

PHYSICALLY BASED SIMULATION OF RAINBOWS

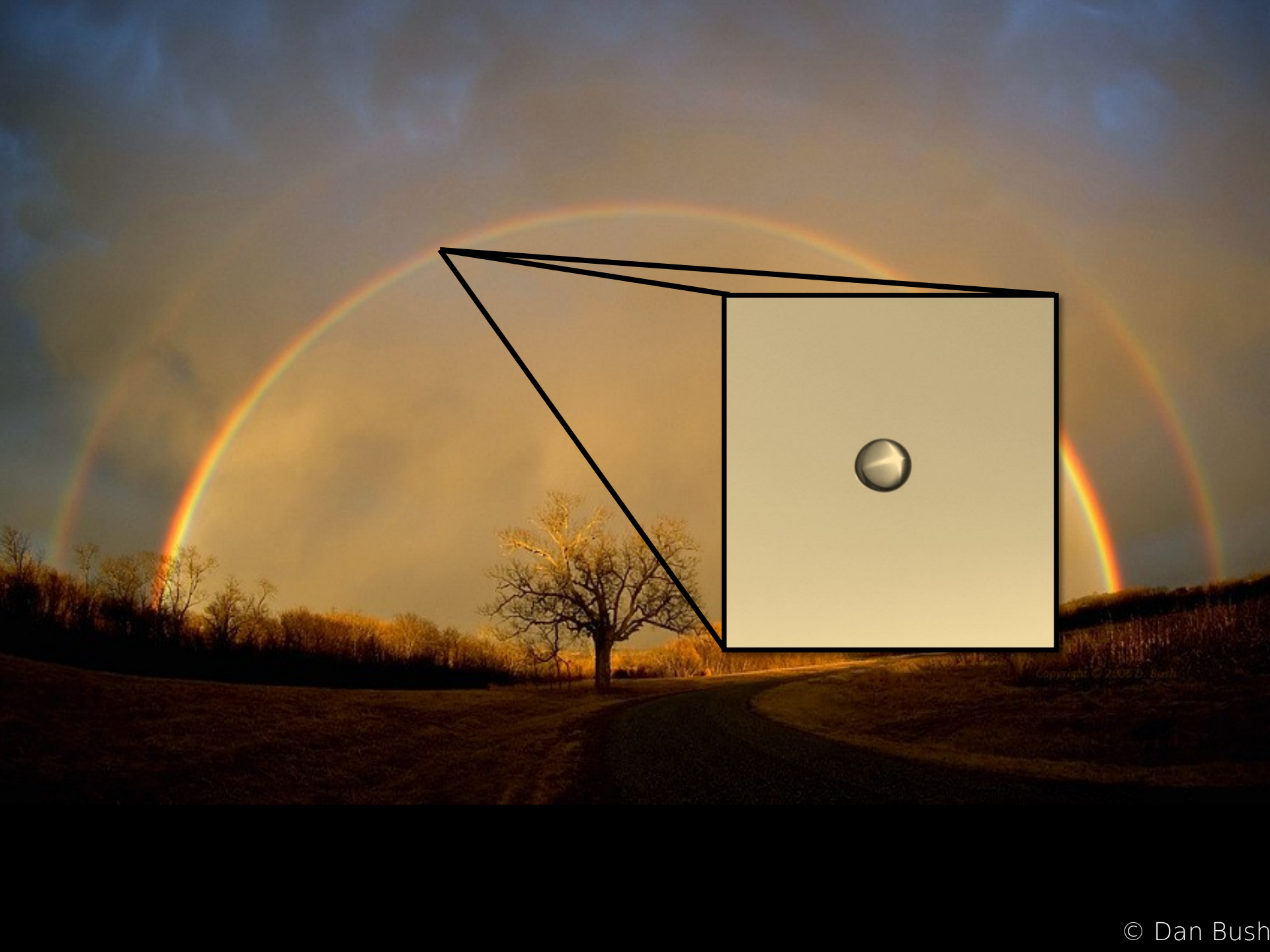
Iman Sadeghi¹, Adolfo Munoz², Philip Laven³, Wojciech Jarosz⁴, Francisco Seron², Diego Gutierrez², Henrik Wann Jensen¹

¹ UC San Diego

² Universidad de Zaragoza

³ Horley, UK

⁴ Disney Research Zurich



$$\begin{aligned}
L_\lambda(x, \vec{\omega}) &= e^{-\tau_\lambda(x_0, x)} L_\lambda(x_0, \vec{\omega}) \\
&+ \int_{x_0}^x e^{-\tau_\lambda(x', x)} \alpha_\lambda(x') L_{e, \lambda}(x', \vec{\omega}) dx' \\
&+ \int_{x_0}^x e^{-\tau_\lambda(x', x)} \sigma_\lambda(x') \int_{\Omega} p_\lambda(x', \vec{\omega}', \vec{\omega}) L_\lambda(x', \vec{\omega}') d\vec{\omega}' dx'
\end{aligned}$$

APPEARANCE OF RAINBOWS



Primary Bow

APPEARANCE OF RAINBOWS



© Wojciech

Double Rainbow

APPEARANCE OF RAINBOWS



© Diego Cutierrez
Double Rainbow / Alexander Dark Band

APPEARANCE OF RAINBOWS



Red Bow at Sunset

APPEARANCE OF RAINBOWS



Multiple Supernumerary Arcs

APPEARANCE OF RAINBOWS



Fog Bow

©Les Cowley

APPEARANCE OF RAINBOWS

© Aravind Krishnaswami



Twinned Bow

APPEARANCE OF RAINBOWS

© Aravind Krishnaswami



Twinned Bow

APPEARANCE OF RAINBOWS



Twinned Bows

WATER DROP SHAPES

- Gravity vs. surface tension vs. air resistance

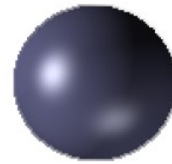
WATER DROP SHAPES

- Gravity vs. surface tension vs. air resistance



WATER DROP SHAPES

- Gravity vs. surface tension vs. air resistance



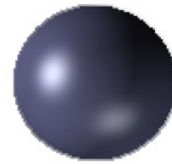
Misconception Small Drops

WATER DROP SHAPES

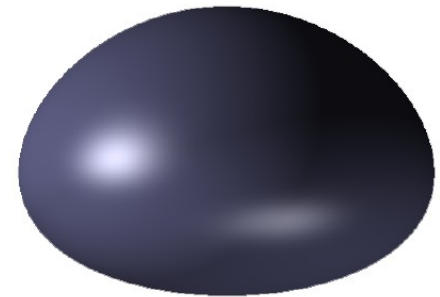
- Gravity vs. surface tension vs. air resistance



Misconception



Small Drops



Large Drops

APPEARANCE OF RAINBOWS

- Refraction
- Dispersion
- Interference
- Diffraction



APPEARANCE OF RAINBOWS

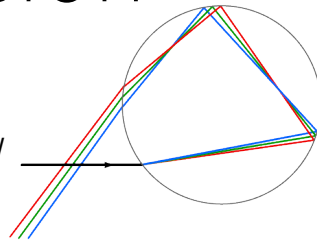
- Refraction
- Dispersion
- Interference
- Diffraction



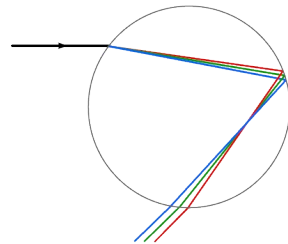
APPEARANCE OF RAINBOWS

- Refraction
- Dispersion

Secondary bow



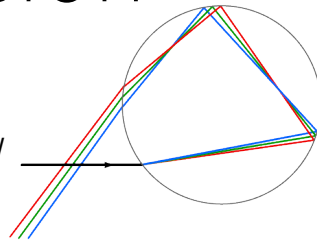
Primary bow



APPEARANCE OF RAINBOWS

- Refraction
- Dispersion

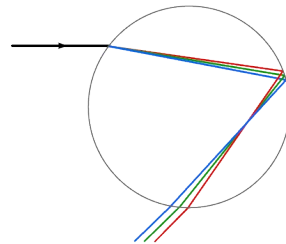
Secondary bow



Alexander Dark Band

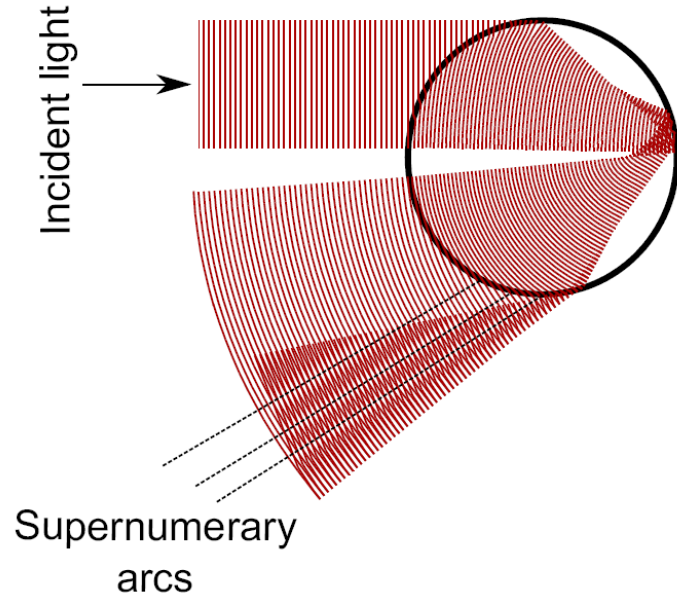


Primary bow



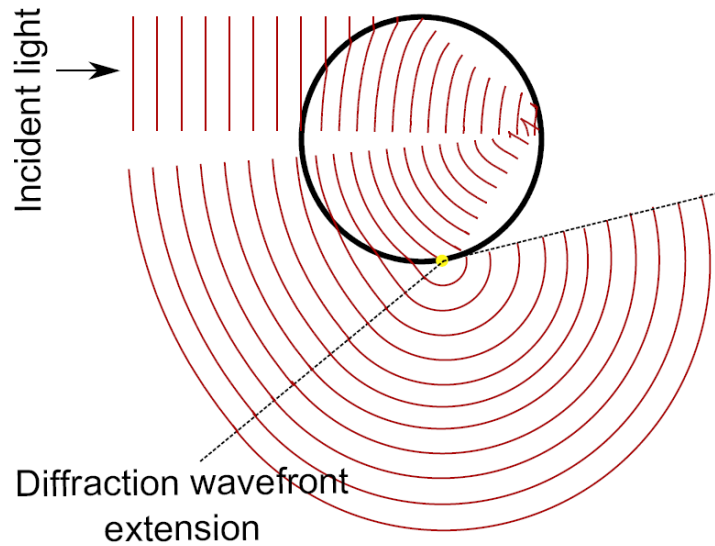
APPEARANCE OF RAINBOWS

➤ Interference



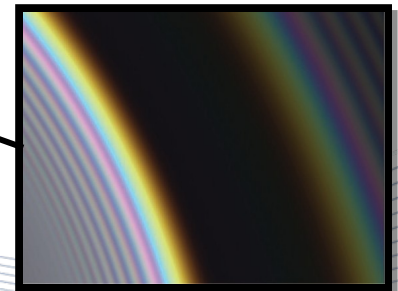
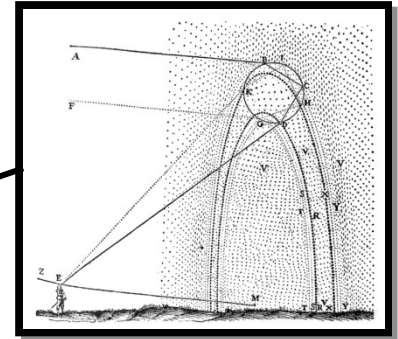
APPEARANCE OF RAINBOWS

