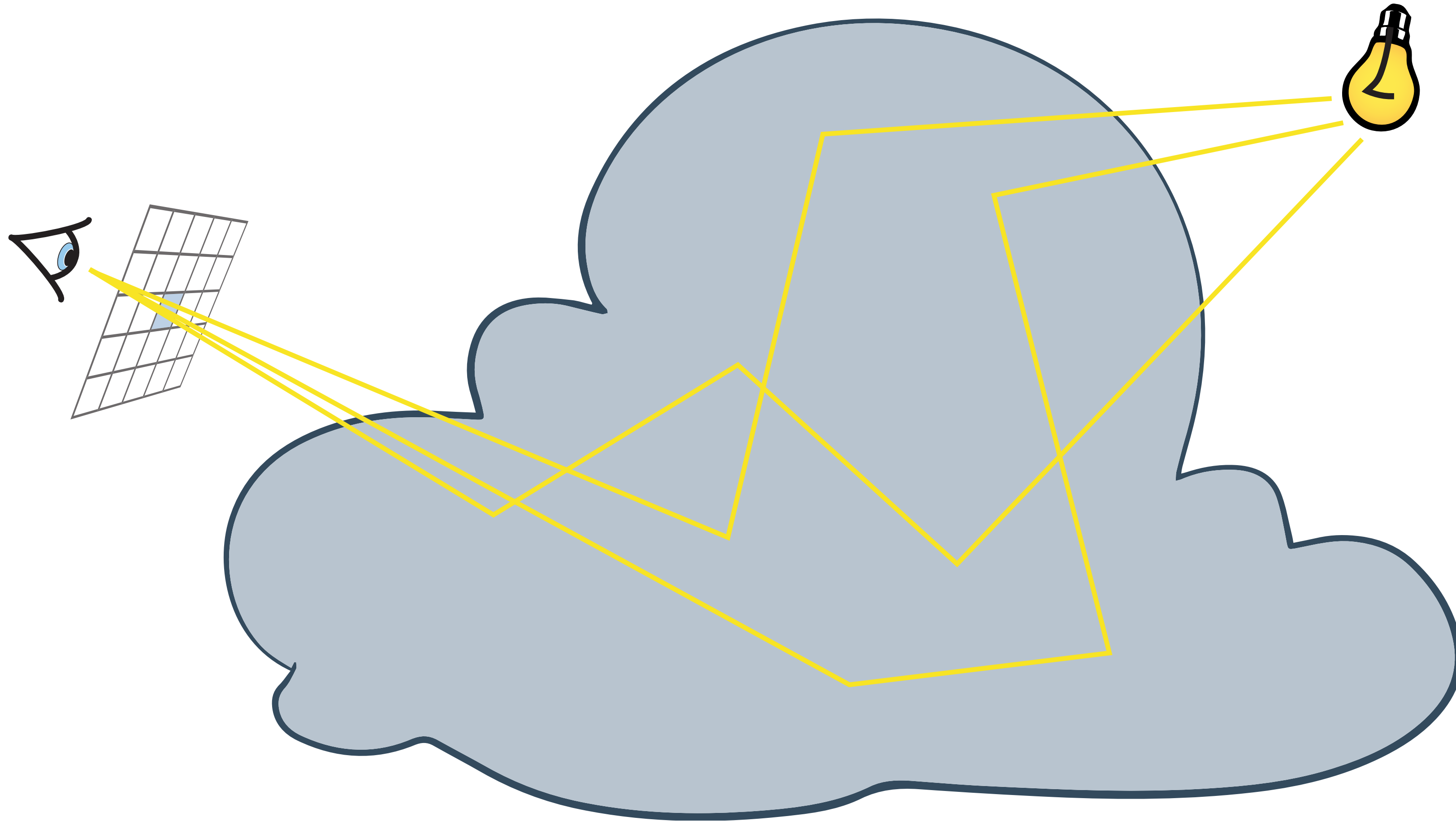


# PATH CONSTRUCTION

Iliyan Georgiev  
Solid Angle

# PATH INTEGRAL FRAMEWORK



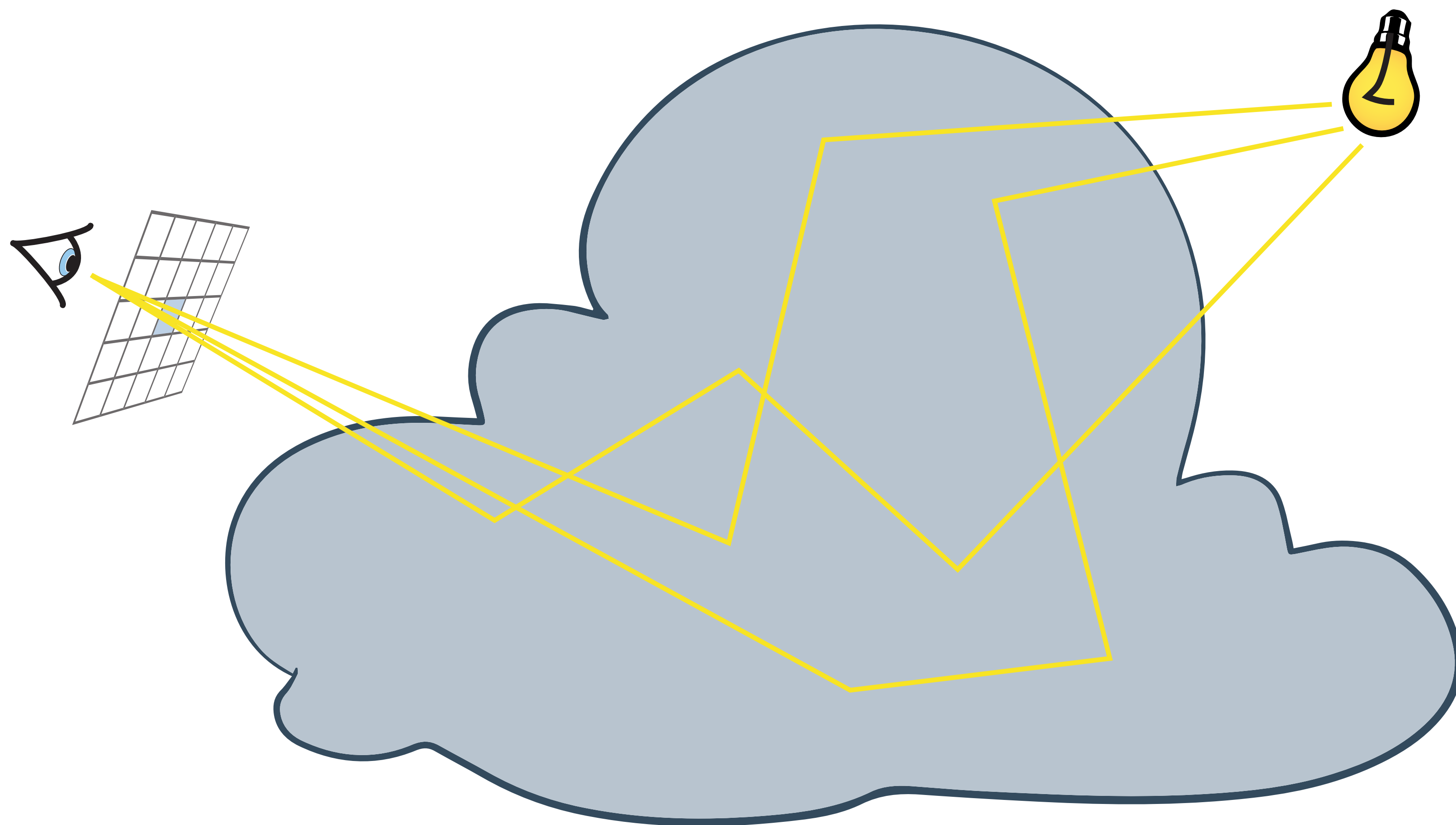
Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

Pixel estimator

$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_j(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$$

# PATH INTEGRAL FRAMEWORK



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

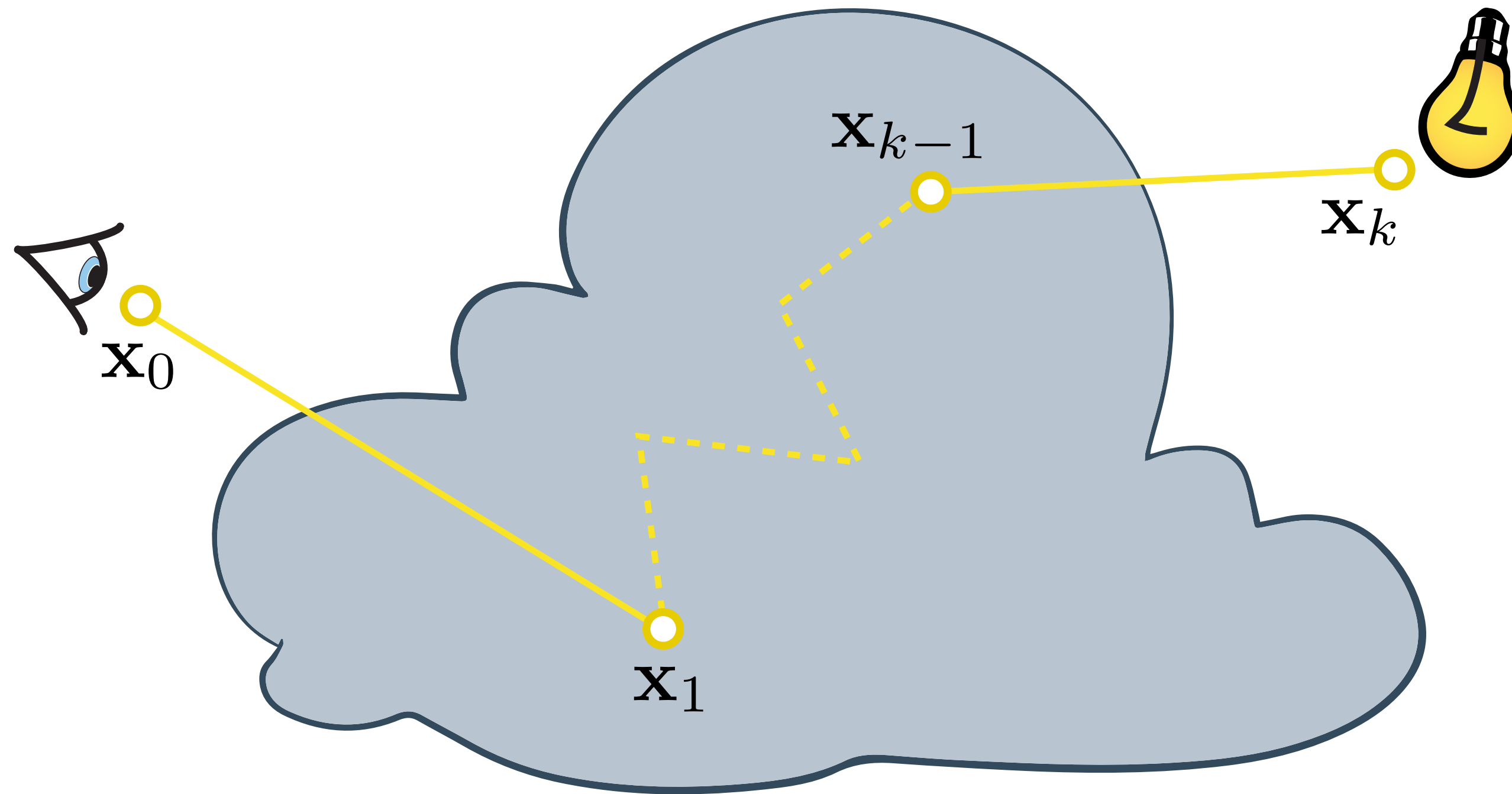
Pixel estimator

$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_j(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$$

path contribution

path pdf

# PATH INTEGRAL FRAMEWORK



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

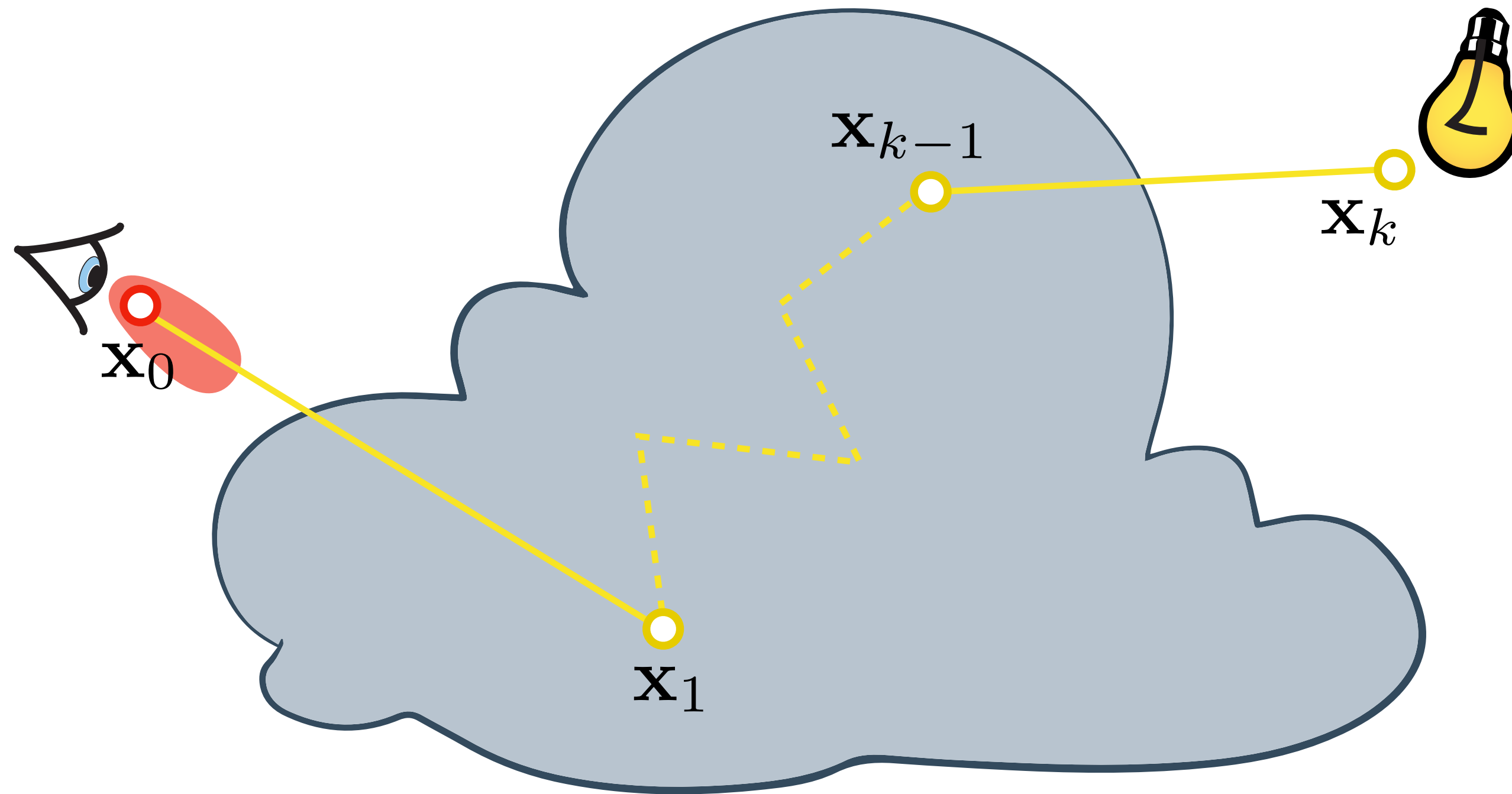
Pixel estimator

$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_j(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$$

Path contribution

$$f_j(\bar{\mathbf{x}}) = W_j(\mathbf{x}_0, \mathbf{x}_1) \left[ \prod_i f_s(\mathbf{x}_i) G(\mathbf{x}_i, \mathbf{x}_{i+1}) T(\mathbf{x}_i, \mathbf{x}_{i+1}) \right] L_e(\mathbf{x}_k, \mathbf{x}_{k-1})$$

# PATH INTEGRAL FRAMEWORK



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

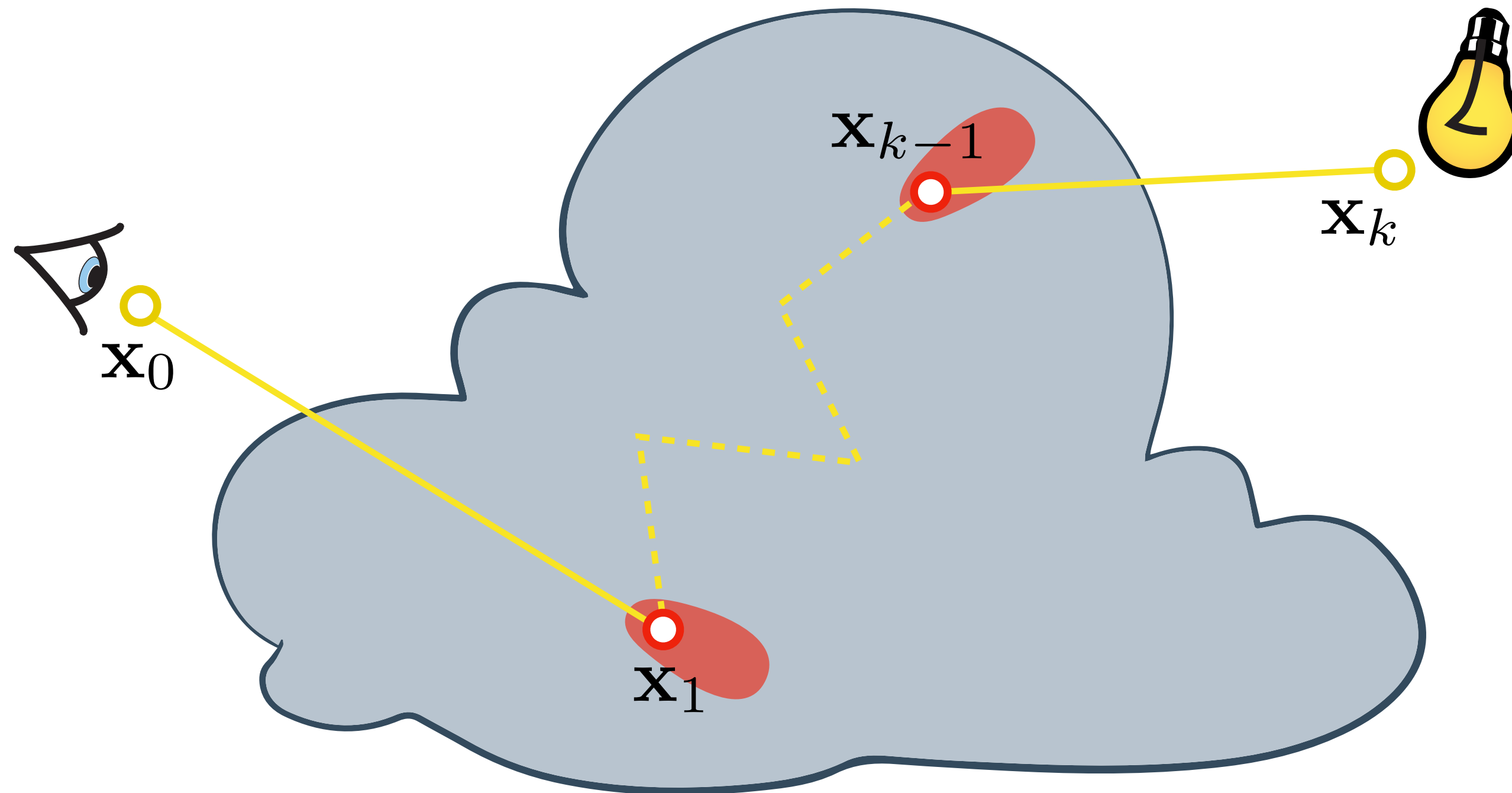
Pixel estimator

$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_j(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$$

Path contribution

$$f_j(\bar{\mathbf{x}}) = \underbrace{W_j(\mathbf{x}_0, \mathbf{x}_1)}_{\text{camera response}} \left[ \prod_i f_s(\mathbf{x}_i) G(\mathbf{x}_i, \mathbf{x}_{i+1}) T(\mathbf{x}_i, \mathbf{x}_{i+1}) \right] L_e(\mathbf{x}_k, \mathbf{x}_{k-1})$$

# PATH INTEGRAL FRAMEWORK



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

Pixel estimator

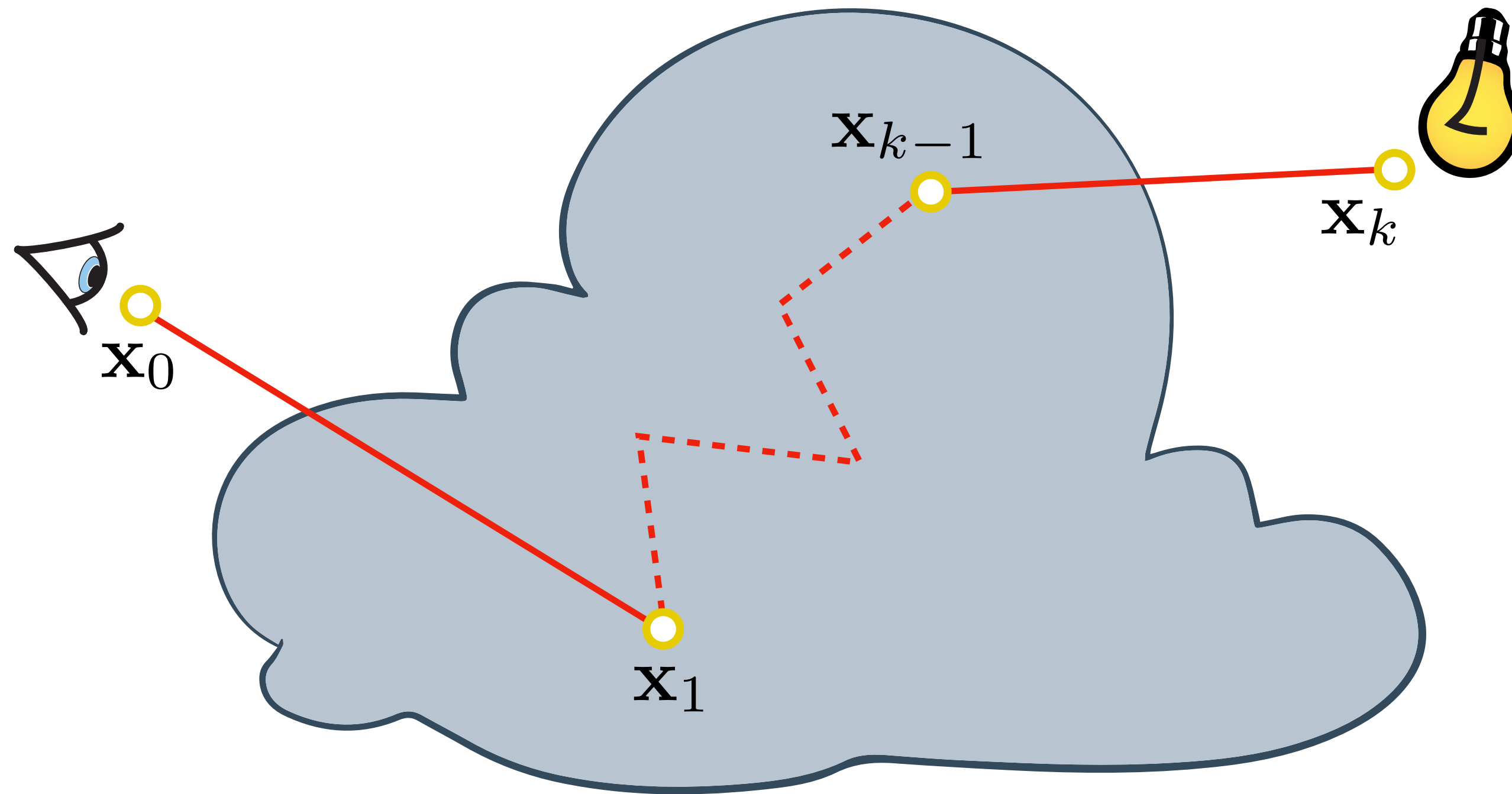
$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_j(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$$

