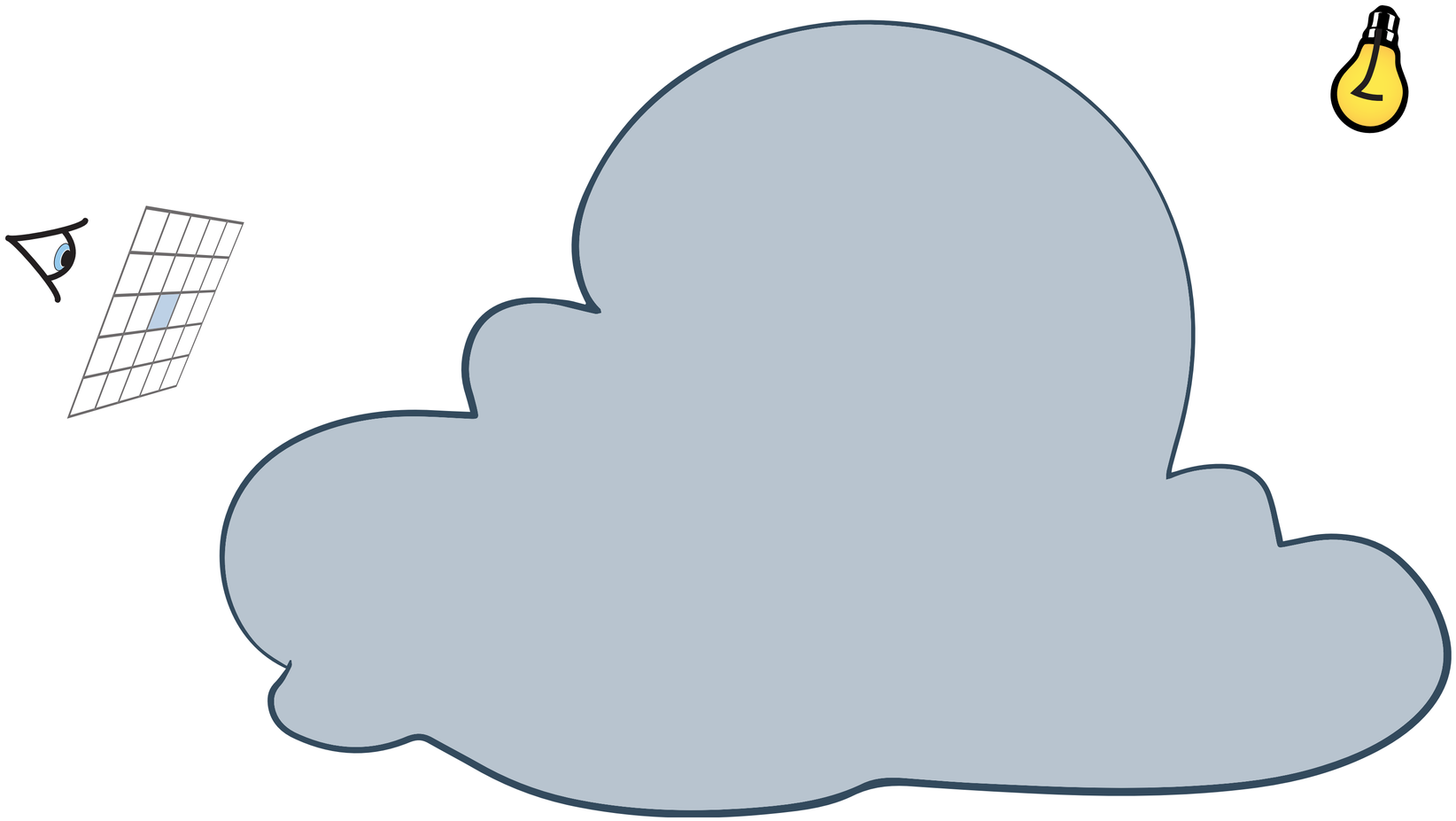
# PATH CONSTRUCTION

Iliyan Georgiev
Solid Angle

# PATH INTEGRAL FRAMEWORK



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\overline{\mathbf{x}}) d\overline{\mathbf{x}}$$

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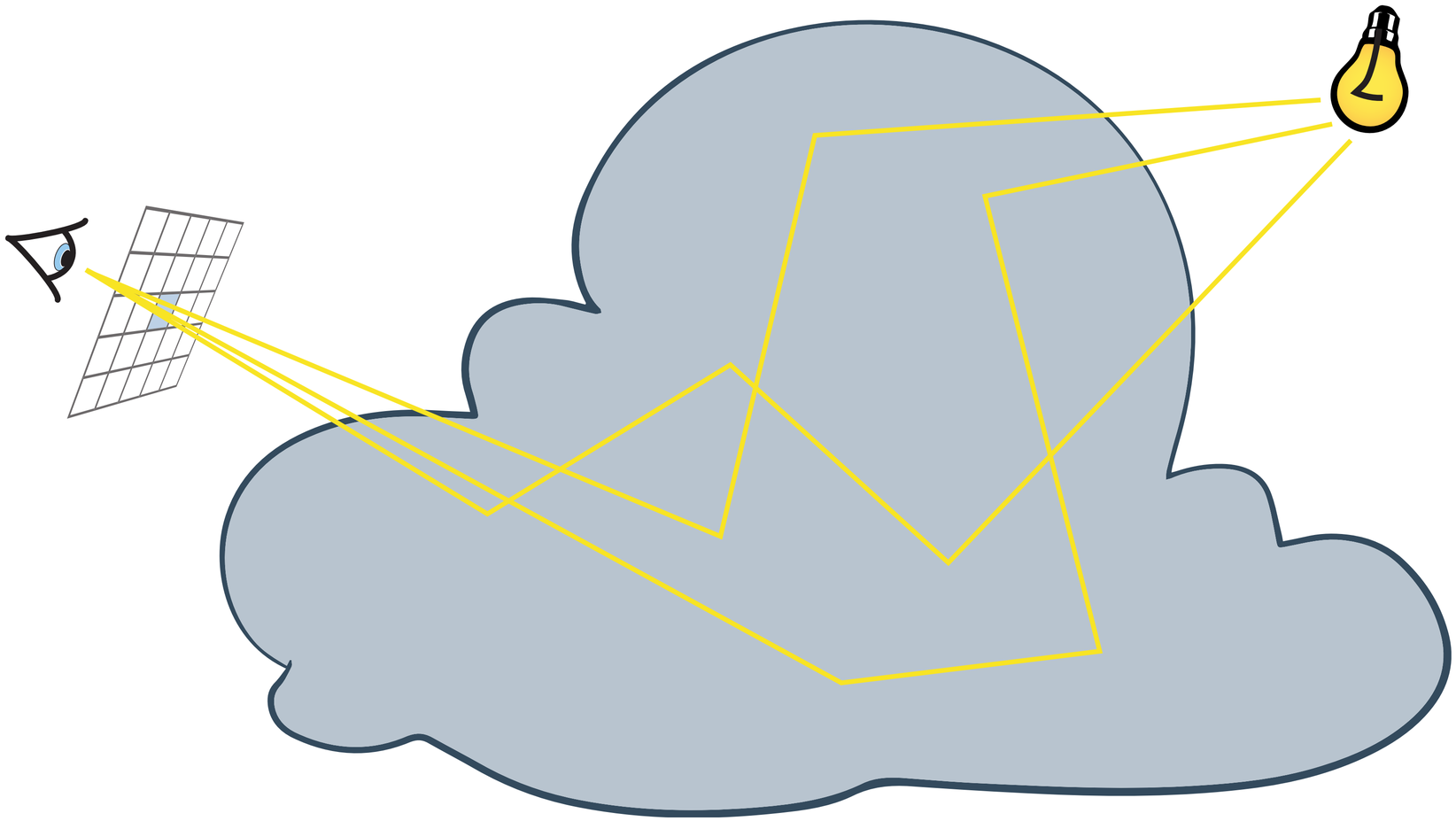

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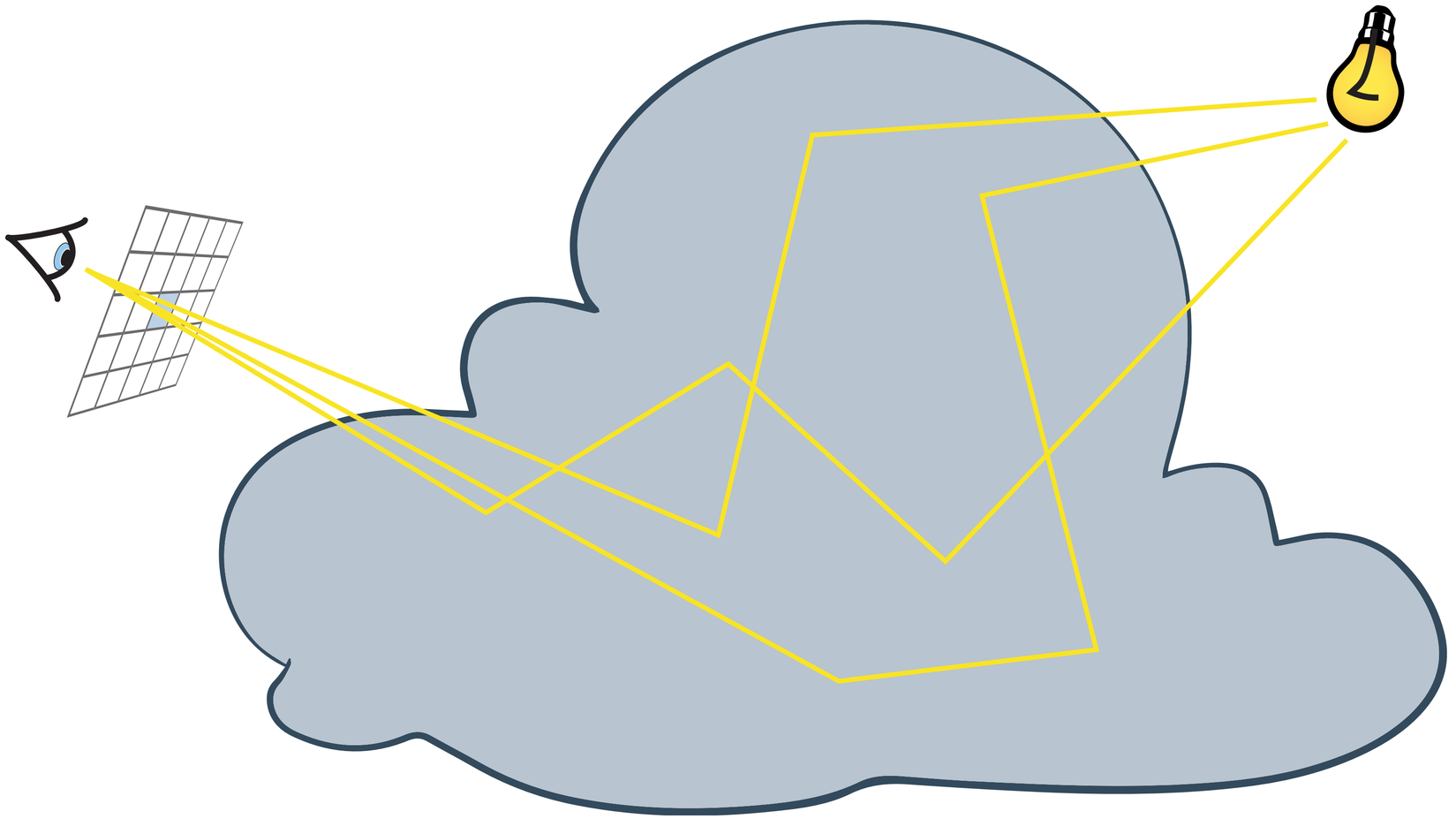

Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\overline{\mathbf{x}}) d\overline{\mathbf{x}}$$

Pixel estimator

$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f_j(\overline{\mathbf{x}}_i)}{p(\overline{\mathbf{x}}_i)}$$

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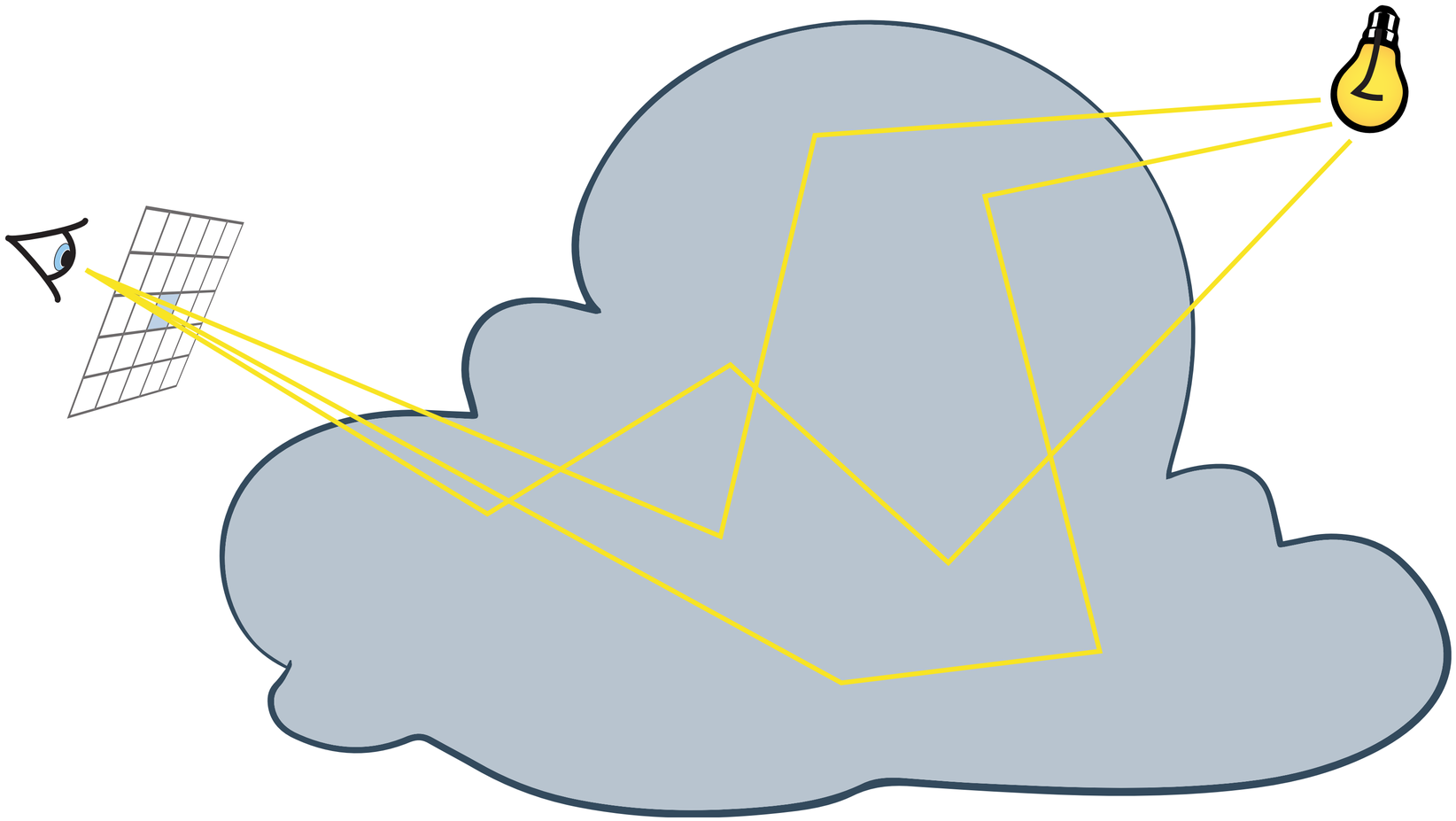

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path contribution

path pdf

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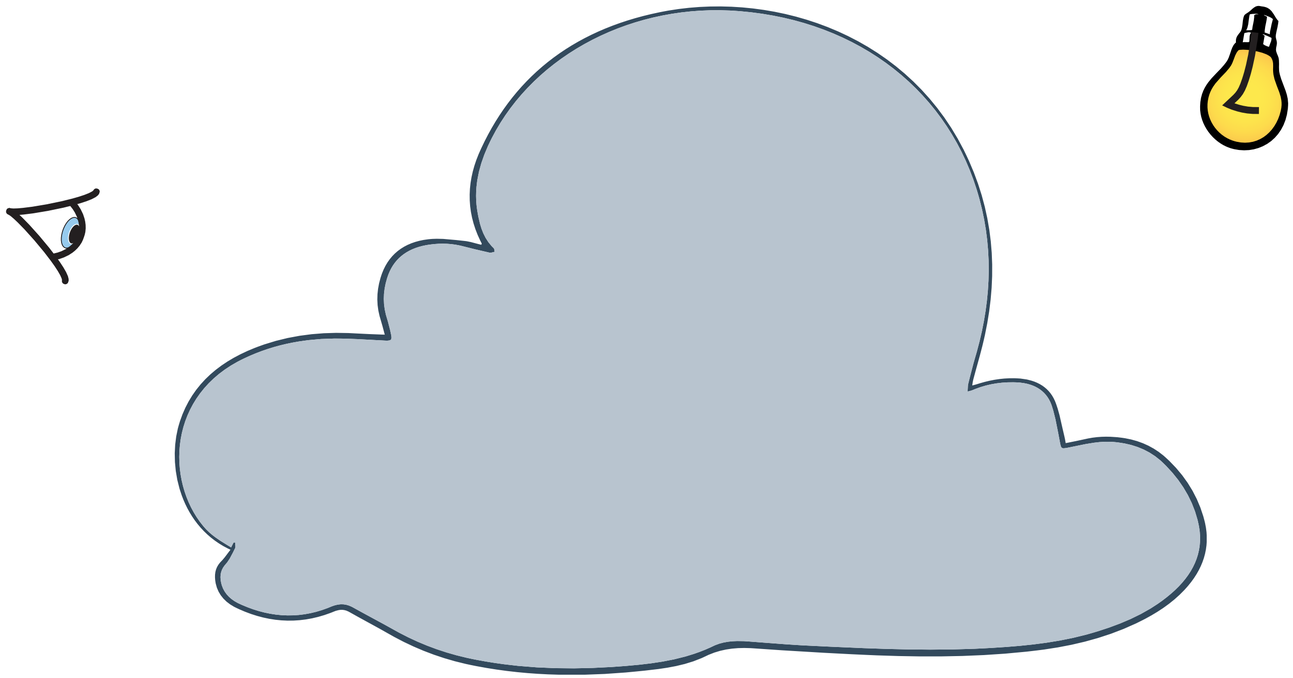

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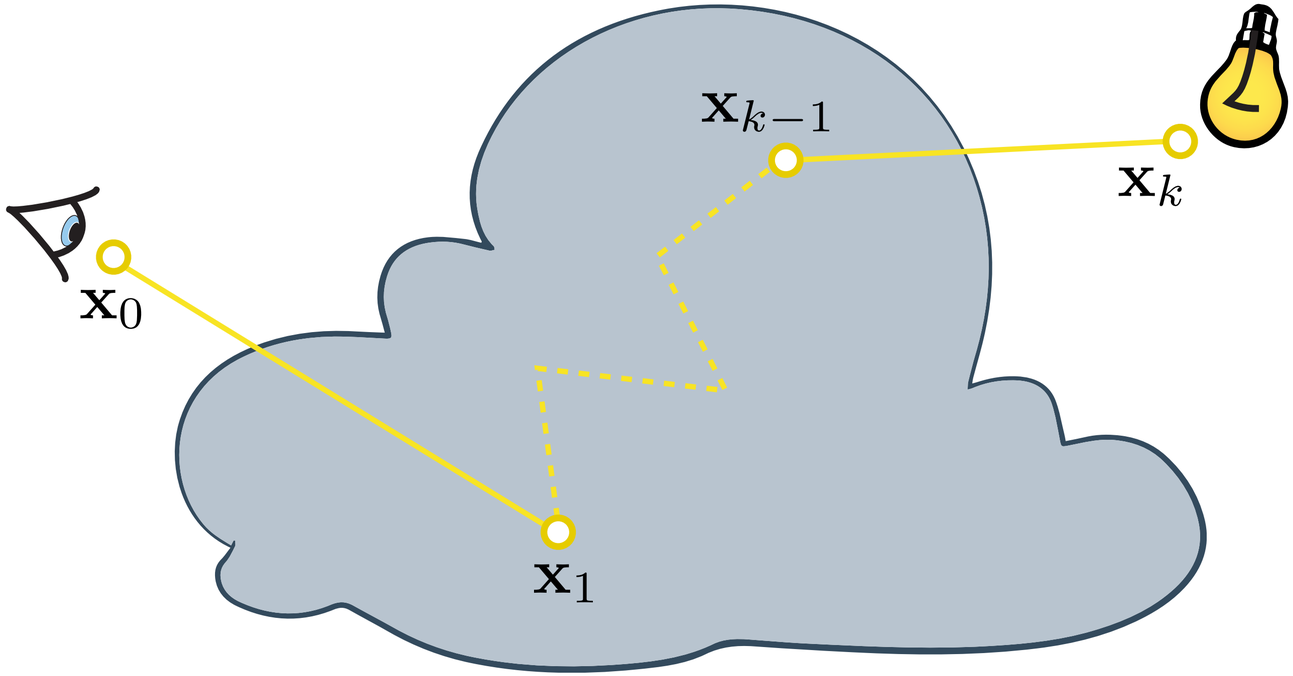


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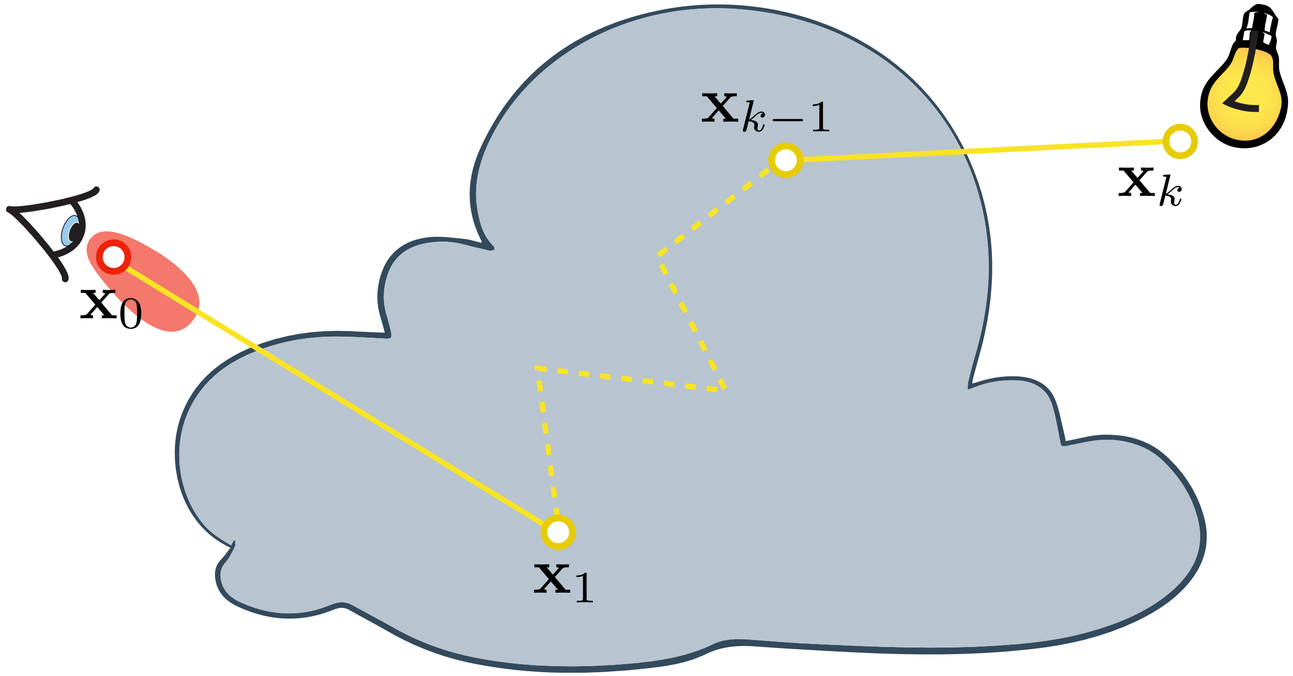


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camera response

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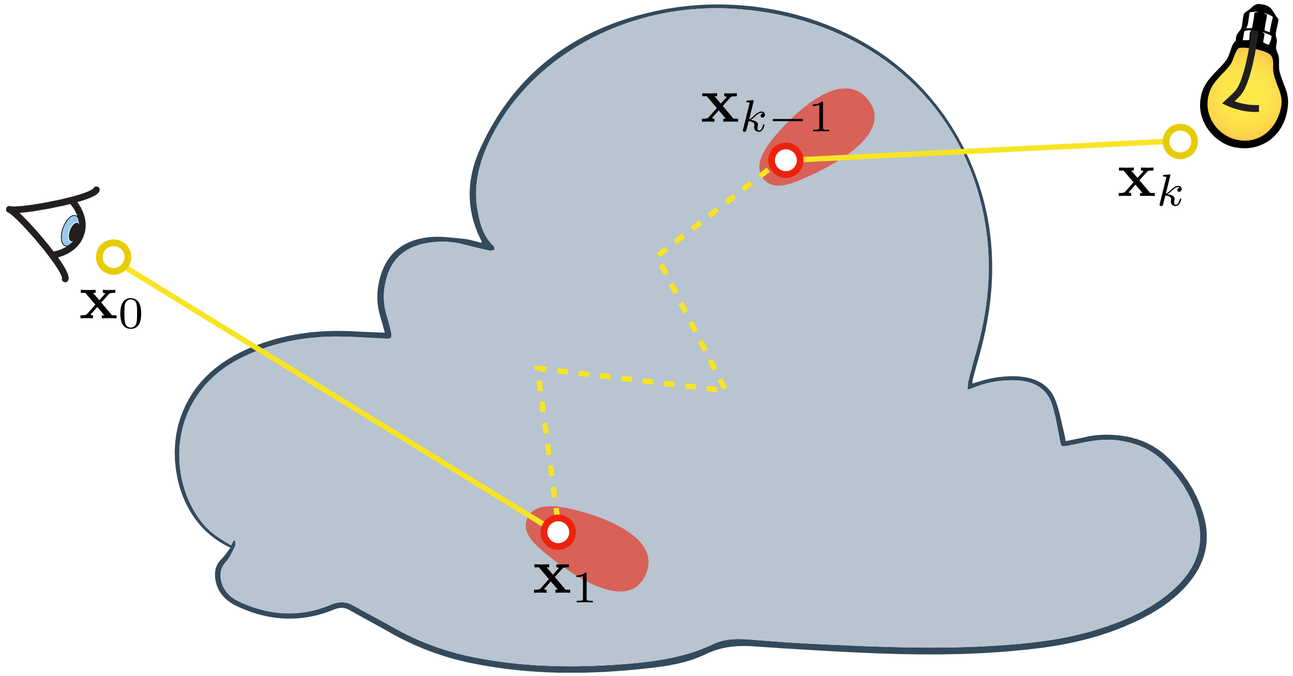


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BSDF/ phase

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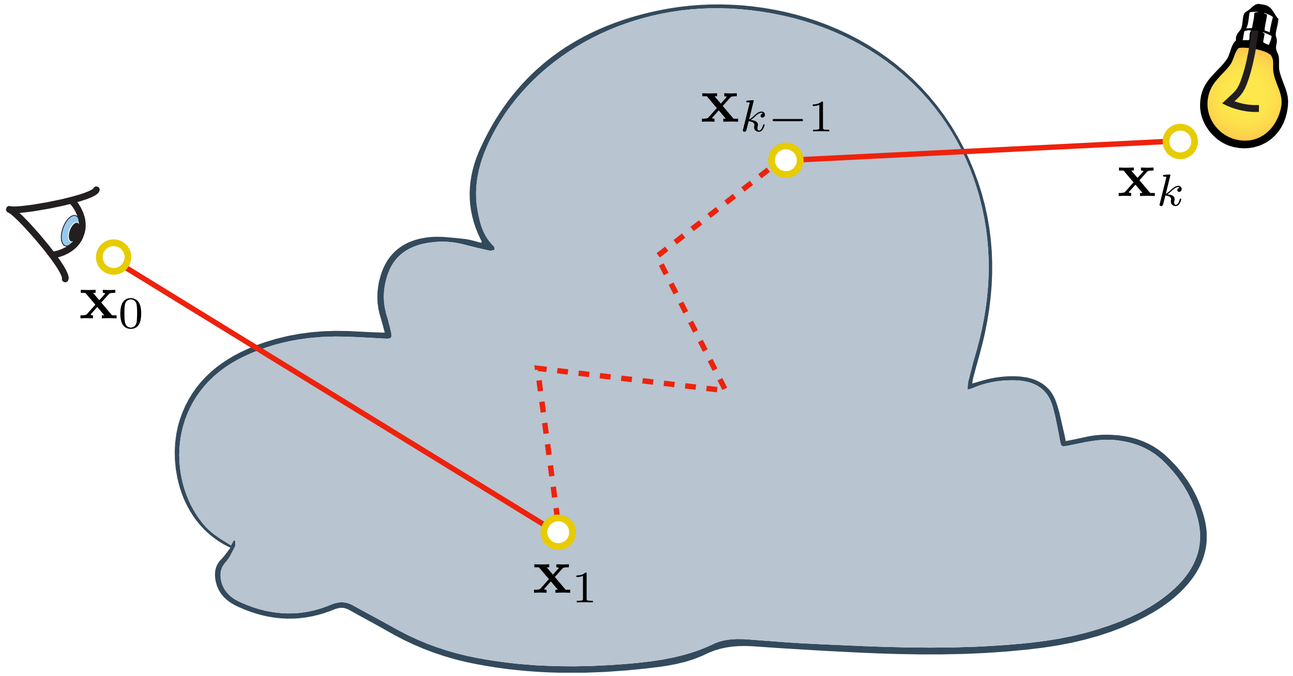


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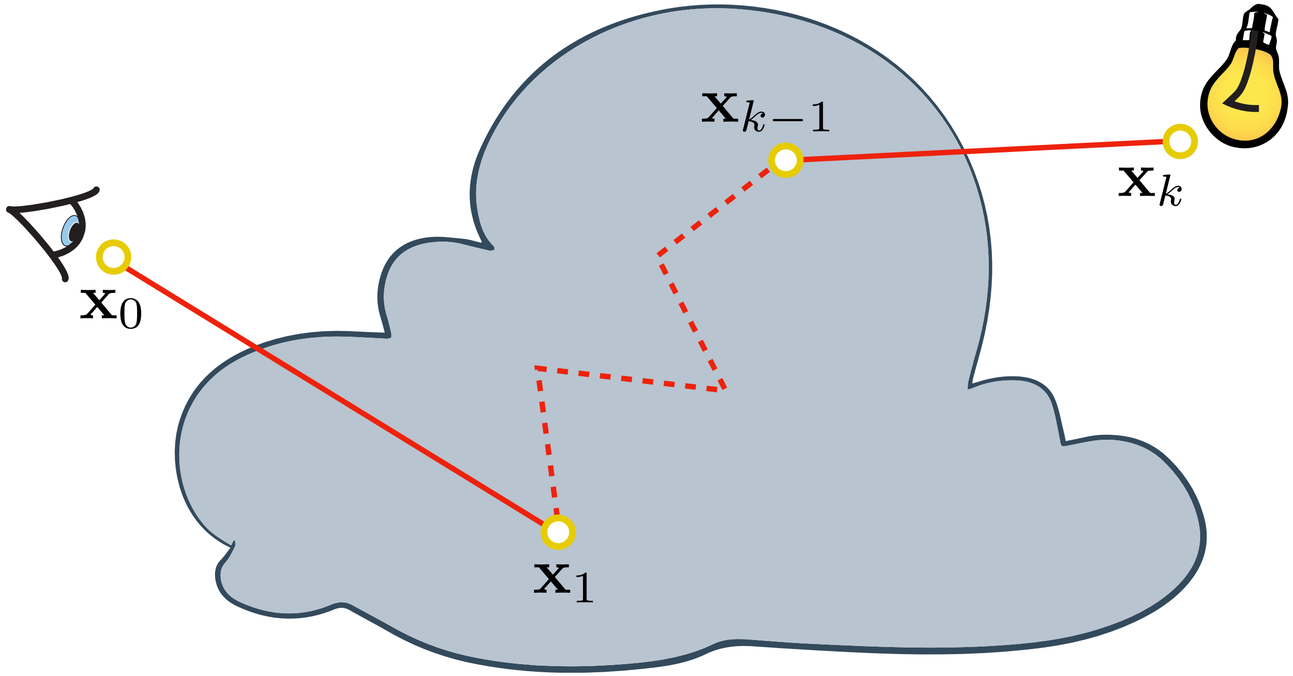


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camera response — BSDF/phase — geometry — transmittance

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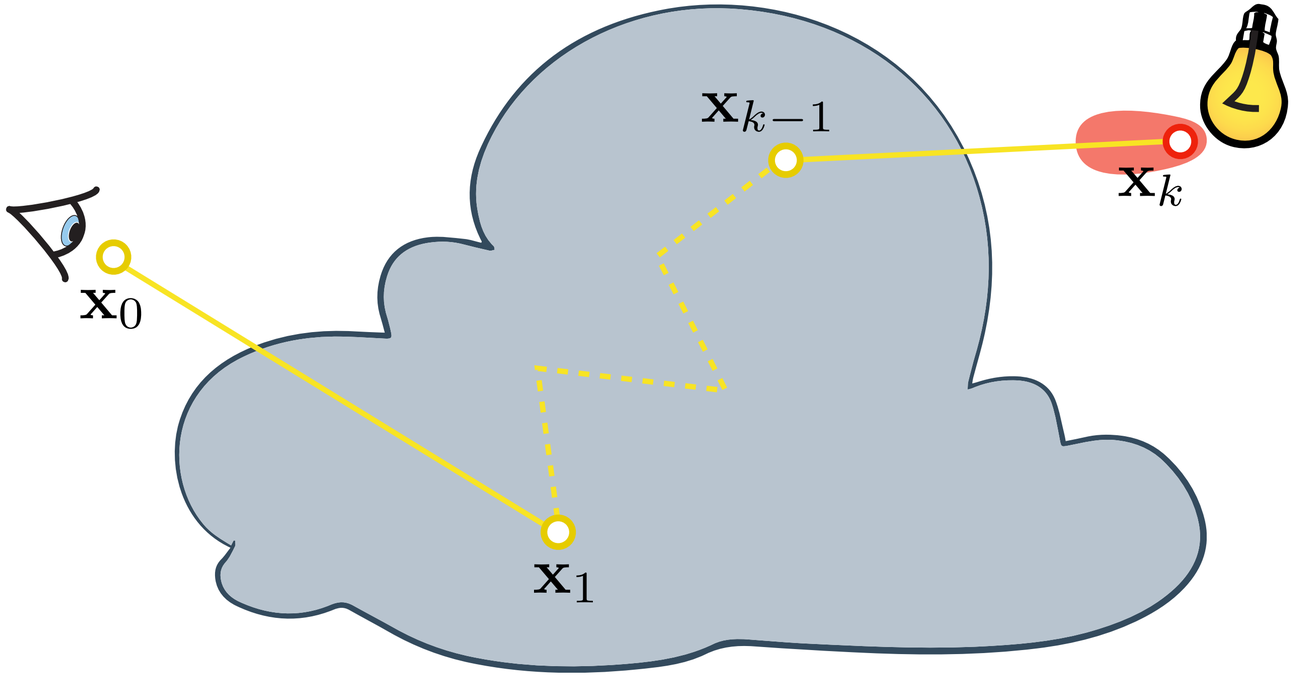


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camera response — BSDF/phase — geometry — transmittance — emitted radiance

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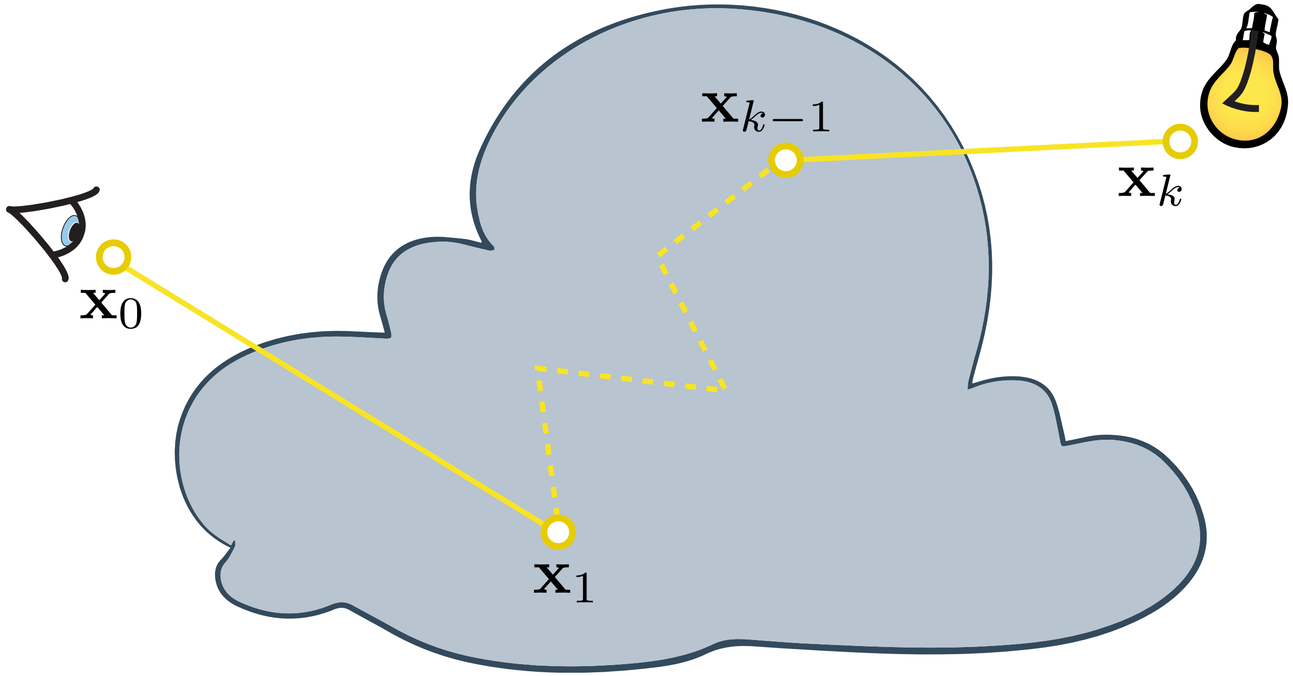


