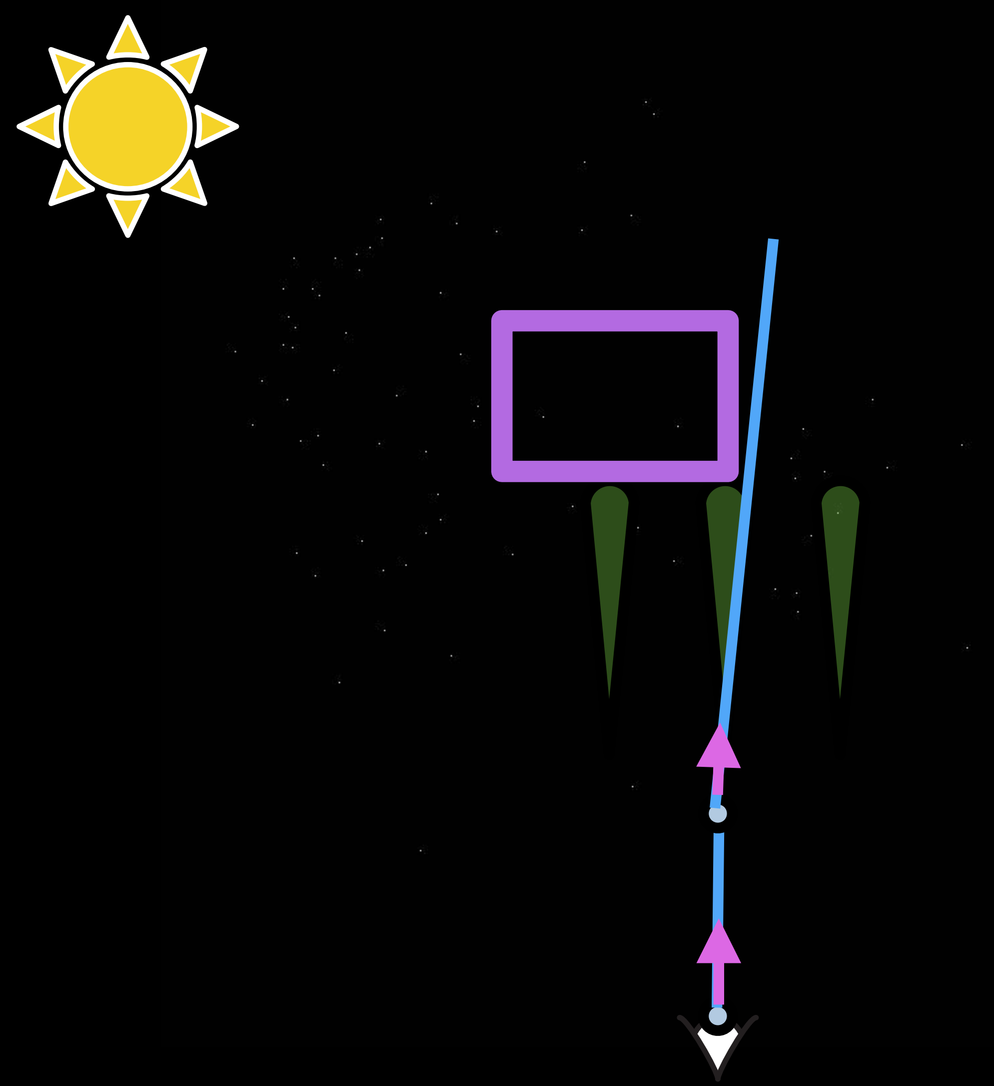


Spatially Varying Media

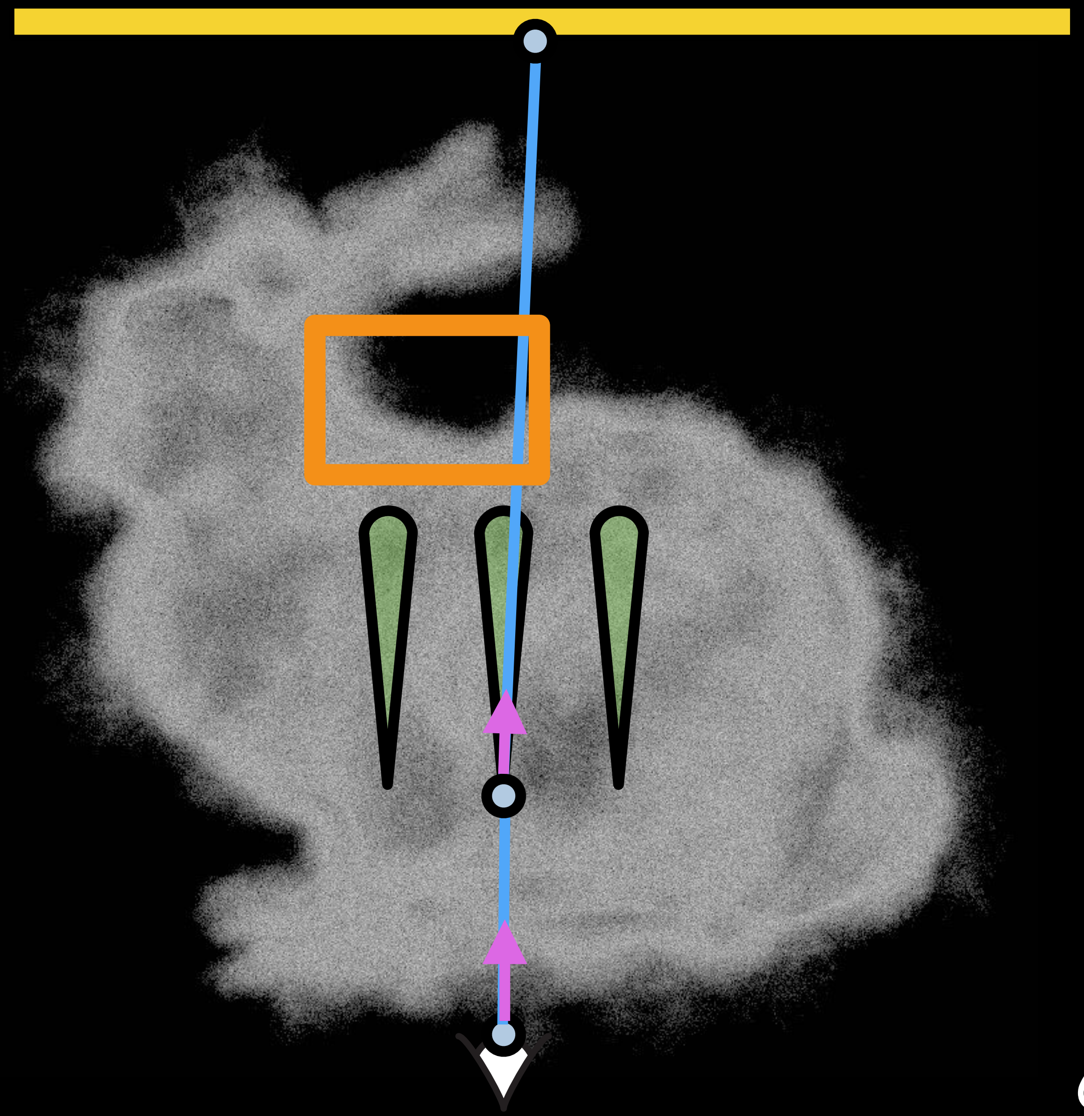


Unidirectional

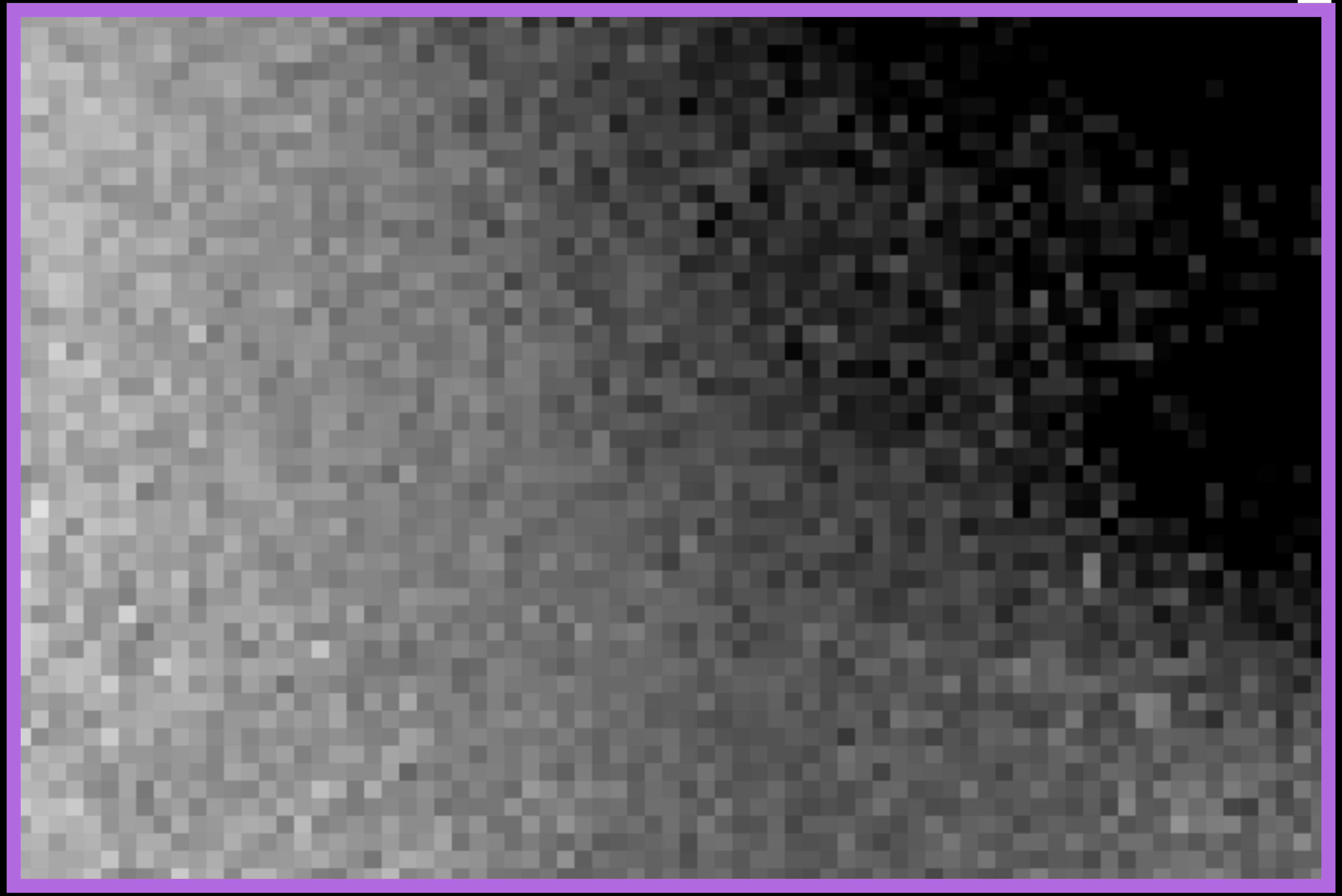


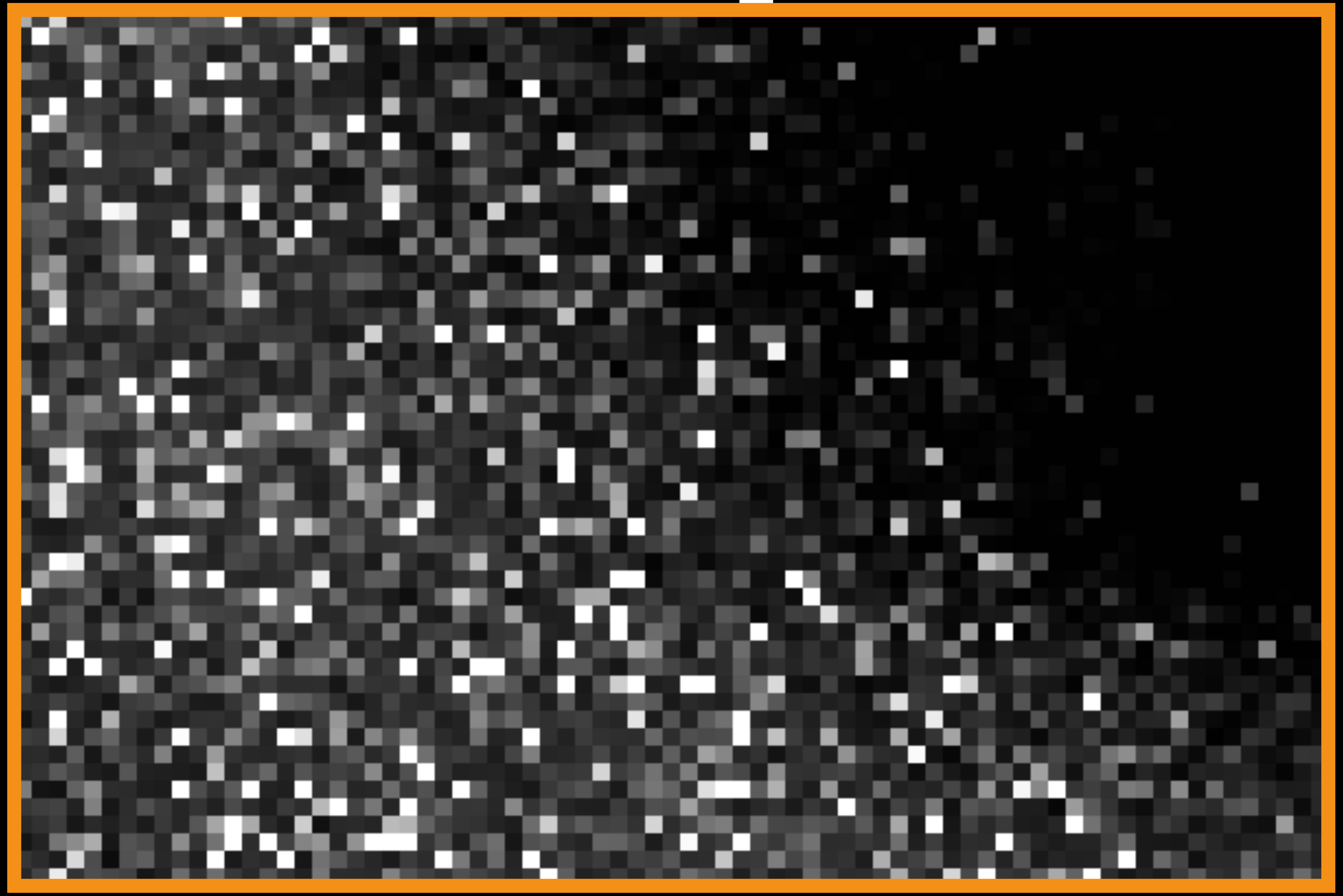


LTUV	Sampler	LTUV
10.3 B	Unidirectional	0.87 M

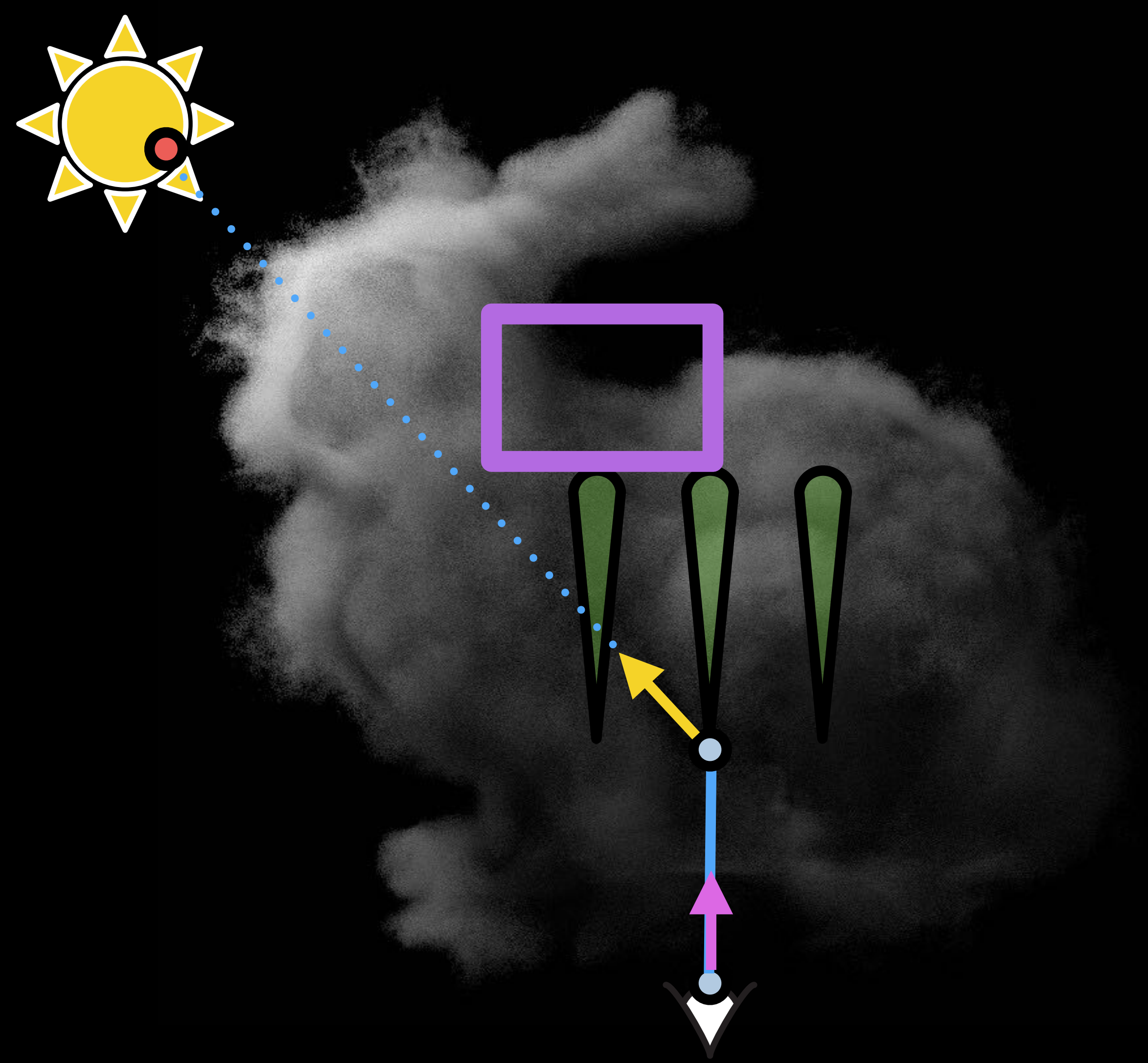


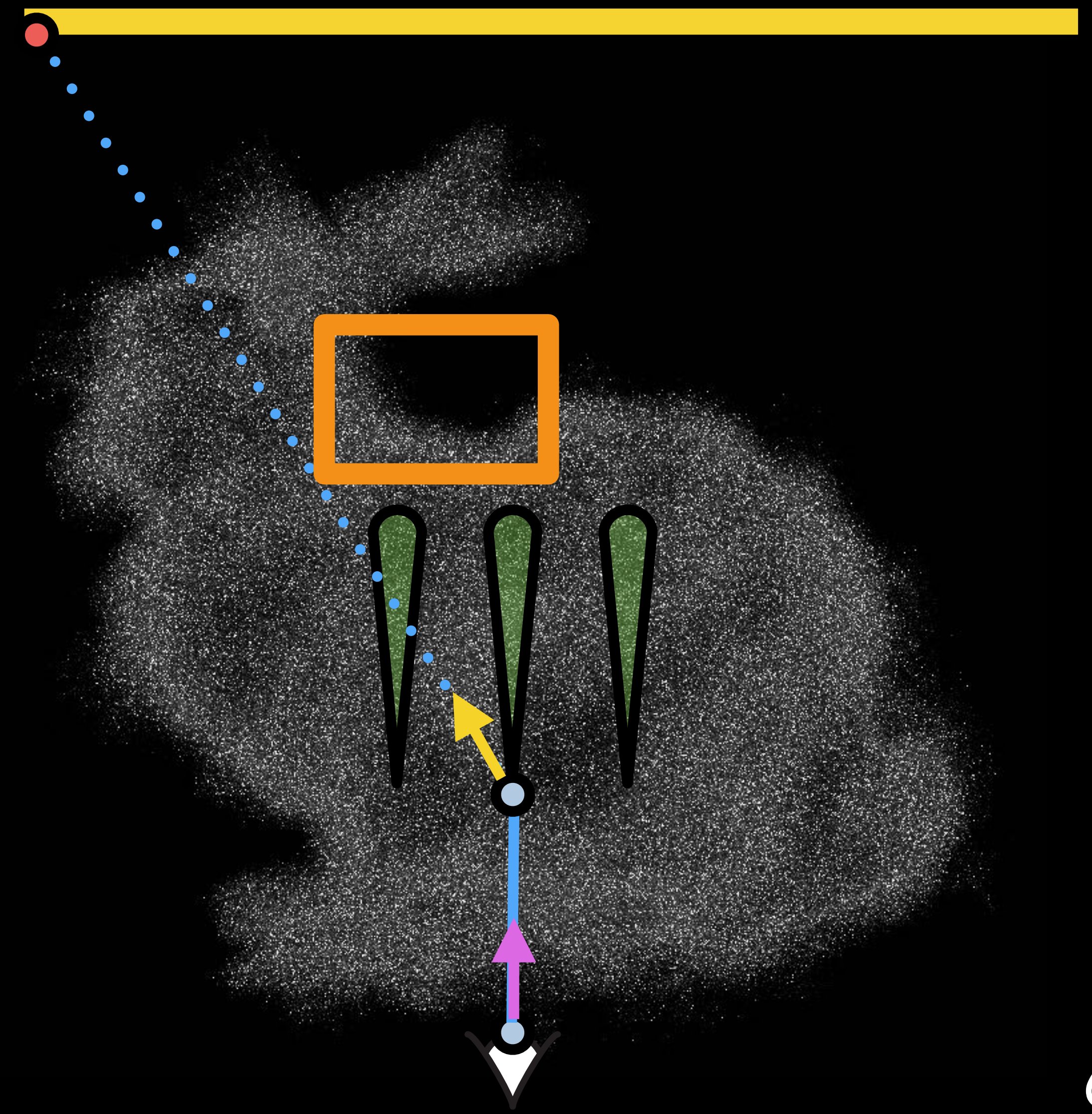
Ratio NEE [Novak 14]



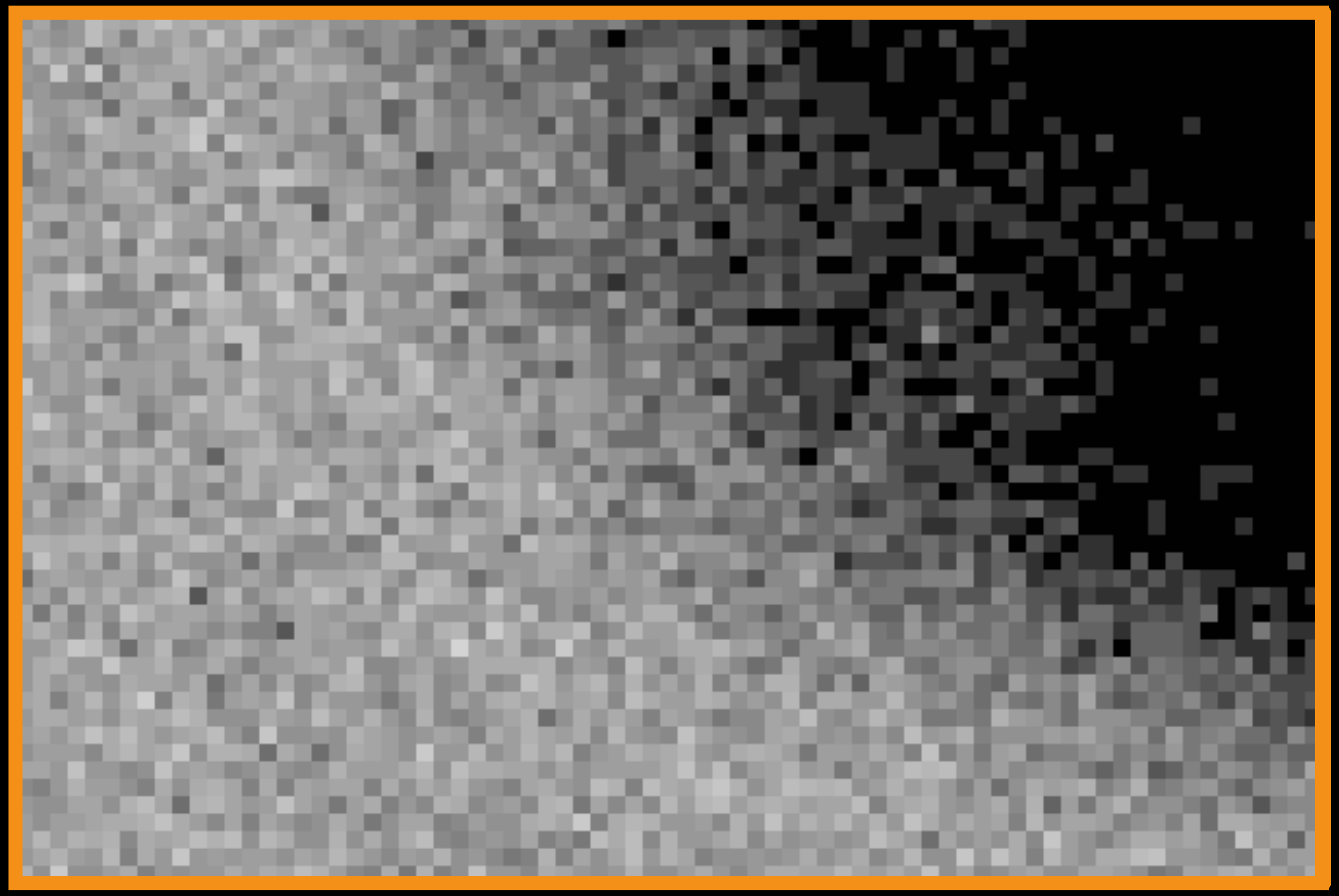


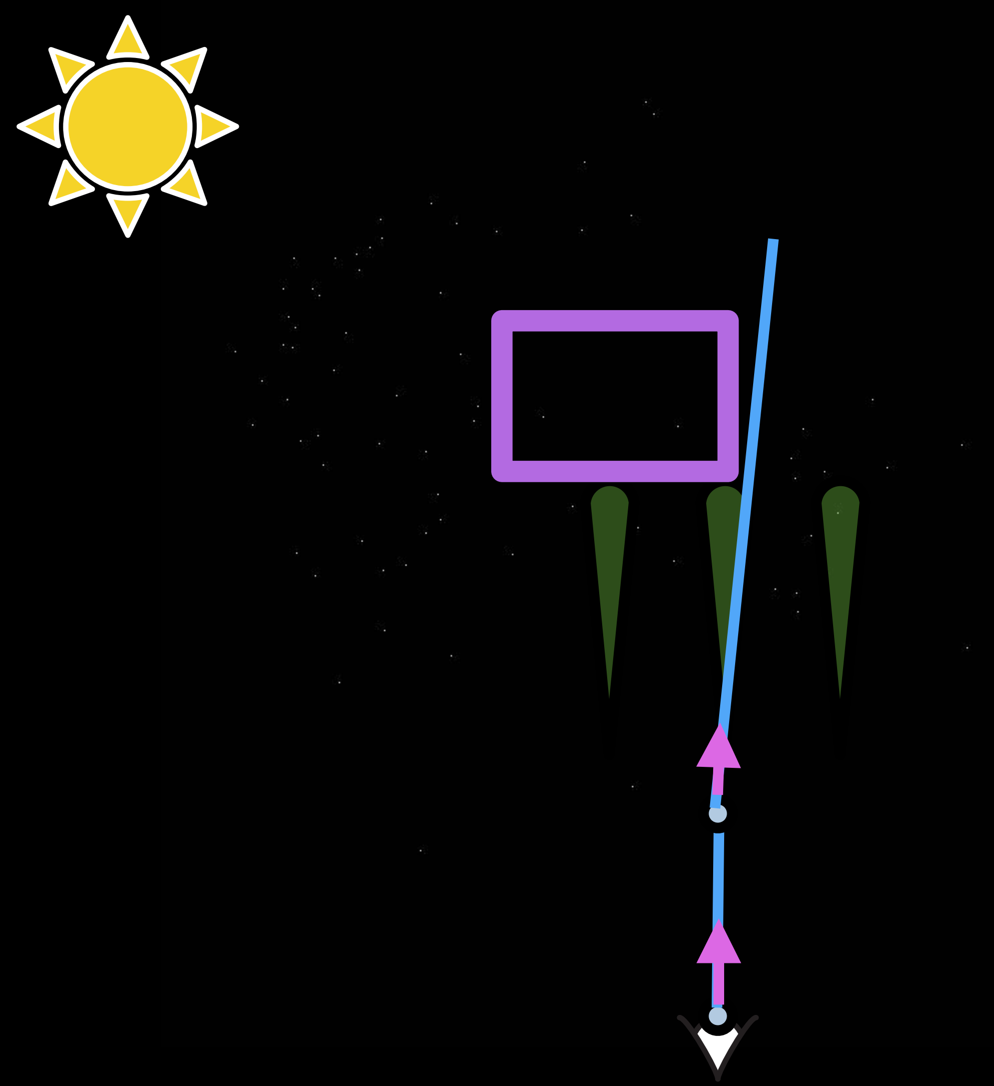
LTUV	Sampler	LTUV
10.3 B	Unidirectional	0.87 M
0.14 M	Ratio NEE [Novak 14]	519 M