Path contribution

$$f_j(\bar{\mathbf{x}}) = W_j(\mathbf{x}_0, \mathbf{x}_1) \left[ \prod_i \underbrace{f_s(\mathbf{x}_i)}_{\text{BSDF/phase}} G(\mathbf{x}_i, \mathbf{x}_{i+1}) T(\mathbf{x}_i, \mathbf{x}_{i+1}) \right] L_e(\mathbf{x}_k, \mathbf{x}_{k-1})$$

camera response

# PATH INTEGRAL FRAMEWORK



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

Pixel estimator

$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_j(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$$

Path contribution

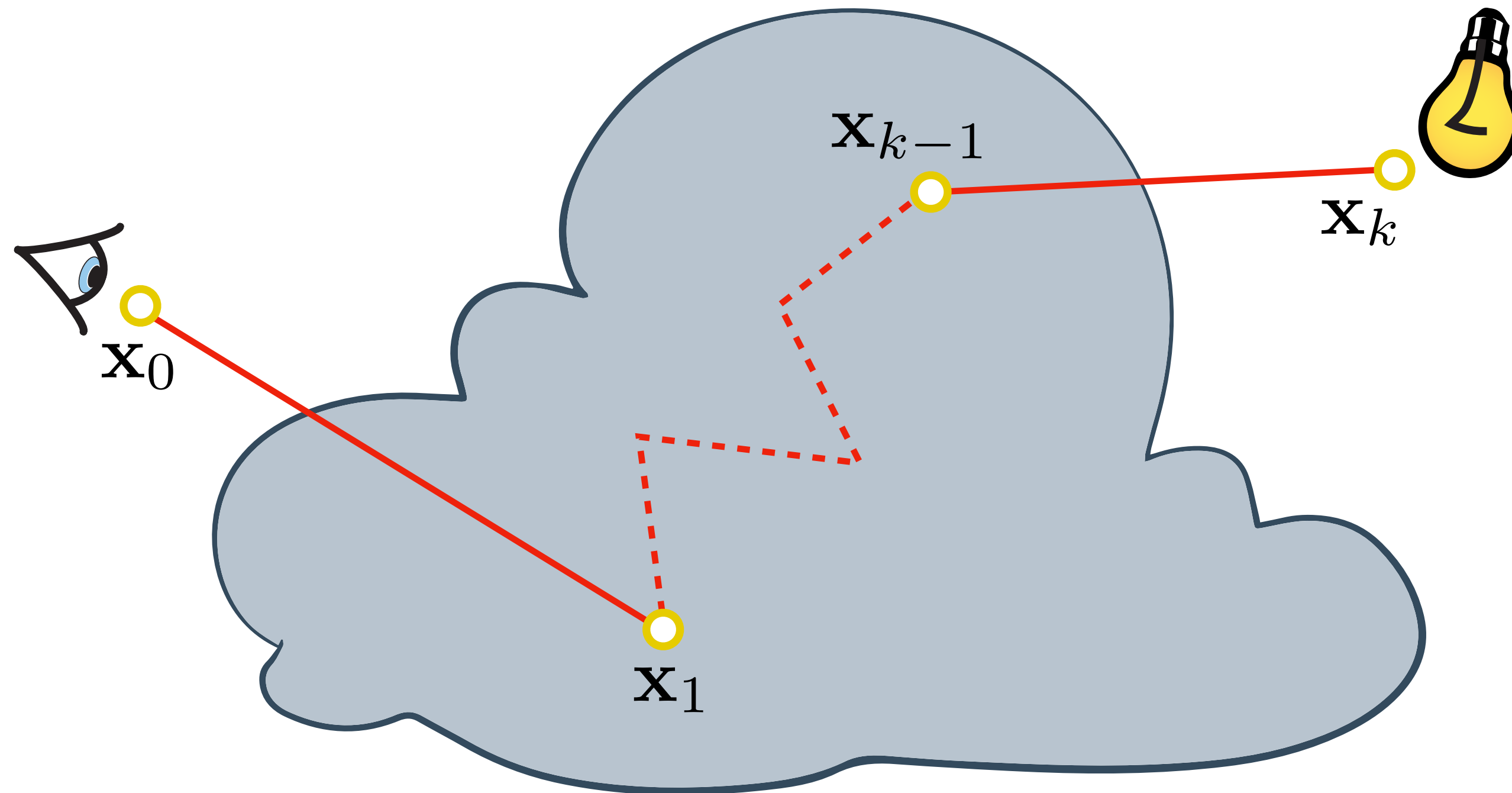
$$f_j(\bar{\mathbf{x}}) = W_j(\mathbf{x}_0, \mathbf{x}_1) \left[ \prod_i f_s(\mathbf{x}_i) G(\mathbf{x}_i, \mathbf{x}_{i+1}) T(\mathbf{x}_i, \mathbf{x}_{i+1}) \right] L_e(\mathbf{x}_k, \mathbf{x}_{k-1})$$

camera response

BSDF/  
phase

geometry

# PATH INTEGRAL FRAMEWORK



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

Pixel estimator

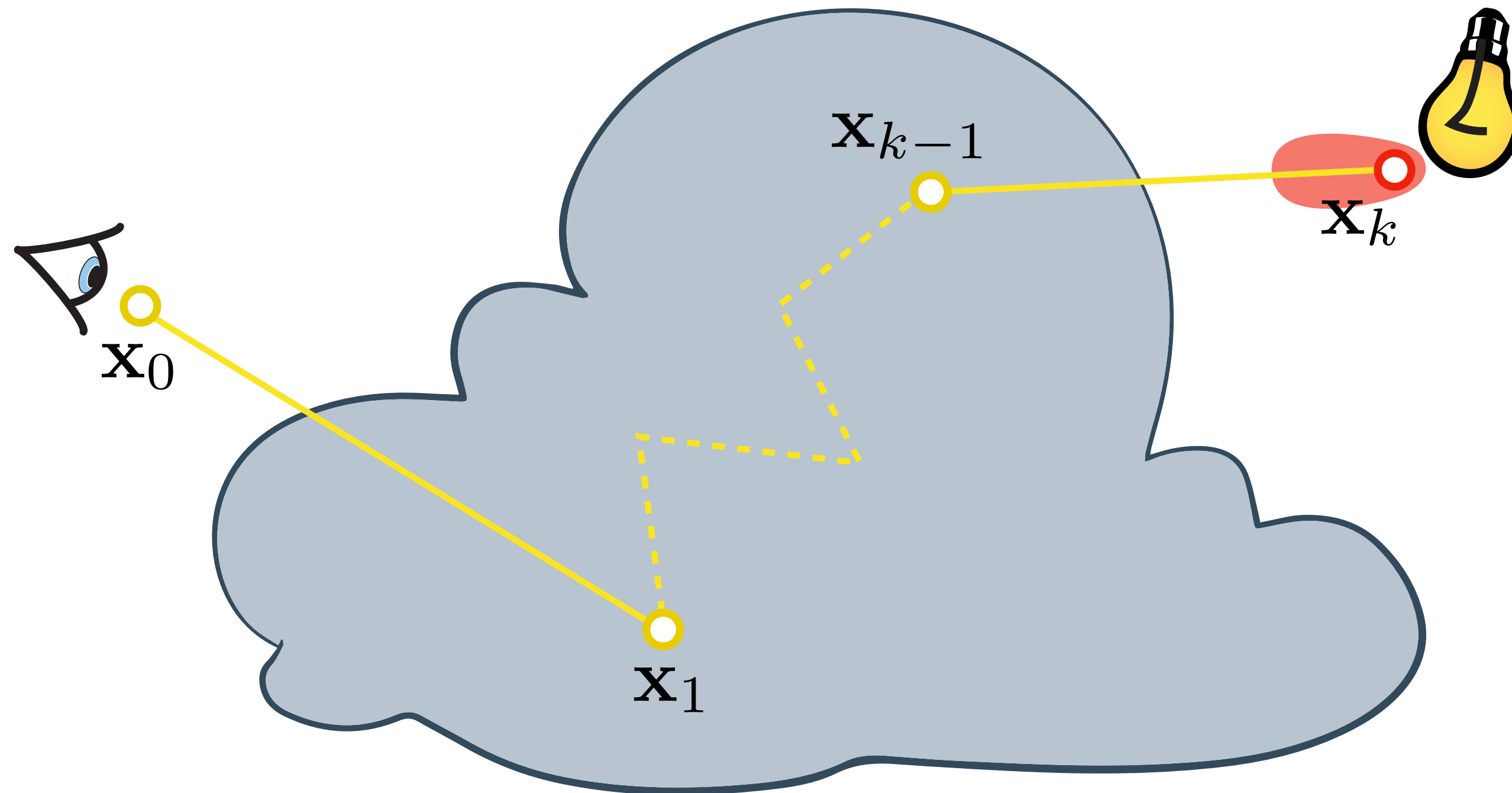
$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_j(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$$

Path contribution

$$f_j(\bar{\mathbf{x}}) = W_j(\mathbf{x}_0, \mathbf{x}_1) \left[ \prod_i \underbrace{f_s(\mathbf{x}_i)}_{\text{BSDF/phase}} \underbrace{G(\mathbf{x}_i, \mathbf{x}_{i+1})}_{\text{geometry}} \underbrace{T(\mathbf{x}_i, \mathbf{x}_{i+1})}_{\text{transmittance}} \right] L_e(\mathbf{x}_k, \mathbf{x}_{k-1})$$



# PATH INTEGRAL FRAMEWORK



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

Pixel estimator

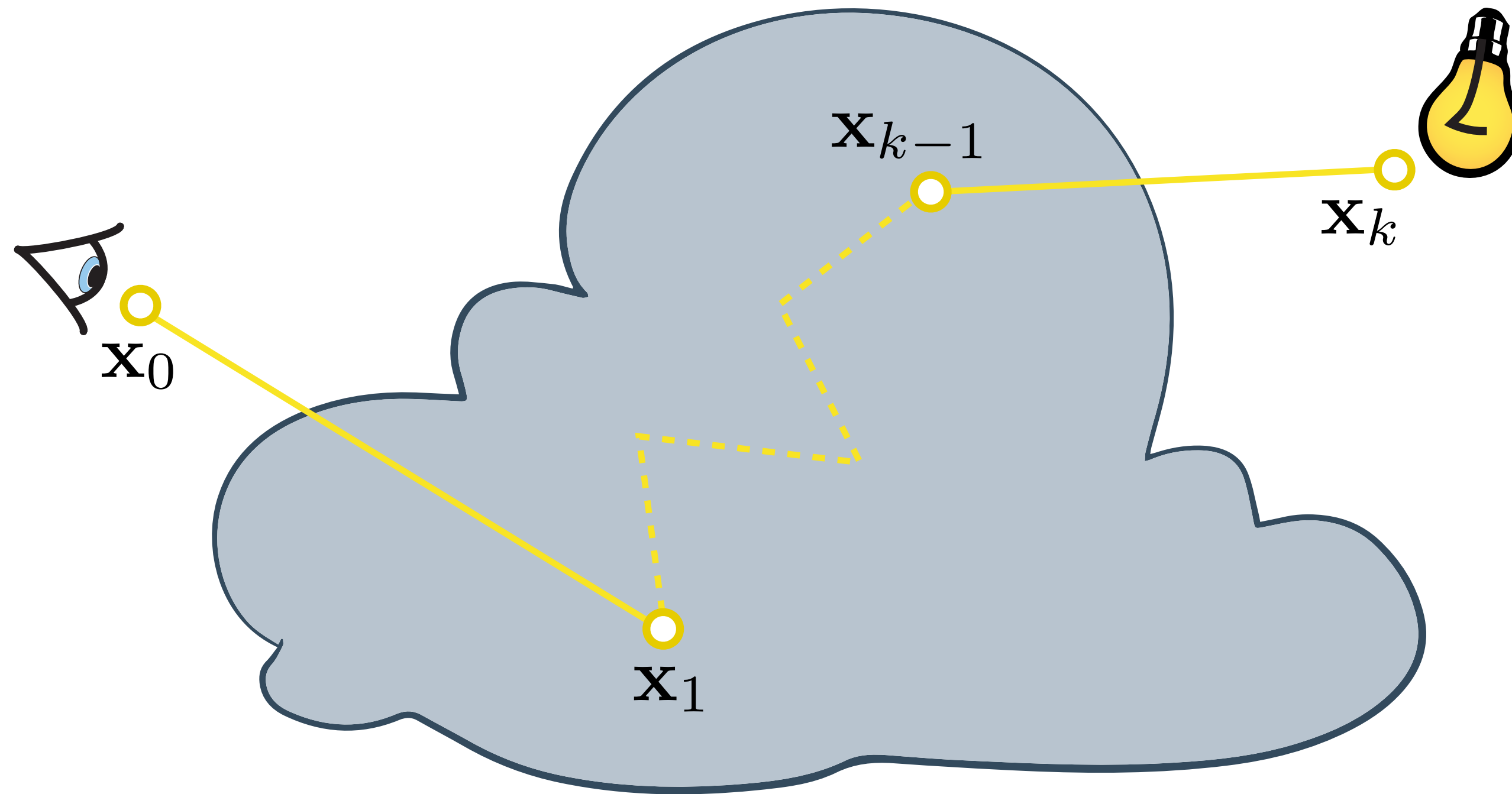
$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_j(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$$

Path contribution

$$f_j(\bar{\mathbf{x}}) = W_j(\mathbf{x}_0, \mathbf{x}_1) \left[ \prod_i \underbrace{f_s(\mathbf{x}_i)}_{\text{BSDF/phase}} \underbrace{G(\mathbf{x}_i, \mathbf{x}_{i+1})}_{\text{geometry}} \underbrace{T(\mathbf{x}_i, \mathbf{x}_{i+1})}_{\text{transmittance}} \right] L_e(\mathbf{x}_k, \mathbf{x}_{k-1})$$

camera response
emitted radiance

# PATH INTEGRAL FRAMEWORK



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

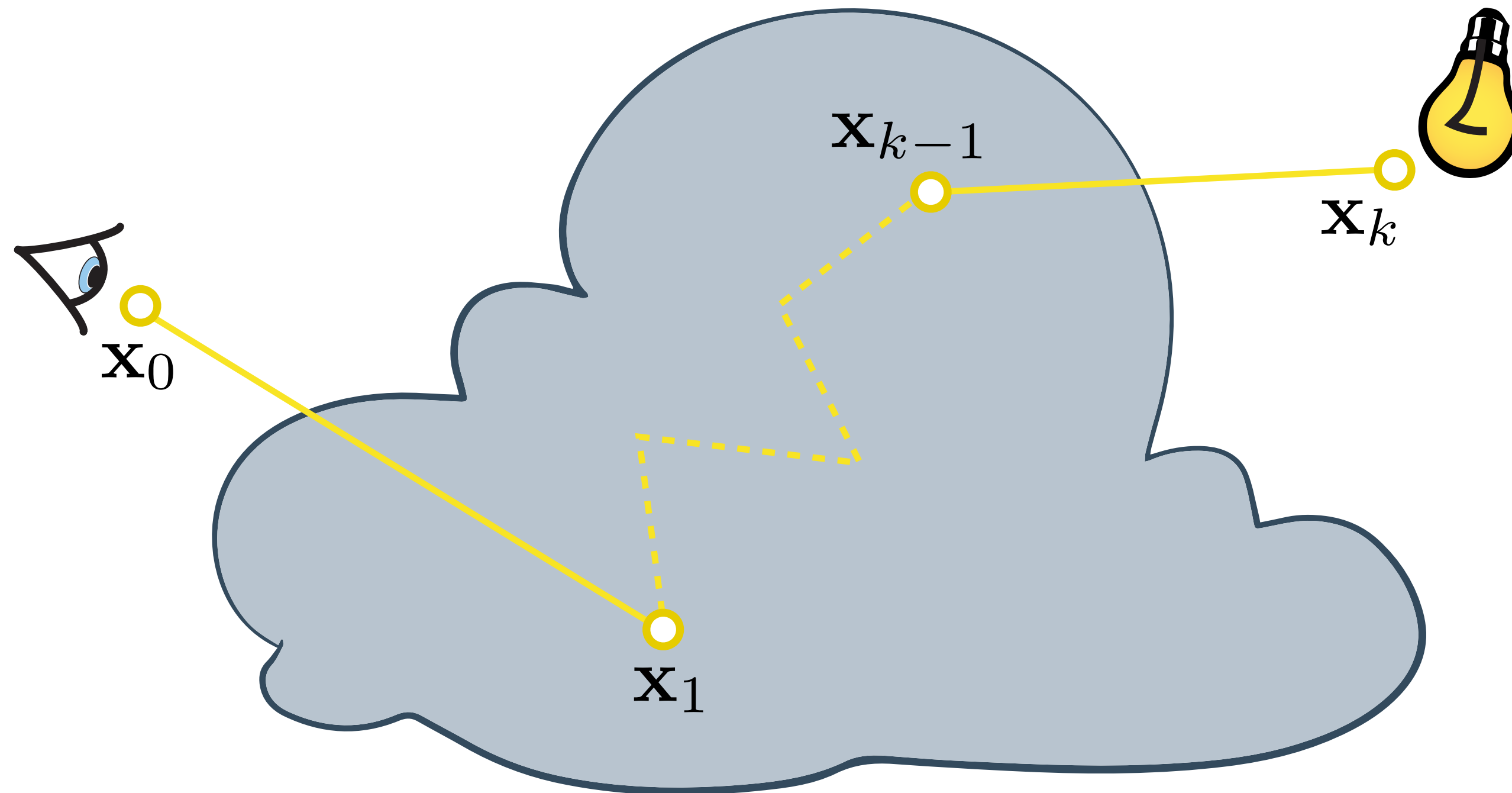
Pixel estimator

$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_j(\bar{\mathbf{x}}_i)}{p(\bar{\mathbf{x}}_i)}$$

Path contribution

$$f_j(\bar{\mathbf{x}}) = \underbrace{W_j(\mathbf{x}_0, \mathbf{x}_1)}_{\text{camera response}} \left[ \prod_i \underbrace{f_s(\mathbf{x}_i)}_{\text{BSDF/phase}} \underbrace{G(\mathbf{x}_i, \mathbf{x}_{i+1})}_{\text{geometry}} \underbrace{T(\mathbf{x}_i, \mathbf{x}_{i+1})}_{\text{transmittance}} \right] \underbrace{L_e(\mathbf{x}_k, \mathbf{x}_{k-1})}_{\text{emitted radiance}}$$

# PATH INTEGRAL FRAMEWORK



Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

Pixel estimator

$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^N f_j(\bar{\mathbf{x}}_i) p(\bar{\mathbf{x}}_i)$$

*ideally proportional*

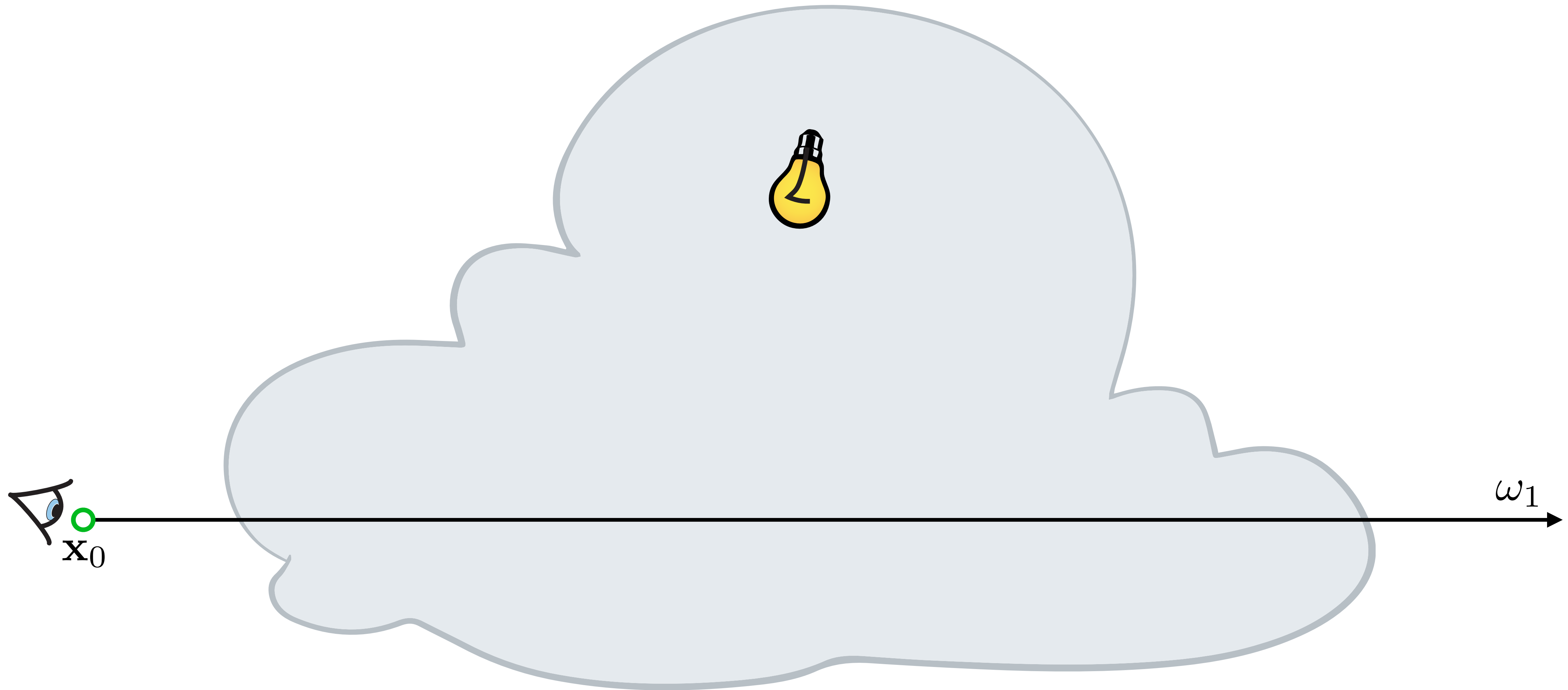
$\propto$

Path contribution

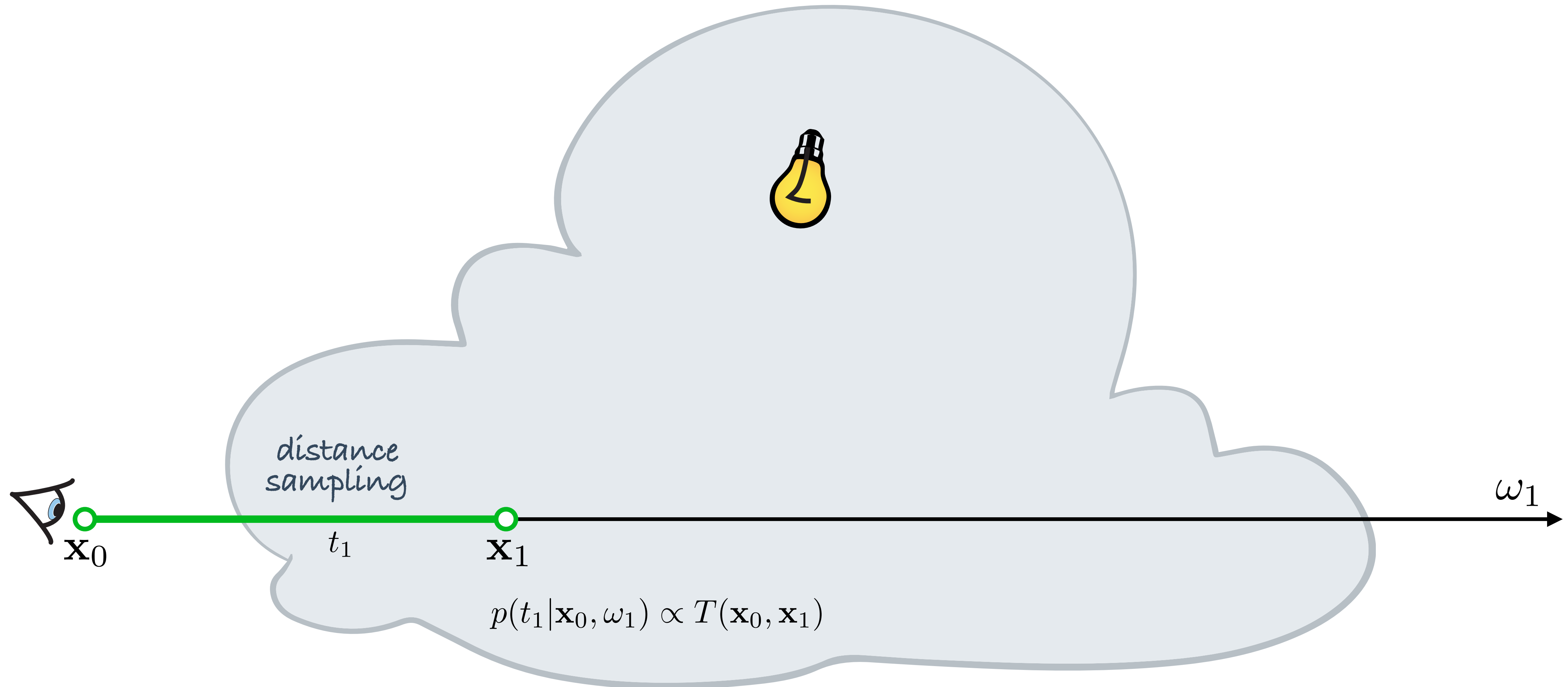
$$f_j(\bar{\mathbf{x}}) = W_j(\mathbf{x}_0, \mathbf{x}_1) \left[ \prod_i f_s(\mathbf{x}_i) G(\mathbf{x}_i, \mathbf{x}_{i+1}) T(\mathbf{x}_i, \mathbf{x}_{i+1}) \right] L_e(\mathbf{x}_k, \mathbf{x}_{k-1})$$