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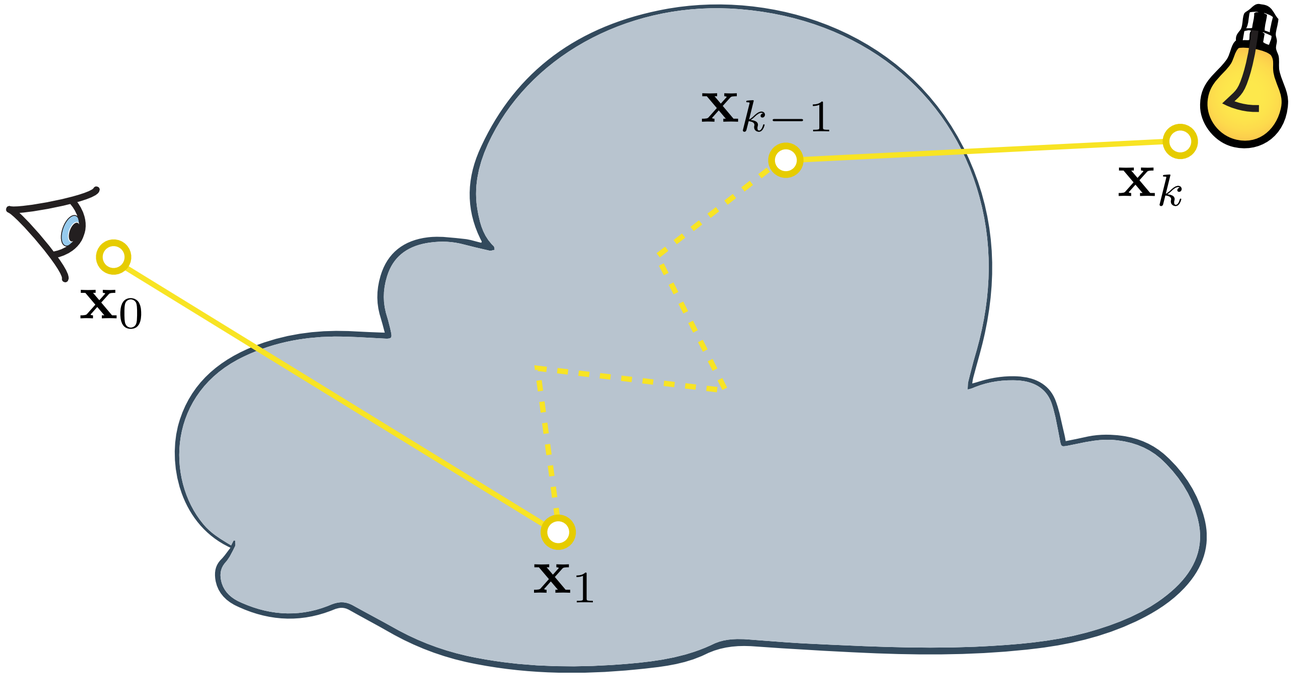


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ideally proportional

$$\propto$$

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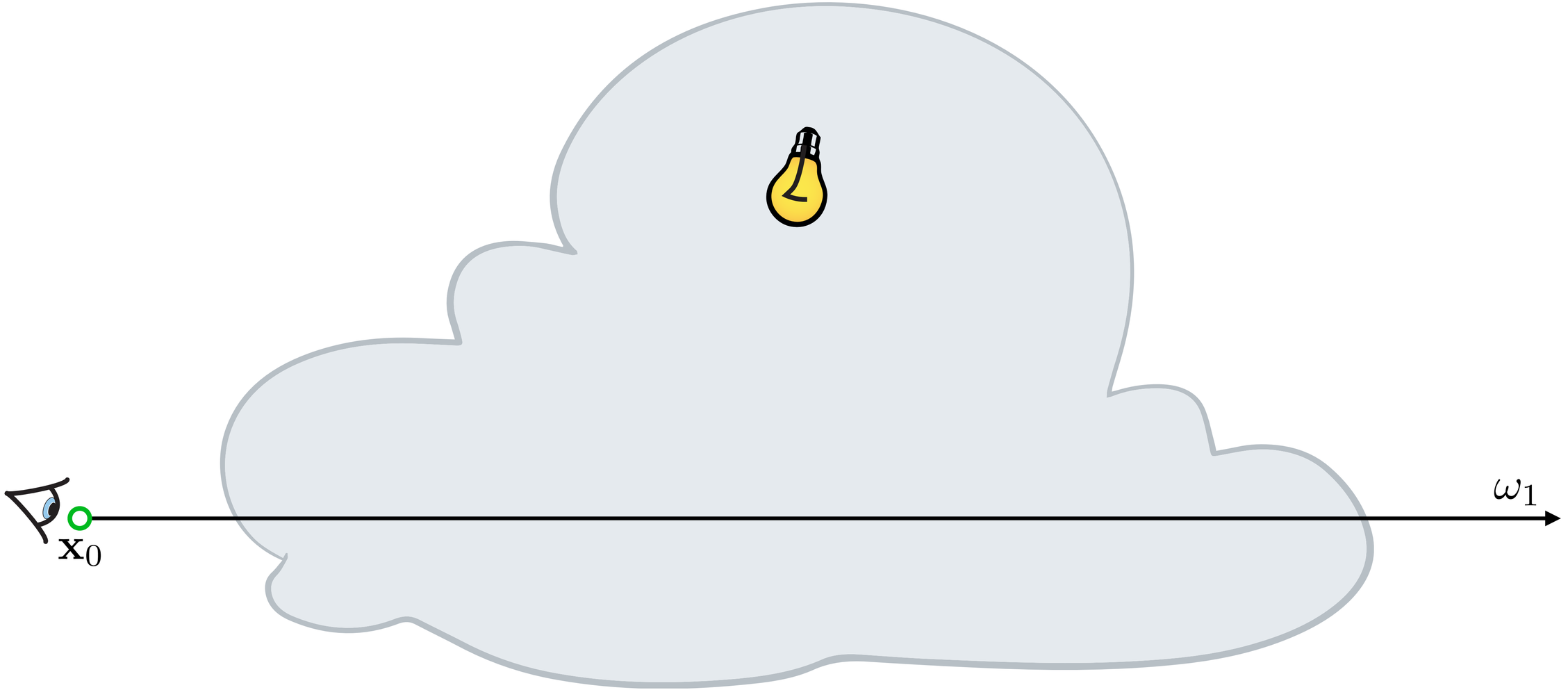

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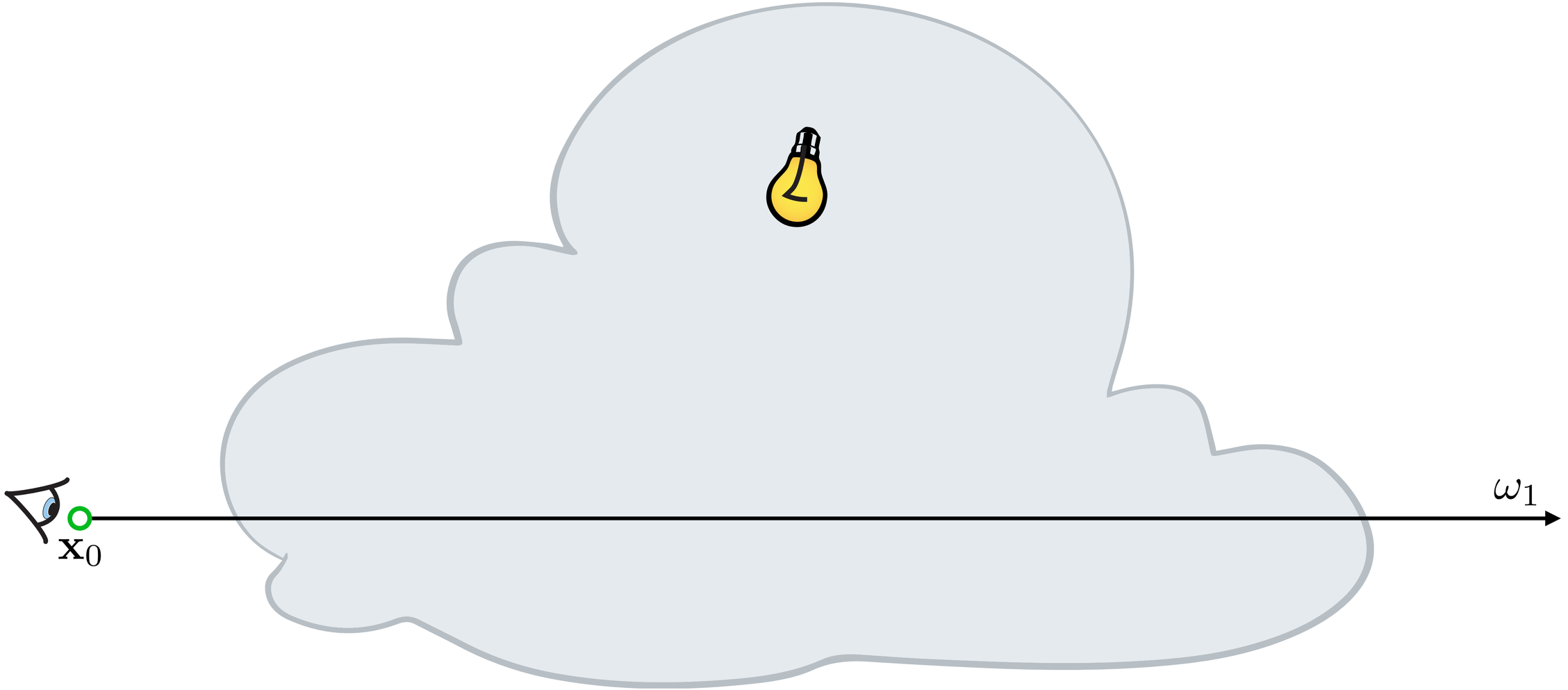

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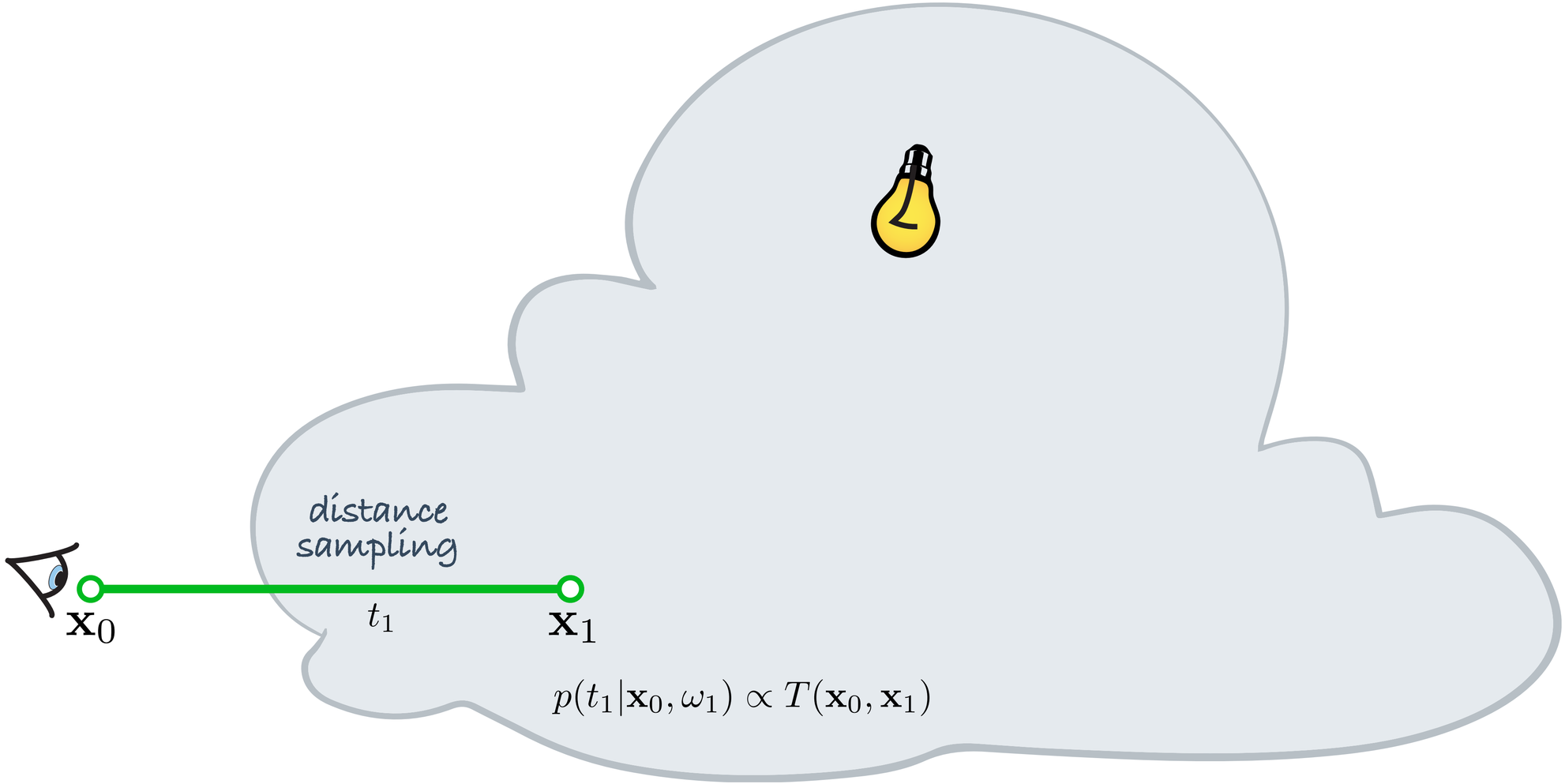

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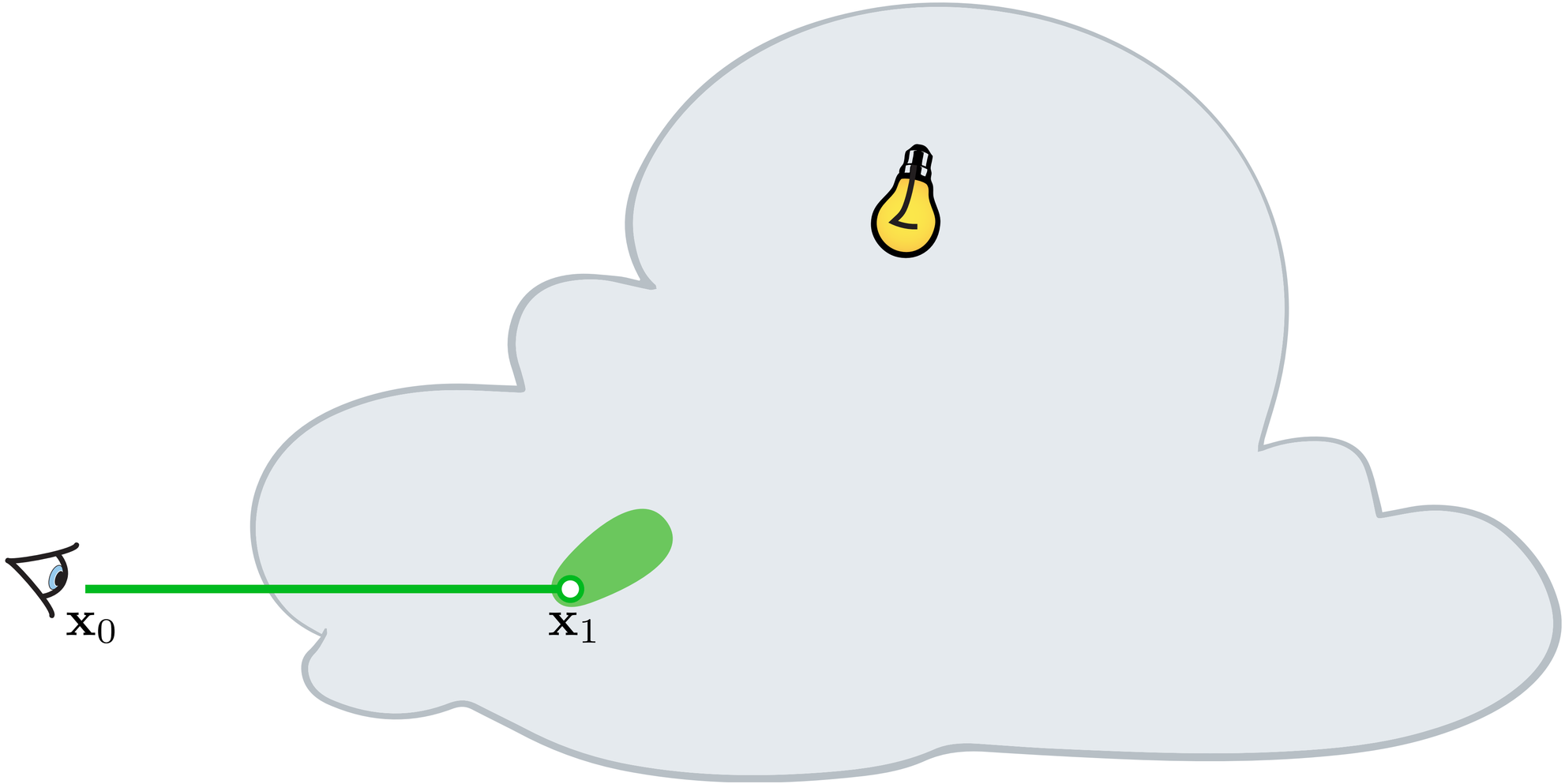

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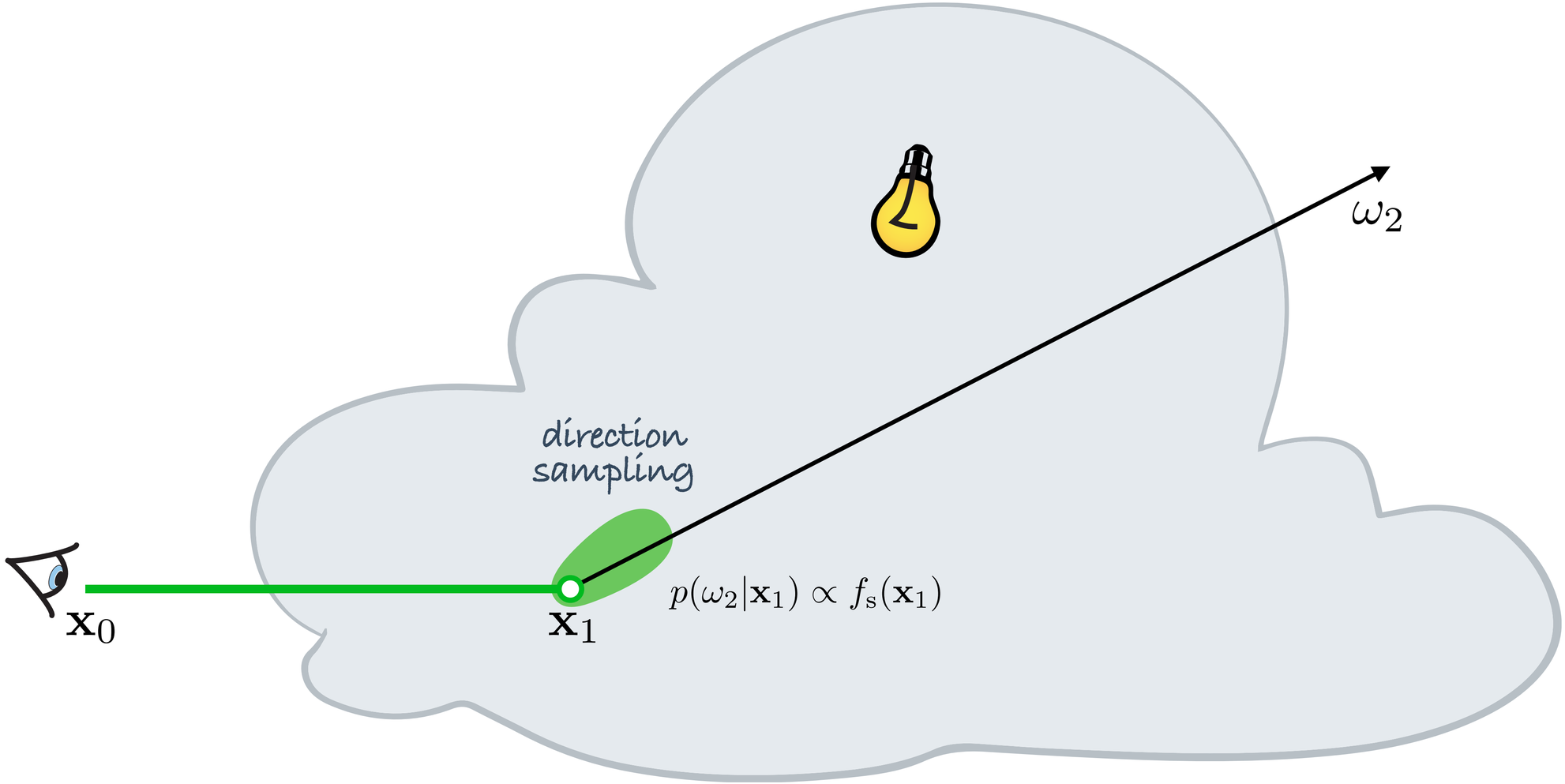

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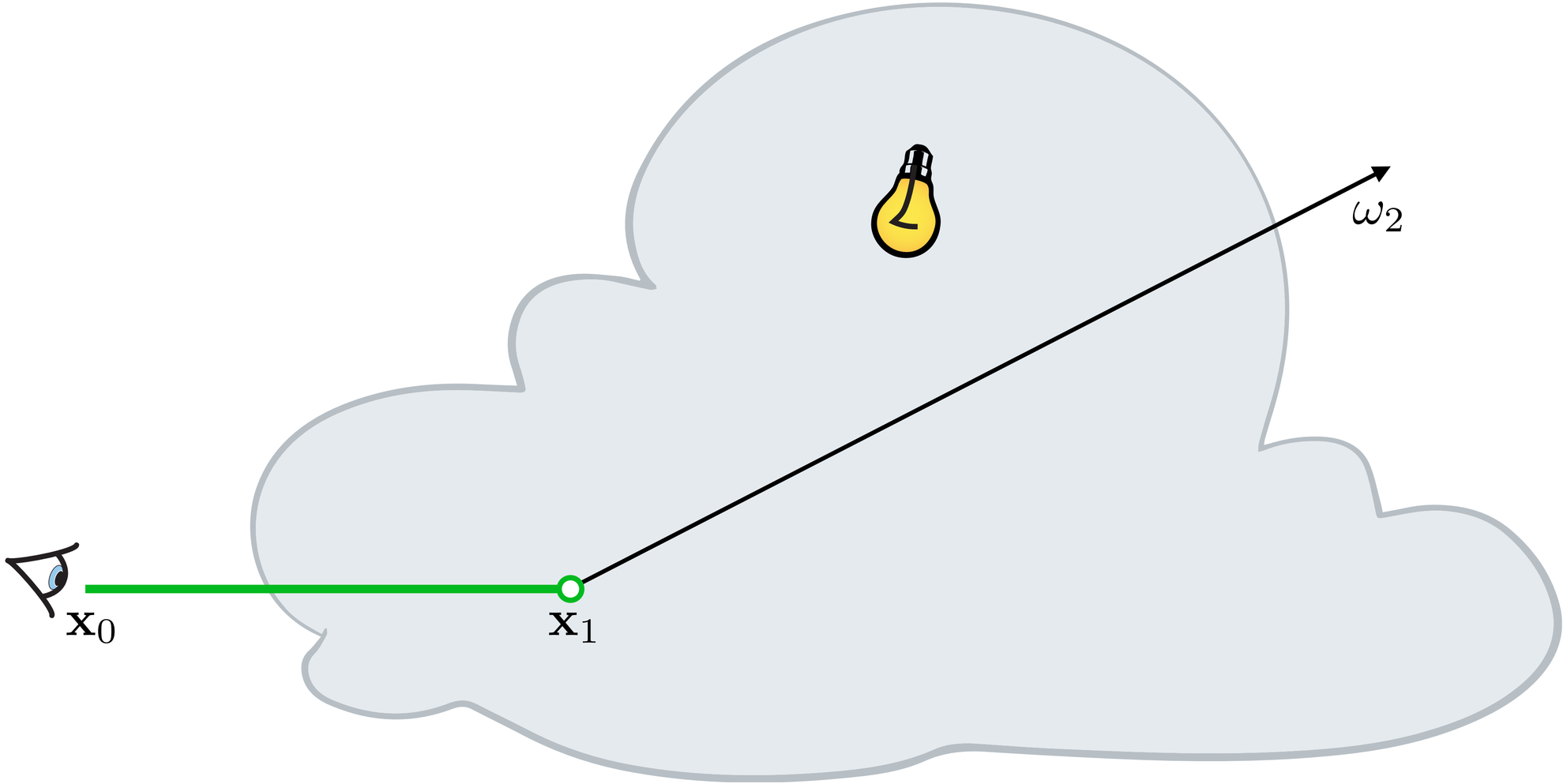

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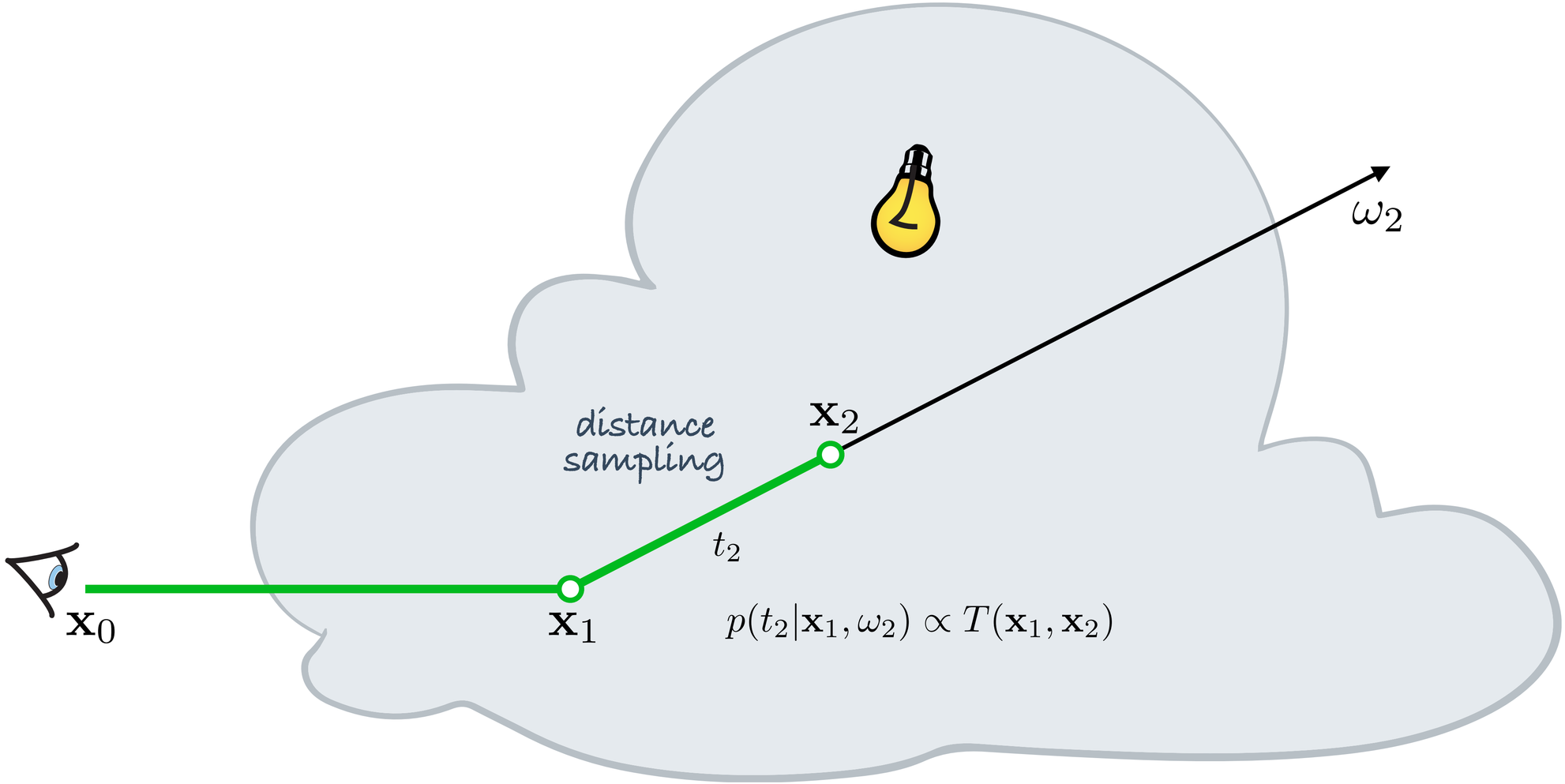

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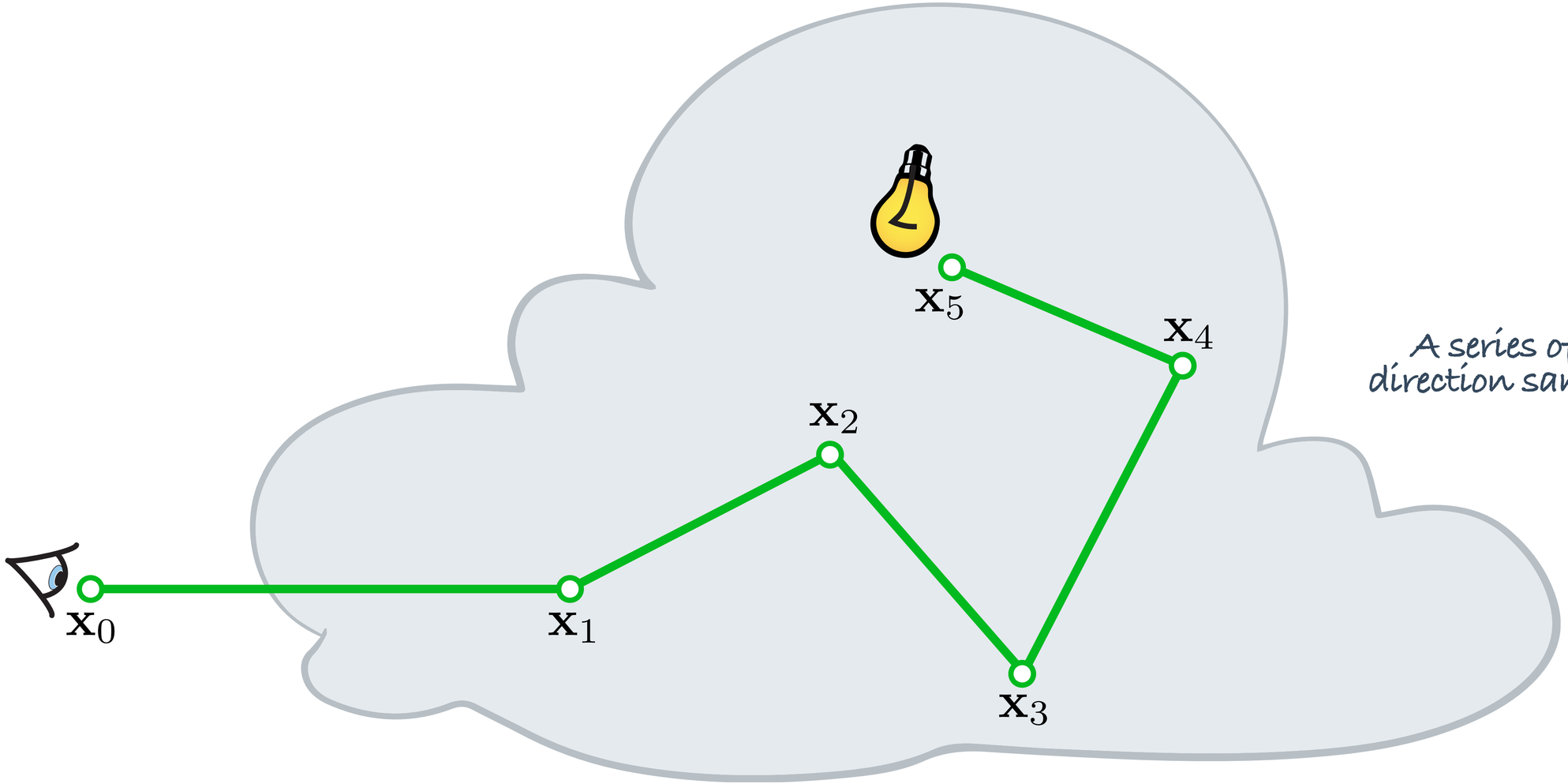


A series of distance and direction sampling decisions

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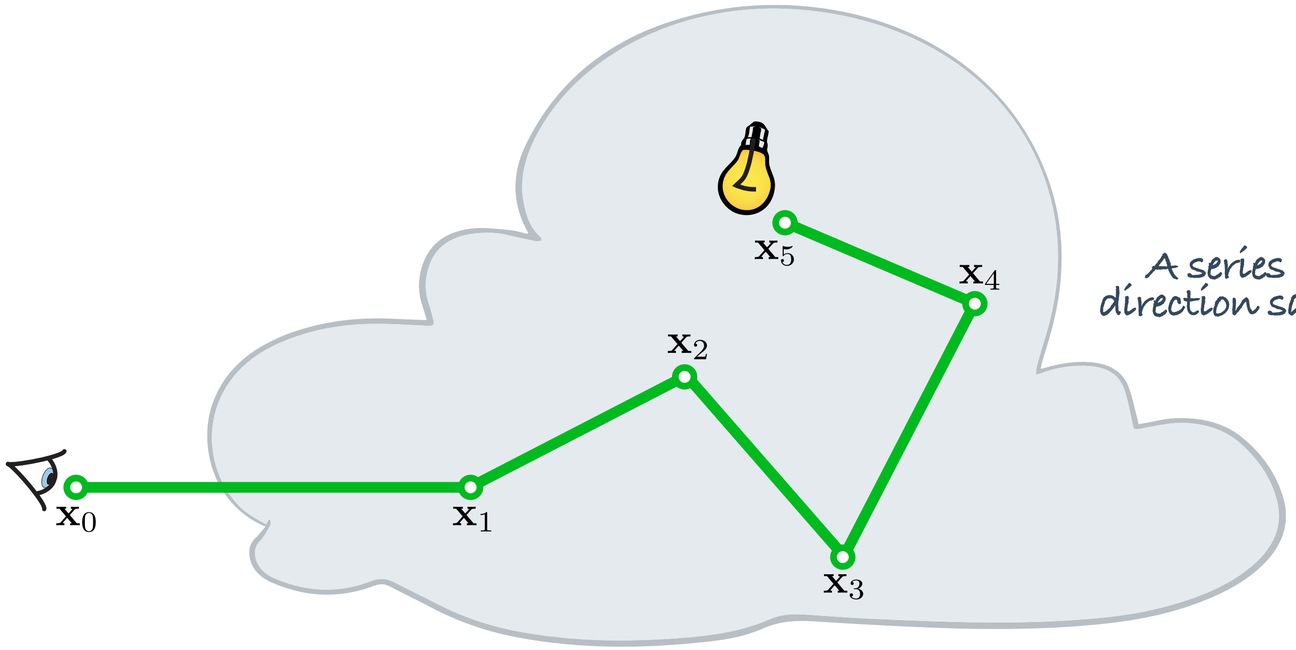


A series of distance and direction sampling decisions

$$p(\overline{\mathbf{x}}) \propto W_j(\mathbf{x}_0, \mathbf{x}_1) \left[ \prod_i f_\mathrm{s}(\mathbf{x}_i) G(\mathbf{x}_i, \mathbf{x}_{i+1}) T(\mathbf{x}_i, \mathbf{x}_{i+1}) \right]$$

