Reversible Jump Metropolis Light Transport

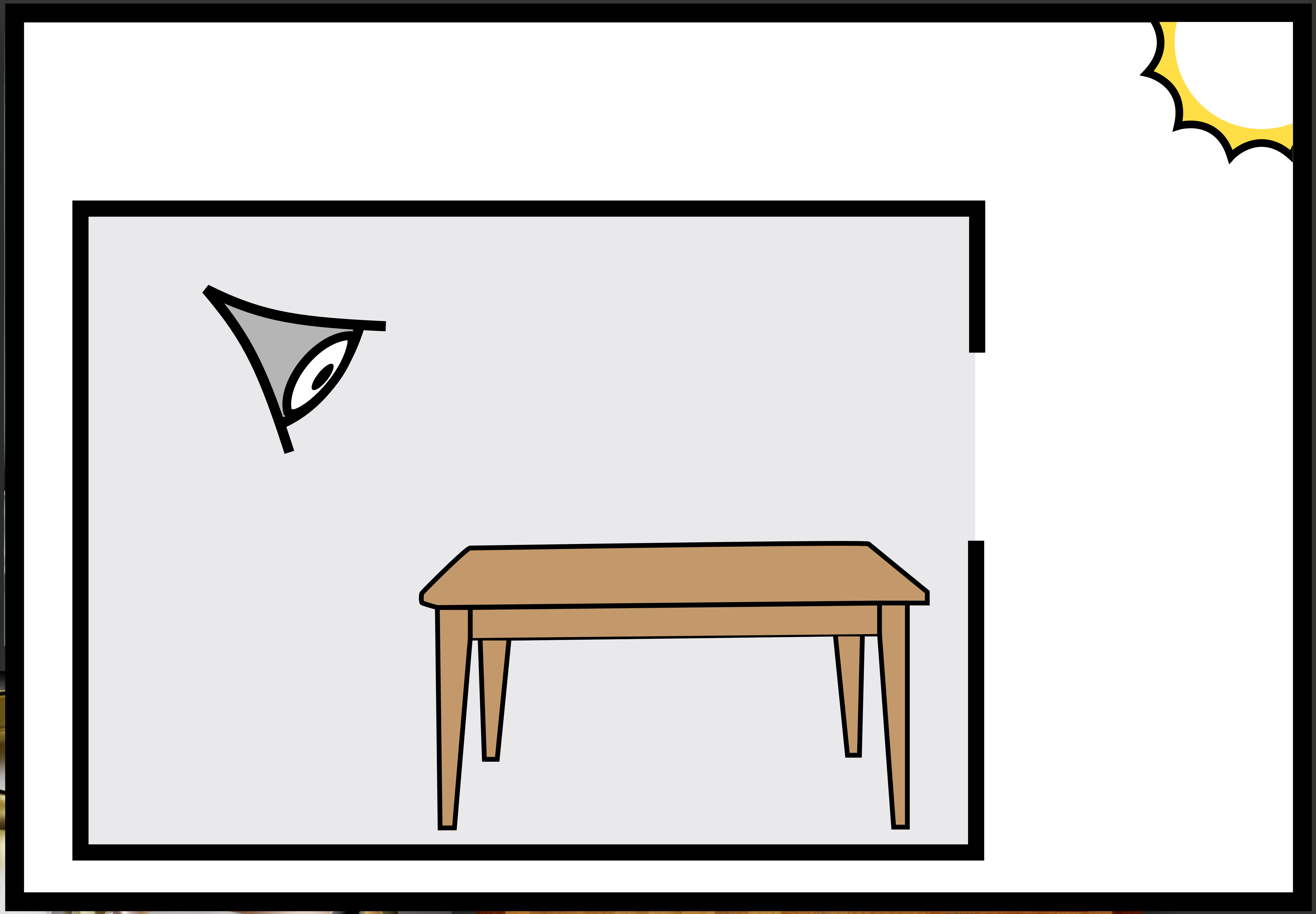
Benedikt Bitterli [1 2 3] Wenzel Jakob [4] Jan Novak [3] Wojciech Jarosz [1 3]

[1] Dartmouth College [2] ETH Zurich [3] Disney Research [4] EPFL

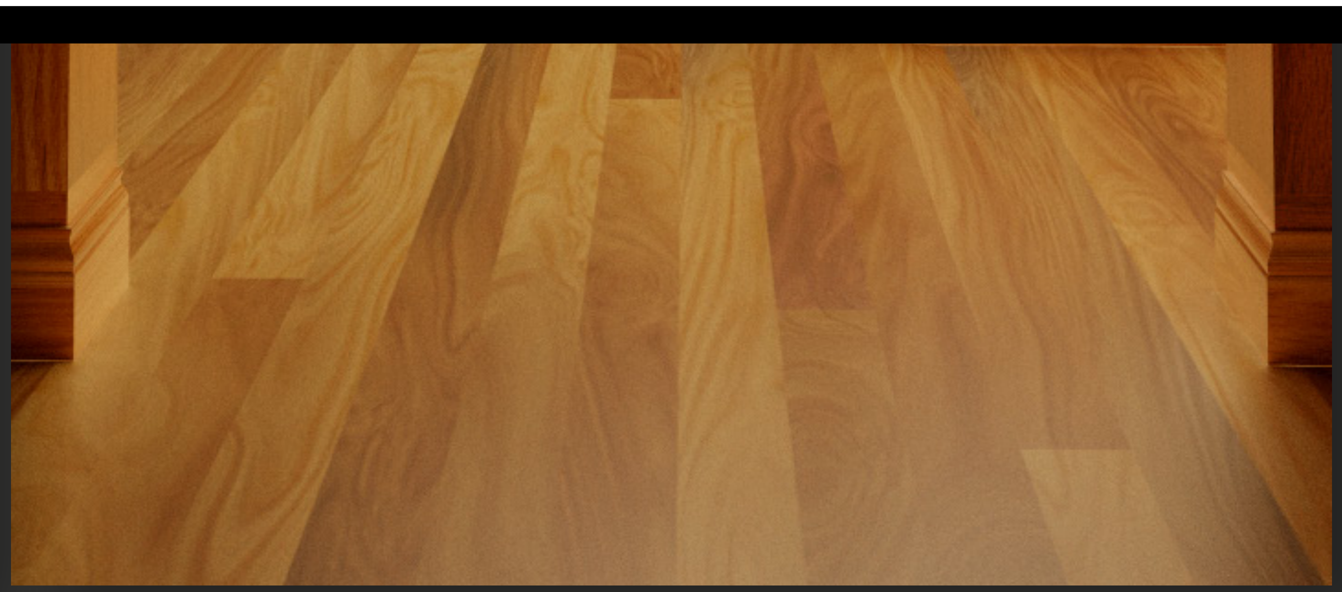
DARTMOUTH
VISUAL COMPUTING LAB

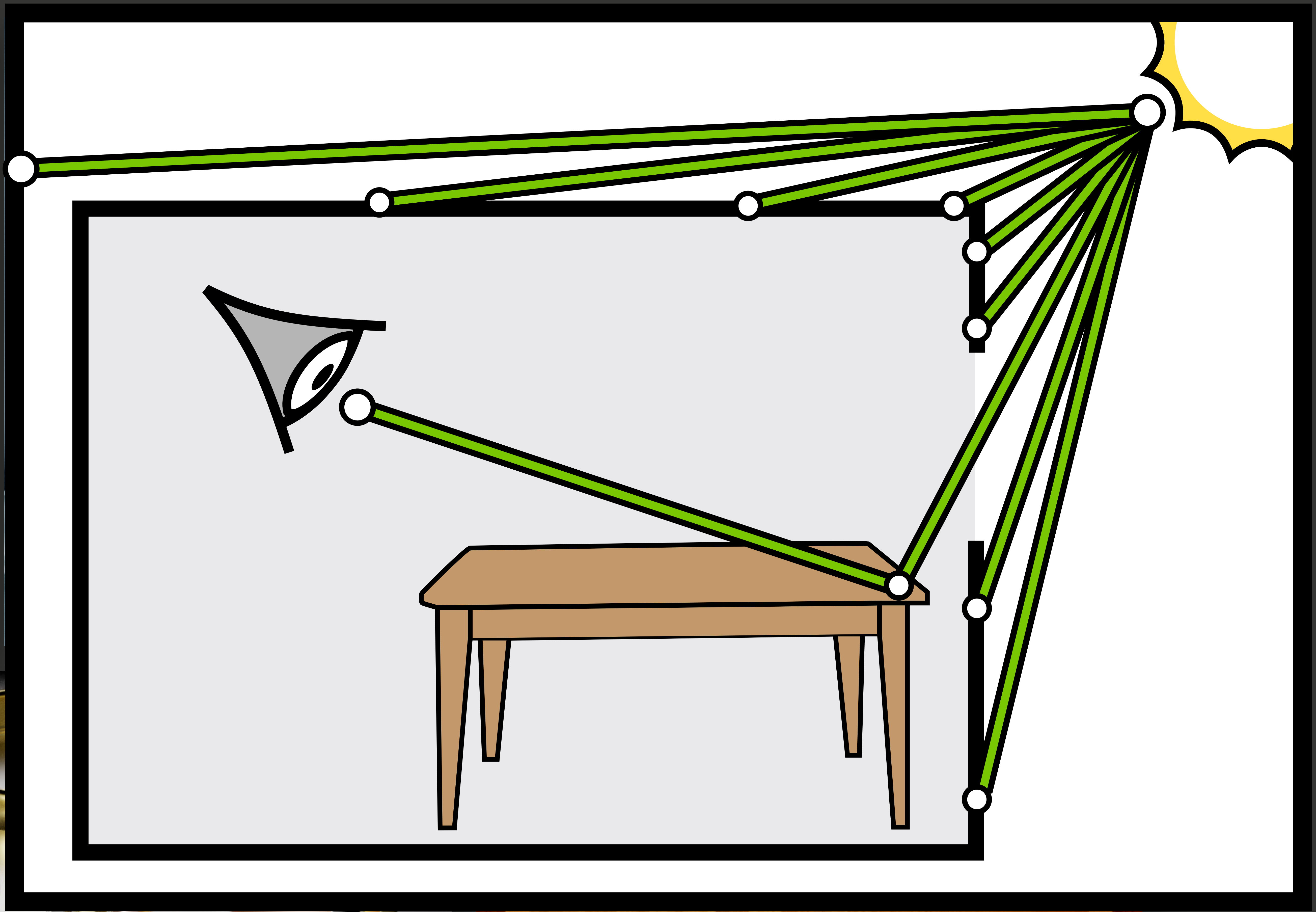
Motivation

Motivation



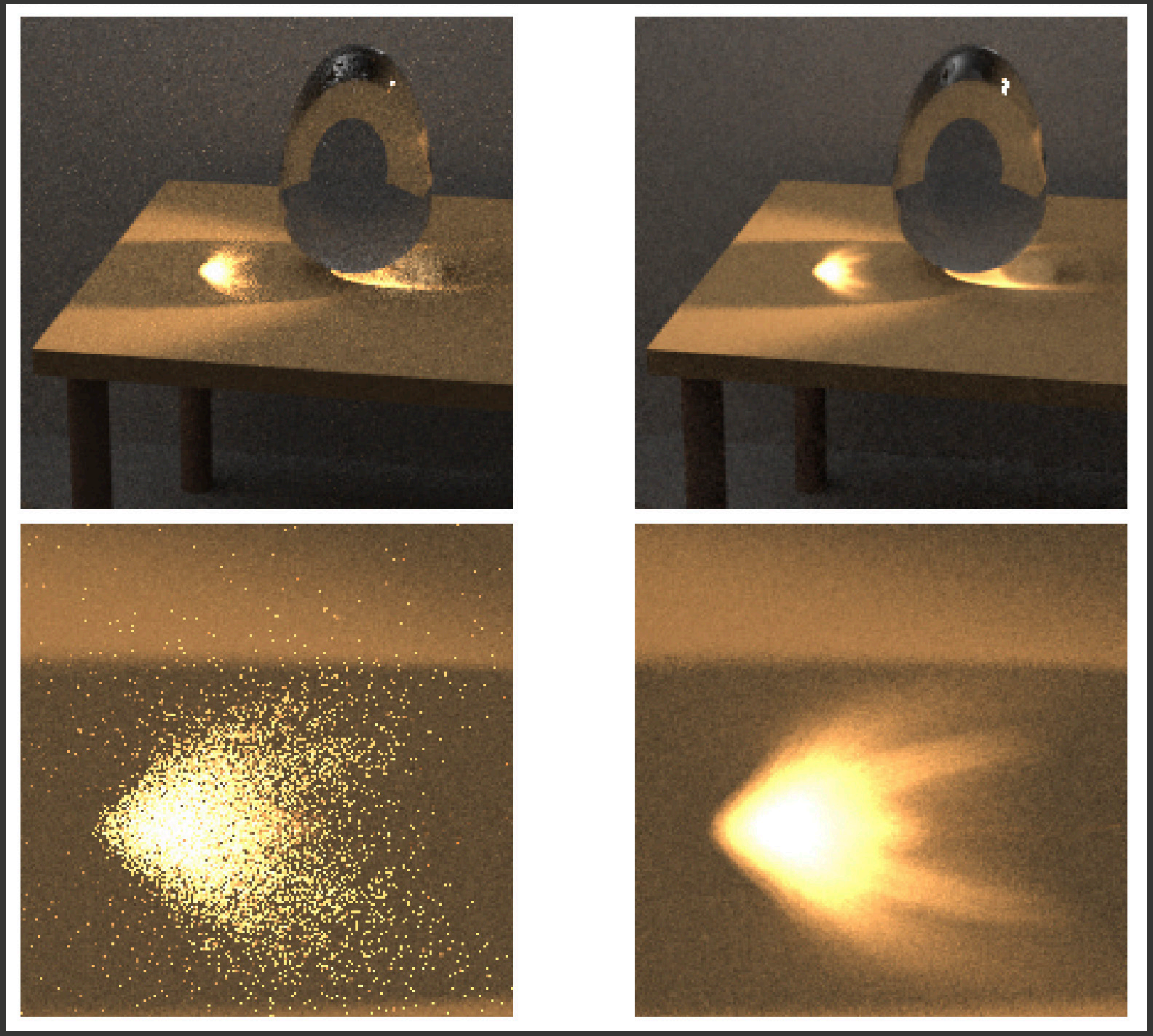
Motivation





Motivation

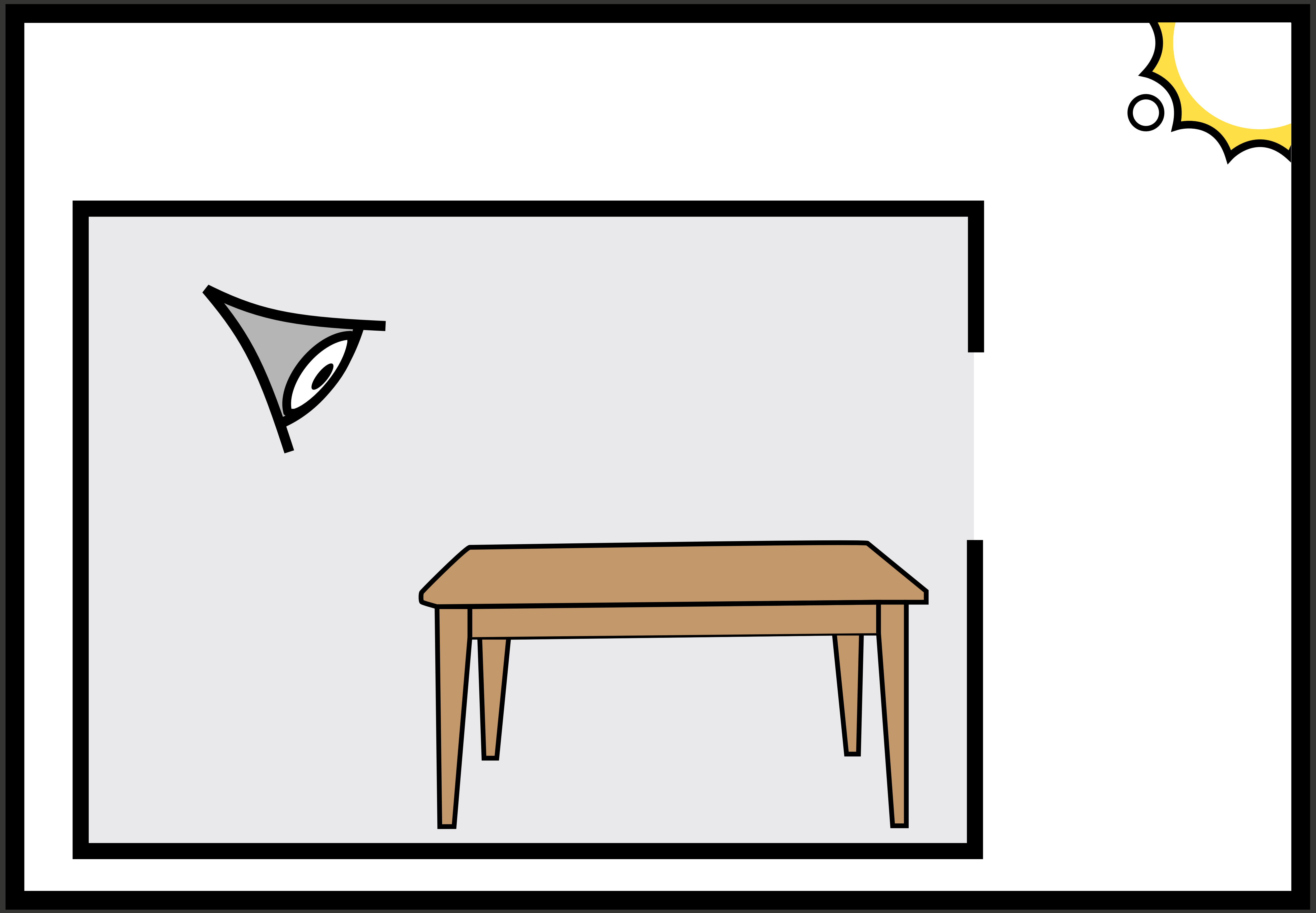
Metropolis Light Transport (MLT)
[Veach & Guibas ’97]



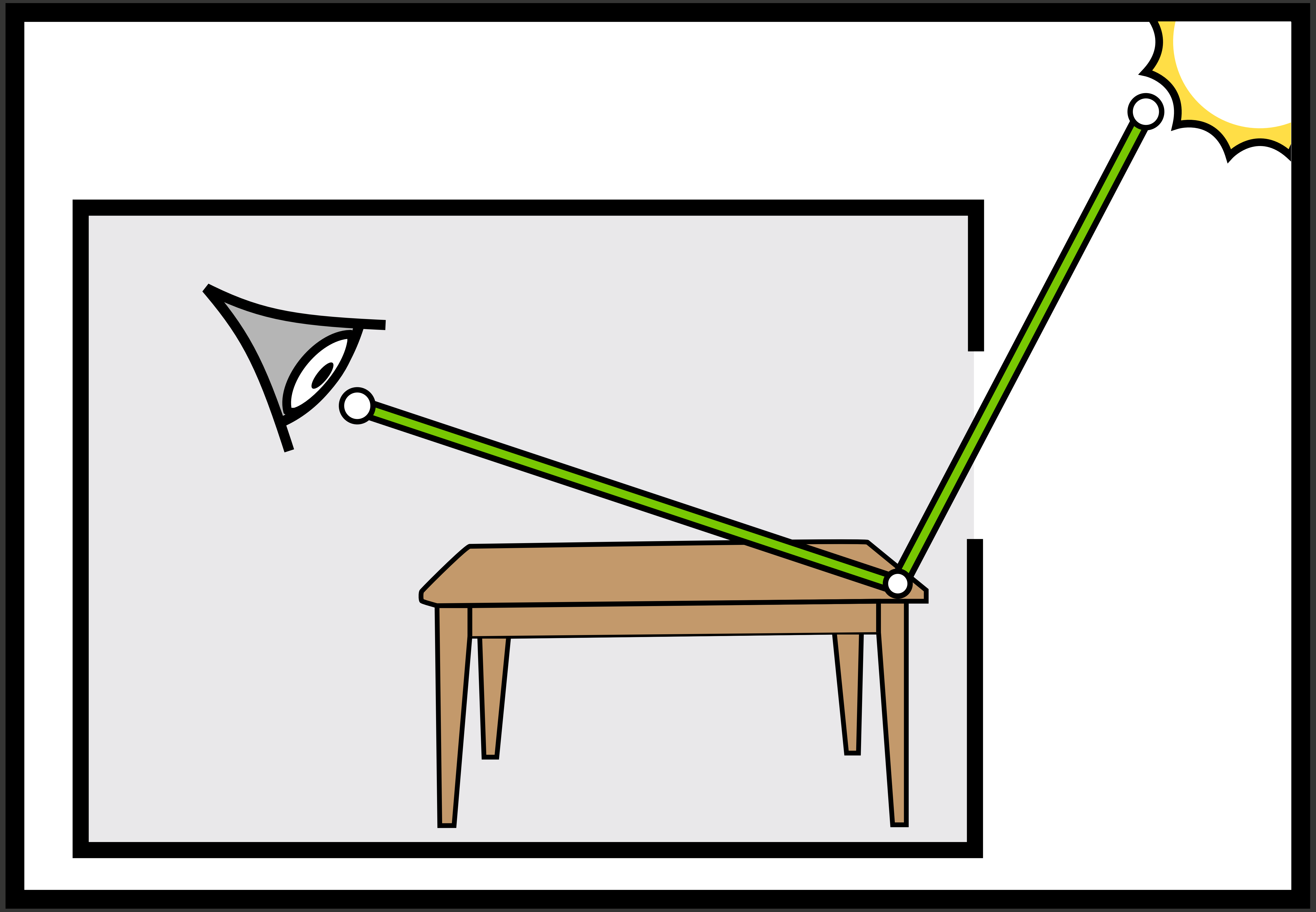
BDPT MLT

Metropolis Light Transport

[Veach & Guibas ’97]



Metropolis Light Transport
[Veach & Guibas ’97]



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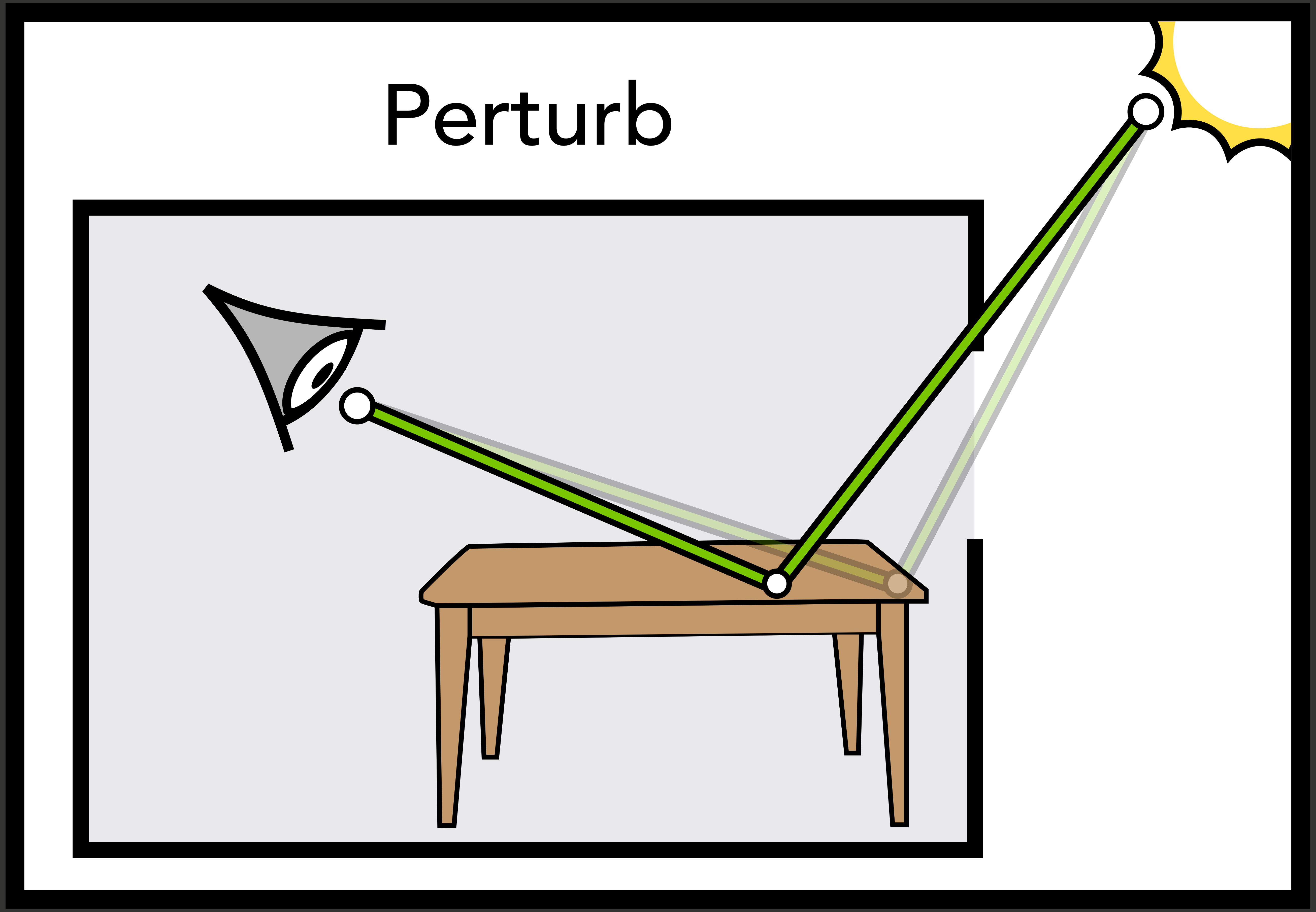

Metropolis Light Transport
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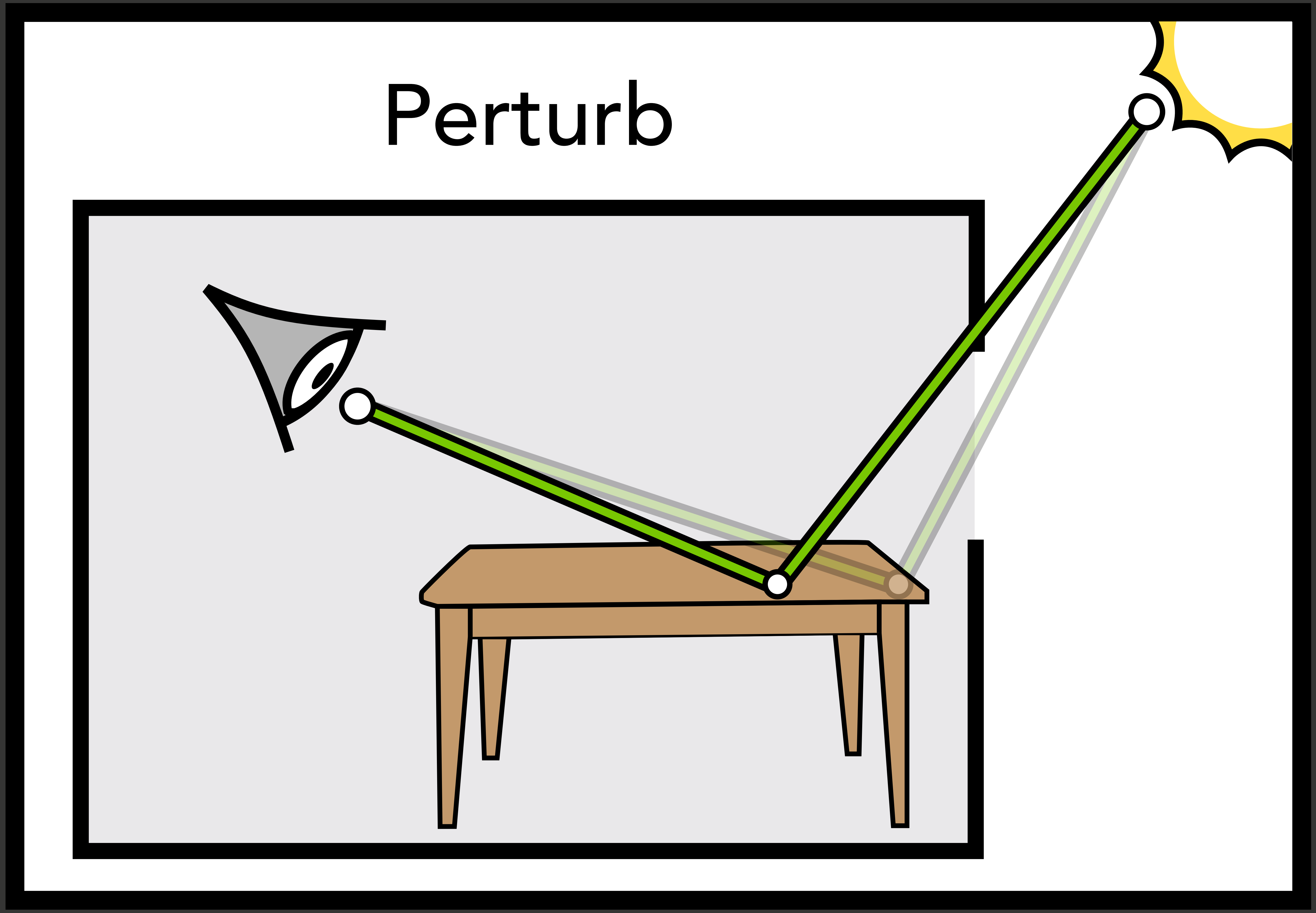
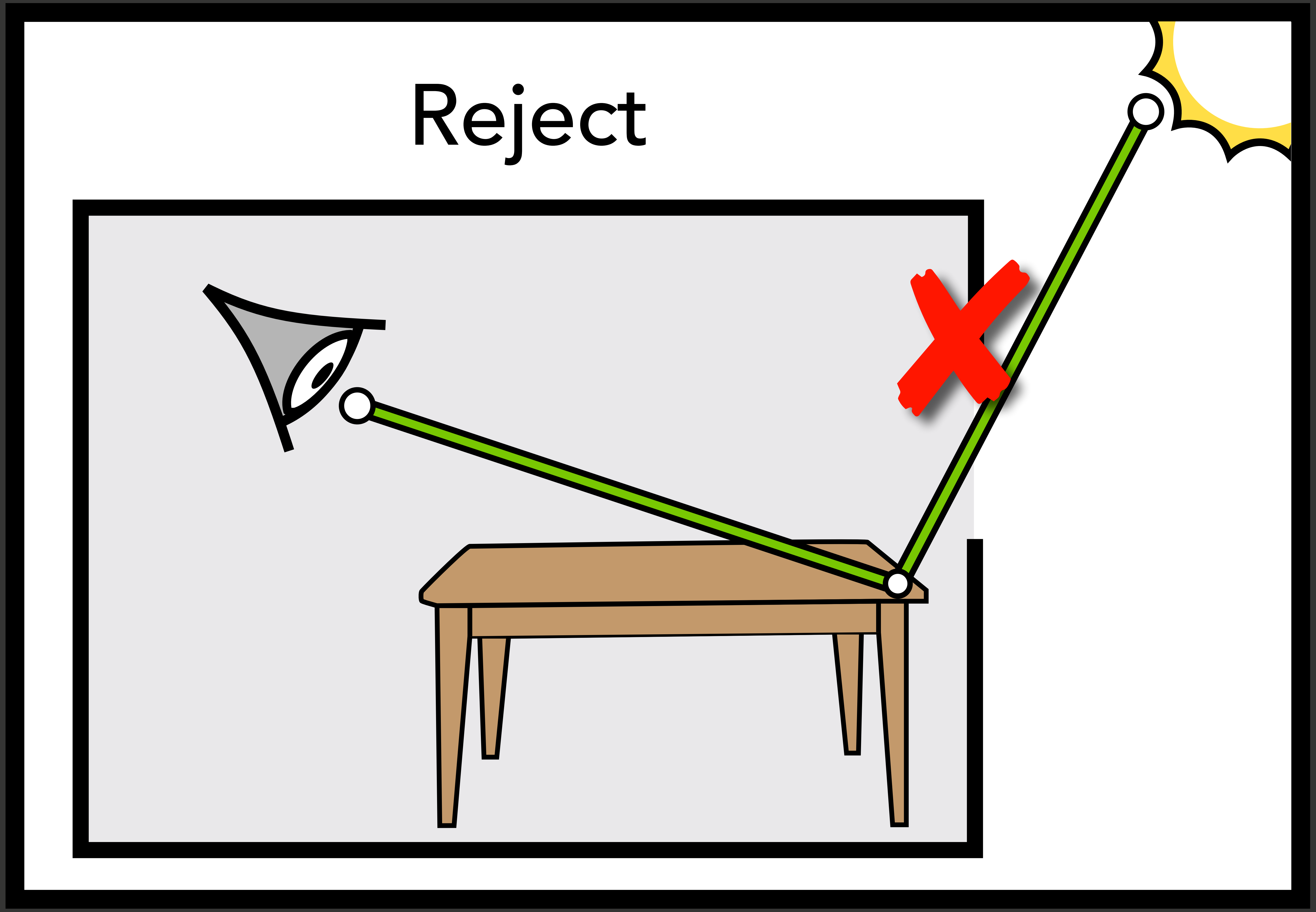


$$\Pr = \frac{f(\bar{\mathbf{v}})T(\bar{\mathbf{v}}, \bar{\mathbf{u}})}{f(\bar{\mathbf{u}})T(\bar{\mathbf{u}}, \bar{\mathbf{v}})}$$

Metropolis Light Transport
[Veach & Guibas ’97]



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Metropolis Light Transport

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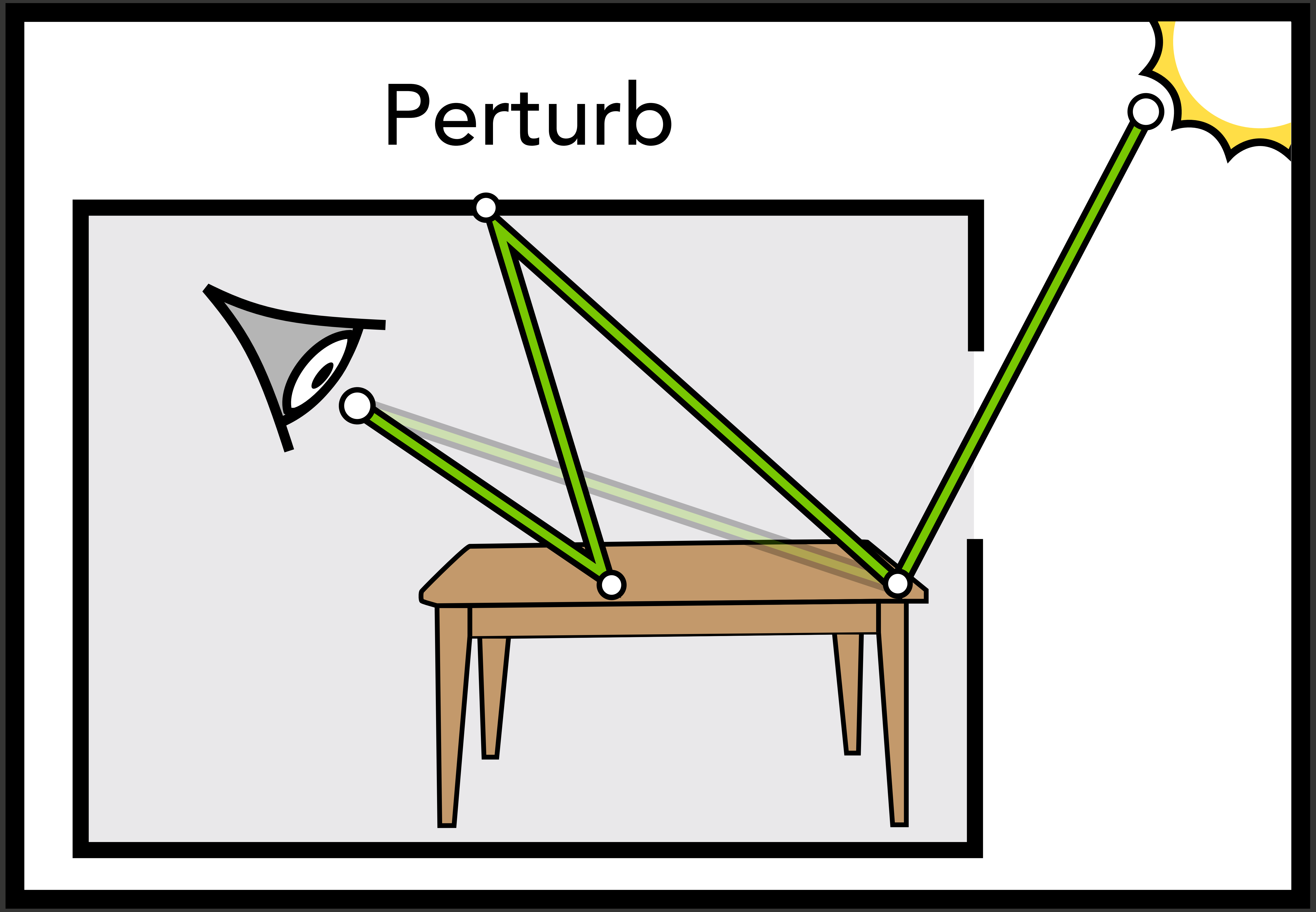