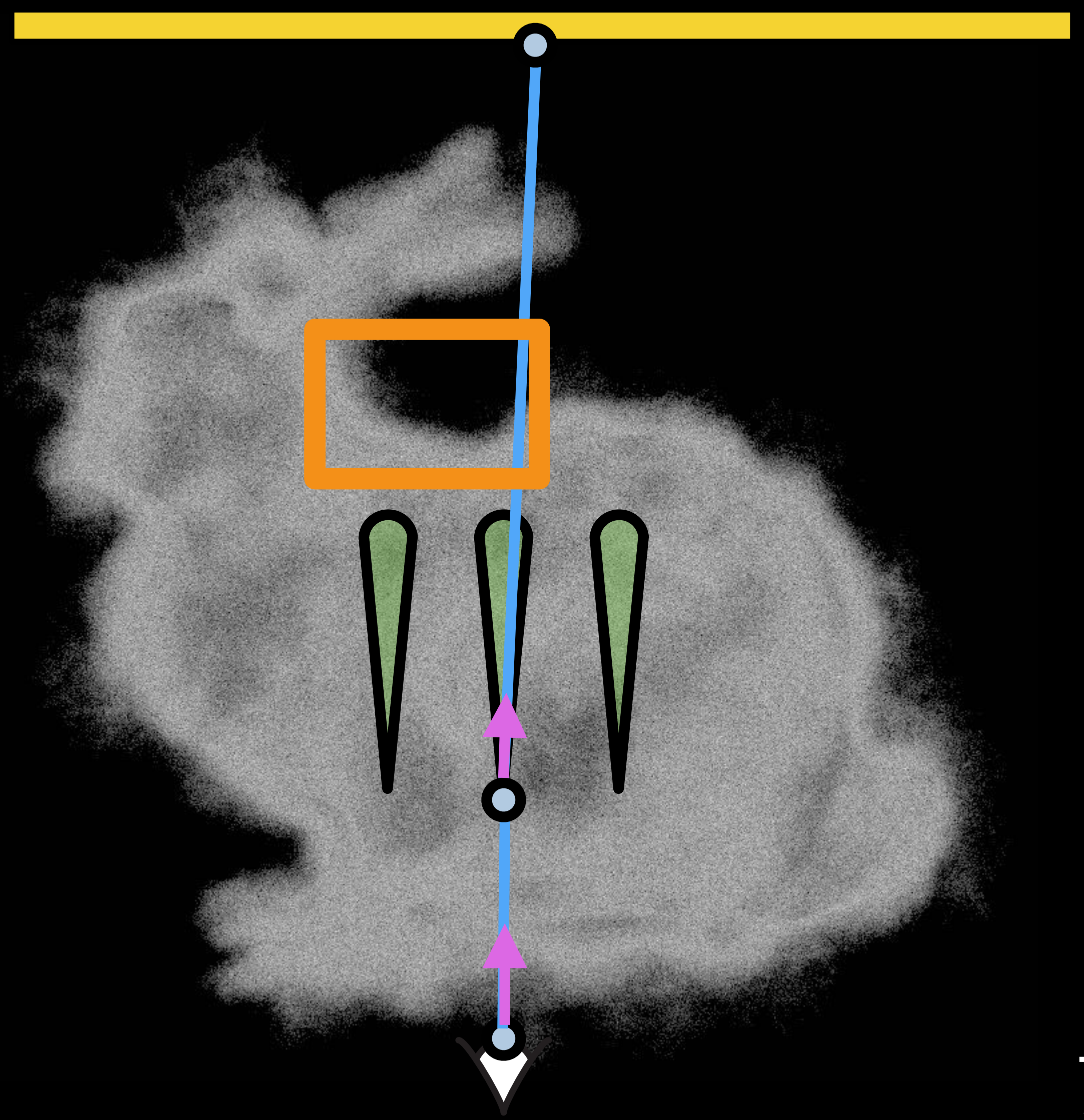


Unidirectional

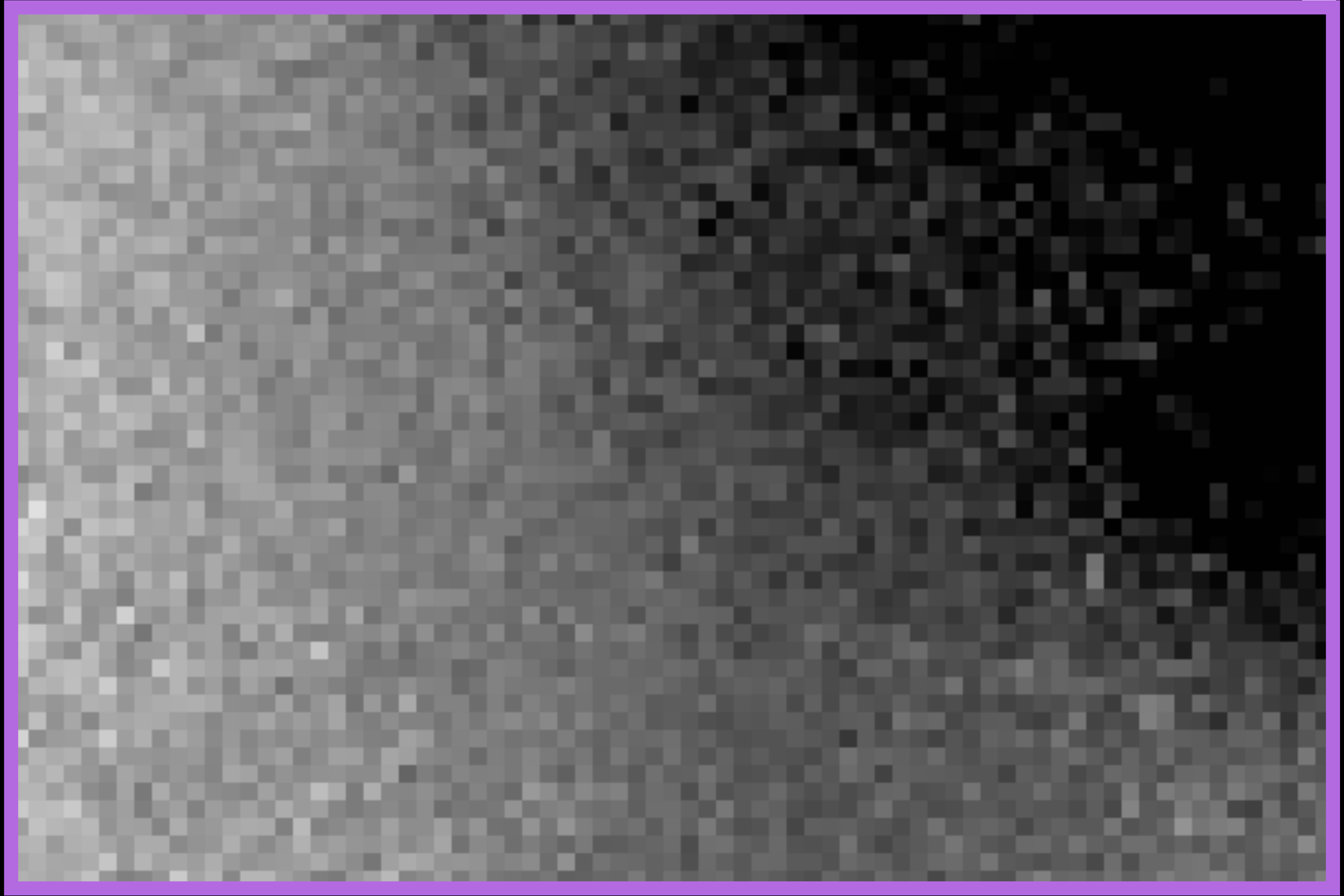


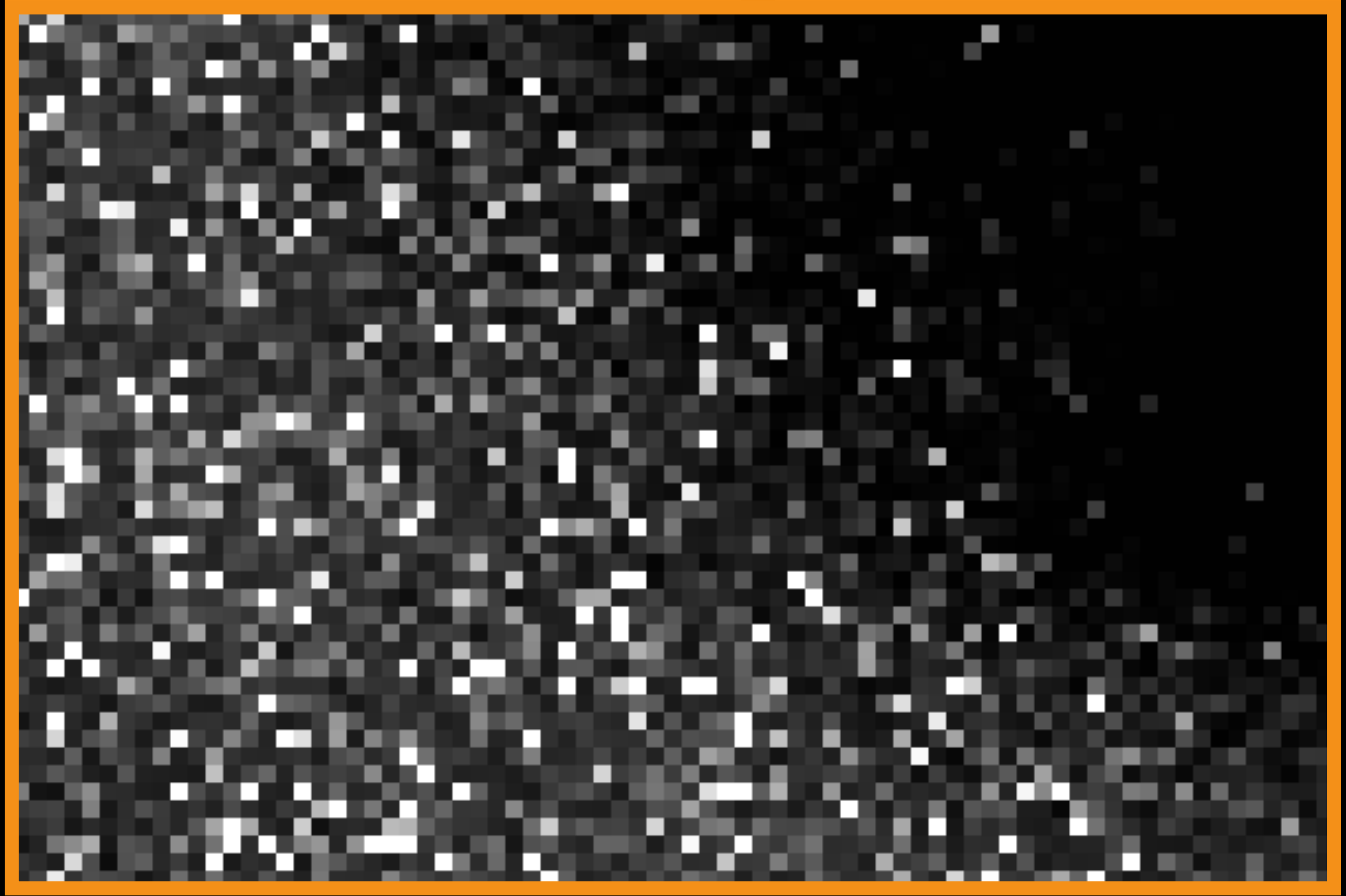


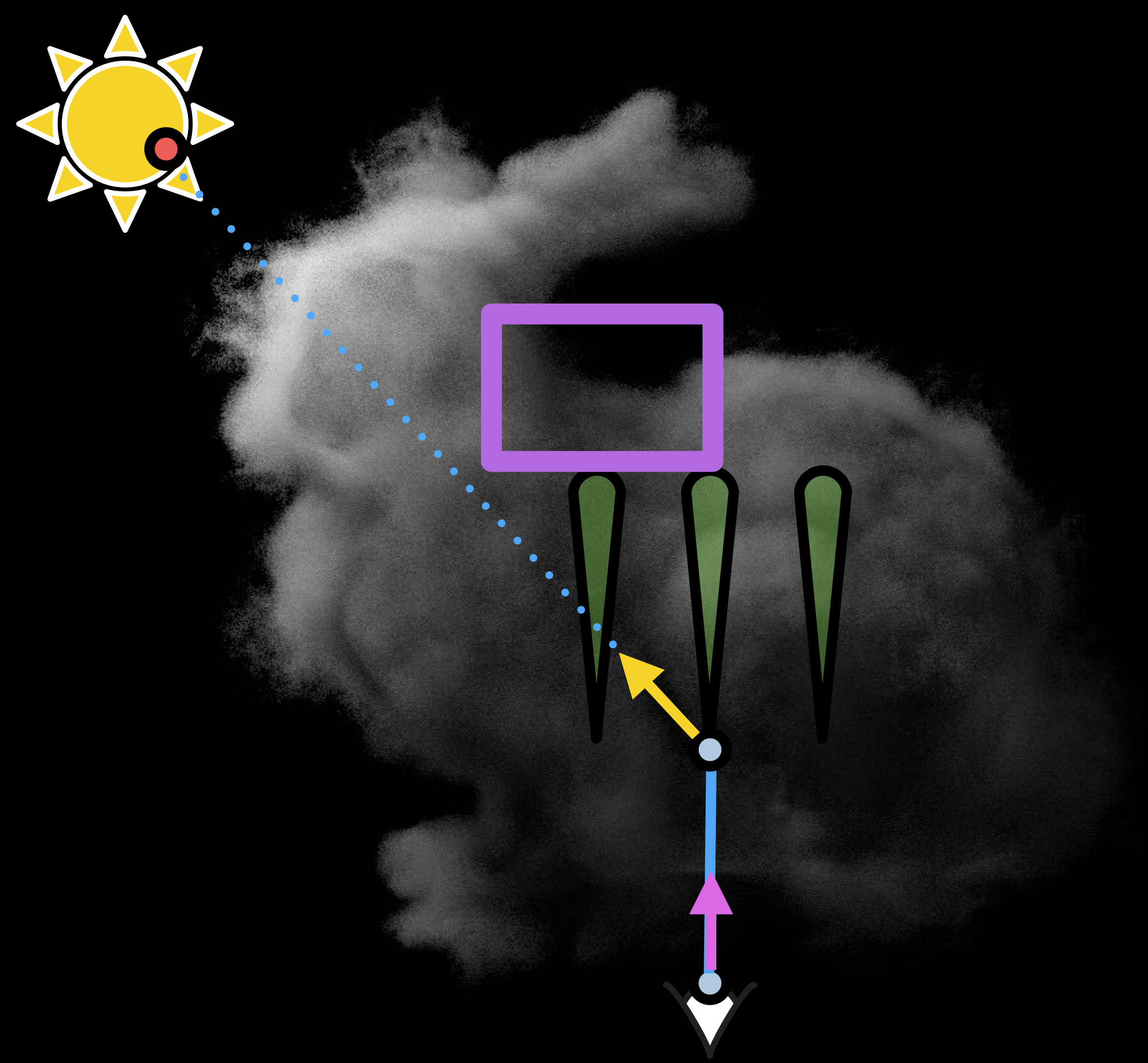
LTUV	Sampler	LTUV
10.3 B	Unidirectional	0.87 M
0.14 M	Ratio NEE [Novak 14]	519 M



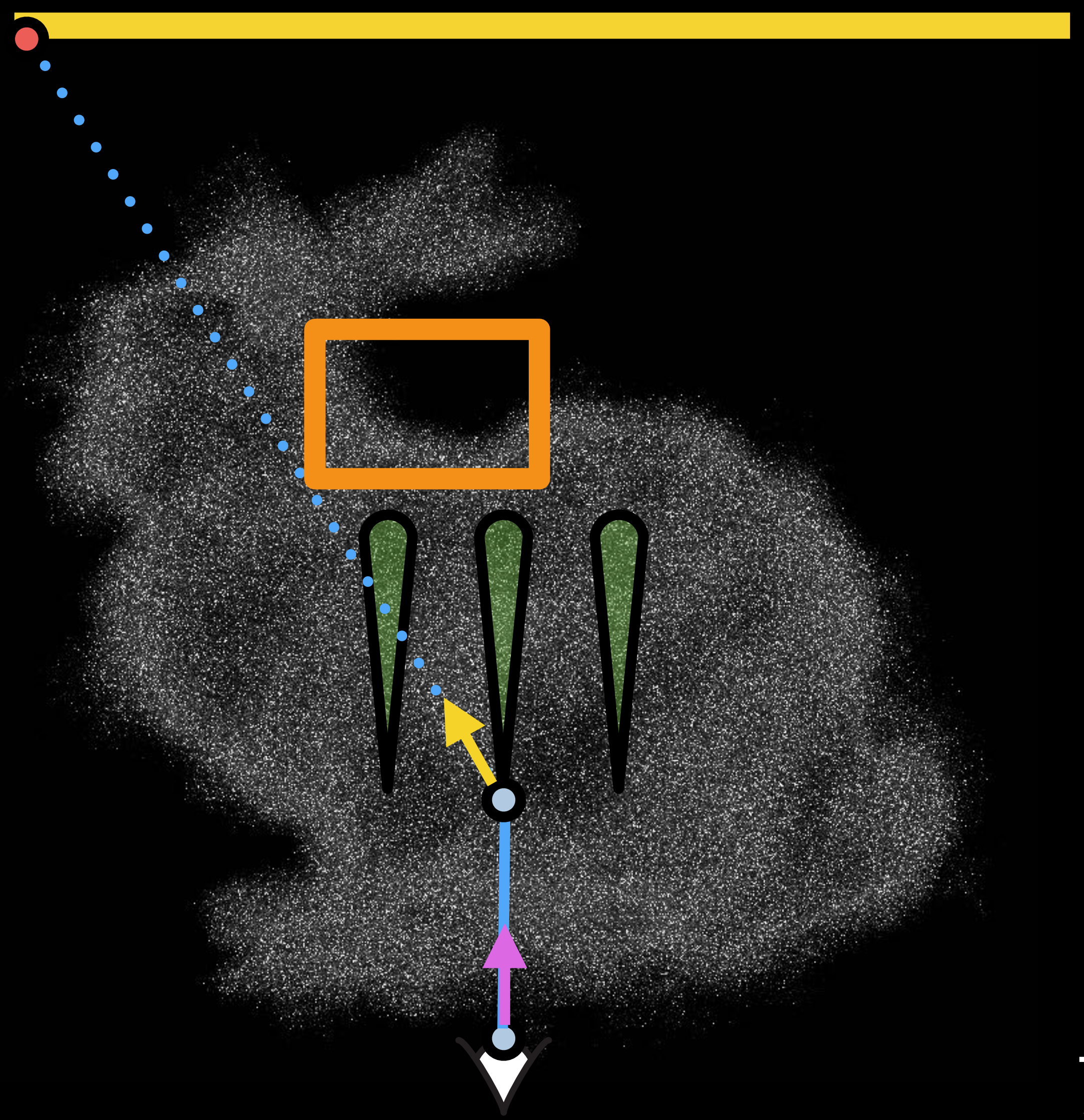
Ratio NEE [Novak 14]



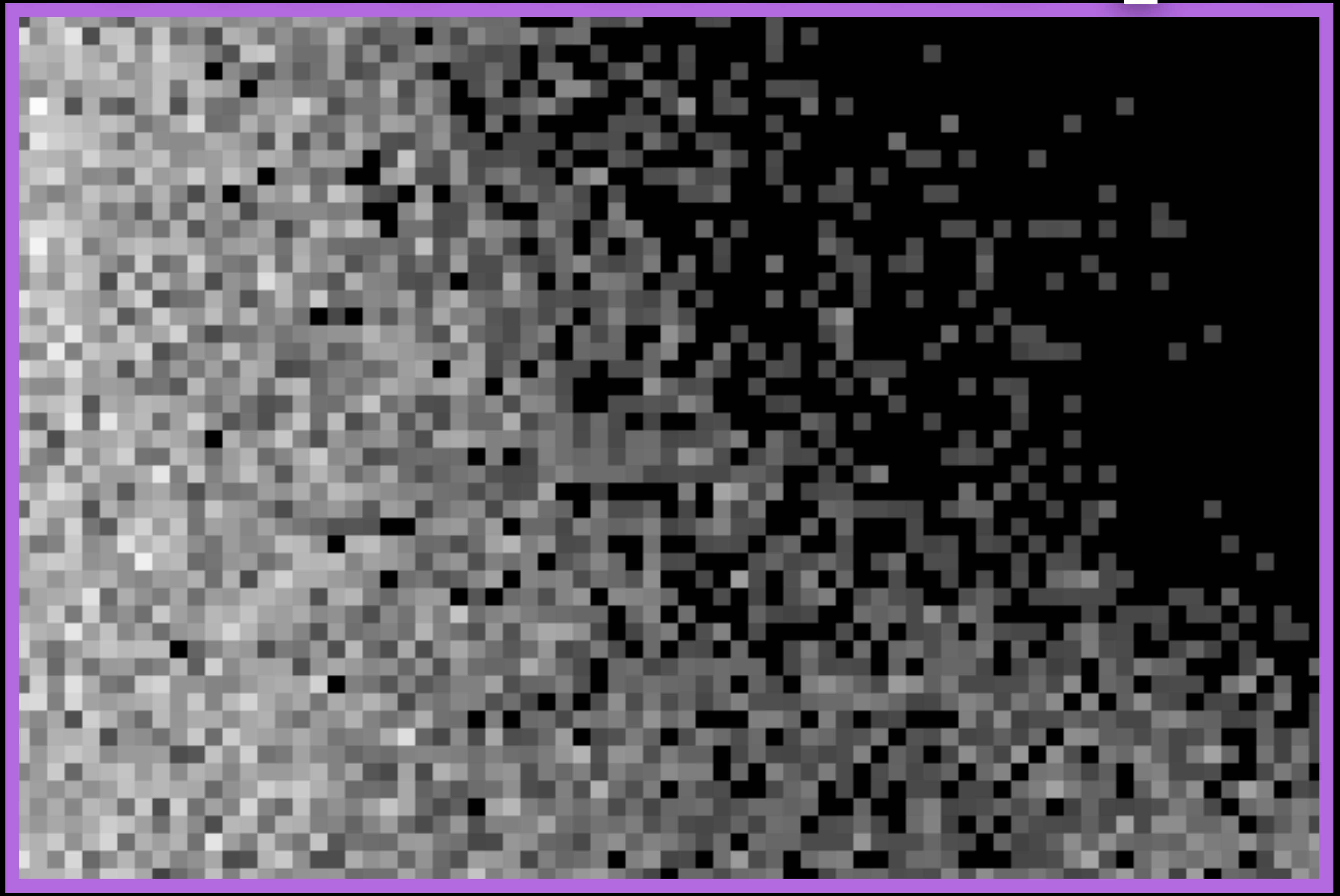


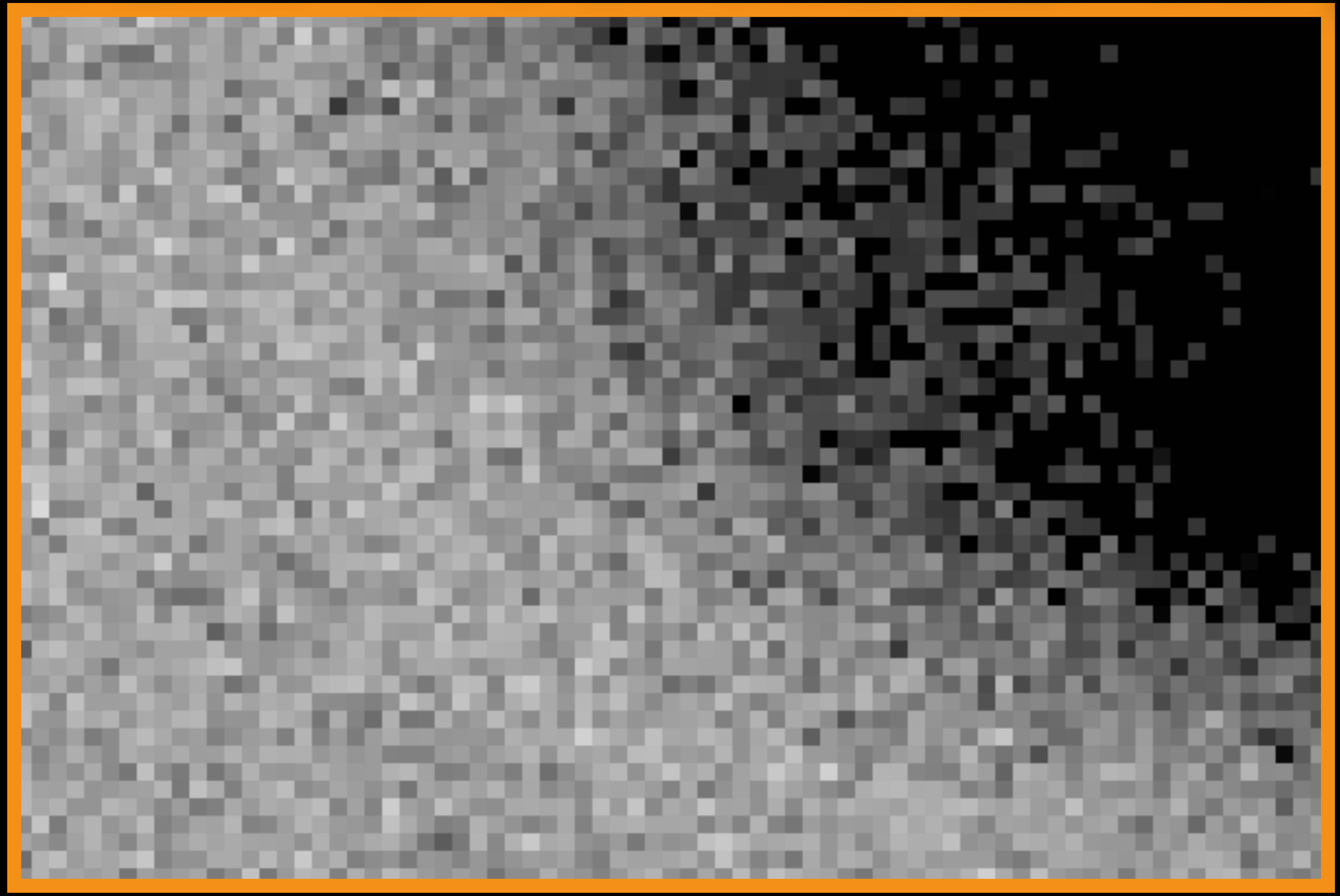


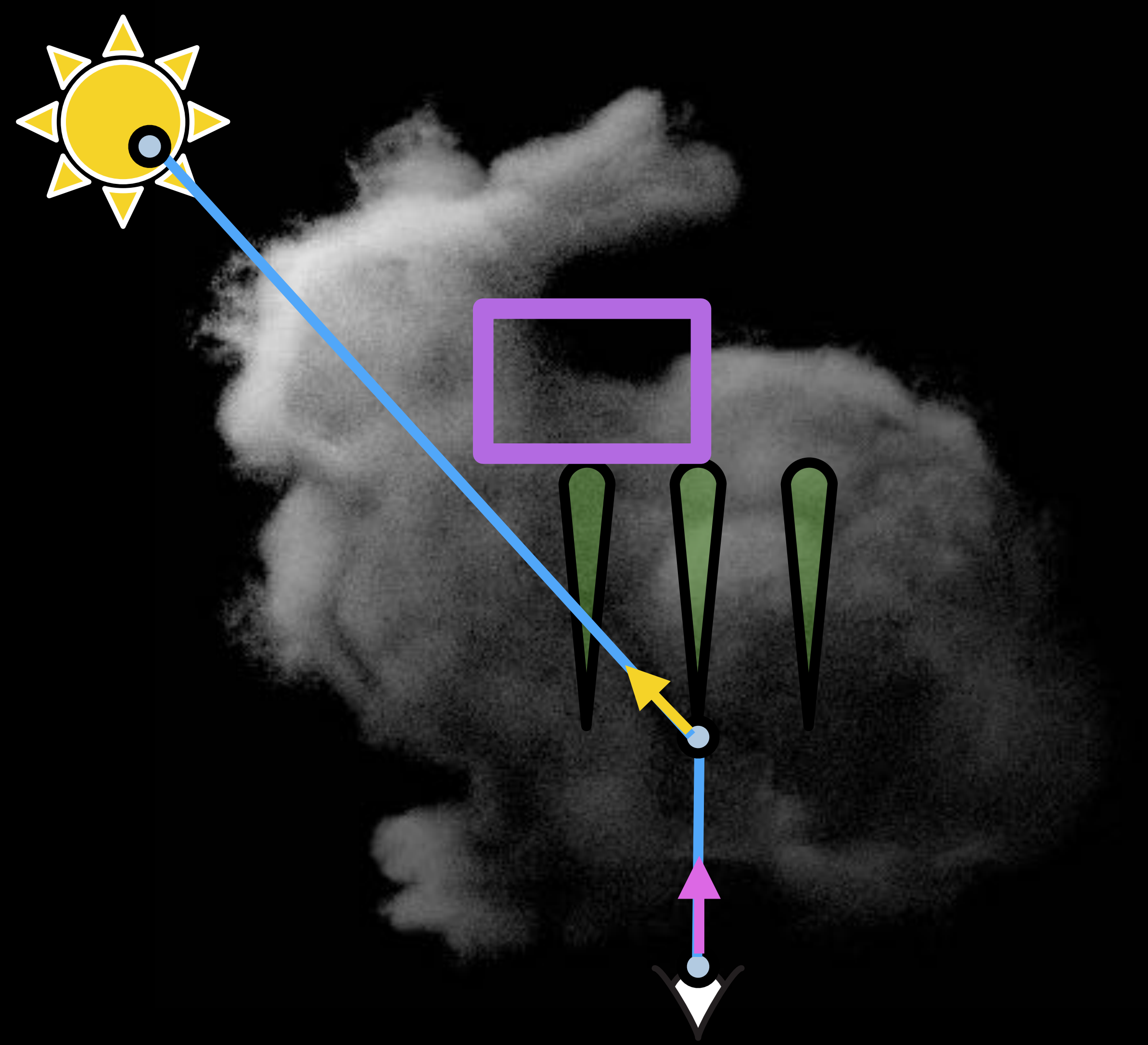
LTUV	Sampler	LTUV
10.3 B	Unidirectional	0.87 M
0.14 M	Ratio NEE [Novak 14]	519 M



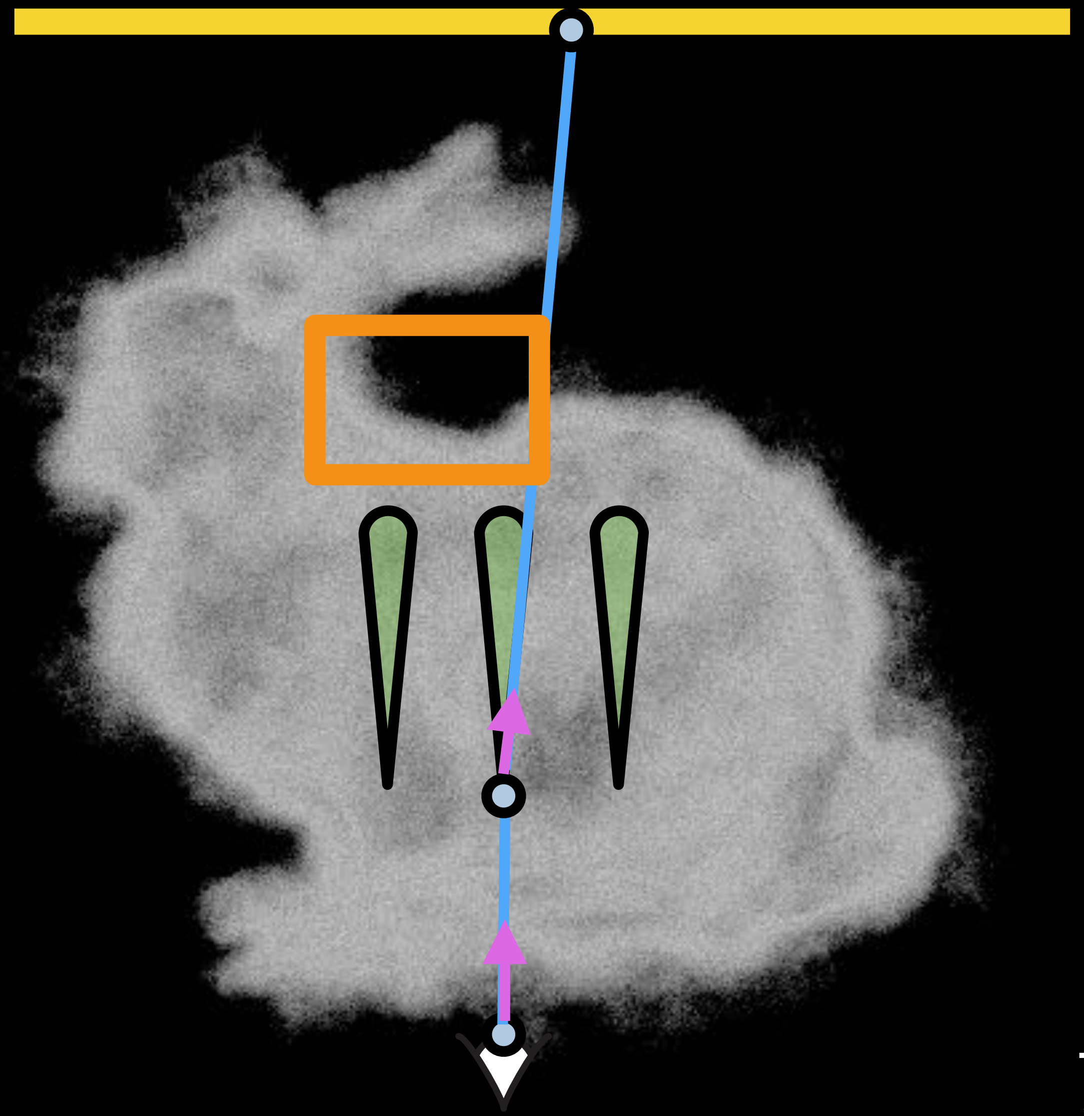
Directional MIS [Novak 14 / Kutz 17]



