CHAPTER IV.

ON THE VIBRATIONS OF COLUMNS OF AIR IN TUBES.

114. Incorrect Opinions of Early Writers. Until about the middle of the last century, almost absolute ignorance prevailed concerning the vibrations of columns of air in tubes. The trumpet-marine which had set men thinking, and had assisted them in the discovery of the theory of the harmonics of strings, only led them hopelessly astray when they compared its notes with those of the real trumpet. It was known that the notes of the two instruments were the same; it was also known that the law concerning the dependence of the pitch of a note upon the length of the string producing it, applied also

to the tubes of wind-instruments, bnt the nature of the vibrations in these was totally misunderstood.

115. The Hon. Francis Roberts, in an interesting paper (1692), called attention to the sounds of the trumpet-marine, which he termed "Trumpet Notes," and he instituted a comparison between those of the string and those of the trumpet, but he contributed little to the existing knowledge of the subject, and the ideas of Sauveur, always clear and accurate with regard to the vibrations of strings, were quite erroneous as to those of columns of air.

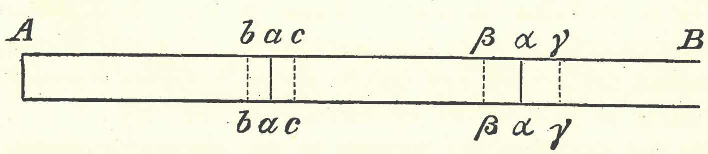
116. Bernoulli's Theories. The illustrious Daniel Bernoulli was certainly the first to solve the problem of the formation of the sonorous vibrations of columns of air. This he effected in the first instance by means of a flute. He thus describes his early experiments: (1753A) "Closing all the holes of a transverse [now the ordinary] flute, one can, simply by the change of *embouchure*, obtain at first the lowest sound, or fundamental, and then successively its *octave*, its *twelfth*, its *double octave* and its *major seventeenth*, which are as 1, 2, 3, 4, and 5 [see table of harmonics, § 46], but it must not be supposed from this, that the progression is always the same. After having formed for myself a good theory on the vibrations of the air in wind-instruments, I concluded from it that one could only obtain from tubes stopped [at one end] the sounds which follow according to the progression of the uneven numbers, namely, 1, 3, 5, 7, etc., and my conclusion was confirmed by experiment: for, having taken the head-joint of a flute, and stopped the end of it with the hand, I produced at first the lowest sound, and then, by blowing more strongly, the *twelfth*, missing the octave, then the *major seventeenth*, and lastly a note which is not received in music [the sound numbered 7: see §46], and which approached the *twenty-first* of the fundamental."

117. Bernoulli gave a promise, at the close of this paper, to write a *mémoire* when he should "have reduced to calculation the vibrations of air formed in open and closed tubes." The promised communication did not appear until nine years later: in the meantime La Grange presented a paper to the *Philosophical and Mathematical Society of Turin* (1759) in which, after alluding to Bernoulli's writings, he demonstrated the futility of close comparisons between the vibrations of stringed and wind-instruments. He subsequently contributed another and most voluminous paper to the same society (1760-1). He was at that time in correspondence with Euler, and they appear to have been comparing notes, with the result of a general agreement.

118. In 1762 Daniel Bernoulli produced his promised *mêmoire*, which contributed greatly to the knowledge of the theory of wind-instruments. In this paper (see the above date in chron. list), he undertook "the examination of the manner in which the air performs its vibrations in inflated tubes; then the explanation of the mechanical laws, following which each particle of air moves to and fro, and lastly the reduction of this movement to calculation, in order to find the duration of each vibration, and hence to appreciate the note formed by these continuous and sustained vibrations." He then proceeded to show that it was necessary to know, before all, the nature of the vibrations of a column of air, and as I do not think that this has been so well explained by any other writer, I translate the original French as nearly as possible.

119. Vibrations of the Air in Cylindrical Tubes Stopped at one End. "Let *A. B.* (*fig.* 8) be a cylindrical tube, closed at the end, *A*, and open at the other end, *B*. By whatever

FIG. 8.



means this tube may be sounded, the sound will be formed by the vibrations of the air enclosed in the tube *A B*, and these

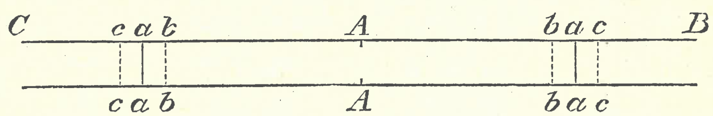
vibrations will be formed in the following manner. Let *a a* be any superficies (*couche*) of air. This superficies will alternately approach and recede from the end *A*. In its nearest approach, the superficies *a a* will reach to *b b*, and in its farthest removal, to *c c*; the entire excursion will be *b c*; these reciprocal vibrations from *b* to *c*, and from *c* to *b* will be made according to the laws of the movement of the oscillations of a simple pendulum. The larger these entire vibrations, *b c*, the louder will be the tone, but the duration of a vibration, strong or weak, will always be the same, and consequently the same note will always be the result. The movement will be accelerated from *b* towards *a*, and retarded from *a* towards *c*. If another superficies of air be taken, such as α α, further removed from the end *A*, this superficies will make similar vibrations, β γ, but these vibrations, if they are of the first order, will be wider, in proportion to their distance from the end *A*; they will be the widest at *B*, and *nil* at *A*. The vibrations of each superficies, taken apart, will be isochronous, whether great or small, whence results the identity of the notes formed by the same tube, and all the movements will be synchronous, ... otherwise the vibrations could not be sustained so as to form a sound. . . . An insensible portion of air must alternately enter and leave the tube at the mouth *B*.