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Metropolis Light Transport
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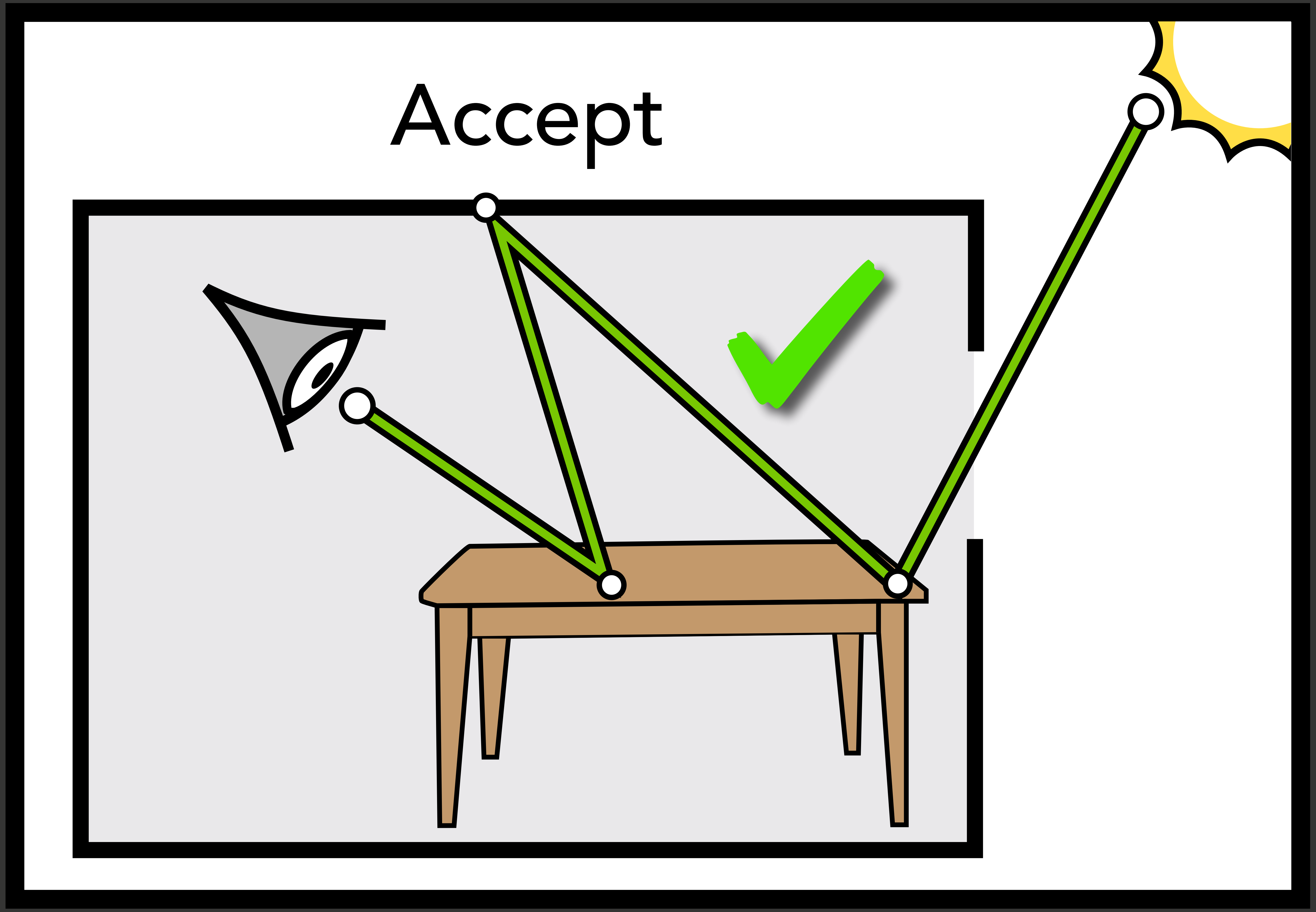

$$\mathrm{Pr} = \frac{f(\bar{\mathbf{v}})T(\bar{\mathbf{v}}, \bar{\mathbf{u}})}{f(\bar{\mathbf{u}})T(\bar{\mathbf{u}}, \bar{\mathbf{v}})}$$

Perturbations

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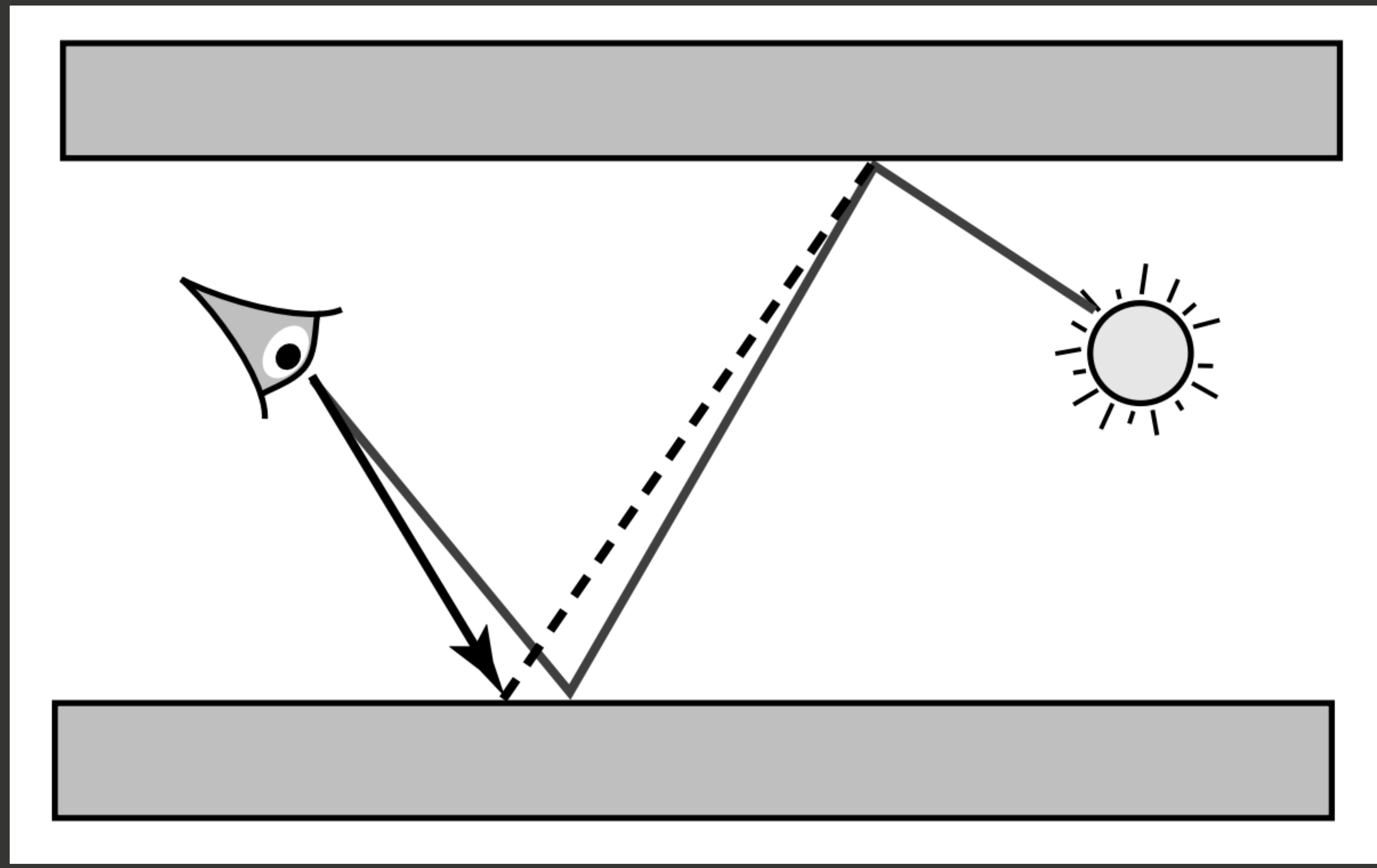
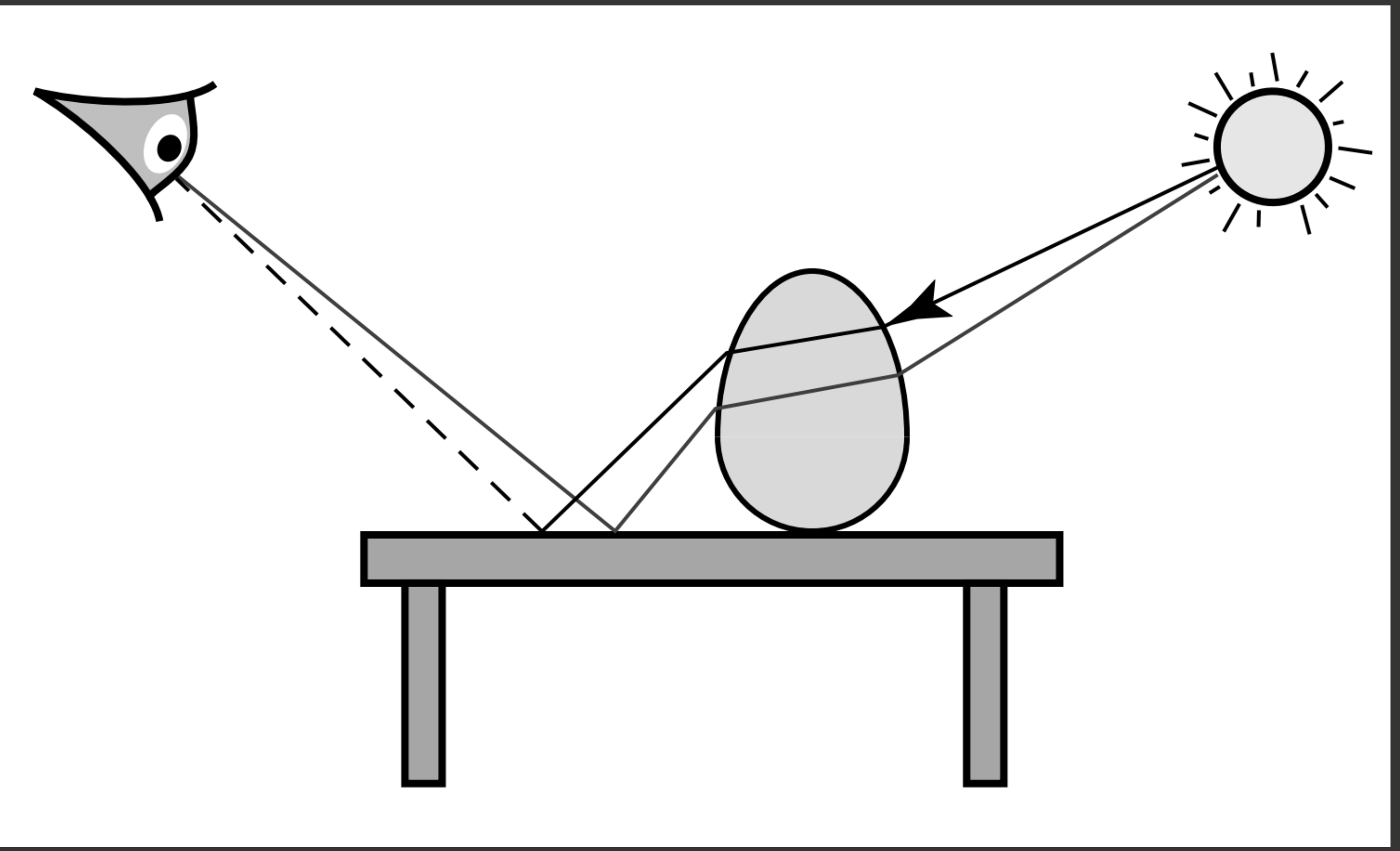
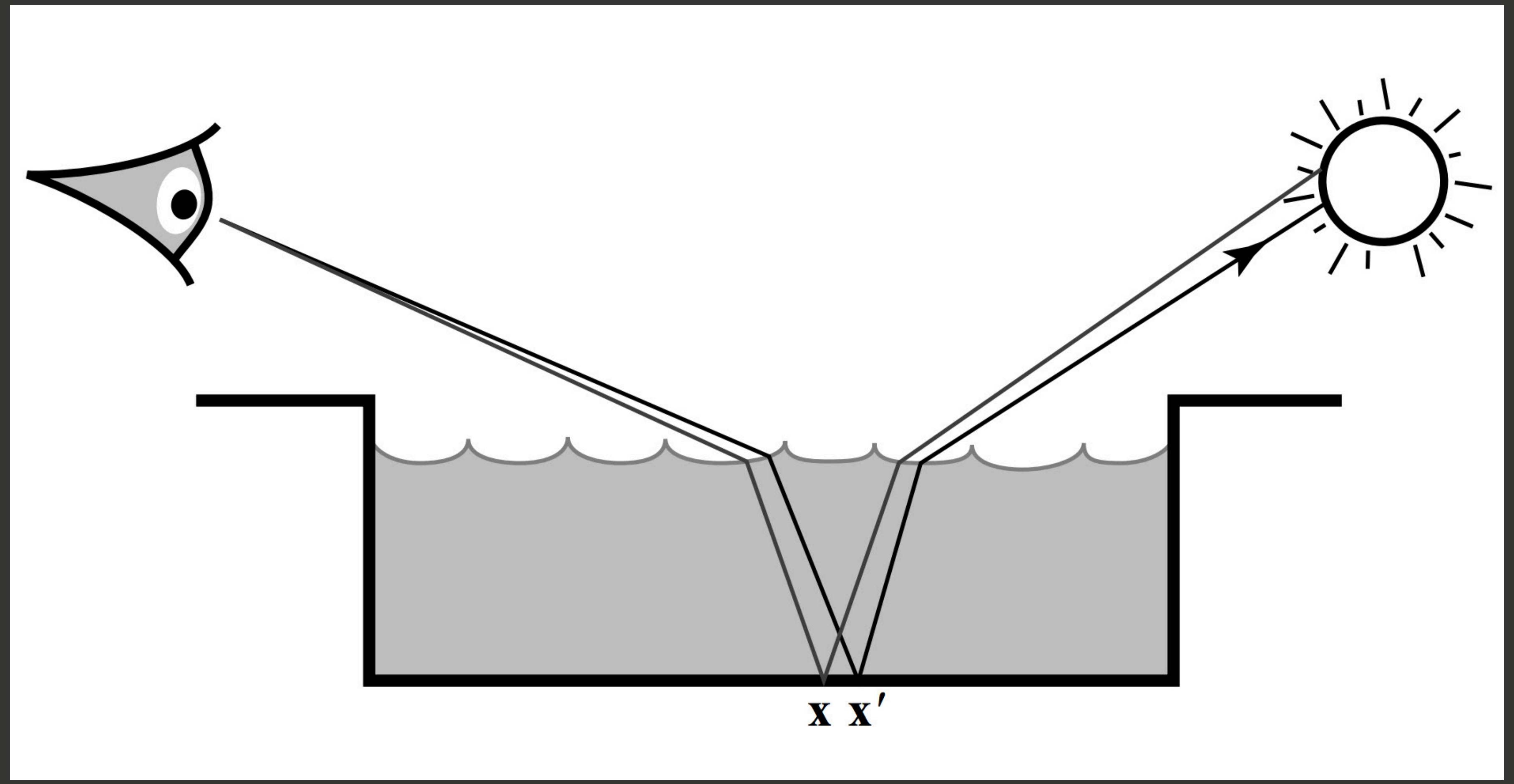


Lens Perturbation

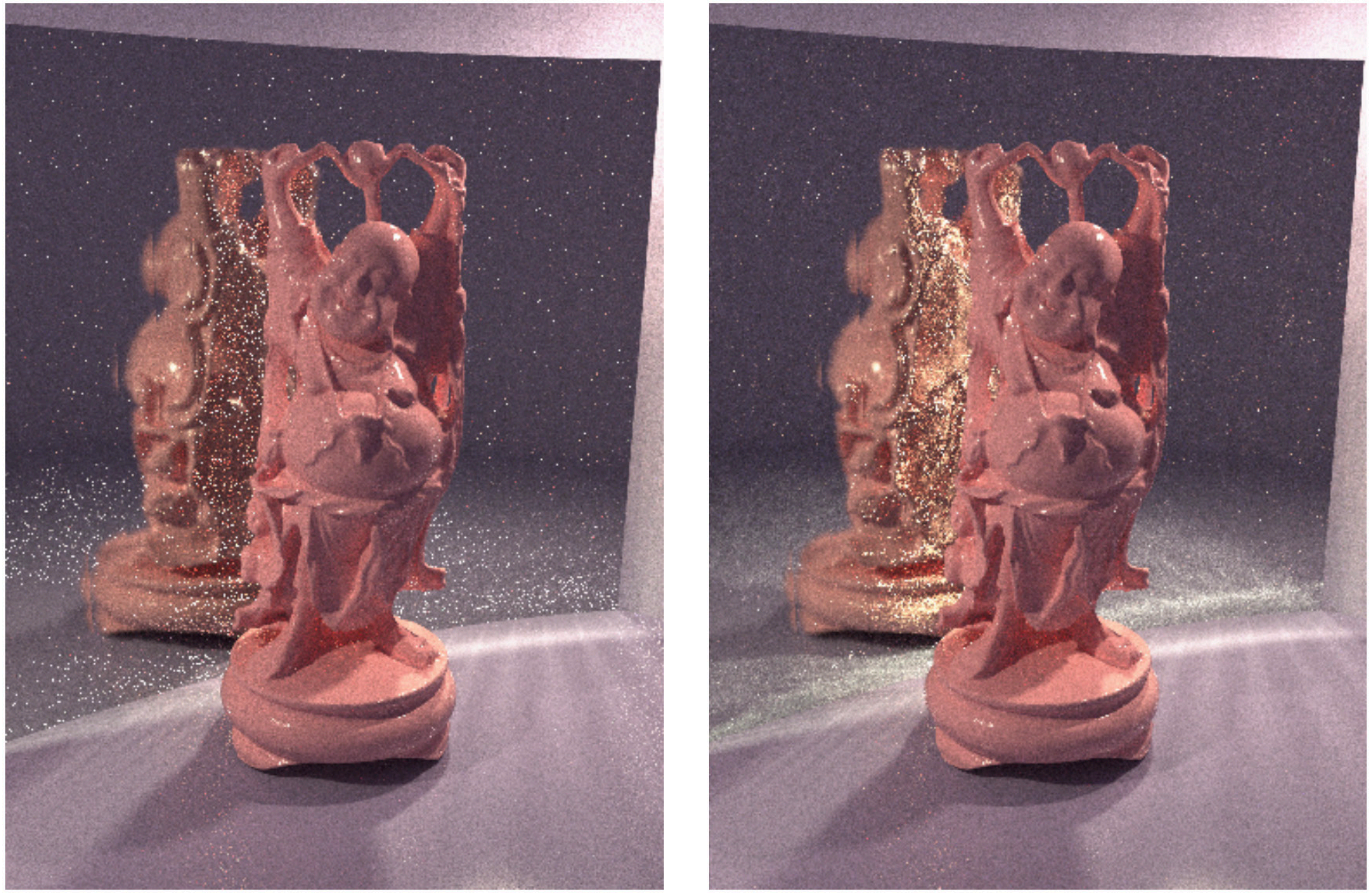


Caustic Perturbation



Multichain Perturbation

Primary Sample Space MLT
[Kelemen et al. ’02]



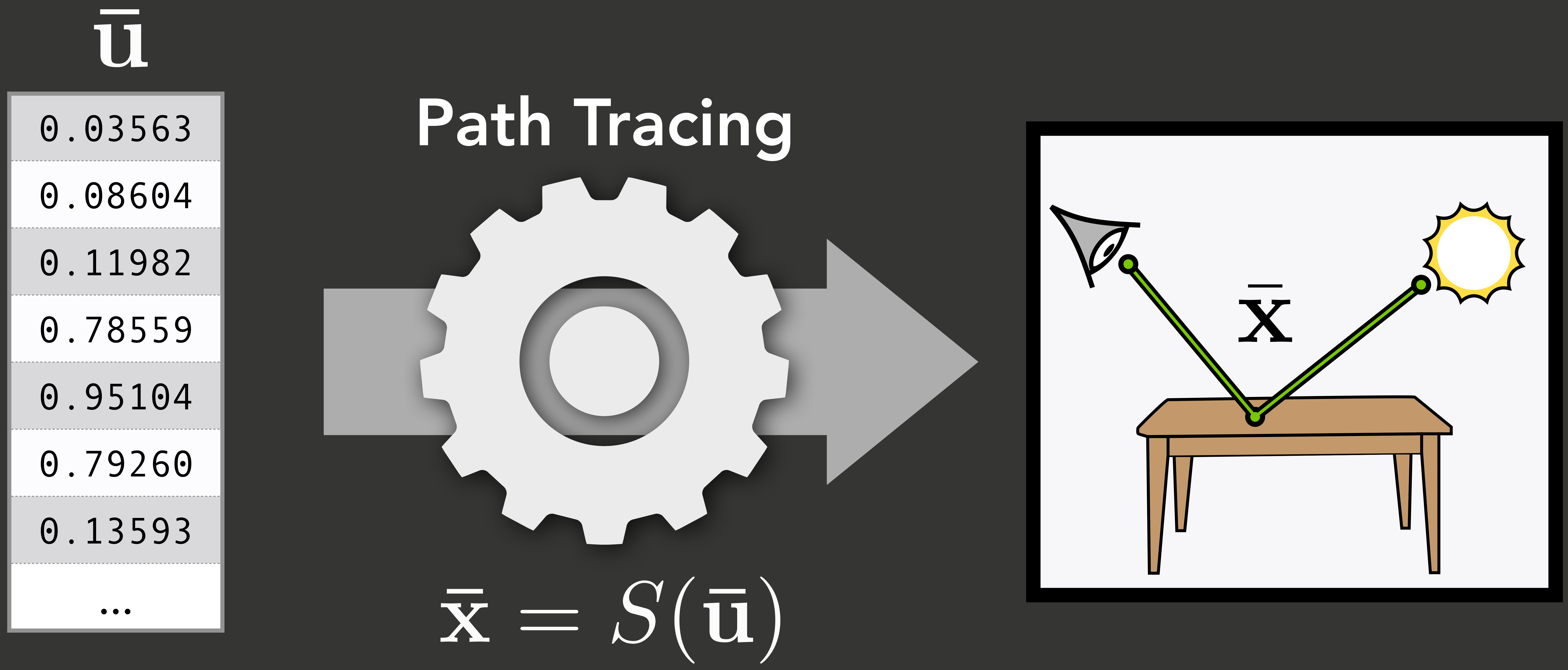
BDPT PSSMLT

Primary Sample Space MLT
[Kelemen et al. ’02]

Path Tracing

Primary Sample Space MLT

[Kelemen et al. '02]



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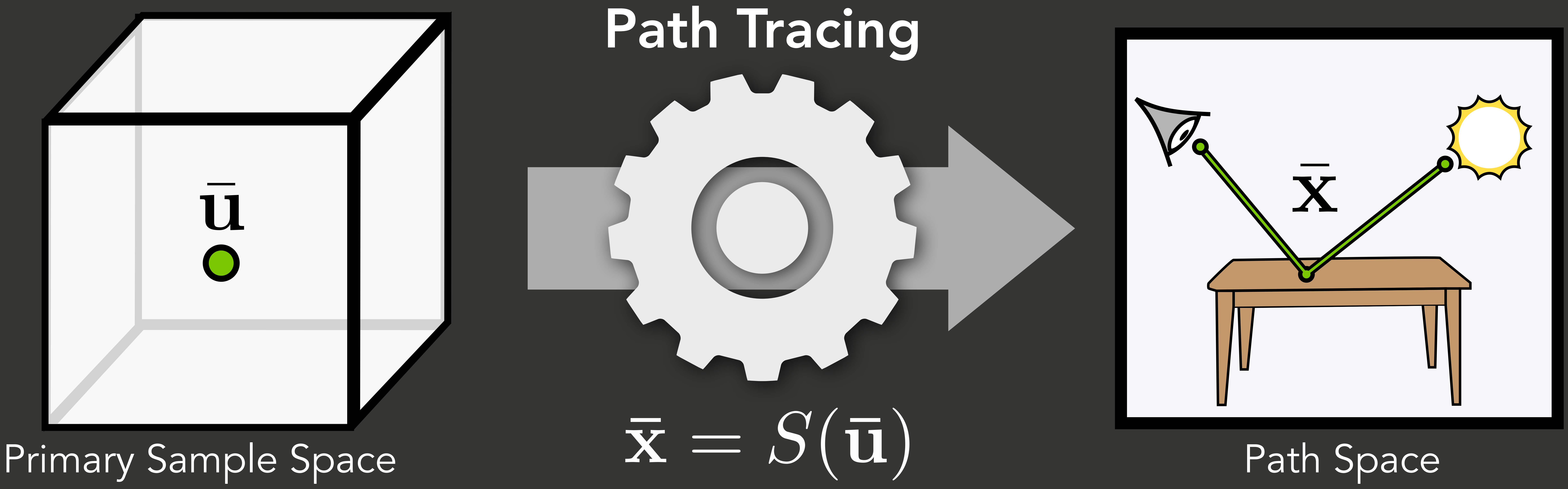

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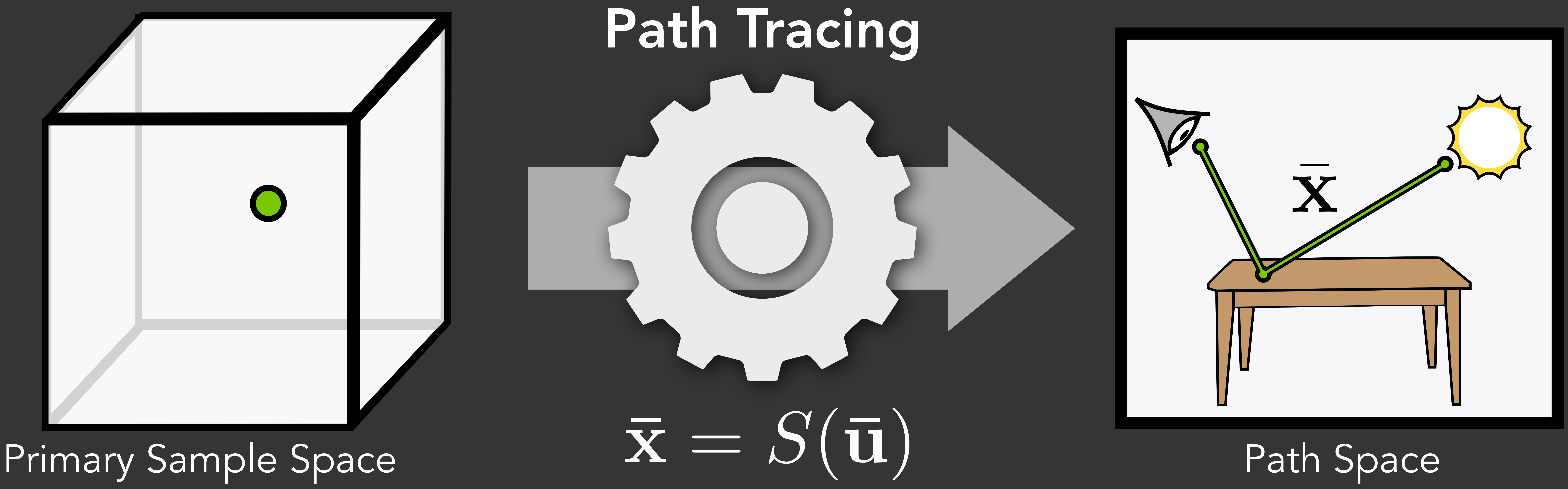

Primary Sample Space MLT
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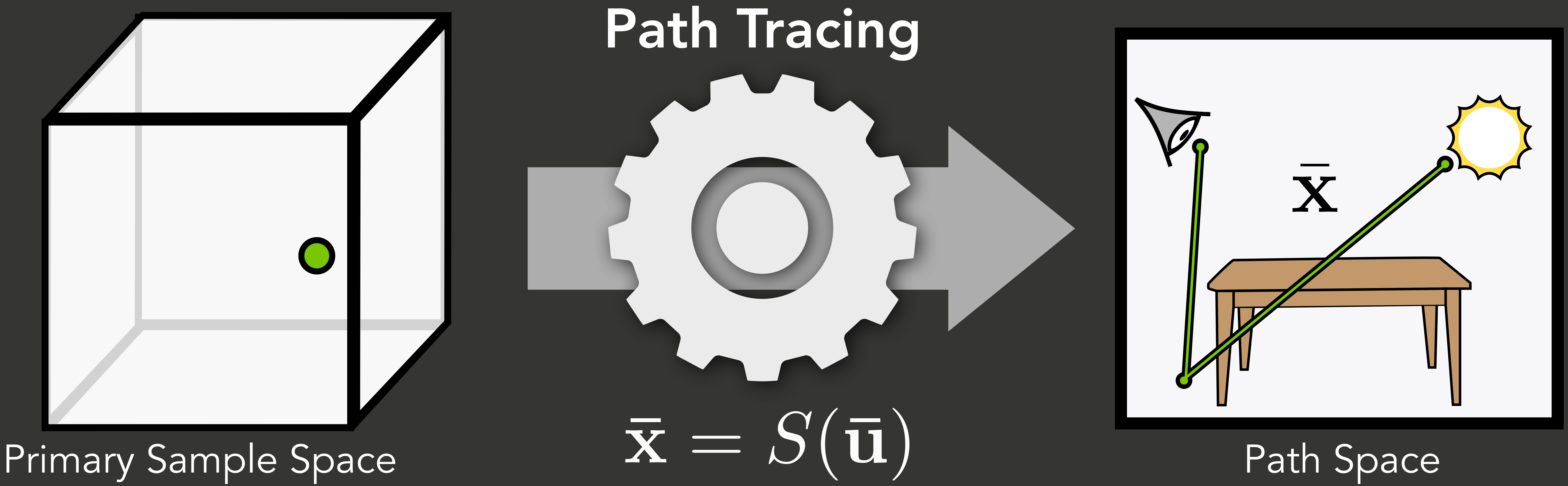
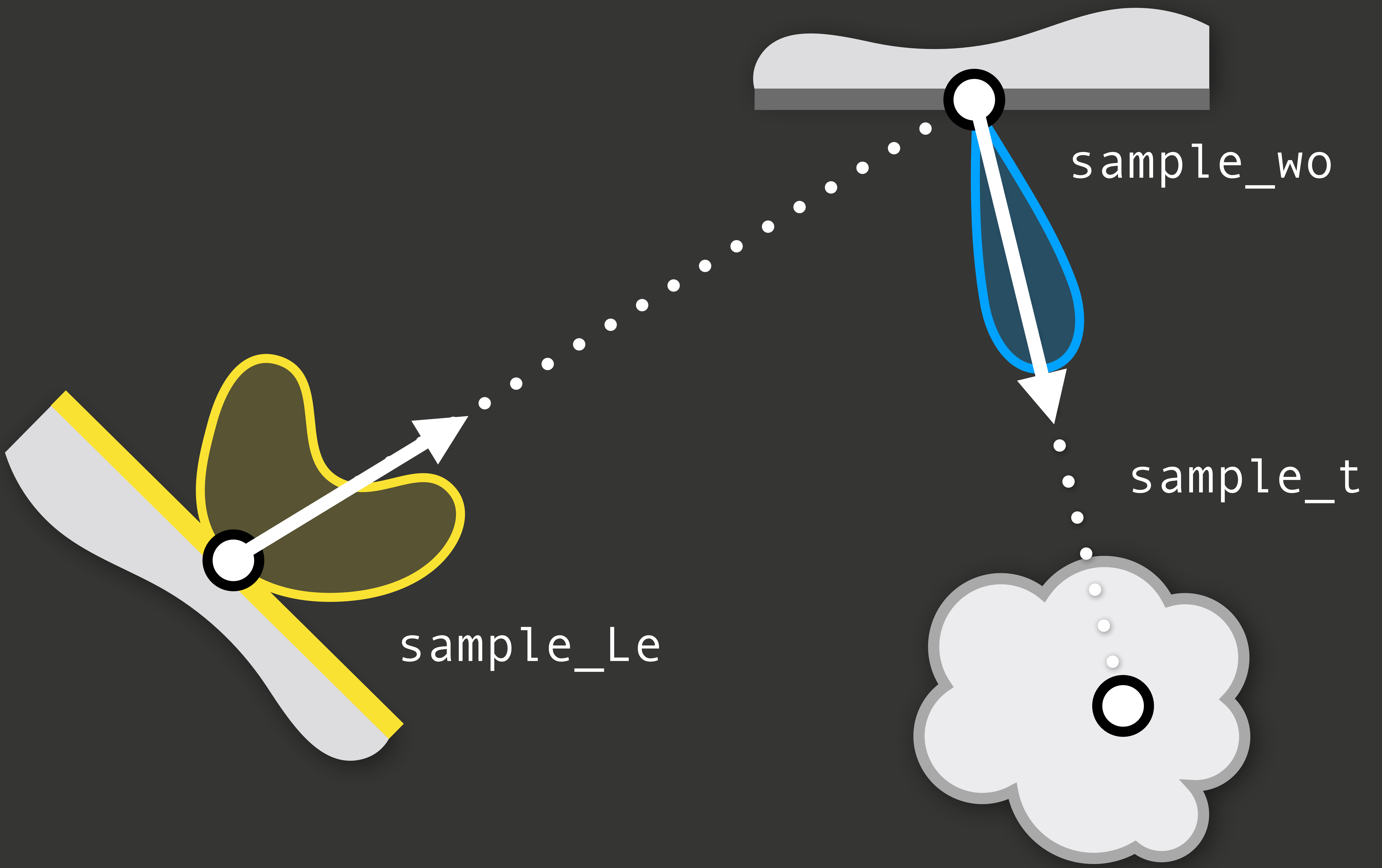


Inside $S(\bar{\mathbf{u}})$: Path Tracing

Inside $S(\bar{\mathrm{u}})$: Path Tracing

$\bar{\mathrm{u}}$
0.03563
0.08604
0.11982
0.78559
0.95104
0.79260
0.13593
...

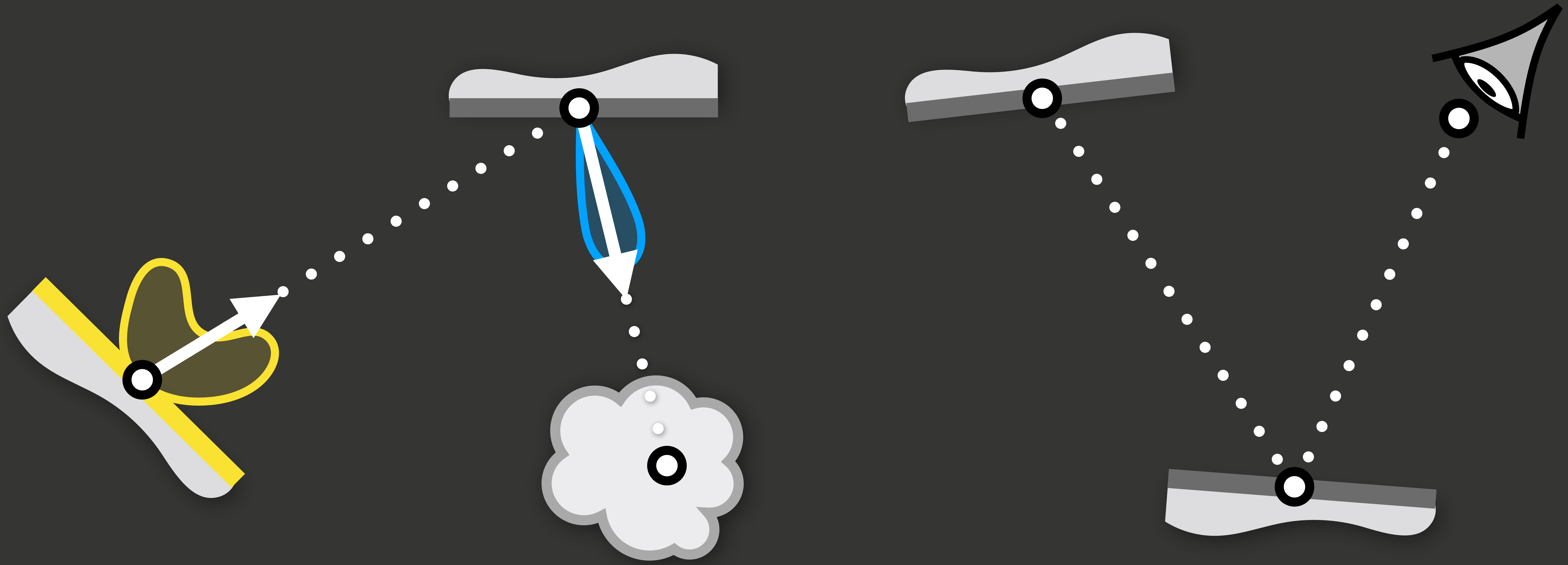
Inside $S(\bar{\mathrm{u}})$: Path Tracing



$\bar{\mathrm{u}}$

0.03563
0.08604
0.11982
0.78559
0.95104
0.79260
0.13593
…

Inside $S(\bar{\mathrm{u}})$: BDPT



$\bar{\mathrm{u}}$
0.03563
0.08604
0.11982
0.78559
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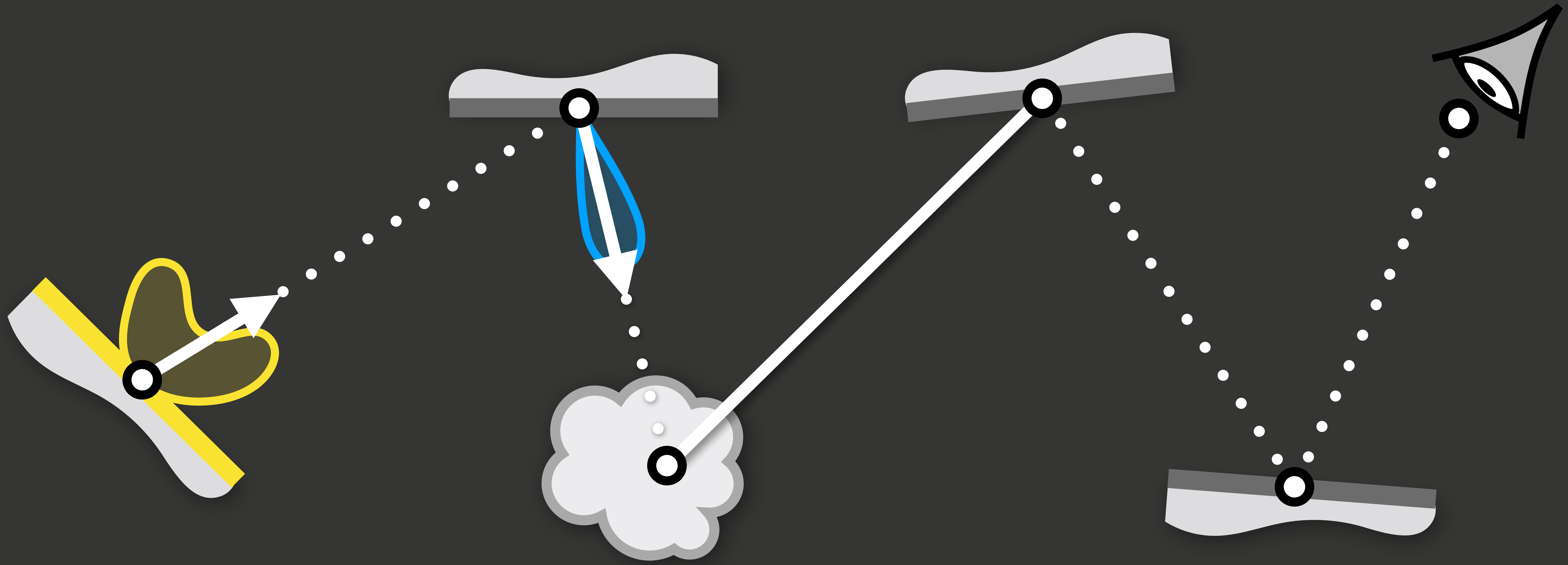
0.13593
...

