Zürich 2015

Eurographics 2015

The 36th Annual Conference of the European Association for Computer Graphics

Recent Advances in Adaptive Sampling and Reconstruction for Monte Carlo Rendering

Derivative Analysis

Wojciech Jarosz

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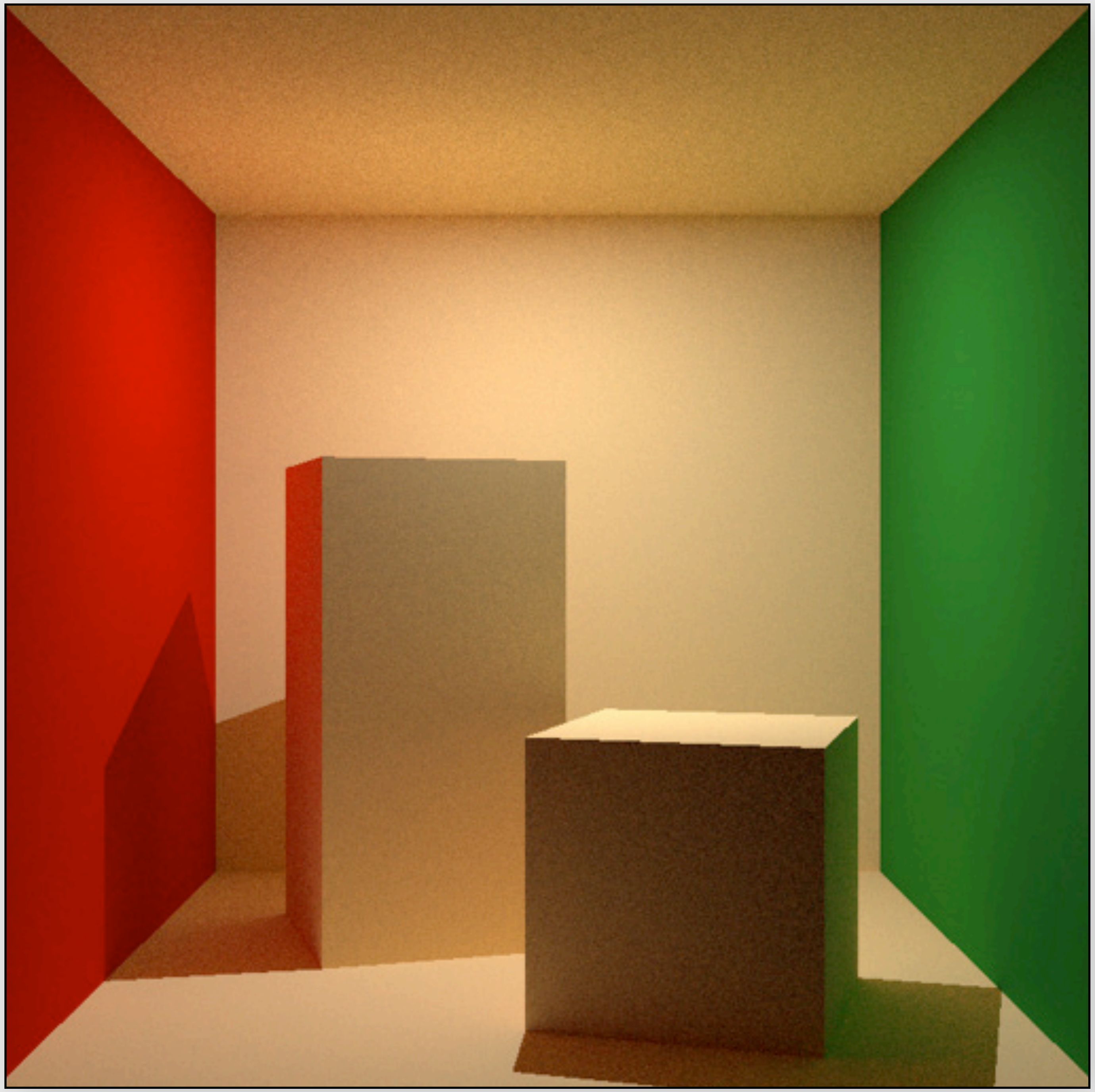
Path tracing - diffuse scene



128 paths/pixel

Diffuse indirect illumination is smooth

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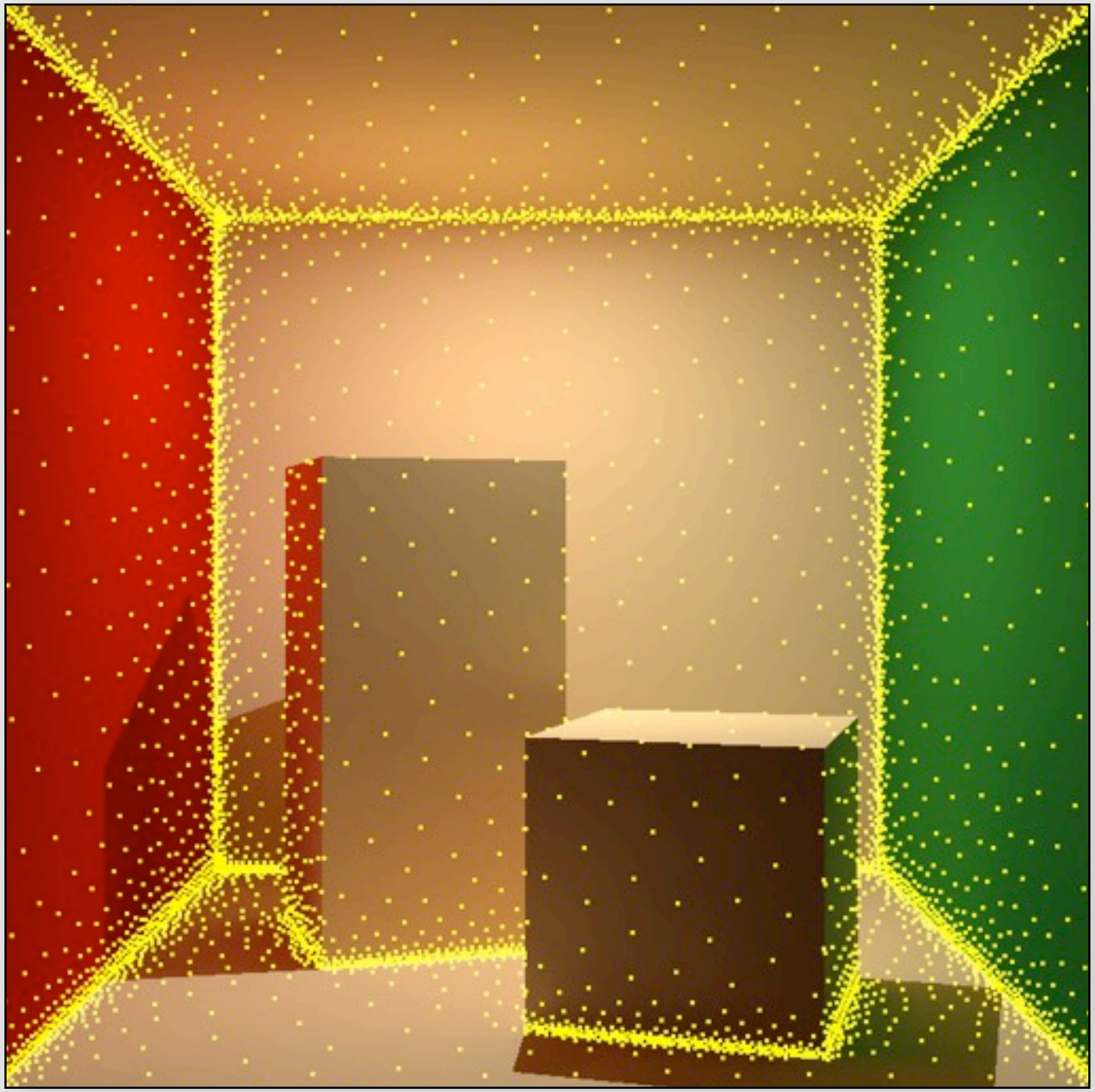
Perfect candidate for sparse sampling and interpolation

Interpolated indirect illumination

Irradiance Caching
[Ward et al. 1988]



1M pixels - 4K cache points

Irradiance Caching Algorithm

[Ward et al. 1988]

Irradiance Caching Algorithm

`if` at least one cached illumination value near **`x`** **`then`**

[Ward et al. 1988]

Irradiance Caching Algorithm

if at least one cached illumination value near **x** **then**
Interpolate illumination from the cached value(s).

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Irradiance Caching Algorithm

```
if at least one cached illumination value near x then
    Interpolate illumination from the cached value(s).
else
    Compute and cache a new illumination value at x.
```


[Ward et al. 1988]

Irradiance Caching Algorithm

`if` at least one cached illumination value near **`x`** **`then`**
  Interpolate illumination from the cached value(s).
`else`
  Compute and cache a new illumination value at **x**.

- Some questions that remain:
  - What do we cache?
  - What makes a cache point "nearby"?
  - How do we interpolate the nearby cached values?

[Ward et al. 1988]

Lambertian assumption

- Indirect illumination on a Lambertian surface:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i)(\vec{\mathbf{n}} \cdot \vec{\omega}_i)\, \mathrm{d}\vec{\omega}_i$$

[Ward et al. 1988]

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$$= \frac{\rho}{\pi} \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i)(\vec{\mathbf{n}} \cdot \vec{\omega}_i)\, \mathrm{d}\vec{\omega}_i$$

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$E(\mathbf{x},\vec{\mathbf{n}})$ → Irradiance

[Ward et al. 1988]

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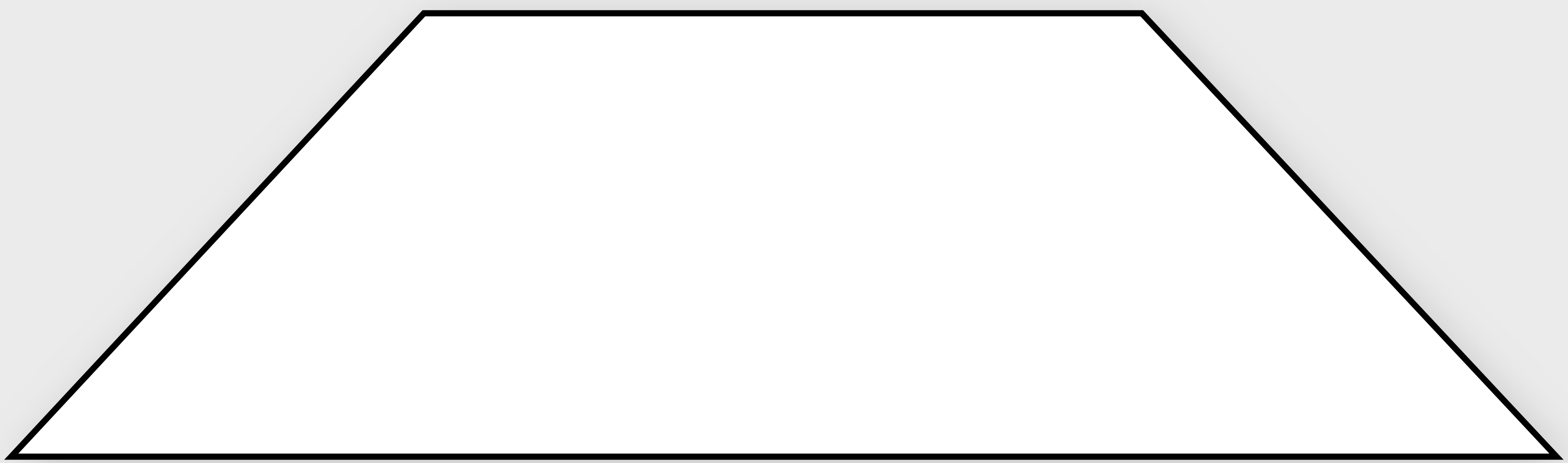
$E(\mathbf{x},\vec{\mathbf{n}})$ → Irradiance

$$E(\mathbf{x},\vec{\mathbf{n}}) \approx \frac{\pi}{N}\sum_{j=1}^{N} L_i(\mathbf{x},\vec{\omega}_{i,j})$$

[Ward et al. 1988]

Interpolating Irradiance

- Irradiance computation costly, reuse whenever possible
- How far away can we reuse a cached value?



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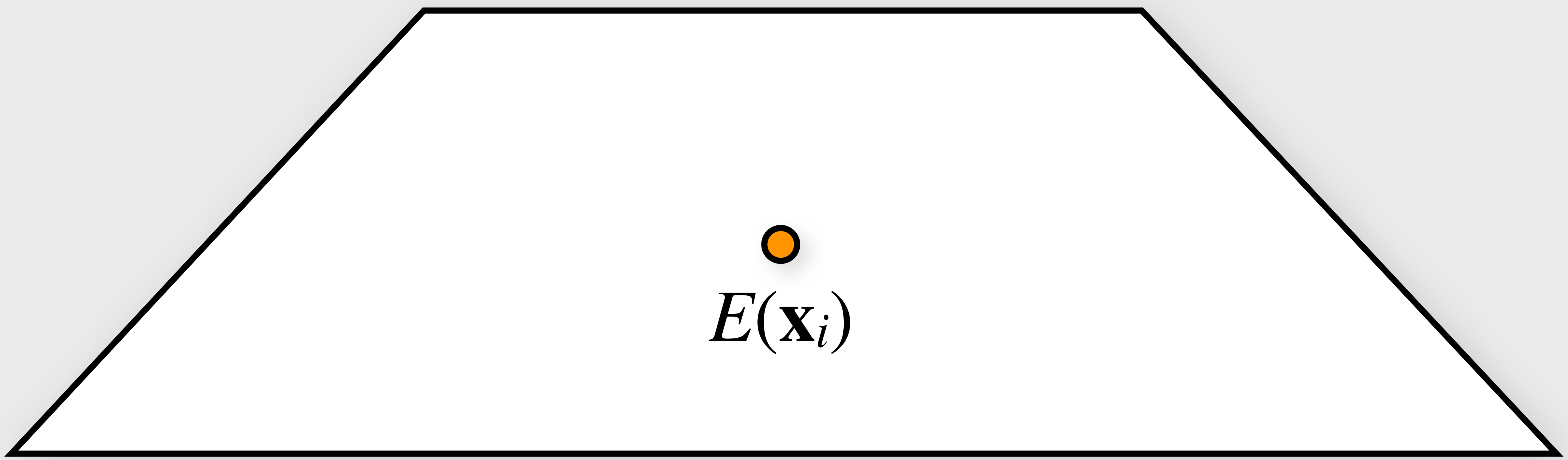

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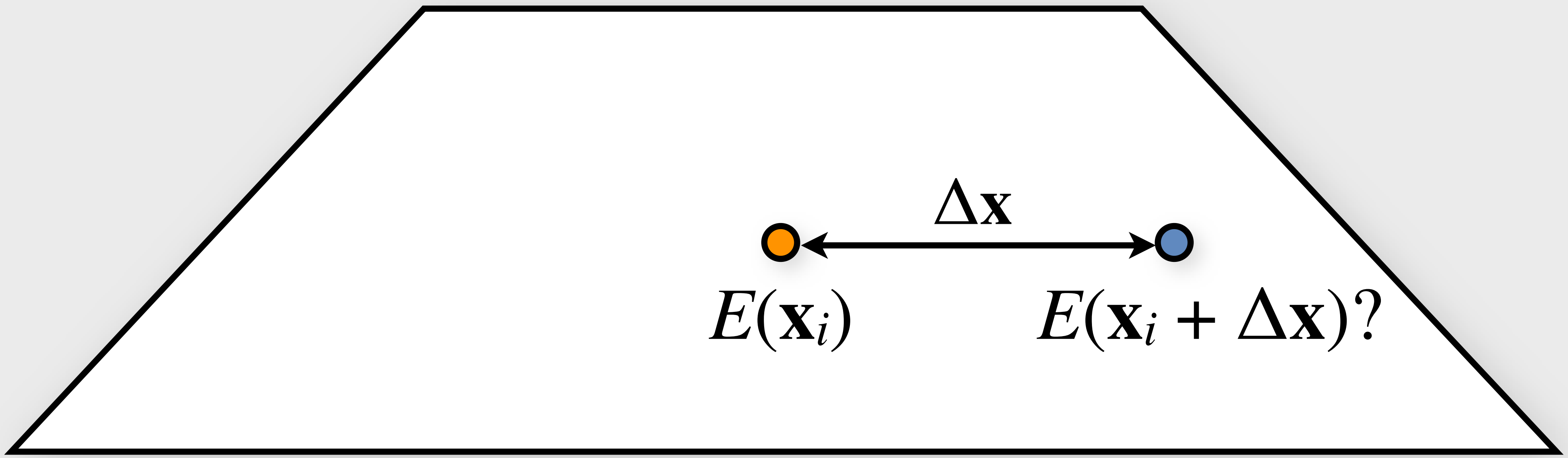

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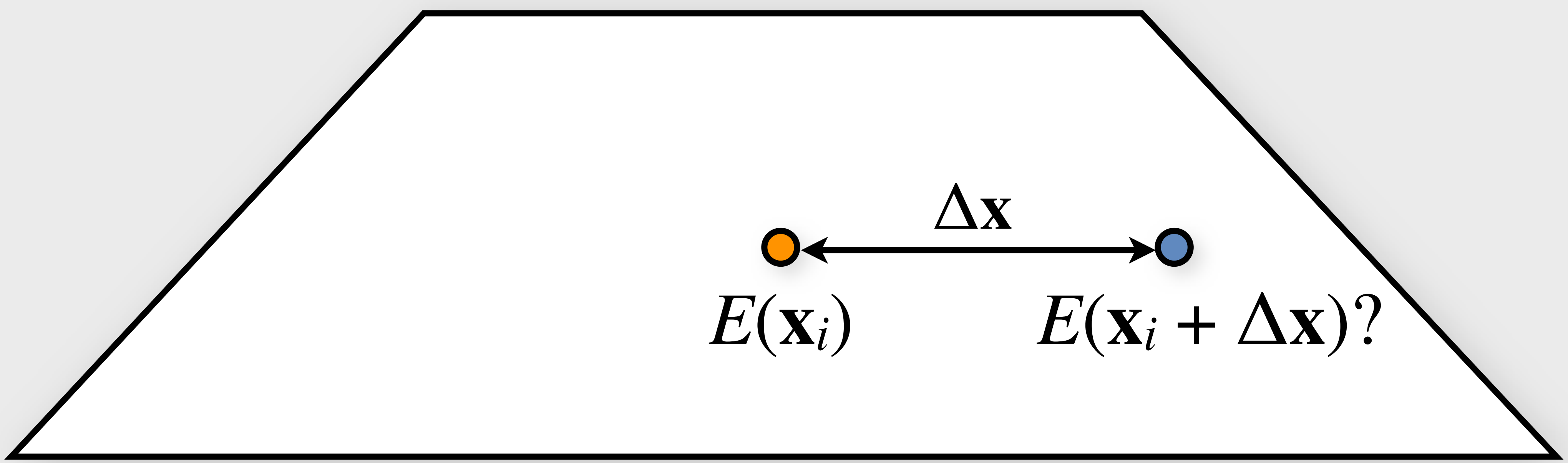

$$E(\mathbf{x}) \approx E(\mathbf{x}_i) + \left(\frac{\partial E}{\partial \mathbf{x}}\Delta\mathbf{x} + \frac{\partial E}{\partial \vec{\mathbf{n}}}\Delta\vec{\mathbf{n}}\right)$$

[Ward et al. 1988]

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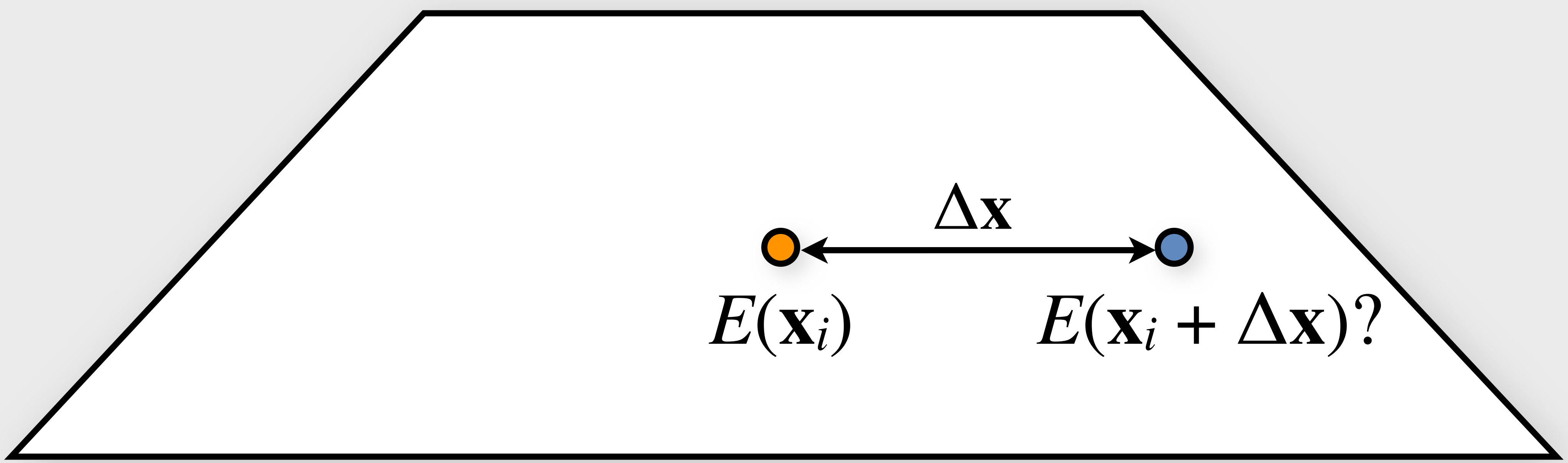

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error

[Ward et al. 1988]

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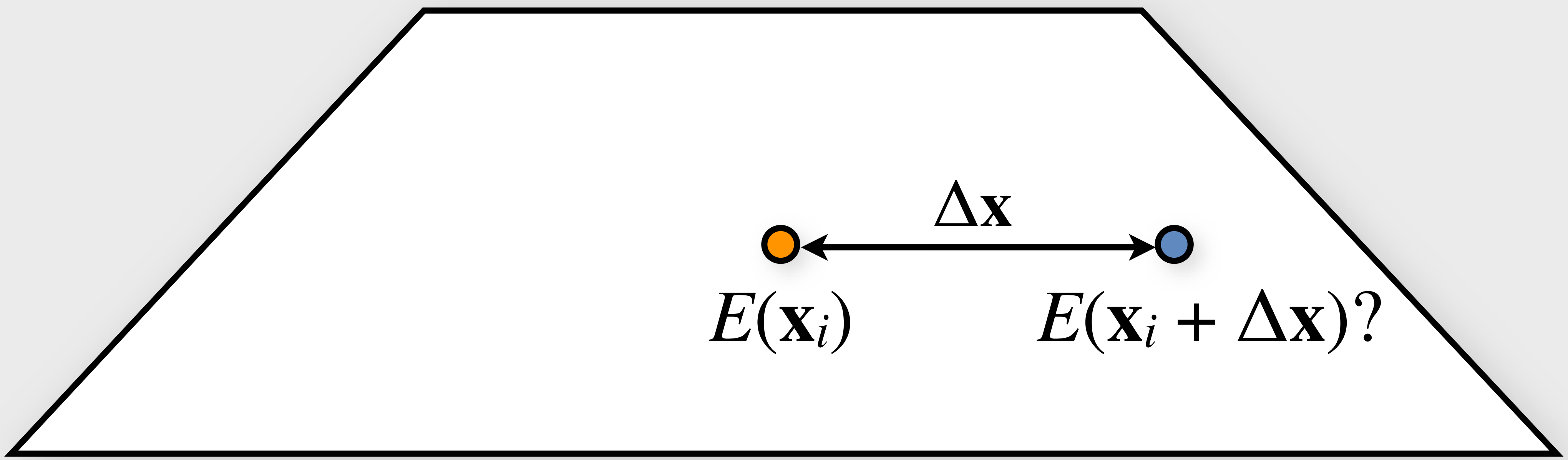

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[Ward et al. 1988]

Interpolating Irradiance

- To compute valid region, need to estimate change in irradiance

$$\frac{\partial E}{\partial \mathbf{x}} \Delta \mathbf{x} + \frac{\partial E}{\partial \vec{\mathbf{n}}} \Delta \vec{\mathbf{n}}$$

- Consider hypothetical,

  worst-case scene:

  the “**Split-Sphere**”

[Ward et al. 1988]

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- To compute valid region, need to estimate change in irradiance

$$\varepsilon_i \lesssim \left| \frac{\partial E}{\partial \mathbf{x}} \Delta \mathbf{x} + \frac{\partial E}{\partial \vec{\mathbf{n}}} \Delta \vec{\mathbf{n}} \right|$$

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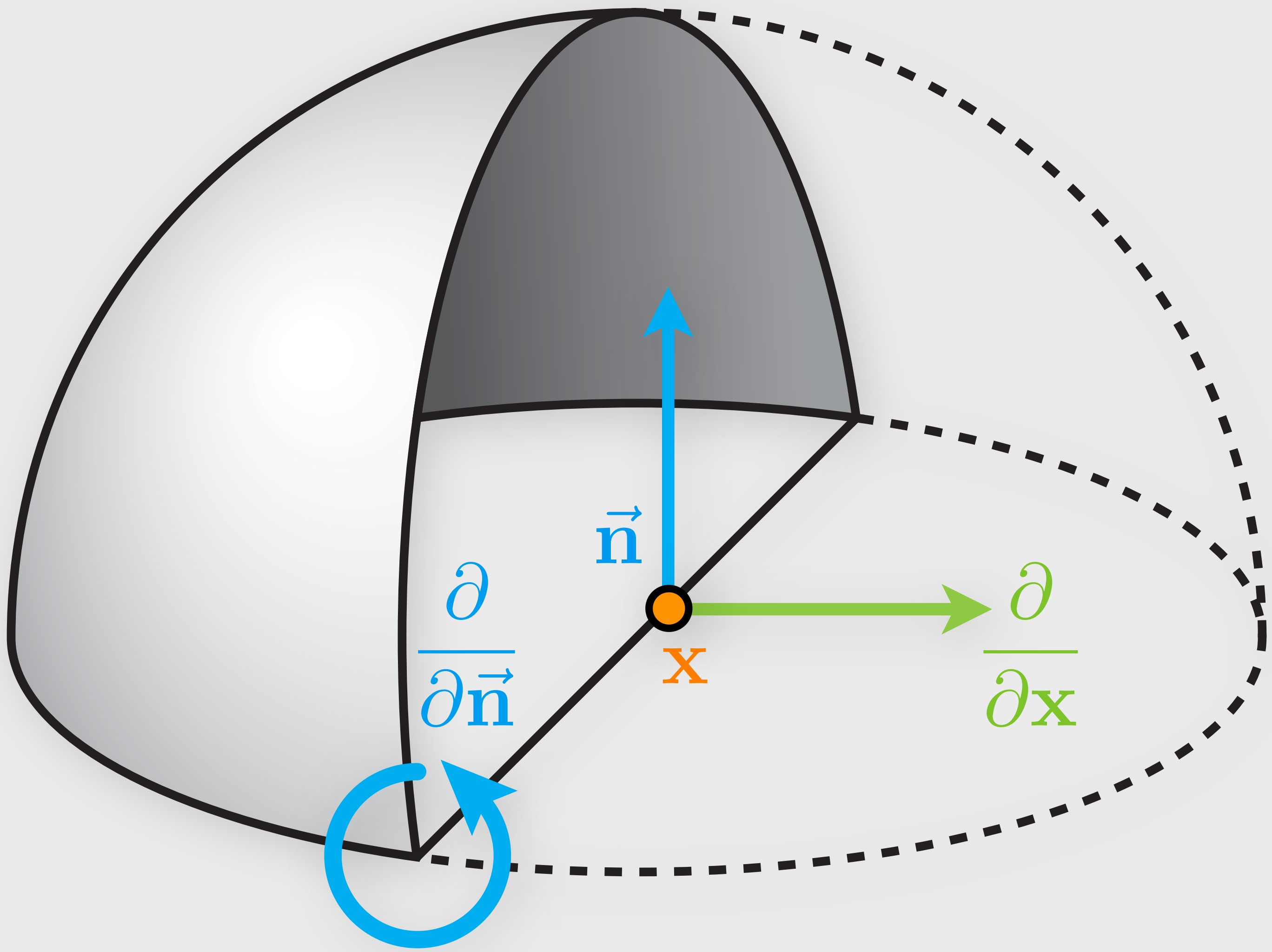

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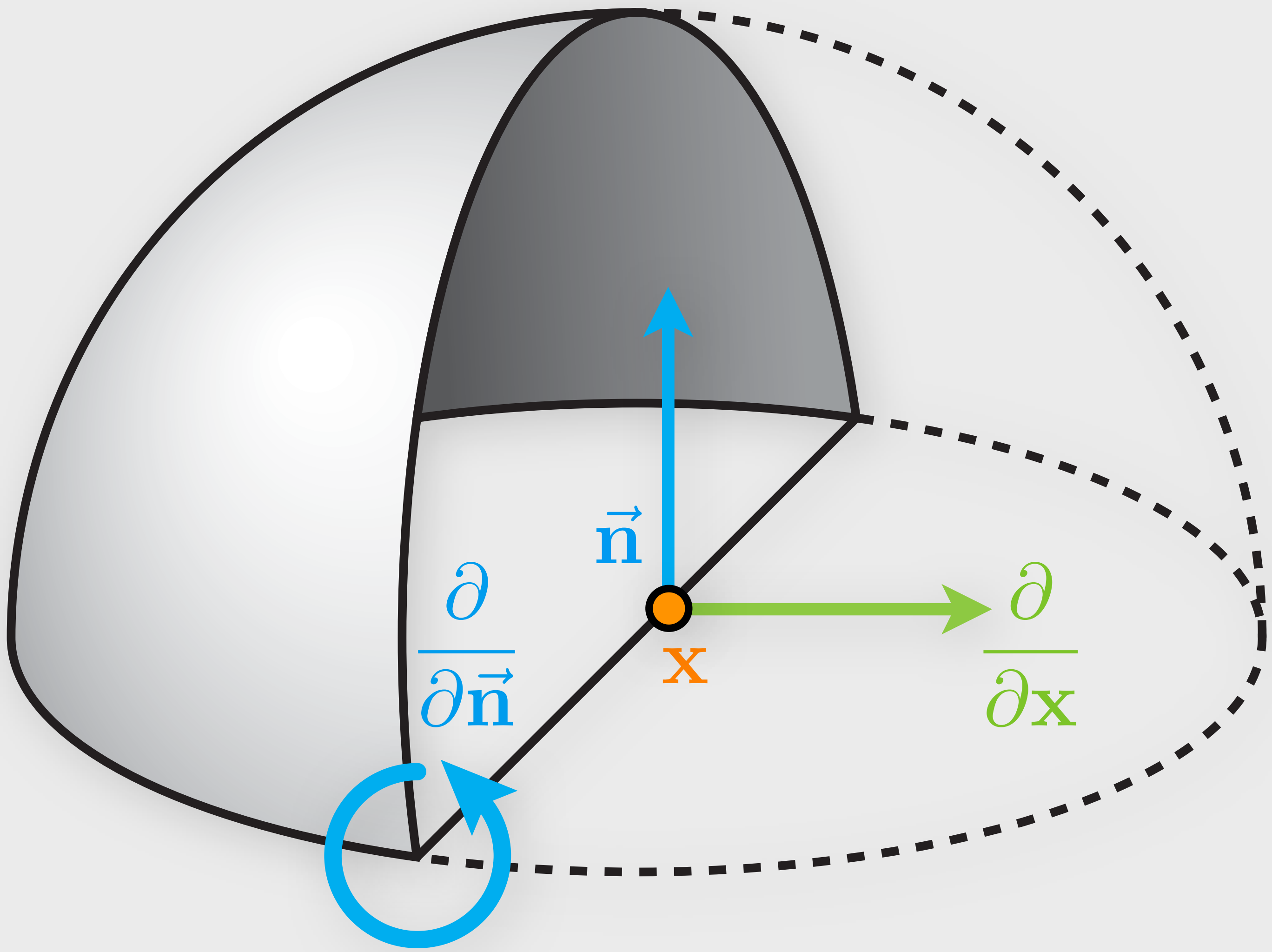

