Unbiased and consistent rendering using biased estimators

Zackary Misso[1] Benedikt Bitterli[1,2] Iliyan Georgiev[3] Wojciech Jarosz[1]

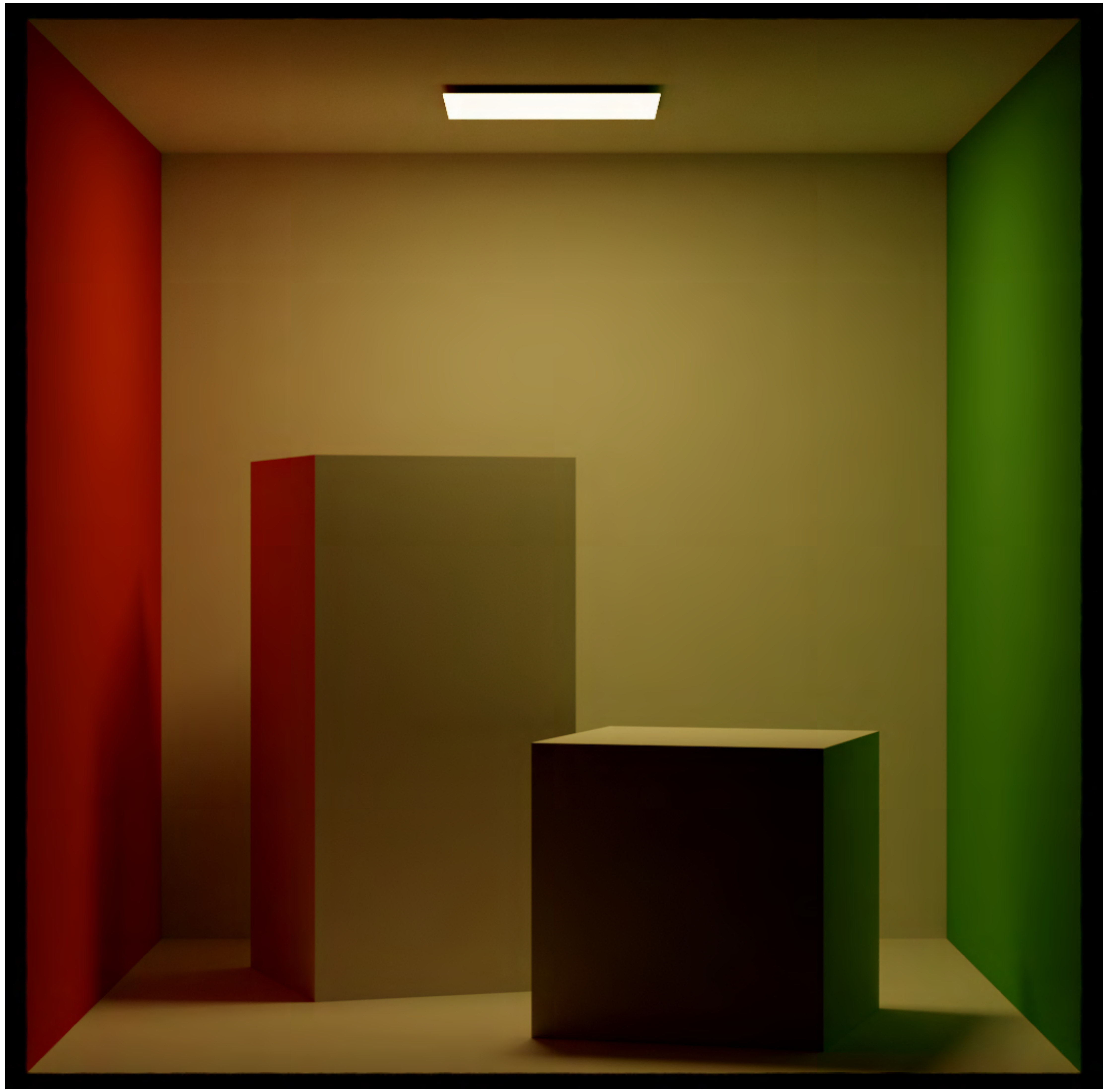
[1]Dartmouth College, [2]NVIDIA, [3]Autodesk


[Bitterli et. al. 2018]