120. "It follows, from what has just been said, that the air enclosed in the tube *A B*, is alternately condensed and rarefied, but these small condensations and rarefactions are very unequal in different places; they are greatest at *A*, because the superficies of air approaching each other from *A*, they tend by their action to condense the air which is near *A*, while the air at *a a* or at α α must be less condensed by the action of the anterior superficies. It is also clear that the air at *B* is neither condensed nor rarefied, and that its density is always precisely the same as that of the air surrounding the tube. . . . When the vibrations are formed in the manner described, they give the lowest sound that a stopped pipe is capable of producing. . . . ."

121. Positive and Negative Reflection. A pulse of condensation reflected from a stationary barrier returns as a pulse of condensation, and under similar circumstances a pulse of rarefaction returns as a pulse of rarefaction. Reflection of this kind is termed *positive*. A pulse reflected from the open end of a tube returns in an opposite condition as regards condensation and rarefaction. Reflection of this kind is termed *negative*.

122. Vibrations of the Air in Cylindrical Tubes open at both Ends. Bernoulli thus continues his demonstration: "Let us now see what takes place in cylindrical tubes open at both ends. For this we have only to imagine, in the middle of the tube, *C B* (*fig*. 9), a separation *A A* and there will be, as it were,

FIG. 9.



two equal stopped tubes, namely *C A* and *B A*; but as this separation is only imaginary, the action that the superficies of air perform during their vibrations against the separation *A A* on both sides, must be constantly and perfectly contrary and equal, and this condition demands that the two superficies *a a*, *a a*, equidistant from *A A*, should make, on both sides, the vibrations *c a b* perfectly equal and entirely opposed: then the superficies of air at *A A* will remain in repose and will perform the function of the separation that we have supposed. . . . It results from this, that the sound of an open cylindrical tube must be the same as that of a stopped tube of only half the length, and experience confirms this."

123. The Extension of the Air-column beyond the End of the Tube. There can hardly be any doubt that Bernoulli intentionally omitted to mention the fact that a stopped pipe does not give a sound quite an octave lower than that of an open one of the same length. He probably wished to avoid

confusing his theory by introducing an uncertain element, but it should not be thought, as many have thought, that he was ignorant of a simple fact that must have been known to every organ-builder; that had been certainly known to Salomon de Caus (1614), and on which considerable stress had been laid by Perrault (1680). Moreover his remark, at the close of §119, proves that he was cognizant of the fact. This subject will be better understood after the perusal of §124.

124. The reason that a closed pipe gives a sound less than an octave lower than an open one of the same length is not difficult to find. The air at the open end of a pipe does not form a rigid barrier, hence the vibrating column extends beyond this end. A stopped pipe, to give a full octave lower, must therefore be double the length of the actual column of air of an open one, not of the tube only. Riccati (1767) treated this subject at considerable length, but mathematicians appear to be still at variance, as to the precise difference between the length of the pipe and that of the air-column. Numerous experiments have convinced me that this difference is not constant, but that it depends much on the strength of the exciting blast. It is easy, by sounding a pipe very gently, to produce nearly an octave lower on the end being closed. On blowing as hard as the pipe will endure, without the sound rising to the octave, the closing of the end will make a difference of less than a major seventh.

Another element of uncertainty on this question is the spreading of the air-column at the terminal opening. That the vibrations do extend laterally the moment they escape is easily proved. While sounding an open pipe of any kind, place a book in contact with the side of the pipe so that it shall project beyond the end, without covering any portion of the opening: the effect on the pitch of the sound will be at once apparent. For further details of this subject, see § 137.

125. As to Nodes being points of absolute Rest there appears to be still some difference of opinion, at all events they are often described as such. It seems, however, impossible to believe that one segment of any vibrating body is able to communicate motion to another segment through an always stationary node.

126. Savart's experimental Proof of Bernoulli's Theory. Savart constructed and described (1823) an ingenious instrument which proved experimentally the truth of Bernoulli's theory. It consisted at first of an organ-pipe placed on a bellows. Loosely suspended in the pipe by means of threads, was a membrane extended on a ring, and covered with sand. When the membrane was lowered into the vibrating air-column, the sand could be seen in a state of agitation which ceased when the membrane reached the place of the node, and was again renewed after that place was passed. Savart afterwards improved his apparatus by substituting an open tube of glass for the organ-pipe, and a plate, which vibrated at the same rate as the column of air in the tube (see §56), for the sound producing apparatus of the organ-pipe, thus the disturbing influence of the current of air passing through the pipe was avoided. William Hopkins slightly improved this apparatus, and wrote a valuable paper (1833) on its use. A translation of this paper afterwards appeared in *Poggendorff's Annalen*, and from this, no doubt, the late Theobald Boehm, of Munich (1847) was led to conclude that Hopkins was the inventor of the apparatus. Professor Tyndall has fallen into the same error, one of the few inaccuracies in his admirable book, previously quoted.

