Fourier Analysis for Sampling and Reconstruction

Cyril Soler

INRIA, LJK, Grenoble University

Outline

- Motivation
- Fourier spectrum along light paths
- Representations of the spectrum

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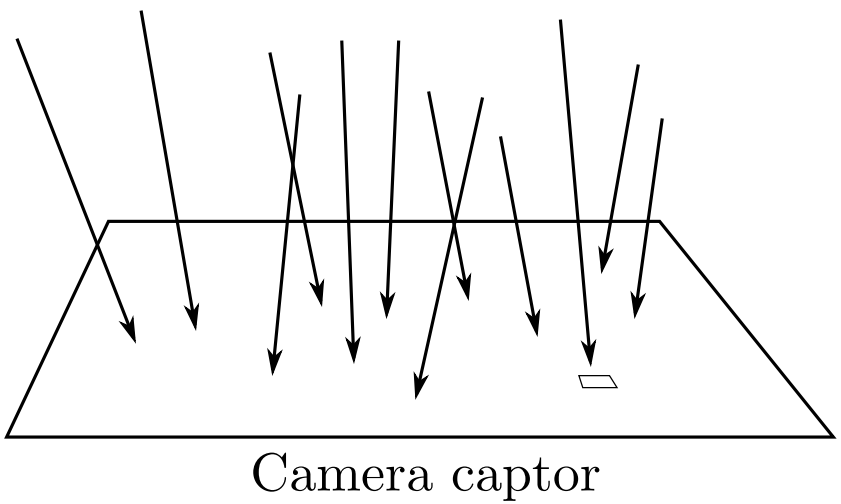

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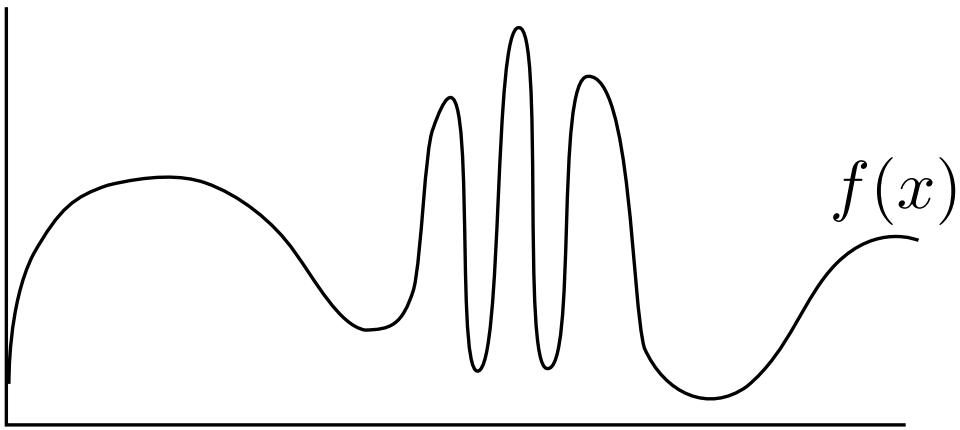

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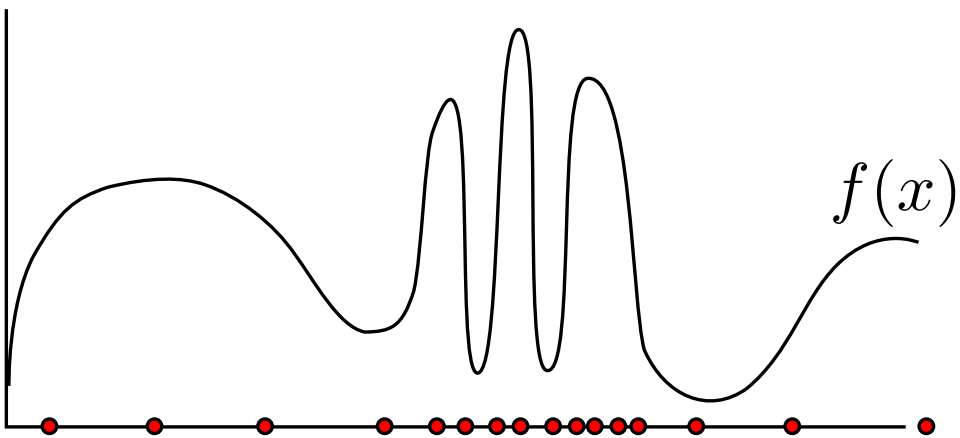

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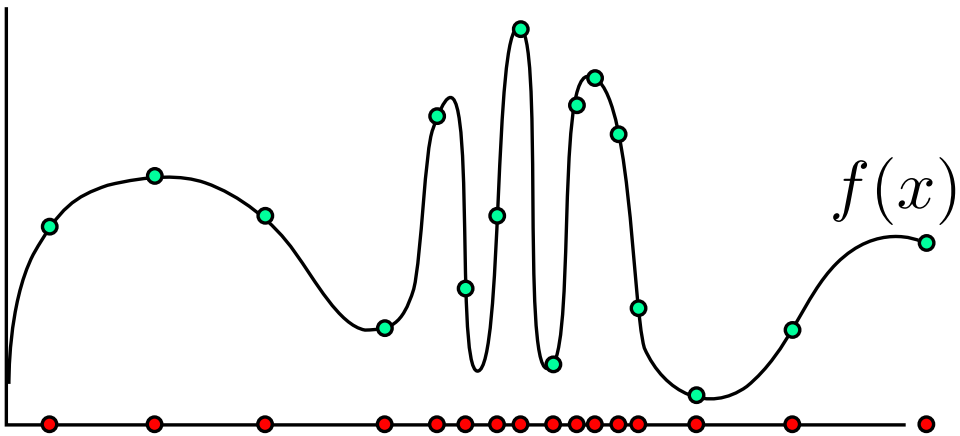

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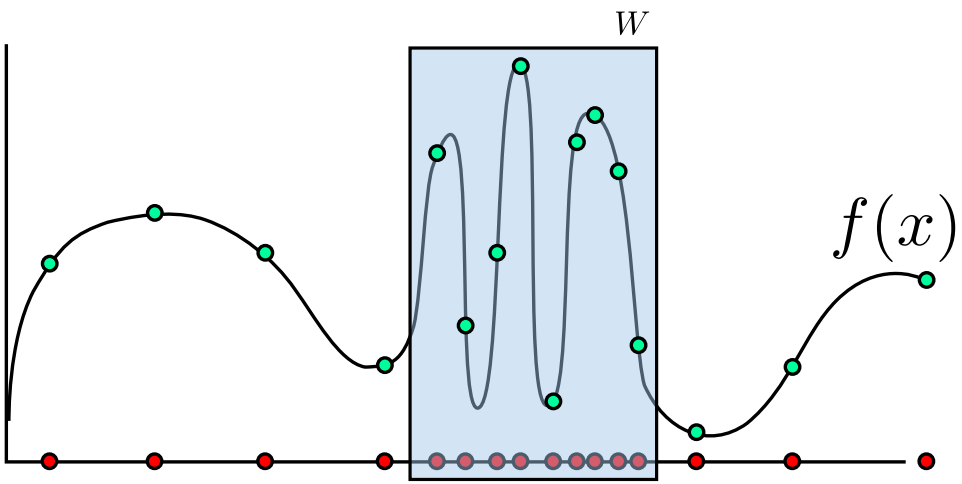

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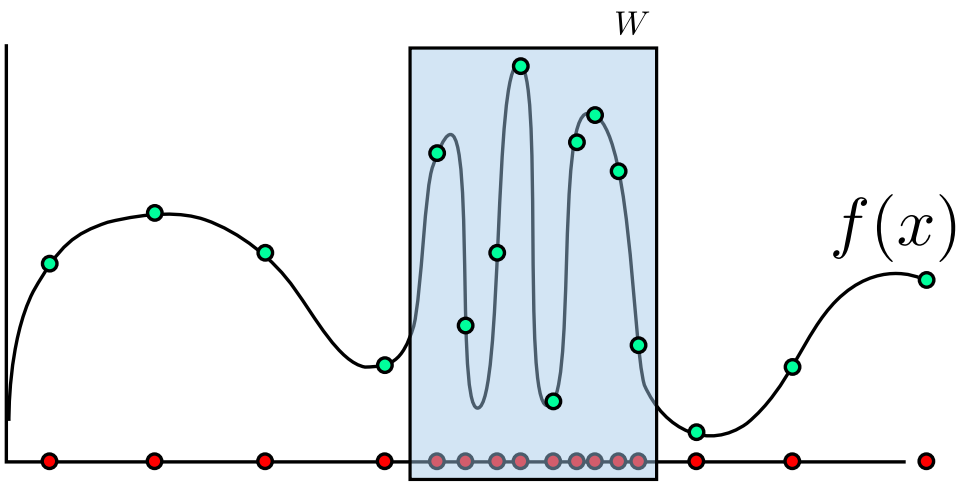