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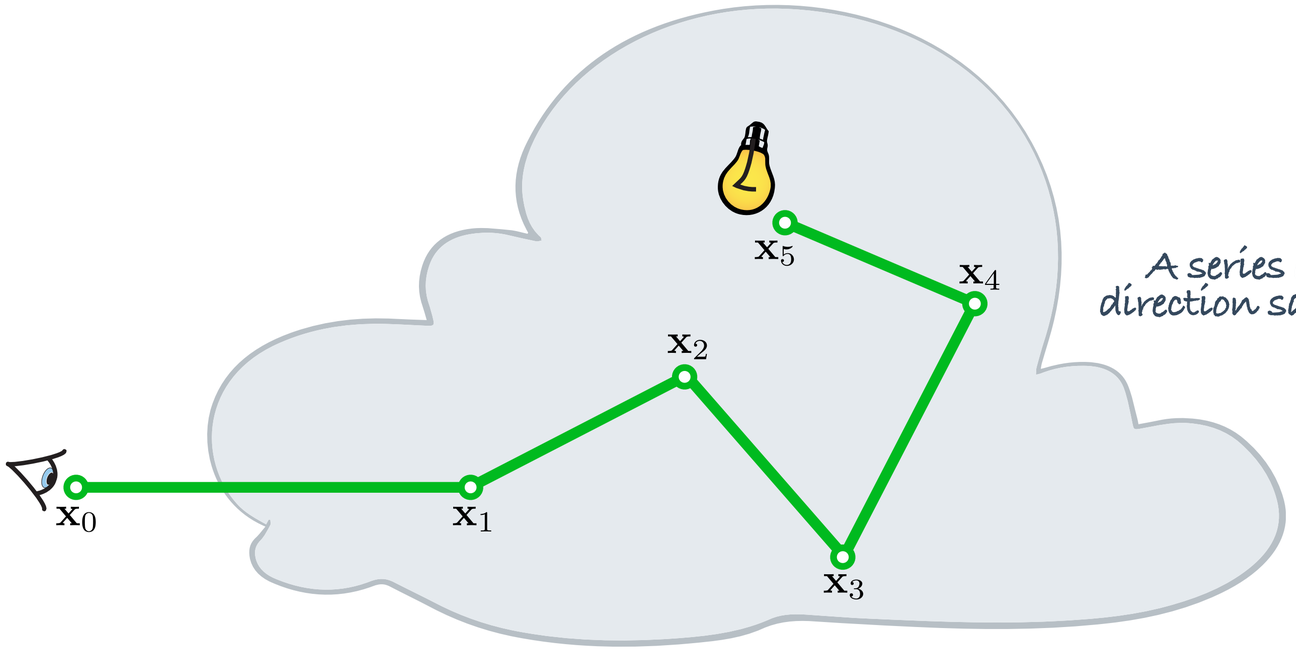


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not importance sampled

$$L_\mathrm{e}(\mathbf{x}_k, \mathbf{x}_{k-1})$$

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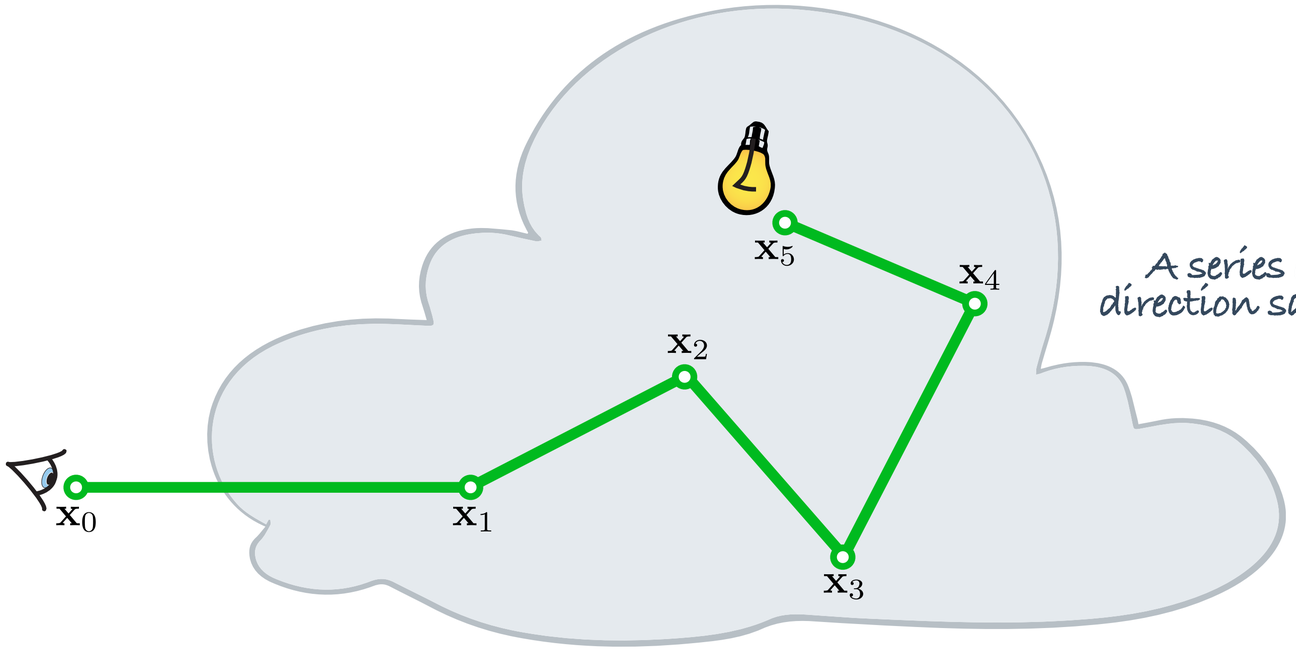


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high variance when light sources are small

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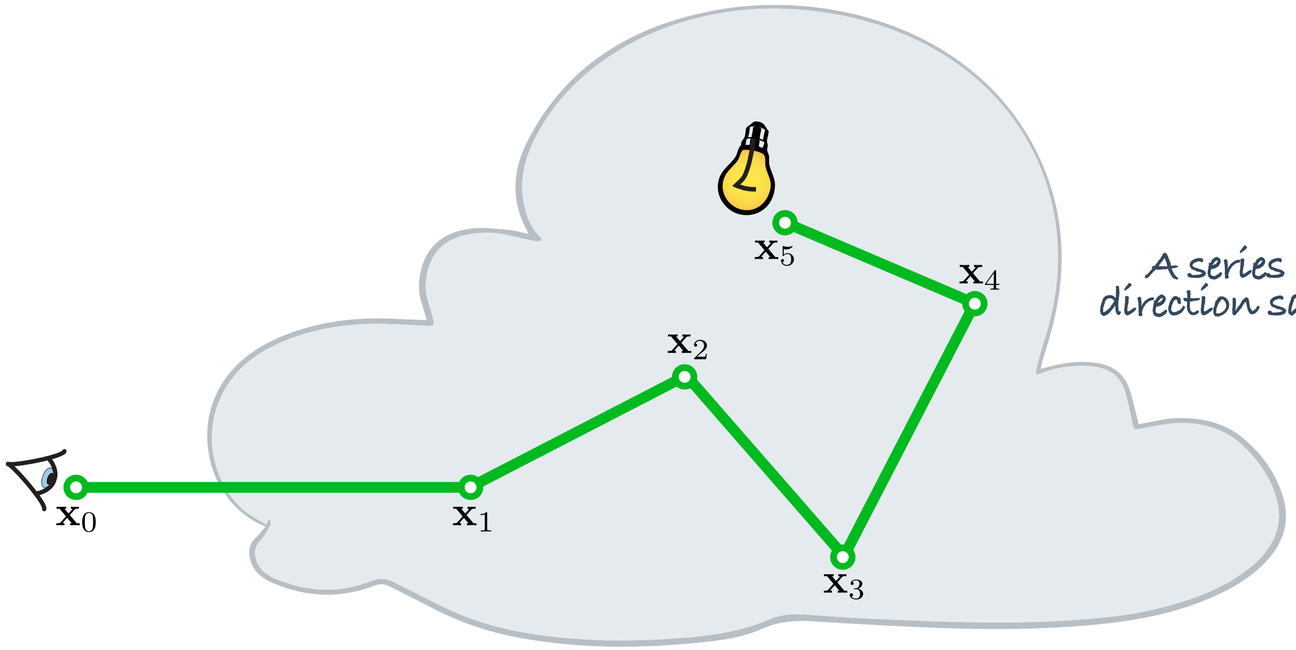


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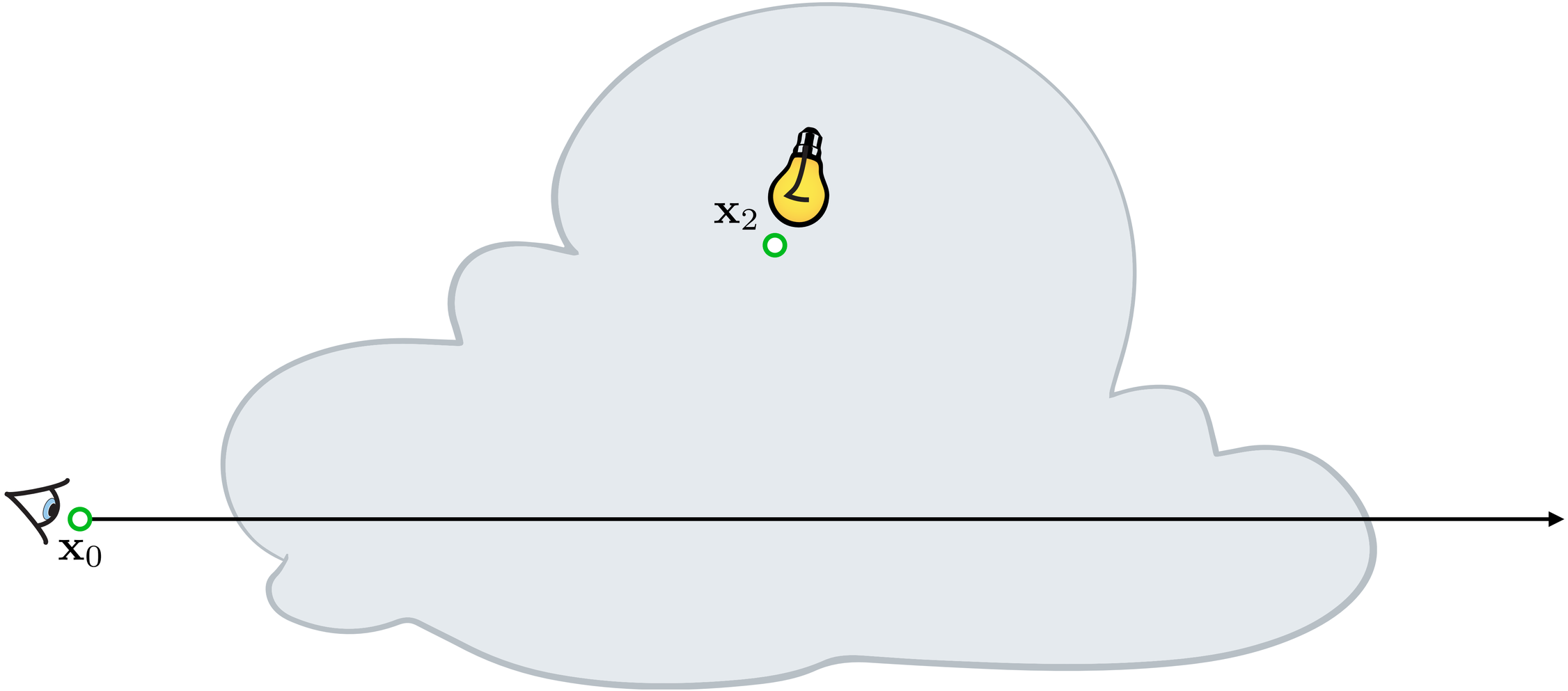
not importance sampled

$$L_\mathrm{e}(\mathbf{x}_k, \mathbf{x}_{k-1})$$

high variance when light sources are small

cannot render illumination from point light sources

# EXPLICIT LIGHT SAMPLING



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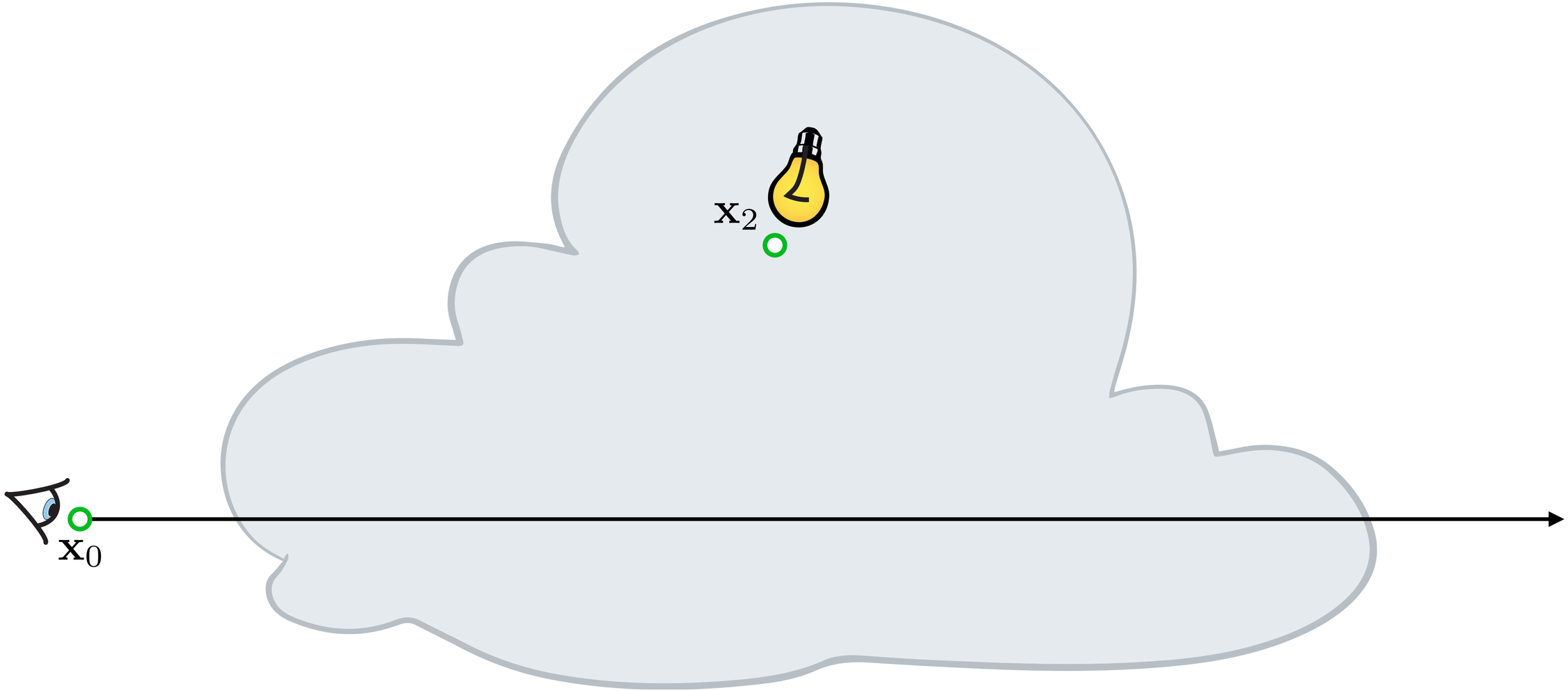

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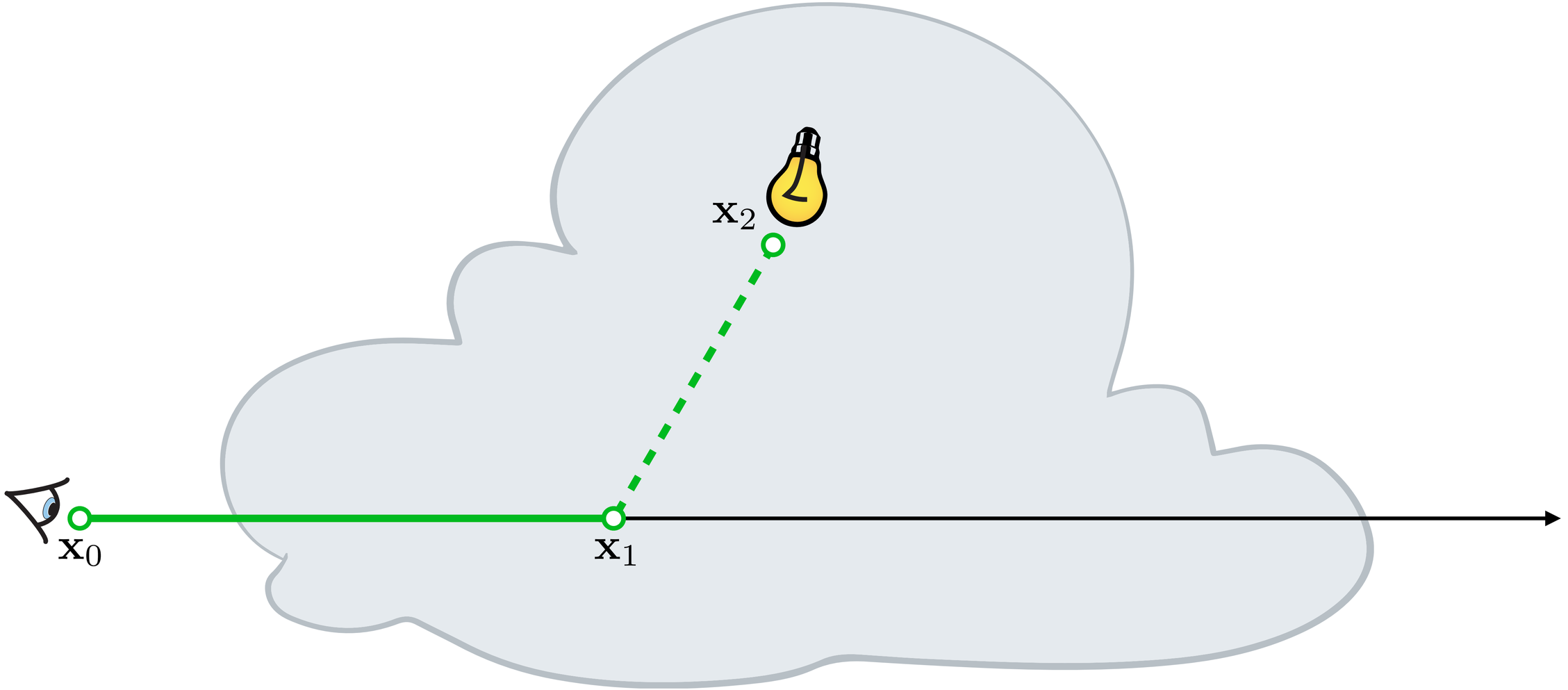
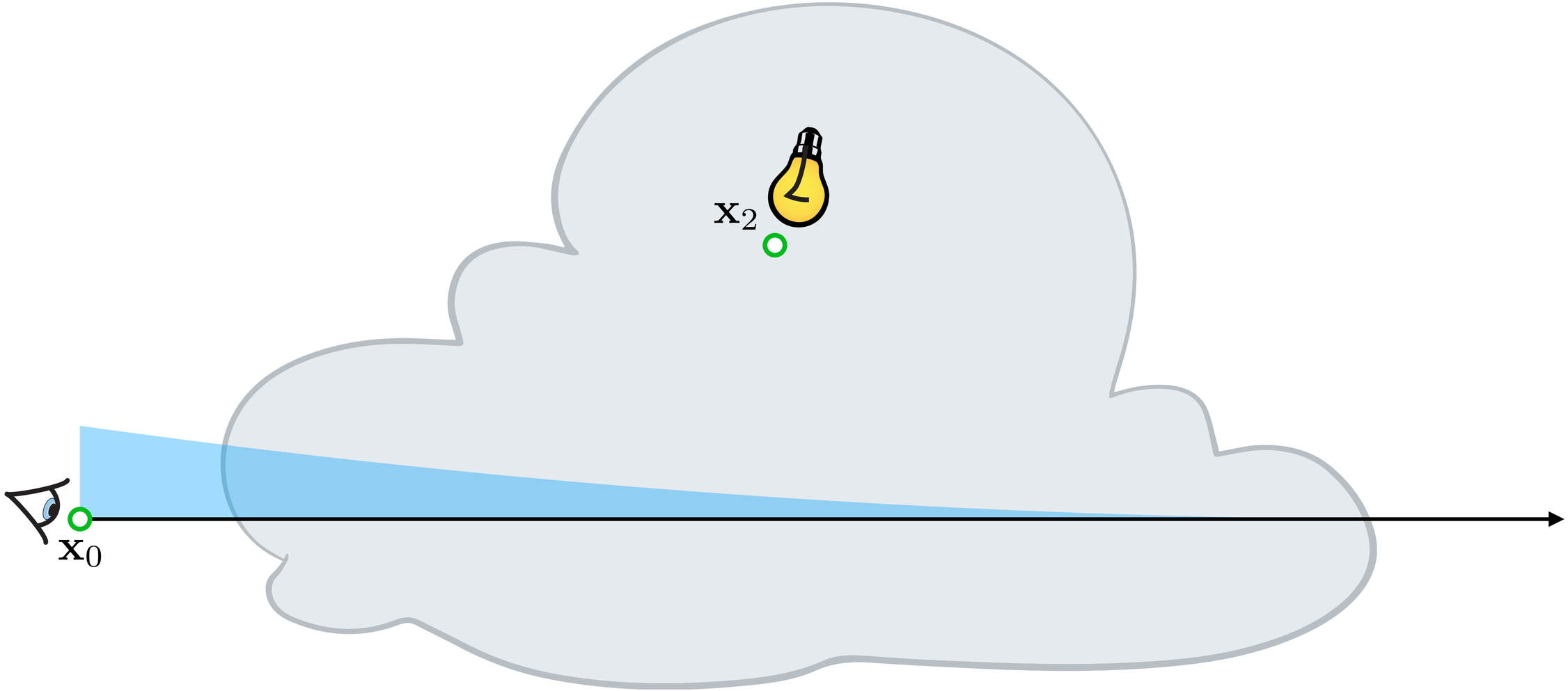


# EXPLICIT: TRANSMITTANCE



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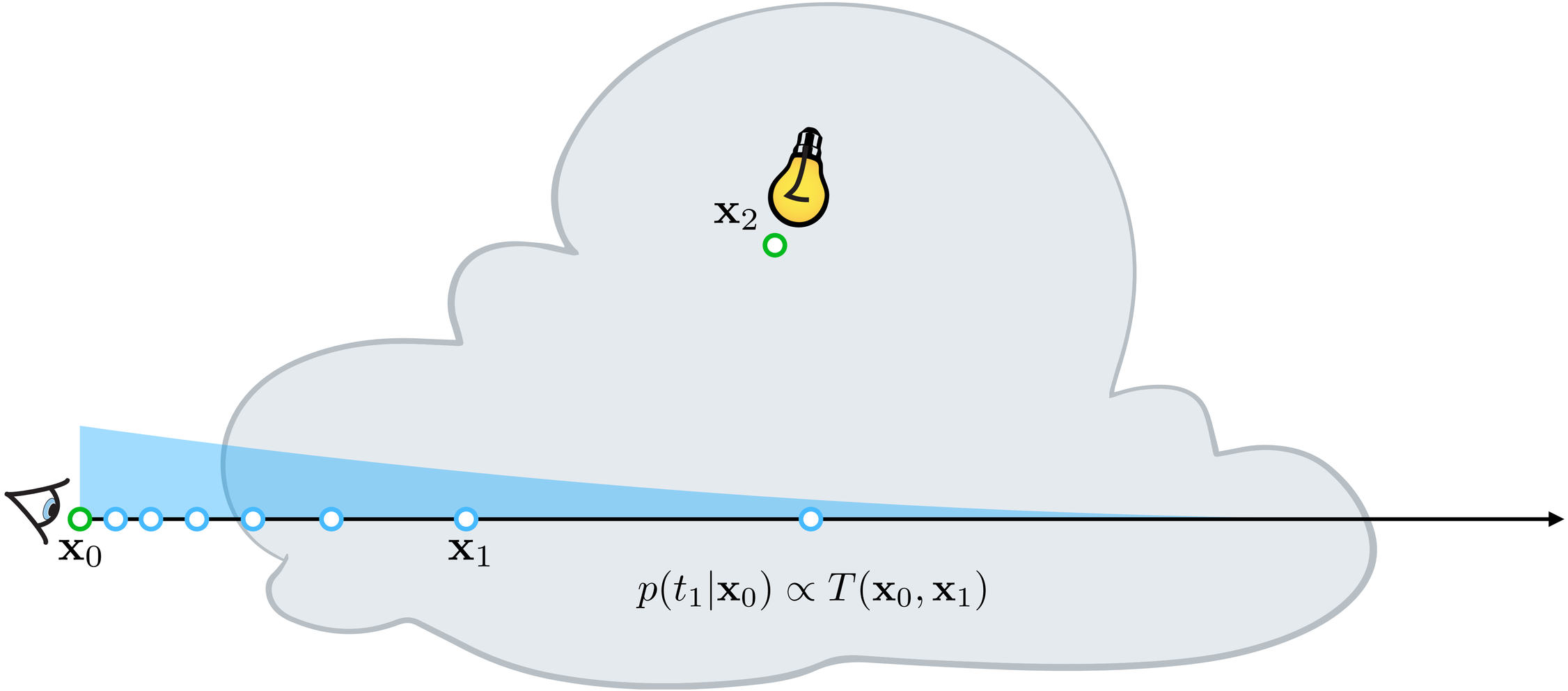

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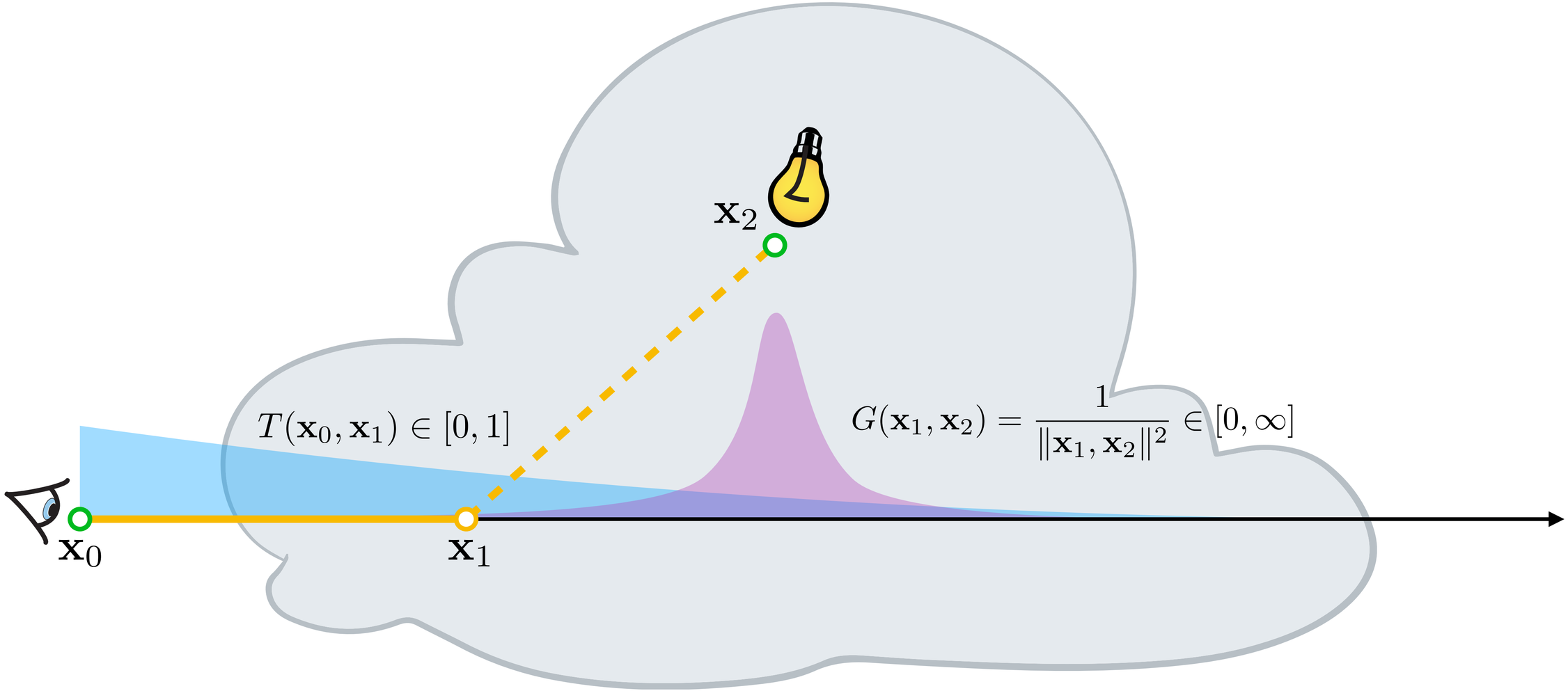

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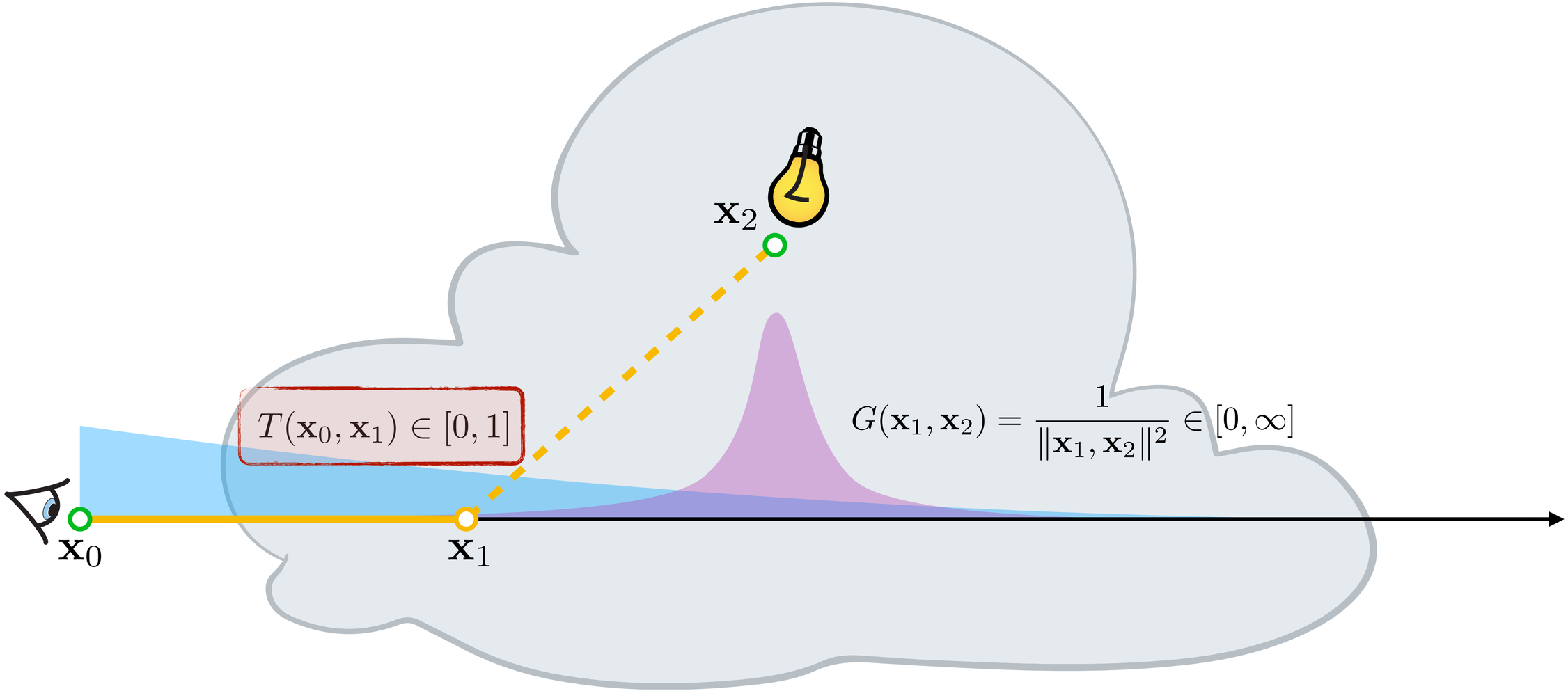

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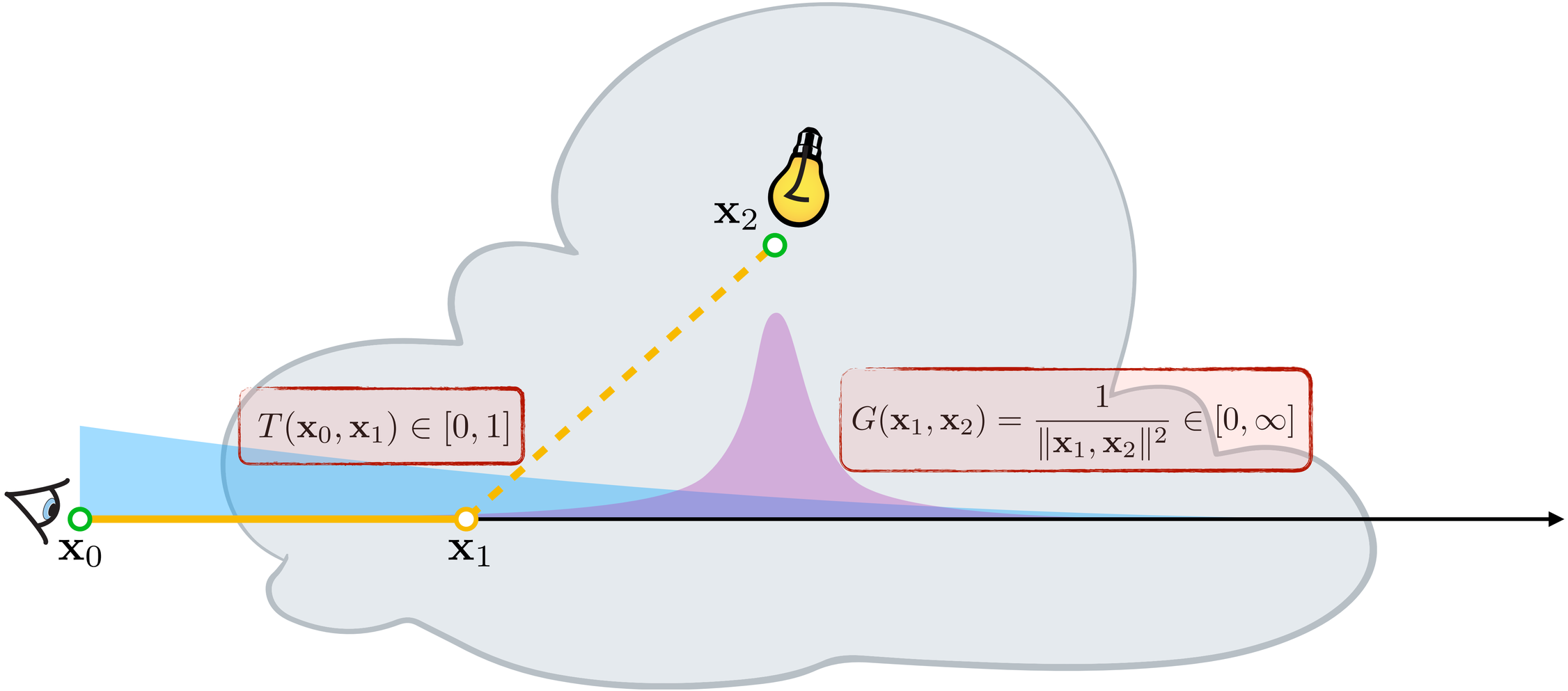

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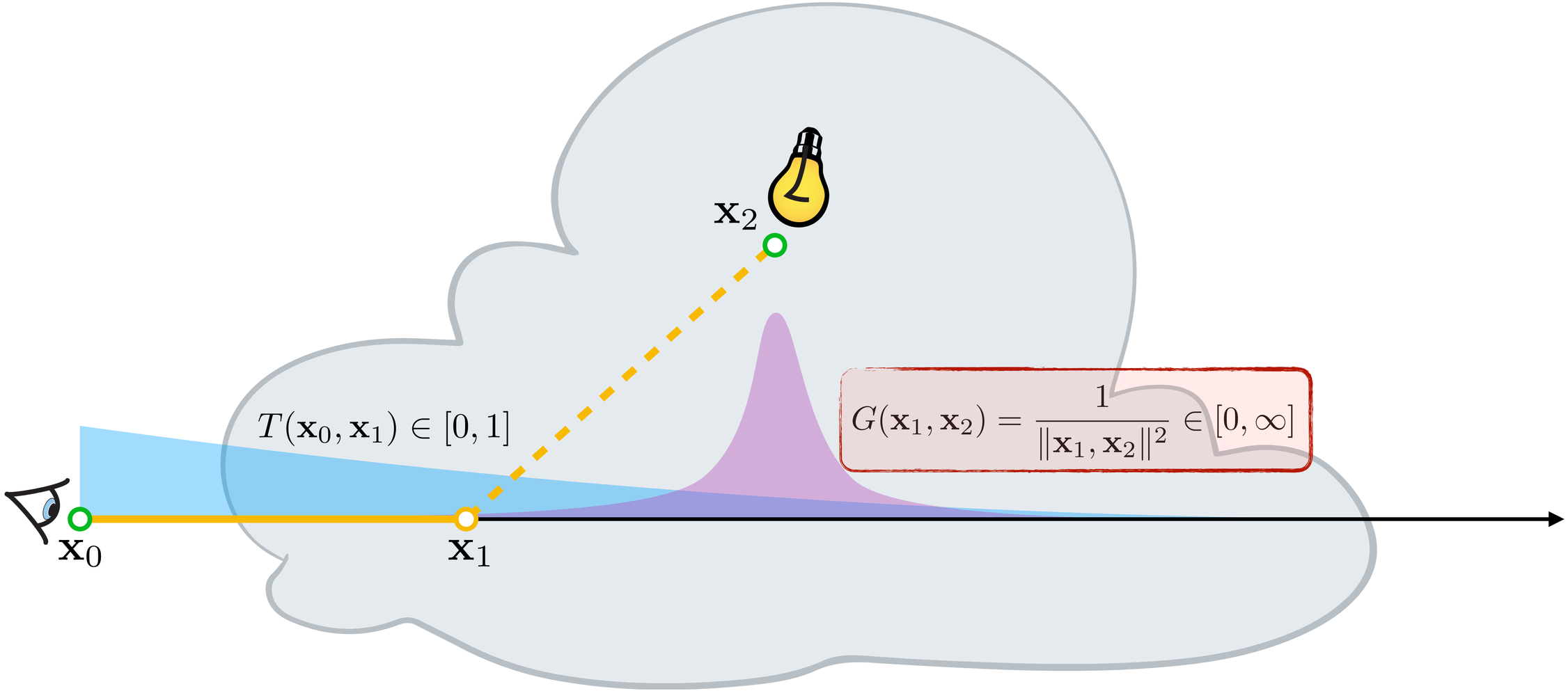