127. Wheatstone's Proof of Bernoulli's Theory. A still more ingenious method was employed by Sir Charles Wheatstone to demonstrate the same truths. His apparatus consisted of a leaden tube, about an inch in diameter, and thirteen inches long, bent nearly into a circle, so that its two ends were near, and opposite to, each other. Between these ends was held a vibrating glass plate. By this arrangement the plate, advancing in its vibrations towards one end of the tube, and receding at the same instant from the other, the effects neutralised each other, and no augmentation of the original sound took place. In the middle of the tube was a joint, which allowed each half

to move independently round the axis of the tube: by this means the two orifices were brought opposite to different parts of the plate (see *fig.* 5, §56) so that the impulses were made at the same instant towards or from both orifices. The column of air in the tube then resounded powerfully. The above account is condensed from the two reports (1832A, 1832B).

Other clever contrivances have been employed for the same purpose, but space will not admit of their being described.

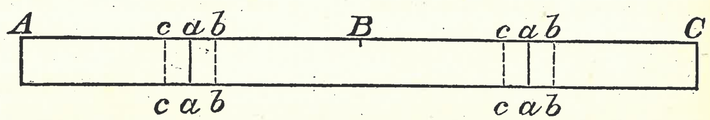
128. Errors of Past and Recent Writers. I have been tempted considerably beyond my proposed limits, in my anxiety to guard against the possibility of error in the mind of the reader on the important subject of the vibrations of columns of air, the very foundation of the knowledge of a wind-instrument, and this caution is rendered the more necessary on account of the loose way in which some really scientific authors have expressed themselves. As an instance: the able and generally accurate Riccati (1767) uses these words. "The air in a cylindrical pipe makes its vibrations in the same manner as a stretched string." *(L'aria in una canna cilindrica fa le sue vibrazioni nello stesso modo come una corda tesa.)* This, of course, is the opposite of the truth, yet Riccati knew and accepted Bernoulli's theory.

129. A peculiarly regrettable case of erroneous statement, occurring in our own time, is to be found in *A Lecture delivered at the Royal Academy of Music, on November 15th*, 1882, printed in *Musical Opinion, February 1st*, 1883, *p.* 188. These are the lecturer's *ipsissima verba*, as presented to the students of the Royal Academy of Music: "As a string or an open organ-pipe can vibrate in one length, so it can vibrate as if it consisted of two separate lengths, producing its octave." This might be taken for a reporter's error, were it not that on the above assertions hang a foot-note and a whole chain of false reasoning. It would be useless to multiply instances: the reader is aware that it is impossible for a column of air, in an open tube, to vibrate musically "in one length," and he will see, in §133, that the harmonic octave cannot be produced without the division of the column into three segments, he will, therefore, not be likely to be misled by such glaring errors of statement.

130. Before proceeding further with the description of Bernoulli's theories, it will be well to state that many of his conclusions are only theoretically accurate, although they are universally acknowledged as laws. Wind-instruments are subject to many disturbing influences which prevent the perfect agreement of theory and practice. When these discrepancies occur, it should not be thought that there is any room for doubt as to the truth of the broad principles which Bernoulli laid down, but only that some slight qualification is necessary in order to bring the practice precisely into accordance with the theory, as in the case of the stopped and open pipes.

131. Vibrations of the Air in Cylindrical Tubes closed at both Ends. Bernoulli proves that the air vibrates in a tube closed at both ends at the same rate, though not in the same manner, as in a tube of similar length open at both ends, and consequently gives a sound of the same pitch. He explains

FIG. 10.



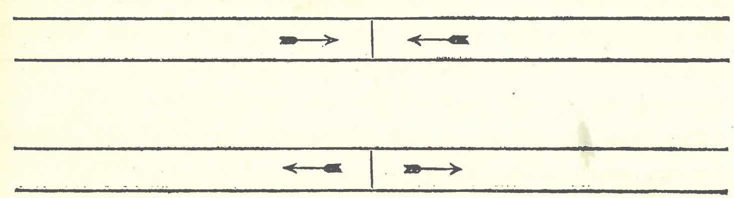
this by supposing the tube represented in *fig.* 10 to consist of two equal parts, *A B* and *C B*, each part being closed at one end, *A* or *C*, and open at *B*. We can understand that "the vibrations of each of these halves may be performed as shown in *fig.* 8, without interfering with those of the other half, provided we suppose all the air, in the whole tube *A C*, to move always in the same direction [at the same time]. Thus the state of condensation in the part *A B* will respond to the state of rarefaction in the part *C B*, and so on reciprocally. The superficies of air at *B* will constantly preserve its natural density, and taking the superficies of air *a a*, *a a*, one in each

part, and equally removed from the point *B*, these two superficies will always perform their corresponding vibrations, *c a b*, in the same direction, with perfect equality in their excursions." Thus we see that the vibrations of the two halves of such a column of air are performed at the same rate as those of a column of the same length in a tube open at both ends, the only difference being that in the former case the oscillations of the two halves of the column are in opposite directions, while in the latter case the oscillations of both halves are in the same direction. In both cases the pulsations of the two halves of the column are simultaneous.

132. The Harmonics of Columns of Air in Cylindrical Tubes. As the correctness of Bernoulli's laws of the harmonics of columns of air has never been authoritatively disputed, there is no need for any prolonged discussion of them. The following explanations, with the accompanying diagrams, will afford all the information likely to be required. The arrows indicate the directions of the oscillations when the regular periodic pulsation is established. The nodes are shown by the perpendicular lines; the antinodes are at the open ends, and also midway between the nodes. It must be remembered that the vibrations of columns of air are isochronous and synchronous.