Inside $S(\bar{\mathrm{u}})$: BDPT



$\bar{\mathrm{u}}$
0.03563
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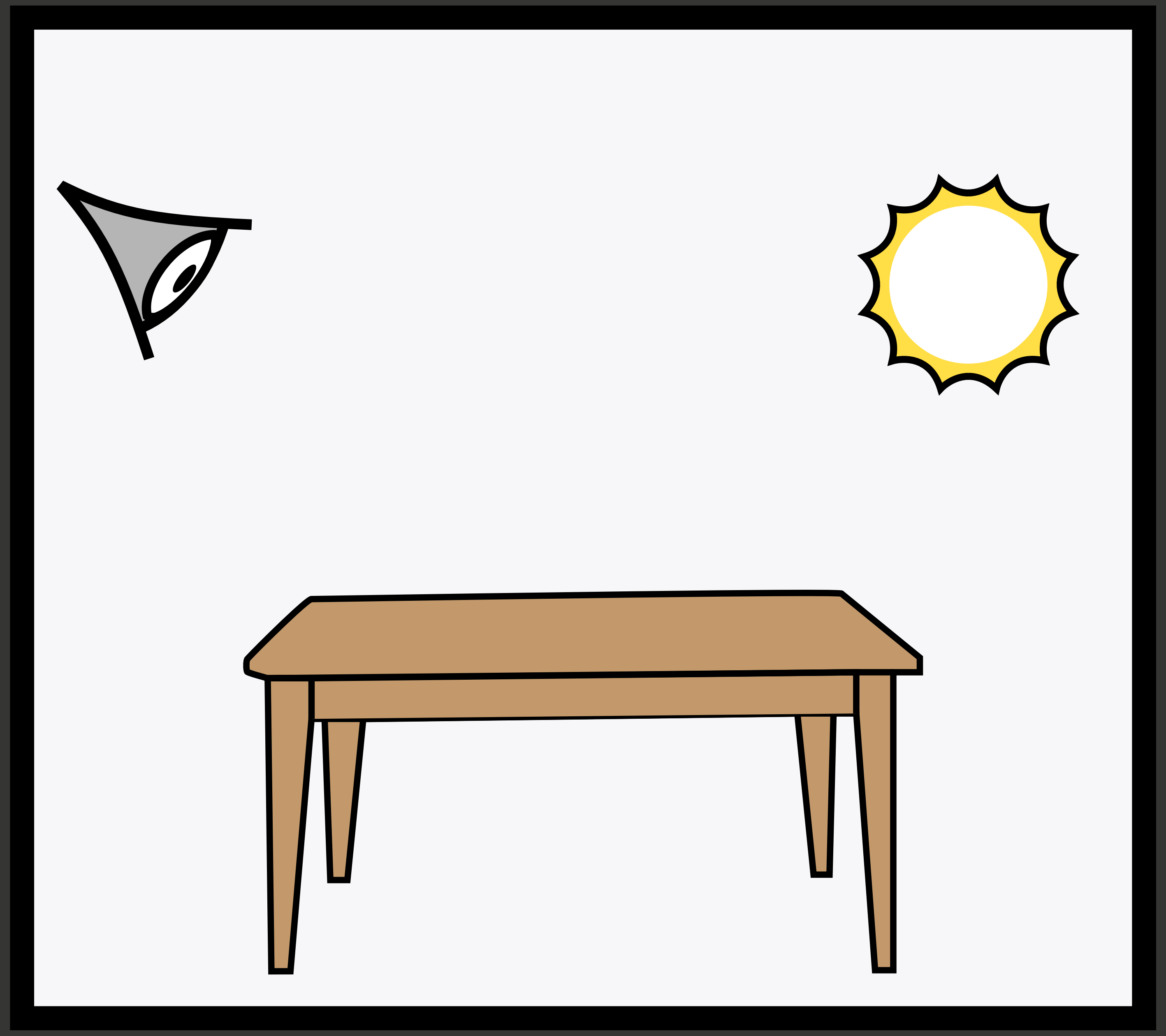
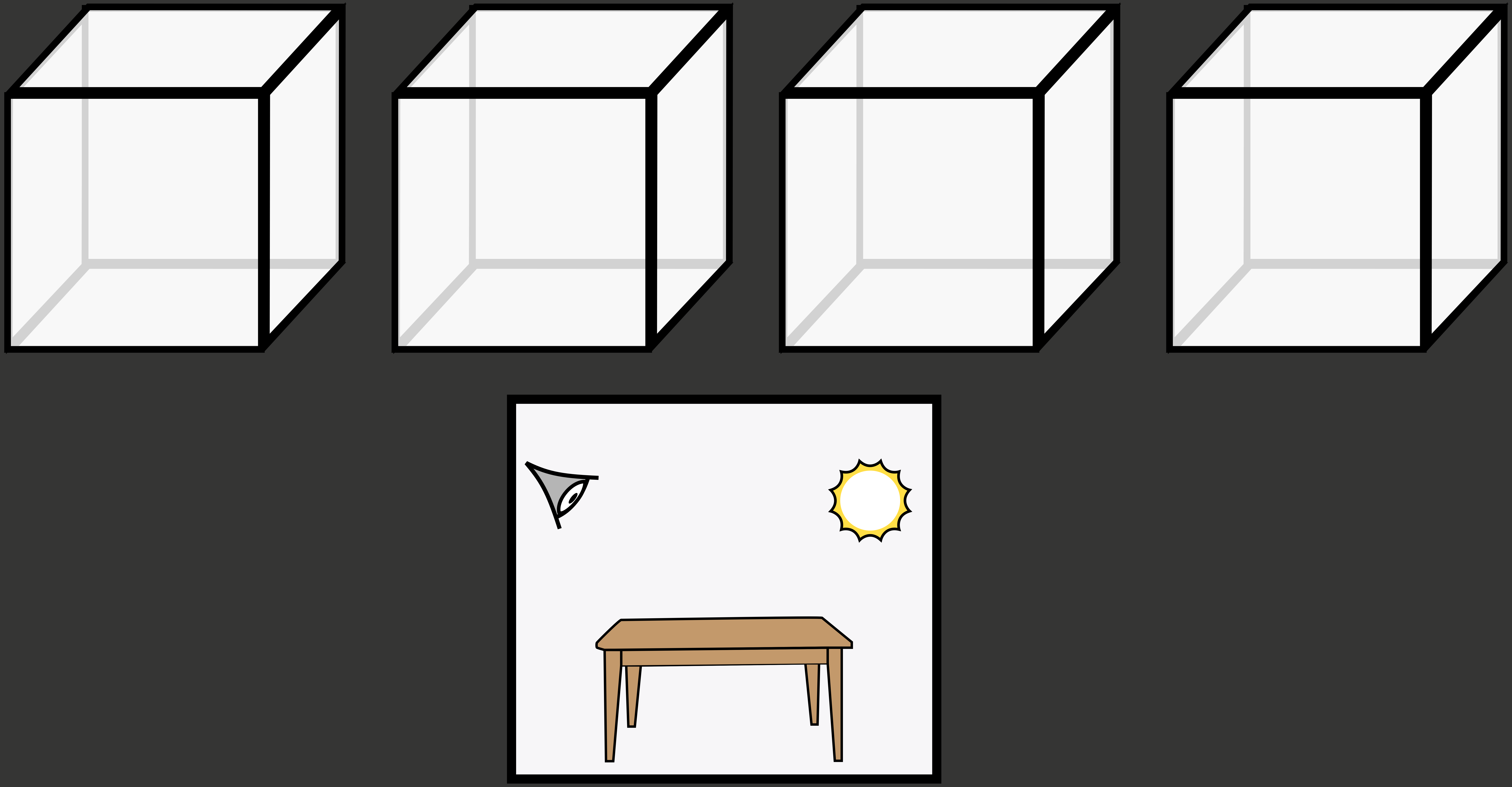
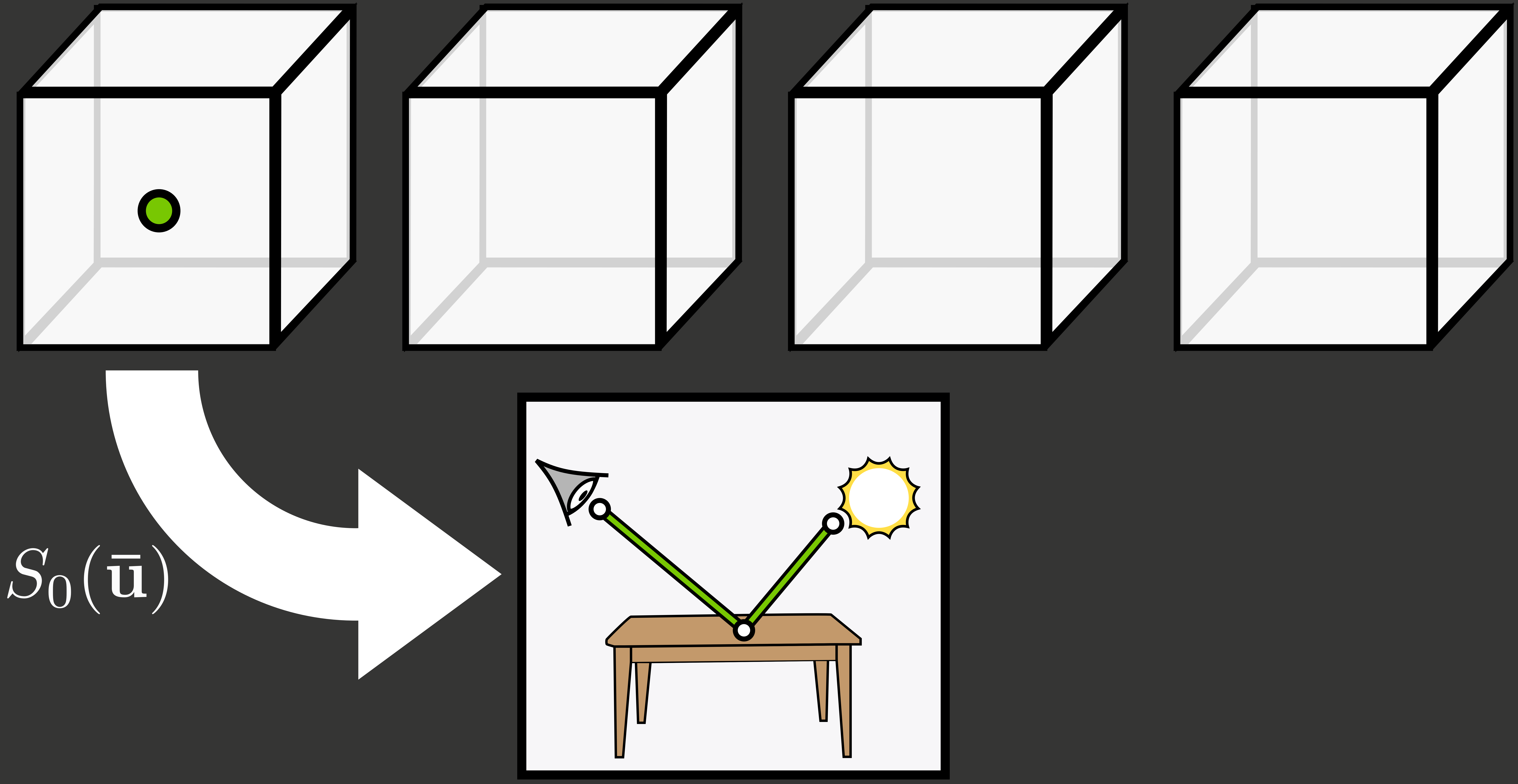
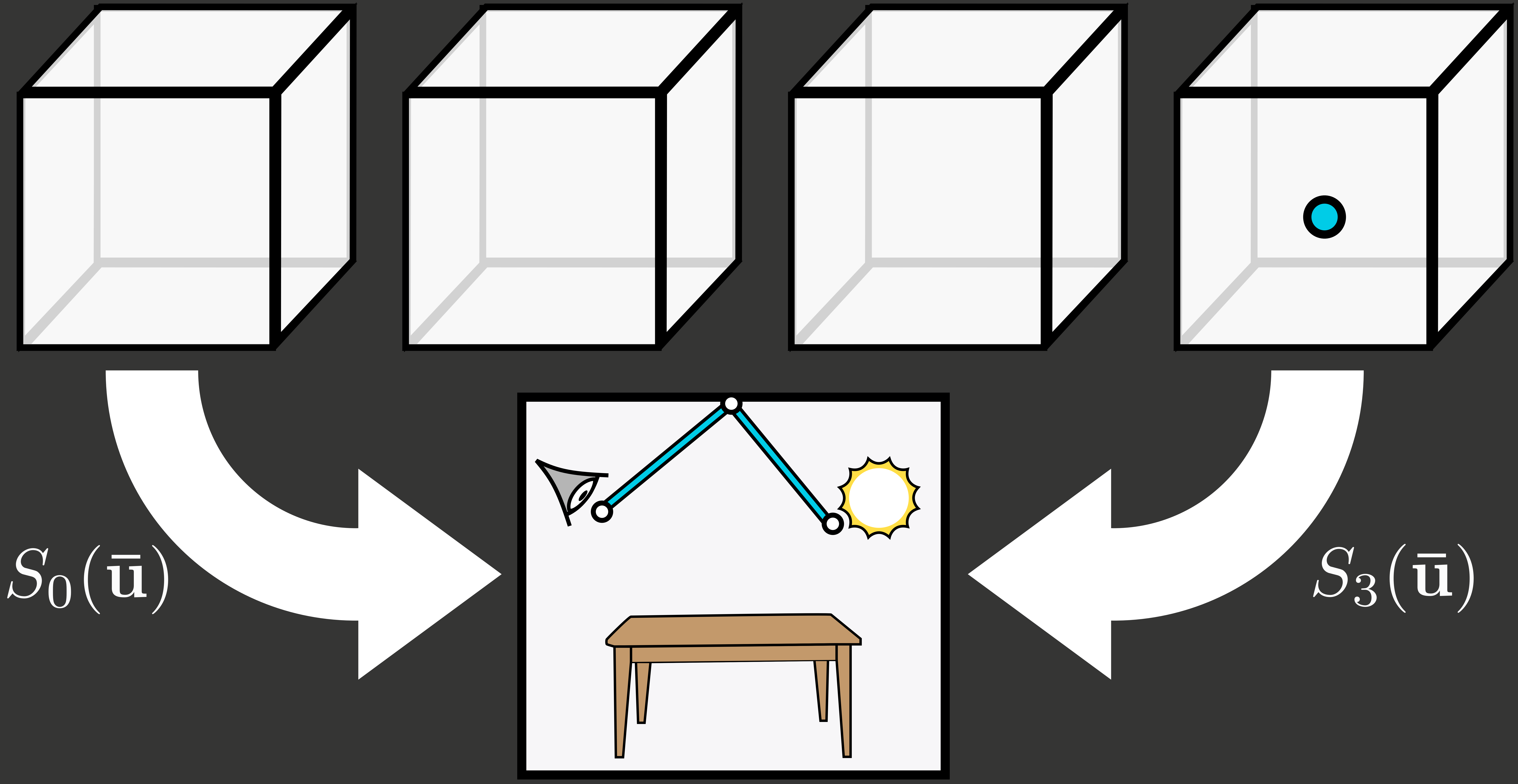
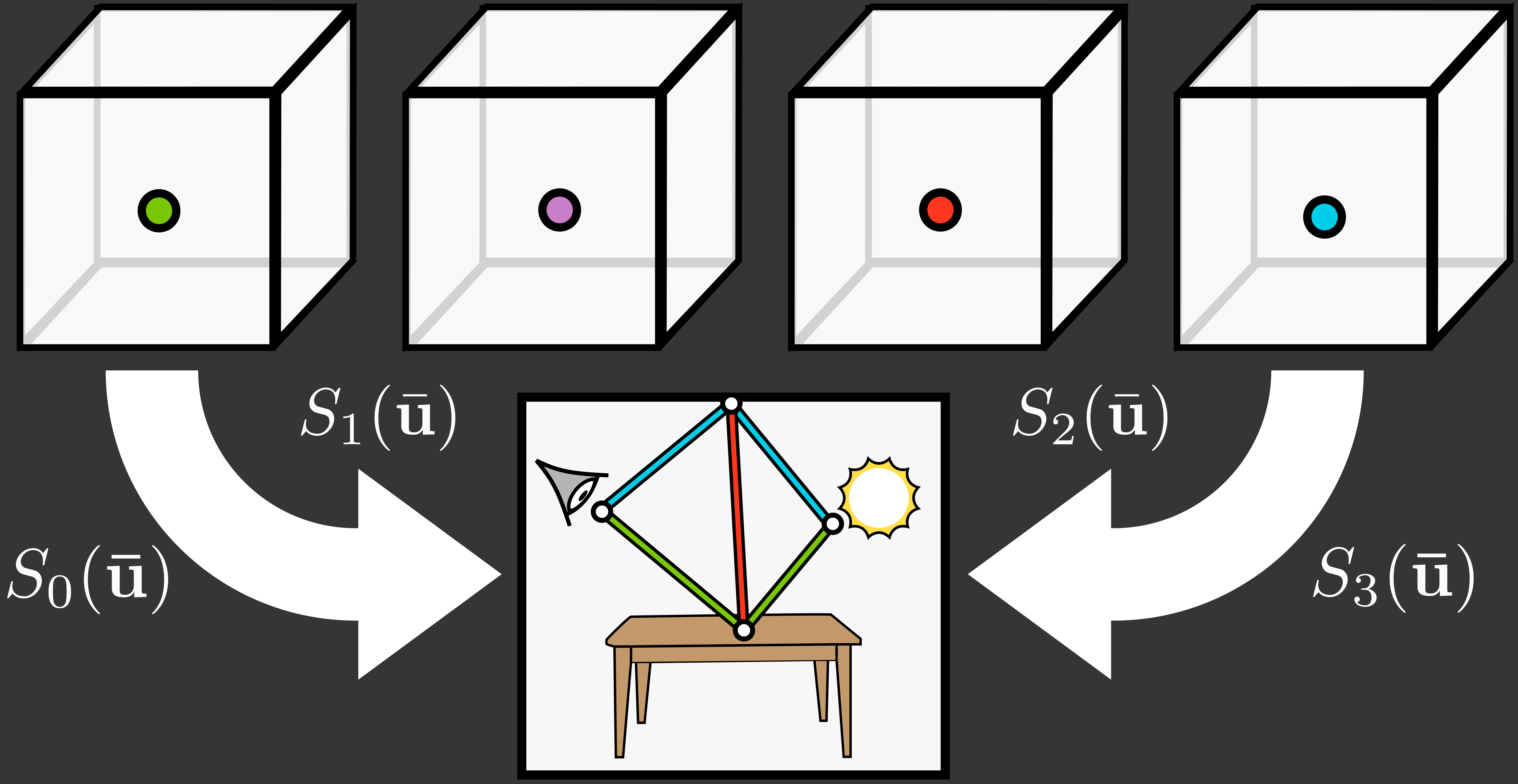





$S_0(\bar{\mathbf{u}})$

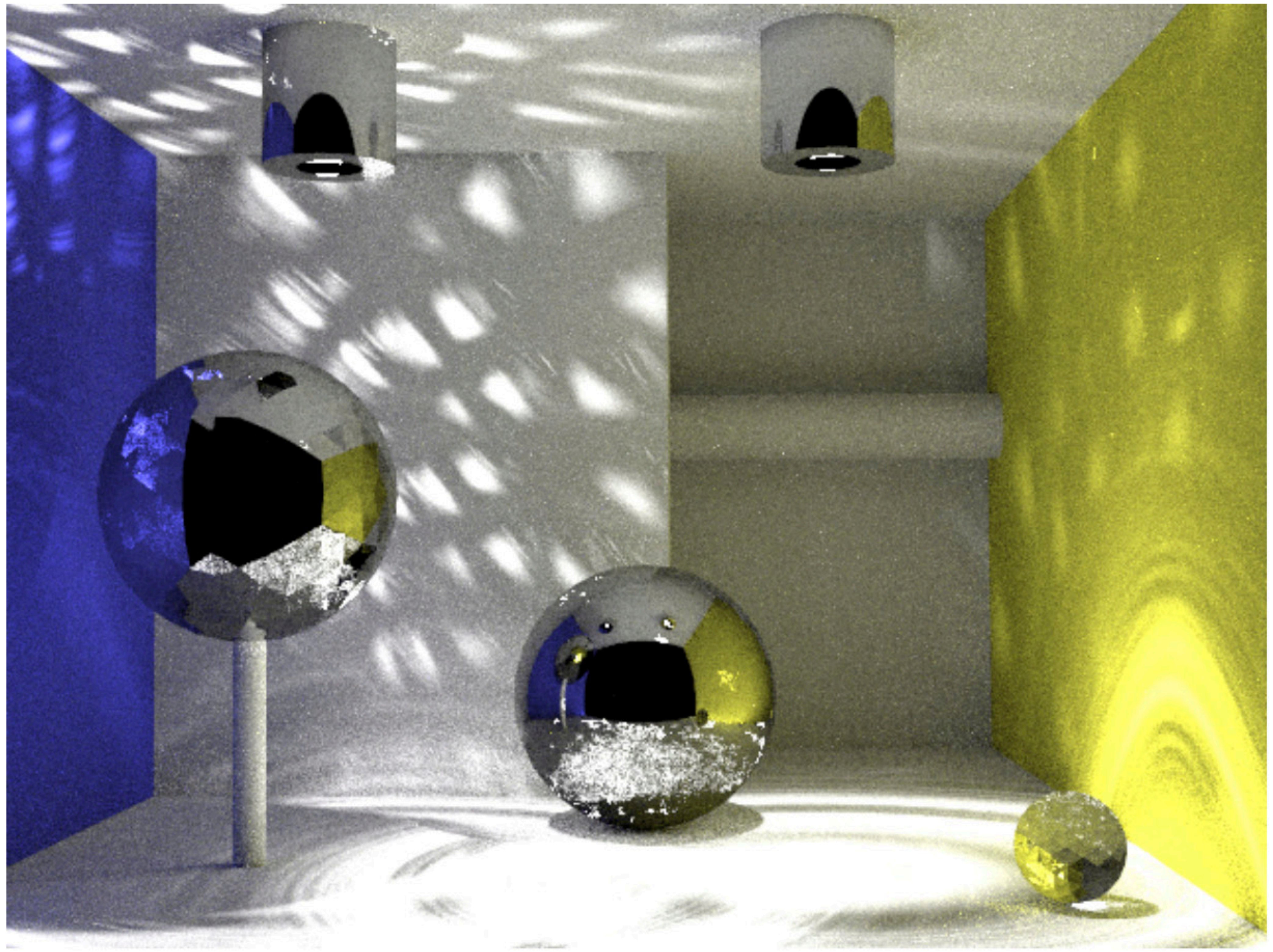

$S_0(\bar{\mathbf{u}})$
$S_3(\bar{\mathbf{u}})$

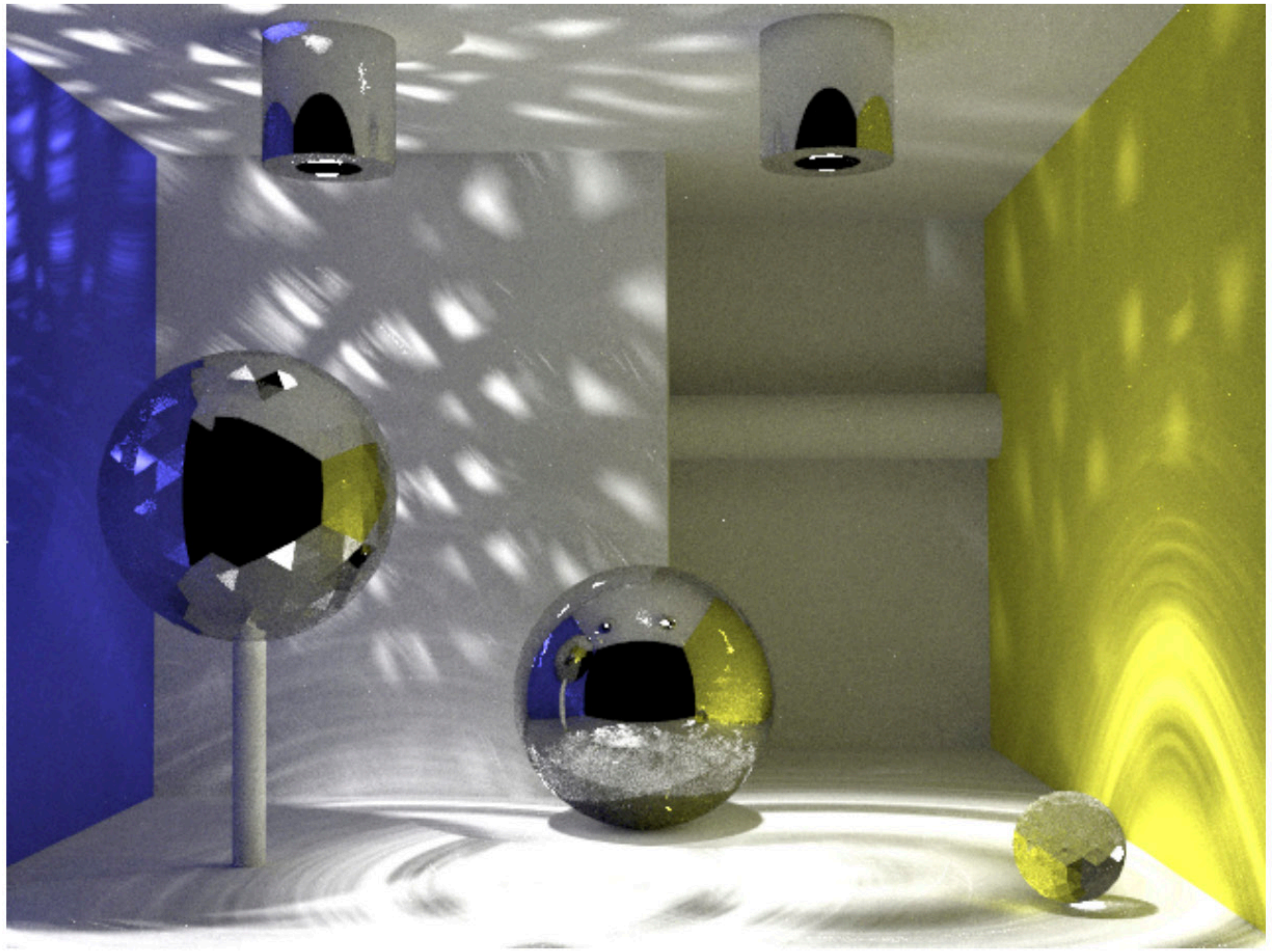


$S_1(\bar{\mathbf{u}})$
$S_2(\bar{\mathbf{u}})$
$S_0(\bar{\mathbf{u}})$
$S_3(\bar{\mathbf{u}})$

Multiplexed MLT

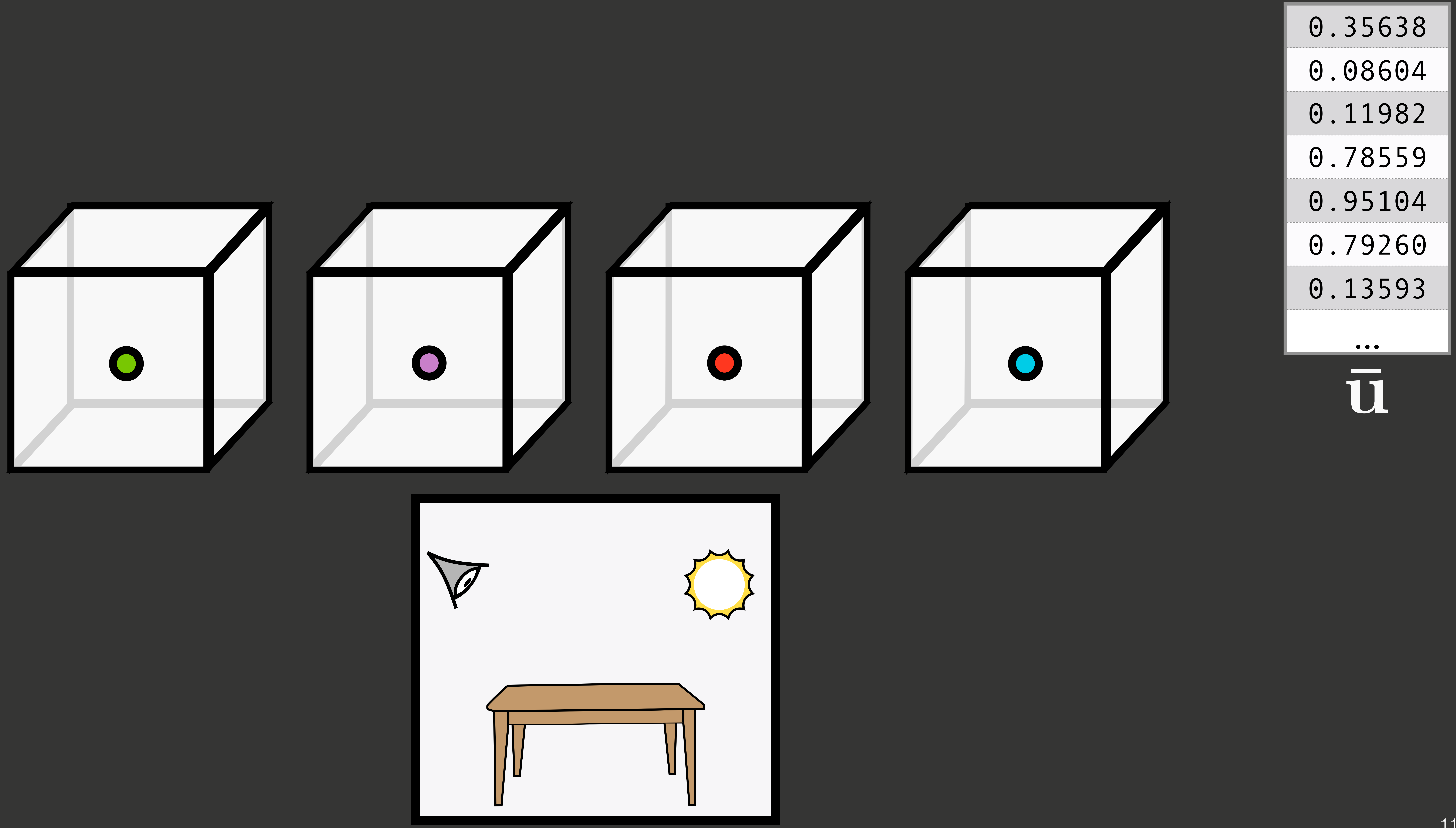
[Hachisuka et al. ’14]





MLT

Multiplexed MLT


0.35638
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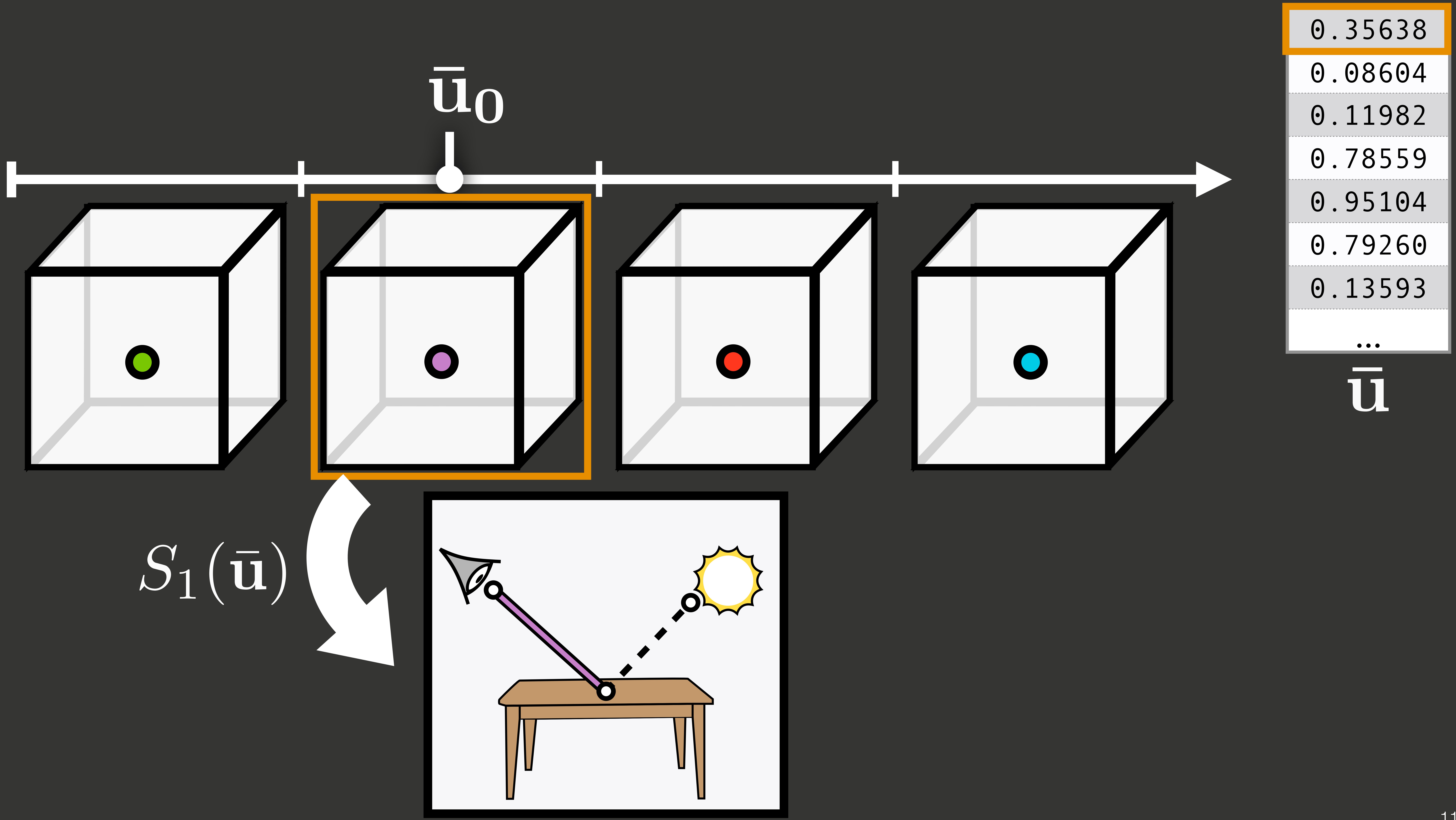
...
$\bar{u}$


$\bar{u}_0$
0.35638
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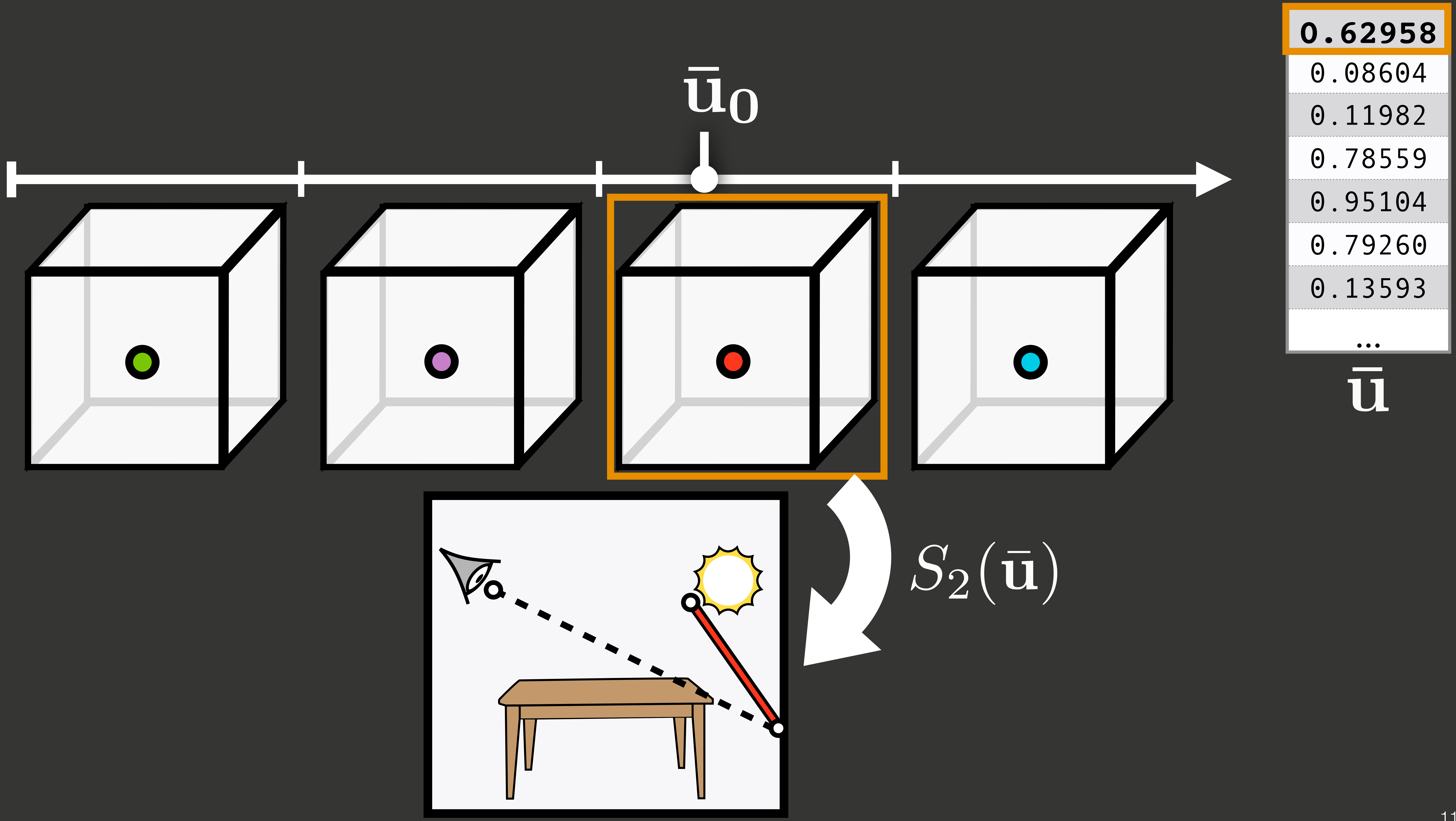
0.78559
0.95104
0.79260
0.13593
...
$\bar{u}$
$S_1(\bar{u})$