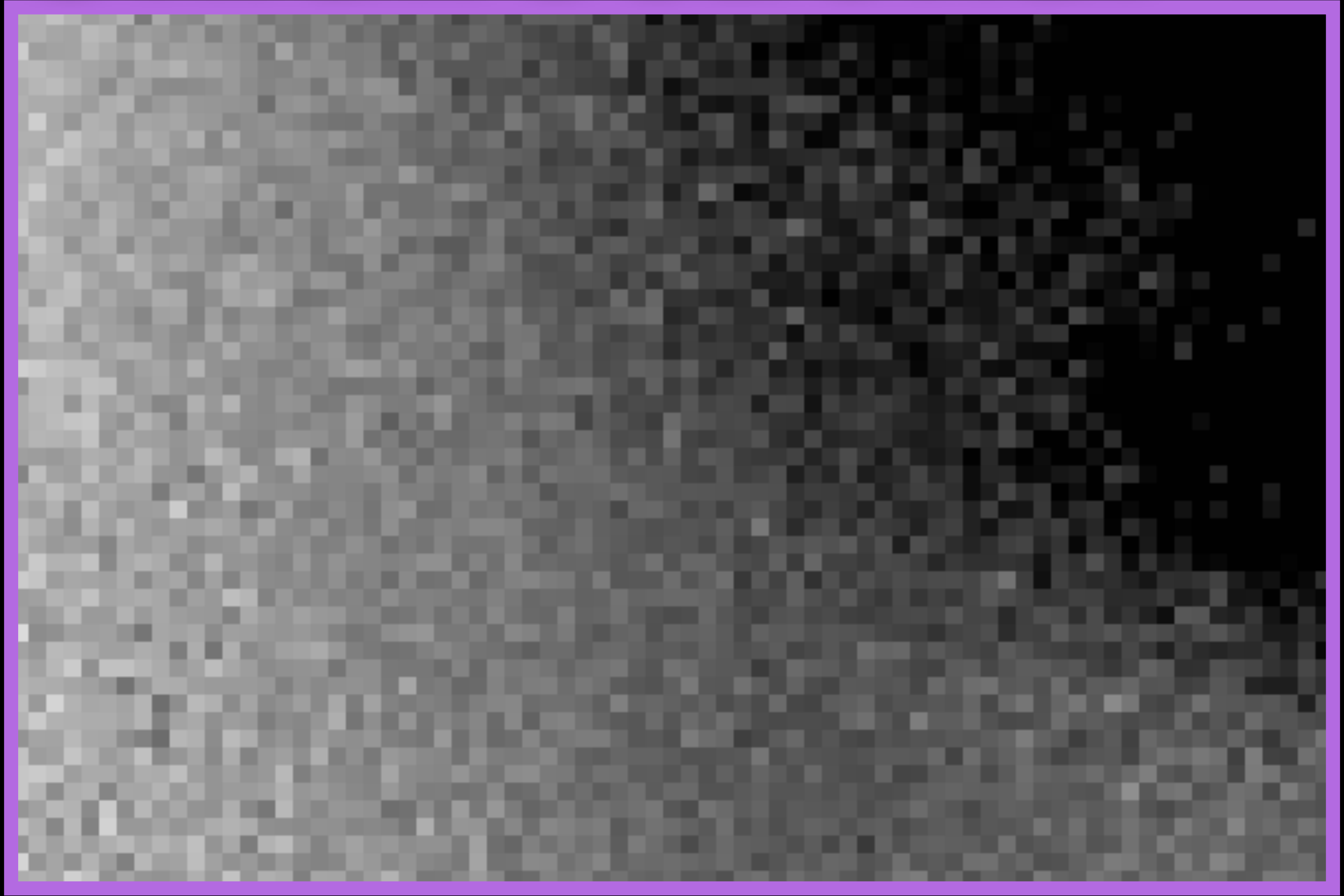


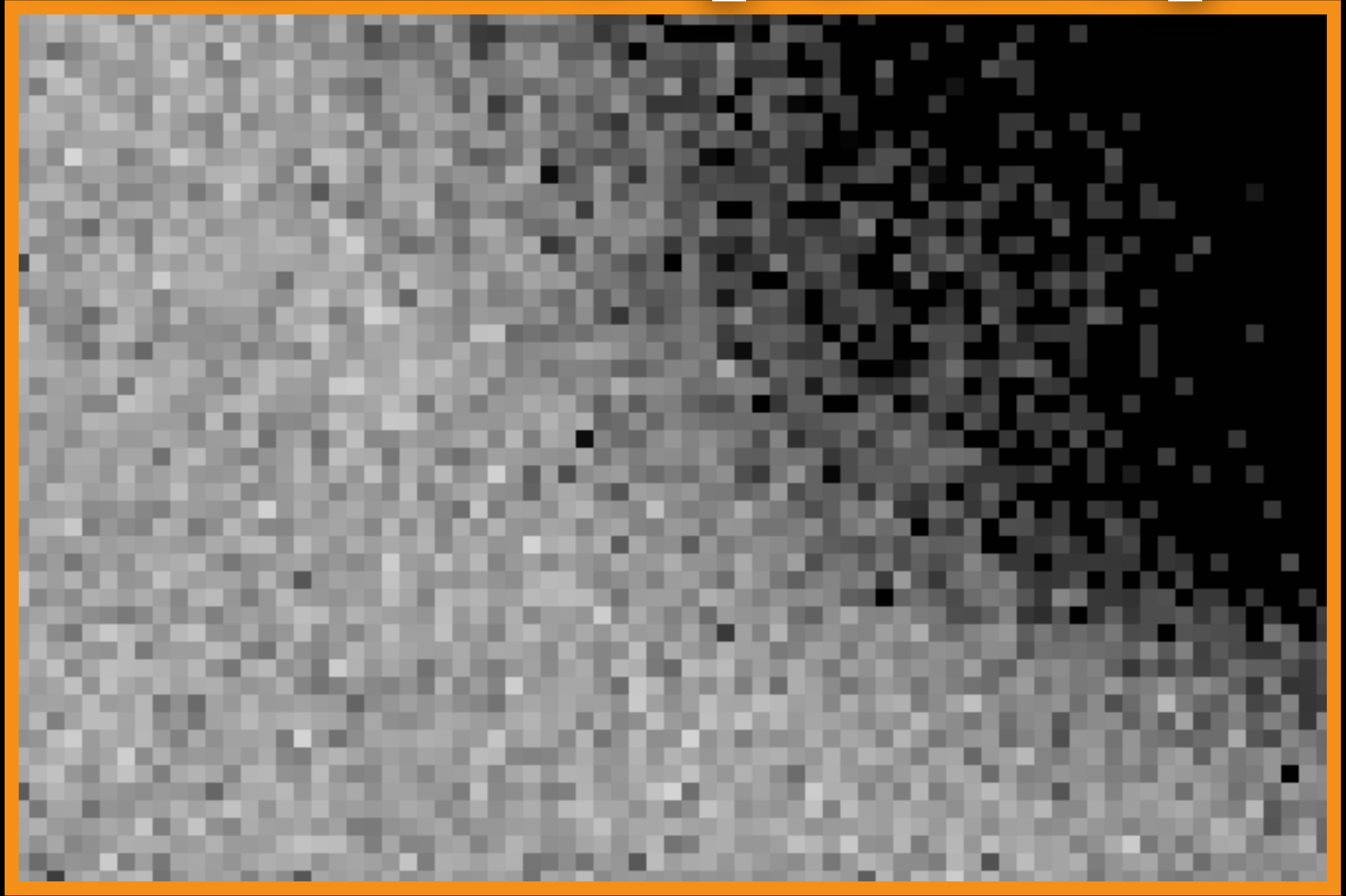


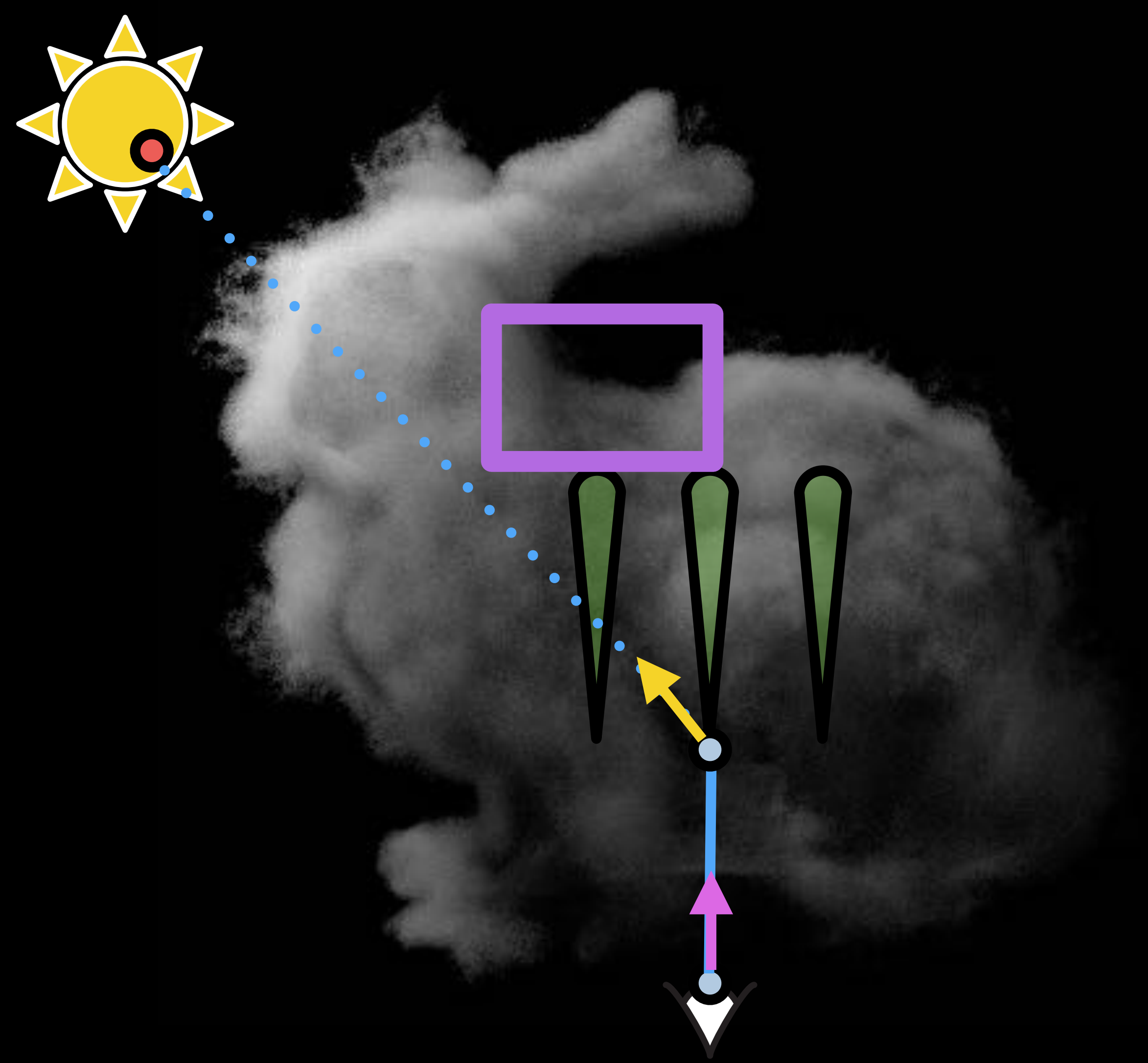
LTUV	Sampler	LTUV
10.3 B	Unidirectional	0.87 M
0.14 M	Ratio Next Event [Novak 14]	519 M
0.27 M	Directional MIS [Novak 14 / Kutz 17]	0.90 M



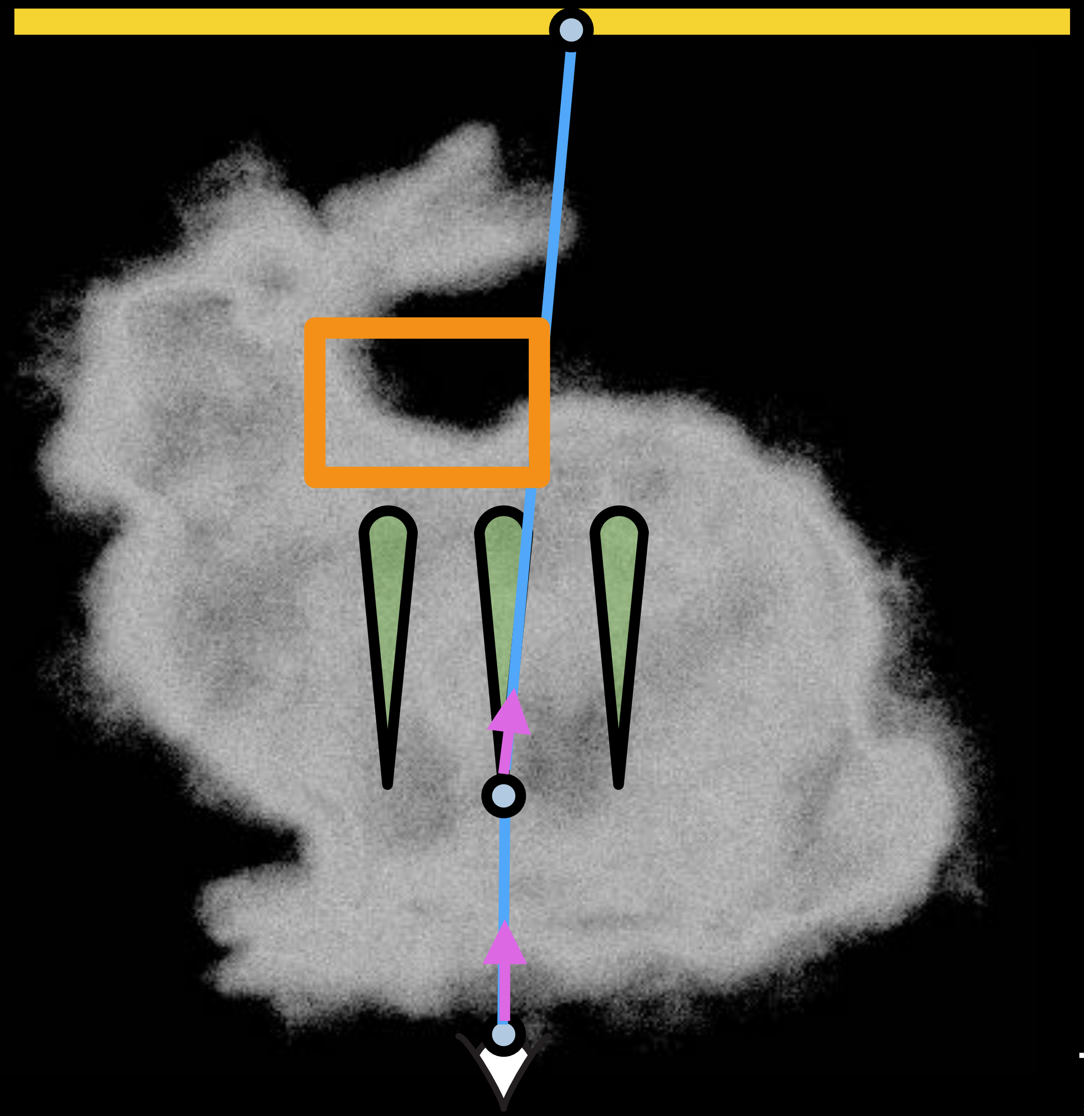
Unidirectional + NEE MIS [Ours]



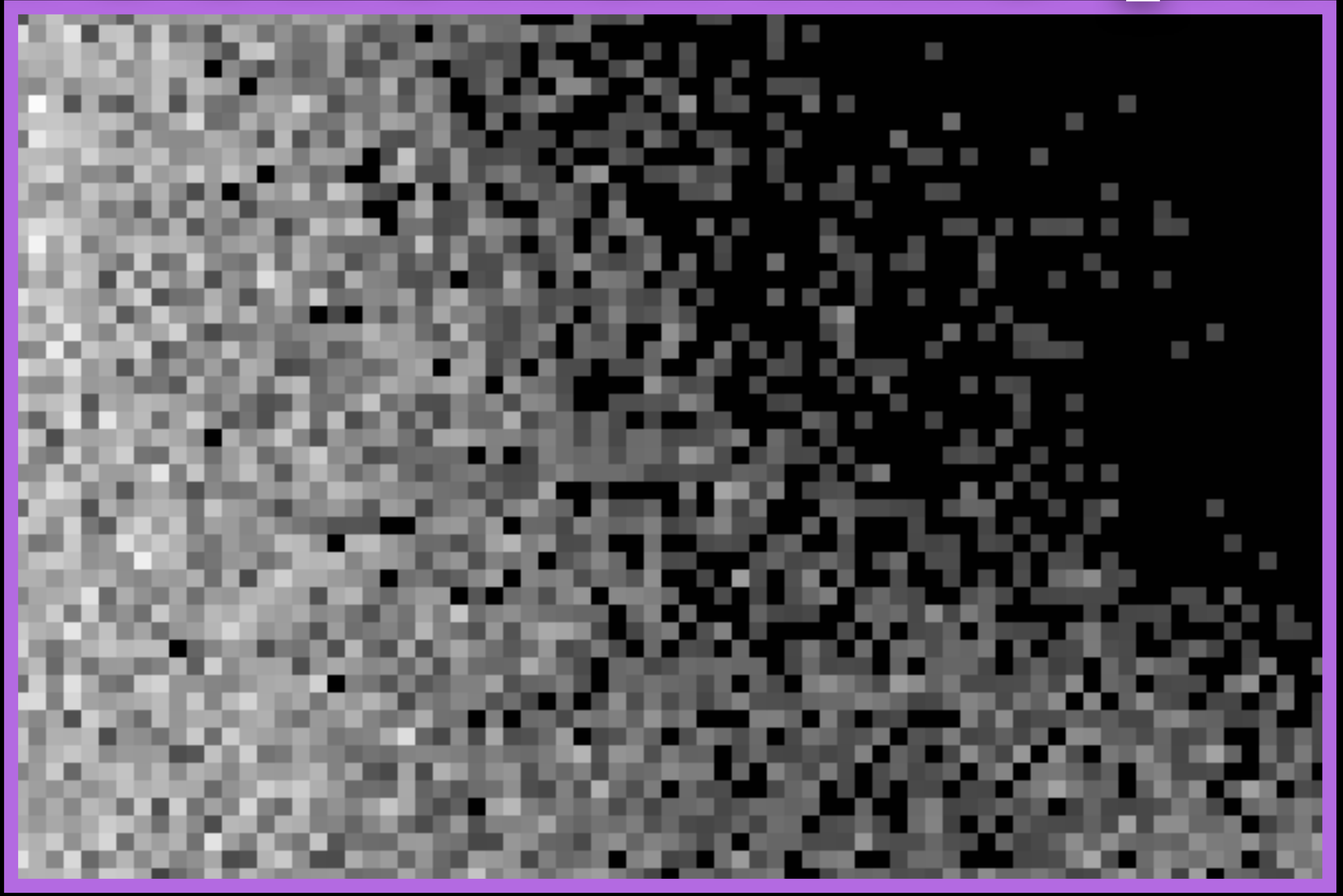


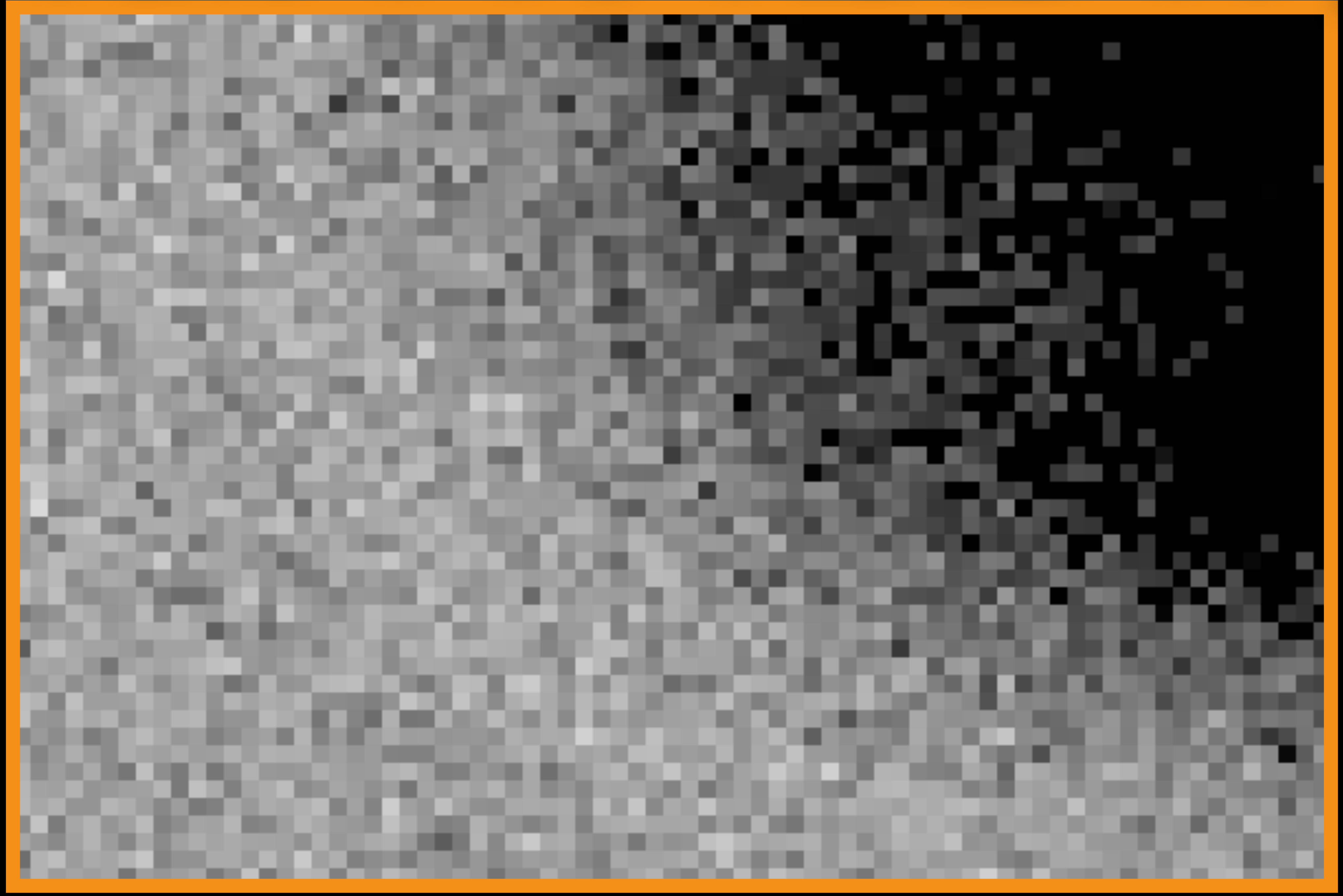


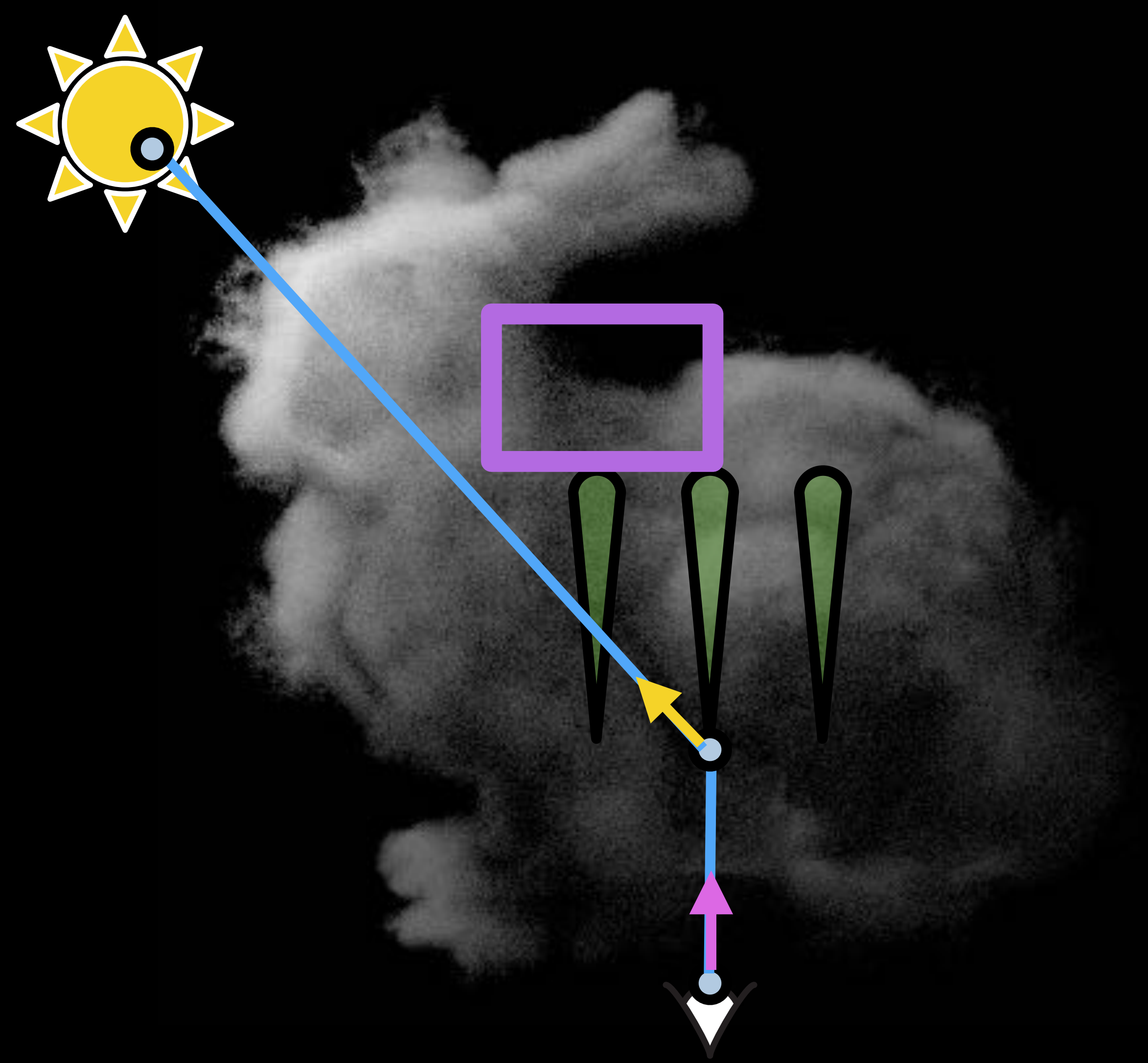
LTUV	Sampler	LTUV
10.3 B	Unidirectional	0.87 M
0.14 M	Ratio Next Event [Novak 14]	519 M
0.27 M	Directional MIS [Novak 14 / Kutz 17]	0.90 M
0.16 M	Unidir. + NEE MIS [Ours]	0.94 M



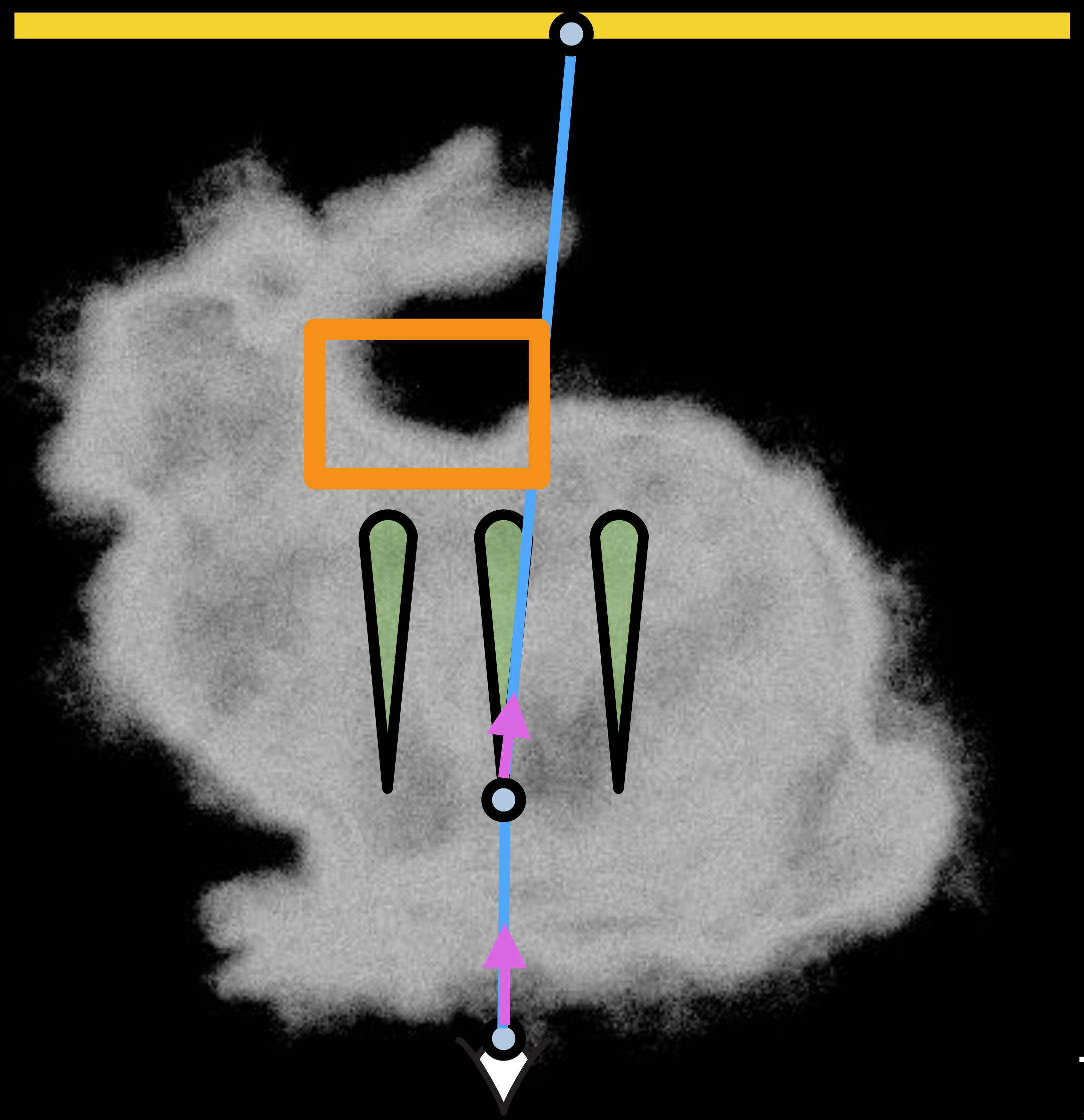
Directional MIS [Novak 14 / Kutz 17]