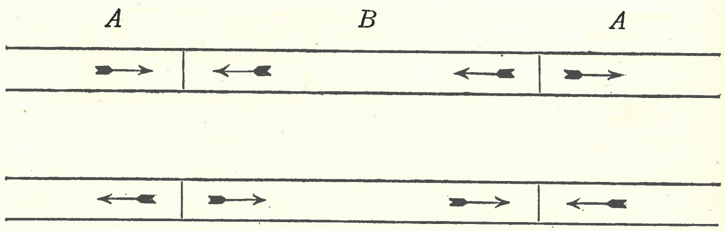
133. *The Harmonics of Columns of Air in Open Cylindrical Tubes*, as has been before stated, bear the same relation to the fundamental, in regard to sound, as those of strings, but the mode of division is entirely different. *Fig.* 11 shows the action of the oscillations of a column of air in an open tube during the production of the fundamental note.

FIG. 11.



For the first harmonic, the octave, the column divides and oscillates as shown below. The interior segment B is equal in length to the terminal segments A and A combined.

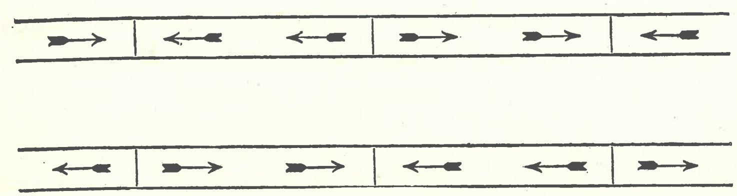
FIG. 12.



134. The perfect isochronism of the vibrations of all the segments of a column of air is one of the most important of Bernoulli's laws. Without this, the sounds of wind-instruments would be unendurable, even if they could be obtained. It has already been clearly established that the portions A A of the column of air represented in the above figure, would vibrate after the manner of those in tubes stopped at one end, the nodes performing the functions of stopped ends: see §122. The nodes being able to act as stopped ends for the portions A A, must also serve to convert the portion B into a column stopped at both ends. Now, the air in a tube closed at one end vibrates at the same rate as that in a tube of double the length, open at both ends. The air in a tube closed at both ends vibrates at the same rate as that in a tube of equal length open at both ends: therefore the column of air in the doubly closed tube B, would vibrate at the same rate as the columns in the tubes A A, which are half the length of B and closed at one end. If we suppose the column A B A to be divided at the central antinode, it will not be difficult to see that there would be two columns vibrating in the same manner, and that the oscillations of the separate halves of B would occur constantly in the same direction and in the same time.

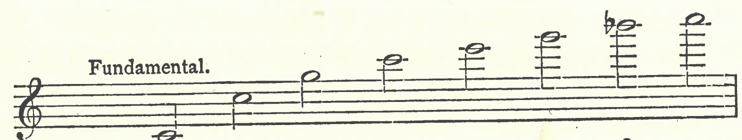
135. The next figure shows the oscillations of the air during the sounding of the second harmonic, *the twelfth*.

FIG. 13.



136. The harmonic divisions of open columns of air may be continued indefinitely, like those of strings, and will, as before stated, bear the same relation, in point of pitch, to the fundamental, but of course the figures denoting the numbers of the segments, as given in §46, require modification in this case.

Harmonics from an Open Pipe.



Number of Segments.	2	3	4	5	6	7	8	9
Theoretical Vibration Numbers.	271.2	542.4	813.6	1084.8	1356	1627.2	1898.4	2169.6

137. It follows, from Bernoulli's law of isochronism, that whenever the number of segments, into which an open cylindrical column of air divides, exceeds two, the terminal segments (of the air-column, not of the tube) are only half the length of the interior segment or segments; it also follows that all interior segments of the same cylindrical air-column, at the same time, are equal to each other.

The discrepancy between the length of the air-column and that of the tube is much increased during the production of the harmonics, and it may be stated that *the extension of the vibrating air-column beyond the end of the tube is directly proportional to the number of segments into which the column is divided, and also to the amplitude of the vibrations:* see §124. These facts have a most important bearing on the theory of wind-instrument fingering, but they appear to have been hitherto disregarded or overlooked.

138. *Harmonics of Columns of Air in Stopped Cylindrical Tubes.* The expression *stopped tube* is always understood to mean a tube stopped at one end only. No musical instrument consists of a tube completely stopped at both ends: such tubes can only exist in a musical capacity, somewhat hypothetically, as segments. *Fig.* 14 shows the action of the oscillations in a stopped tube during the production of the fundamental sound.

FIG 14.

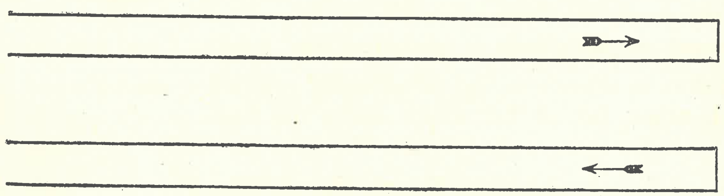


FIG. 15.