*camera response*
*BSDF/phase*
*geometry*
*transmittance*
*emitted radiance*

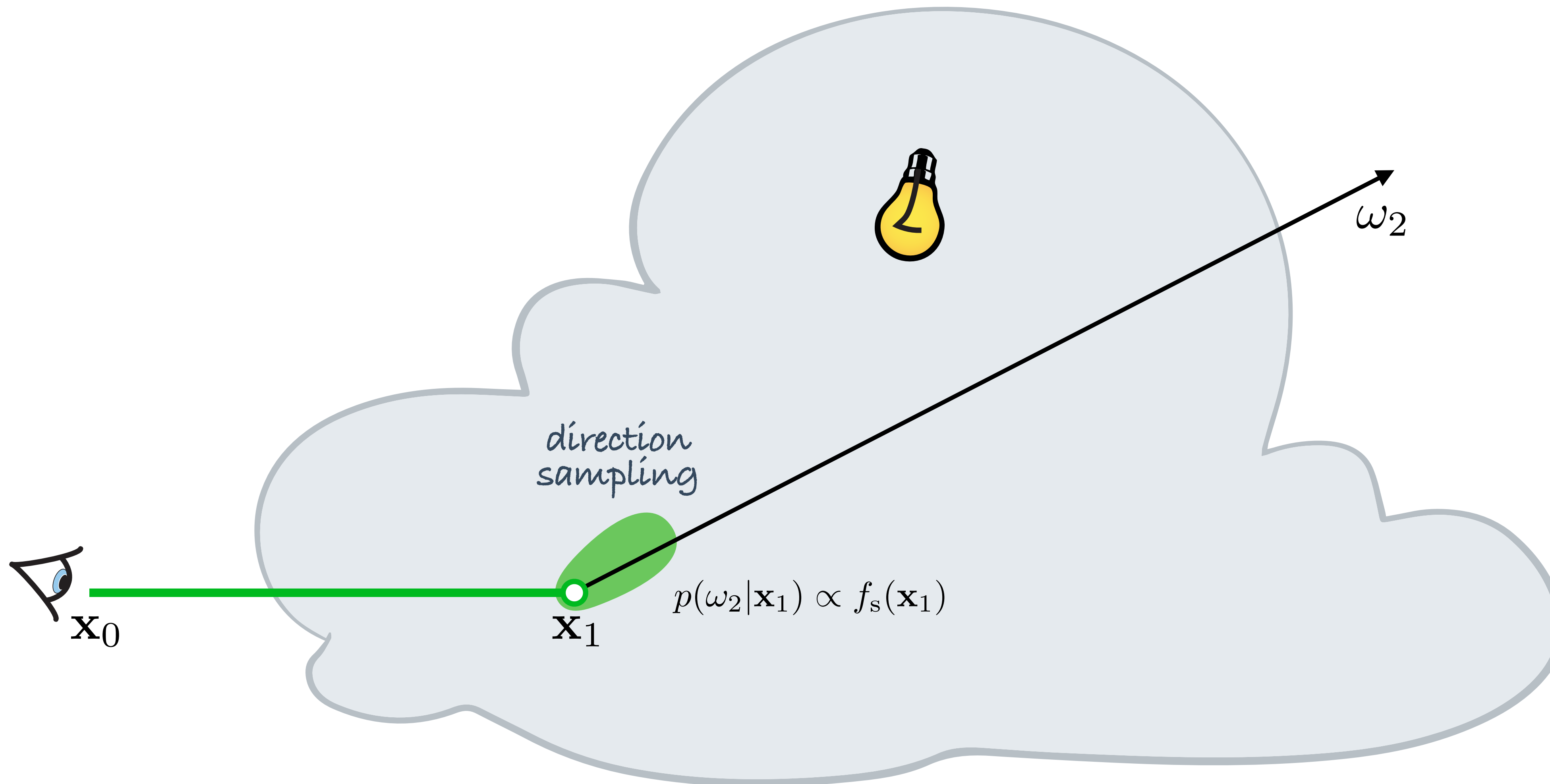
# UNIDIRECTIONAL PATH SAMPLING



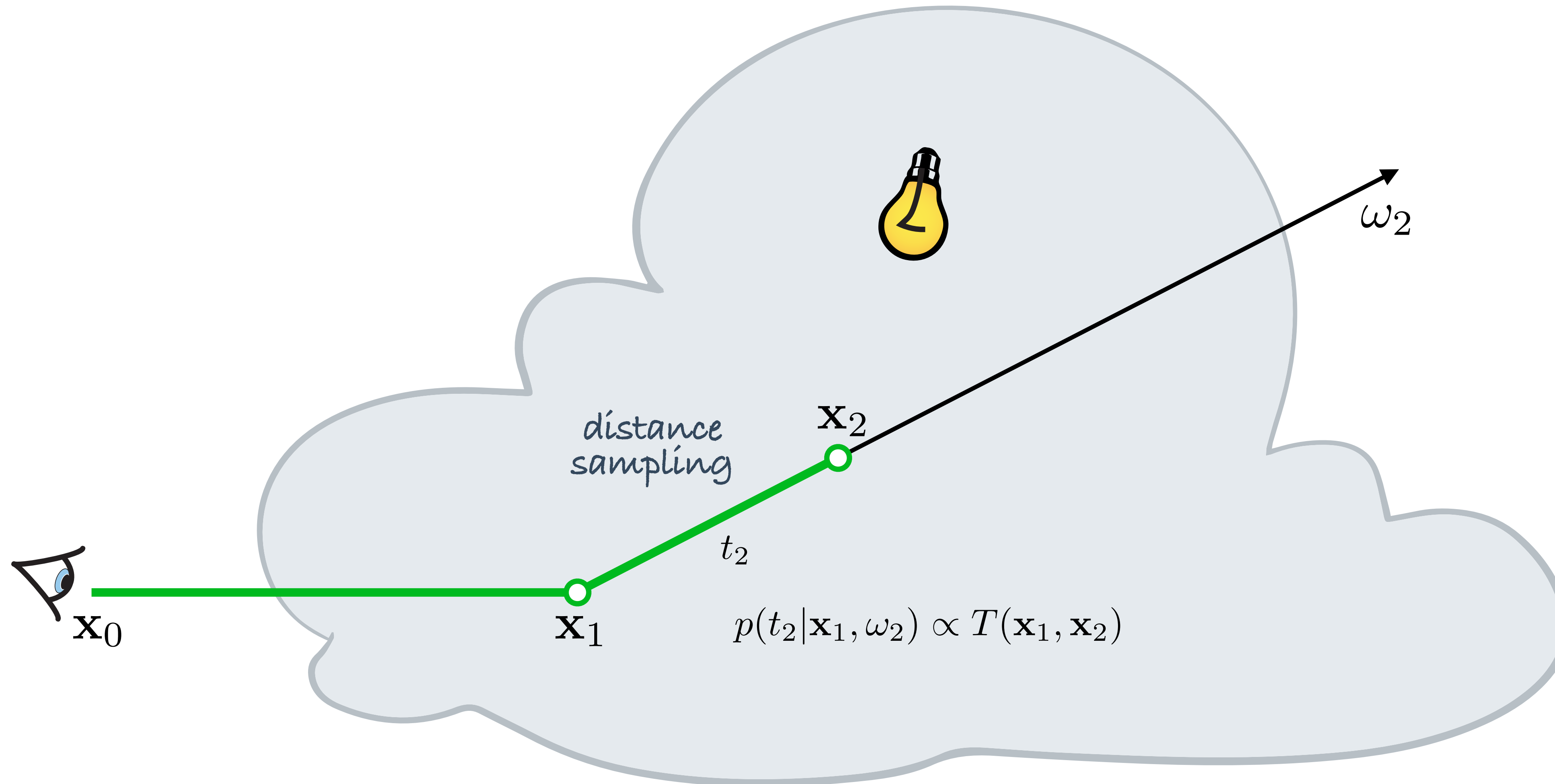
# UNIDIRECTIONAL PATH SAMPLING



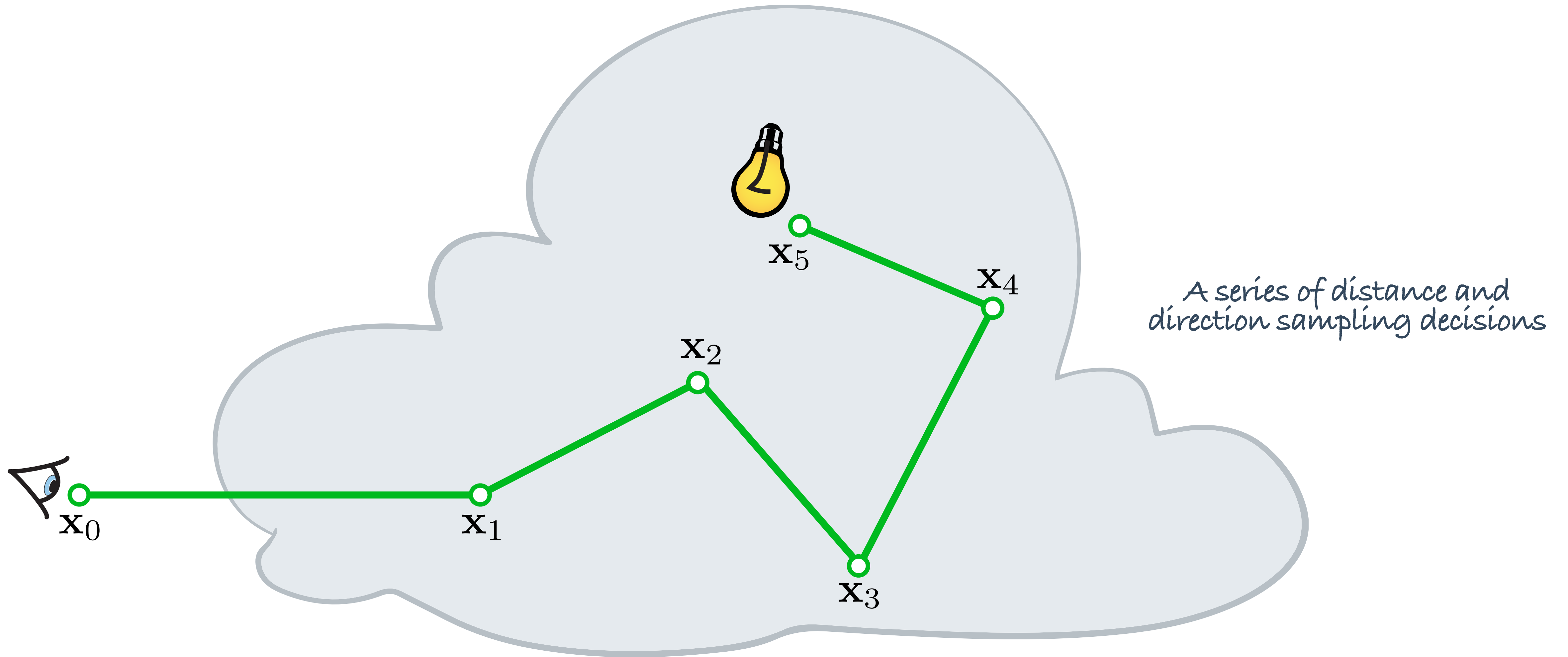
# UNIDIRECTIONAL PATH SAMPLING



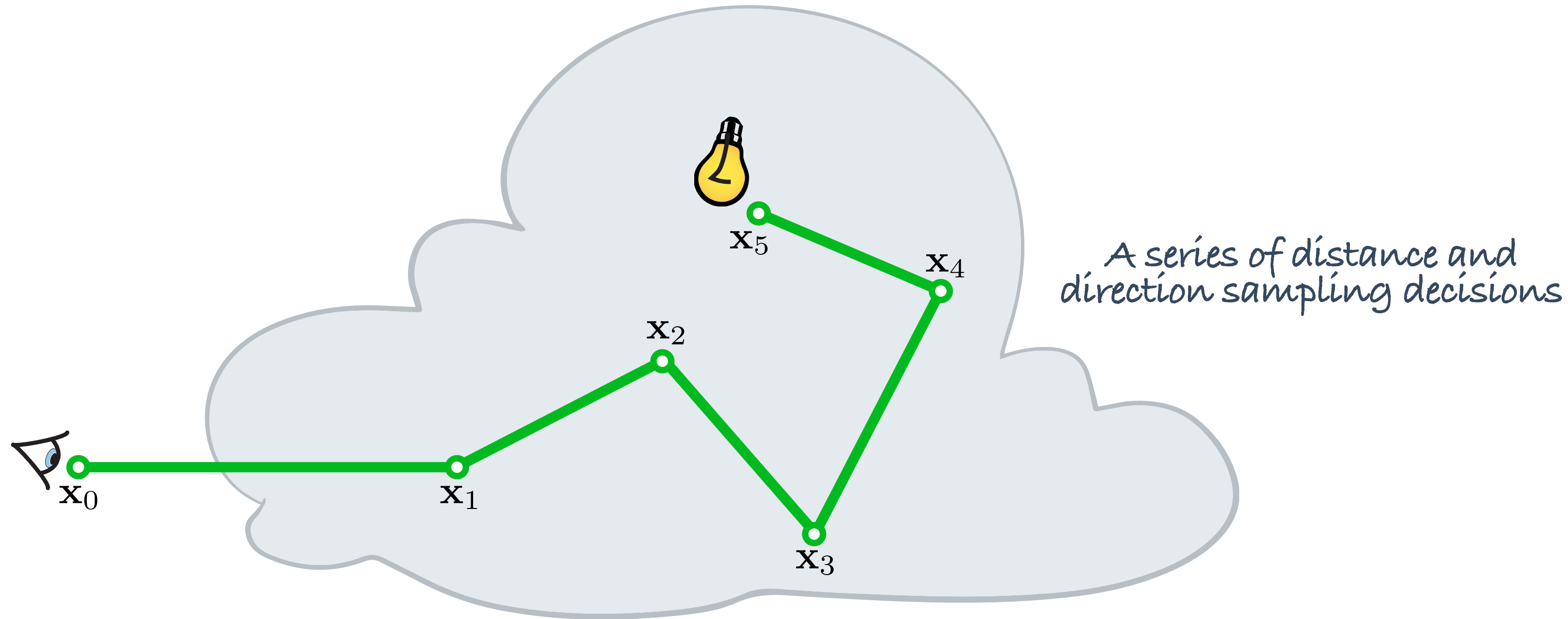
# UNIDIRECTIONAL PATH SAMPLING



# UNIDIRECTIONAL PATH SAMPLING







cannot render illumination from point light sources

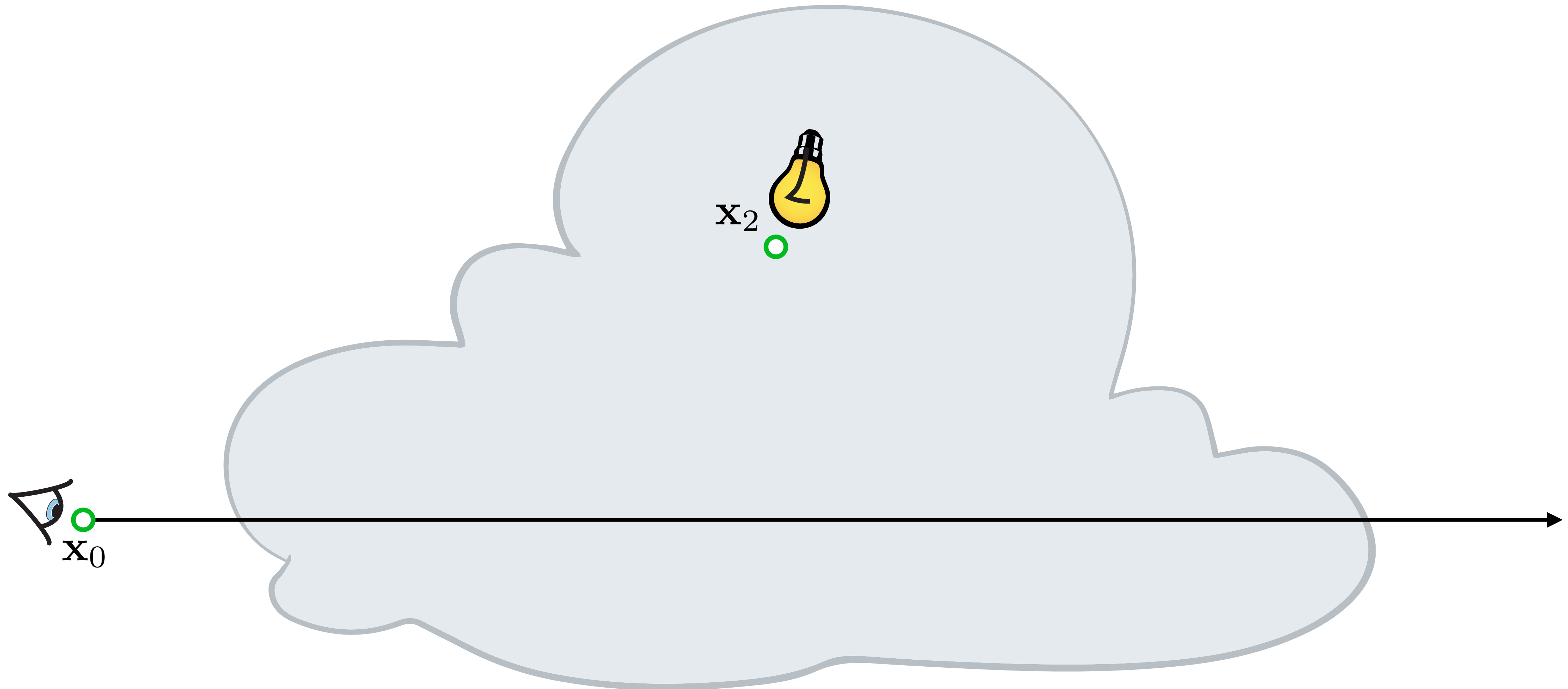
high variance when light sources are small

$$p(\bar{\mathbf{x}}) \propto W_j(\mathbf{x}_0, \mathbf{x}_1) \left[ \prod_i f_s(\mathbf{x}_i) G(\mathbf{x}_i, \mathbf{x}_{i+1}) T(\mathbf{x}_i, \mathbf{x}_{i+1}) \right]$$

