

# Image-space Control Variates for Rendering

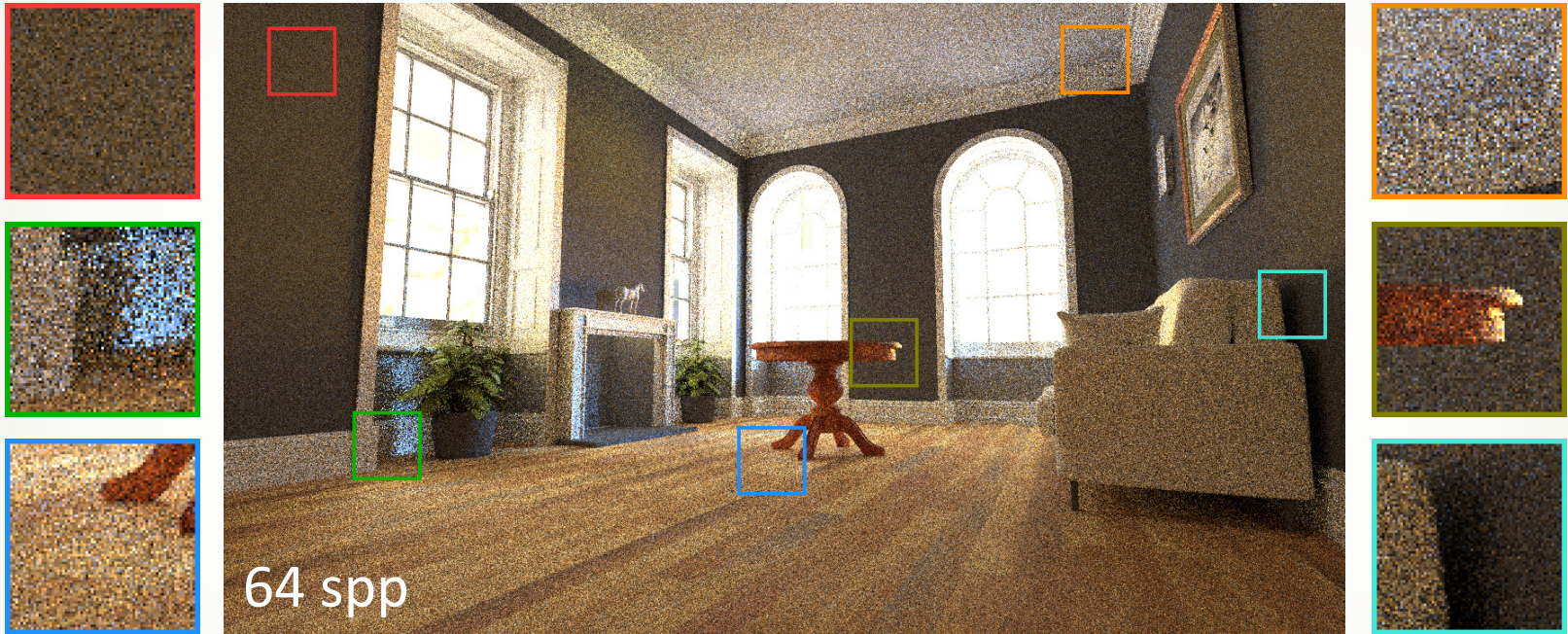
Fabrice Rousselle<sup>1</sup> Wojciech Jarosz<sup>2</sup> Jan Novák<sup>1</sup>

<sup>1</sup>Disney Research <sup>2</sup>Dartmouth College

[SA2016.SIGGRAPH.ORG](http://SA2016.SIGGRAPH.ORG)



# SPATIO-TEMPORAL COHERENCE



Original scene by Wig42, downloaded from [blendswap.com](http://blendswap.com)





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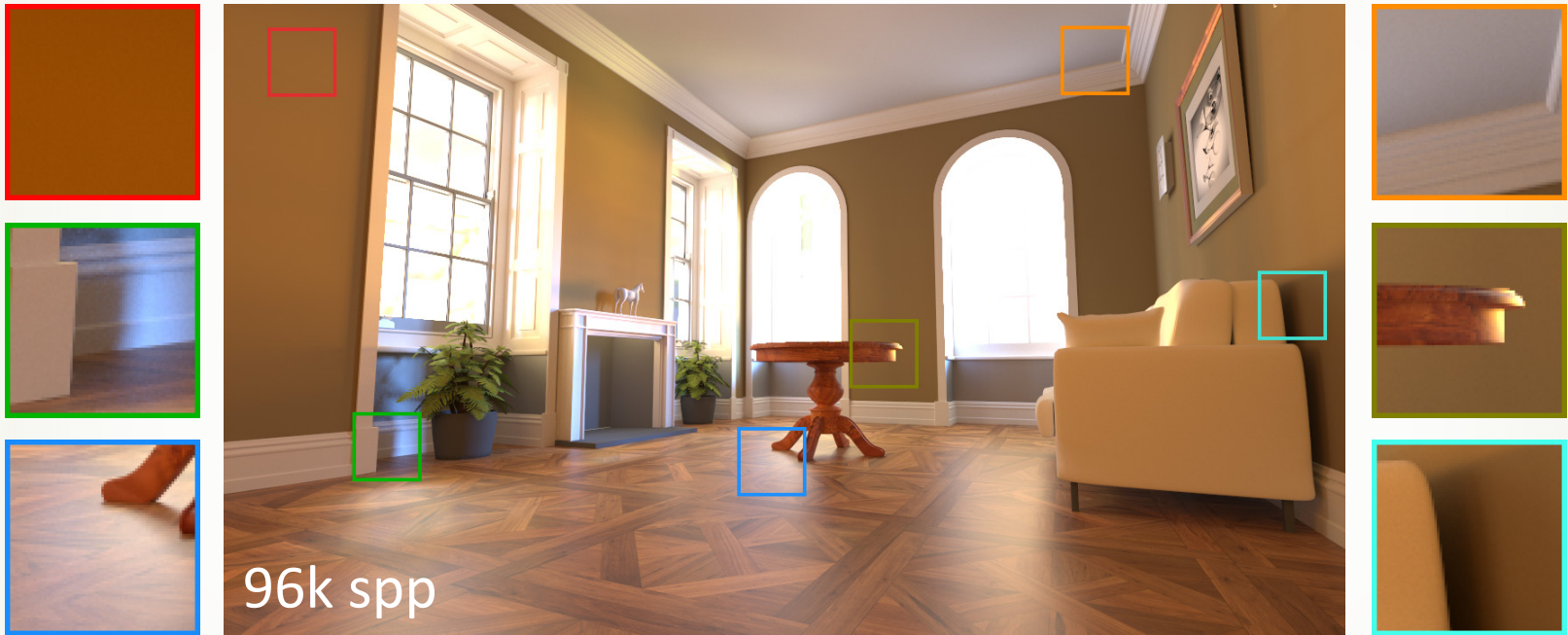


Original scene by Wig42, downloaded from [blendswap.com](https://www.blendswap.com)





# SPATIO-TEMPORAL COHERENCE

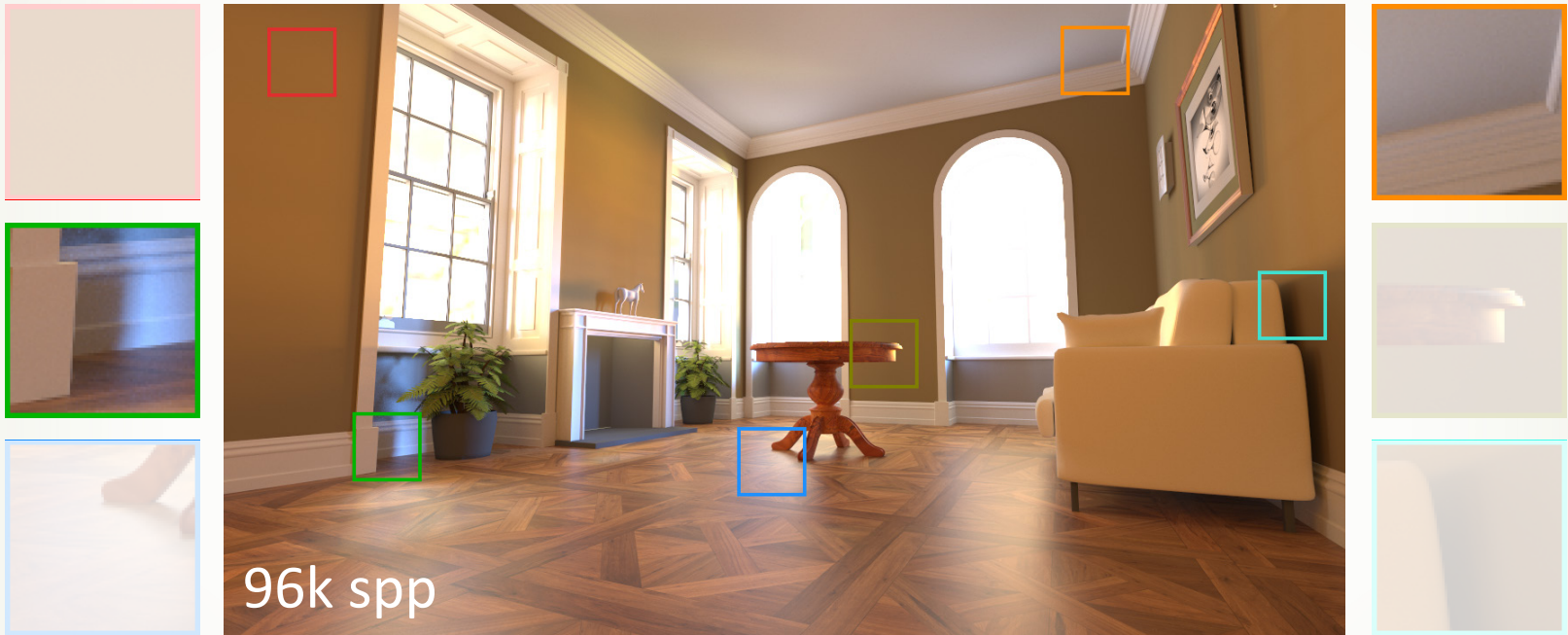


Original scene by Wig42, downloaded from [blendswap.com](https://www.blendswap.com)





# SPATIO-TEMPORAL COHERENCE



Original scene by Wig42, downloaded from [blendswap.com](https://www.blendswap.com)





## PREVIOUS WORKS

- Irradiance caching  
[Ward et al. 1988, ...]
- Photon mapping  
[Jensen 1995, ...]
- Image-space denoising  
[Rushmeier and Ward 1994, ...]
- Many more...