Unbiased solutions

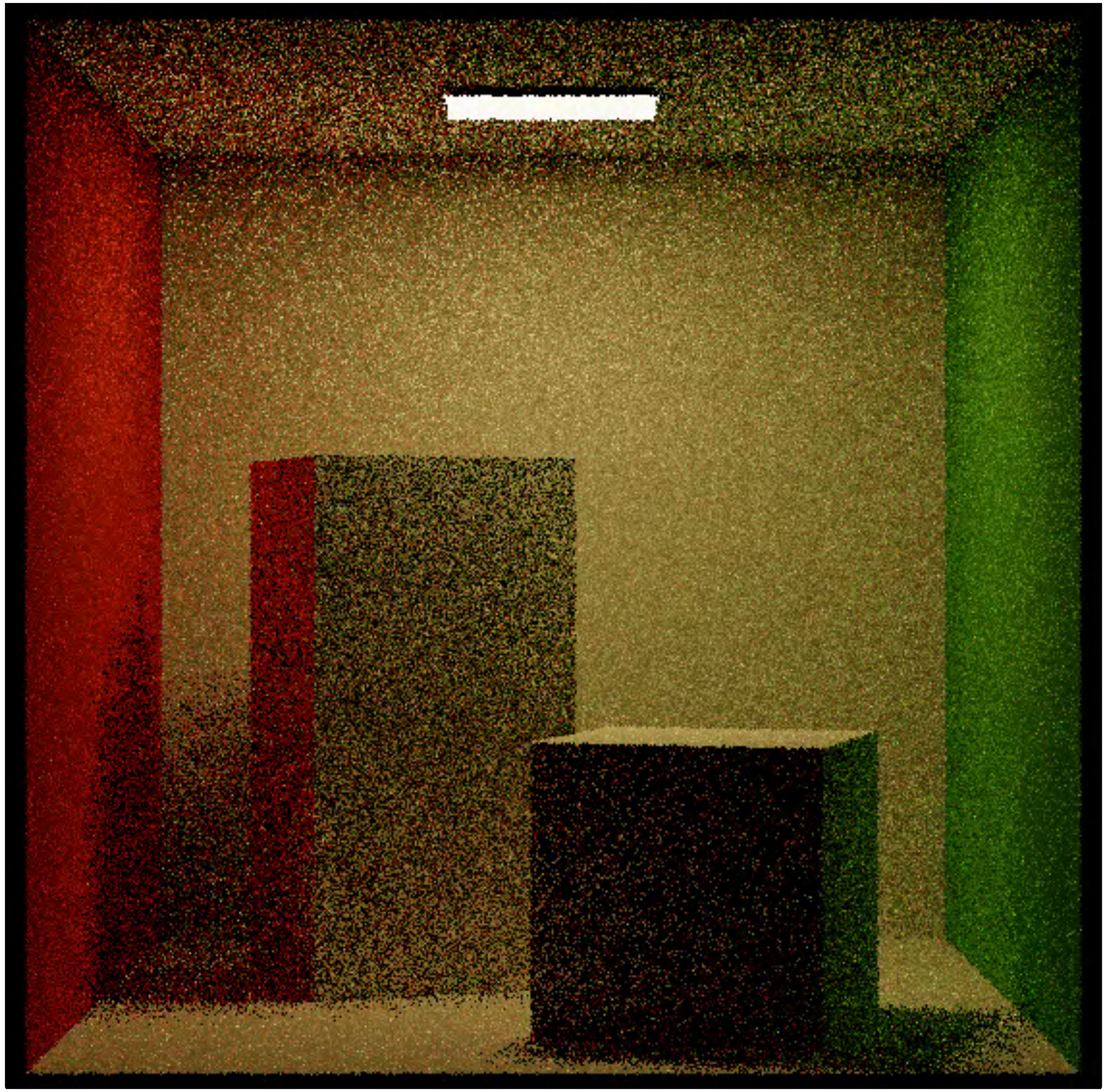
Unbiased solutions

$$I$$



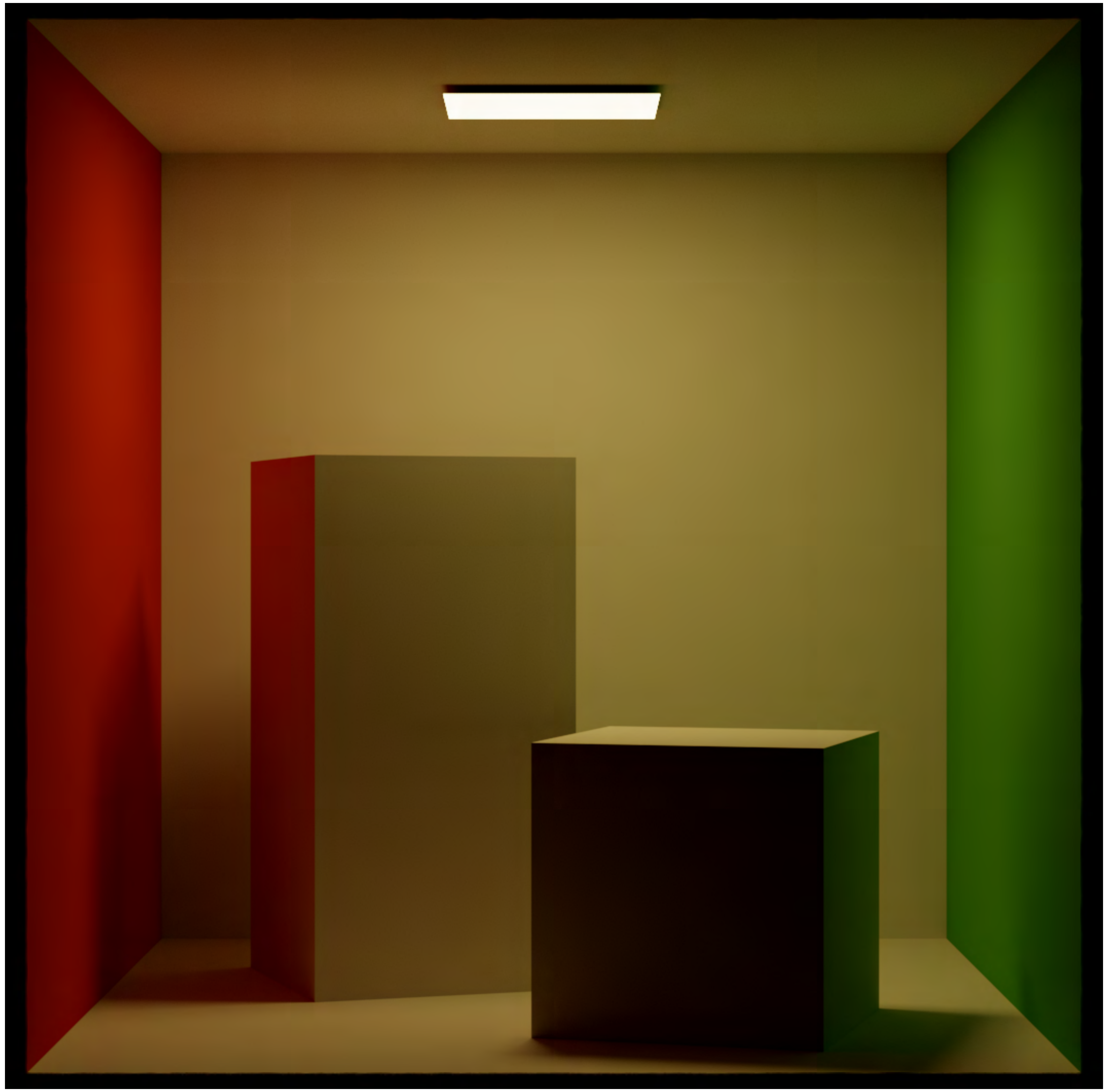
Unbiased solutions

$$\langle I \rangle$$



Unbiased solutions

$$\mathrm{E}\left[\langle I \rangle\right] = I$$

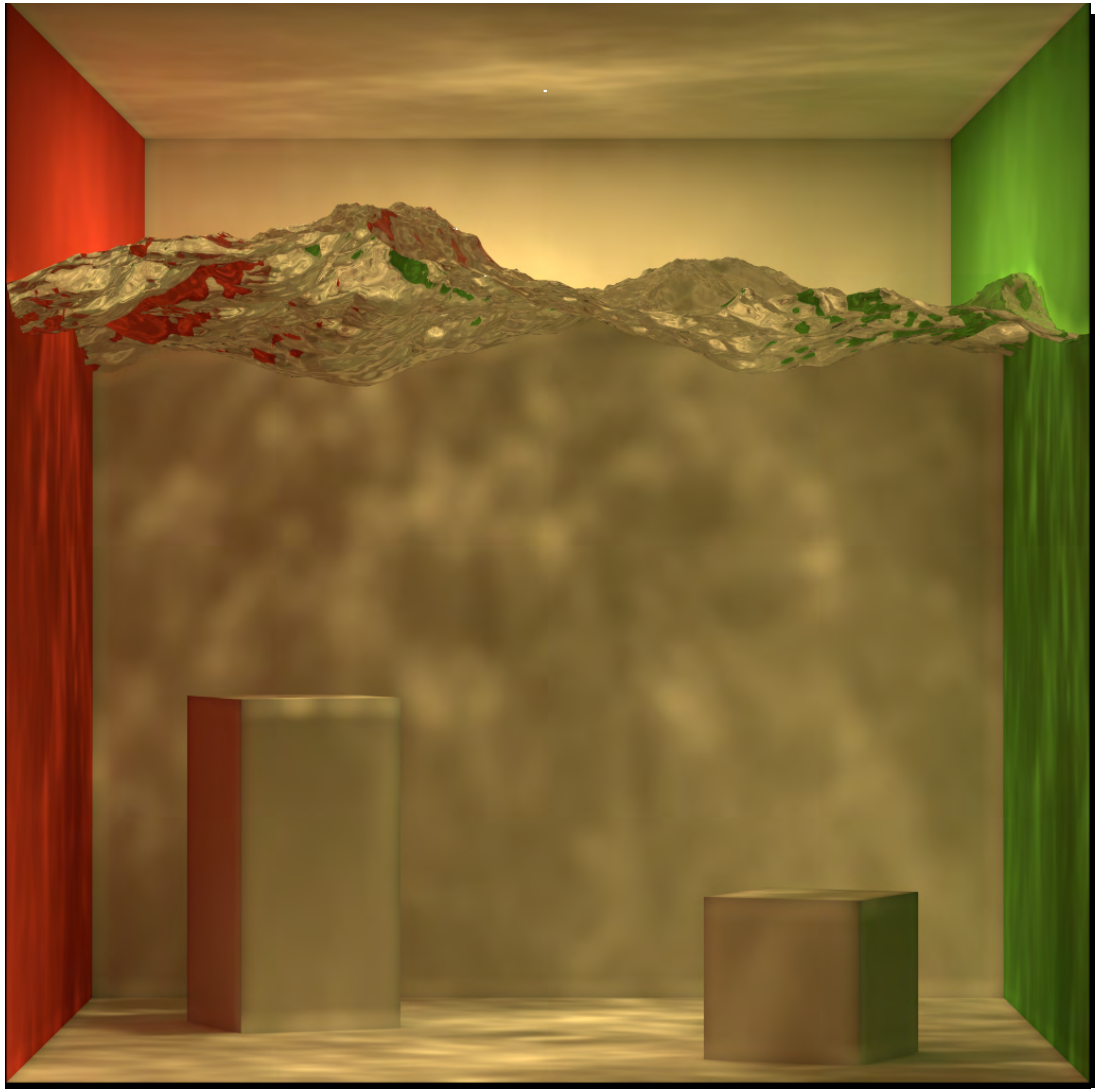


Biased solutions

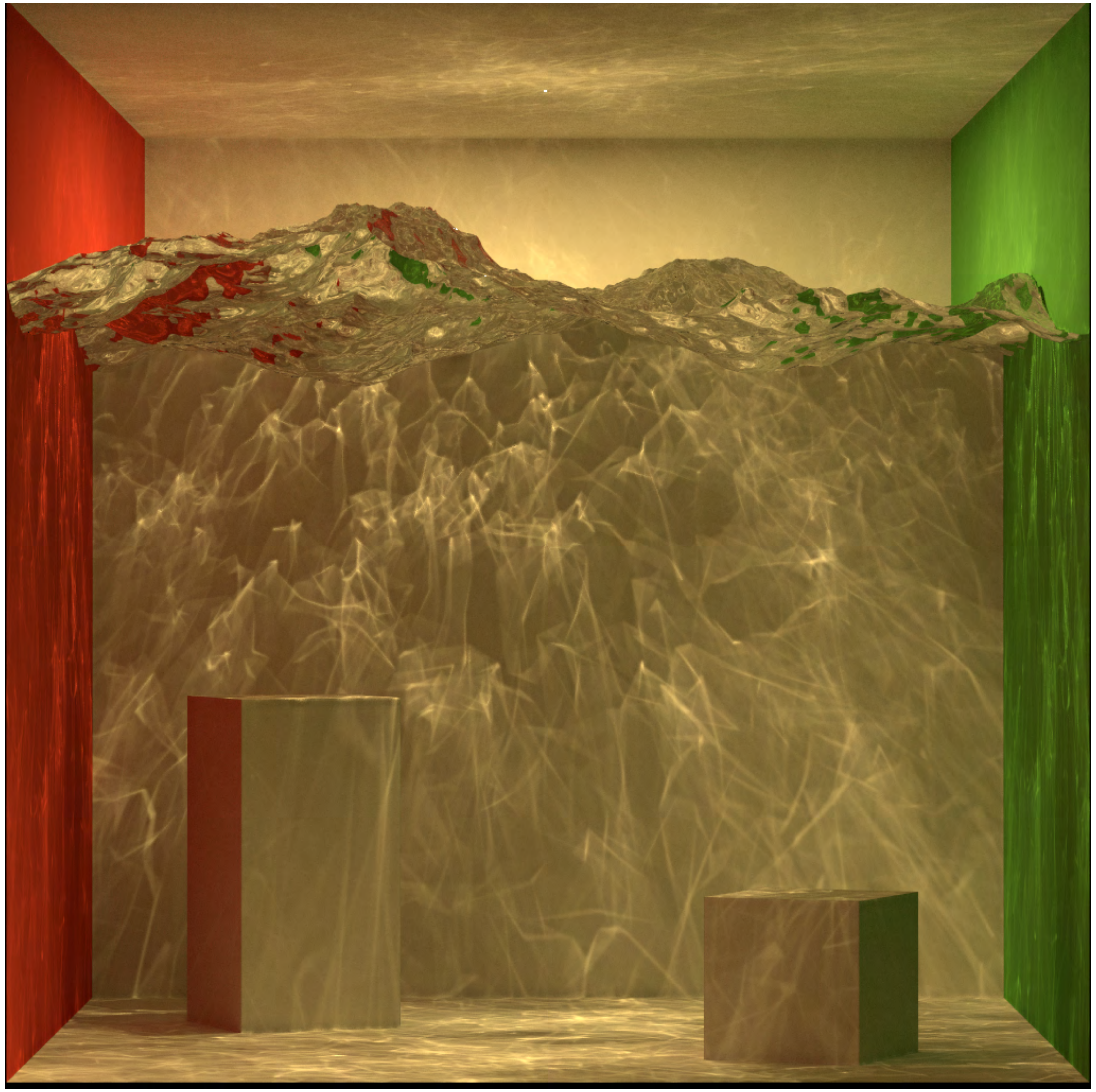
$$\mathrm{E}\left[\langle I \rangle\right] \neq I$$

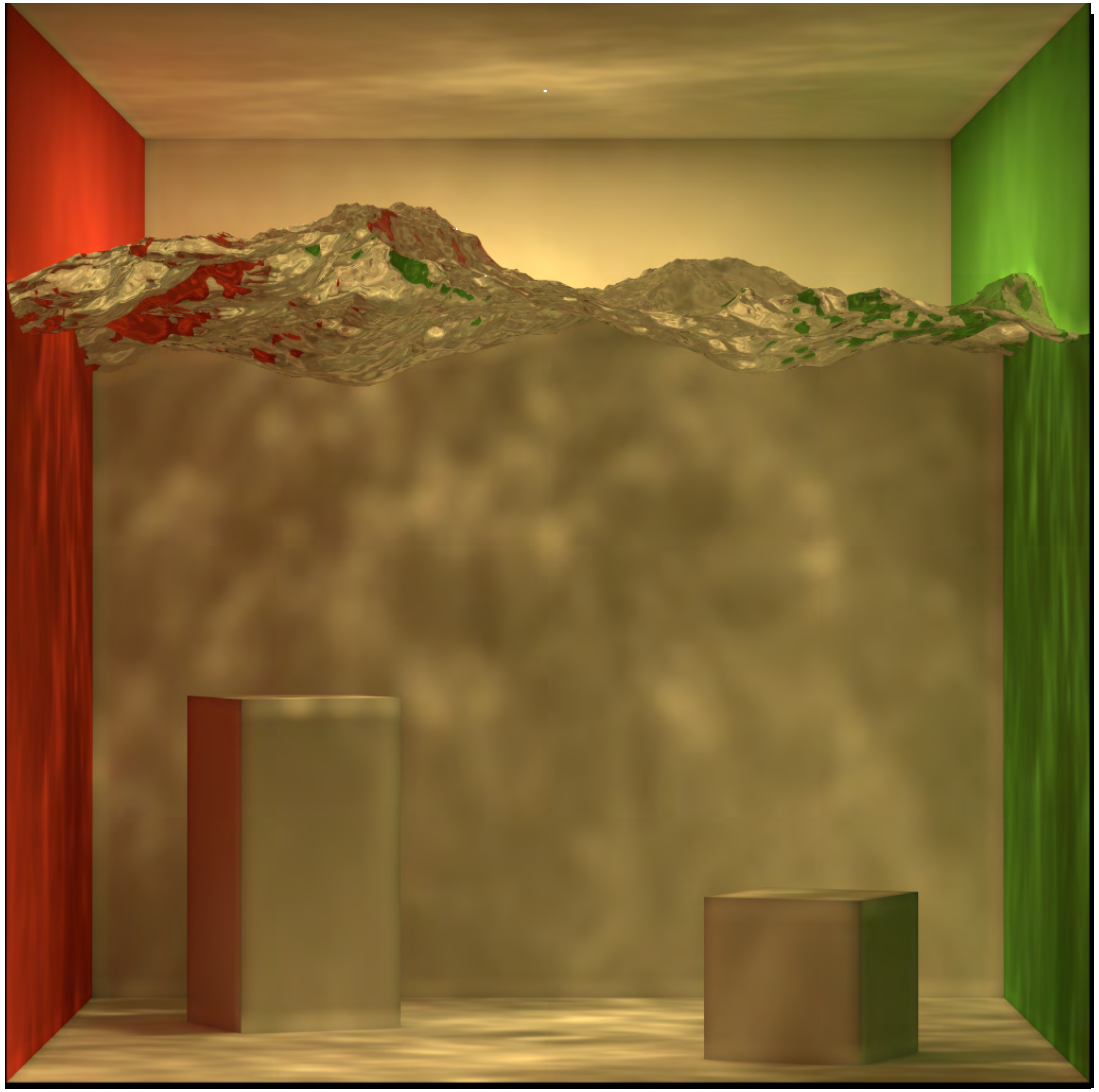
Biased solutions

$$\mathrm{E}\left[\langle I \rangle\right] \neq I$$



Biased solutions



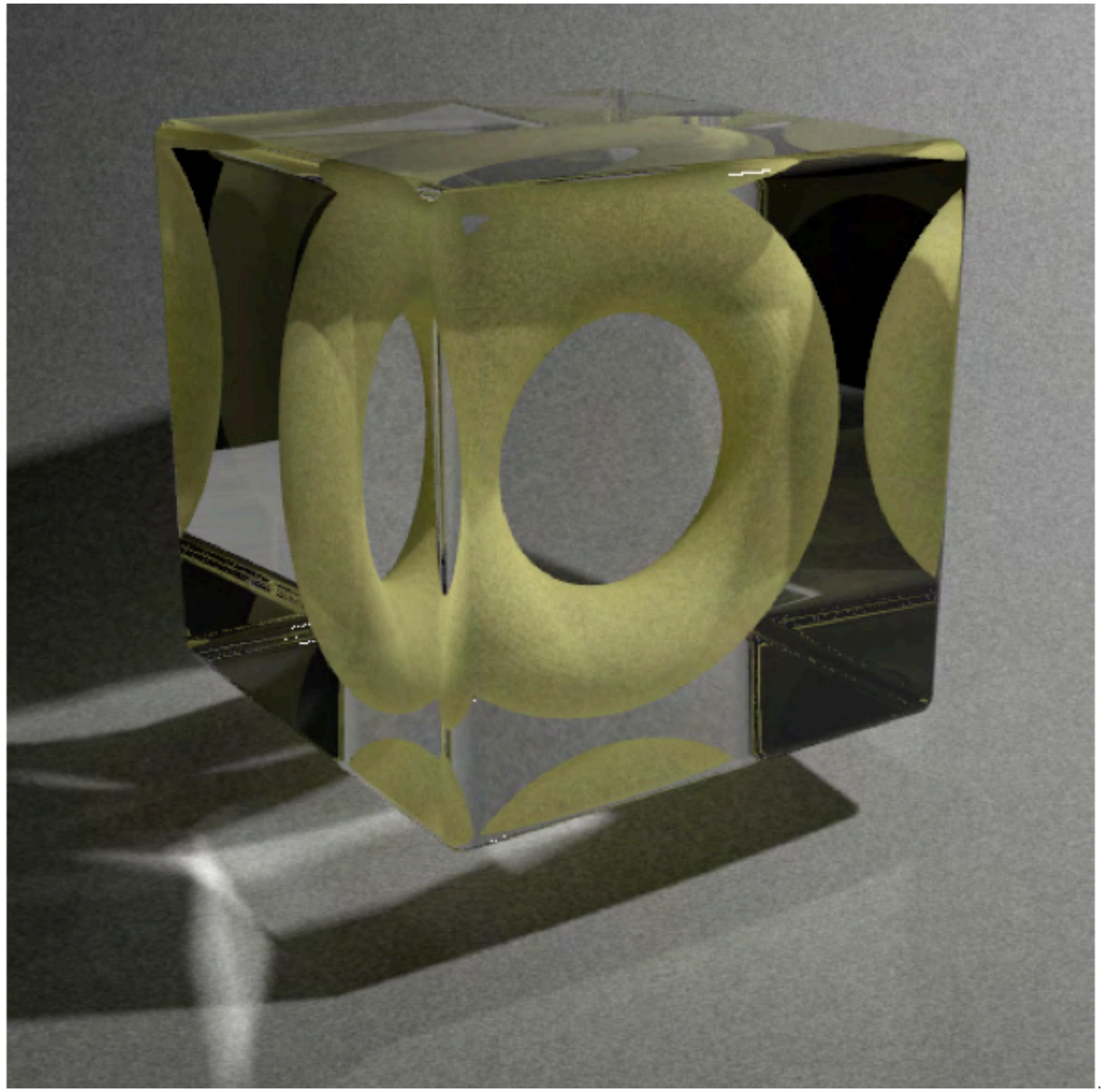


Motivation

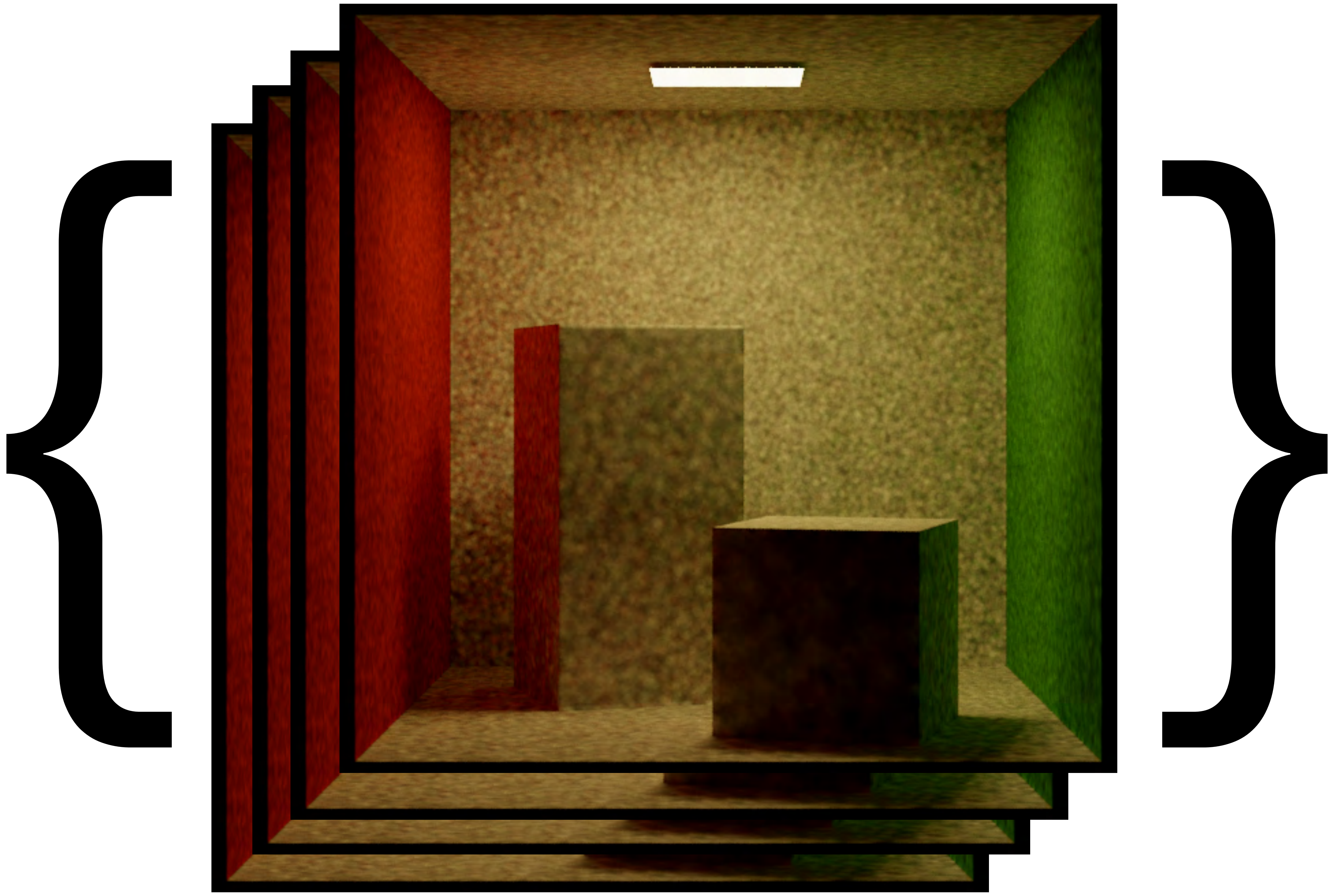
Progressive photon mapping

[Hachisuka et. al. 2008]

[Knaus et. al. 2011]