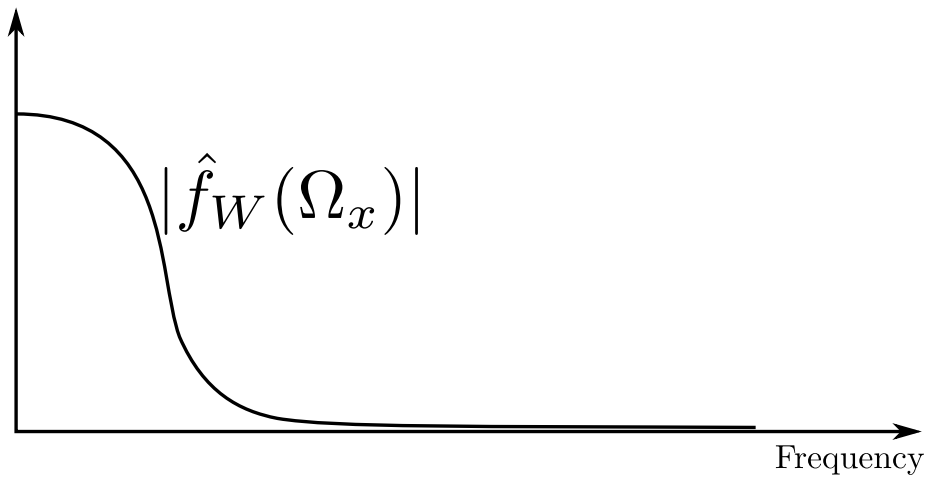

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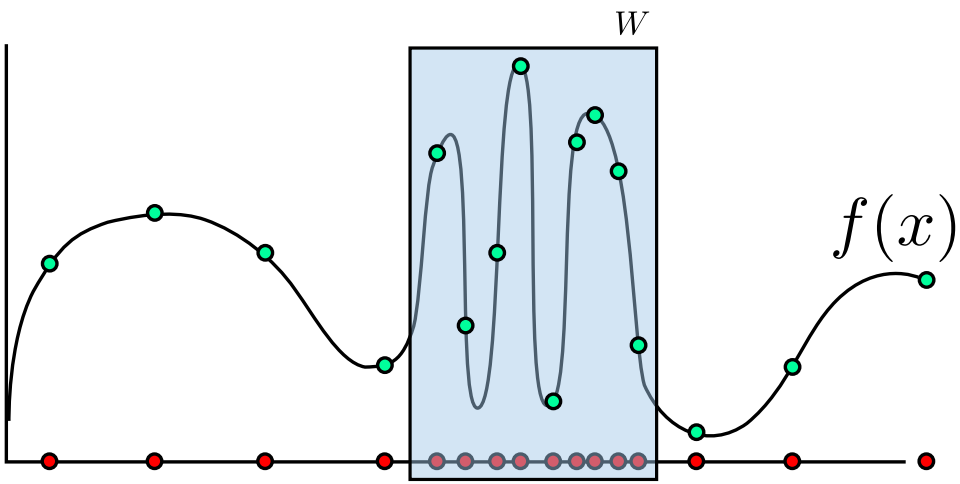

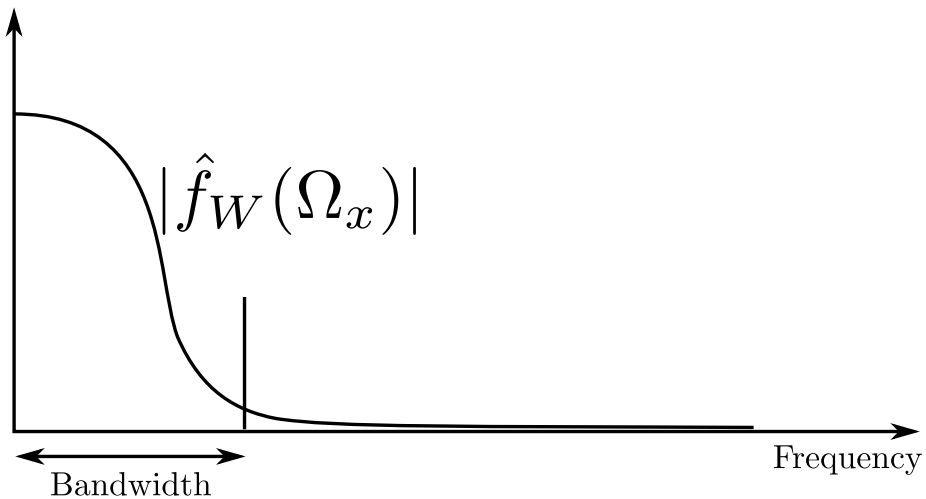

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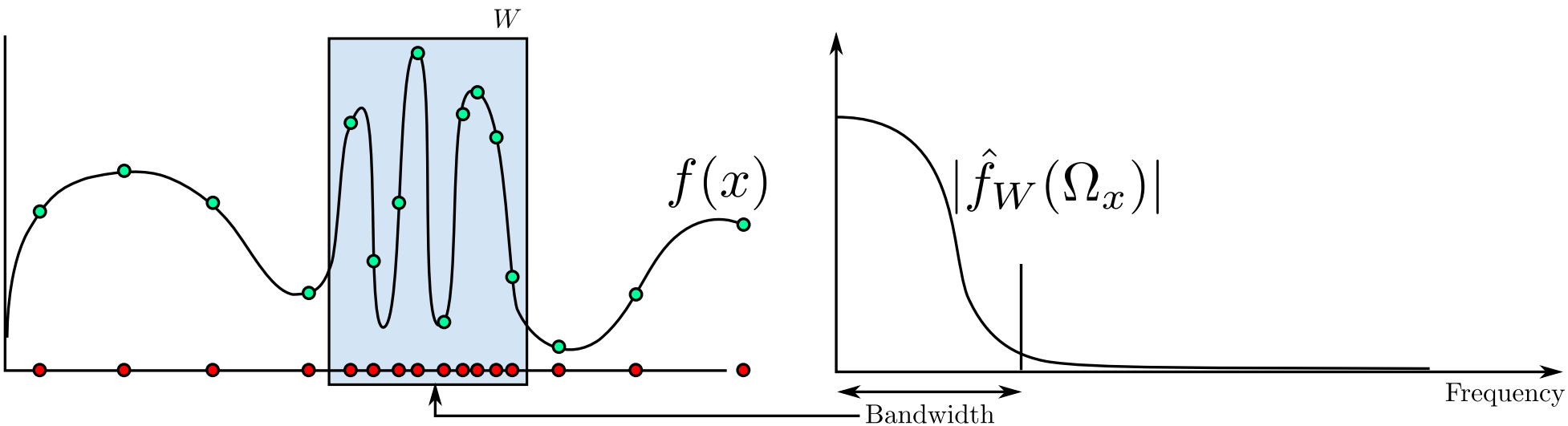

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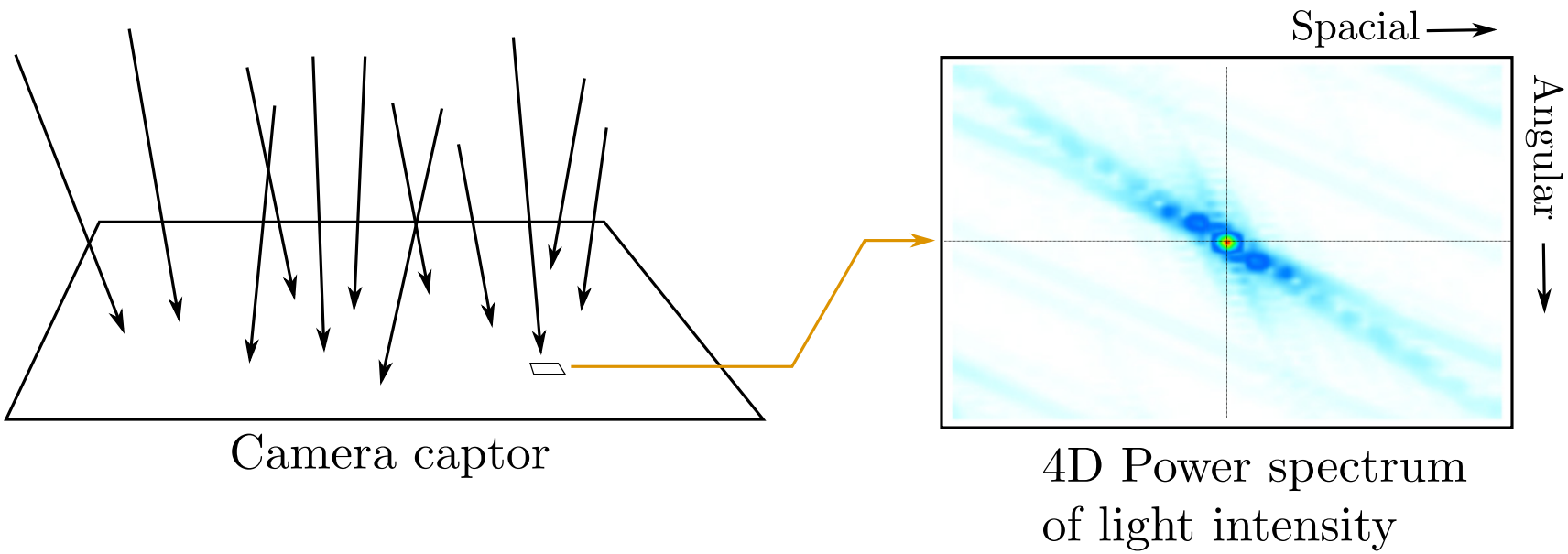

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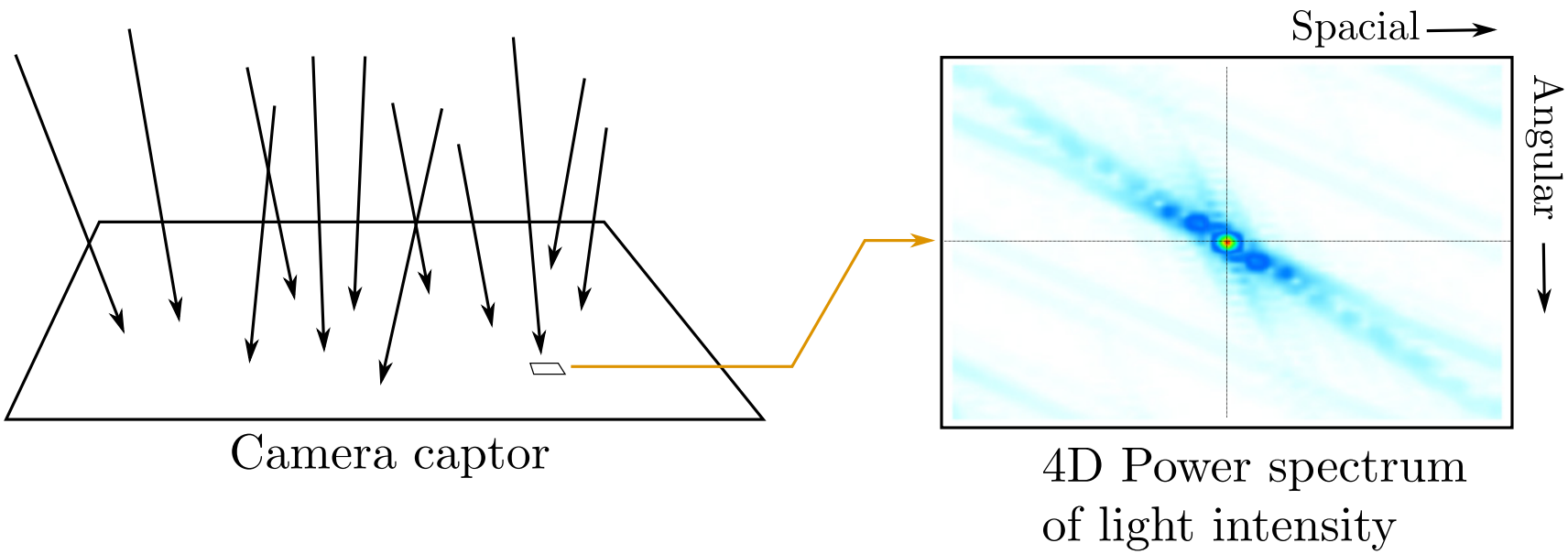