$\bar{u}_0$
0.62958
0.08604
0.11982
0.78559
0.95104
0.79260
0.13593
...
$\bar{u}$
$S_2(\bar{u})$

Summary

- Good perturbations are key
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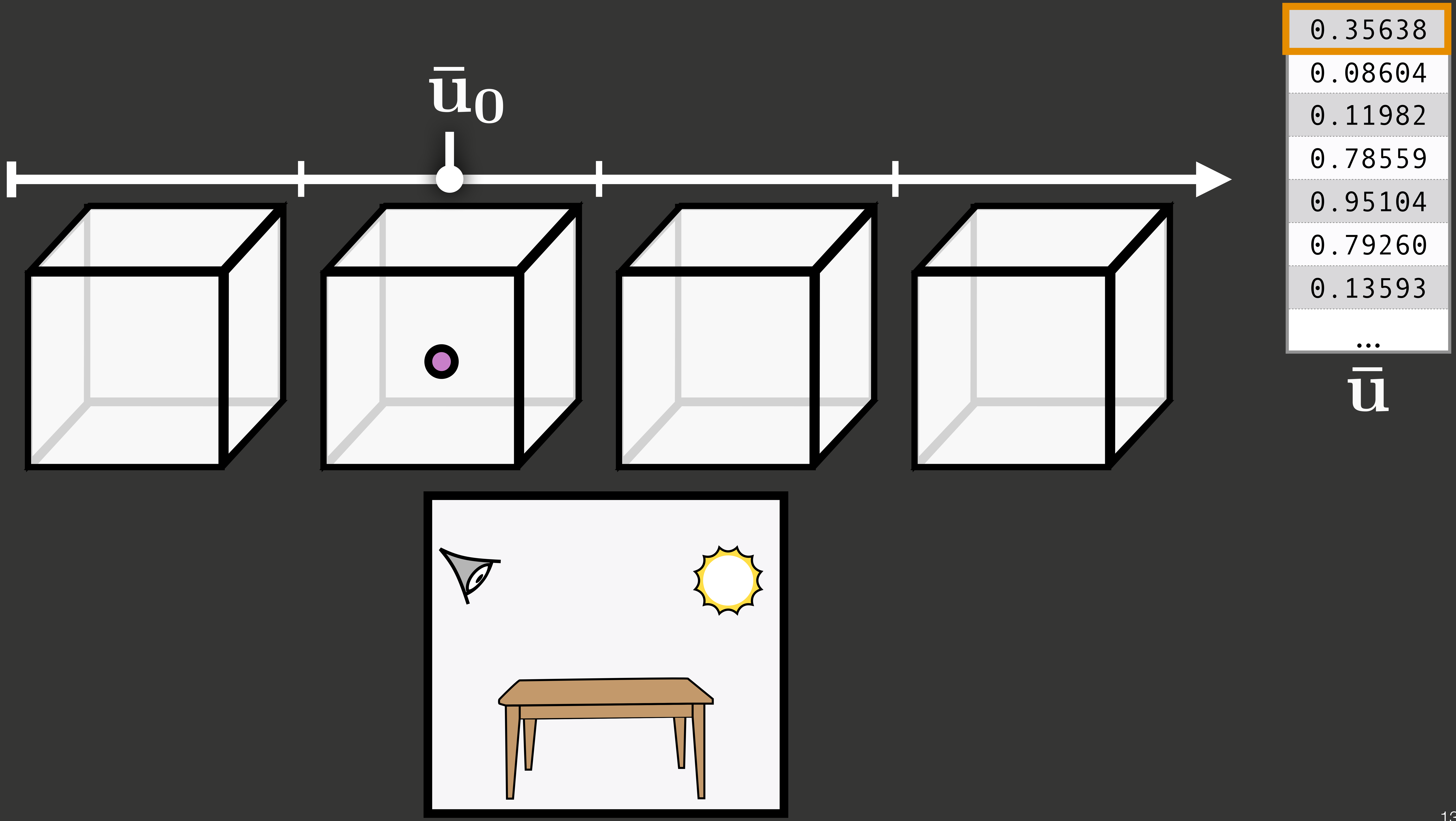

$\bar{u}_0$
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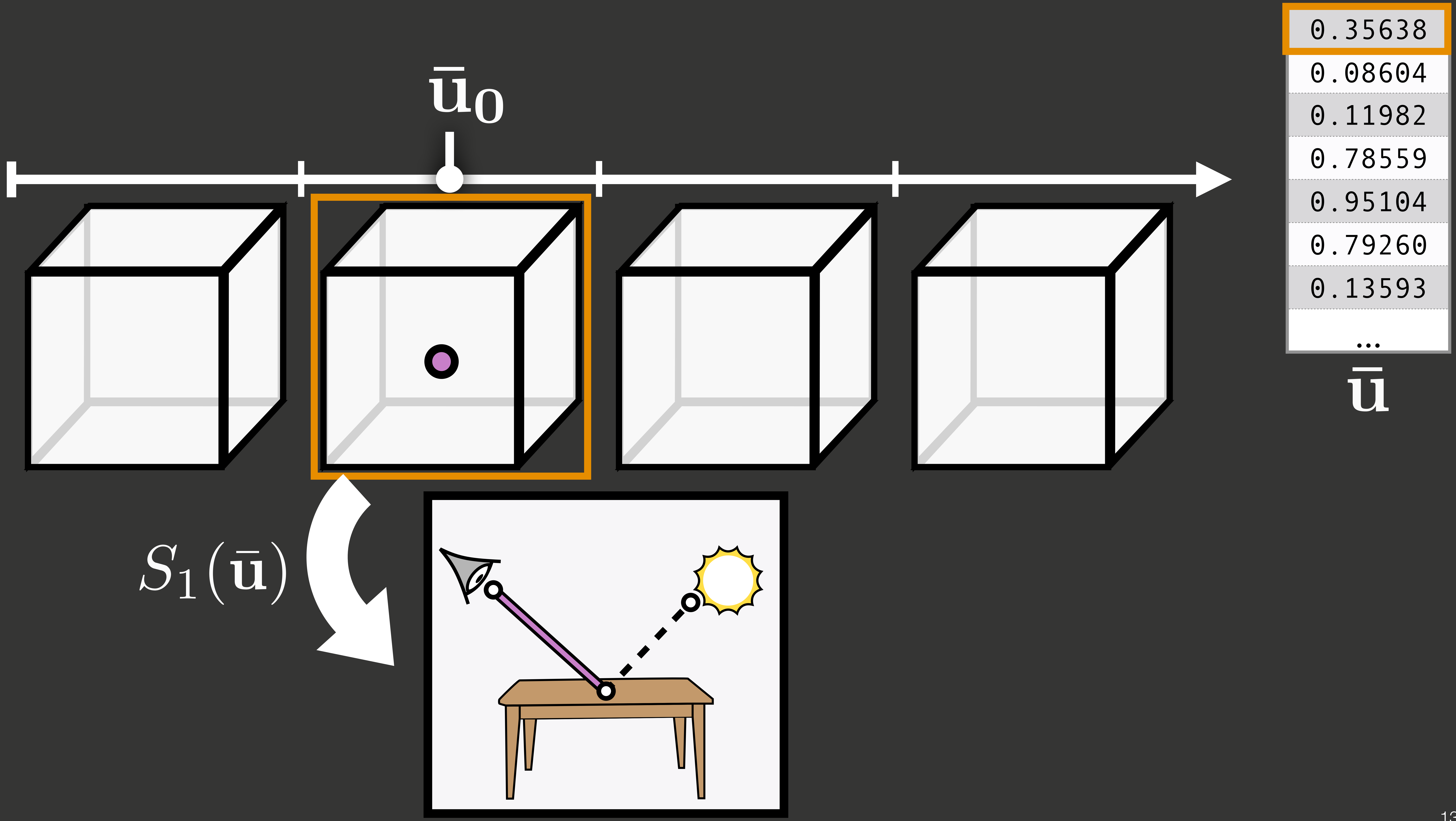
$\bar{u}_0$
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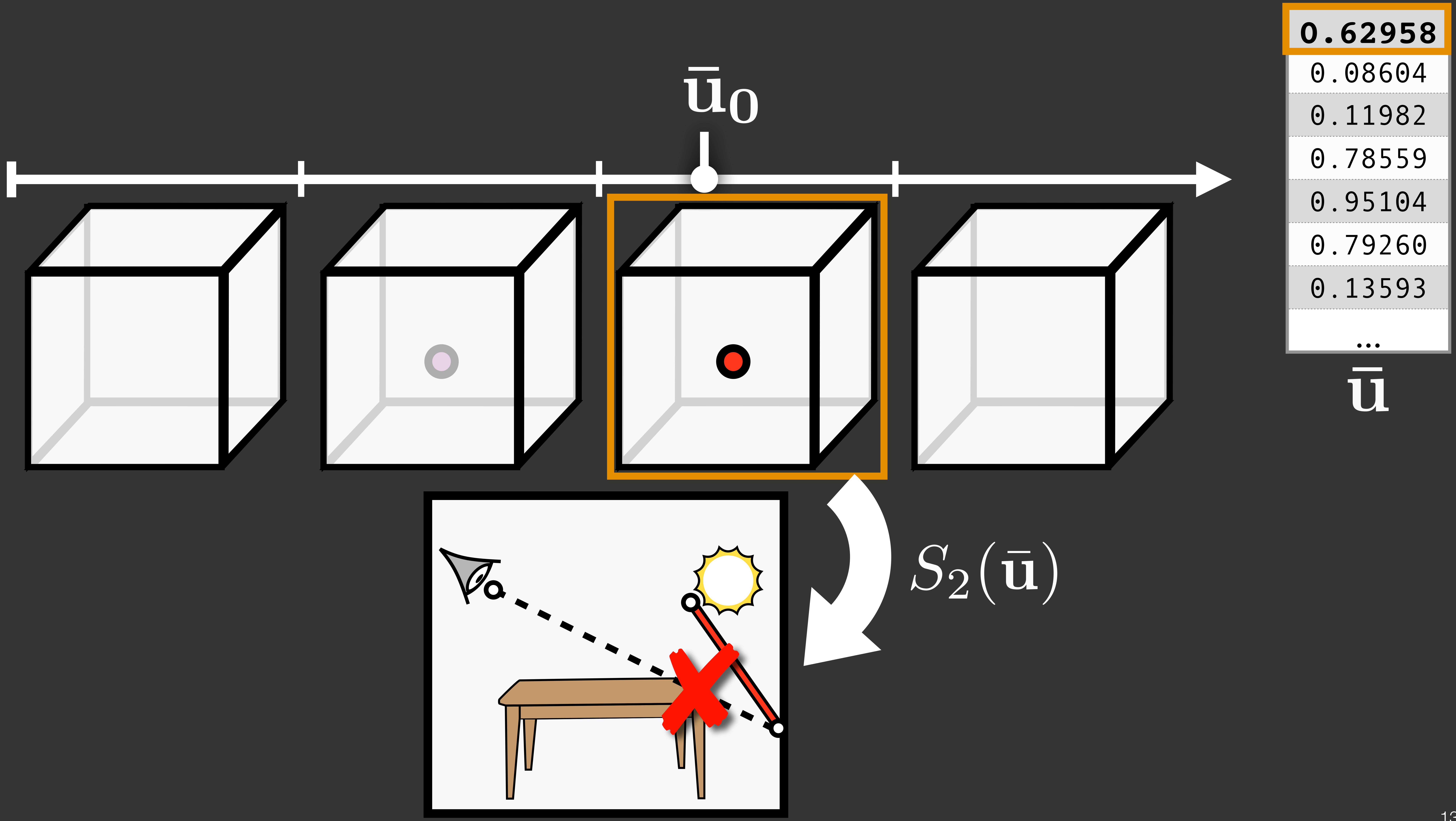
...
$\bar{u}$
$S_1(\bar{u})$


0.62958
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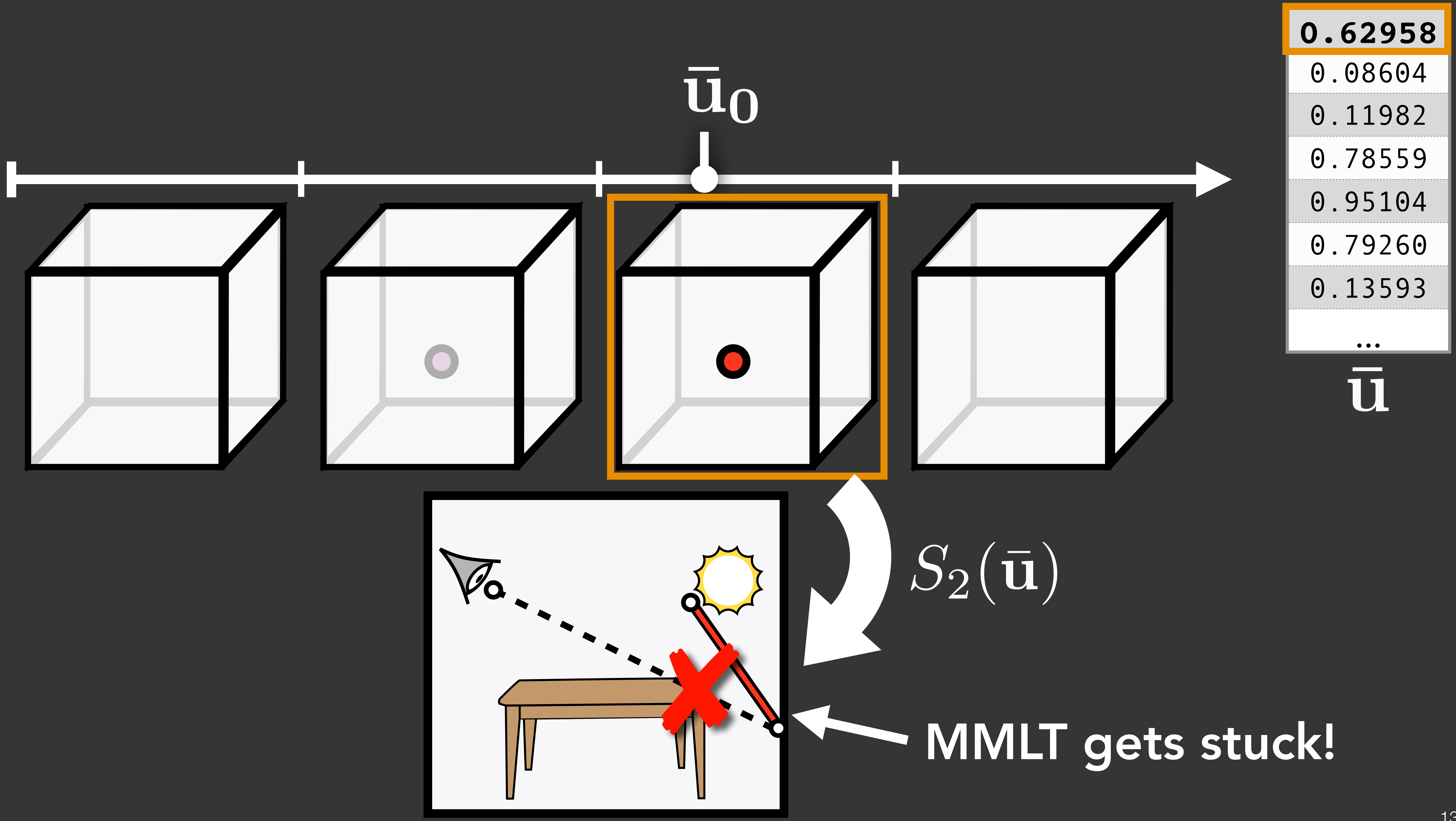
$\bar{u}$
$\bar{u}_0$
$S_2(\bar{u})$


$\bar{u}_0$
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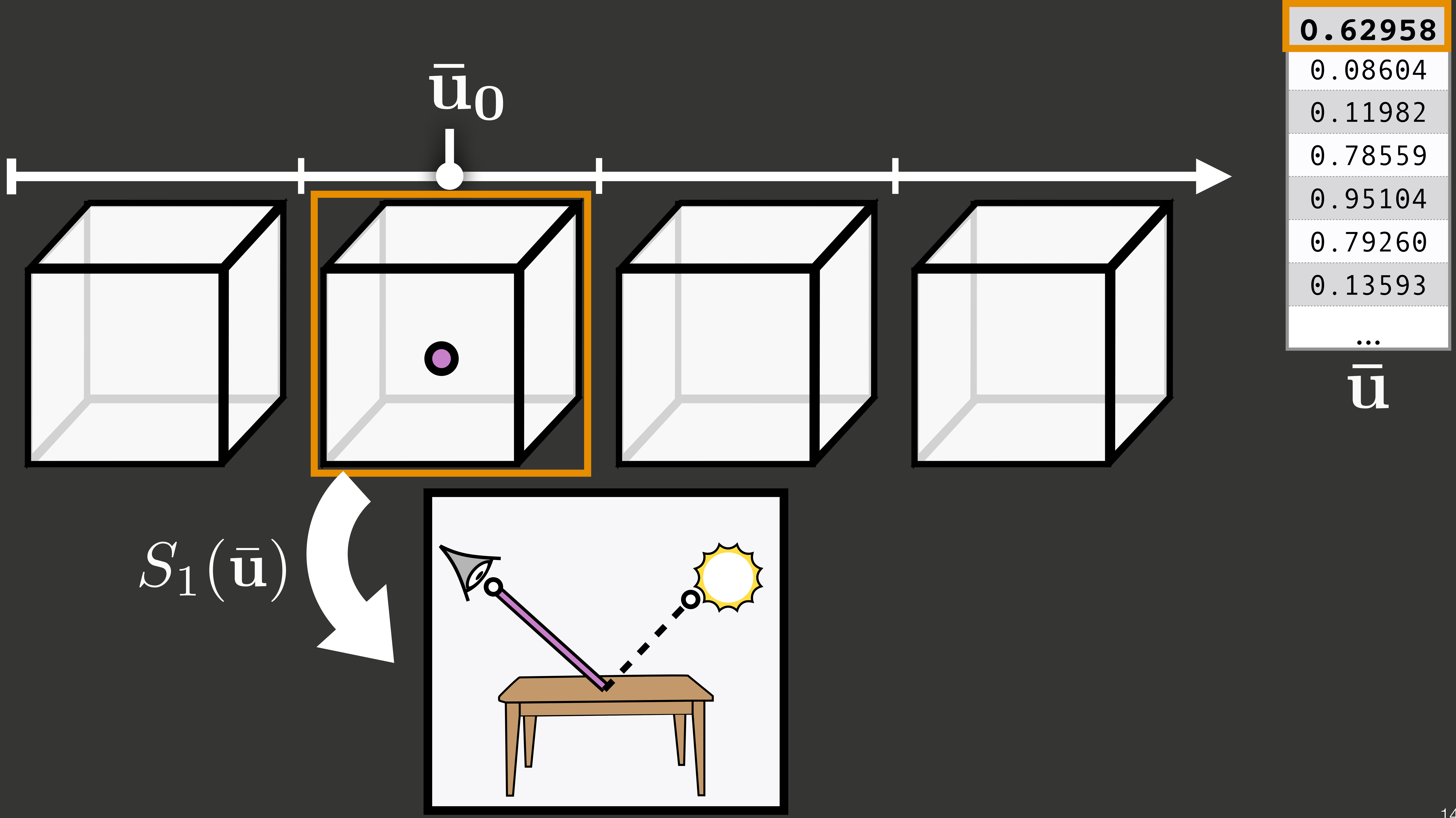
...
$\bar{u}$
$S_2(\bar{u})$
MMLT gets stuck!



$\bar{u}_0$
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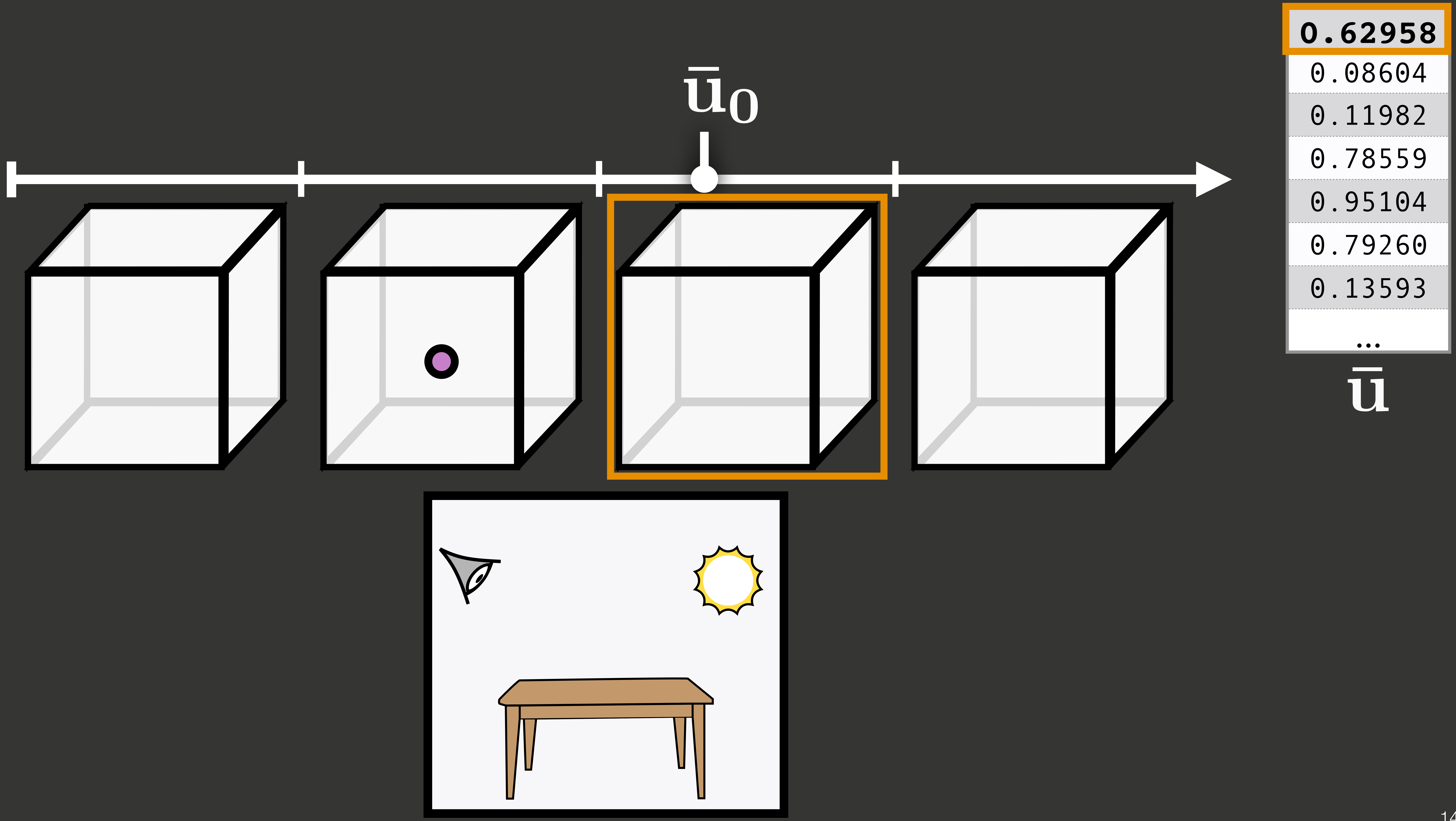
...
$\bar{u}$
$S_1(\bar{u})$



$\bar{u}_0$
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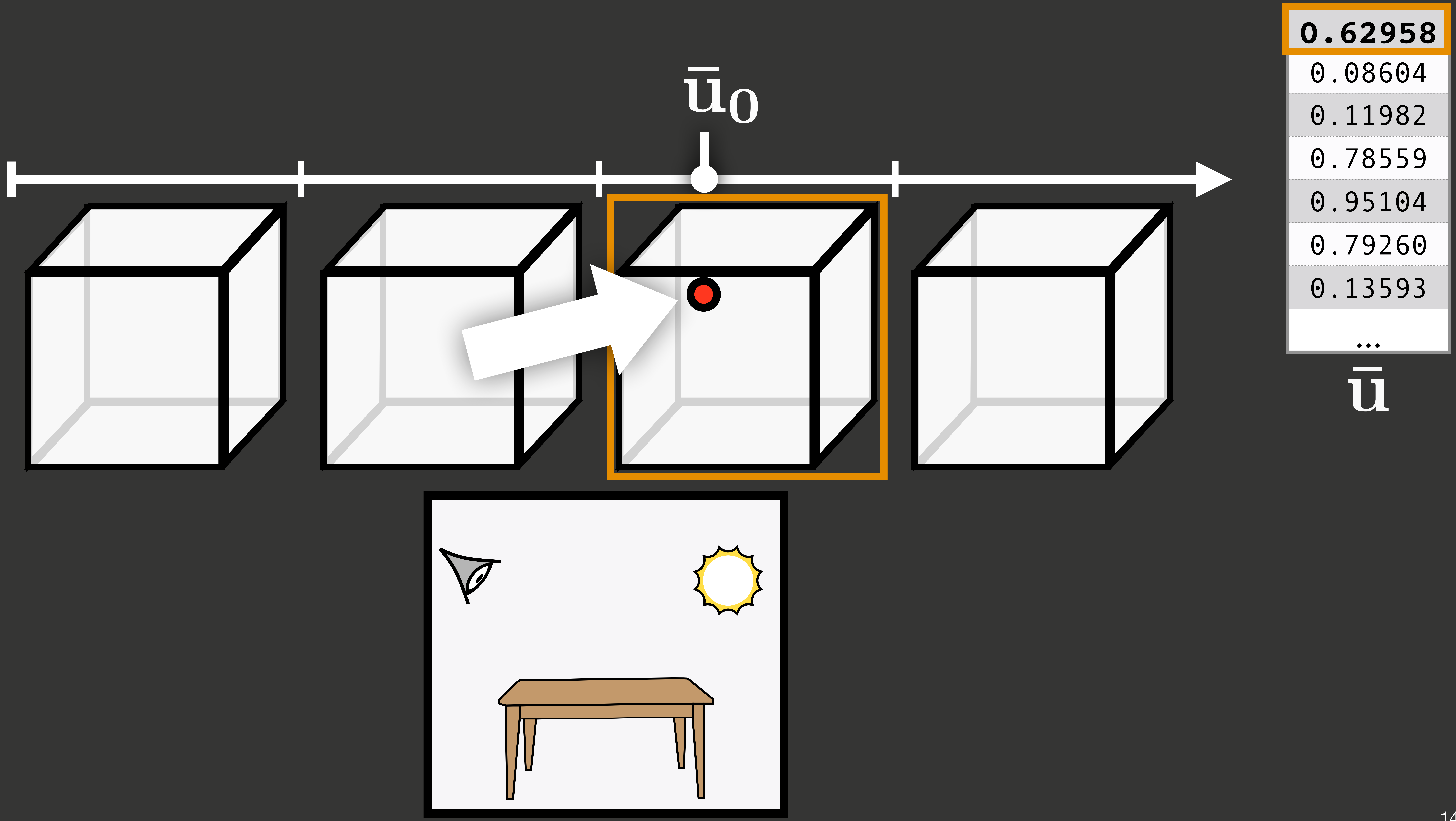

$\bar{u}_0$
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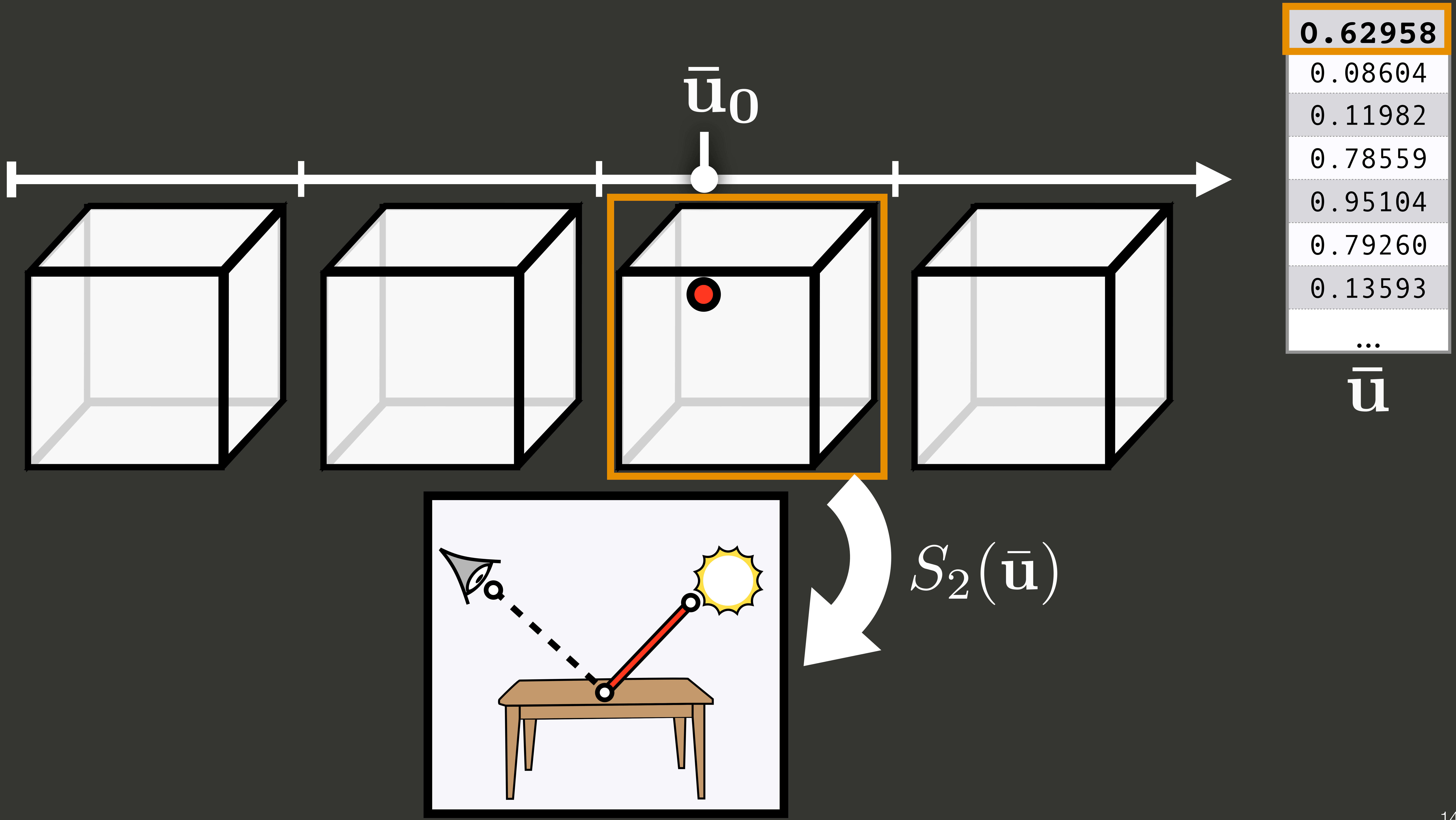

$\bar{u}_0$
0.62958
0.08604
0.11982
0.78559
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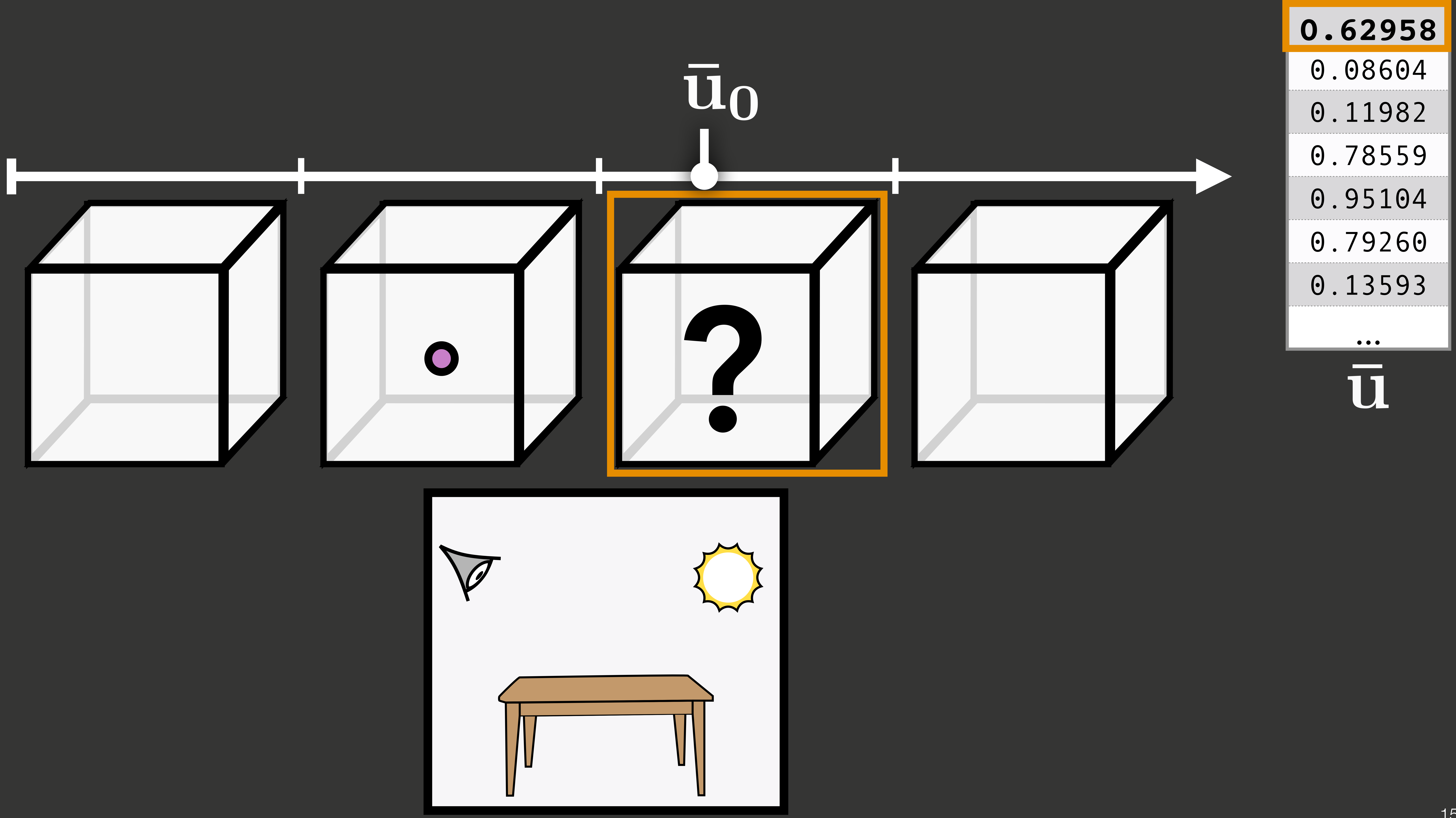
0.79260
0.13593
...
$\bar{u}$
$S_2(\bar{u})$


$\bar{u}_0$
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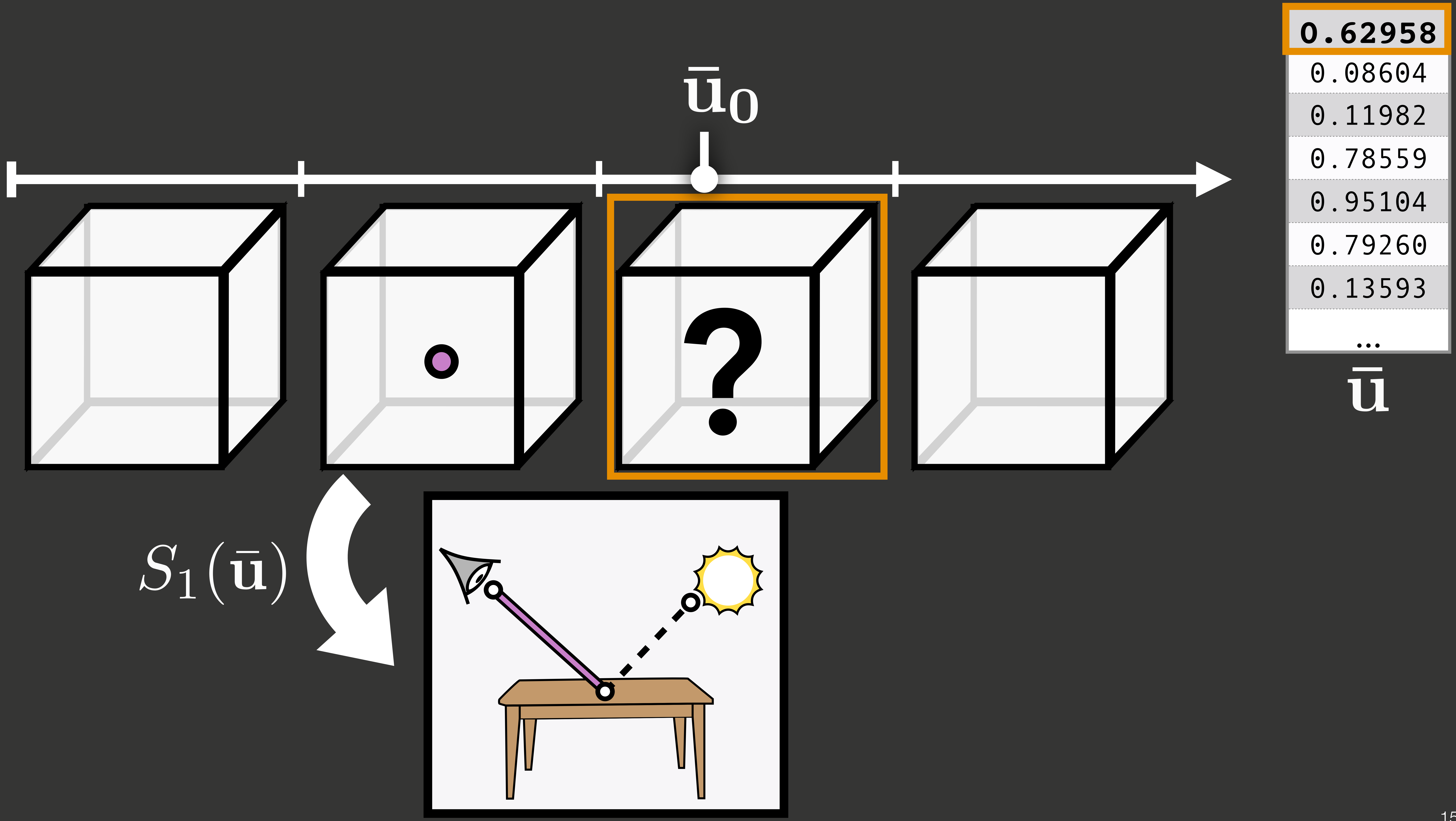