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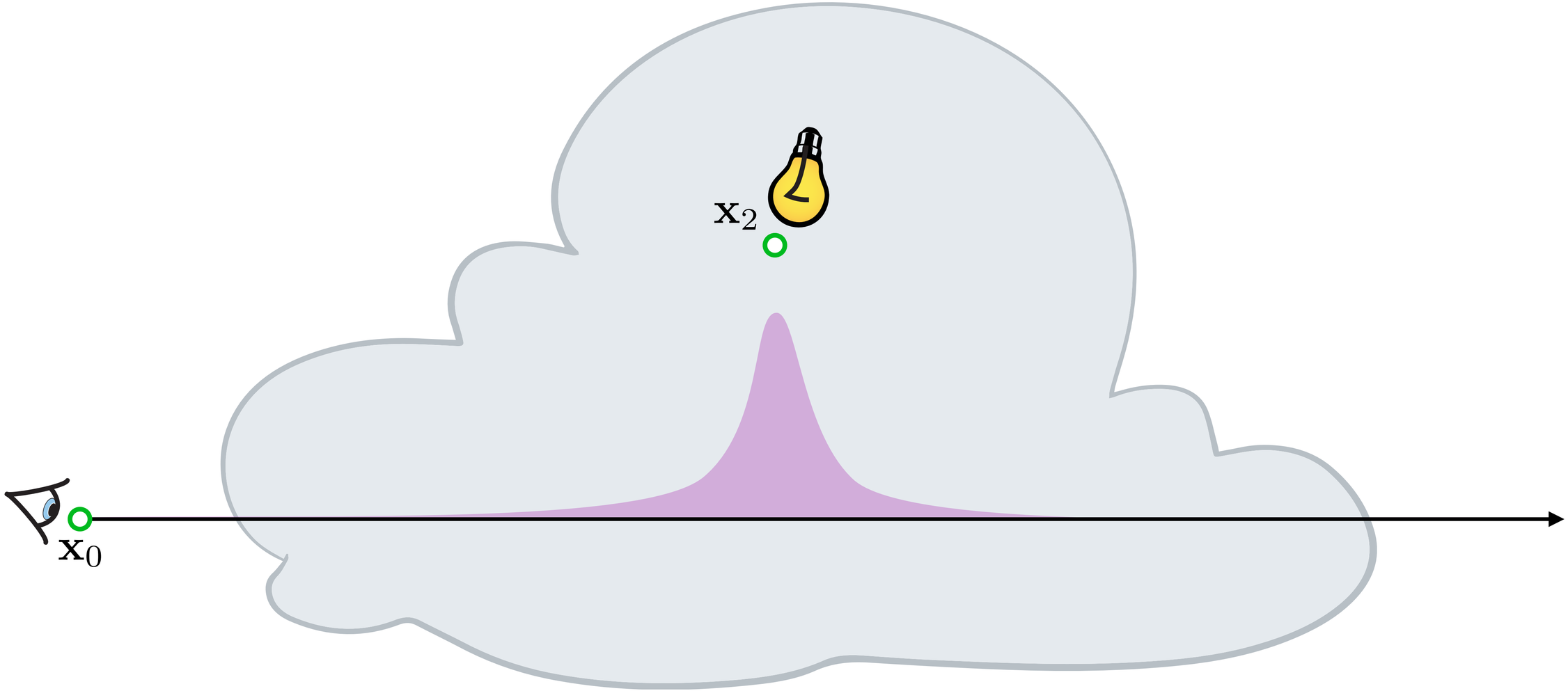

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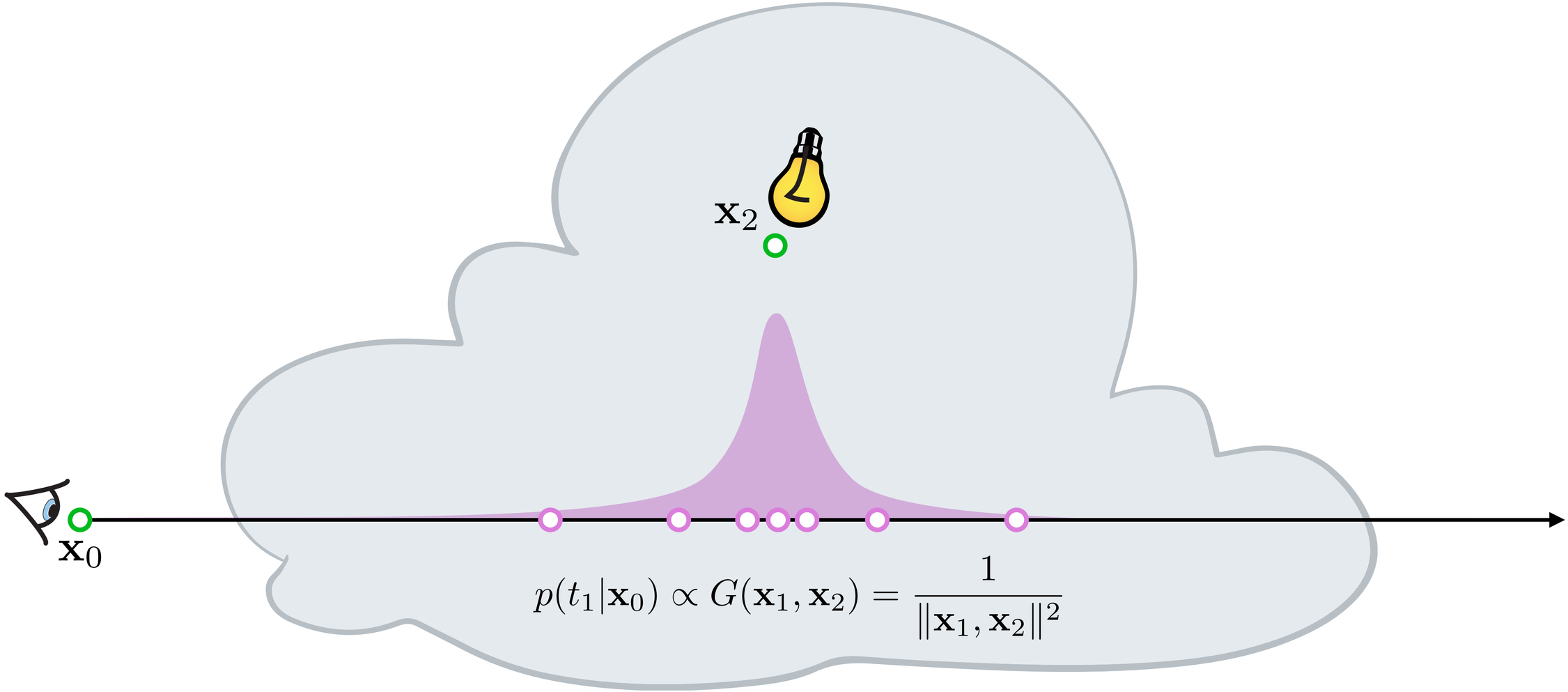
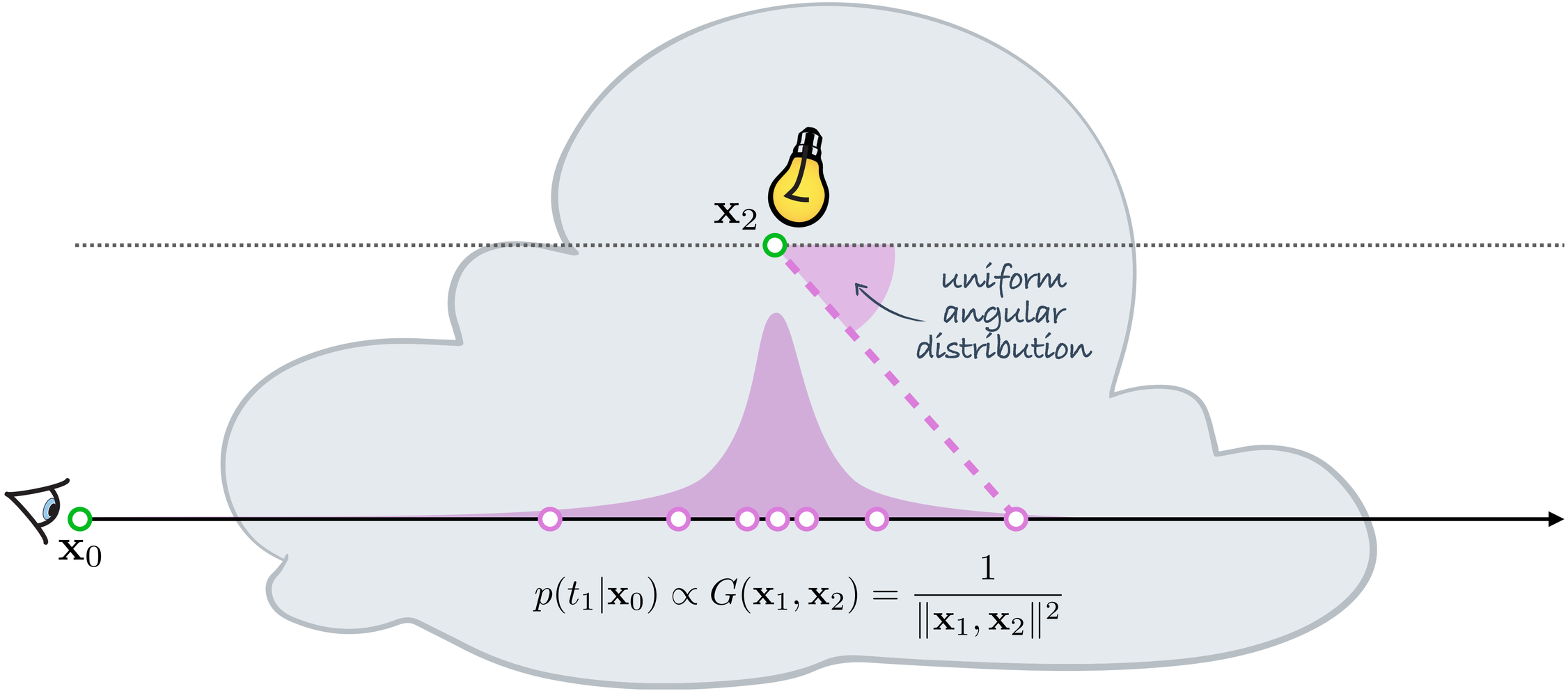
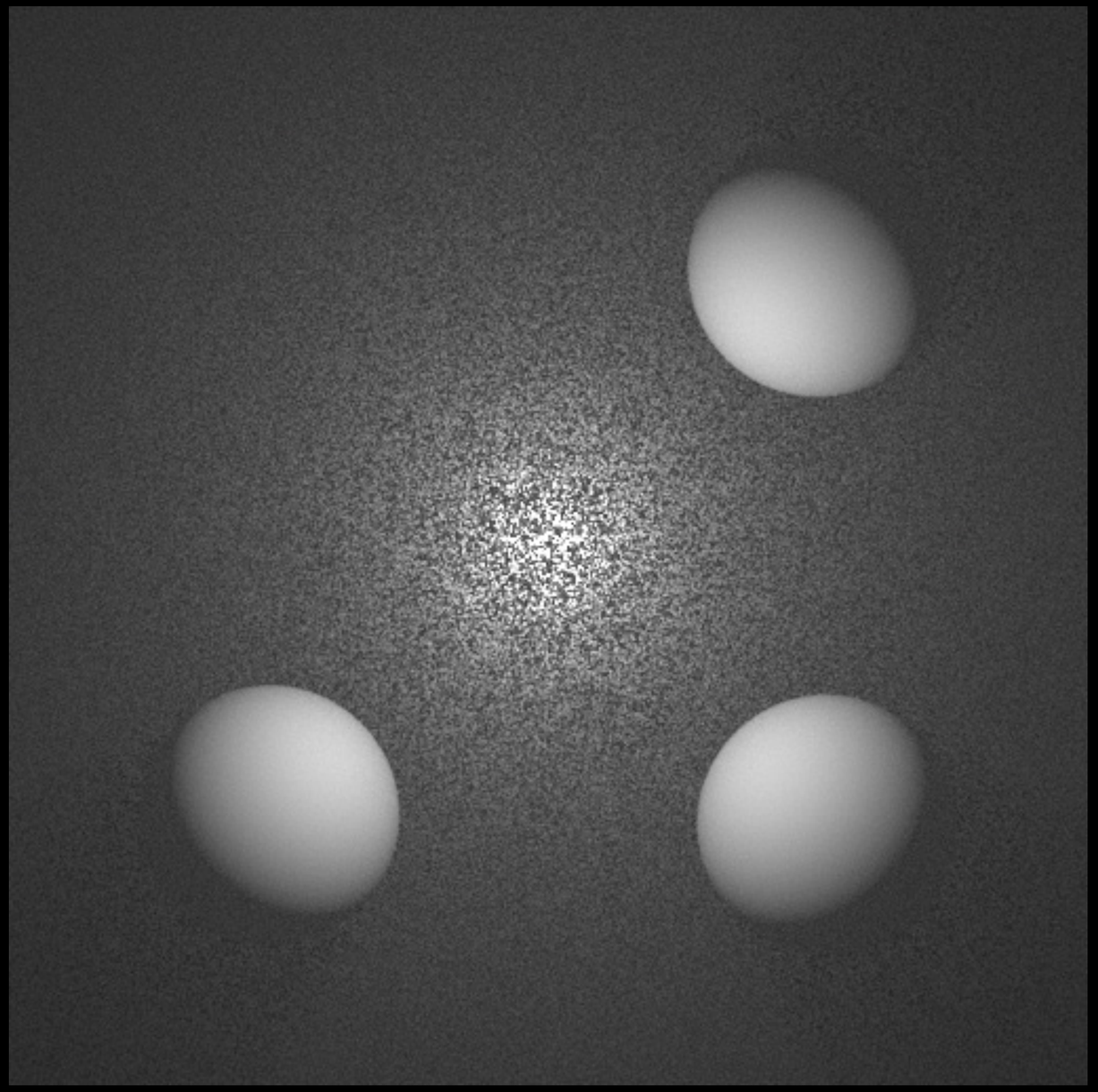
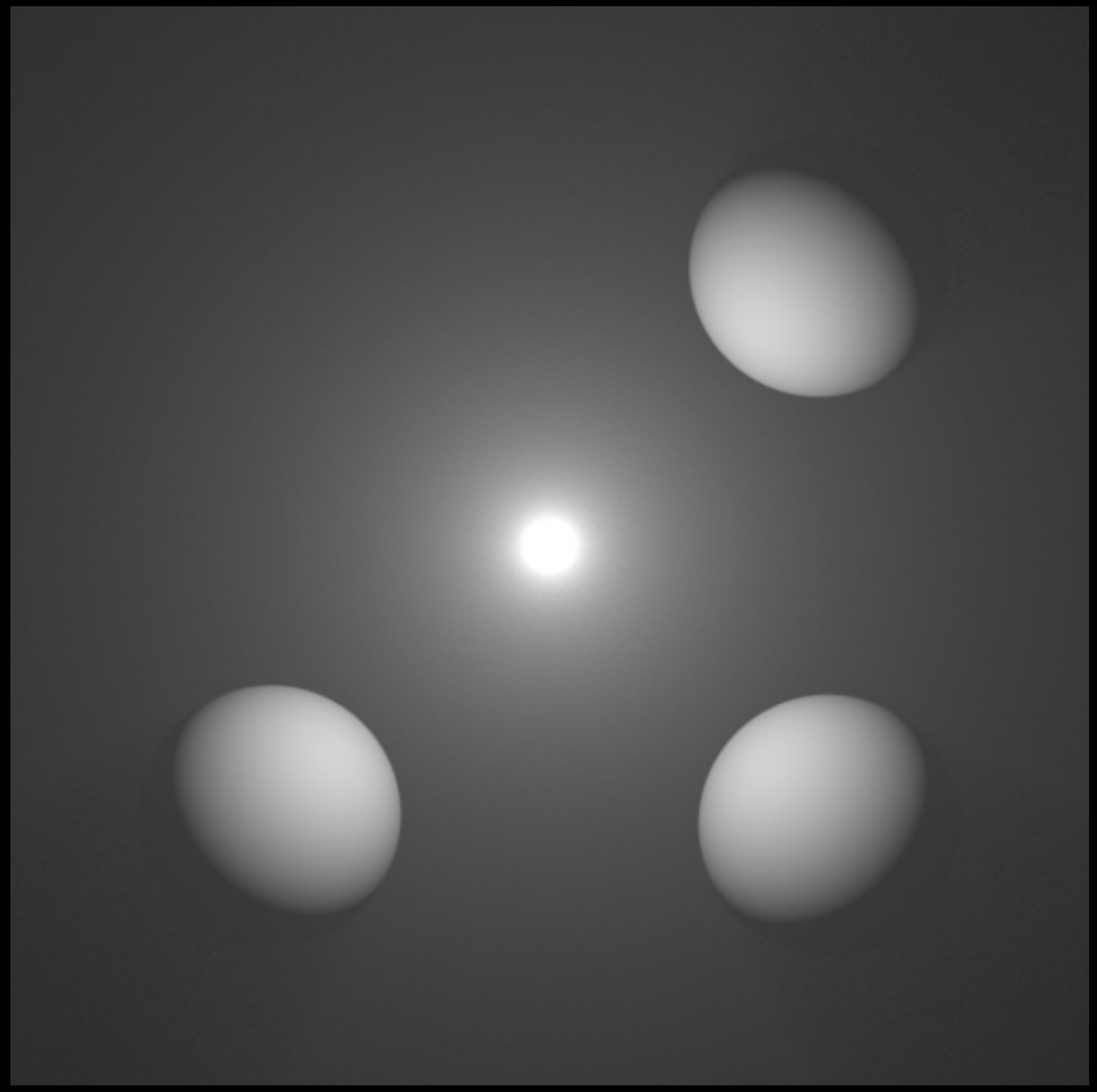