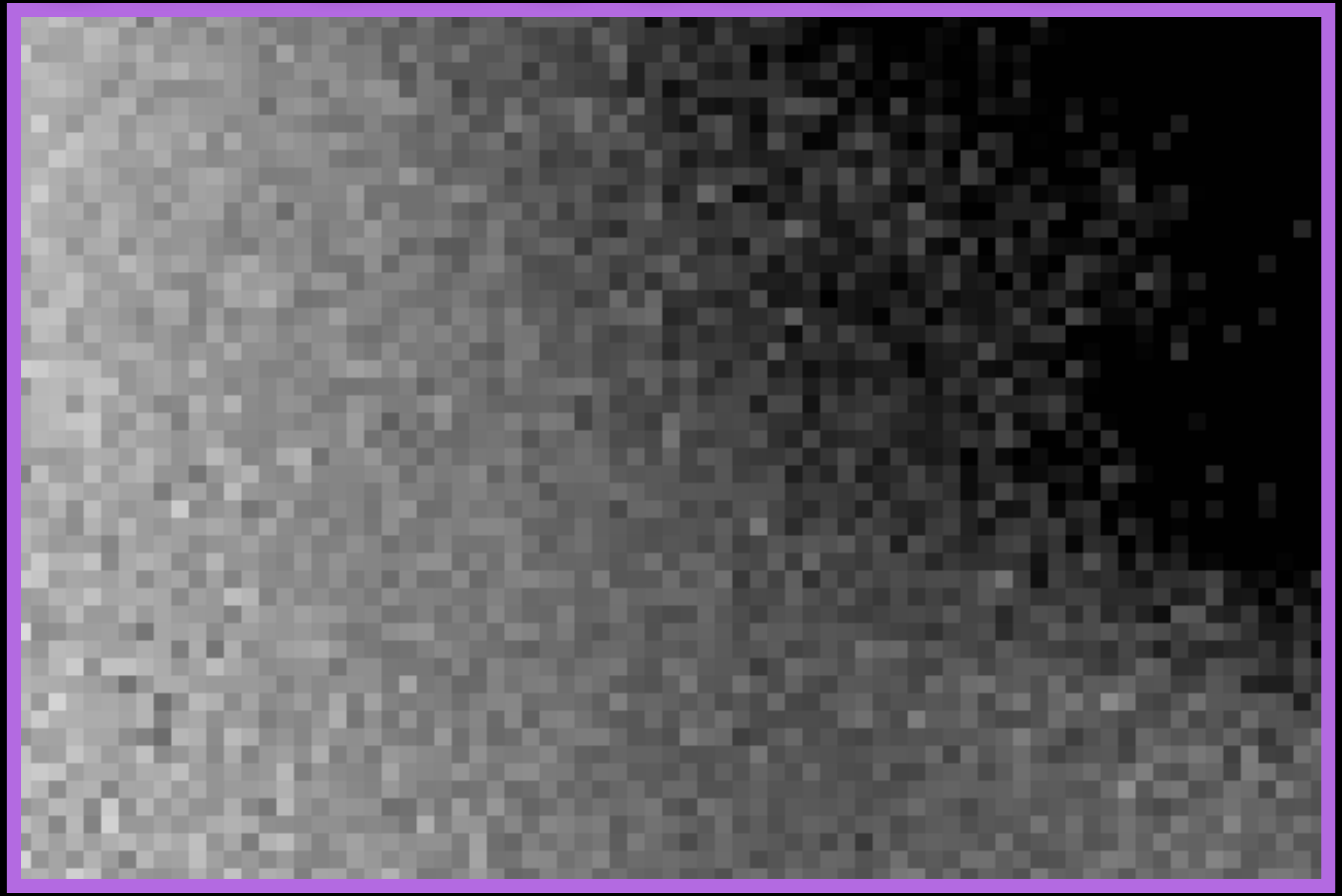


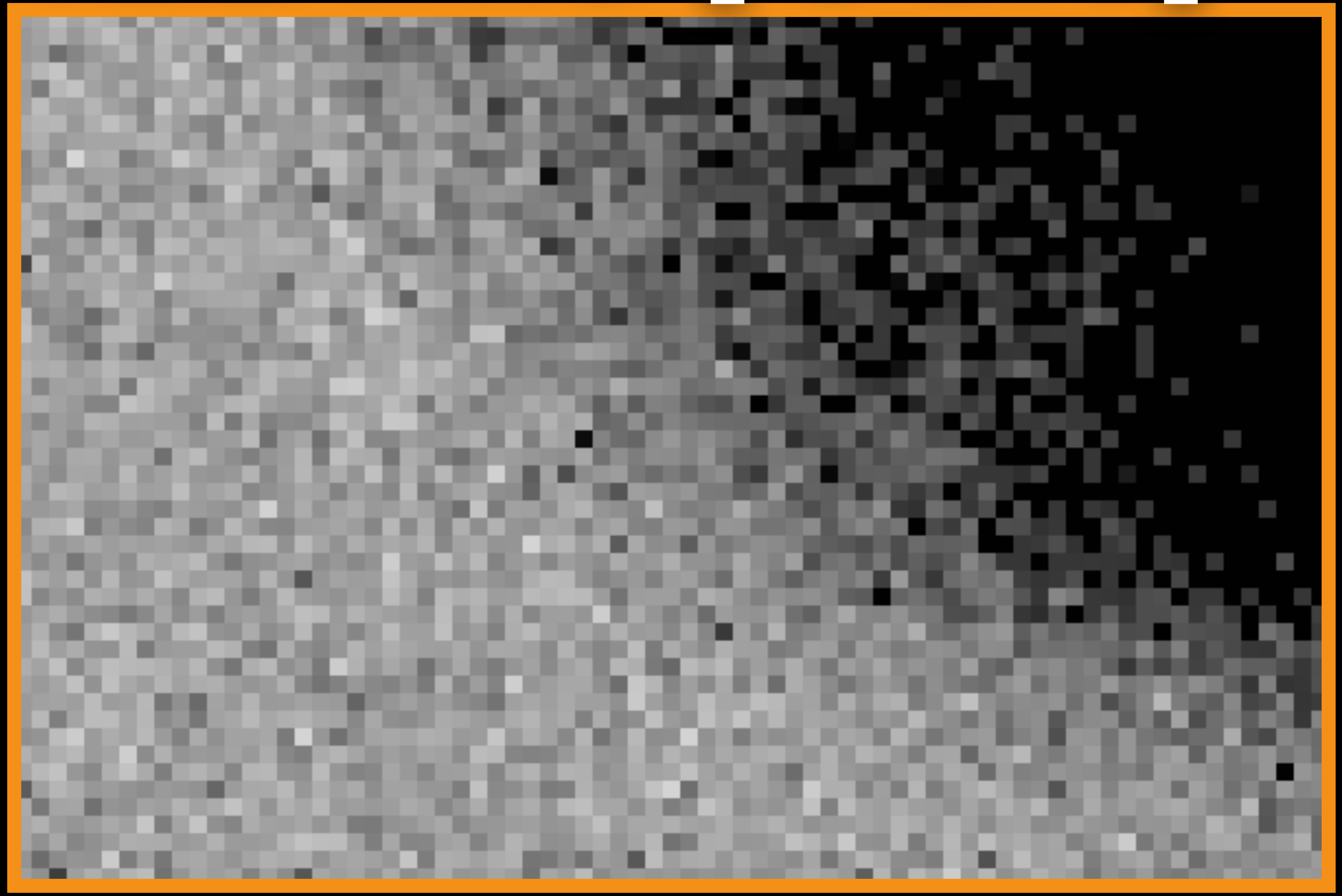


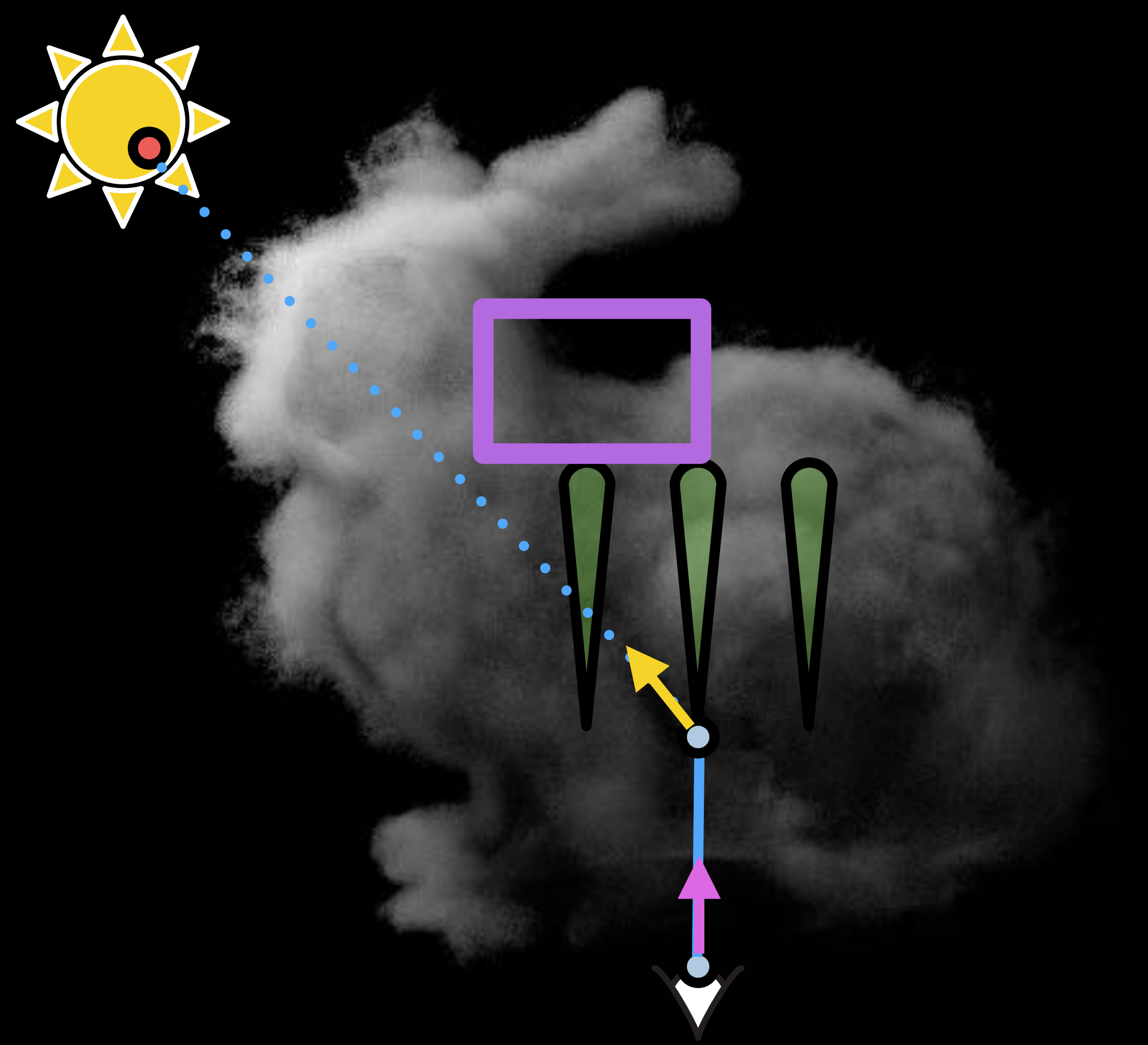
LTUV	Sampler	LTUV
10.3 B	Unidirectional	0.87 M
0.14 M	Ratio Next Event [Novak 14]	519 M
0.27 M	Directional MIS [Novak 14 / Kutz 17]	0.90 M
0.16 M	Unidir. + NEE MIS [Ours]	0.94 M



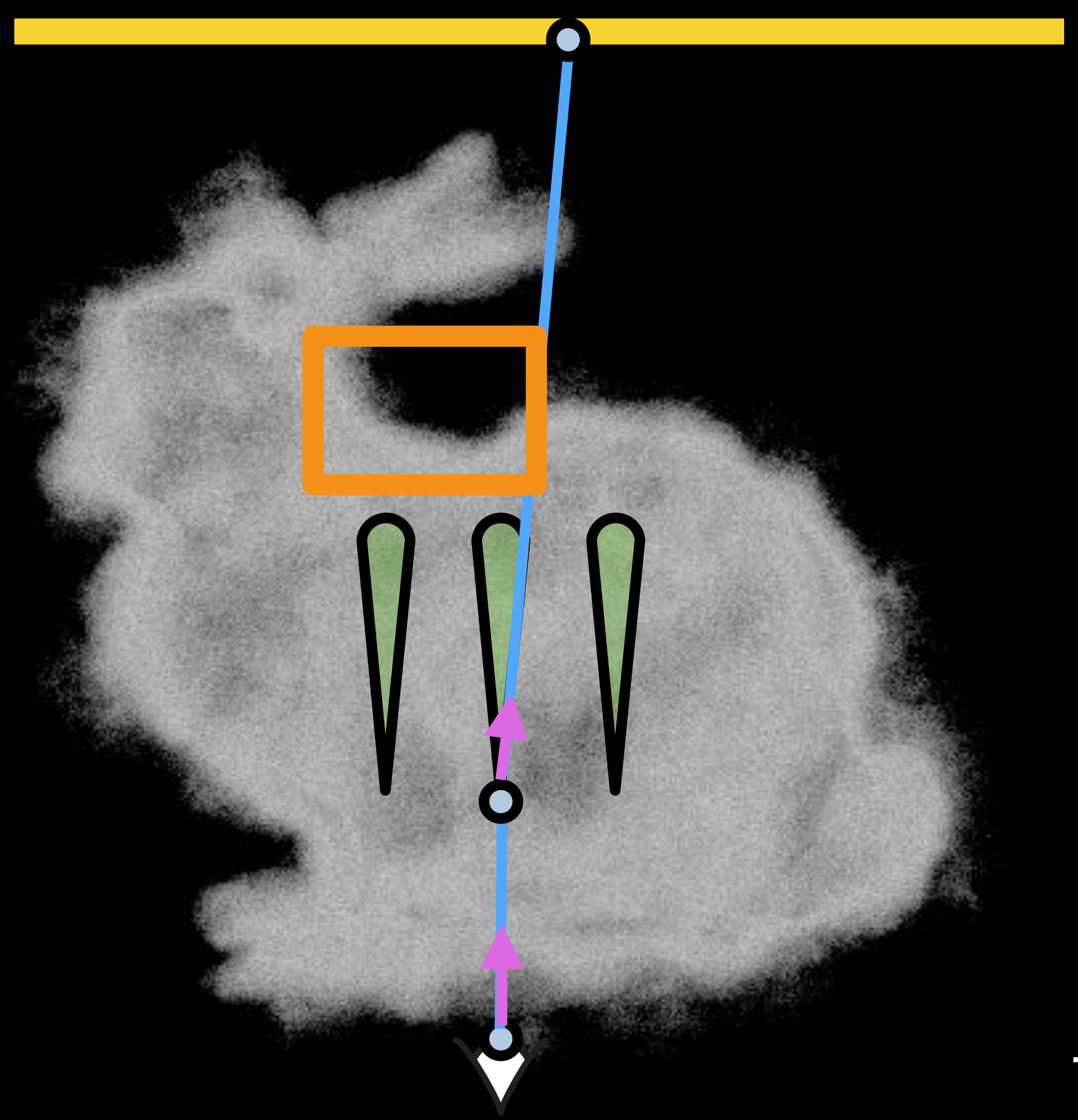
Unidirectional + NEE MIS [Ours]



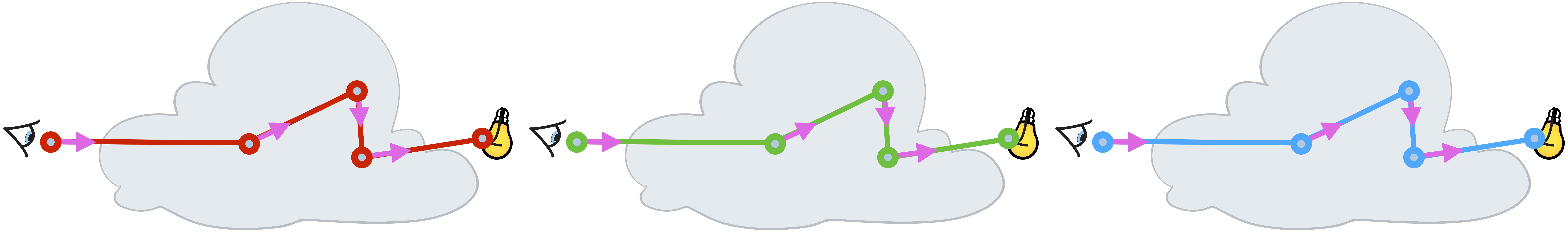




LTUV	Sampler	LTUV
10.3 B	Unidirectional	0.87 M
0.14 M	Ratio Next Event [Novak 14]	519 M
0.27 M	Directional MIS [Novak 14 / Kutz 17]	0.90 M
0.16 M	Unidir. + NEE MIS [Ours]	0.94 M