➤ Diffraction



PREVIOUS WORK

- Ray Optics
 - Aristotle 384-322 BC
 - Descartes 1637
 - ...
 - Musgrave 1989
 - Frisvad et al. 2007
- Lorenz-Mie Theory
 - Jakel and Walter 1997
 - Lee 1998
 - Laven 2003, 2004
 - Riley et al. 2004
 - Gedzelman 2008
- FDTD Methods
 - Yang and Liou 1995 (Ice halos)



PREVIOUS WORK

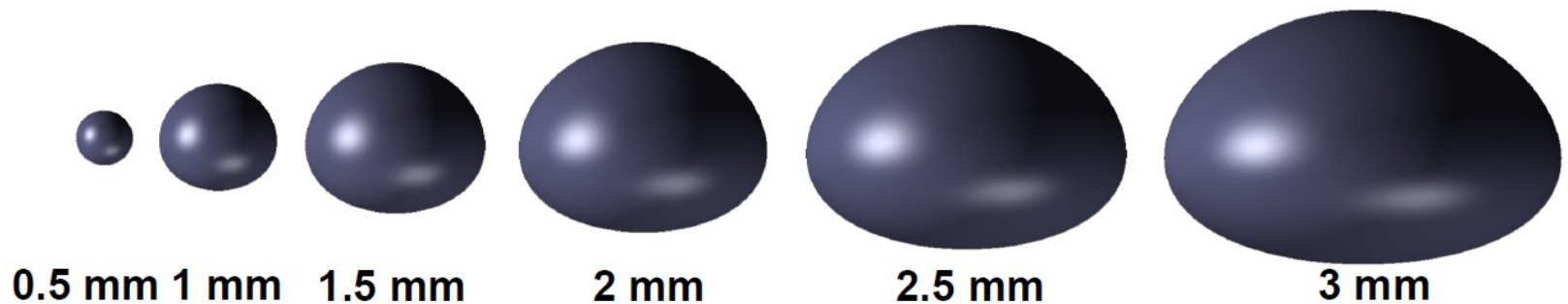
	Interference/ Diffraction	Physically Based Drops	Practical
Ray Optics	✗	✗	✓
Lorenz-Mie	✓	✗	✓
FDTD	✓	✓	✗
Our Goal	✓	✓	✓

OUR APPROACH

- Model physically based shape of water drops
- Compute the scattering profile of a water drop
- Use the scattering profile in rendering

PHYSICALLY BASED SHAPES

- Based on [Beard and Chuang 1987]



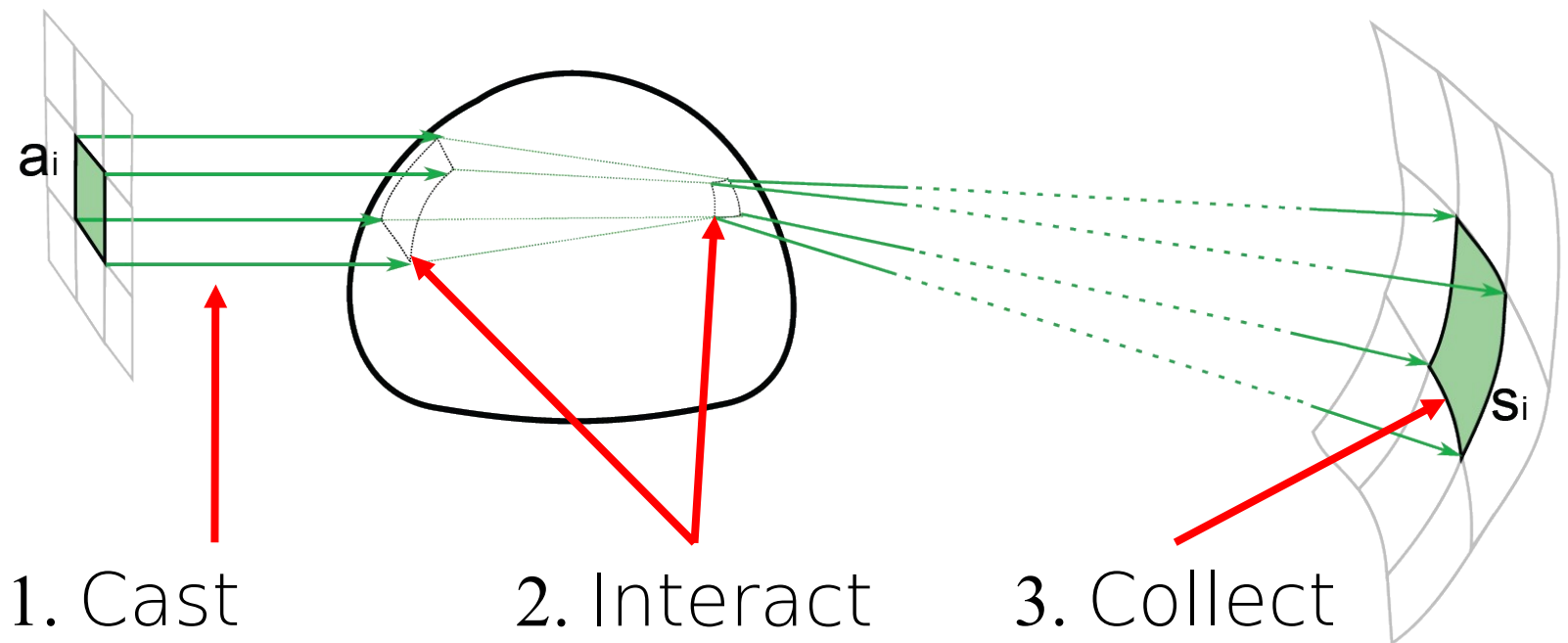
- Considered:
 - Surface tension
 - Hydrostatic pressure
 - Aerodynamic pressure

SCATTERING PROFILE SIMULATION

- Key ideas
 - Ray tracing
 - ✓ Arbitrary geometry
 - ✗ Interference
 - Keep track of the phase of light
 - ✓ Interference

SCATTERING PROFILE SIMULATION

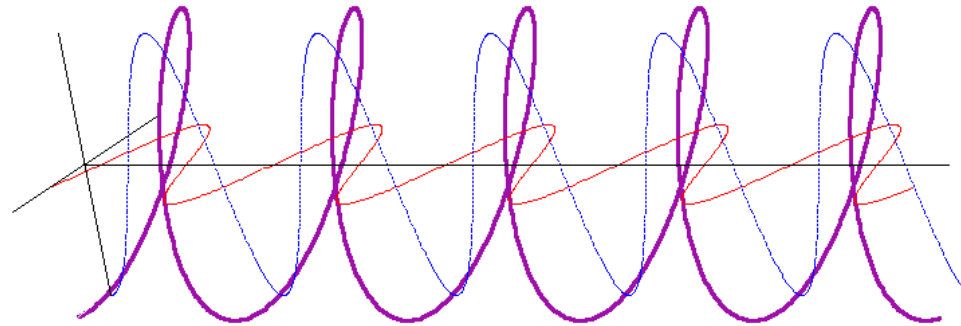
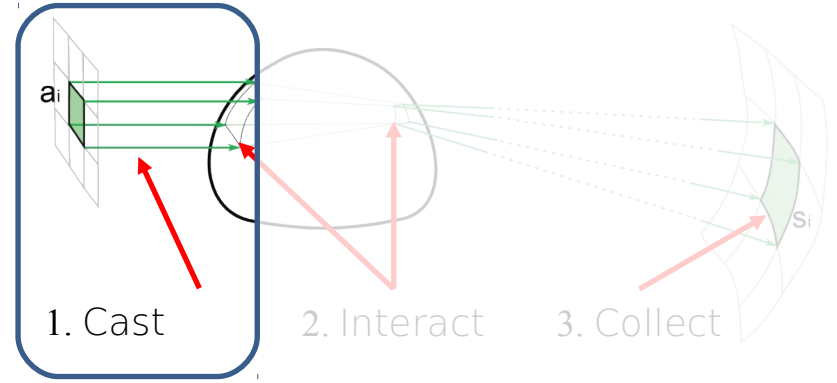
- Algorithm (per wavelength)



SCATTERING PROFILE SIMULATION

➤ 1. Cast

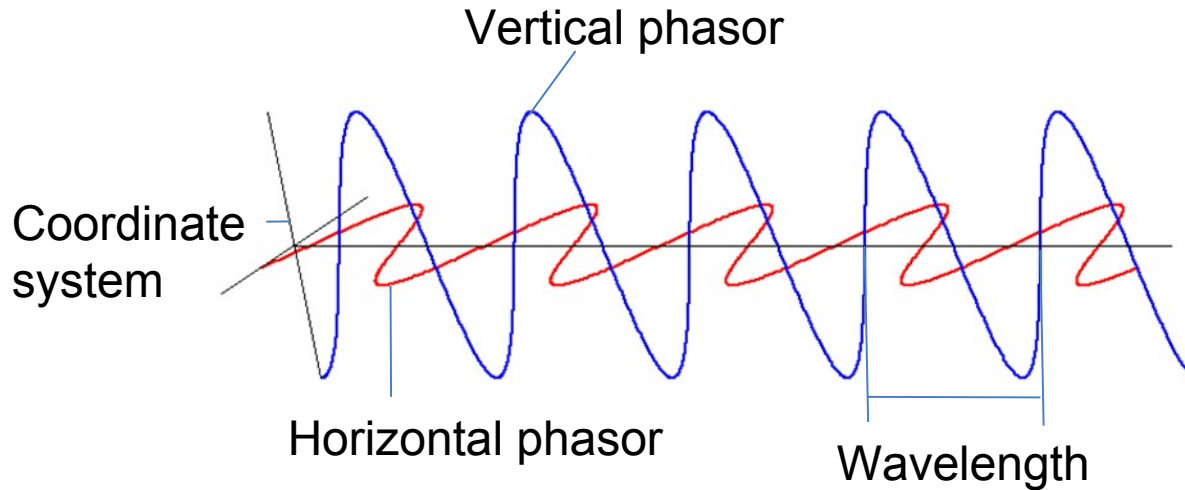
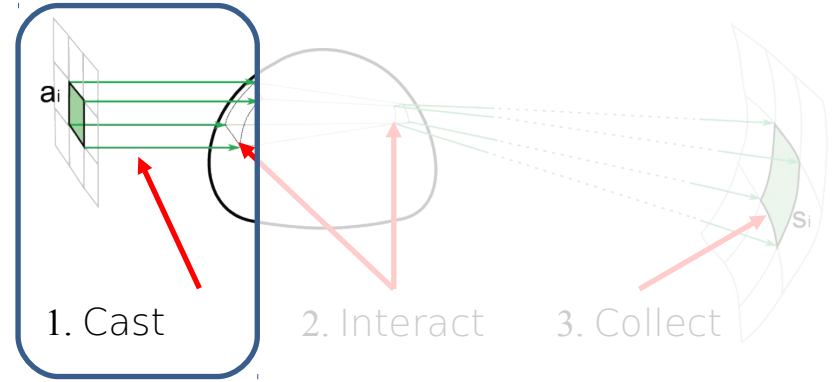
- Electromagnetic wave information