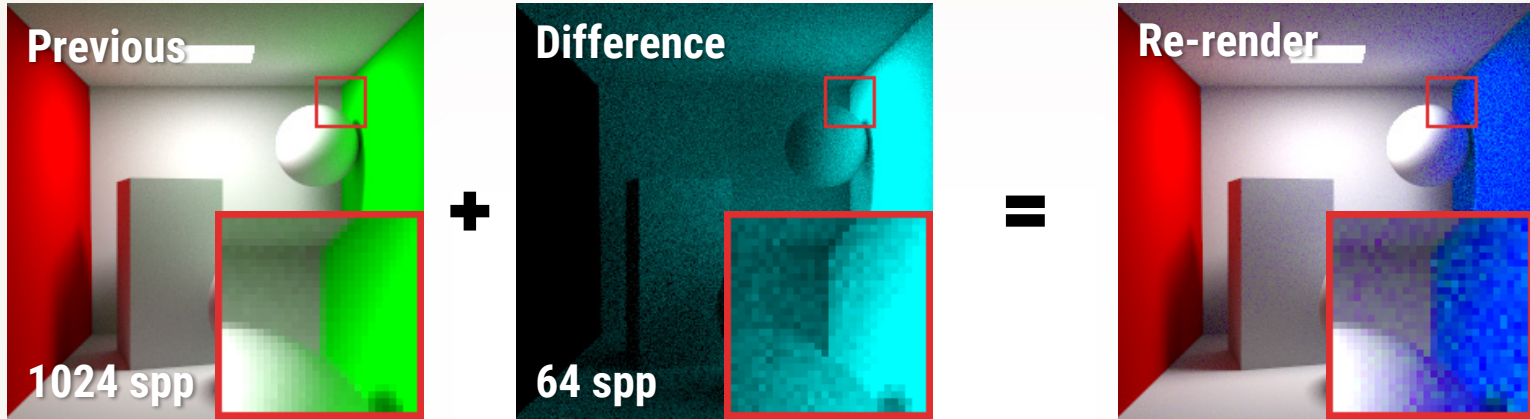




# PREVIOUS WORKS



Consistent scene editing [CSE], Günther and Grosch, EGSR 2015

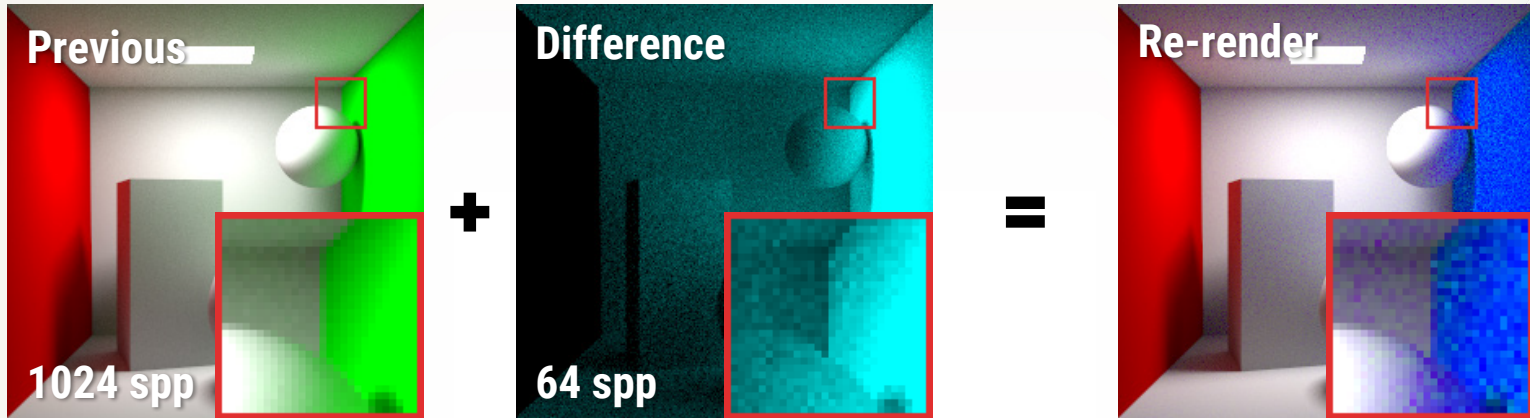




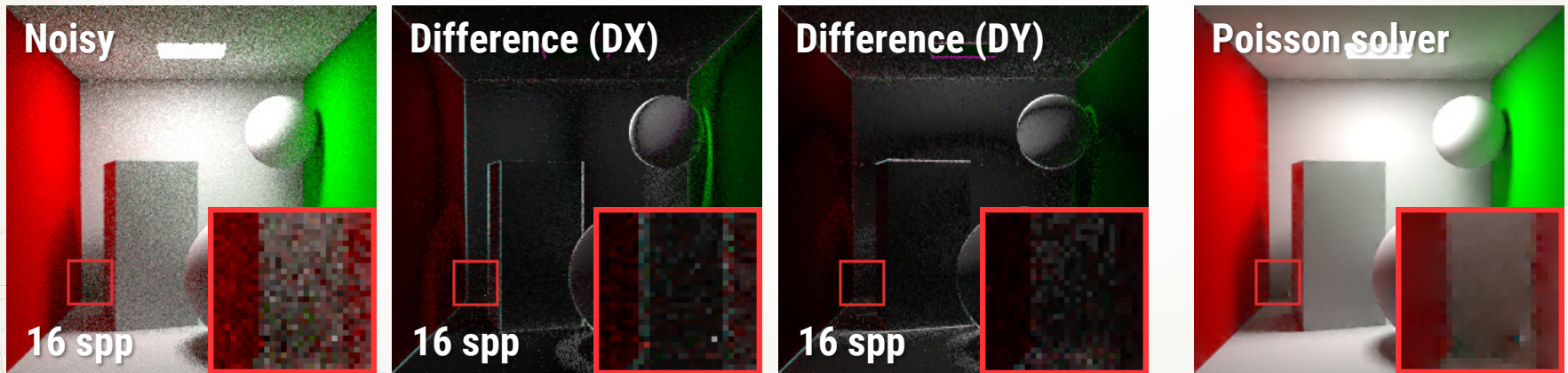
# PREVIOUS WORKS



Consistent scene editing [CSE], Günther and Grosch, EGSR 2015

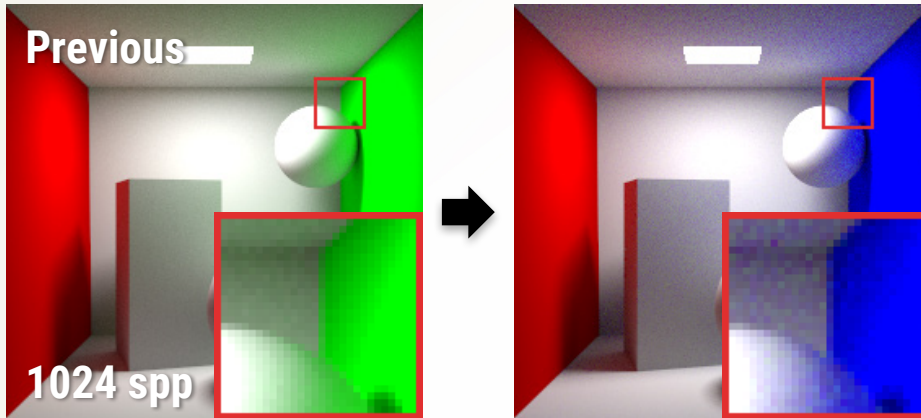


Gradient-domain path tracing [GDPT], Kettunen et al., SIGGRAPH 2015



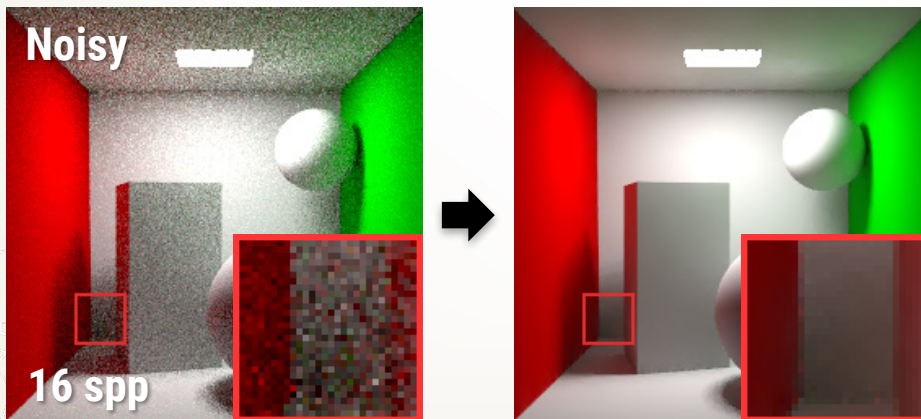


## Scene editing



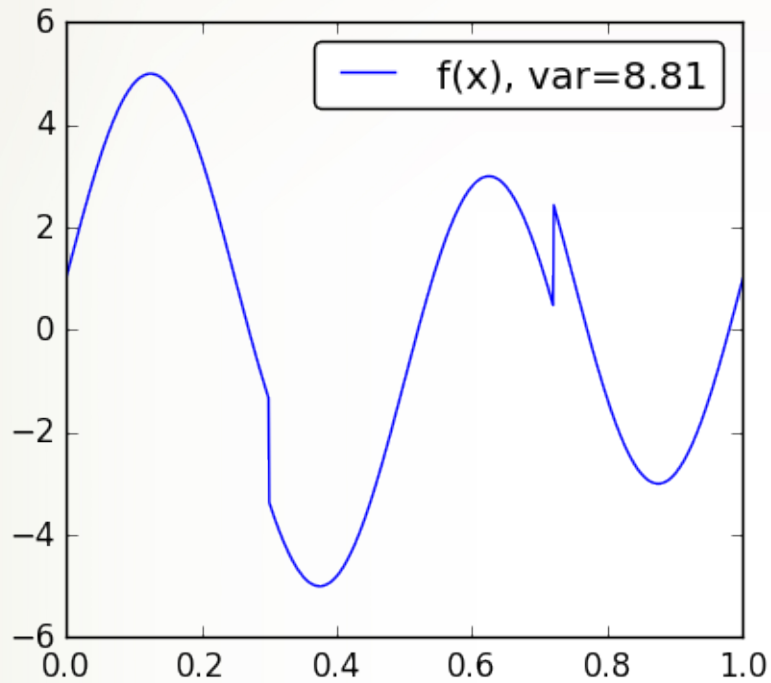
- Unified framework
- Control variates
- Provably optimal variance

## Gradient-domain rendering





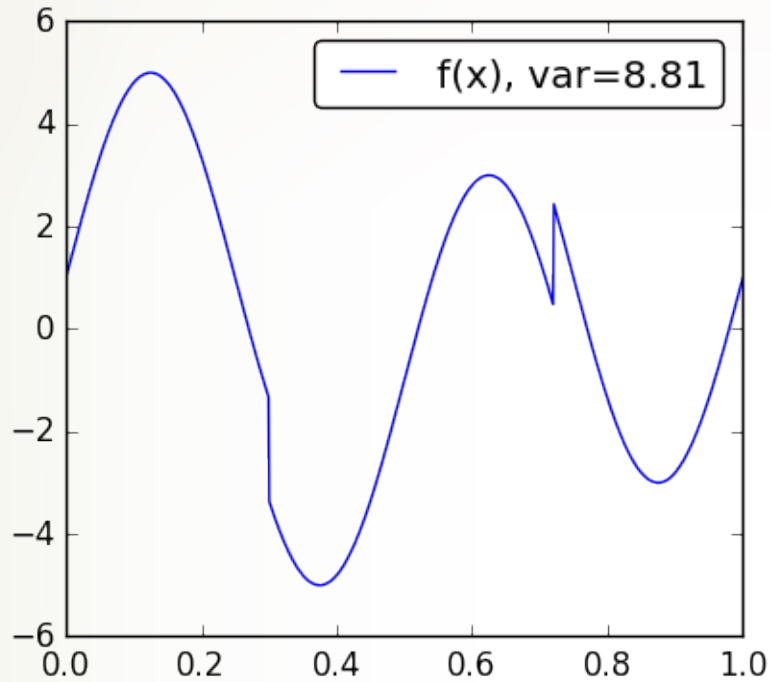
# INTEGRATION WITH CONTROL VARIATES



$$\int_{\Omega} f(x) dx$$



# INTEGRATION WITH CONTROL VARIATES

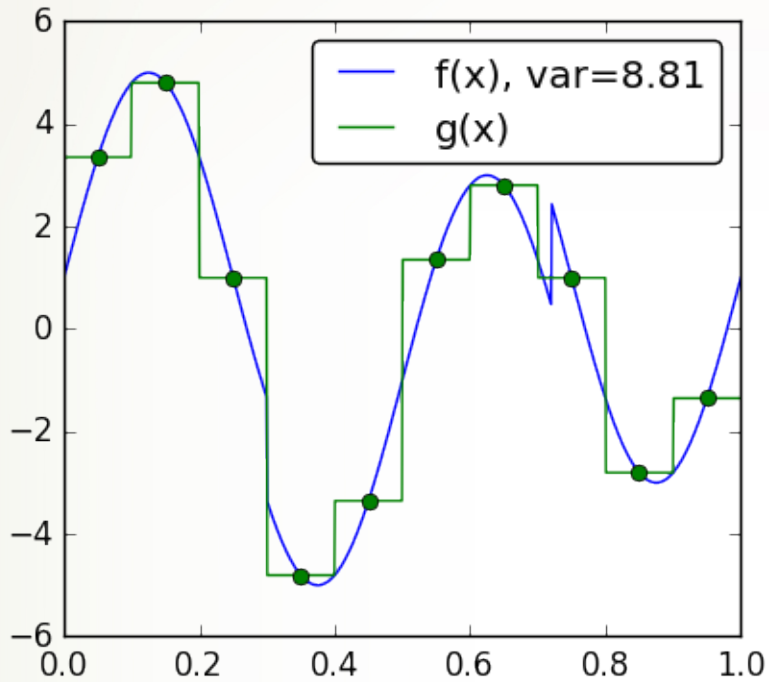


$$\int_{\Omega} f(x) - g(x) dx + G$$

Control variate with known integral



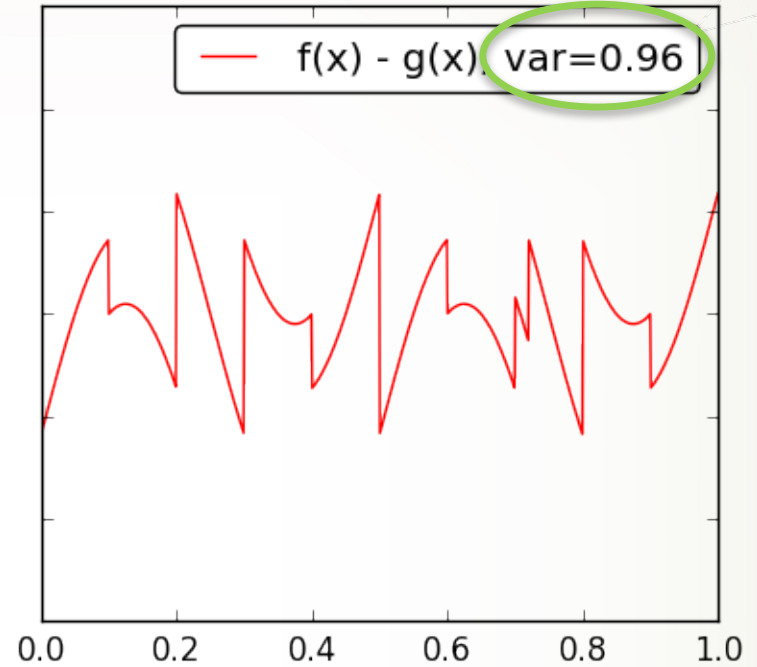
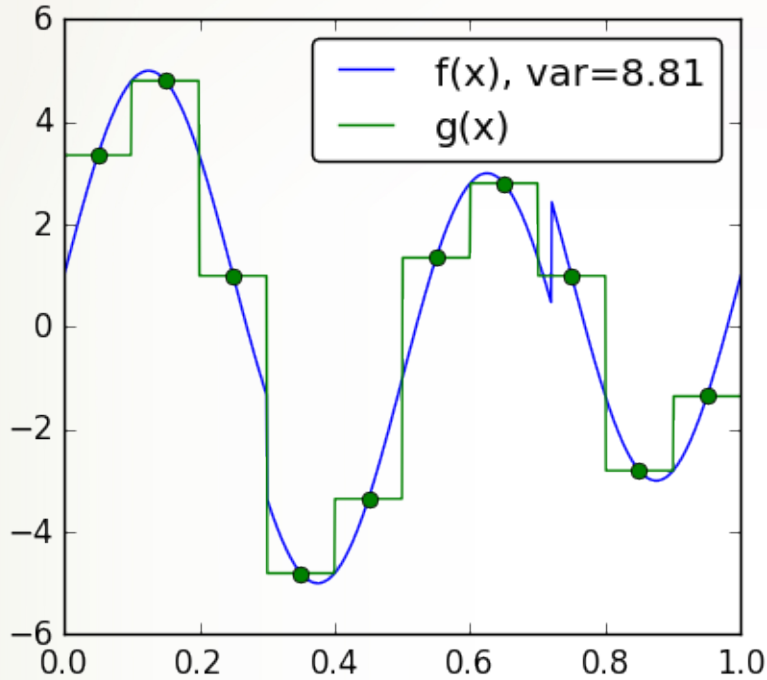
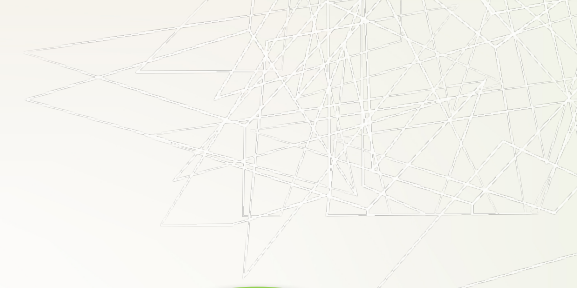
# INTEGRATION WITH CONTROL VARIATES