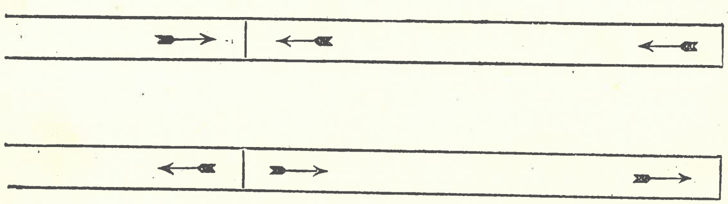
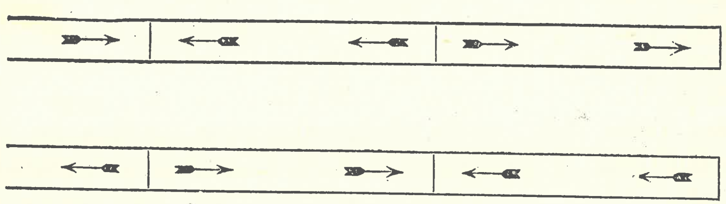


FIG. 16.



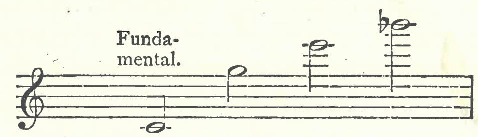
139. The harmonics of columns of air in stopped cylindrical tubes have already been stated to be fewer than those in open ones. This is to be explained by the immoveable character of

the closed end, which necessitates the limitation of the number of segments into which the column can be divided. The closed end *must* form one of the nodes, therefore no sound can be produced from the tube, which does not admit of a node being in that position.

The first harmonic, *fig.* 15, is formed as in one half of the column shown in *fig.* 13, and similarly produces the *twelfth* of the fundamental, the octave being necessarily missed. The second harmonic of a stopped cylindrical tube, *fig.* 16, is the major *seventeenth*, or double *octave* and major *third.*

In obedience to the law of isochronism, during the production of any harmonic from a closed cylindrical tube, the terminal segment of the column of air is half the length of the interior segment or segments, and the interior segments are equal to each other, for all harmonics above the first.

Harmonics from a Stopped Pipe.



Number of Segments.		2	3	4
Theoretical Vibration Numbers.	271.2	813.6	1356	1898.4

140. For the interesting and instructive experiments of Dr. August Kundt, who investigated and rendered visible the lengths of the sound-waves in pipes, by means of the powder of lycopodium or of calcined silica strewed over the interior surfaces, the reader must be referred to the original papers (1866A, 1866B, 1868); to the condensed account by Professor Tyndall (1883, p. 209), or to that by Lord Rayleigh (1878, p. 53).

141. *The Sonorous Vibrations of an Air-column are independent of any Rush of Wind that may pass through the Tube.* Whether the sound be produced by means of a reed blown by the breath in such a manner that every particle of the exciting air travels straight through the instrument, or by means of some vibrating body near the mouth of the tube, as in the experiments before cited, the sound-pulse behaves in precisely the same manner, that is, it moves backwards and forwards, either with or against the current of air, in the same way as when there is none.

142. The Relation of the Exterior to the Interior Sound-wave. On the production of a fundamental note in a column of air, a pulse of condensation is first projected down the pipe, and is then reflected from the closed end if the pipe be a stopped one, or from the central node if the pipe be an open one, and the pulse returns in its original condensed state, that is, the reflection is positive: see §121. When the pulse returns to the open end it suffers negative reflection, and it therefore performs its third excursion in a state of rarefaction, to be again reflected positively, and having thus completed four excursions of a whole stopped tube, or of half an open one, it is restored to the condition of the surrounding atmosphere. During the four excursions of the pulse, the air-reed, or other exciting cause of sound, makes one complete vibration.

The length of the exterior sound-wave is, in general terms, equal to the distance which the pulse traverses in these four excursions within the tube.

A harmonic sound generates an atmospheric sound-wave approximately four times the length of the terminal segment, or segments, of the air-column.

No definite reliance can be placed upon the rule which is supposed to govern the relation of the exterior to the interior sound-wave, for it is subject to numerous interferences, and must always be subservient to the laws given in §22: these may be accepted with less qualification.

143. Lateral Perforations in Musical Tubes. It has already been roughly stated, at the beginning of chapter III., that under certain circumstances the opening of a lateral aperture in a wind-instrument is equivalent to shortening its length, and that the pitch of the note produced is accordingly

raised thereby. It will now be necessary to consider this subject more closely.

144. *Note-holes.* An aperture which determines the length of a tube for the production of a particular note is correctly termed a *note-hole.* The opening of such a hole may be regarded as analogous to shortening a musical string.

The depth of the note varies directly as the distance of the opening from the opposite end of the air-column. The primary function of a note-hole is to allow the external air to enter the tube, and thus to shorten the air-column. It will be evident that there must always be a *terminal* antinode at or near the place of a note-hole.

145. If a note-hole be as large as the bore of the tube, the effect of the opening of that hole on the sound, whether fundamental or harmonic, will be nearly the same as that of cutting off the portion of the tube below the opening. See §124, especially the last paragraph, and §137.

When the note-hole is much smaller than the bore, the external air cannot enter in sufficient quantity to separate efficiently the upper from the lower part of the tube, consequently the vibrating column extends to a greater distance beyond a small opening than it does beyond a large one, in other words: *the distance which the air-column extends beyond the note-hole varies inversely as the diameter of the opening;* but it may sometimes happen that a small perforation will prevent the production of any fundamental note: see §151.