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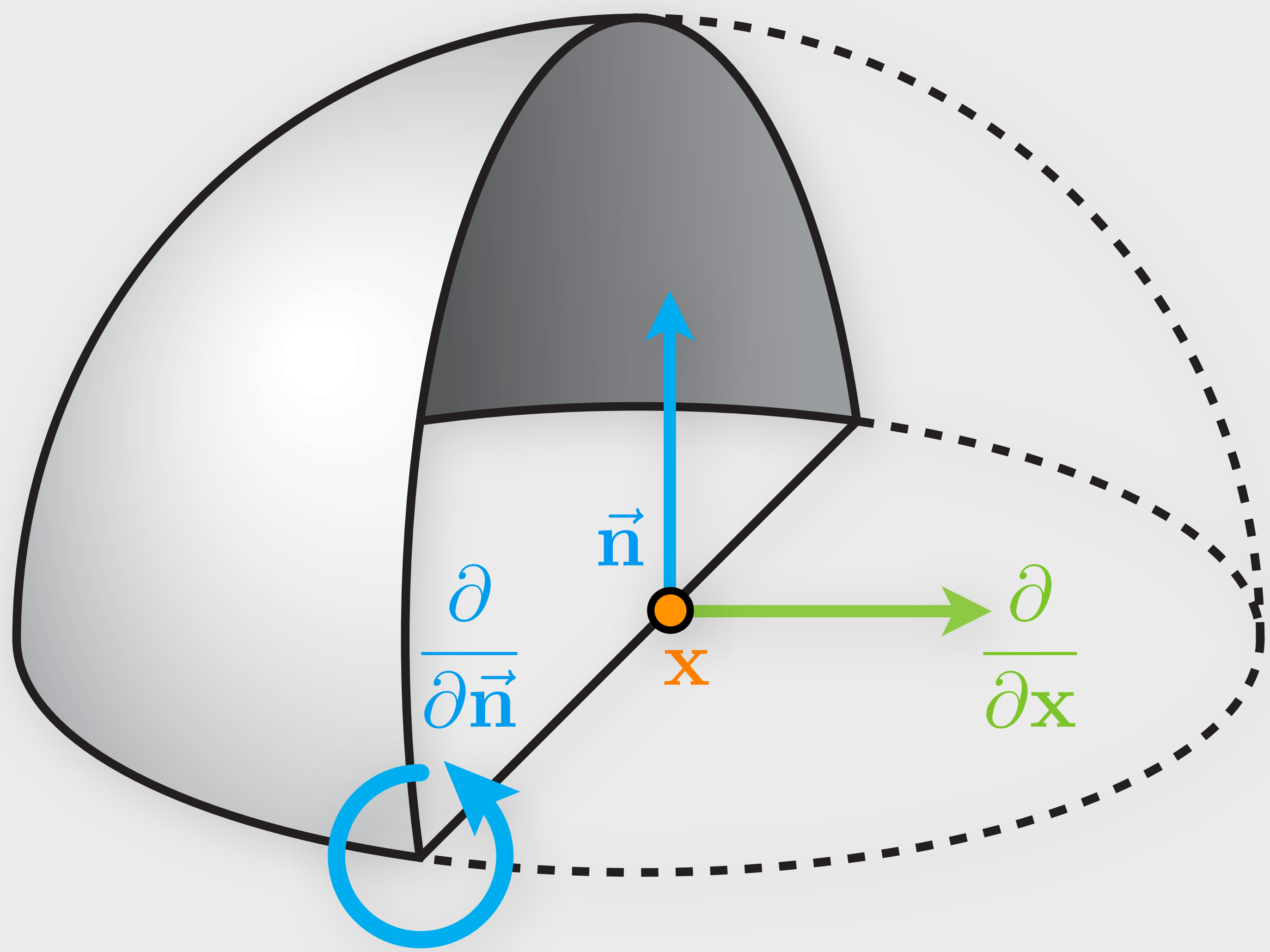

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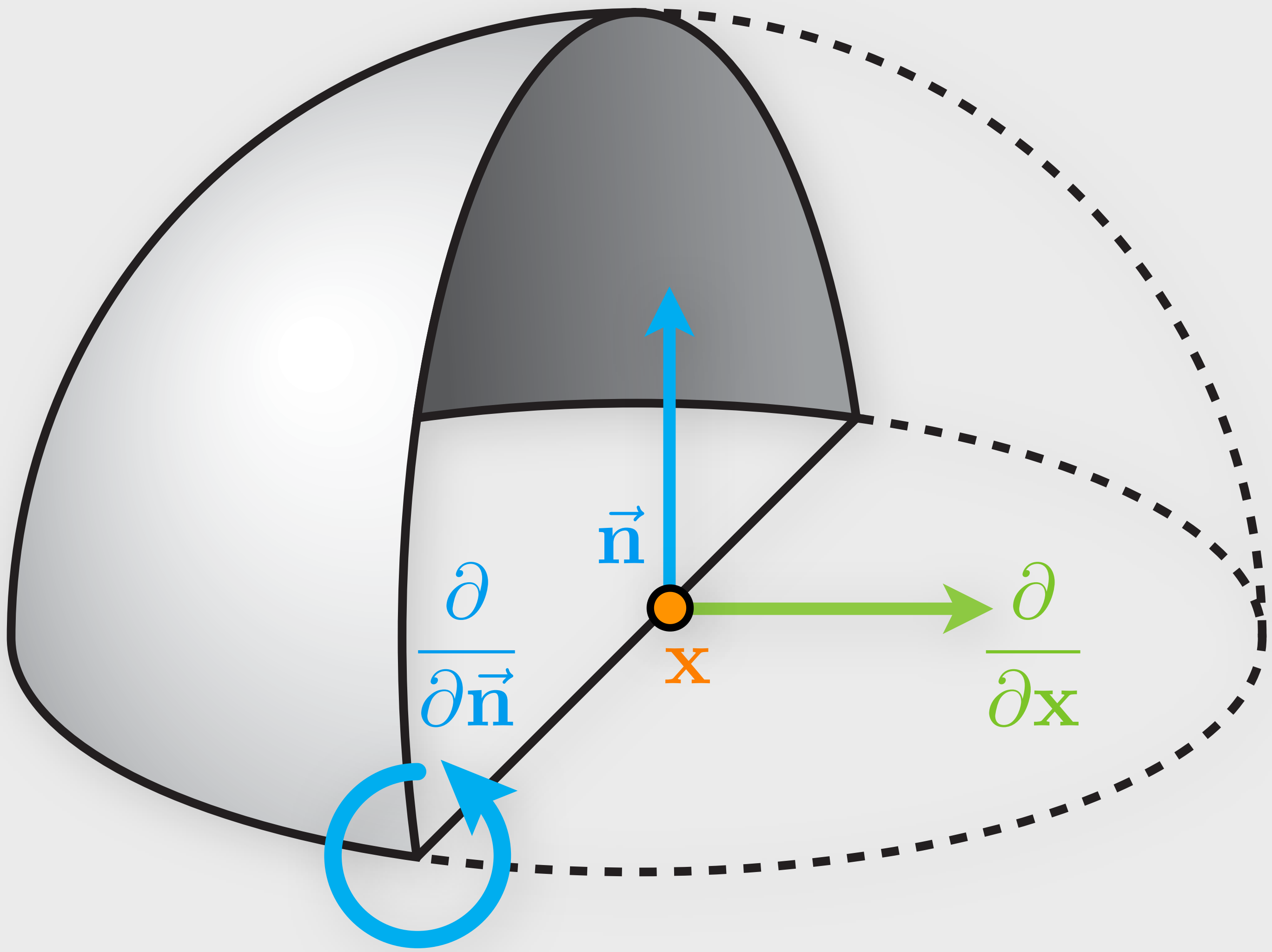

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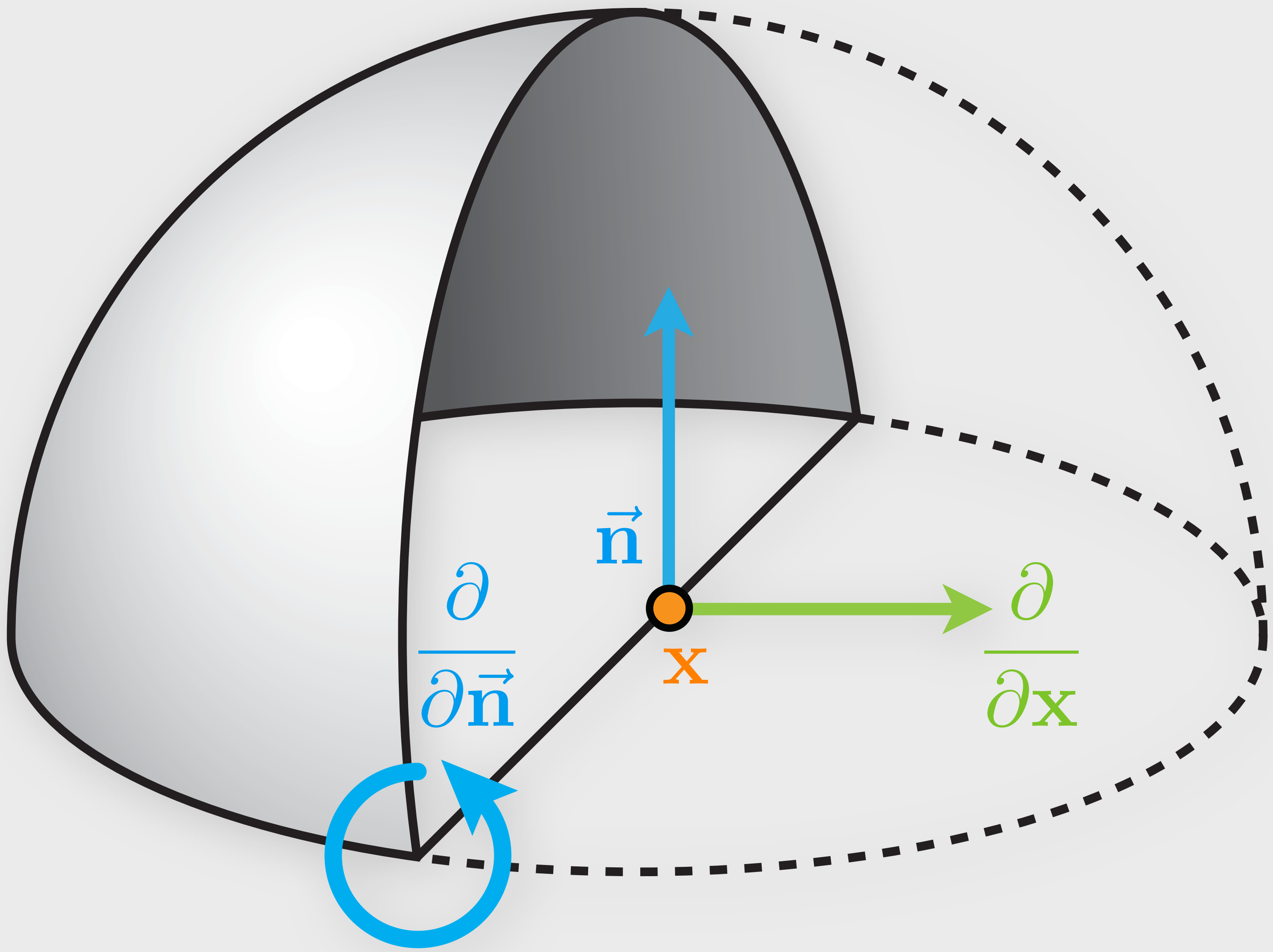

[Ward et al. 1988]

Interpolating Irradiance

- In the "Split-Sphere" environment the error becomes:

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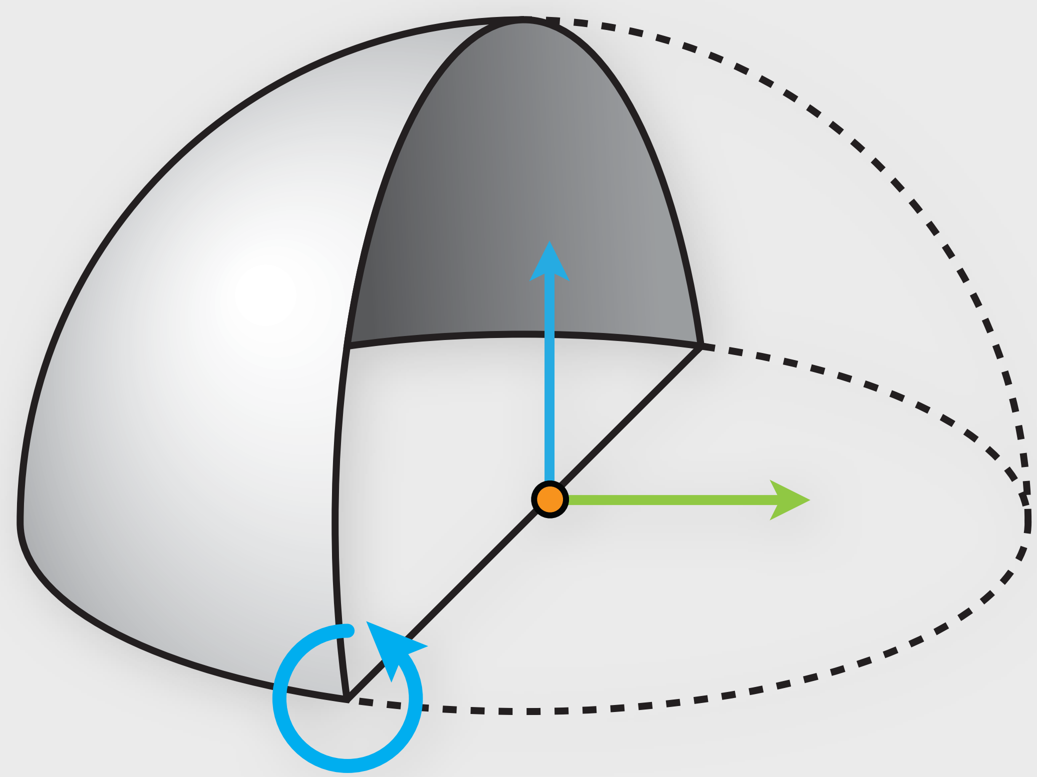

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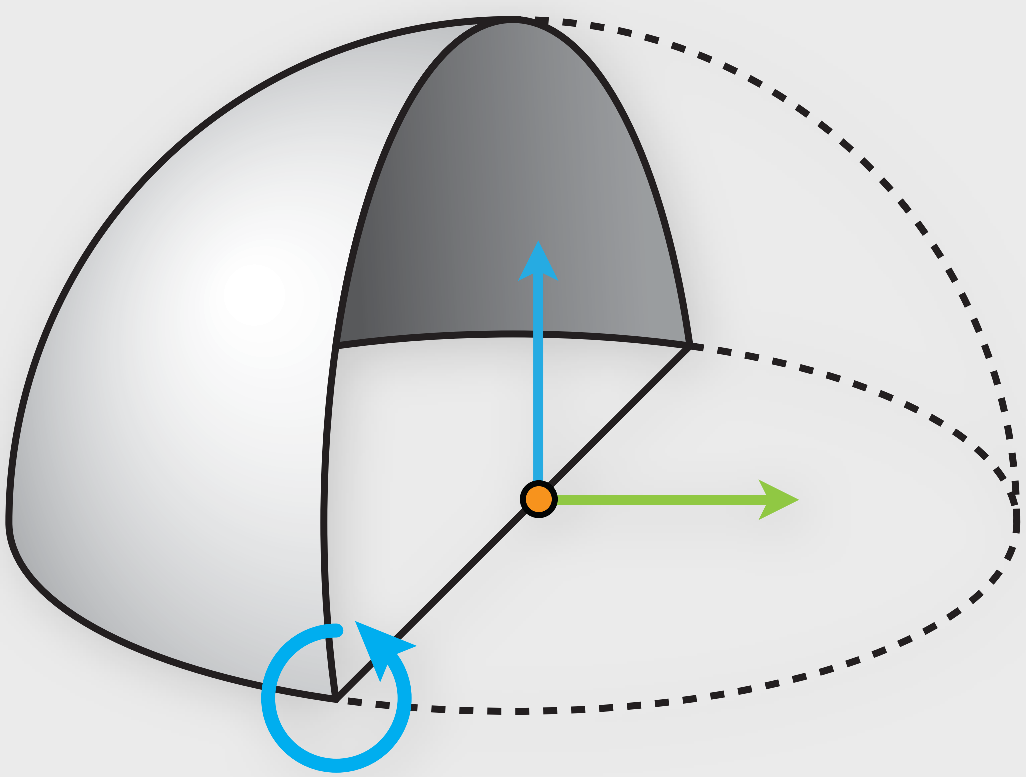
Interpolating Irradiance

- In the "Split-Sphere" environment the error becomes:

$$\varepsilon_i \lesssim E_i \left(\frac{4}{\pi} \frac{\|\mathbf{x} - \mathbf{x}_i\|}{R_i} + \sqrt{1 - (\vec{\mathbf{n}} \cdot \vec{\mathbf{n}}_i)} \right)$$



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Interpolating Irradiance

- In the "Split-Sphere" environment the error becomes:

orientation difference

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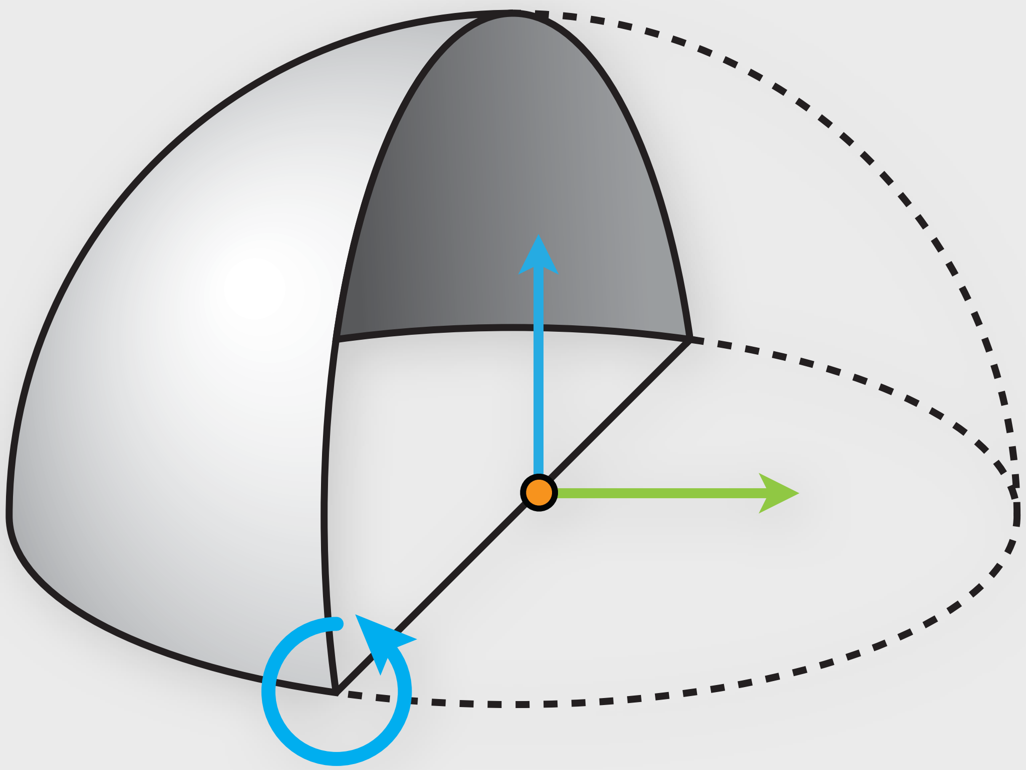

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Interpolating Irradiance

- In the "Split-Sphere" environment the error becomes:

position difference, relative to radius of sphere

orientation difference

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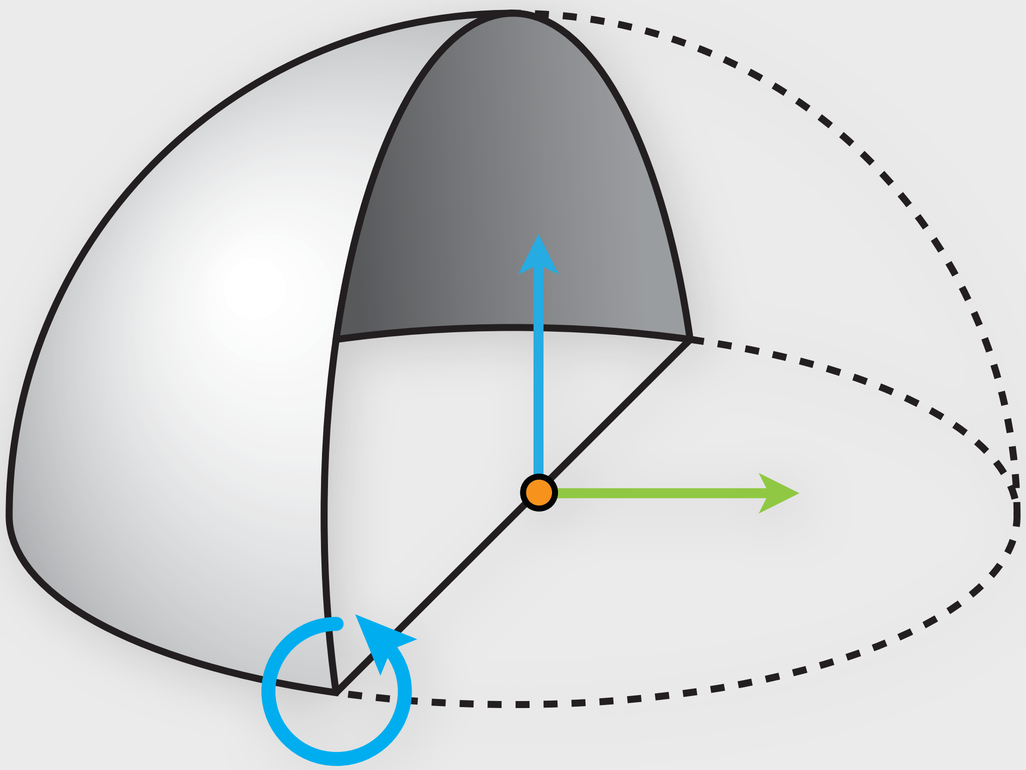

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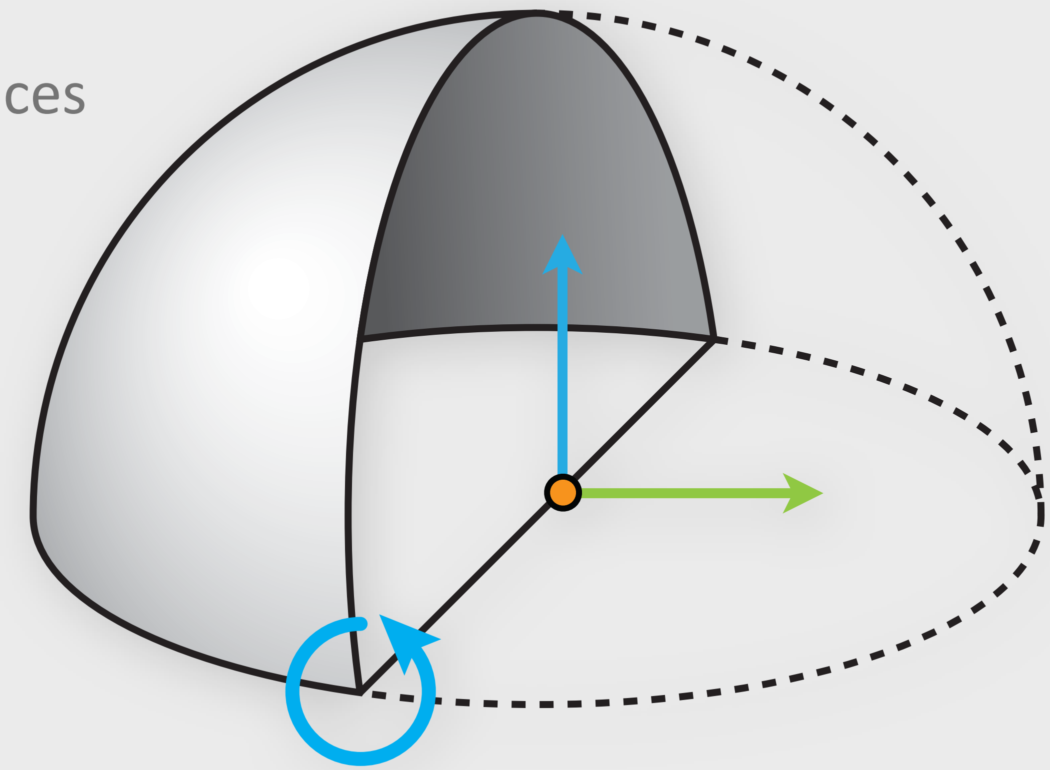
“average” distance to visible surfaces

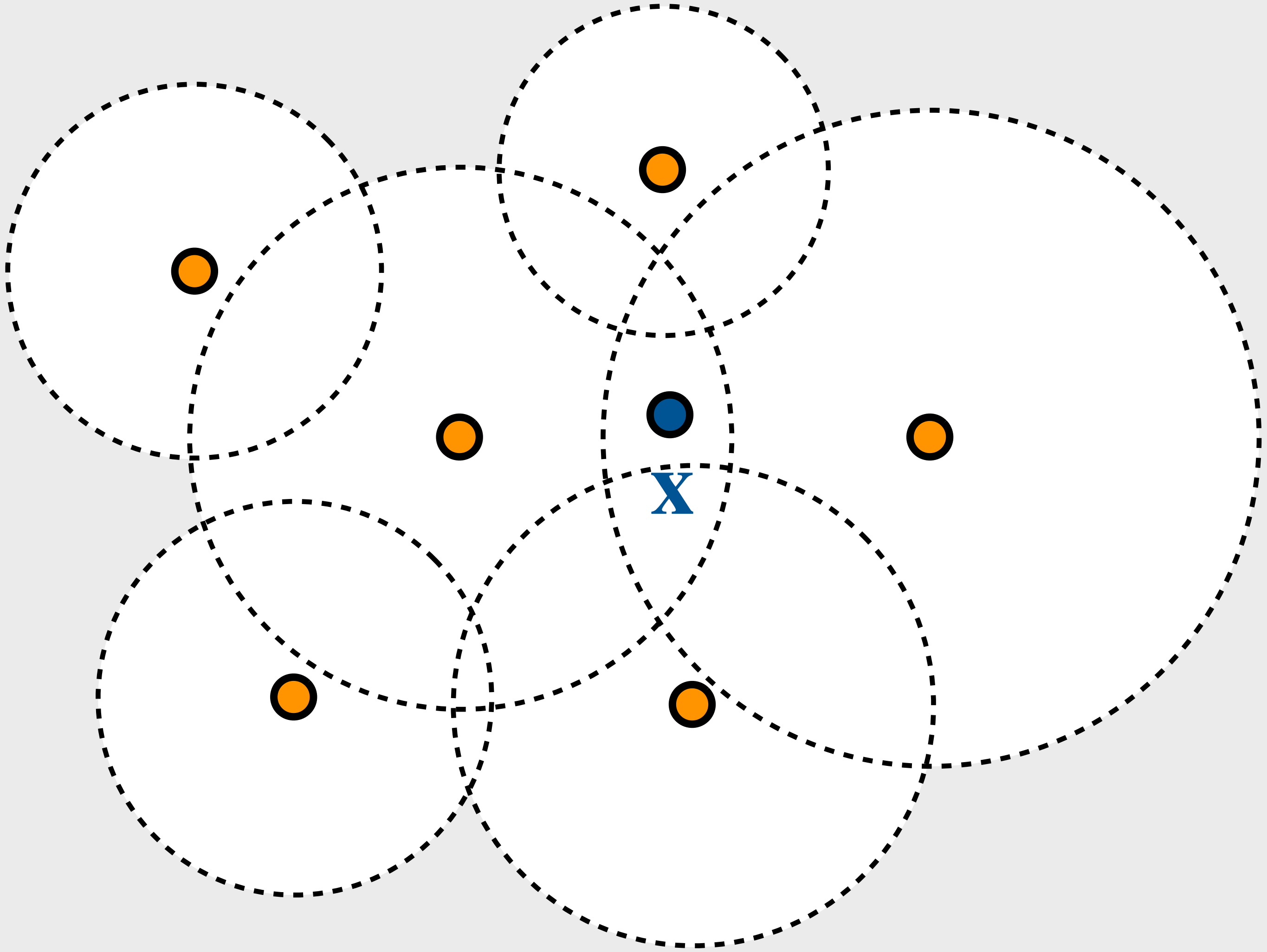


[Ward et al. 1988]

Interpolating Irradiance

- At each shading location, perform a weighted average of all cached values which have an error below some threshold.

$$E(\mathbf{x}, \vec{\mathbf{n}}) \approx \frac{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})\, E_i}{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})}$$



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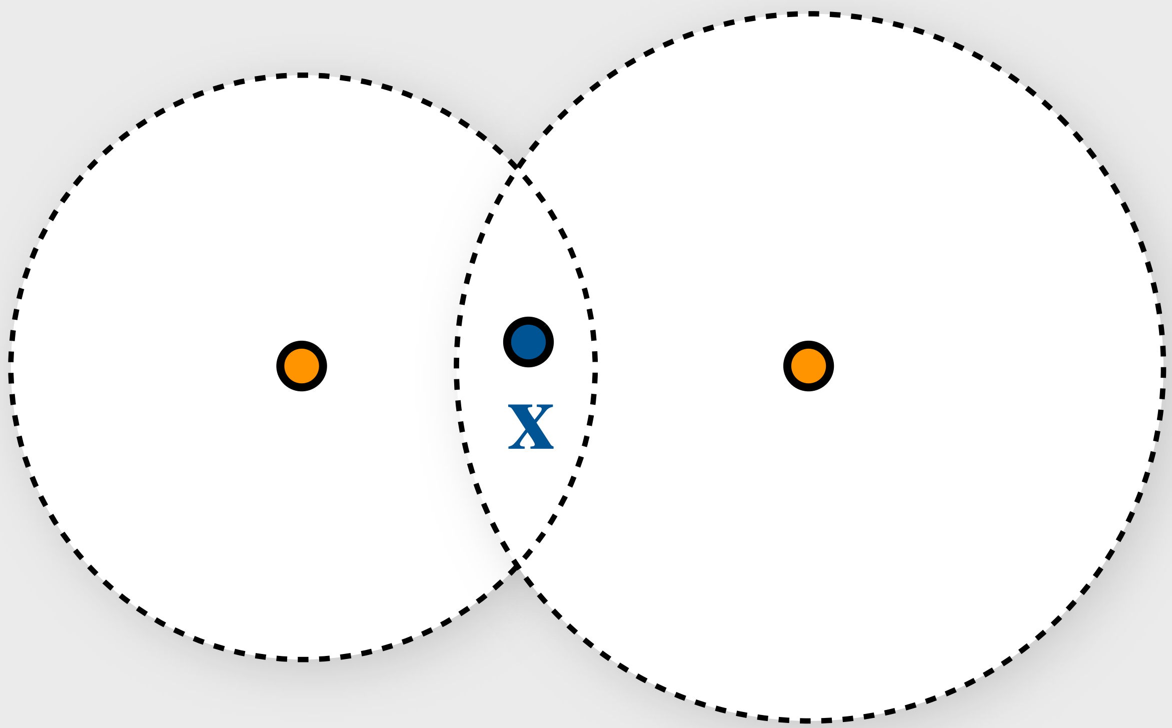
where:

$$S = \{i : \epsilon_i(\mathbf{x}, \vec{\mathbf{n}}) < a\}$$



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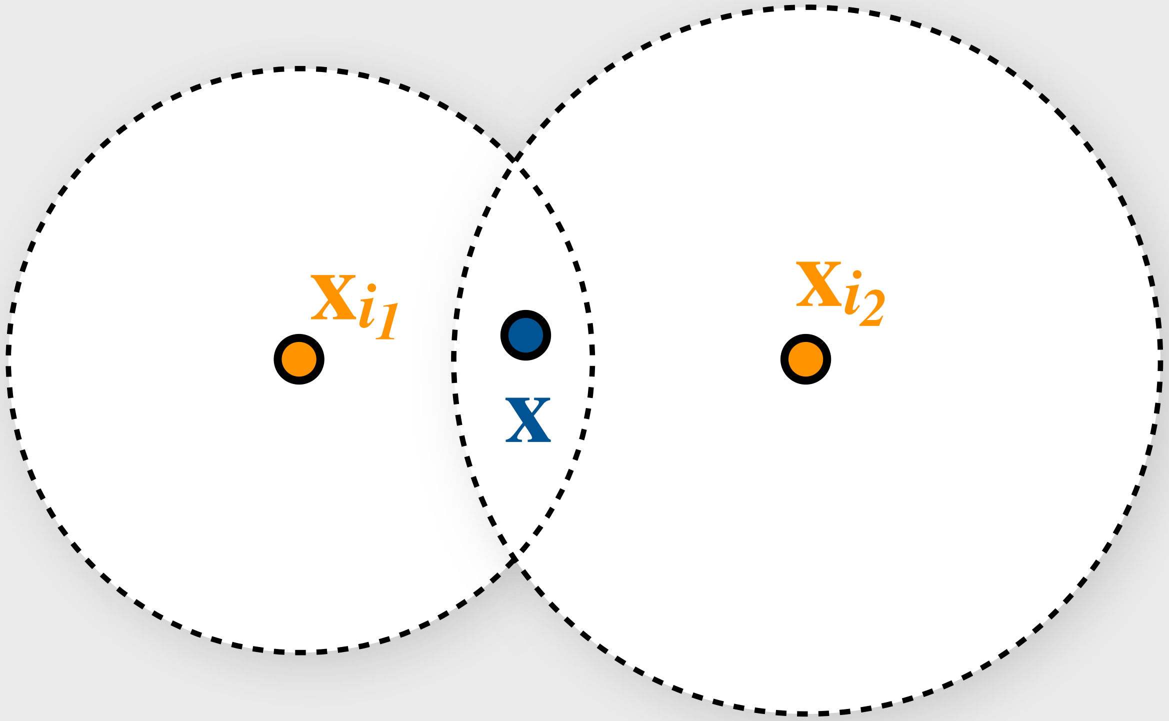