$$\int_{\Omega} f(x) - g(x) dx + G$$



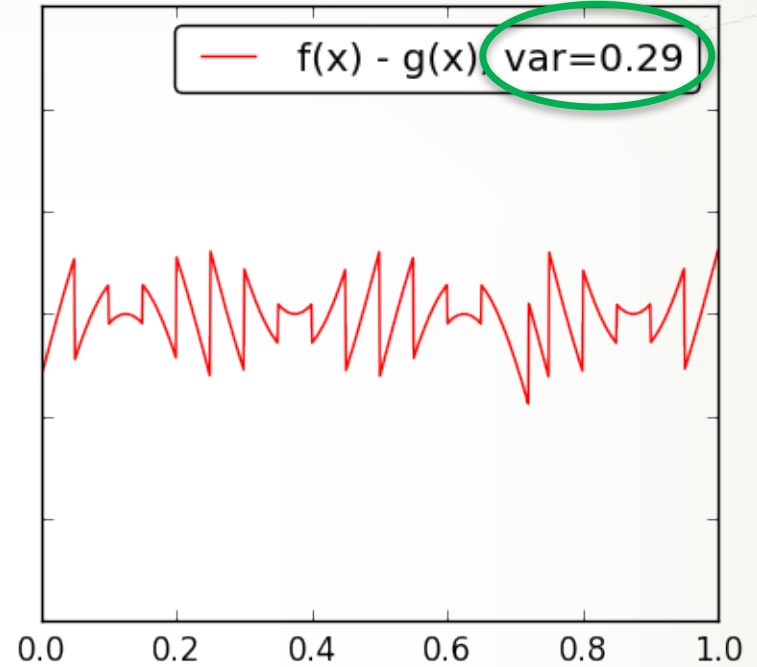
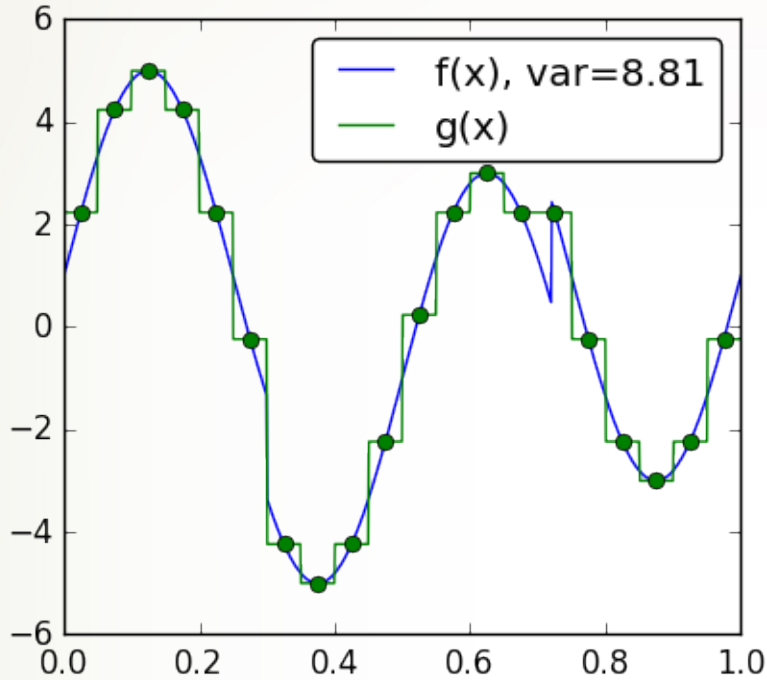
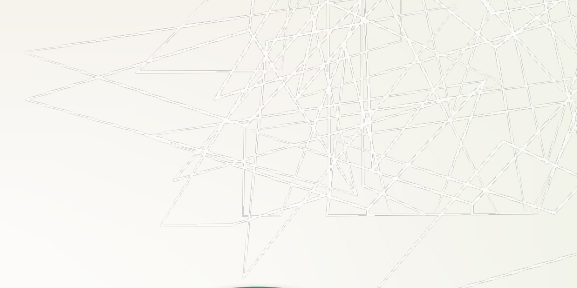
# INTEGRATION WITH CONTROL VARIATES



$$\int_{\Omega} f(x) - g(x) dx + G$$



# INTEGRATION WITH CONTROL VARIATES

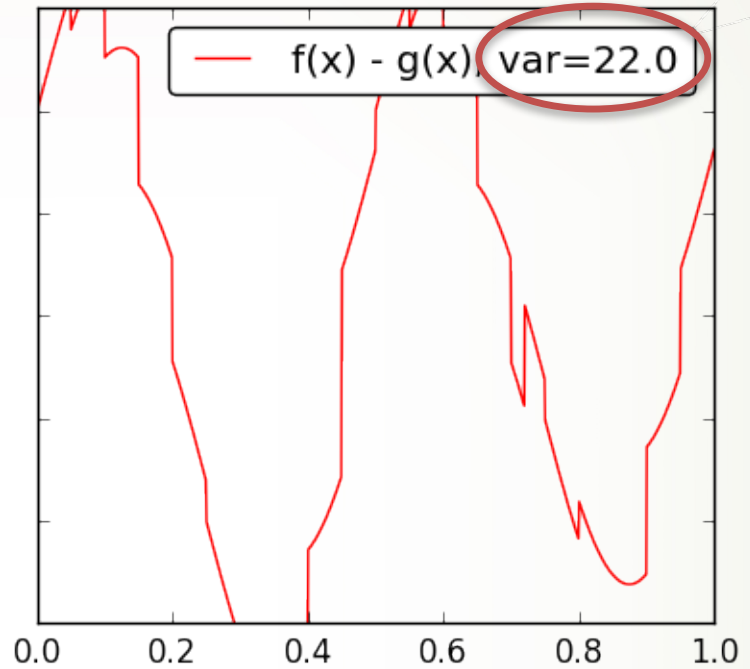
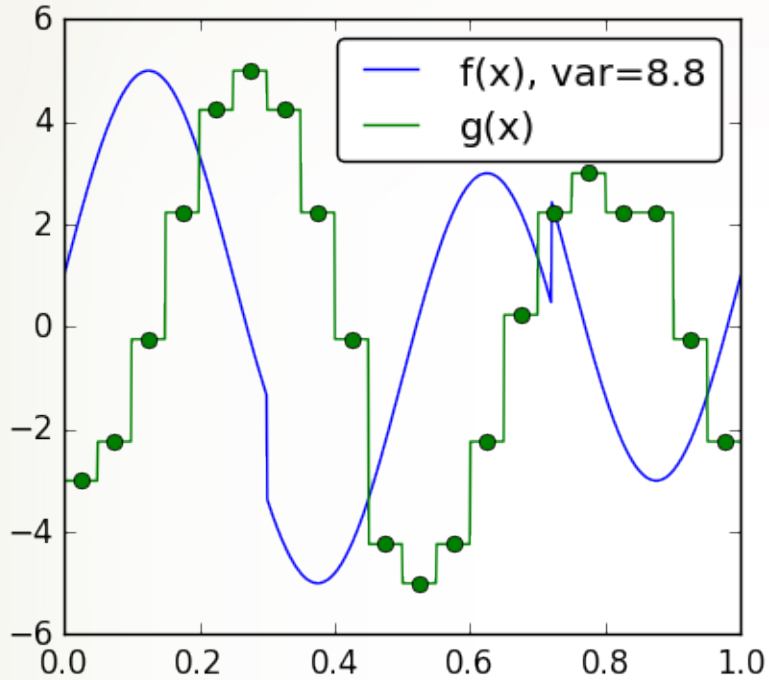
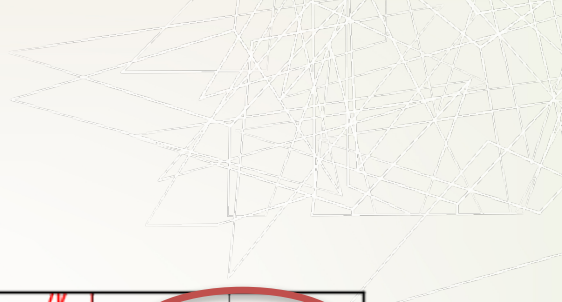


$$\int_{\Omega} f(x) - g(x) dx + G$$





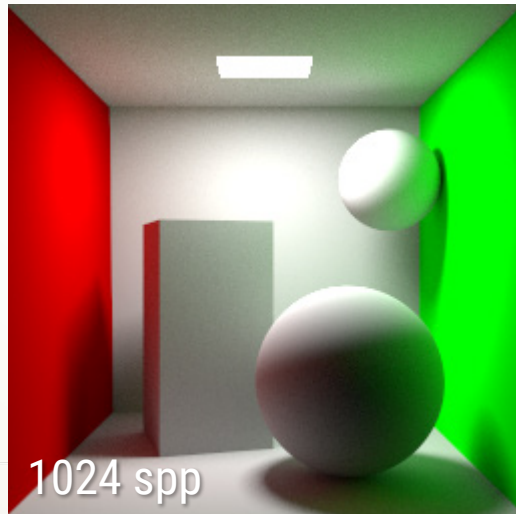
# INTEGRATION WITH CONTROL VARIATES



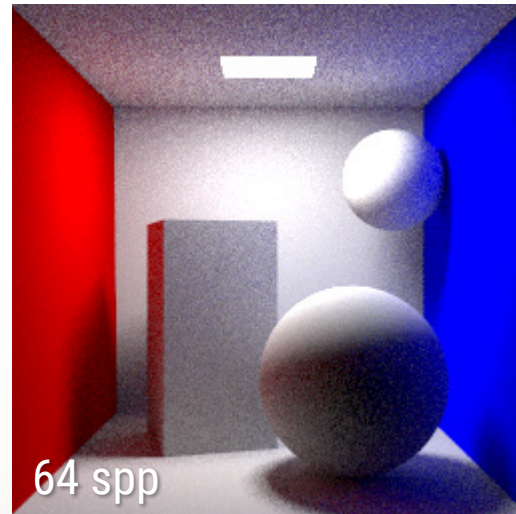
$$\int_{\Omega} f(x) - g(x) dx + G$$



Previous



Re-render



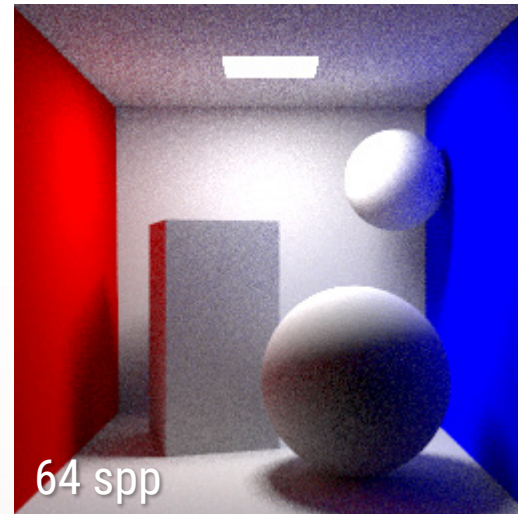


Previous



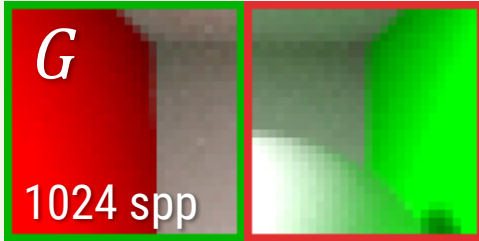
$$\int_{\Omega} f(x) - g(x) dx + G$$

Re-render



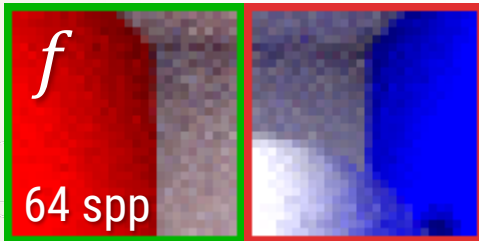


Previous



$$\int_{\Omega} f(x) - g(x) dx + G$$

Re-render



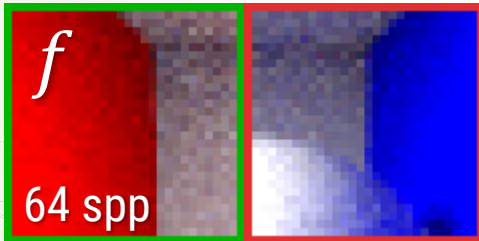
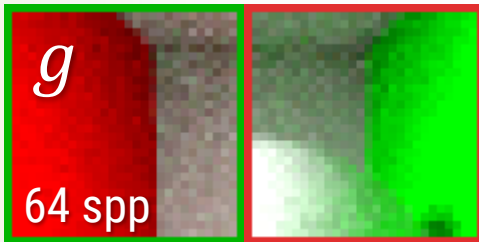