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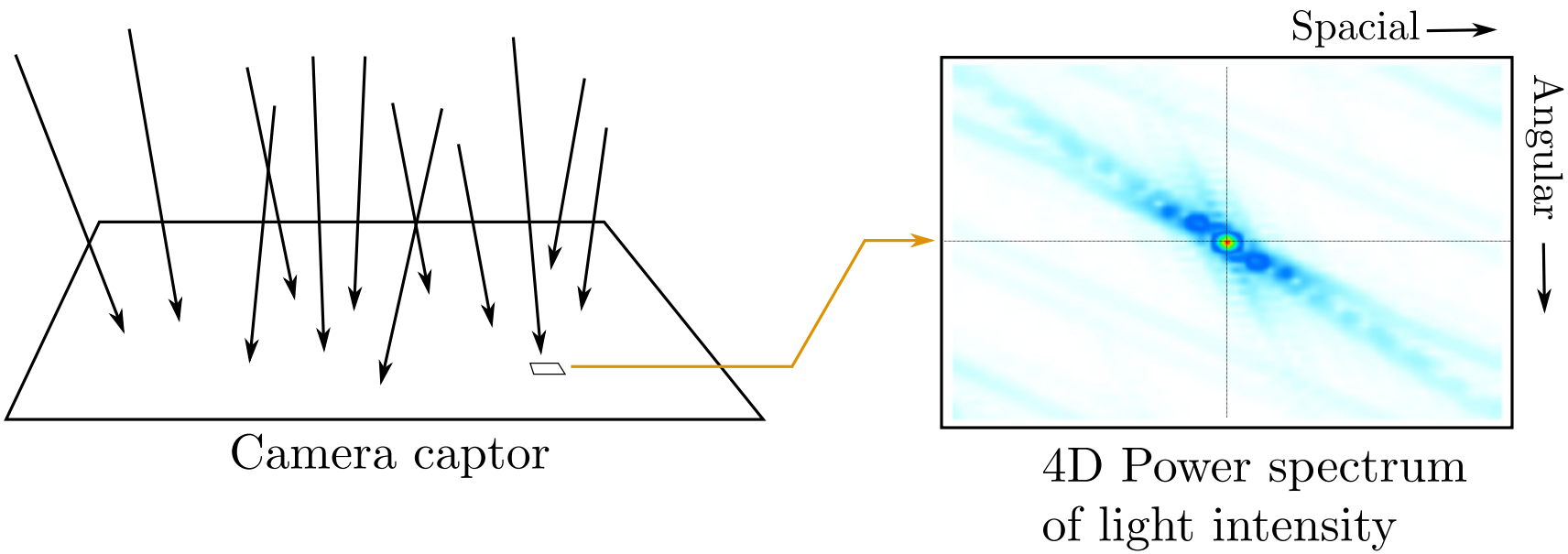

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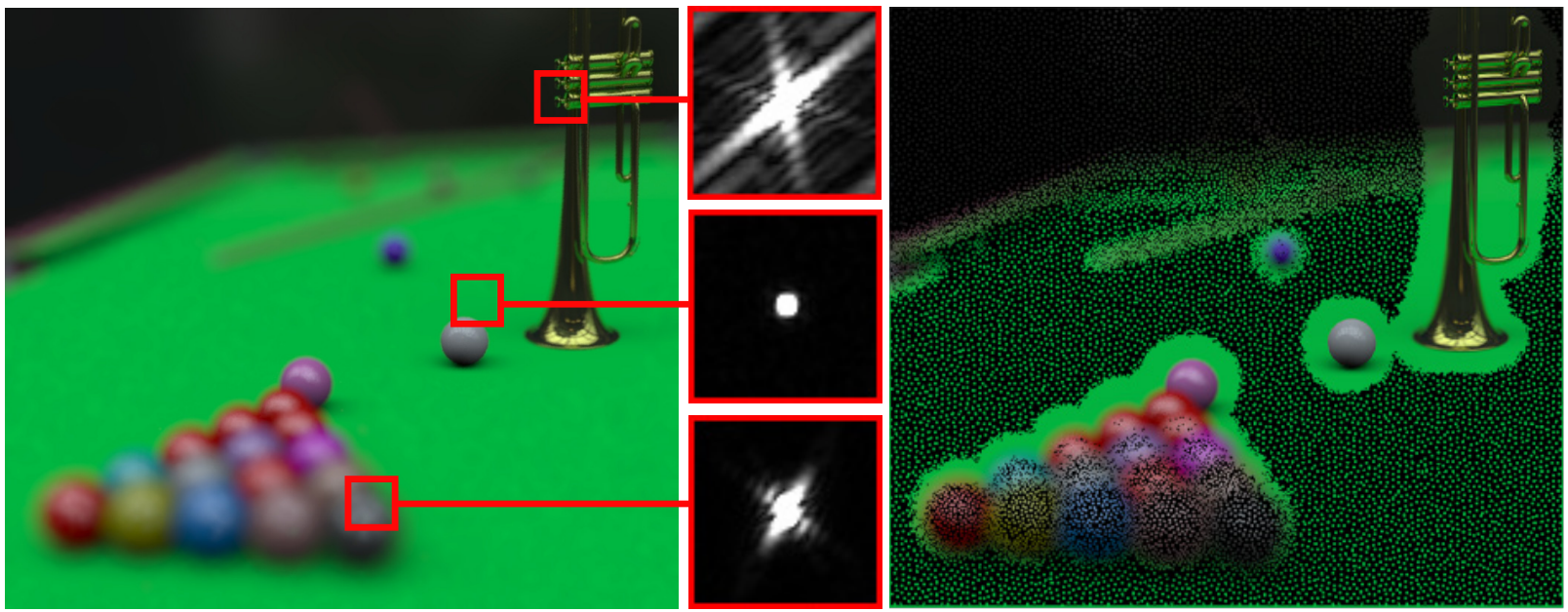

[Soler, Subr, *et al.* TOG'2009]

So, can we predict the local image spectrum??

→ add up spectra of local light fields along light paths

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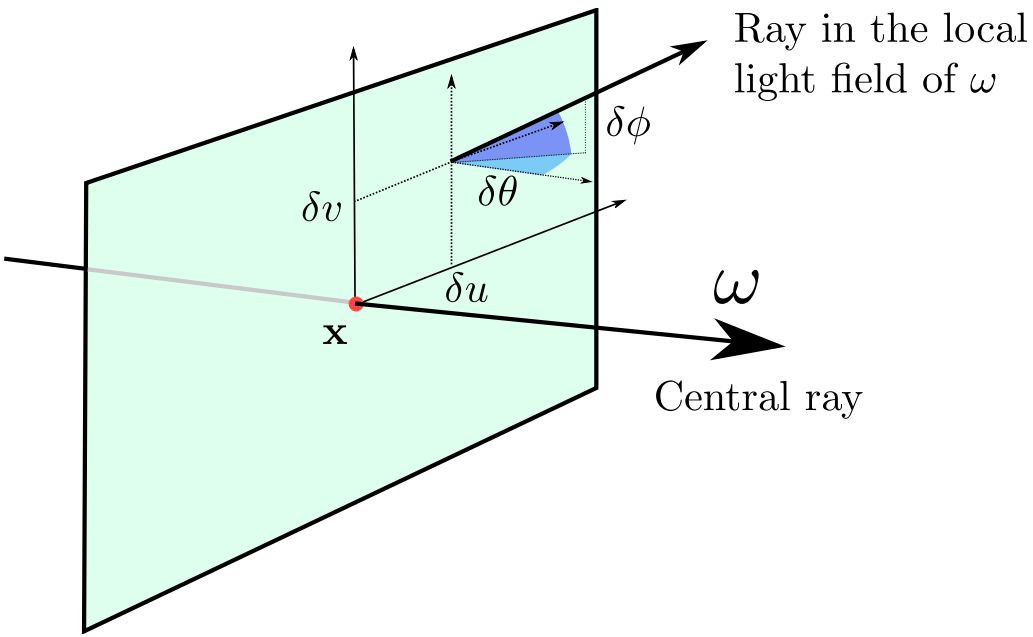
- use of a 4D paraxial parameterization
- ...and analyze the effect of the various light transport events
- accumulate at captor