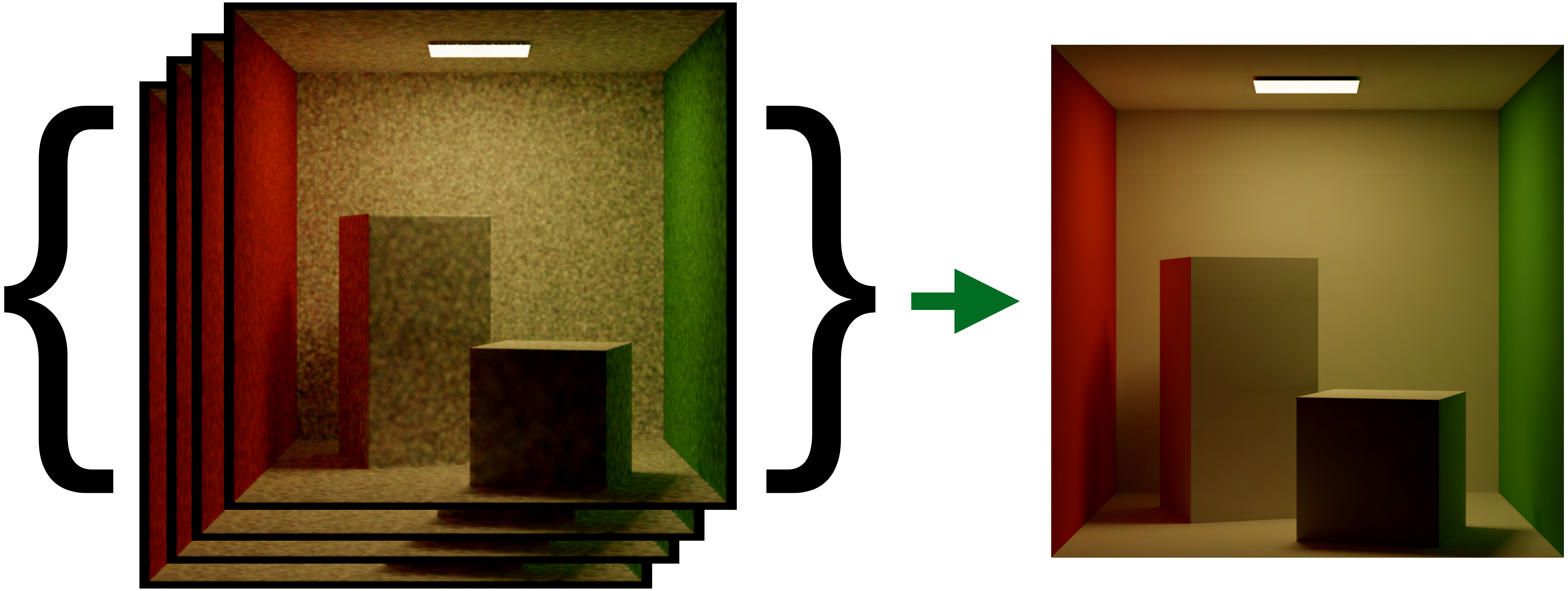


Progressive photon mapping



[Knaus et. al. 2011]

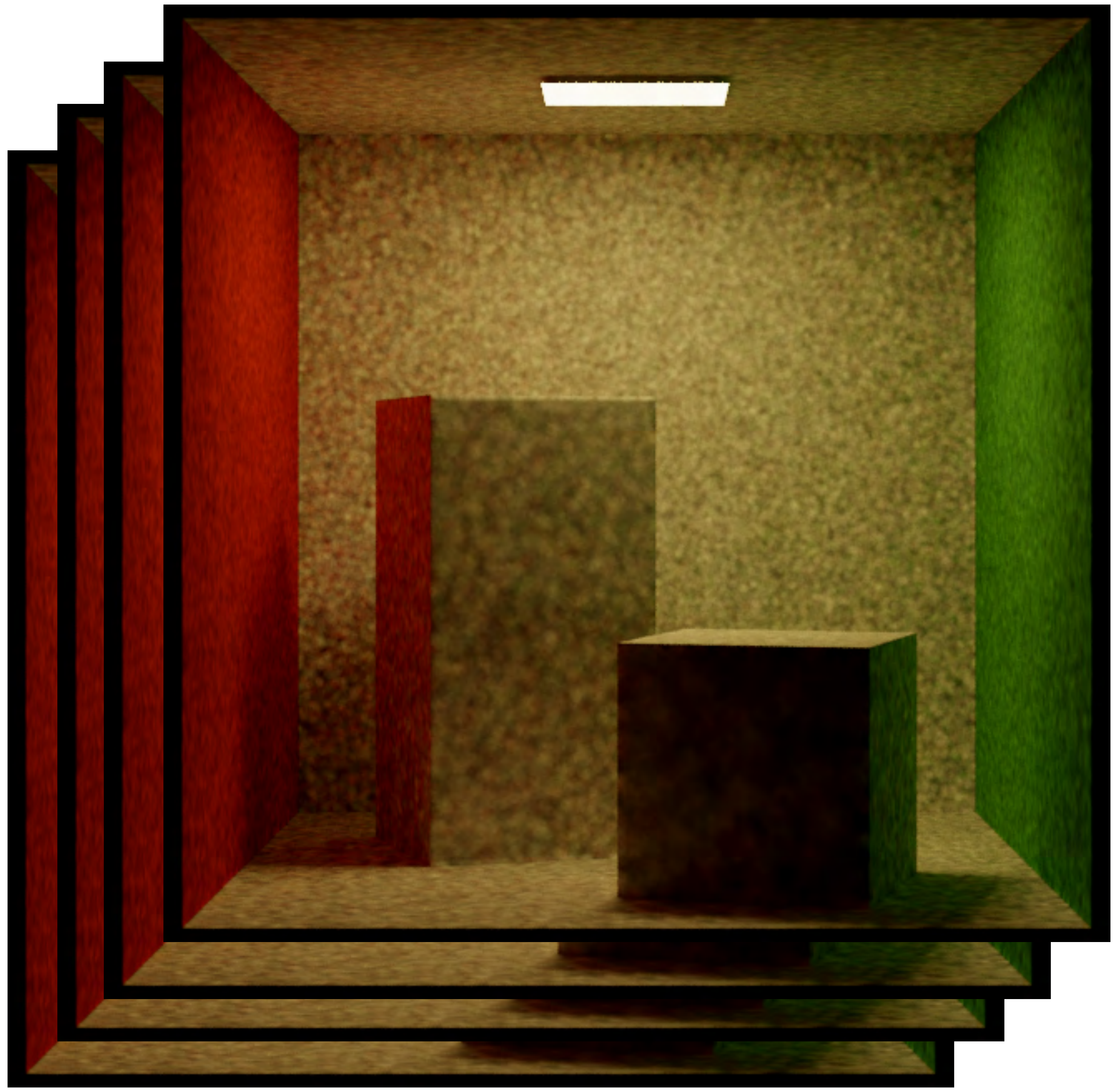
Progressive photon mapping



[Knaus et. al. 2011]

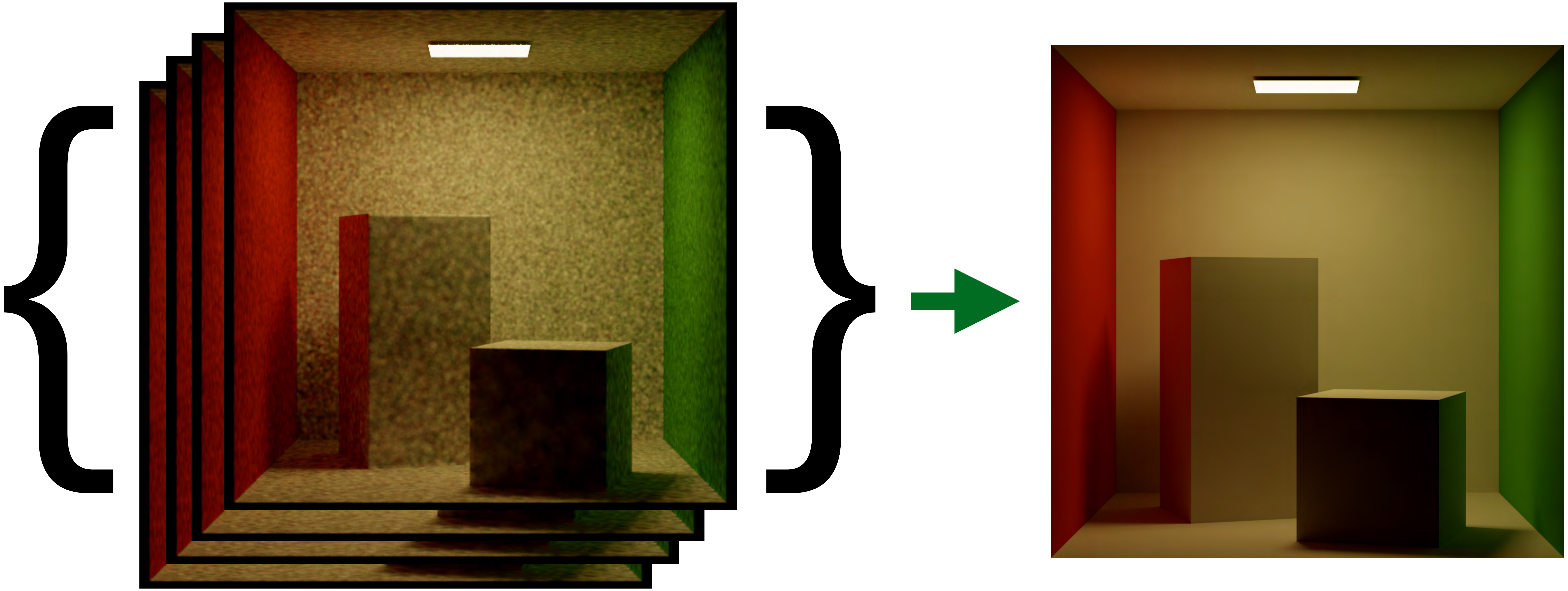
Progressive photon mapping

$$\lim_{k\to\infty} \left\{ \quad \right\} \rightarrow$$

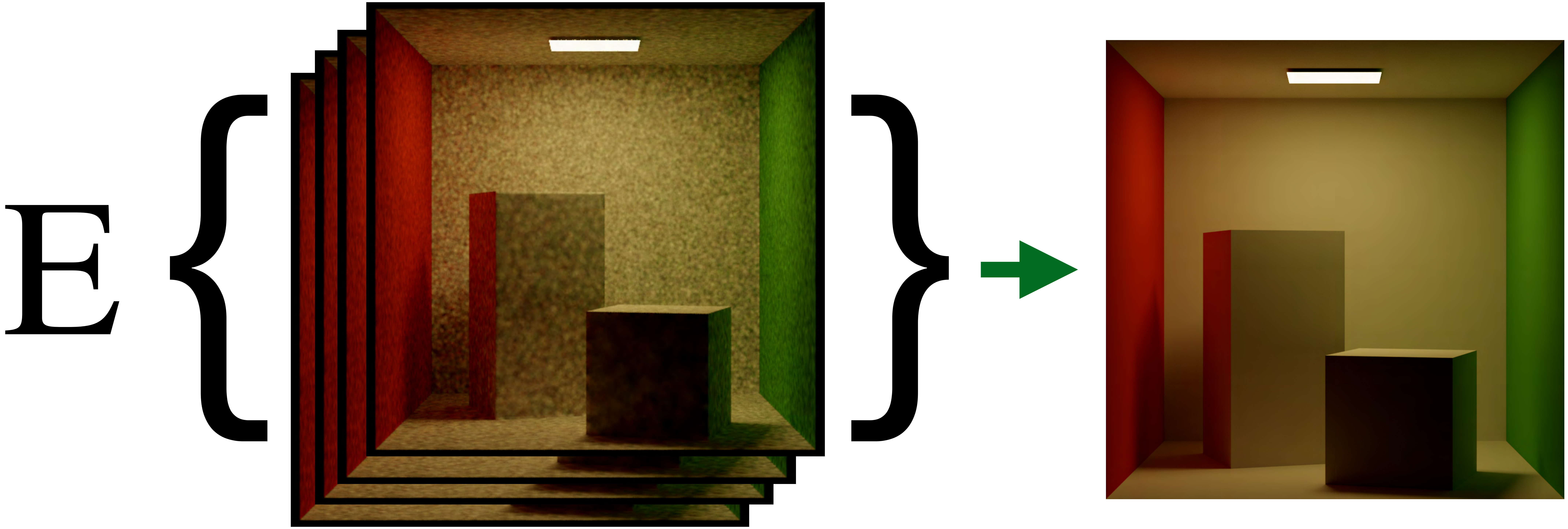


[Knaus et. al. 2011]

Our framework



Our framework



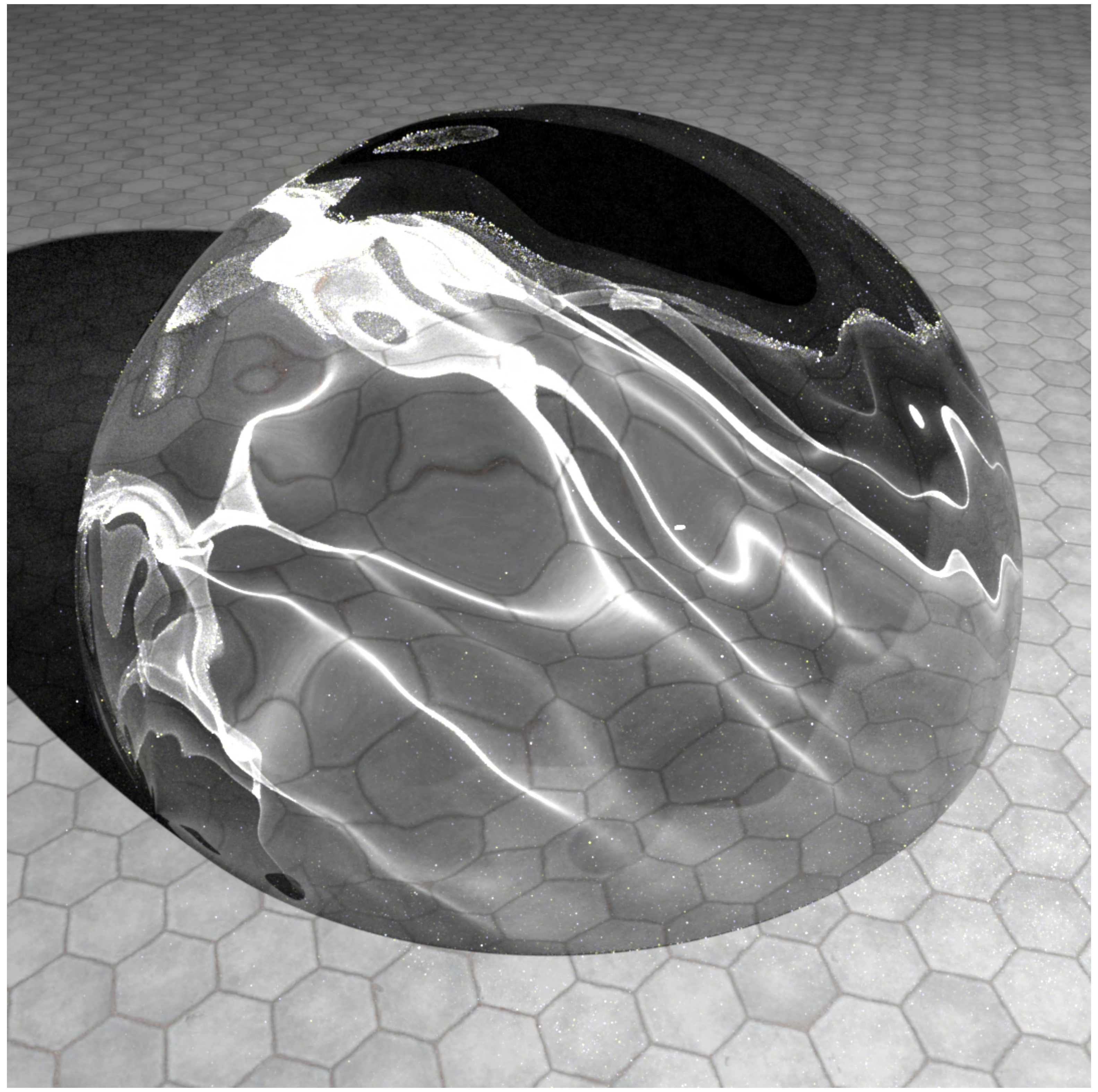
Related work

Reciprocal Estimation

[Booth 2007]

[Qin et. al. 2015]

[Zeltner et. al. 2020]



Related work

Null Collision

[Novak et. al. 2014] [Georgiev et. al. 2019]