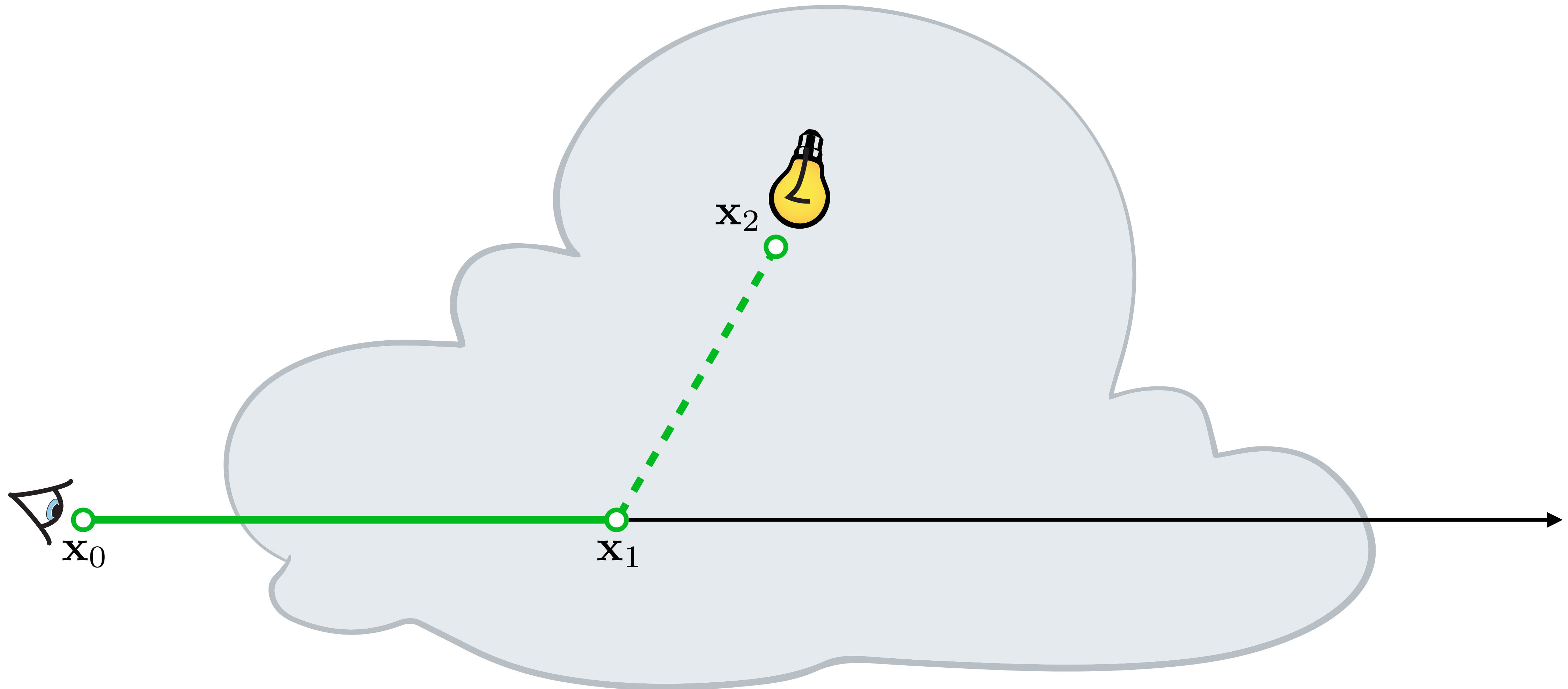
not importance sampled

$L_e(\mathbf{x}_k, \mathbf{x}_{k-1})$

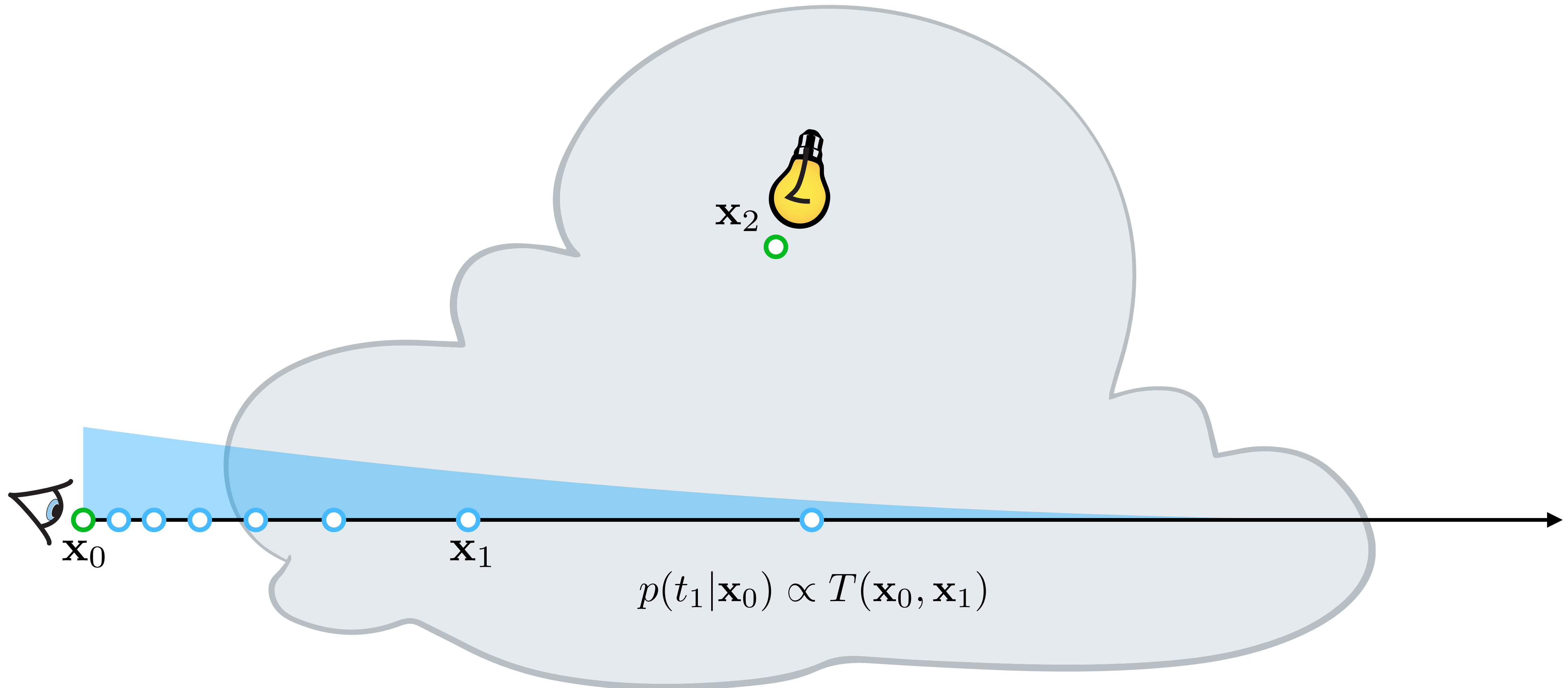
# EXPLICIT LIGHT SAMPLING



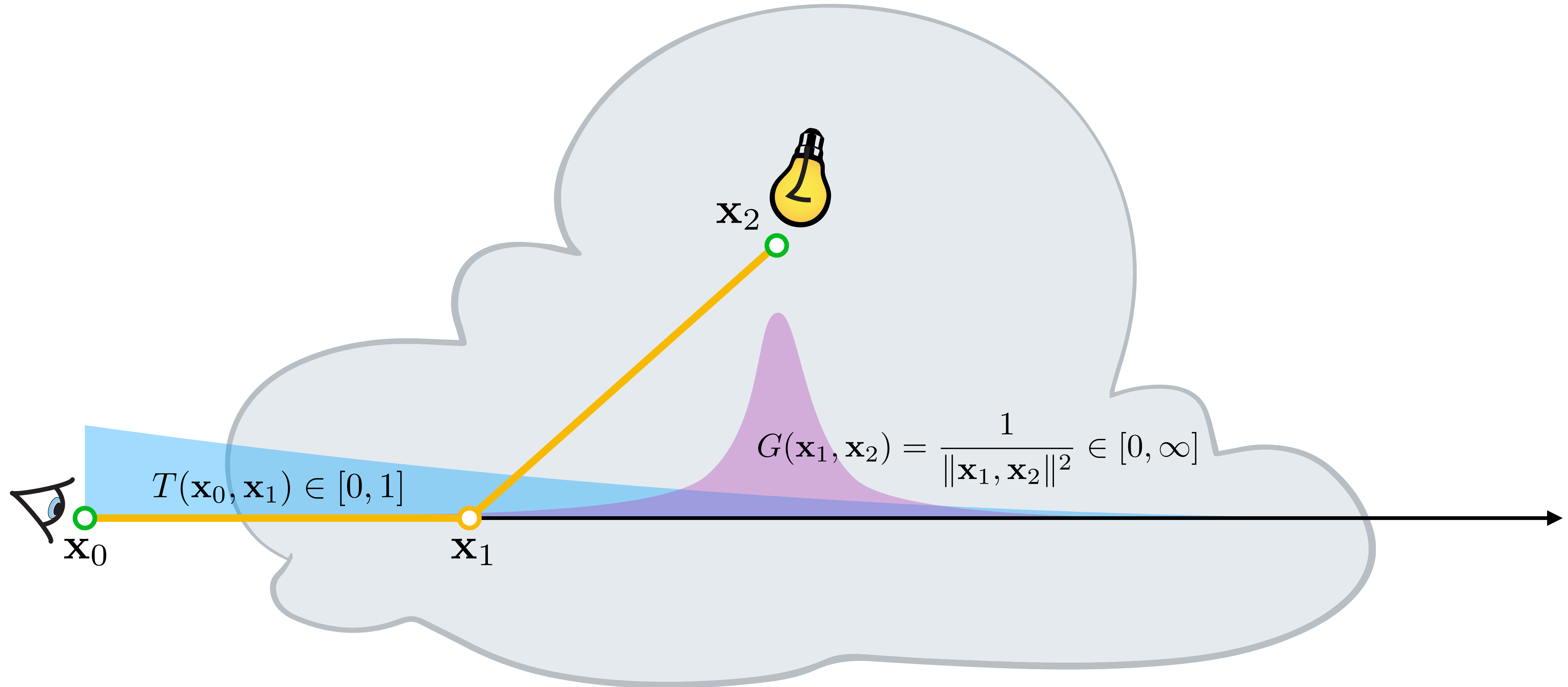
# EXPLICIT LIGHT SAMPLING



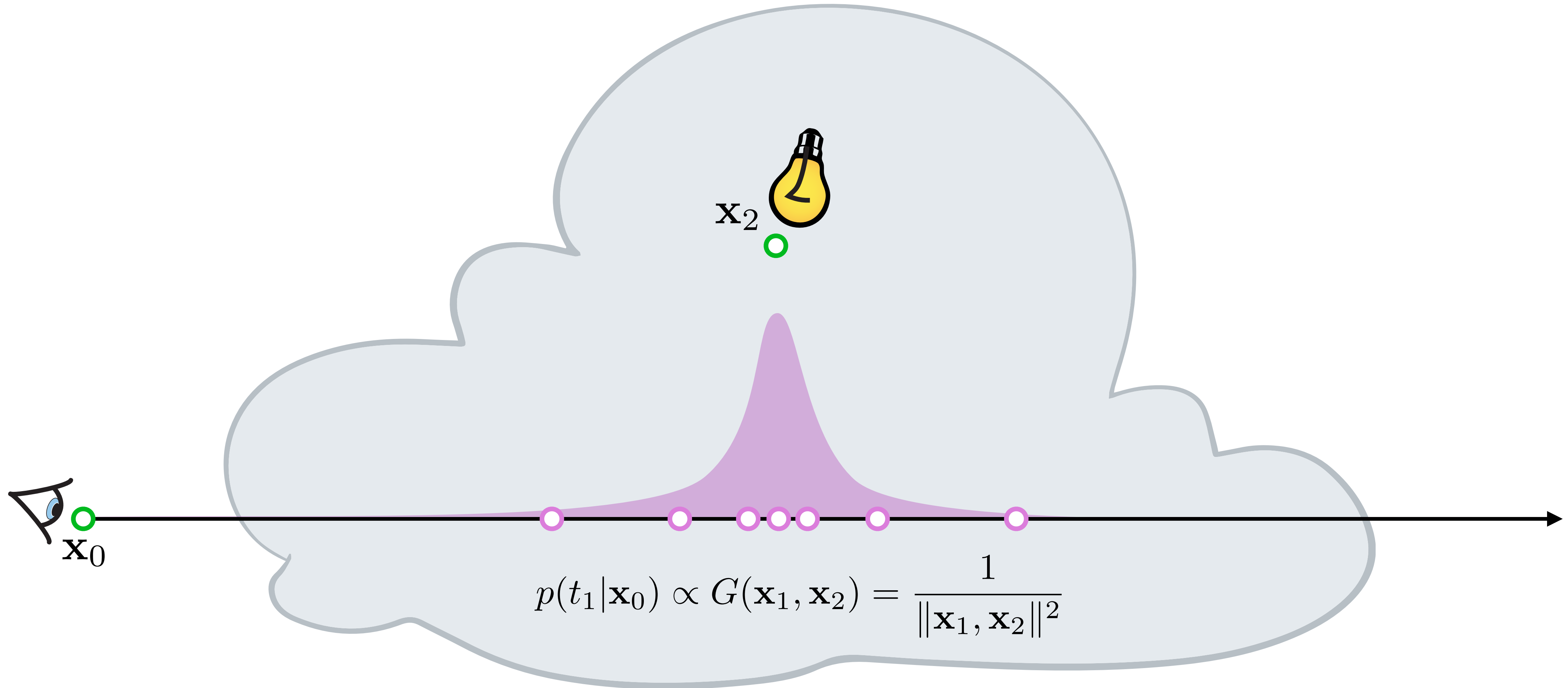
# EXPLICIT: TRANSMITTANCE



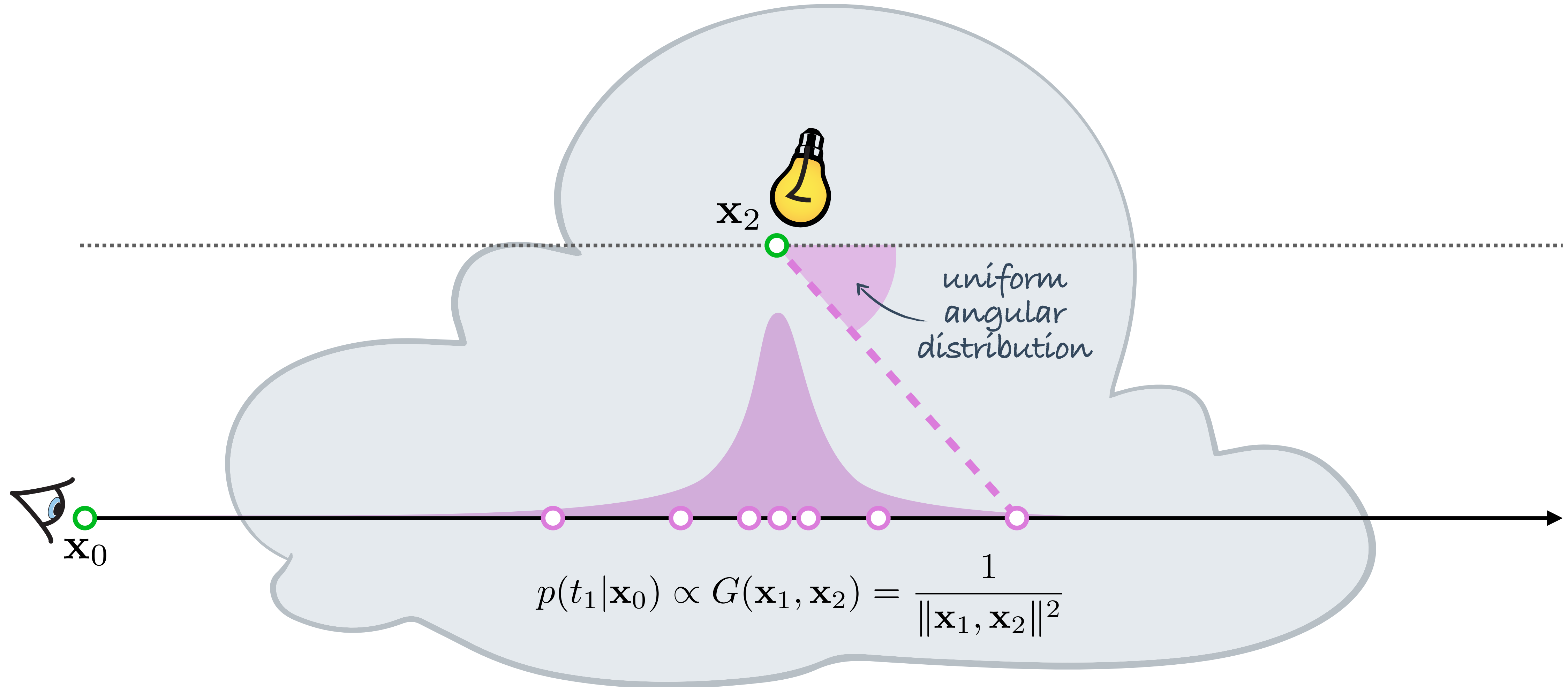
# EXPLICIT: TRANSMITTANCE

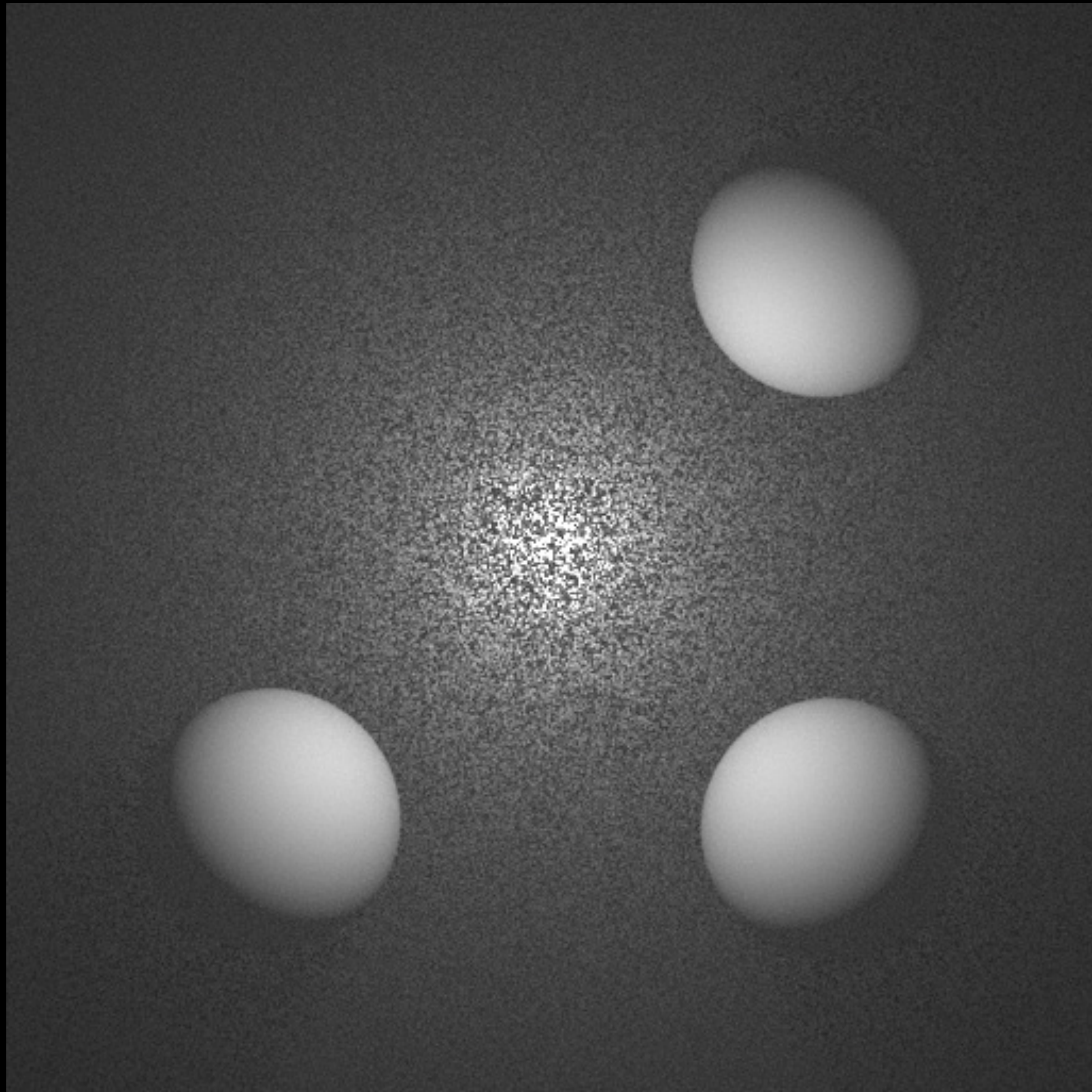


# EXPLICIT: EQUIANGULAR

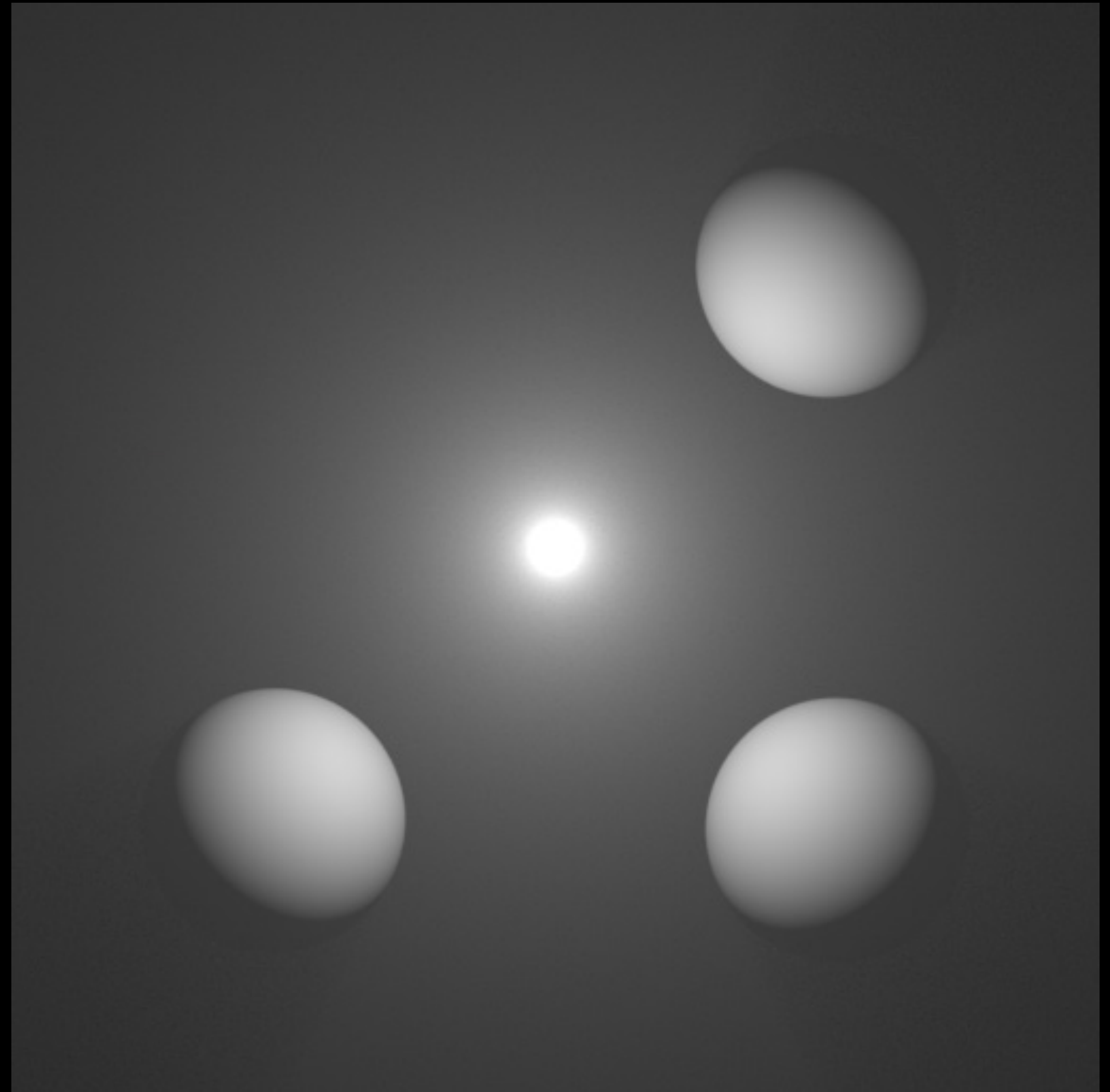


# EXPLICIT: EQUIANGULAR



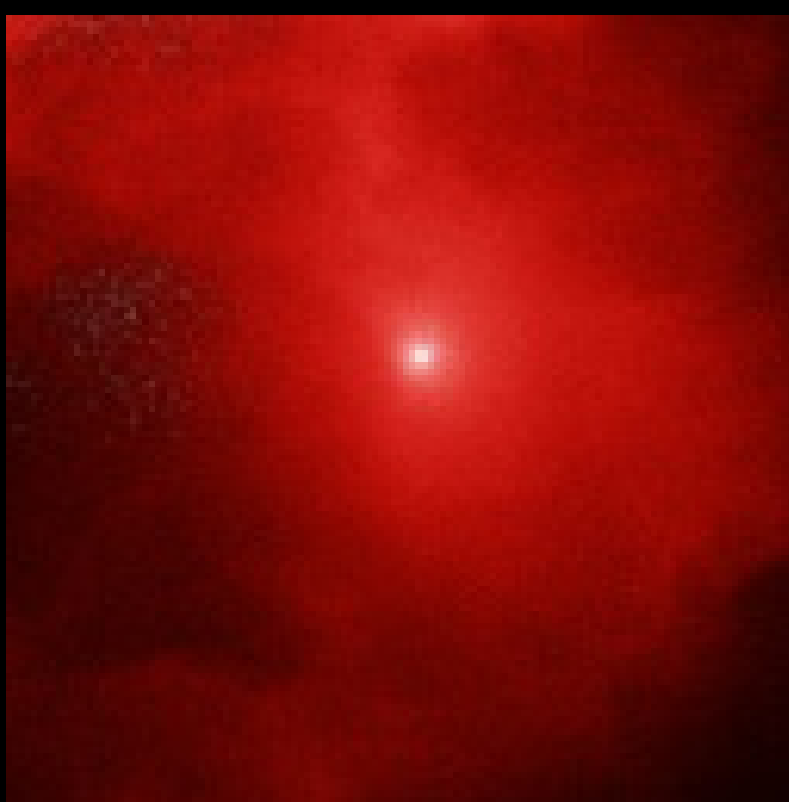
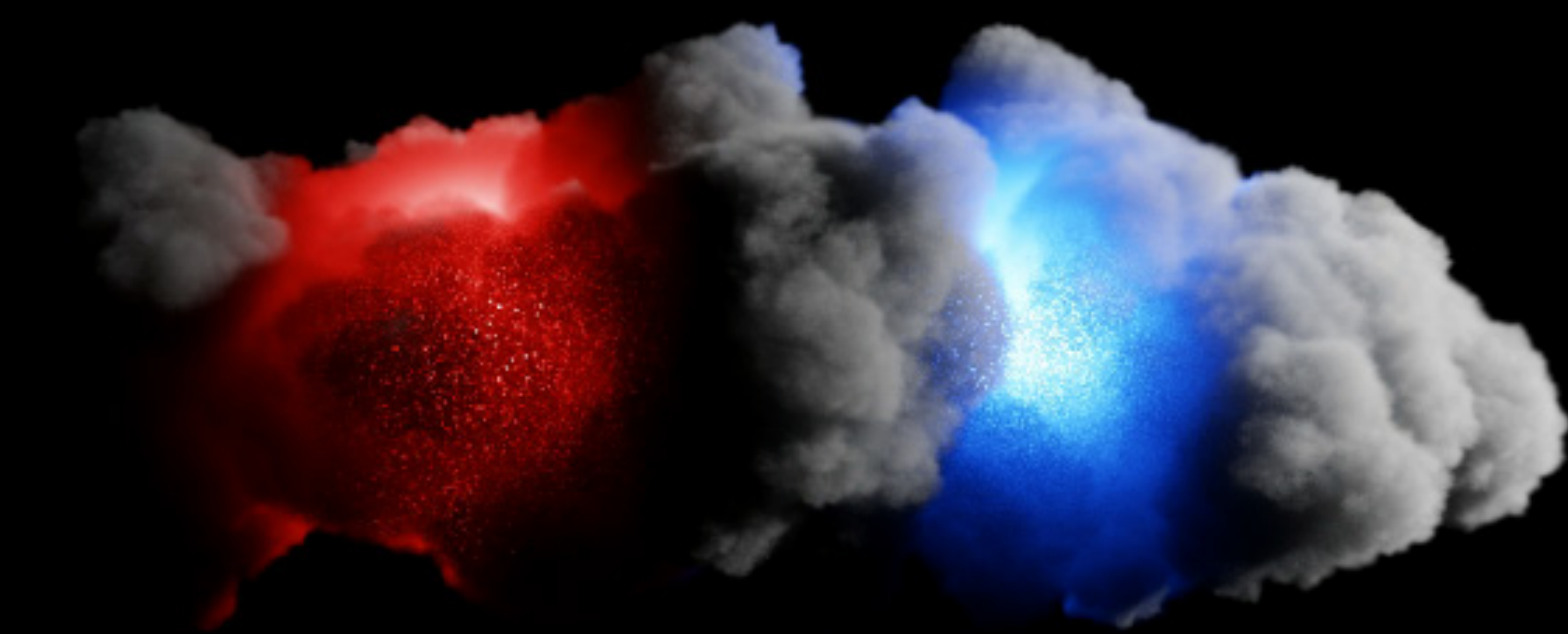