# EXPLICIT: EQUIANGULAR

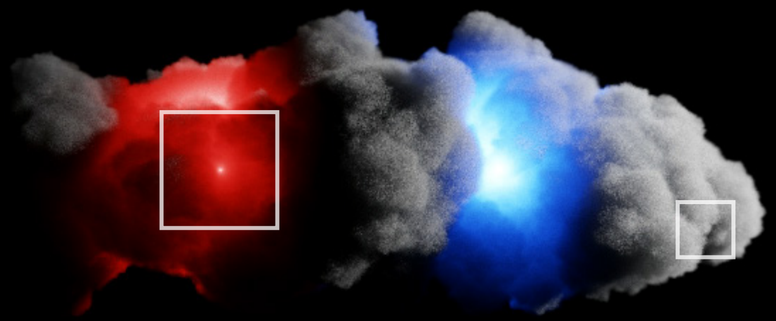


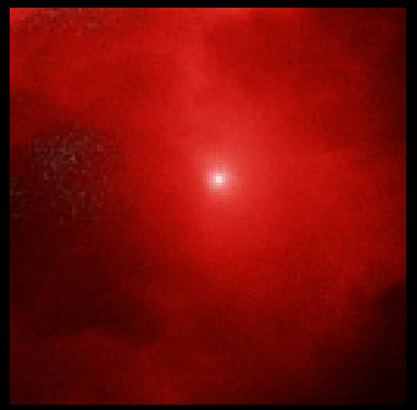


Transmittance sampling, 16 spp

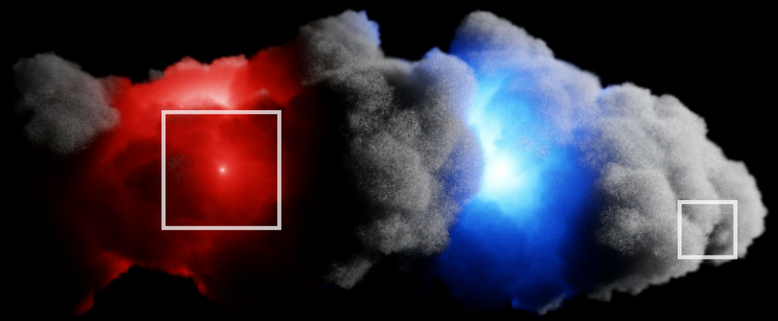


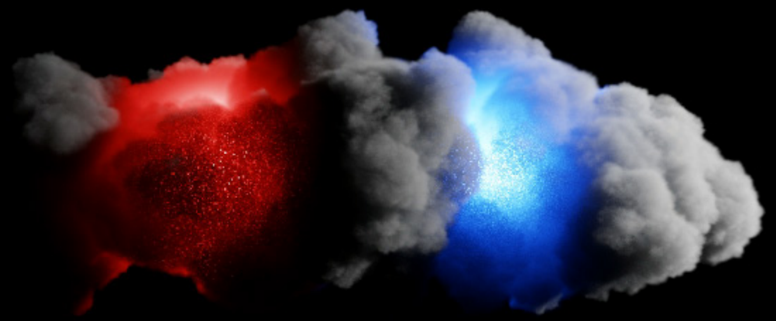
**Equiangular sampling, 16 spp**

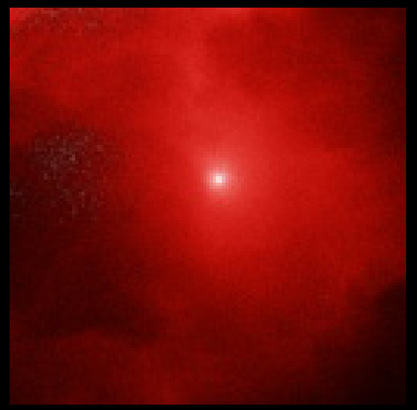




Equiangular sampling

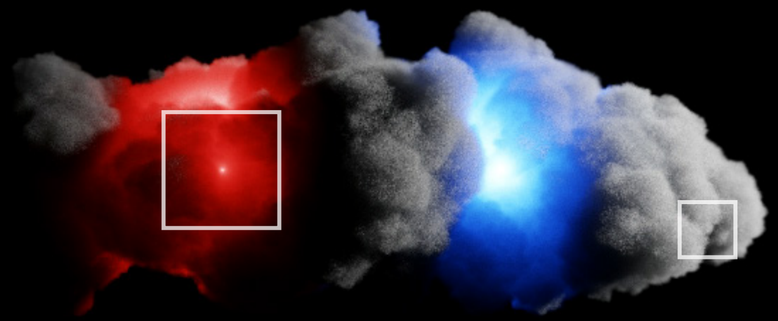


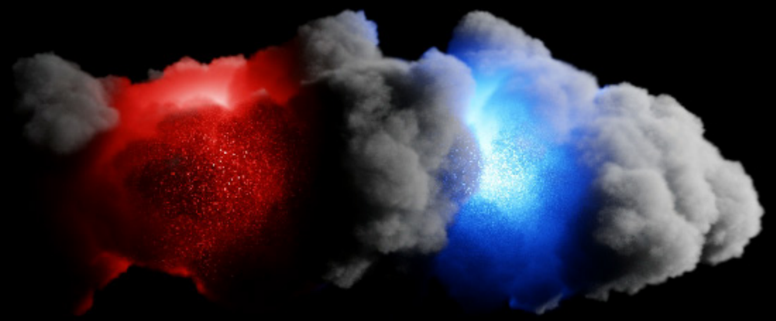


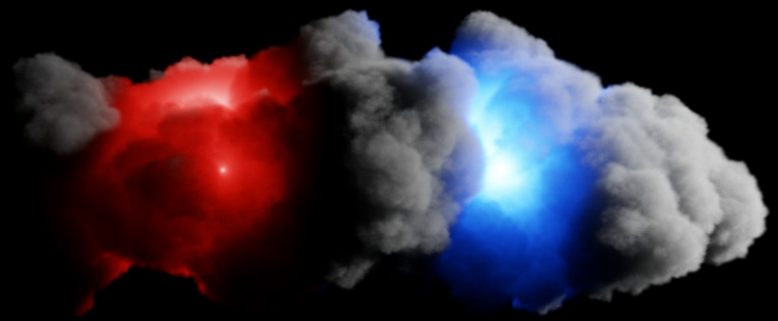


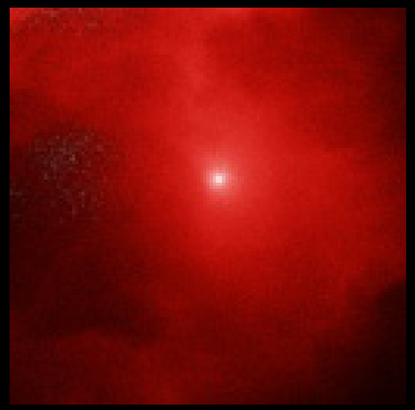
Equiangular sampling

Transmittance sampling







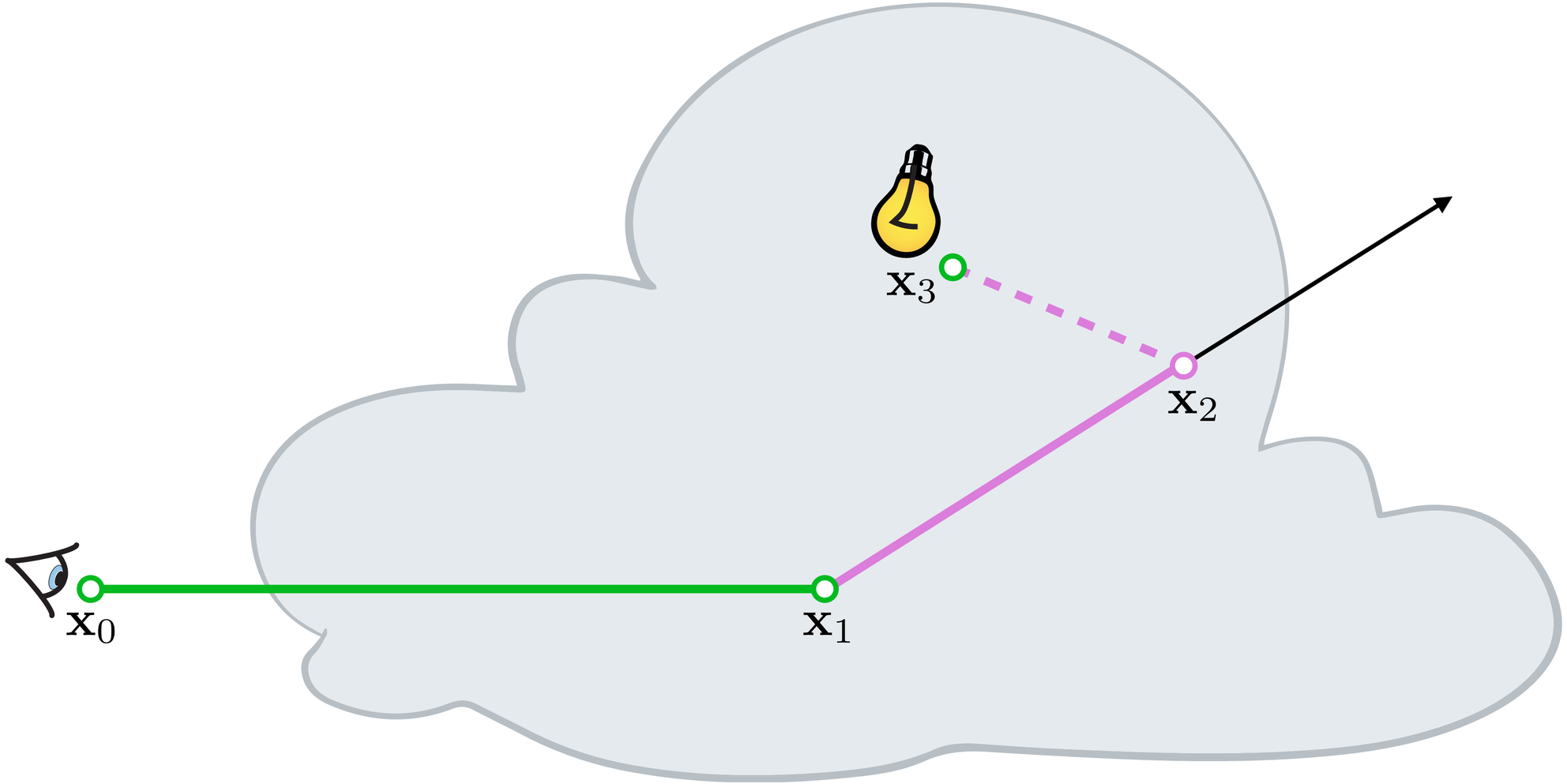


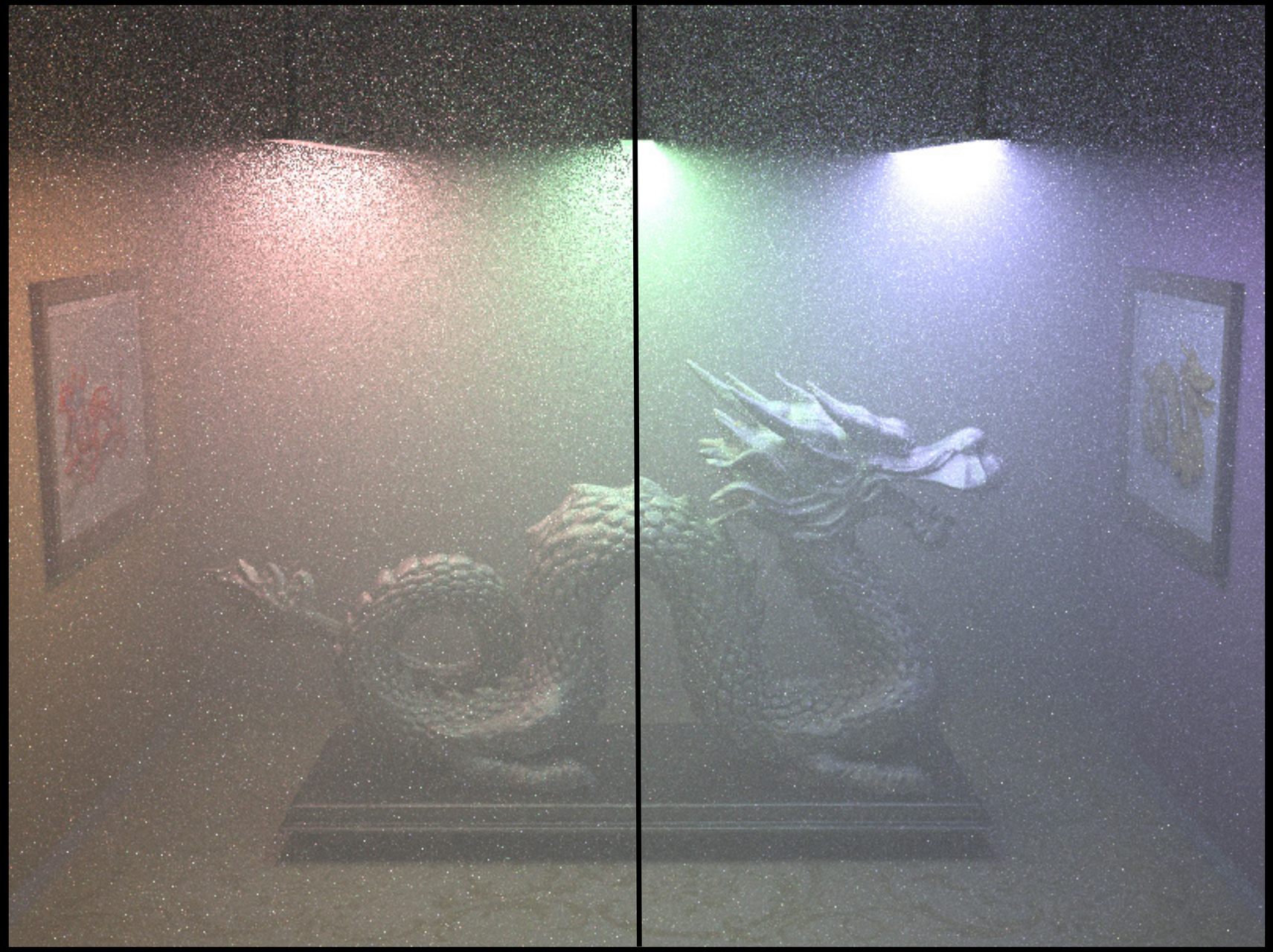
Equiangular sampling

Transmittance sampling

MIS combination

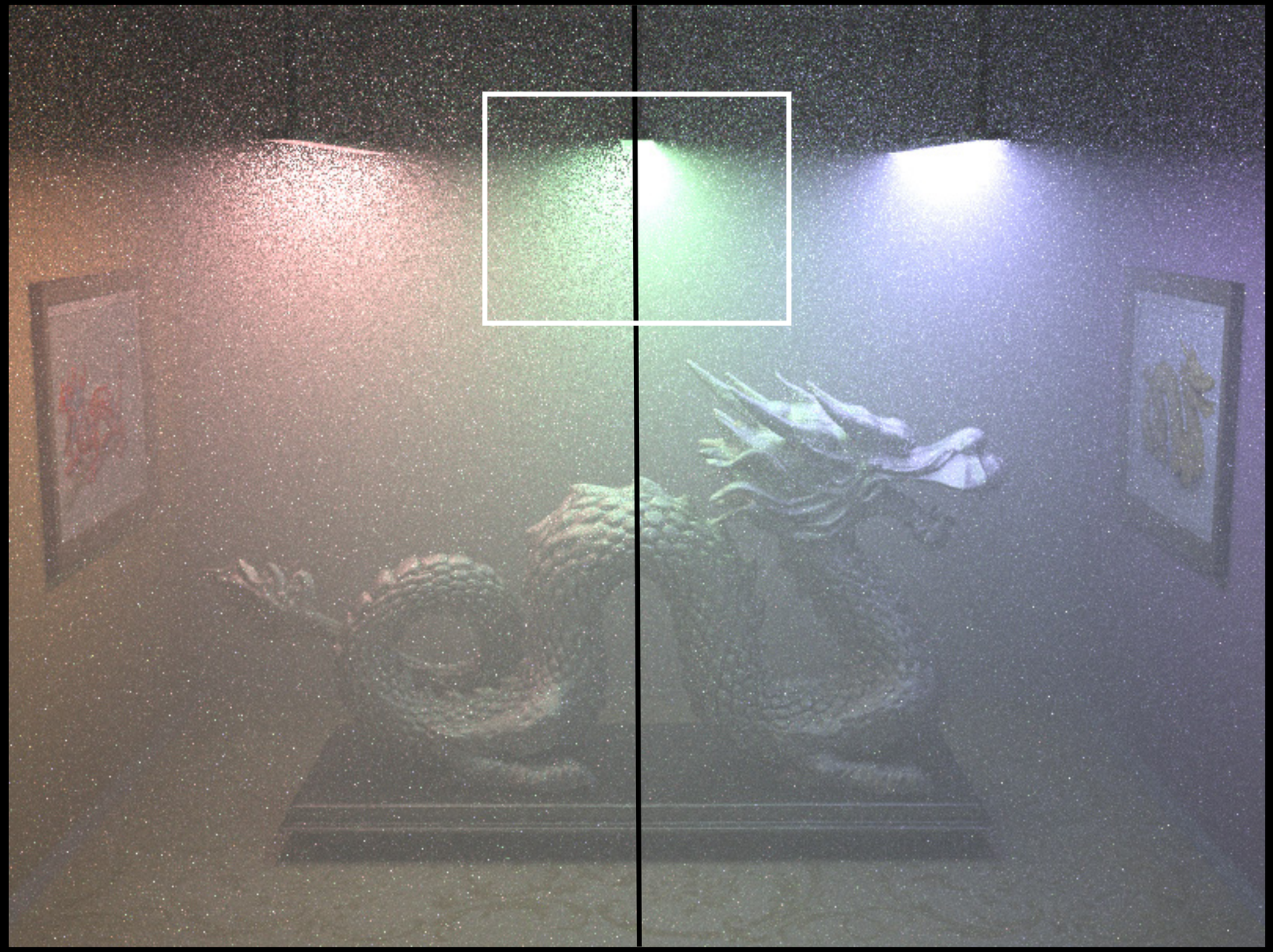
# UNIDIRECTIONAL + NEXT EVENT





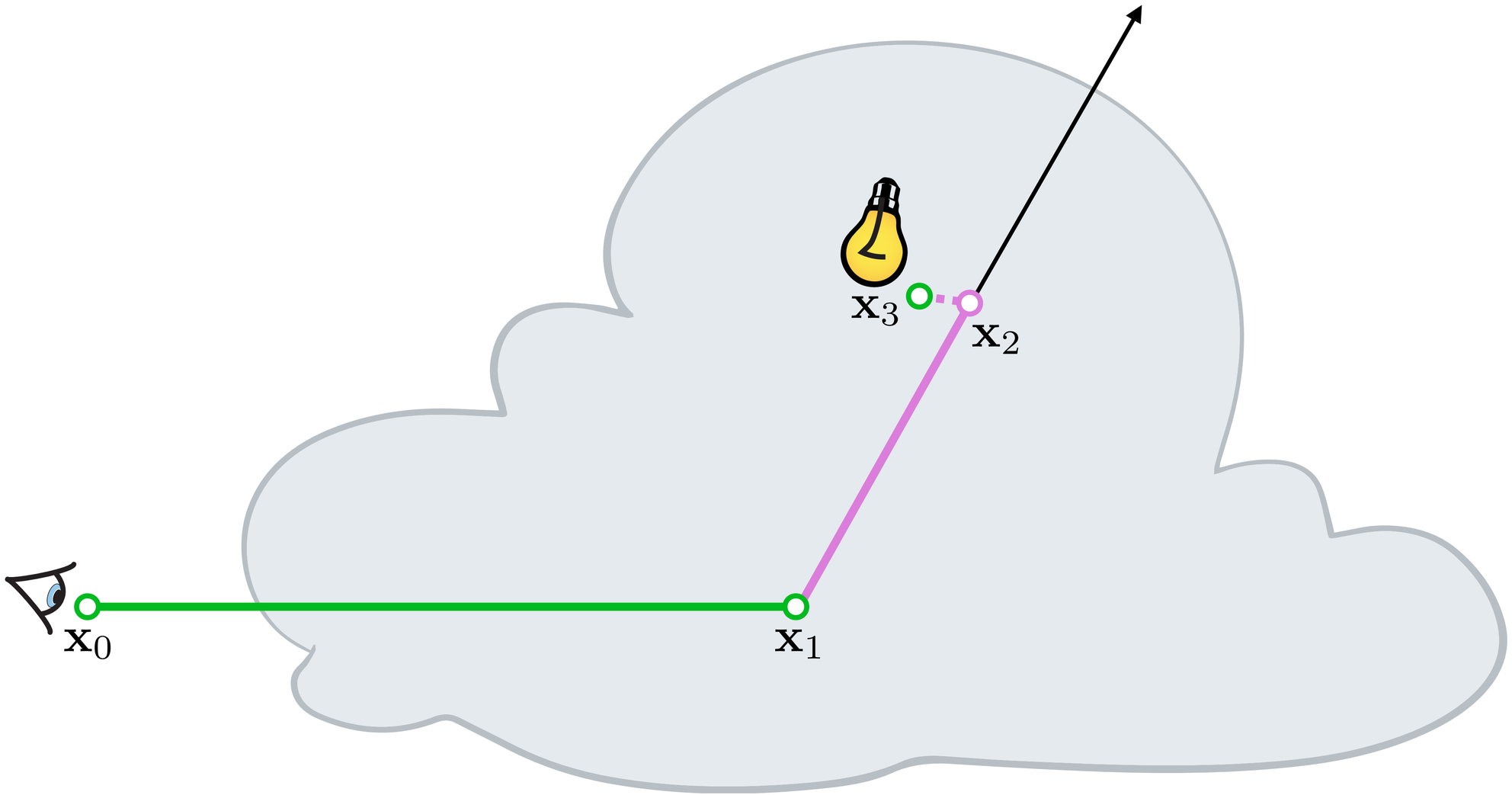
Transmittance connections

**Equiangular connections**

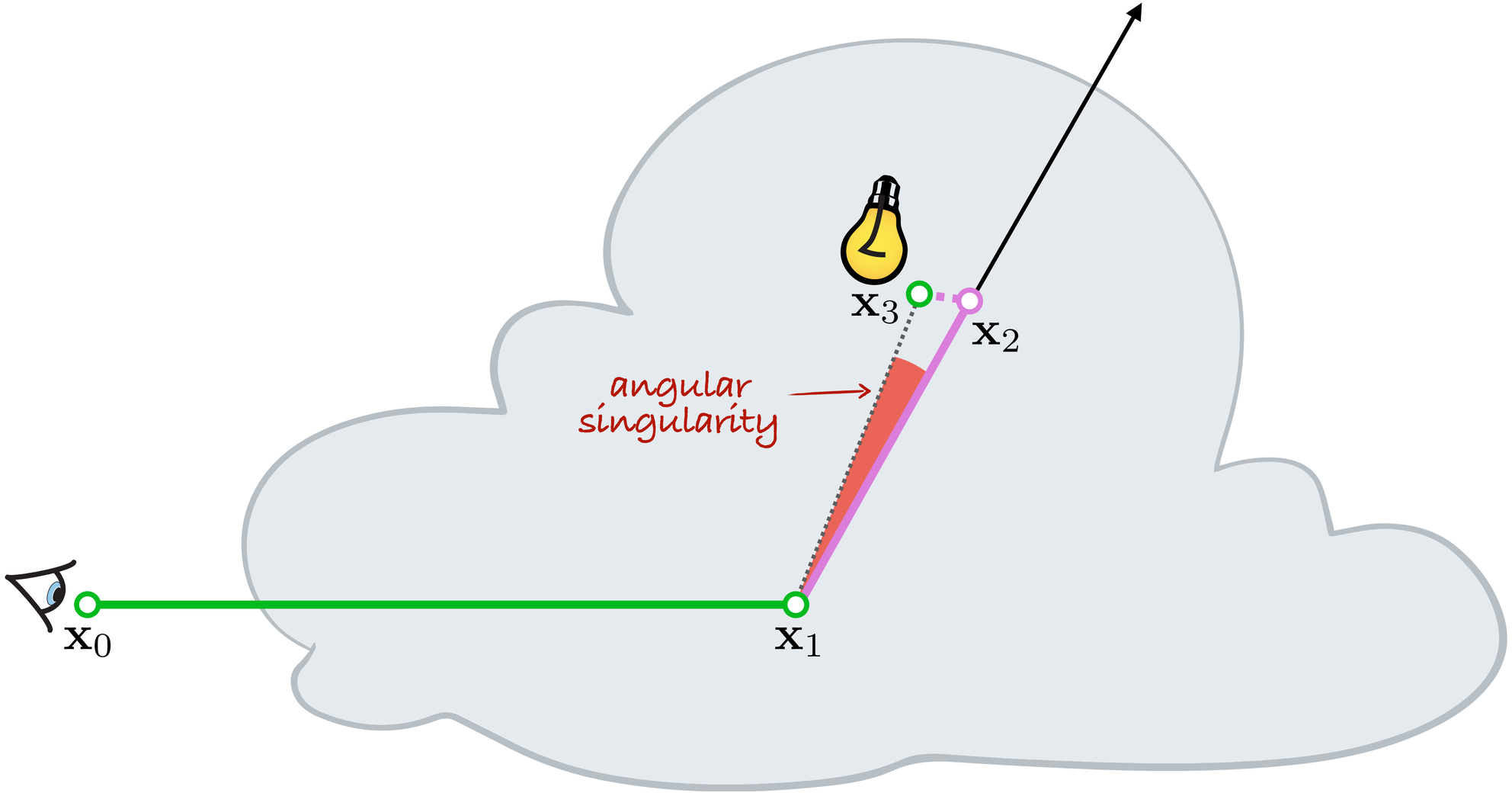


Transmittance connections **Equiangular connections**

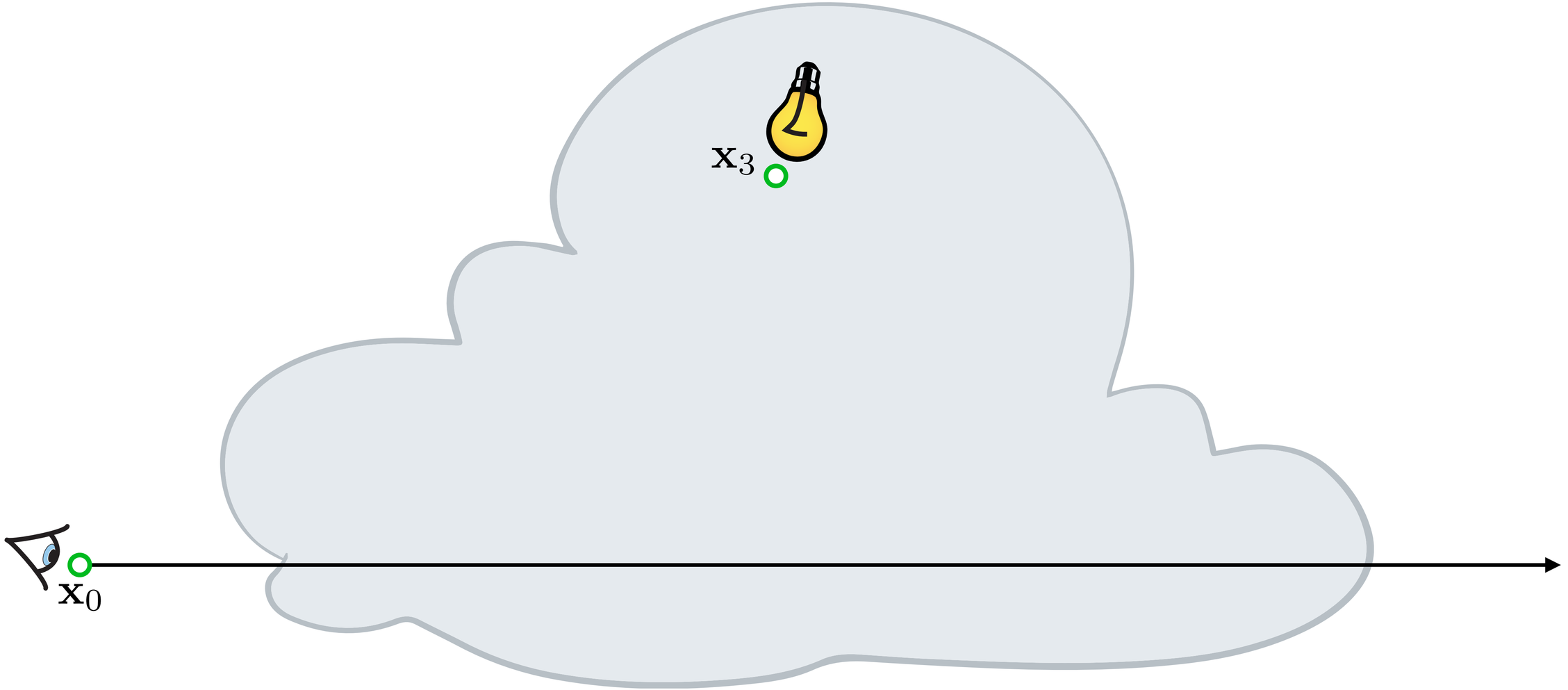
# UNIDIRECTIONAL + NEXT EVENT



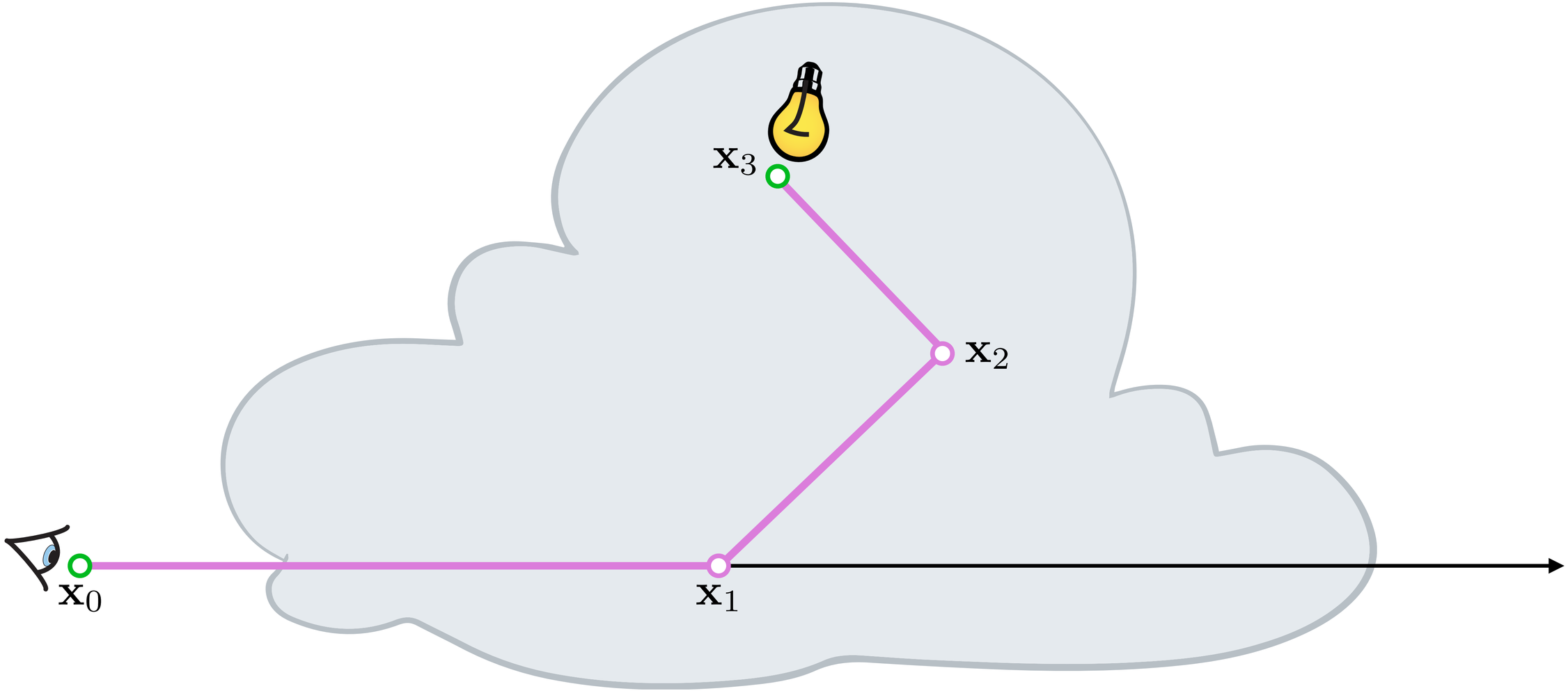
# UNIDIRECTIONAL + NEXT EVENT



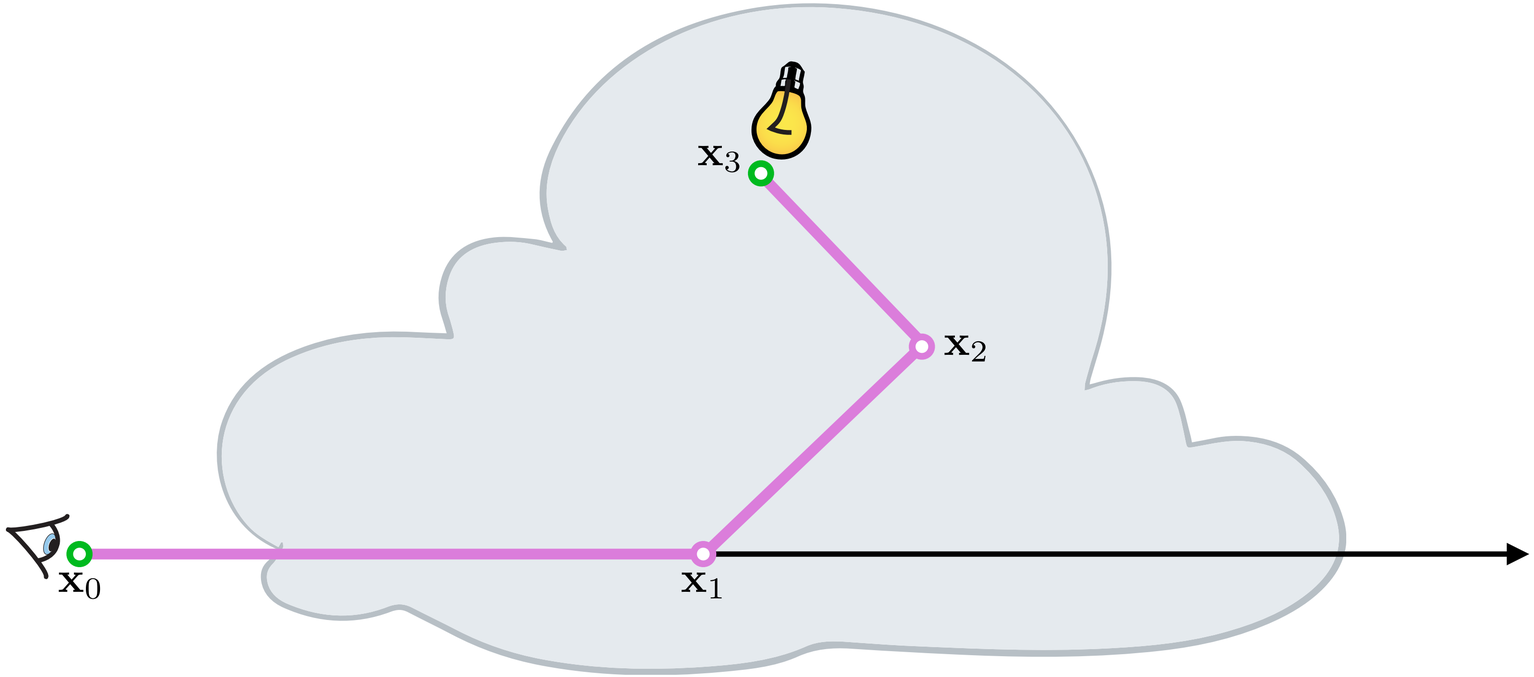
# JOINT PATH SAMPLING



# JOINT PATH SAMPLING

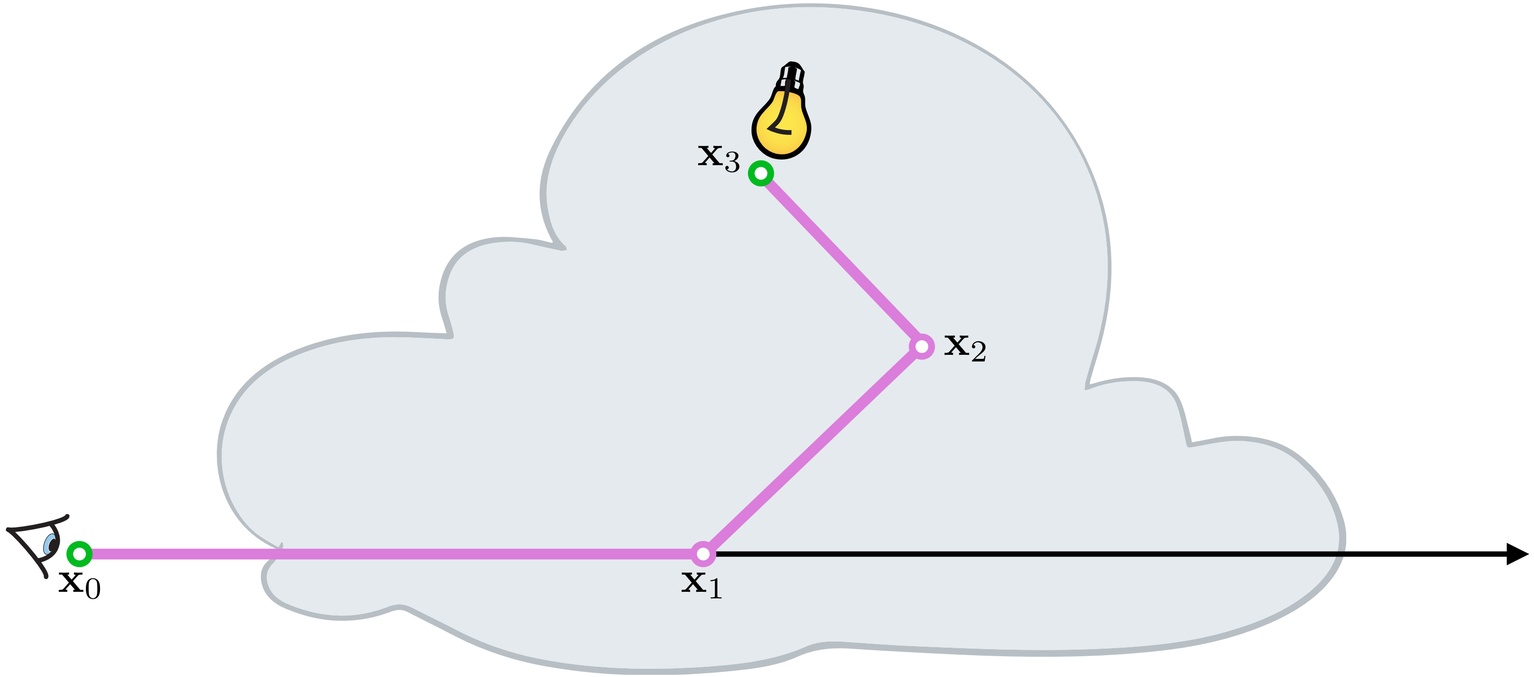


# JOINT PATH SAMPLING




**TRADITIONAL**: prescribes conditional pdfs, no explicit control over joint pdf

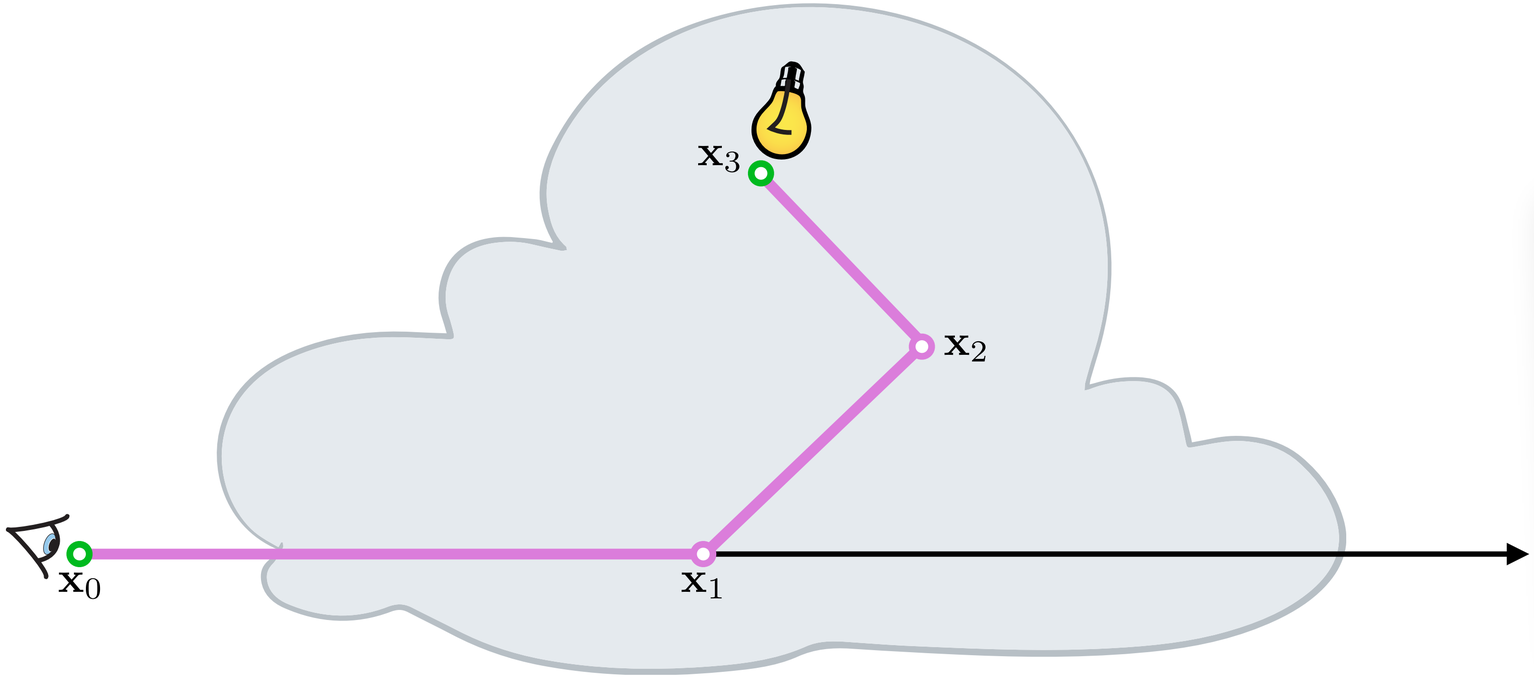
# JOINT PATH SAMPLING




**TRADITIONAL**: prescribes conditional pdfs, no explicit control over joint pdf

**JOINT SAMPLING**: prescribe joint pdf, conditional pdfs derived from it

# JOINT PATH SAMPLING



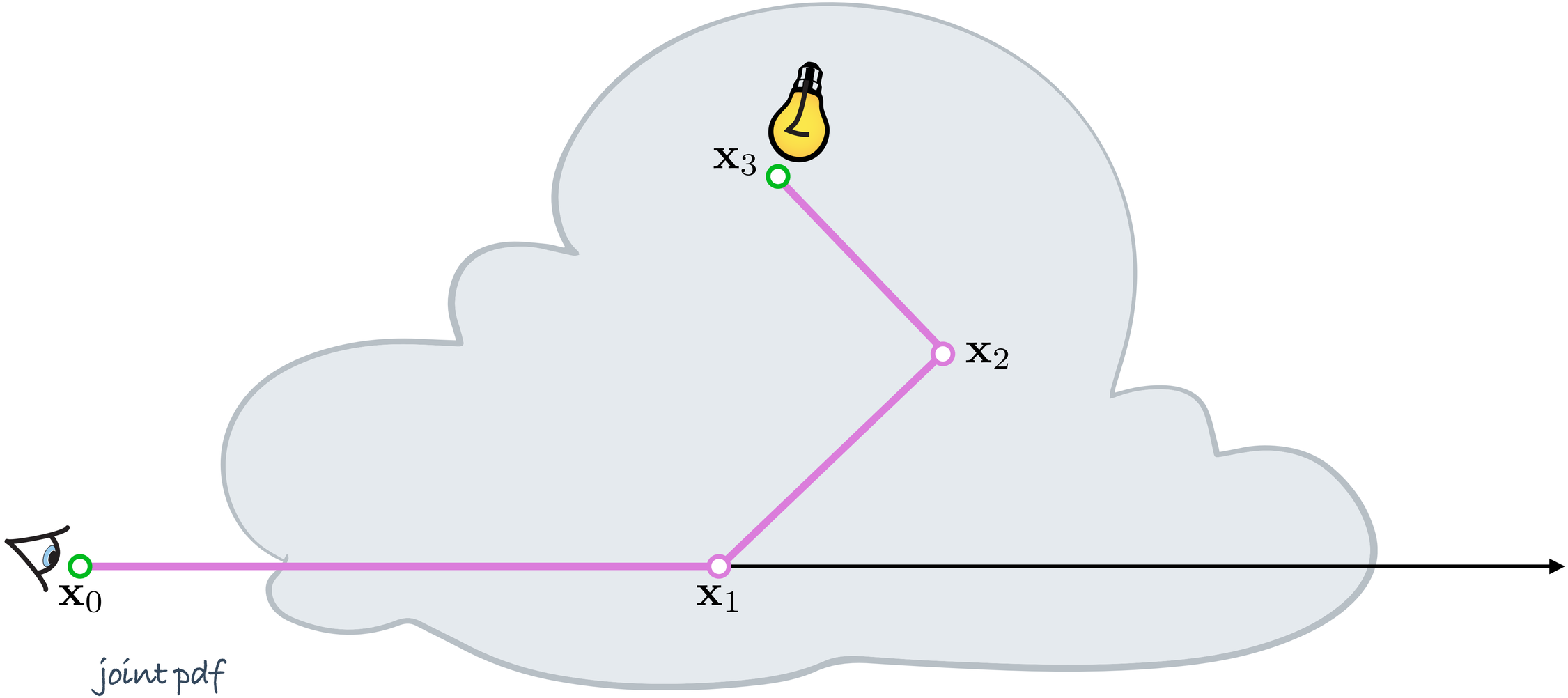

Joint path sampling:

1) Prescribe joint pdf
2) Derive conditional pdfs via successive joint pdf marginalization
3) Conditionals are obtained in reverse order

**TRADITIONAL**: prescribes conditional pdfs, no explicit control over joint pdf

**JOINT SAMPLING**: prescribe joint pdf, conditional pdfs derived from it

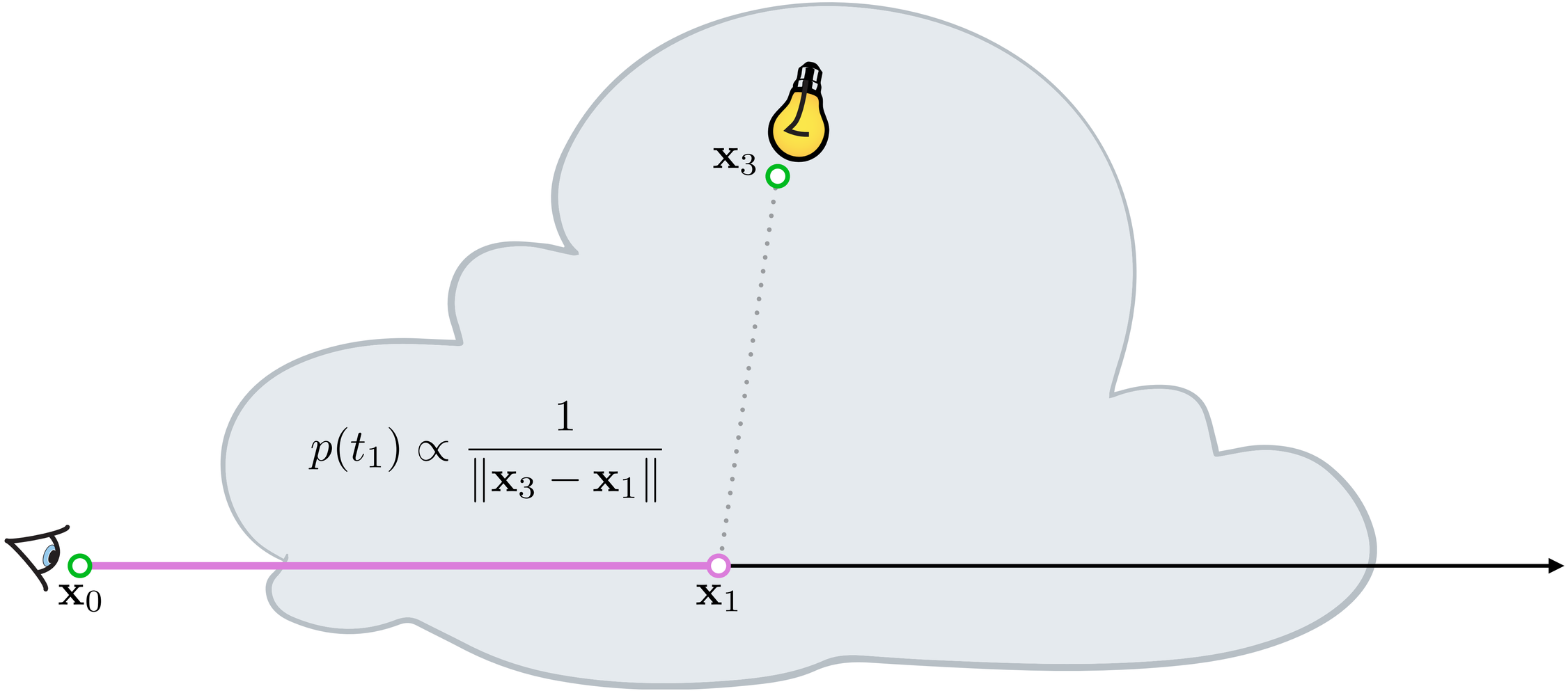
# JOINT PATH SAMPLING



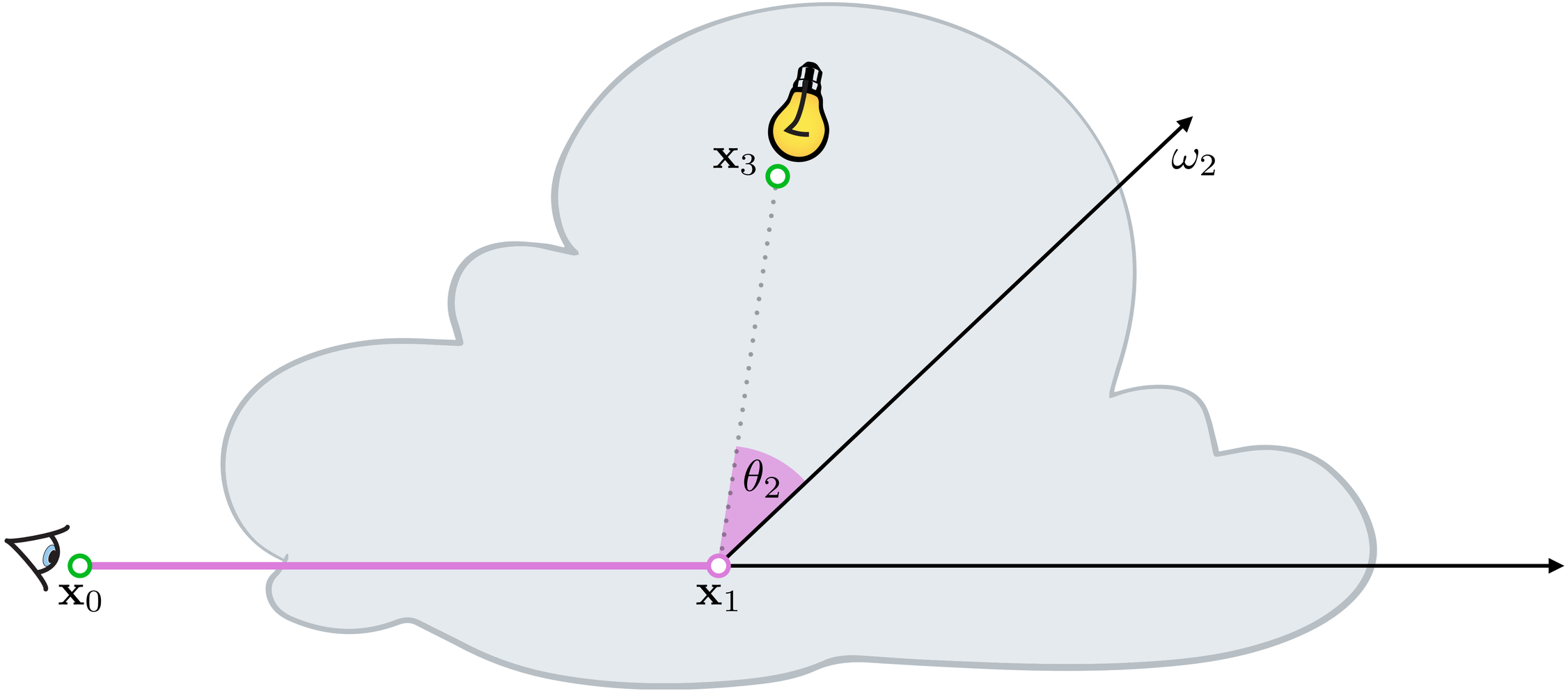

joint pdf

$$p(\mathbf{x}_1, \mathbf{x}_2) \propto G(\mathbf{x}_0, \mathbf{x}_1) G(\mathbf{x}_1, \mathbf{x}_2) G(\mathbf{x}_2, \mathbf{x}_3)$$