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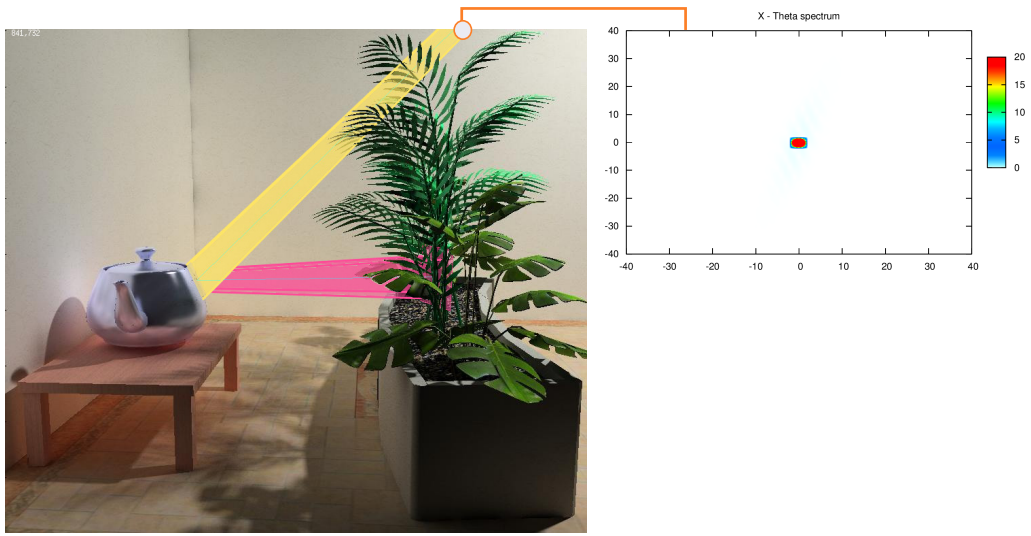

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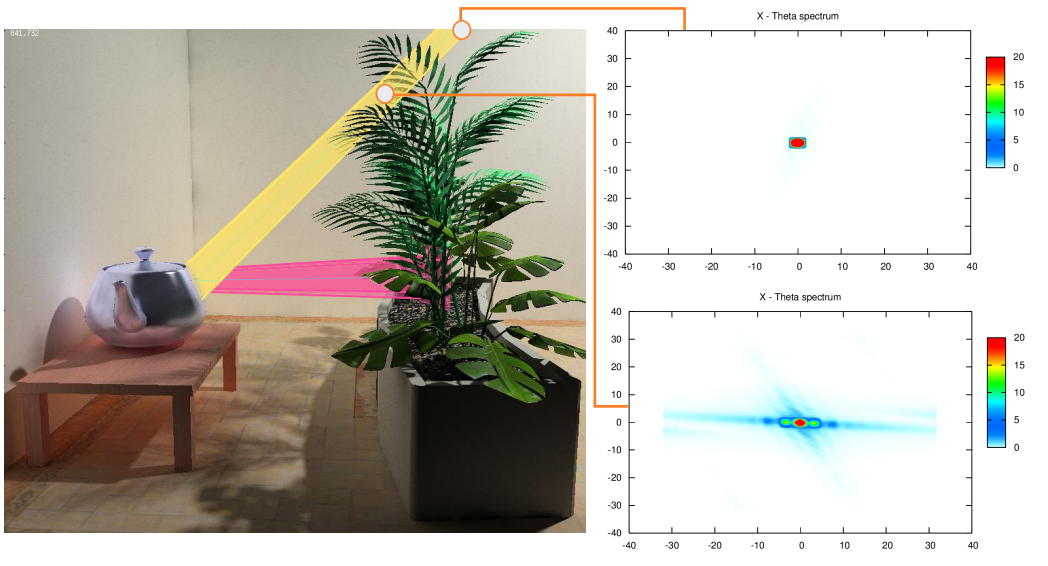

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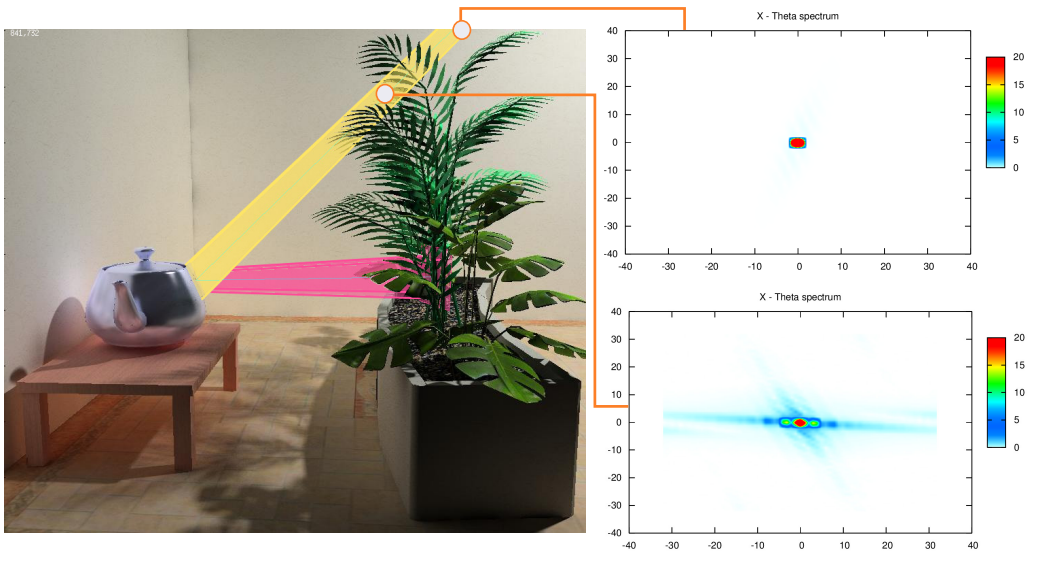

Occlusion ⇨ Convolution

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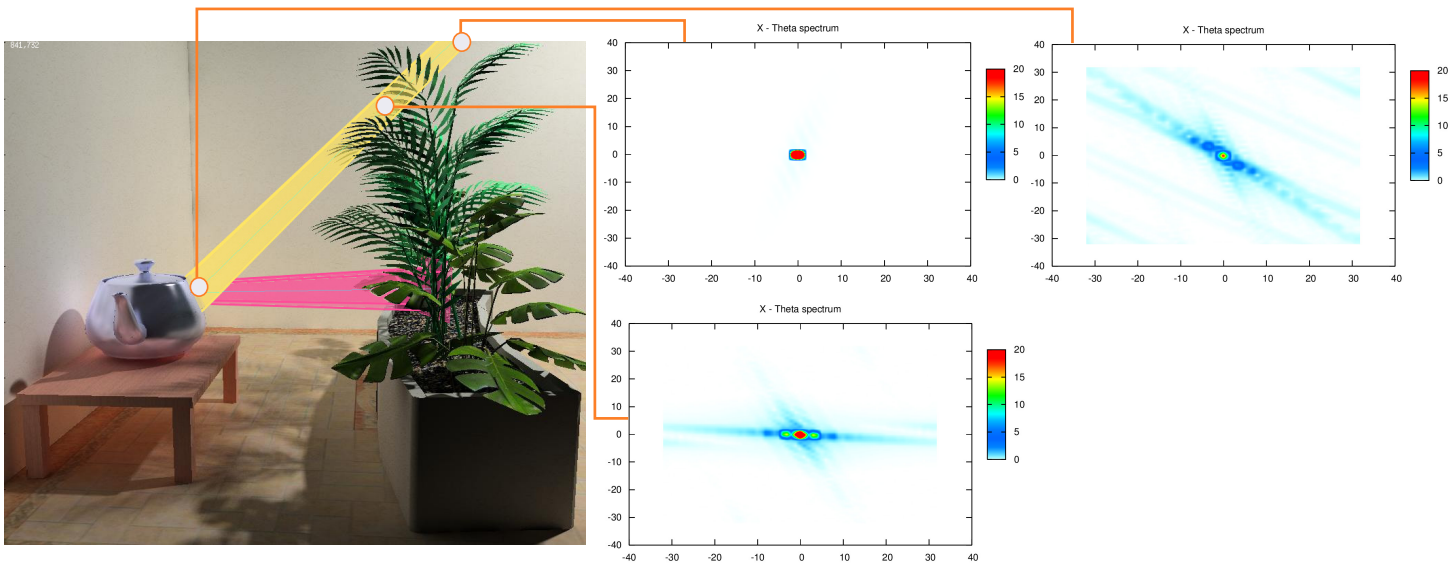

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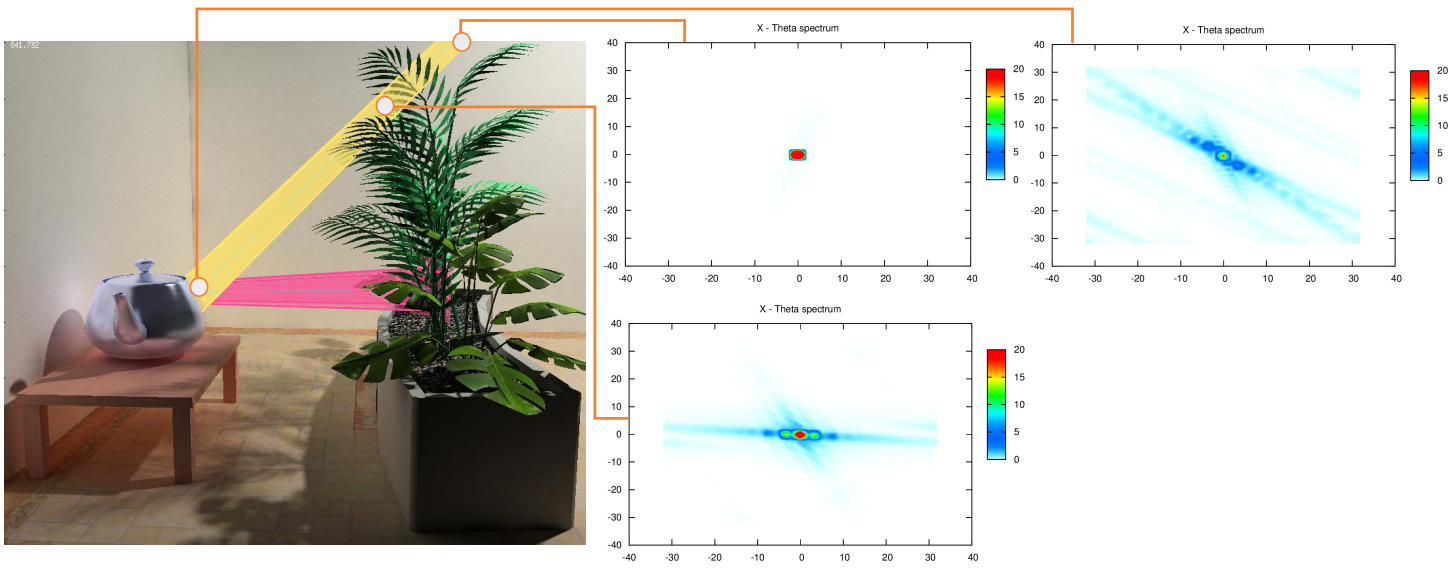