Applicable problems

Applicable problems

$$I(k)$$

Applicable problems

$$I(k)$$

Applicable problems

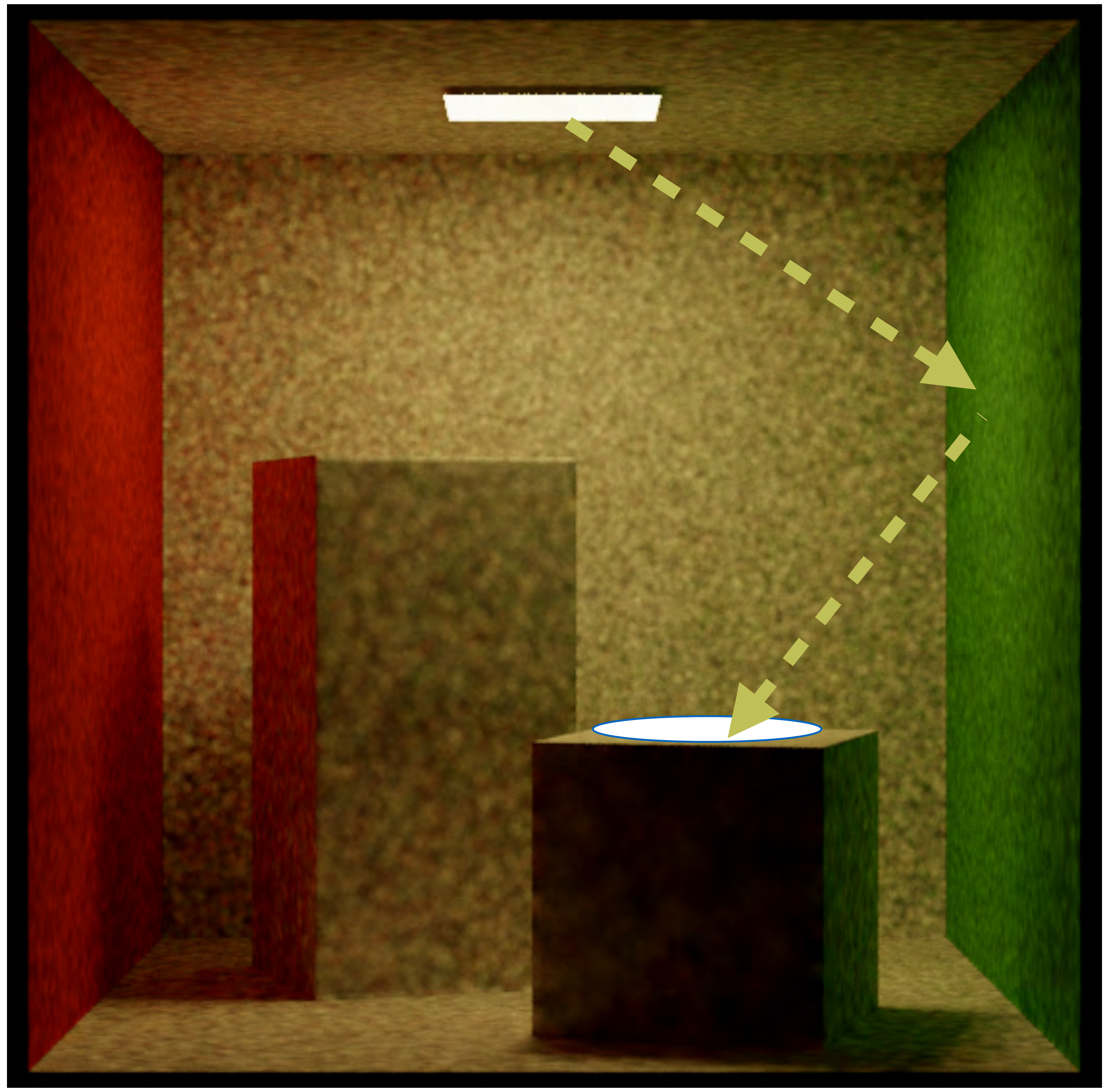
$$\lim_{k \to \infty} I(k) = I$$

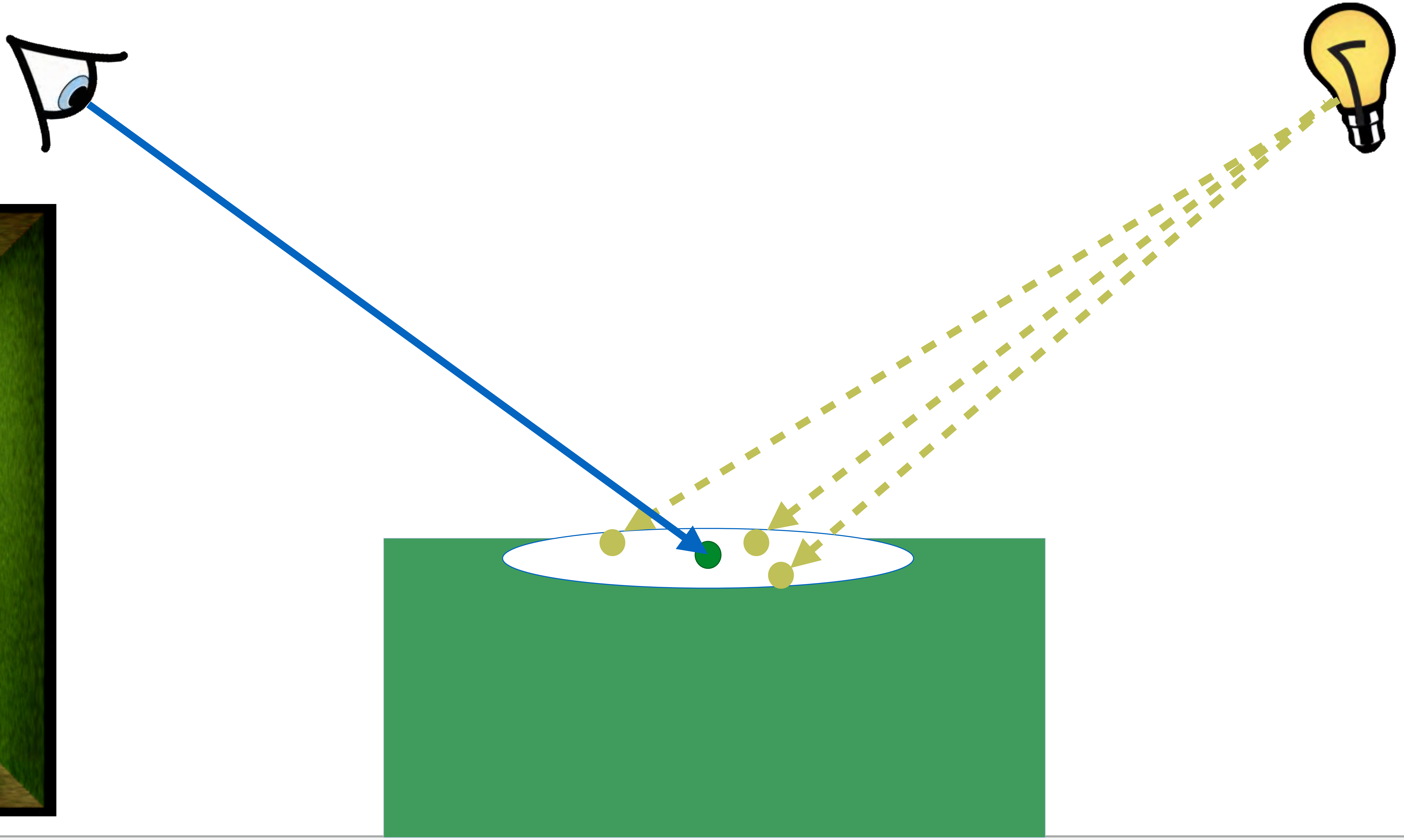
Path Tracing - max path depth


I(1)