SCATTERING PROFILE SIMULATION

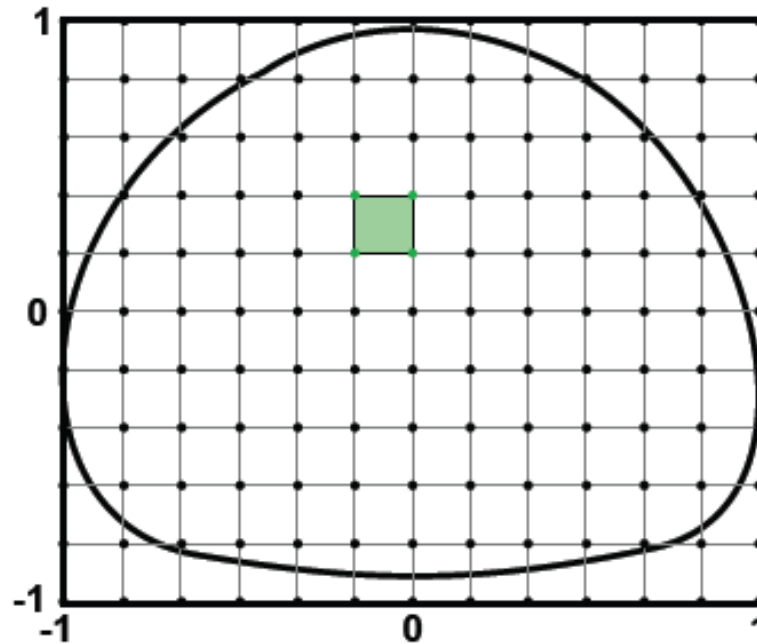
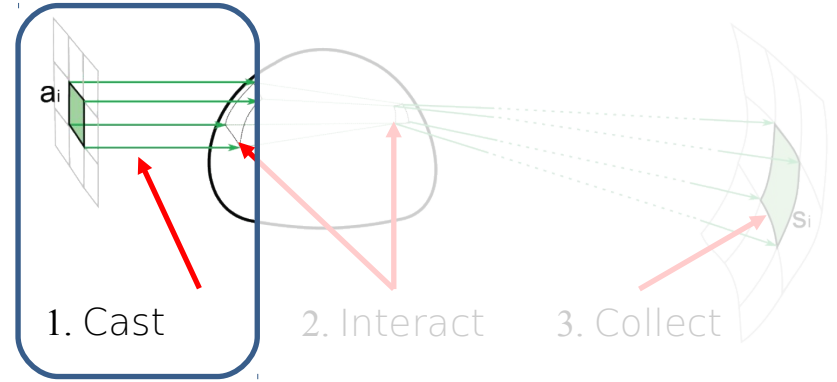
➤ 1. Cast

- Electromagnetic wave information



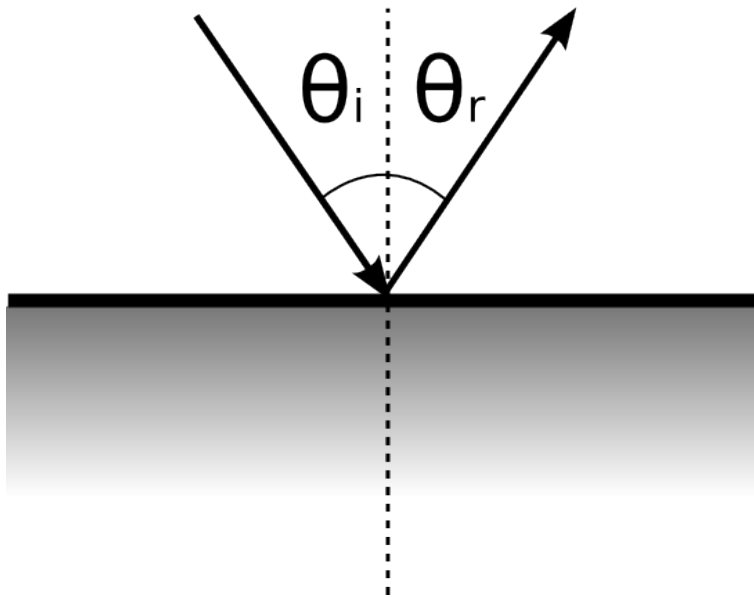
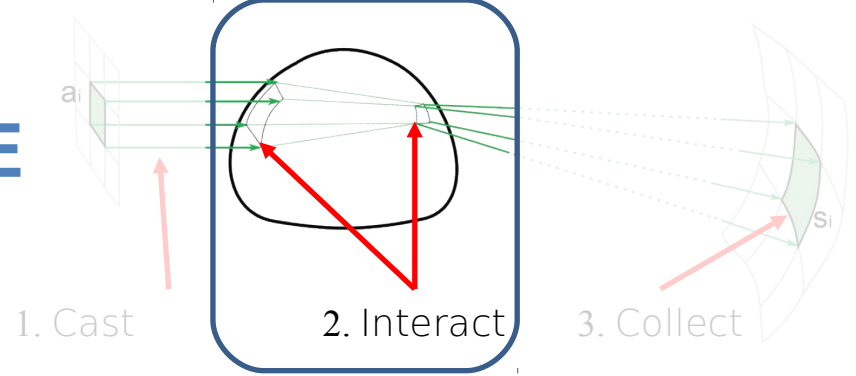
SCATTERING PROFILE SIMULATION

- 1. Cast
 - From a reference plane
 - Grid of rays

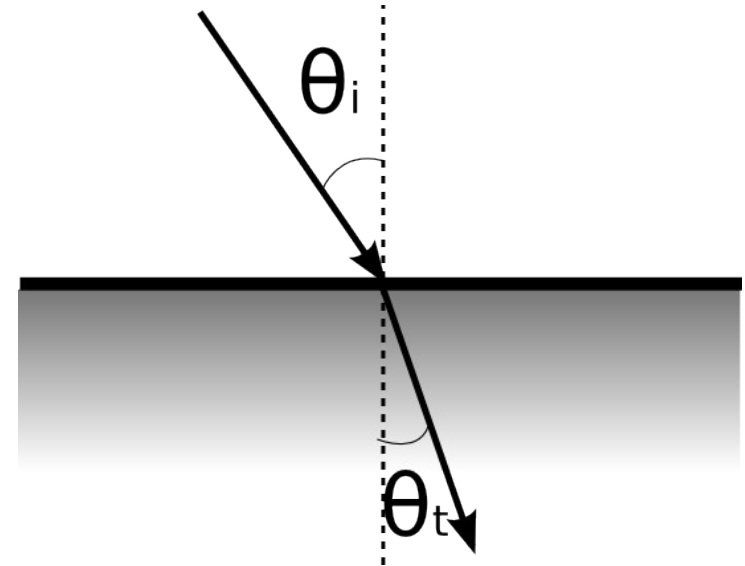


SCATTERING PROFILE SIMULATION

- 2. Interact
 - Geometric optics



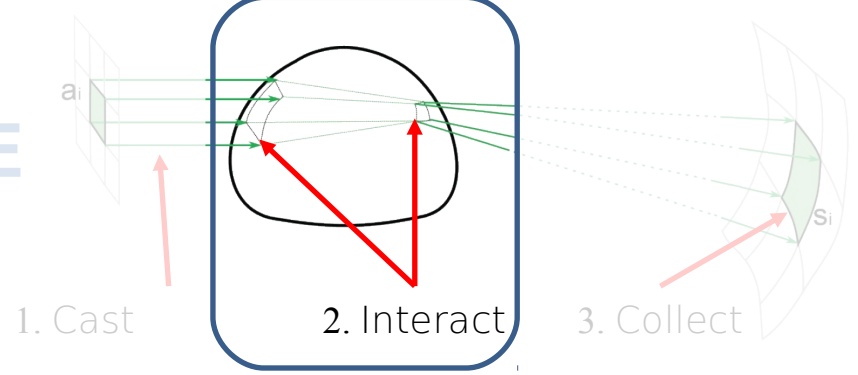
Reflection: law of reflection



Refraction: Snell's law

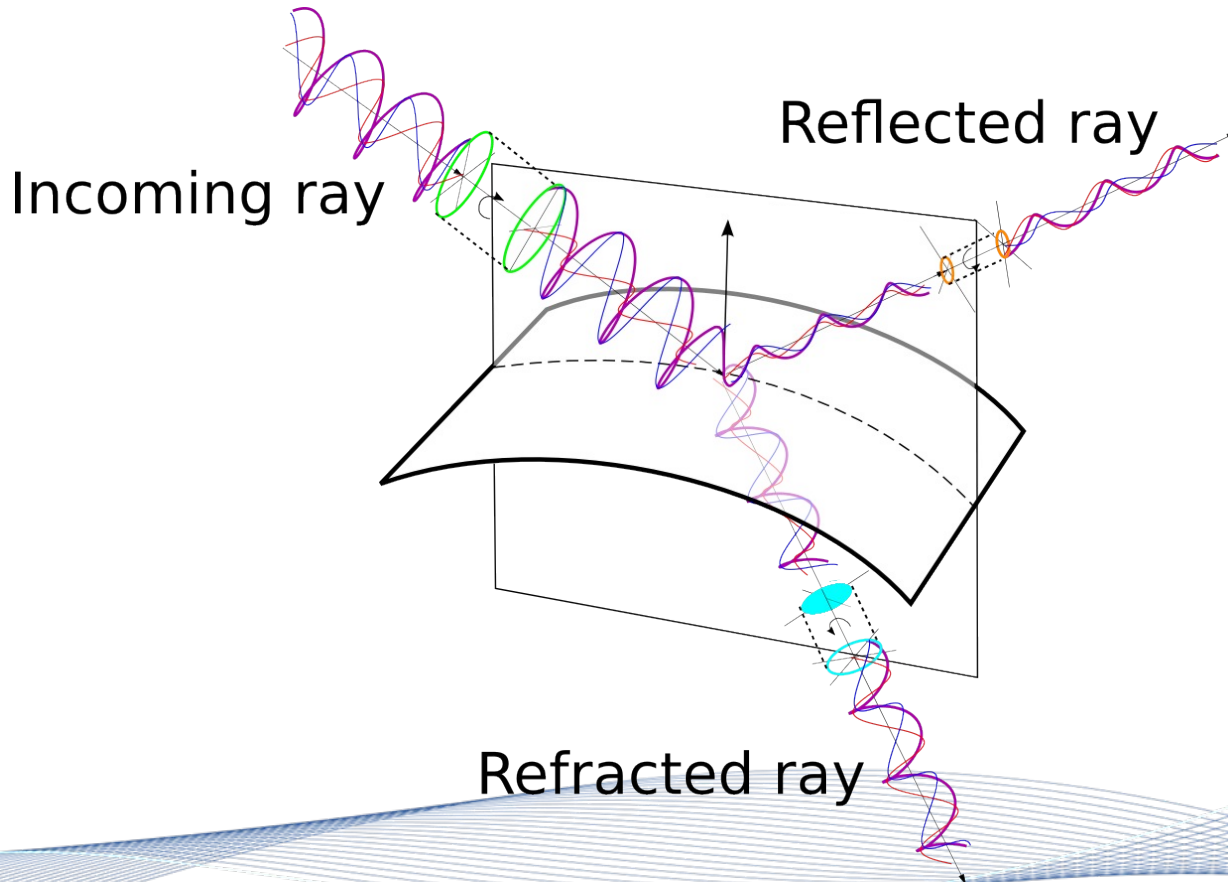
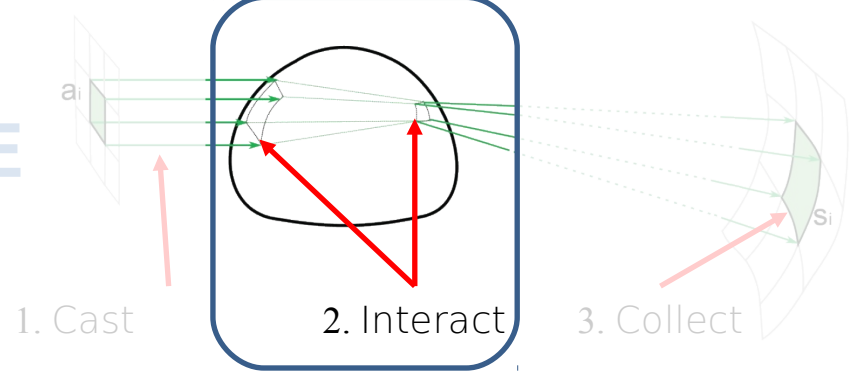
SCATTERING PROFILE SIMULATION