Interpolating Irradiance

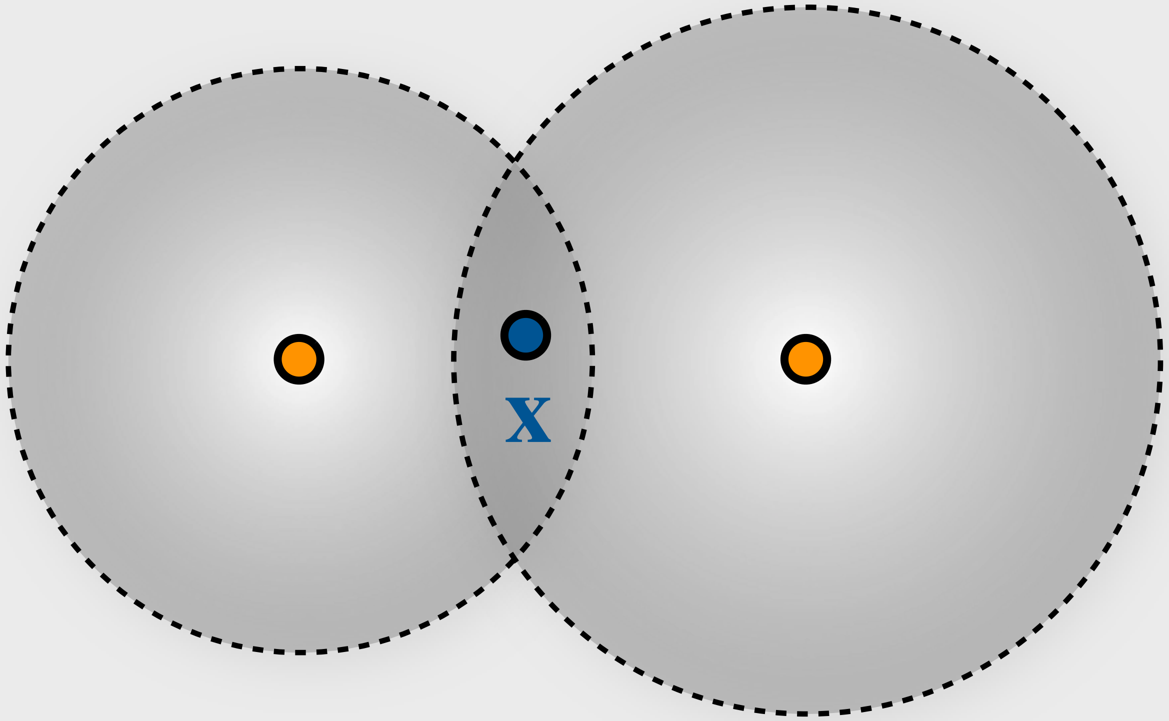
- At each shading location, perform a weighted average of all cached values which have an error below some threshold.
- Reciprocal of the error is used as the weight

$$E(\mathbf{x}, \vec{\mathbf{n}}) \approx \frac{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}}) E_i}{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})}$$

where:

$$S = \{i : \epsilon_i(\mathbf{x}, \vec{\mathbf{n}}) < a\}$$

$$\mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}}) = \frac{1}{\epsilon_i(\mathbf{x}, \vec{\mathbf{n}})} - \frac{1}{a}$$



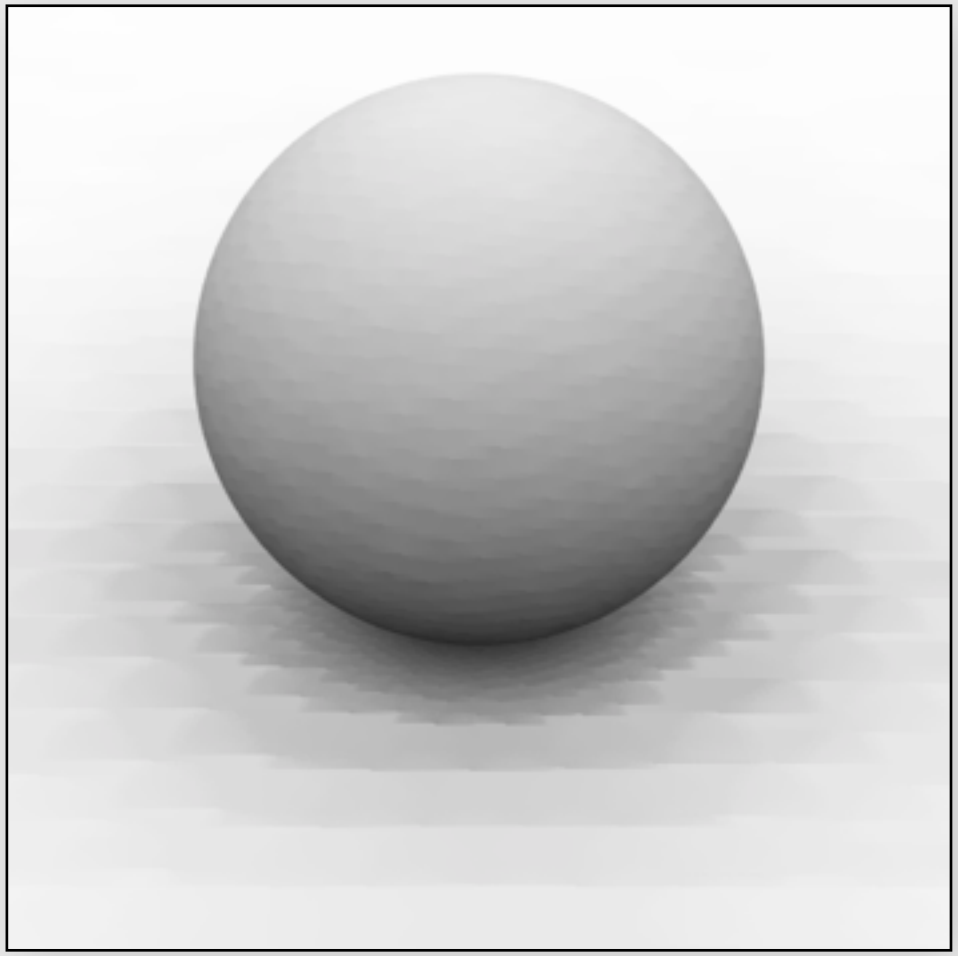
Irradiance Caching

- Pros:
  - Independent of resolution.
  - Computation amortized across many pixels
  - Concentrates computation in visible regions were illumination changes rapidly

[Ward et al. 1988]

Irradiance Caching

- Cons:
  - Interpolation/extrapolation can introduce visible artifacts
  - Valid radius metric not always robust
  - Limited to Lambertian (matte) surfaces



[Ward et al. 1988]

Improvements/Extensions

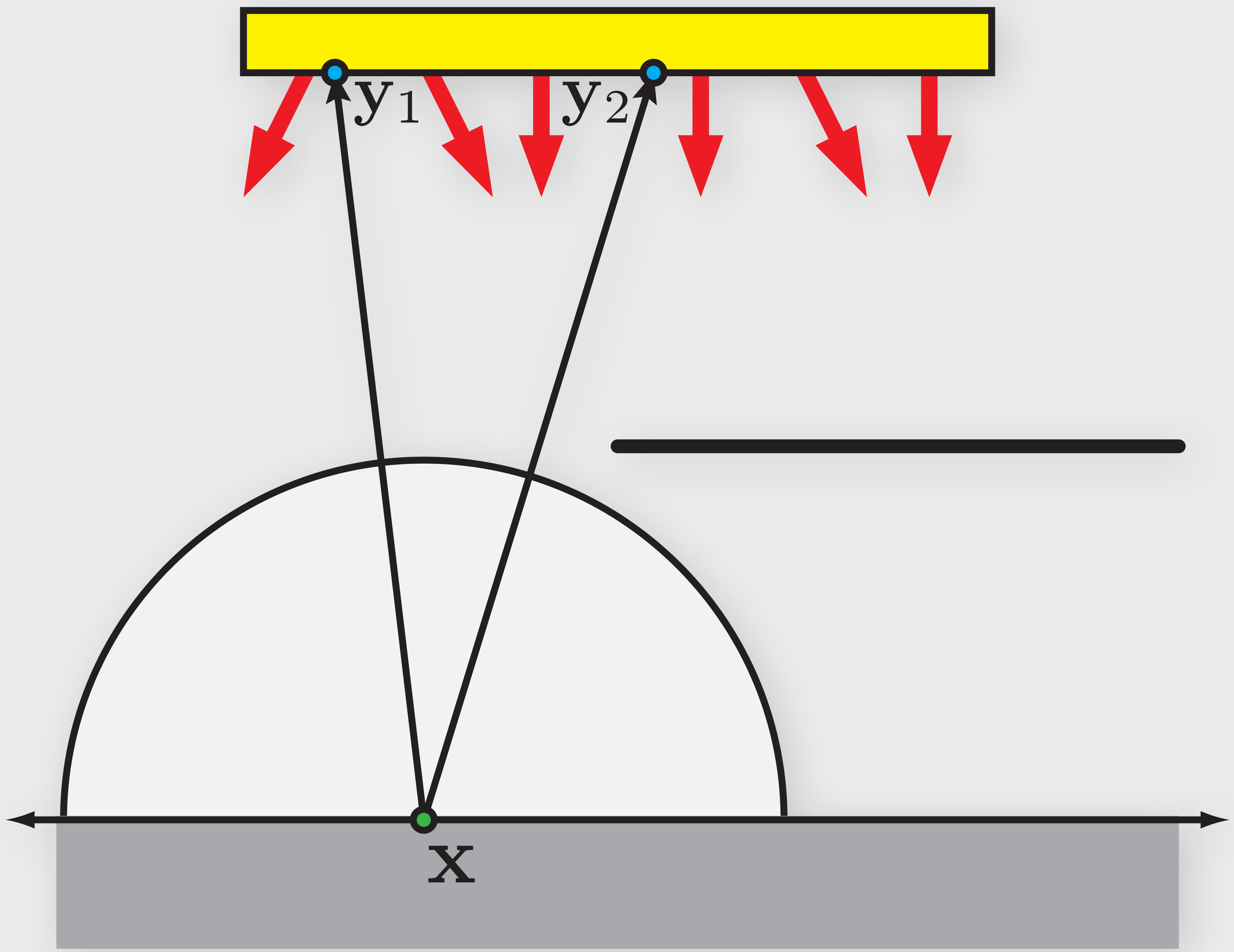
- Many extensions:
  - Ward and Heckbert '92 - better interpolation
  - Křivánek et al. '05a, '05b - glossy surfaces
  - Jarosz et al. '08 - participating media
  - Jarosz et al. '12 - irradiance Hessians
  - Schwarzhaupt et al. '12 - better error control
  - ...

Irradiance gradients

- Improve interpolation/extrapolation quality using gradients
- Irradiance Gradients [Ward and Heckbert 1992]
  - Estimate an actual derivative to the irradiance
  - Apply this derivative to the weighted average

[Ward and Heckbert 1992]

Gradients (surface-area formulation)



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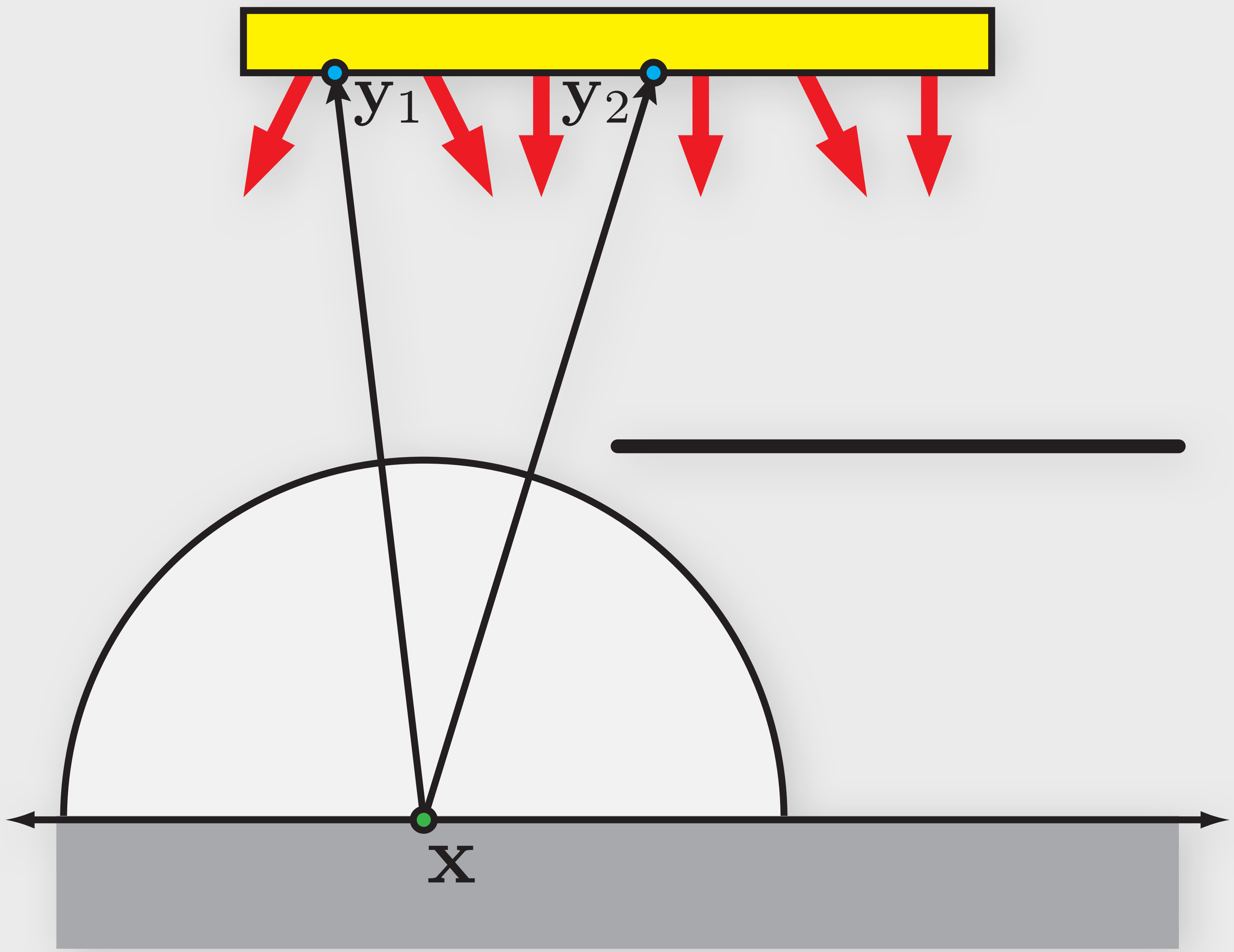
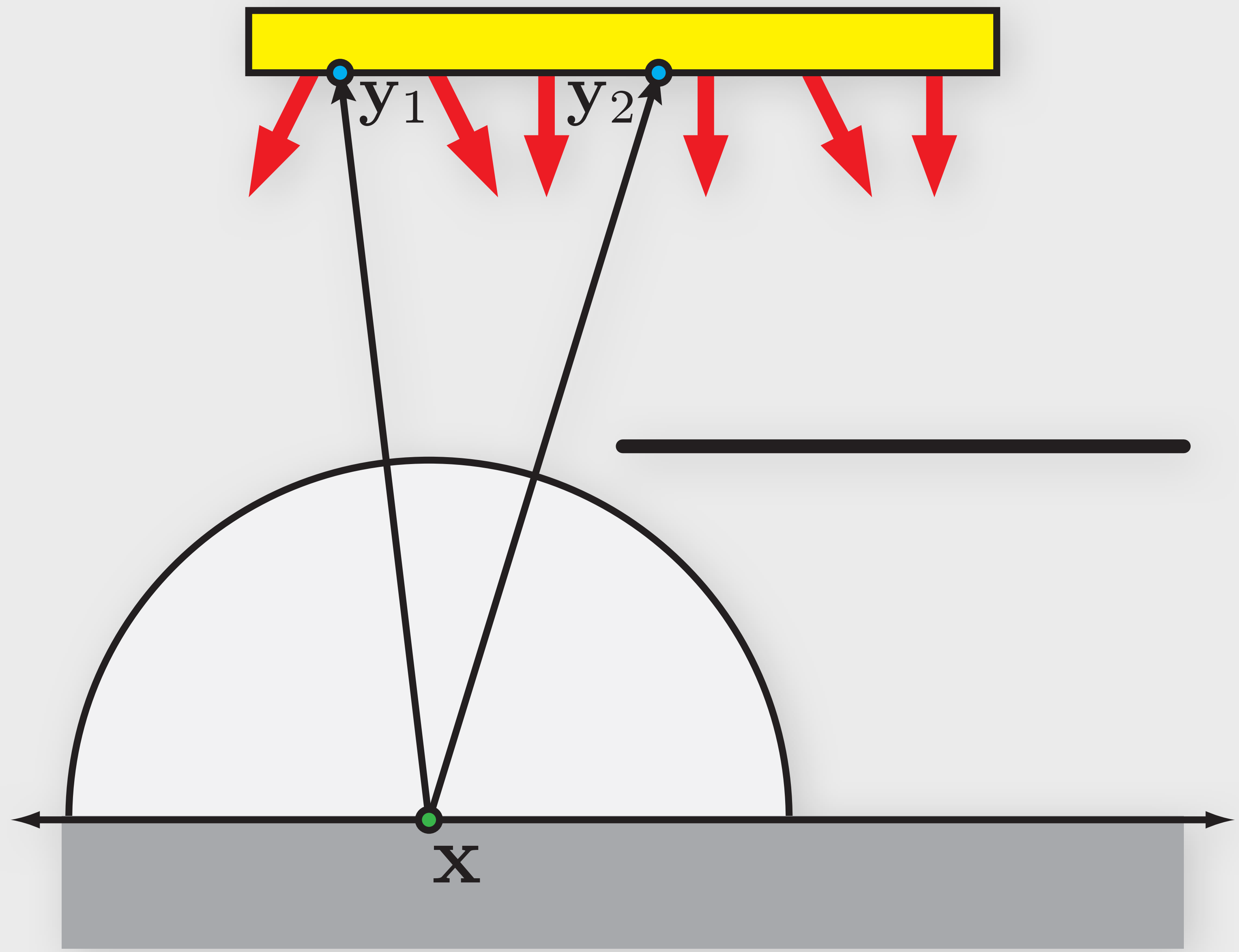


$$\nabla E(\mathbf{x}) = \nabla \int_A L(\mathbf{x} \leftarrow \mathbf{y})\, V(\mathbf{x}, \mathbf{y})\, G(\mathbf{x}, \mathbf{y})\; \mathrm{d}\mathbf{y}$$

Gradients (surface-area formulation)



$$\nabla E(\mathbf{x}) = \int_A \nabla L\, V\, G + L\, \nabla V\, G + L\, V\, \nabla G \,\mathrm{d}\mathbf{y}$$

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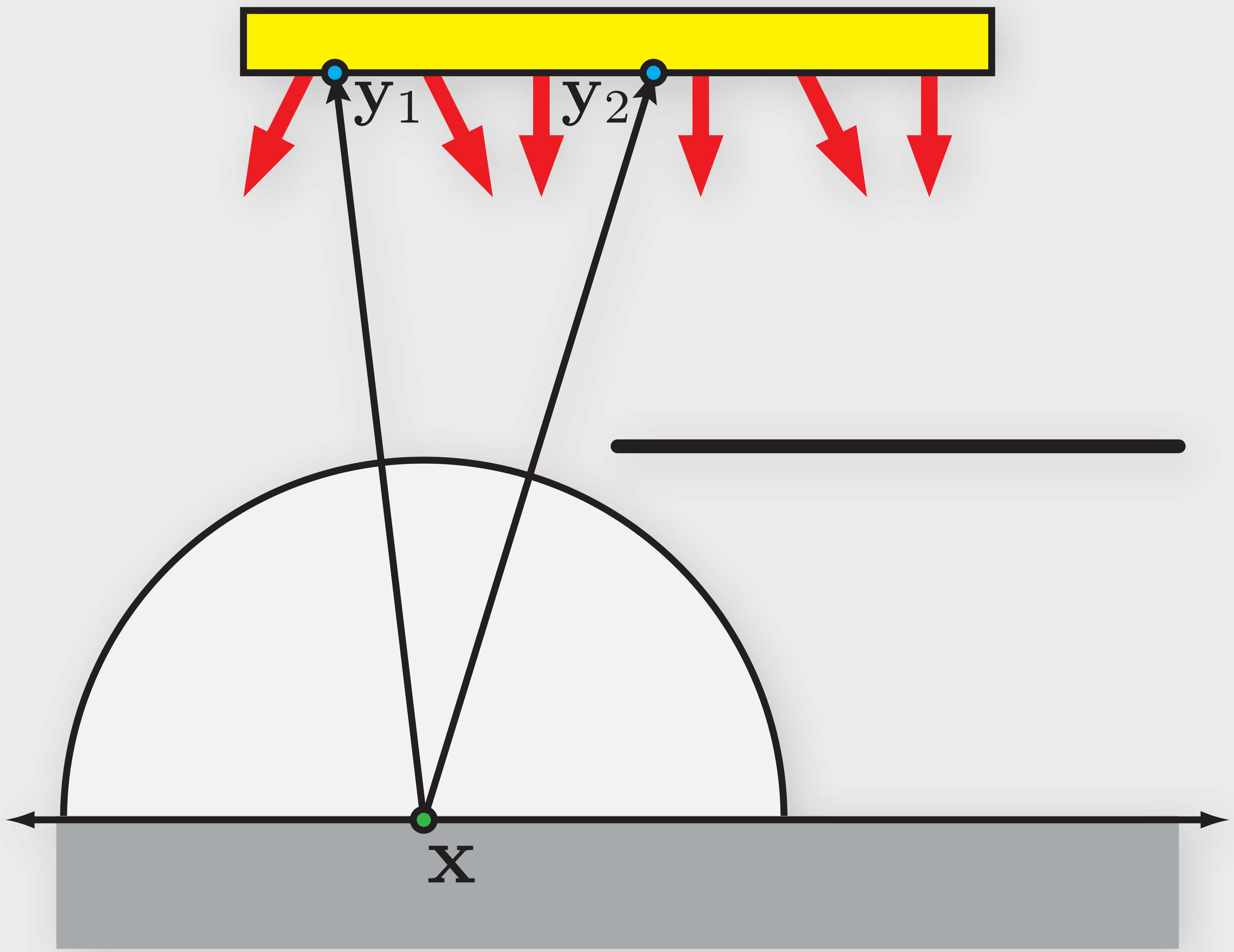

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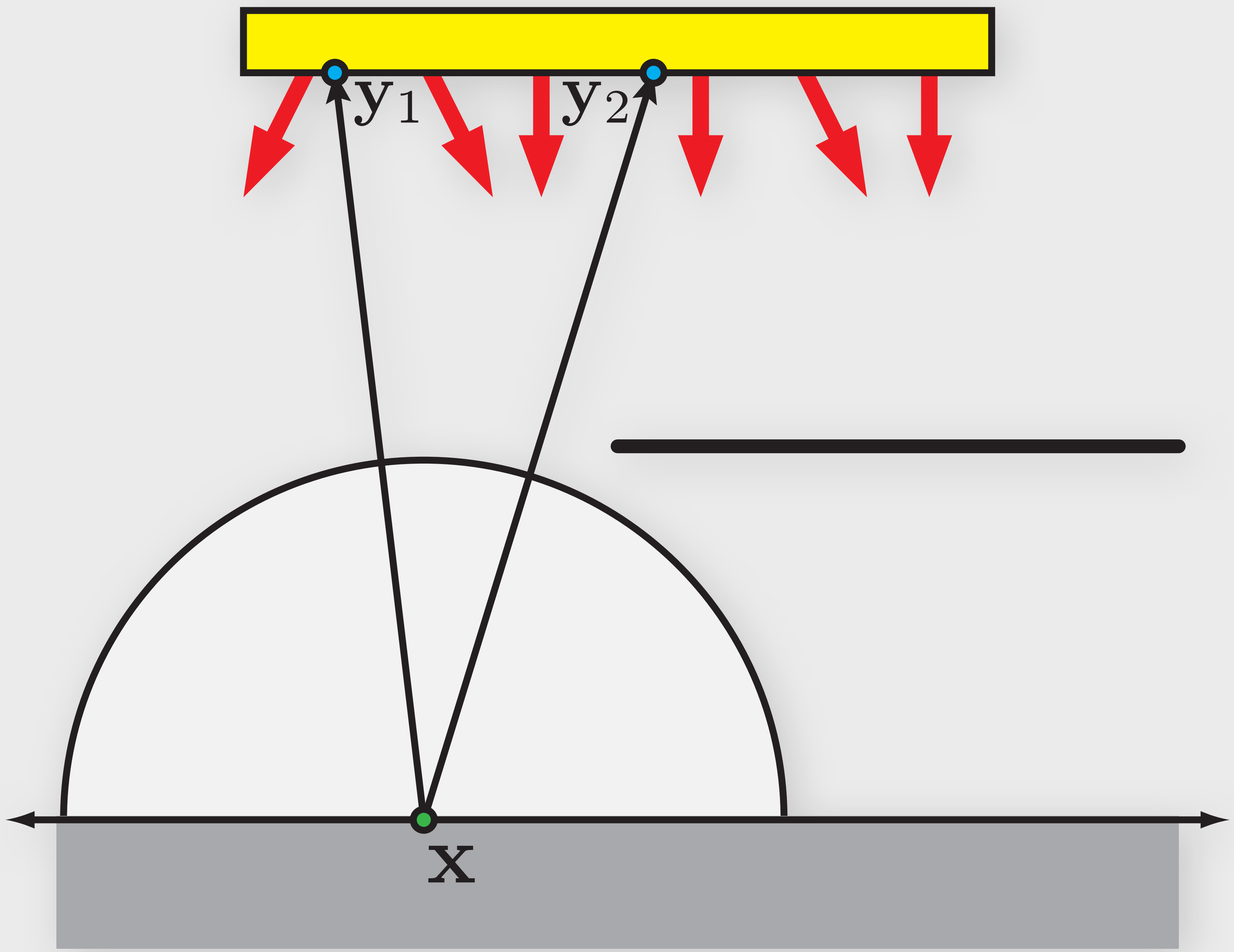

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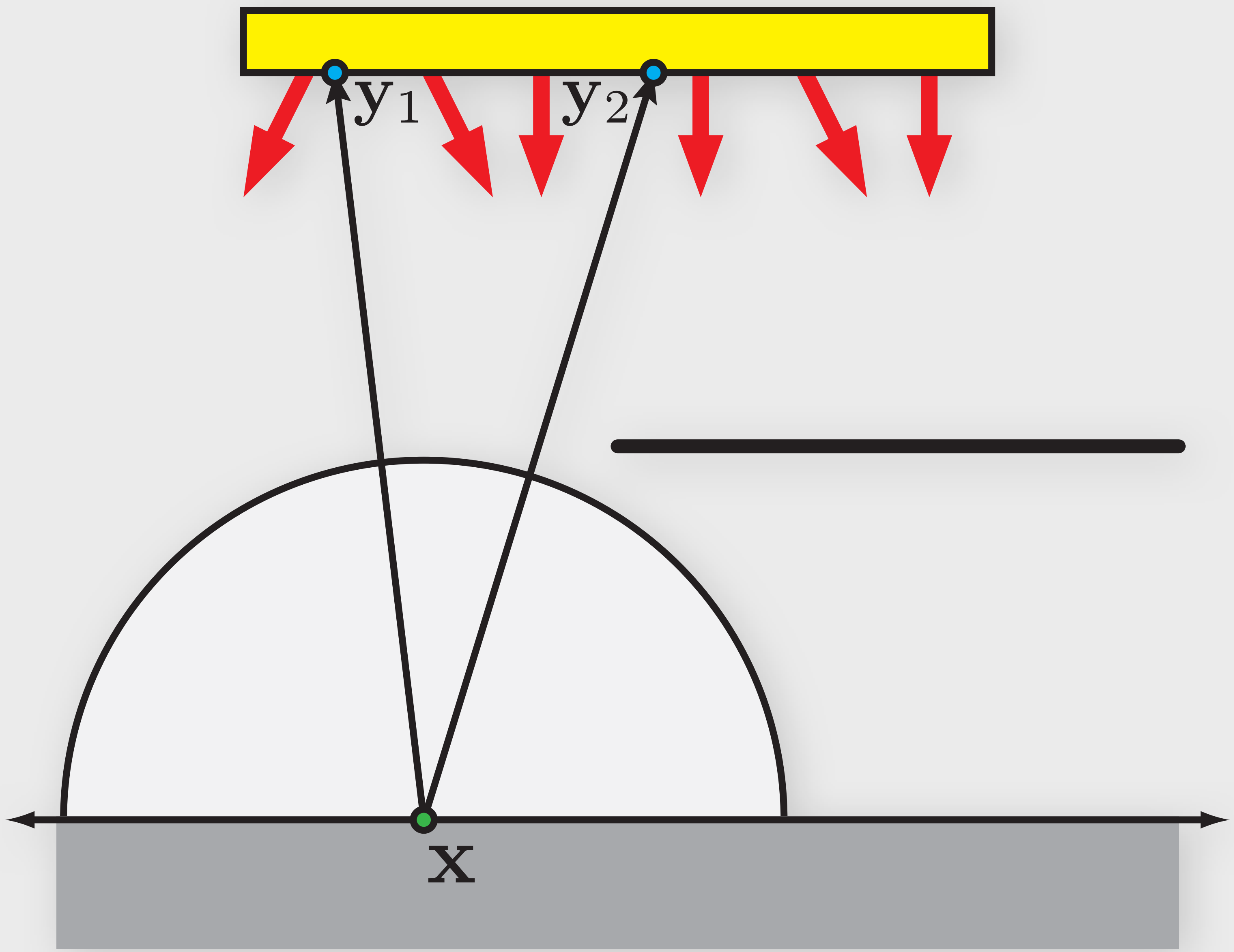

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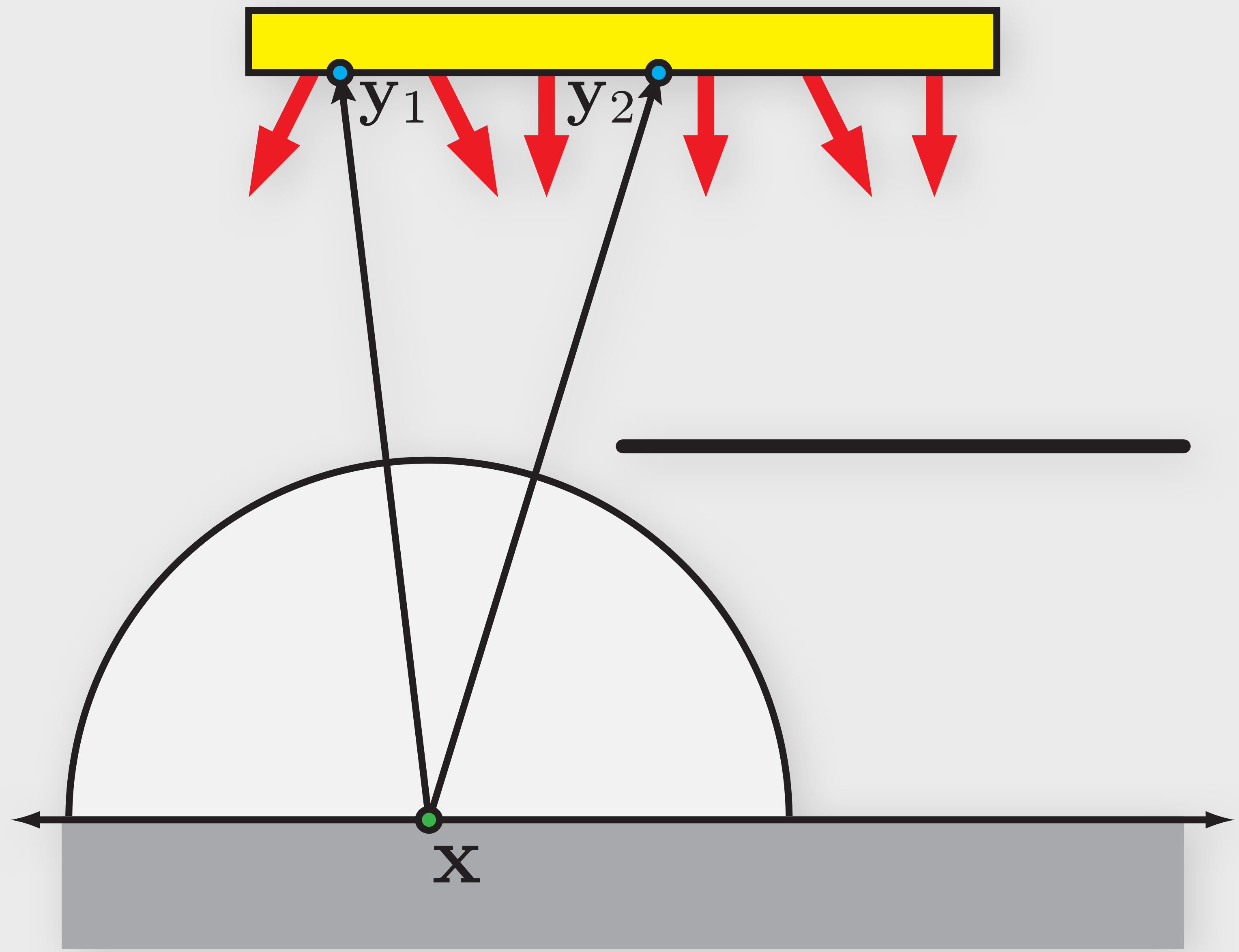