Space travel
Occlusion

⇝

Shear
Convolution

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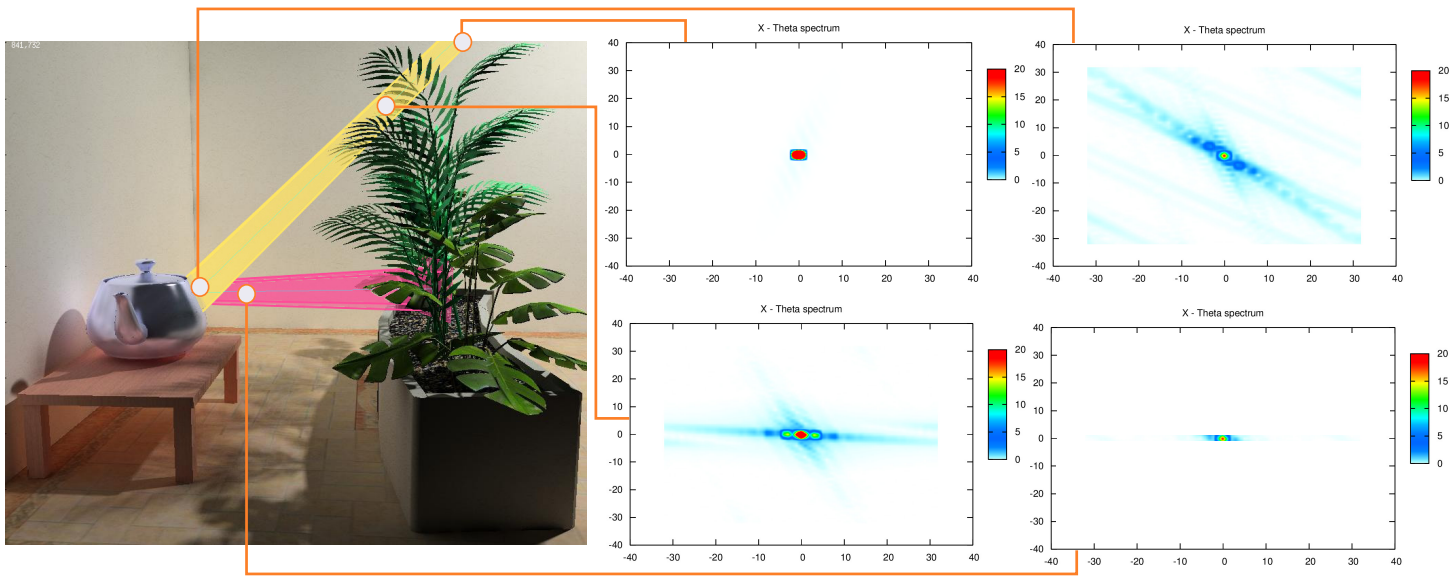

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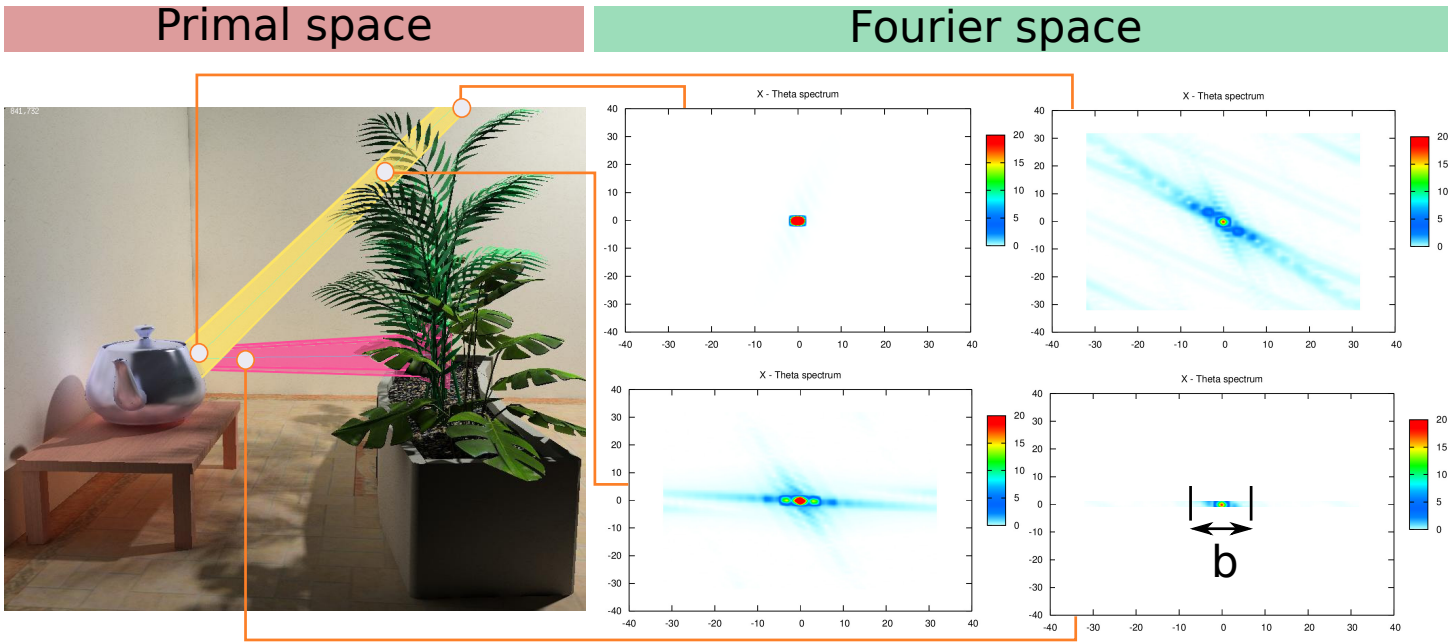

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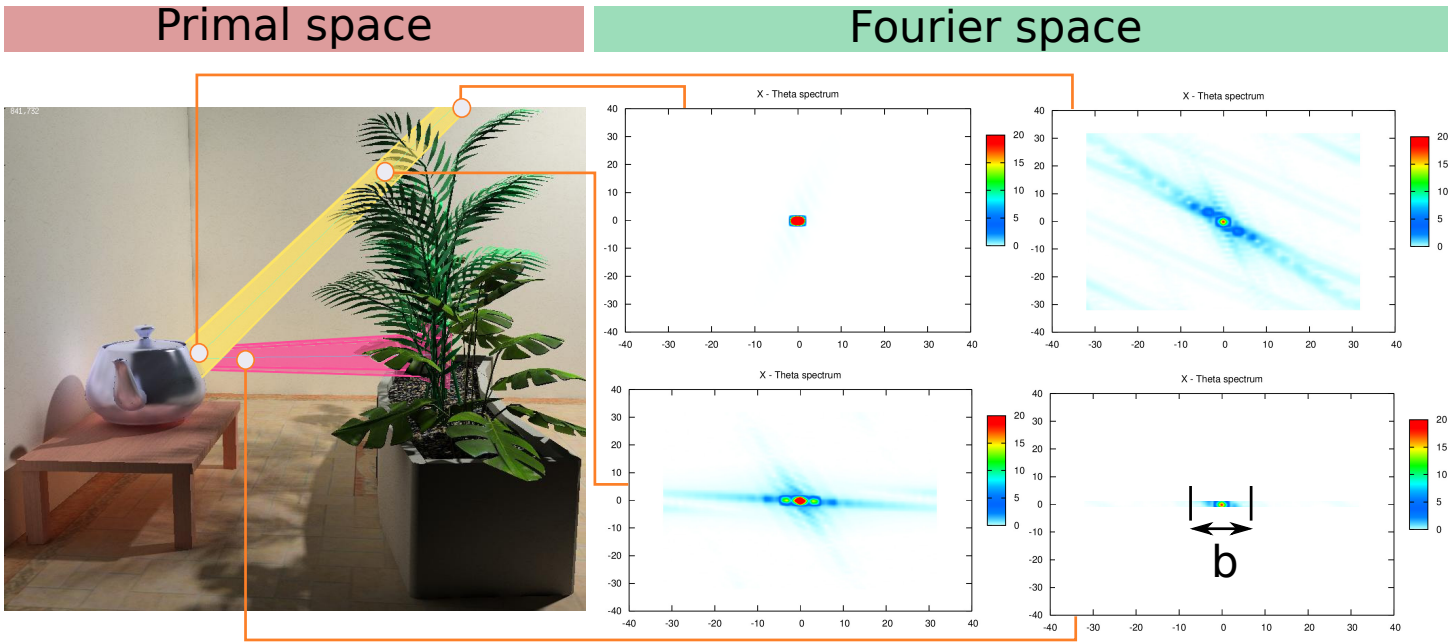