- 2. Interact
 - Fresnel coefficients
 - Focal lines
 - Optical path



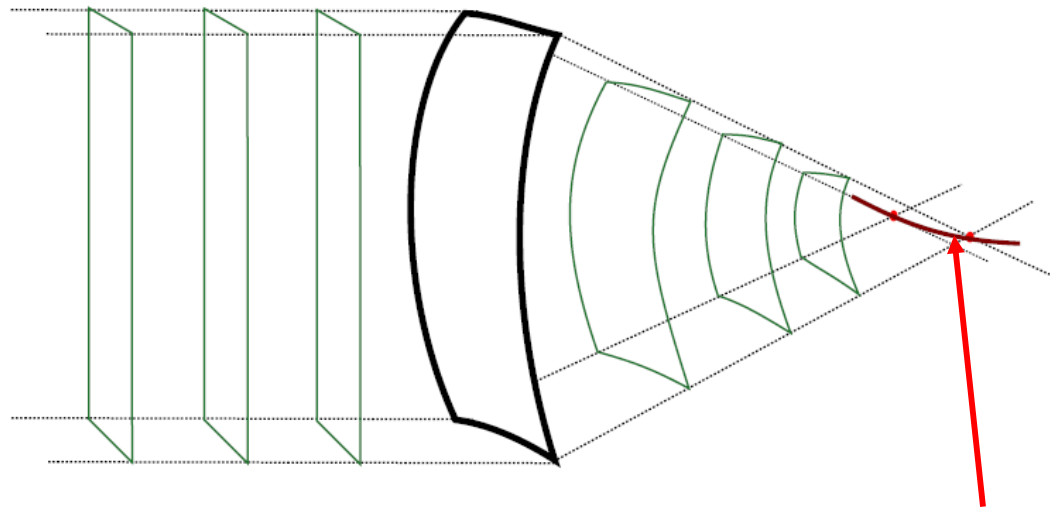
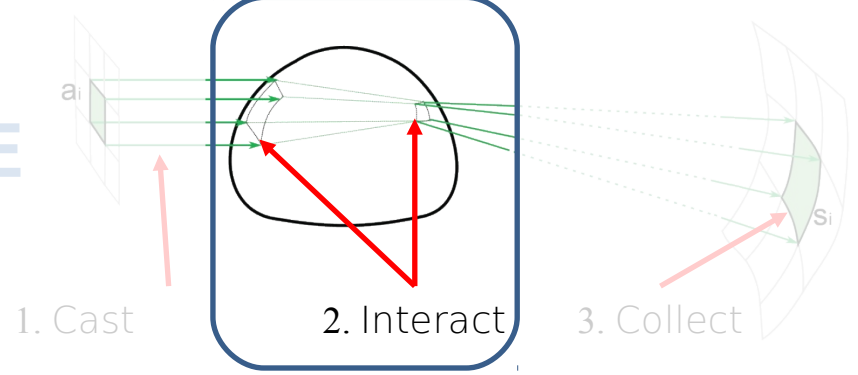
SCATTERING PROFILE SIMULATION

- 2. Interact
 - Fresnel coefficients



SCATTERING PROFILE SIMULATION

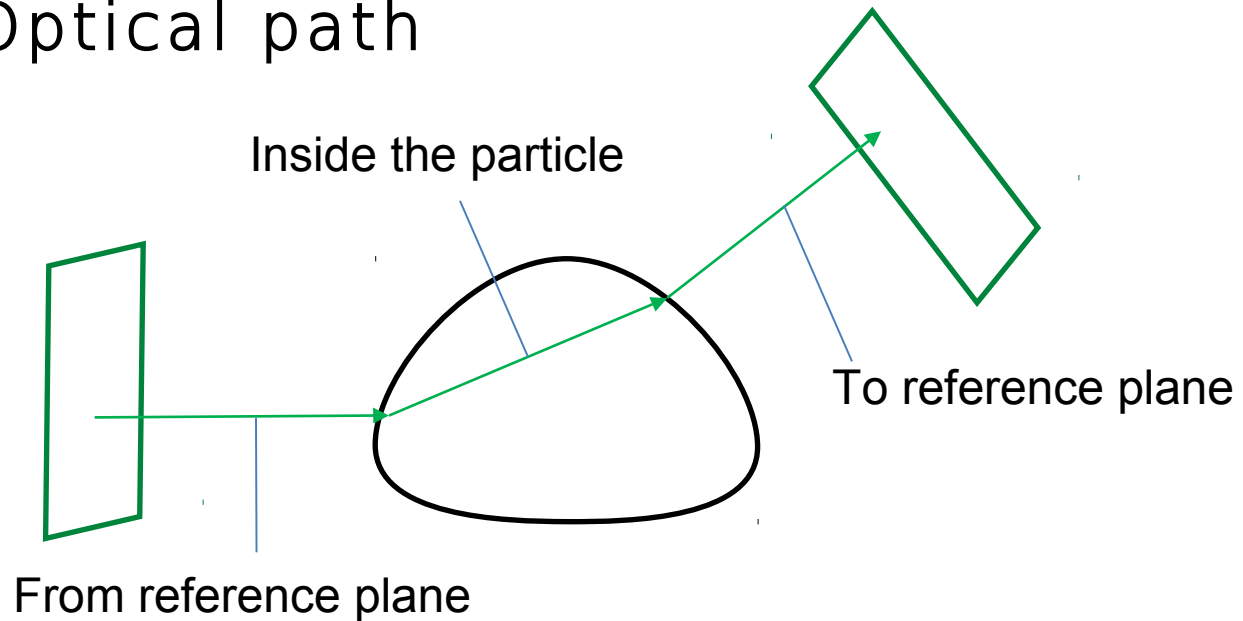
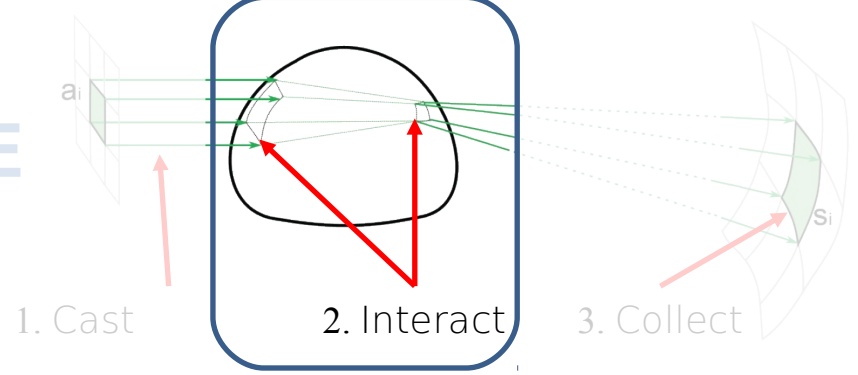
- 2. Interact
 - Fresnel coefficients
 - Focal lines



phase advance of $\pi/2$

SCATTERING PROFILE SIMULATION

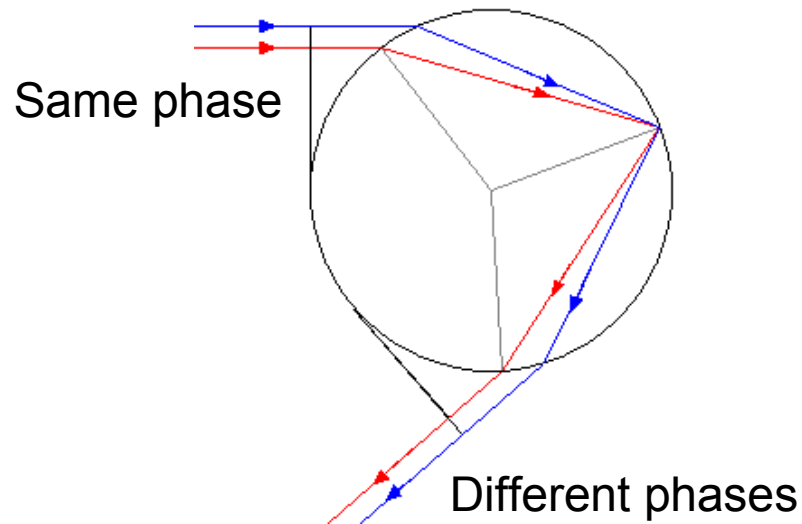
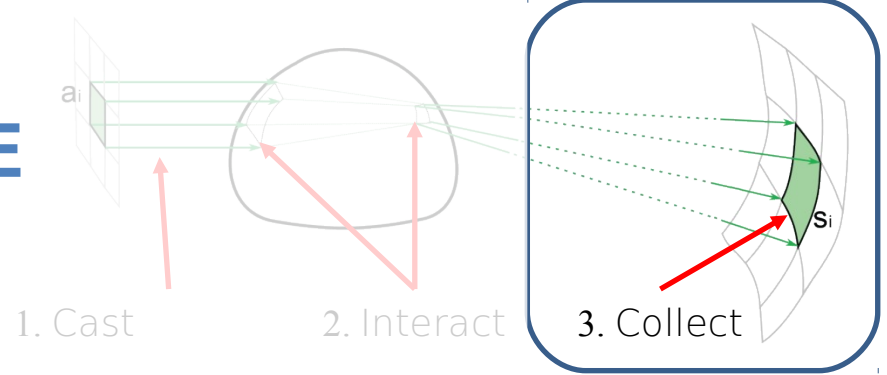
- 2. Interact
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 - Optical path