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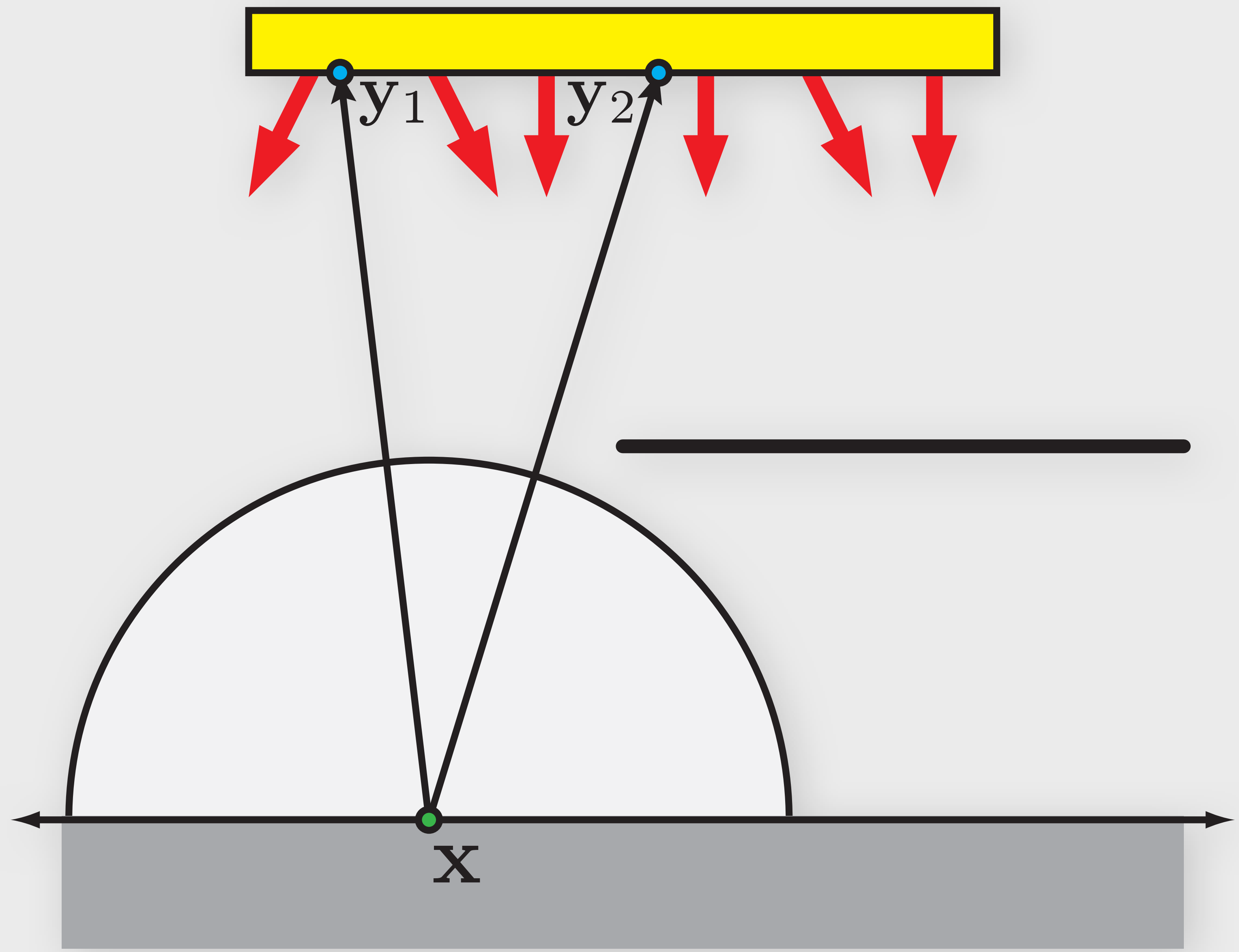

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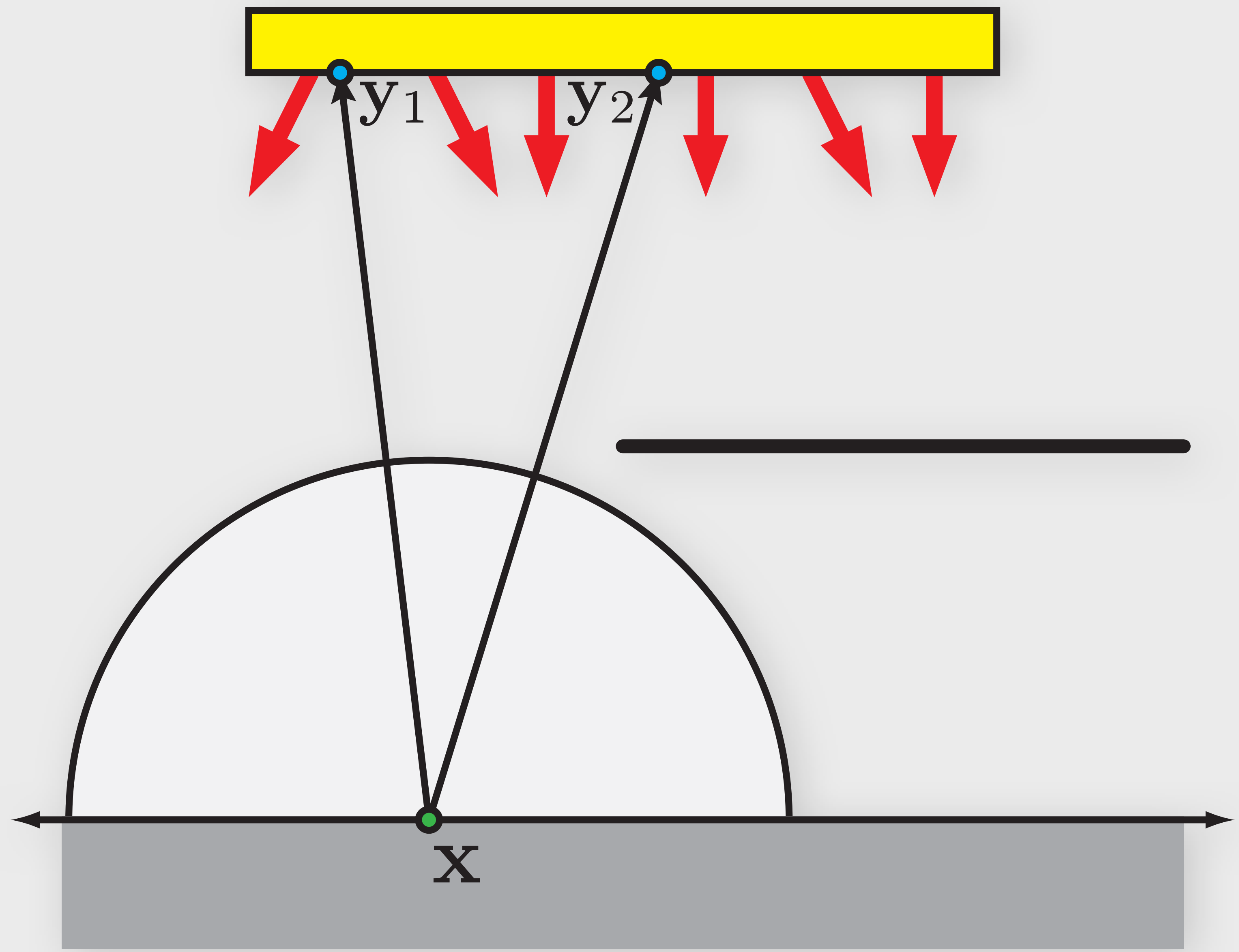

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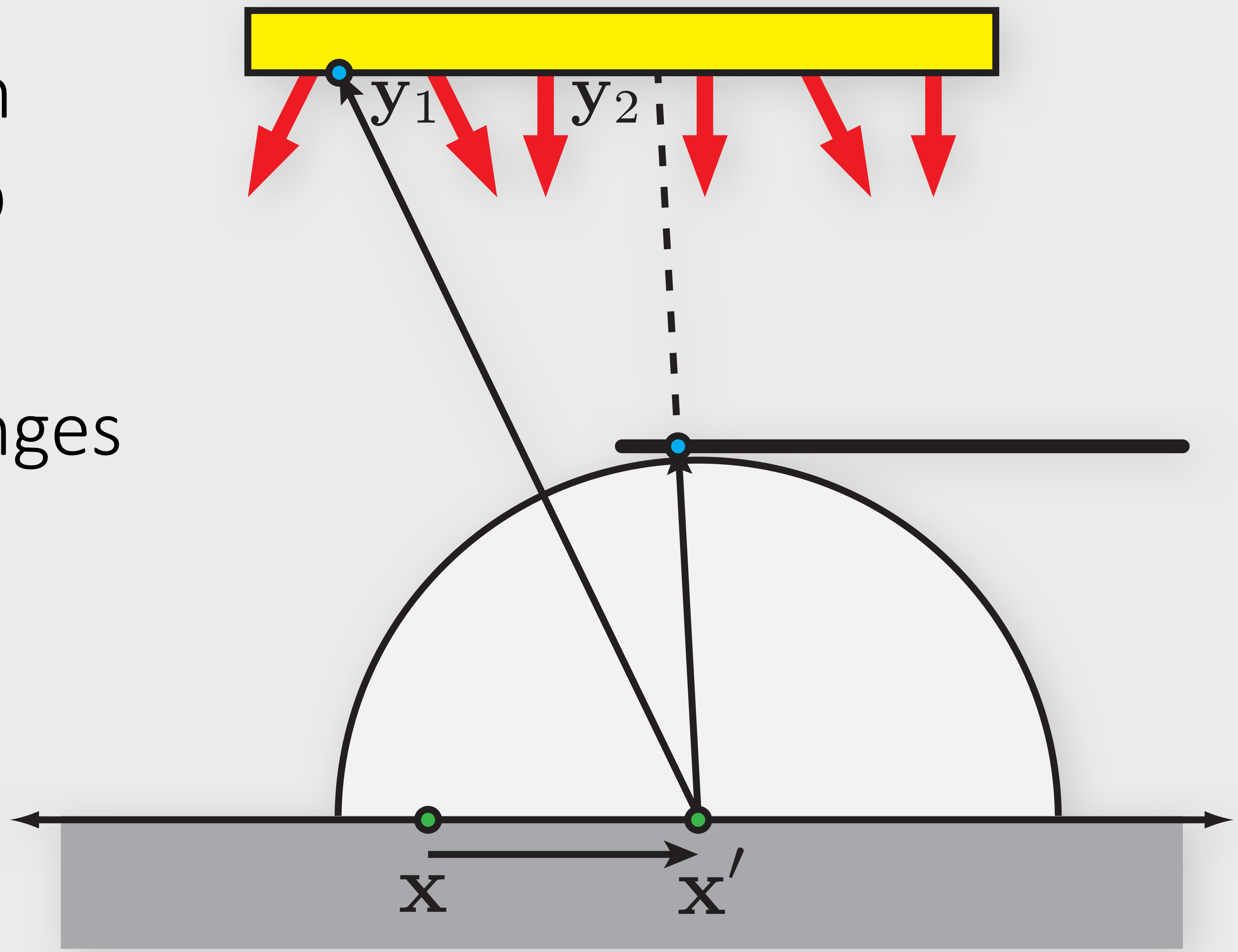
Gradients (surface-area formulation)

- Accounts for change in geometric relationship between x & y
- Ignores occlusion changes



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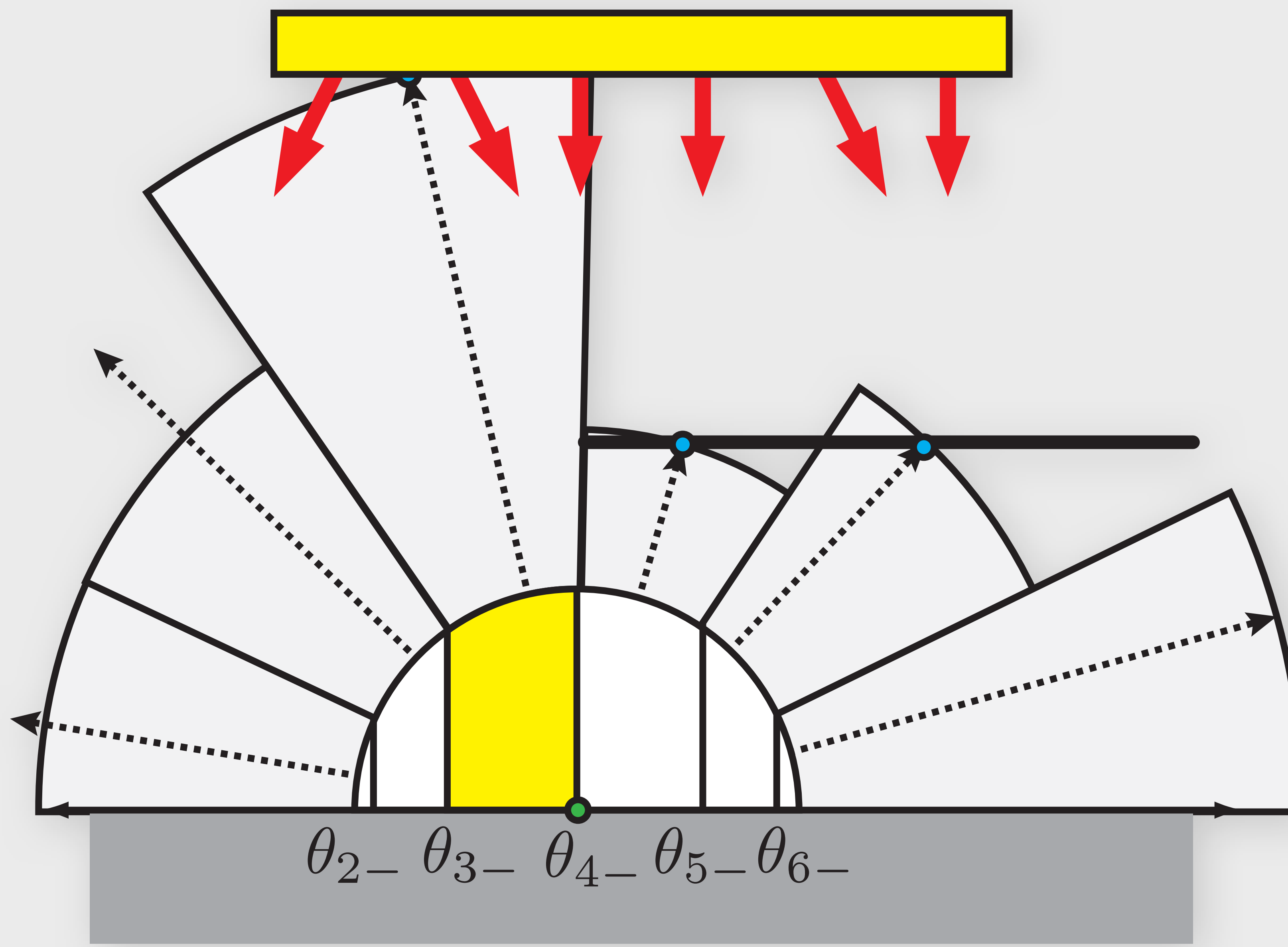
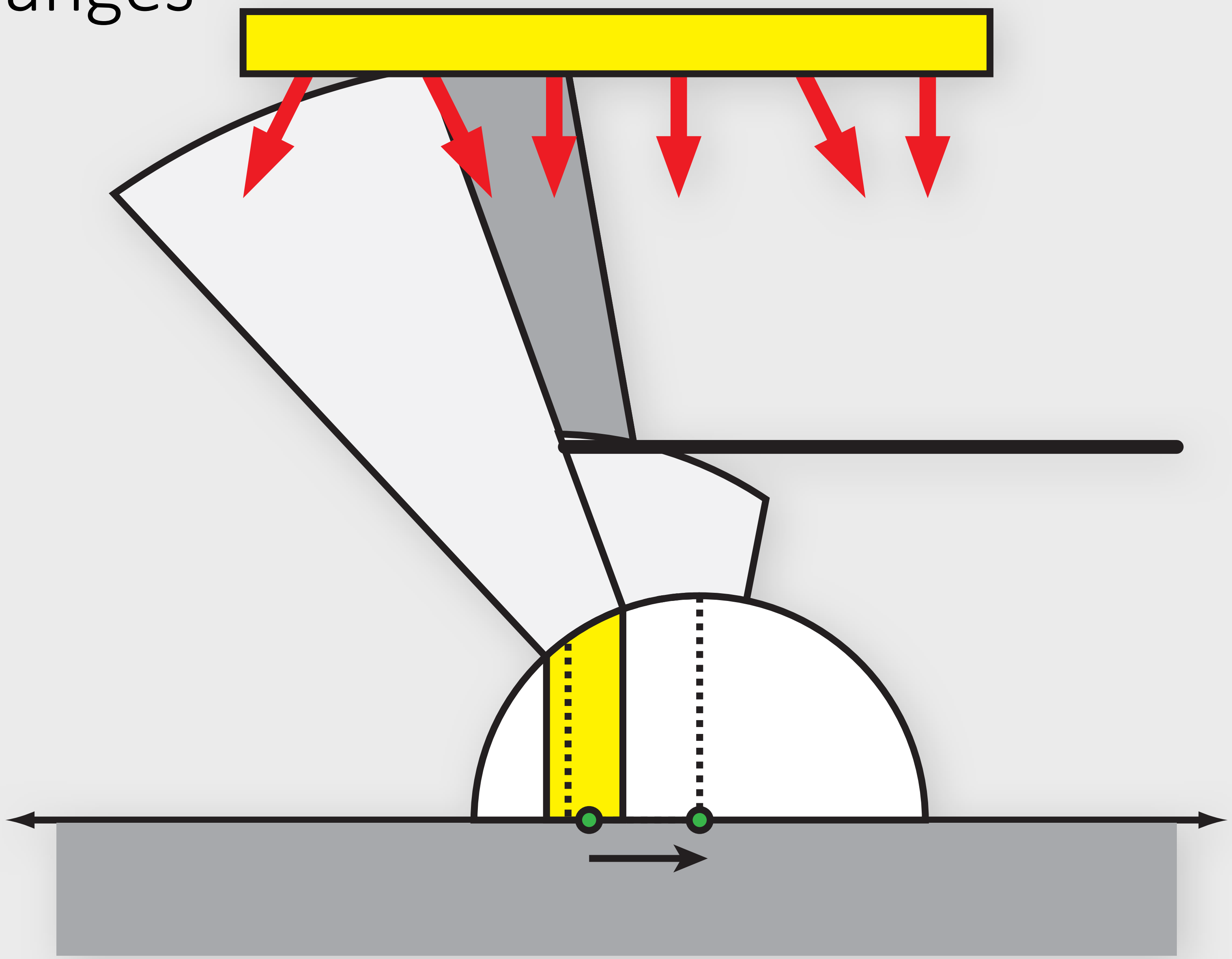
Gradients (stratified formulation)



[Ward and Heckbert 1992]

Gradients (stratified formulation)

- Considers occlusion changes

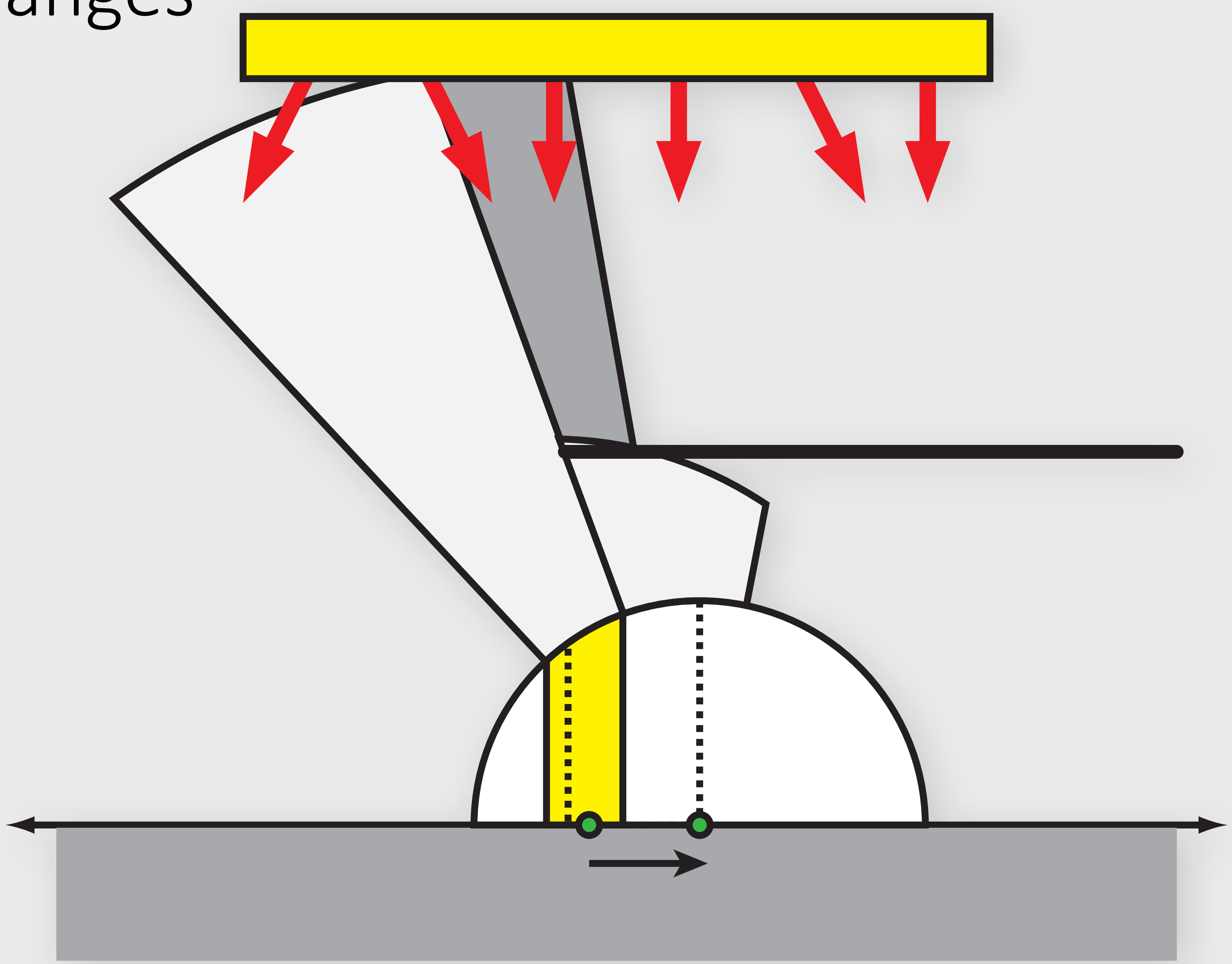


[Ward and Heckbert 1992]

Gradients (stratified formulation)

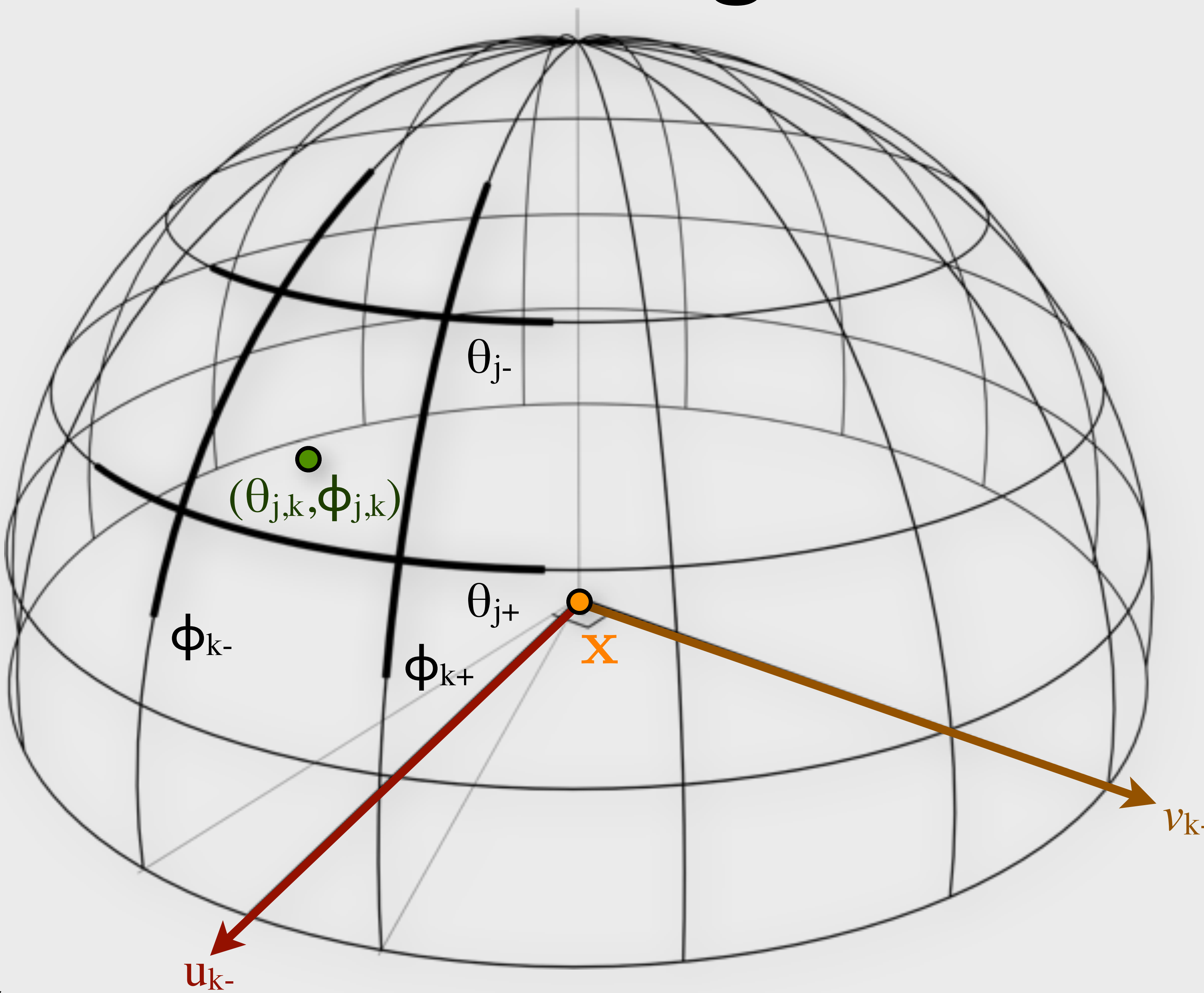
- Considers occlusion changes

Very Important!



[Ward and Heckbert 1992]

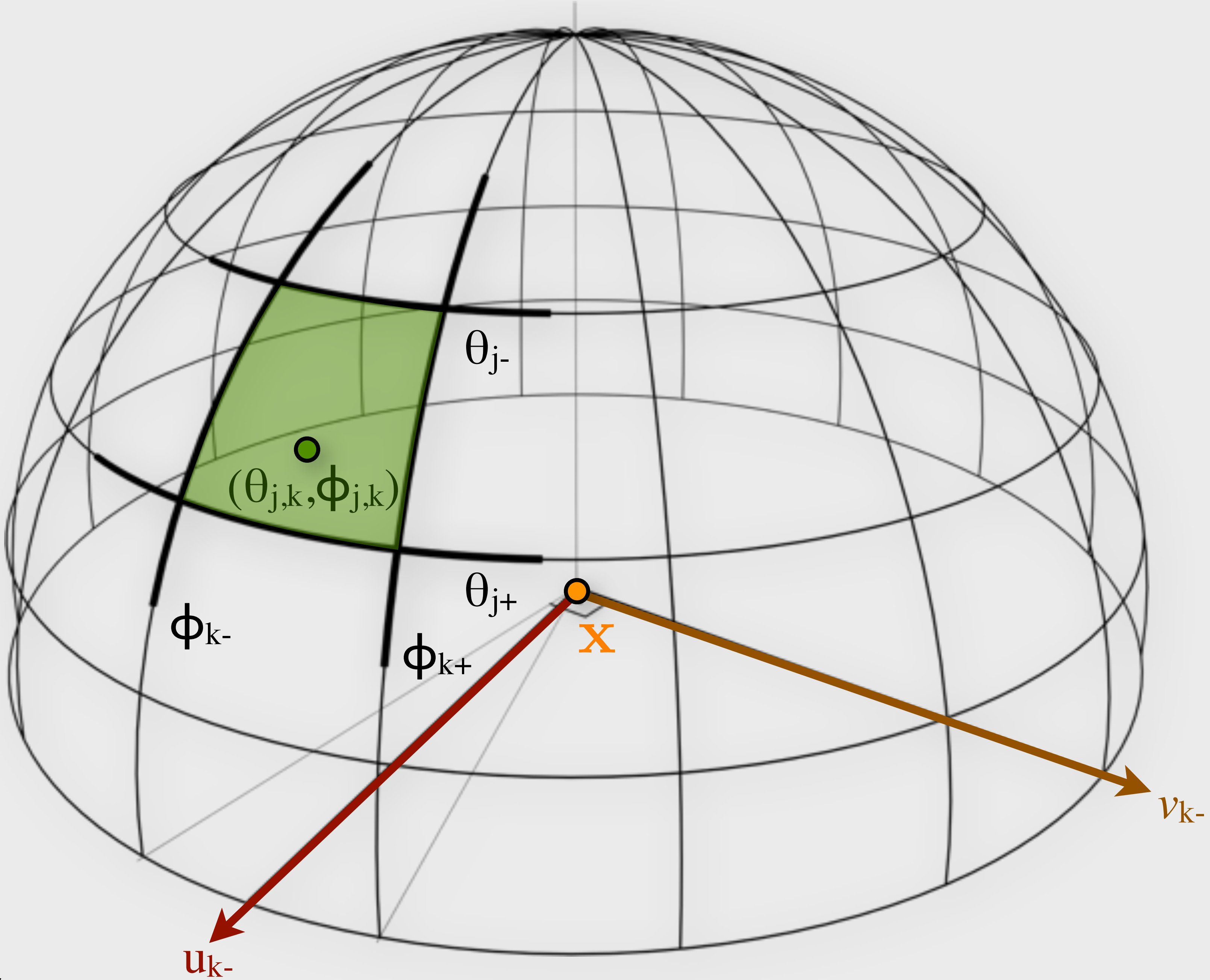
Stratified irradiance gradient



$$\nabla_t E(\mathbf{x}) = \sum_{k=1}^{N_1} \left(\hat{u}_k \sum_{j=2}^{N_2} \nabla_{\hat{u}_k} A_{j_-,k} (L_{j,k} - L_{j\text{-}1,k}) \cos\theta_{j_-} + \hat{v}_{k_-} \sum_{j=1}^{N_2} \nabla_{\hat{v}_{k_-}} A_{j,k_-} (L_{j,k} - L_{j,k\text{-}1}) \cos\theta_j \right)$$

[Ward and Heckbert 1992]

Stratified irradiance gradient



$$\nabla_t E(\mathbf{x}) = \sum_{k=1}^{N_1} \left(\hat{u}_k \sum_{j=2}^{N_2} \nabla_{\hat{u}_k} A_{j\text{-},k} (L_{j,k} - L_{j\text{-}1,k}) \cos\theta_{j\text{-}} + \hat{v}_{k\text{-}} \sum_{j=1}^{N_2} \nabla_{\hat{v}_{k\text{-}}} A_{j,k\text{-}} (L_{j,k} - L_{j,k\text{-}1}) \cos\theta_j \right)$$

[Ward and Heckbert 1992]