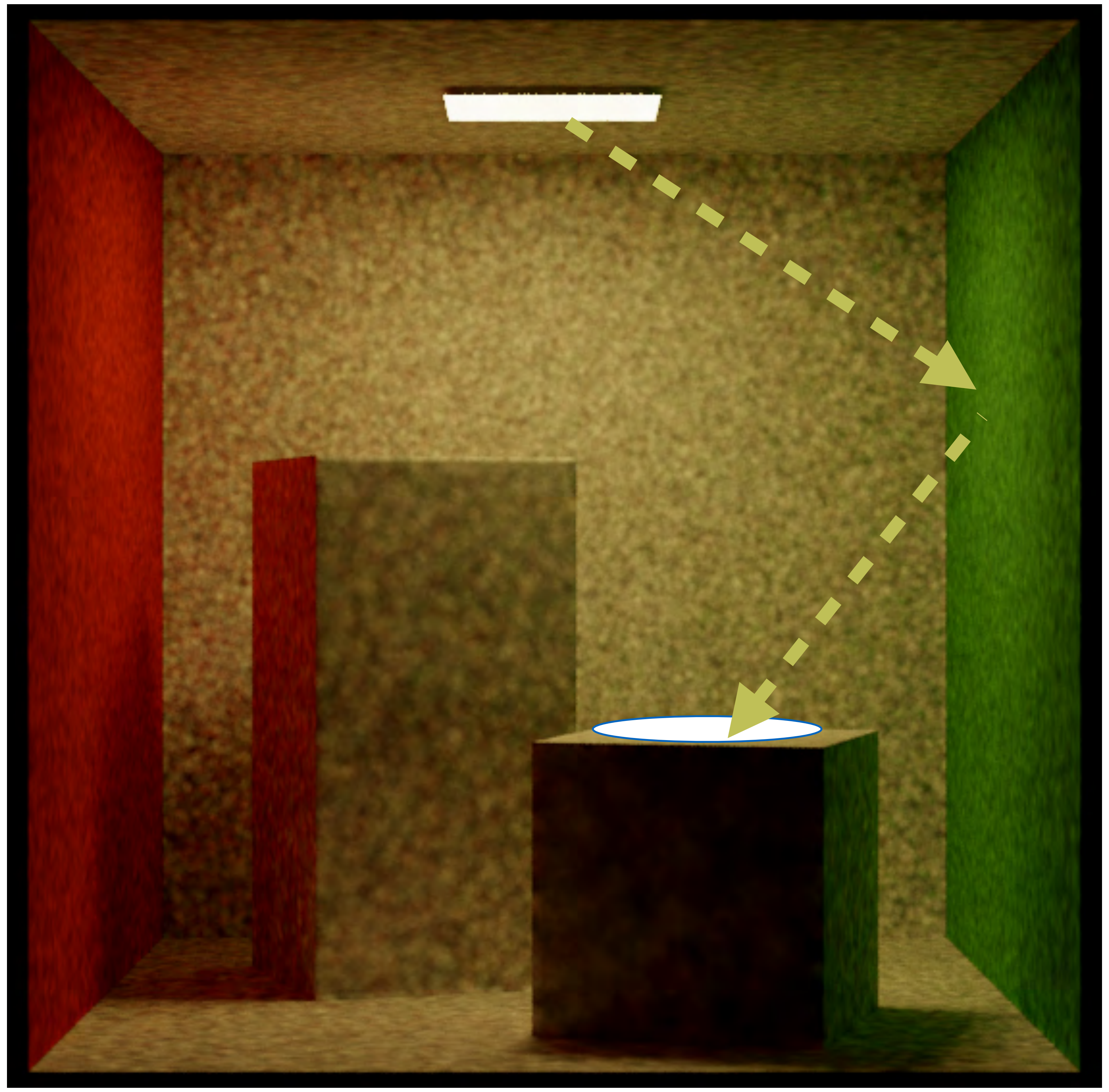

$I(1)$
$I(\infty)$

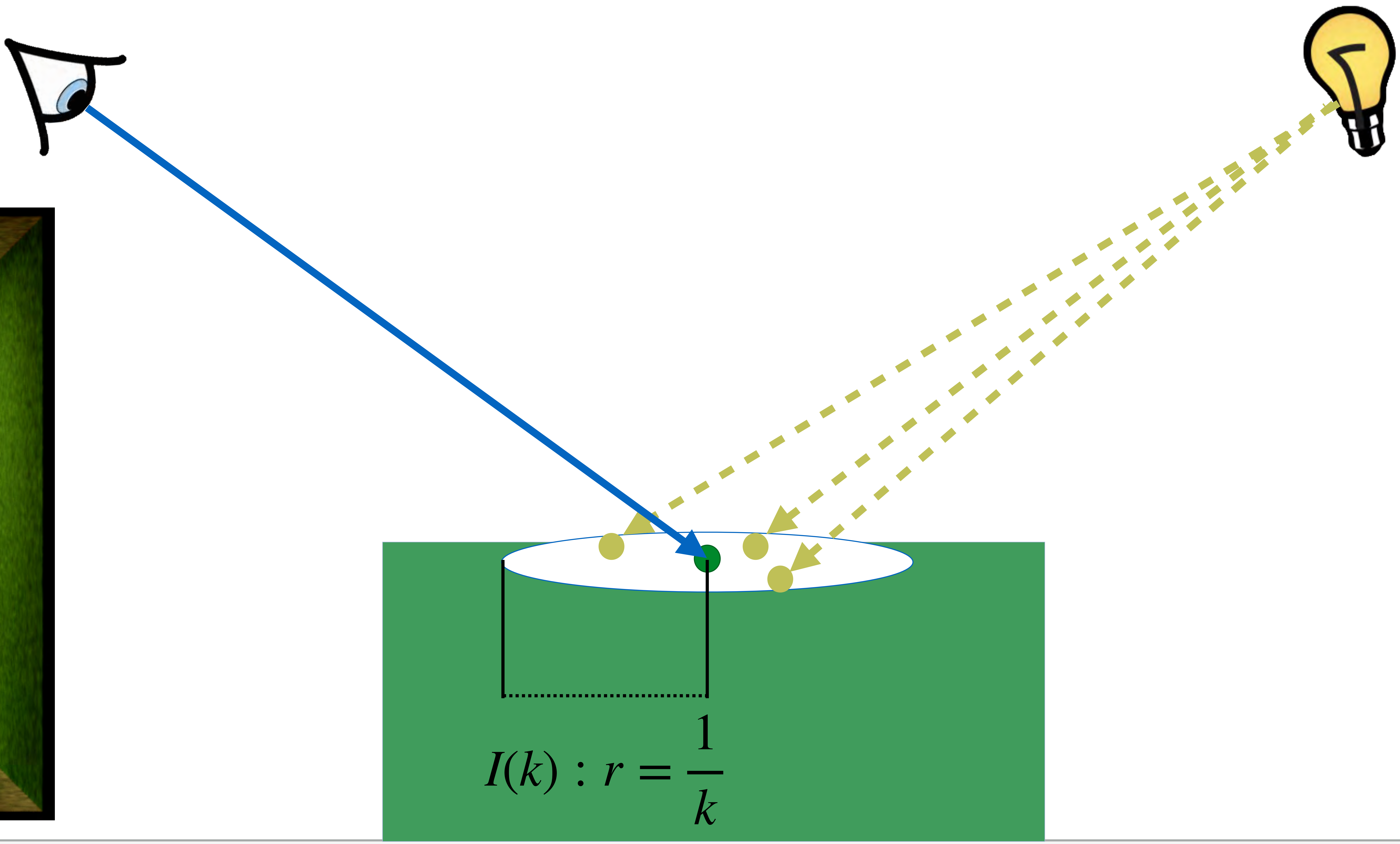
Photon mapping





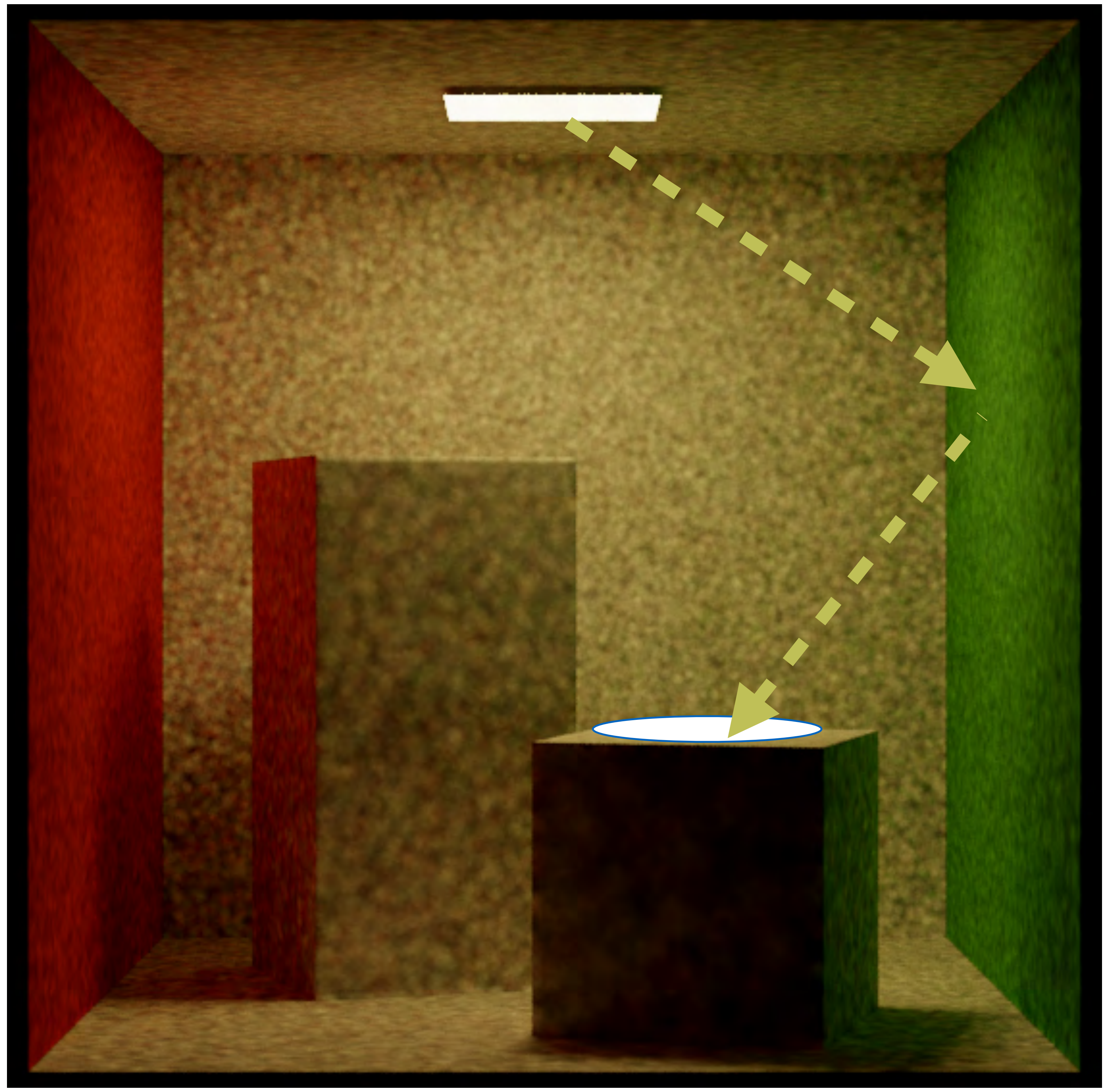
Photon mapping

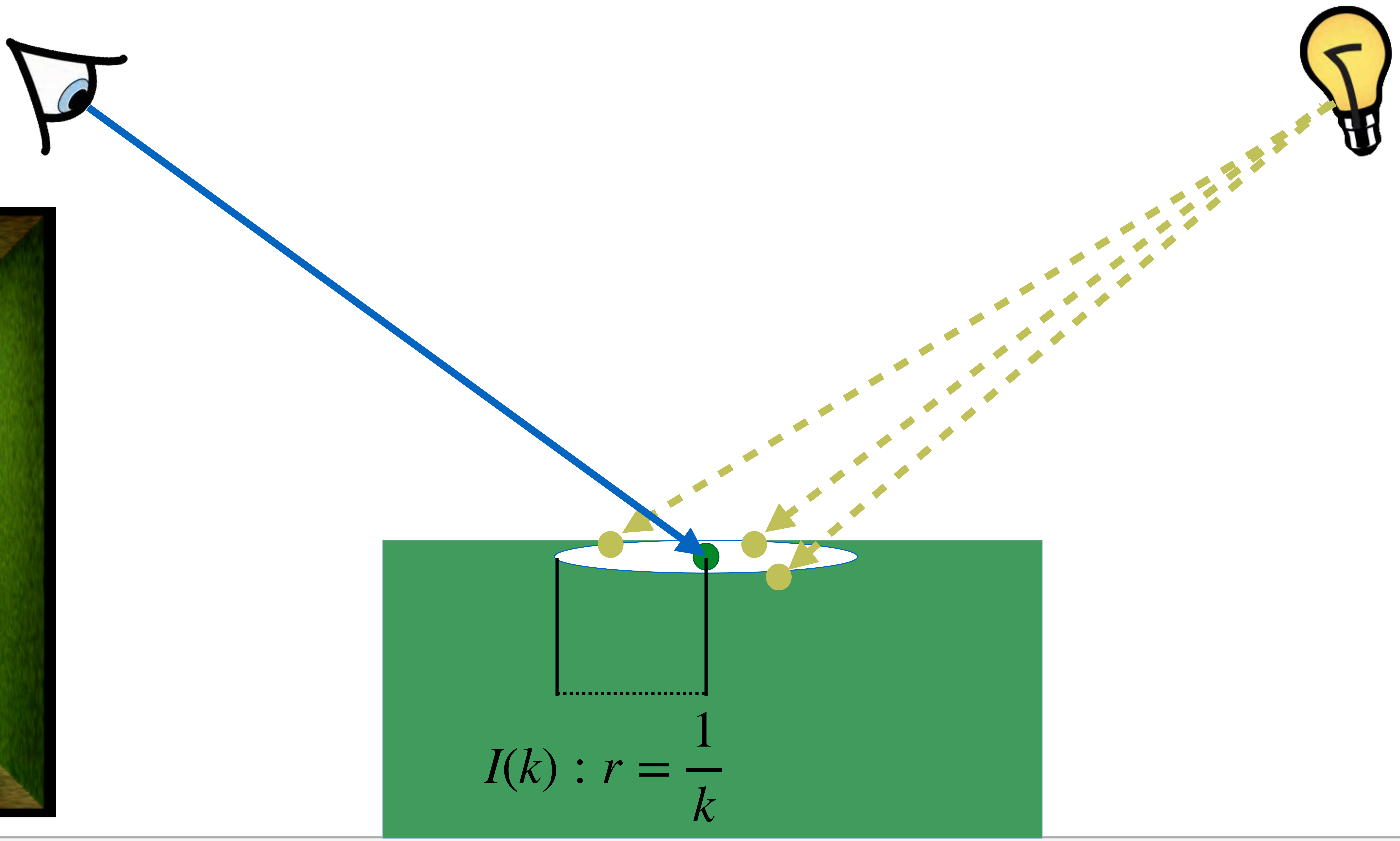




$$I(k) : r = \frac{1}{k}$$

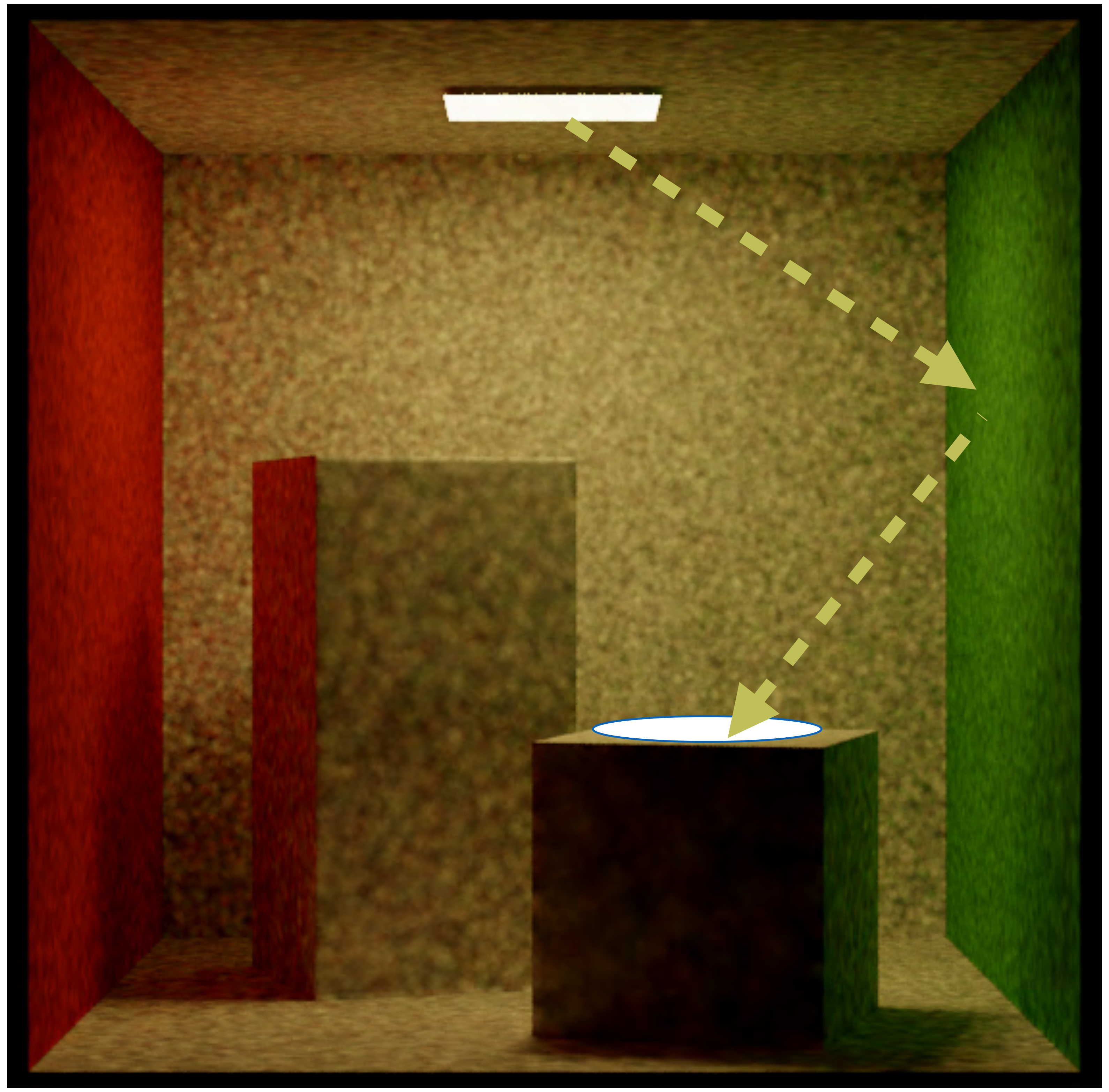
Photon mapping

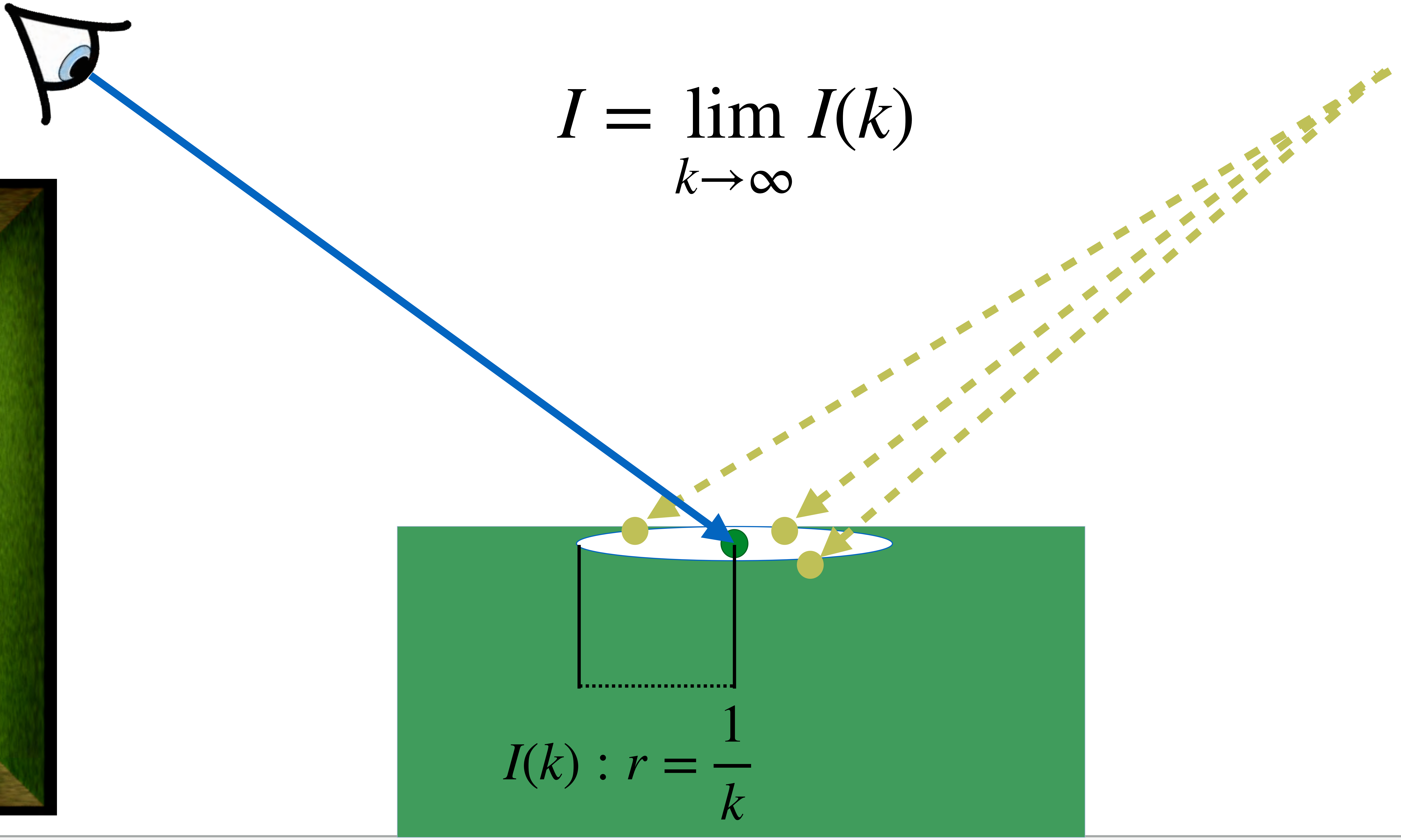




$$I(k) : r = \frac{1}{k}$$

Photon mapping





$$I = \lim_{k \to \infty} I(k)$$

$$I(k) : r = \frac{1}{k}$$

Debiasing

$$I = \lim_{k \to \infty} I(k)$$

Debiasing

$$I(\infty) = \lim_{k \to \infty} I(k)$$

Debiasing

$$I(\infty) = I(k) + \left[I(\infty) - I(k) \right]$$

Debiasing

$$I(\infty) = \boxed{I(k)} + \left[I(\infty) - \boxed{I(k)} \right]$$

Debiasing

$$\overbrace{I(\infty)}^{\text{GT}} = I(k) + \left[I(\infty) - I(k)\right]$$

Debiasing

GT　　　Biased

$$\underbrace{I(\infty)}_{\text{GT}} = \underbrace{I(k)}_{\text{Biased}} + \left[I(\infty) - I(k) \right]$$

Debiasing

GT Biased Bias Correction

$$\overbrace{I(\infty)}^{\text{GT}} = \overbrace{I(k)}^{\text{Biased}} + \overbrace{\left[I(\infty) - I(k)\right]}^{\text{Bias Correction}}$$

Debiasing

GT Biased Bias Correction

$$\underbrace{I(\infty)}_{\text{GT}} = \underbrace{I(k)}_{\text{Biased}} + \underbrace{\left[\boxed{I(\infty)} - I(k) \right]}_{\text{Bias Correction}}$$