SCATTERING PROFILE SIMULATION

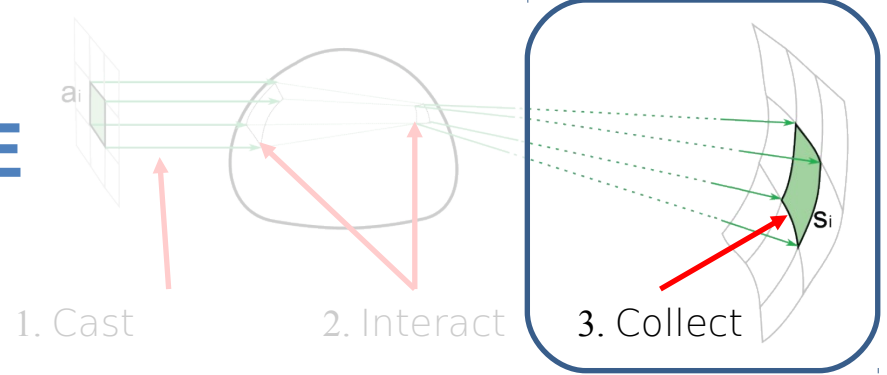
➤ 3. Collect

- Computing interference



- Probability of finding interfering paths is ~ 0

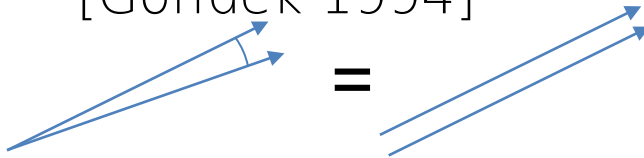
SCATTERING PROFILE SIMULATION



➤ 3. Collect

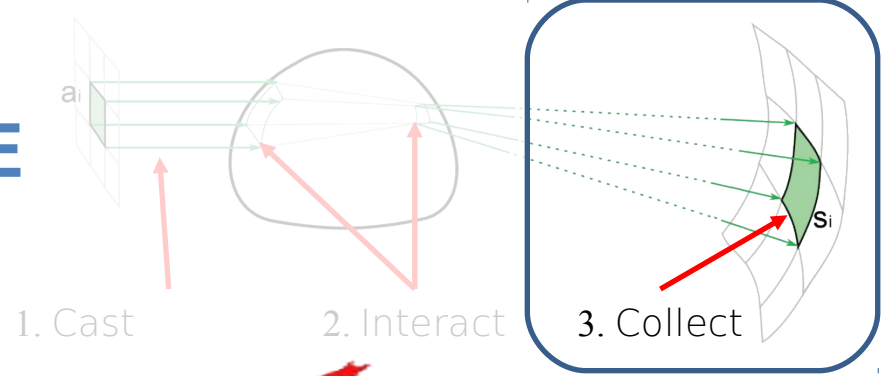
– Finding rays exiting the same direction:

- Idea: Establish a small angular threshold [Gondek 1994]



- Another idea: Use a density estimation kernel (as in Photon Mapping)

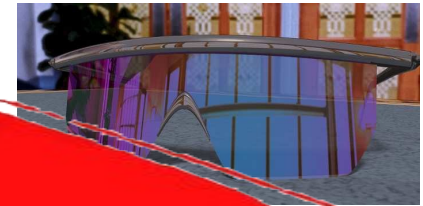
SCATTERING PROFILE SIMULATION



➤ 3. Collect

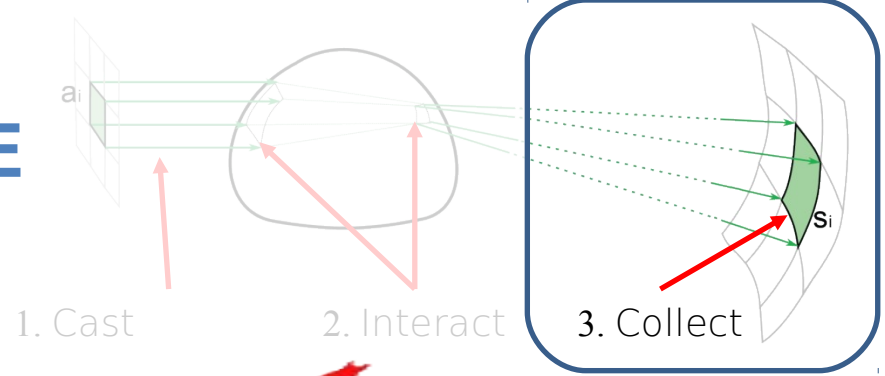
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[Gonder 1998]



Another idea: Use a density estimation kernel
(as in Photon Mapping)

SCATTERING PROFILE SIMULATION



➤ 3. Collect

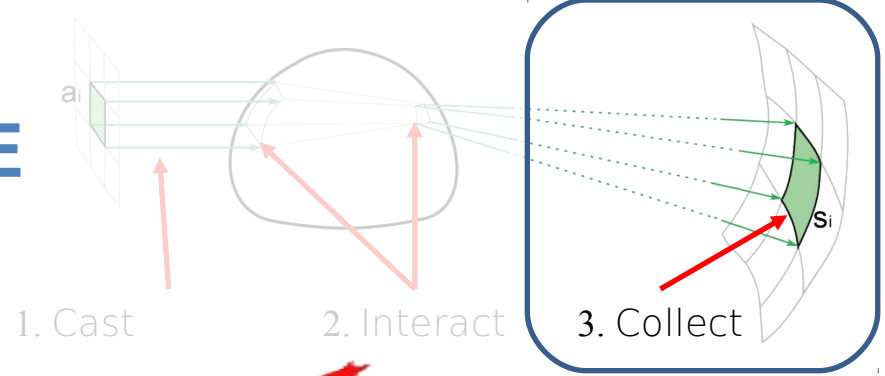
– Finding rays exiting the same direction:

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[Gondek 1998]



Better idea: Use a density estimation kernel (as in Photon Mapping)

SCATTERING PROFILE SIMULATION



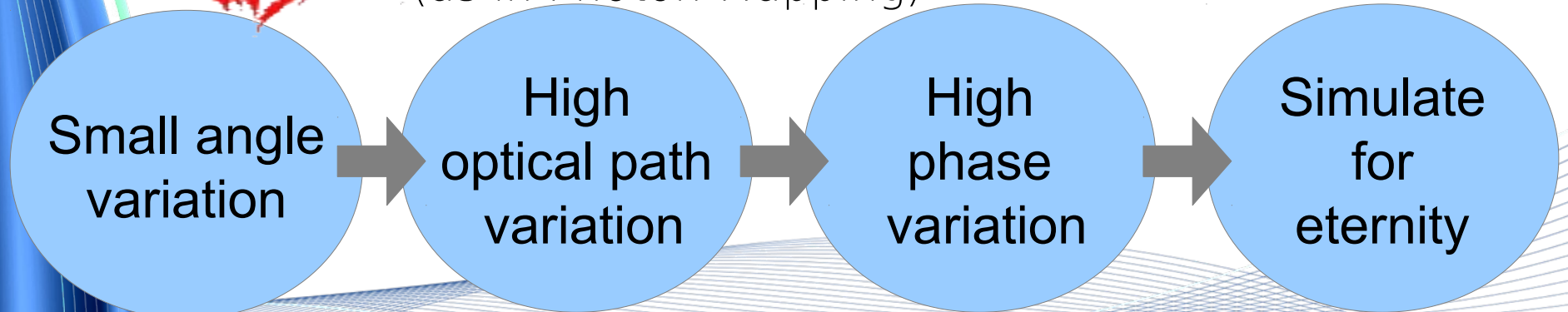
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– Finding rays exiting the same direction:

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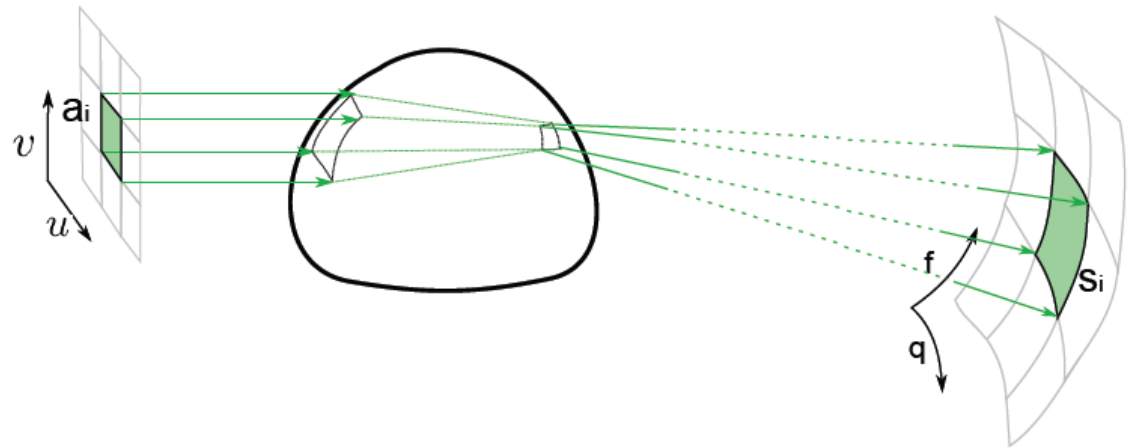
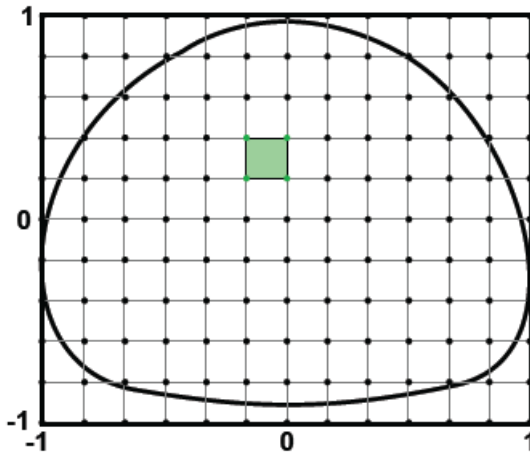
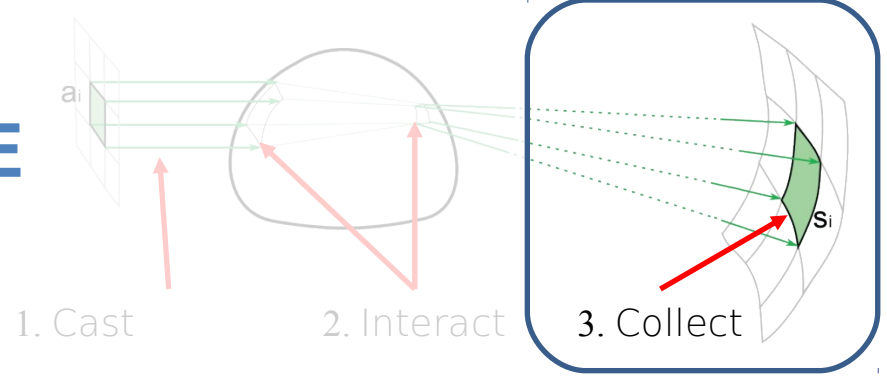


Another idea: Use a density estimation kernel (as in Photon Mapping)