146. There are exceptions to some of the above rules in their application to instruments of the flute type, which, though not of much practical importance, are yet worth notice. In this class of instruments a small hole above the highest node, and within a certain distance from the *embouchure*, virtually increases the area of the opening at the upper antinode. *The influence of such a hole varies directly as the distance of the hole from the* embouchure, *and inversely as the length of the tube.* There has never been, to my knowledge, any practical application of this principle to wind-instruments, except in the case of the French flageolet, (in which the highest hole gives a note a semitone lower than that given by the hole next below it) but a suggestion to that effect was made with regard to flutes by William Close (1802), an apothecary of Dalton in Lancashire, a man of considerable ingenuity, and the first to apply the principle of the now popular piston to brass instruments. Close states that he never carried out his idea of opening a hole near the mouth-hole of a flute; had he done so, the experiment must have been musically unsuccessful.

147. Other things being equal, *the loudness of any note of a wind-instrument varies directly as the diameter of the note-hole*, but the influence of the size of the note-hole on the power, and also on the pitch, of the sound, will be much less if there be another hole at a short distance below the one in question, particularly if the lower one be larger than the upper one.

A recent application of this principle to the flute, by myself, has been attended with excellent results, which will be found particularly described in part II.

148. The contraction of the open end of a tube has the same results as those of reducing the size of a note-hole. The effect of closing lateral openings below the one that immediately determines a note, is similar to that of reducing the size of that aperture. *The effect of this reduction, on pitch, is directly proportional to the number of the segments of the column of air.* See §§124 and 137. *The effect on power of sound is inversely proportional to the rapidity of the vibrations.* This is partly due to the natural penetrating power of high notes, according to their degree of elevation, and, in the case of harmonics, to there being a greater number of uninjured segments above that which is spoiled by the inadequacy of the opening.

149. *Vent-holes.* Bernoulli proved that the air, at any interior antinode of a cylindrical tube, is in the same condition as that at the open end, that is, the same as that of the surrounding atmosphere, consequently opening a hole, or even cutting completely across the tube, at an antinode would cause no disturbance of the vibrations, as "the interior air

would make no effort to escape, nor the exterior air to enter." The opening of an aperture at a node is, on the contrary, productive of most important consequences, as it prevents the pressure which is necessary for the existence of a node, from taking effect, and thus causes the node to vanish instantly, an antinode taking its place, while, if the air-column be set in sufficiently rapid motion, other nodes will be formed, and a higher note will necessarily be the result. Apertures such as this are called *vent-holes*. *The primary function of a vent-hole is to allow the internal air to escape from the tube; a vent-hole does not shorten the air-column.* It will be evident that there must always be an *interior* antinode at or near the place of a vent-hole.

150. Vent-holes are of great use in assisting to determine the harmonics of a wind-instrument, but they will not always suffice for that purpose unless there be a corresponding increase in the rate of the vibrations, caused by what may be termed the motive power of the instrument, the breath.

151. When the size of a lateral aperture exceeds a certain limit, the opening cannot act properly as a vent-hole, and may refuse to serve otherwise than as a note-hole. On the other hand, a small opening will not always answer as a note-hole, and may permit only the production of a harmonic, or of mere wind-rush. Generally, the influence of the greater or less superficies of a vent-hole is of the same nature as that of similar variation in the size of a note-hole, though in a much less degree.

152. The influence of the position of a vent-hole, on the pitch of the sound, is generally greater than that of the position of a note-hole, and the influence of a note-hole is much reduced when a vent-hole is opened. It may be taken as a rule that *when a note-hole and a vent-hole are open at the same time the influence of the note-hole varies inversely as the influence of the vent-hole.*

A vent-hole will assist in the determination of a harmonic, even if the hole be considerably above its true position, but if it be much lower the correct harmonic will not sound. The extent to which this deviation may be carried in a downward direction, consistently with the production of the desired harmonic, varies inversely as the size of the opening. The limit of upward deviation may be considerably extended if one or more holes be opened below and near to the vent-hole.

Further information on the subject of vent-holes will be found in §§ 357 to 364, and there are numerous examples of their various applications in chapters XIV., XV., and XIX.

For the sake of convenience, and to avoid complication of mechanism, there are no special vent-holes in the flute, certain of the note-holes being employed in that capacity.

153. It should now be clear to the reader that the opening of a vent-hole in a tube, has an immediate effect precisely the reverse of that produced by shading a string; the former operation causing the removal of a node from the position it previously occupied; the latter establishing a node at the spot touched. It should be equally clear that the placing of an air-tight partition across a vibrating column of air at a node, would give rise to no change in the vibrations, though there would, of course, be a reduction in the number of vibrating segments.

154. The Influence of Diameter on the Sounds of an Air-column. Precise rules cannot be laid down for the regulation of the comparative width and length of a musical tube, but it is necessary that there should be some reasonable proportion, according to the nature of the instrument and its requirements. Within certain limits, other things being equal, *the depth and the power of a fundamental note vary directly as the width of the air-column from which it is produced.* All this was worked out by Mersenne (1636. *Livre IV. Prop. XII.*), and was well-known to the early organ builders. For the production of any fundamental note, a certain width of bore is absolutely necessary. A wide tube will give a fundamental note of large volume of sound, which is not liable to "fly off" to a harmonic, and is therefore more steady under considerable wind pressure than one given by a narrow tube. Such a bore is consequently only

suitable for organ-pipes which are intended to give none but fundamental notes.

155. On the other hand, a bore narrow in proportion to its length will allow of the production of the harmonics with ease, but may render the sounding of the fundamental difficult or even impossible. Lambert (1775) says that "a very long and narrow tube will give the *octave* or the *twelfth* rather than the sound due from the entire length." Some important experiments throwing light on this subject were conducted by Chladni (1802) and by Savart (1823). As a result of direct observation, I have found that the comparative width of a tube, in relation to its length, has great influence on the pitch of the harmonics, too great width causing them to be too sharp in comparison to the fundamental. The converse of this rule follows of course.