Previous



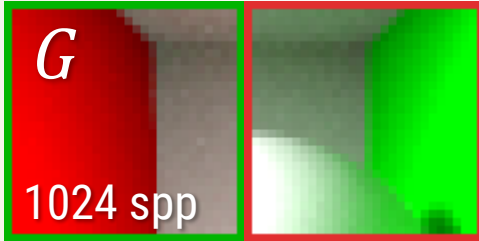
$$\int_{\Omega} f(x) - g(x) dx + G$$

Re-render



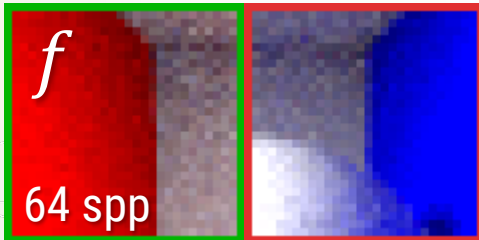
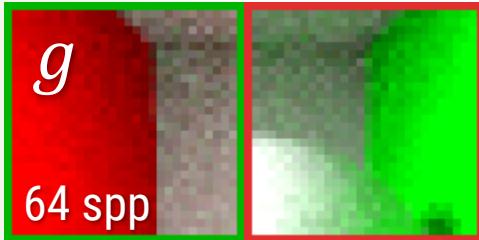


Previous



$$\int_{\Omega} f(x) - g(x) dx + G$$

Re-render *same random seed*





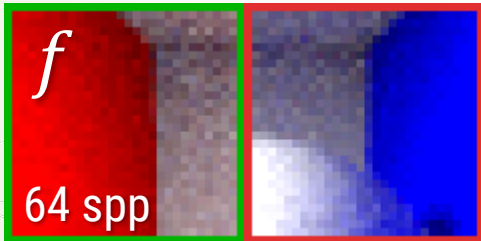
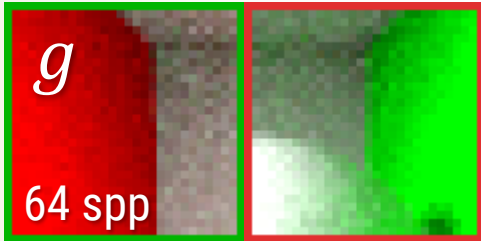
# SCENE EDITING



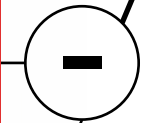
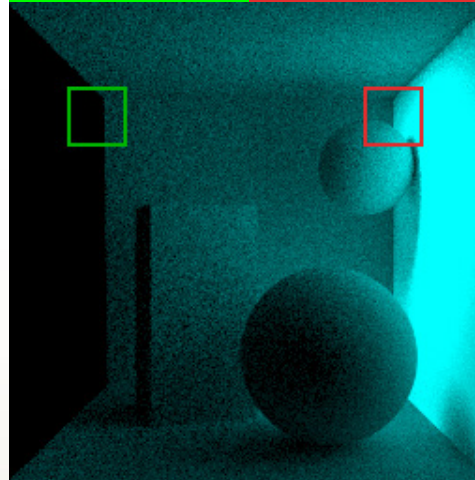
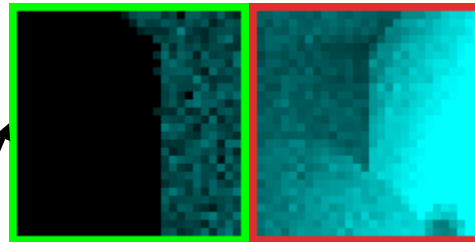
Previous



Re-render *same random seed*



Difference

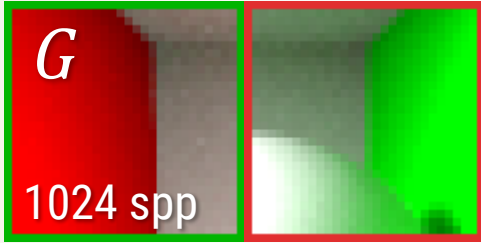




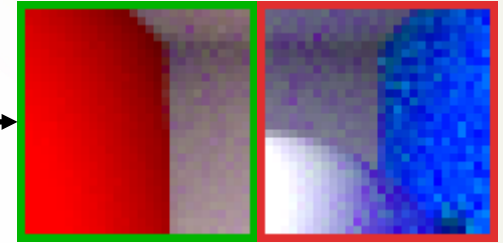
# SCENE EDITING



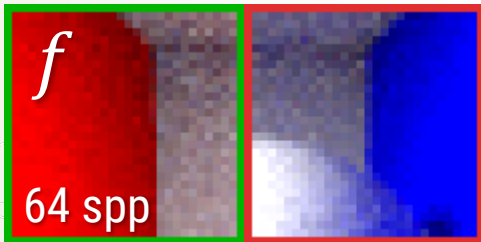
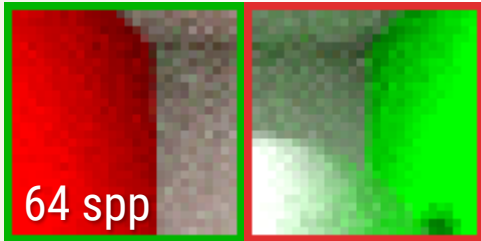
Previous



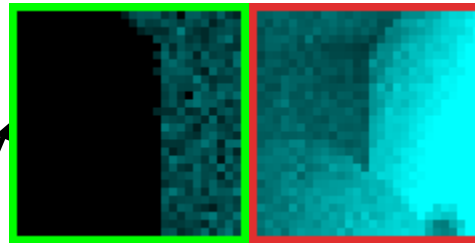
Reuse



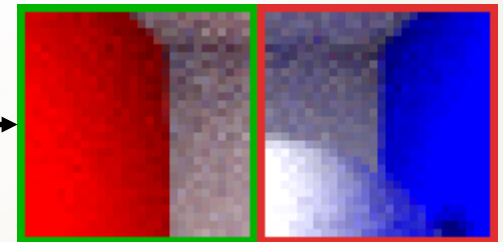
Re-render twice  
*same random seed*



Difference



Re-render



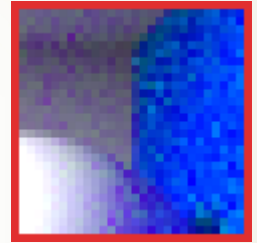
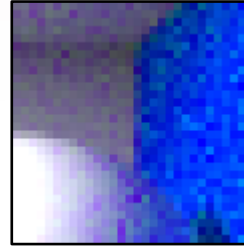




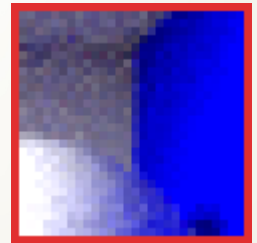
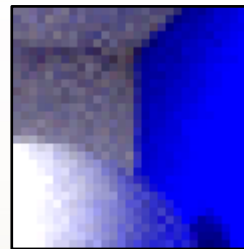
# OPTIMAL COMBINATION OF ESTIMATORS



Reuse

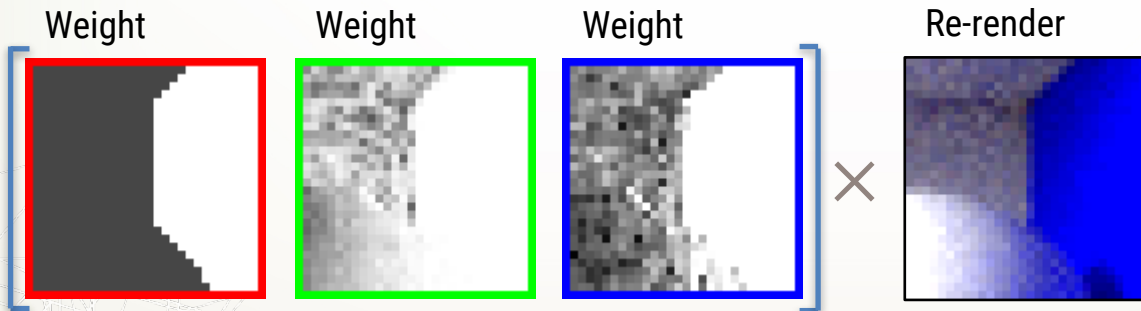
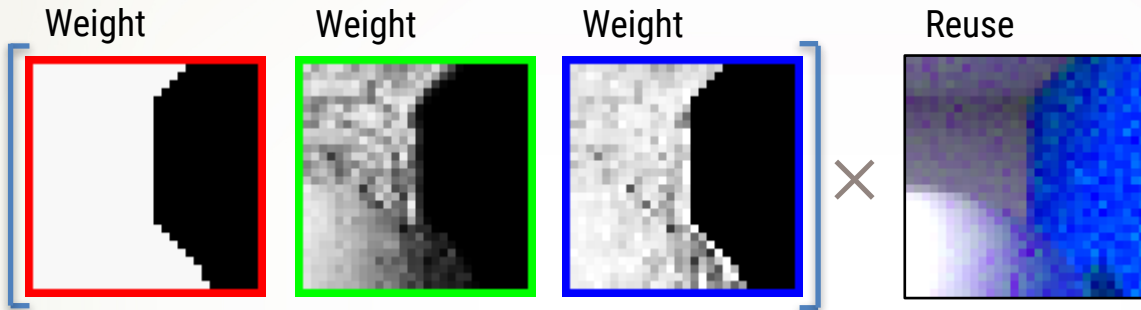