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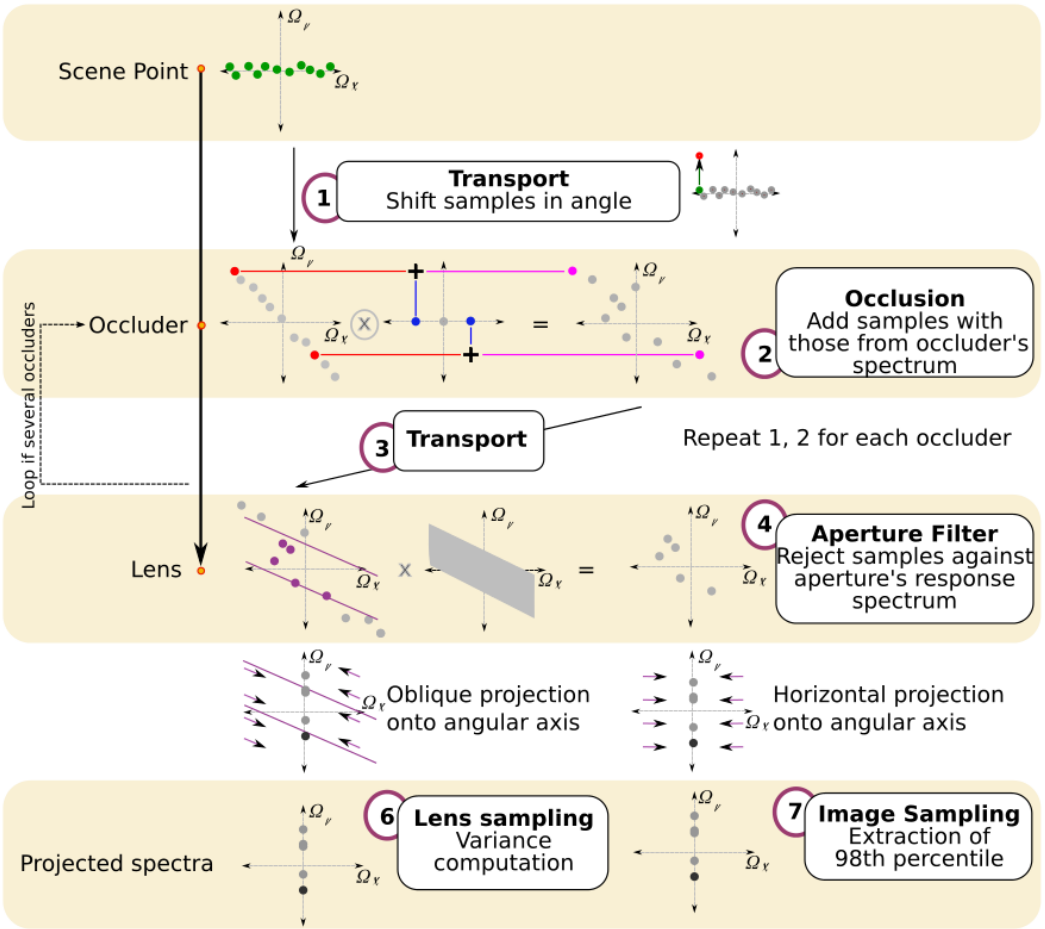
Representations of the power spectrum

- sampling **[Soler'2009]**
- explicit calculations **[Egan'2009] [Egan'2011] [Mehta'2013+2014]**
- bounding box **[Bagher'2011]**
- covariance **[Belcour'2013+2014][Munkberg'2014]**

Spectrum representations: sampling

Idea: represent the spectrum by a collection of 4D/2D samples, transform them individually, and extract the bandwidth



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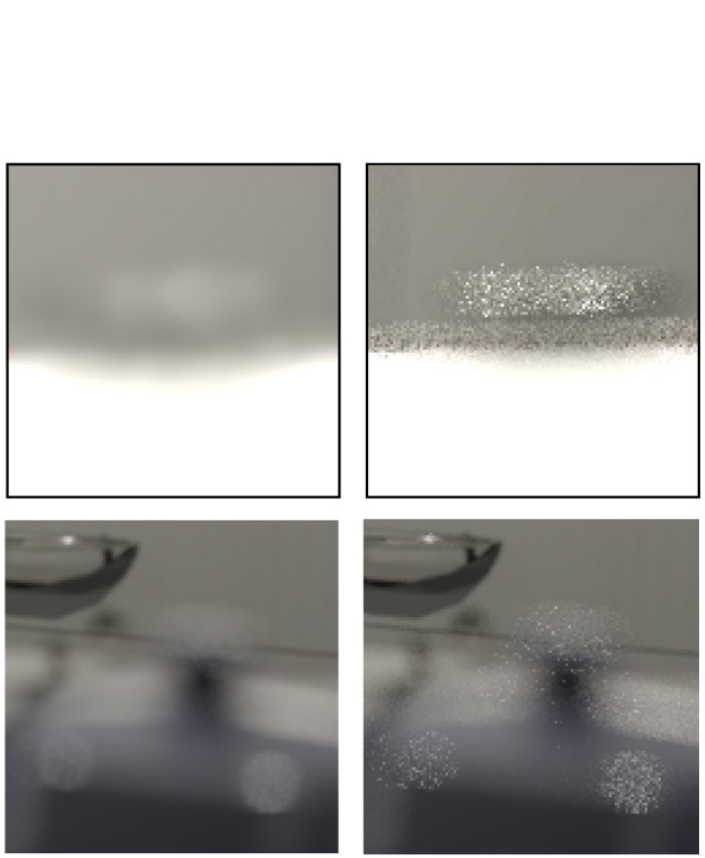
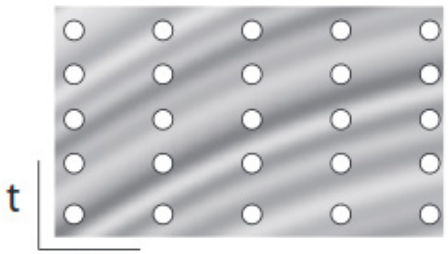


Adaptive Uniform (same time)

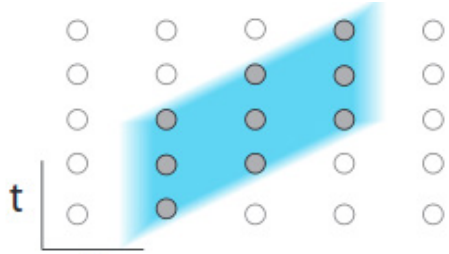
[Soler'2009]

Spectrum representations: explicit calculations

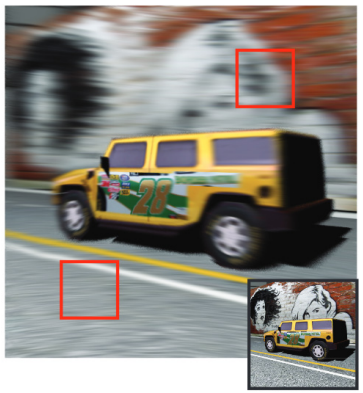
Idea: perform explicit Fourier analysis in 2D of each phenomena (visibility, motion blur, ...) and translate to 4D.

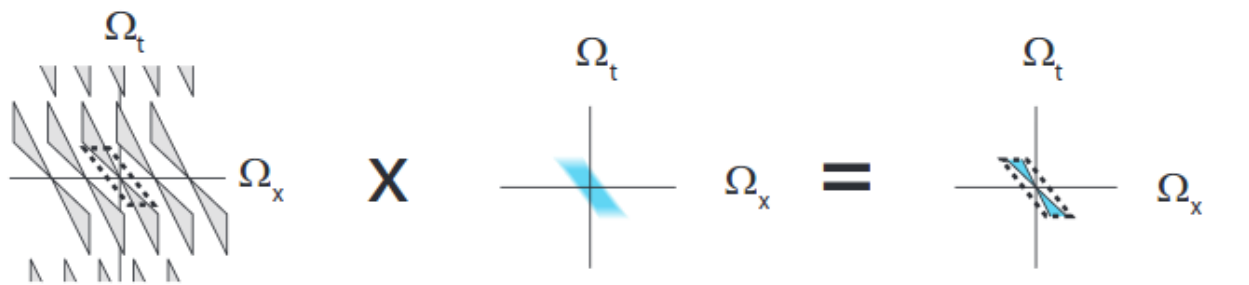


sparse sampling
moving signal



sheared filter,
no aliasing





sparse sampling
produces dense replicas

sheared
reconstruction filter

signal is bandlimited
with no aliasing

[Egan'2009]

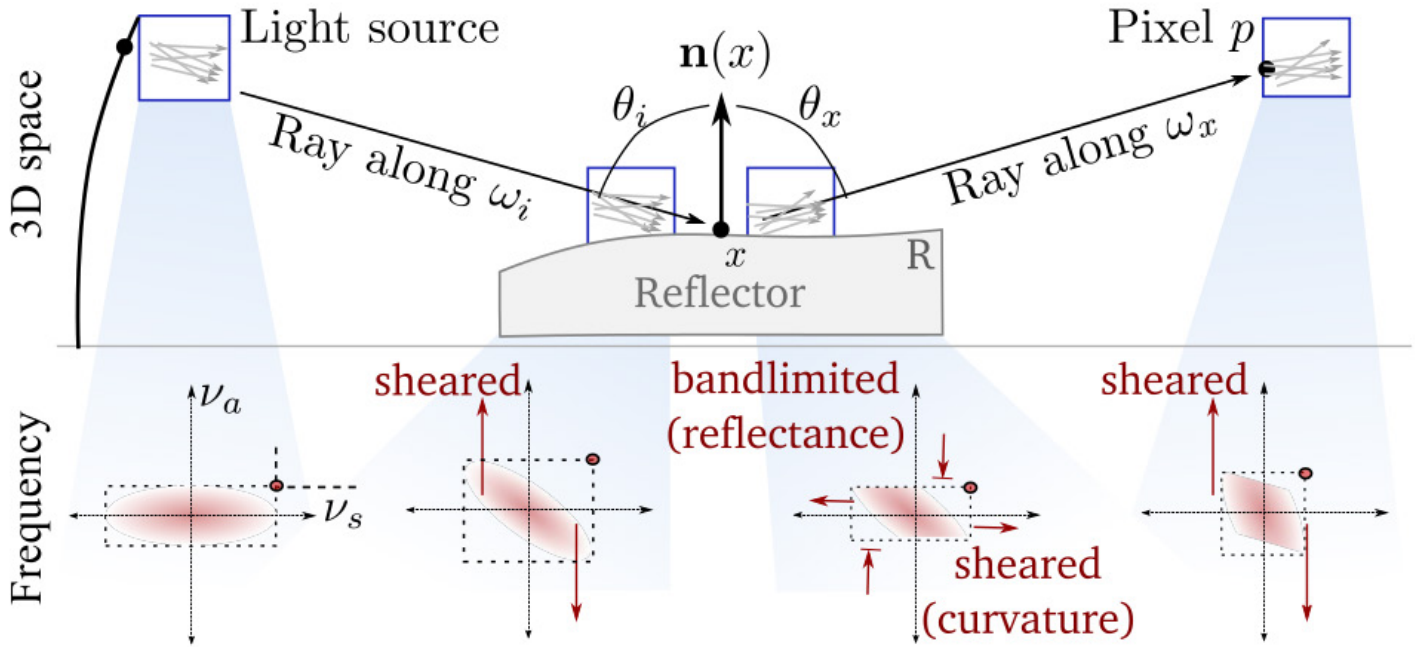
Spectrum representations: explicit calculations

Idea: perform explicit Fourier analysis in 2D of each phenomena (visibility, motion blur, ...) and translate to 4D.