156. Prismatic columns of air differ very little in their behaviour from those of cylindrical form. Their discussion would be out of place in this work.

157. Varying Positions of Nodes, and of the Origin of Sound, in Tubes. A very important fact, first proved by Bernoulli (1762) and further investigated by Savart (1823), Wheatstone (1832), William Hopkins (1833), G. S. Ohm (1853), Zamminer (1855), Mr. Hermann Smith (1873, 1874A), Lord Rayleigh (1878), and others, is the variability in the positions of the nodes of air-columns. Bernoulli's law concerning the perfect unison of all the segments of an air-column, see §134, is so far immutable, that if it be subjected to any serious interference, no musical sound will result.

158. If a cylindrical tube be perfectly open at both ends, the node of the fundamental will be in the exact centre of the tube, and its position will be unchanged if the openings at the ends of the tube be equally reduced in size, but if only one opening be reduced or if the two be reduced unequally, the node will move further from the more open end. Supposing that this did not occur, the segment with the freer opening would necessarily vibrate more rapidly than the other. The law of isochronism will not generally permit this incongruity, therefore the node alters its position, and thus brings the two segments of the air-column into perfect unison. The harmonical divisions of the column obey the same law.

159. Savart very forcibly drew attention to the fact that no musical wind-instrument can be entirely open at both ends (1825). Nevertheless, in any instrument of the flute type, the end at or near to which the sound is produced, is open in an acoustical sense, and is therefore the place of an antinode. All other wind-instruments are necessarily sounded at the stopped end, which is, on that account, none the less the place of the node. Sir Charles Wheatstone was the first to point this out in a lecture delivered at the Royal Institution (1832B).

160. "The lecturer then proceeded to show the erroneousness of the prevailing opinion, stated by Chaldni and others, 'that the end at which the tube is excited into vibration, must always be considered as an open end, even if it be placed immediately to the mouth, as in the horn and trumpet.' He showed that a cylindrical tube gave the same fundamental sound, and the same series of harmonics, when it was excited as a horn, or with a reed at one end, the other end being open, as when it was excited, like a flute or flageolet, at one end, the other end being shut. In proof of this he adduced the Cremona pipe of the organ, which is a cylindrical tube, one half the length of the open diapason pipe which gives the same note; and the clarionet, which is also a cylindrical tube (the conical bell which terminates it being merely a useless appendage), giving a fundamental sound an octave below that of a flute of equal length, and the series of harmonics of a tube closed at one end. He then adverted to the circumstance that, in all cases of the production of sound at the closed end of the tube, the tone is invariably more powerful than when the sound is produced at the open end of the same tube; and explained that in the one case the impulses are made at that part of the air where the condensations and dilatations are greatest, and in the other case where these variations of density are least. This point was

illustrated by some experiments with the flame of hydrogen gas, by which means a column of air can be excited into vibration at any point between the open end and the node, with a corresponding alteration of intensity. At the orifice of the tube, the smallest possible flame is sufficient to excite the sound, which, however, ceases if the flame be made to move towards the node (*i.e.*, the centre of a tube open at both ends, or the closed end of a tube stopped at one end); but if, at the same time that the flame is advanced in the tube, it be also enlarged in volume, the sound continues; and with increased intensity. By continuing to move the flame towards the node, and at the same time to enlarge proportionally the volume, the sound progressively increases in loudness until it attains its maximum at the node."

161. The transition of the fundamental note of a stopped pipe to that of an open one cannot, according to my experience, be effected gradually. If a very small hole be bored in the cover of a stopped pipe, a note may still be produced from the pipe, and the pitch will be slightly raised. The pipe may then be considered to be *imperfectly stopped.* If the hole be gradually enlarged, a point will soon be reached at which the pipe will refuse to "speak." The node will then have left the partially stopped end of the pipe, but in so doing it will have become broken up, and will, in fact, have ceased to exist.

162. The period of silence, or rather wind-rush, may be deferred, or may not occur at all, if the stopper of the pipe be long, so that the hole may form a small pipe at the end of the main one. The union of the two pipes will then form what is termed a "*tuyau à cheminée.*" These curious pipes appear to have somewhat puzzled the ingenious Salomon de Caus (1615) as they have also puzzled one of the best popular writers on musical acoustics of our own time, Mr. Hermann Smith. The main tube of this combination may be regarded as being imperfectly stopped, the opening being modified, according to a well-known custom amongst wind-instrument makers, by the addition of a small tube, which will often convert an otherwise destructive hole into a useful one. Such is the extreme delicacy with which this experiment may be conducted, that .01 inch added to, or taken from, the length of the smaller tube, may be found to permit or prevent the production of a sound.

163. To resume the process of converting a stopped pipe into an open one: on the enlargement of the hole in the stopper being continued, the period of the rushing noise will pass, and a note, somewhat sharper than the former one, will gradually come into hearing. The pipe may then be considered to be *imperfectly open*. By that time the node will have succeeded in establishing itself once more, but it will be at first perilously near the place of its late departure, and the pipe must be blown very gently, or the note will again disappear. The operation of opening being continued, the node will move further from the end of the tube, and the tone will become stronger by degrees, but no rule can be laid down for the determination of the size of the opening which will produce the best note.