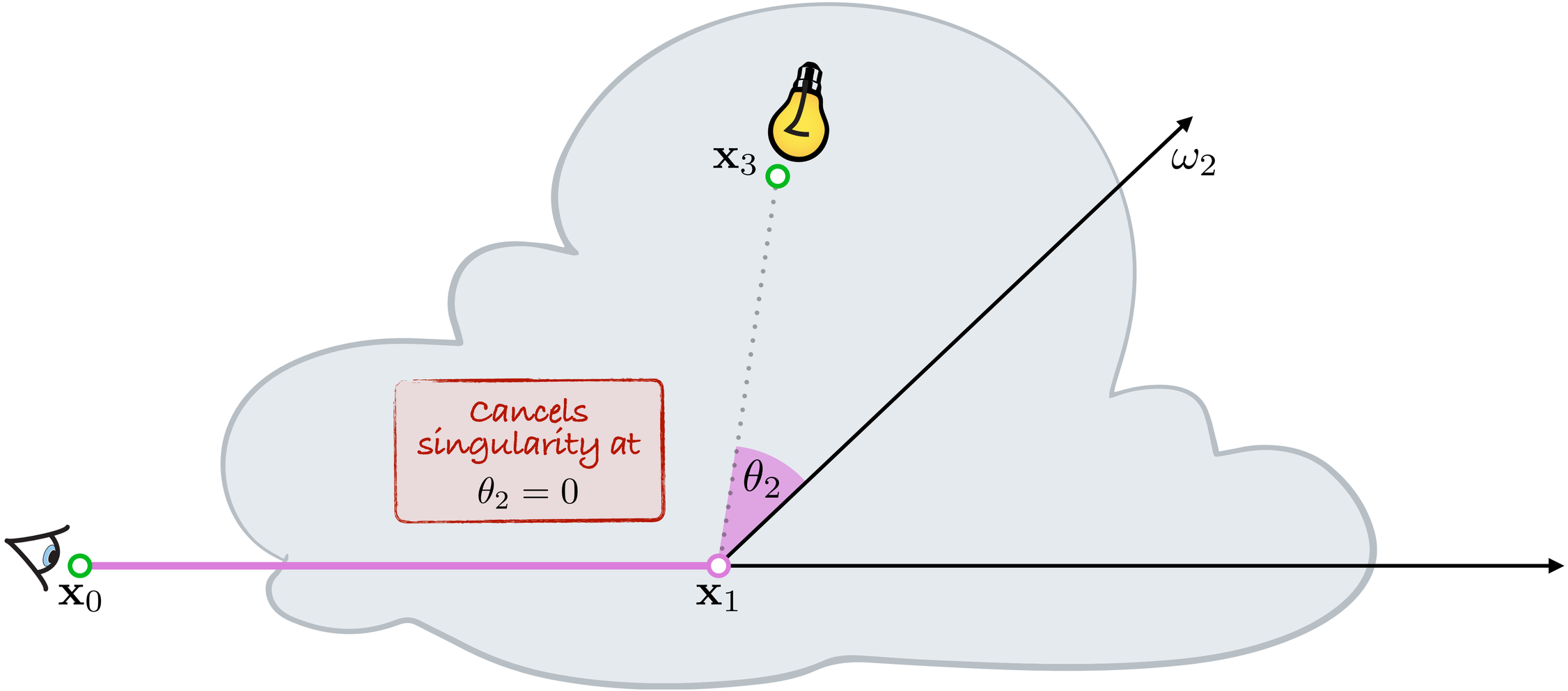
# JOINT PATH SAMPLING



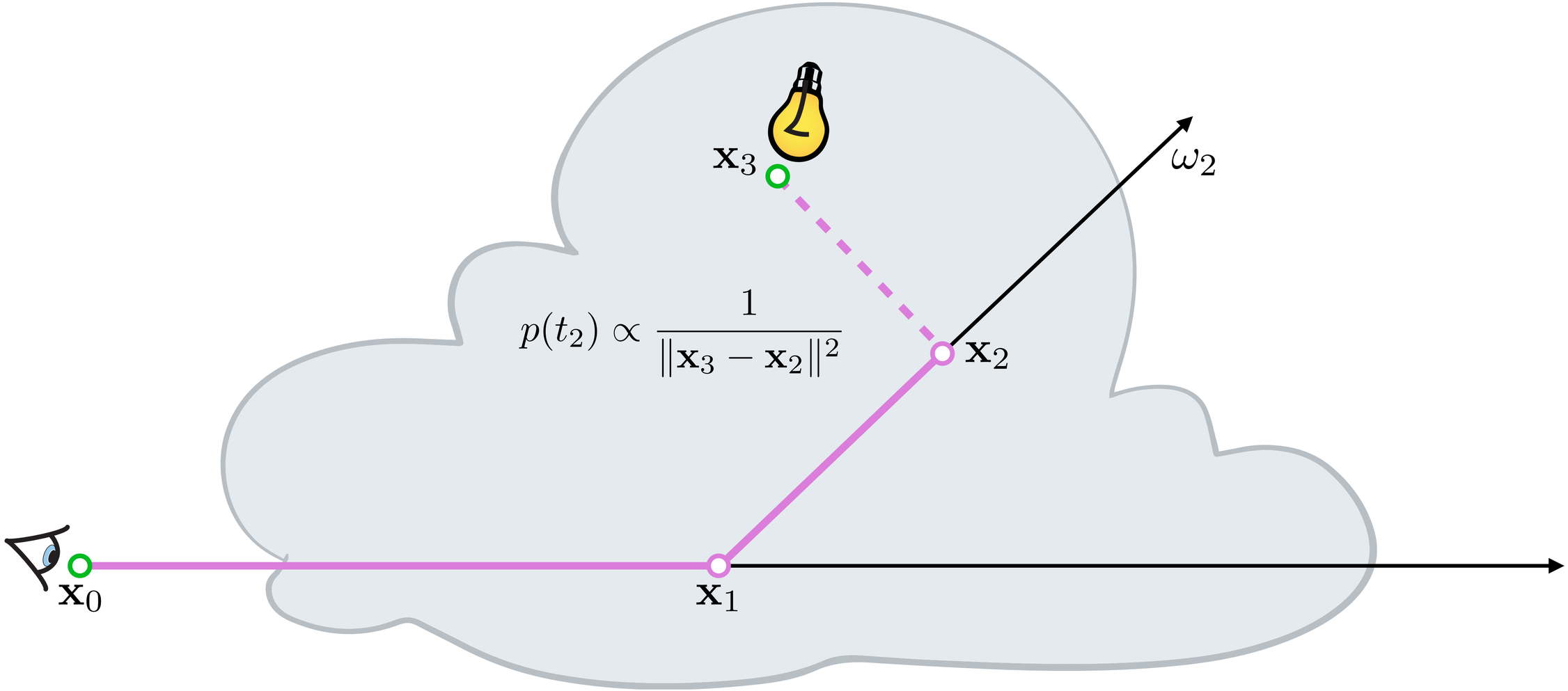
# JOINT PATH SAMPLING



# JOINT PATH SAMPLING



# JOINT PATH SAMPLING



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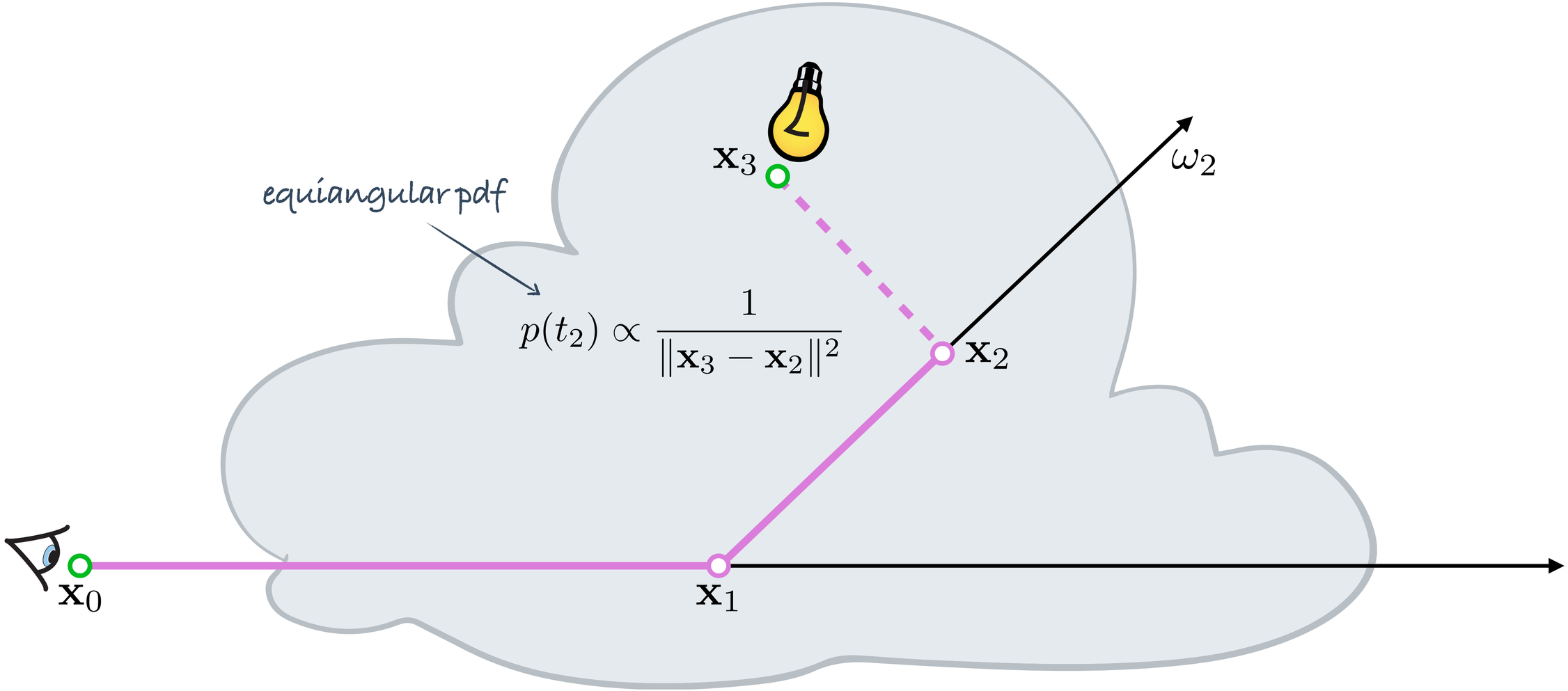
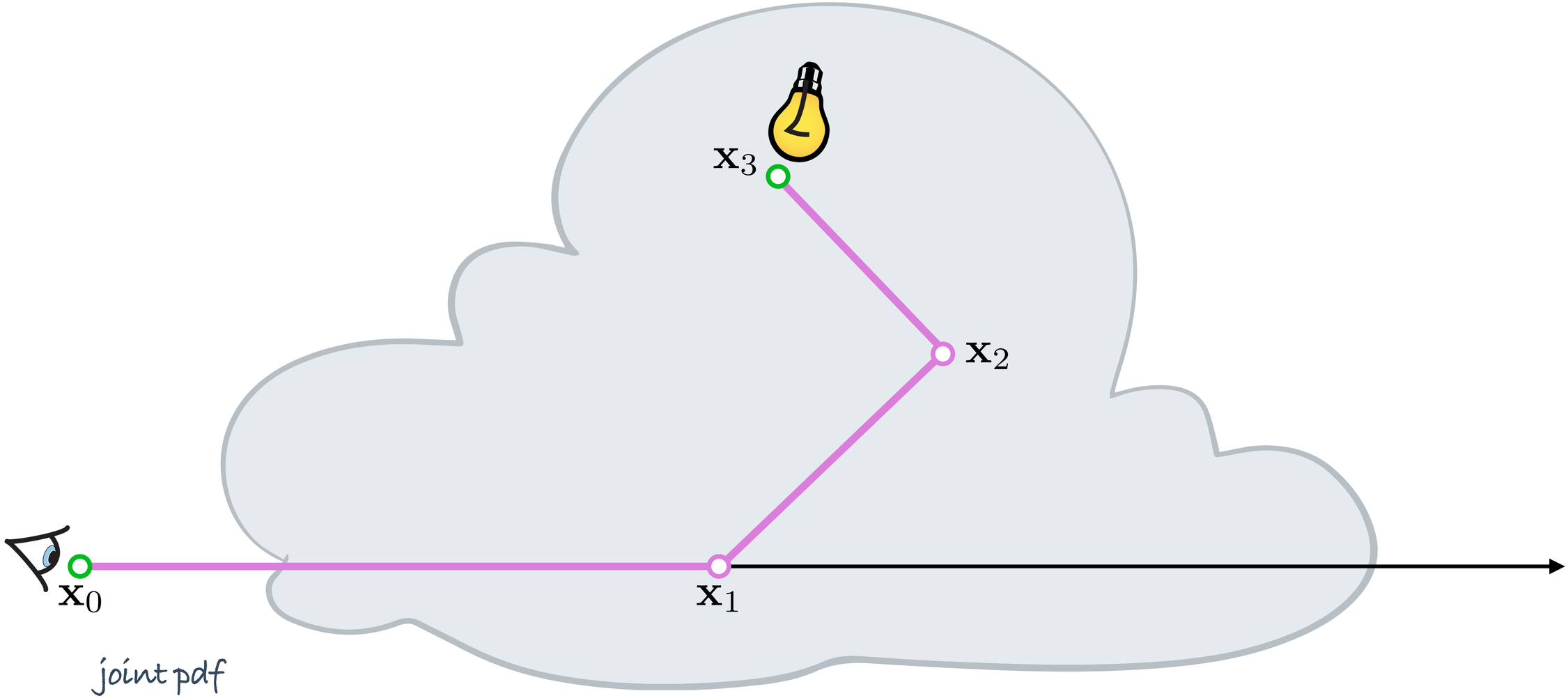


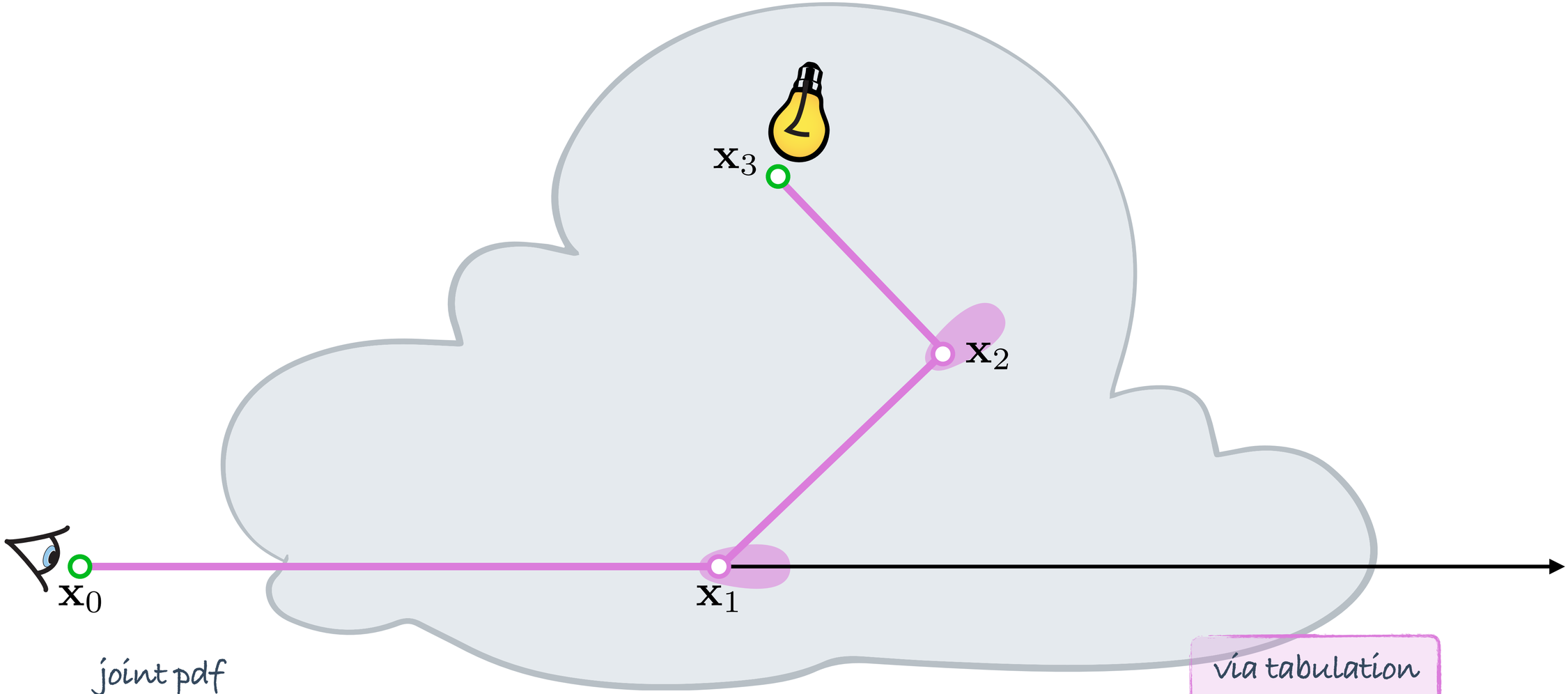
# JOINT PATH SAMPLING




joint pdf

$$p(\mathbf{x}_1, \mathbf{x}_2) \propto G(\mathbf{x}_0, \mathbf{x}_1) G(\mathbf{x}_1, \mathbf{x}_2) G(\mathbf{x}_2, \mathbf{x}_3)$$

# JOINT PATH SAMPLING



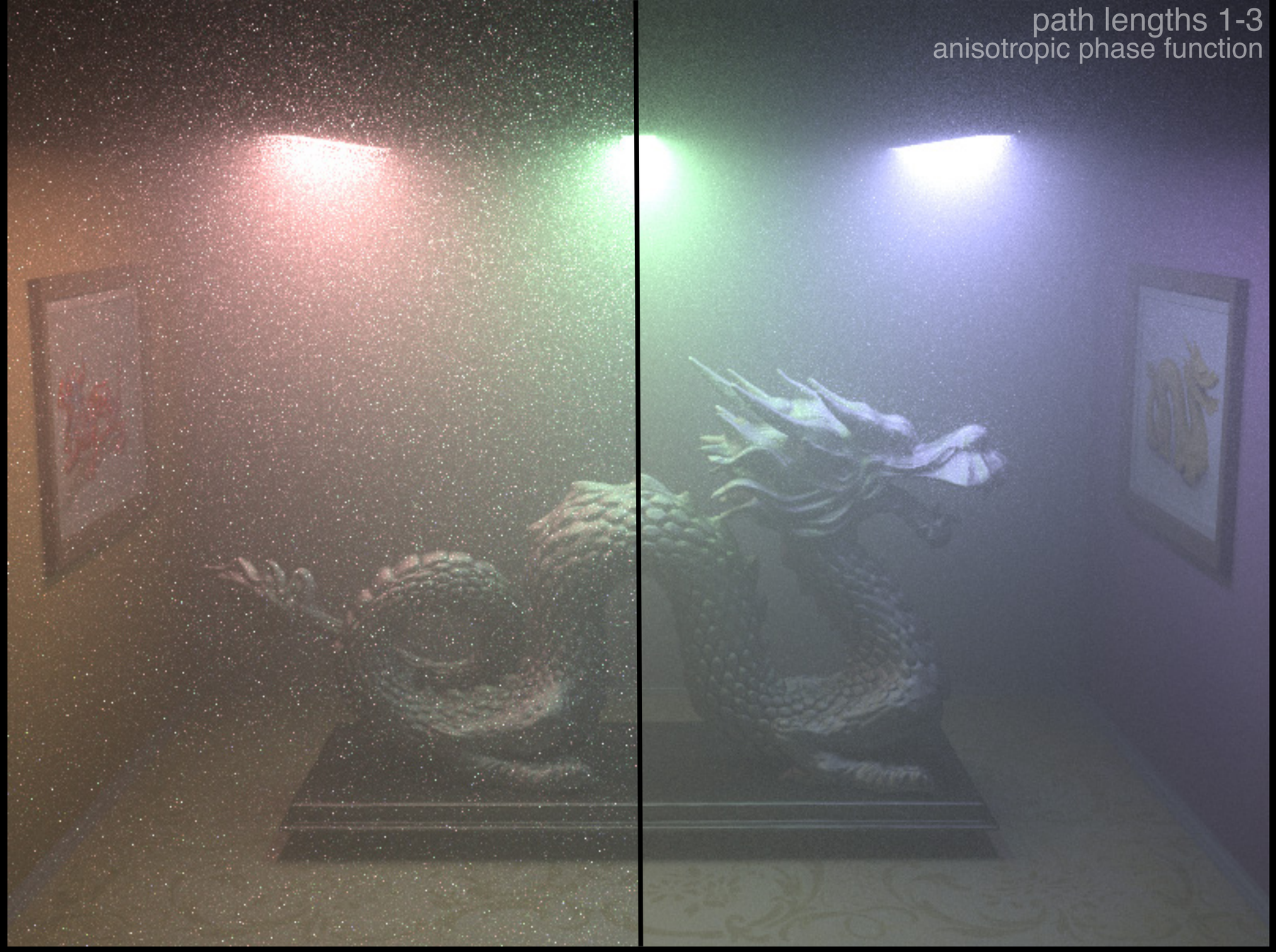

joint pdf

$$p(\mathbf{x}_1, \mathbf{x}_2) \propto G(\mathbf{x}_0, \mathbf{x}_1) G(\mathbf{x}_1, \mathbf{x}_2) G(\mathbf{x}_2, \mathbf{x}_3) f_s(\mathbf{x}_1) f_s(\mathbf{x}_2)$$

via tabulation

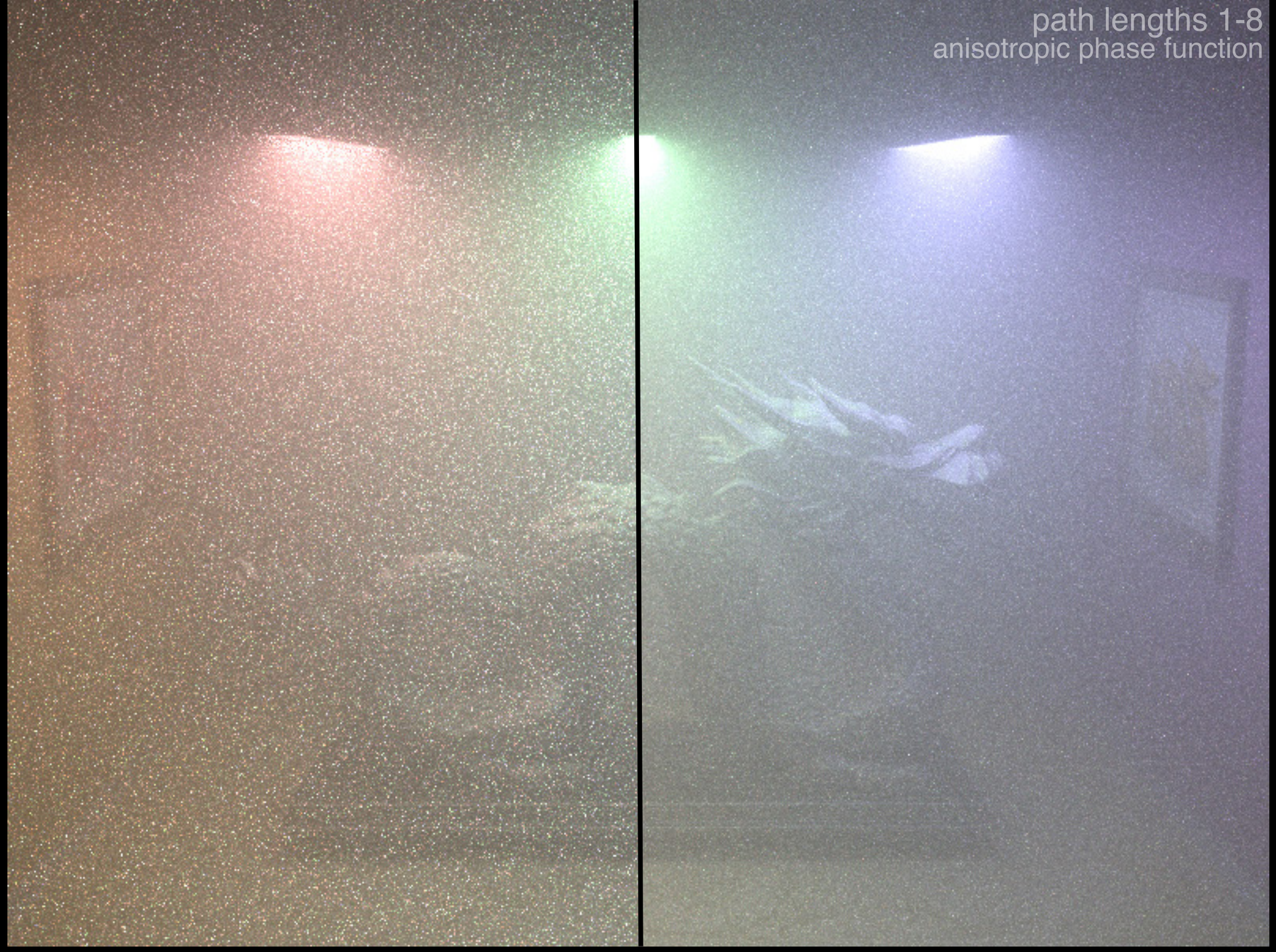
Transmittance | Equiangular | **Joint sampling**