Interpolating with gradients

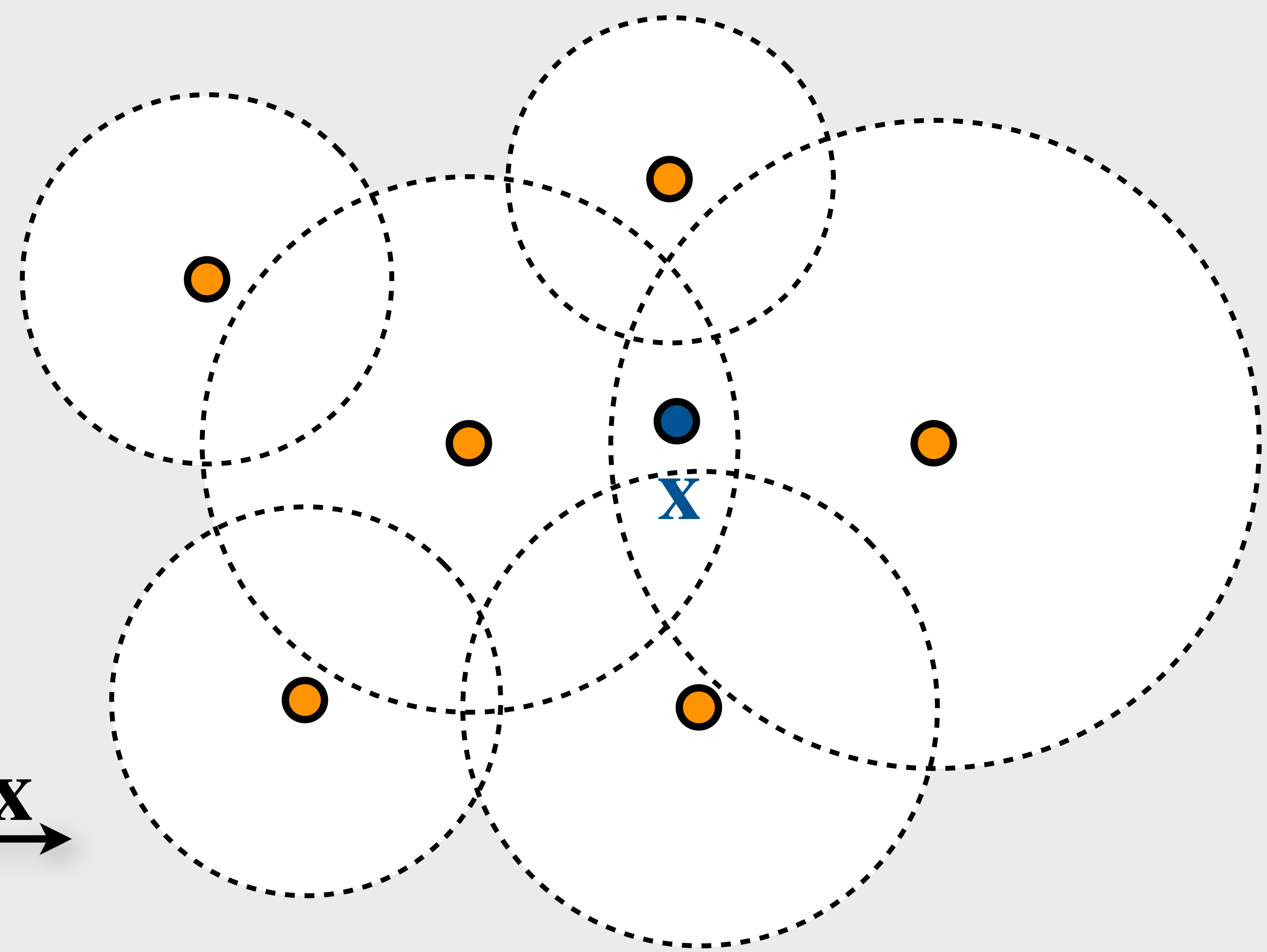
$$E(\mathbf{x}, \vec{\mathbf{n}}) \approx \frac{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})\, E_i}{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})}$$

Interpolating with gradients

$$E(\mathbf{x}, \vec{\mathbf{n}}) \approx \frac{\sum_{i \in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})\Big(E_i + (\vec{n}_i \times \vec{\mathbf{n}}) \cdot (\vec{\nabla}_r E_i) + (\mathbf{x} - \mathbf{x}_i) \cdot (\vec{\nabla}_t E_i)\Big)}{\sum_{i \in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})}$$

Interpolating with gradients

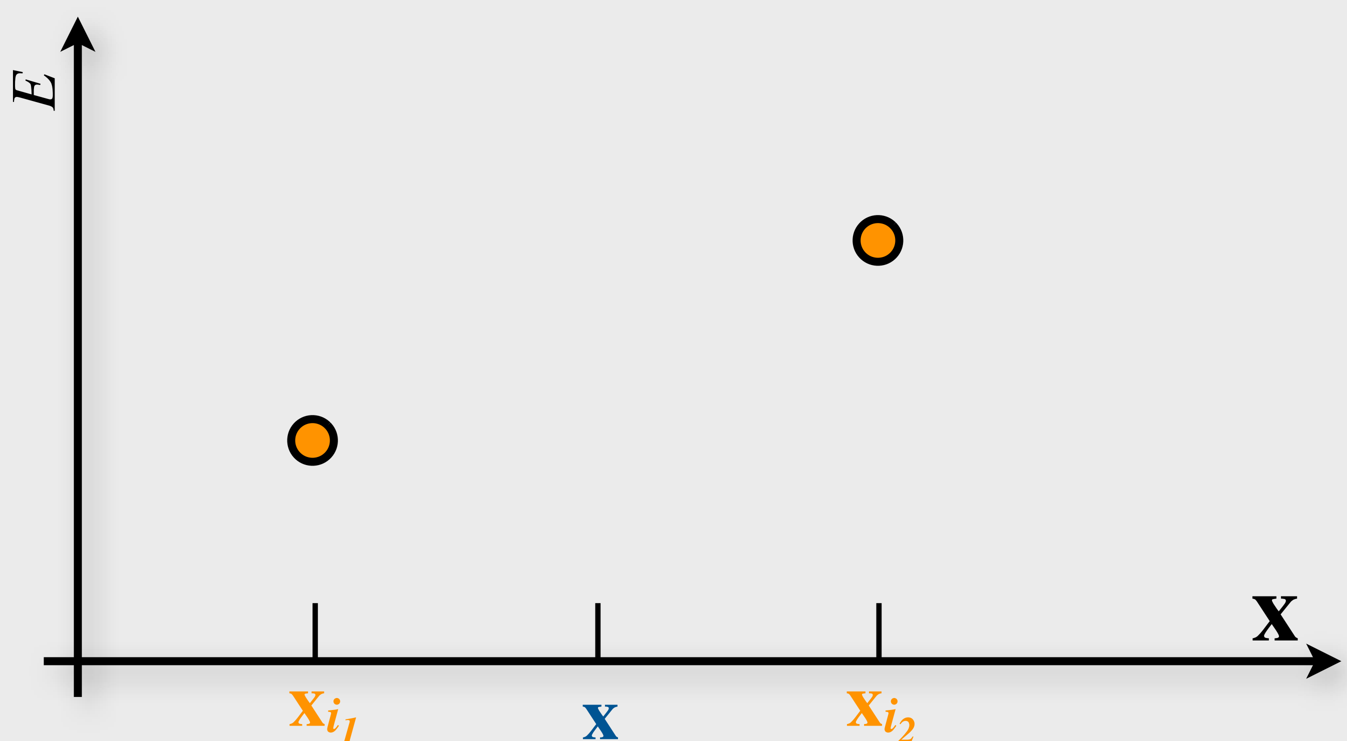
$$E(\mathbf{x}, \vec{\mathbf{n}}) \approx \frac{\sum_{i \in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})\Big(E_i + (\vec{n}_i \times \vec{\mathbf{n}}) \cdot (\vec{\nabla}_r E_i) + (\mathbf{x} - \mathbf{x}_i) \cdot (\vec{\nabla}_t E_i)\Big)}{\sum_{i \in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})}$$

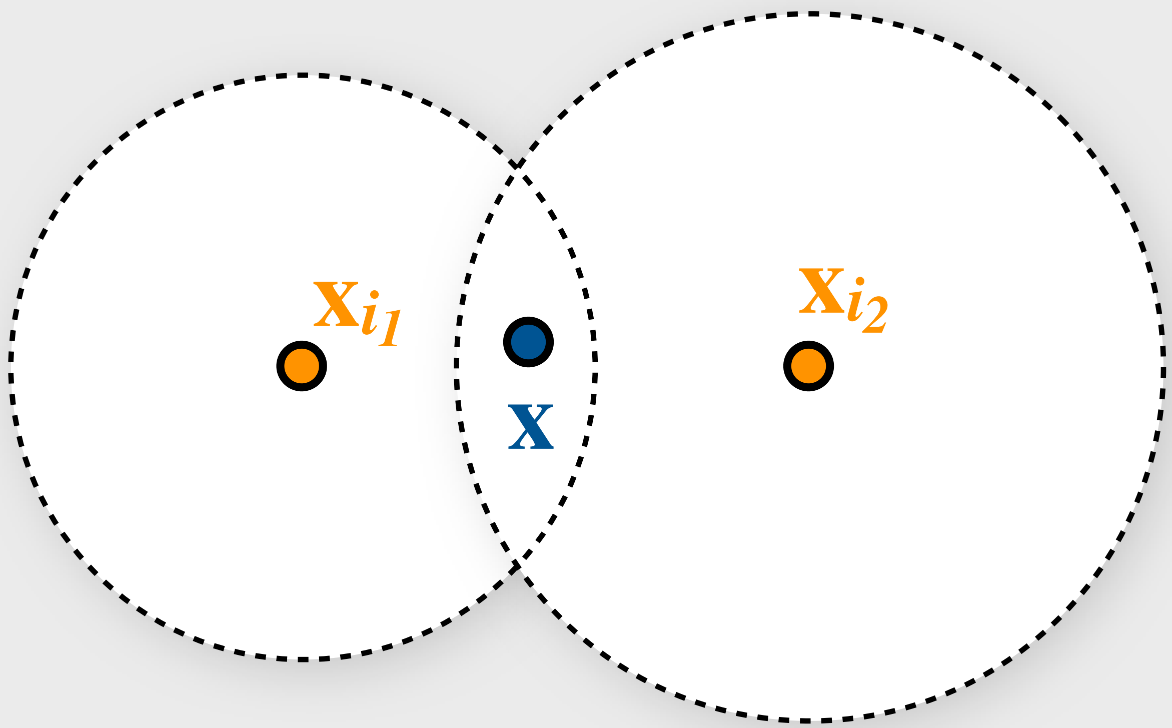


Interpolating with gradients

$$E(\mathbf{x},\vec{\mathbf{n}}) \approx \frac{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})\big(E_i+(\vec{n}_i\times\vec{\mathbf{n}})\cdot(\vec{\nabla}_r E_i)+(\mathbf{x}-\mathbf{x}_i)\cdot(\vec{\nabla}_t E_i)\big)}{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})}$$

Find overlapping cache records

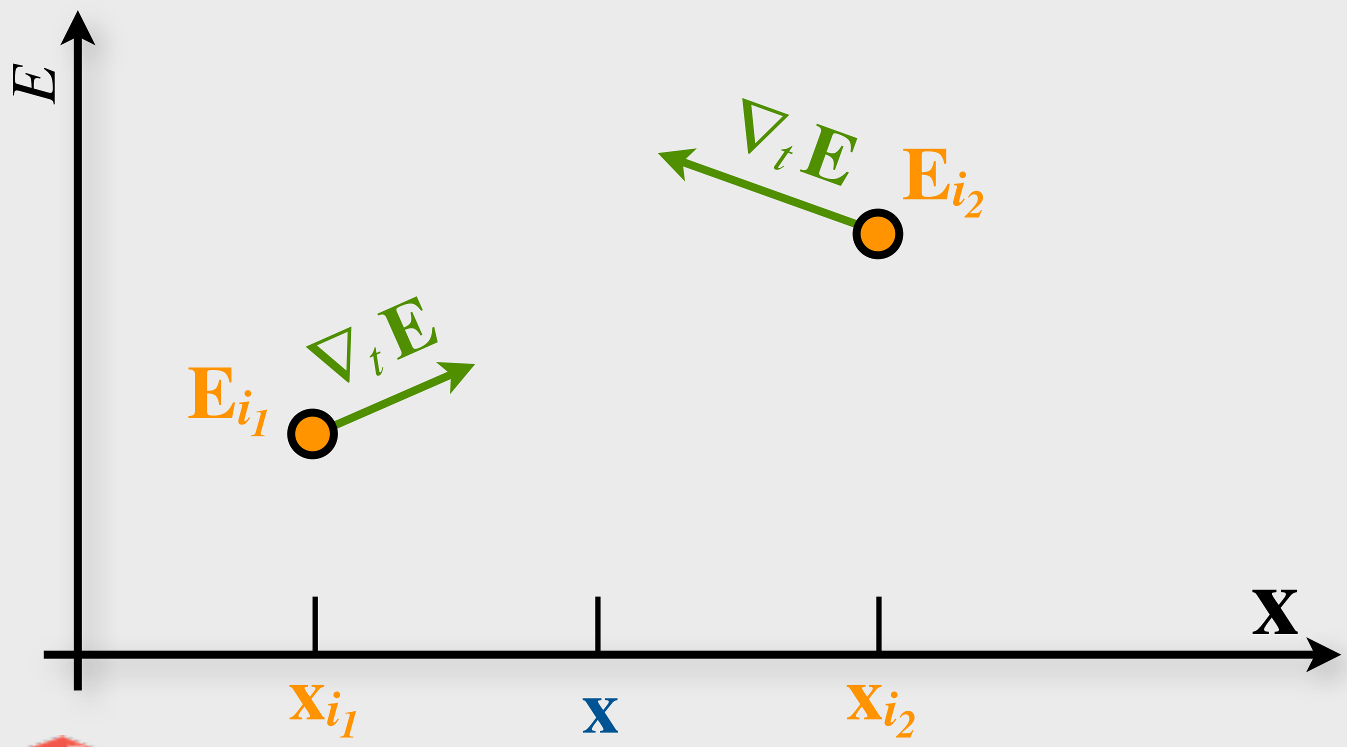


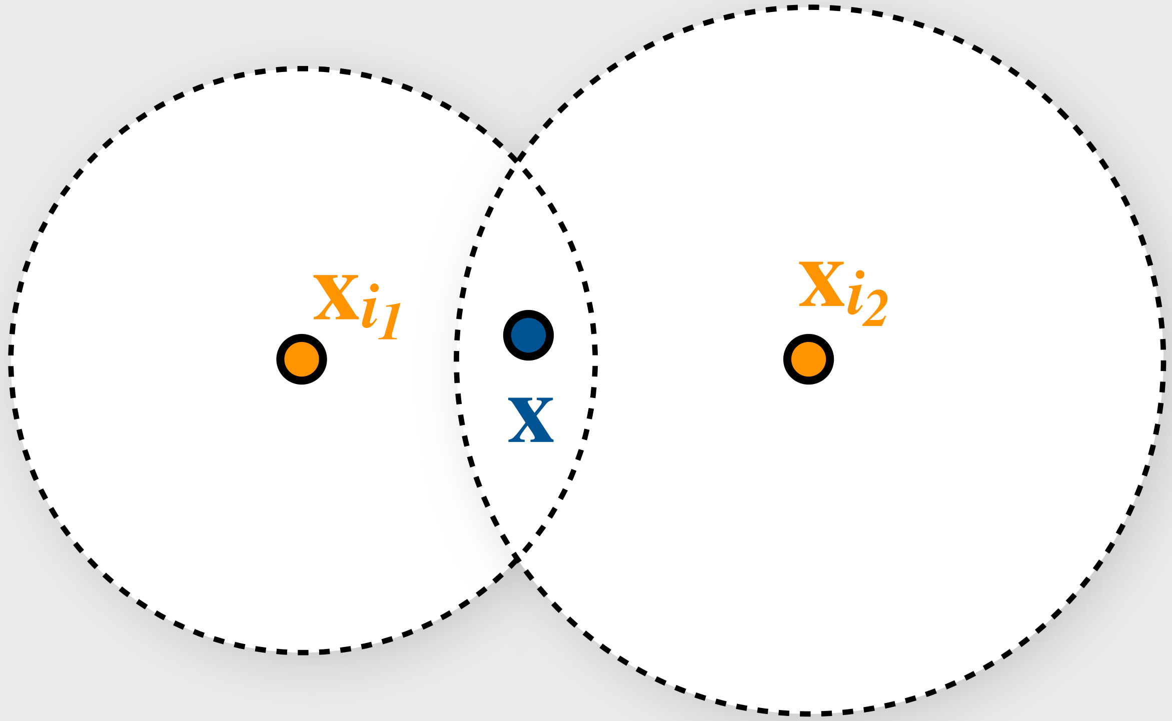


Interpolating with gradients

$$E(\mathbf{x},\vec{\mathbf{n}}) \approx \frac{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})\left(E_i + (\vec{n}_i \times \vec{\mathbf{n}})\cdot(\vec{\nabla}_r E_i) + (\mathbf{x}-\mathbf{x}_i)\cdot(\vec{\nabla}_t E_i)\right)}{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})}$$

Extrapolate along gradients

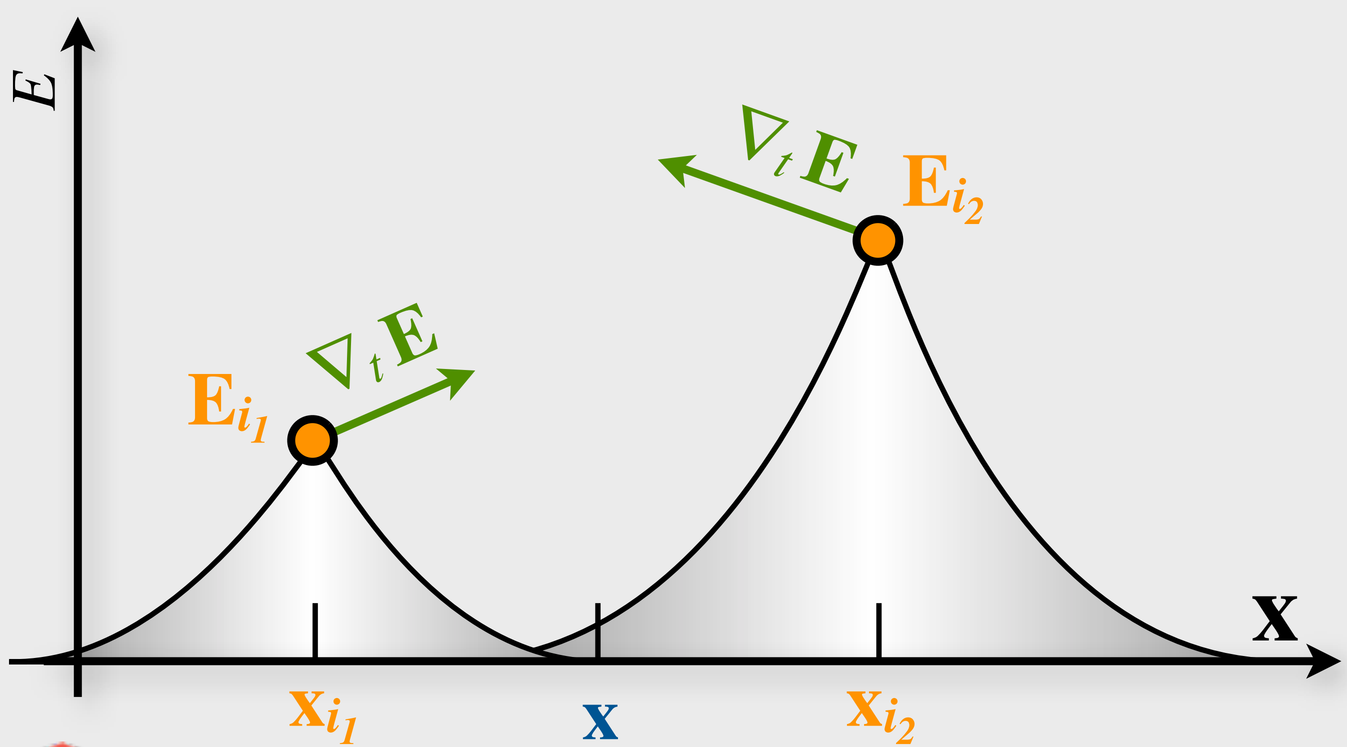


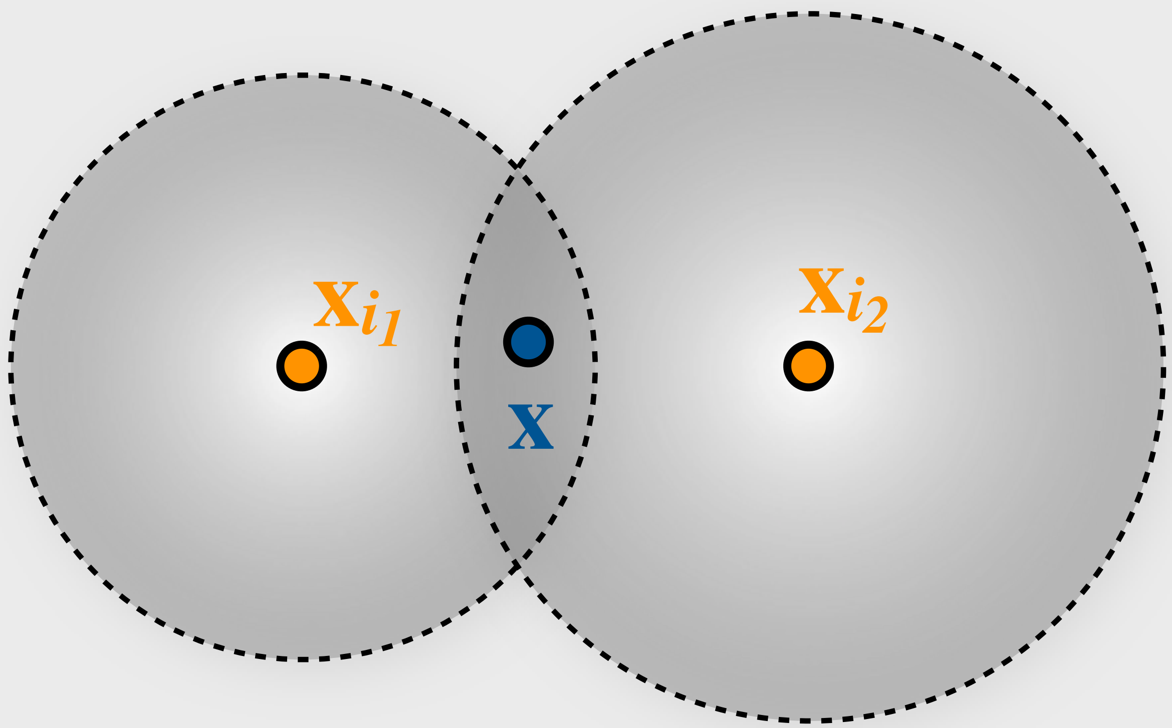


Interpolating with gradients

$$E(\mathbf{x},\vec{\mathbf{n}}) \approx \frac{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})\left(E_i+(\vec{n}_i\times\vec{\mathbf{n}})\cdot(\vec{\nabla}_r E_i)+(\mathbf{x}-\mathbf{x}_i)\cdot(\vec{\nabla}_t E_i)\right)}{\sum_{i\in S} \mathrm{w}_i(\mathbf{x},\vec{\mathbf{n}})}$$

Weight contributions

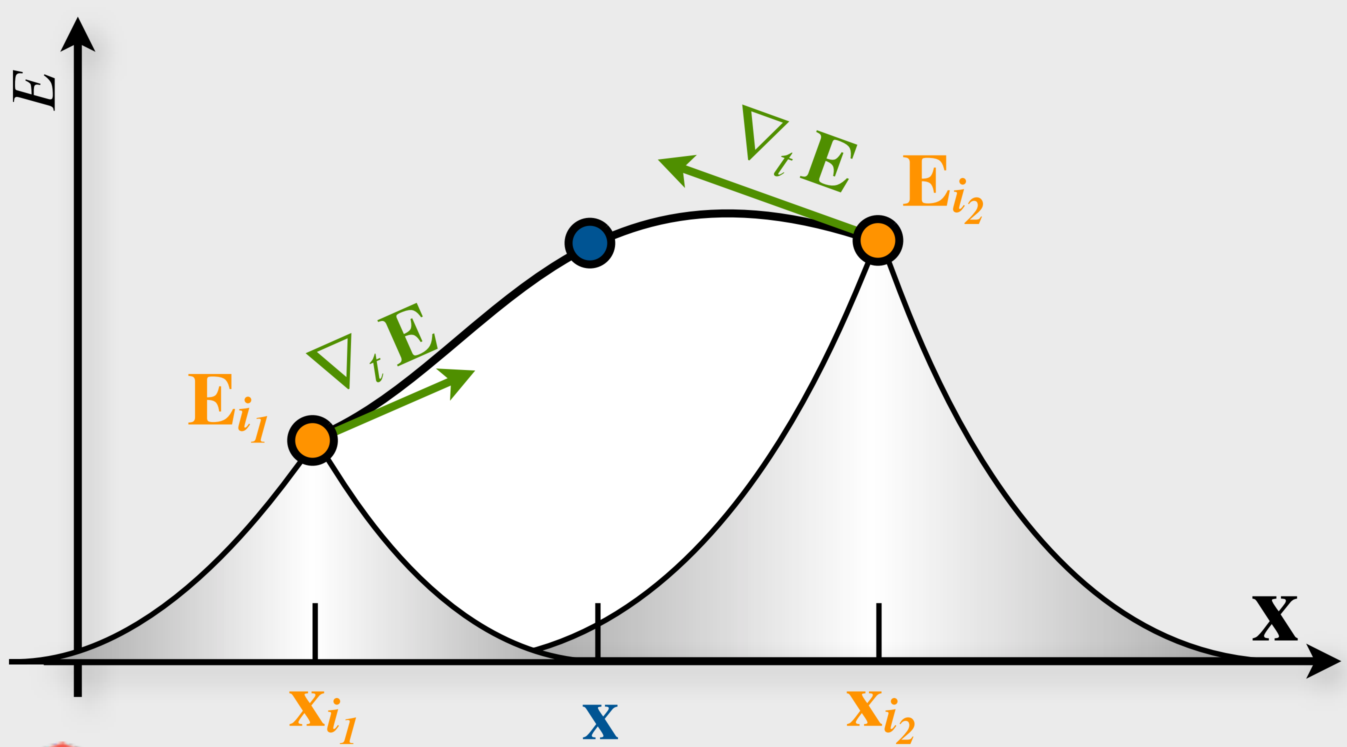


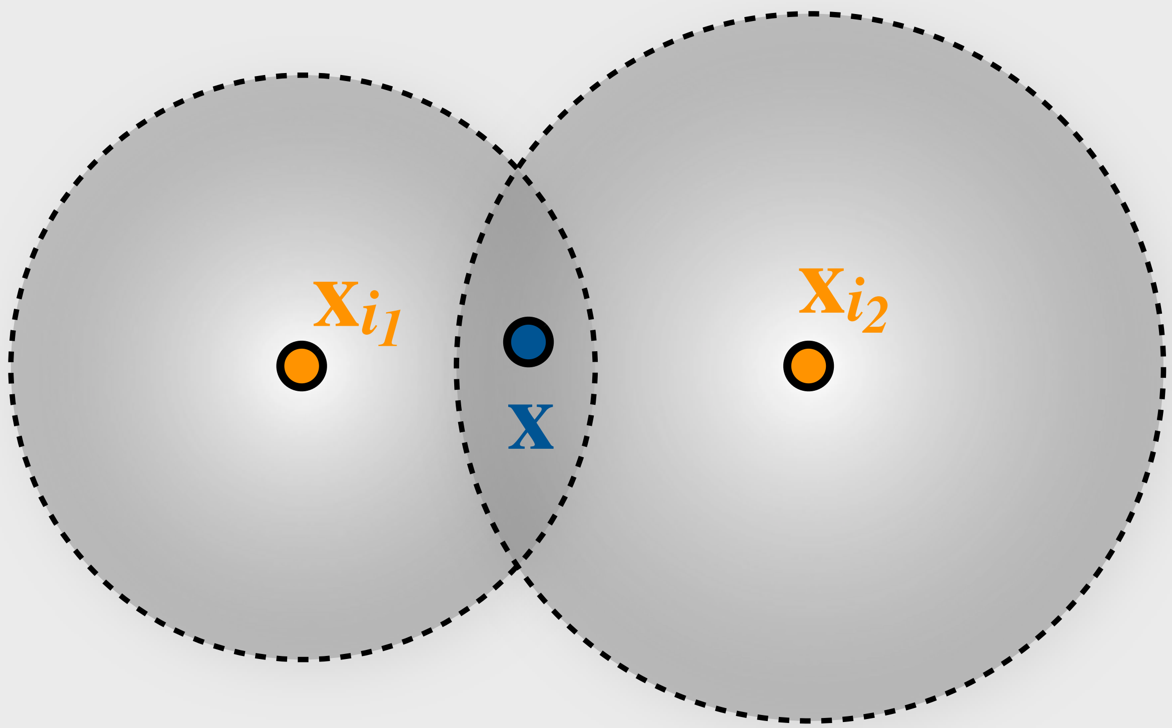


Interpolating with gradients

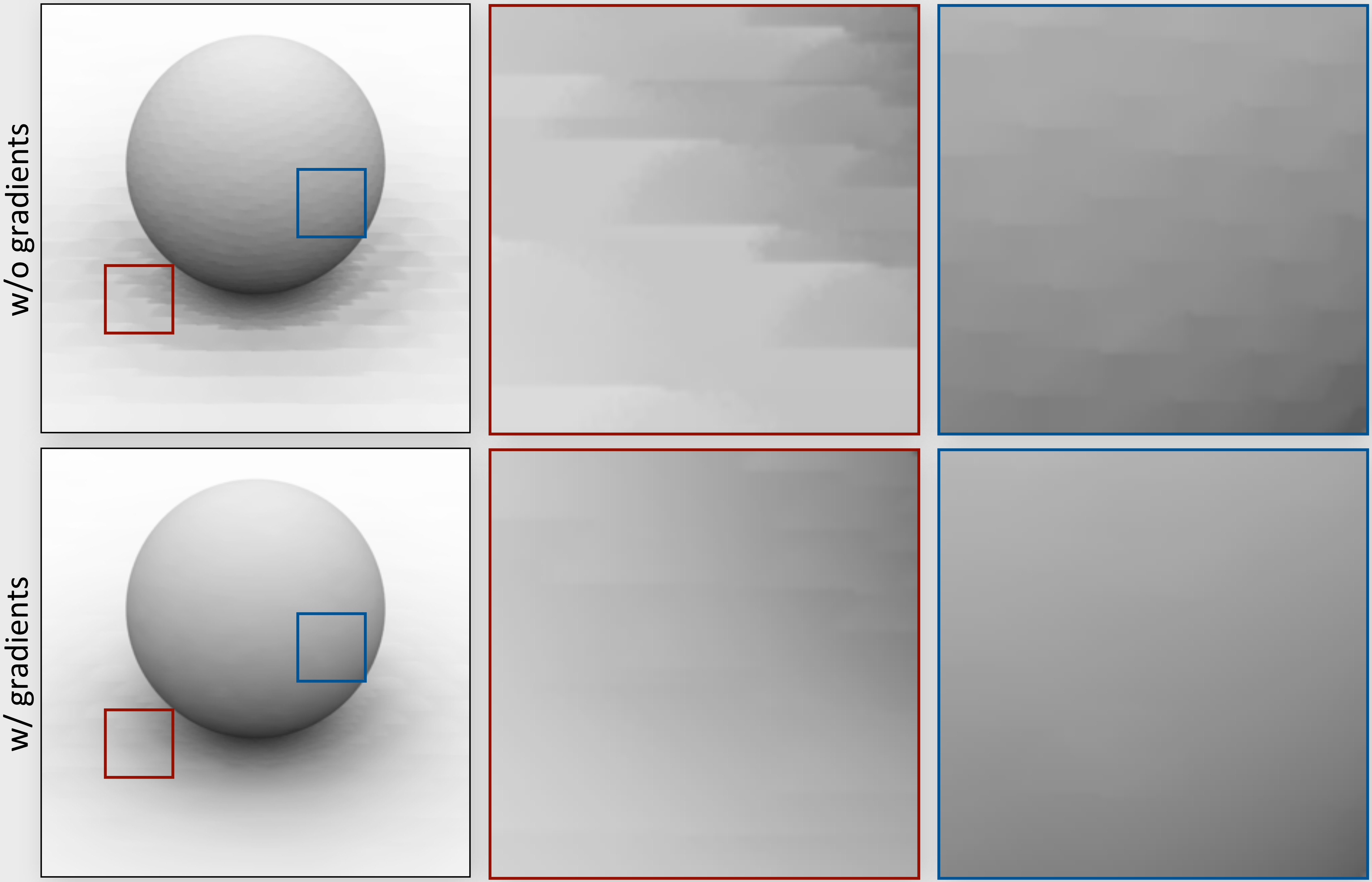
$$E(\mathbf{x}, \vec{\mathbf{n}}) \approx \frac{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})\Big(E_i + (\vec{n}_i \times \vec{\mathbf{n}}) \cdot (\vec{\nabla}_r E_i) + (\mathbf{x} - \mathbf{x}_i) \cdot (\vec{\nabla}_t E_i)\Big)}{\sum_{i \in S} \mathrm{w}_i(\mathbf{x}, \vec{\mathbf{n}})}$$