164. During the gradual opening or stopping of a pipe, the behaviour of the harmonics is very different from that of the fundamental: they depend upon other nodes besides the primary one, and therefore sound uninterruptedly during the entire process, even if the covering of the pipe be no thicker than a sheet of paper. I am not aware that these interesting facts have ever been mentioned before, and that is the chief reason for their intrusion here, though it will eventually be seen that they have some bearing on the subject of the notes of the flute.

165. Vibrations of the Air in Tubes of varying Diameter. Next to the cylindrical, the most simple form of a column of air is the conical. Bernoulli (1762) proved that a conical tube open at the base, produced the same complete series of harmonics as a cylindrical tube open at both ends, but according to a different system. "For the fundamental sound," he says, "all the air in the tube moves constantly in one direction, but

alternately towards the summit and towards the base. For the second sound the air divides into two parts, as by a diaphragm. . . . For the third sound two nodes are formed, which divide the tube into three parts, and so on."

166. The segments of such a conical air-column are not of equal length, but the node recedes from the open end according to the amount of divergence of the sides of the cone, otherwise the isochronism of the vibrations would not be maintained: see §158. Of course the sound of the air-column in a perfect hollow cone could only be induced by some such method as that adopted by Sir Charles Wheatstone: see §160.

167. The so-called conical wind-instruments are generally closed at the smaller end. They may be roughly estimated to give the same notes as open cylindrical tubes of the same length, but Wheatstone (1832B) proved that in order to render possible the production of a perfect series of harmonics from a closed tube, the diameters at the ends of the tube must be as four to one. The numerous interferences that are constantly at work in all wind-instruments appear invariably to necessitate some departure from the true cylinder or cone. I am not aware of one, at present in use, except the organ-pipe (which is intended to sound but one note), without some curvature in the lines of the bore, or what amounts in practice to the same thing.

168. The old-fashioned military fife was cylindrical, but that has been long out of date. Amongst instruments of the present day, with the exception above mentioned, the clarionet has the nearest approach to a cylindrical bore, but, passing over the question of the bell, one side of its mouth-piece is curved and the other is oblique. The so-called conical instruments would be more correctly described as truncated conoids. A detailed account of the various bores of flutes will be found in part II.

169. I have observed some very curious and interesting phenomena which occur when portions of the bore of an open flute-pipe, are abruptly altered in diameter. The sound from the tube is not only lowered in pitch by the reduction of the bore at the lower extremity, but the depression goes on increasing, as the bore is reduced progressively from this end, until the place of mean amplitude of vibration is reached. After that, the pitch gradually rises until the size of the tube is reduced from the node downwards, when the sound is exactly the same as that given by the wide open tube. On continuing the reduction of the bore, the pitch rises constantly until the tube becomes smaller from end to end.

This experiment produces effects which are rather startling. It may be conveniently performed in either a cylindrical or a prismatic tube, by the insertion of a rod of sufficient thickness to reduce the diameter of the bore appreciably. The above account refers to the fundamental sound; the behaviour of the harmonics is still more curious. The same experiment may be so conducted as to afford conclusive proof of the truth of the statements given in §§ 154 and 155, and also of the following important facts: *the enlargement of the upper part of an open flute-tube flattens the pitch: the enlargement of the lower part has a contrary effect.* This rule applies to harmonics as well as to fundamentals.

170. The last subject of this chapter is a very obscure one, and its theoretical explanation has never, I believe, been attempted. Under certain circumstances, the natural law of isochronism is unable to assert itself; an ill-proportioned tube may set it at defiance, and cause the segments of the air-column to vibrate at different rates. The flute is peculiarly liable to this defect, as the delicate air-reed has not the power to control the vibrations of the column of air to the same extent as can the more ponderous reeds of other instruments: see §§ 82, 84, 112 and 134.

171. Falseness in a string (§ 47) affects the harmonics more than the fundamental, but deviation from the true proportions of the bore of a flute has often a contrary effect: the harmonics may even be improved, while the fundamental may be destroyed. The "horrid sound" of an ill-bored flute is technically termed a *rattle*, and when once heard, in full development, is not likely to be forgotten.

CHAPTER V.

ON SIMPLE AND COMPOSITE SOUNDS.

172. Simplicity and Complexity of Sound. There appears to be such good reason to doubt the possibility of the existence of an absolutely simple sound, that we may, perhaps, be justified in considering that all the sounds we hear are of necessity more or less complex, although the ear may sometimes fail to distinguish their component parts. Rameau (1737) went so far as to assert that a perfectly simple sound would be inappreciable to the ear, an assertion which he afterwards modified (1749) by saying that a simple sound is only noise, while musical tone is invariably composite. With these opinions which, however, rest on but a shadowy basis, Estève (1750) generally agreed.

173. Lord Rayleigh (1877) says that a tuning-fork gives a sound which becomes *nearly* simple, after the cessation of certain higher sounds which die rapidly away. Professor Helmholtz (1885, p. 69) tells us that "such musical tones as are not decomposable, but consist of a single, simple tone, are most readily and purely produced by holding a struck tuning-fork over the mouth of a resonance tube." On the next page we read: "Simple tones, accompanied only by the noise of rushing wind, can also be produced . . . . . . by blowing over the mouths of bottles with necks." He also speaks of wide stopped organ-pipes giving simple sounds. Probably the nearest approach to perfect simplicity is attained by the method of Lord Rayleigh, above mentioned. The first plan of Professor Helmholtz could