Debiasing

$$\overbrace{I(\infty)}^{\text{GT}} = \overbrace{I(k)}^{\text{Biased}} + \overbrace{\left[I(\infty) - I(k)\right]}^{\text{Bias Correction}}$$

$$I(\infty) = I(k) + \sum_{j=k}^{\infty} \left[I(j+1) - I(j)\right]$$

Debiasing

$$I(\infty) = I(k) + \sum_{j=k}^{\infty} \left[I(j+1) - I(j) \right]$$

Debiasing

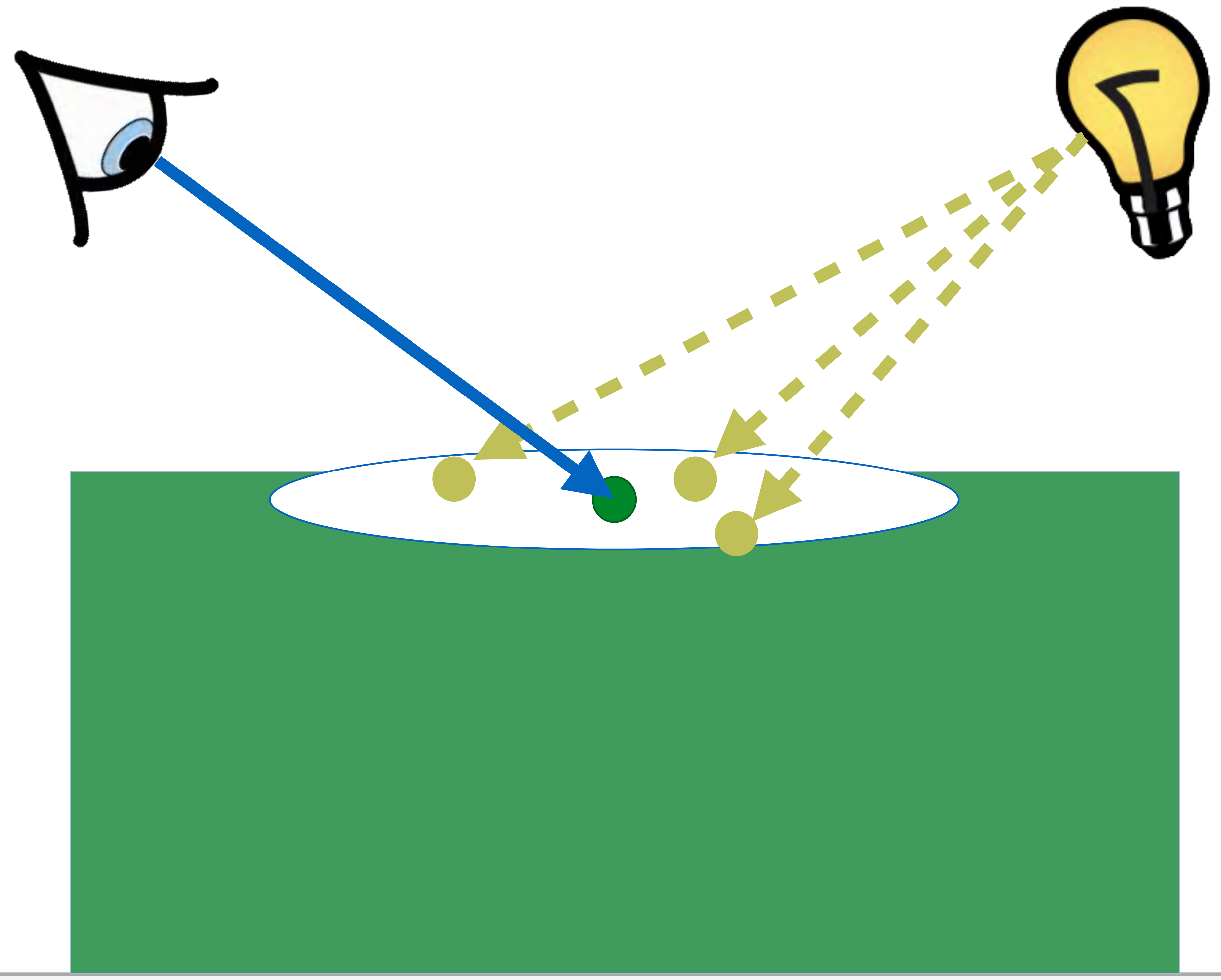
$$I(\infty) = I(k) + \frac{I(j+1) - I(j)}{p(j)}$$

Debiasing

$$\langle I(\infty)\rangle = \langle I(k)\rangle + \frac{\langle I(j+1)\rangle - \langle I(j)\rangle}{p(j)}$$

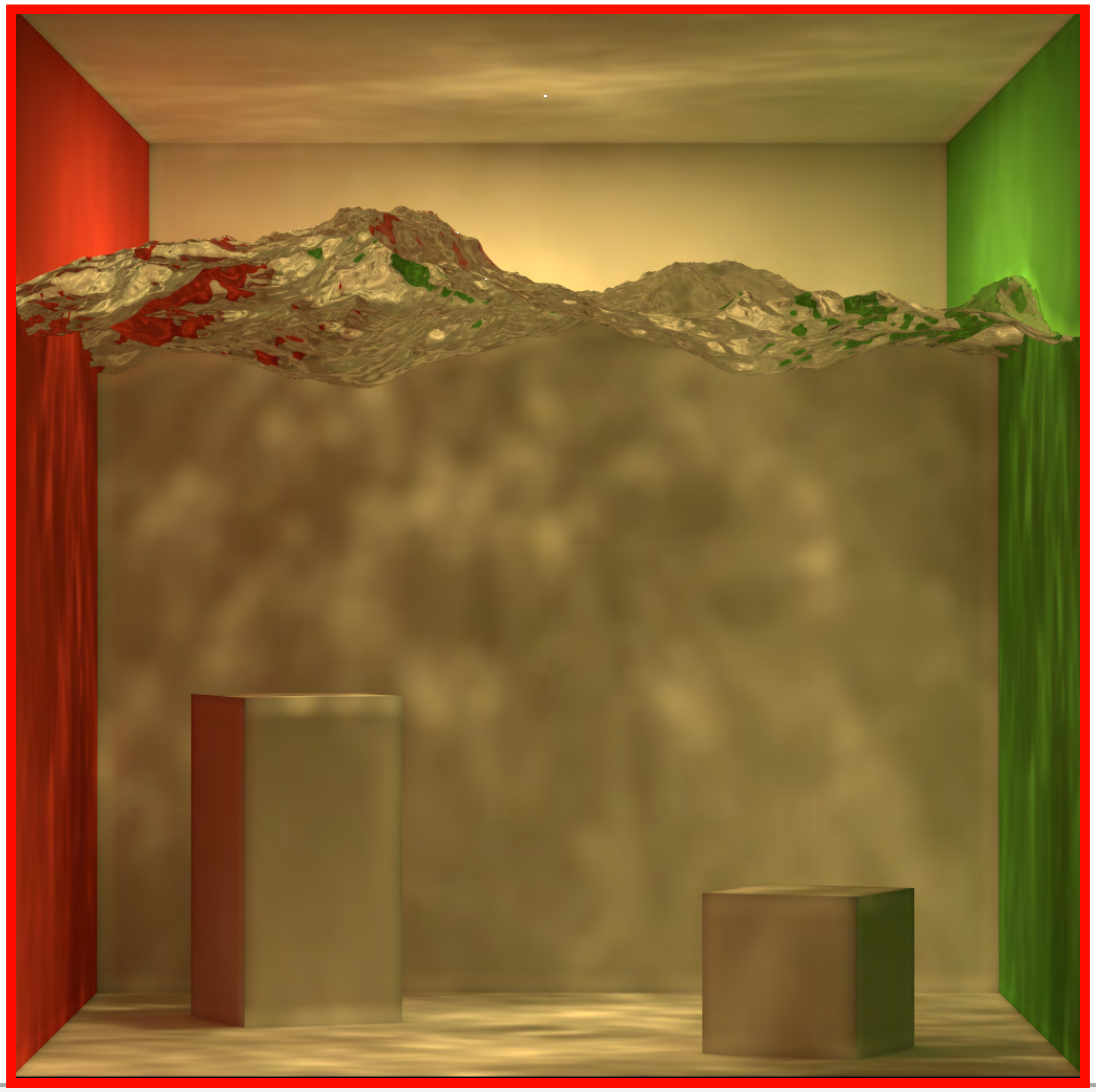
Unbiased Photon-mapping

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$



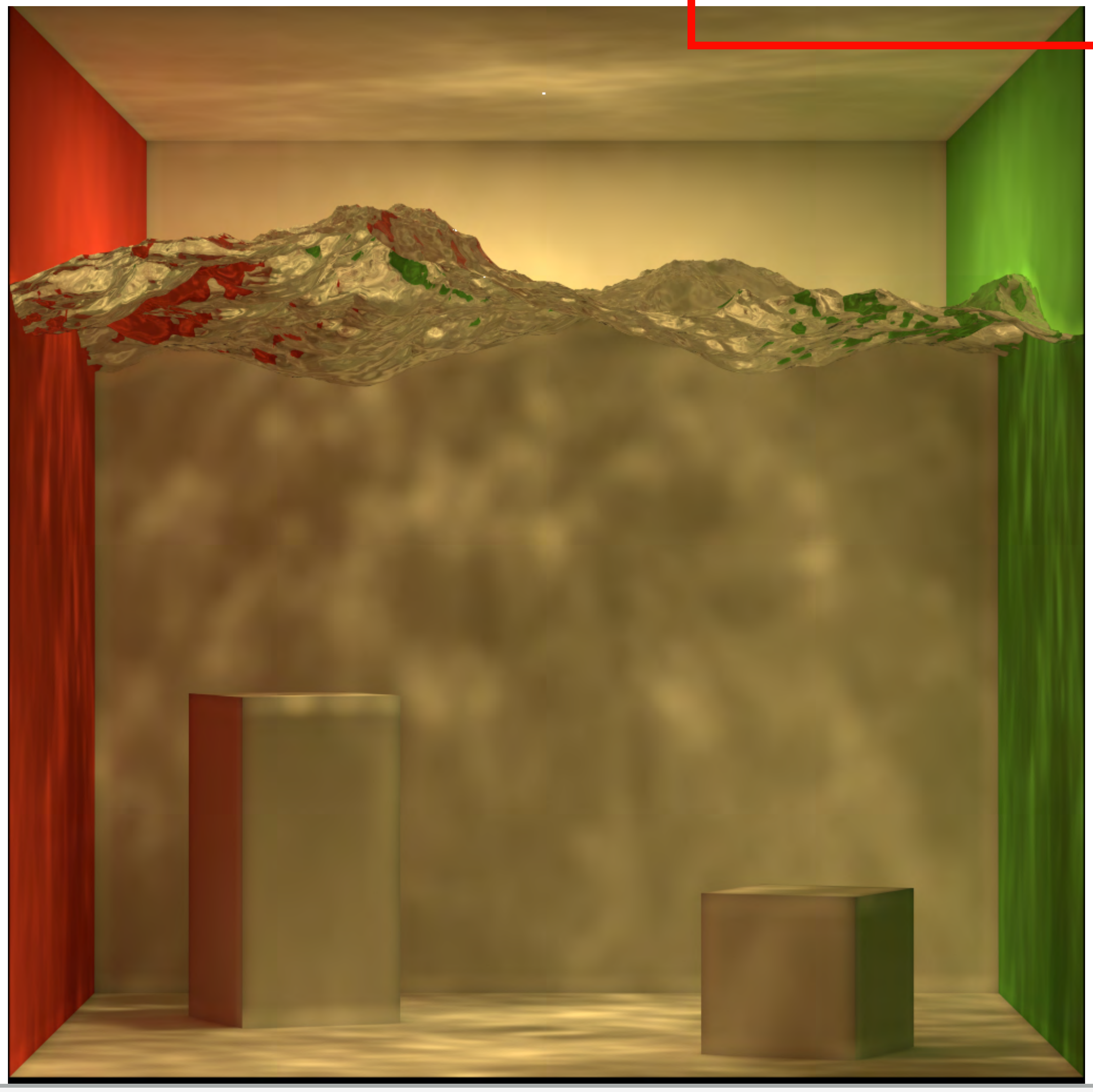
Unbiased Photon-mapping

$$\langle I \rangle = \boxed{\langle I(k) \rangle} + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$