Transmittance Equiangular **Joint sampling**




Transmittance connections

**Joint tabulated path sampling**

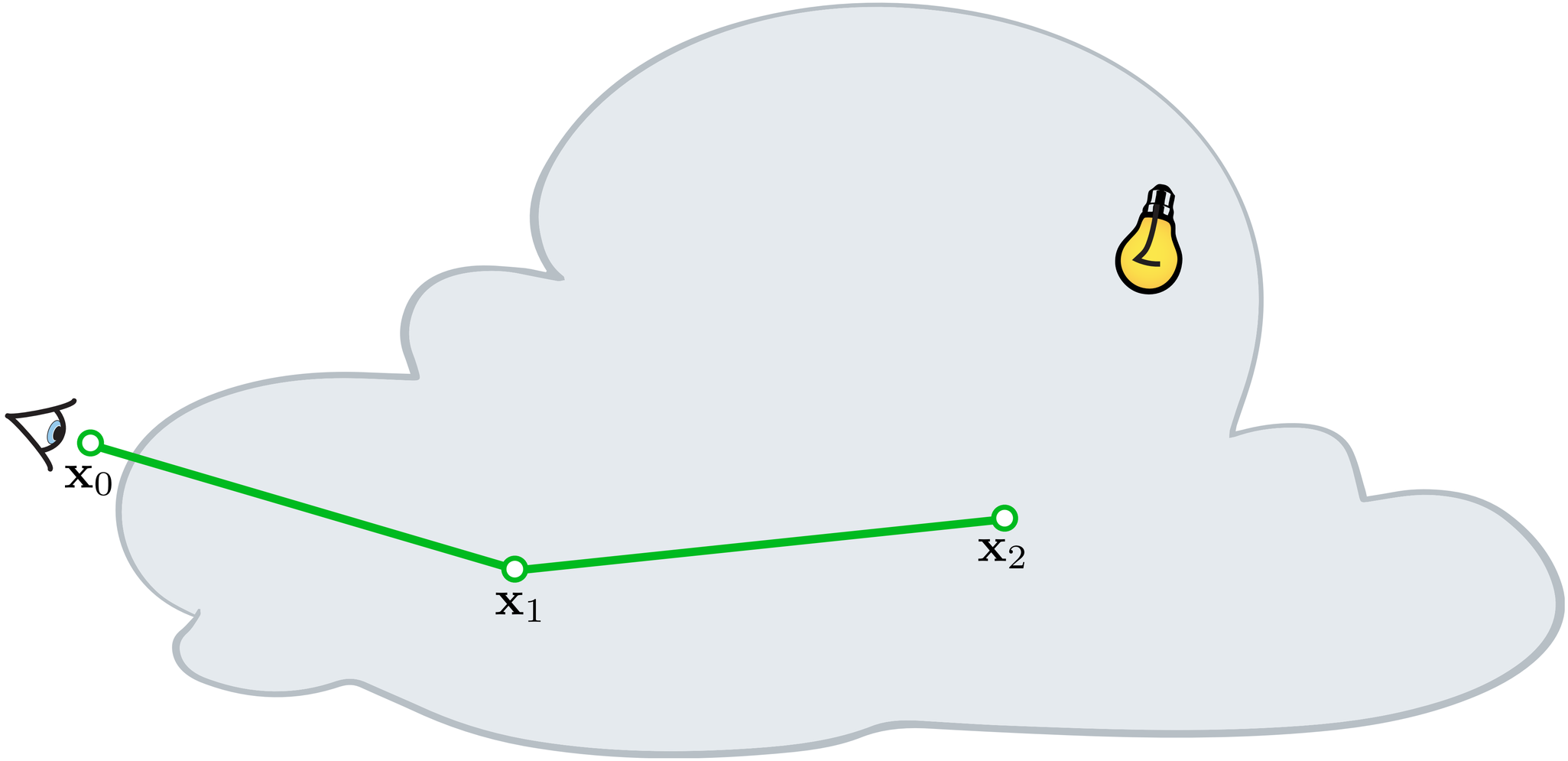



Transmittance connections

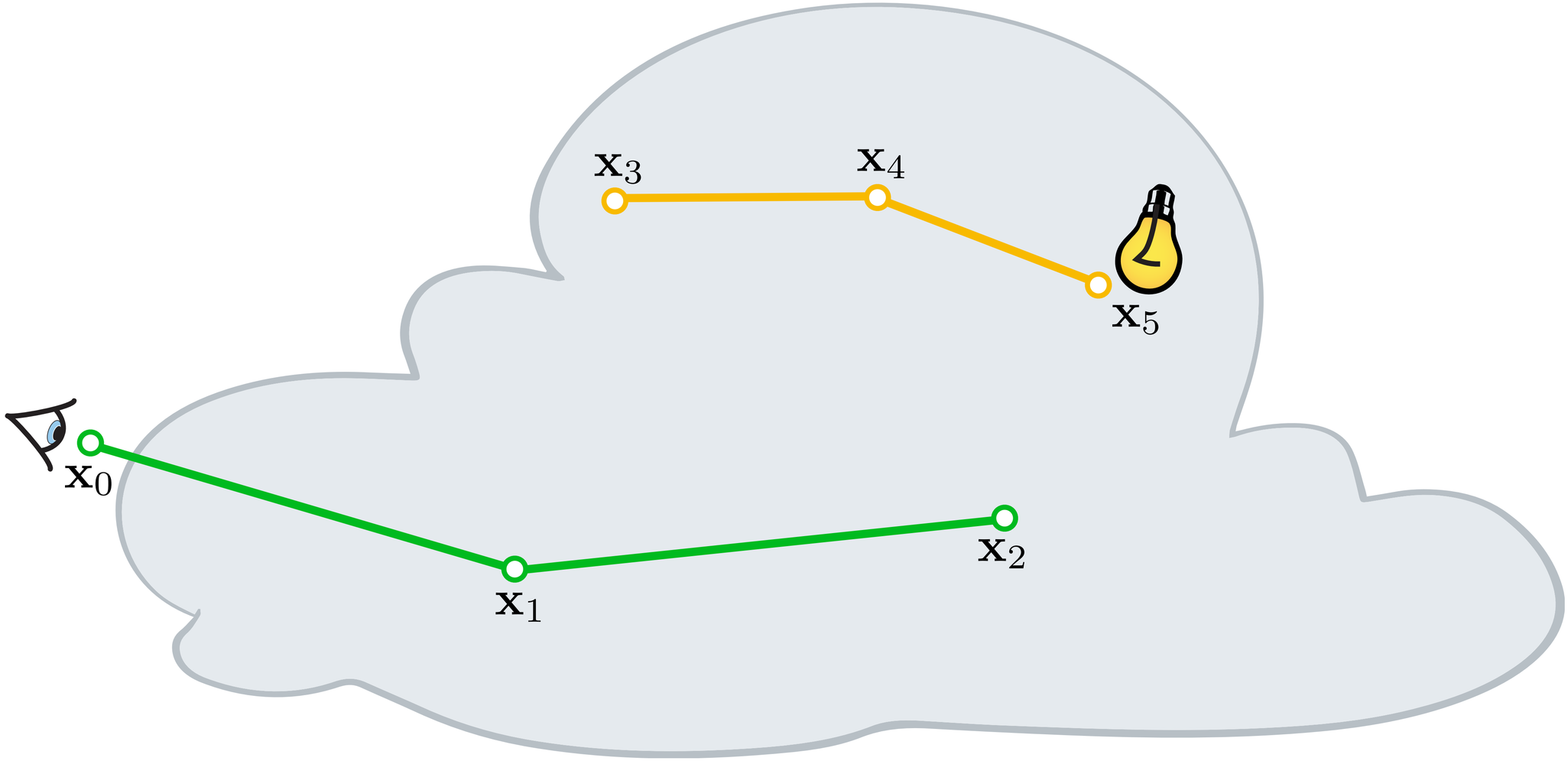
**Joint tabulated path sampling**

# BIDIRECTIONAL PATH SAMPLING

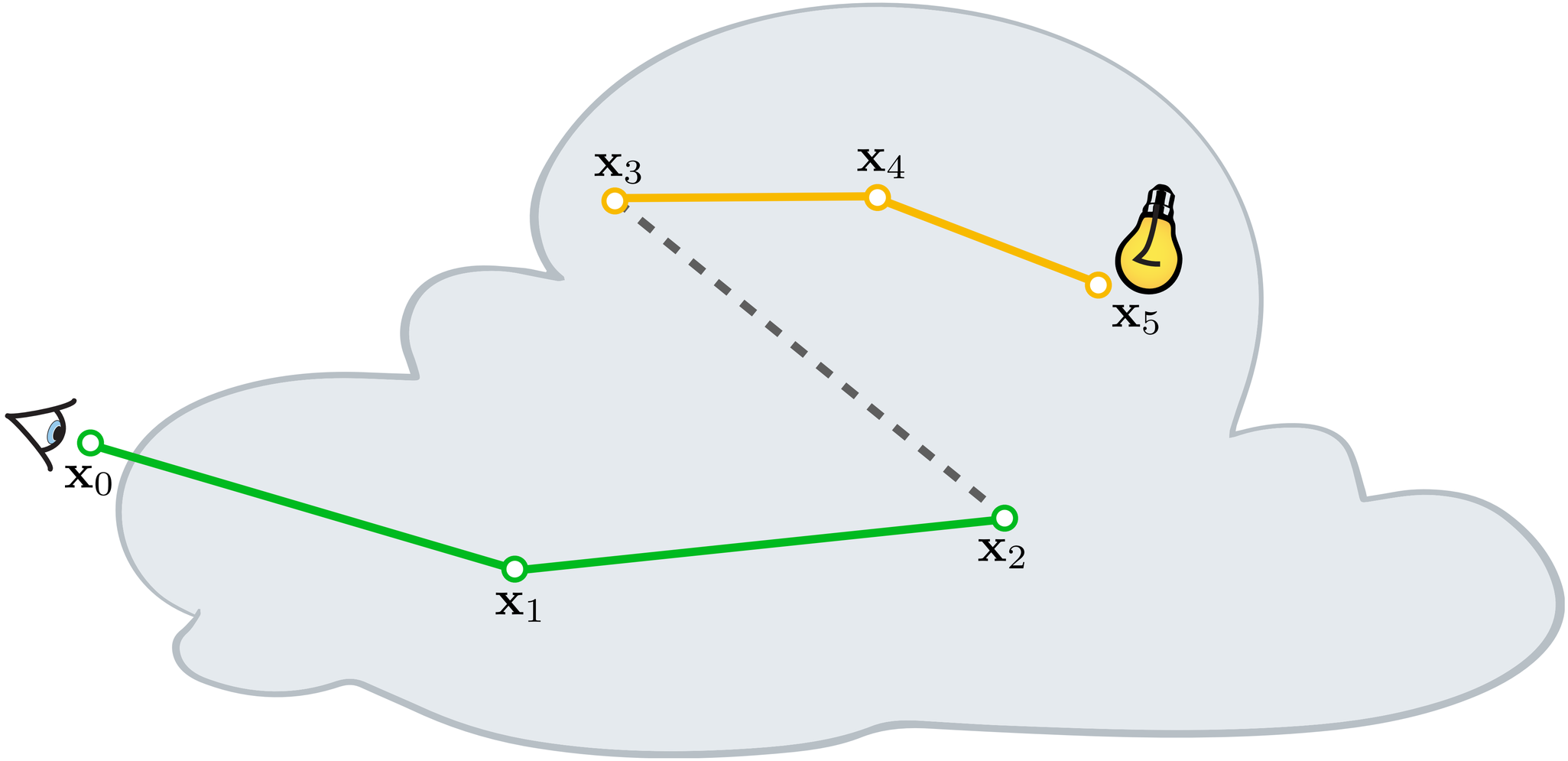
# BIDIRECTIONAL PATH SAMPLING



# BIDIRECTIONAL PATH SAMPLING



# BIDIRECTIONAL PATH SAMPLING



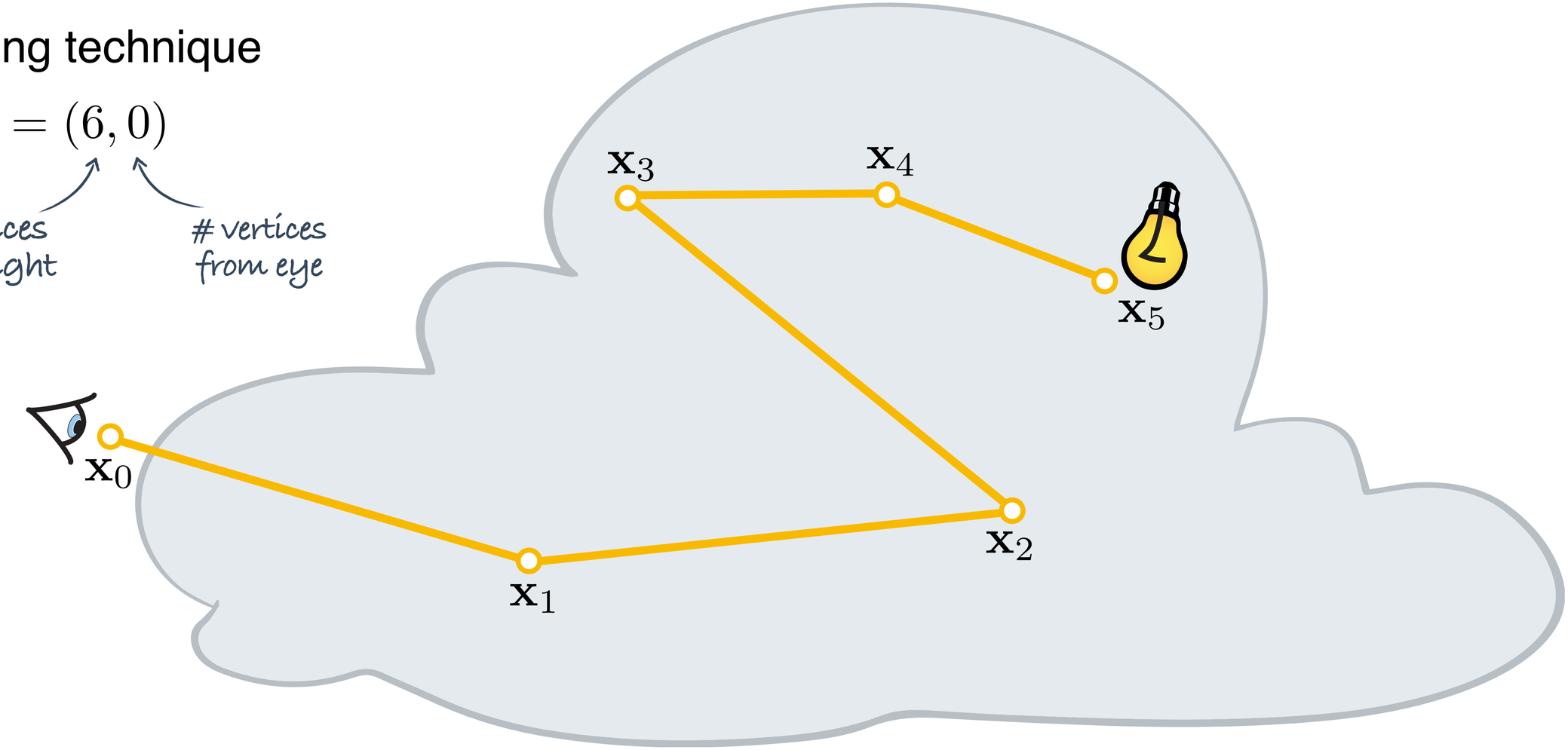
# BIDIRECTIONAL PATH SAMPLING

Sampling technique

$$(s, t) = (6, 0)$$

# vertices from light

# vertices from eye



# BIDIRECTIONAL PATH SAMPLING

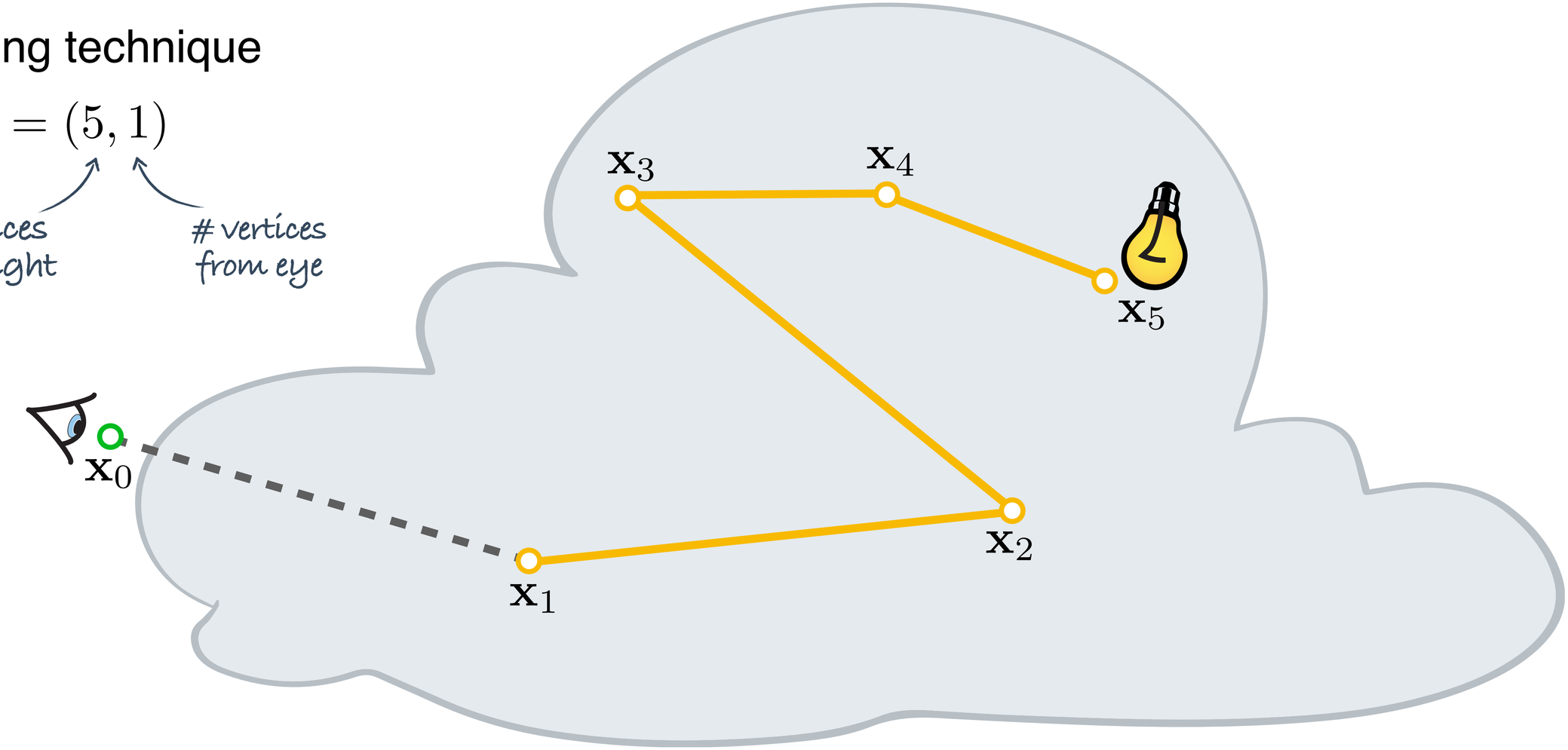


Sampling technique

$$(s, t) = (5, 1)$$

# vertices from light

# vertices from eye



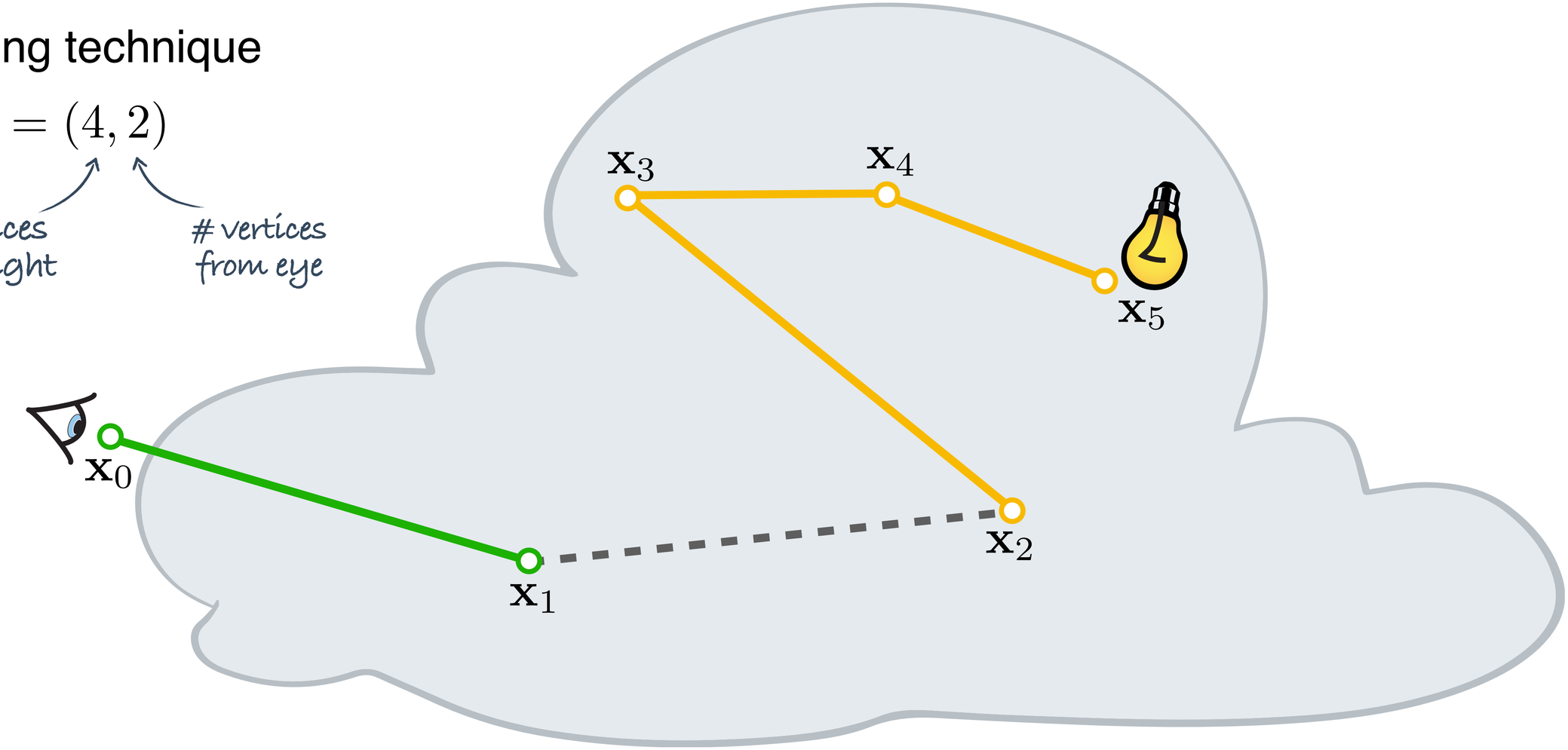
# BIDIRECTIONAL PATH SAMPLING

Sampling technique

$$(s, t) = (4, 2)$$

# vertices from light

# vertices from eye



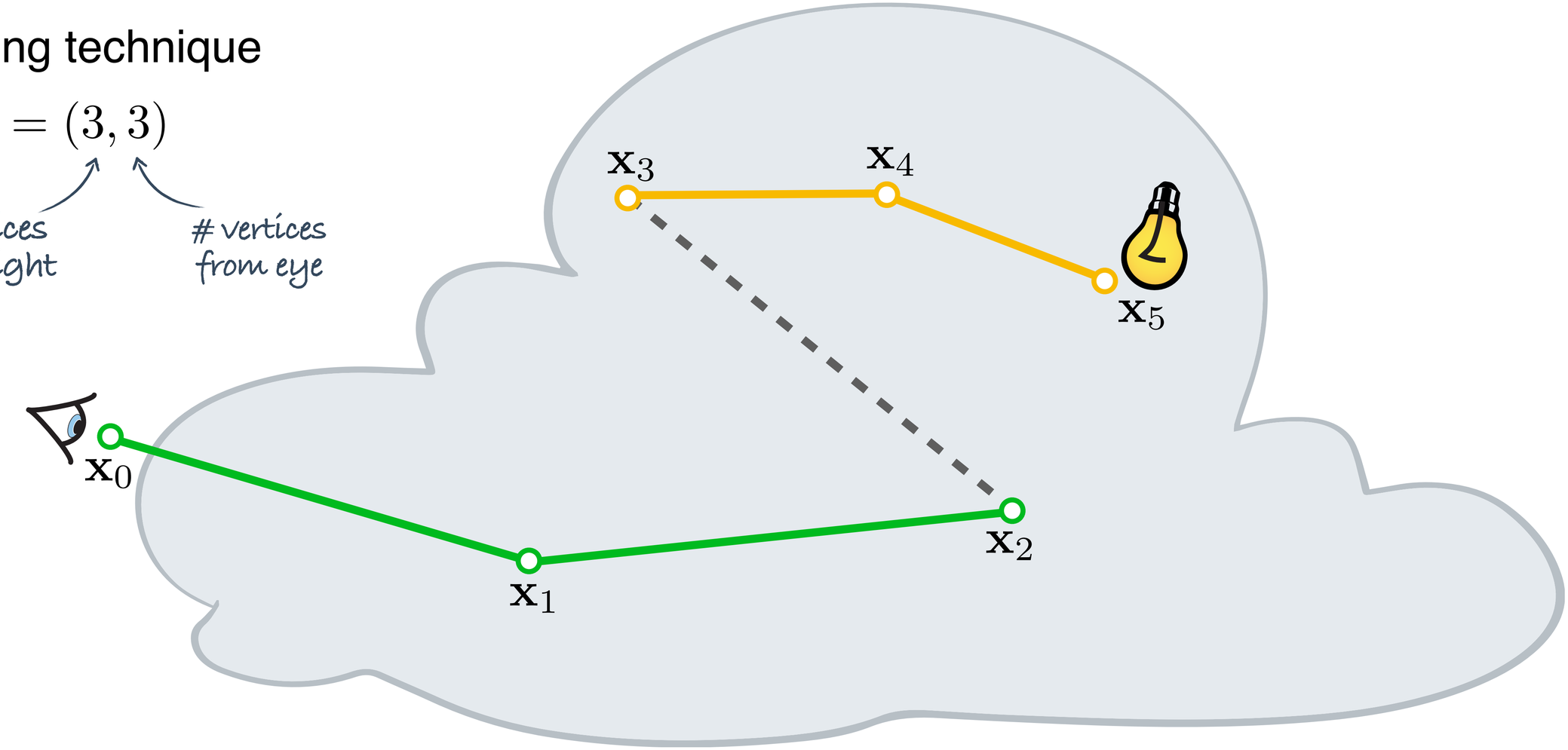
# BIDIRECTIONAL PATH SAMPLING

Sampling technique

$$(s, t) = (3, 3)$$

# vertices from light

# vertices from eye



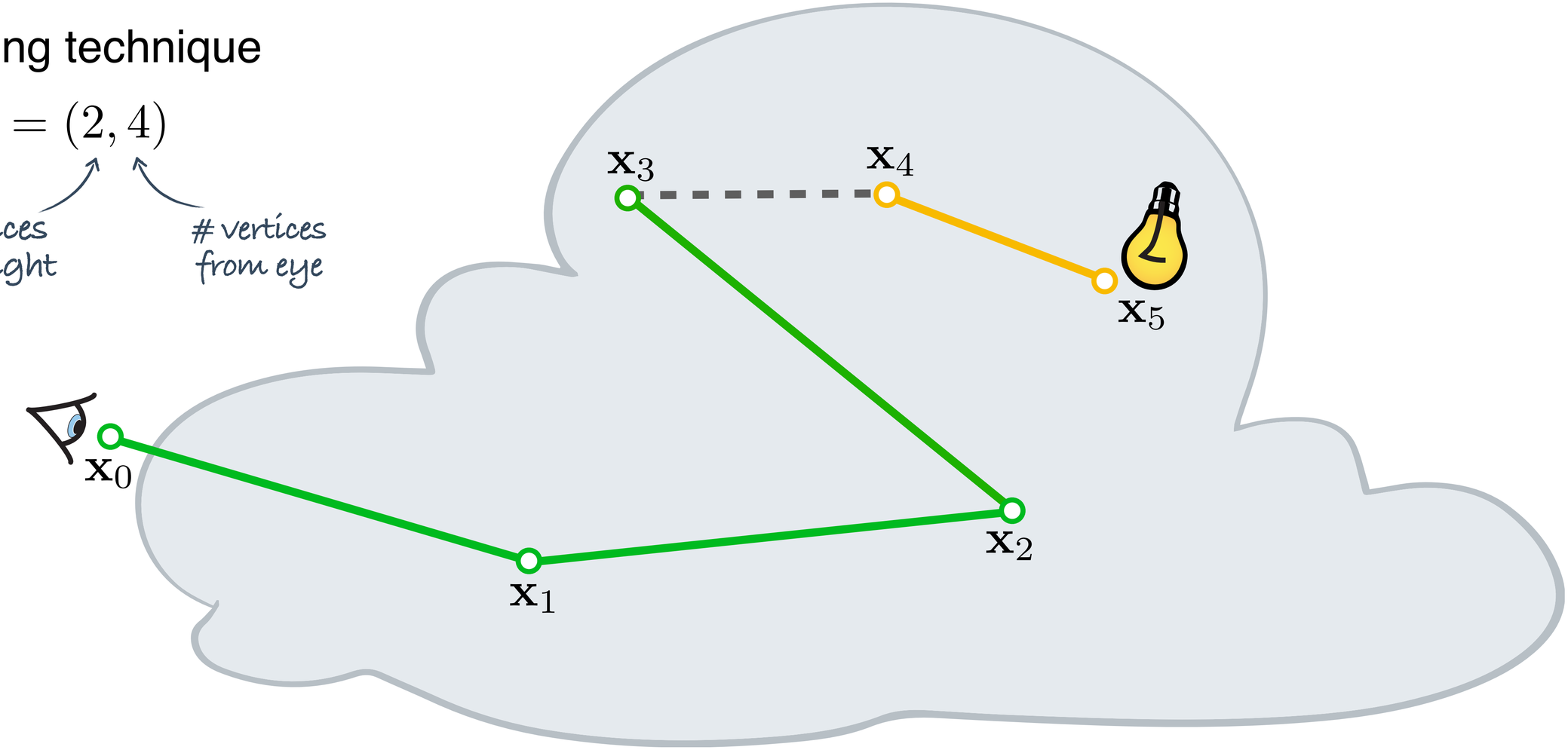
# BIDIRECTIONAL PATH SAMPLING

Sampling technique

$$(s, t) = (2, 4)$$

# vertices from light

# vertices from eye



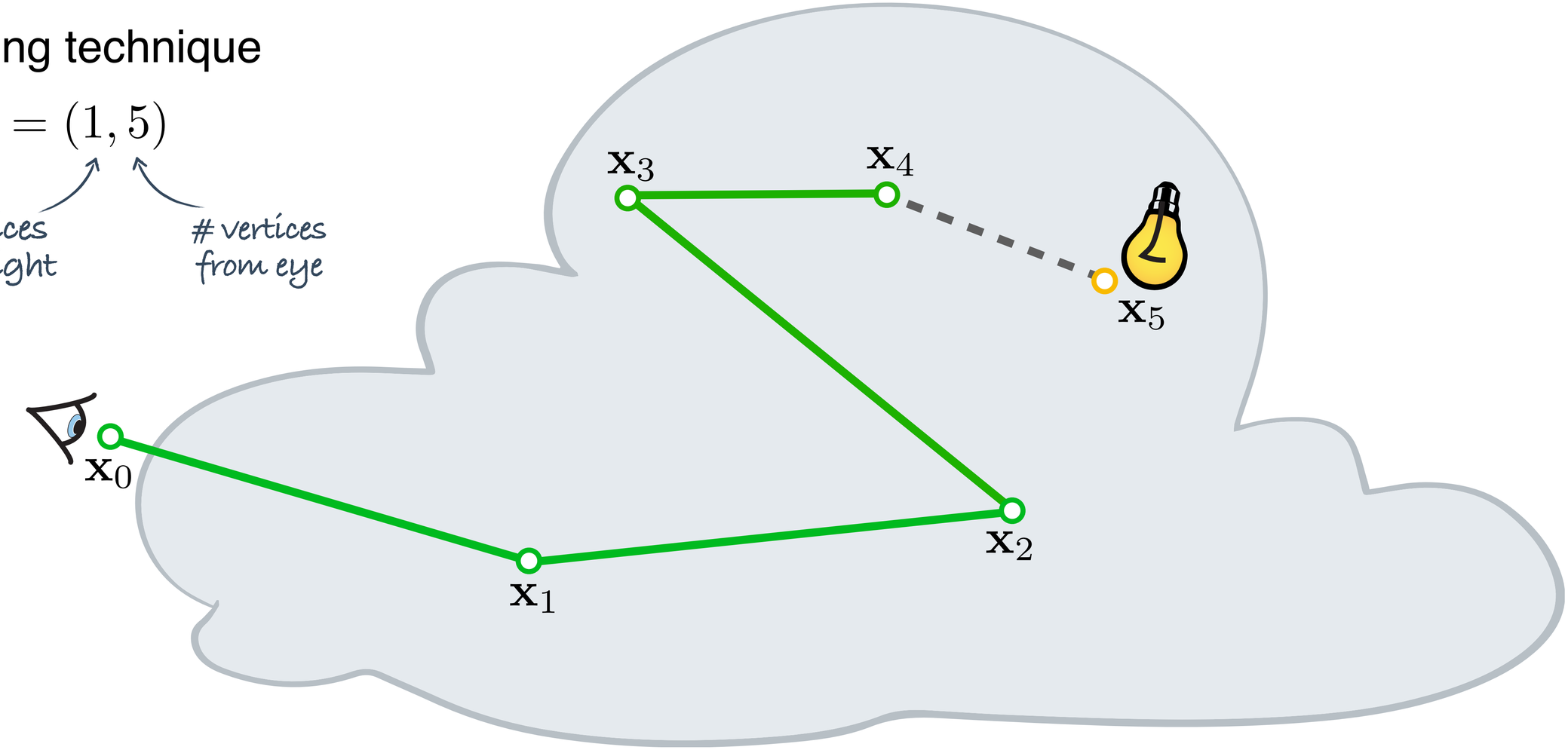
# BIDIRECTIONAL PATH SAMPLING

Sampling technique

$$(s, t) = (1, 5)$$

# vertices from light

# vertices from eye



# BIDIRECTIONAL PATH SAMPLING

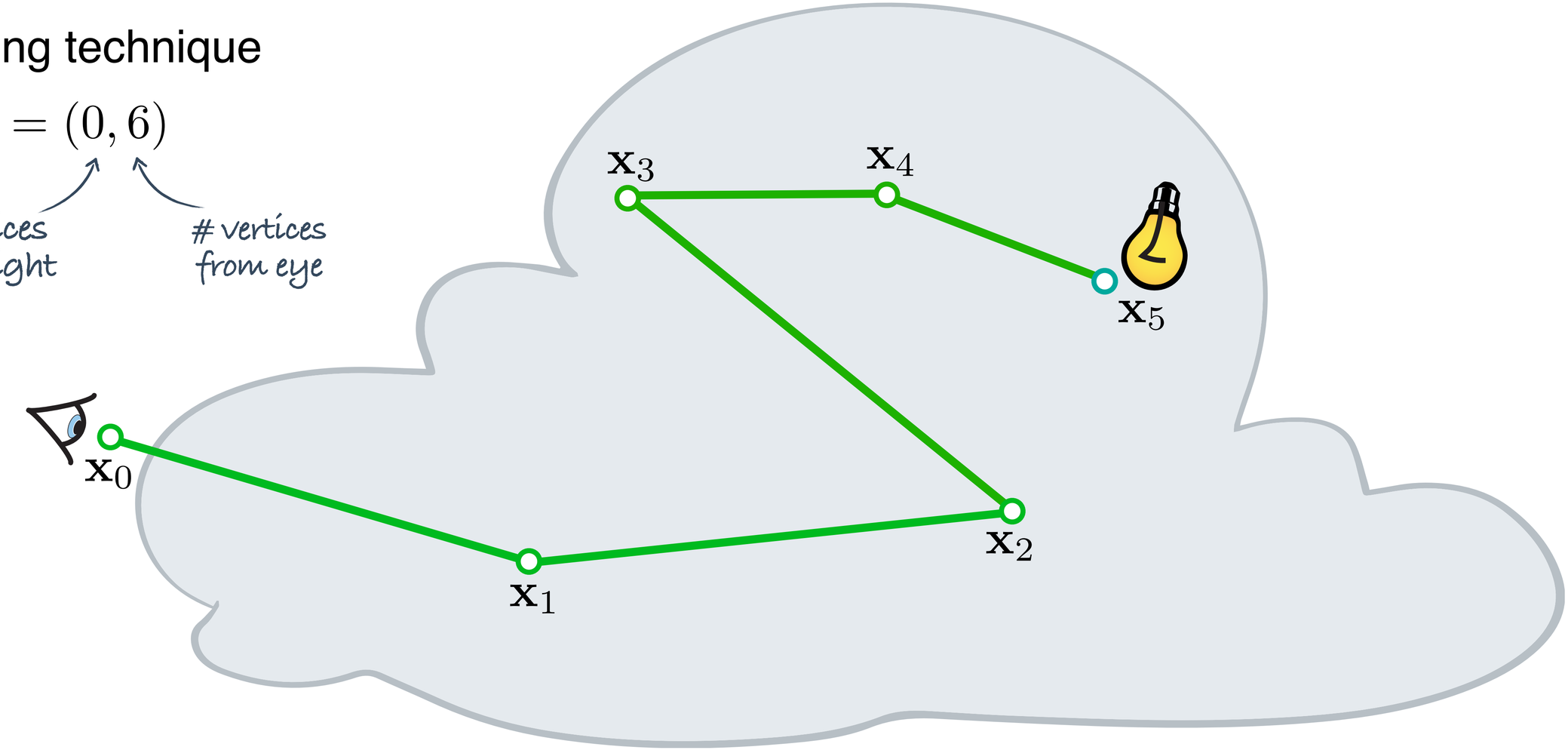


Sampling technique

$$(s, t) = (0, 6)$$

# vertices from light

# vertices from eye



# SINGULARITY

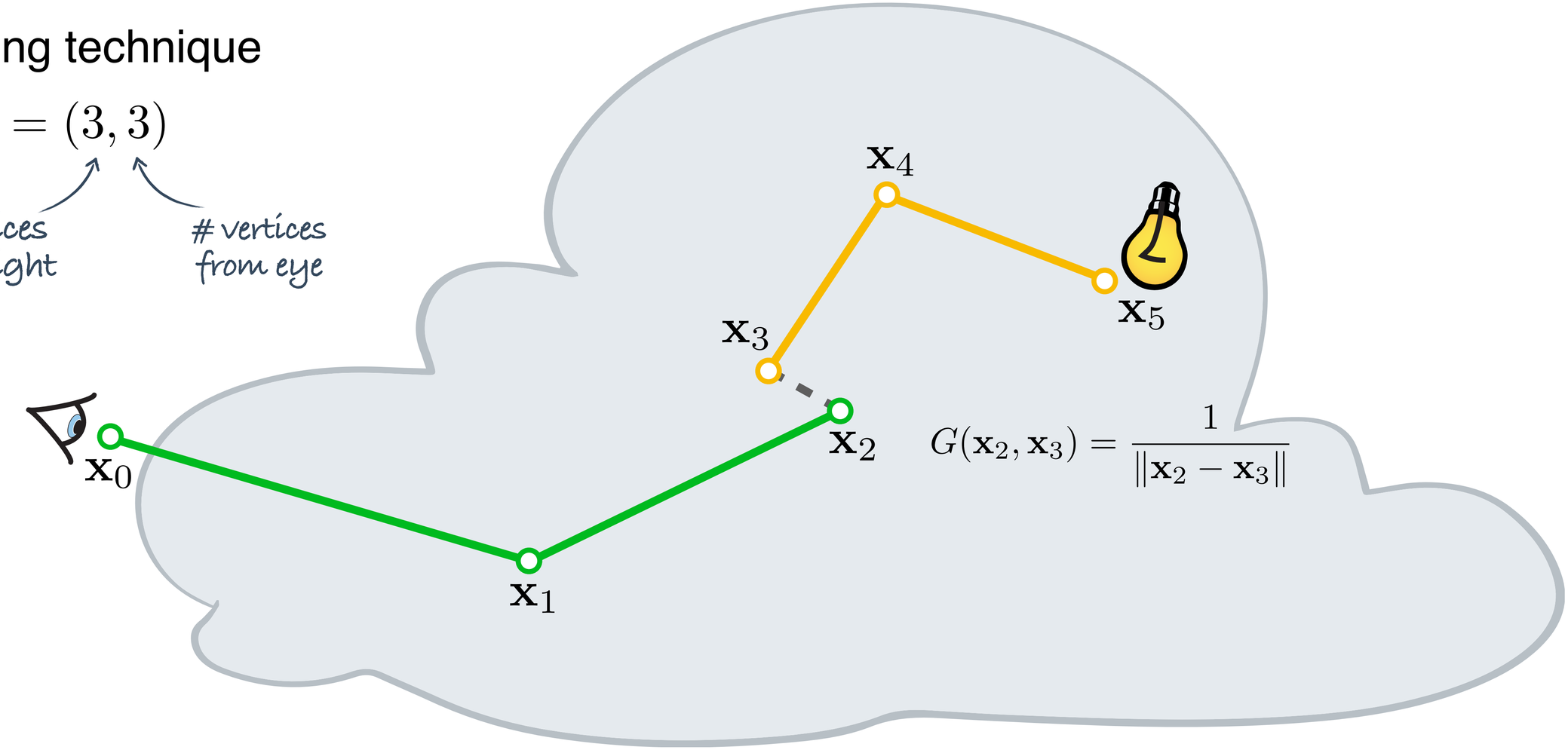