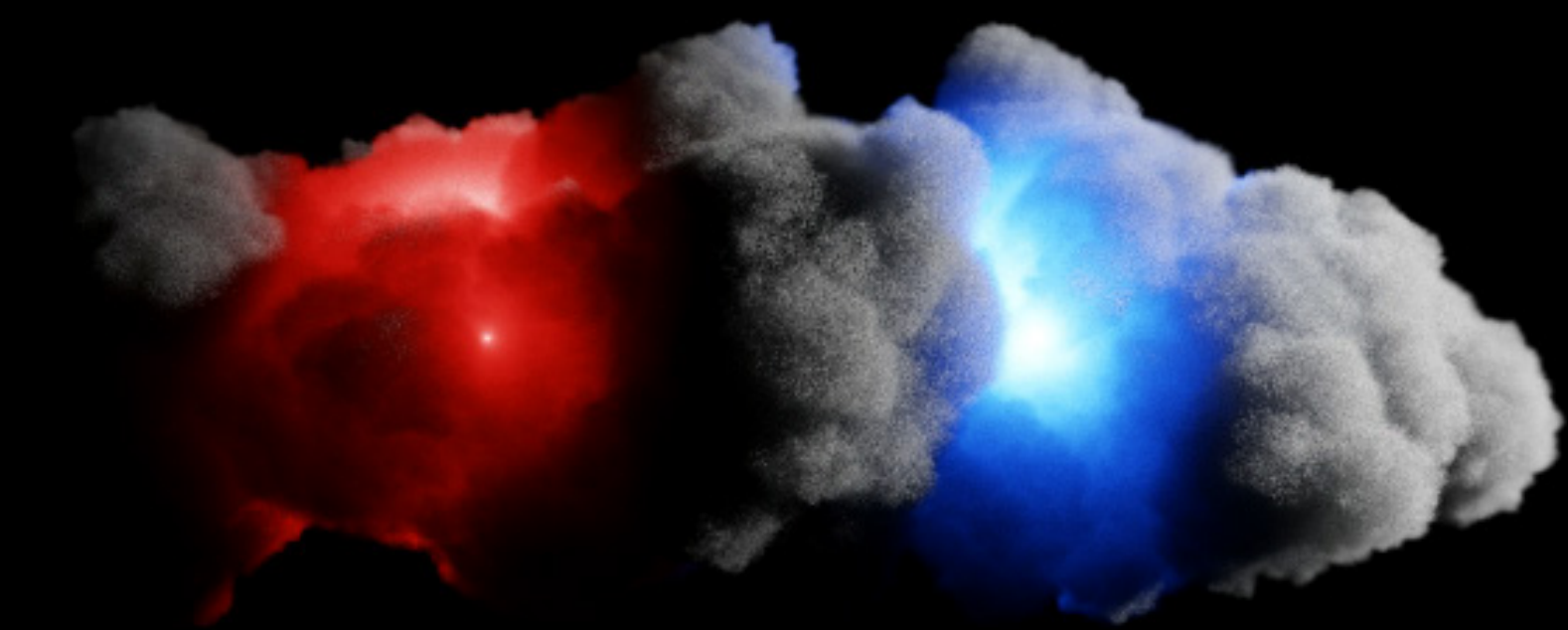


Transmittance sampling, 16 spp



Equiangular sampling, 16 spp



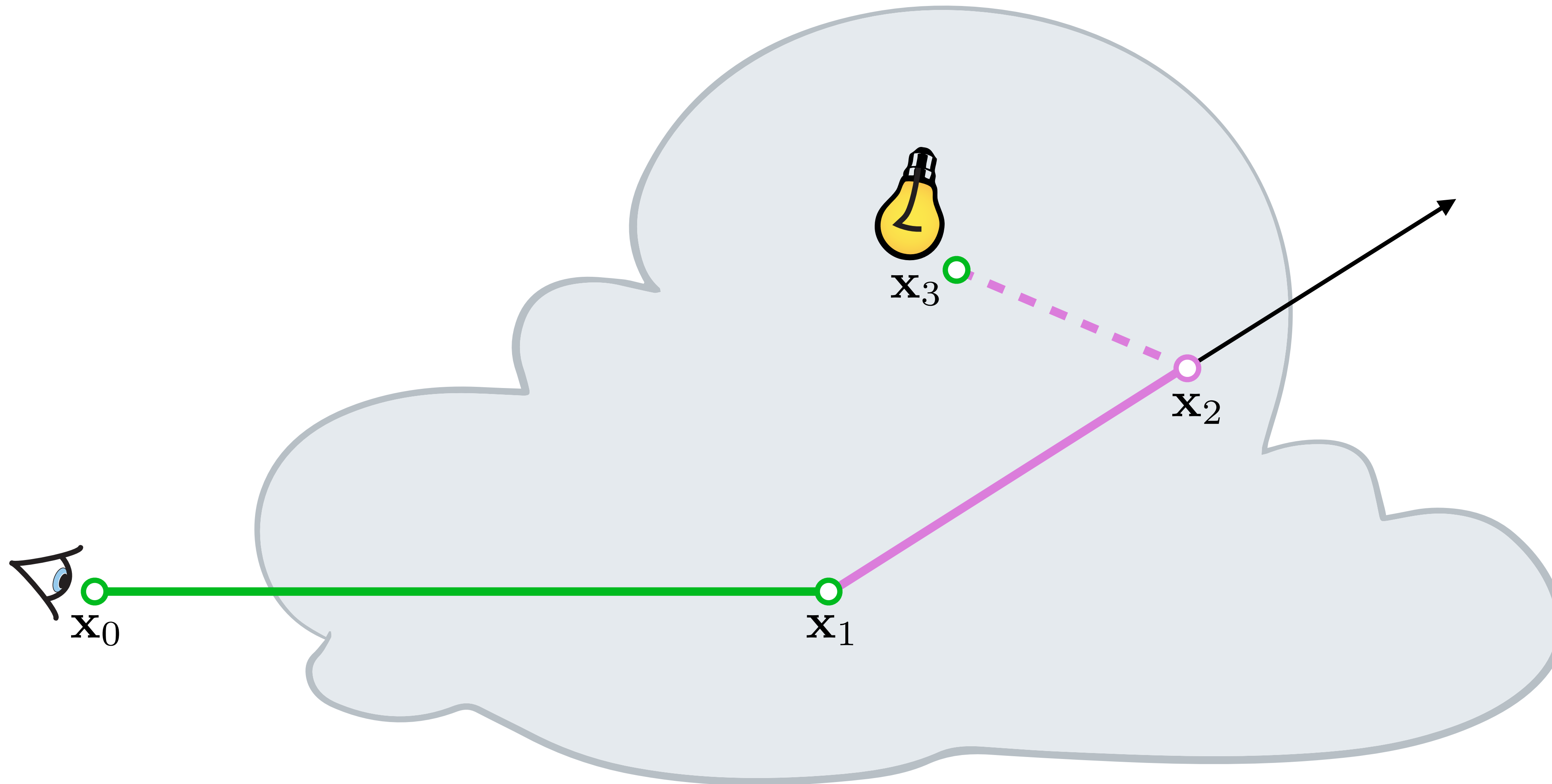


Equiangular sampling

Transmittance sampling

MIS combination

# UNIDIRECTIONAL + NEXT EVENT



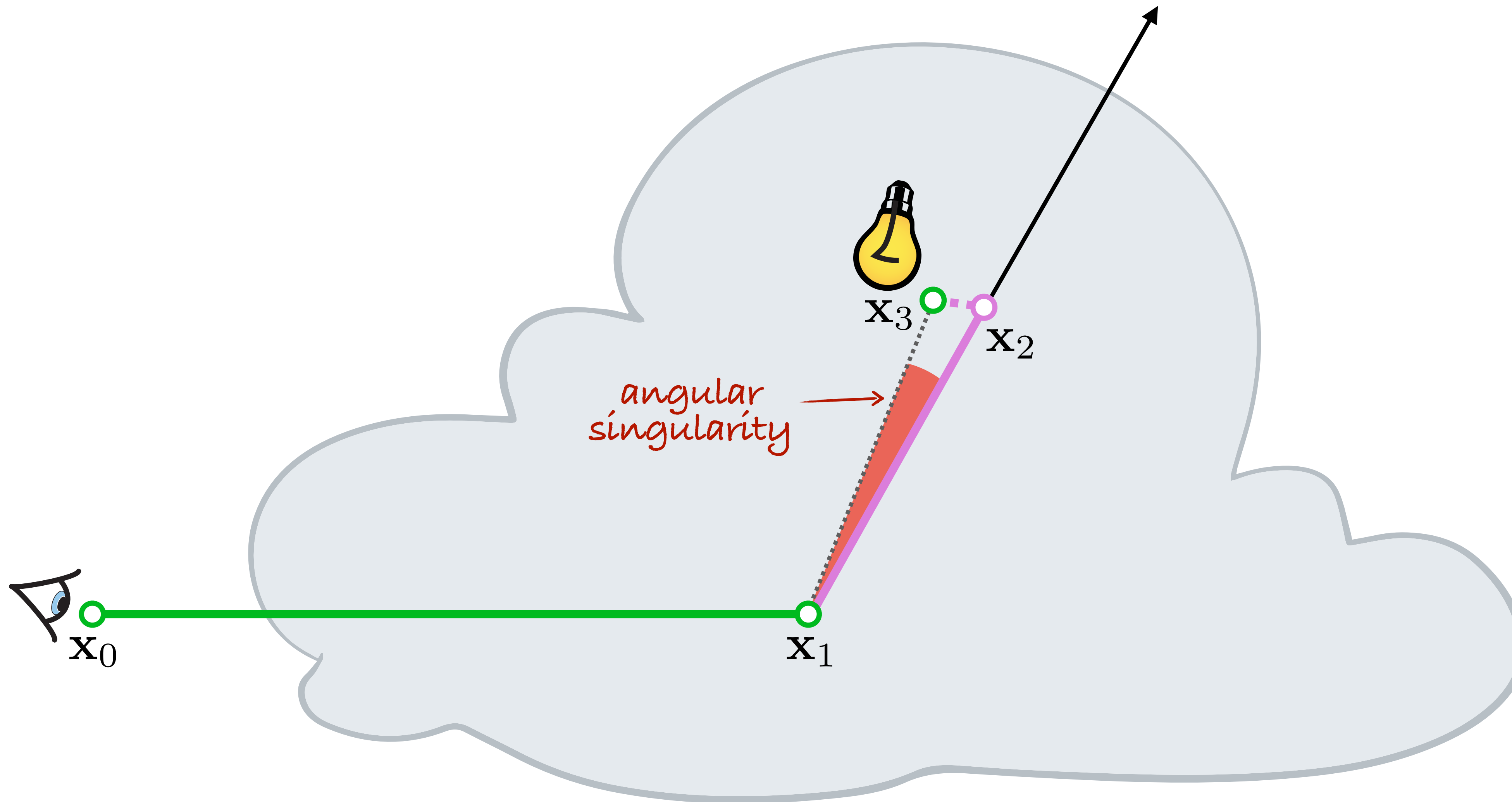


Transmittance connections

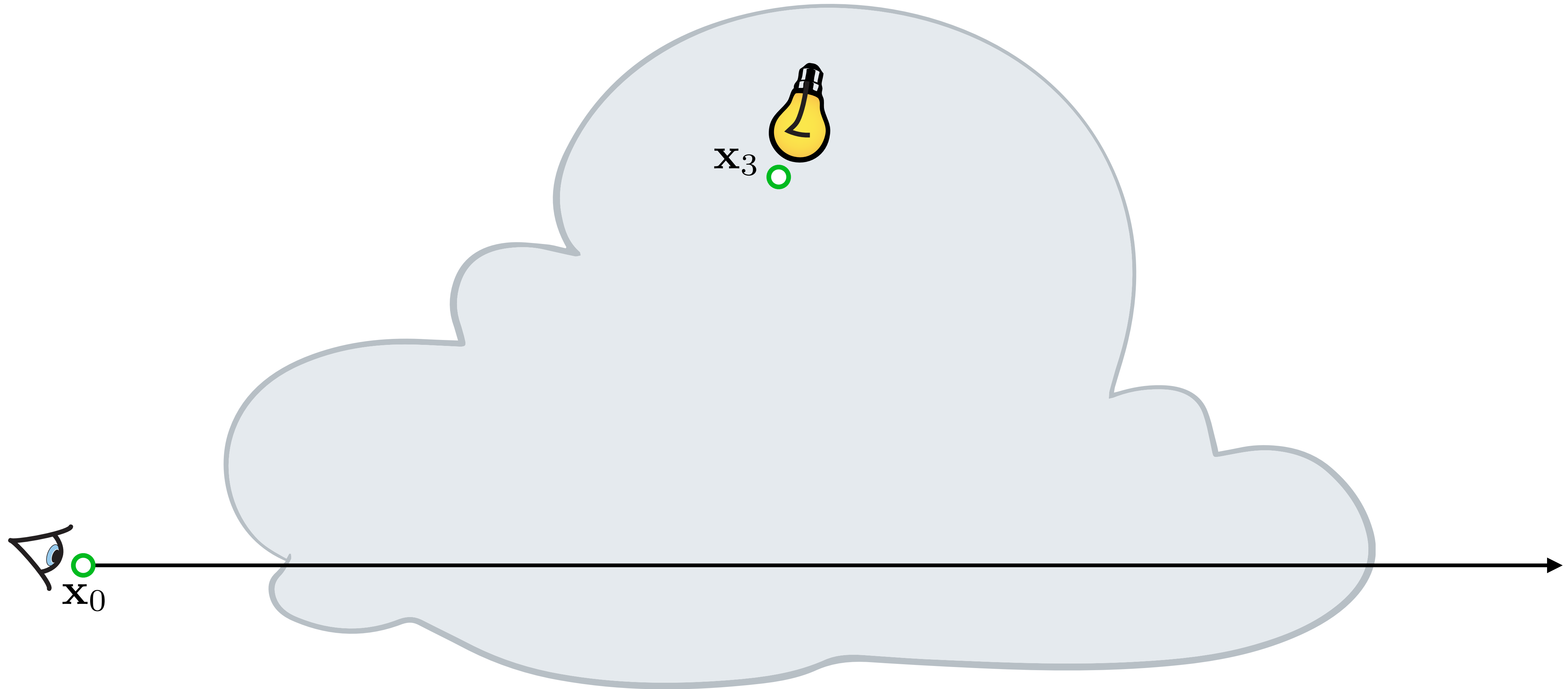


Equiangular connections

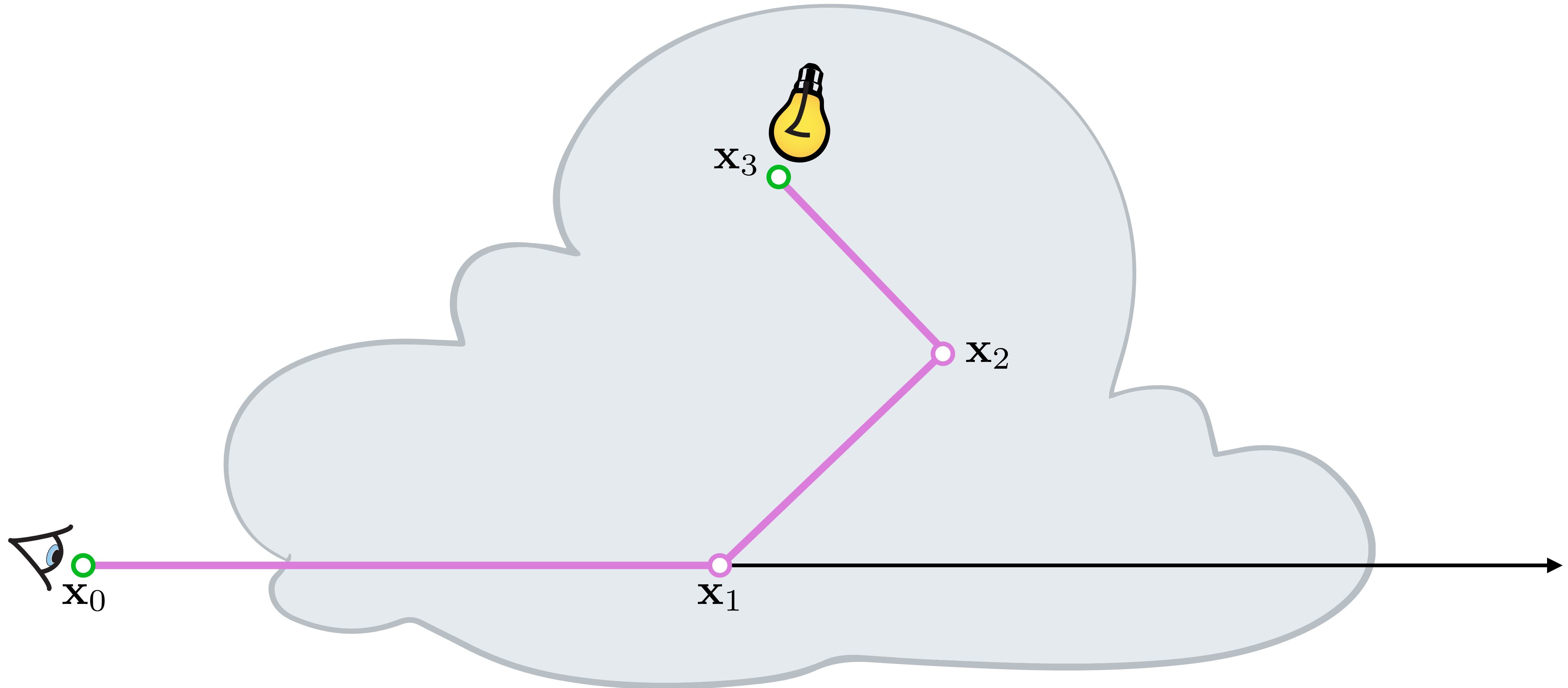
# UNIDIRECTIONAL + NEXT EVENT



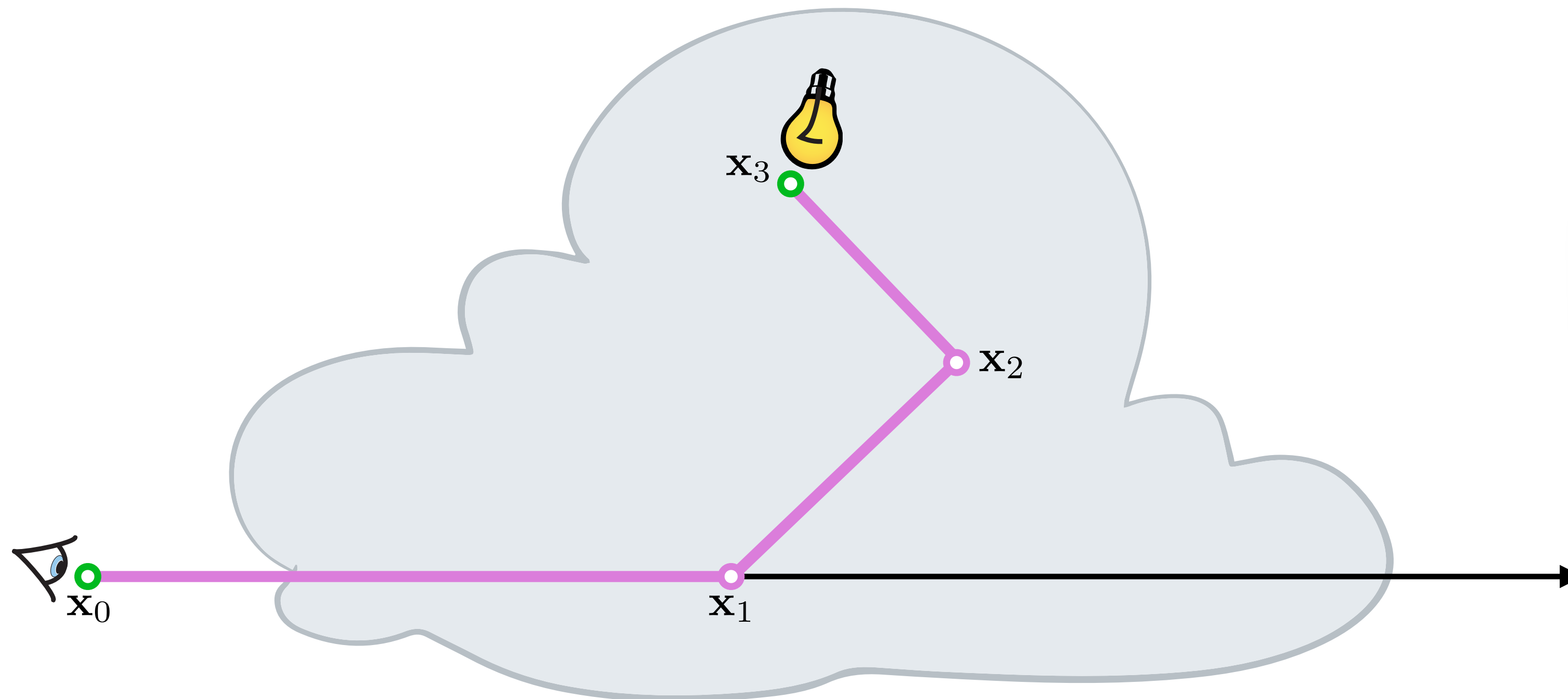
# JOINT PATH SAMPLING



# JOINT PATH SAMPLING



# JOINT PATH SAMPLING



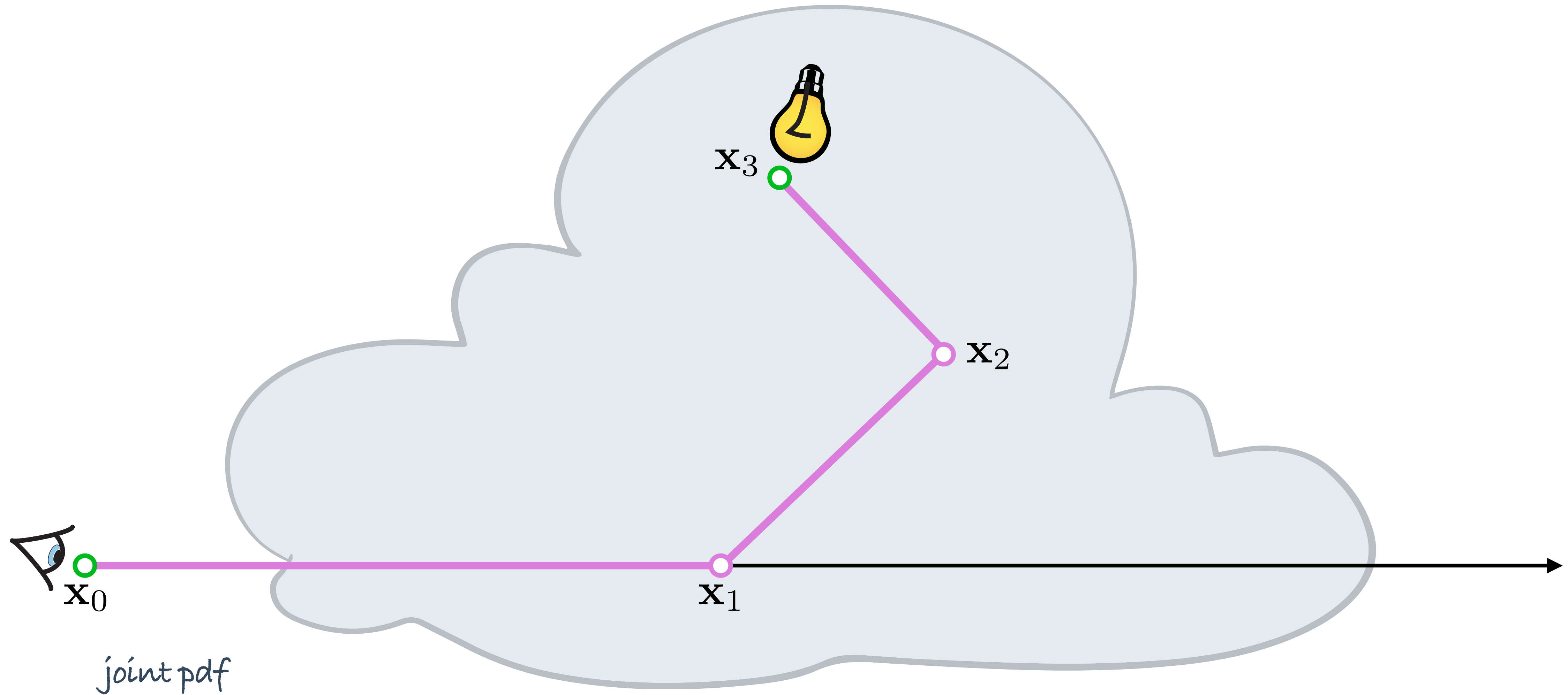
Joint path sampling:

- 1) Prescribe joint pdf
- 2) Derive conditional pdfs via successive joint pdf marginalization
- 3) Conditionals are obtained in reverse order

**TRADITIONAL:** prescribes conditional pdfs, no explicit control over joint pdf

**JOINT SAMPLING:** prescribe joint pdf, conditional pdfs derived from it

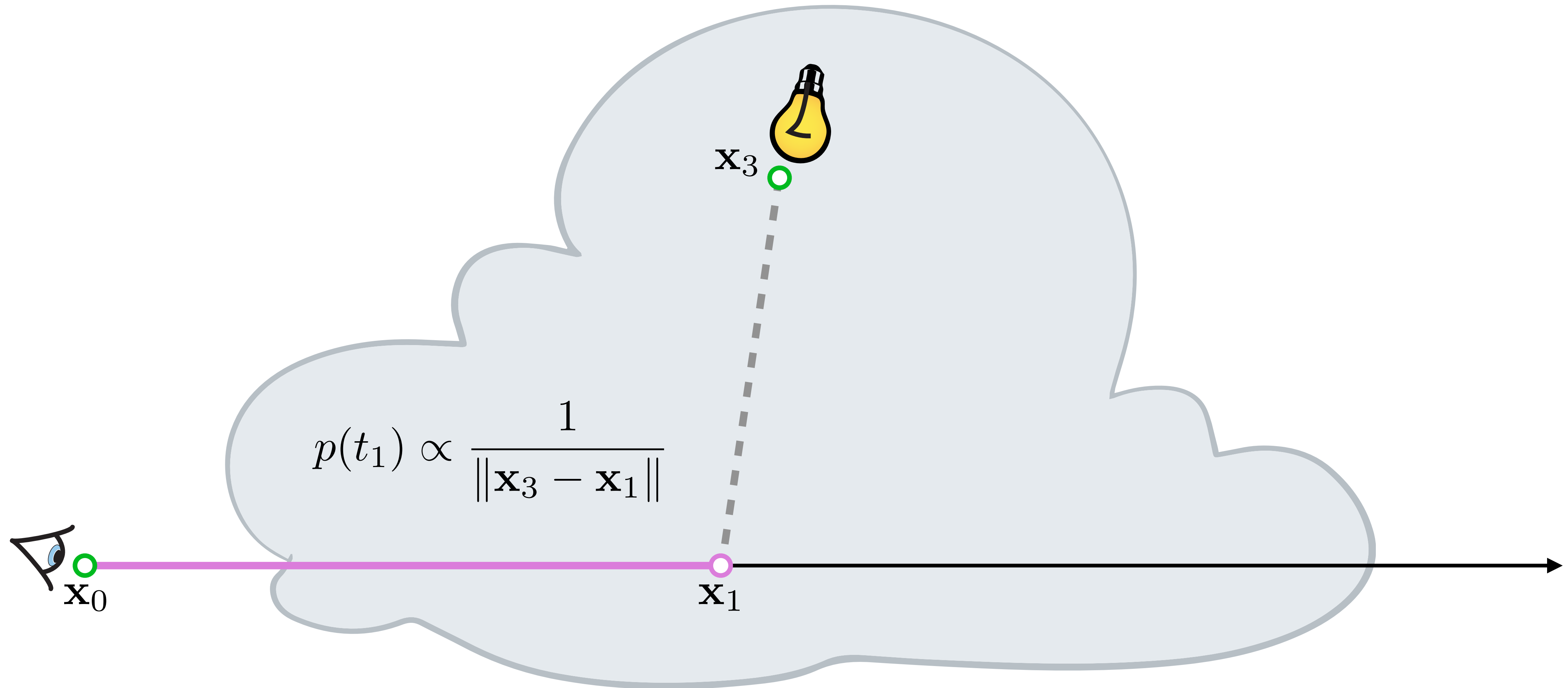
# JOINT PATH SAMPLING



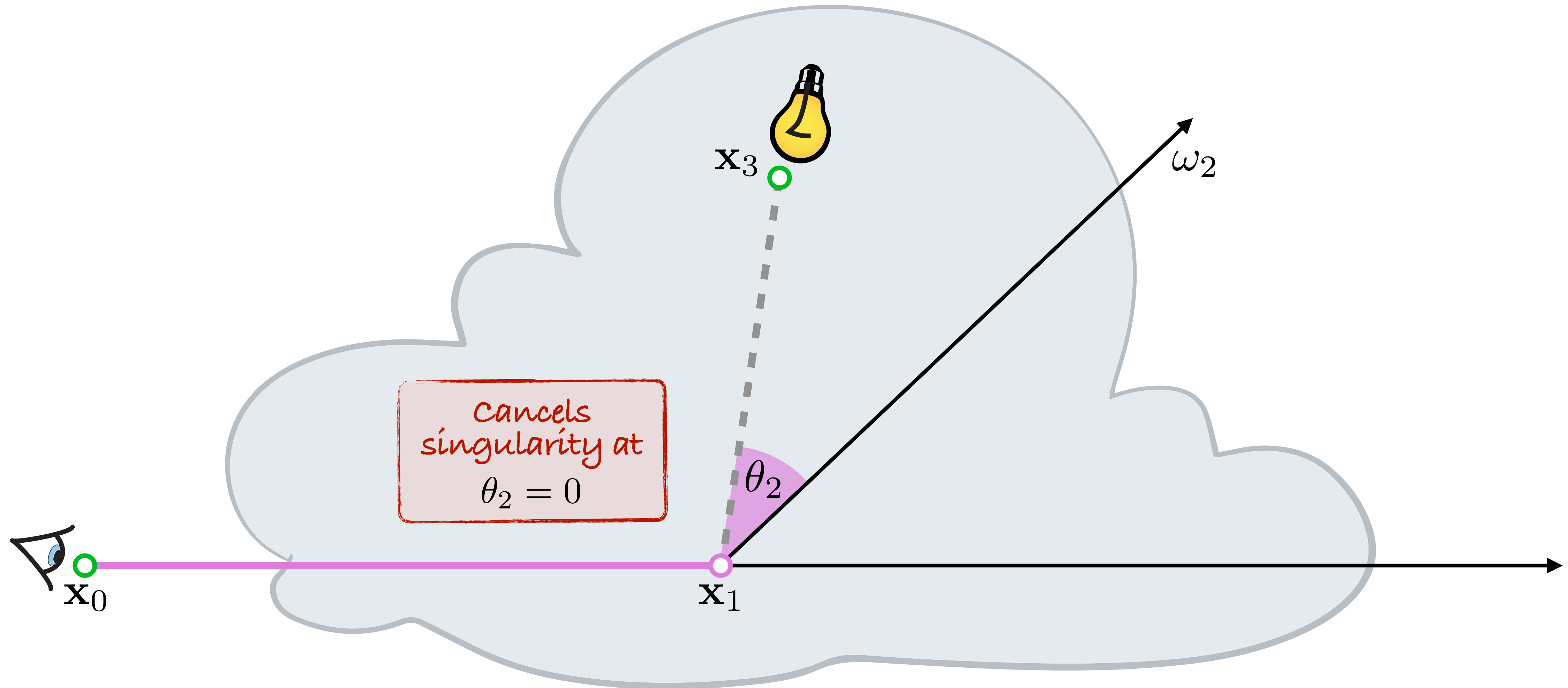
$$p(\mathbf{x}_1, \mathbf{x}_2) \propto G(\mathbf{x}_0, \mathbf{x}_1)G(\mathbf{x}_1, \mathbf{x}_2)G(\mathbf{x}_2, \mathbf{x}_3)$$



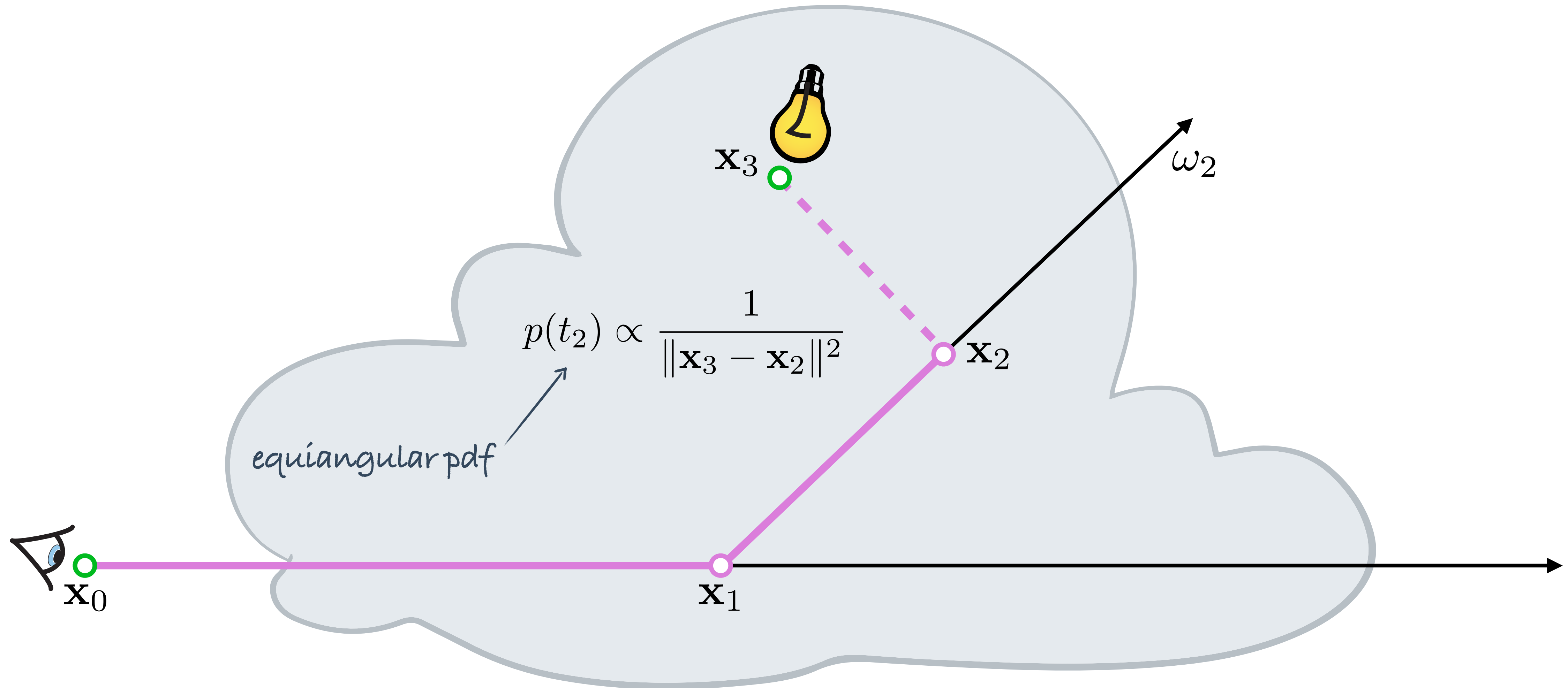
# JOINT PATH SAMPLING



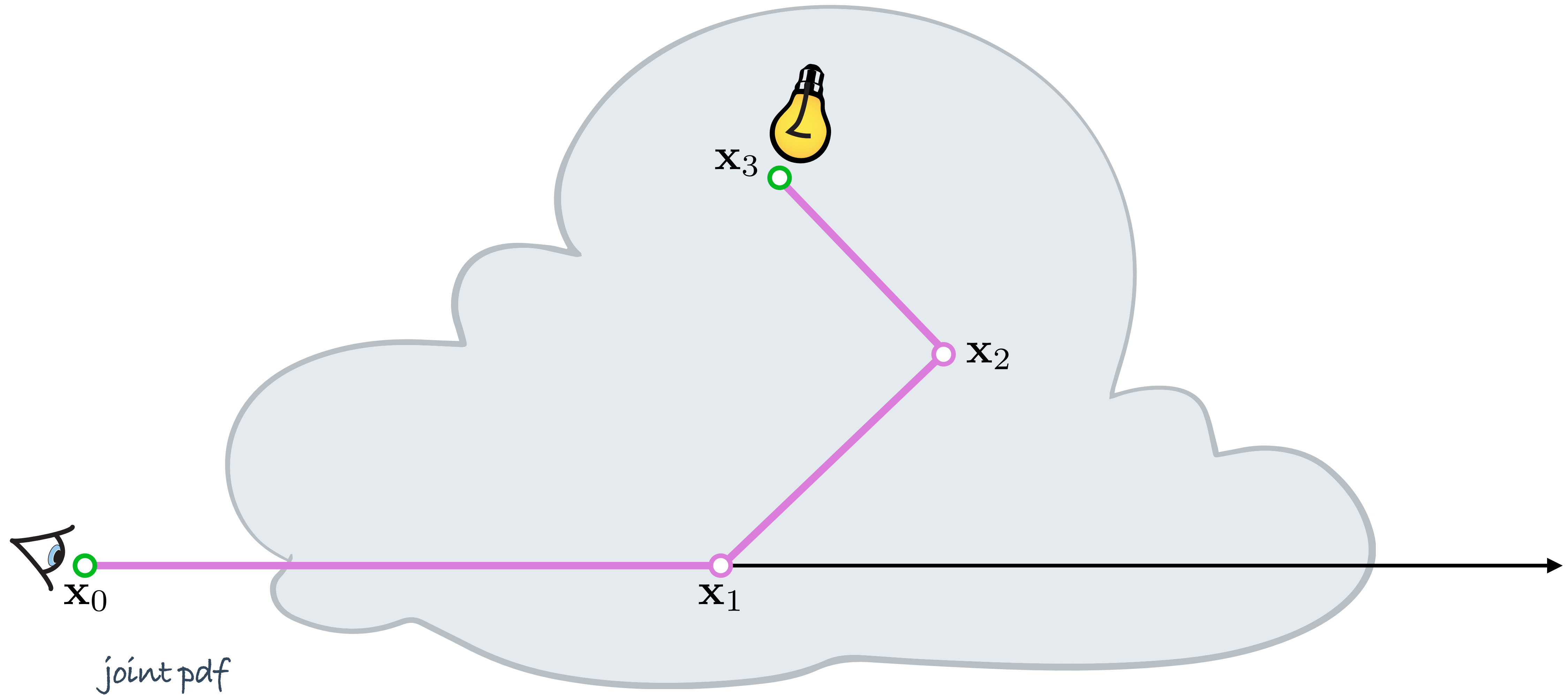
# JOINT PATH SAMPLING



# JOINT PATH SAMPLING



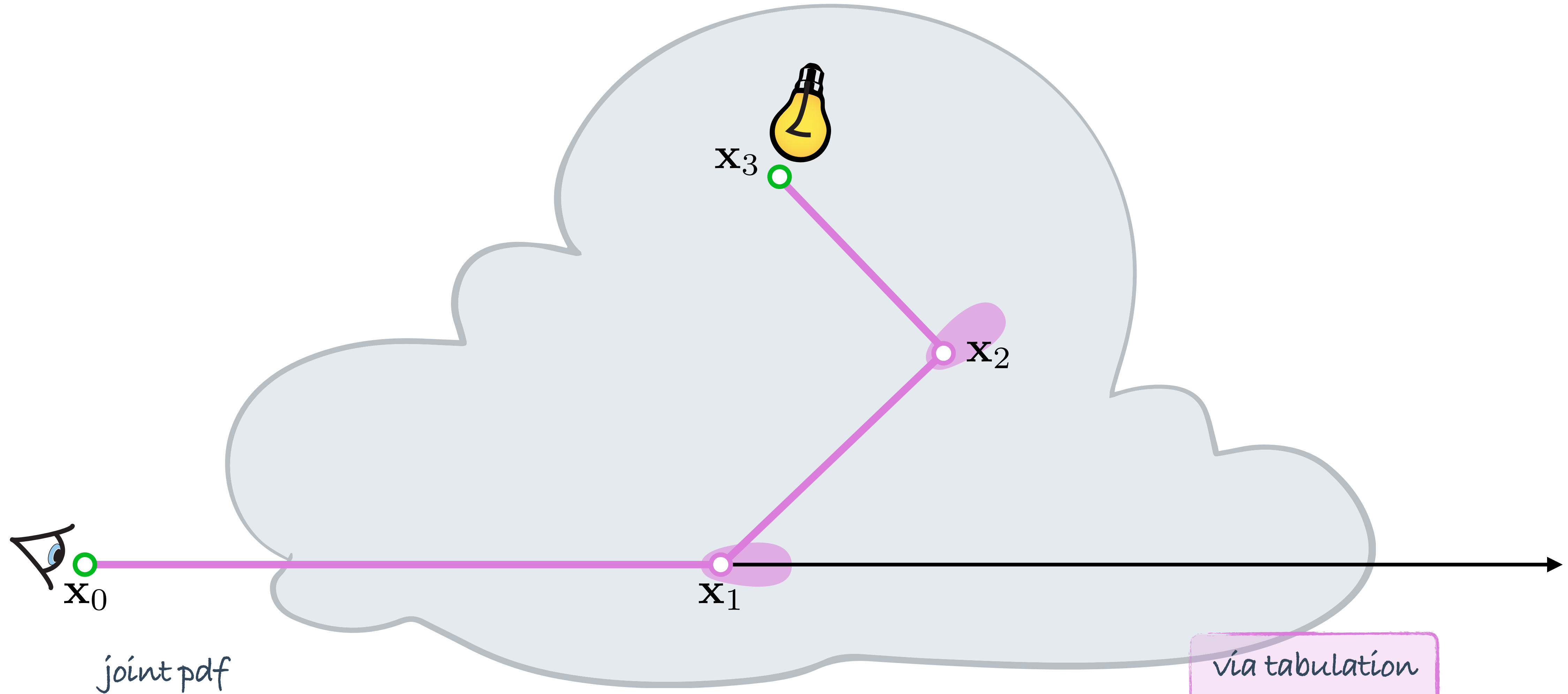
# JOINT PATH SAMPLING



joint pdf

$$p(\mathbf{x}_1, \mathbf{x}_2) \propto G(\mathbf{x}_0, \mathbf{x}_1)G(\mathbf{x}_1, \mathbf{x}_2)G(\mathbf{x}_2, \mathbf{x}_3)$$