$\bar{u}_0$
?
$S_1(\bar{u})$
0.62958
0.08604
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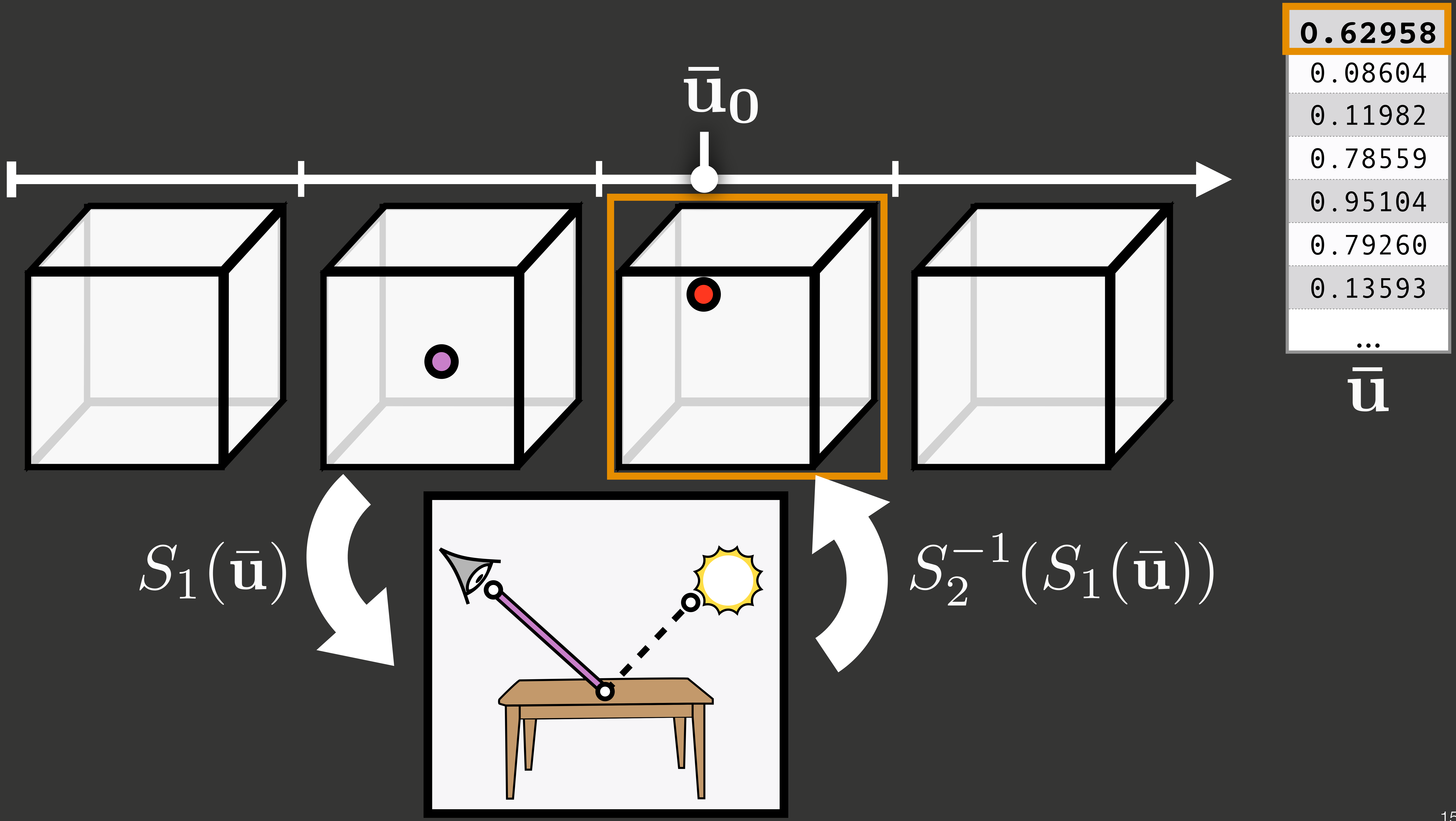
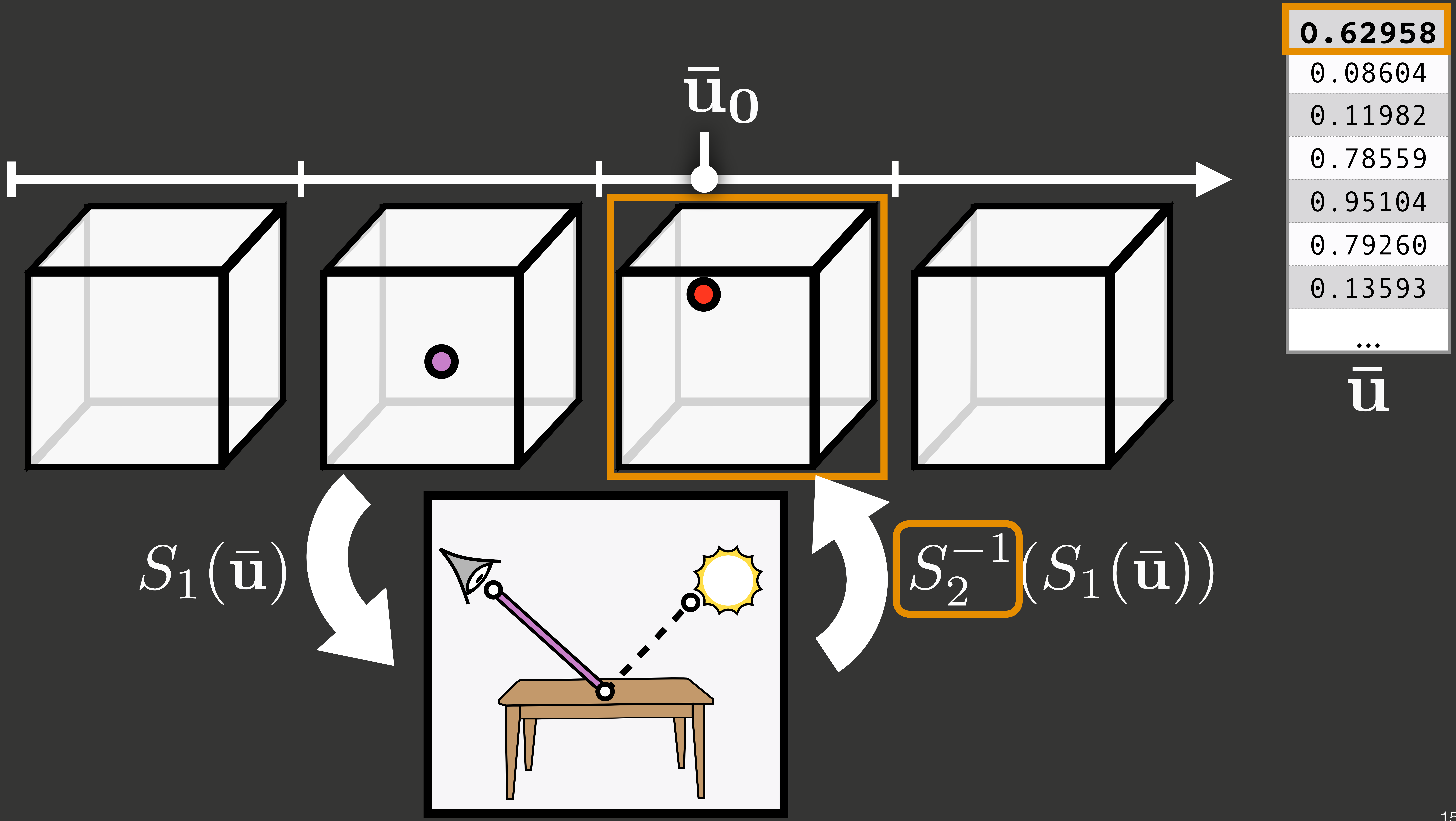
...
$\bar{u}$



$\bar{\mathrm{u}}_0$
0.62958
0.08604
0.11982
0.78559
0.95104
0.79260
0.13593
...
$\bar{\mathrm{u}}$
$S_1(\bar{\mathrm{u}})$
$S_2^{-1}(S_1(\bar{\mathrm{u}}))$


$\bar{u}_0$
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0.11982
0.78559
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0.13593
...
$\bar{u}$
$S_1(\bar{u})$
$S_2^{-1}(S_1(\bar{u}))$

New Perturbation: Reversible Jump

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Algorithm:

New Perturbation: Reversible Jump

Algorithm:

1. Select new technique *j*

New Perturbation: Reversible Jump

Algorithm:

1. Select new technique j
2. Invert current path $\bar{\mathbf{v}} = S_j^{-1}(S_i(\bar{\mathbf{u}}))$

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Reversible Jump MCMC

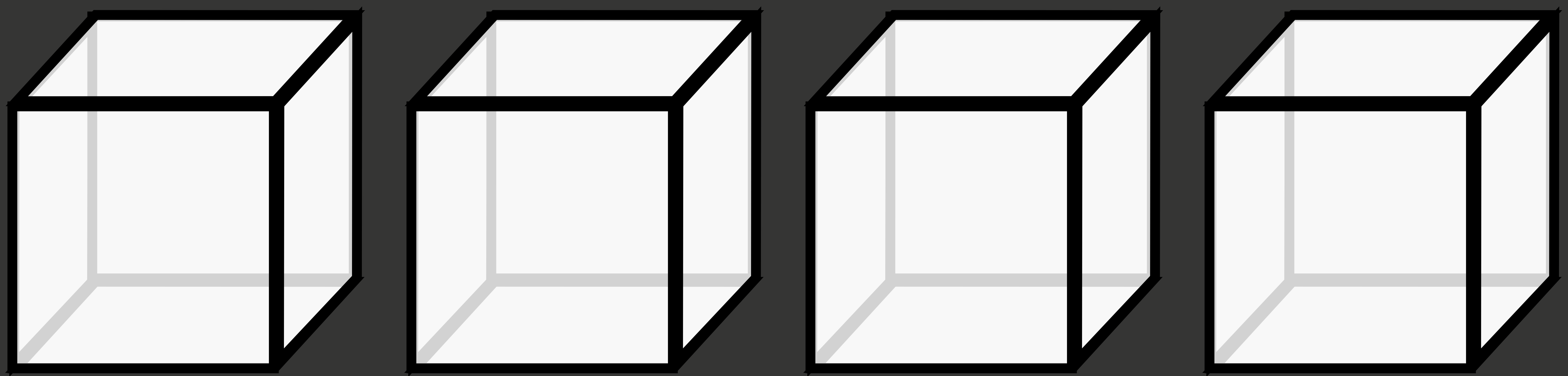
[Green & Hastie ’09]

Reversible Jump MCMC
[Green & Hastie '09]

- Deals with Metropolis across multiple spaces

Reversible Jump MCMC
[Green & Hastie ’09]

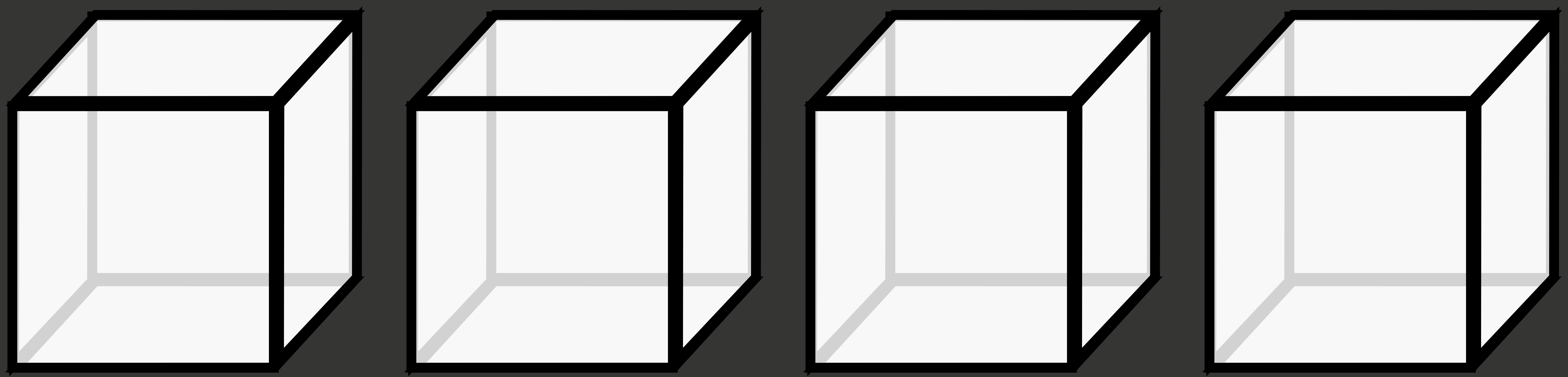
- Deals with Metropolis across multiple spaces



Reversible Jump MCMC
[Green & Hastie '09]

- Deals with Metropolis across multiple spaces

- Reversible Jump: Continuous map between spaces

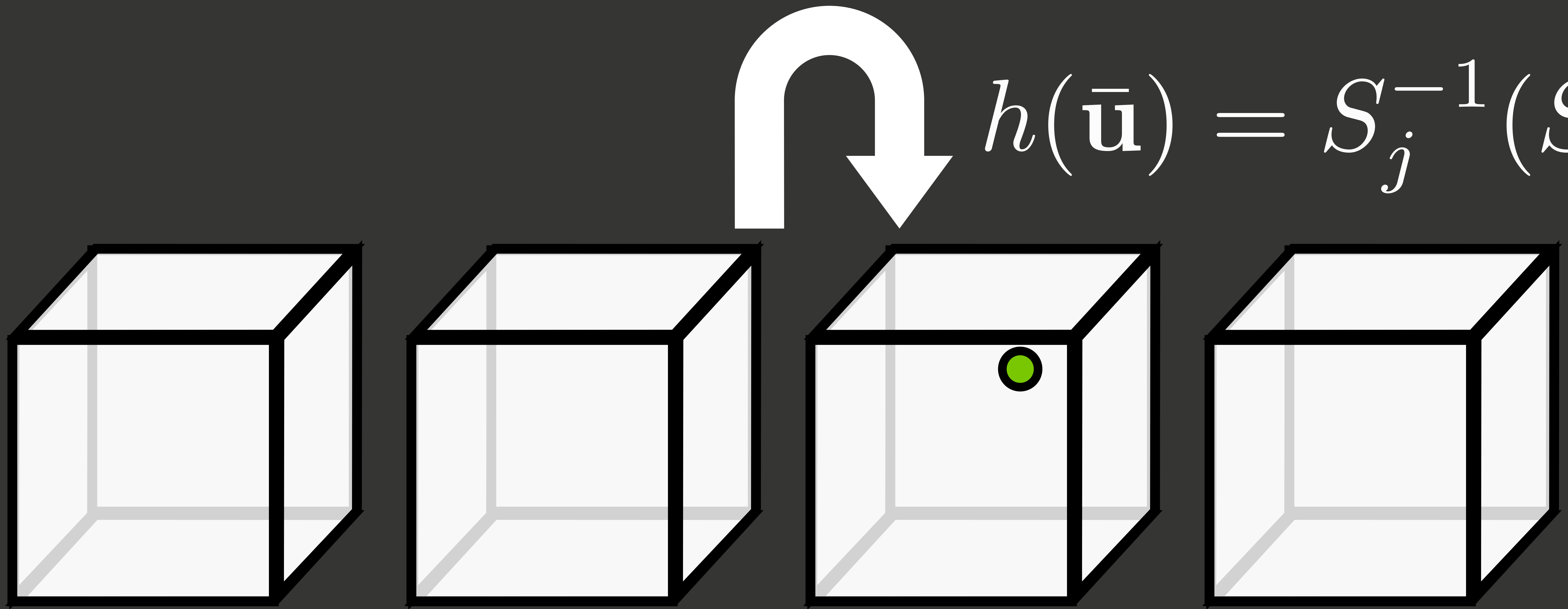


Reversible Jump MCMC

[Green & Hastie '09]

- Deals with Metropolis across multiple spaces
- Reversible Jump: Continuous map between spaces

$$h(\bar{\mathbf{u}}) = S_j^{-1}(S_i(\bar{\mathbf{u}}))$$



New Perturbation: Reversible Jump

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3. Accept $\bar{\mathbf{v}}$ with probability $\mathrm{Pr} = \frac{f(\bar{\mathbf{v}})T(\bar{\mathbf{v}}, \bar{\mathbf{u}})}{f(\bar{\mathbf{u}})T(\bar{\mathbf{u}}, \bar{\mathbf{v}})} ||\partial h||$

New Perturbation: Reversible Jump

Algorithm:

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Algorithm:

1. Select new technique *j*
2. Invert current path $\bar{\mathbf{v}} = h(\bar{\mathbf{u}}) = S_j^{-1}(S_i(\bar{\mathbf{u}}))$
3. Accept $\bar{\mathbf{v}}$ with probability $\Pr = \dfrac{f(\bar{\mathbf{v}})T(\bar{\mathbf{v}},\bar{\mathbf{u}})}{f(\bar{\mathbf{u}})T(\bar{\mathbf{u}},\bar{\mathbf{v}})} ||\partial h||$

$$||\partial h|| = \frac{p_j(\bar{\mathbf{x}})}{p_i(\bar{\mathbf{x}})}$$

New Perturbation: Reversible Jump

Algorithm:

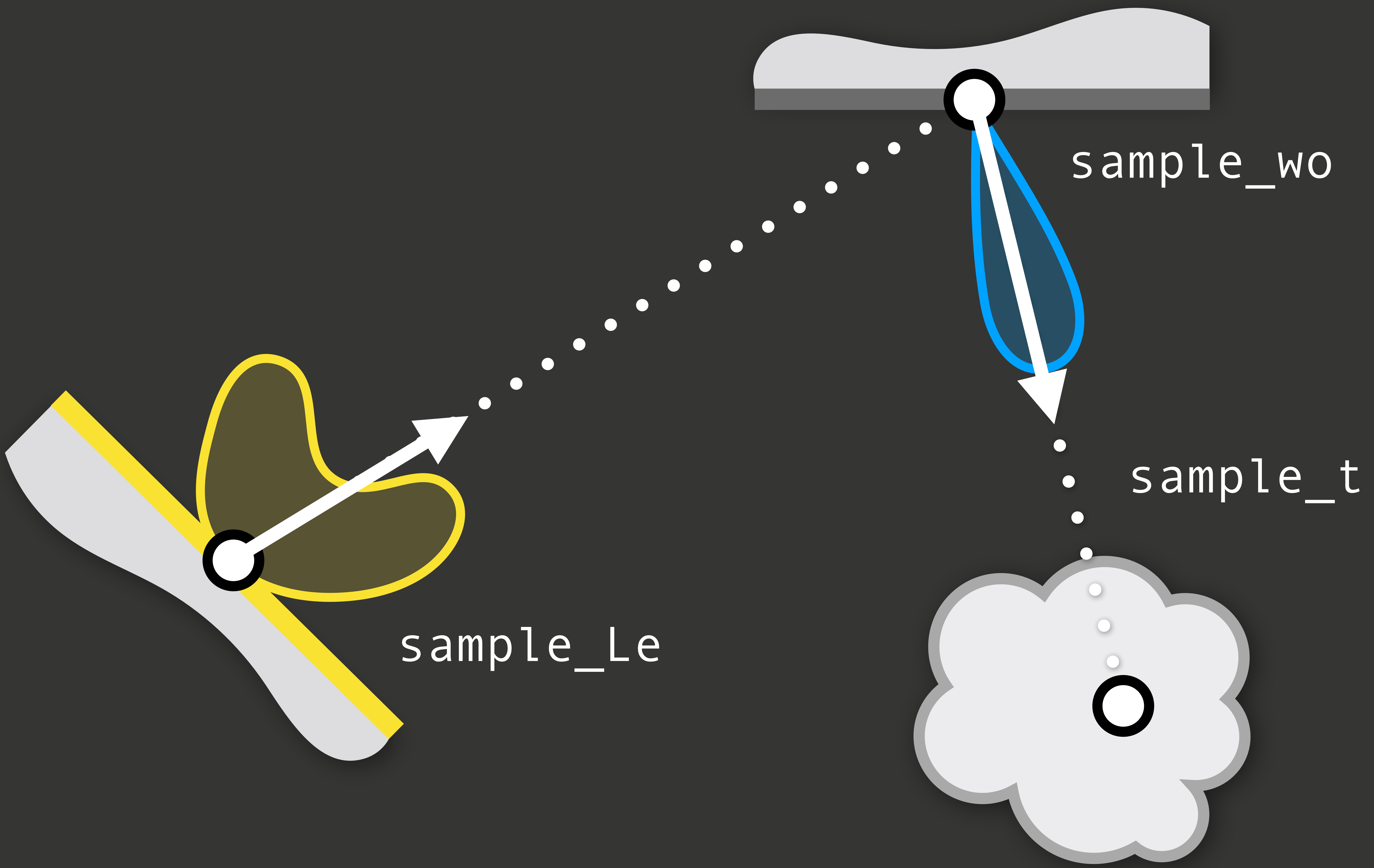
1. Select new technique *j* **with probability w_j**
2. Invert current path $\bar{\mathbf{v}} = h(\bar{\mathbf{u}}) = S_j^{-1}(S_i(\bar{\mathbf{u}}))$
3. Accept $\bar{\mathbf{v}}$ with probability $\mathrm{Pr} = 1$

New Perturbation: Reversible Jump

Algorithm:

1. Select new technique *j* **with probability *w_j***
2. Invert current path $\bar{\mathbf{v}} = h(\bar{\mathbf{u}}) = S_j^{-1}(S_i(\bar{\mathbf{u}}))$
3. Accept $\bar{\mathbf{v}}$ with probability $\Pr = 1$

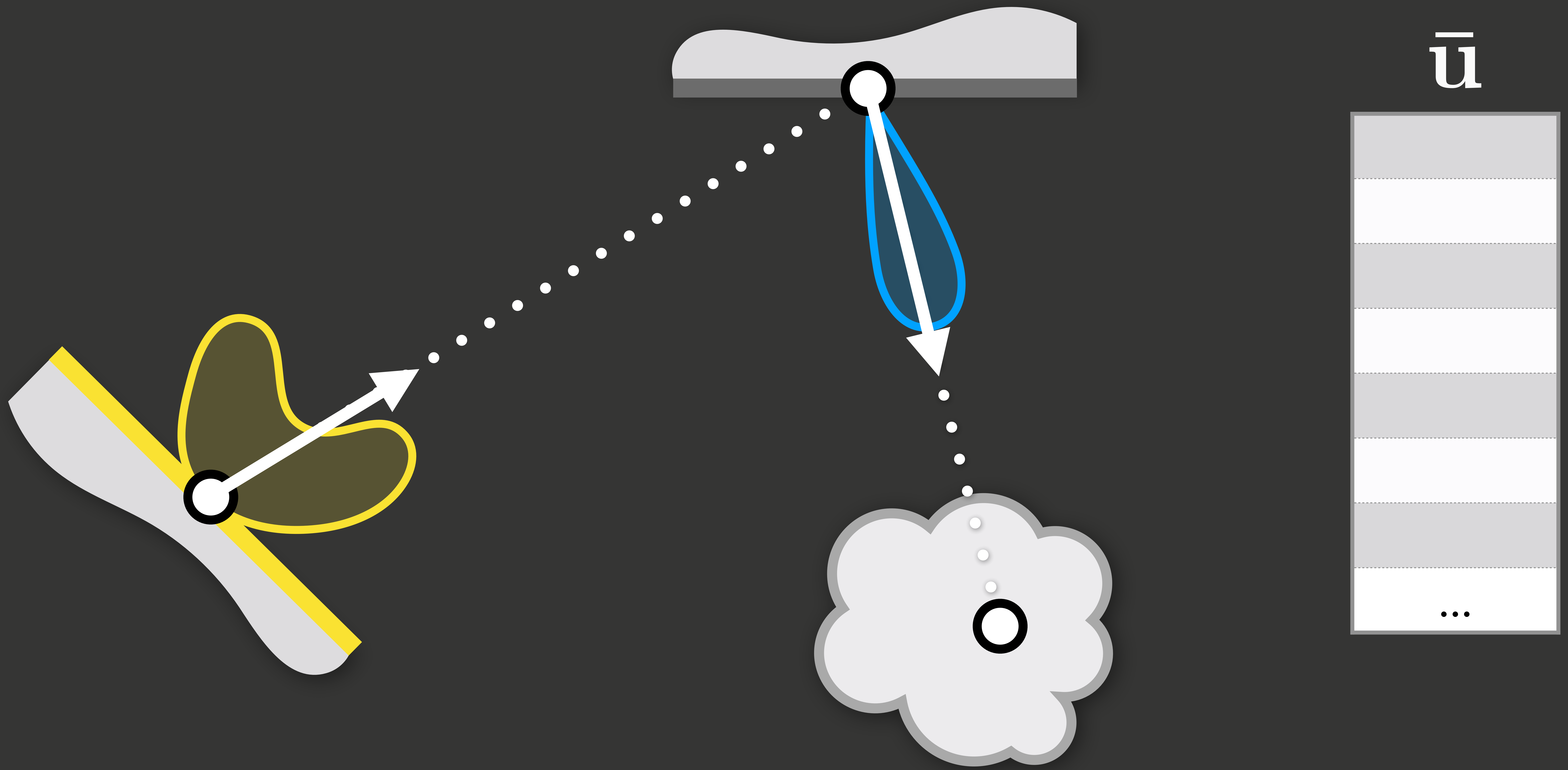
Inside $S(\bar{\mathbf{u}})$



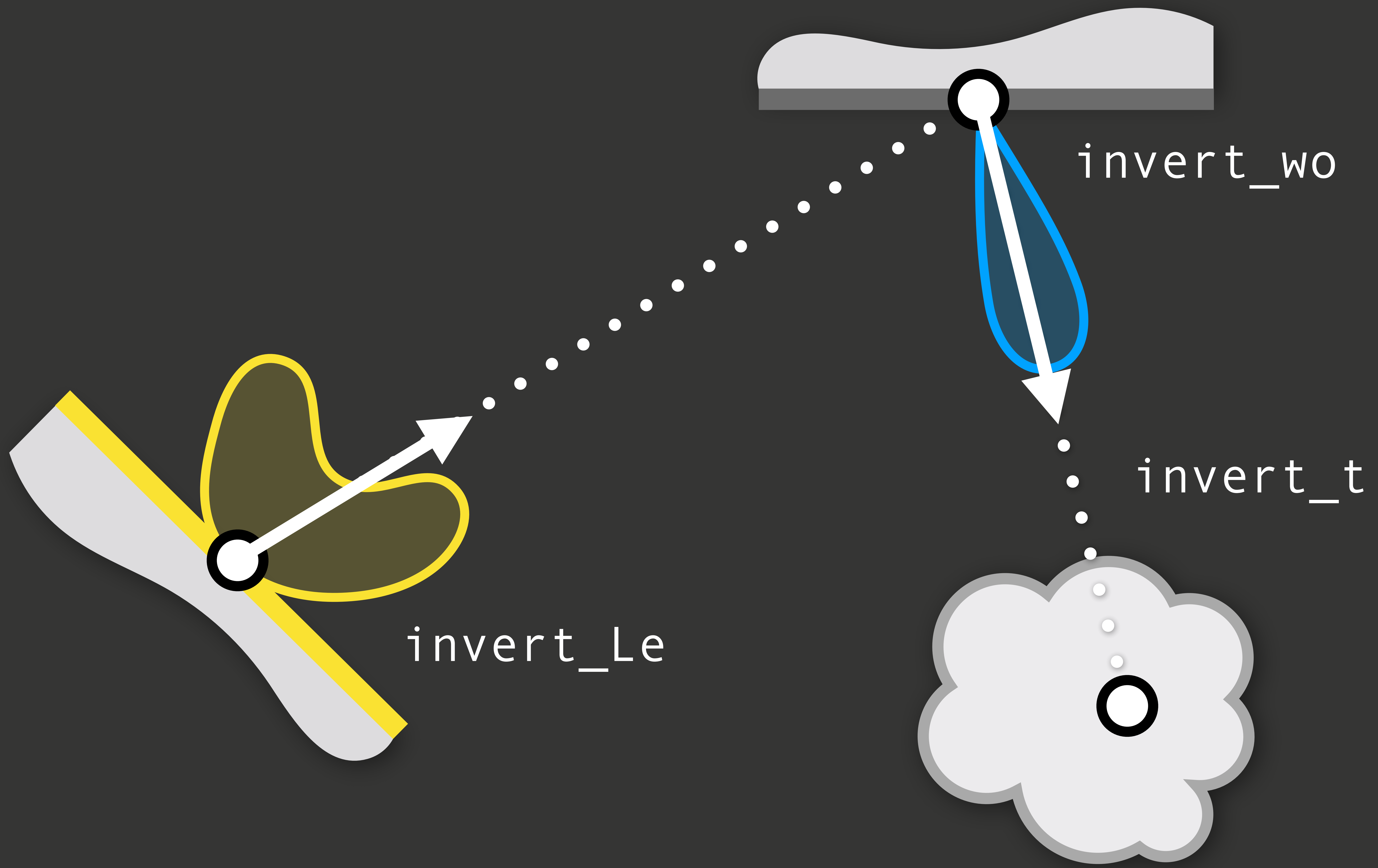
$\bar{\mathbf{u}}$

0.03563
0.08604
0.11982
0.78559
0.95104
0.79260
0.13593
…

Inverse Random Walks



Inverse Random Walks



ū
0.03563
0.08604
0.11982
0.78559
0.95104
...

Inverting Local Sampling

- Usually: Sampling algorithm derived with inversion method

Inverting Local Sampling

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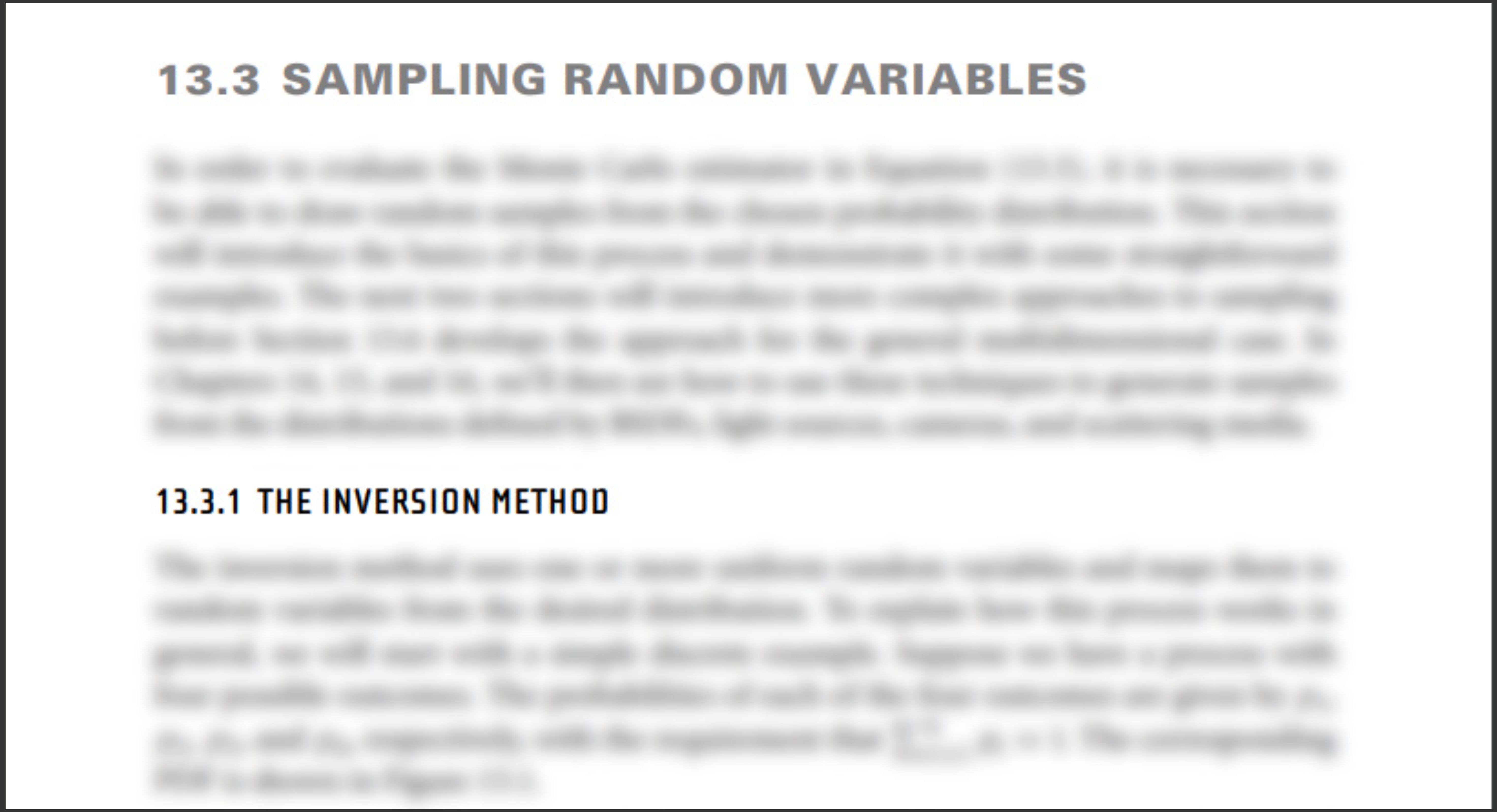

13.3 SAMPLING RANDOM VARIABLES

13.3.1 THE INVERSION METHOD

Inverting Local Sampling

- Usually: Sampling algorithm derived with inversion method

Inverting Local Sampling

- Usually: Sampling algorithm derived with inversion method

Algorithm 1: Inversion Method

Inverting Local Sampling

- Usually: Sampling algorithm derived with inversion method

Algorithm 1: Inversion Method

Input: $\mathrm{pdf}(x)$

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Algorithm 1: Inversion Method

Input: $\mathrm{pdf}(x)$

Step 1) Compute $\mathrm{cdf}(x)$

Inverting Local Sampling

- Usually: Sampling algorithm derived with inversion method

Algorithm 1: Inversion Method

Input: $\mathrm{pdf}(x)$

Step 1) Compute $\mathrm{cdf}(x)$

Step 2) Compute $\mathrm{cdf}^{-1}(u)$

Inverting Local Sampling

- Usually: Sampling algorithm derived with inversion method

Algorithm 1: Inversion Method

Input: $\mathrm{pdf}(x)$

Step 1) Compute $\mathrm{cdf}(x)$

Step 2) Compute $\mathrm{cdf}^{-1}(u)$ ← Sampling Algorithm

Inverting Local Sampling

- Usually: Sampling algorithm derived with inversion method

Algorithm 1: Inversion Method

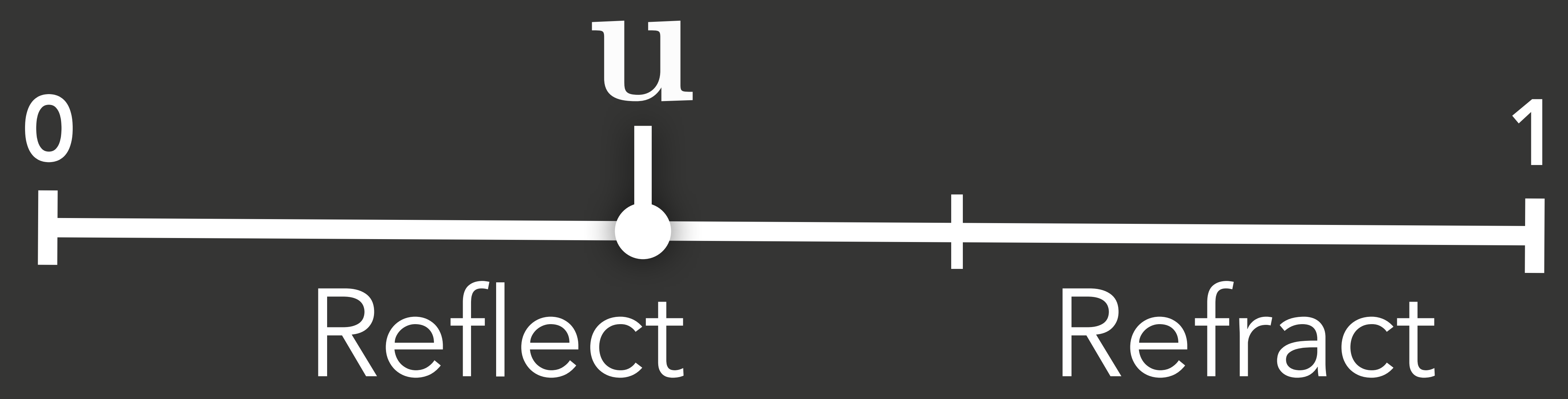
Input: $\mathrm{pdf}(x)$

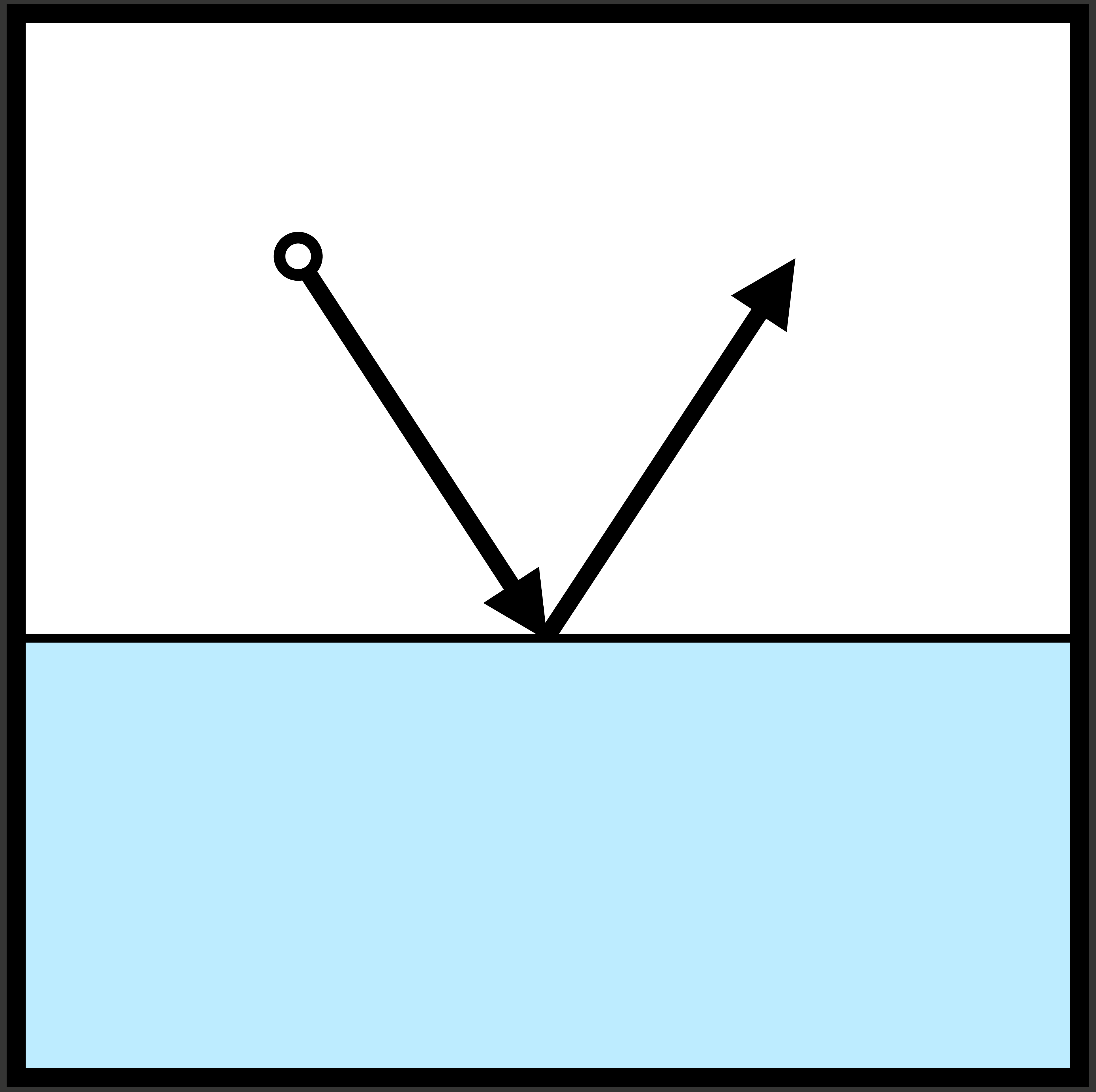
Step 1) Compute $\mathrm{cdf}(x)$ ← Inverse!