[Egan'2009]

Spectrum representations: bounding box

Idea: represent bandwidth as the "bounding box" of the power spectrum. Apply operators to that bounding box.



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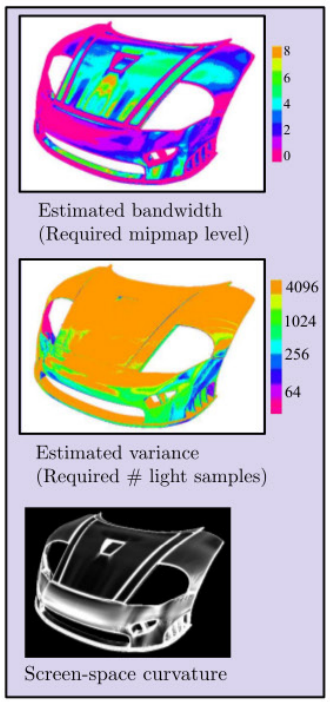
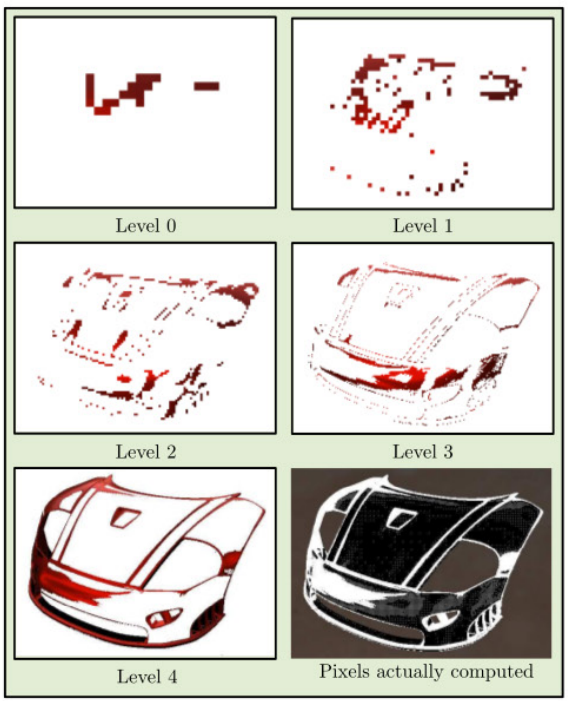
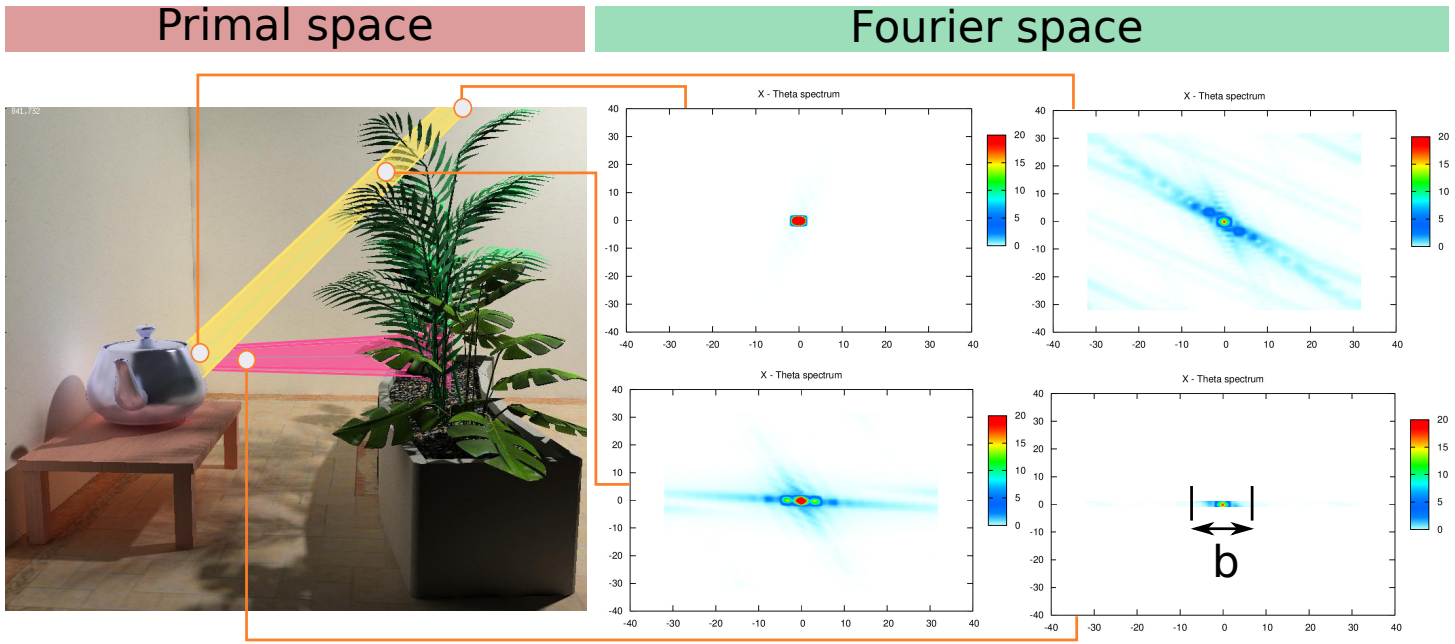




[Bagher'2011]

Spectrum representations: covariance matrices

- the covariance gives anisotropy of the spectrum, shapes for reconstruction filters, and optimal kernels for density estimation
- it's related to second derivatives
- it's easy to update without ever manipulating explicit spectra



Space travel
Occlusion
Reflectance

⇝

Shear
Convolution
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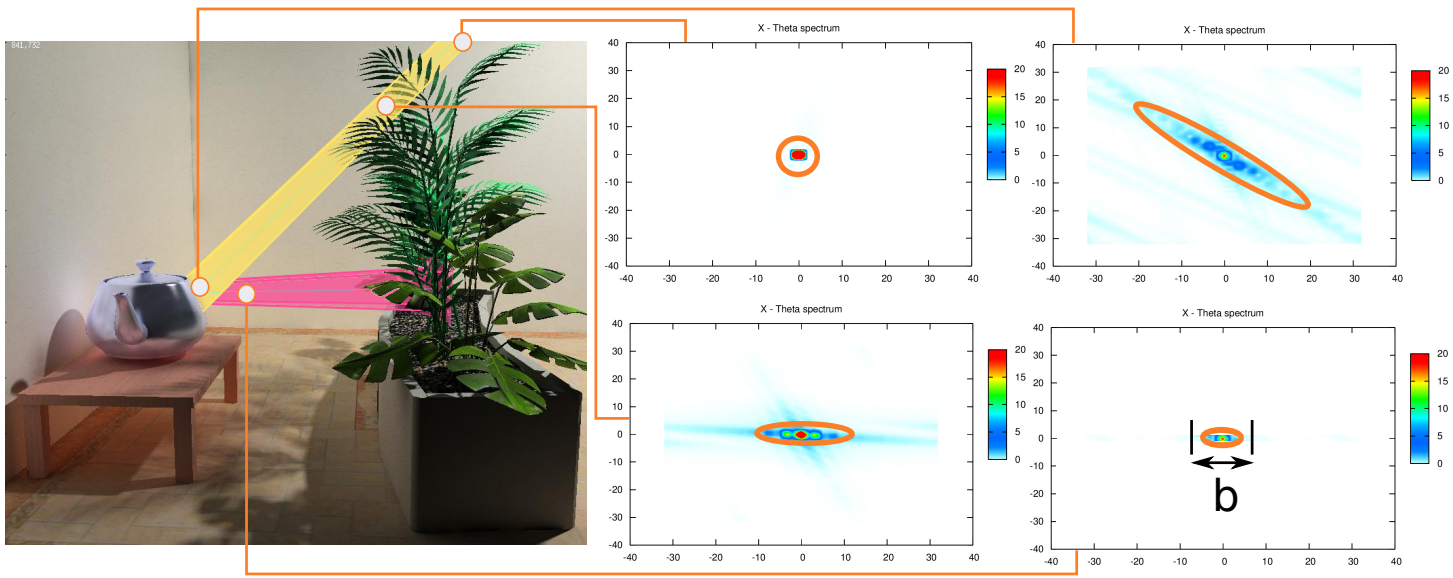

$$\Sigma$$

(4x4 matrix!)