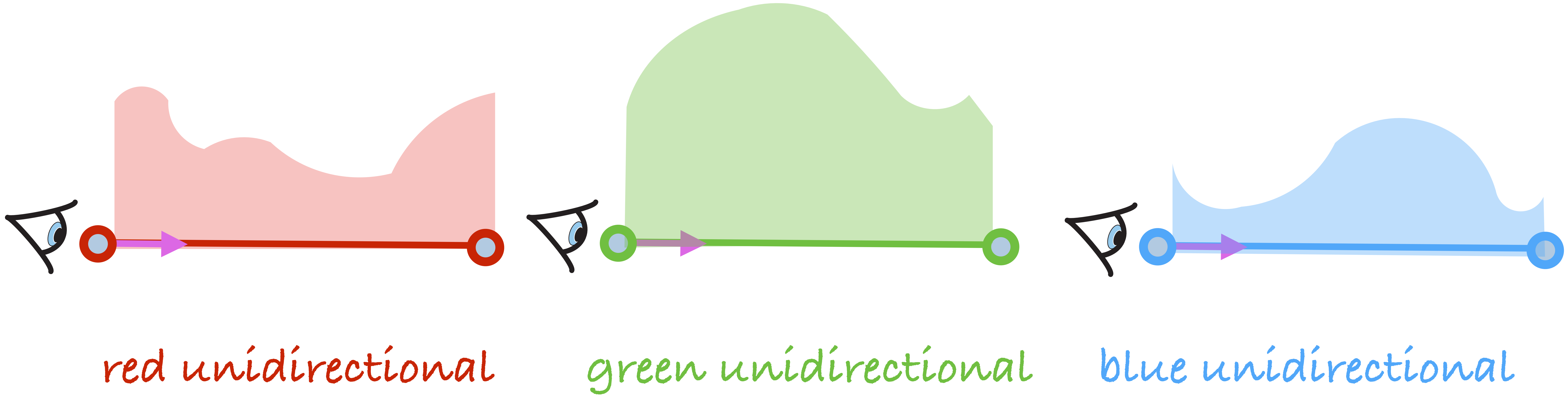


Spatially and Spectrally Varying Media

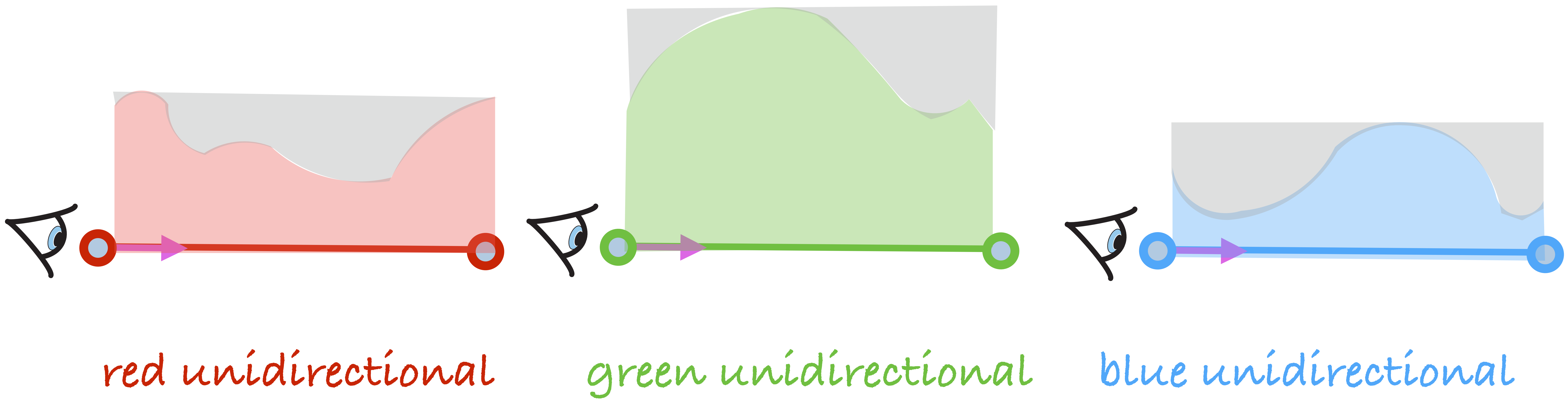


red unidirectional | green unidirectional | blue unidirectional

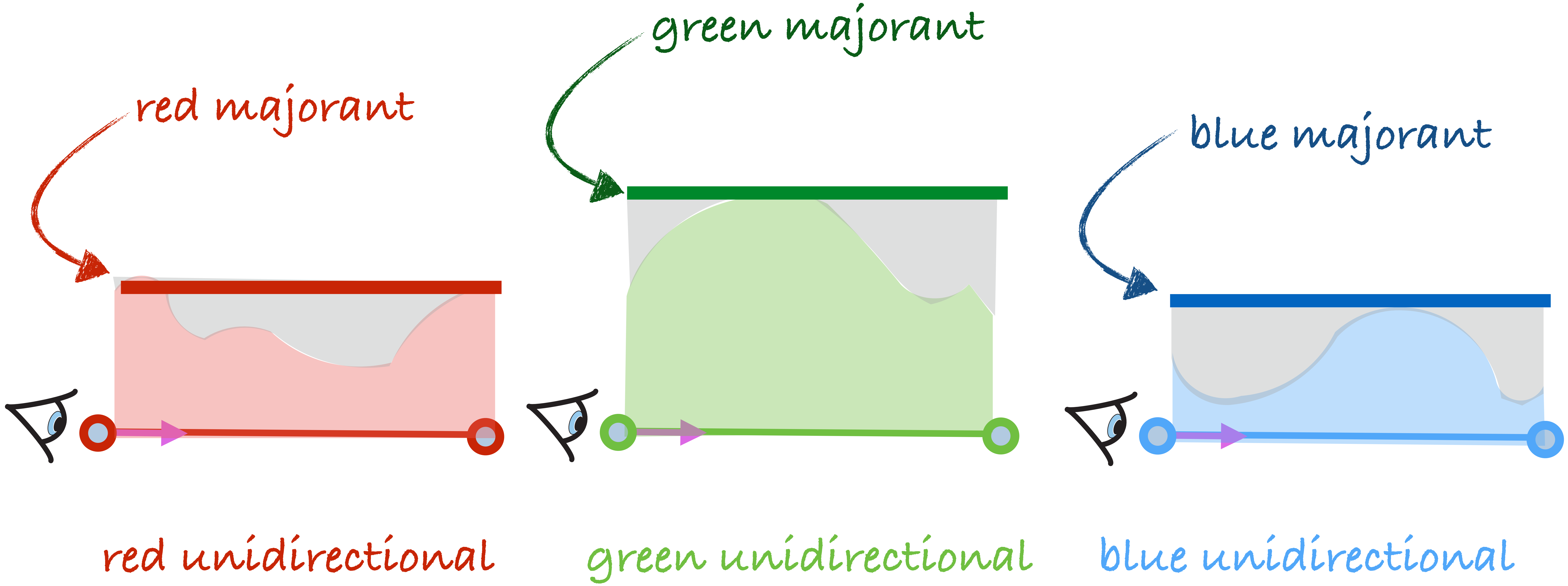
Spatially and Spectrally Varying Media



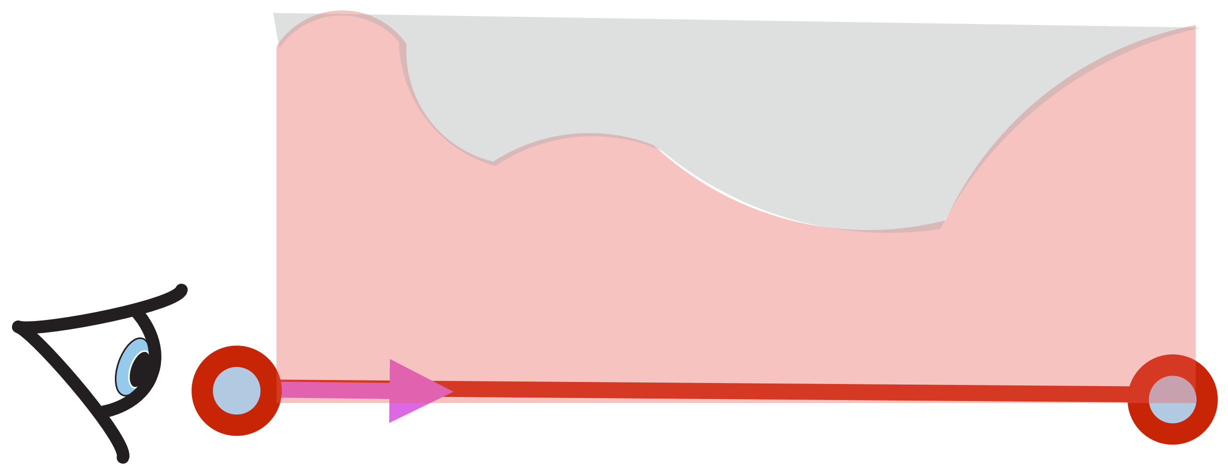
Spatially and Spectrally Varying Media



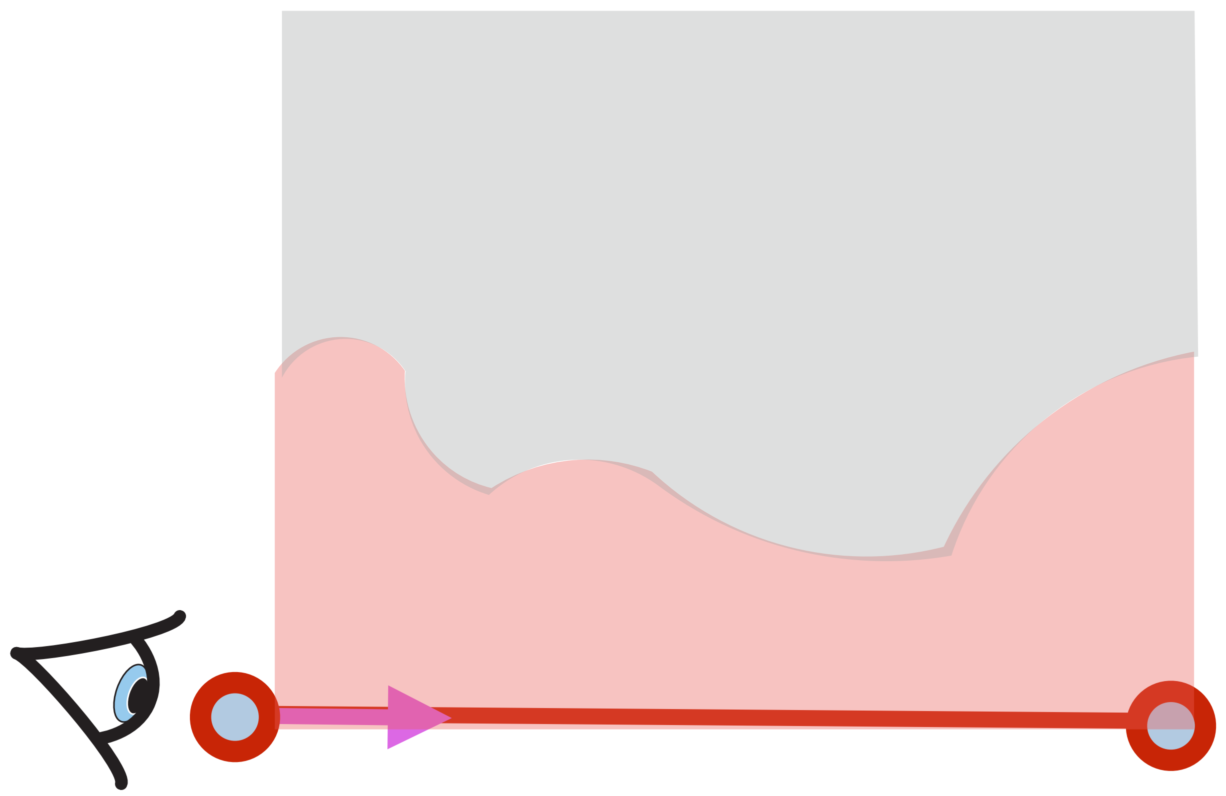
Spatially and Spectrally Varying Media



Spatially and Spectrally Varying Media

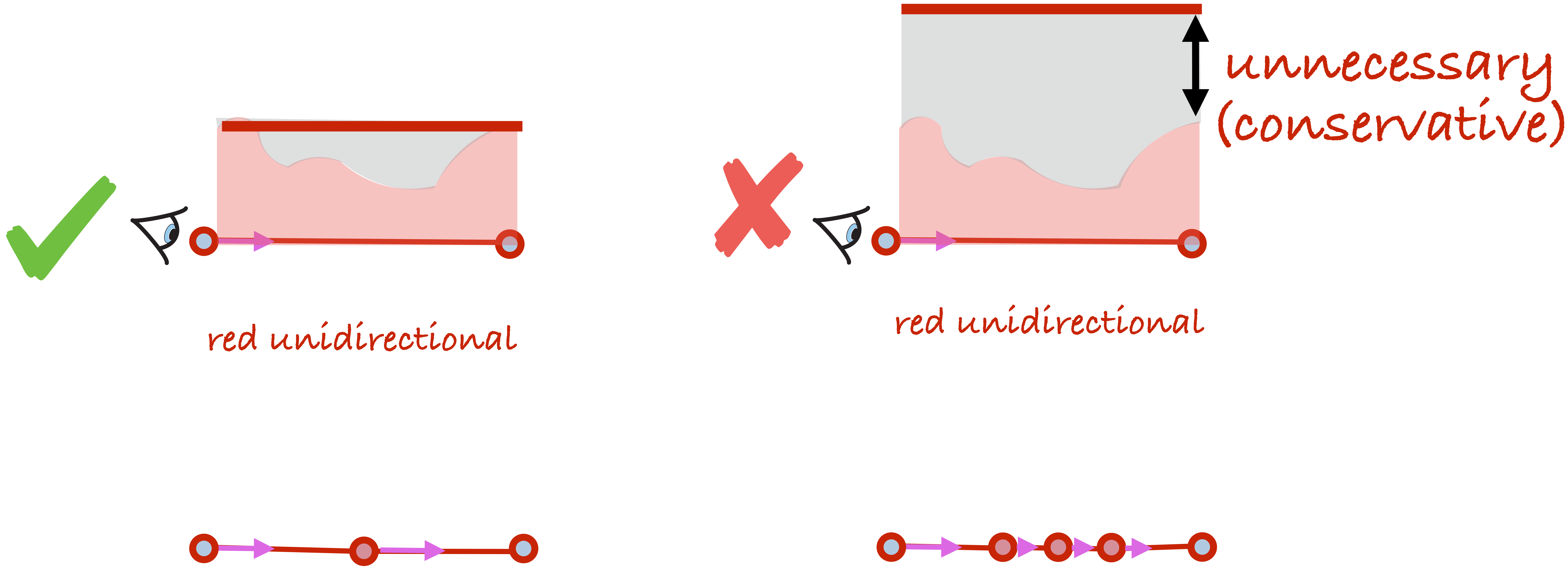


red unidirectional

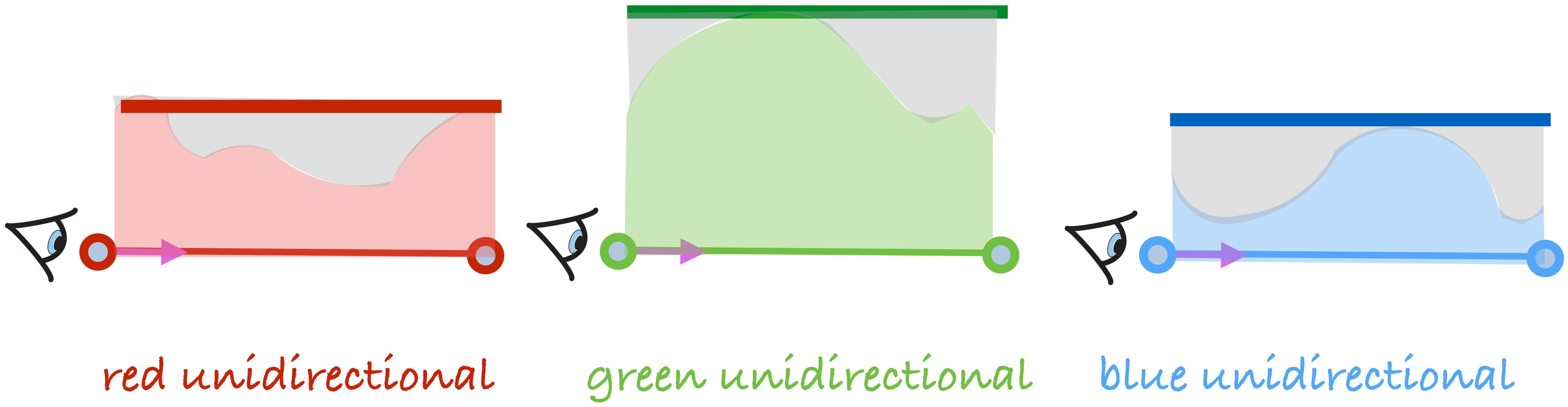


red unidirectional

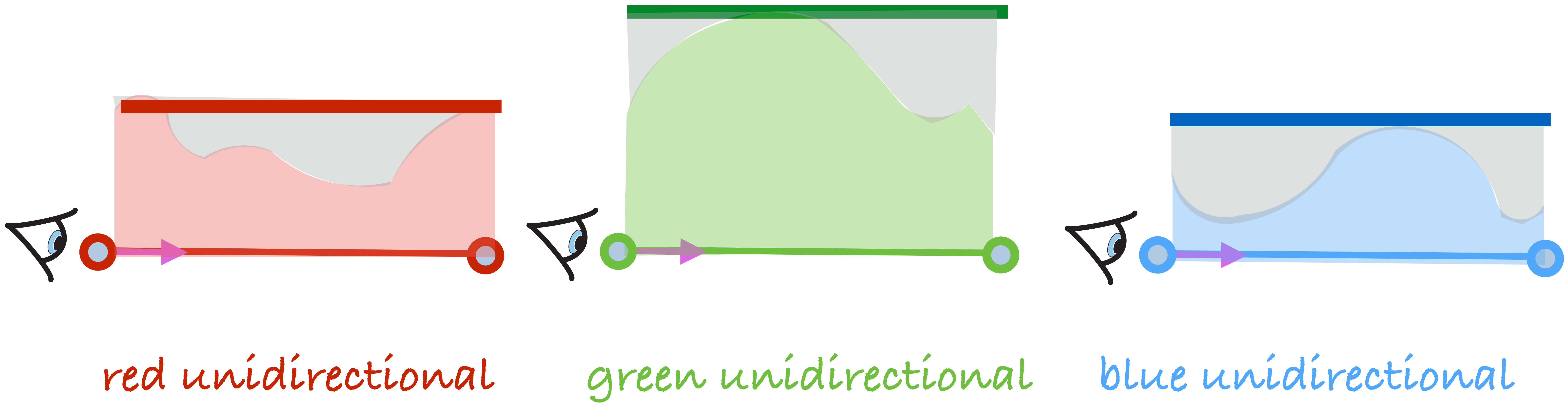
Spatially and Spectrally Varying Media



Independent Tracking

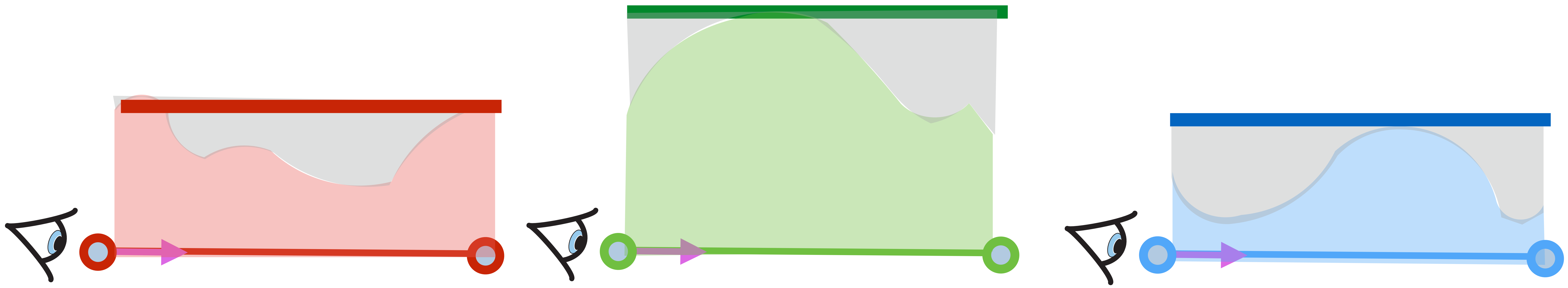


Independent Tracking



✓ tight majorants

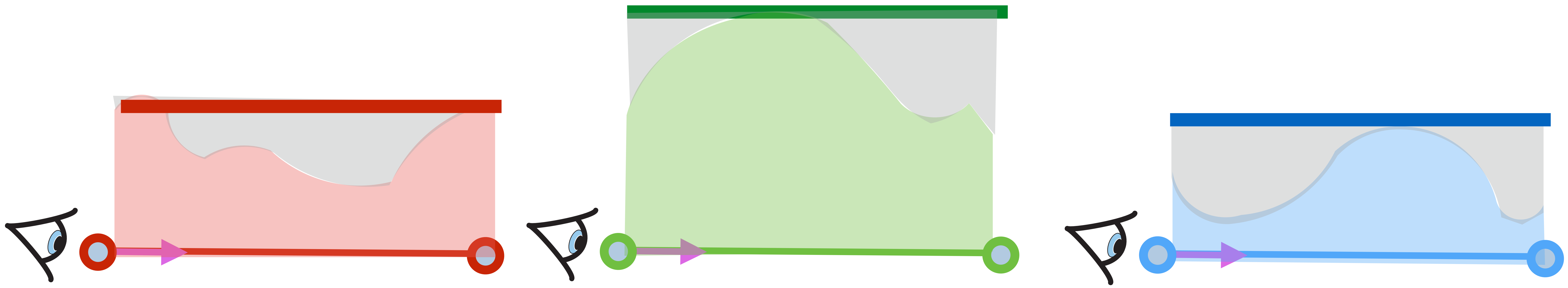
Independent Tracking



✓ tight majorants

✗ render wavelengths separately

Independent Tracking



red unidirectional

green unidirectional

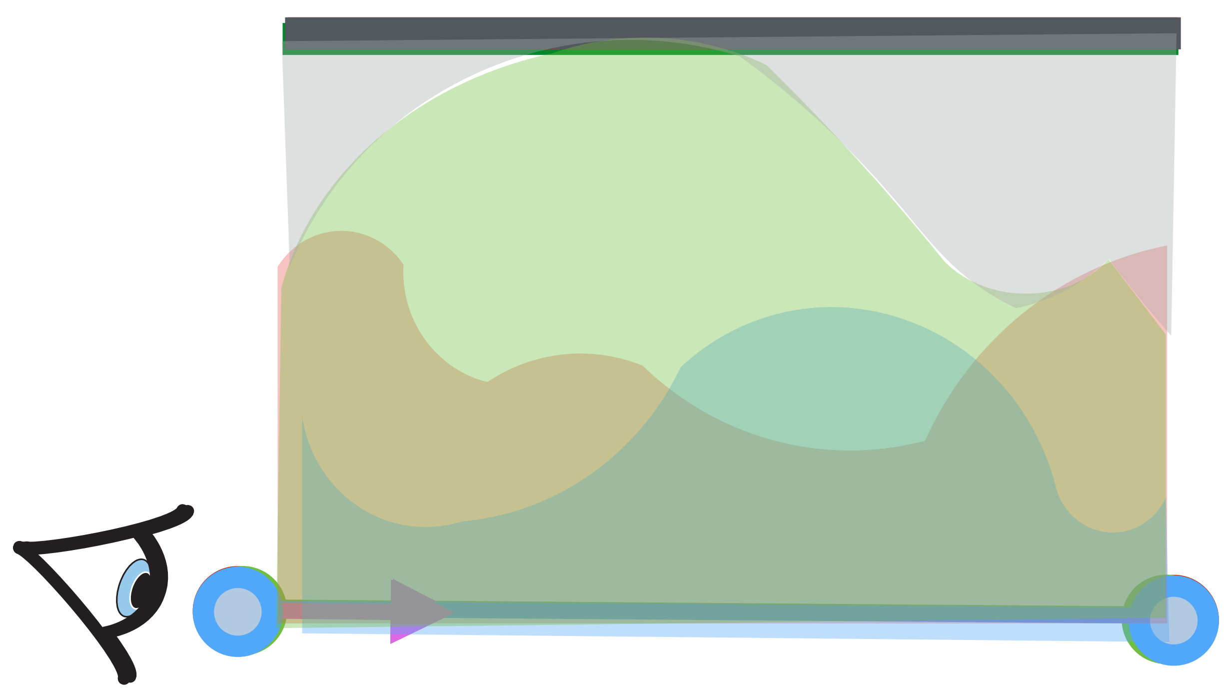
blue unidirectional

✓ tight majorants

✗ render wavelengths separately

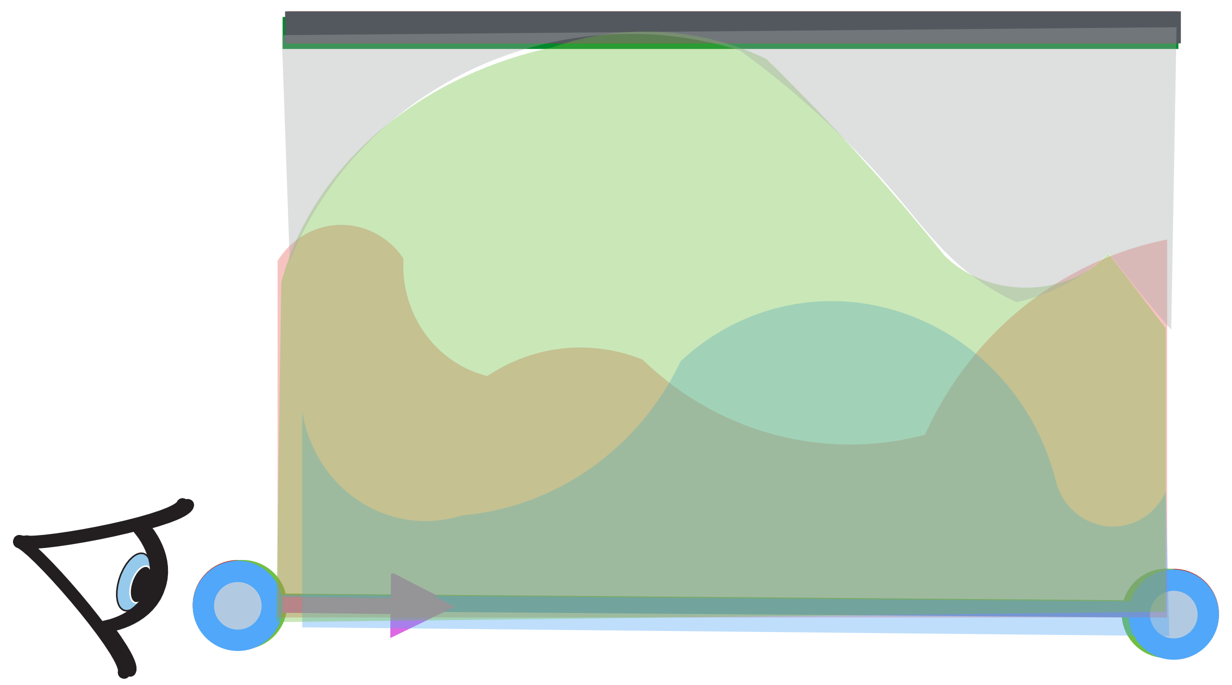
✗ color noise

Spectral Tracking [Kutz 17]



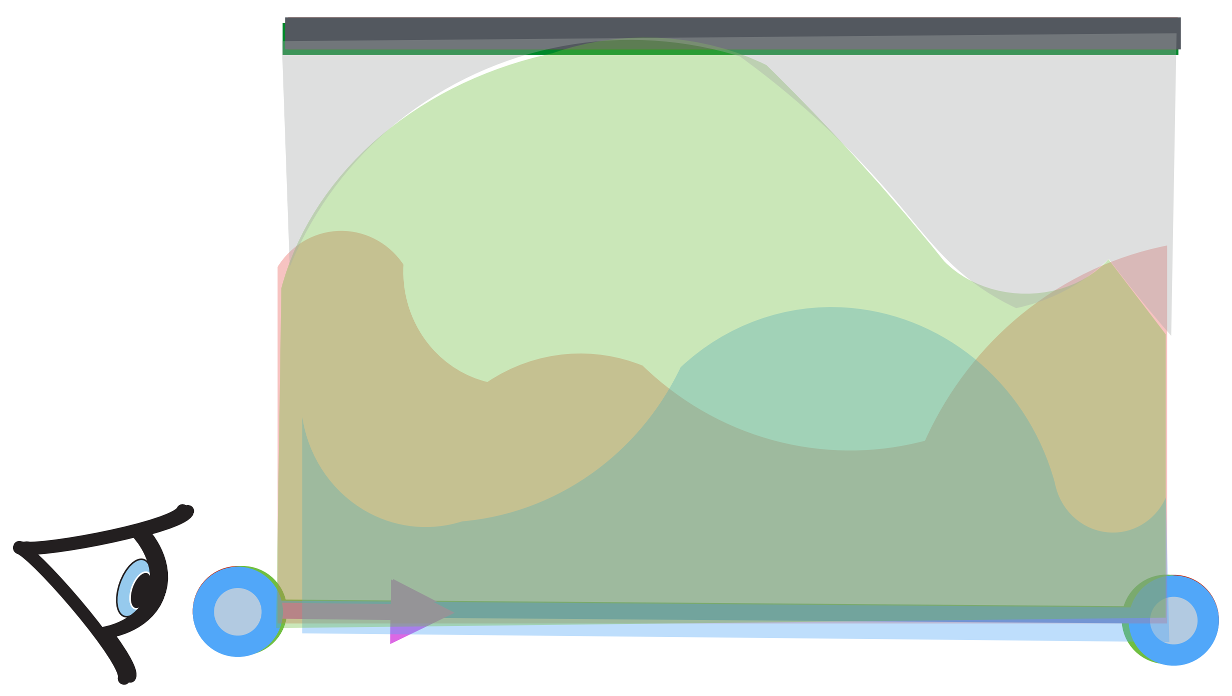
Spectral Tracker

Spectral Tracking [Kutz 17]



Spectral Tracker

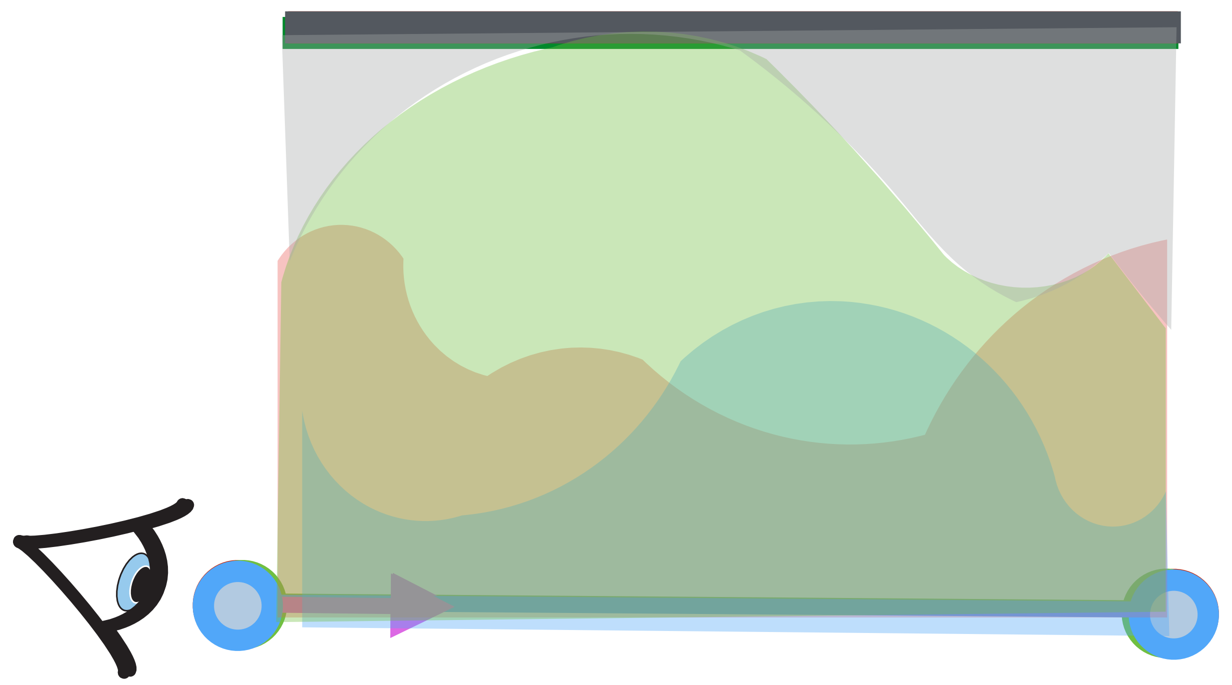
Spectral Tracking [Kutz 17]



Spectral Tracker

✓ render wavelengths together

Spectral Tracking [Kutz 17]

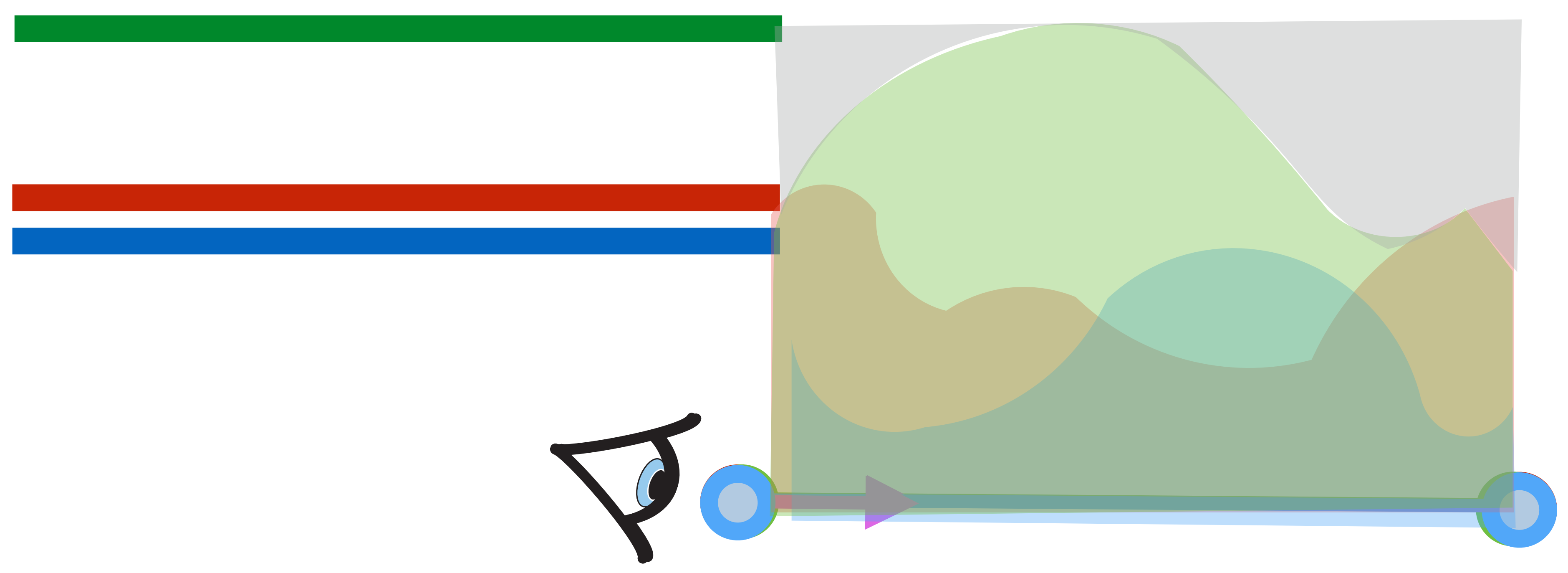


Spectral Tracker

✓ render wavelengths together

✓ avoids color noise

Spectral Tracking [Kutz 17]

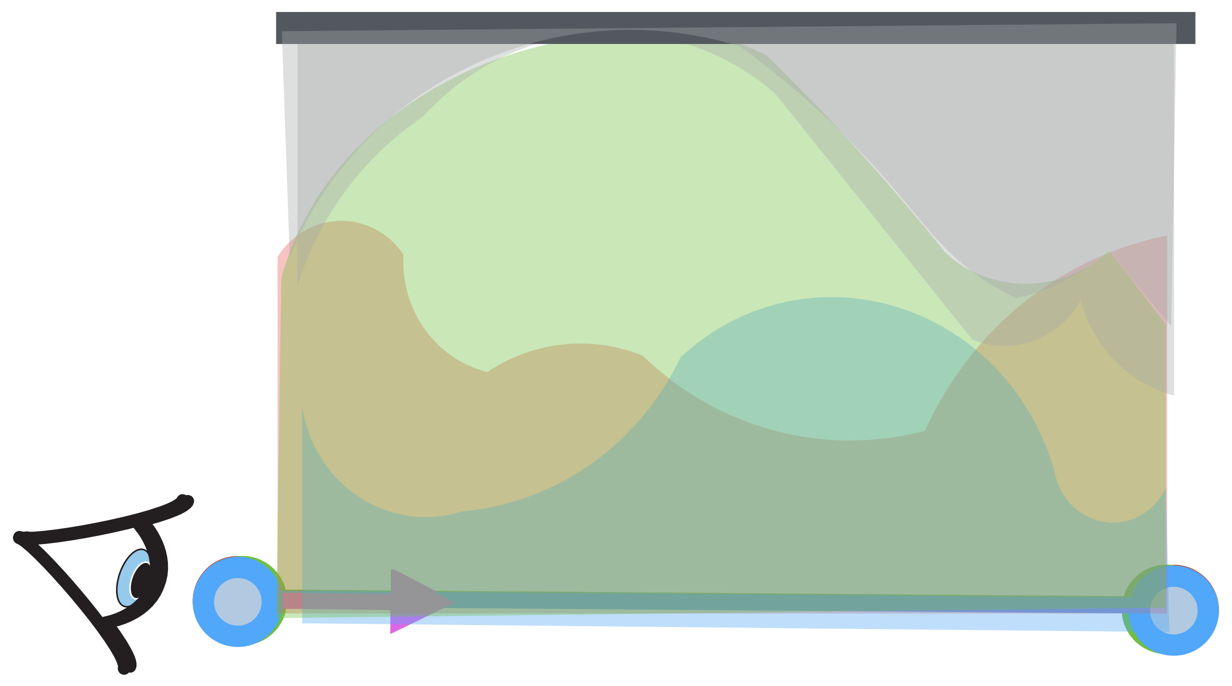


✗ single bounding majorant

✓ render wavelengths together

✓ avoids color noise

Spectral Tracking [Kutz 17]



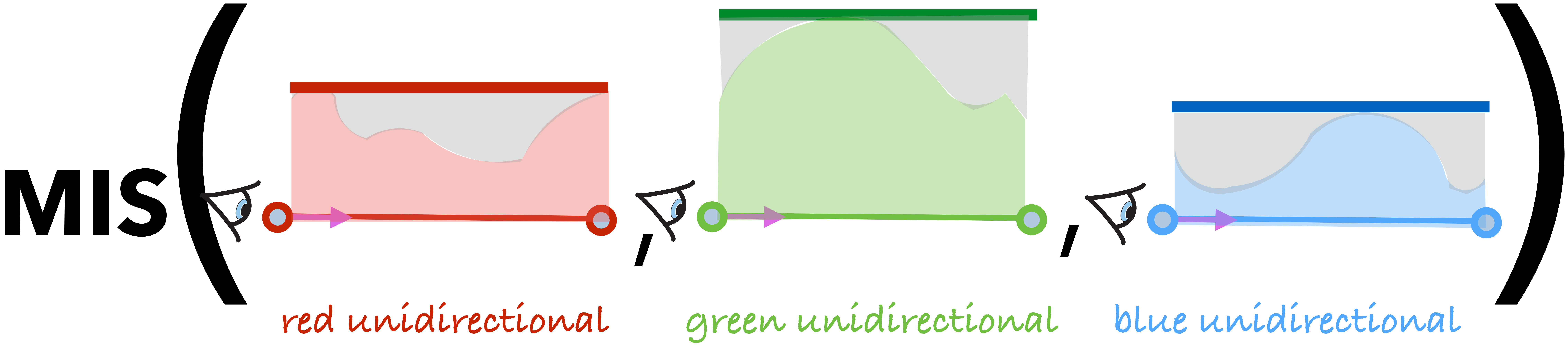
Spectral Tracker

✗ single bounding majorant

✓ render wavelengths together

✓ avoids color noise

Spectral MIS [Ours]

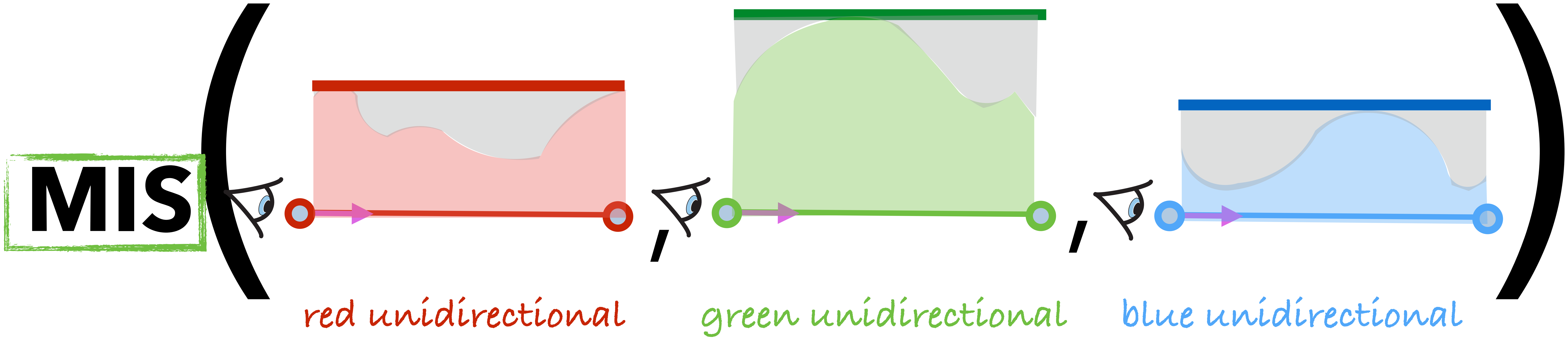


✓ tight majorants (choose any majorants)

✓ render wavelengths together

✓ avoids color noise

Spectral MIS [Ours]

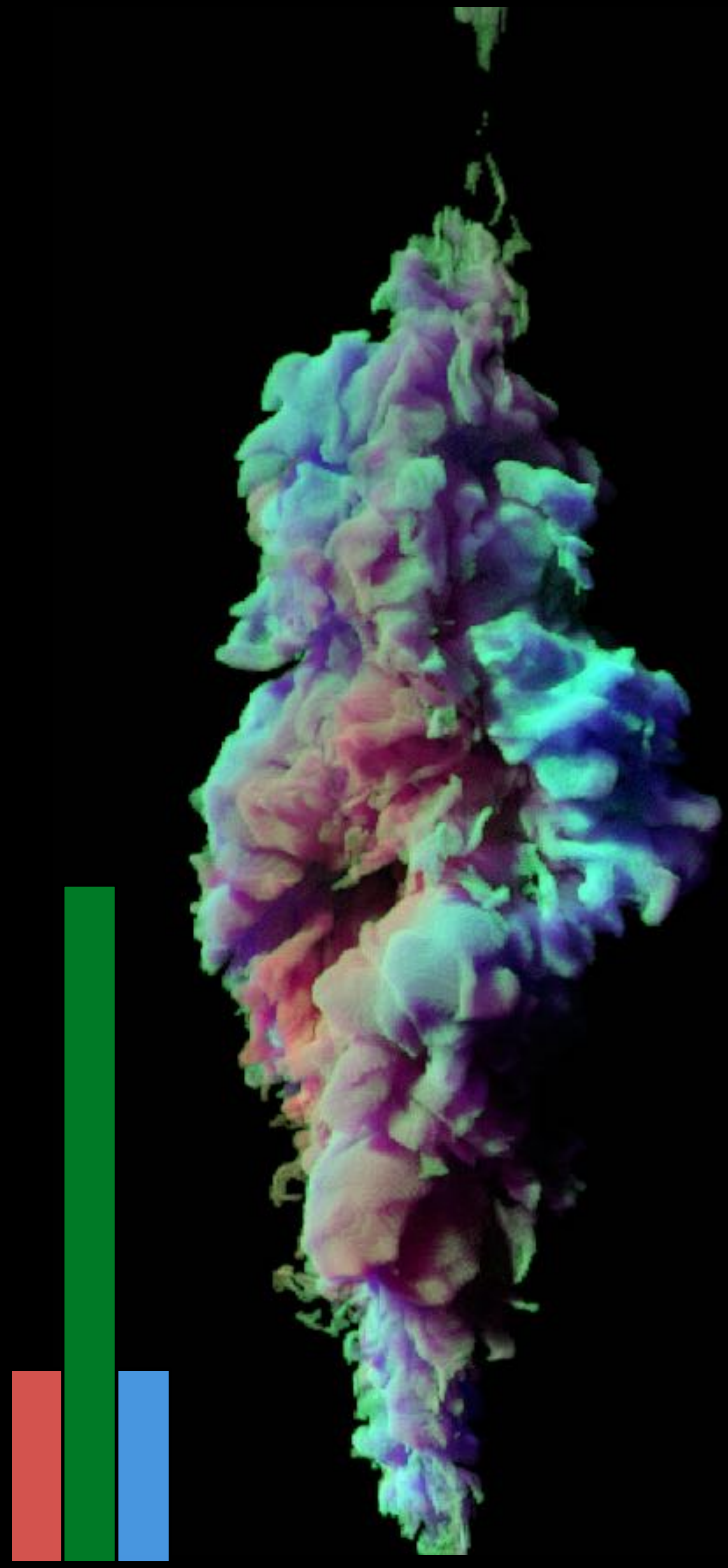


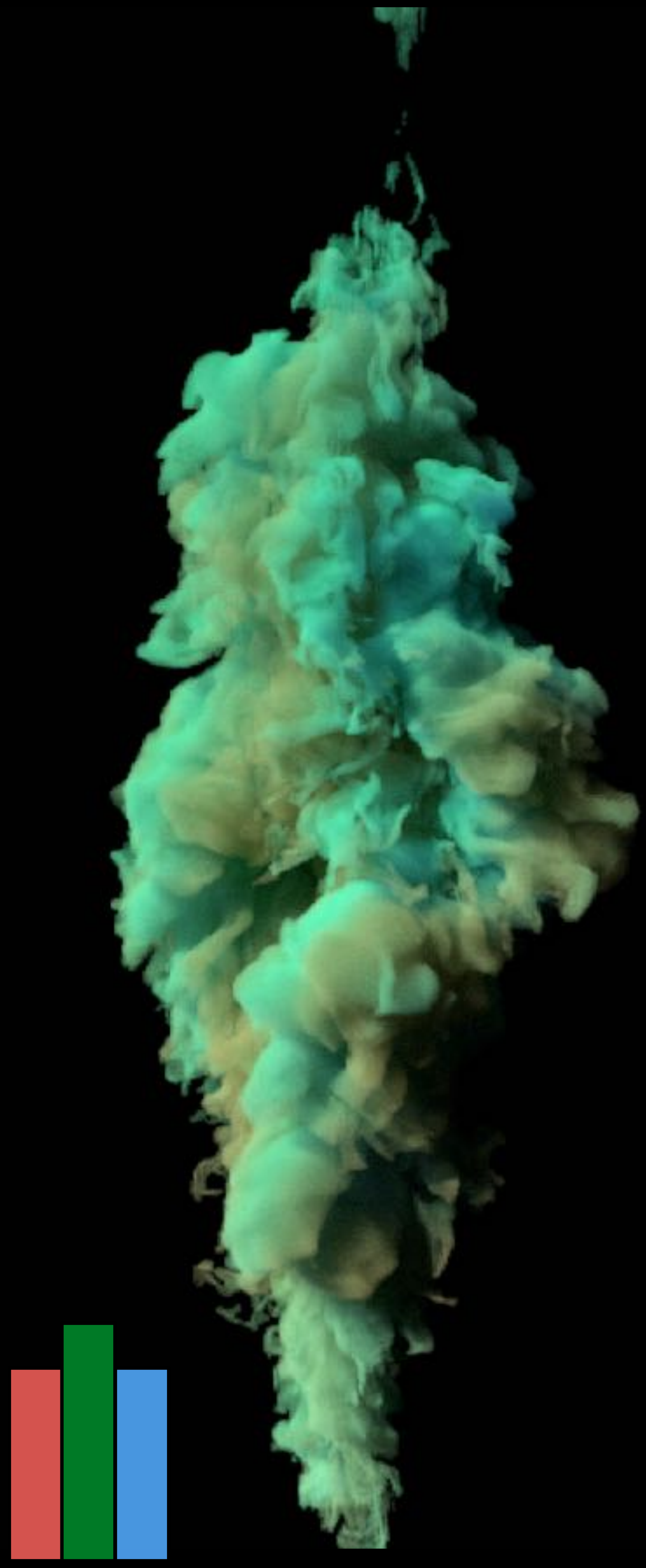
✓ tight majorants (choose any majorants)