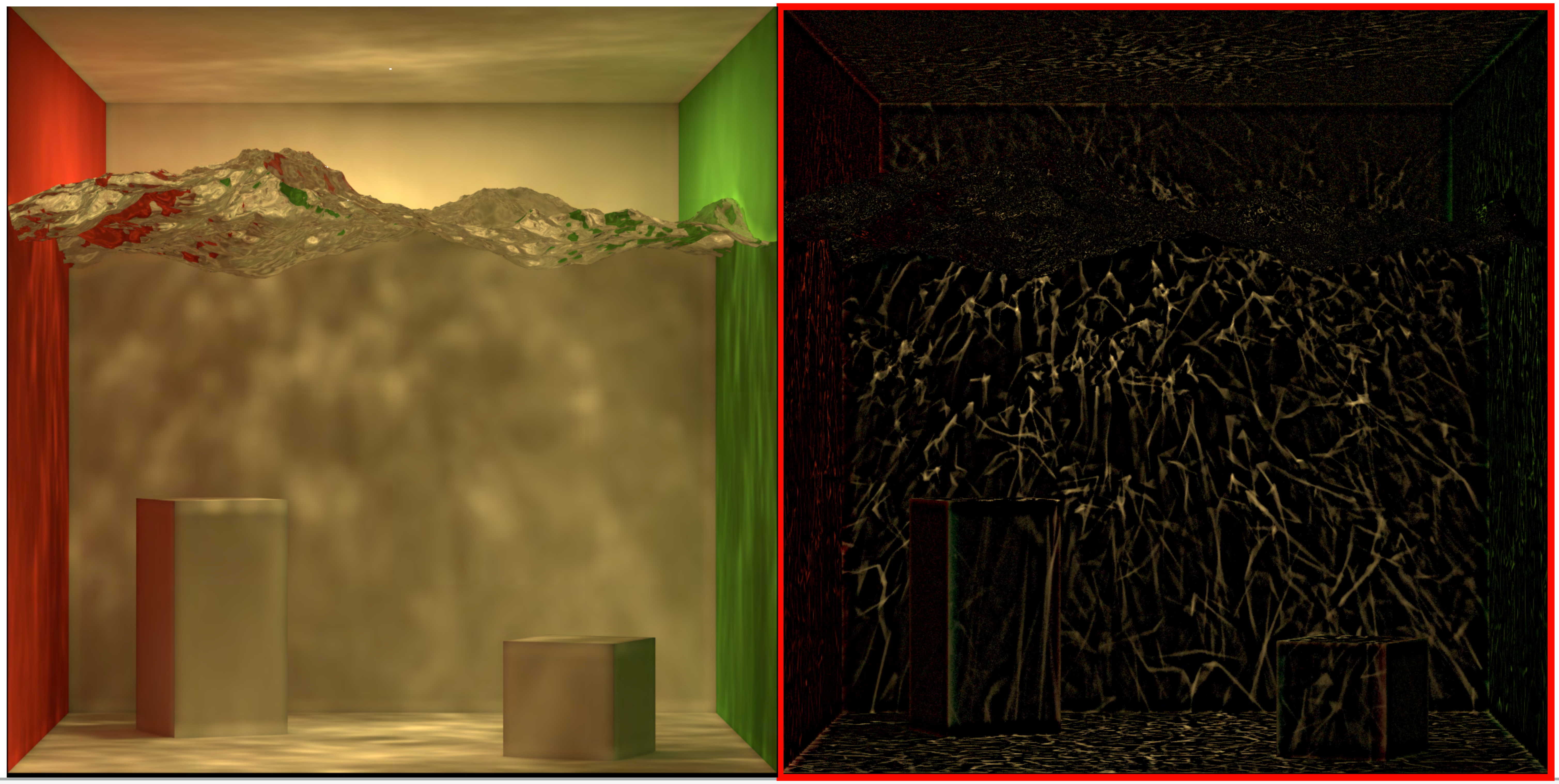
Unbiased Photon-mapping

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$



Unbiased Photon-mapping

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

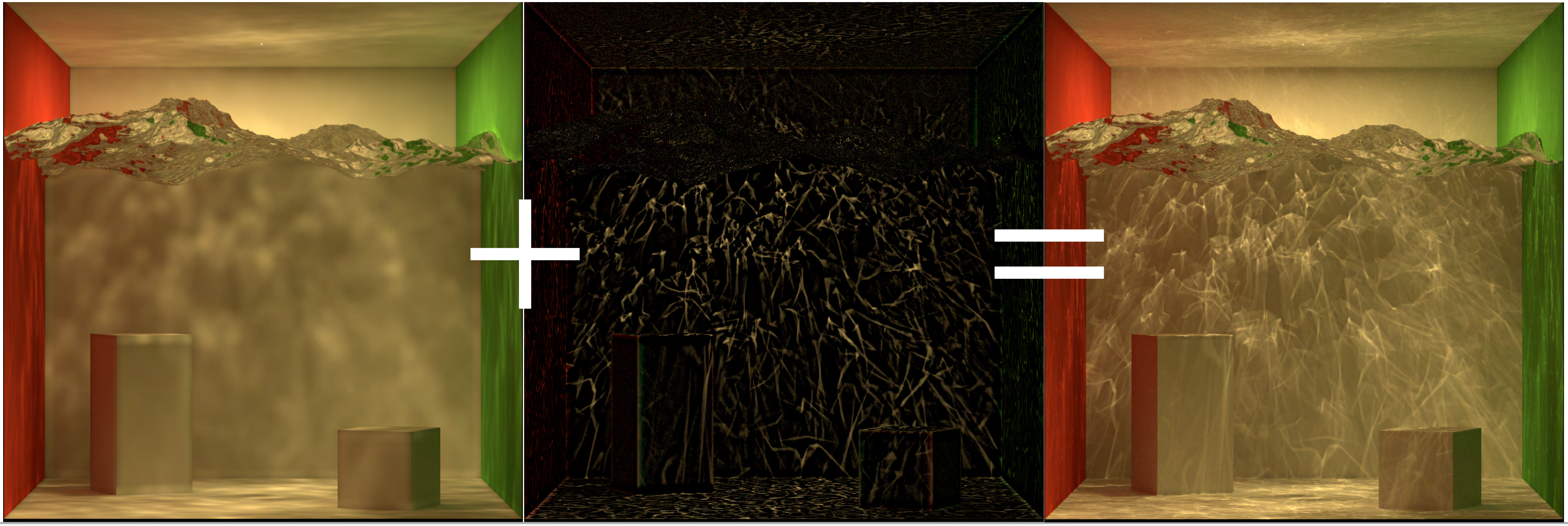


Unbiased Photon-mapping

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

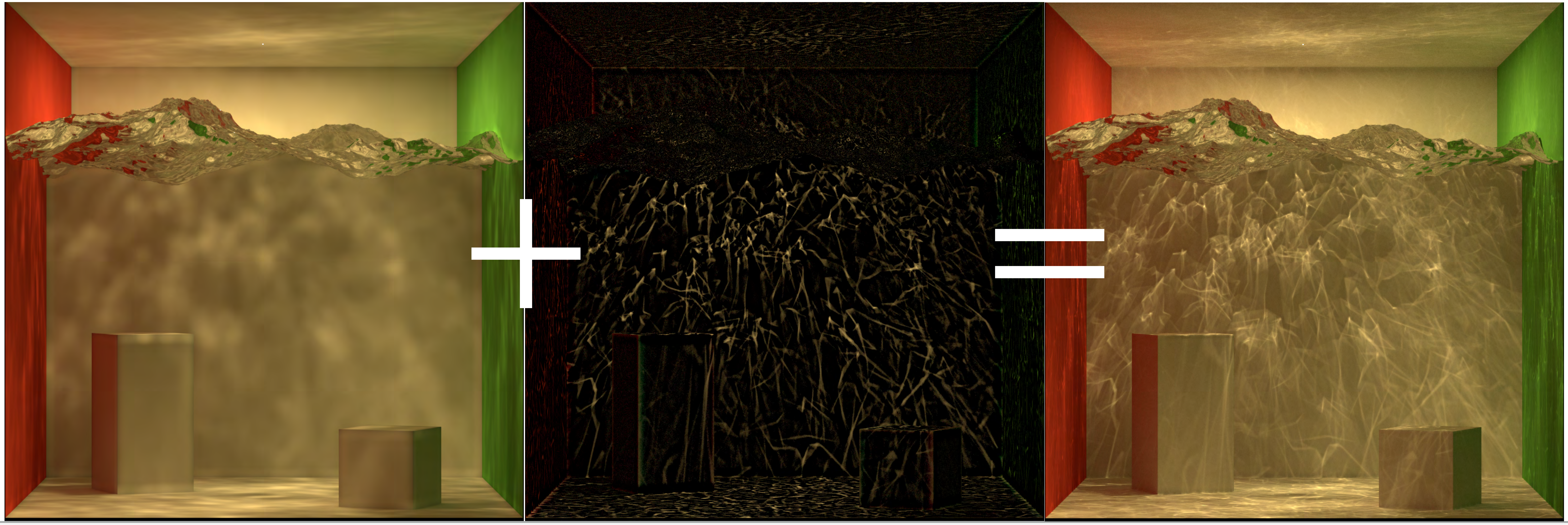
Unbiased Photon-mapping

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$



Unbiased Photon-mapping

$$\langle I \rangle = \left\langle I(k) + \frac{I(j+1) - I(j)}{p(j)} \right\rangle$$



Unbiased Ray-marching

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

Unbiased Ray-marching

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

$$I = g\left(\int f(x)dx\right)$$

Unbiased Ray-marching

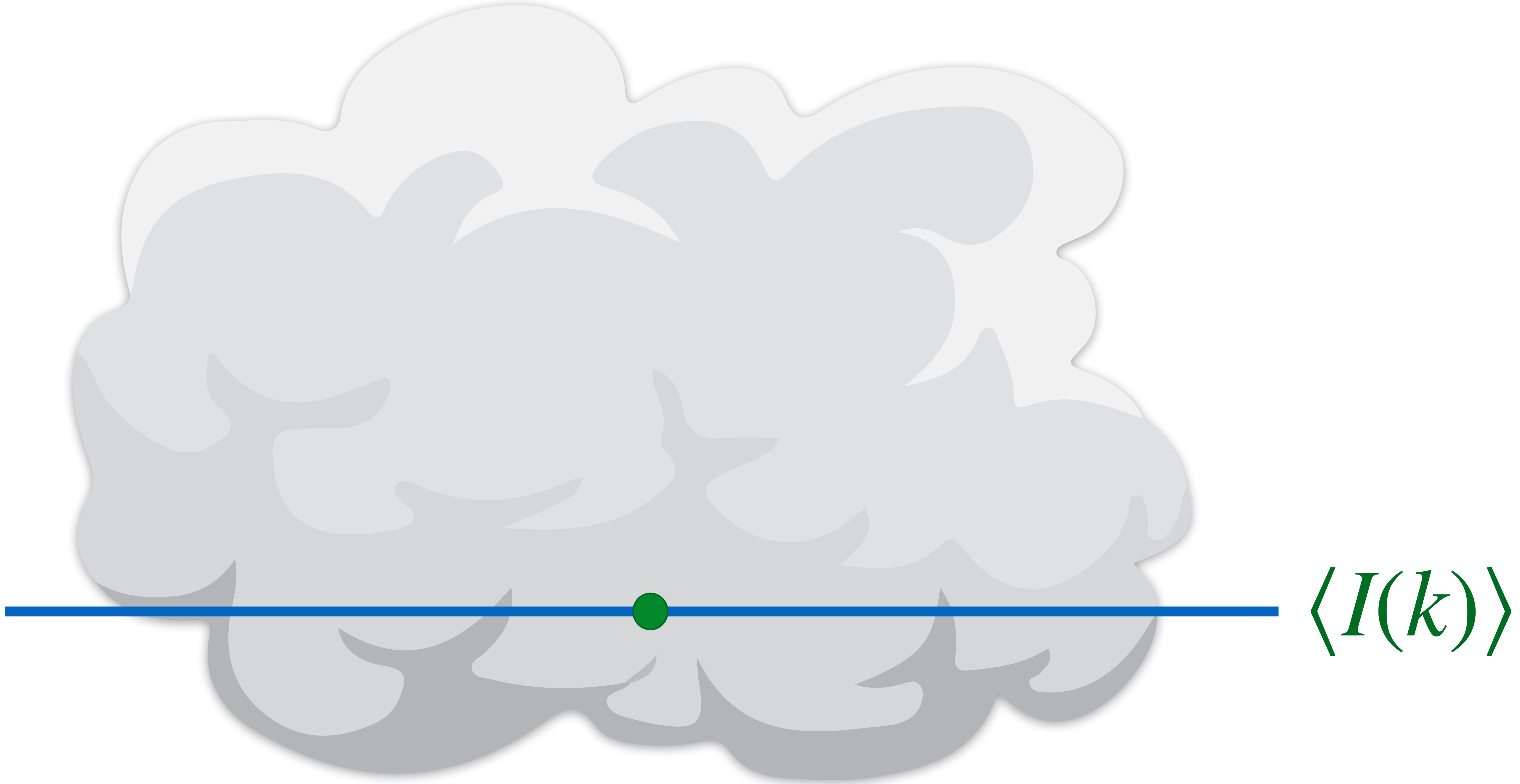
$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

$$\langle I(k) \rangle = g\left(\sum_{j=1}^{k} f(x_j) \Delta x \right)$$

Unbiased Ray-marching

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

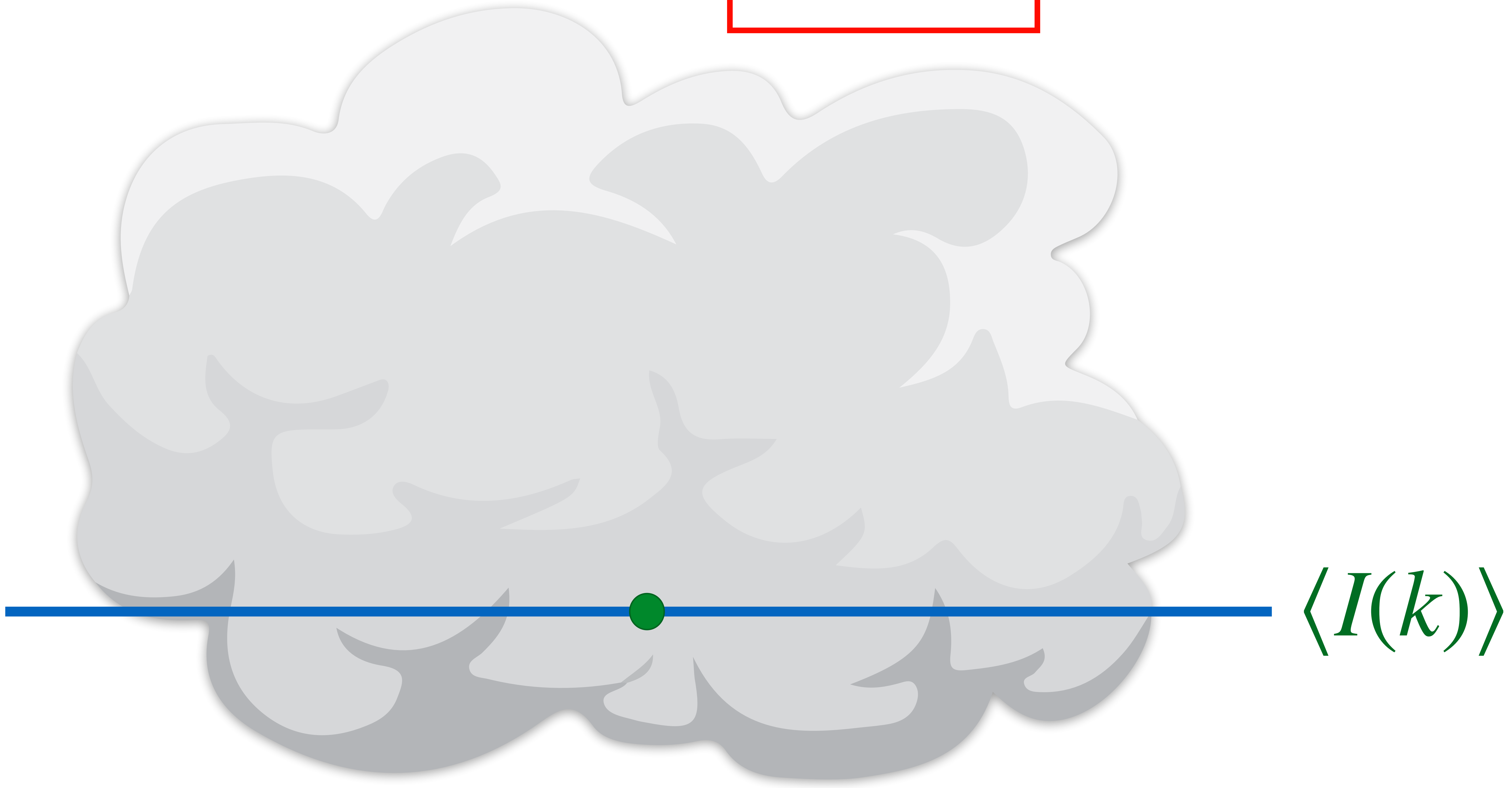
$$\langle I(k) \rangle = g\left(\sum_{j=1}^{k} f(x_j)\Delta x \right)$$