Re-render



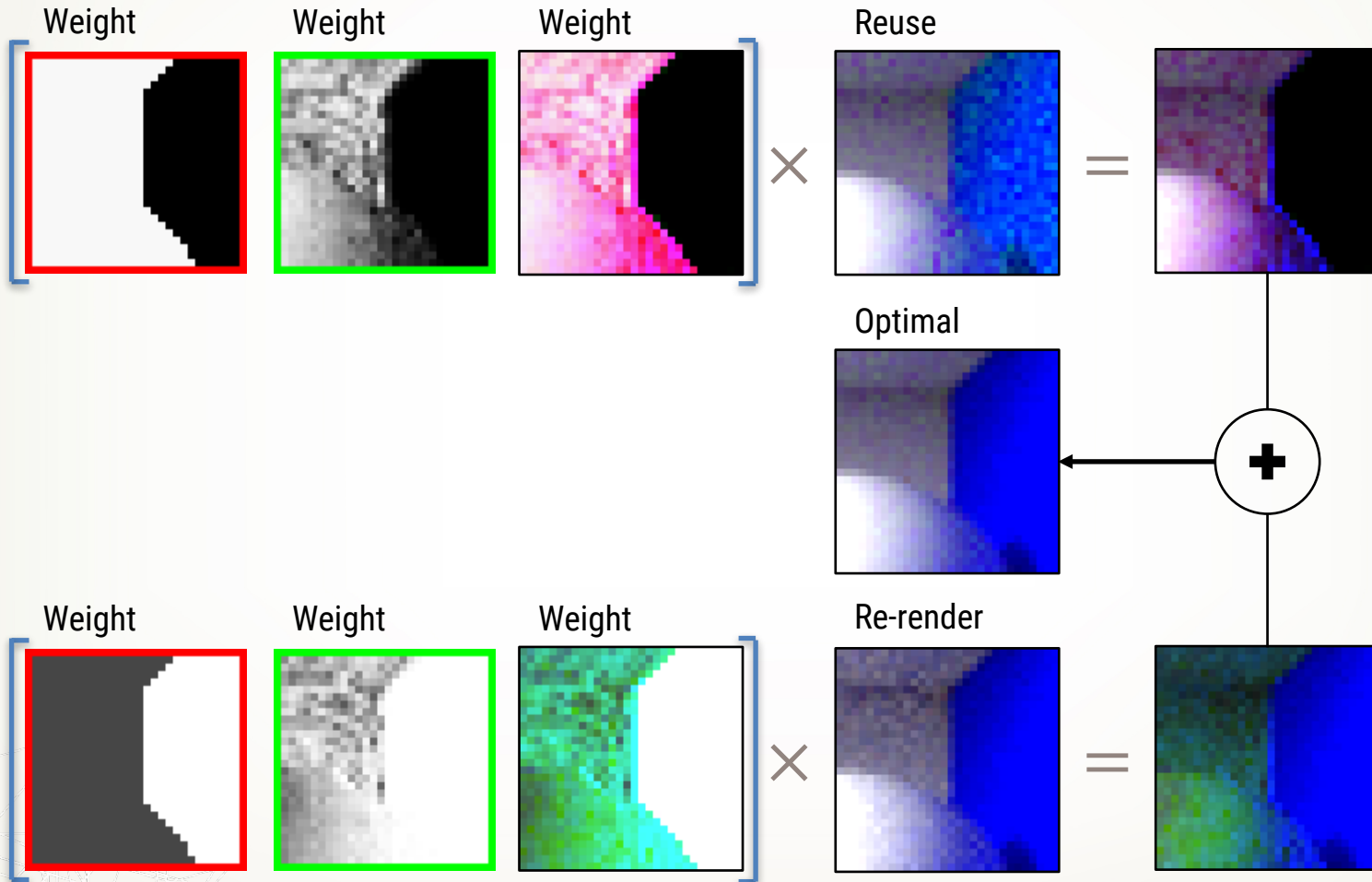


# OPTIMAL COMBINATION OF ESTIMATORS



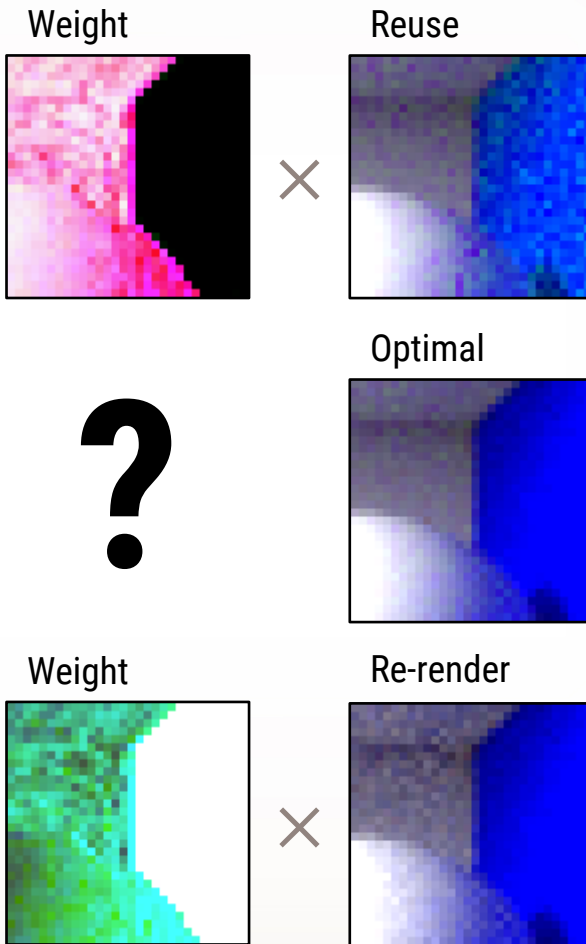


# OPTIMAL COMBINATION OF ESTIMATORS





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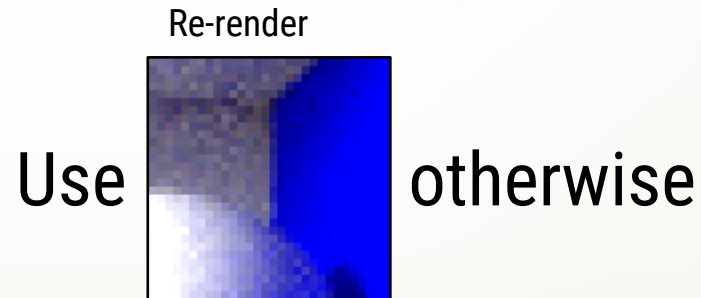
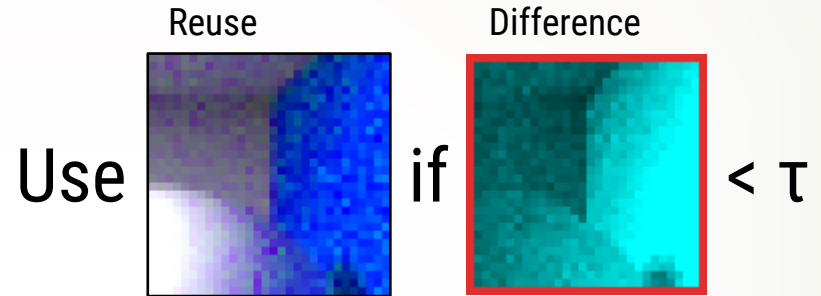




# PREVIOUS WORK: SELECTION HEURISTIC [GG15]

## Limitations

- Assumes hi-quality previous image
- User-defined threshold
- Selection discards data



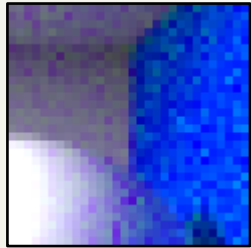


# OPTIMAL COMBINATION OF ESTIMATORS

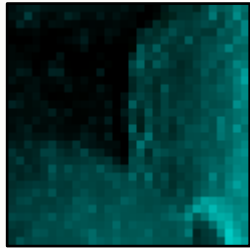


## Independent

Reuse



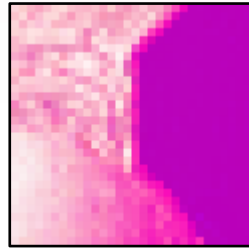
Variance



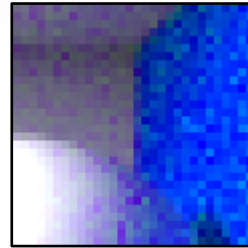
**-1**

=

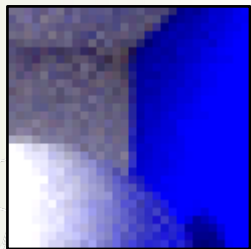
Weight



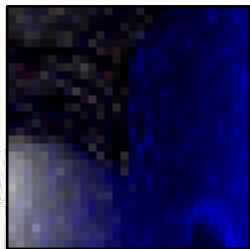
Reuse



Re-render



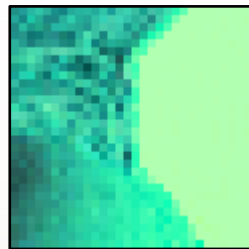
Variance



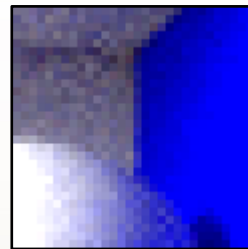
**-1**

=

Weight



Re-render



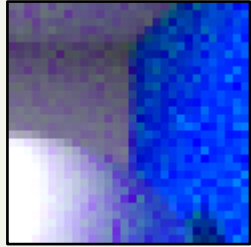


# OPTIMAL COMBINATION OF ESTIMATORS

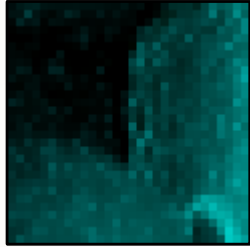


## Independent

Reuse



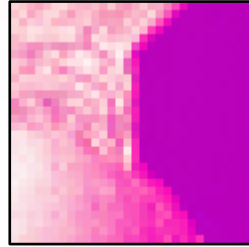
Variance



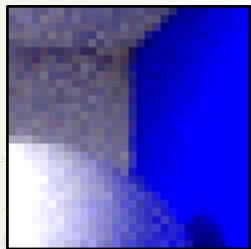
-1

=

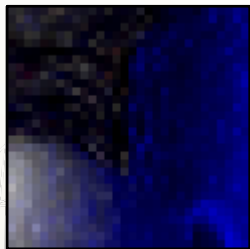
Weight



Re-render



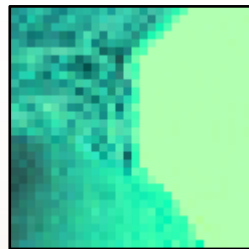
Variance



-1

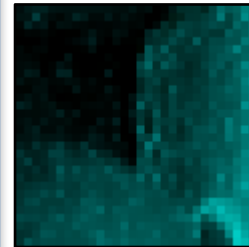
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Weight

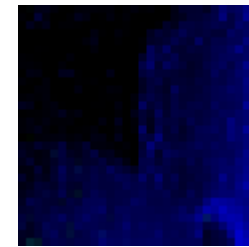


## Correlated

Variance

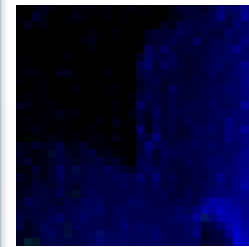


Covariance

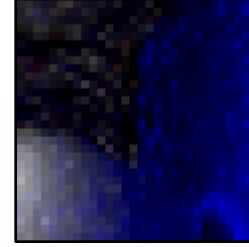


-1

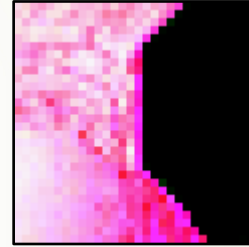
Covariance



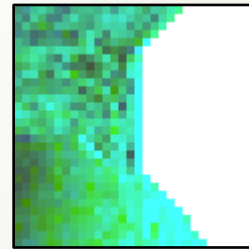
Variance



Weight



Weight



2x2 matrix  
per pixel



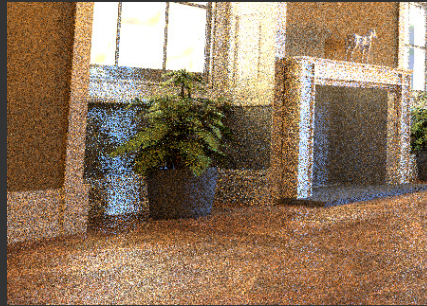
# COMPARISON TO PREVIOUS WORK [GG15] – relative MSE

Previous



1024 spp – *HQ*

Re-render



64 spp – 0.363





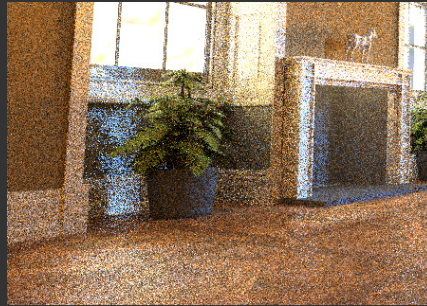
# COMPARISON TO PREVIOUS WORK [GG15] – relative MSE

Previous



1024 spp – *HQ*

Re-render



64 spp – 0.363

[GG15]



0.230

Ours



0.084



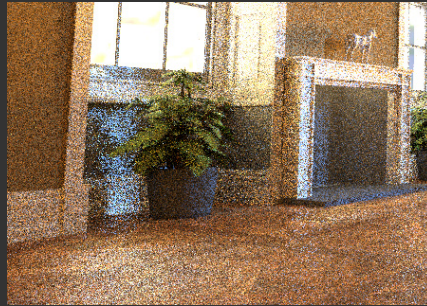
# COMPARISON TO PREVIOUS WORK [GG15] – relative MSE

Previous



1024 spp – *HQ*

Re-render



64 spp – 0.363

[GG15]