✓ render wavelengths together

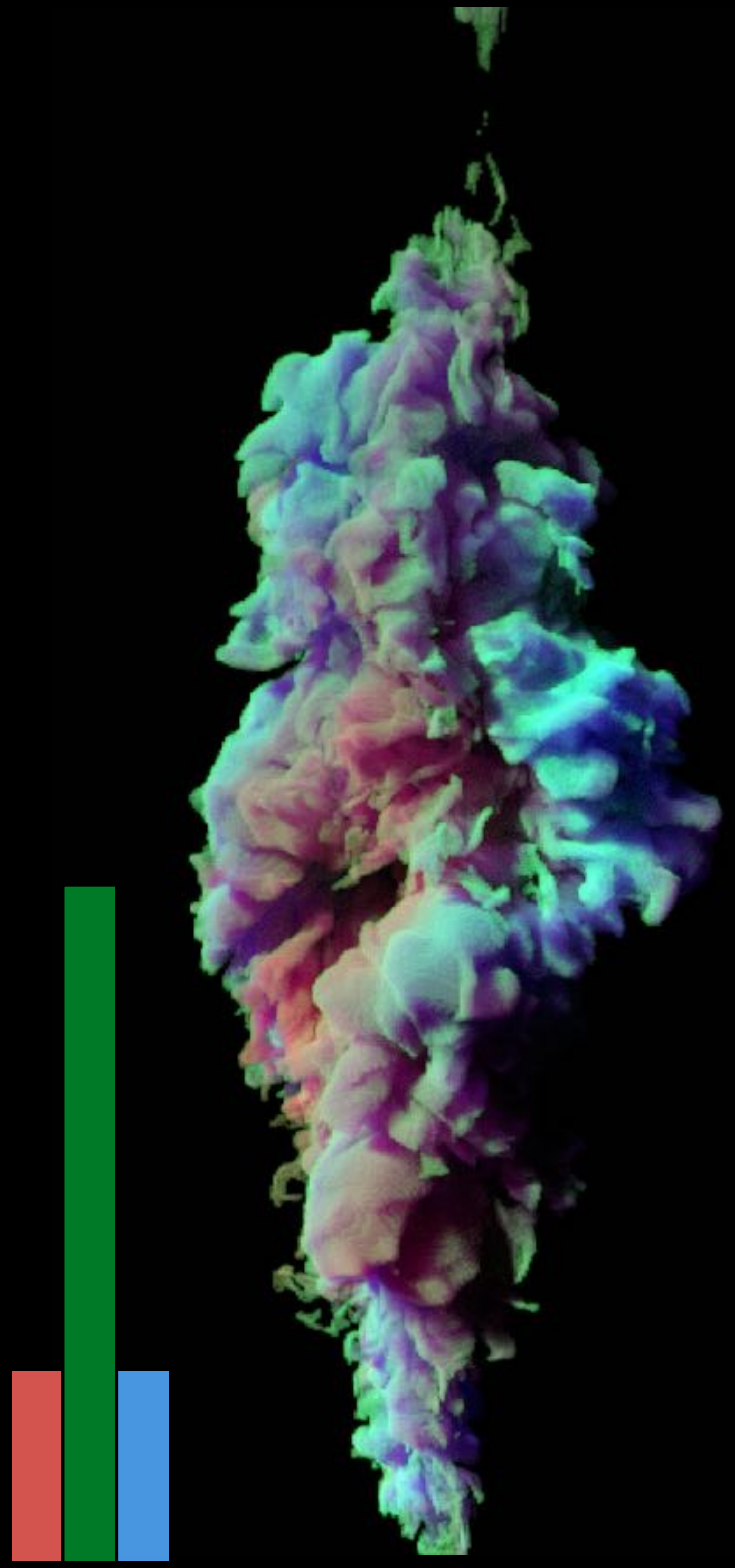
✓ avoids color noise

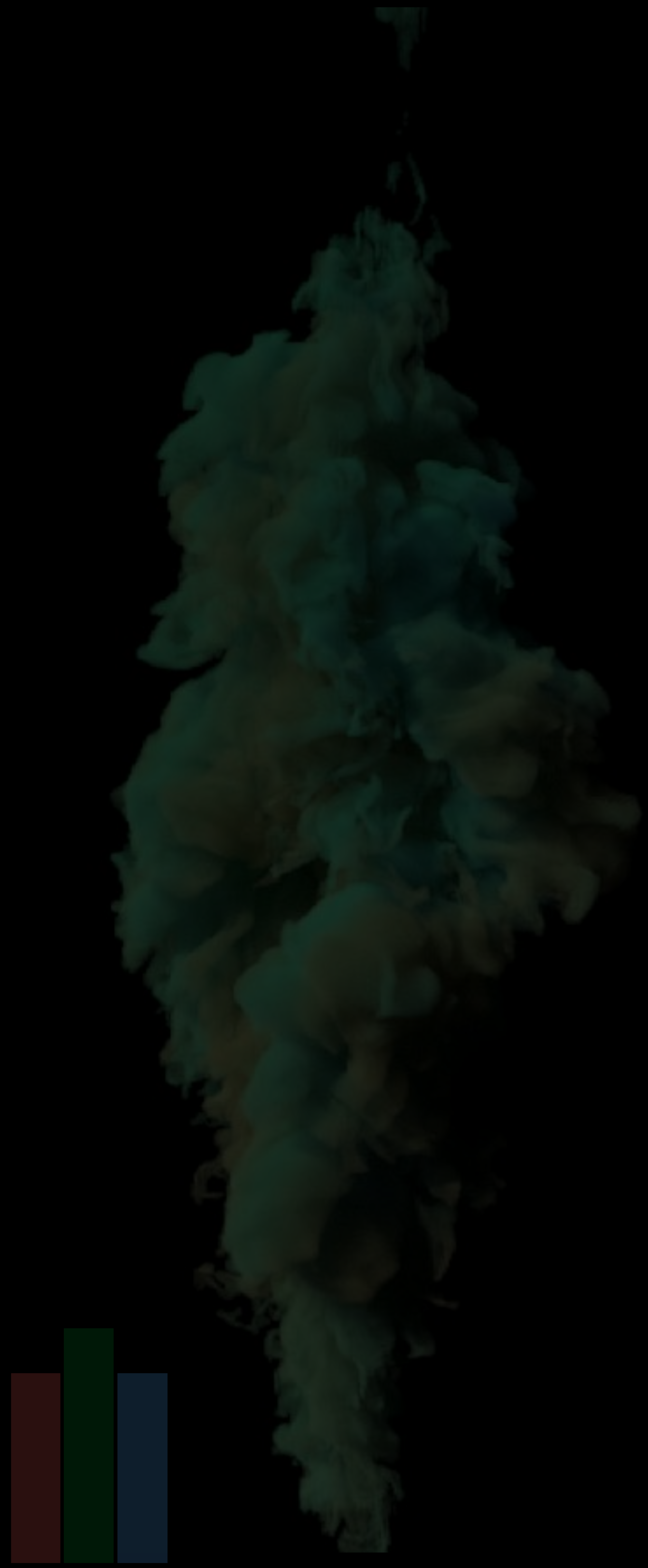
Smoke Scene



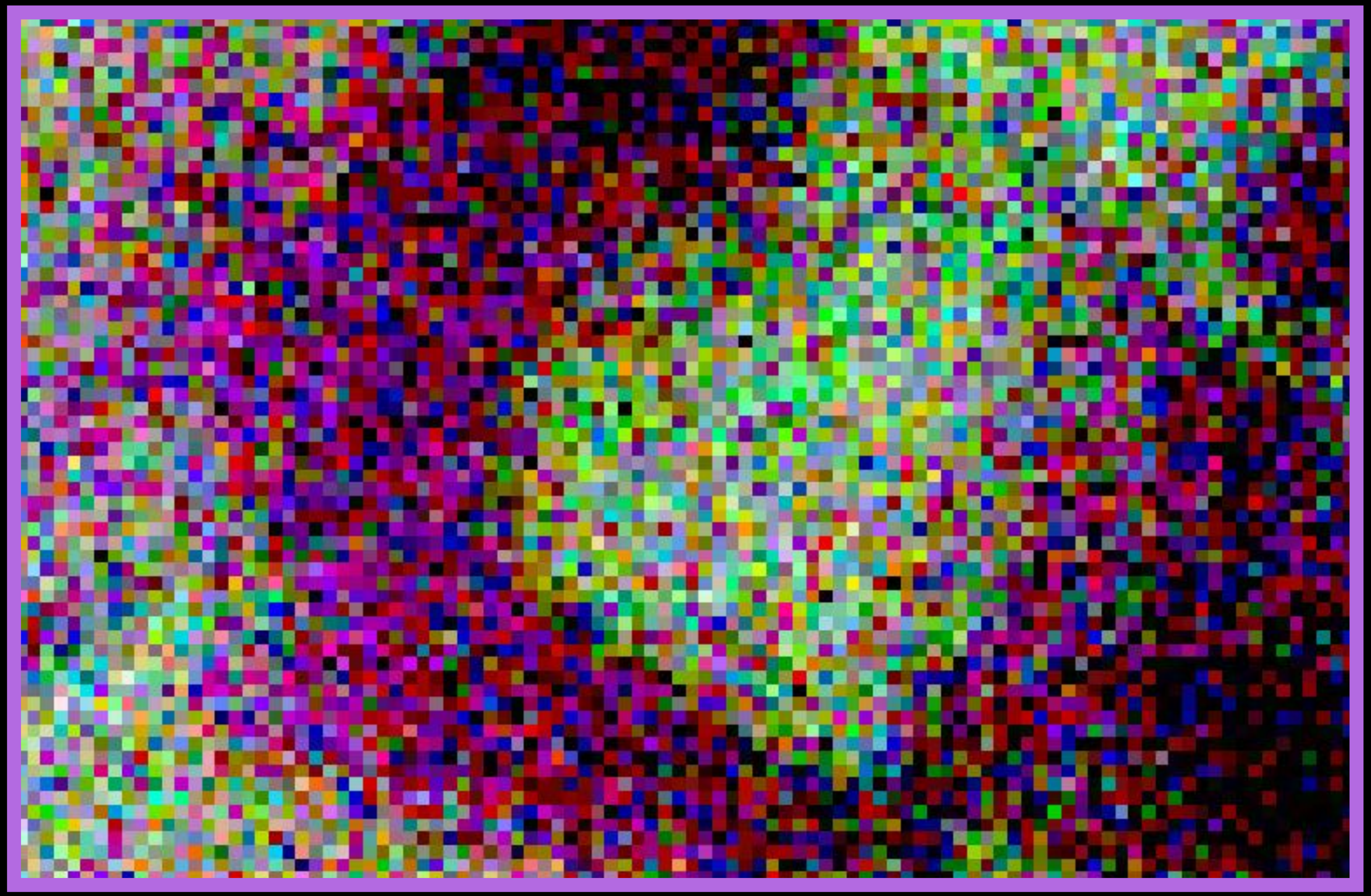


Smoke Scene

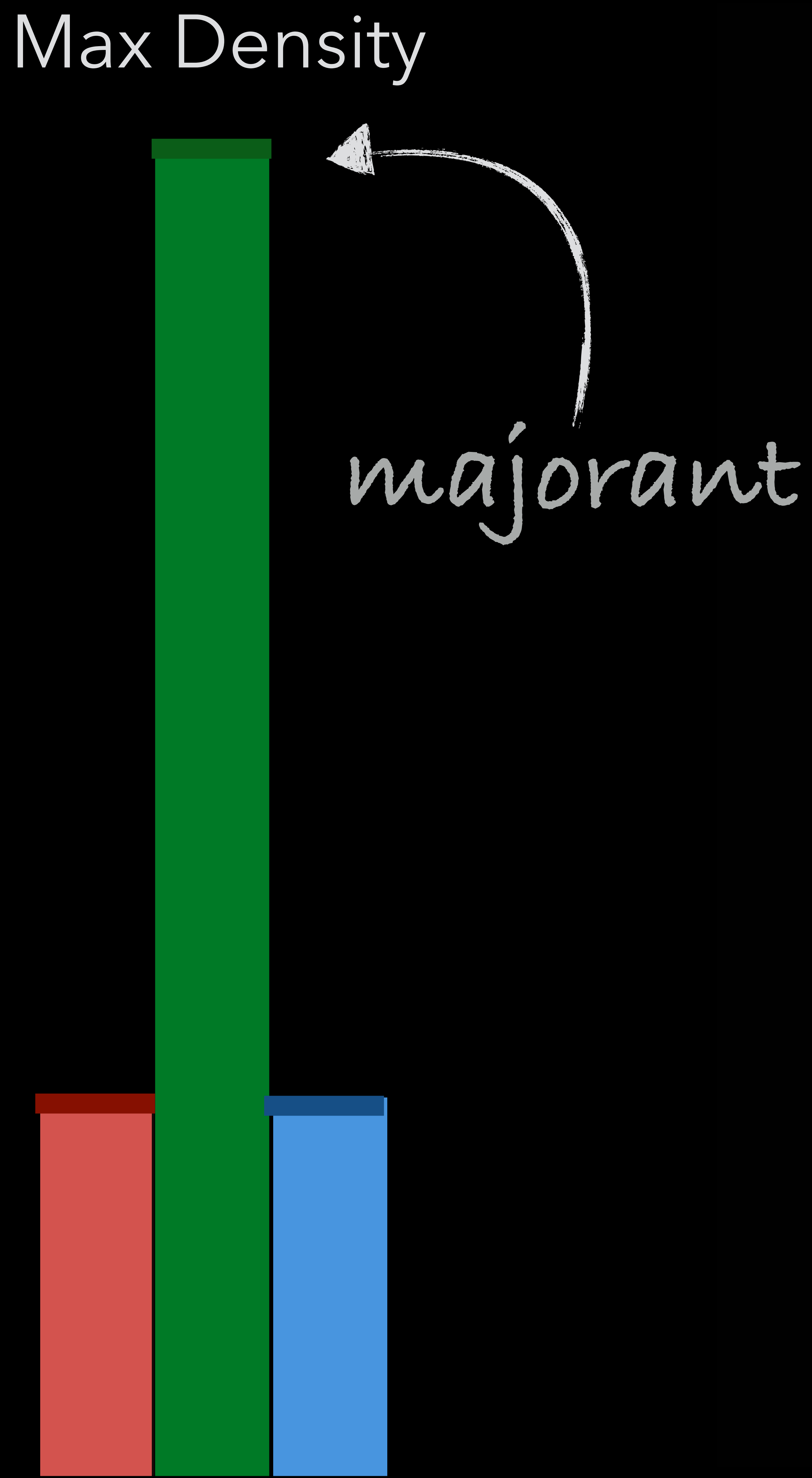


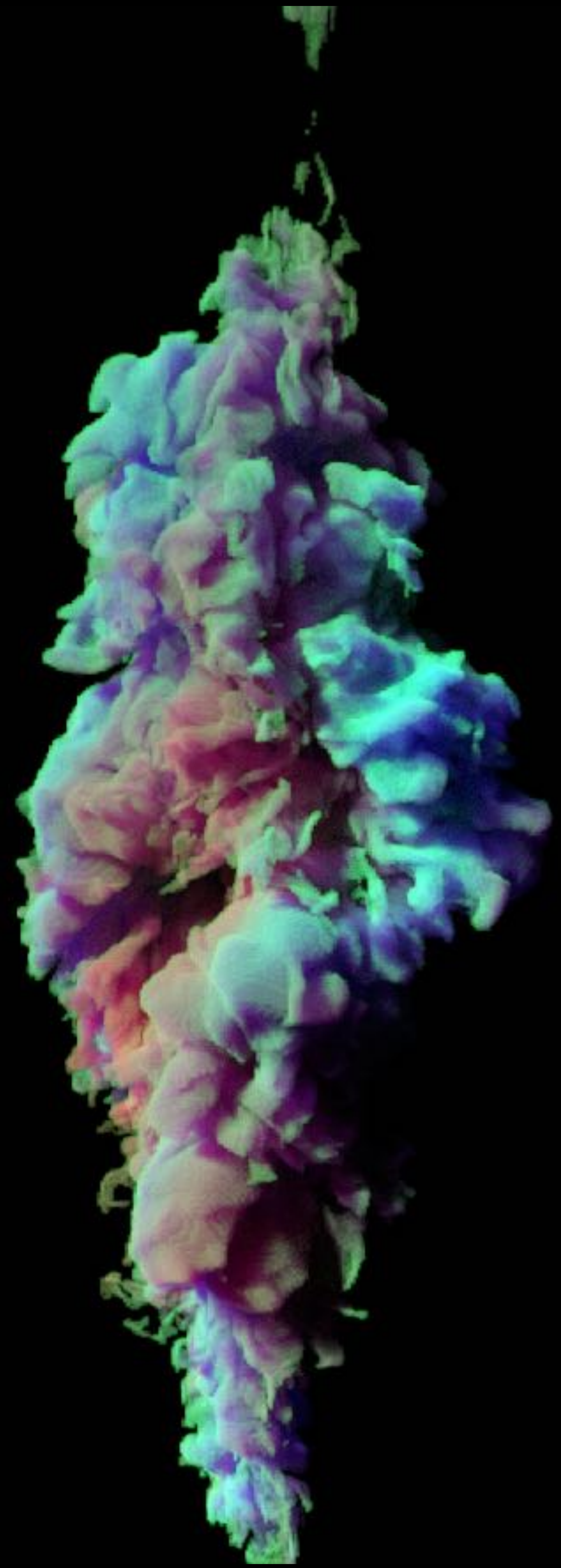


Smoke Scene: Independent Tracking

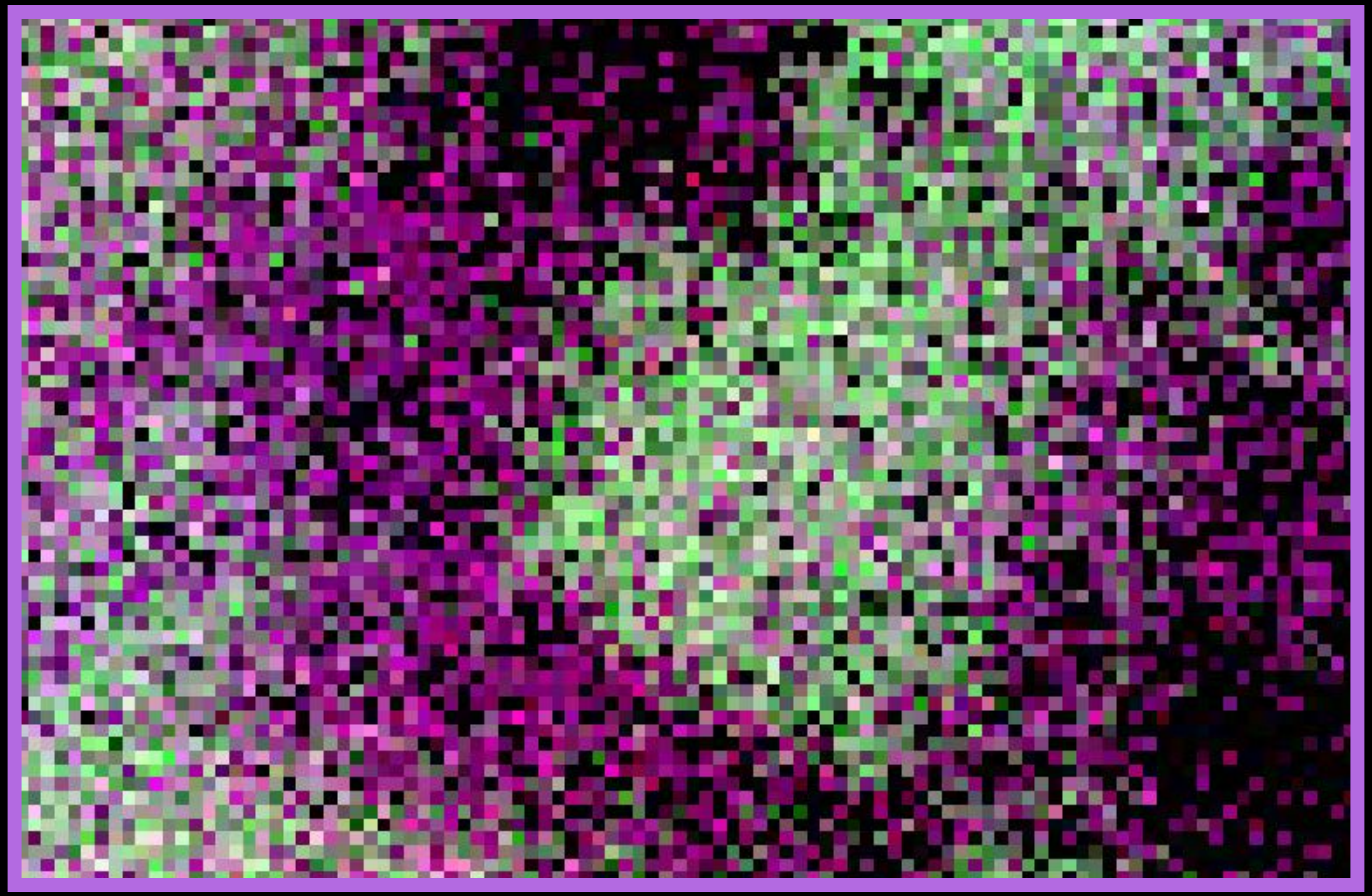


Sampler	LTUV
Independent	16.0 M

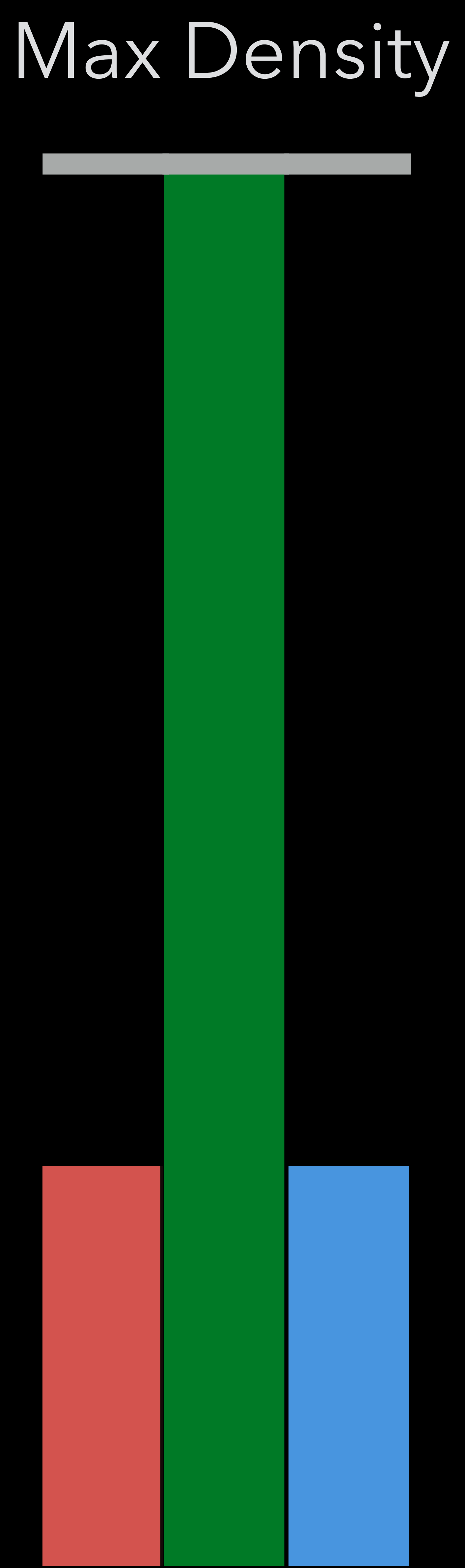


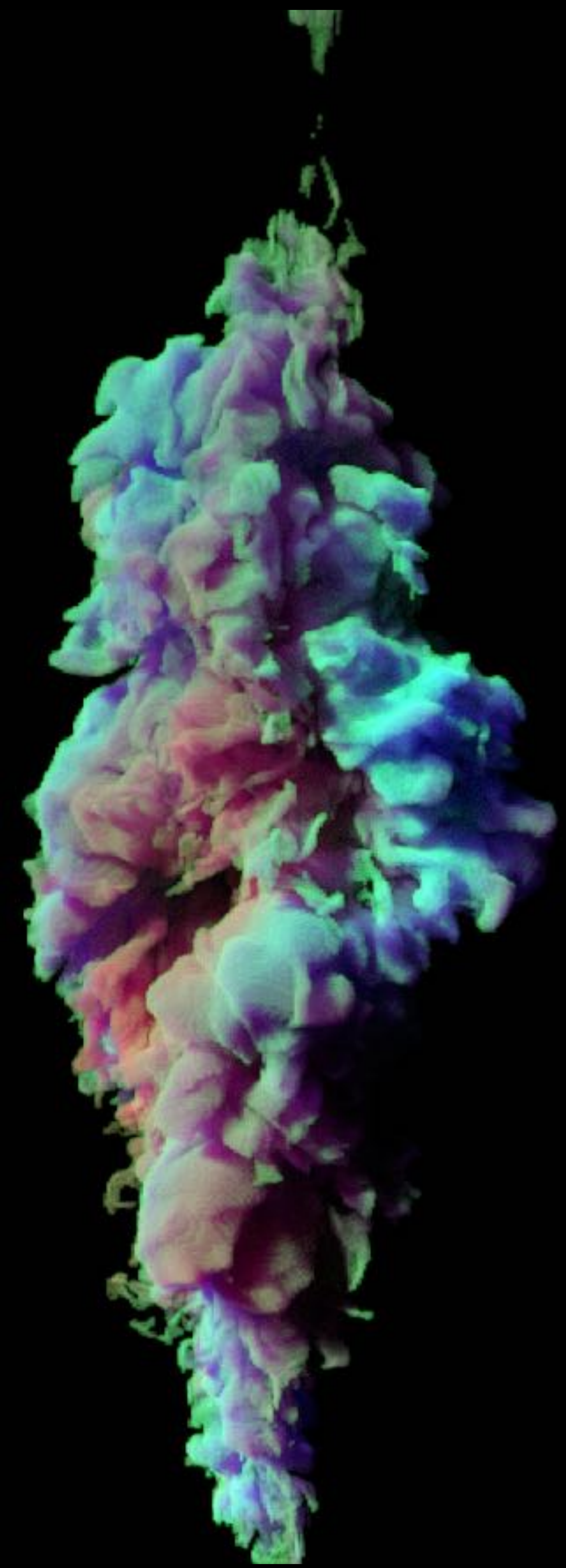


Smoke Scene: Spectral Tracking [Kutz 17]

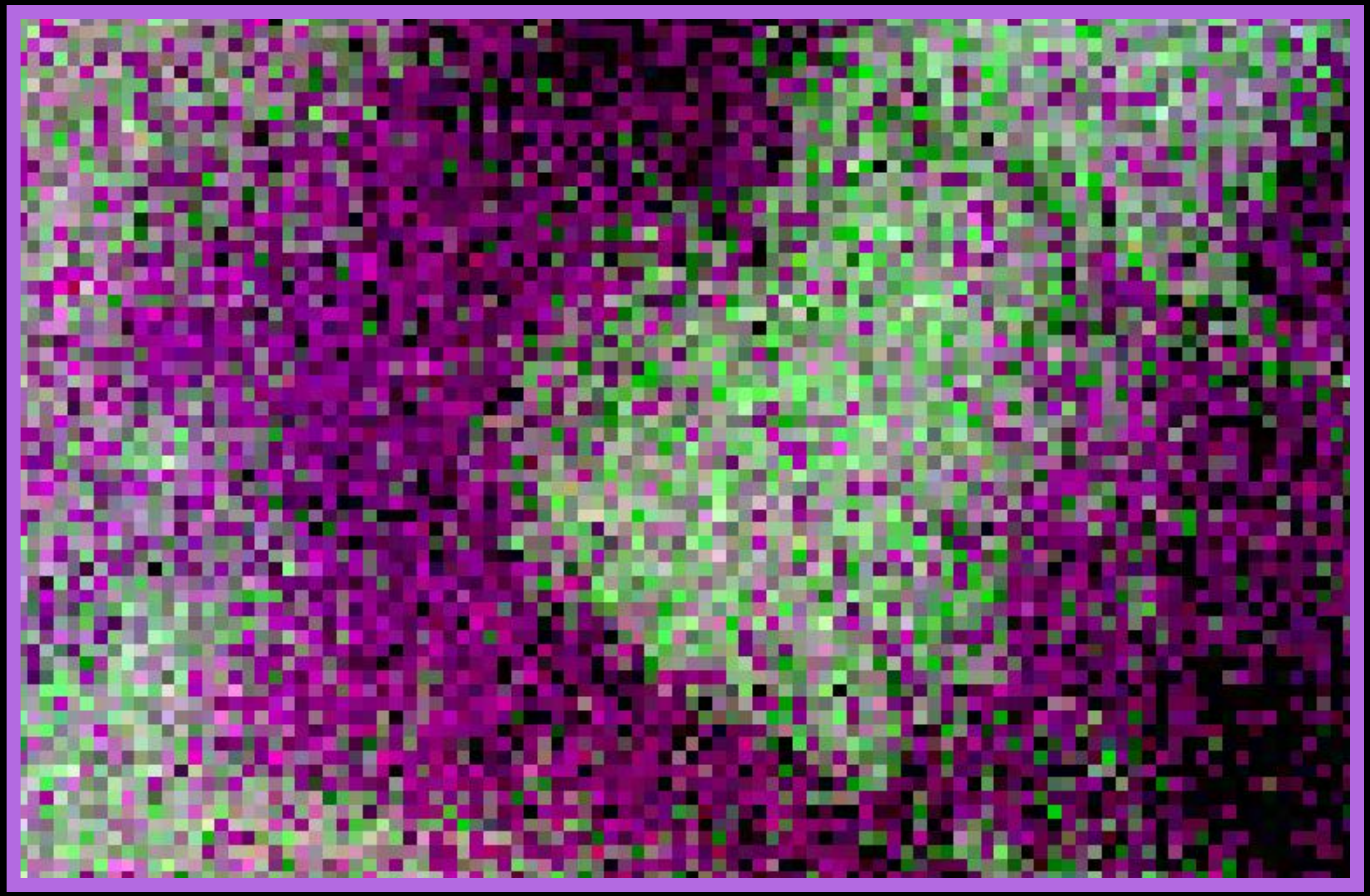


Sampler	LTUV
Independent	16.0 M
Spectral Tracking [Kutz 17]	16.5 M

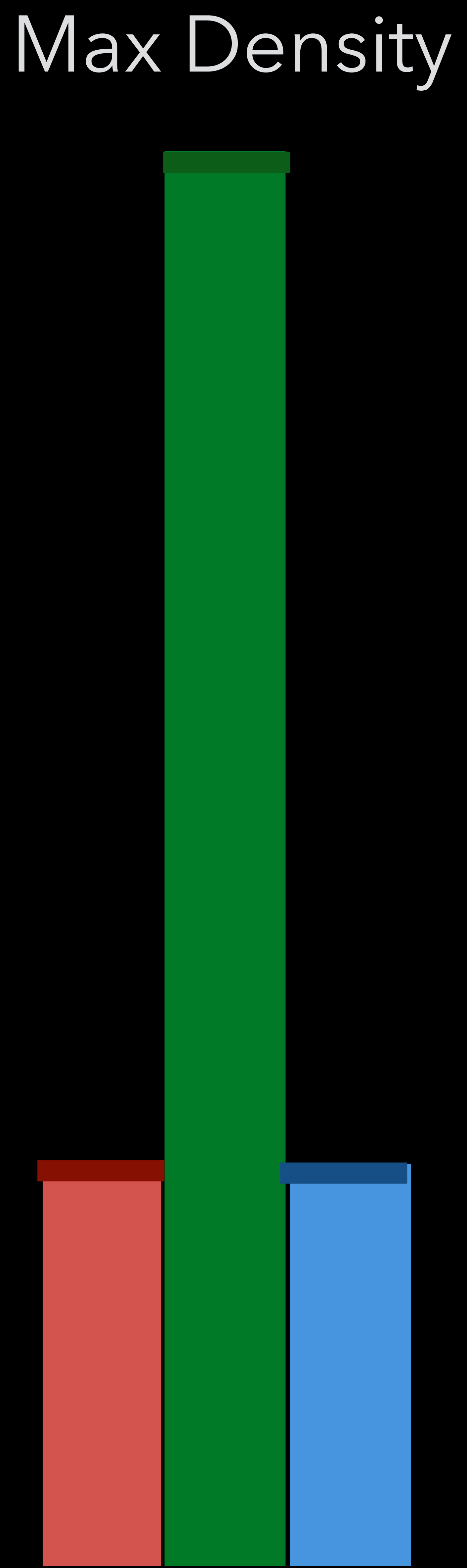


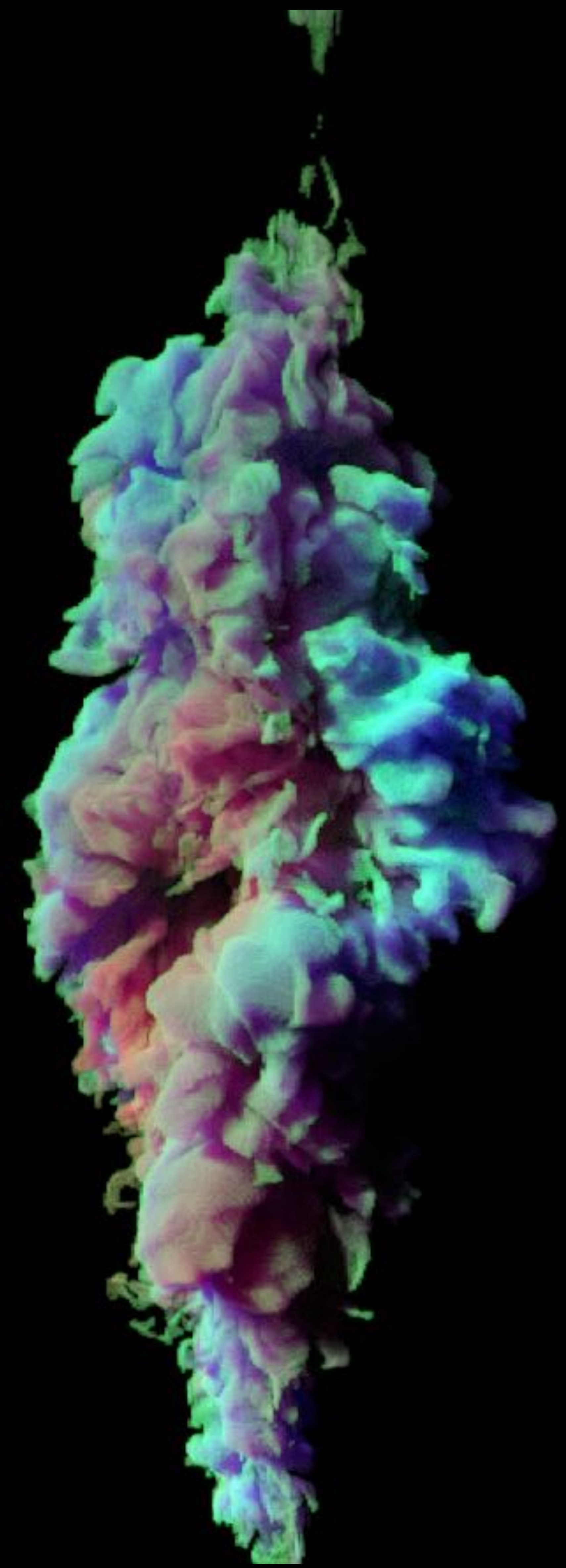


Smoke Scene: Spectral MIS [Ours]