# JOINT PATH SAMPLING



joint pdf

$$p(\mathbf{x}_1, \mathbf{x}_2) \propto G(\mathbf{x}_0, \mathbf{x}_1)G(\mathbf{x}_1, \mathbf{x}_2)G(\mathbf{x}_2, \mathbf{x}_3)f_s(\mathbf{x}_1)f_s(\mathbf{x}_2)$$

via tabulation

path lengths 1-3  
isotropic phase function



Transmittance



Equiangular



Joint sampling

path lengths 1-8  
isotropic phase function



Transmittance



Equiangular



Joint sampling

path lengths 1-3  
anisotropic phase function



Transmittance connections



Joint tabulated path sampling



path lengths 1-8  
anisotropic phase function

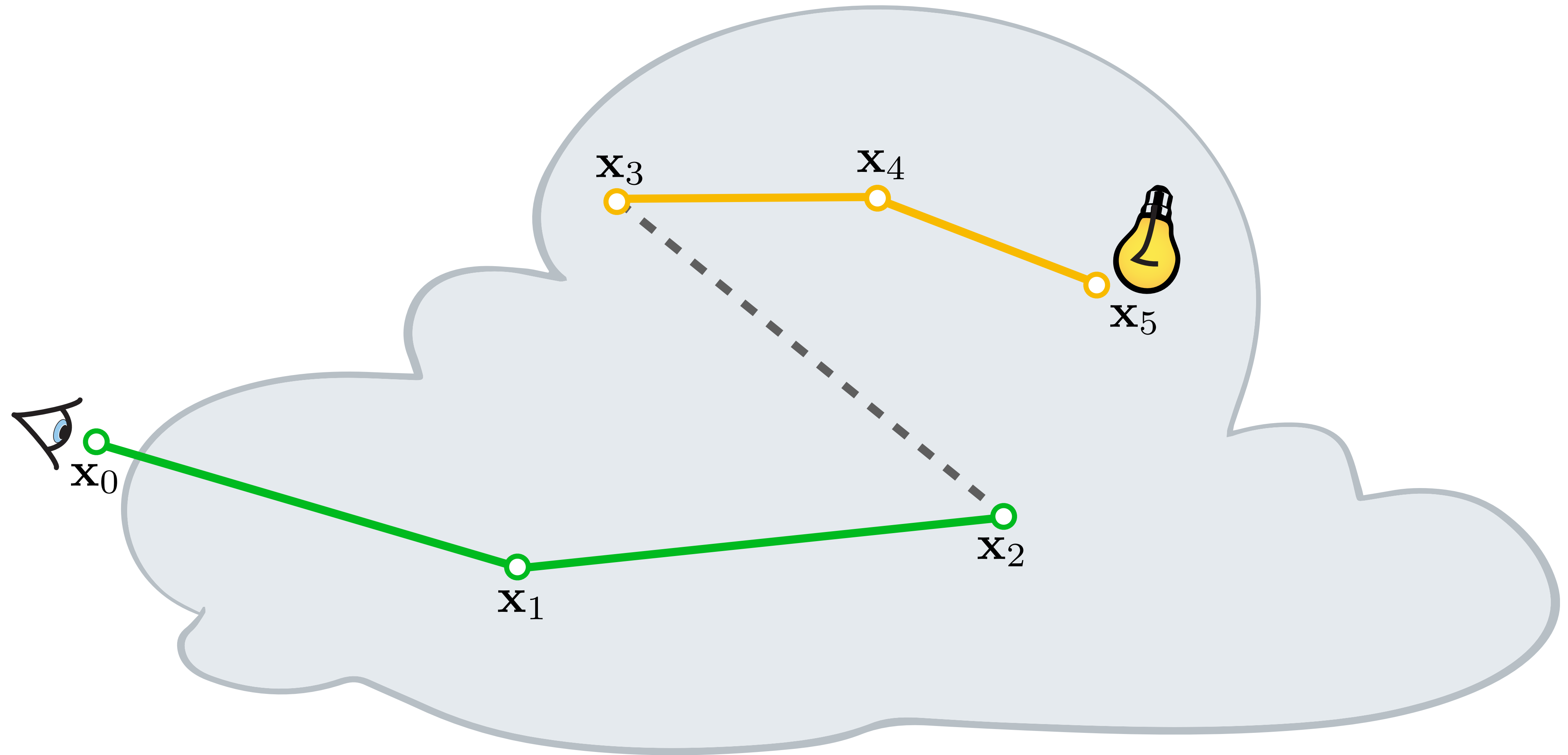


Transmittance connections



Joint tabulated path sampling

# BIDIRECTIONAL PATH TRACING



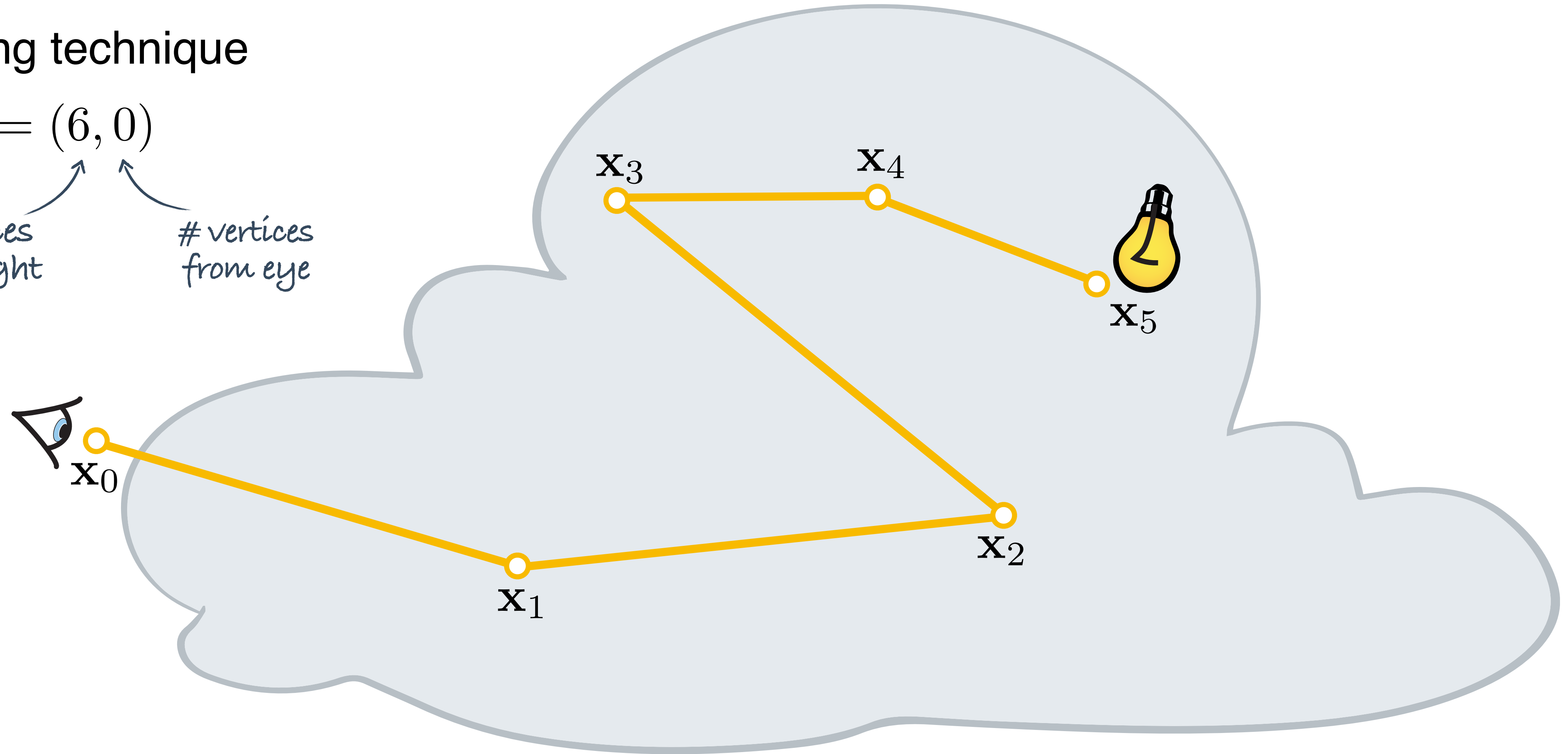
# BIDIRECTIONAL PATH TRACING

## Sampling technique

$$(s, t) = (6, 0)$$

# vertices  
from light

# vertices  
from eye



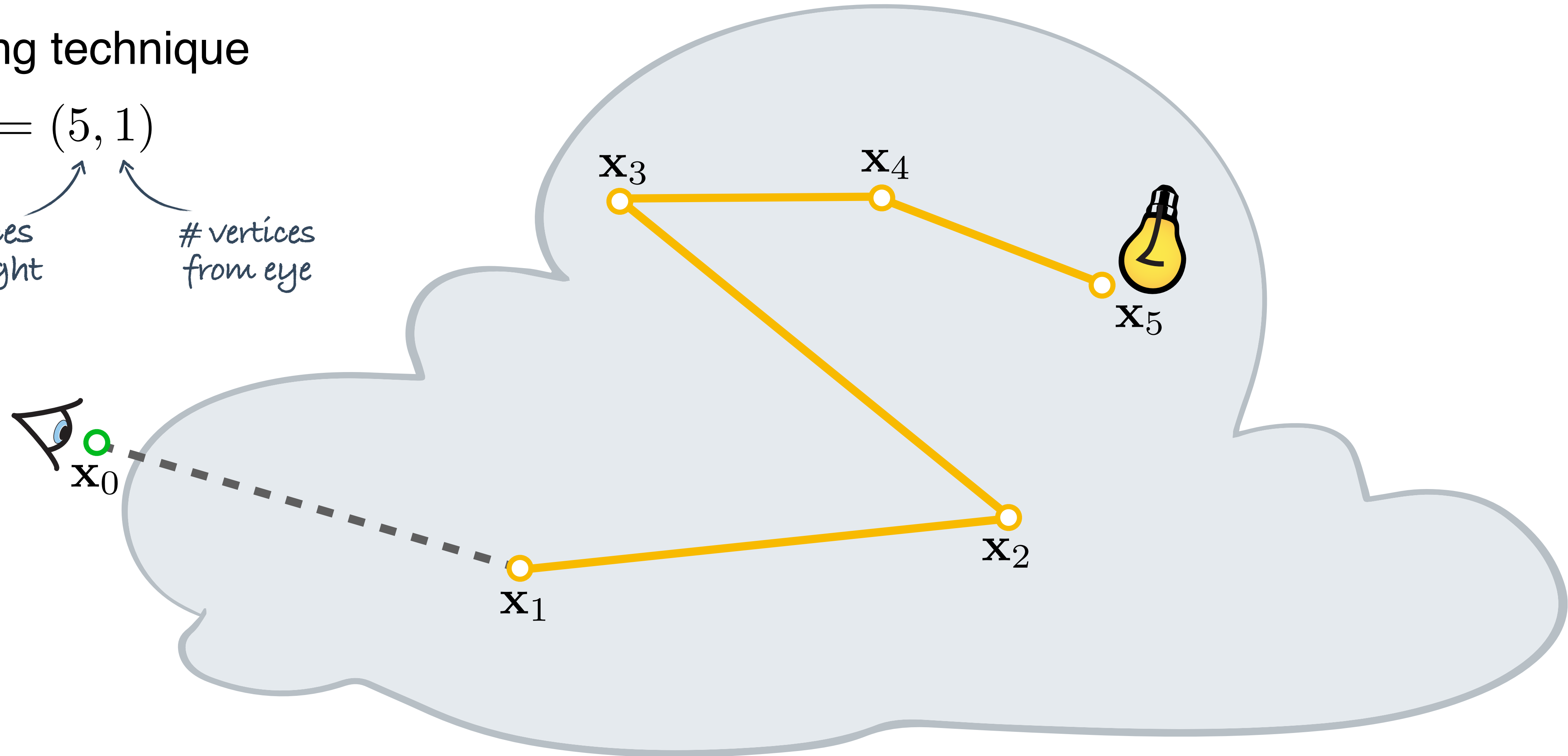
# BIDIRECTIONAL PATH TRACING

## Sampling technique

$$(s, t) = (5, 1)$$

# vertices  
from light

# vertices  
from eye



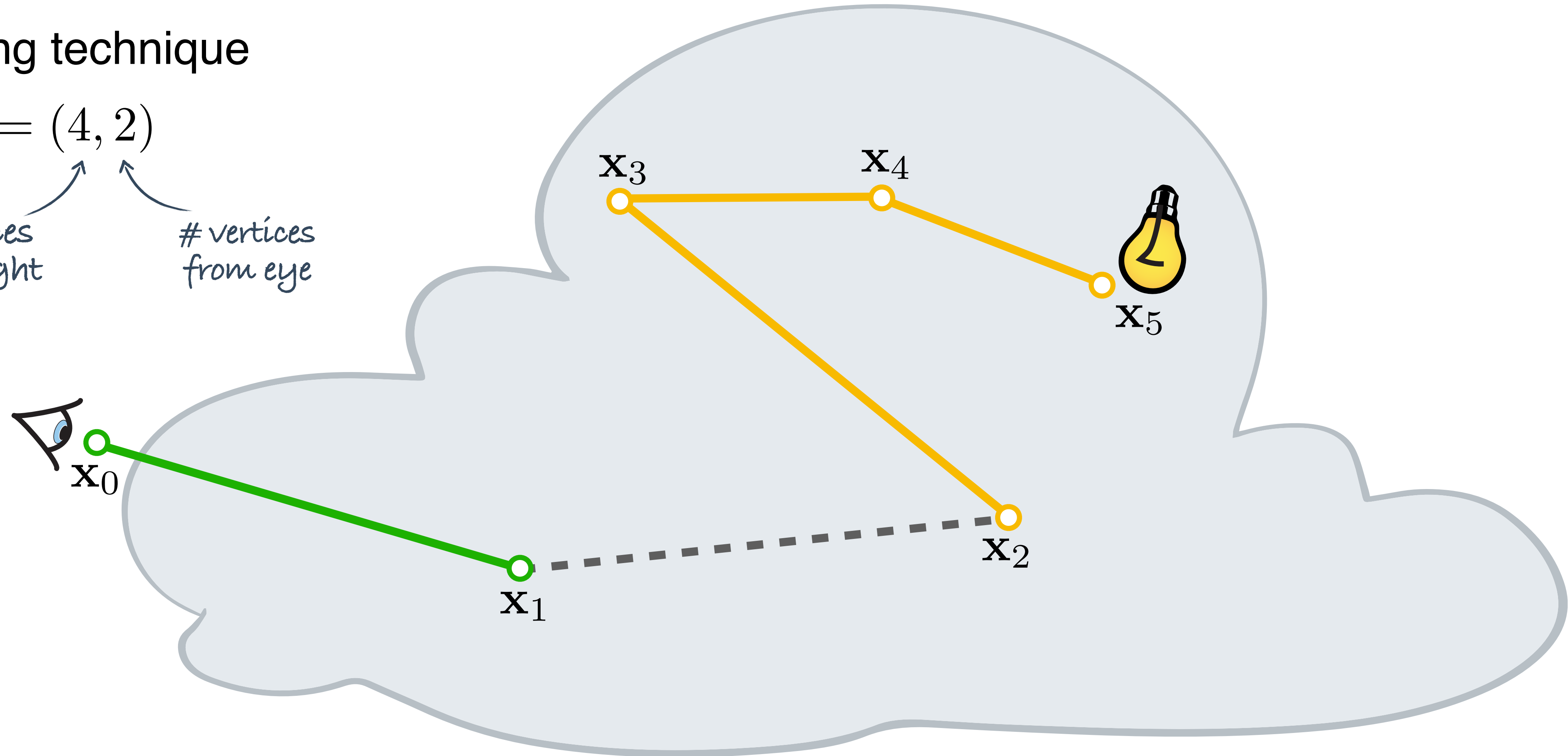
# BIDIRECTIONAL PATH TRACING

## Sampling technique

$$(s, t) = (4, 2)$$

# vertices  
from light

# vertices  
from eye



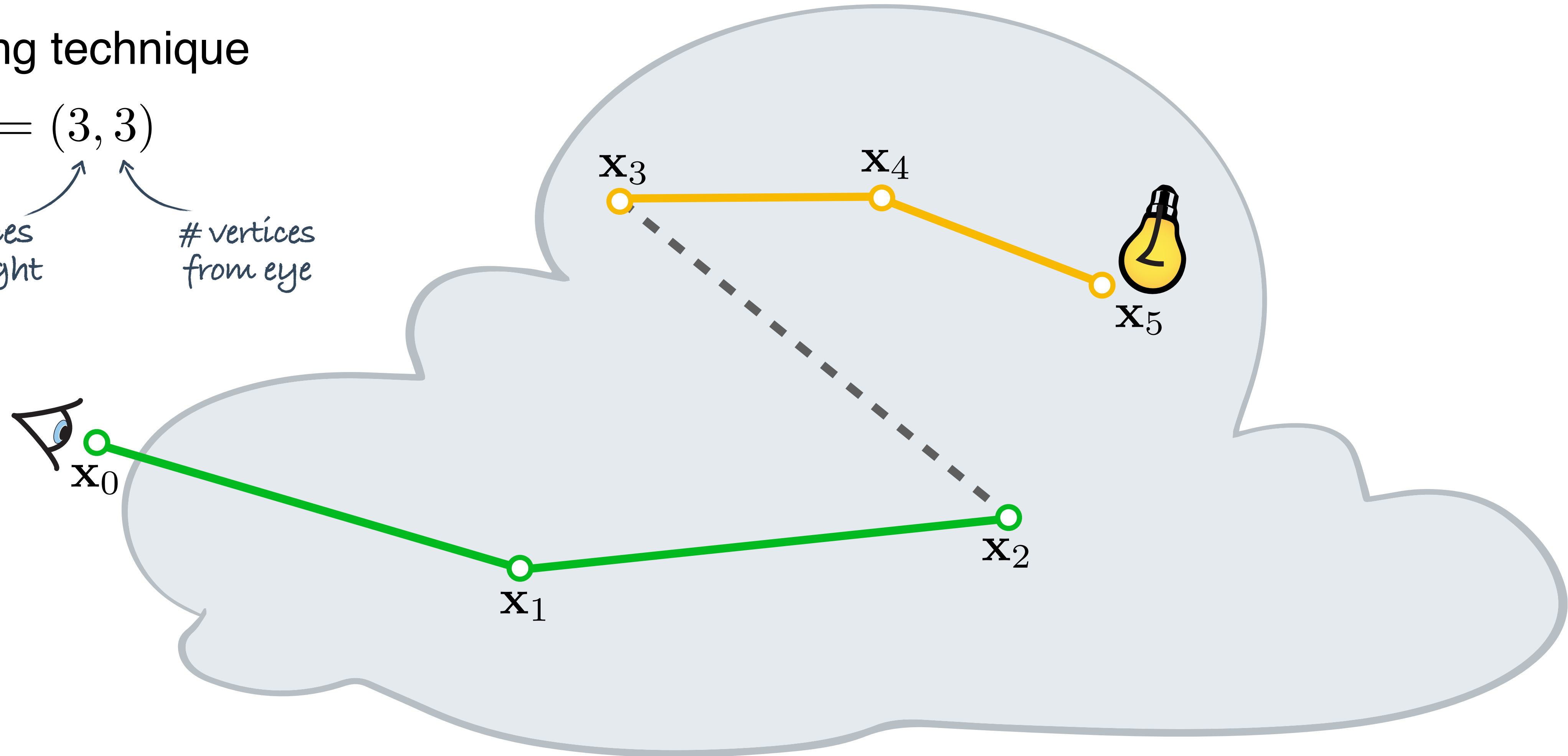
# BIDIRECTIONAL PATH TRACING

## Sampling technique

$$(s, t) = (3, 3)$$

# vertices  
from light

# vertices  
from eye



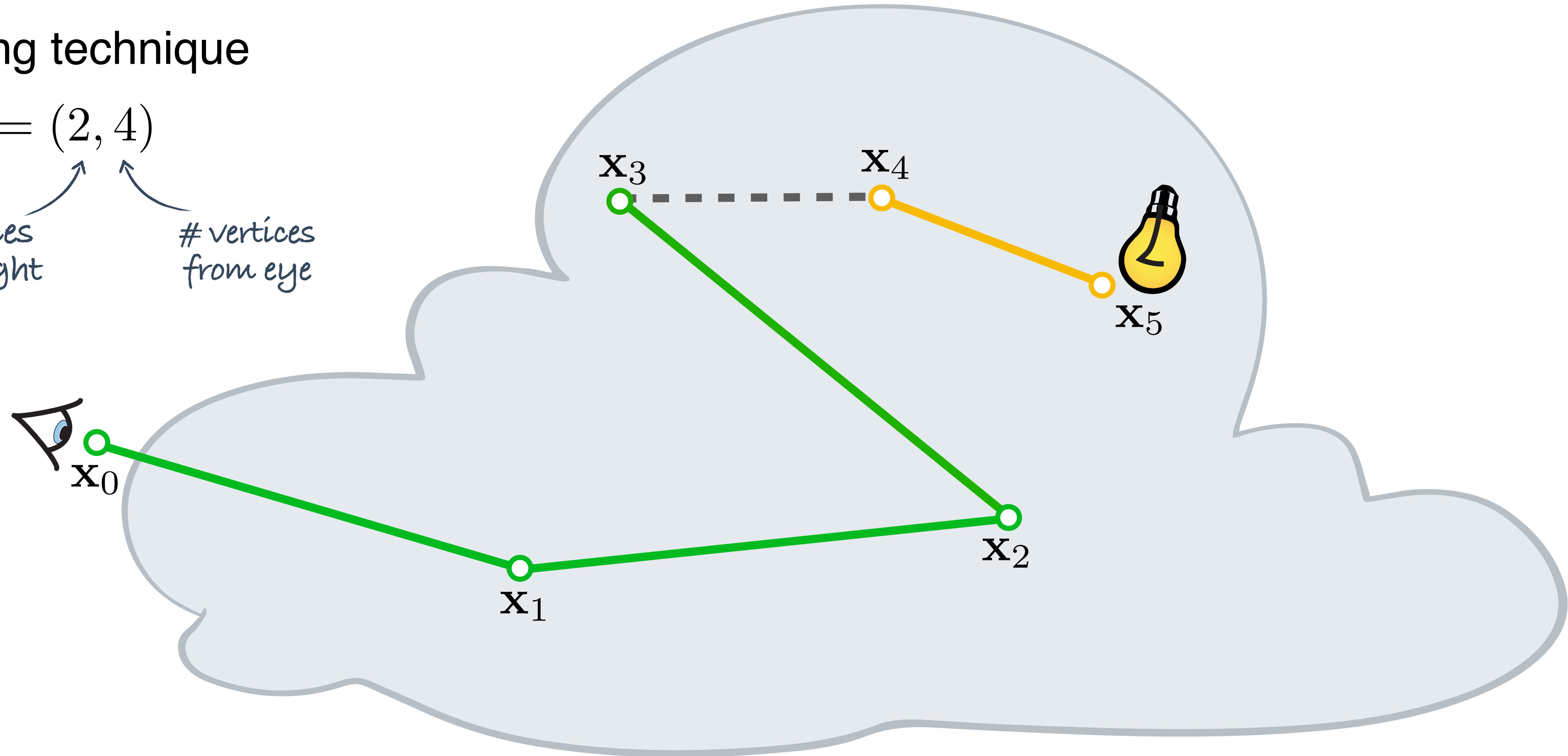