Step 2) Compute $\mathrm{cdf}^{-1}(u)$ ← Sampling Algorithm

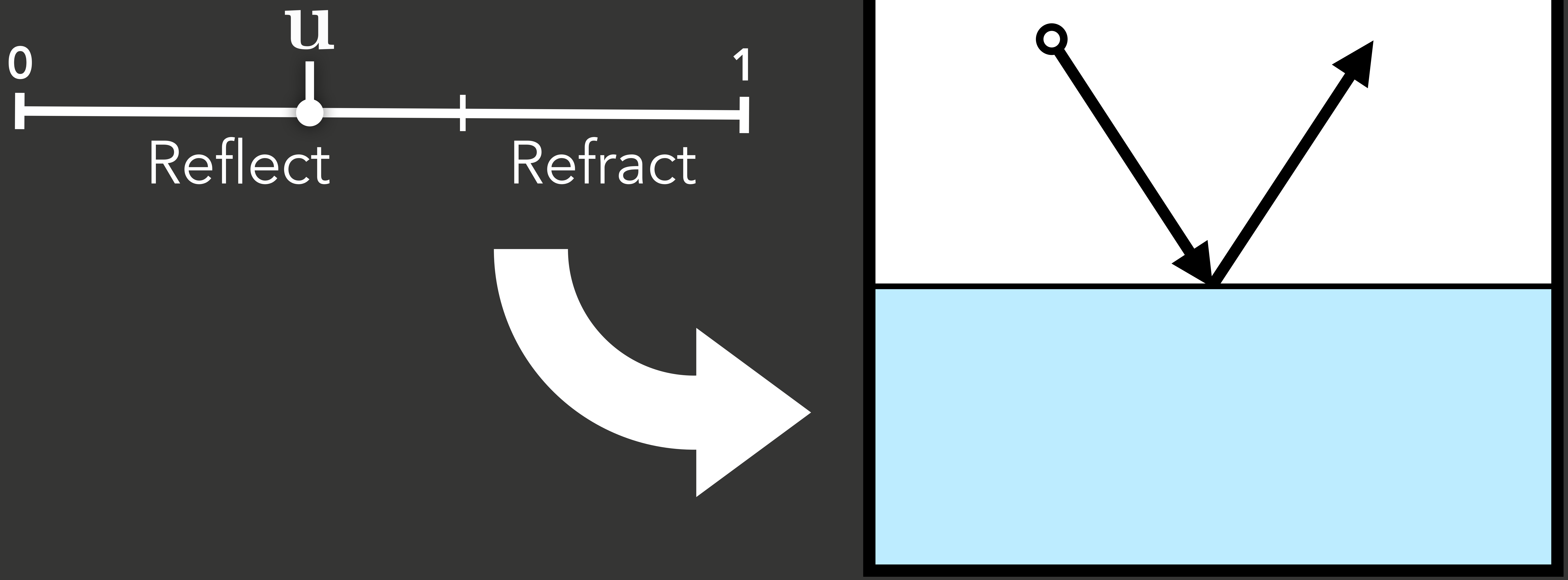
Problems

Problems: Discrete Decisions

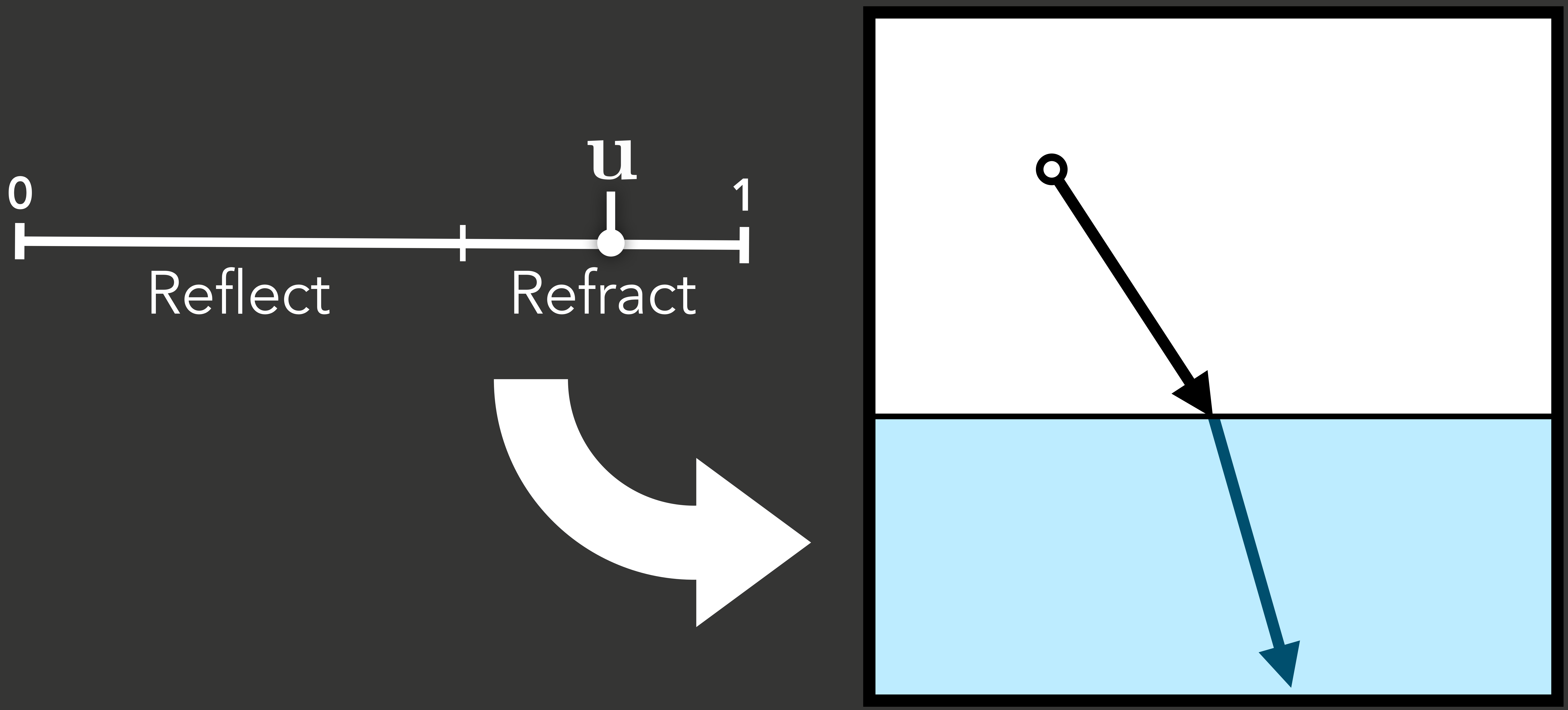




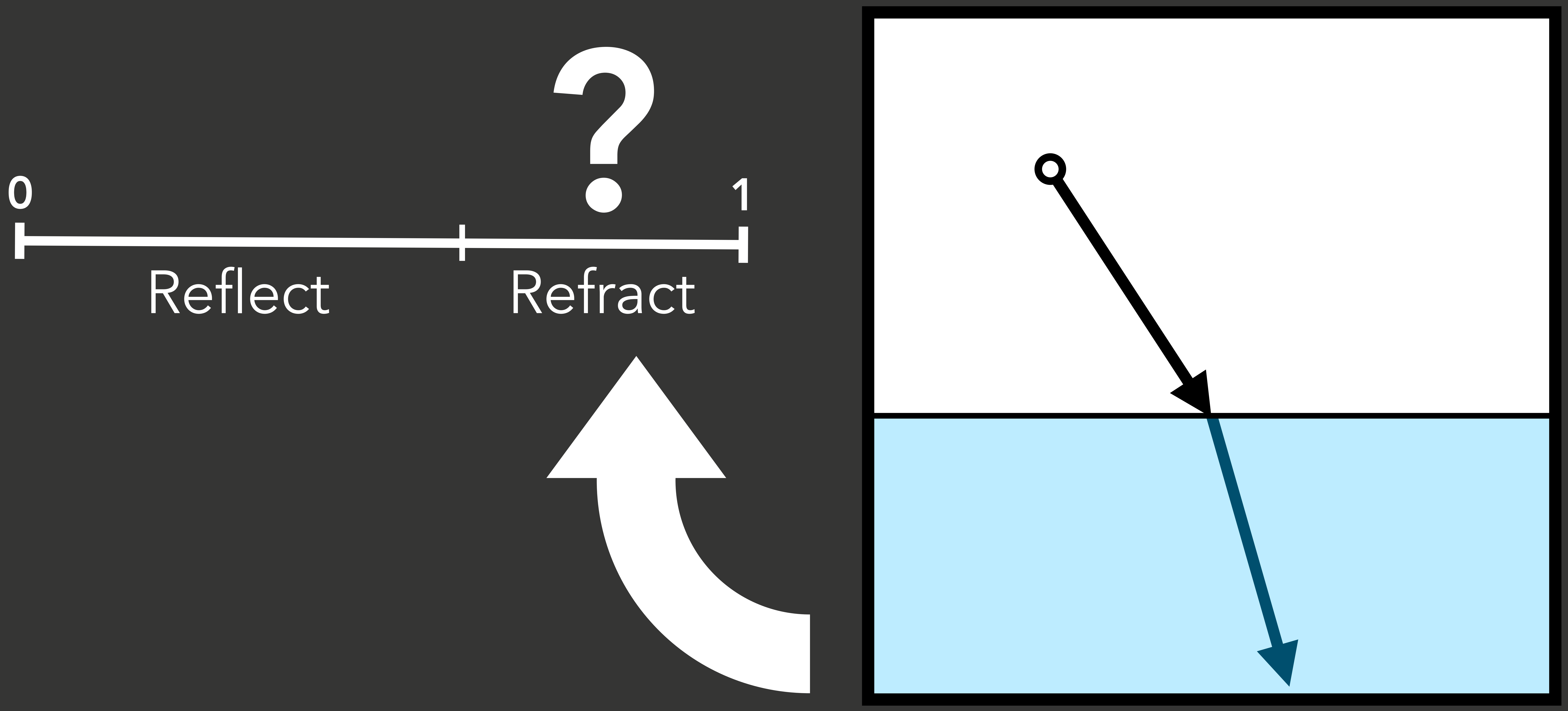
Problems: Discrete Decisions



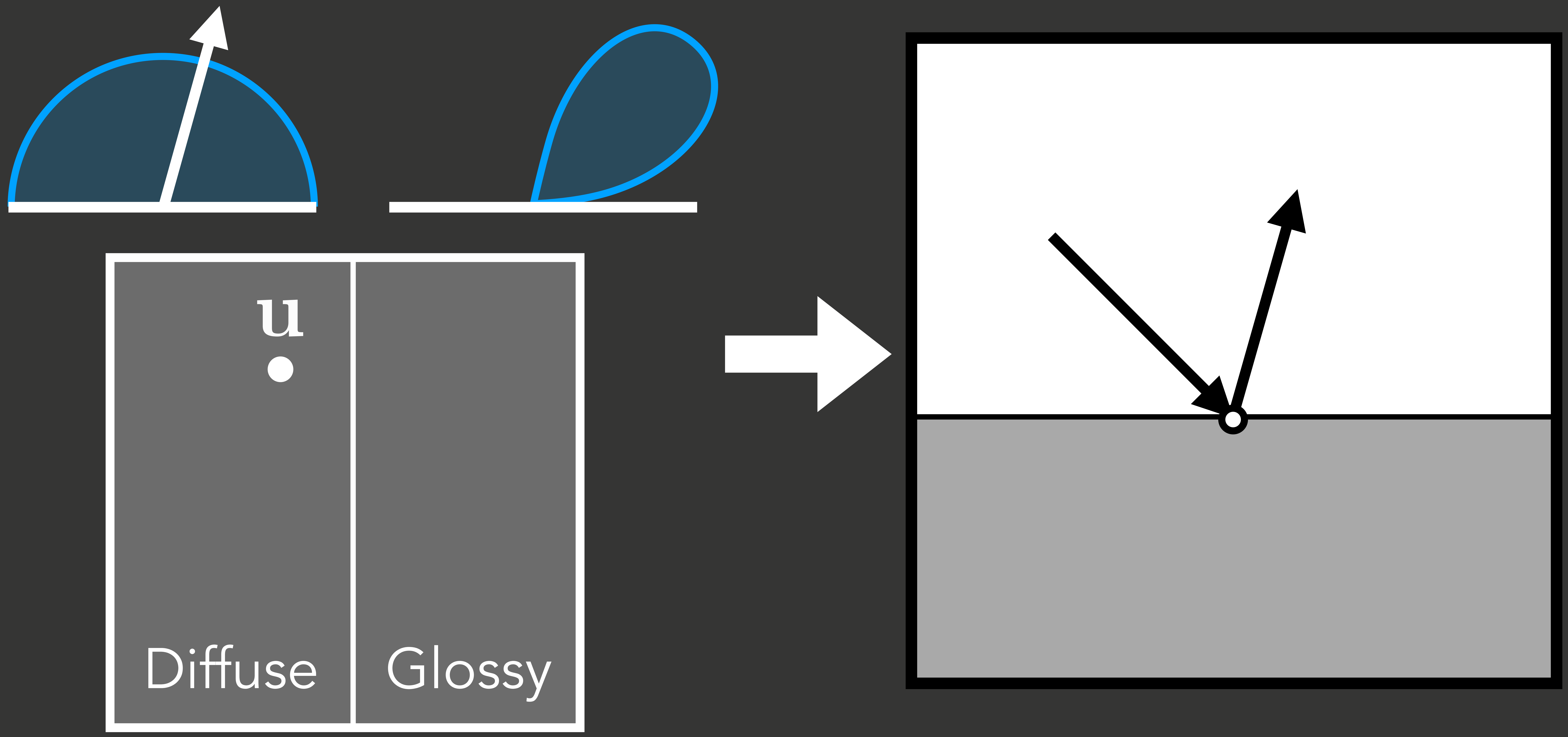
Problems: Discrete Decisions



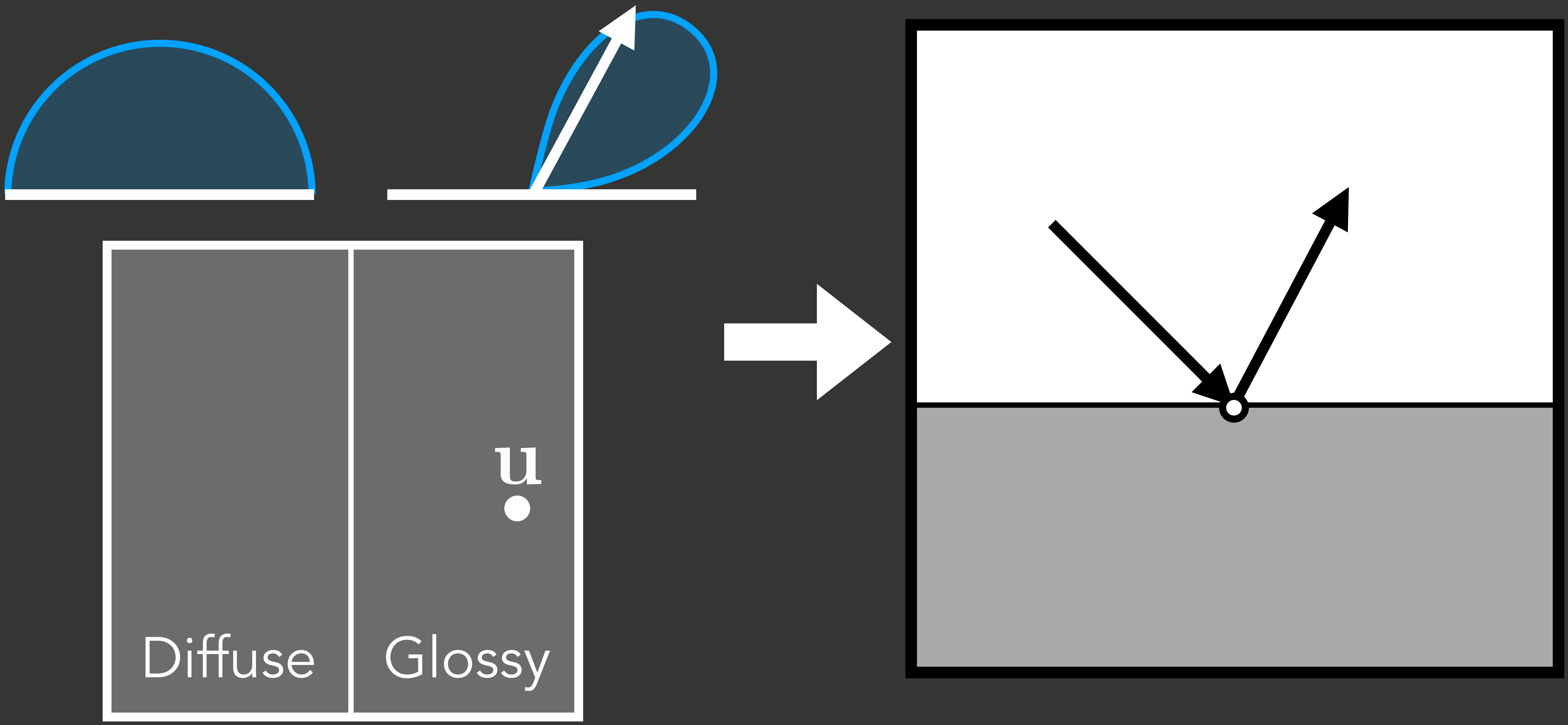
Problems: Discrete Decisions



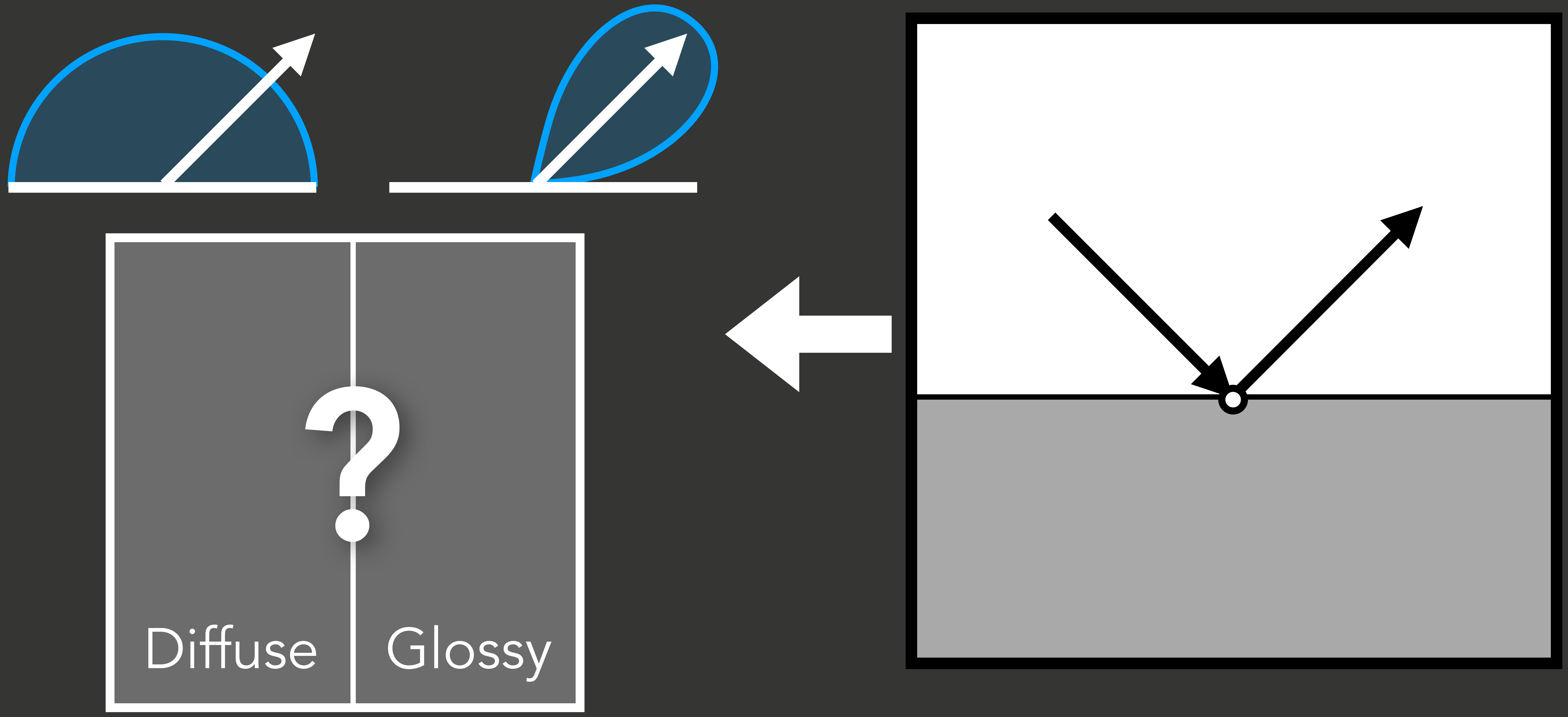
Problems: Ambiguity



Problems: Ambiguity



Problems: Ambiguity



A Problem of Dimensions

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- Primary Sample Space has more dimensions than Path Space!

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- Practical Problem: Can’t reconstruct all random numbers

A Problem of Dimensions

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A Problem of Dimensions

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$$\mathrm{Pr} = \frac{f(\bar{\mathbf{v}})T(\bar{\mathbf{v}}, \bar{\mathbf{u}})}{f(\bar{\mathbf{u}})T(\bar{\mathbf{u}}, \bar{\mathbf{v}})} ||\partial h(\bar{\mathbf{u}})||$$

A Problem of Dimensions

- Primary Sample Space has more dimensions than Path Space!
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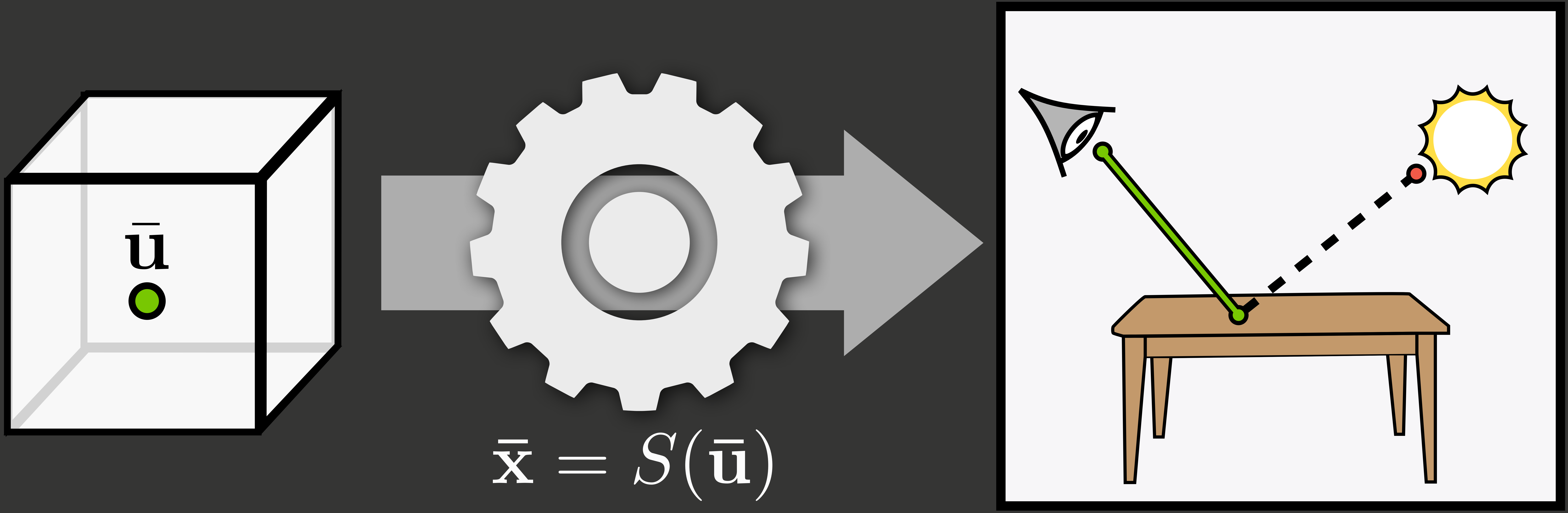
$$\Pr = \frac{f(\bar{\mathbf{v}})T(\bar{\mathbf{v}},\bar{\mathbf{u}})}{f(\bar{\mathbf{u}})T(\bar{\mathbf{u}},\bar{\mathbf{v}})} \boxed{||\partial h(\bar{\mathbf{u}})||}$$

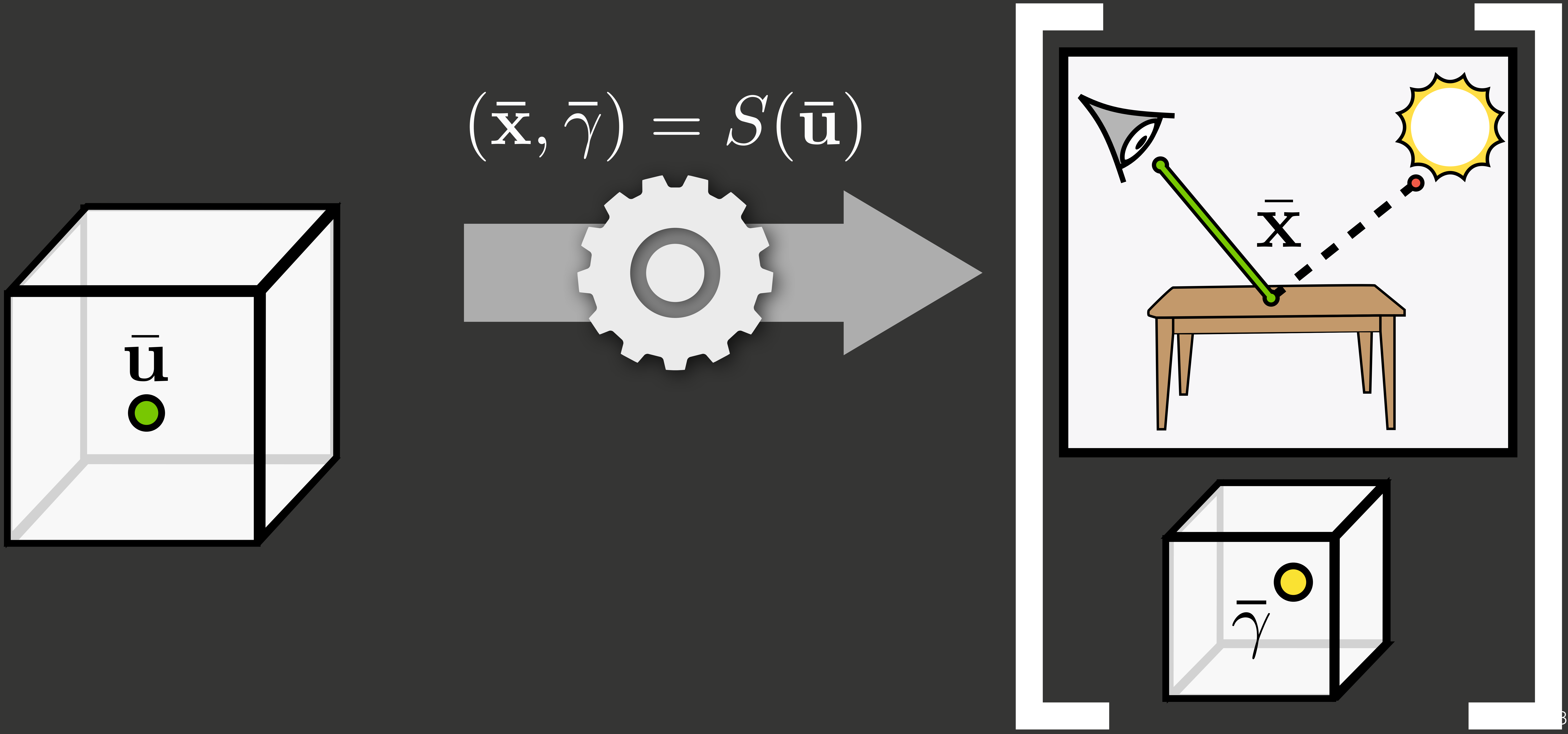
Idea: Augmented Path Space



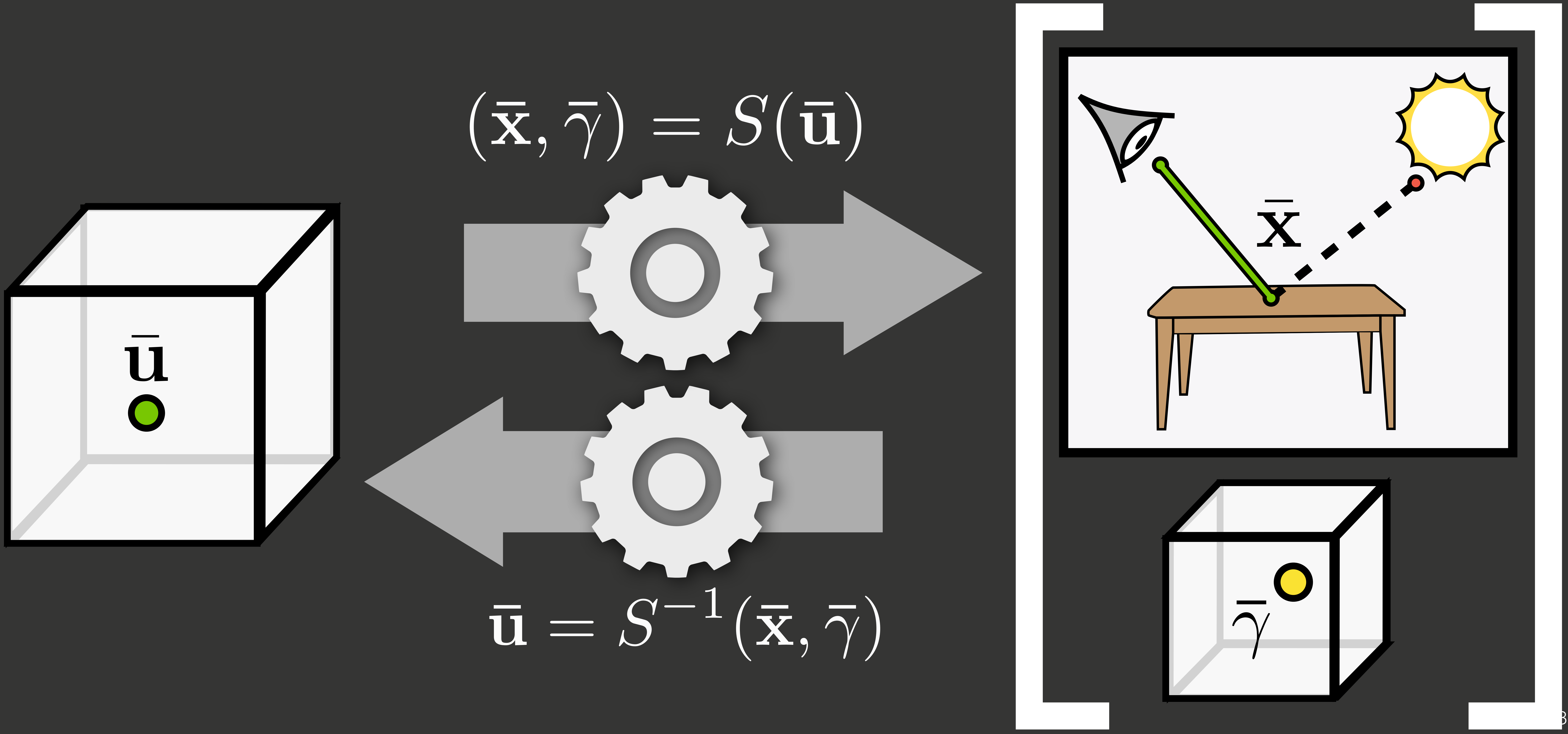
$$\bar{\mathbf{x}} = S(\bar{\mathbf{u}})$$

Idea: Augmented Path Space

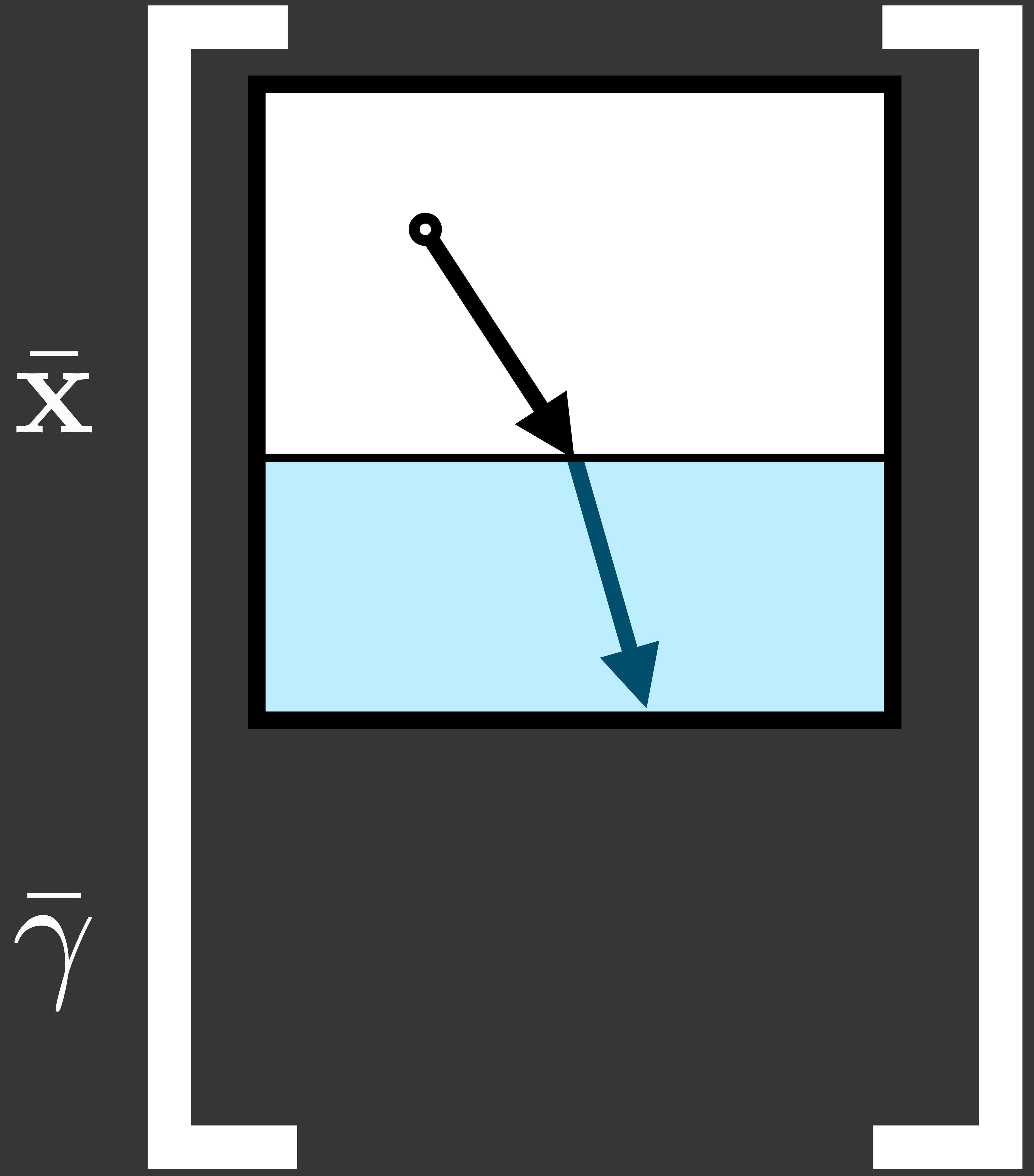
$$(\bar{\mathbf{x}}, \bar{\gamma}) = S(\bar{\mathbf{u}})$$