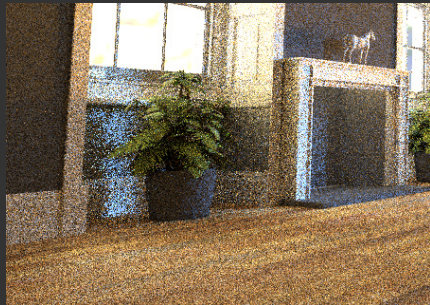


0.230

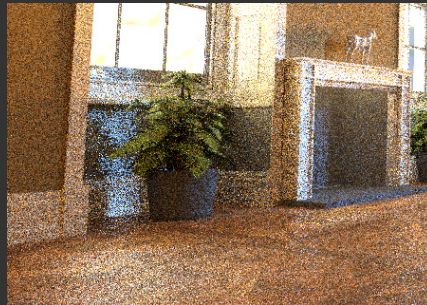
Ours



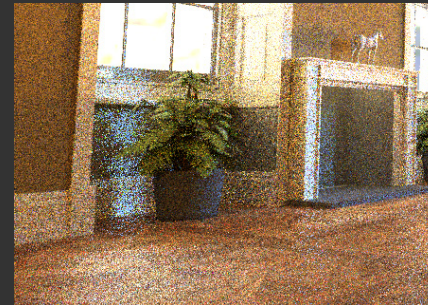
0.084



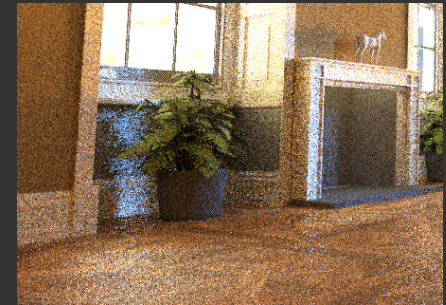
64 spp – *LQ*



64 spp – 0.363



0.384



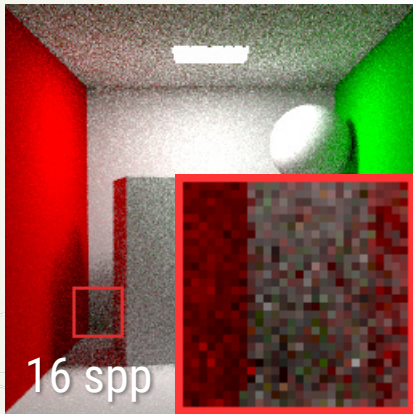
0.153



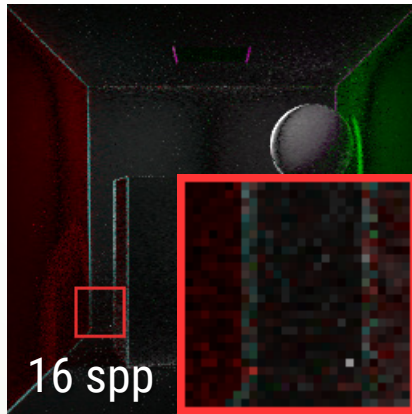
# GRADIENT-DOMAIN PATH TRACING



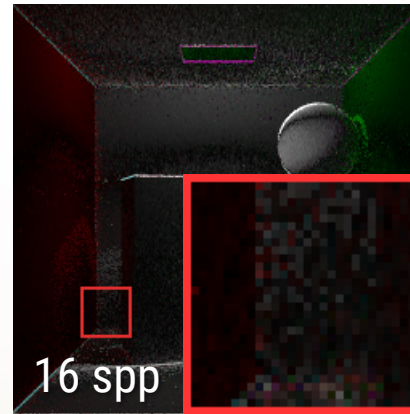
Throughput



Horizontal gradient



Vertical gradient

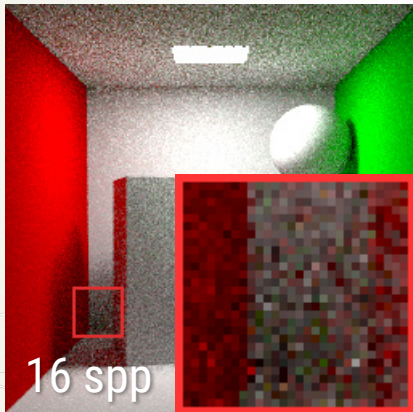




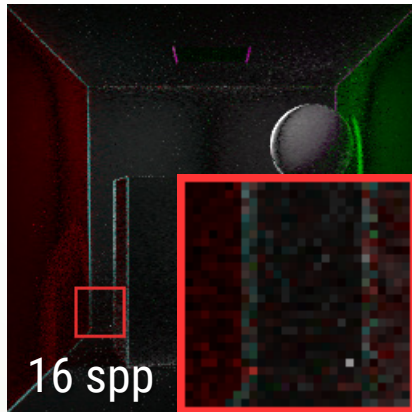
# PREVIOUS WORK: POISSON RECONSTRUCTION



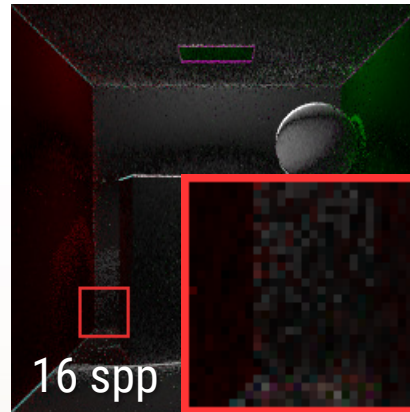
Throughput



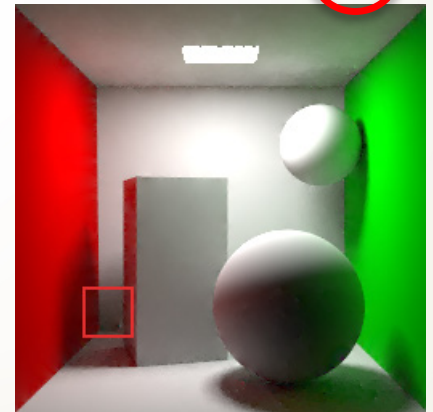
Horizontal gradient



Vertical gradient



Poisson solver (L2)

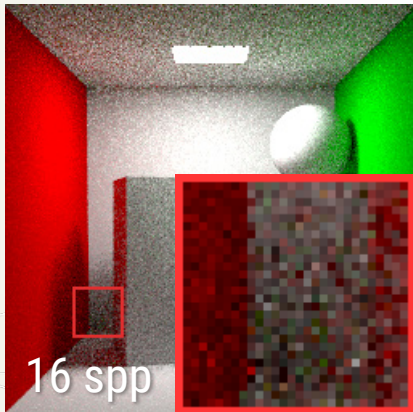




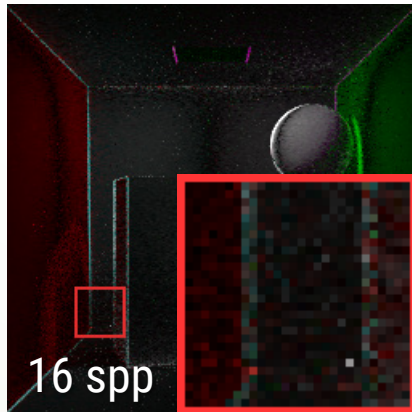
# PREVIOUS WORK: POISSON RECONSTRUCTION



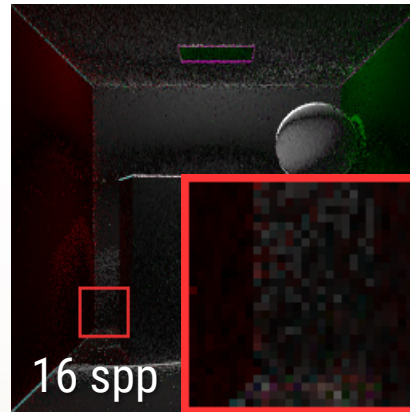
Throughput



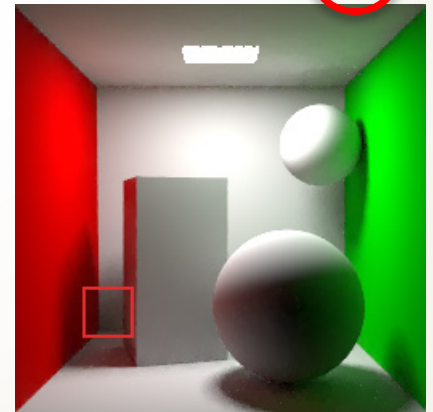
Horizontal gradient



Vertical gradient



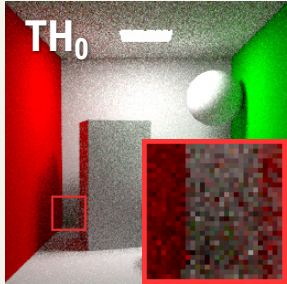
Poisson solver (L1)



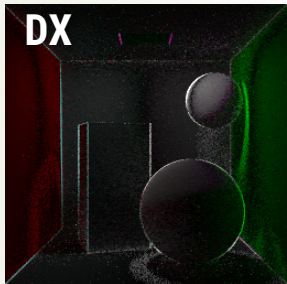


# EXTENSION TO MULTIPLE ESTIMATORS: GDR

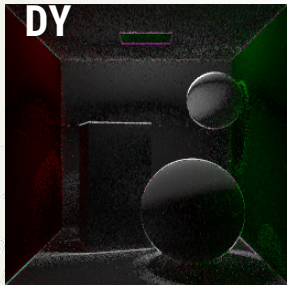
Throughput



Hori. gradient



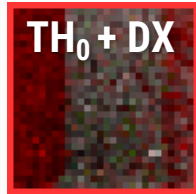
Vert. gradient



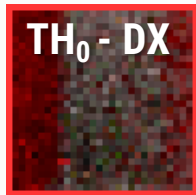
Control Variate



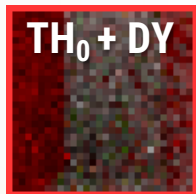
None



Left neighbor



Right neighbor



Bottom neighbor

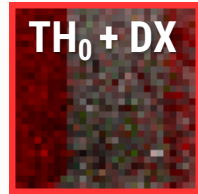
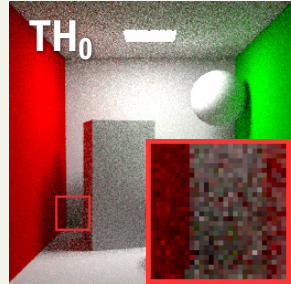


Top neighbor



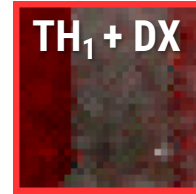
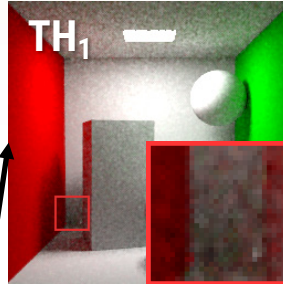
# EXTENSION TO MULTIPLE ESTIMATORS: GDR

Throughput



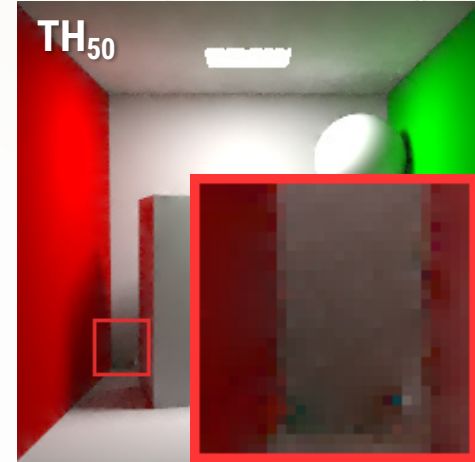
AVG

Throughput

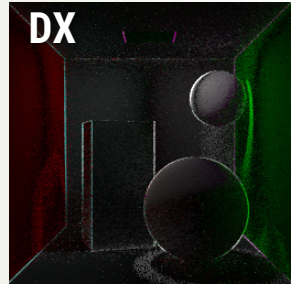


...

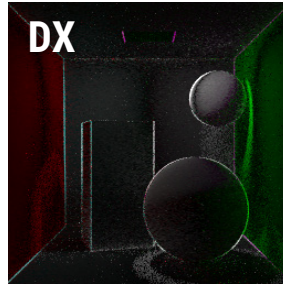
Throughput [CVPT-uni]



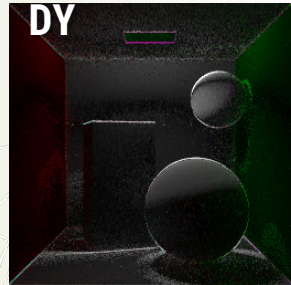
Hori. gradient



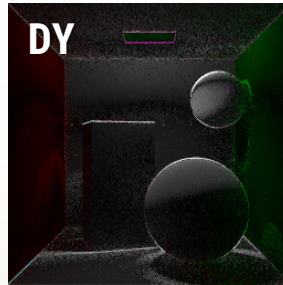
Hori. gradient



Vert. gradient



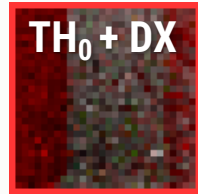
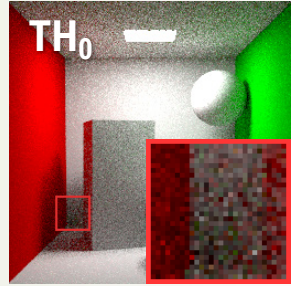
Vert. gradient



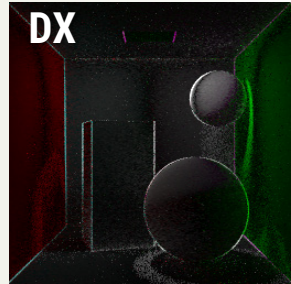


# EXTENSION TO MULTIPLE ESTIMATORS: GDR

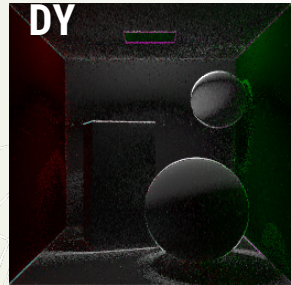
Throughput



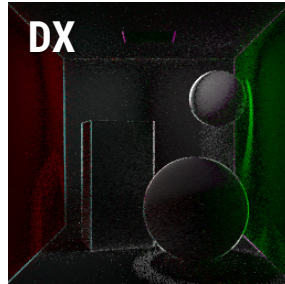
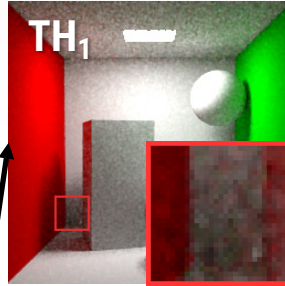
Hori. gradient



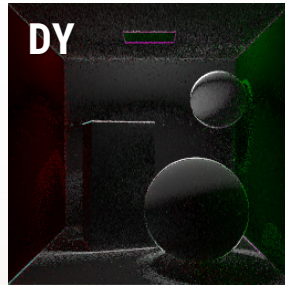
Vert. gradient



Throughput

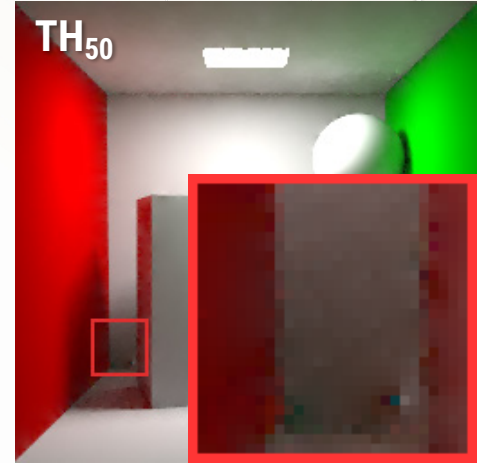


Vert. gradient



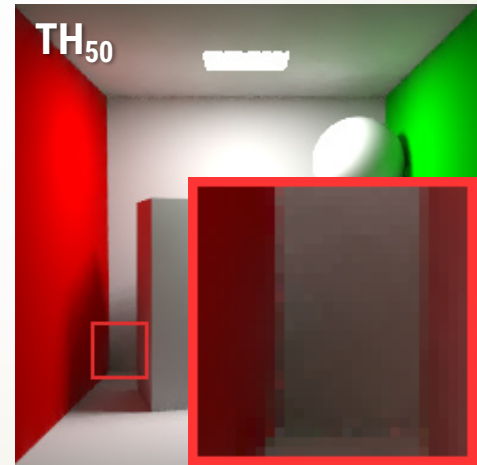
OPT

Throughput [CVPT-uni]



...