$$S(\mathbf{X}) \approx e^{-\mathbf{X}^T \Sigma^{-1} \mathbf{X}}$$

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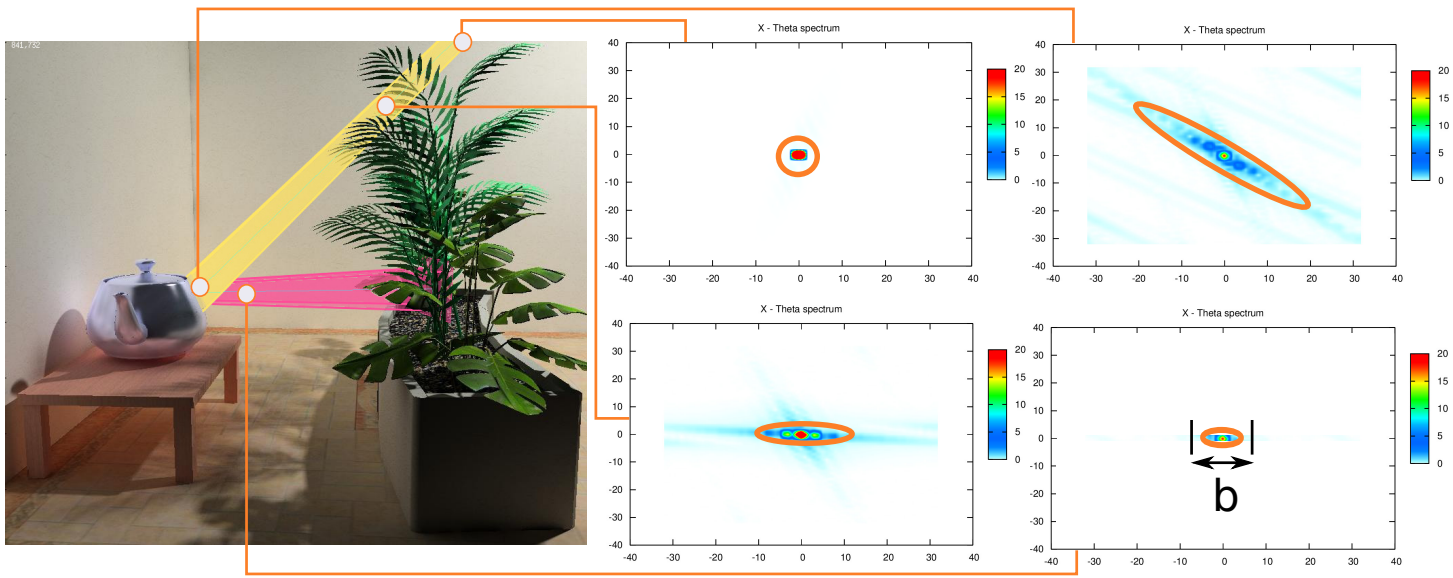

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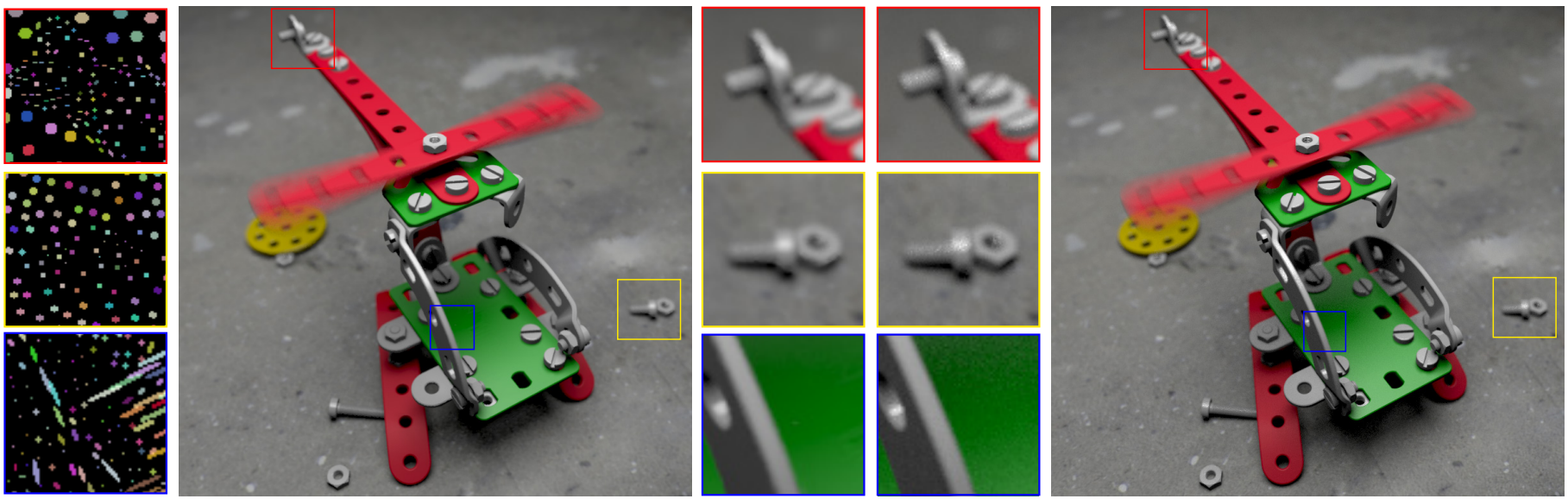
(4x4 matrix!)

$$S(\mathbf{X}) \approx e^{-\mathbf{X}^T \Sigma^{-1} \mathbf{X}}$$

Space travel	⇒	Left-right product
Occlusion	⇒	Sum of matrices
Reflectance	⇒	Product + p-inverse
Shear		
Convolution		
Angular product		

Efficient 5D Image reconstruction

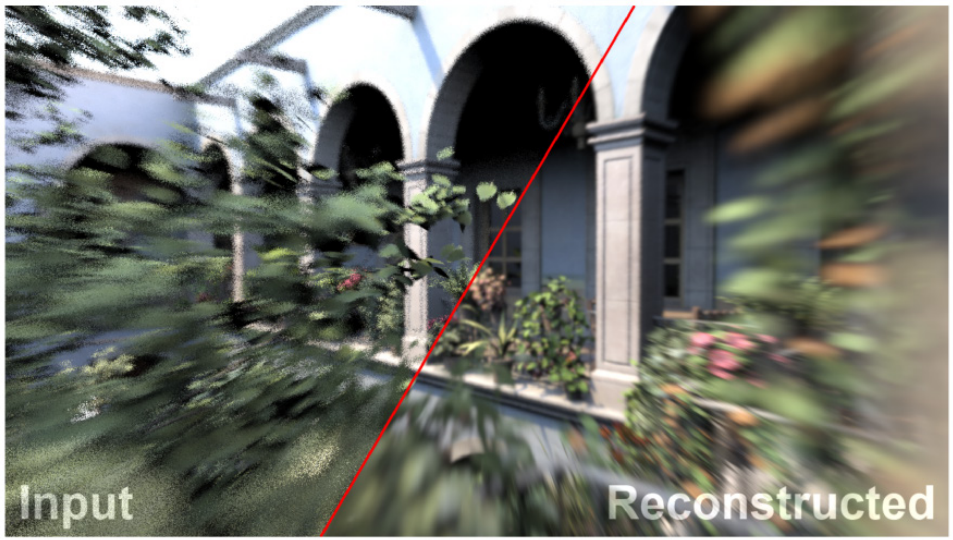
$\Sigma \longrightarrow$ image-space reconstruction filters (4D/5D spectrum, scene)



(a) Filters (b) Our method (c) Equal time path tracing (xx paths per pixel)

[Belcour *et al.* TOG'2011]

Real time reconstruction

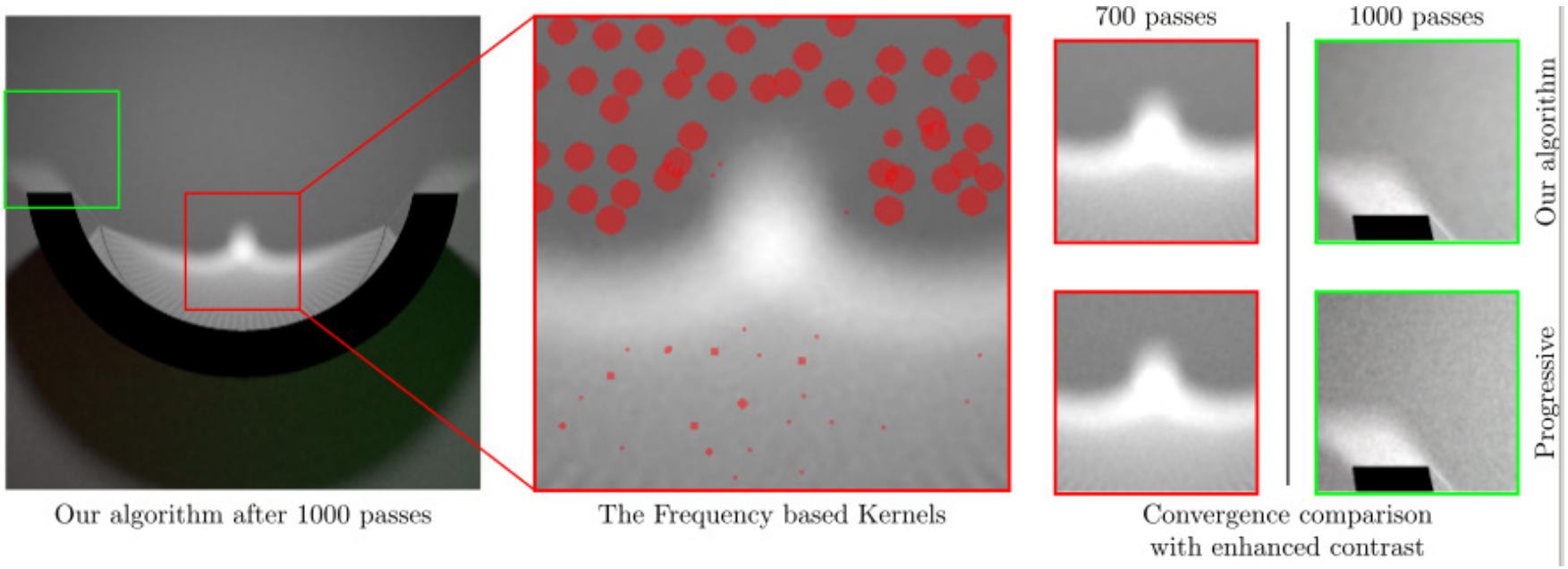


Input samples (8 spp) | Our filter 32×32 80 ms | Our filter 44×44 436 ms | Reference (1024 spp)

[Munkberg'2014]

Density estimation methods

$\Sigma \longrightarrow$ optimal size of density estimation kernels



[Belcour. Siggraph'2012 poster]

Spectrum representations: Summary

	Speed	Generic	Full 4D/5D	Computational complexity
Bounding box	🙂🙂	☹	NO	Very low
Covariance	🙂	🙂🙂	YES	low
Sampling	☹	🙂🙂	YES	☹
Explicit	🙂🙂	☹	NO	low

Conclusion

- It took several years and many people to refine the ideas
- The maths are sometimes scary but actually hide simple concepts
- Even very approximate calculations give useful predictions