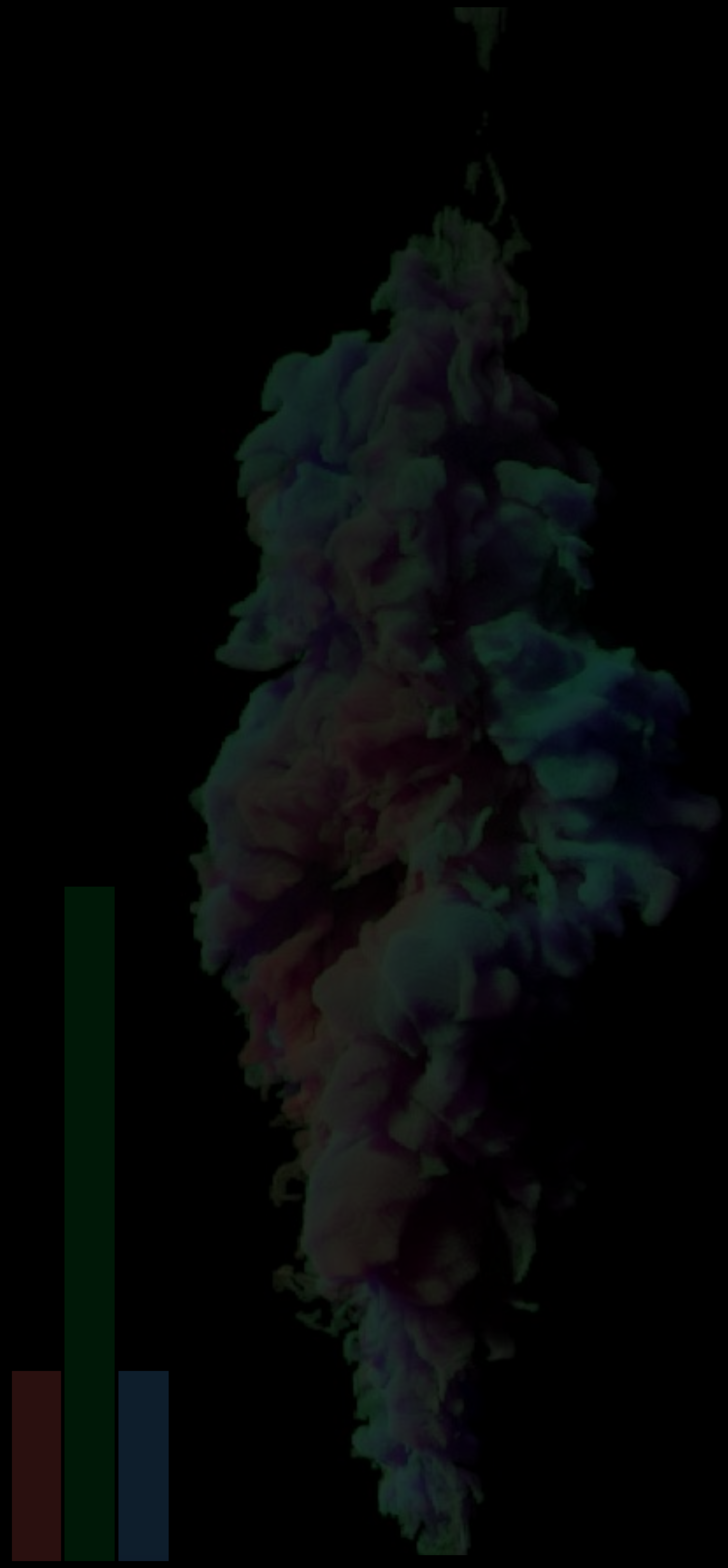


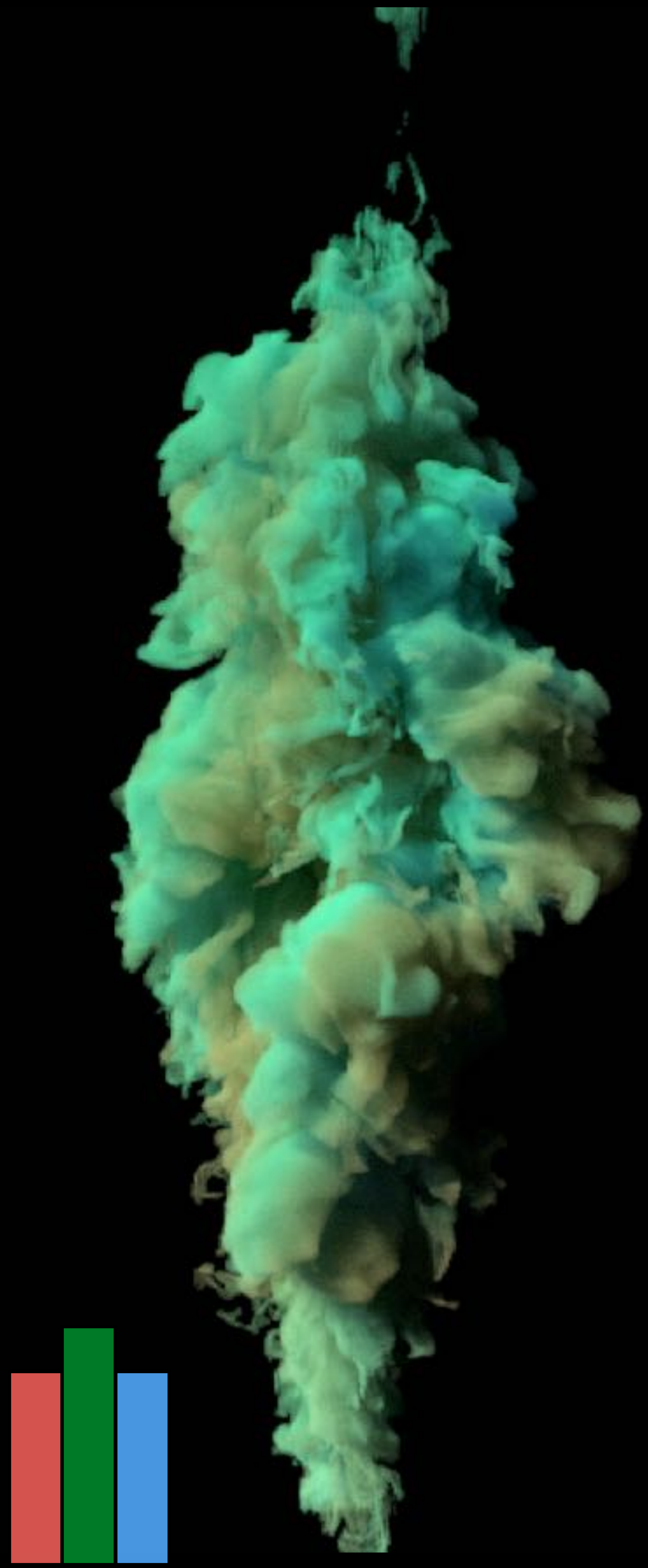
Sampler	LTUV
Independent	16.0 M
Spectral Tracking [Kutz 17]	16.5 M
Spectral MIS [Ours]	12.0 M



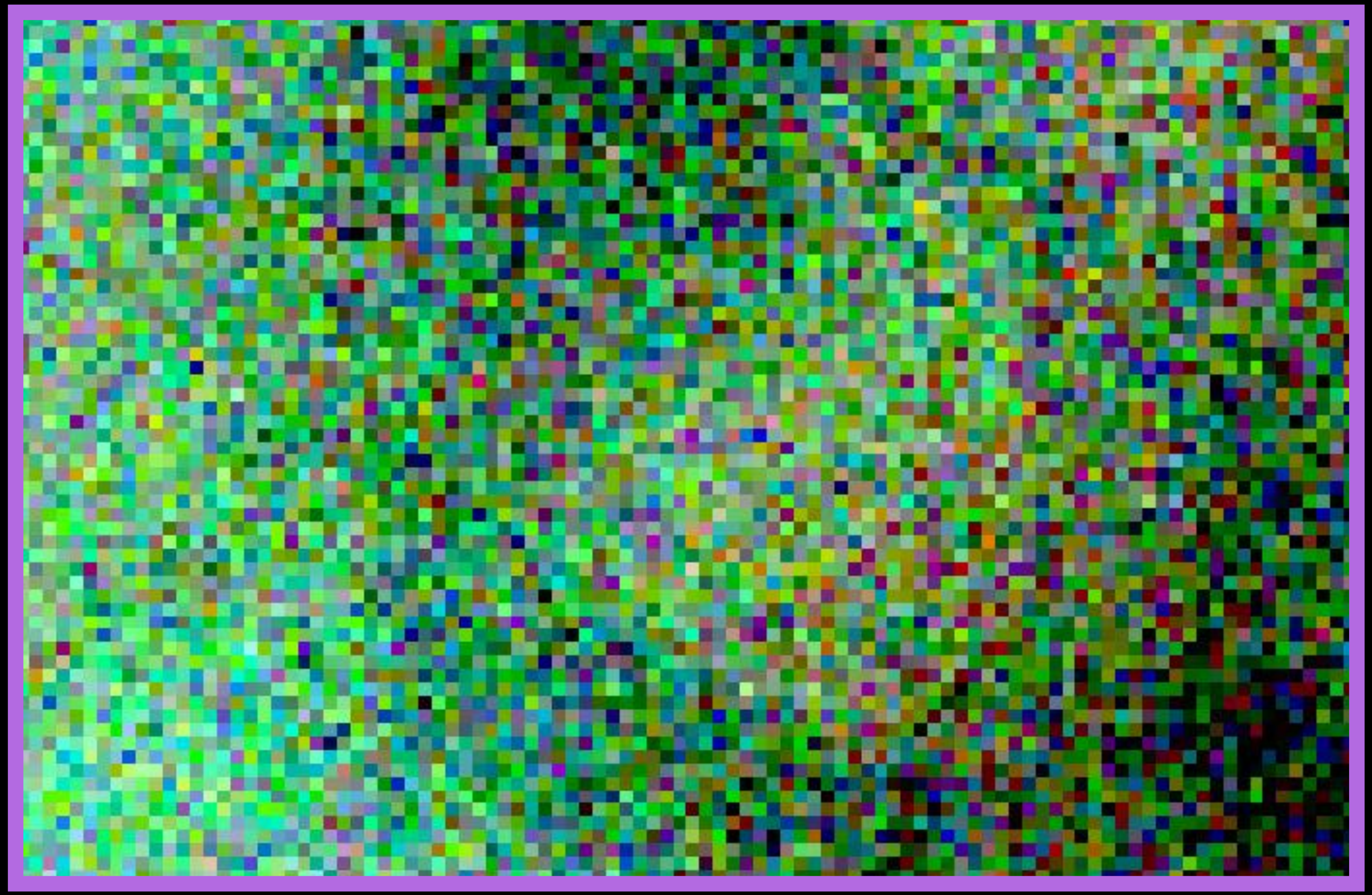


Smoke Scene



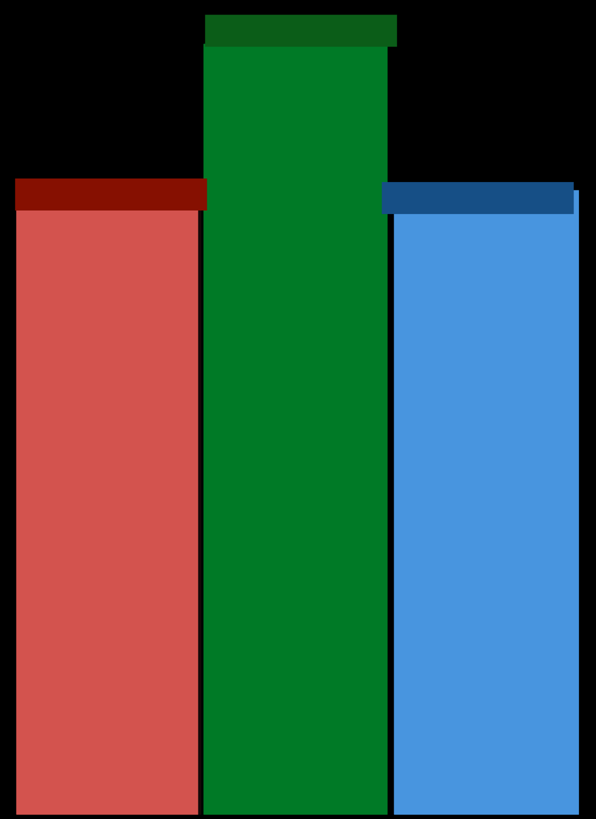


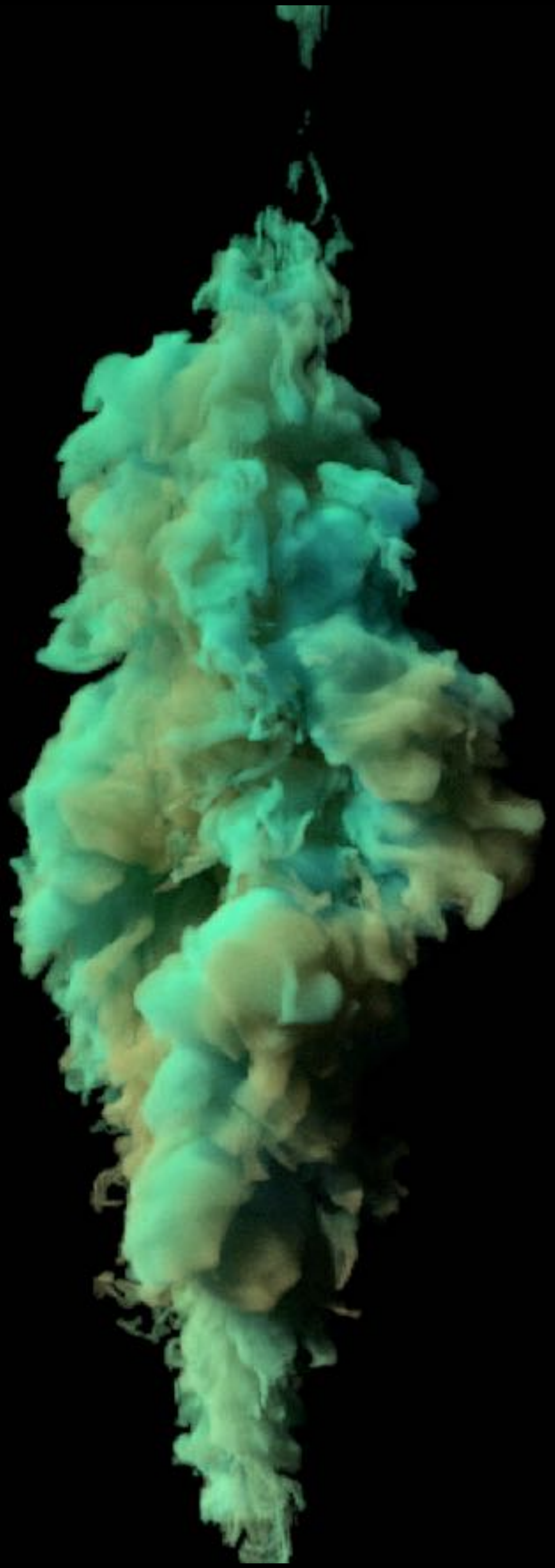
Smoke Scene: Independent Tracking



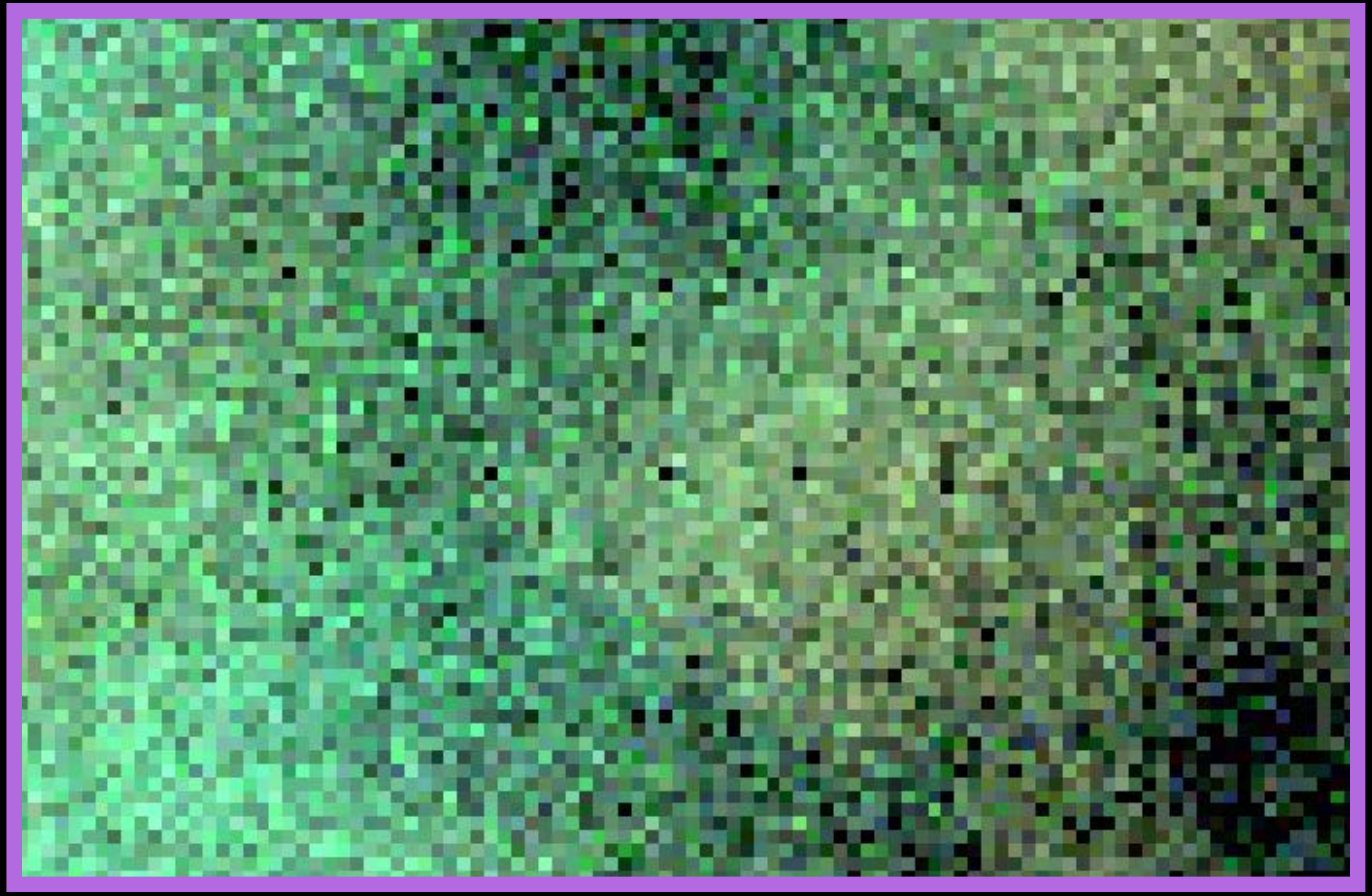
Sampler	LTUV
Independent	3.57 M

Max Density

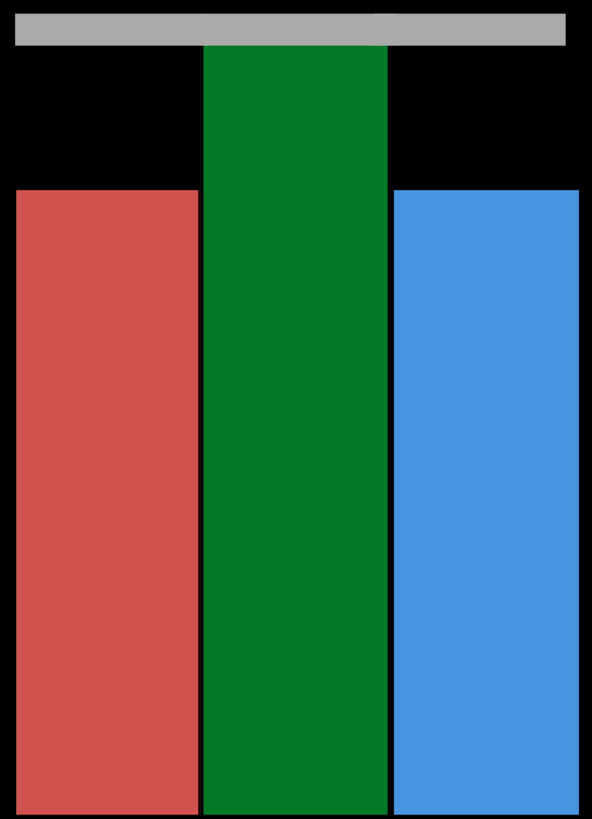


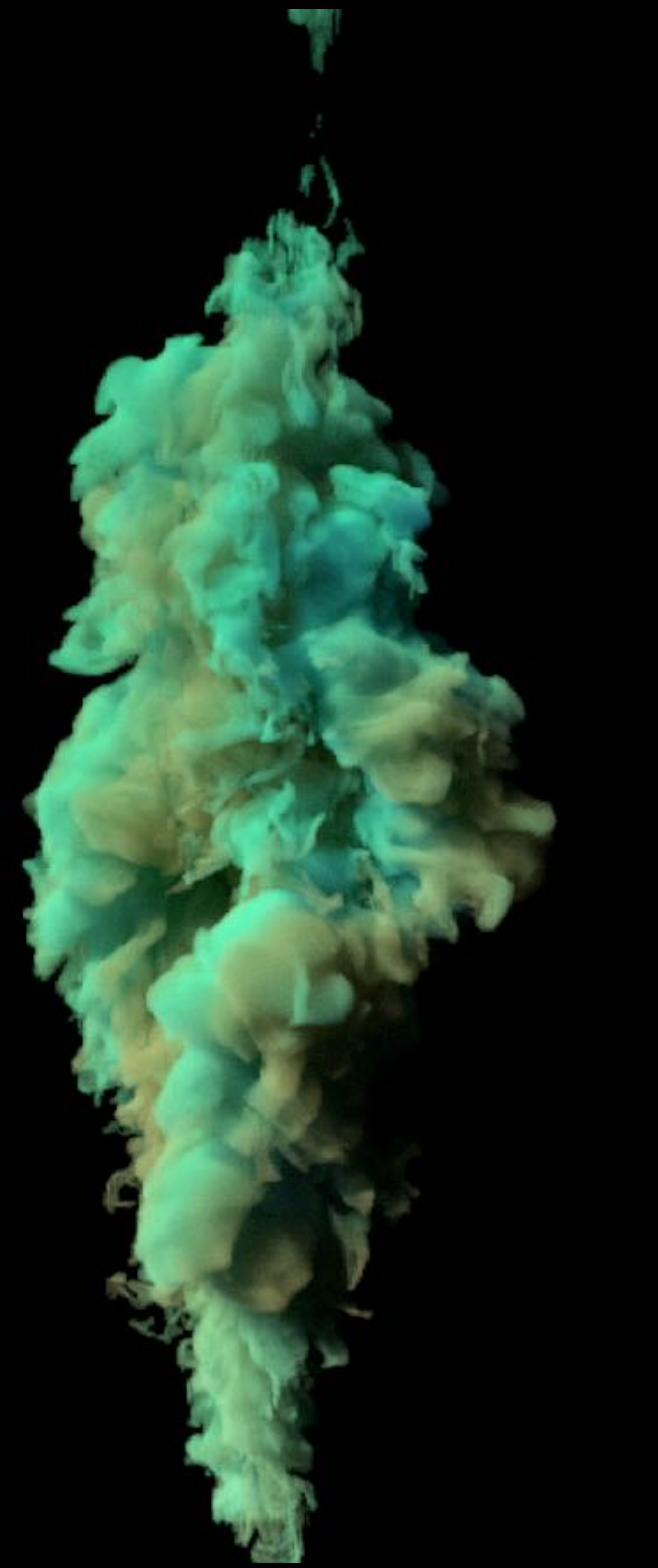


Smoke Scene: Spectral Tracking [Kutz 17]

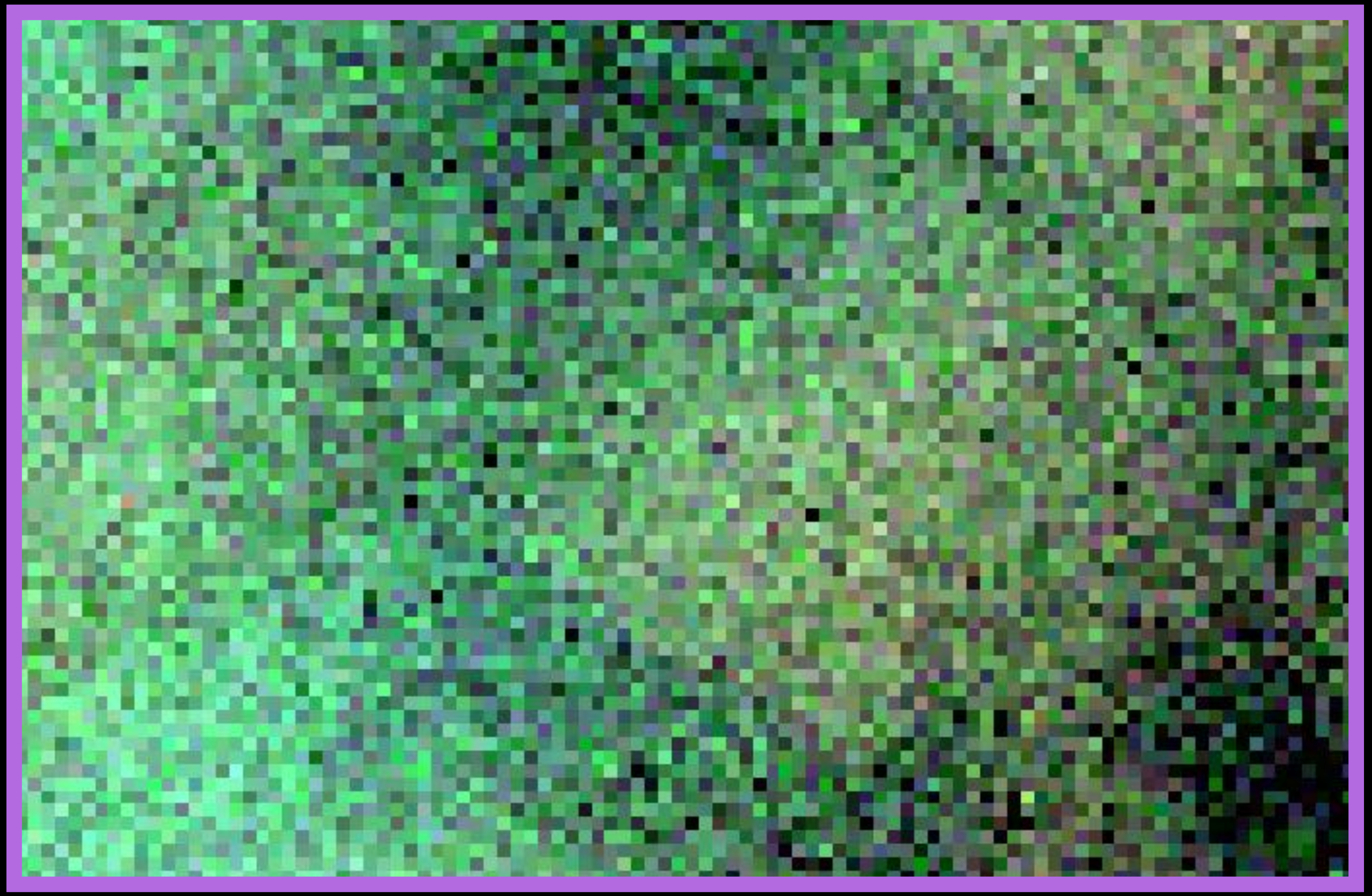


Sampler	LTUV
Independent	3.57 M
Spectral Tracking [Kutz 17]	1.35 M



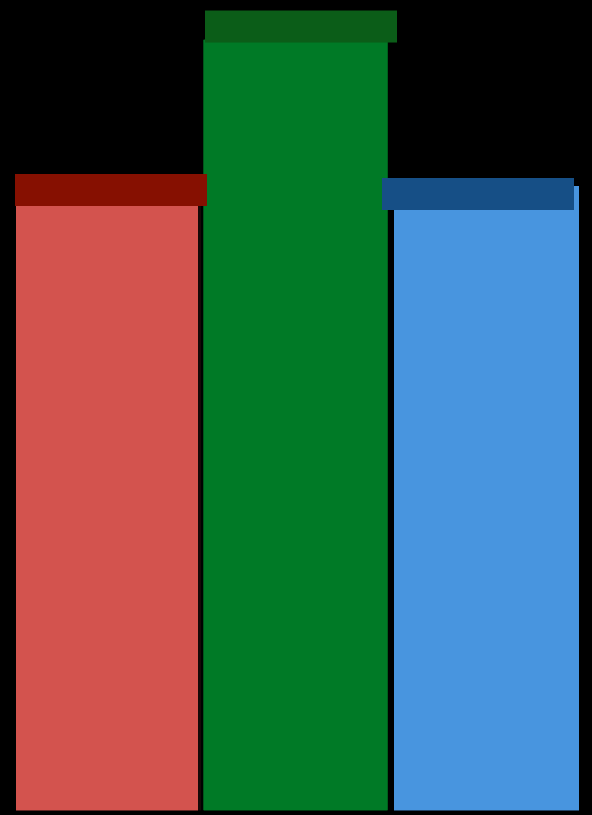


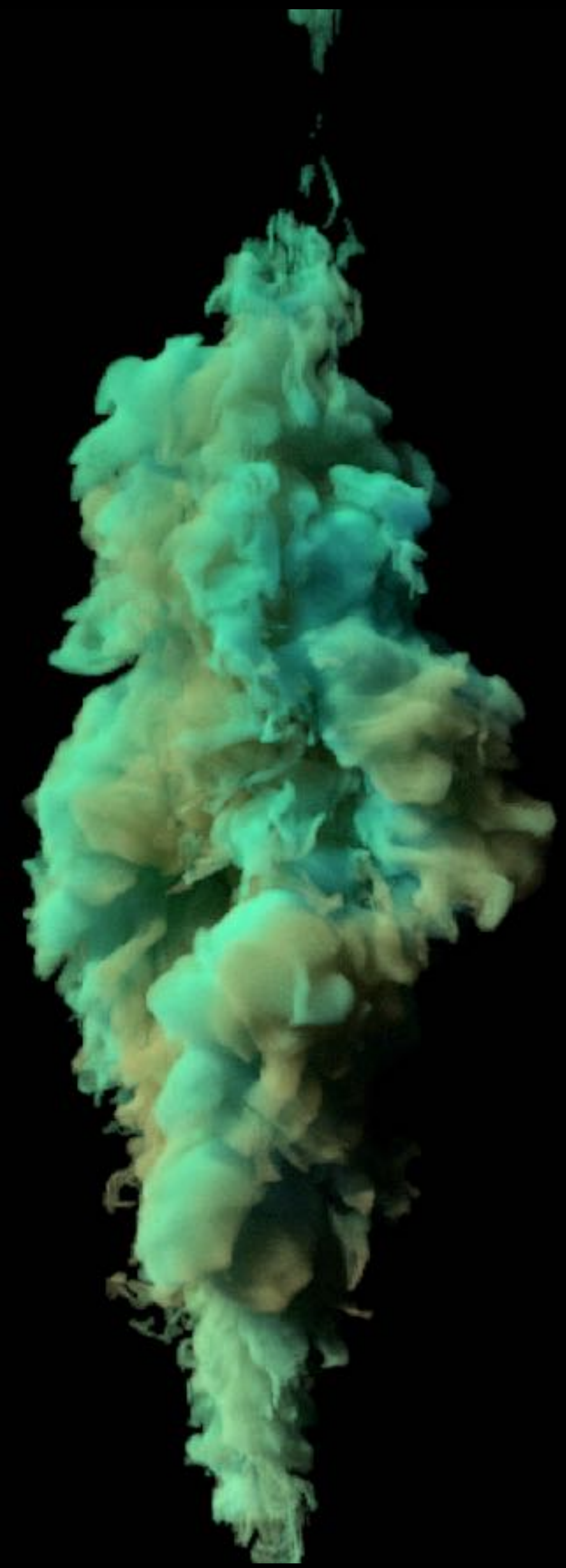
Smoke Scene: Spectral MIS [Ours]