Throughput [CVPT-opt]

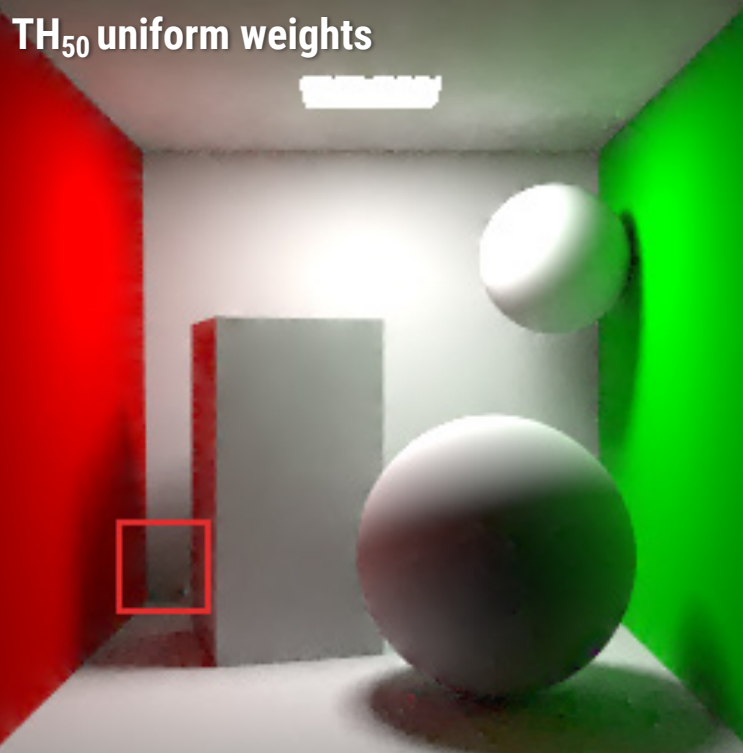




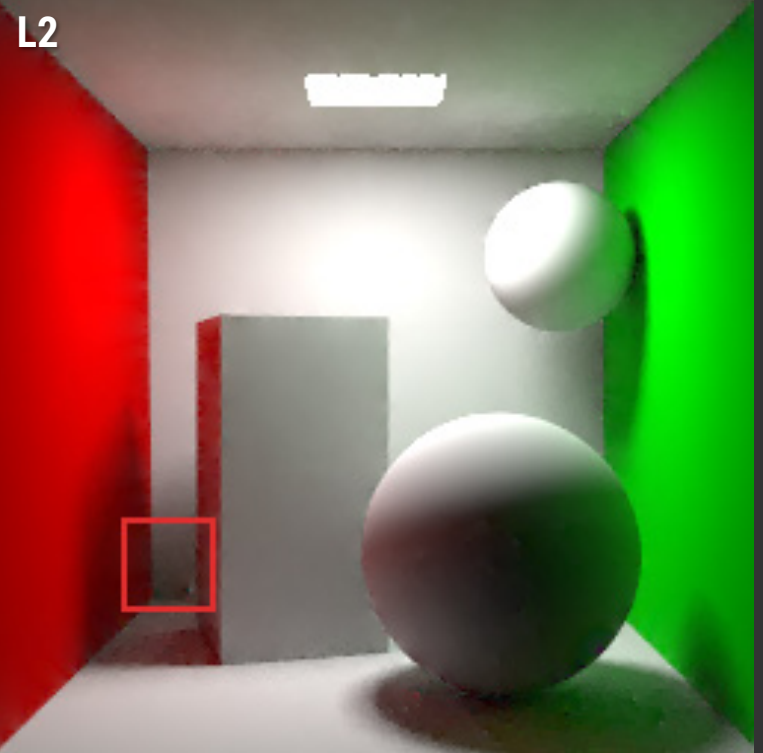


# RELATION TO POISSON SOLVER

Iterated reconstruction (Ours)



Poisson Solver [GDPT]



≈



# COMPARISON TO PREVIOUS WORK [GDPT] – relative MSE



Sponza scene – 16 spp





# COMPARISON TO PREVIOUS WORK [GDPT] – relative MSE

Bookshelf scene – standard path tracing

0.0590





# COMPARISON TO PREVIOUS WORK [GDPT] – relative MSE

Bookshelf scene – L1 Poisson reconstruction

0.0094





# COMPARISON TO PREVIOUS WORK [GDPT] – relative MSE

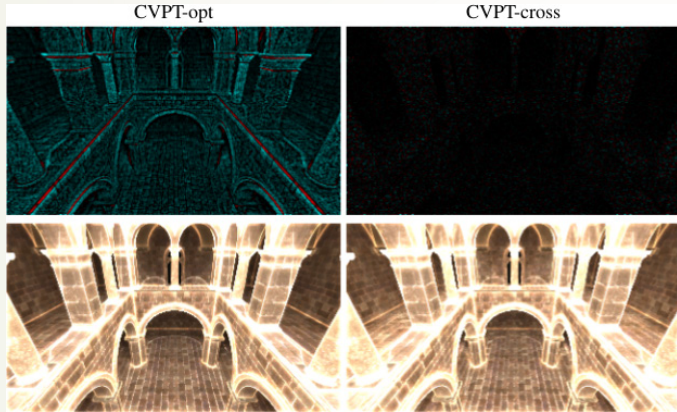
Bookshelf scene – Ours with optimized weights

0.0044



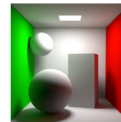


# MORE IN THE PAPER: UNBIASED RECONSTRUCTIONS

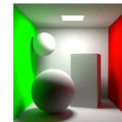


## Re-rendering Test Suite

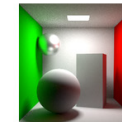
The following scenes were rendered with the [PBRTv3 renderer](#). For each scene, we show multiple images: the control image (Control), the standard path tracer output (PT), the result of using integration with a fully weighted control variate (CVPT), the result of using our optimized weights (CVPT-opt), and the result using our optimized weights based on independently estimated statistics (CVPT-cross). For our optimized weights, we first filter the statistics using an NL-Means filter, and therefore provide results without this prefilter step (CVPT-opt-raw, CVPT-cross-raw). Lastly, we show results using the heuristic of Gunther and Grosch [2015] to pick between the PT and CVPT estimates on a per pixel basis. That heuristic uses a threshold parameter and we show results for two settings of this parameter (including the setting recommended in the original publication, 0.1). For all results, we indicate underneath whether the estimate is biased or not.



Luminance Test



Color Test



Roughness Test



Horse Room Test

## Gradient-domain Reconstruction Test Suite

The following scenes were rendered using the [public implementation](#) of the Gradient-domain Path Tracing method of Kettunen et al. [2015], which uses Wenzel Jakob's [Mitsuba renderer](#). We reconstruct the scenes with an array of techniques, some of which operate in the gradient domain (GDPT-L2, GDPT-L1, GDPT-L1L2, GDPT-WL2, CVPT-uni, CVPT-opt, CVPT-cross), and some of which don't (PT, RDFC, NFOR). Techniques with the prefix GDPT all use a screened Poisson solver, whereas techniques with the prefix CVPT use an iterated reconstruction using control variates. Since computing the gradients incurs a  $\sim 2.5\times$  rendering time overhead, we increased the sampling rate by a factor of 2.5 for the PT, RDFC, and NFOR results. Please see our paper for a description of the CVPT-uni, CVPT-opt, CVPT-cross, and GDPT-WL2 reconstructions. The GDPT-L2 and GDPT-L1 reconstructions were proposed by Kettunen et al. [2015], the GDPT-L1L2 reconstruction by Manzi et al. [2016], the RDFC reconstruction by Rousselle et al. [2013], and the NFOR reconstruction by Bitterli et al. [2016]. The GDPT-L1L2 reconstruction is essentially a weighted screened Poisson solver where only the gradient constraints are reweighted, hence the name L1L2; in their original publication, Manzi et al. regularize the Poisson solver with constraints based on auxiliary buffers, but in our comparison we use a standard screened Poisson solver.



Sponza



Veach Door



Bathroom



Kitchen



Bookshelf





## CONCLUSION

- Integration with control variates...
  - simple yet powerful tool
  - relatively unexplored in rendering
  - should be used with optimal weighting scheme
- Future work
  - animations, stereo rendering, light fields
  - better sampling of the difference buffer



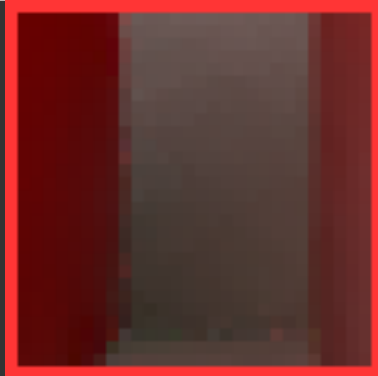
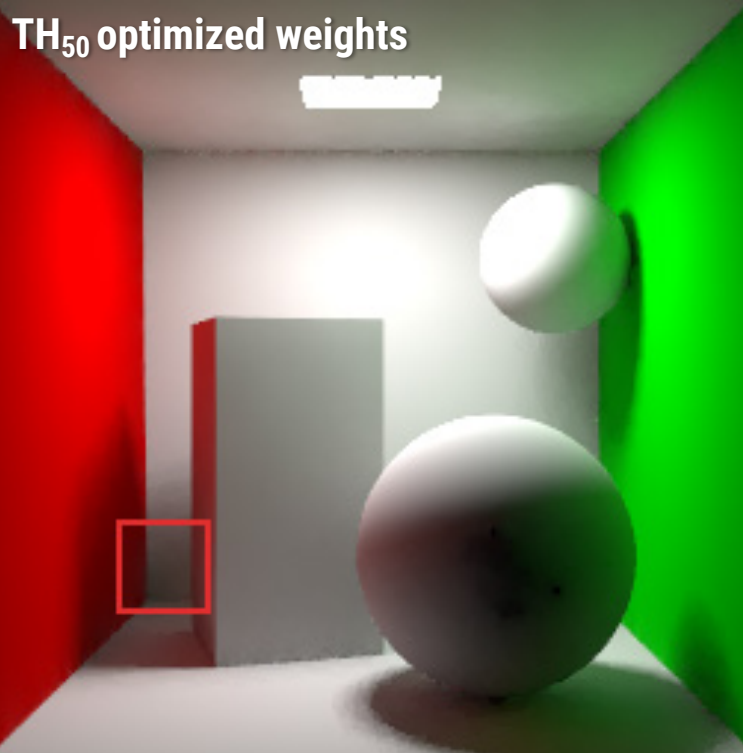




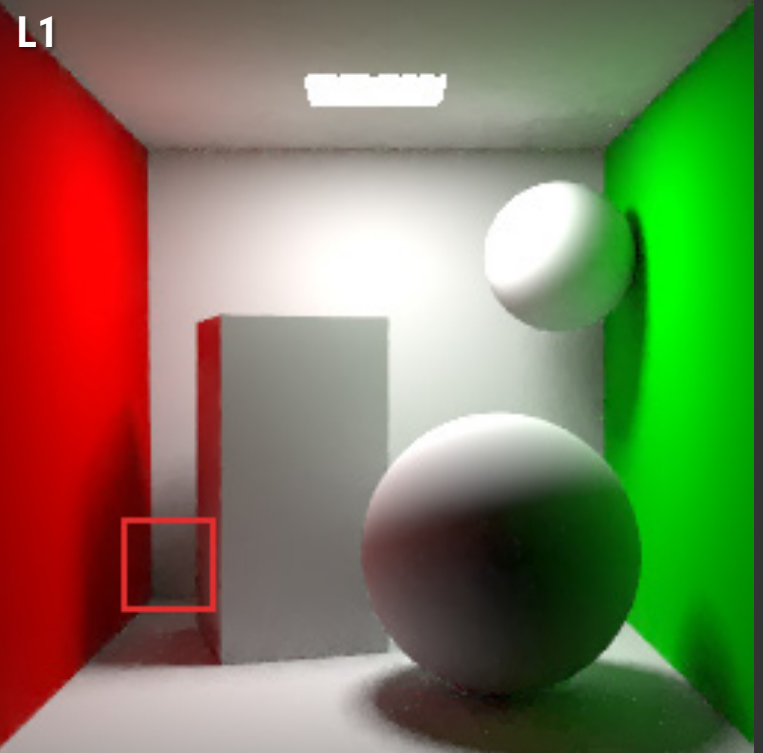


# RELATION TO POISSON SOLVER

Iterated reconstruction (Ours)