Sum extrapolated values





Irradiance Gradients



Beyond Lambertian surfaces

- Generalization to glossy surfaces

[Křivánek et al. 2005a,2005b]

Beyond Lambertian surfaces

- Generalization to glossy surfaces
- **Radiance Caching** [Křivánek et al. 2005a,2005b]

[Křivánek et al. 2005a,2005b]

Beyond Lambertian surfaces

- Generalization to glossy surfaces
- **Radiance Caching** [Křivánek et al. 2005a,2005b]
  - Can no longer cache just the irradiance value

[Křivánek et al. 2005a,2005b]

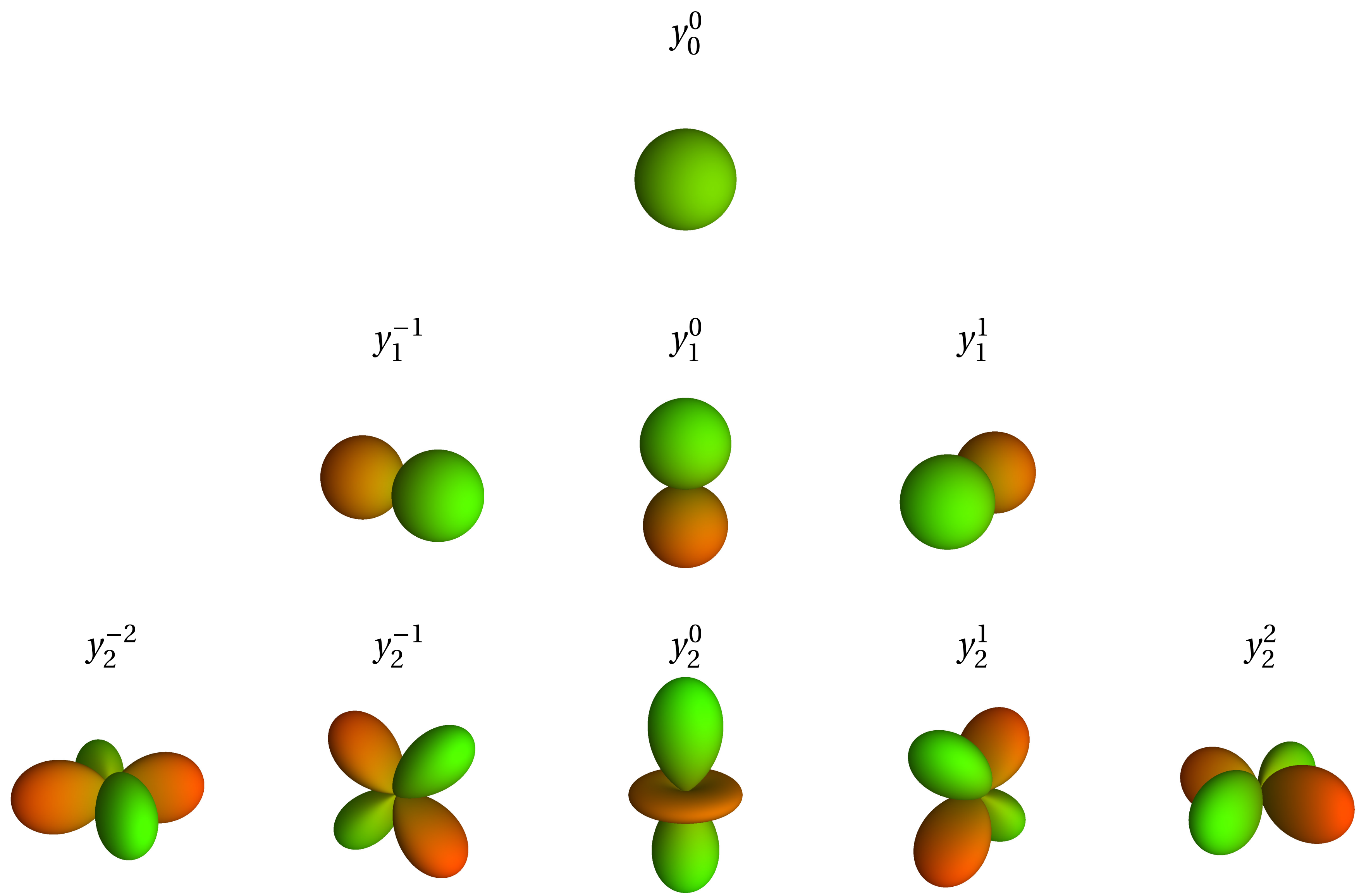
Beyond Lambertian surfaces

- Generalization to glossy surfaces
- **Radiance Caching** [Křivánek et al. 2005a,2005b]
  - Can no longer cache just the irradiance value
  - Cache full hemispherical *radiance* field at sparse locations

[Křivánek et al. 2005a,2005b]

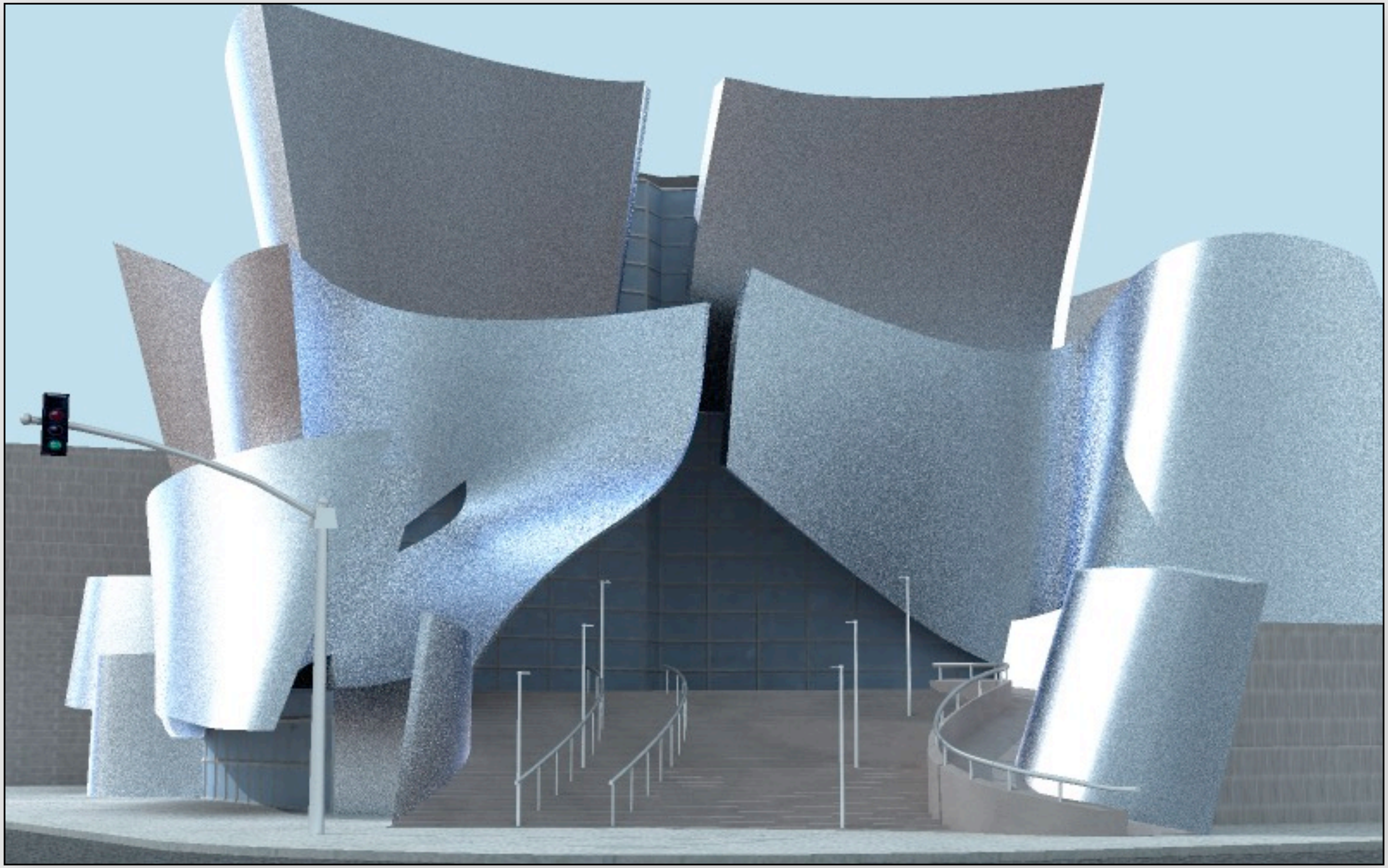
Radiance Storage

- Use spherical or hemispherical harmonics
- Approximates smooth functions with a few coefficients



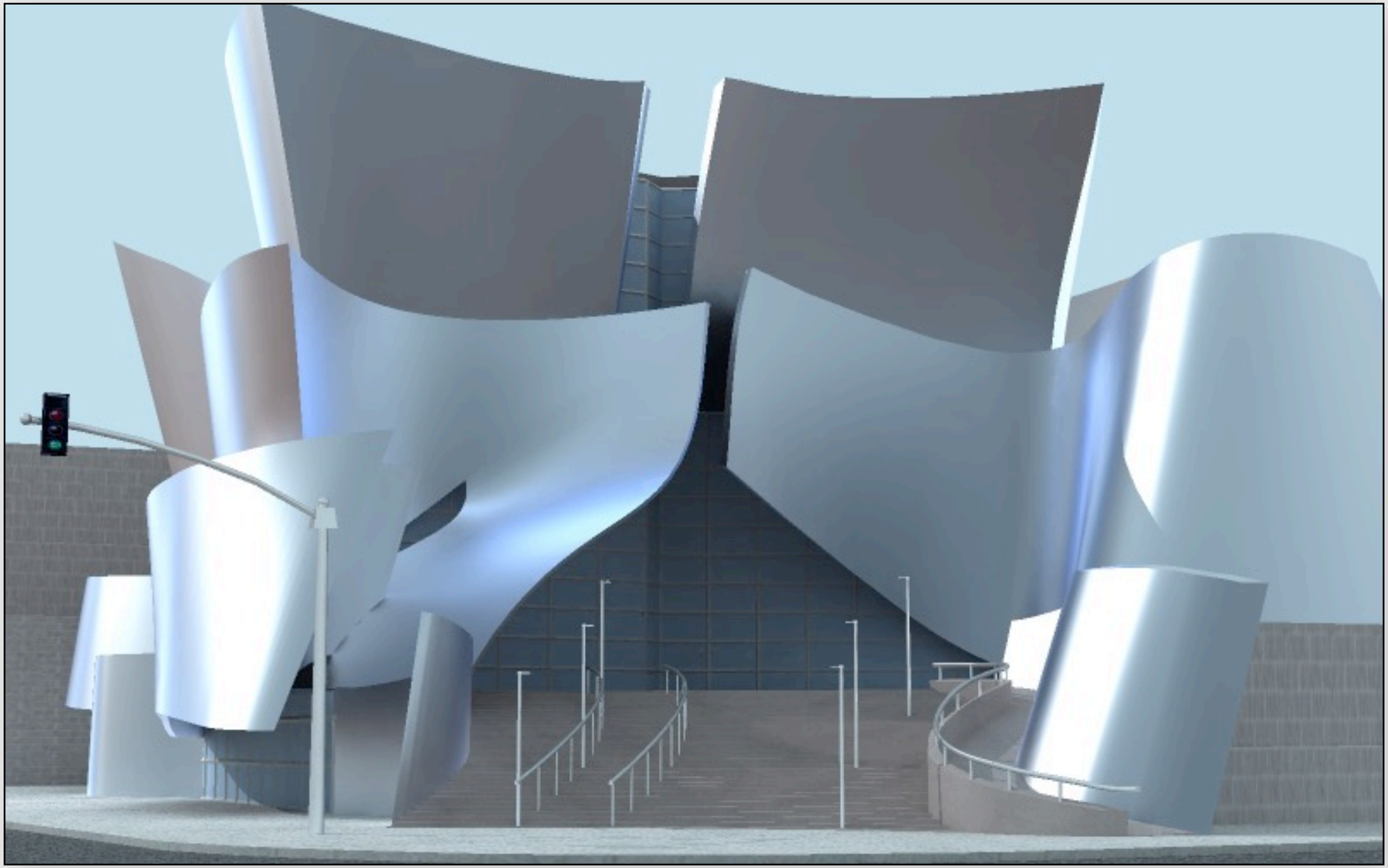
[Křivánek et al. 2005a,2005b]

Monte Carlo



[Křivánek et al. 2005a,2005b]

Radiance Caching

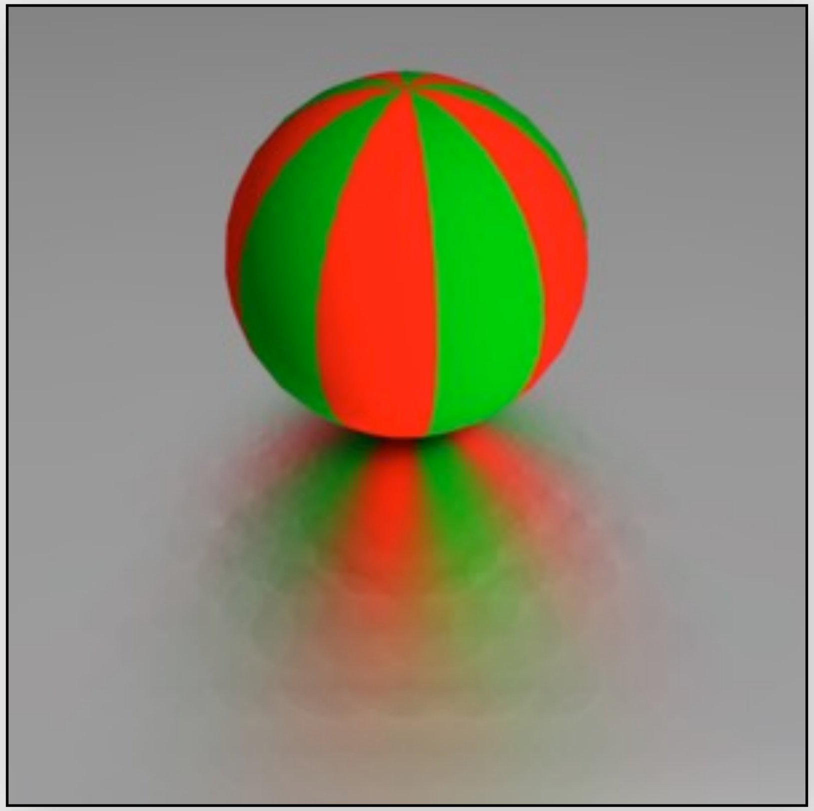


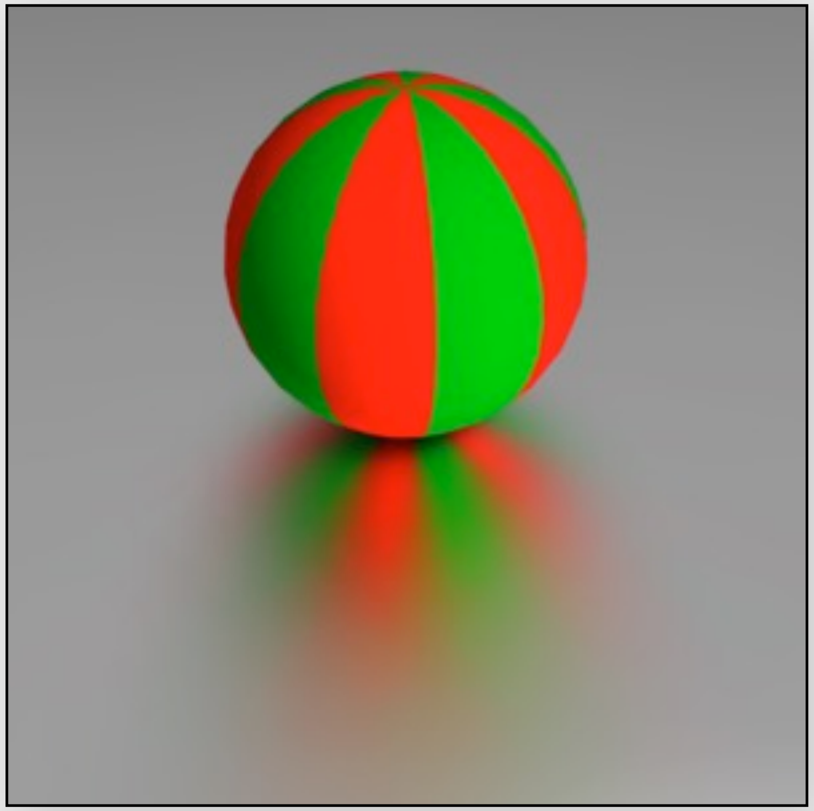
[Křivánek et al. 2005a,2005b]

Radiance Gradients

- Improve interpolation quality by storing gradient of incoming radiance field

[Křivánek et al. 2005a,2005b]





occlusion-unaware

occlusion-aware

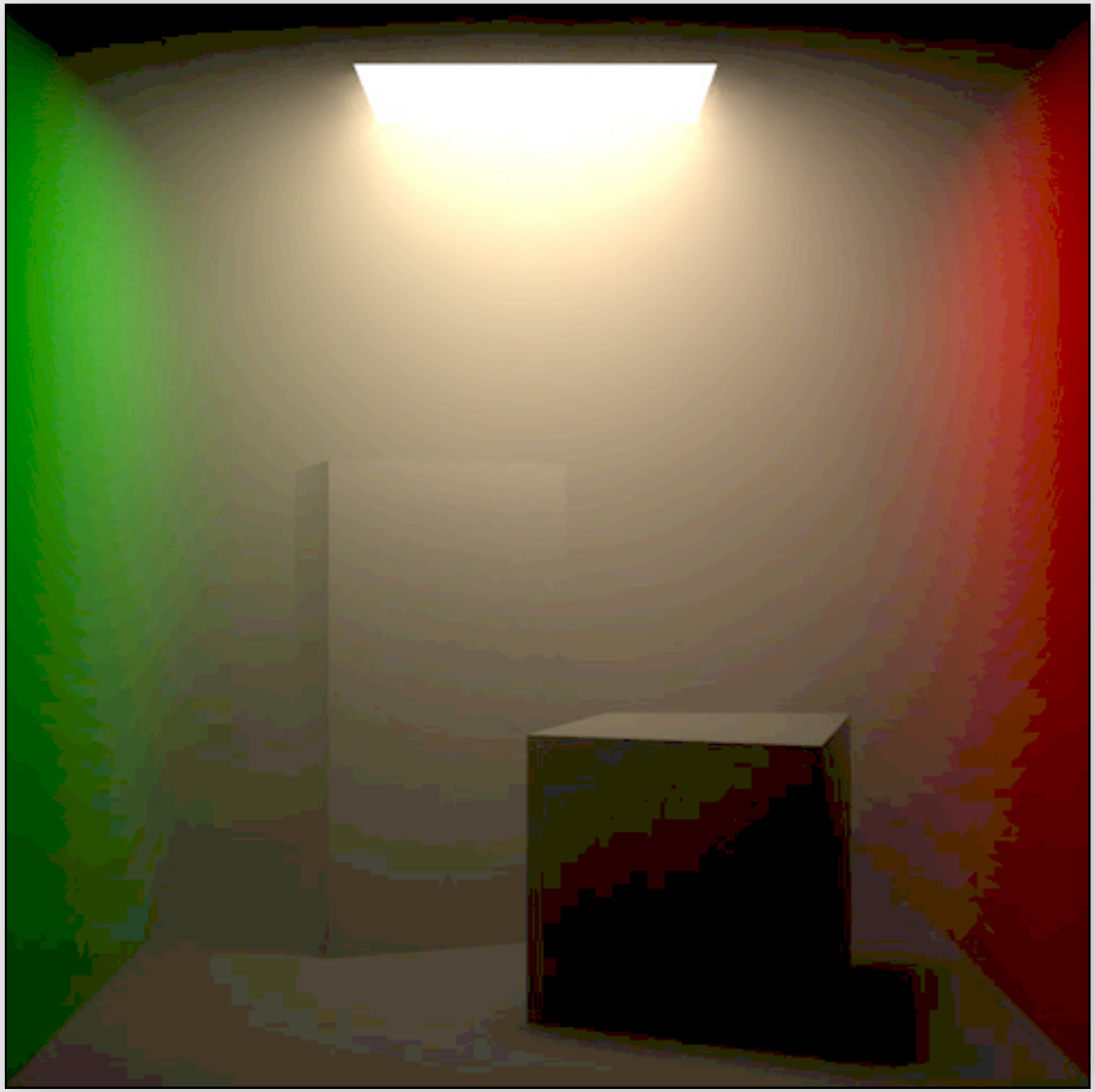
[Křivánek et al. 2005a,2005b]

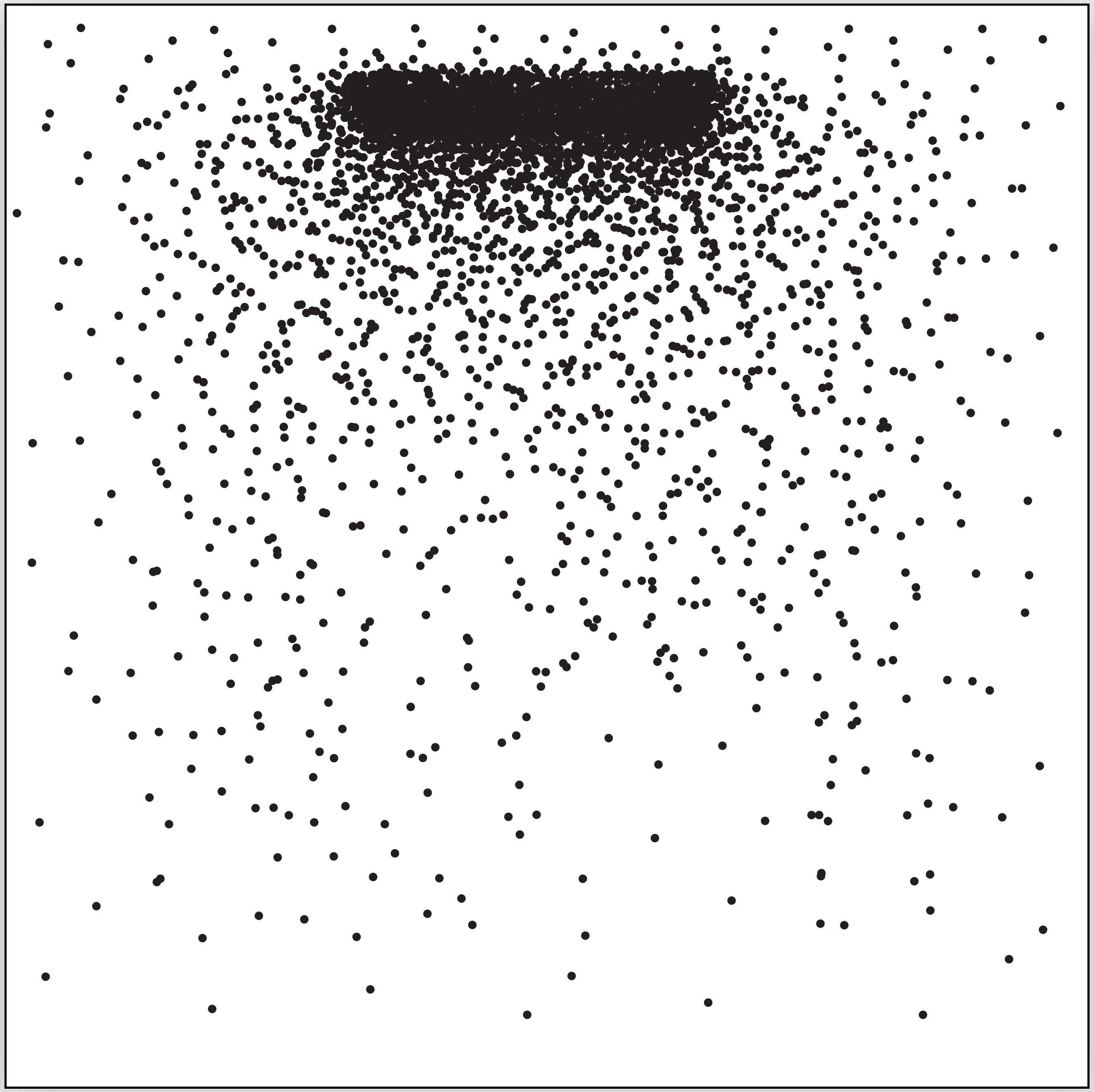
Beyond surfaces

- Generalizations to participating media
- **Volumetric Radiance Caching** [Jarosz et al. 2008a, 2008b]
    - Cache radiance and gradients within volume

[Jarosz et al. 2008a, 2008b]

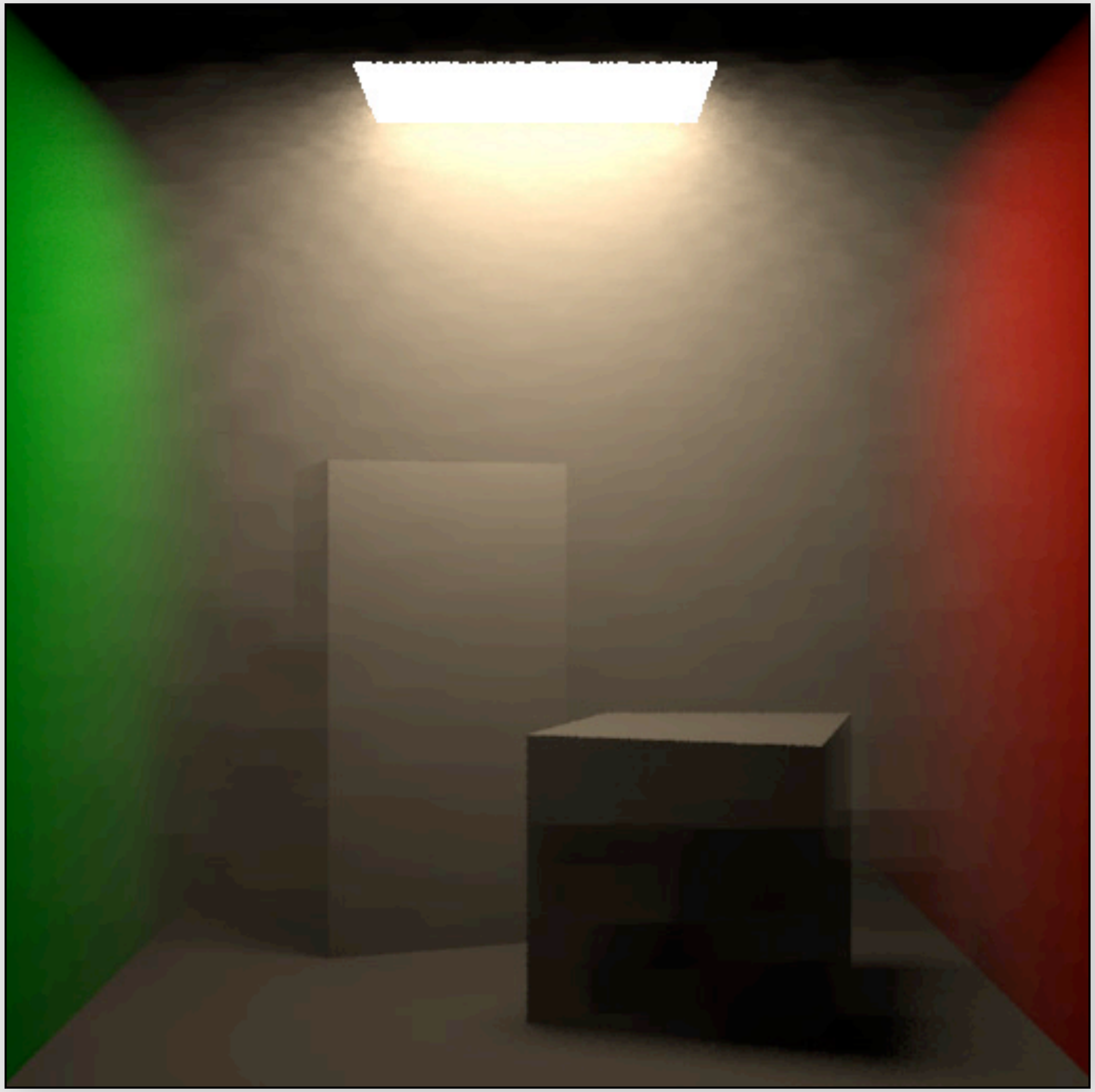
Valid Radius





[Jarosz et al. 2008a, 2008b]

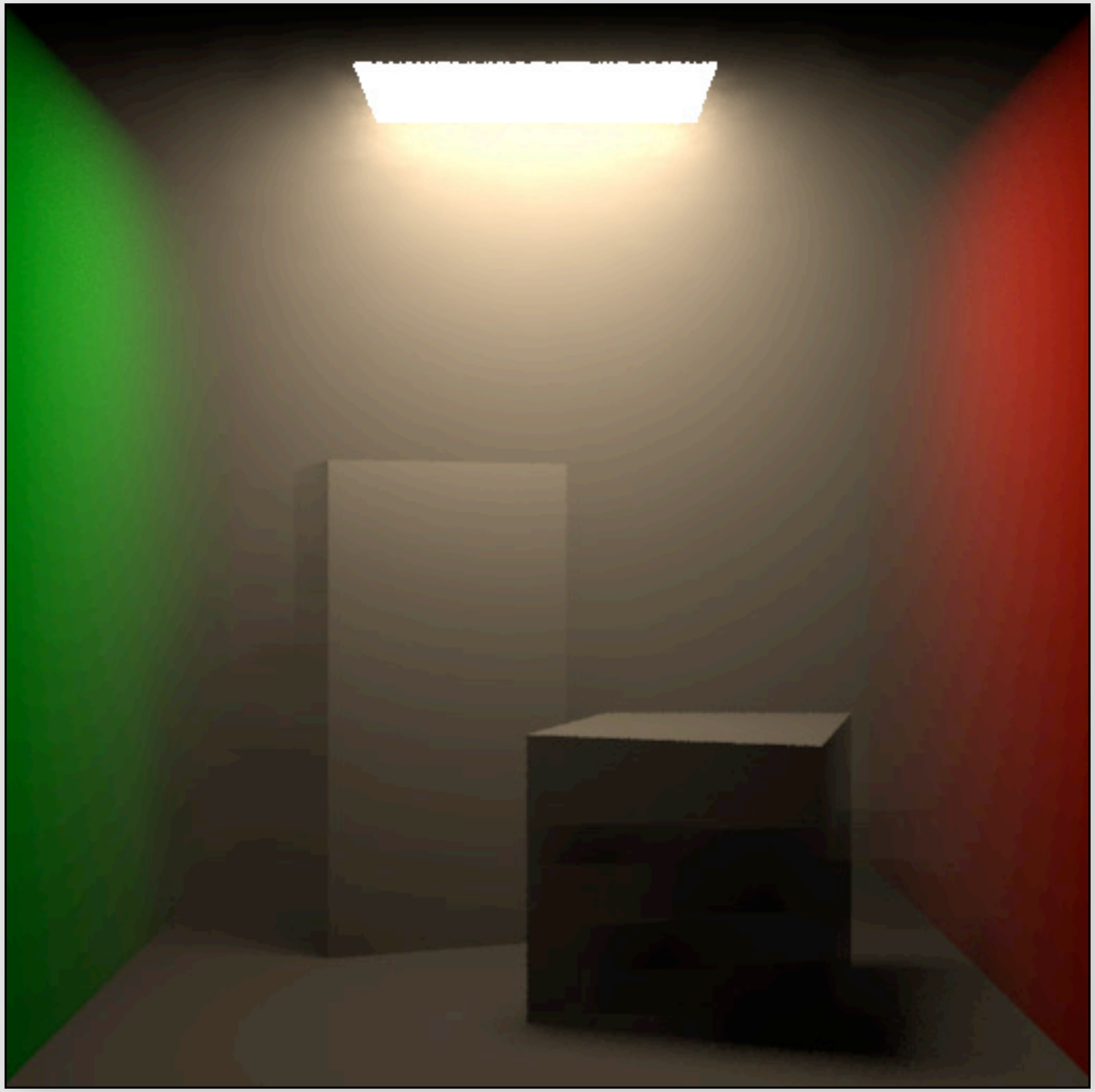
Gradients



no gradients

[Jarosz et al. 2008a]