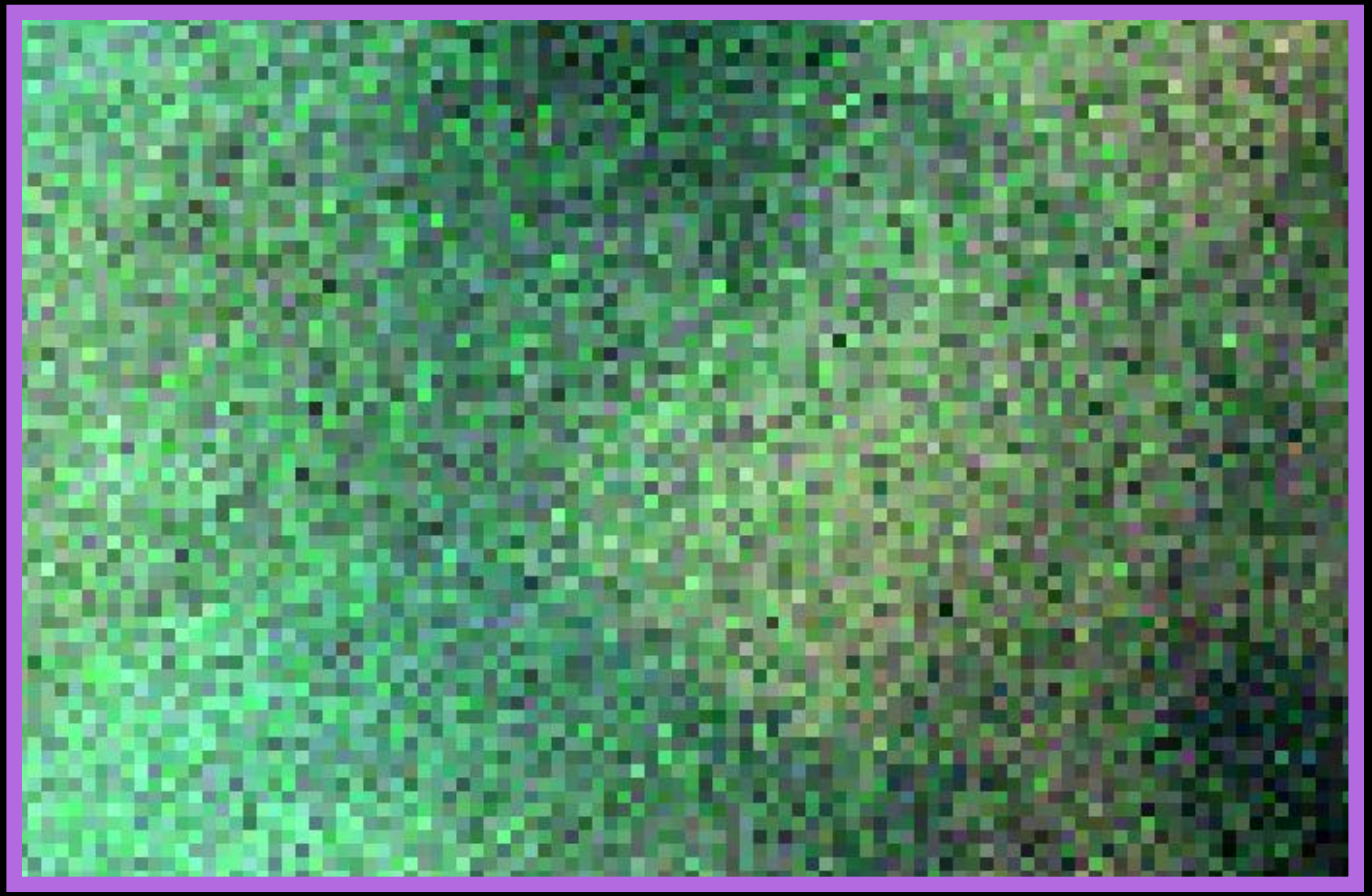
Sampler	LTUV
Independent	3.57 M
Spectral Tracking [Kutz 17]	1.35 M
Spectral MIS [Ours]	1.51 M

Max Density

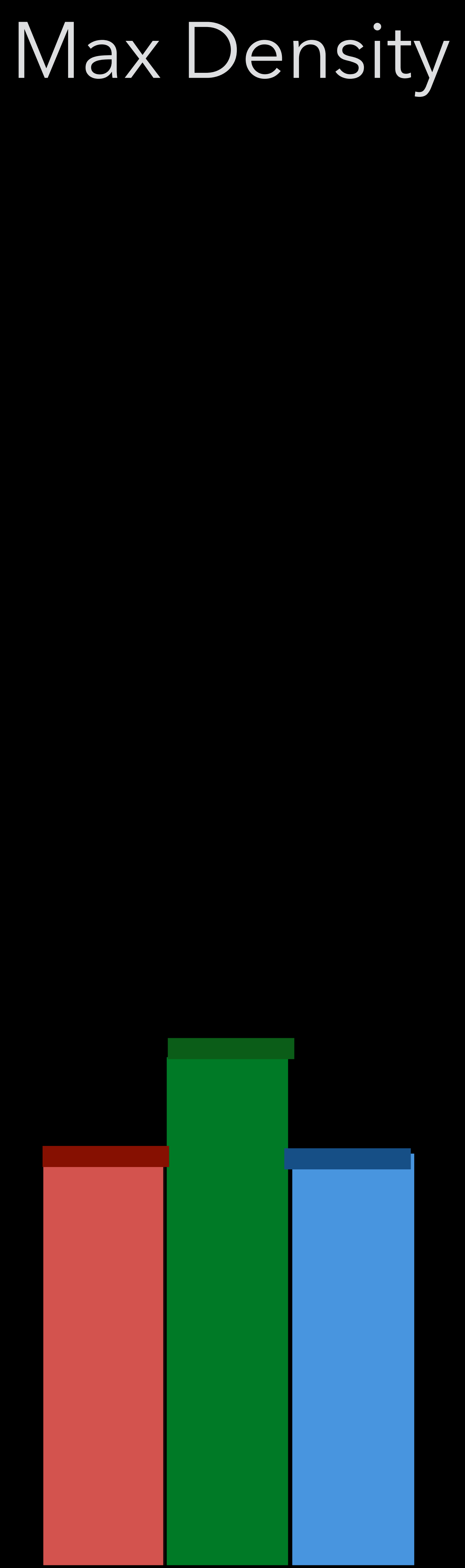


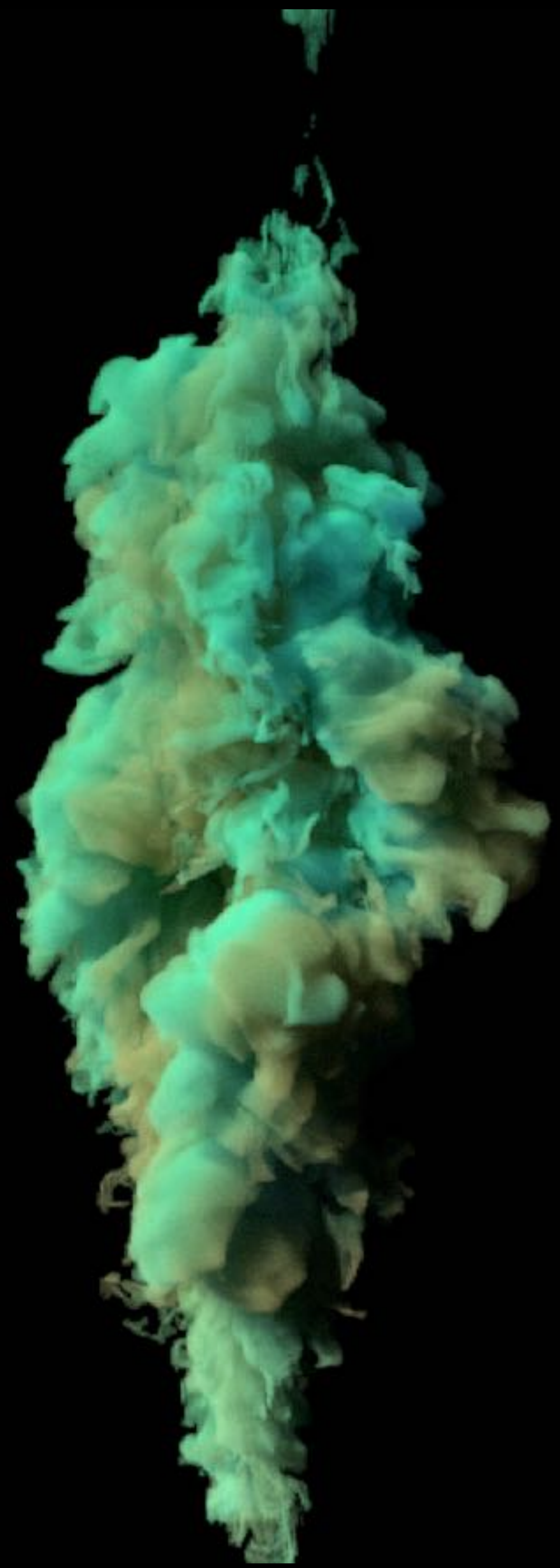


Smoke Scene: Spectral MIS + NEE [Ours]



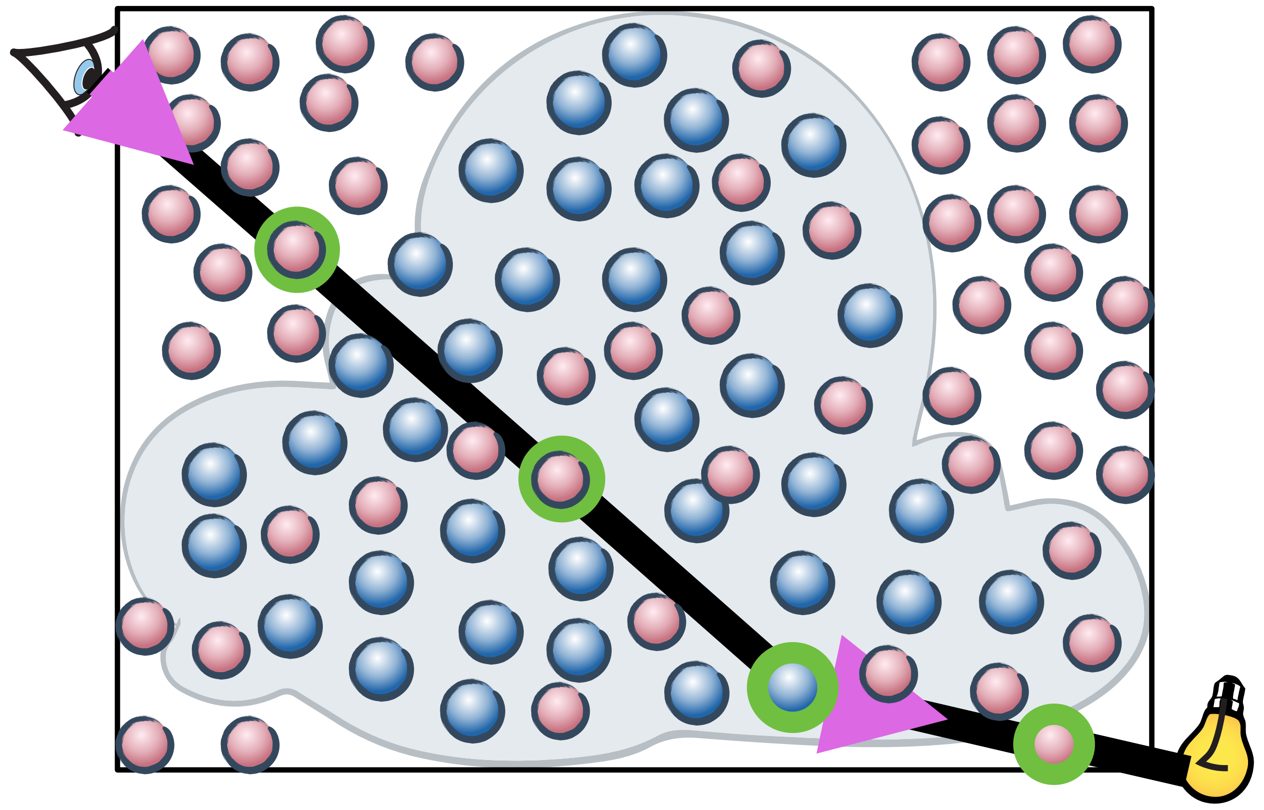
Sampler	LTUV
Independent	3.57 M
Spectral Tracking [Kutz 17]	1.35 M
Spectral MIS [Ours]	1.51 M
Spectral MIS + NEE [Ours]	1.09 M





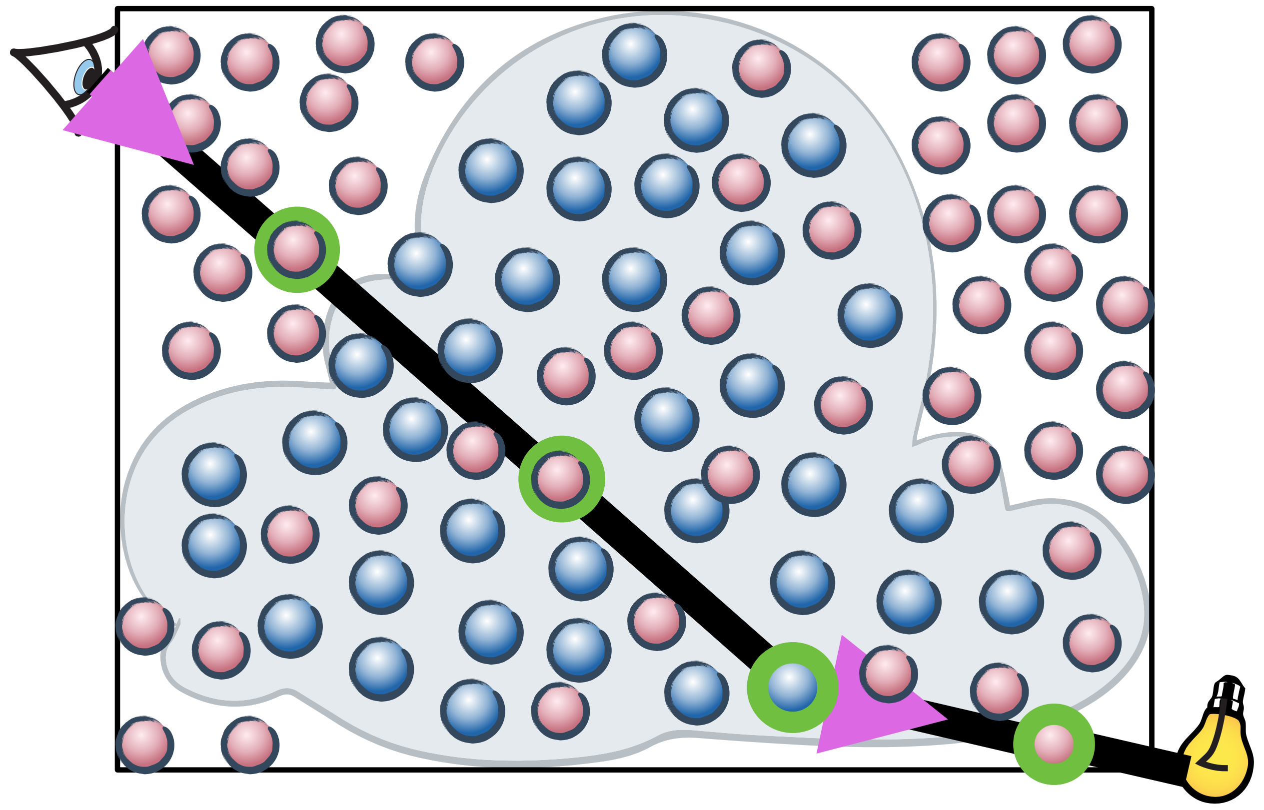
Summary

Summary



null scattering
path integral

Summary



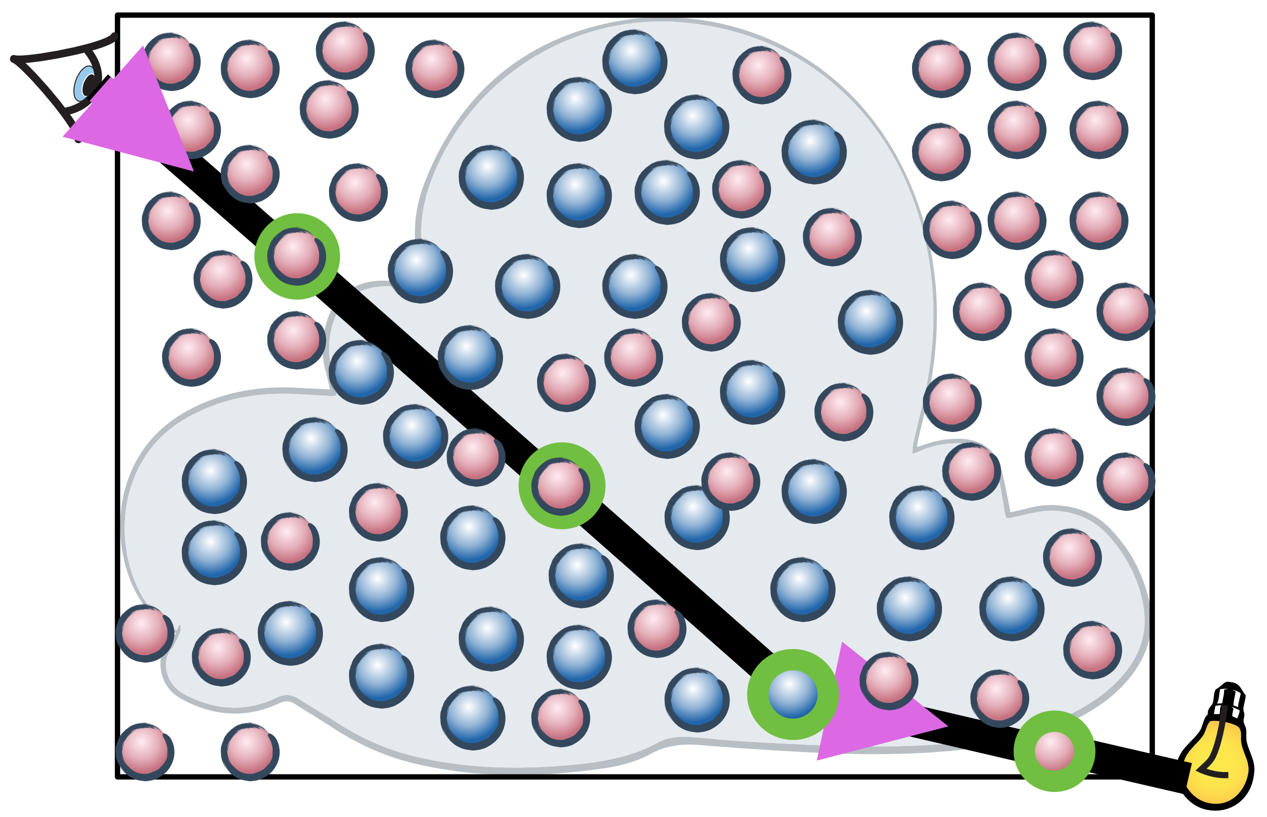
$$p(\overline{x})$$

analytic

null scattering path integral

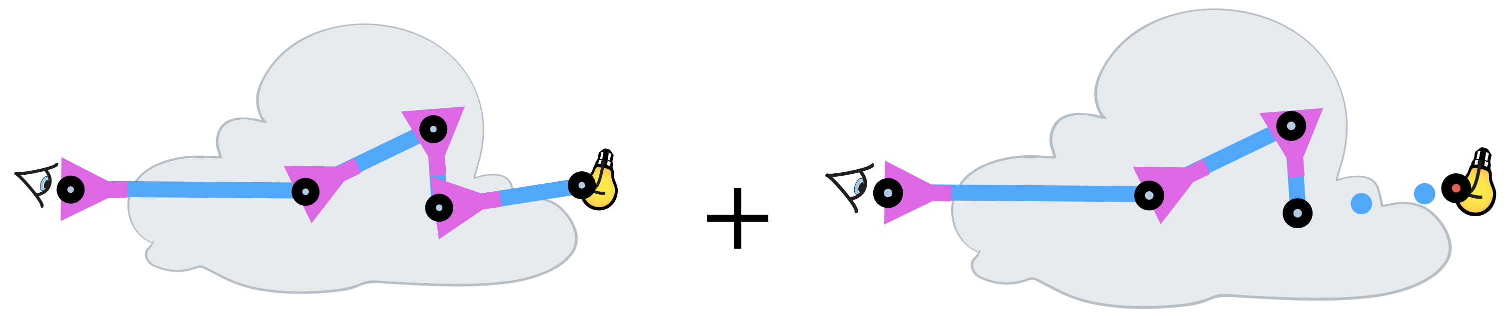
unbiased and analytic path pdf

Summary



$$p(\overline{x})$$

analytic



$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(\overline{\mathbf{x}}_i)}{\frac{1}{2}(p_1(\overline{\mathbf{x}}_i) + p_2(\overline{\mathbf{x}}_i))}$$

null scattering path integral

unbiased and analytic path pdf

MIS in complex media

Future Work

Future Work

- Unified surface + volume null scattering framework

Future Work

- Unified surface + volume null scattering framework
- Similar null-scattering path for other types of media
  - Correlated media / non-exponential media

Future Work

- Unified surface + volume null scattering framework
- Similar null-scattering path for other types of media
  - Correlated media / non-exponential media
- Other light transport algorithms
  - Joint path sampling, MLT, etc.

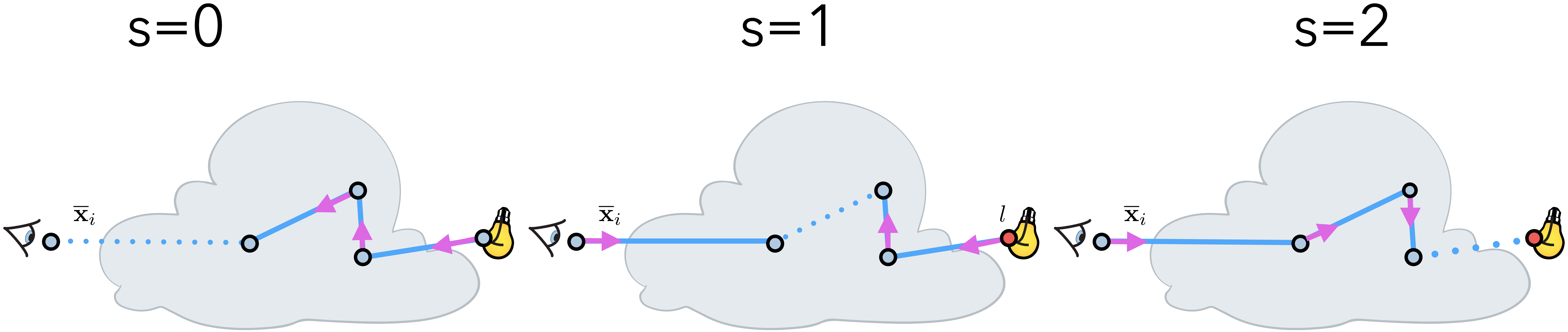
Future Work

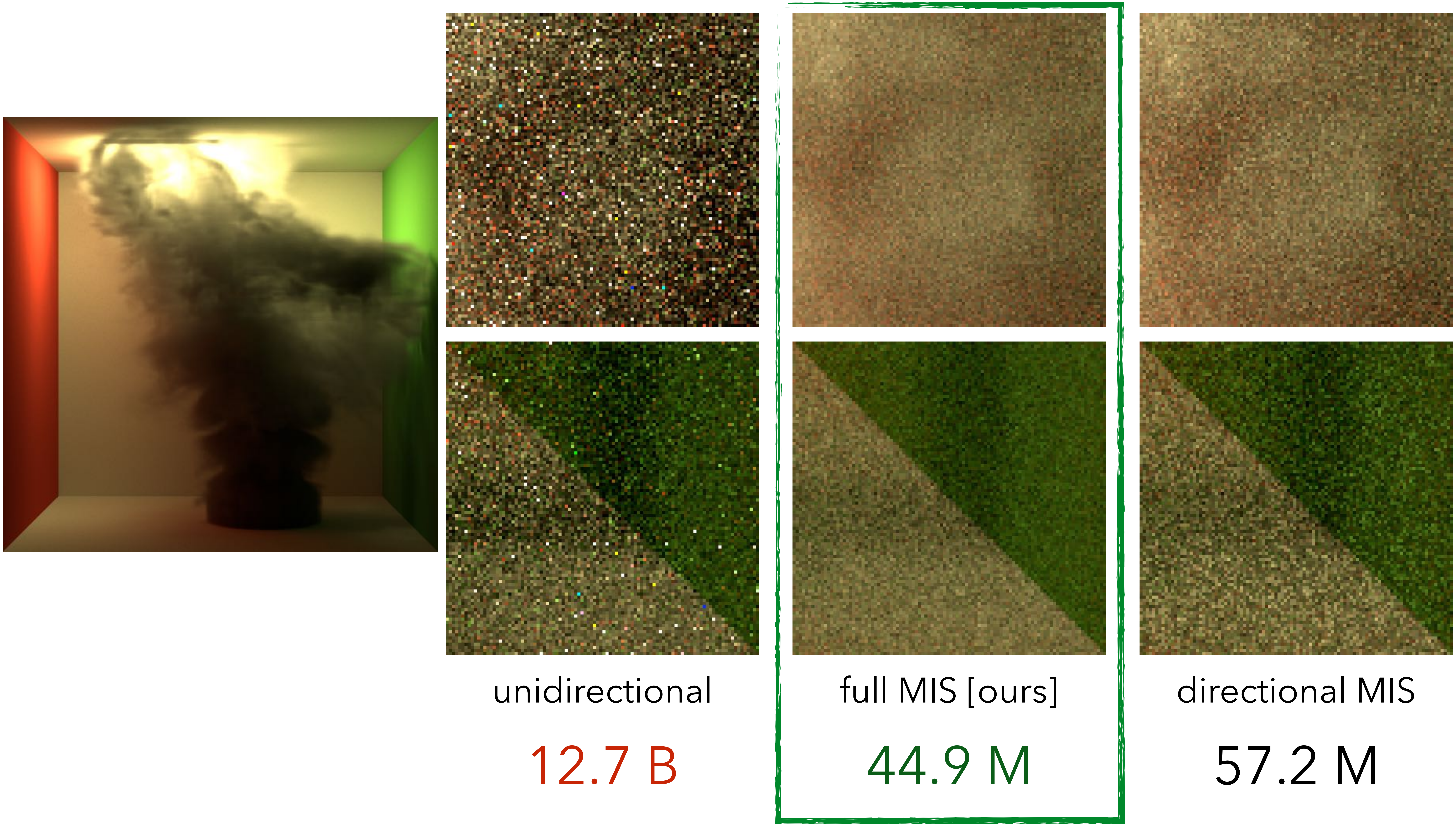
- Unified surface + volume null scattering framework
- Similar null-scattering path for other types of media
  - Correlated media / non-exponential media
- Other light transport algorithms
  - Joint path sampling, MLT, etc.
- Other rendering techniques
  - Photon planes/volumes in heterogeneous media

Thank you for listening!

Question and Answer:

Bidirectional Path Tracing



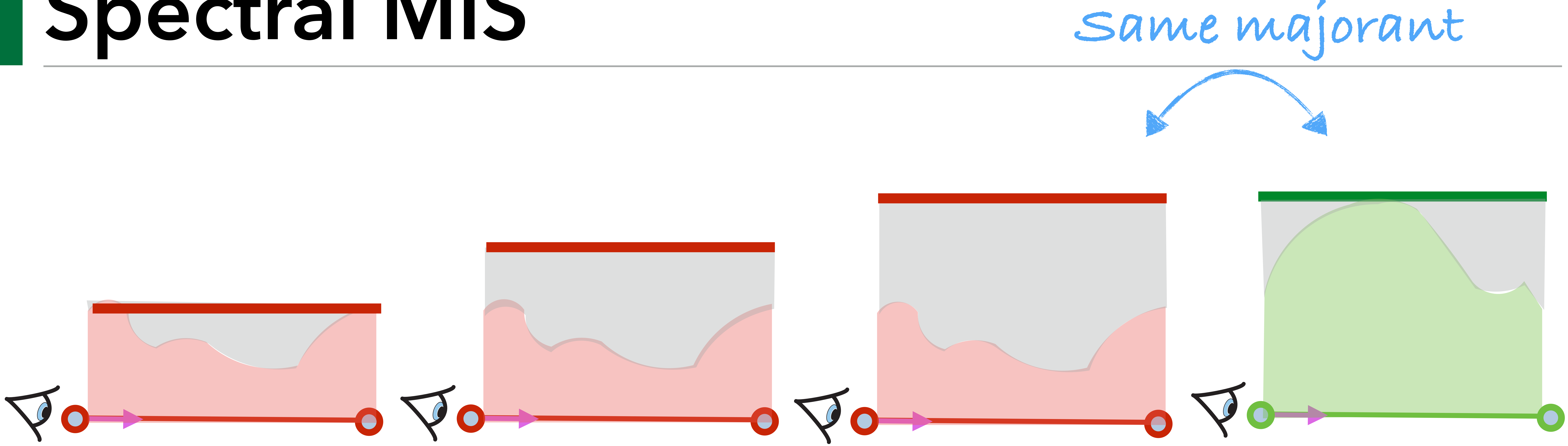

unidirectional
12.7 B
full MIS [ours]
44.9 M
directional MIS
57.2 M


=
classic
ours
≠

Spectral MIS

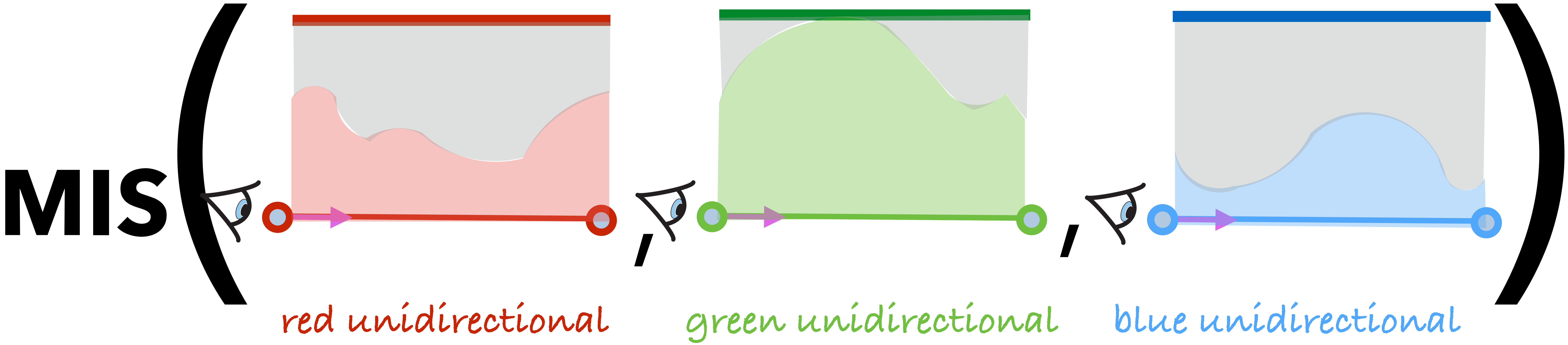
Doesn't under sample
green majorant

Spectral MIS



Doesn't under sample
green majorant

Spectral MIS



Spectral MIS

$$\frac{f_{\mathrm{r}}(\bar{\mathrm{x}})}{\frac{1}{3}(p_{\mathrm{r}}(\bar{x}) + p_{\mathrm{g}}(\bar{x}) + p_{\mathrm{b}}(\bar{x}))}$$

$$\frac{f_{\mathrm{g}}(\bar{\mathrm{x}})}{\frac{1}{3}(p_{\mathrm{r}}(\bar{x}) + p_{\mathrm{g}}(\bar{x}) + p_{\mathrm{b}}(\bar{x}))}$$

$$\frac{f_{\mathrm{b}}(\bar{\mathrm{x}})}{\frac{1}{3}(p_{\mathrm{r}}(\bar{x}) + p_{\mathrm{g}}(\bar{x}) + p_{\mathrm{b}}(\bar{x}))}$$

Spectrally Varying Media

Path Integral

geometry transmittance

$$f(\overline{\mathbf{x}}) = W_{\mathrm{e}}(\mathbf{x}_0, \omega_{\mathbf{x}_1\mathbf{x}_0}) \cdot \left(\prod_{i=0}^{r-1} G(\mathbf{x}_{\mathrm{r}_i}, \mathbf{x}_{\mathrm{r}_{i+1}}) \right) \cdot \left(\prod_{i=0}^{k-1} \overline{T}(\mathbf{x}_i, \mathbf{x}_{i+1}) \right) \cdot \left(\prod_{i=1}^{k-1} \rho(\omega_{\mathbf{x}_i\mathbf{x}_{i-1}}, \mathbf{x}_i, \omega_{\mathbf{x}_{i+1}\mathbf{x}_i}) \right) \cdot L_{\mathrm{e}}(\mathbf{x}_k, \omega_{\mathbf{x}_k\mathbf{x}_{k-1}}), \tag{15}$$

scattering scattering

Path Integral

$$G(\mathbf{x},\mathbf{y}) = \frac{D(\mathbf{x},\omega_{\mathbf{xy}})V(\mathbf{x},\mathbf{y})D(\mathbf{y},\omega_{\mathbf{yx}})}{\|\mathbf{x}-\mathbf{y}\|^2}$$

geometry

$$D(\mathbf{x},\omega) = \begin{cases} |n(\mathbf{x})\cdot\omega|, & \text{if } \mathbf{x}\in\mathcal{A}, \\ 1, & \text{if } \mathbf{x}\in\mathcal{V} \end{cases}$$

$$L_{\mathrm{e}}(\mathbf{x},\omega) = \begin{cases} L_{\mathrm{e}}(\mathbf{x},\omega_{\mathbf{xy}}), & \text{if } \mathbf{x}\in\mathcal{A}, \\ \mu_{\mathrm{a}}(\mathbf{x})L_{\mathrm{e}}(\mathbf{x},\omega_{\mathbf{xy}}), & \text{if } \mathbf{x}\in\mathcal{V} \end{cases}$$

$$\rho(\omega,\mathbf{x},\omega') = \begin{cases} \rho_{\mathrm{s}}(\omega,\mathbf{x},-\omega'), & \text{if } \mathbf{y}\in\mathcal{A}, \\ \mu_{\mathrm{s}}(\mathbf{x})\rho_{\mathrm{m}}(\omega,\mathbf{x},\omega'), & \text{if } \mathbf{x}\in\mathcal{V}, \\ \mu_{\mathrm{n}}(\mathbf{x})H(\omega\cdot\omega'), & \text{if } \mathbf{x}\in\mathcal{V}_\delta, \end{cases}$$

scattering