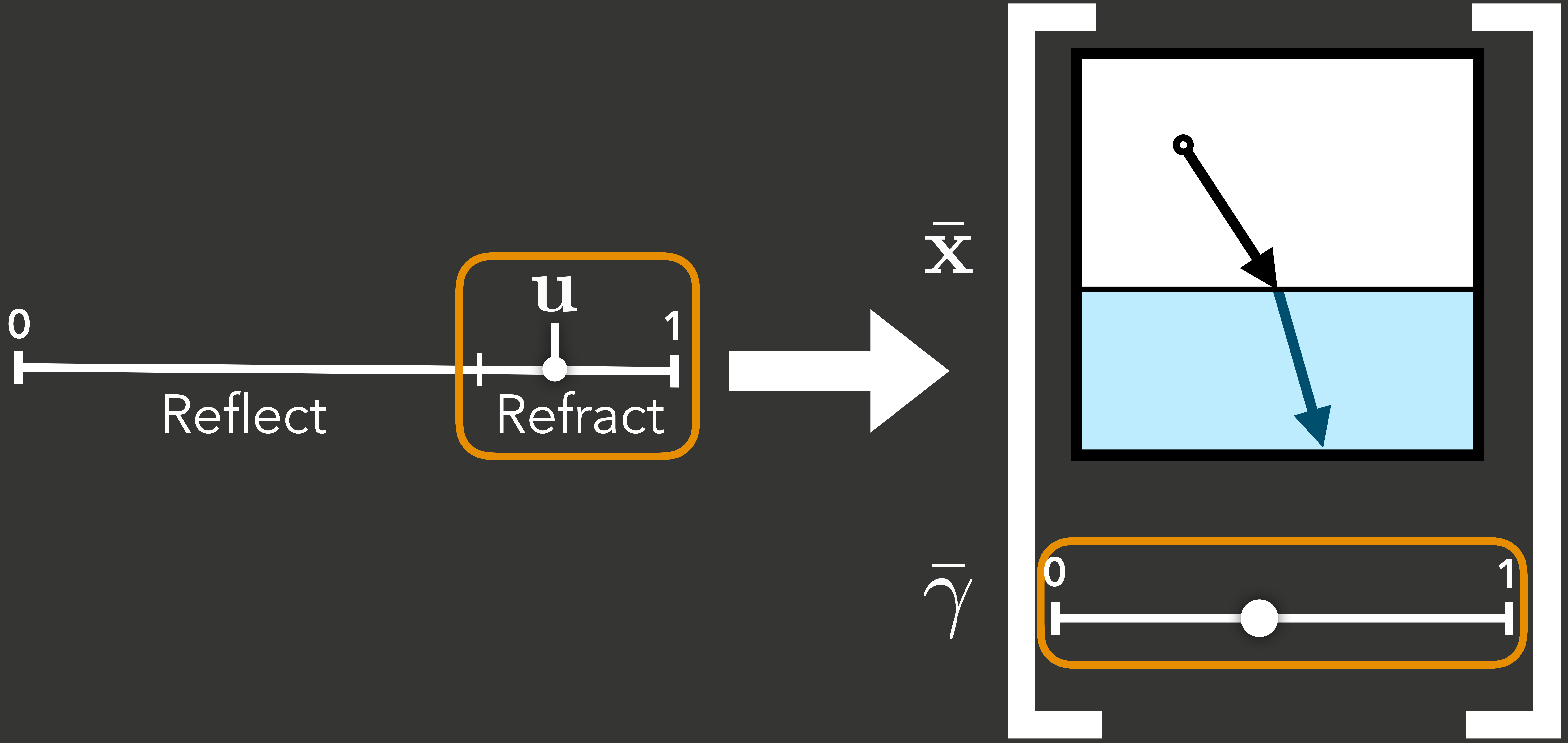
Idea: Augmented Path Space



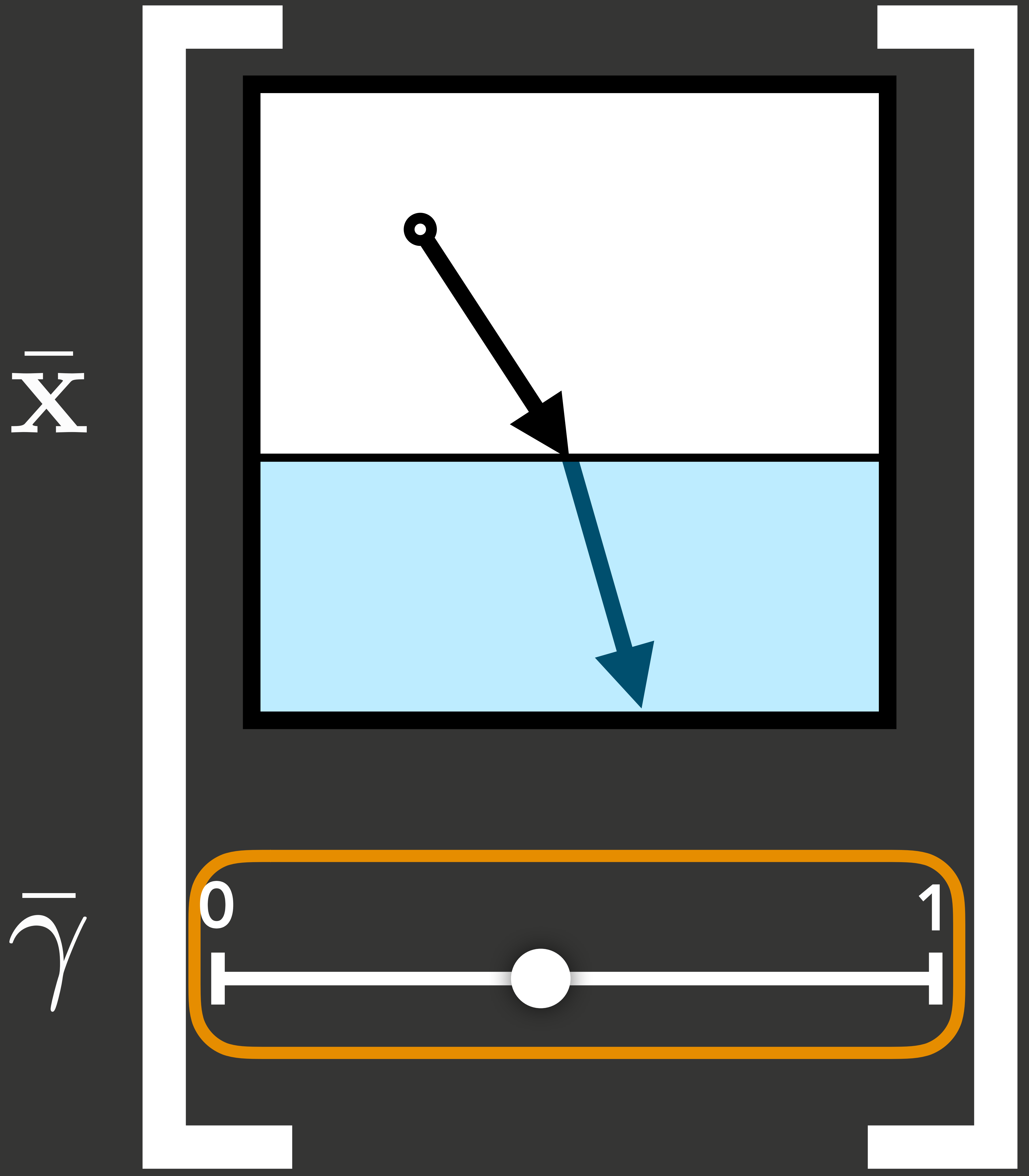
Idea: Augmented Path Space



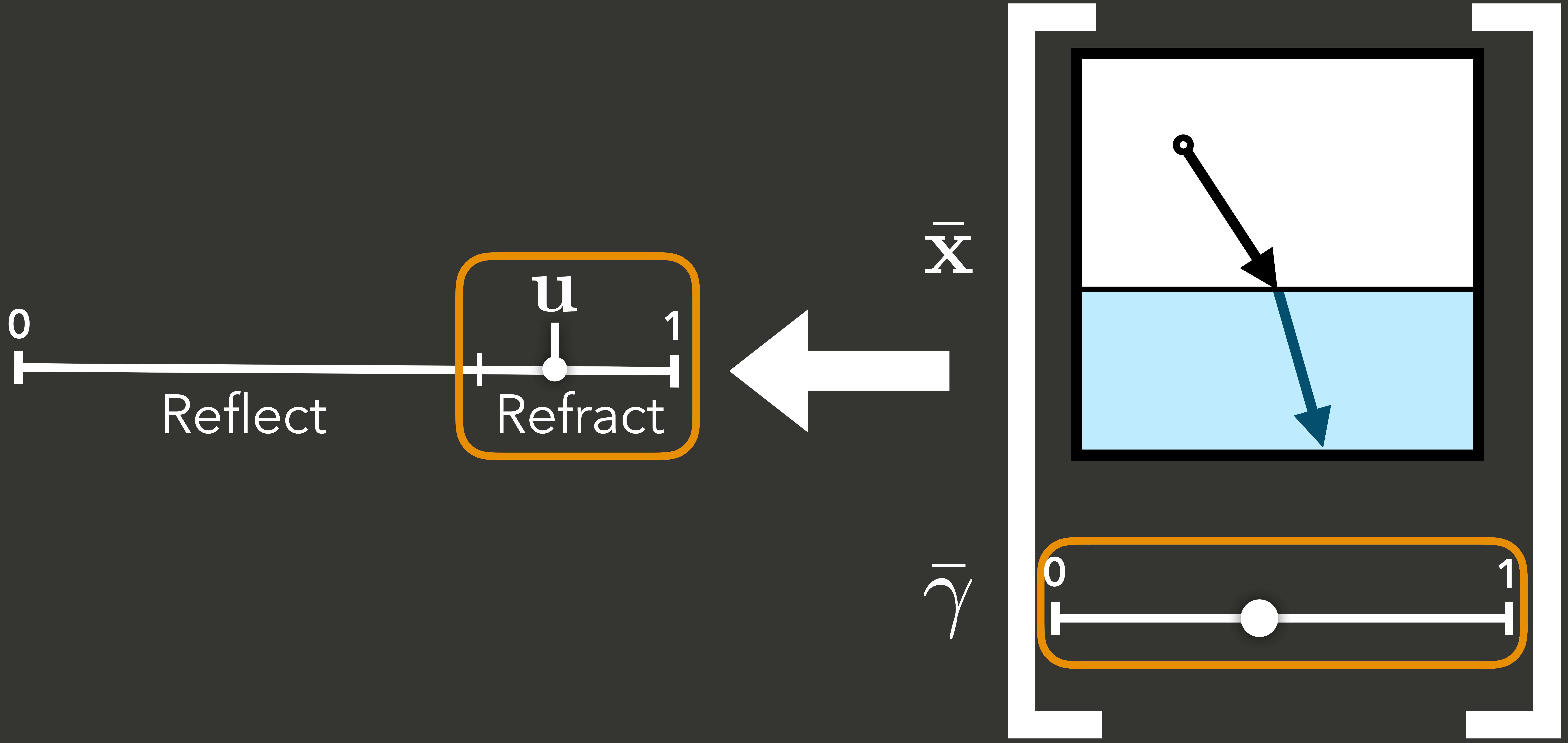
Idea: Augmented Path Space



Idea: Augmented Path Space



Idea: Augmented Path Space



Practical Augmented Path Space

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- Takeaway: Don't actually need to compute $\bar{\gamma}$

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- Invert probabilistically

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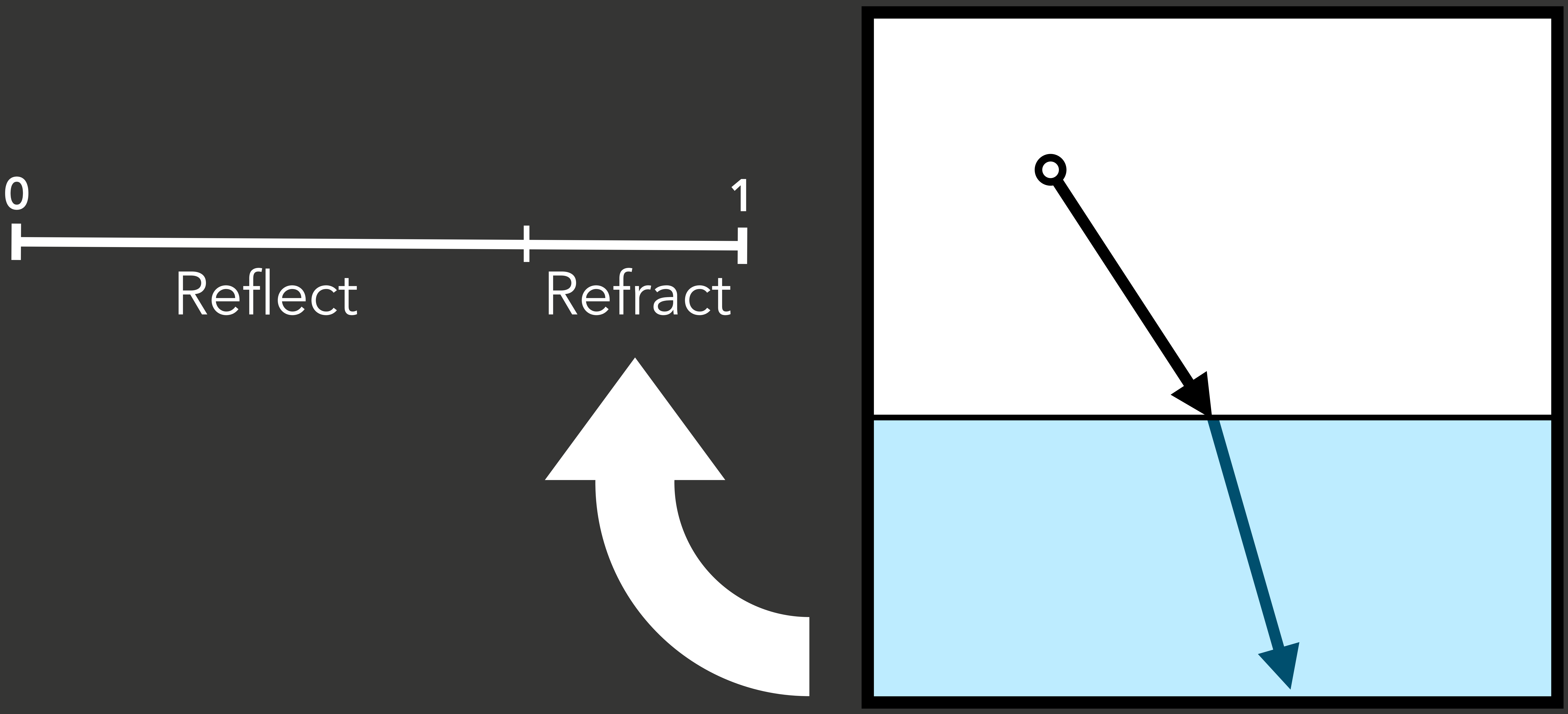
- Theory tells us *how* to sample inverse correctly

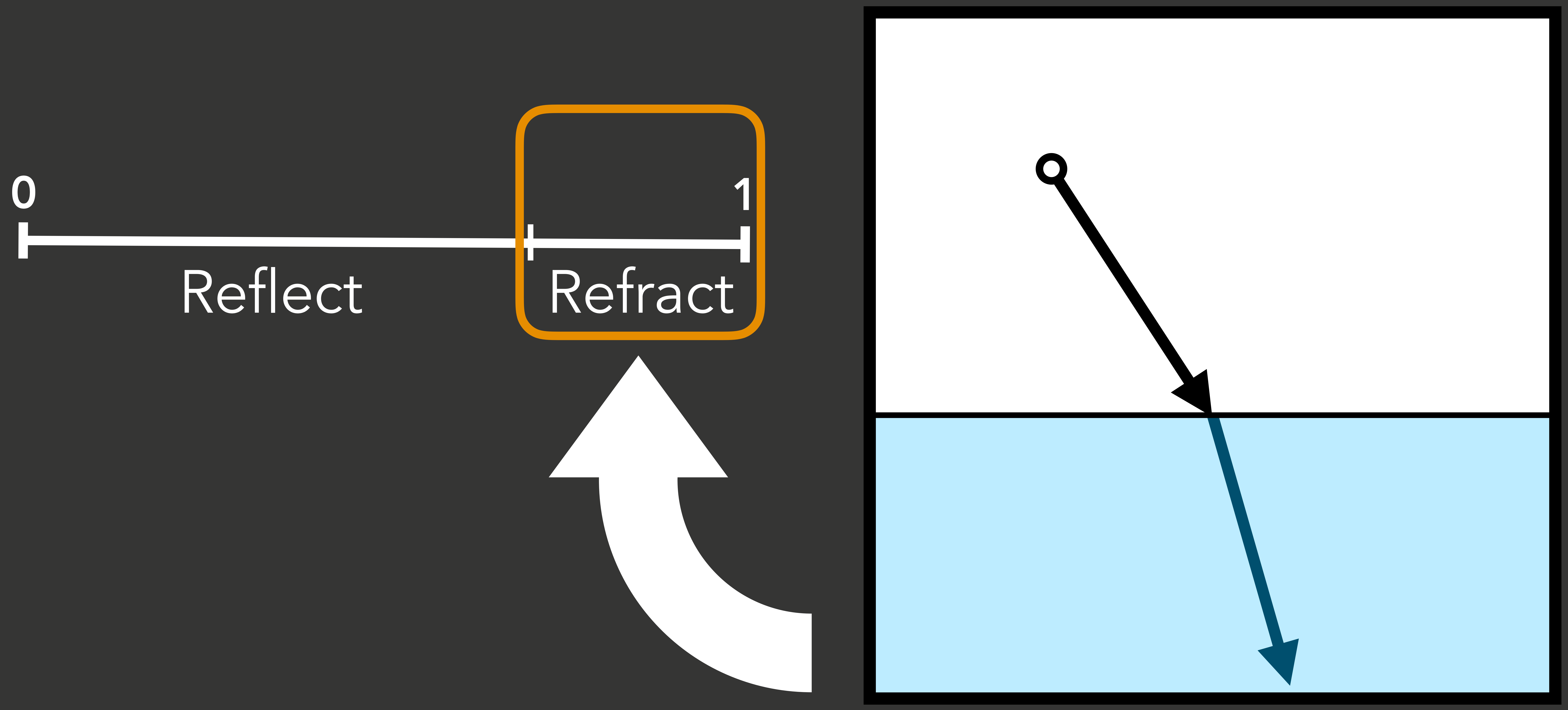
Practical Augmented Path Space

- Takeaway: Don't actually need to compute $\bar{\gamma}$
- Invert probabilistically
- Theory tells us *how* to sample inverse correctly
- Details in paper