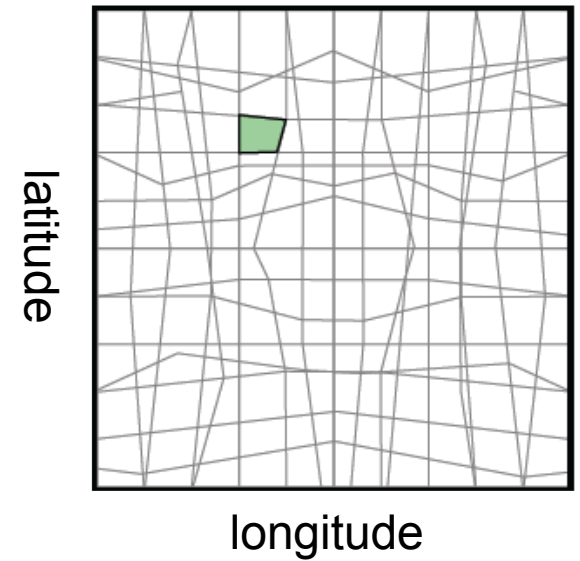
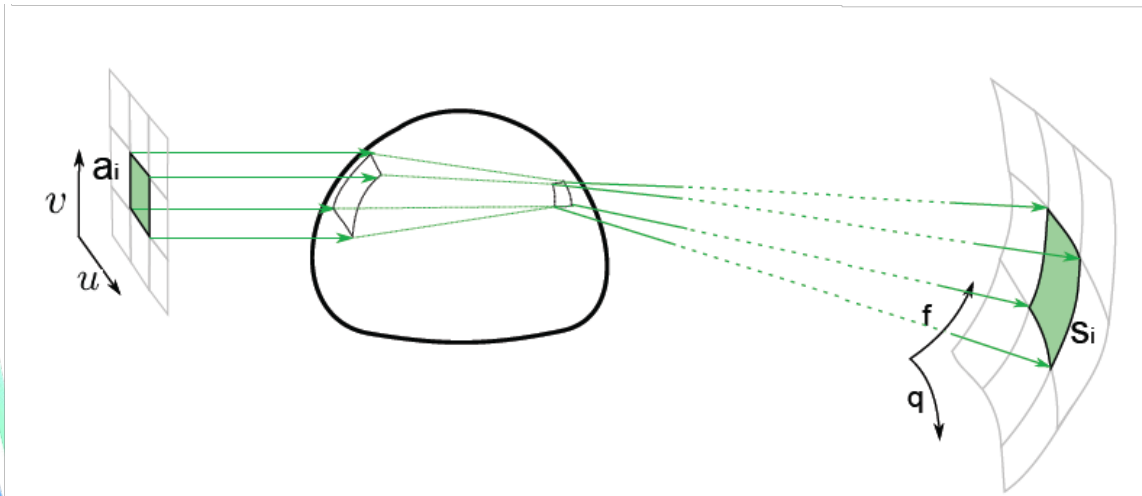
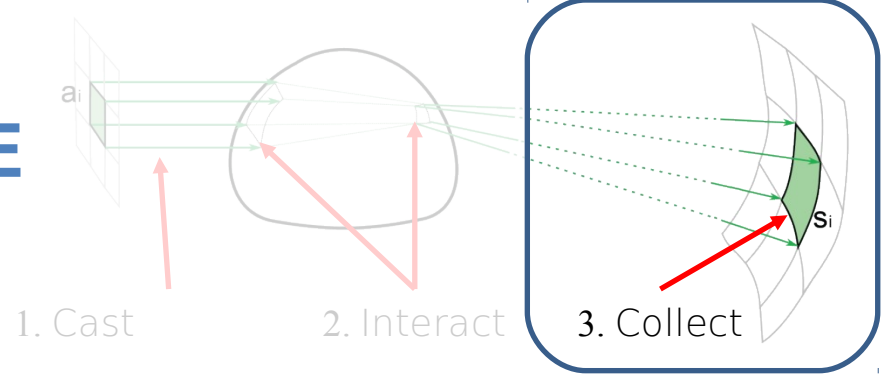
SCATTERING PROFILE SIMULATION

➤ 3. Collect



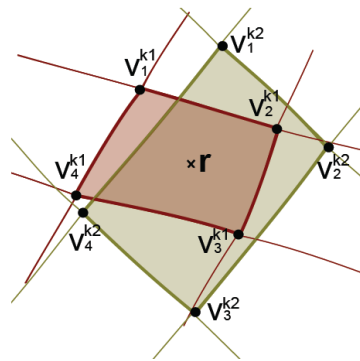
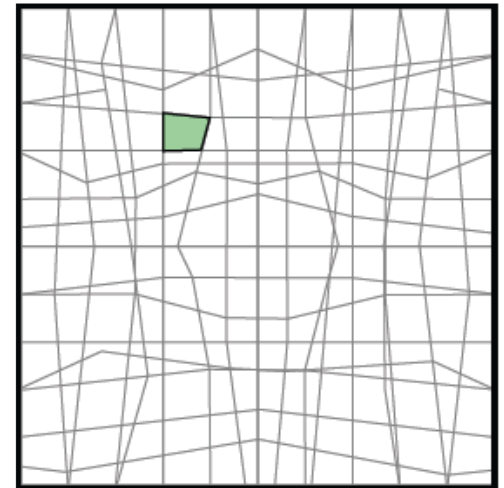
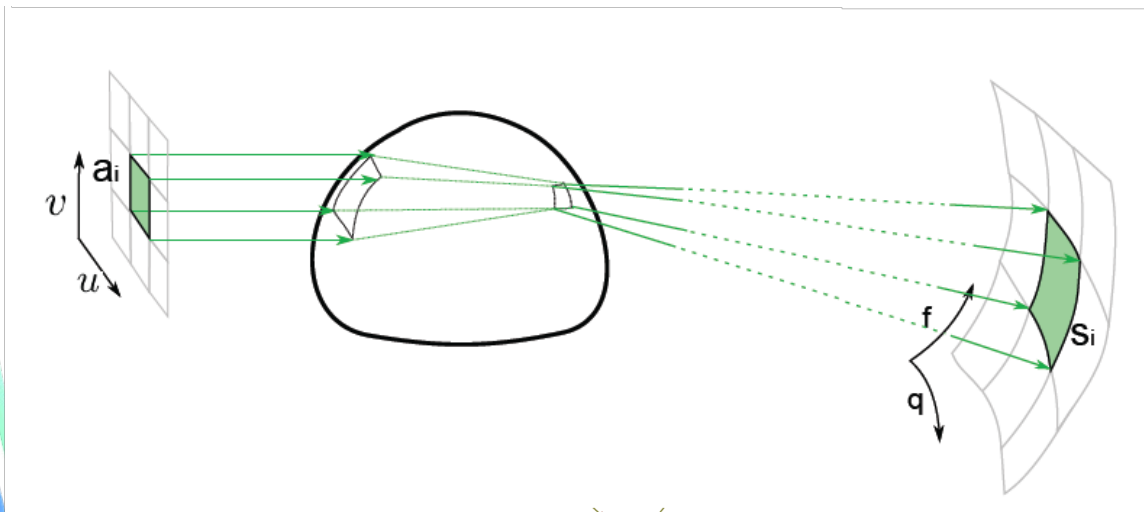
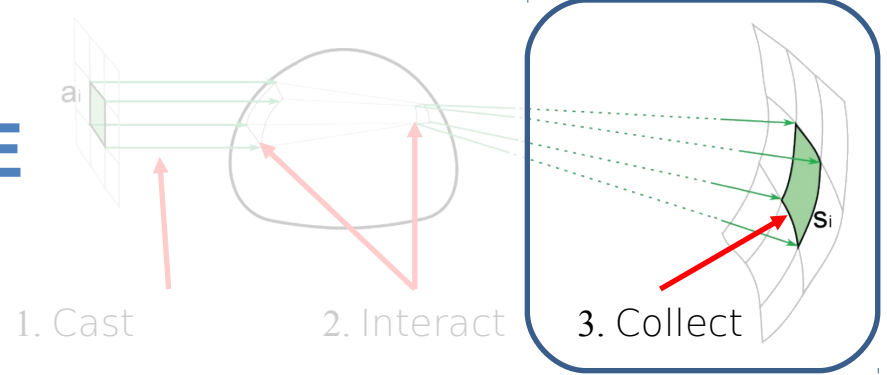
SCATTERING PROFILE SIMULATION

➤ 3. Collect



SCATTERING PROFILE SIMULATION

➤ 3. Collect

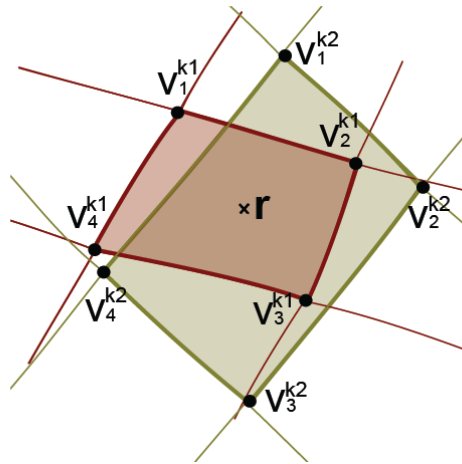
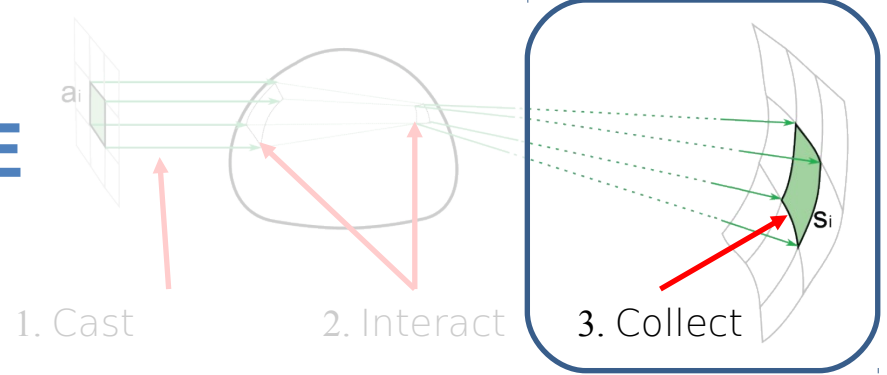


Per-patch interpolation
Phasor addition

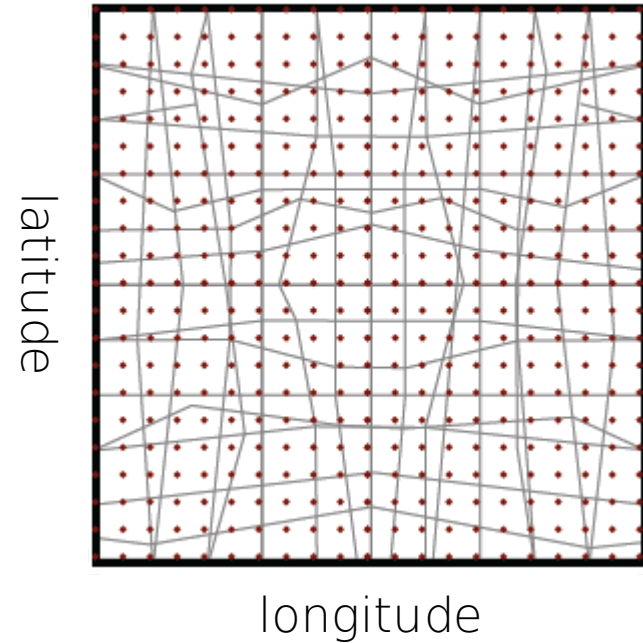
SCATTERING PROFILE SIMULATION

➤ 3. Collect

- Query and tabulate data (per wavelength)