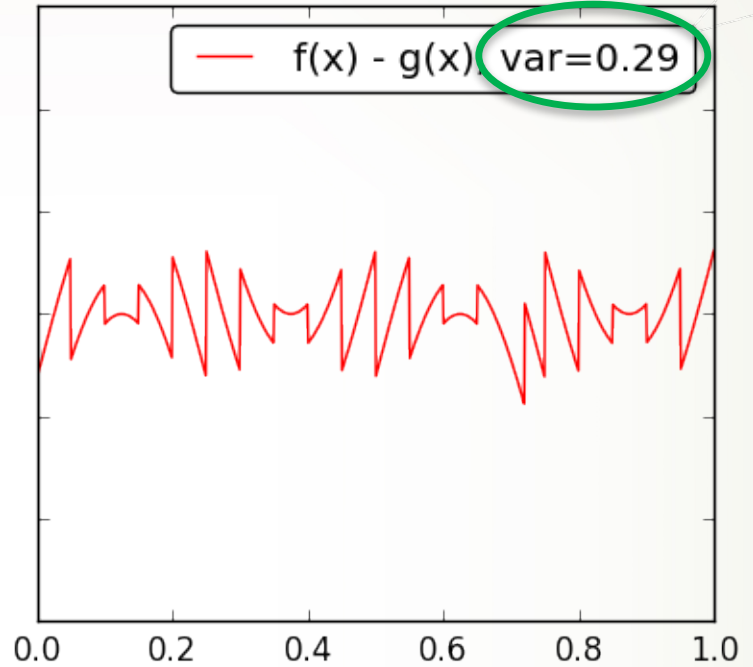
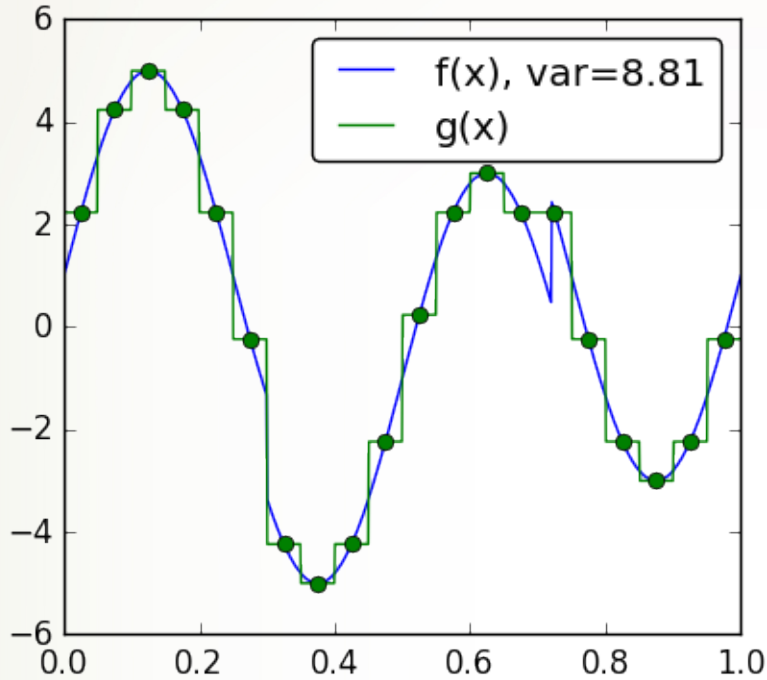
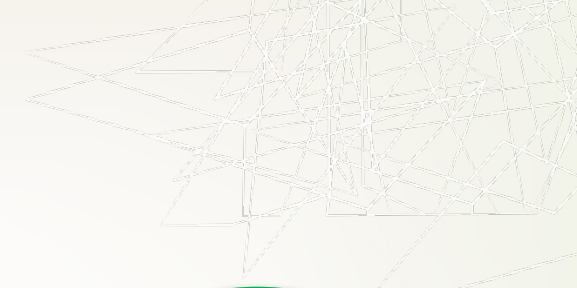


Poisson Solver [GDPT]





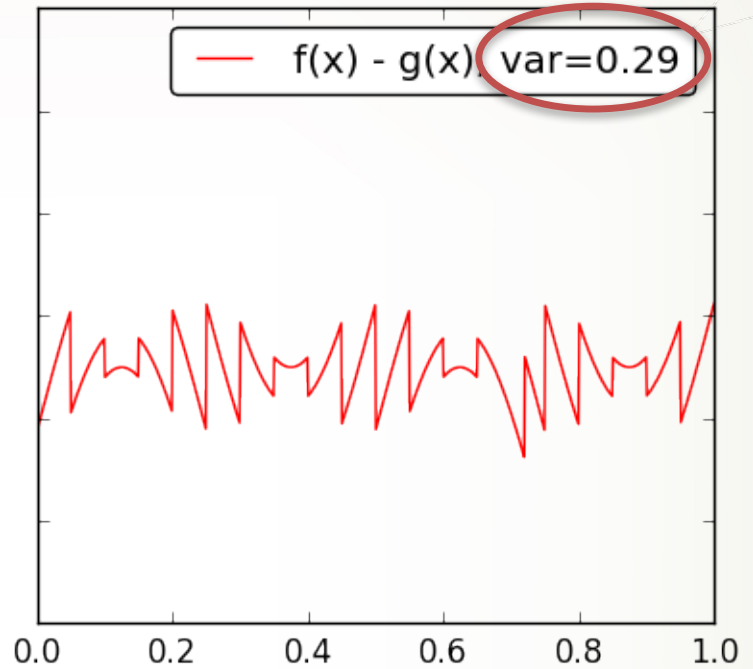
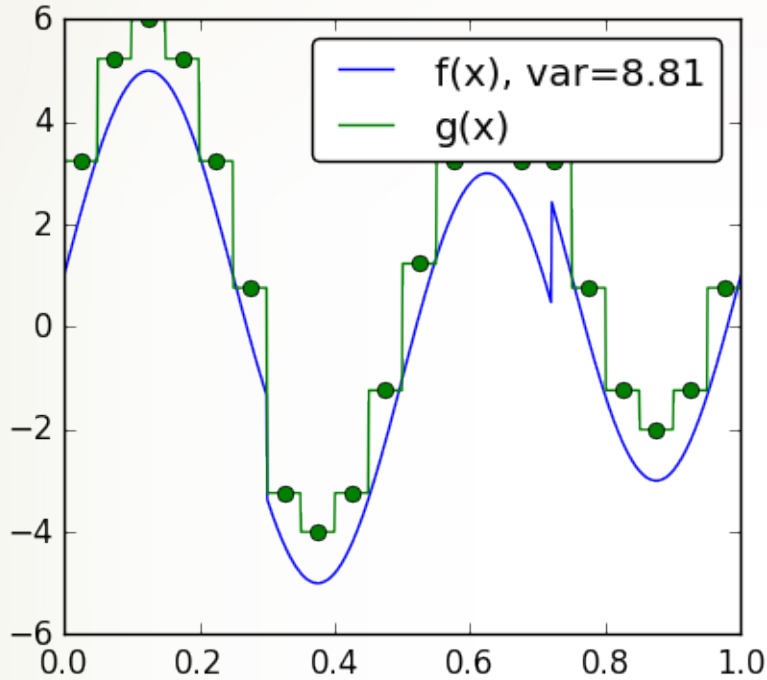
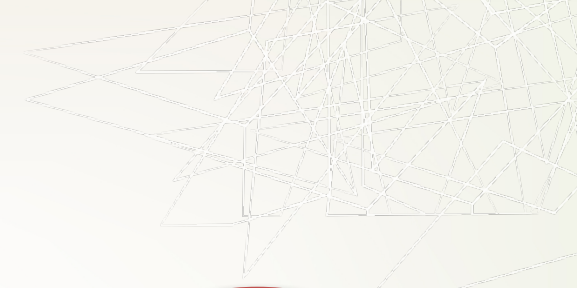
# INTEGRATION WITH CONTROL VARIATES



$$\int_{\Omega} f(x) - g(x) dx + G$$



# INTEGRATION WITH CONTROL VARIATES



$$\int_{\Omega} f(x) - g(x) dx + G$$



# COMPARISON TO PREVIOUS WORK [GDPT] – relative MSE



Bookshelf scene – 256 spp

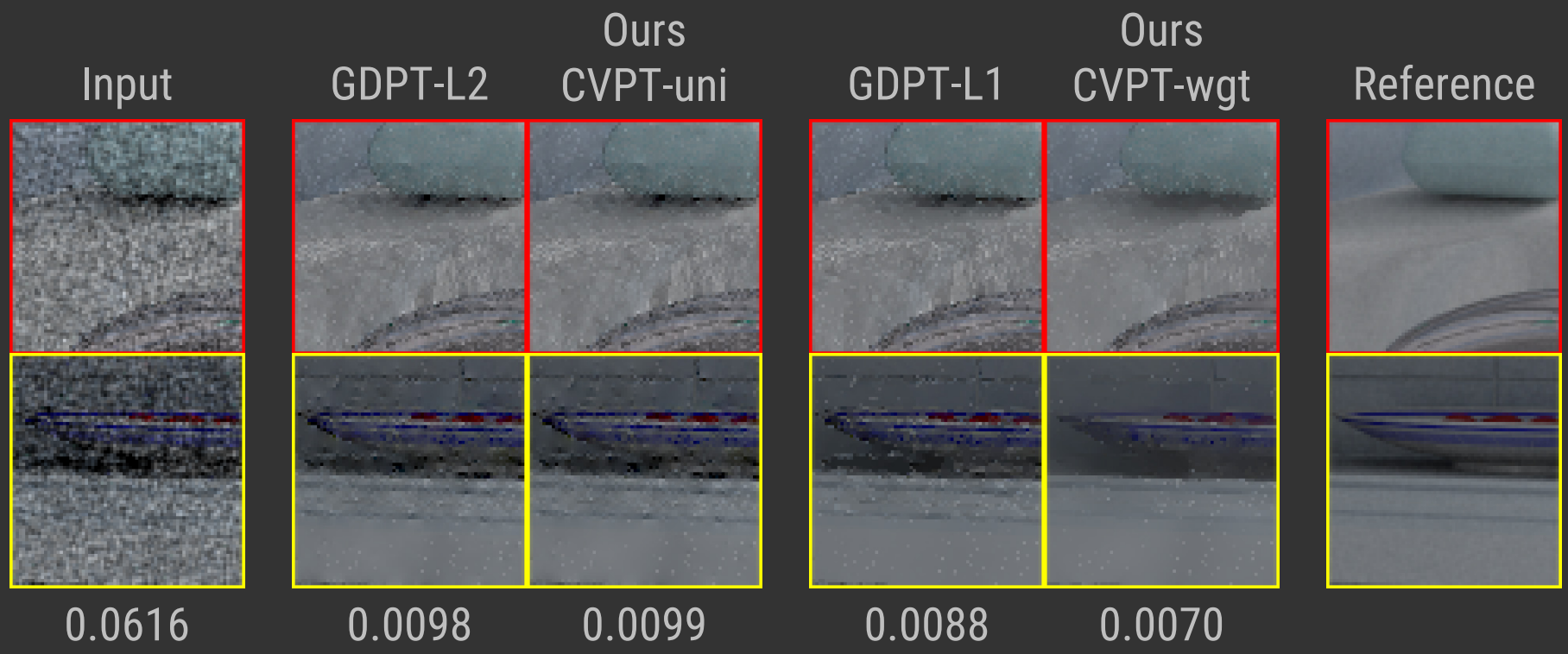
Input	GDPT-L2	Ours CVPT-uni	GDPT-L1	Ours CVPT-wgt	Reference
0.206	0.0365	0.0362	0.0314	0.0129	



# COMPARISON TO PREVIOUS WORK [GDPT] – relative MSE



Bathroom scene – 256 spp





# LEVERAGING COHERENCE: PREVIOUS WORK

- Irradiance caching

A ray tracing solution for diffuse interreflection

Ward et al., SIGGRAPH 1988

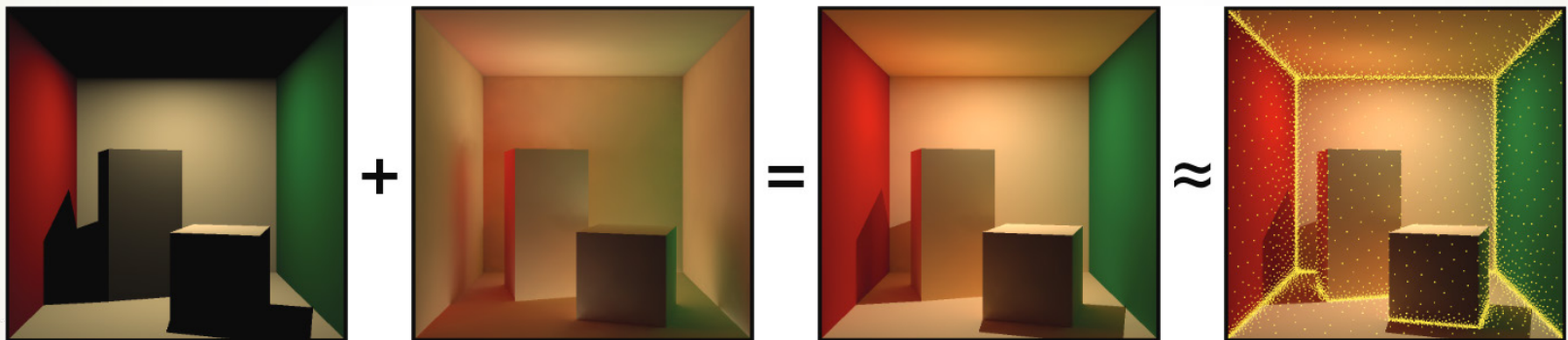


Image by Wojciech Jarosz



## LEVERAGING COHERENCE: PREVIOUS WORK

- Irradiance caching
- Photon mapping

**Global illumination using photon maps**

Jensen, EGSR 1996



Image by Henrik Wann Jensen





# LEVERAGING COHERENCE: PREVIOUS WORK

- Irradiance caching
- Photon mapping
- Image-space denoising

**Energy-preserving non-linear filters**  
Rushmeier and Ward, SIGGRAPH 1994



Scene by Guillermo M. Lean Llaguno  
Image by Benedikt Bitterli