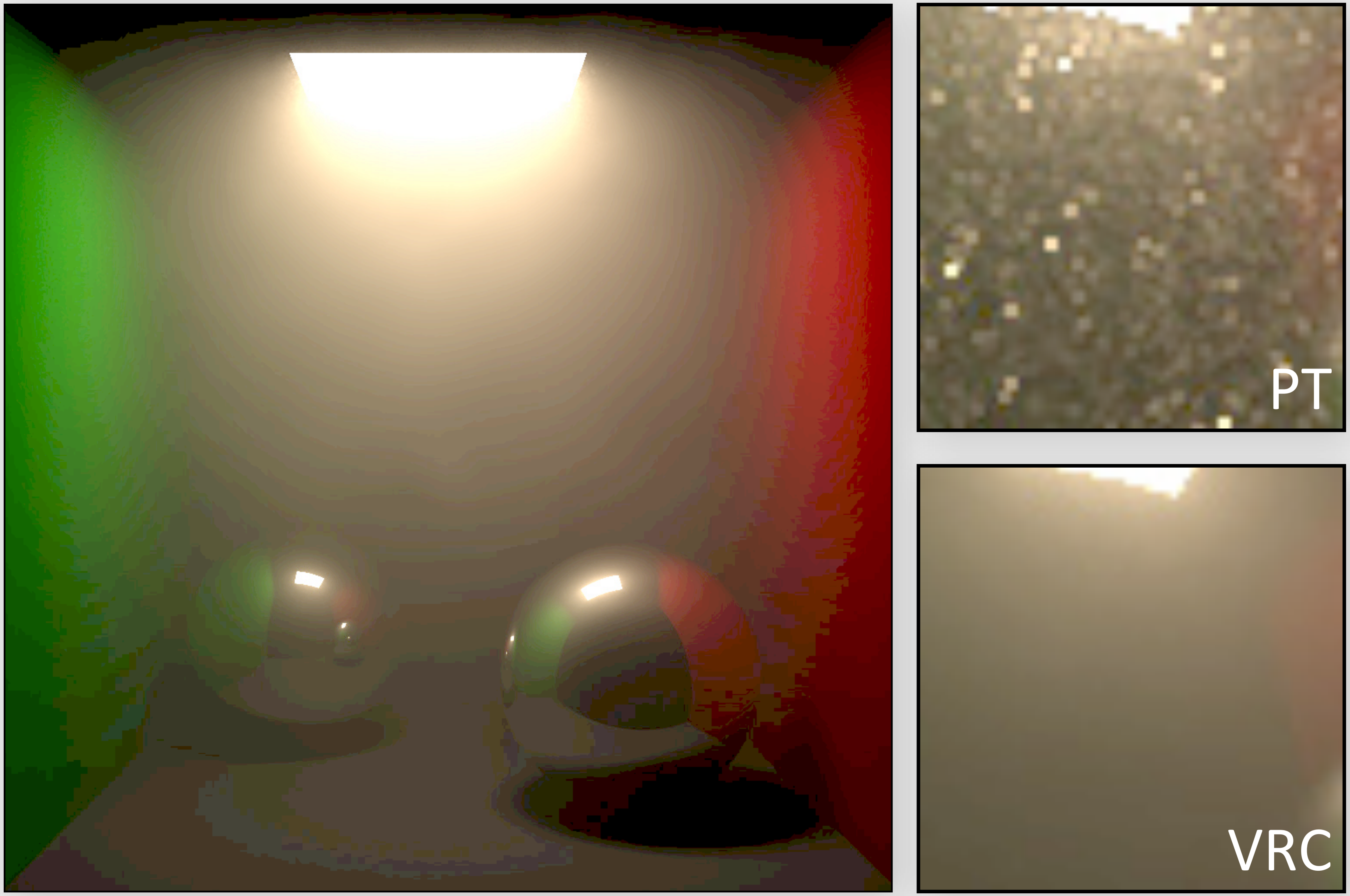
Gradients



with gradients

[Jarosz et al. 2008a]

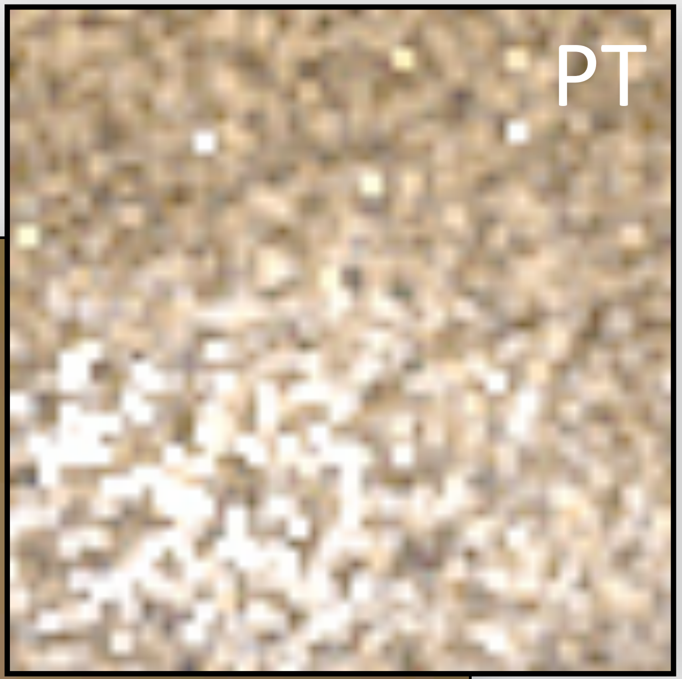
Results



[Jarosz et al. 2008a]

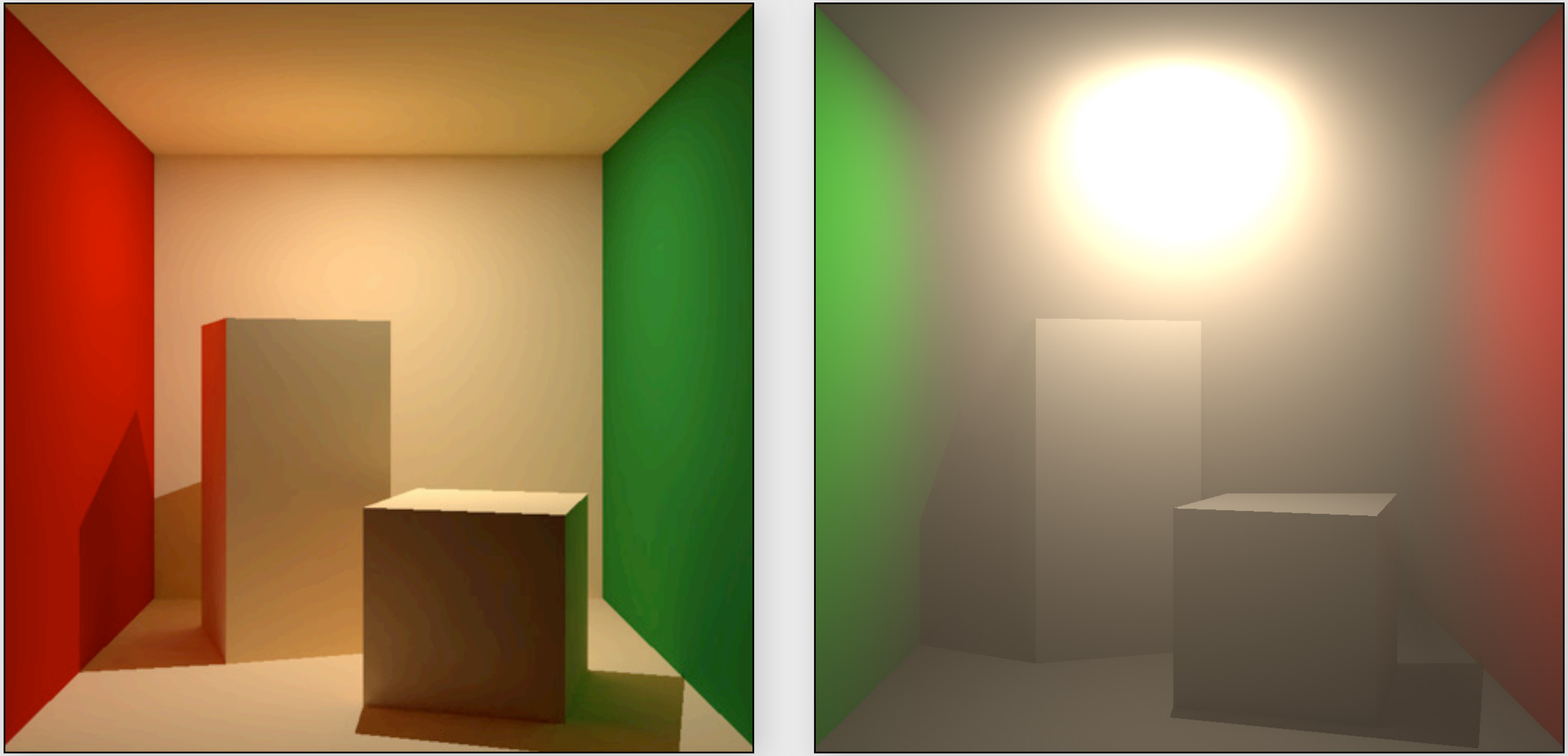
Thursday, July 16, 15

Results



[Jarosz et al. 2008a]

Participating media

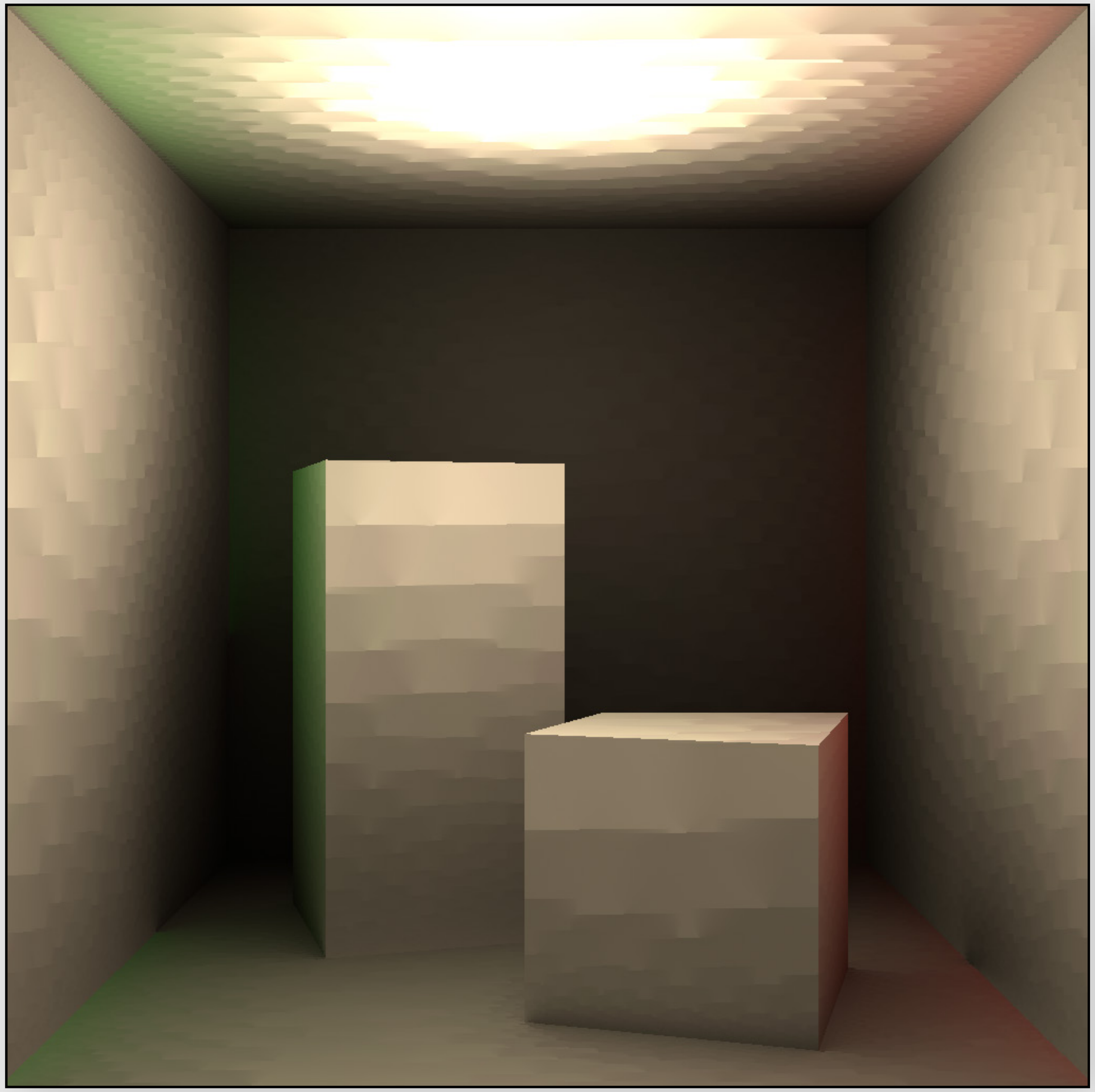


no media | with media

[Jarosz et al. 2008b]

Surfaces in participating media

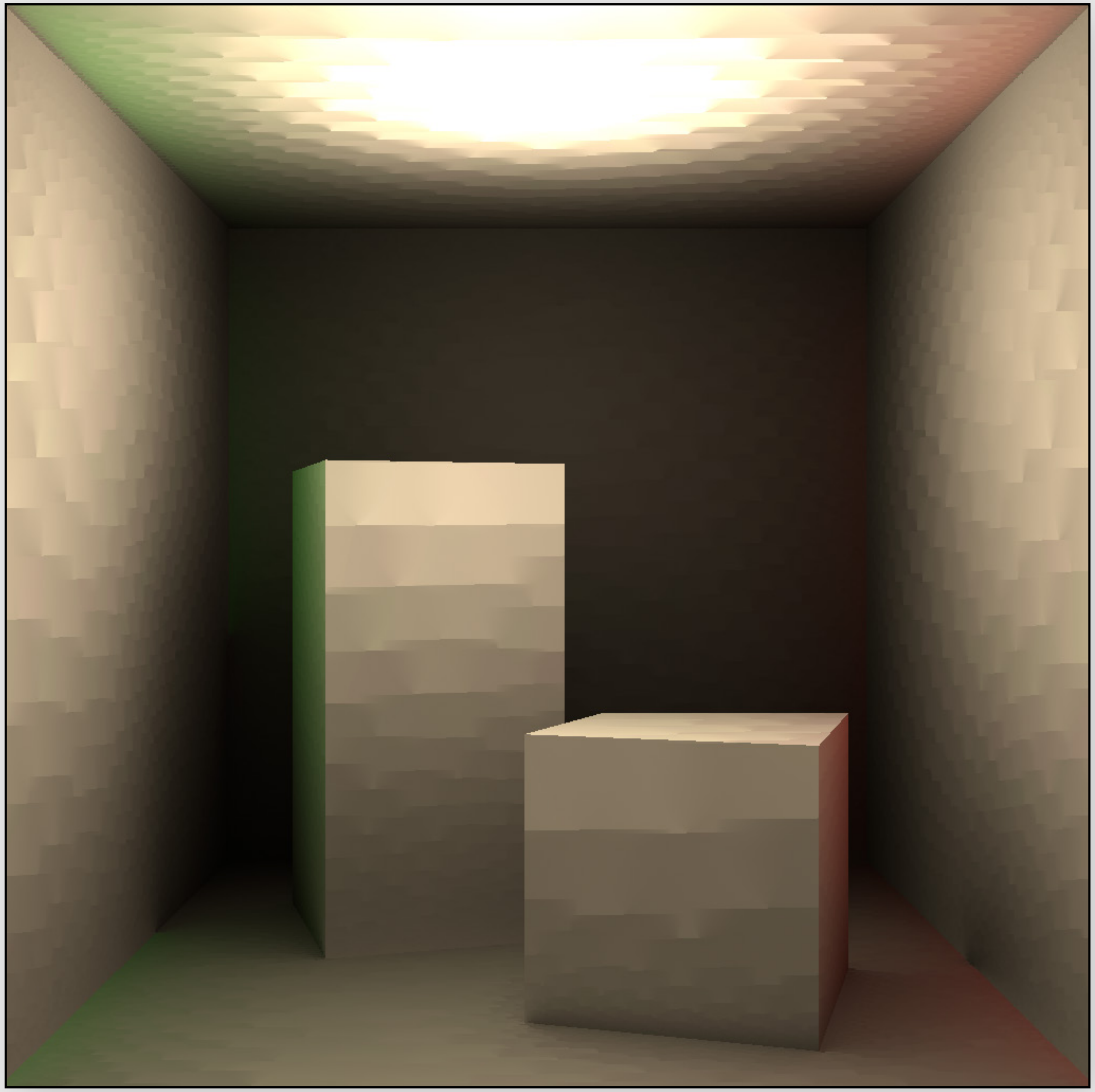
no media
(indirect irradiance)



with media
(indirect irradiance)

[Jarosz et al. 2008b]

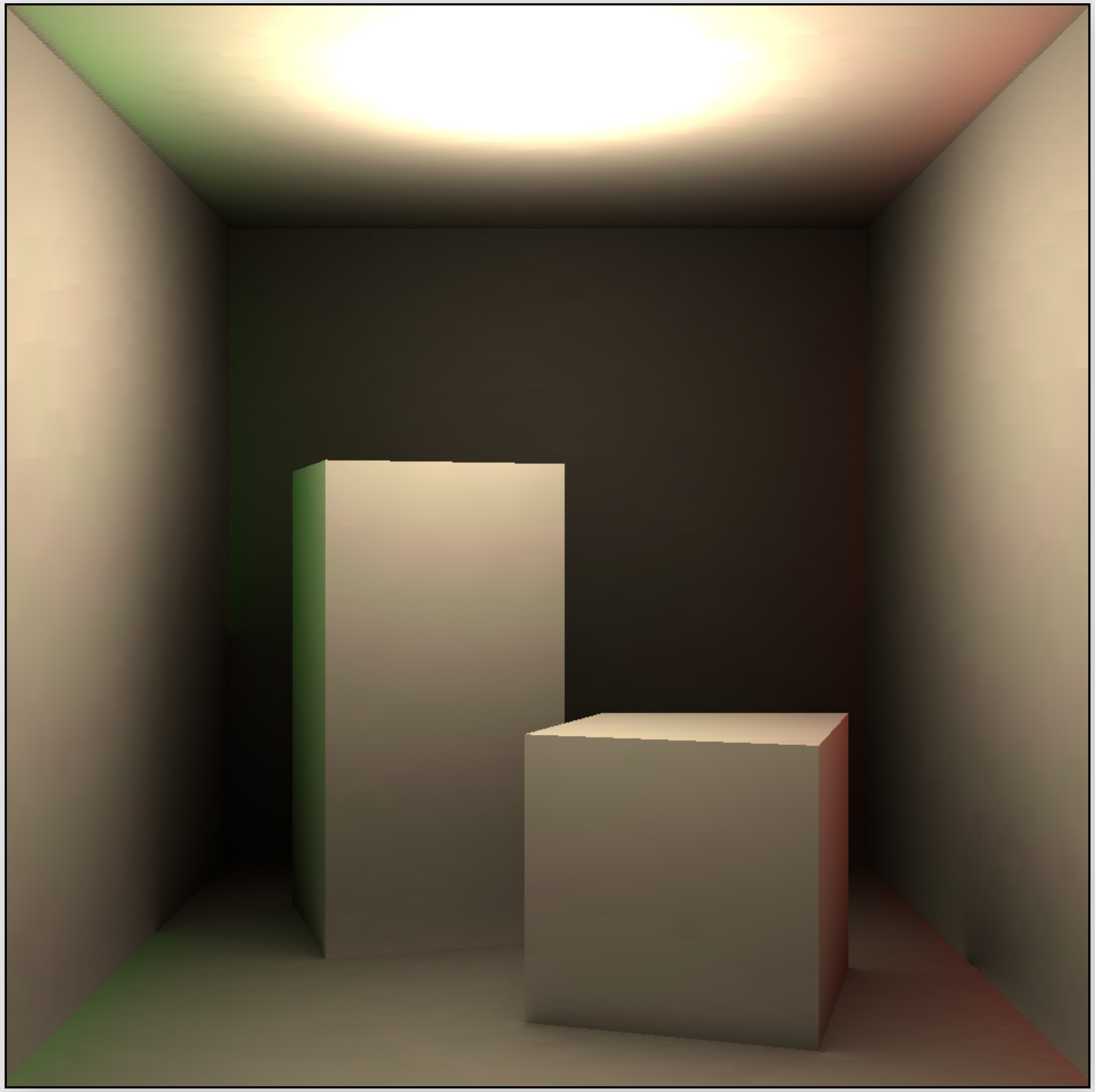
Surfaces in participating media



Occlusion aware, but media unaware gradients
[Ward and Heckbert 92]

[Jarosz et al. 2008b]

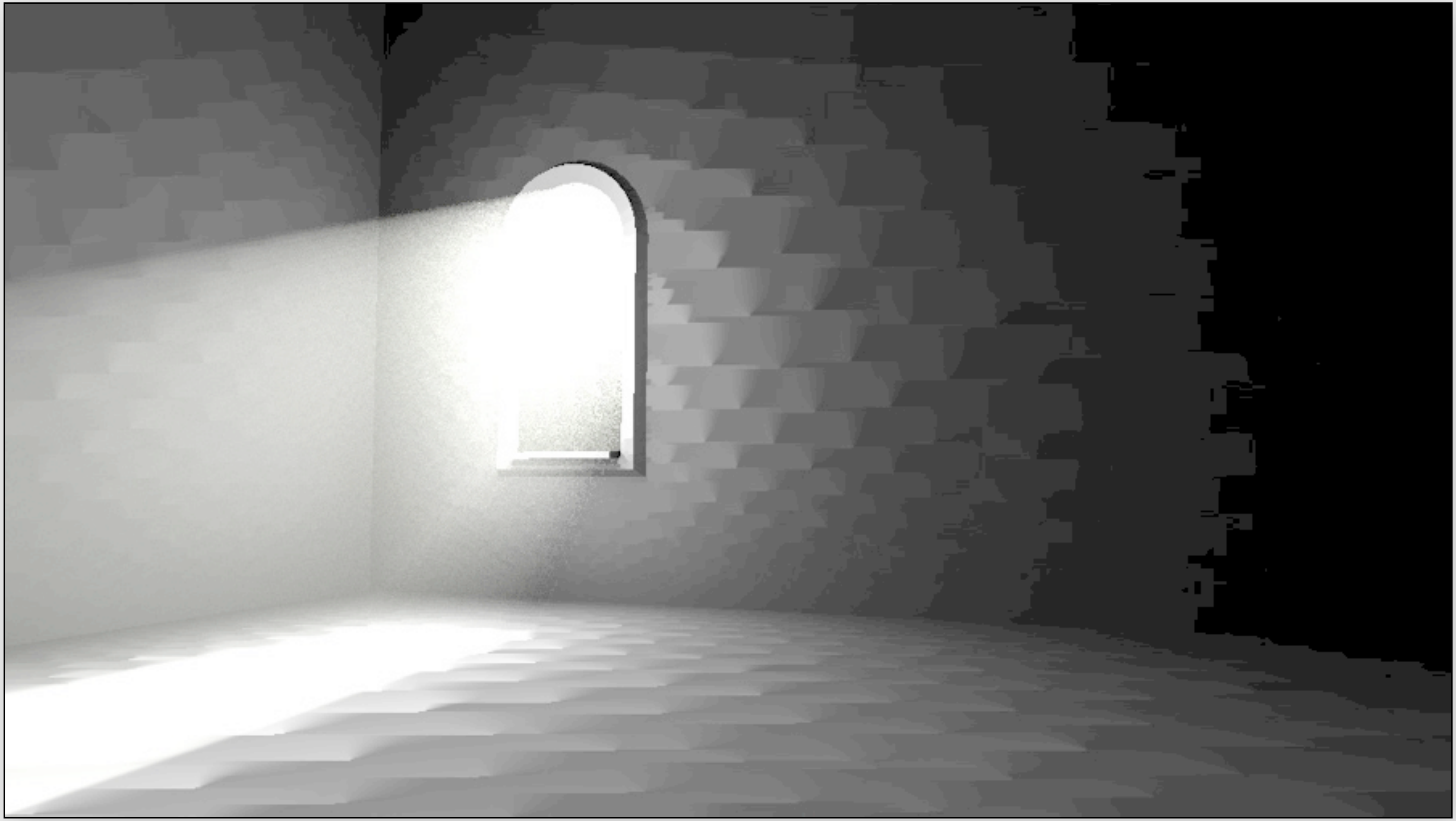
Surfaces in participating media



Occlusion and media aware gradients
[Jarosz et al. 2008b]

[Jarosz et al. 2008b]

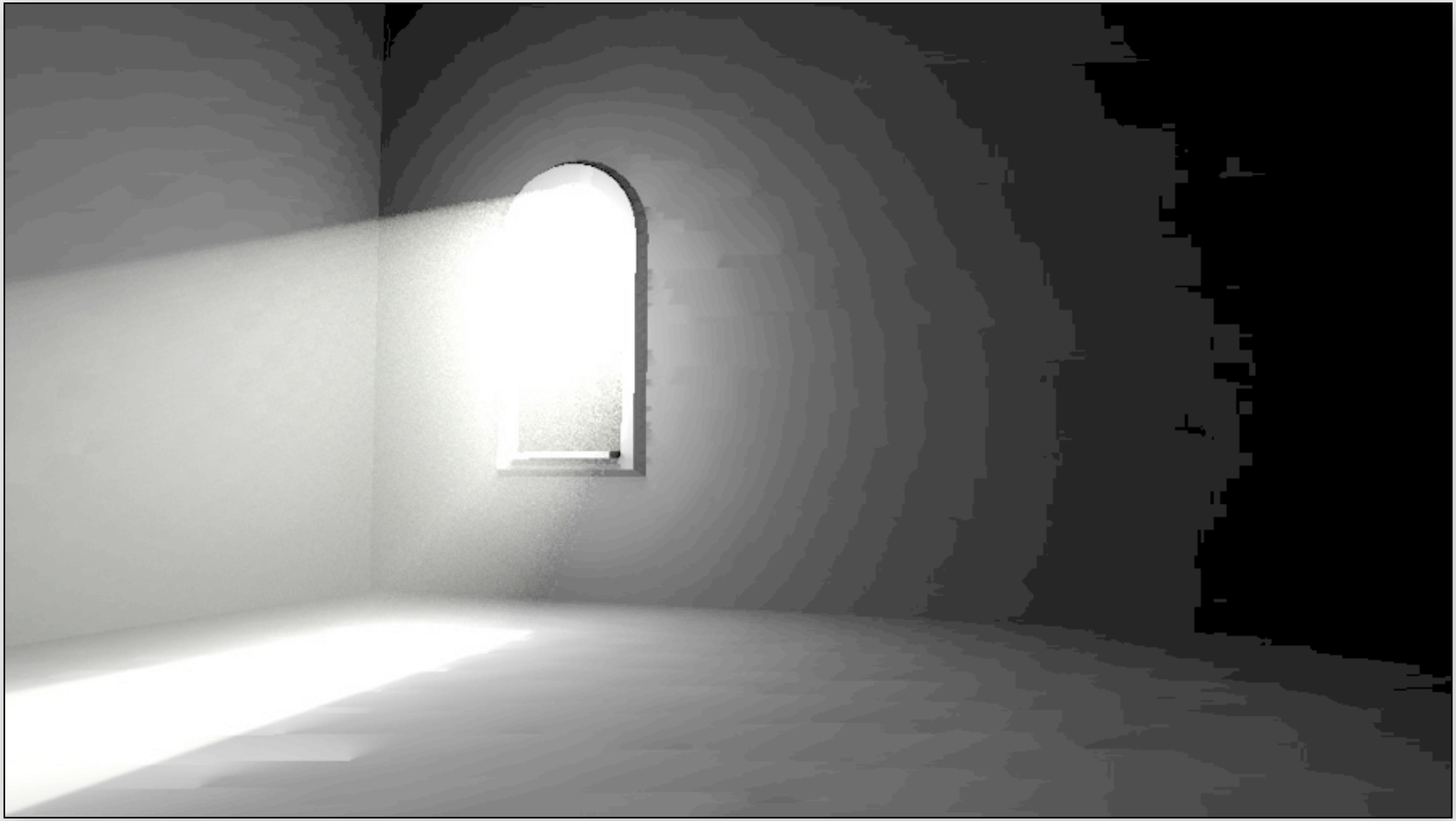
Sun beam through window



Gradients by [Ward and Heckbert 92]

[Jarosz et al. 2008b]

Sun beam through window



Gradients by [Jarosz et al. 2008b]

Higher-order derivatives

- Exploit higher-order derivatives for better error control
  - [Jarosz et al. 2012] - Hessians (occlusion-unaware)
  - [Schwarzhaupt et al. 2012] - occlusion-aware Hessians & practical details

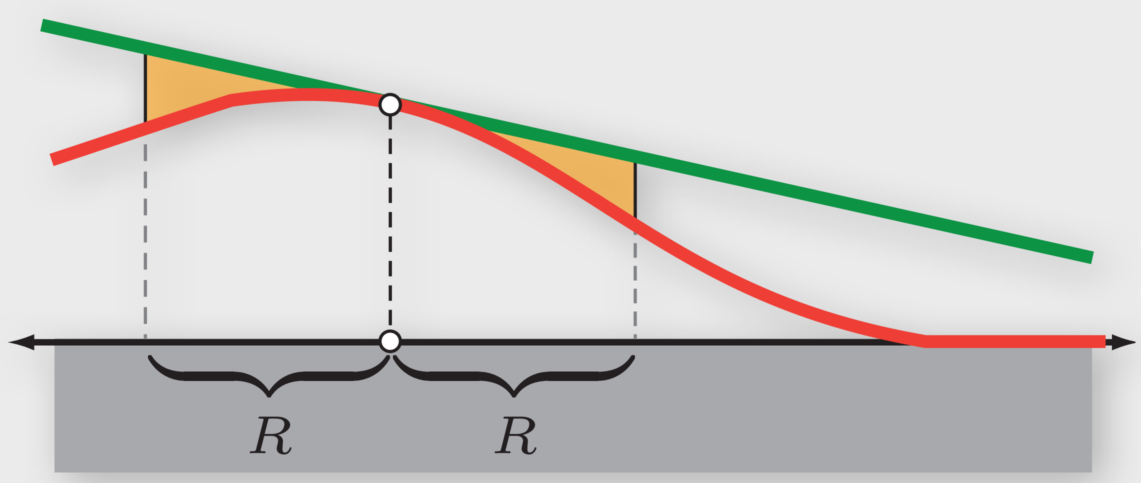
Split-Sphere Heuristic

- Basis for most irradiance caching algorithms for 20+ years
- Fix-ups to original metric lead to many parameters
    - error threshold
    - min/max screen-space radii
    - min/max world-space radii
    - gradient clamping
    - ...
- Hard to control!