Unbiased Ray-marching

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

$$\langle I(k) \rangle = g\left(\sum_{j=1}^{k} f(x_j)\Delta x \right)$$



Unbiased ray-marching

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

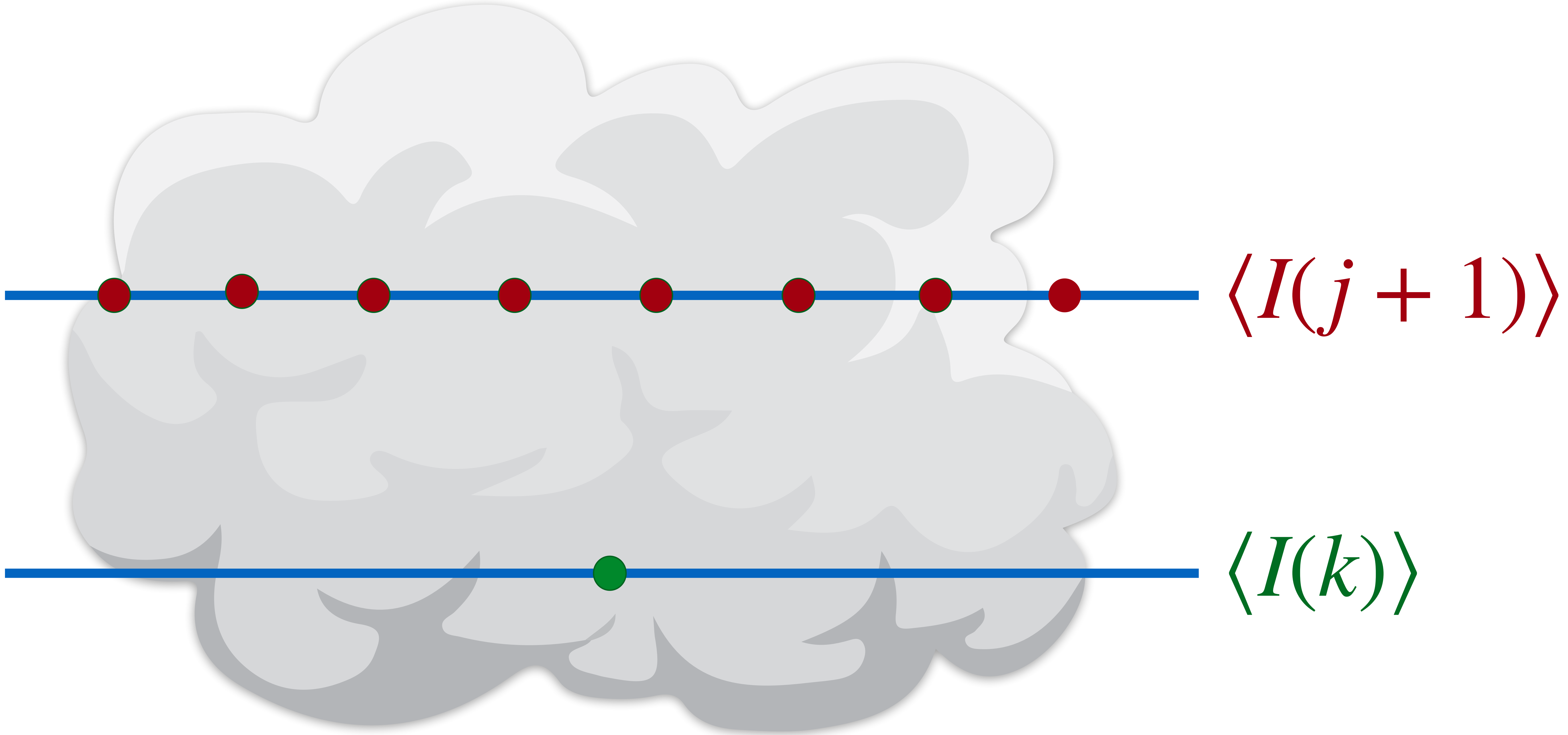
steps $\propto 2^{j+1}$

$$\langle I(k) \rangle = g\left(\sum_{j=1}^{k} f(x_j)\Delta x\right)$$

Unbiased ray-marching

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

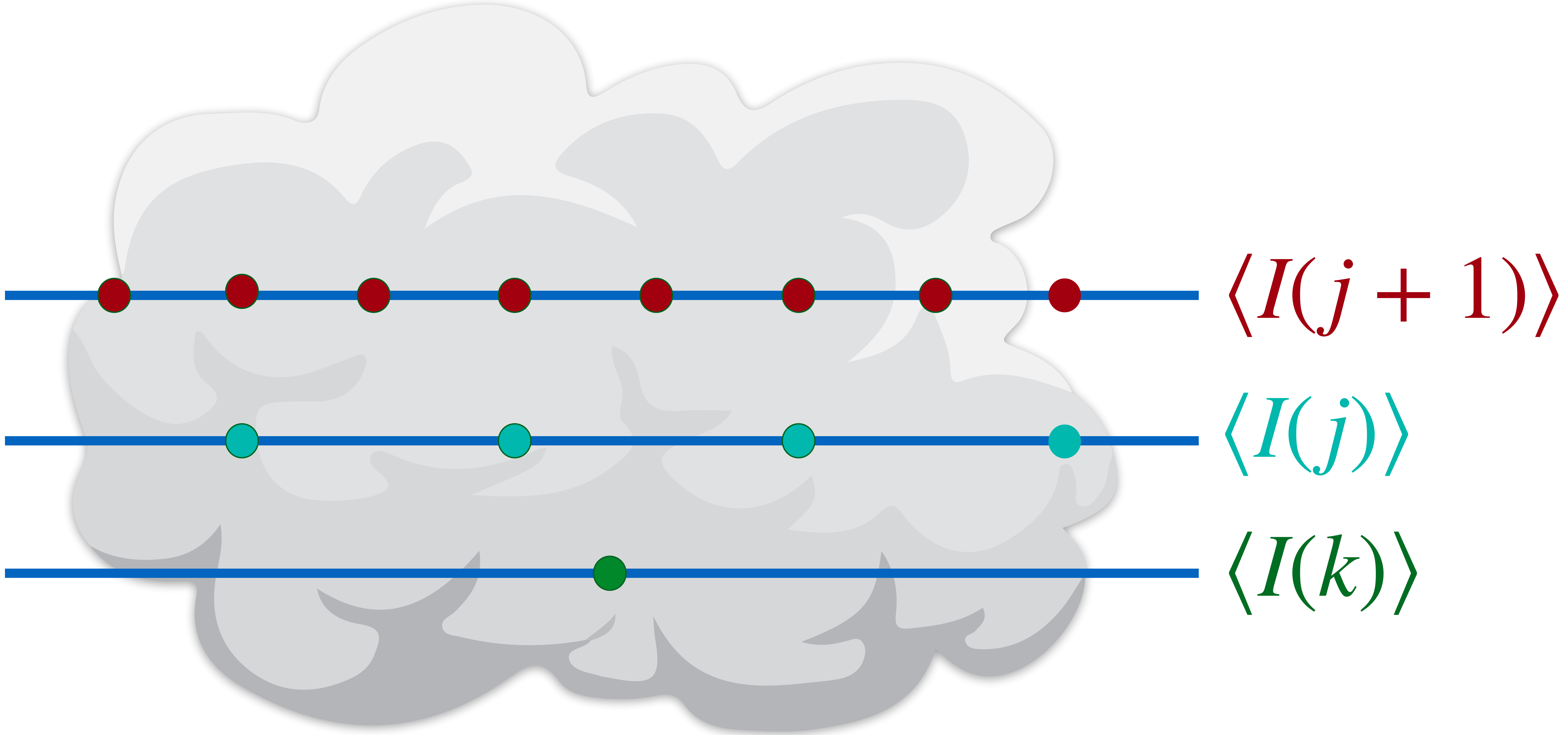
steps = 8



$$\langle I(k) \rangle = g\left(\sum_{j=1}^{k} f(x_j)\Delta x \right)$$

Unbiased ray-marching

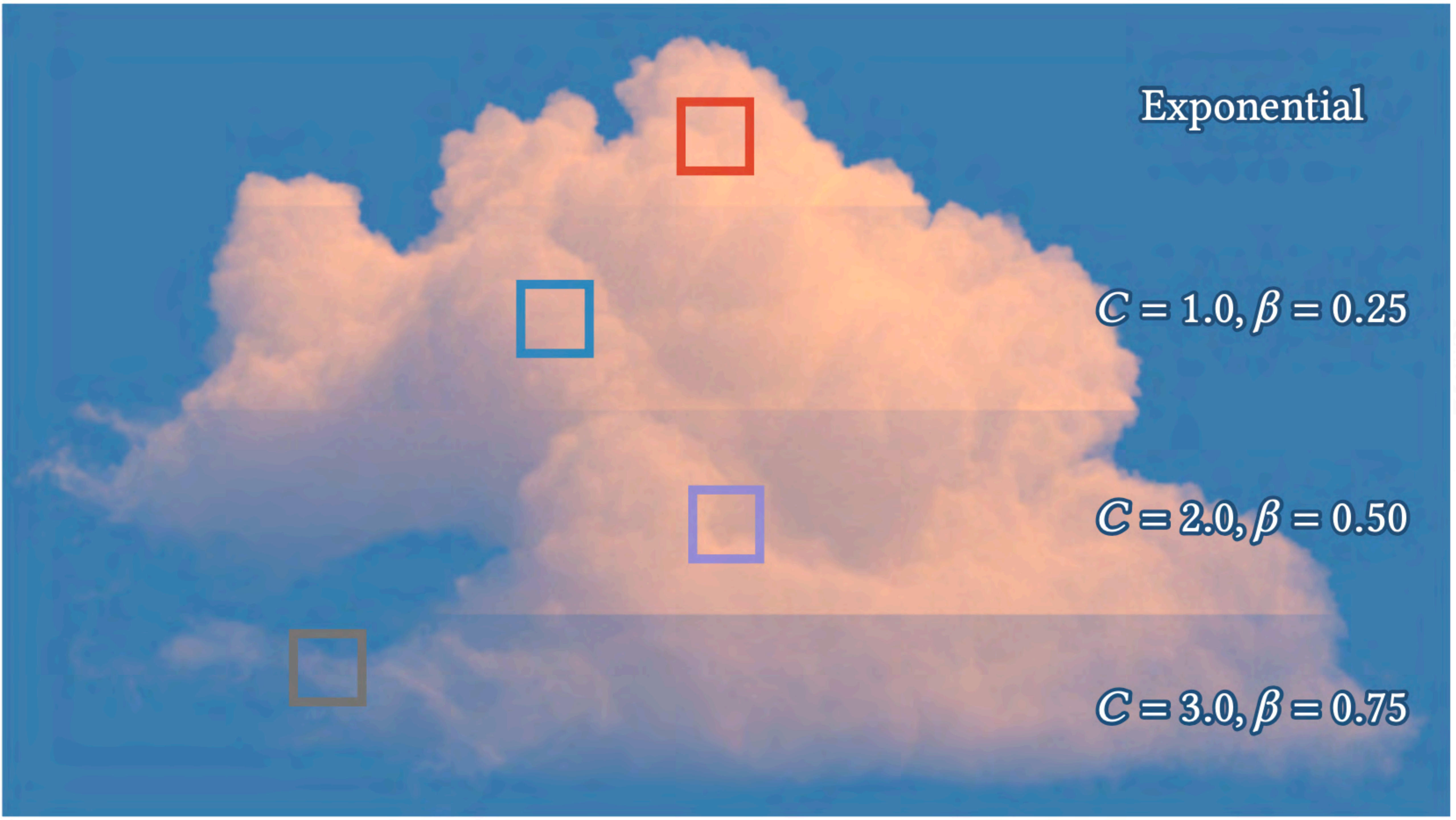
$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

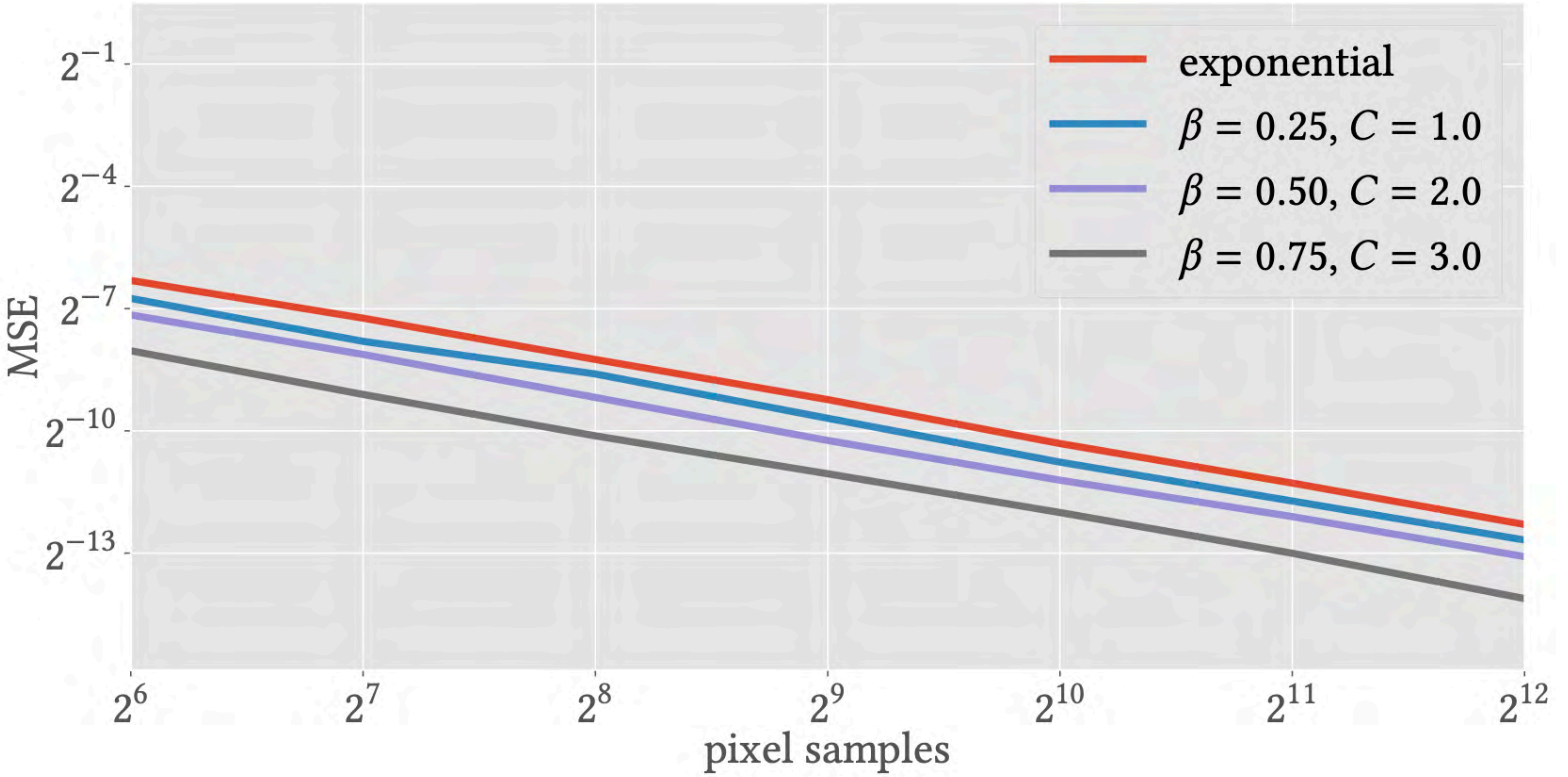


$$\langle I(k) \rangle = g\left(\sum_{j=1}^{k} f(x_j)\Delta x \right)$$

Results

Transmittance estimation





[Bitterli et. al. 2018]

Probability mass function

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

Probability mass function

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

$$\mathrm{E}\left[\langle I \rangle\right] = I$$

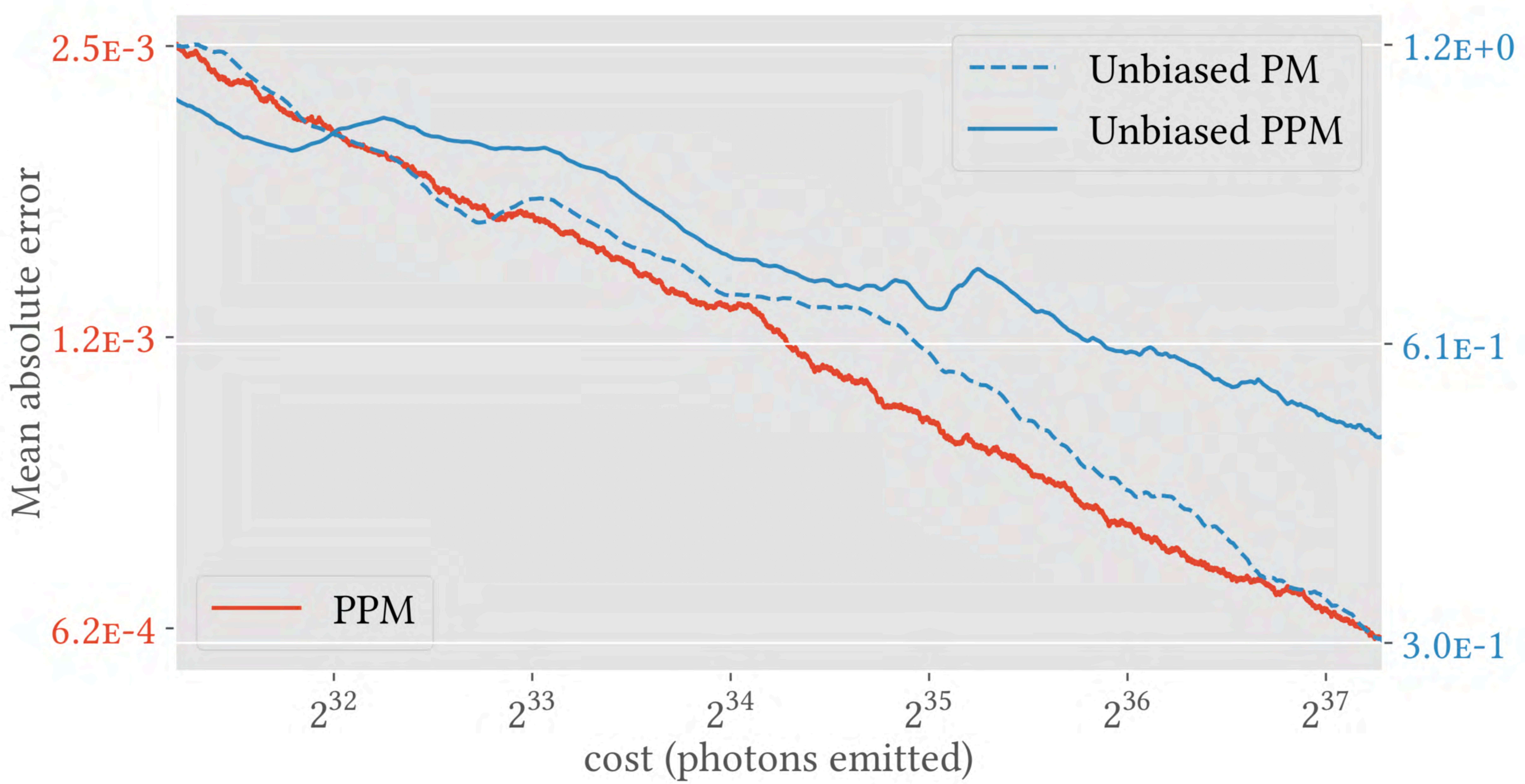
Probability mass function

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$

$$\mathrm{E}\left[\langle I \rangle\right] = I \qquad \mathrm{V}\left[\langle I \rangle\right] = \infty$$

Photon mapping


Progressive PM
Unbiased PM (ours)



2.5E-3
1.2E-3
6.2E-4
1.2E+0
6.1E-1
3.0E-1
Mean absolute error
Unbiased PM
Unbiased PPM
PPM
2^{32}
2^{33}
2^{34}
2^{35}
2^{36}
2^{37}
cost (photons emitted)

Additional Contributions

- Recipe
- Taylor series
- Infinite variance
- Finite differences

Thank you!