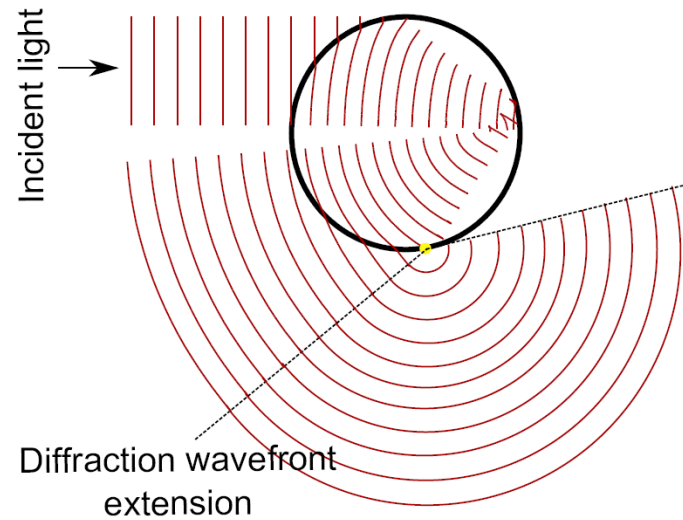


Per-patch interpolation
Phasor addition



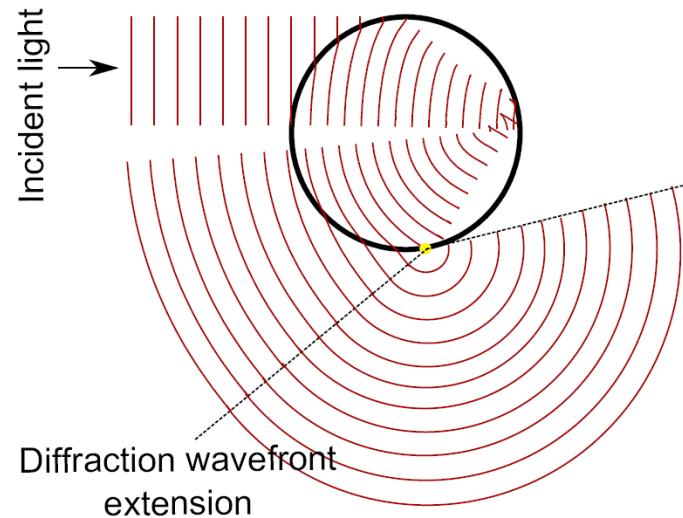
SCATTERING PROFILE SIMULATION

➤ Diffraction



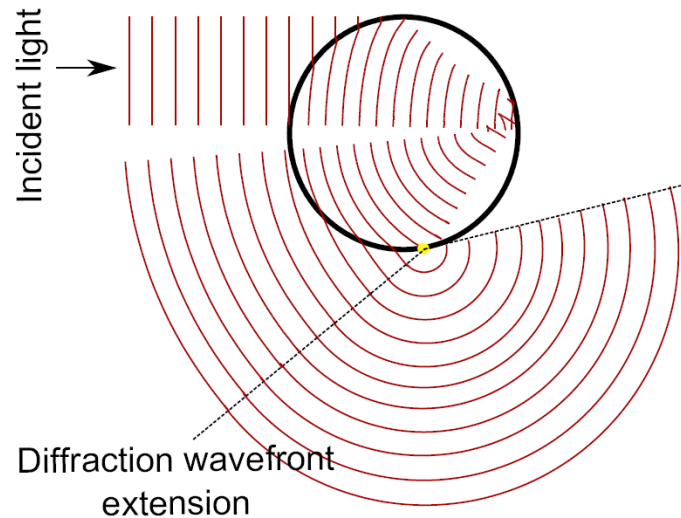
SCATTERING PROFILE SIMULATION

- Diffraction approximation



SCATTERING PROFILE SIMULATION

- Diffraction approximation
 - Smooth sharp transitions



Radius (mm)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
σ (degrees)	0.70	0.45	0.30	0.25	0.22	0.20	0.18	0.17	0.16	0.15

OUR MODEL VS. LORENZ-MIE

Our Model

Dispersion

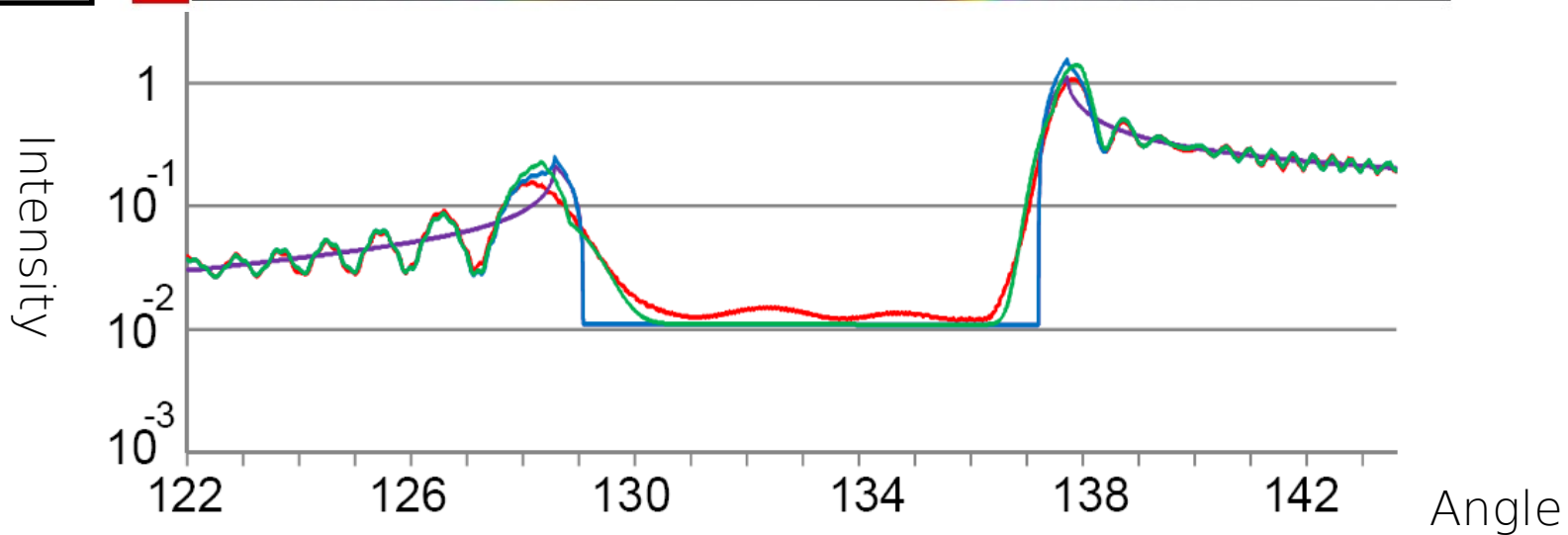
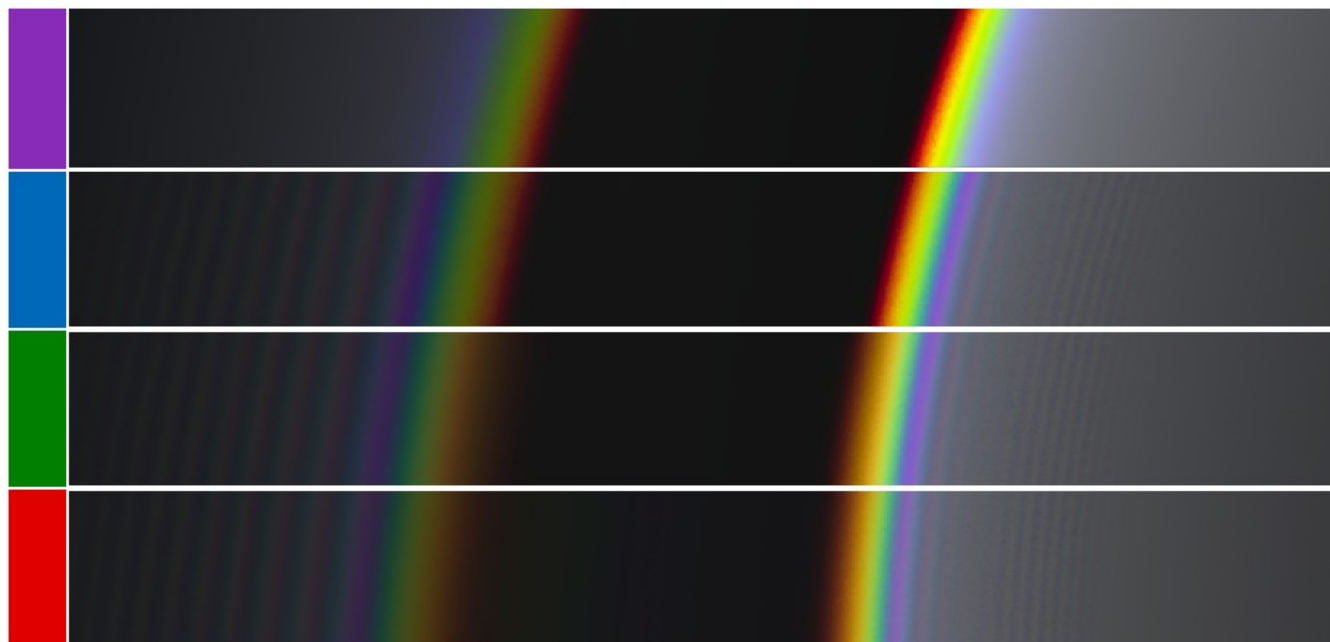
+

Interference

+

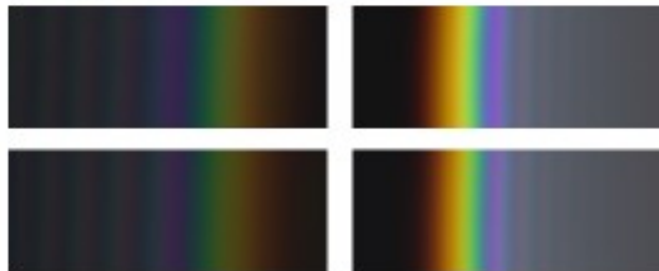
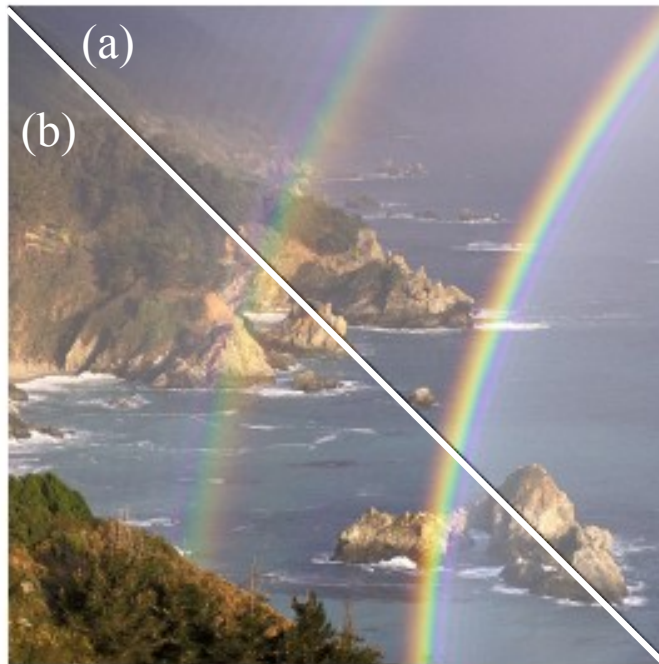
Diffraction