[Jarosz et al. 2012]

Better Error Control

- **total error** ϵ^t = integrated difference between extrapolated and correct irradiance



Better Error Control

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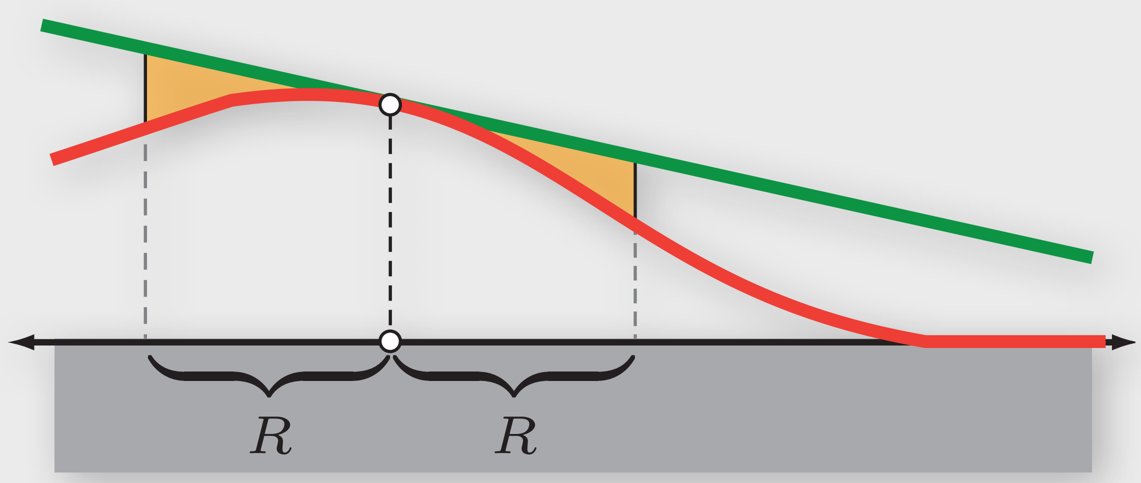
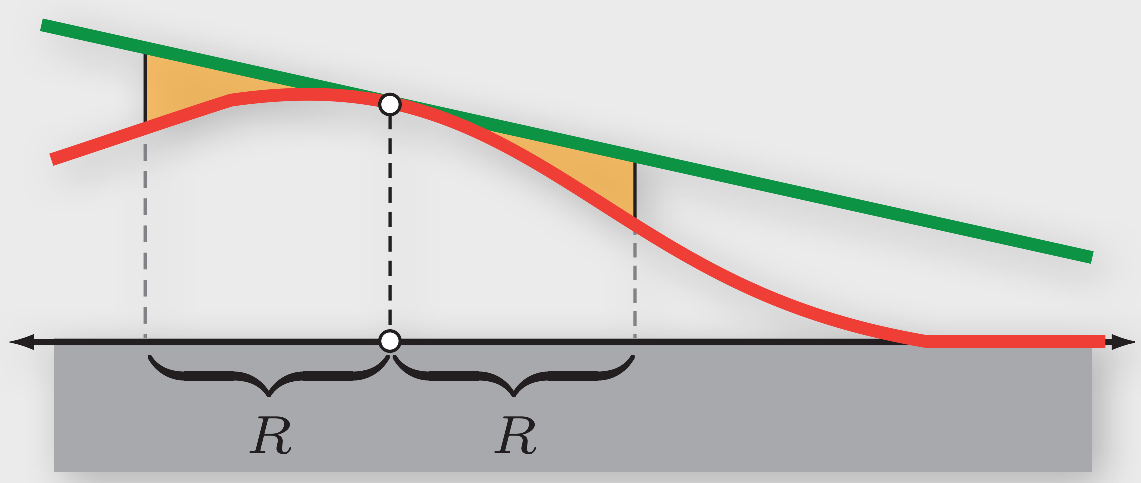


$$\epsilon^t = \int_{-R_i}^{R_i} \left| E(\mathbf{x}_i + x) - E'(\mathbf{x}_i + x) \right| dx$$

Better Error Control

- *E'* is 1st-order Taylor extrapolation

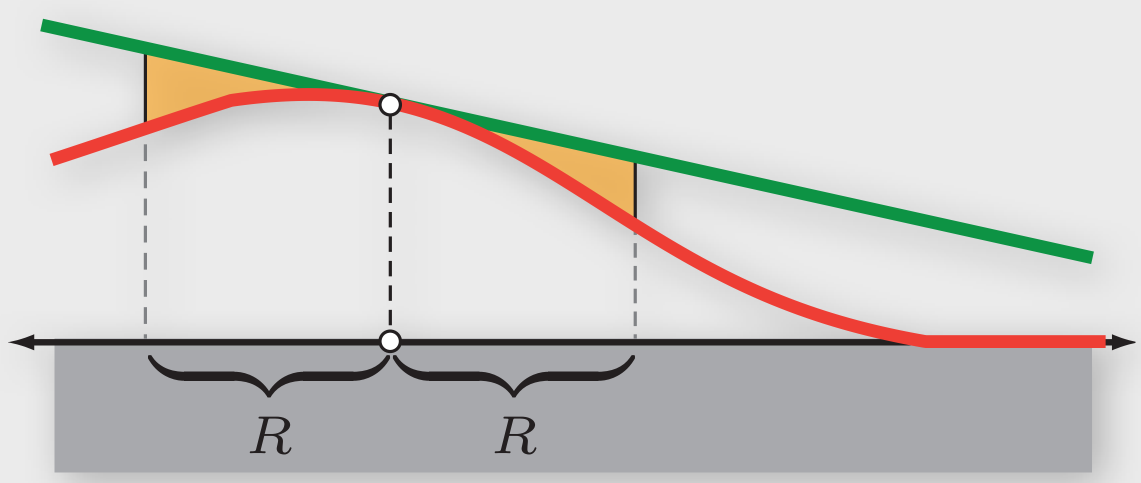


$$\epsilon^t = \int_{-R_i}^{R_i} \left| E(\mathbf{x}_i + x) - E'(\mathbf{x}_i + x) \right| dx$$

[Jarosz et al. 2012]

Better Error Control

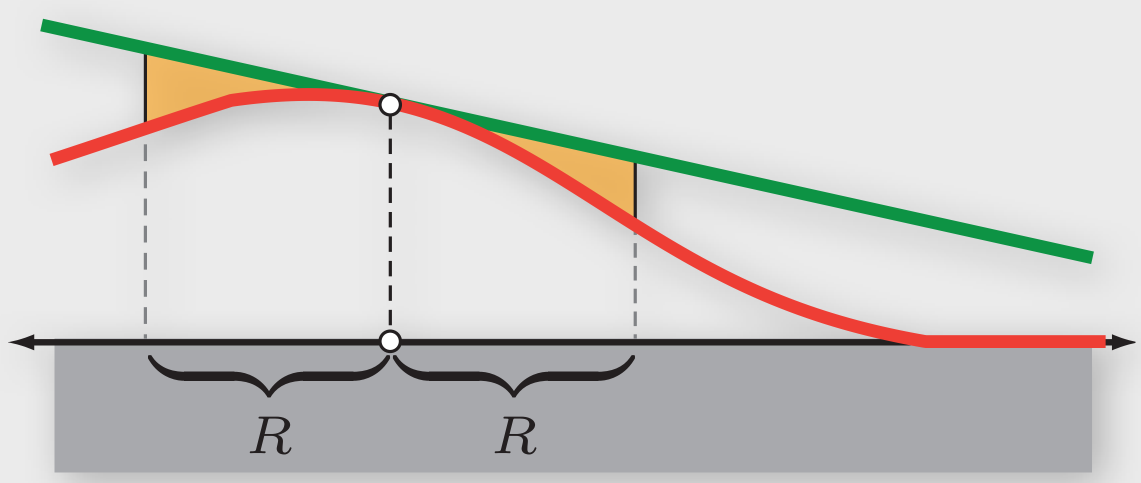
- E' is 1st-order Taylor extrapolation
- E is unknown!



$$\epsilon^t = \int_{-R_i}^{R_i} \left| E(\mathbf{x}_i + x) - E'(\mathbf{x}_i + x) \right| dx$$

Better Error Control

- E' is 1st-order Taylor extrapolation
- E is unknown!



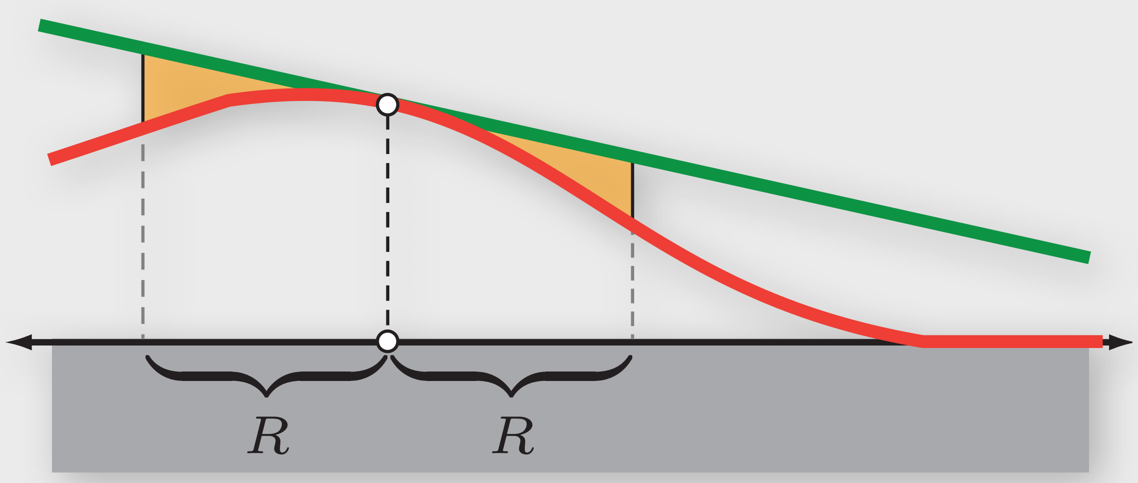
$$\epsilon^t = \int_{-R_i}^{R_i} \left| E(\mathbf{x}_i + x) - E'(\mathbf{x}_i + x) \right| dx$$

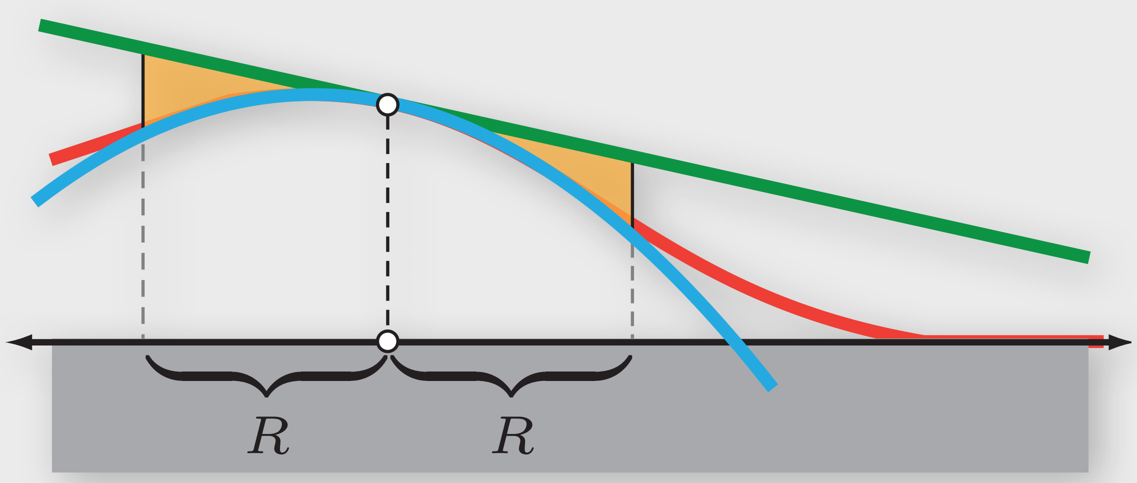
[Jarosz et al. 2012]

Better Error Control

- *E'* is 1st-order Taylor extrapolation
- *E* is unknown!

2nd-order Taylor extrapolation

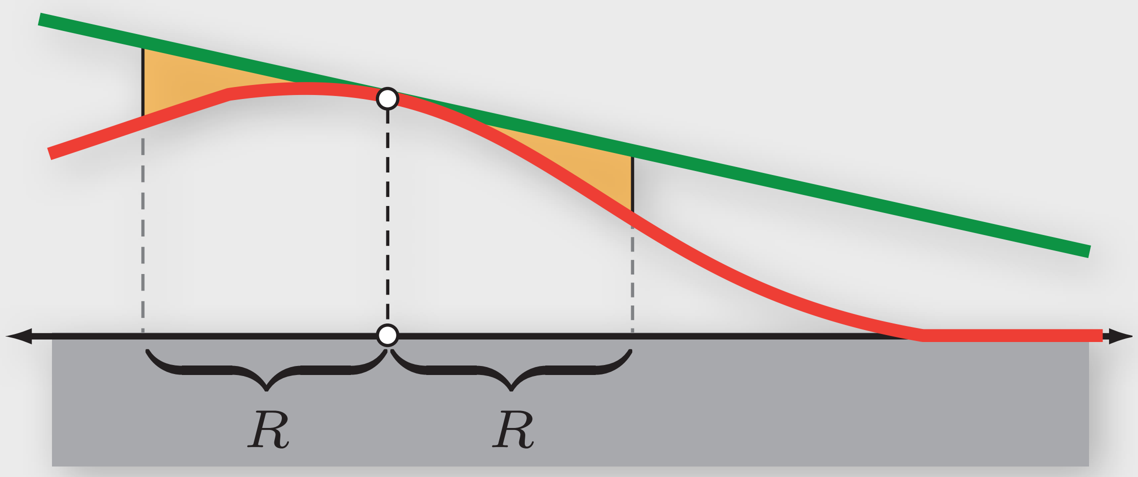


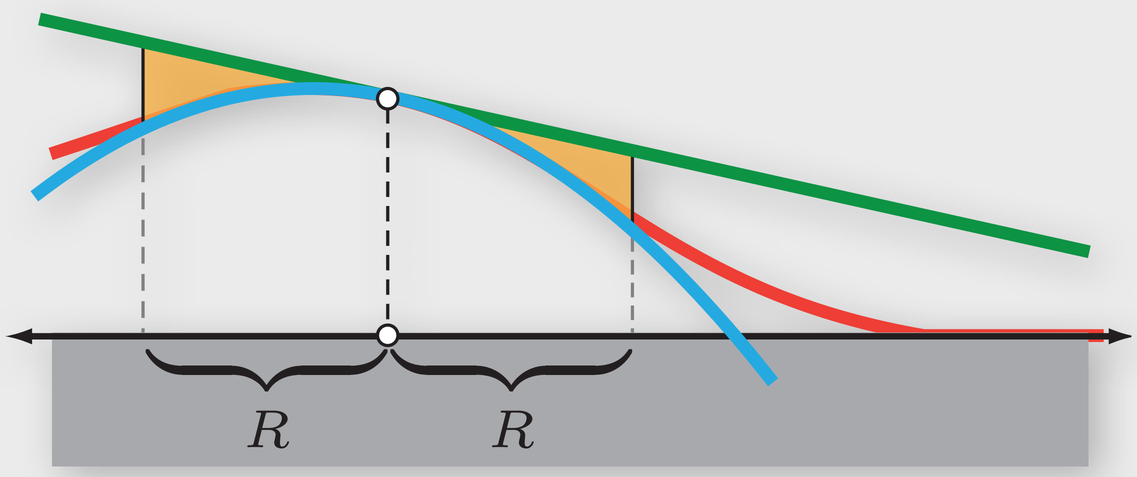


$$\epsilon^t = \int_{-R_i}^{R_i} \left| E(\mathbf{x}_i + x) - E'(\mathbf{x}_i + x) \right| dx$$

Hessian-based Error Control

- *E'* is 1st-order Taylor extrapolation
- 2nd-order Taylor extrapolation approximates *E*



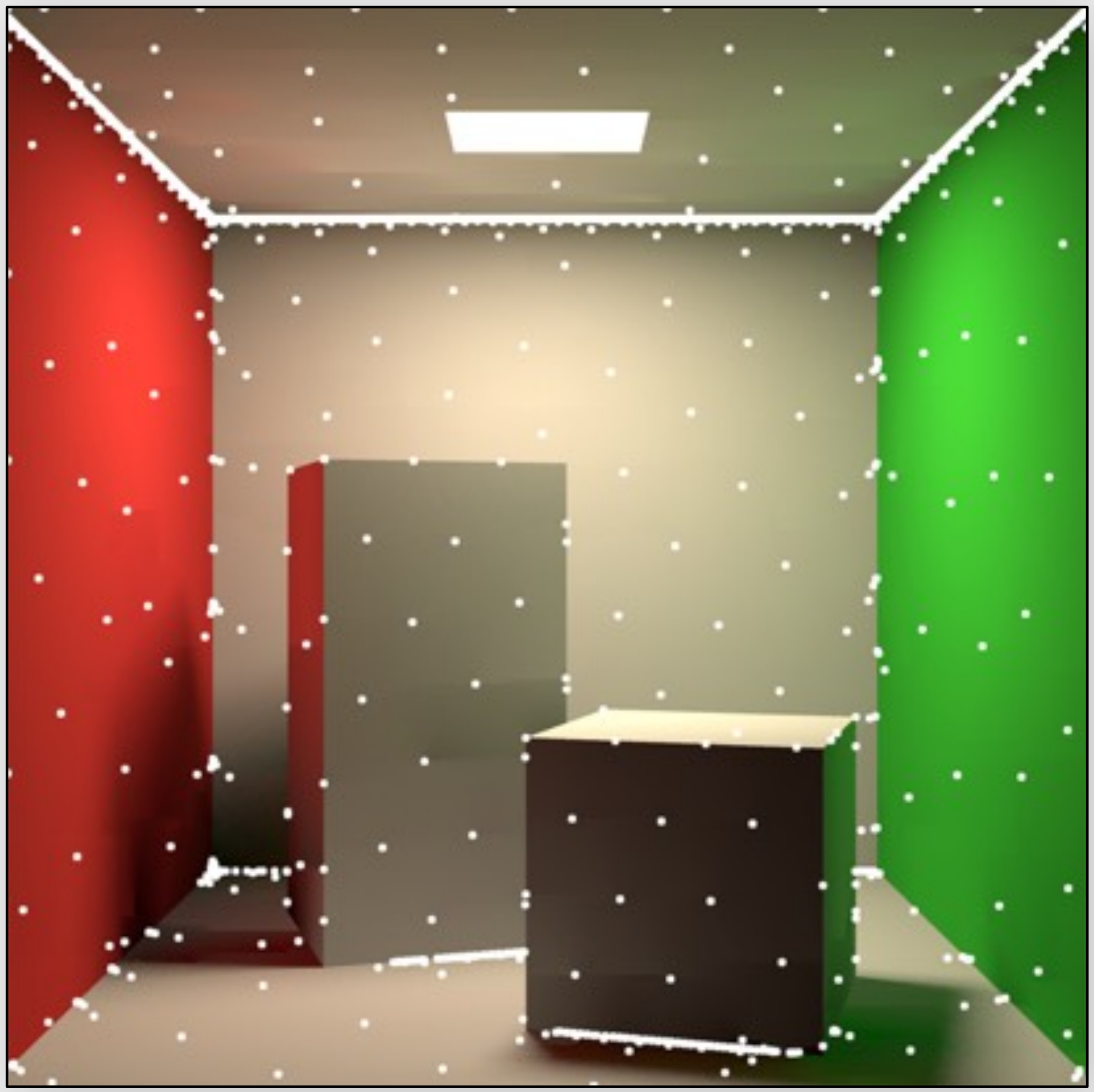


$$\epsilon^t = \int_{-R_i}^{R_i} \left| E(\mathbf{x}_i + x) - E'(\mathbf{x}_i + x) \right| dx \quad \approx \quad \hat{\epsilon}^t = \frac{1}{2} \int_{-R_i}^{R_i} \left| x \, \mathbf{H}_{\mathbf{x}}(E_i) \, x \right| dx$$

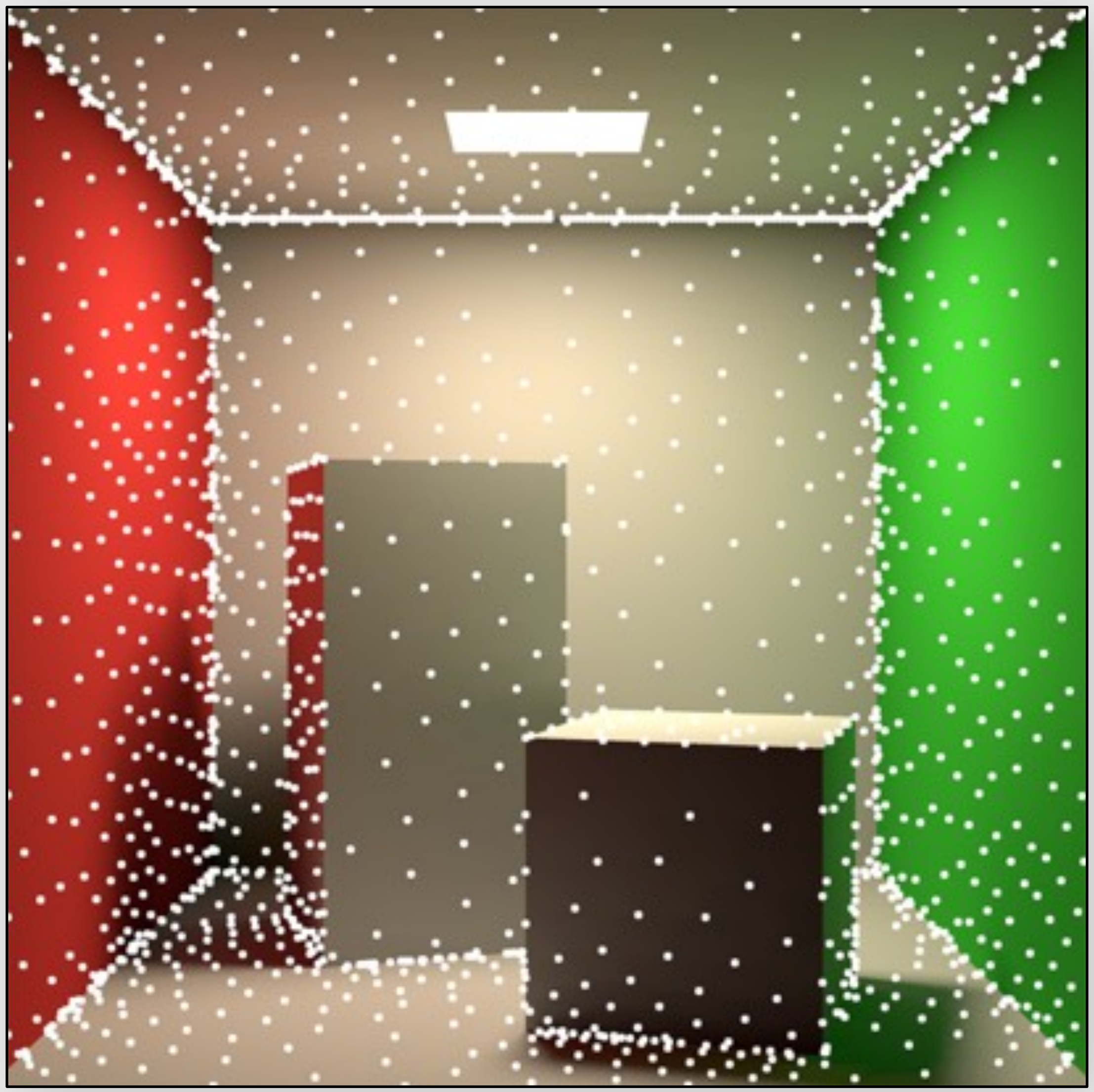
[Jarosz et al. 2012]

Beyond the Split-Sphere

~1,700 Cache Points



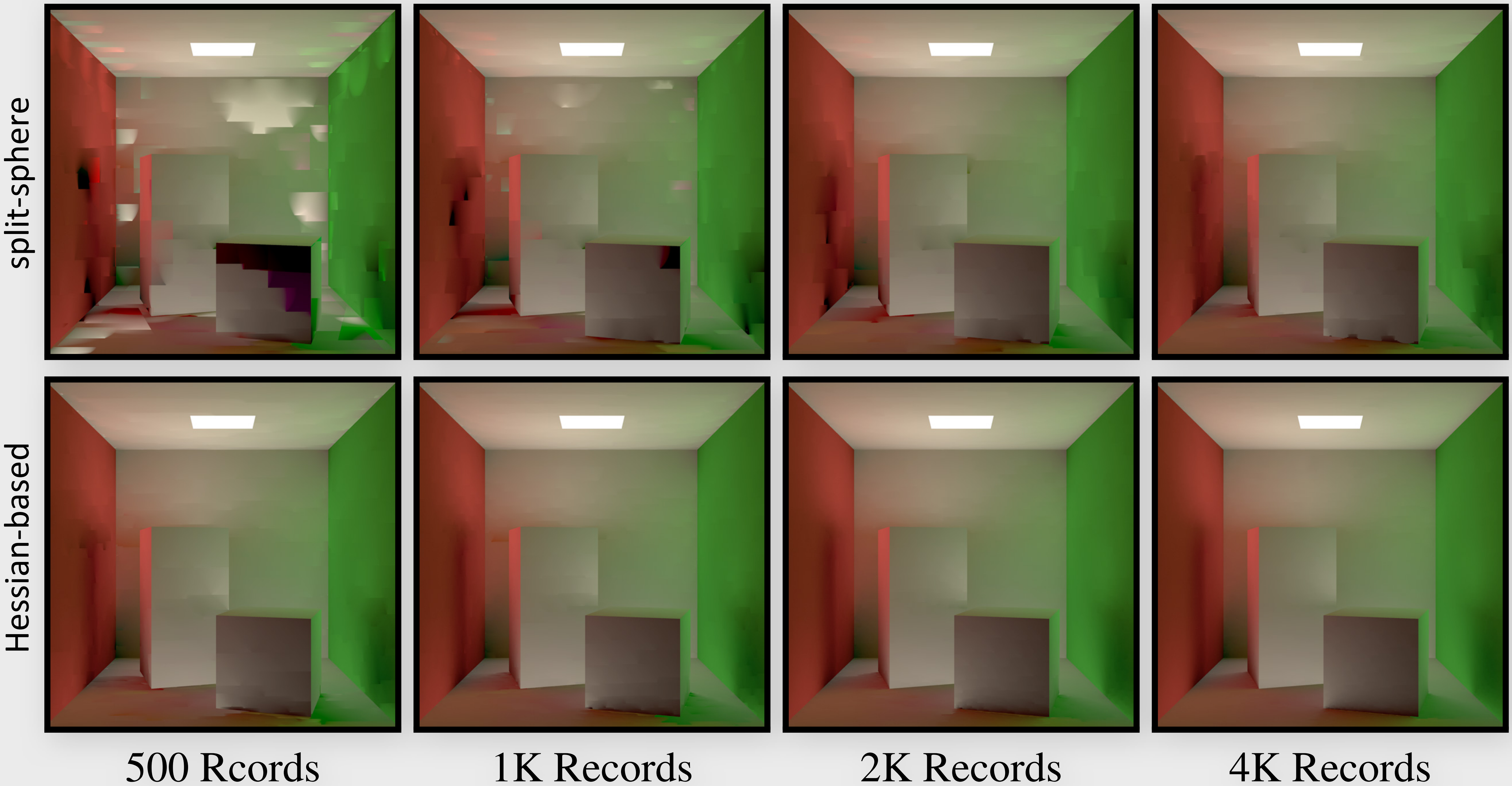
Split-Sphere



Hessian-based

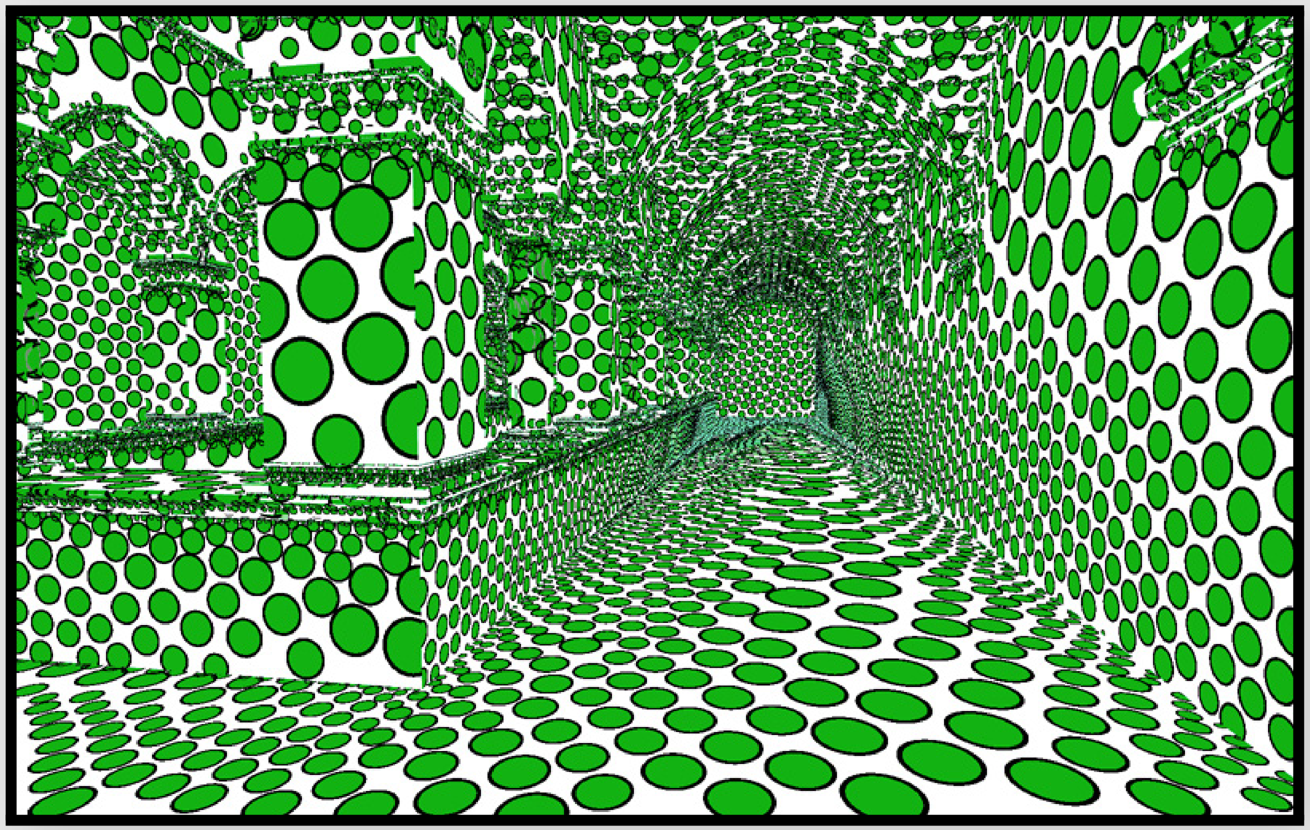
[Schwarzhaupt et al. 2012]

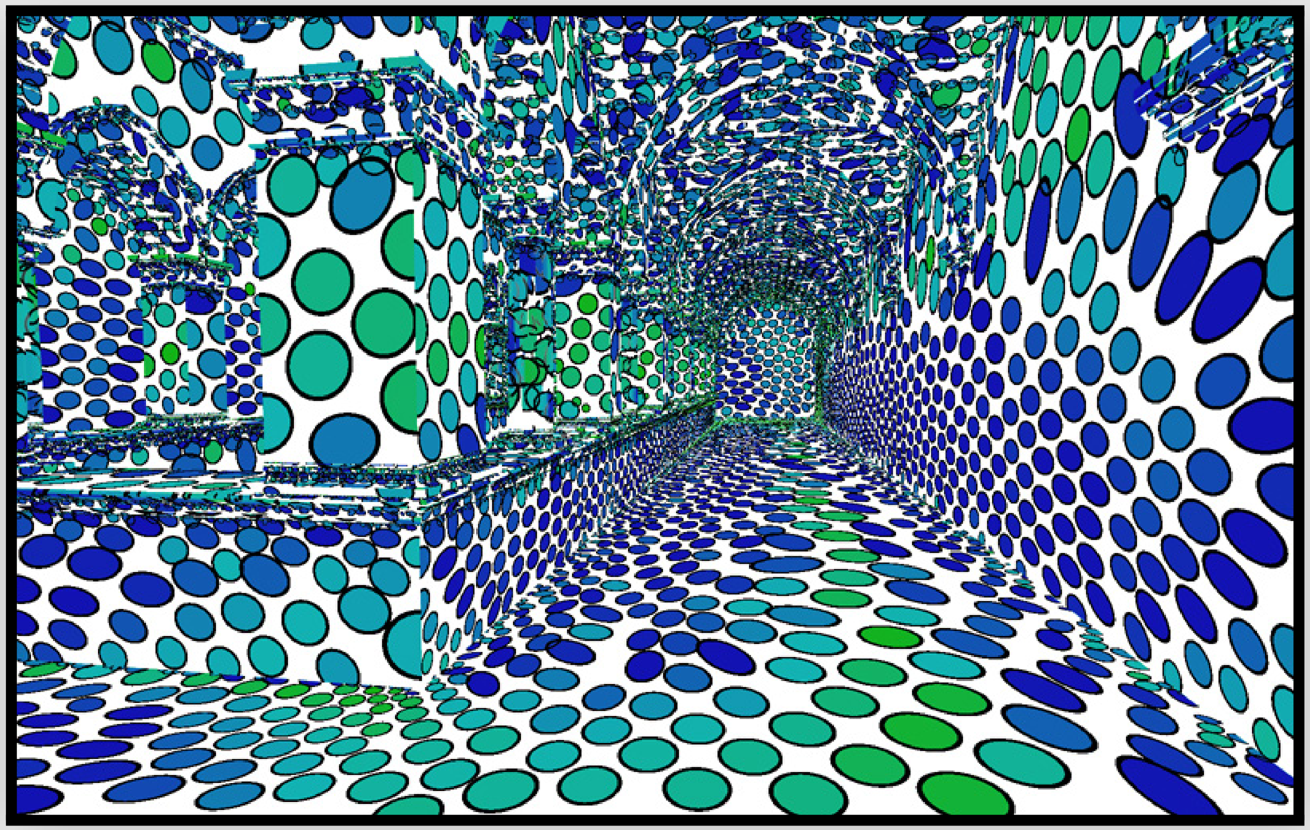
Split-Sphere vs Hessian-based



[Schwarzhaupt et al. 2012]

Anisotropic Cache Records





Reference Bounded Split-Sphere Occlusion Hessian

Summary

- Derivatives can estimate local function smoothness
- Amortize illumination computation across many pixels
- Accounting for occlusions is challenging but critical
- Specialized techniques for diffuse or moderately glossy