Sampling technique

$$(s, t) = (3, 3)$$

# vertices from light

# vertices from eye



# SINGULARITY

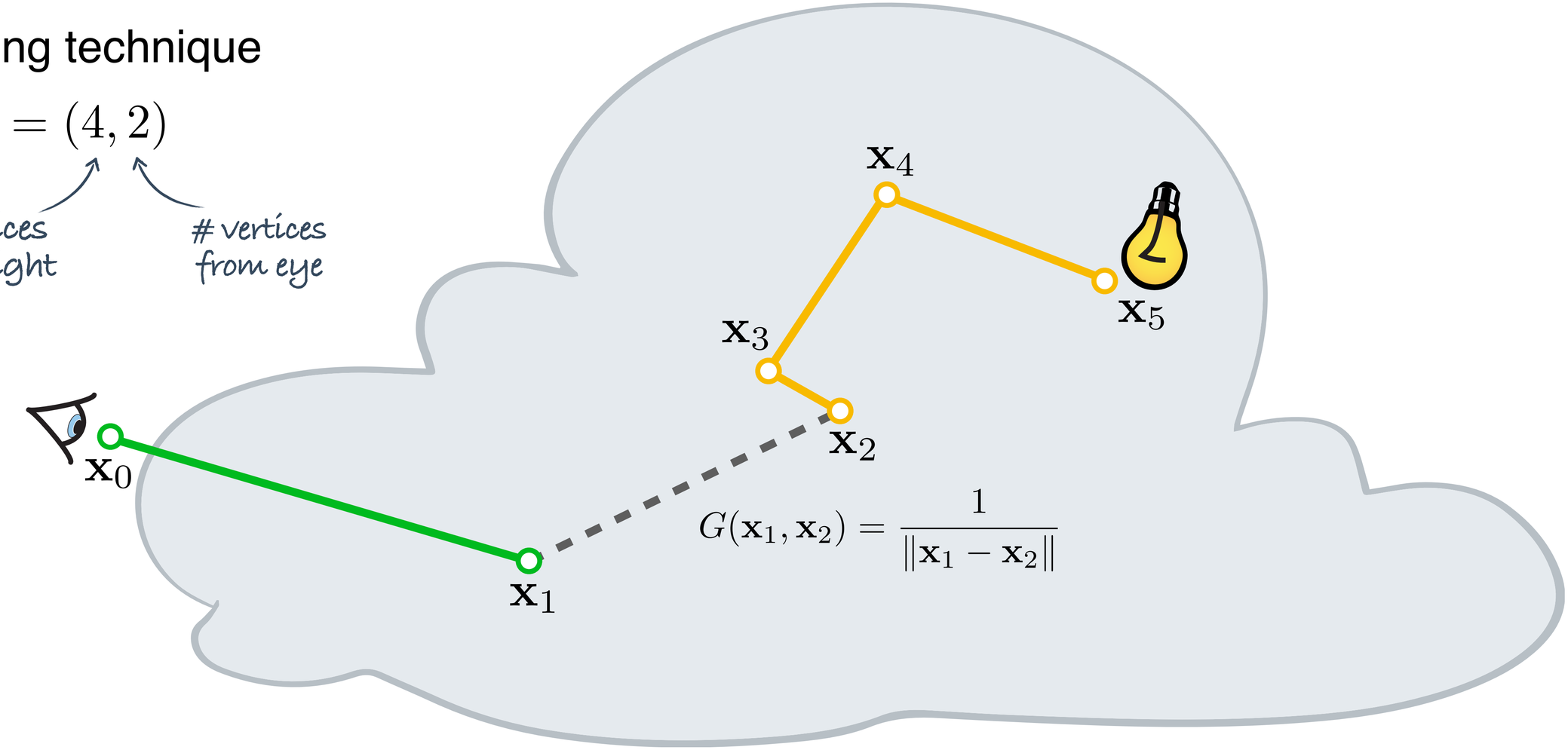


## Sampling technique

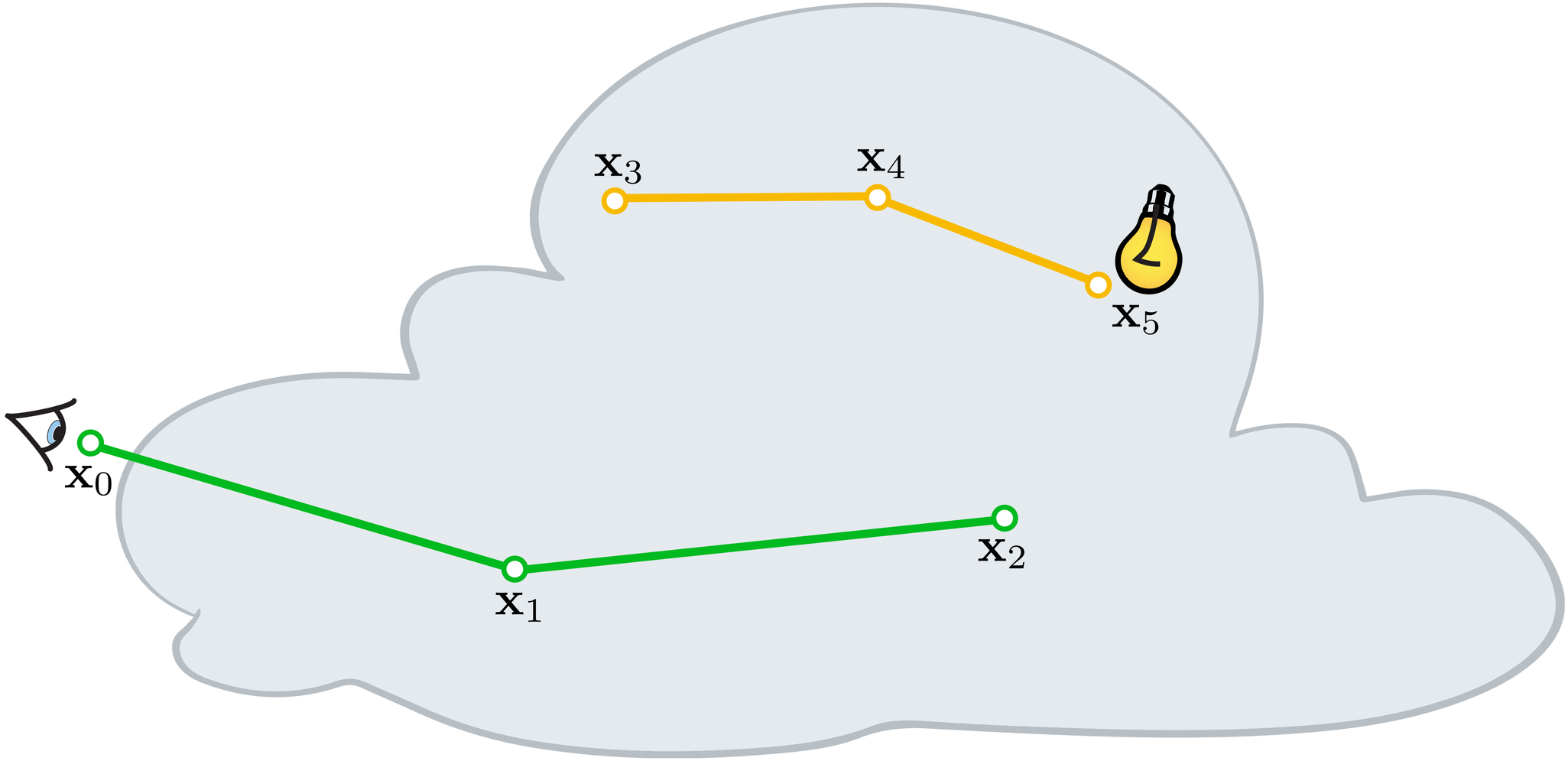
$$(s, t) = (4, 2)$$

# vertices from light

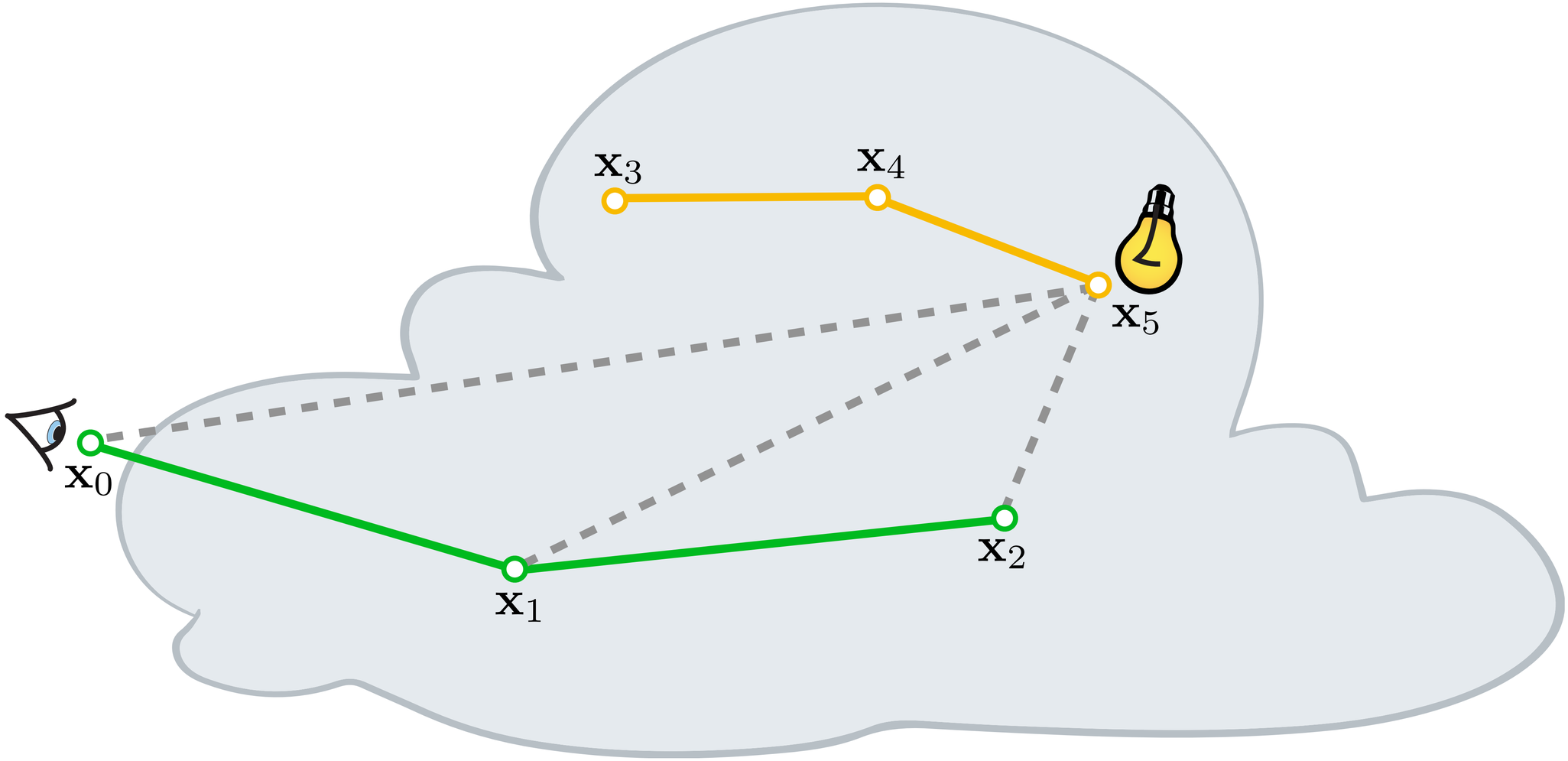
# vertices from eye



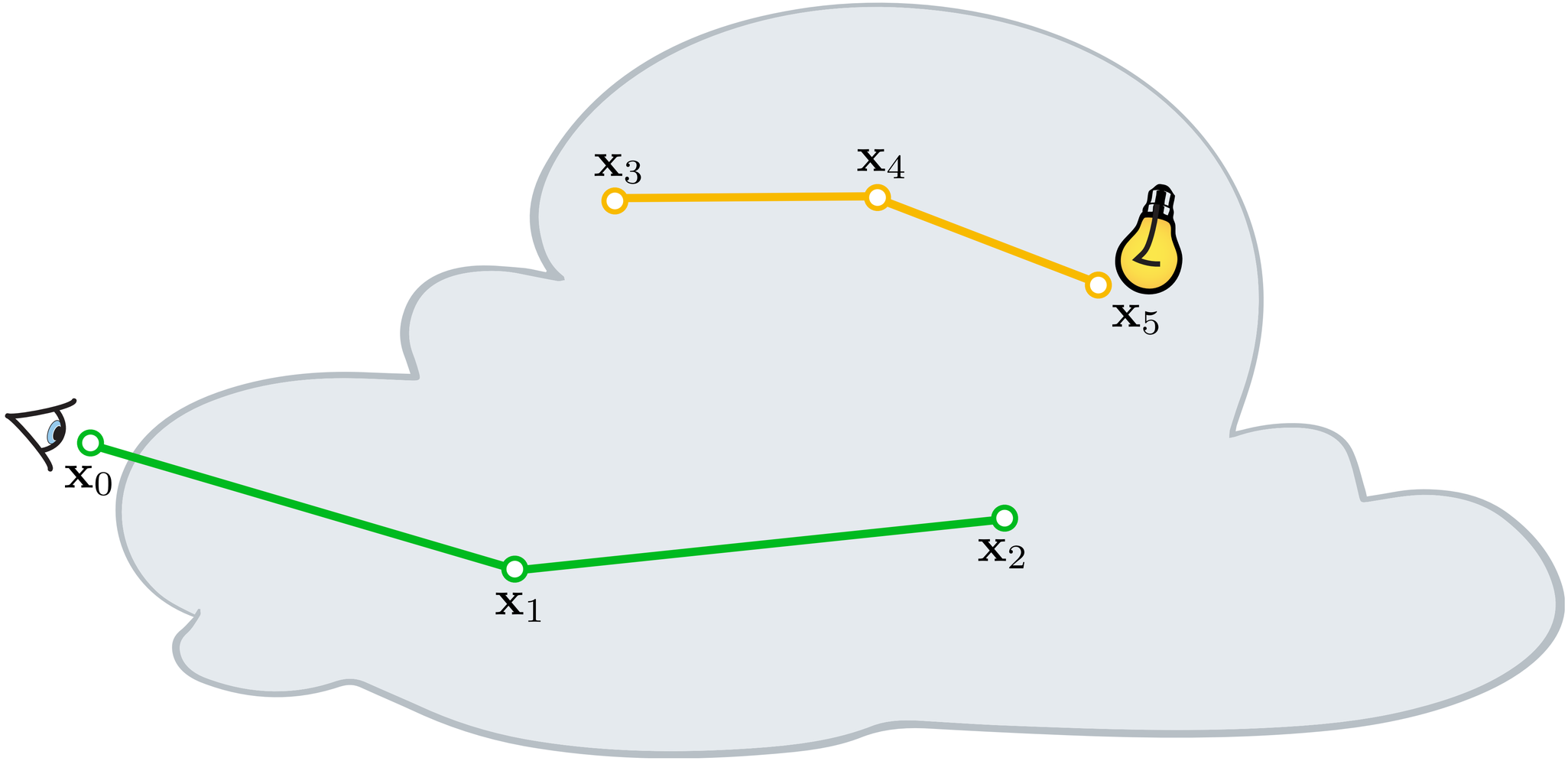
# BIDIRECTIONAL PATH TRACING



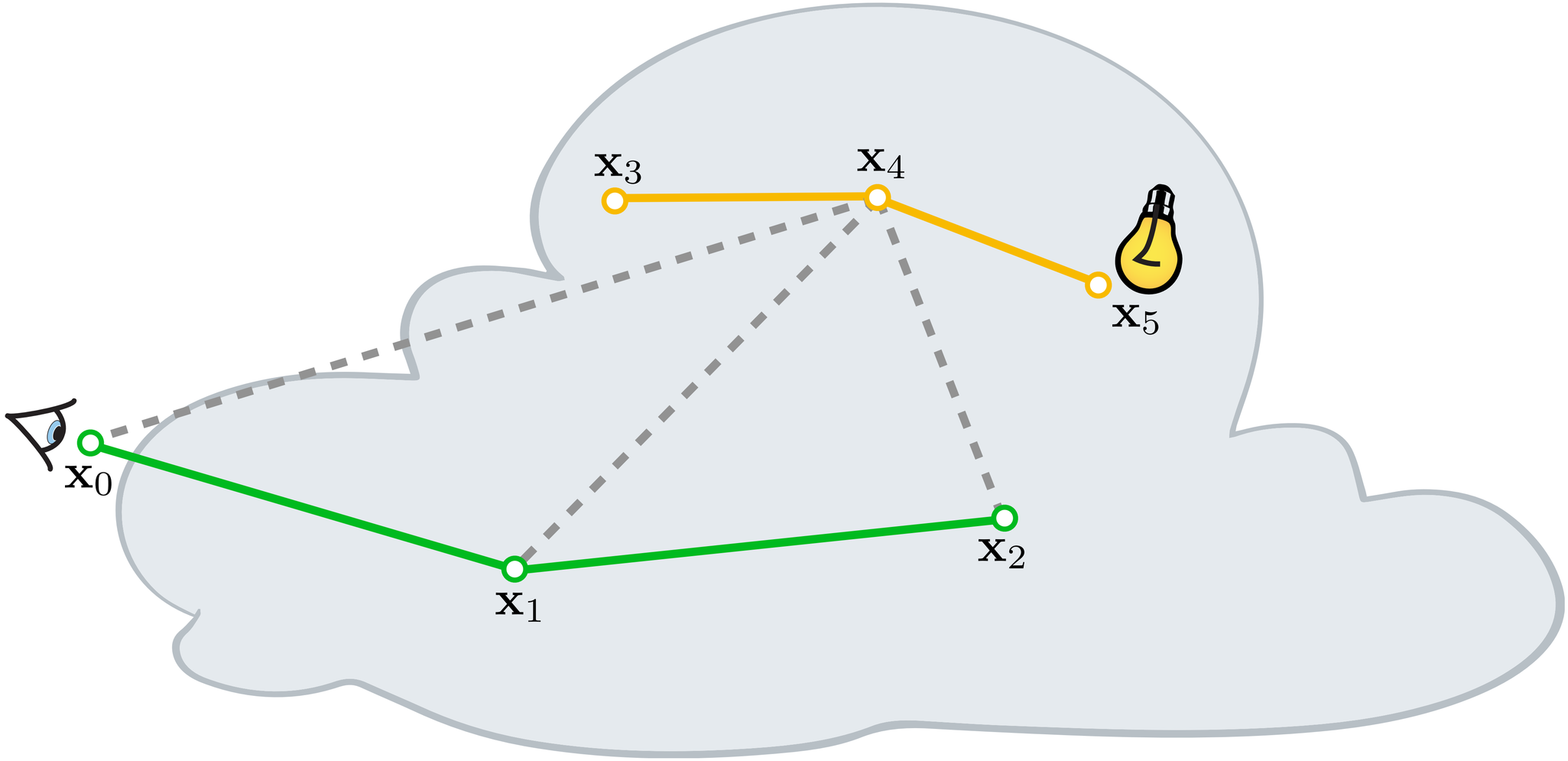
# BIDIRECTIONAL PATH TRACING



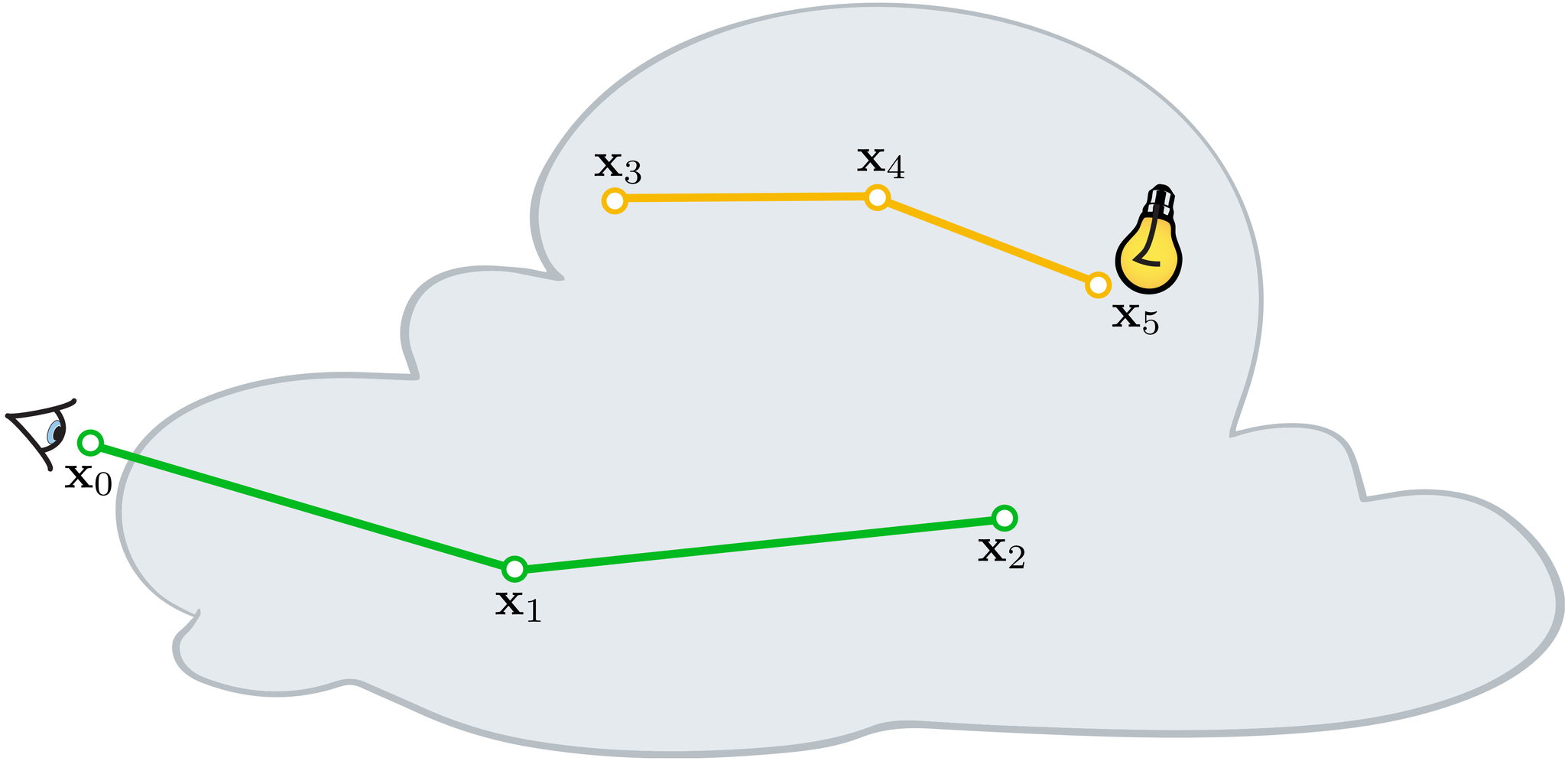
# BIDIRECTIONAL PATH TRACING



# BIDIRECTIONAL PATH TRACING



# BIDIRECTIONAL PATH TRACING



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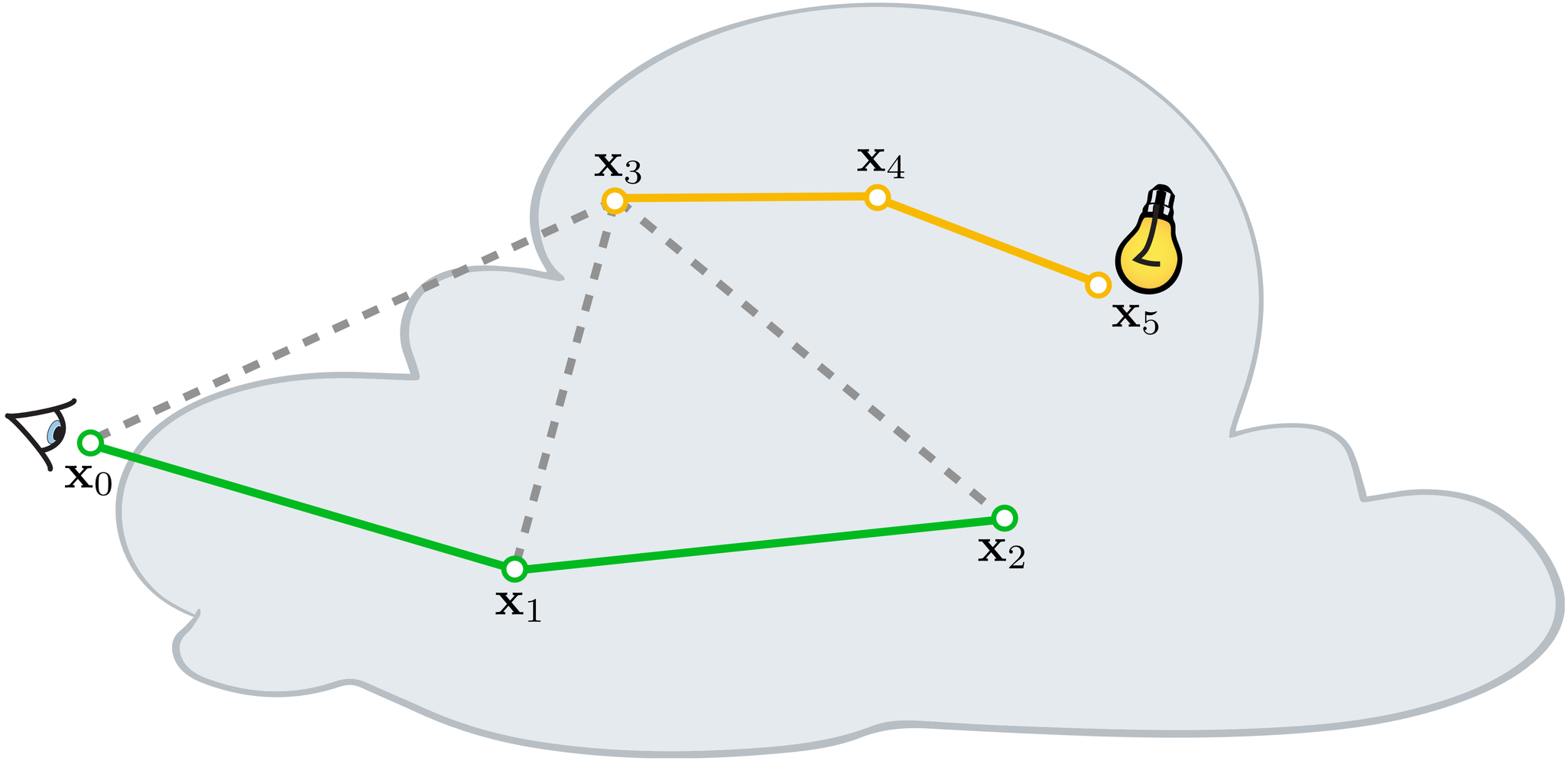

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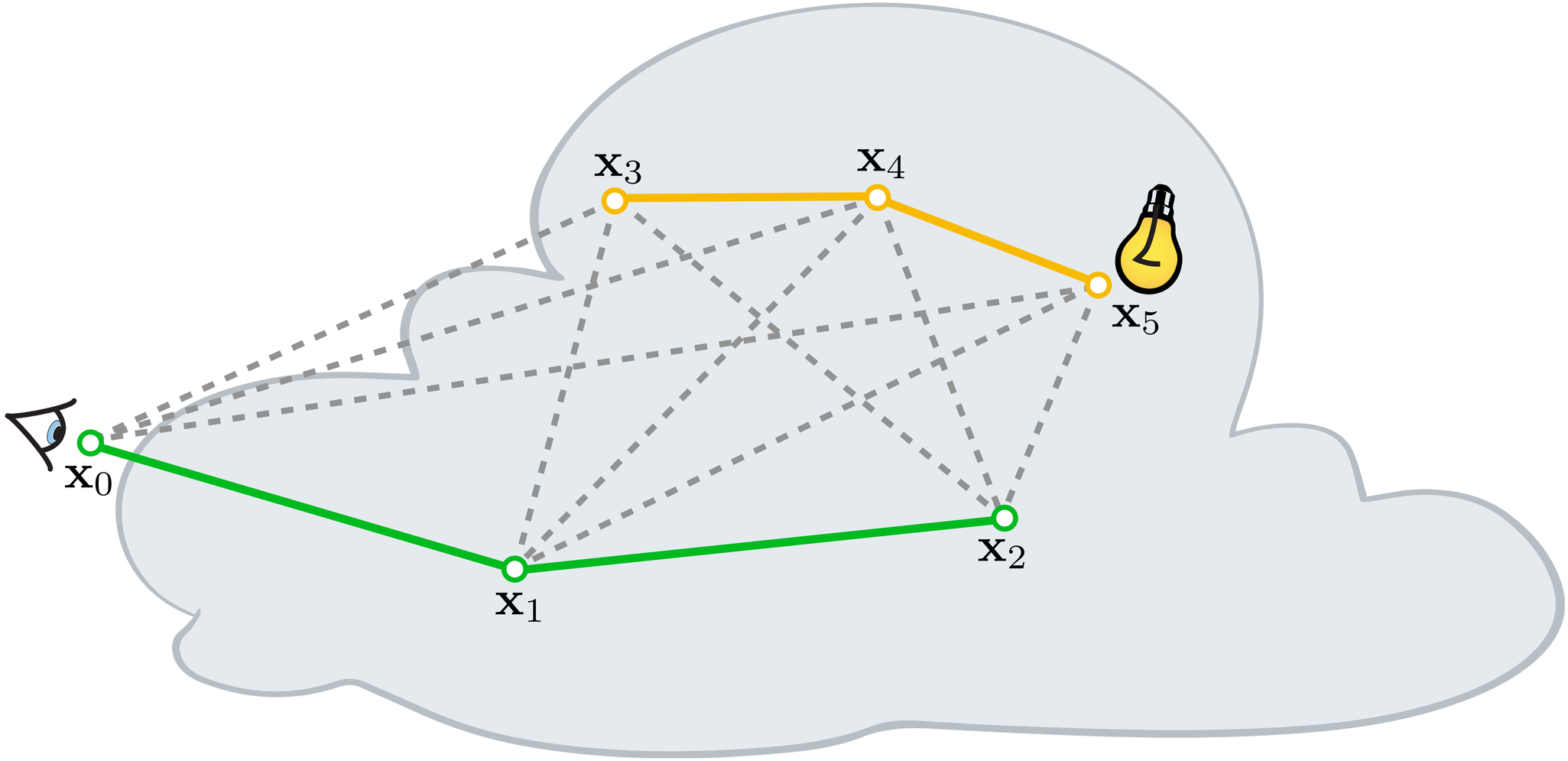


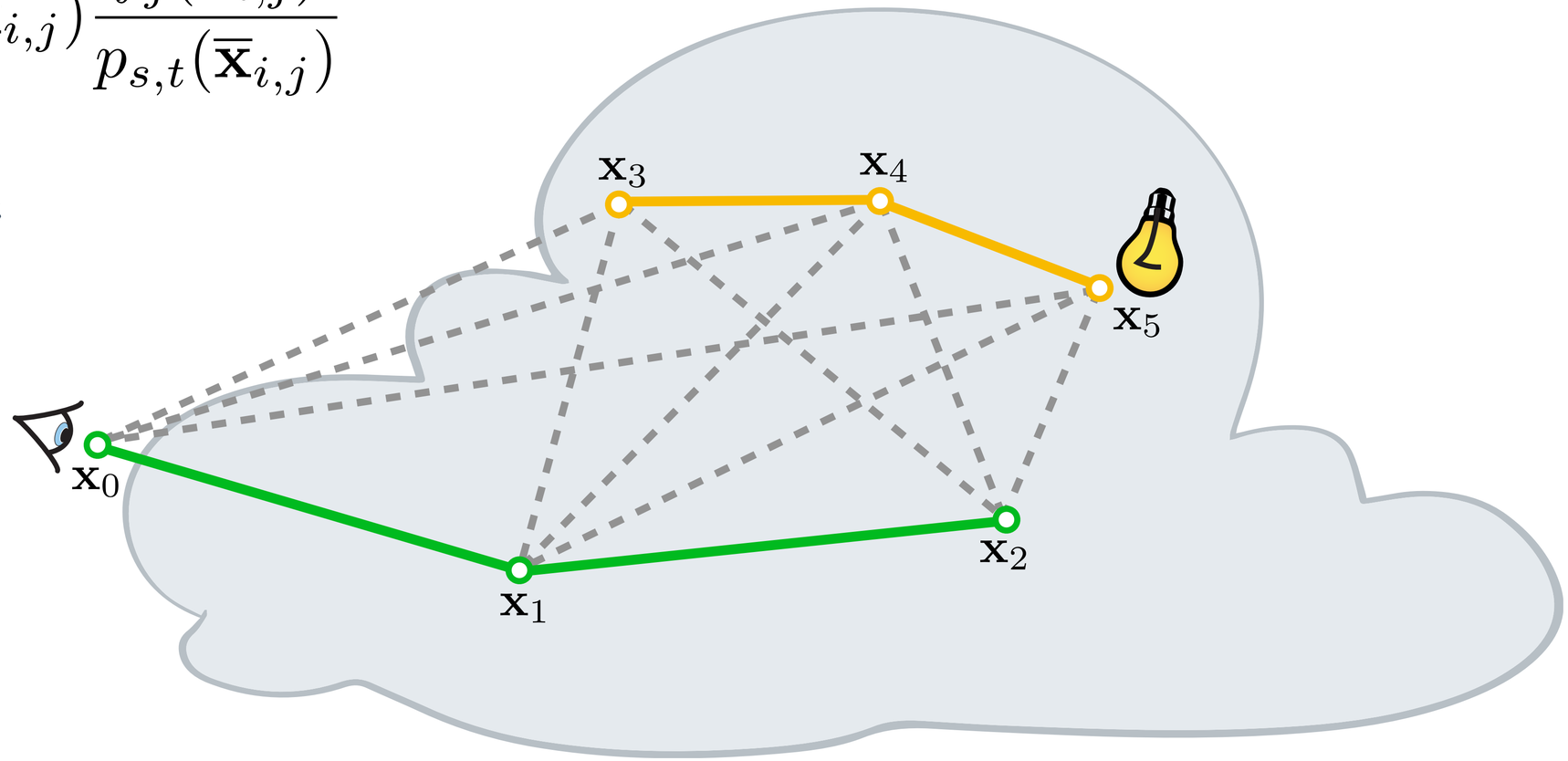
# BIDIRECTIONAL PATH TRACING

Combined MIS pixel estimator:

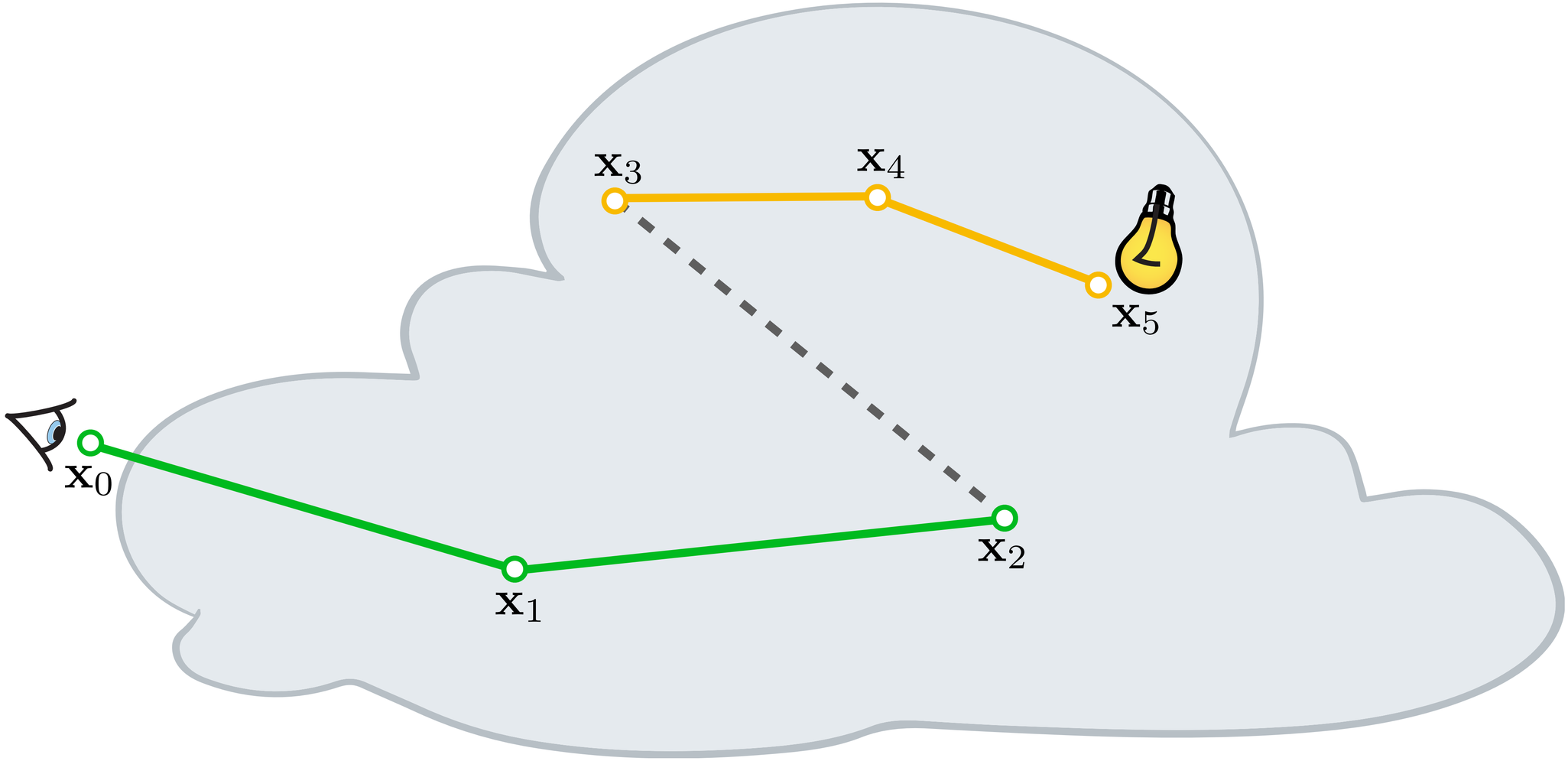
$$\langle I_j \rangle = \sum_s \sum_t w_{s,t}(\overline{\mathbf{x}}_{i,j}) \frac{f_j(\overline{\mathbf{x}}_{i,j})}{p_{s,t}(\overline{\mathbf{x}}_{i,j})}$$

# vertices from light

# vertices from eye



# METROPOLIS LIGHT TRANSPORT



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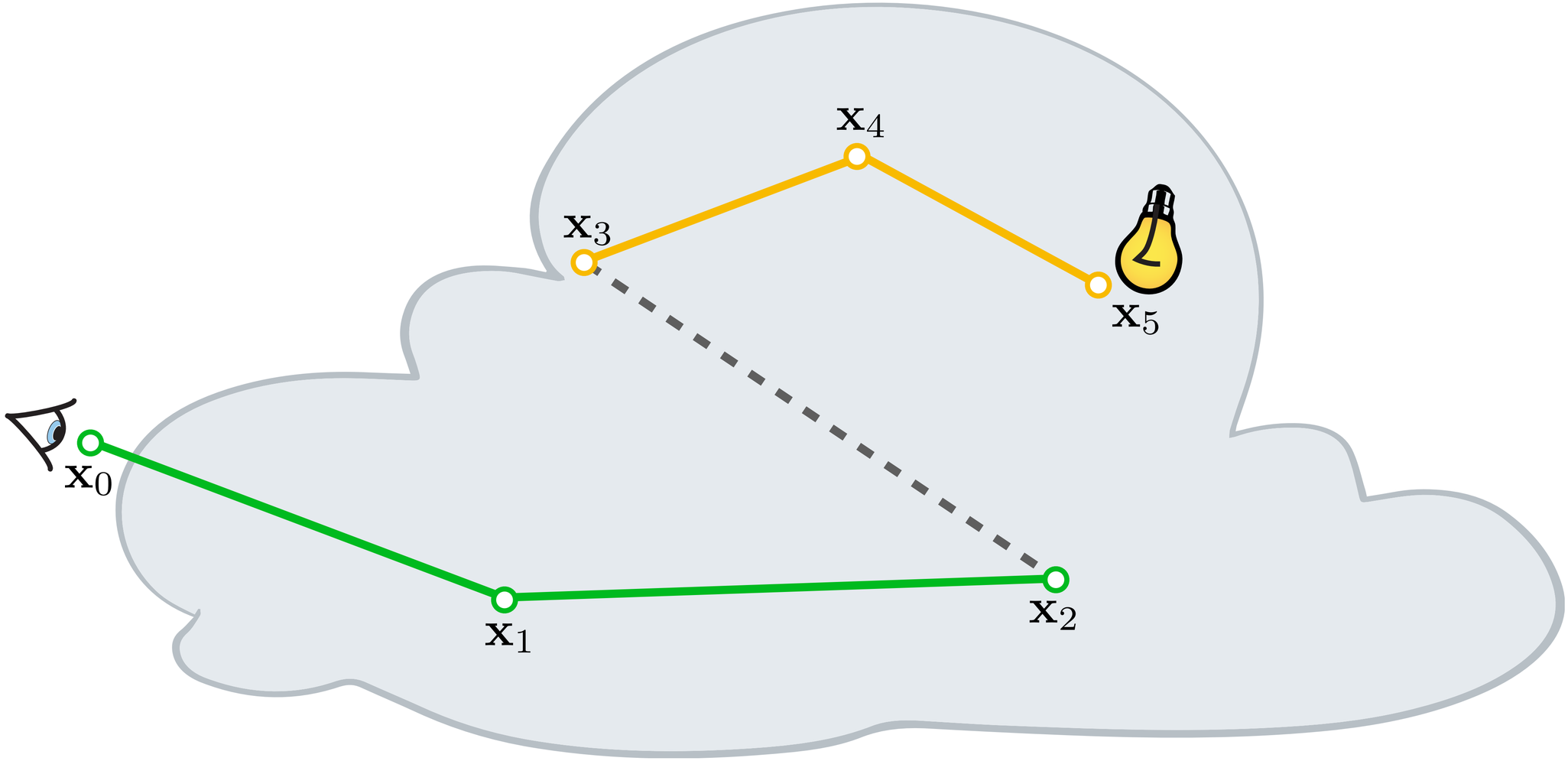

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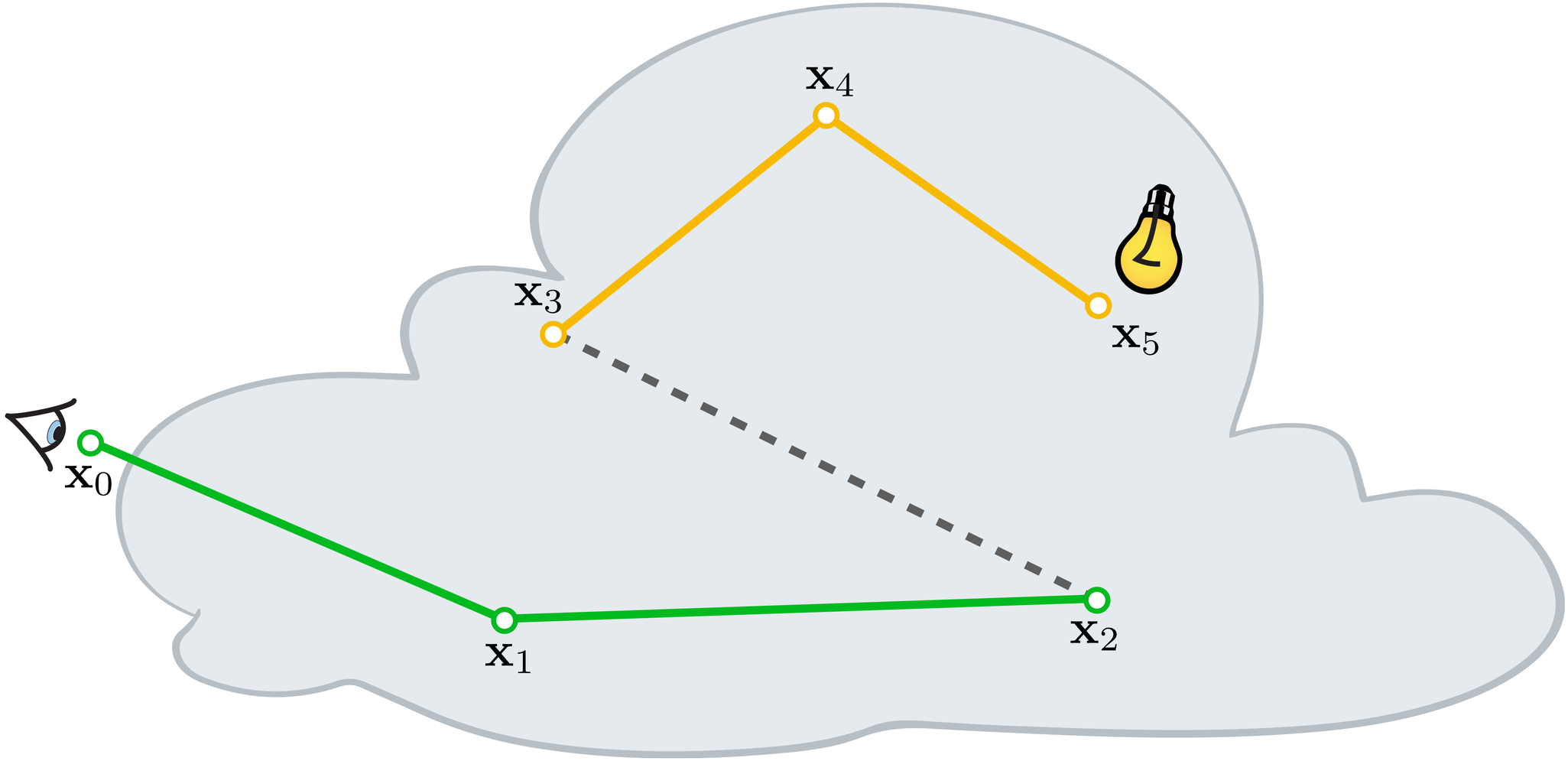

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SIGGRAPH 2018

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## METROPOLIS SAMPLING

- Apply on top, great for very difficult illumination
- (Difficult to implement)$^2$