Lorenz-Mie

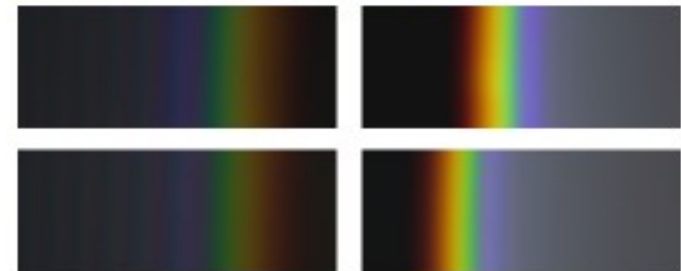
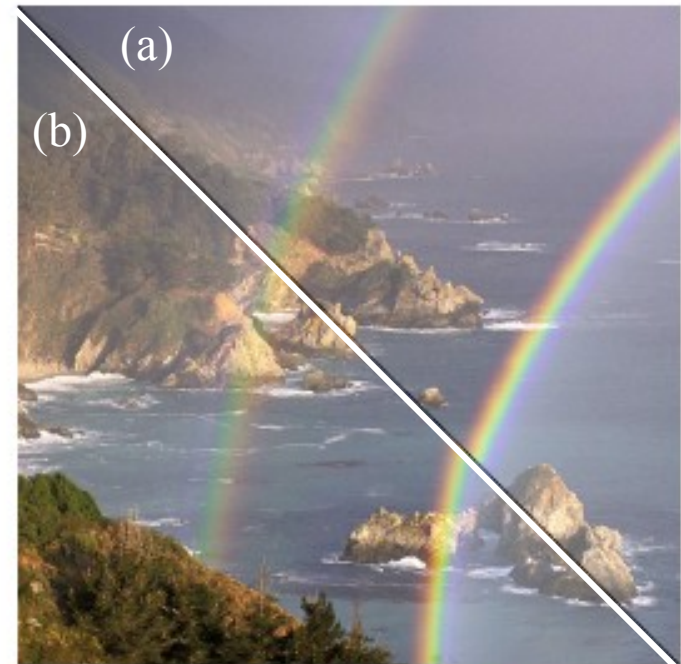


OUR MODEL VS. LORENZ-MIE

(a) Our model
(b) Lorenz-Mie

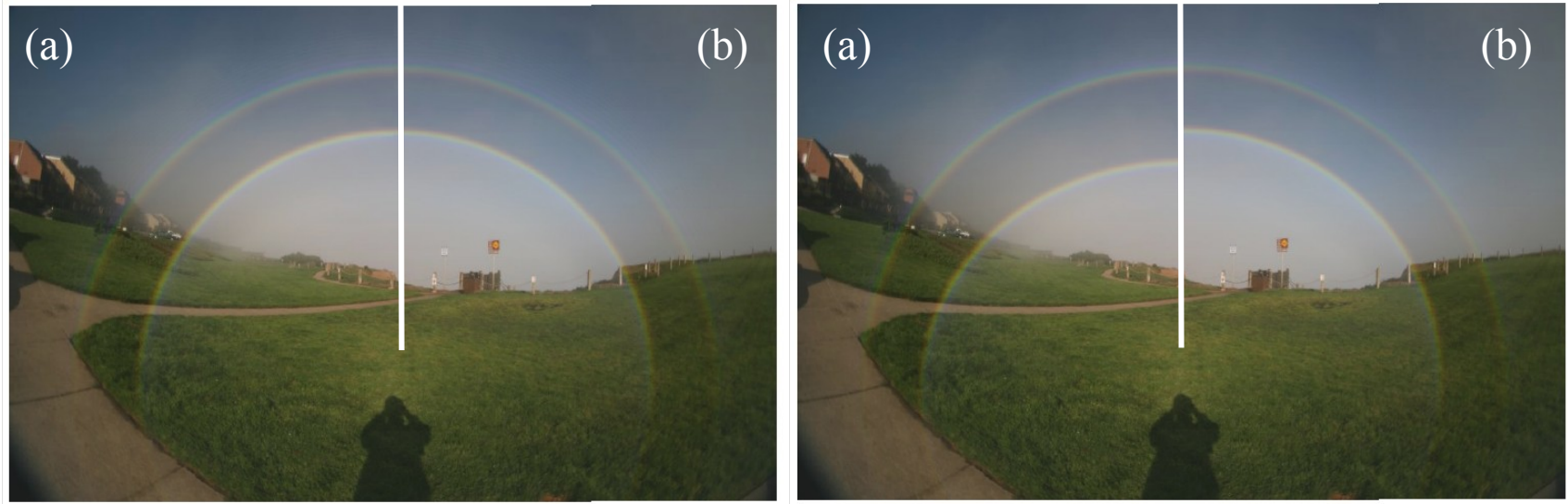


Spherical (0.4mm)



Nonspherical (0.5mm)

OUR MODEL VS. LORENZ-MIE



Spherical (0.4mm)

Nonspherical (0.5mm)

(a) Our model
(b) Lorenz-Mie

MATCHING PHOTOGRAPHS



Double Rainbow

MATCHING PHOTOGRAPHS



Double Rainbow / Alexander Dark Band

MATCHING PHOTOGRAPHS



Double Rainbow with Supernumerary Arcs

MATCHING PHOTOGRAPHS



Red Bow at Sunset

MATCHING PHOTOGRAPHS



© Ian Goddard

Multiple Supernumerary Arcs

MATCHING PHOTOGRAPHS



©Les Cowley

Fog Bow

MATCHING PHOTOGRAPHS

Photograph



Rendering
(0.4 mm & 0.45 mm)



Twinned Bow

SUMMARY

- Rainbow simulation from physically based shapes
 - Match Lorenz-Mie for spheres.
 - Not limited to spheres.
- Matched photographs of real rainbows
 - Double rainbows, supernumerary arcs, fogbows, red bows, etc.
 - First comprehensive simulation of twinned bows

FUTURE WORK

- Automate photo matching
- Simulation of other phenomena
- Diffraction Approximation
- GPU implementation



THANK YOU!

Physically-Based Simulation of Rainbows

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Diego Gutierrez²

Henrik Wann Jensen¹

¹University of California, San Diego

²Universidad de Zaragoza

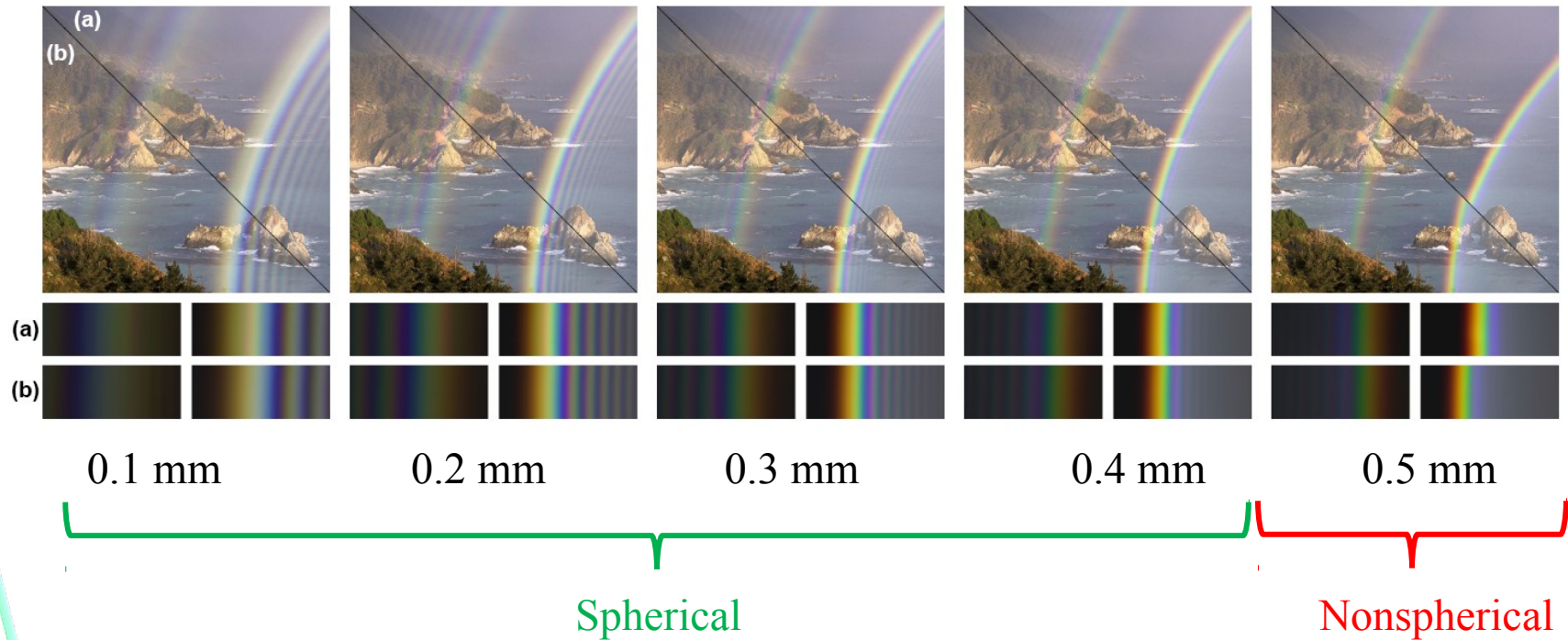
³Geneva

⁴Disney Research, Zürich



OUR MODEL VS. LORENZ-MIE

(a) Our model (b) Lorenz-Mie



OUR MODEL VS. LORENZ-MIE

(left half) Lorenz-Mie

(right half) Our model



0.4 mm



0.5 mm



0.6 mm



0.7 mm



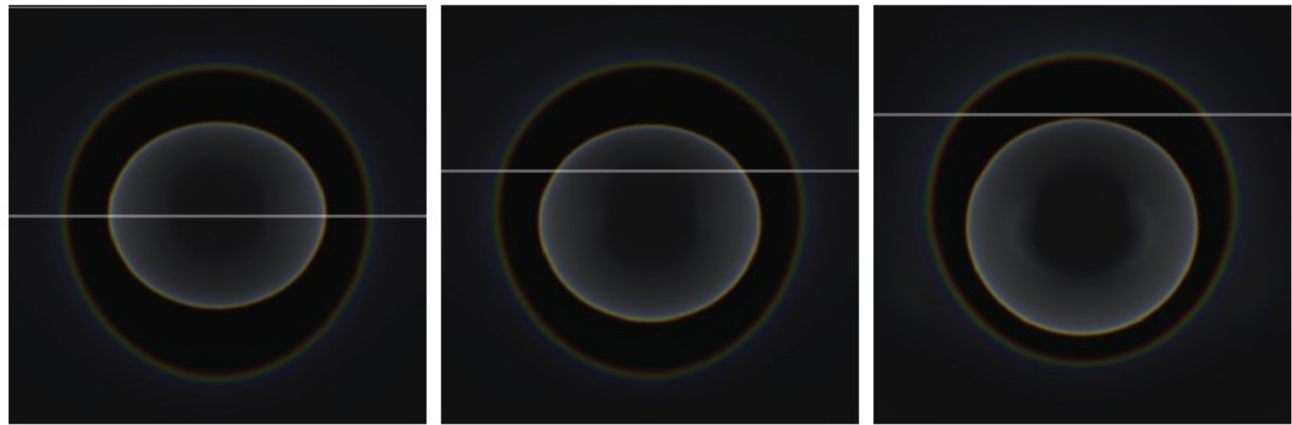
Spherical



Nonspherical

Sun's inclination 25°

SUN'S INCLINATION



0°

20°

40°

Sun's inclination

NON-SPHERICAL WATER DROPS



0.4 mm

0.5 mm

0.6 mm

0.7 mm

Sun's inclination 0°