# BIDIRECTIONAL PATH TRACING

## Sampling technique

$$(s, t) = (2, 4)$$

# vertices  
from light

# vertices  
from eye



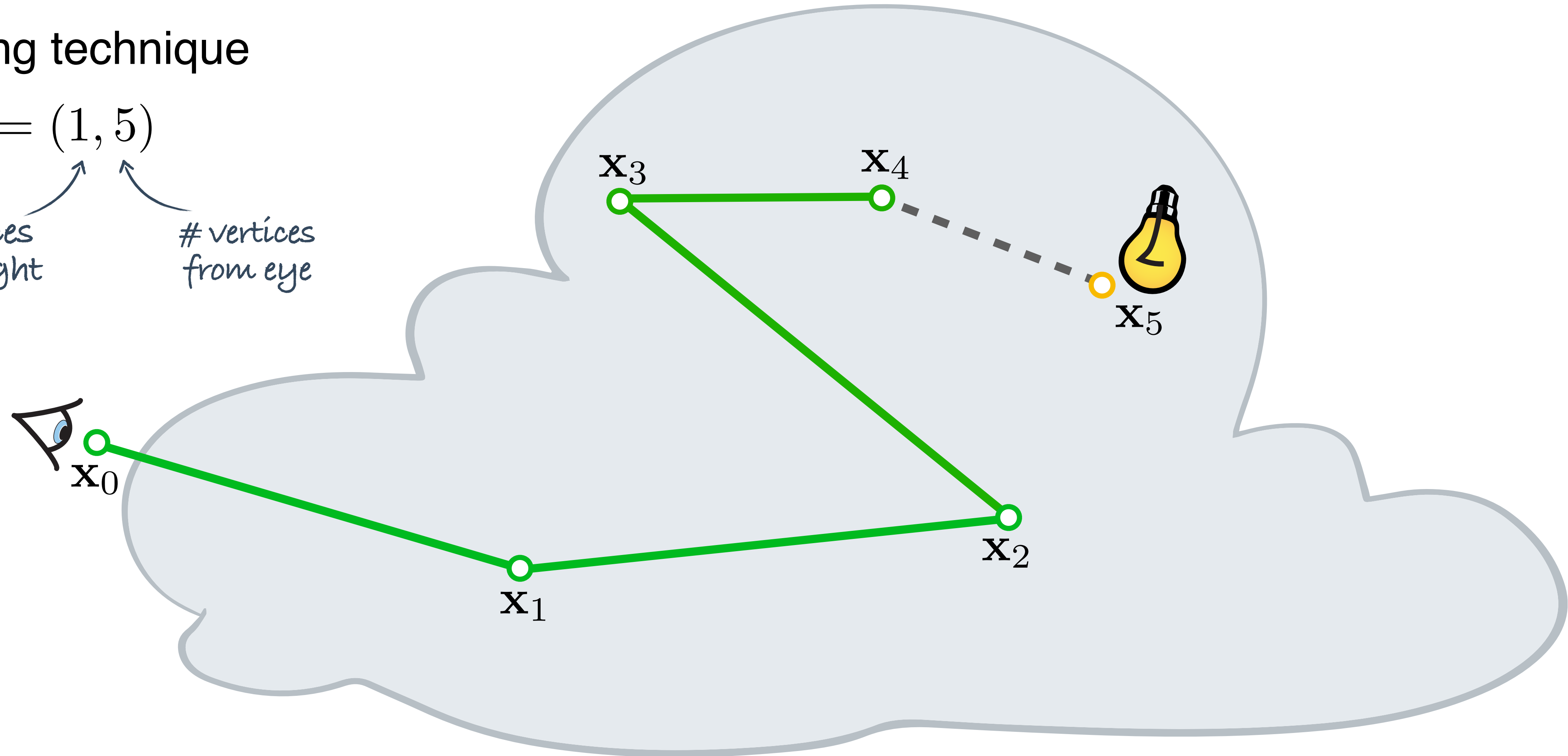
# BIDIRECTIONAL PATH TRACING

## Sampling technique

$$(s, t) = (1, 5)$$

# vertices  
from light

# vertices  
from eye





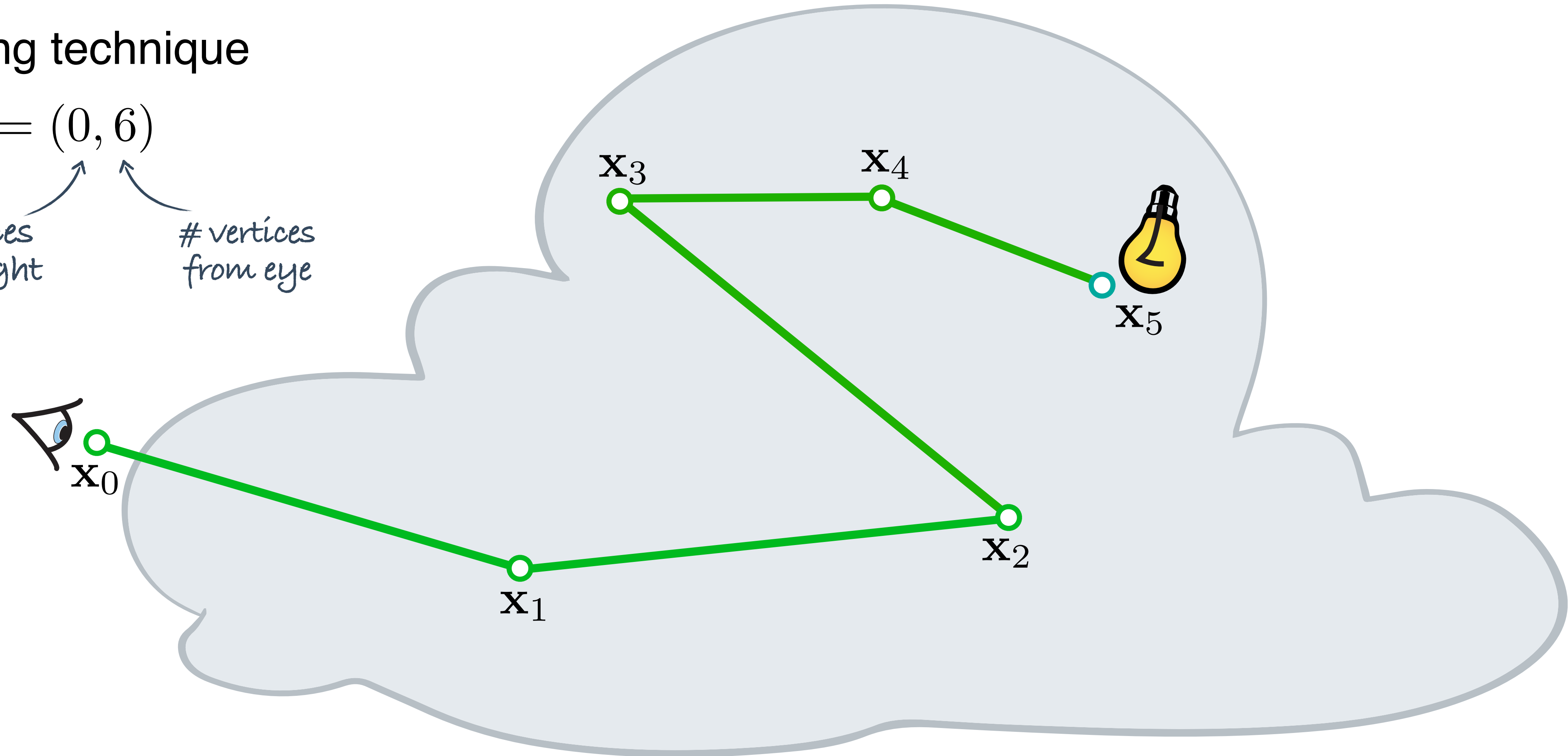
# BIDIRECTIONAL PATH TRACING

## Sampling technique

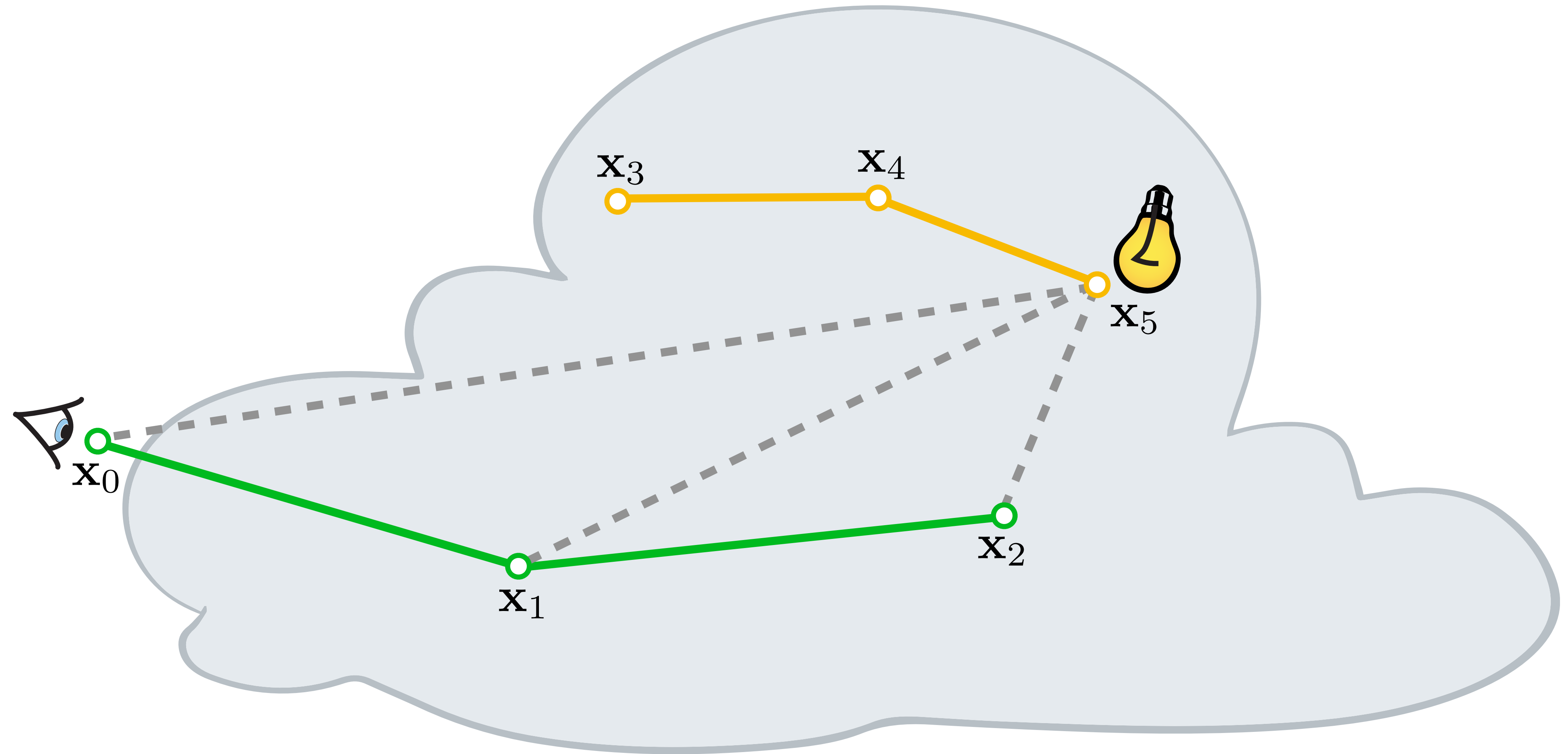
$$(s, t) = (0, 6)$$

# vertices  
from light

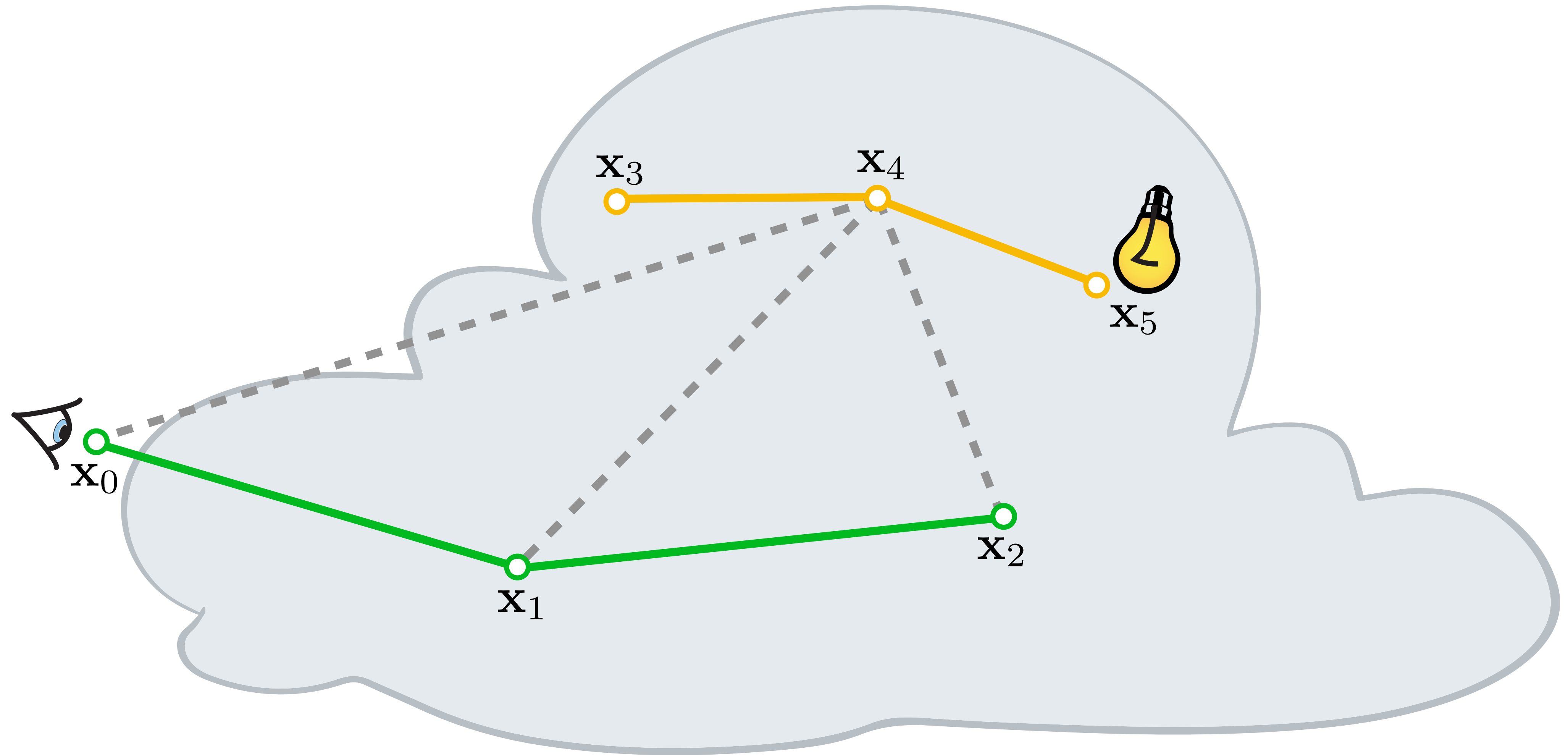
# vertices  
from eye



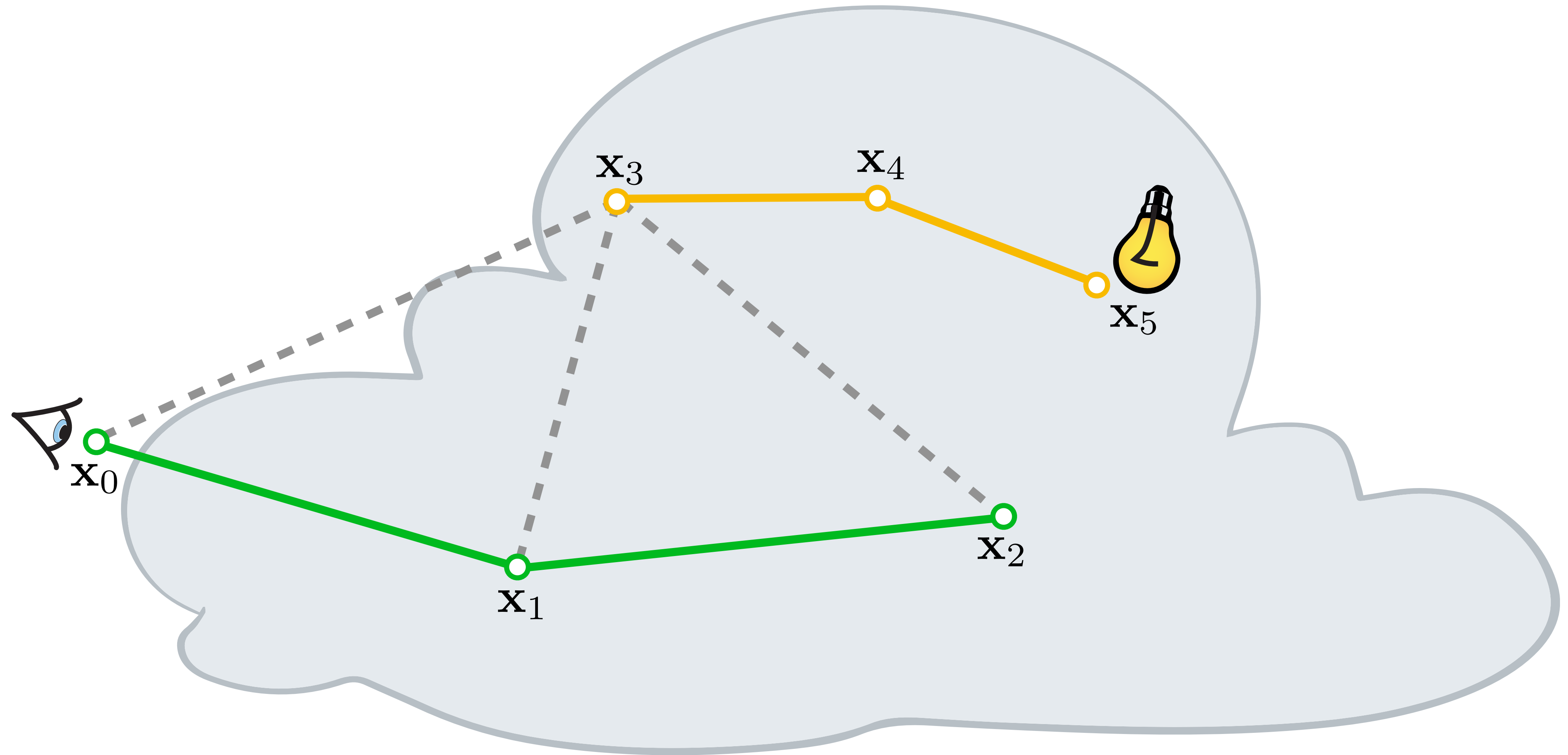
# BIDIRECTIONAL PATH TRACING



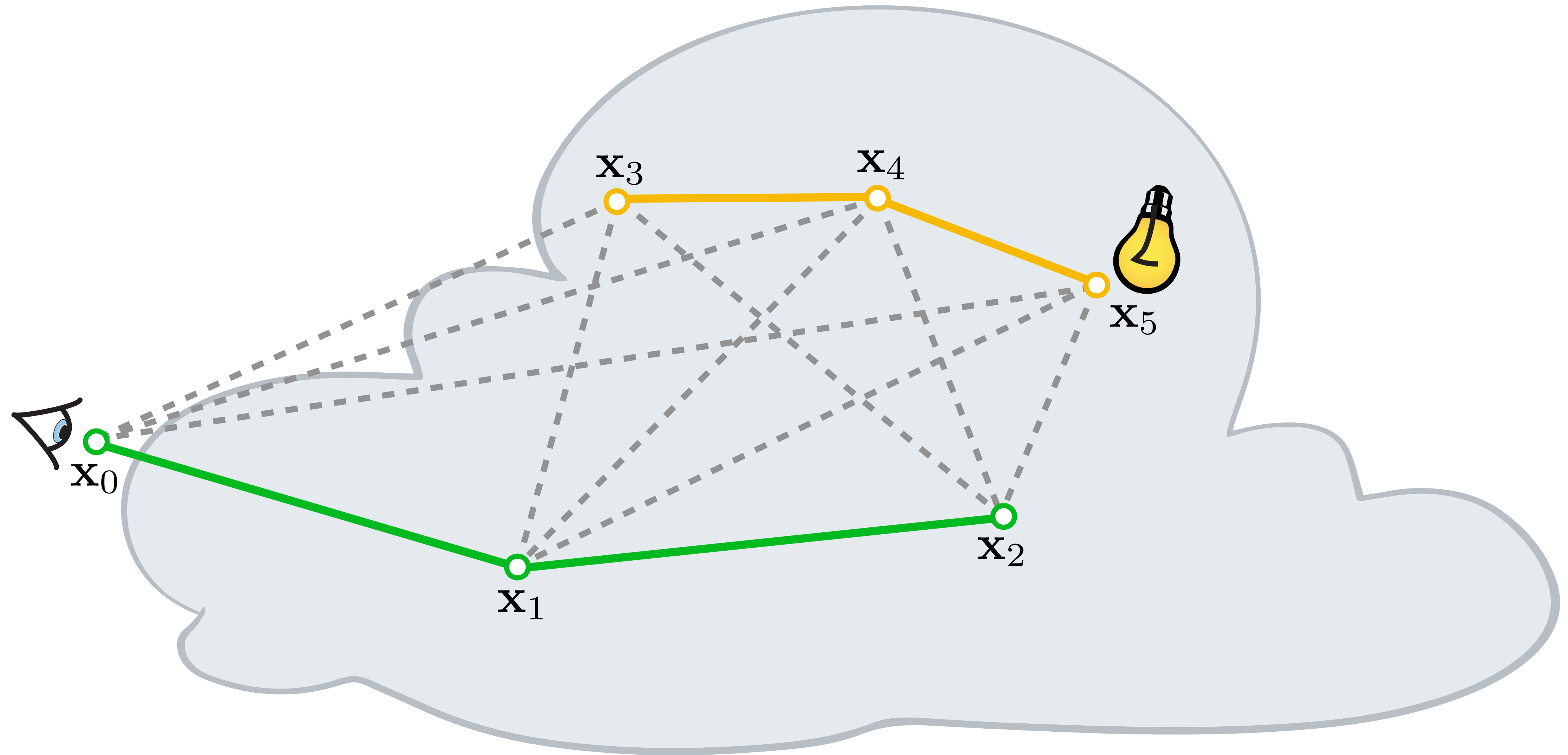
# BIDIRECTIONAL PATH TRACING



# BIDIRECTIONAL PATH TRACING



# BIDIRECTIONAL PATH TRACING



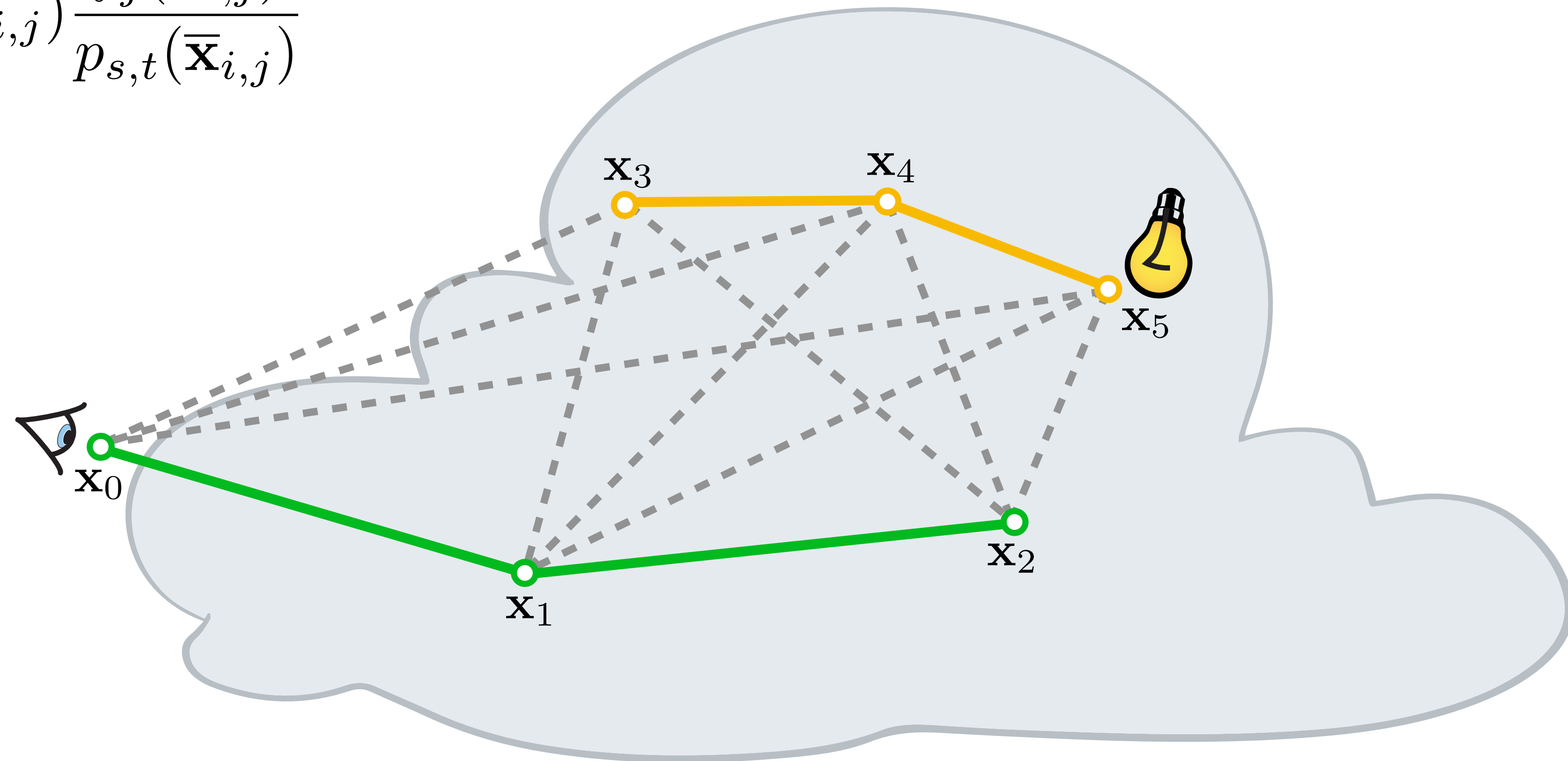
# BIDIRECTIONAL PATH TRACING

Combined MIS pixel estimator:

$$\langle I_j \rangle = \sum_{s \geq 0} \sum_{t \geq 0} w_{s,t}(\bar{\mathbf{x}}_{i,j}) \frac{f_j(\bar{\mathbf{x}}_{i,j})}{p_{s,t}(\bar{\mathbf{x}}_{i,j})}$$

# vertices  
from light

# vertices  
from eye



## UNIDIRECTIONAL SAMPLING

- ▶ Almost ideal on paper, rarely useful in practice

## NEXT EVENT ESTIMATION

- ▶ Improvement, but singularity in indirect lighting (reduced convergence rate)

## JOINT PATH SAMPLING

- ▶ Substantial improvement in the presence of singularities

## BIDIRECTIONAL PATH TRACING

- ▶ Avoids singularities, more robust thanks to mixing many sampling techniques
- ▶ Difficult to implement