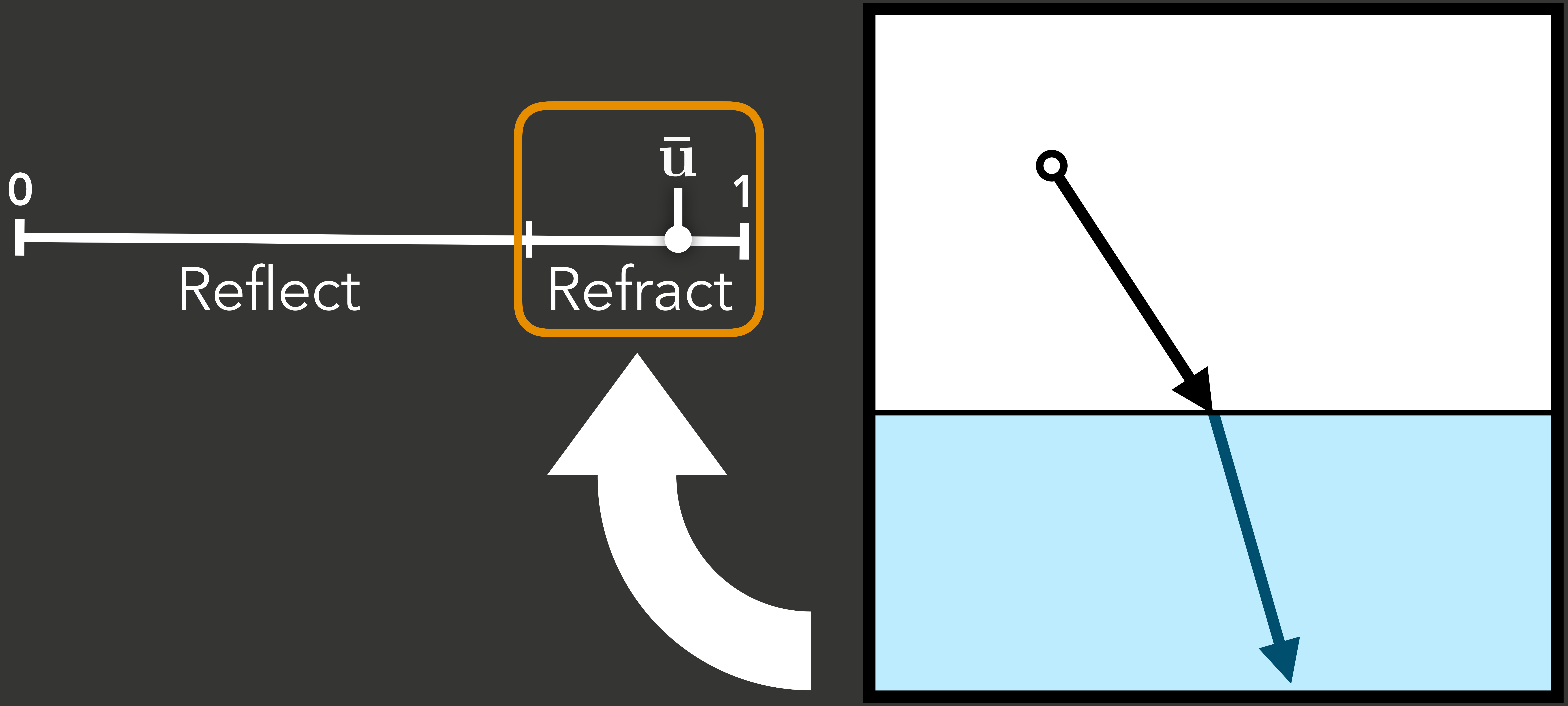
Practical Guidelines



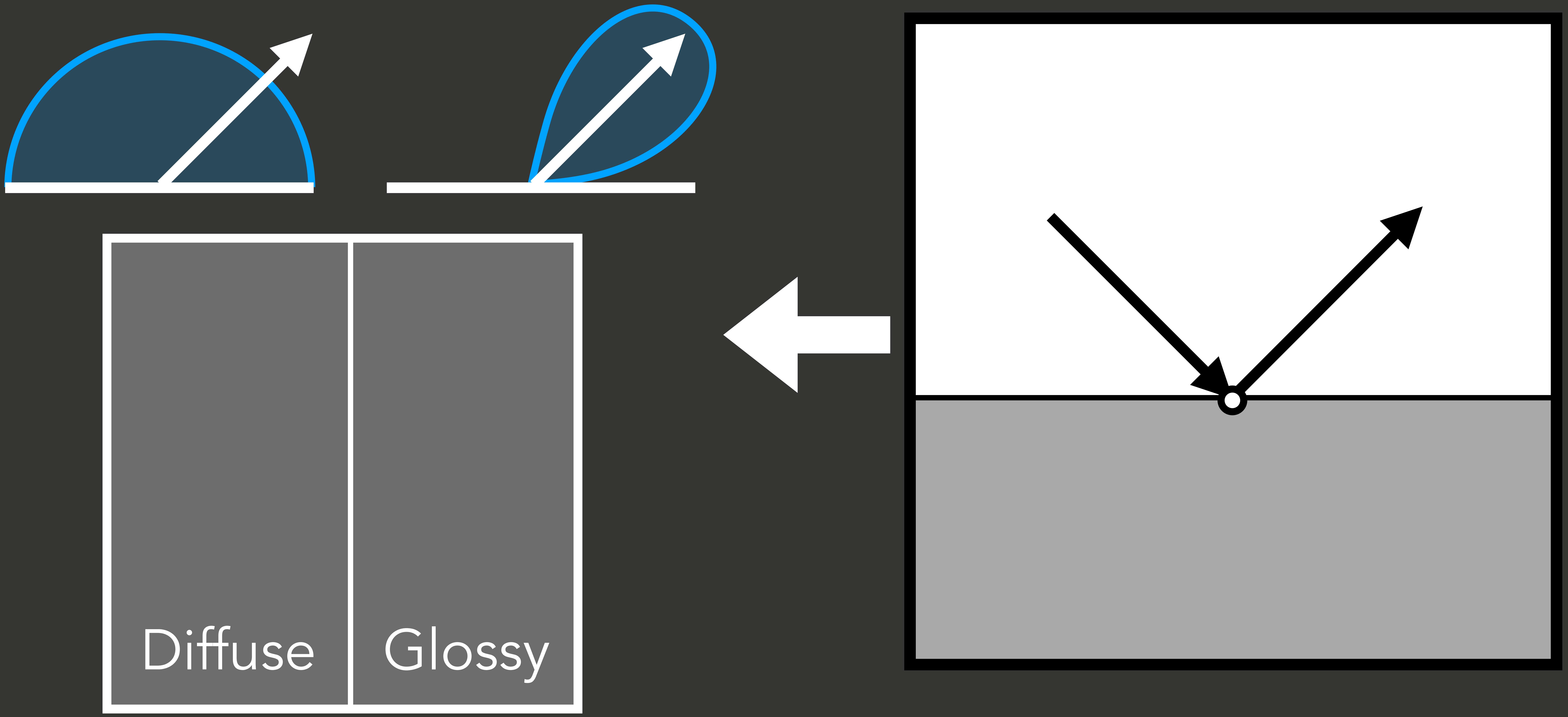
Practical Guidelines



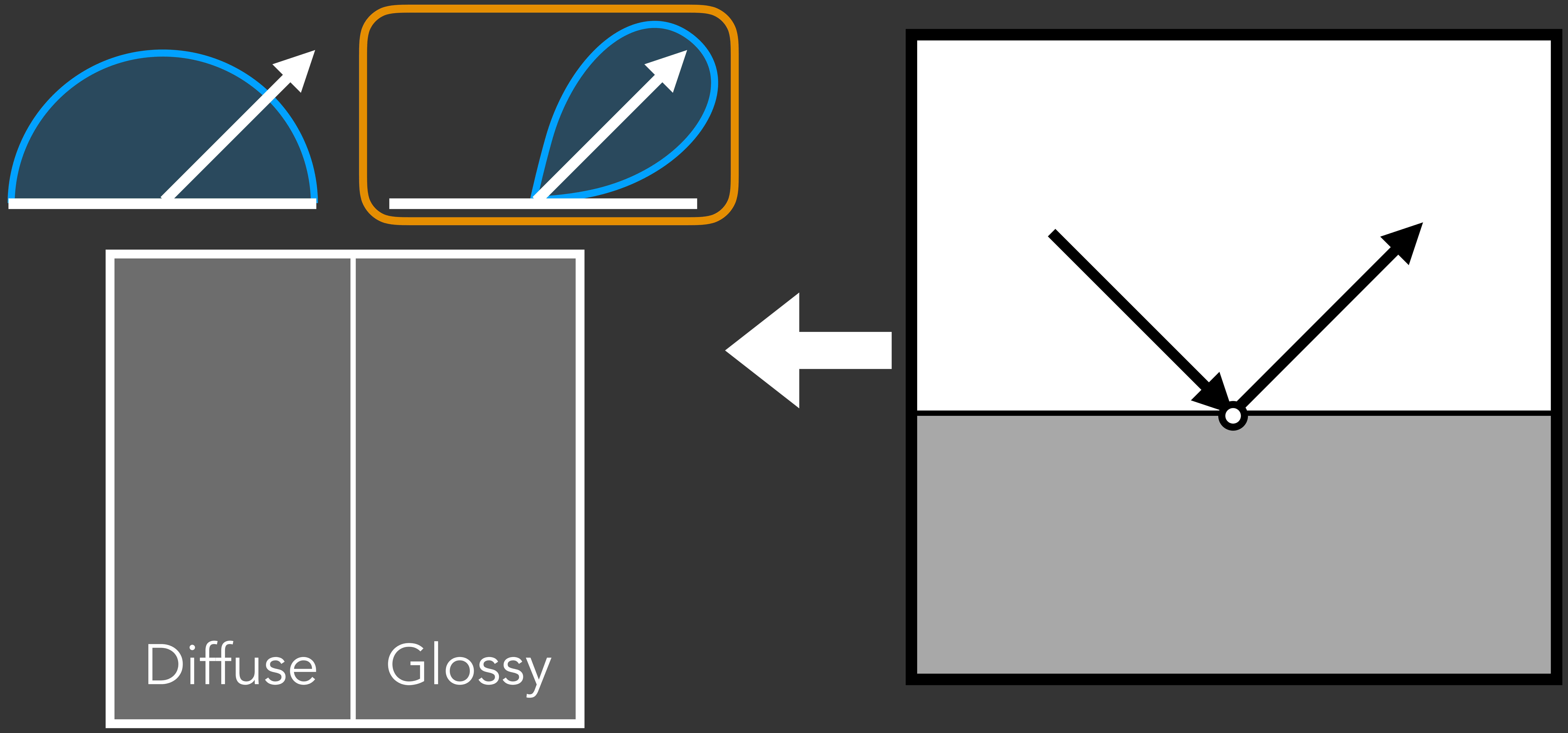
Practical Guidelines



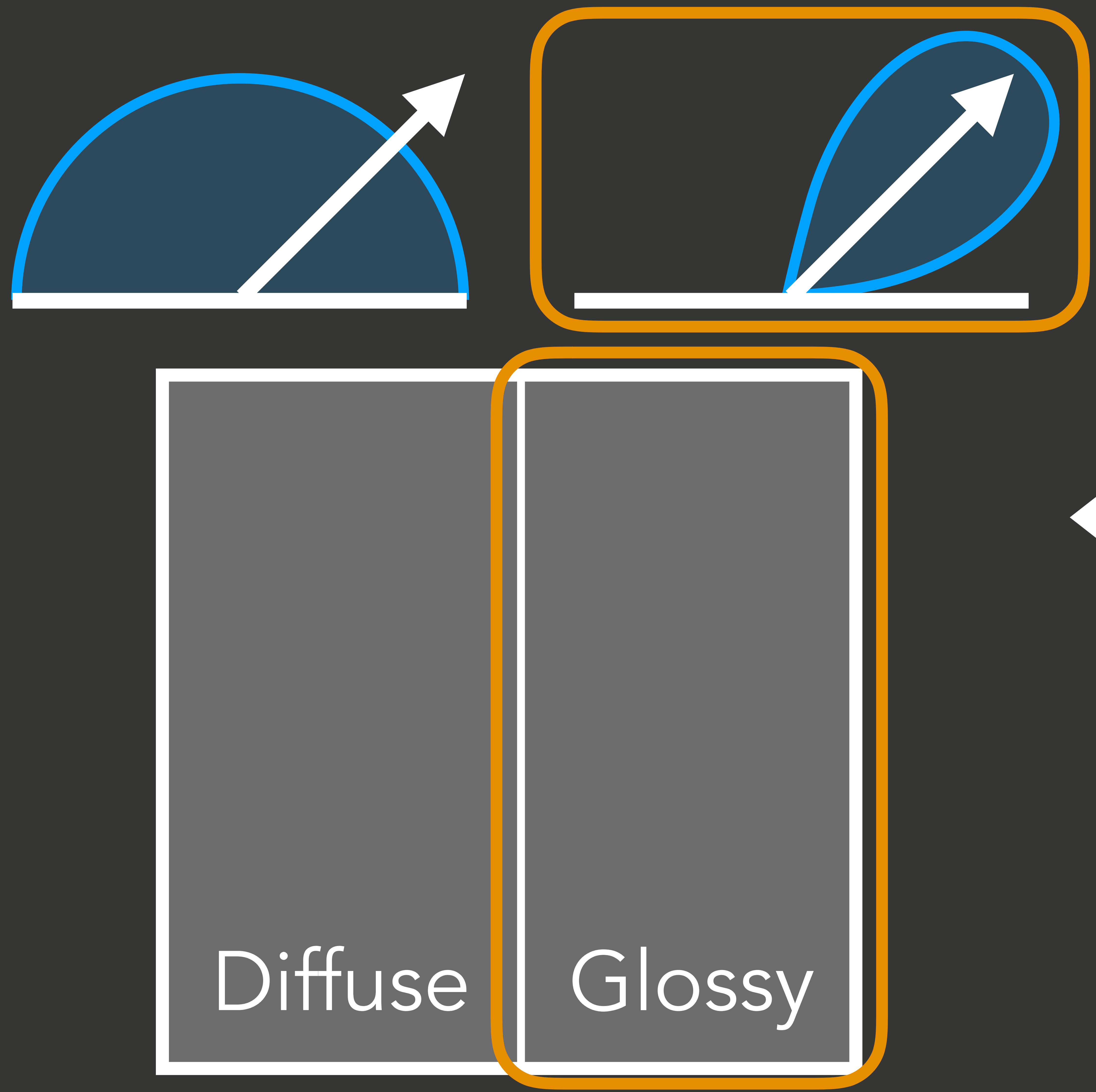
Practical Guidelines

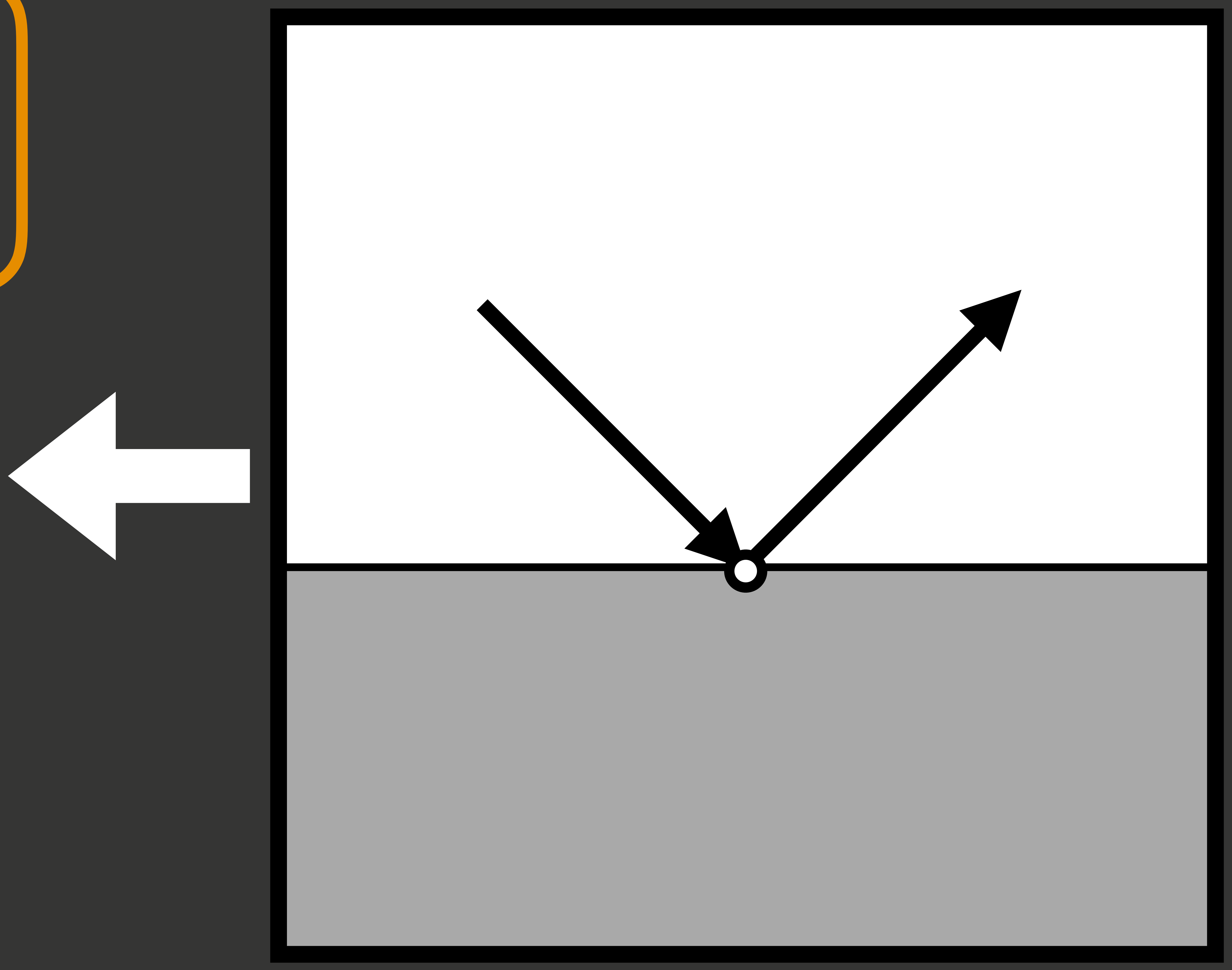


Practical Guidelines

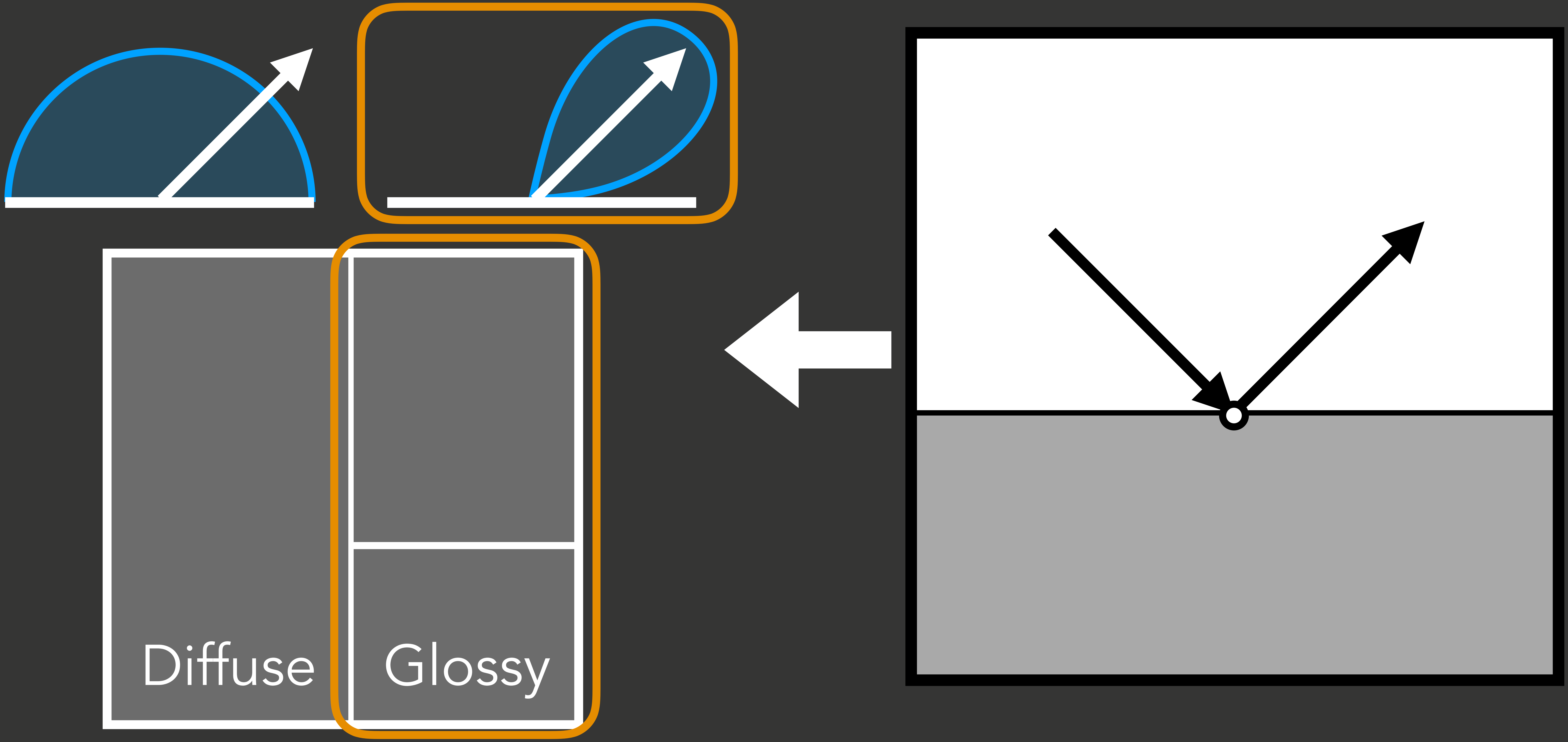


Practical Guidelines

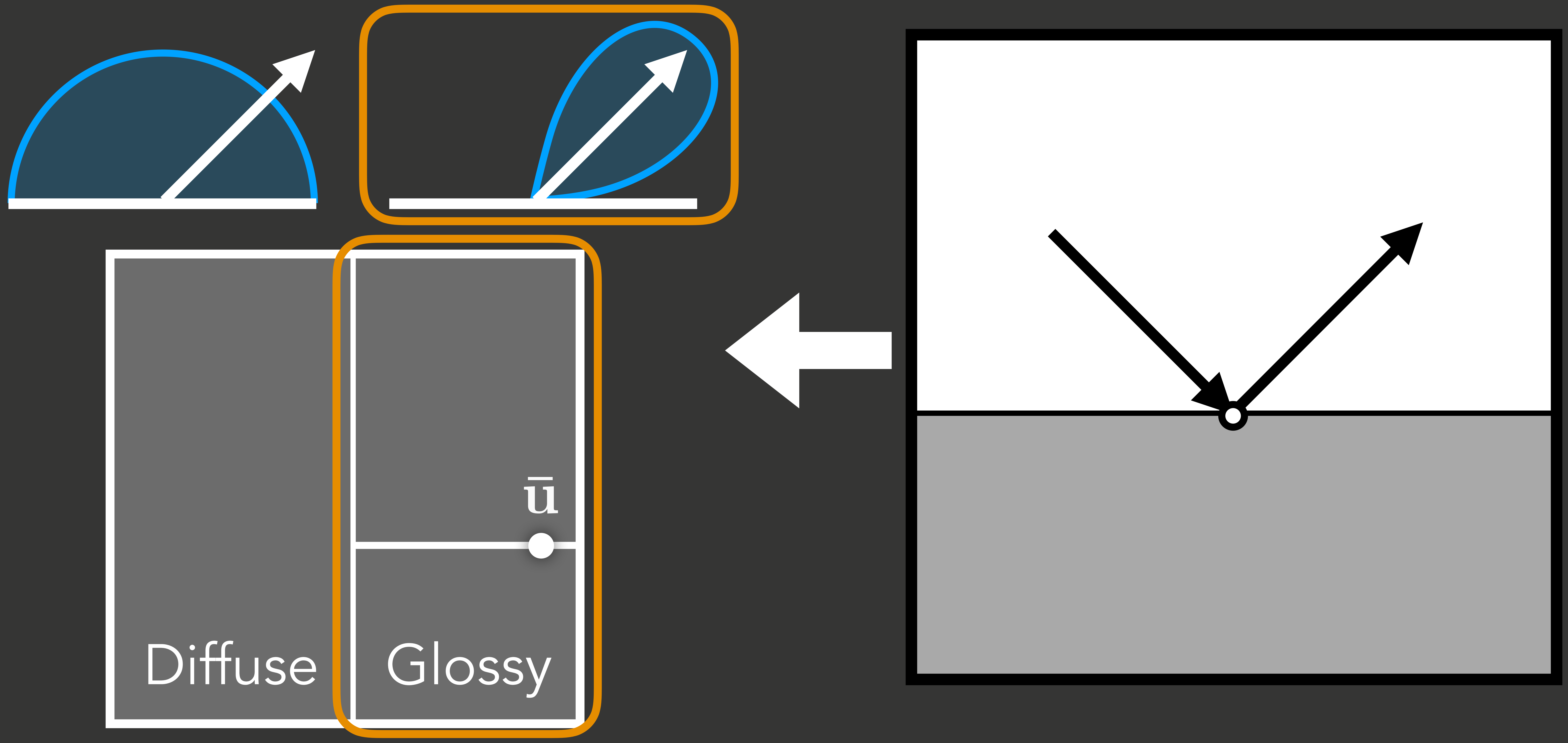




Practical Guidelines



Practical Guidelines

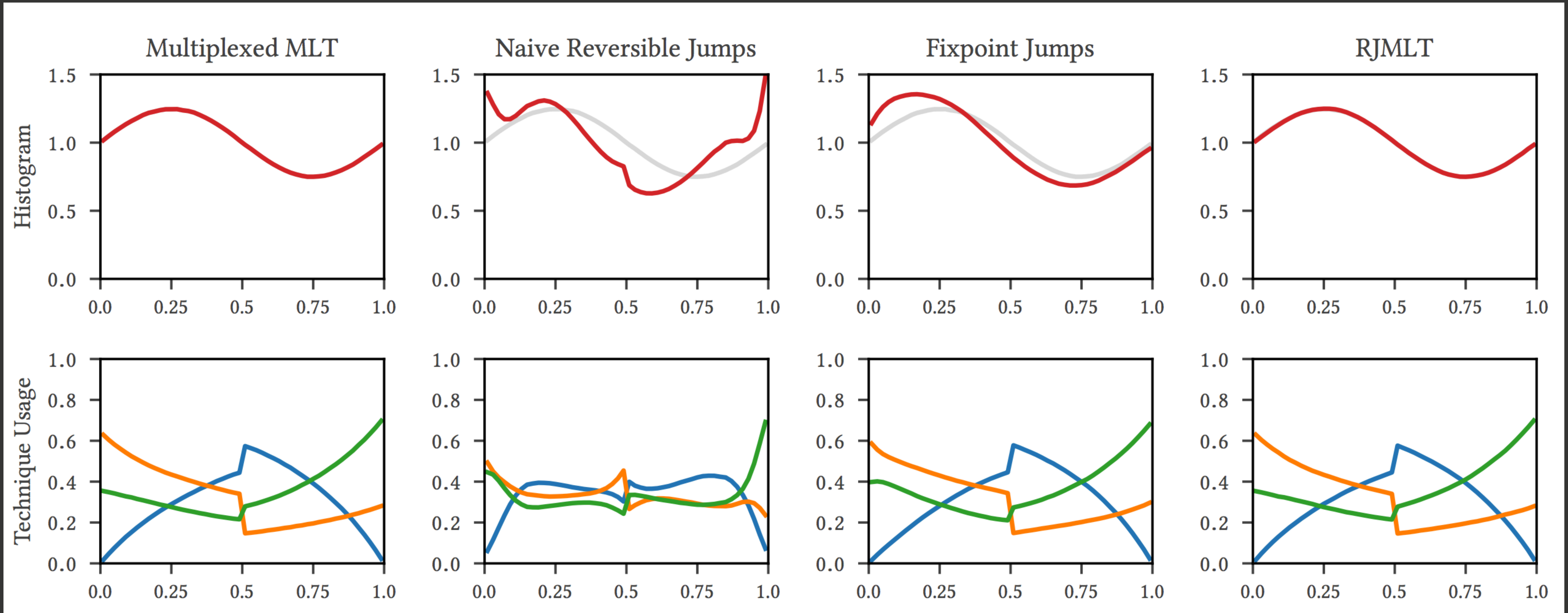


Implementation

- Implemented on top of Multiplexed MLT
- All code released online!

https://github.com/tunabrain/tungsten

Validation

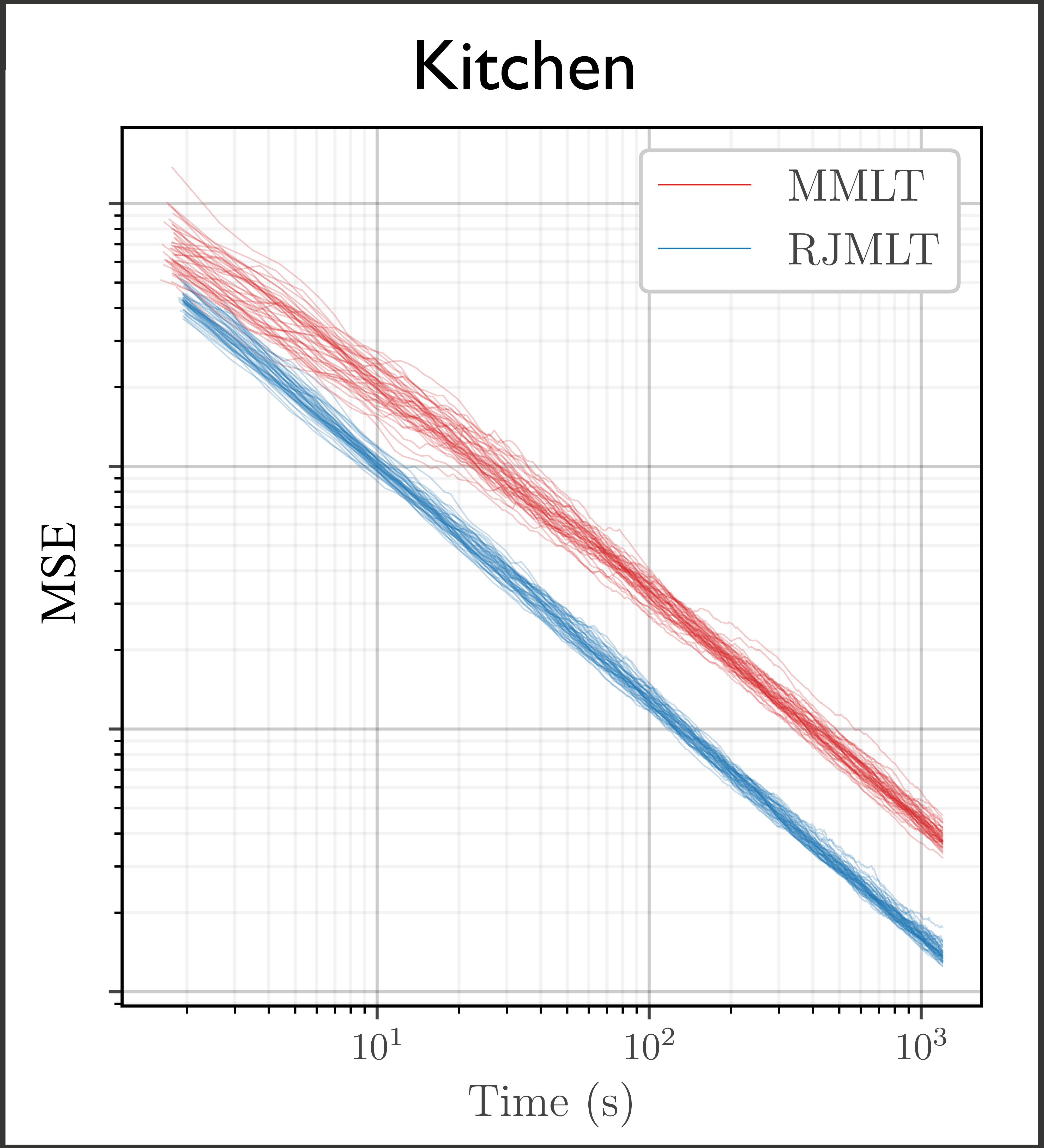


Multiplexed MLT
Naive Reversible Jumps
Fixpoint Jumps
RJMLT
Histogram
Technique Usage
1.5
1.0
0.5
0.0
0.8
0.6
0.4
0.2
0.25
0.75

Results

Convergence

Temporal Convergence



MMLT

RJMLT

MMLT

RJMLT

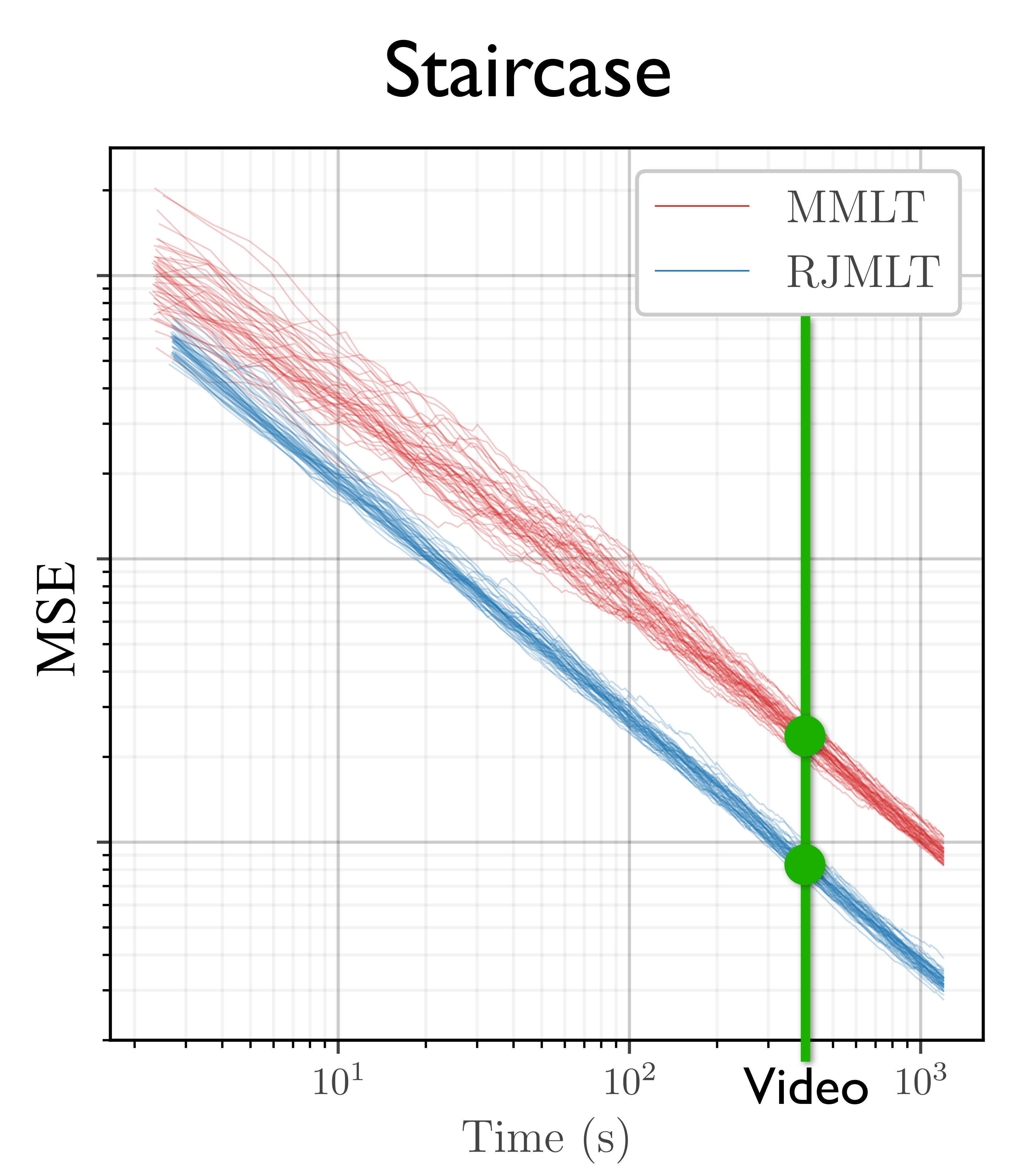
MMLT

RJMLT

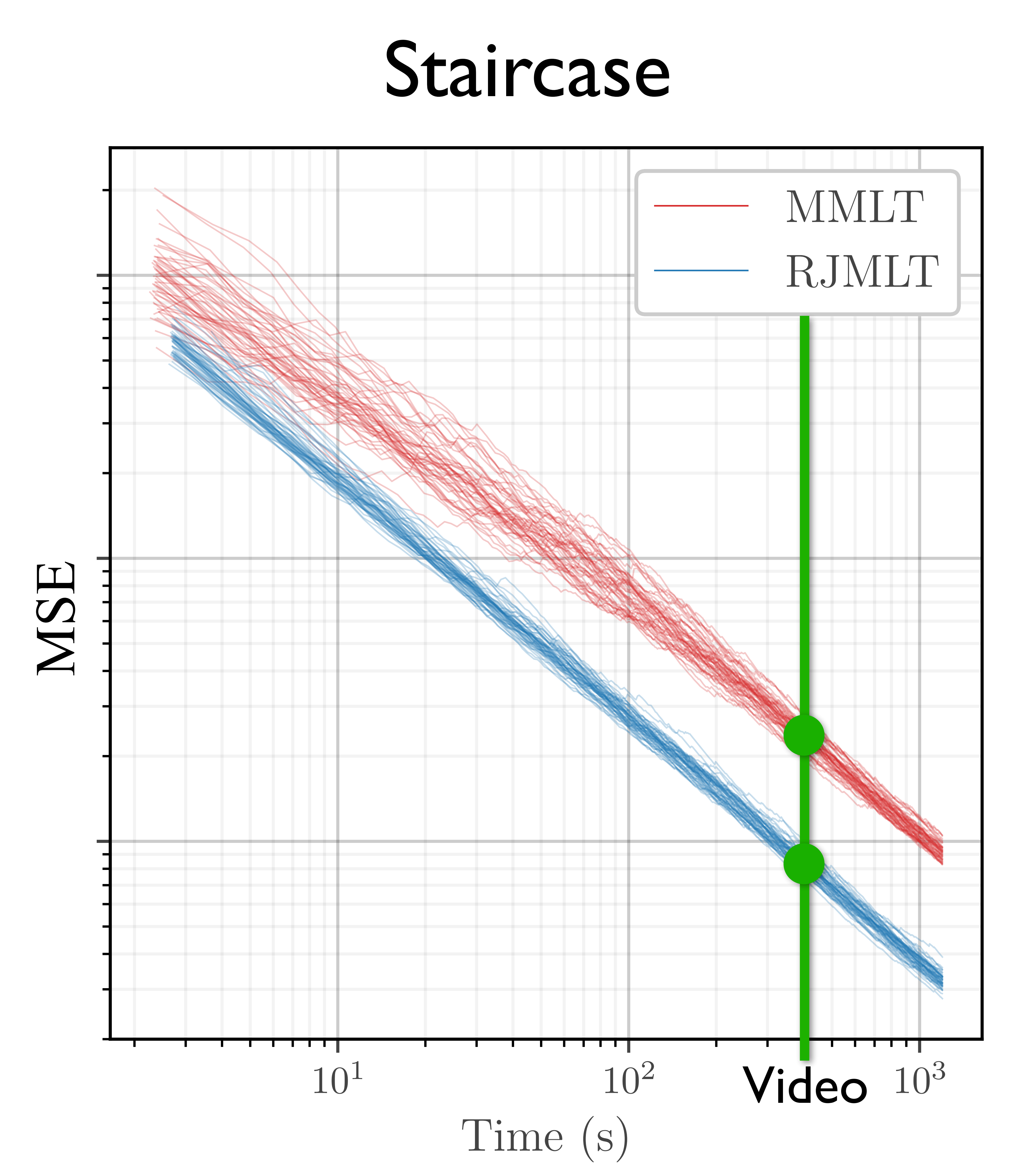
MMLT

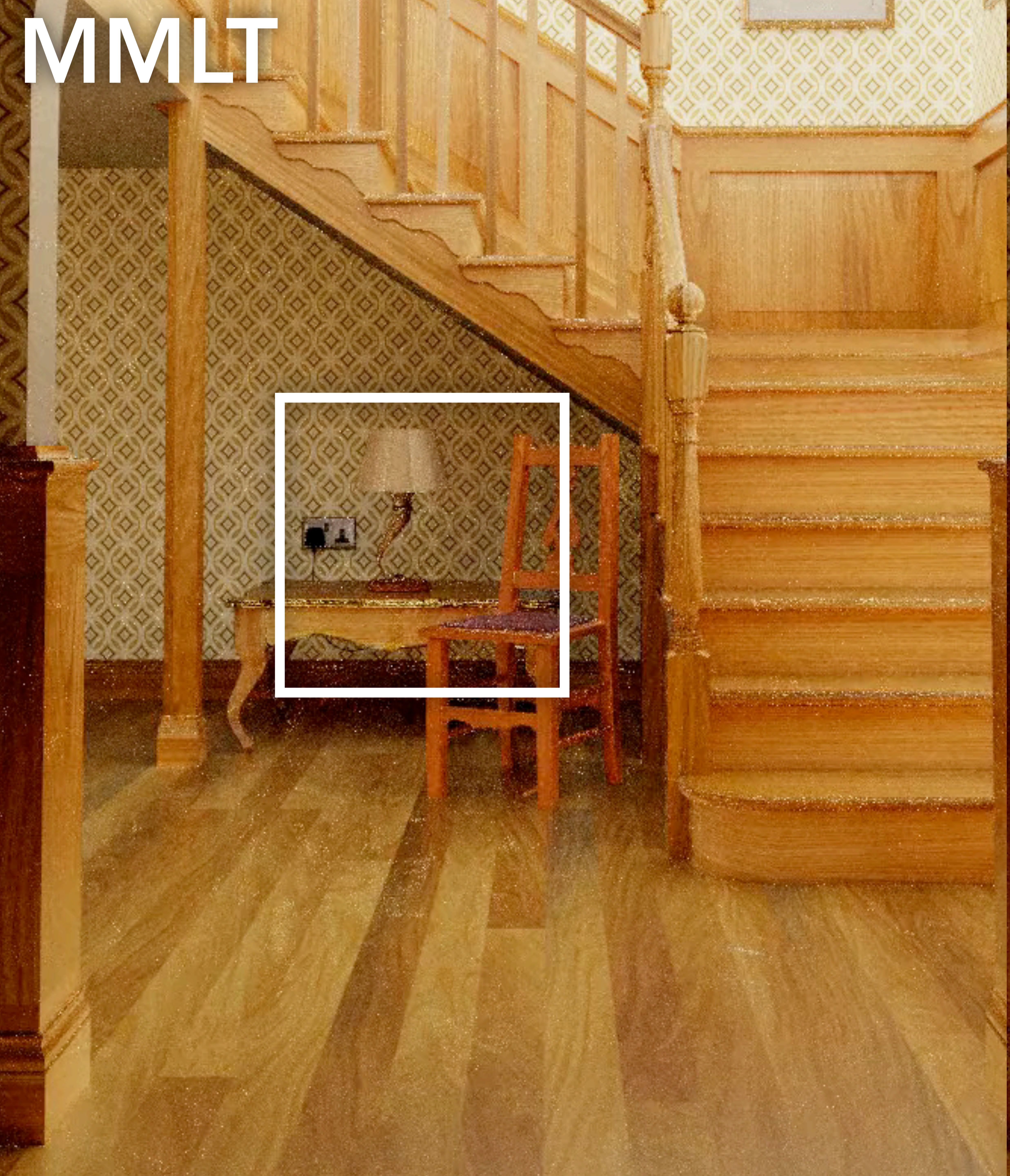
RJMLT

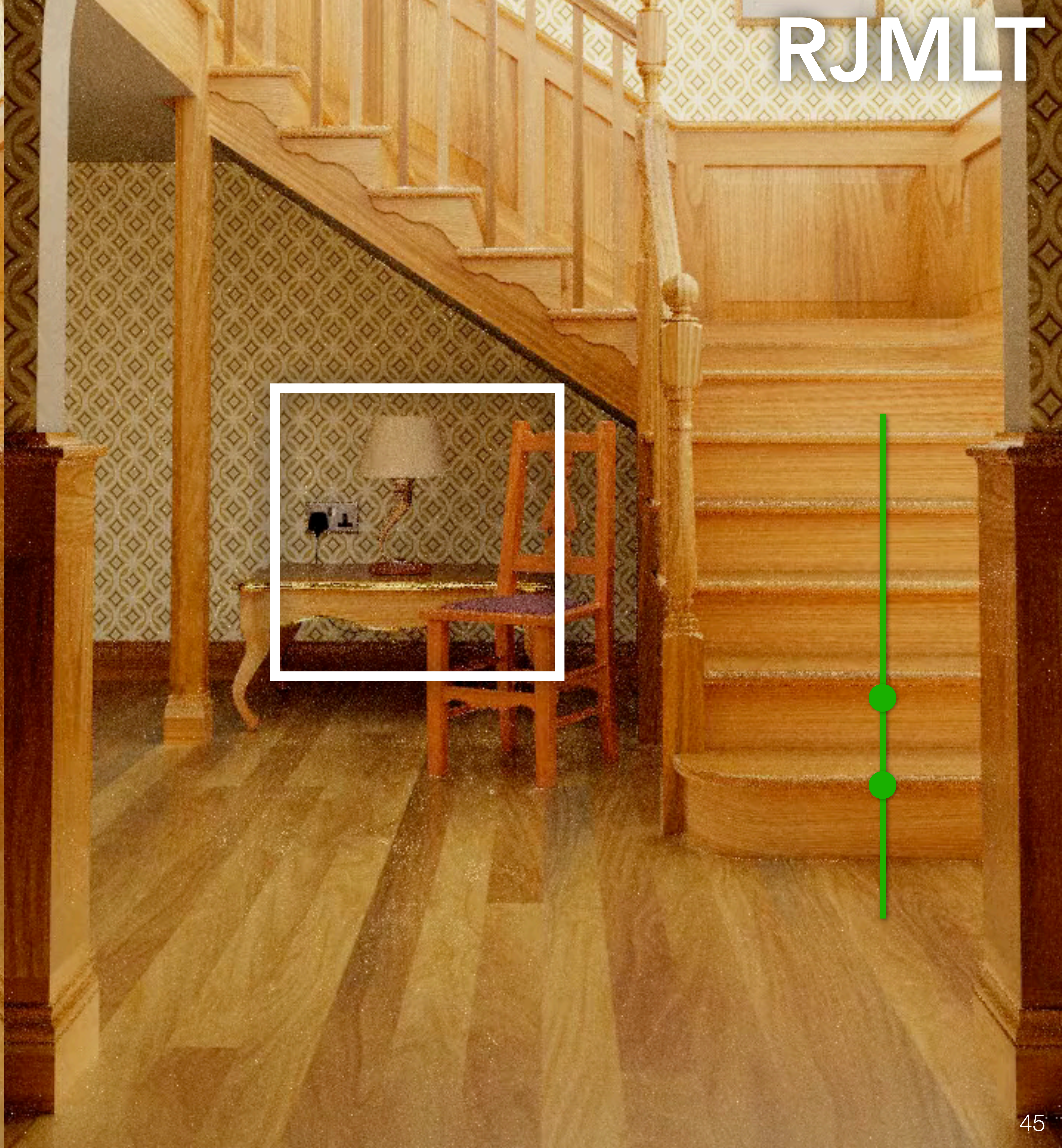

MMLT
RJMLT


Staircase
MMLT
RJMLT
MSE
10^1
10^2
Video
10^3
Time (s)

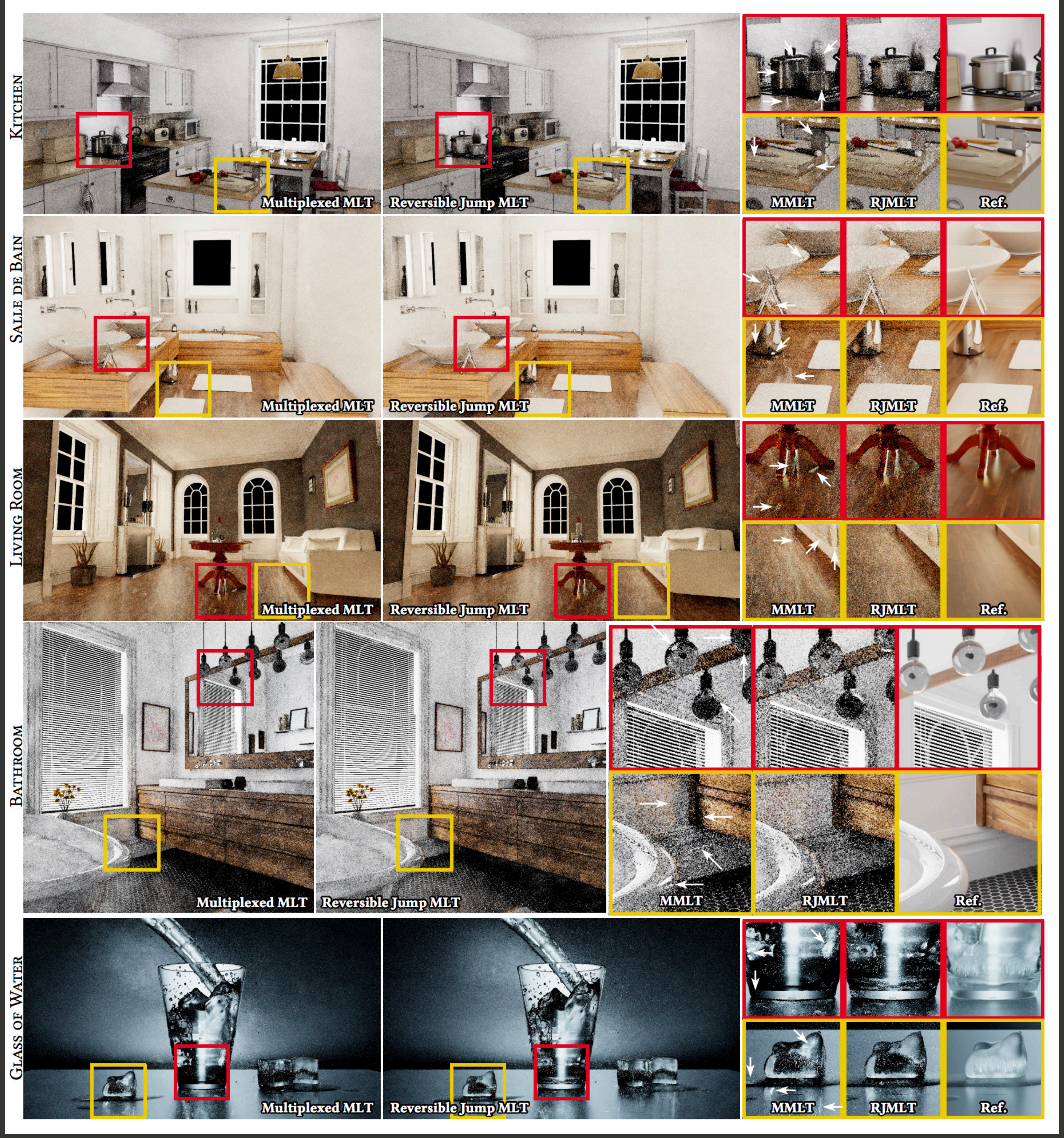

MMLT
RJMLT

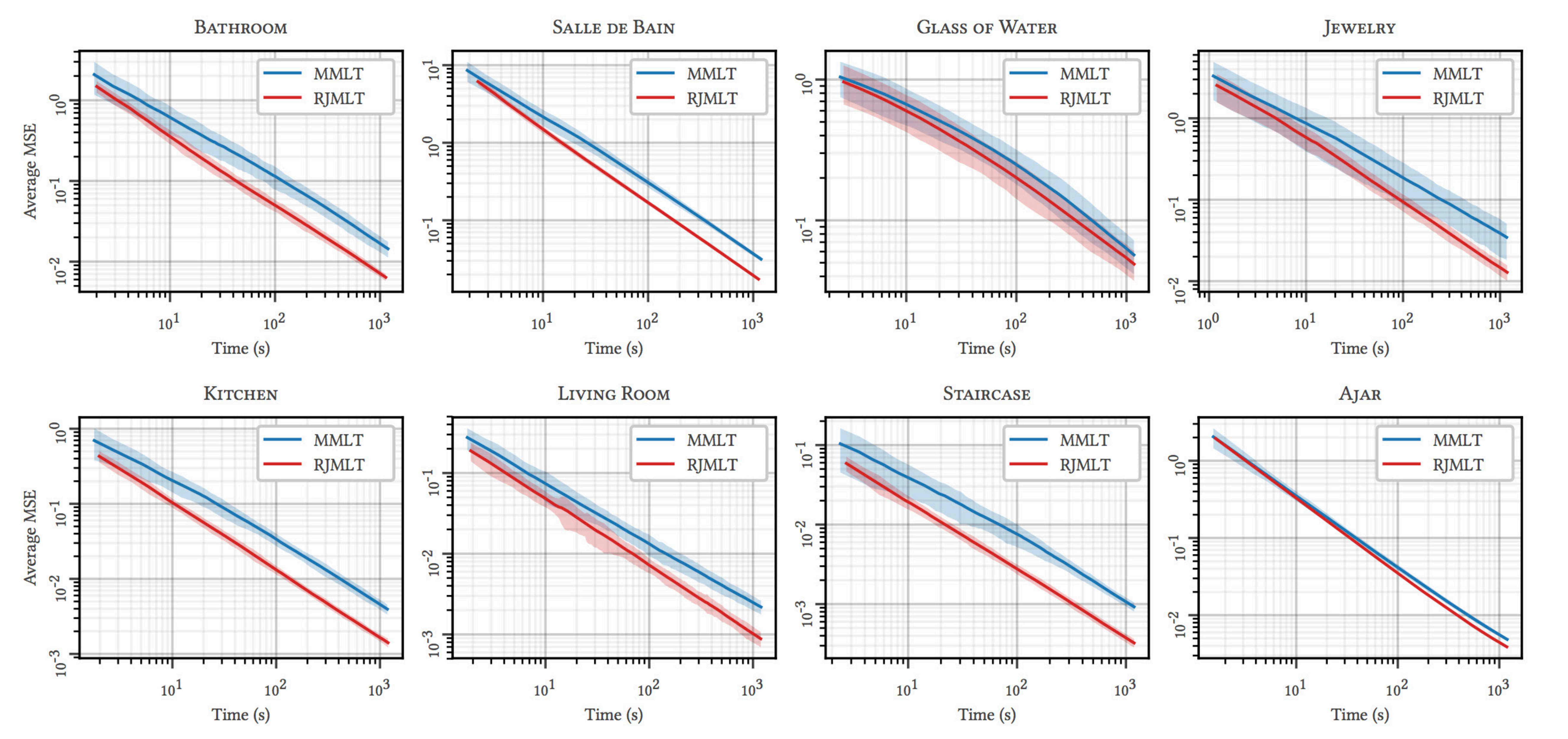

Staircase
MMLT
RJMLT
MSE
10^1
10^2
Video
10^3
Time (s)


MMLT


RJMLT

More results in paper!





Limitations

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  - Rejection sampling (Woodcock tracking, …)

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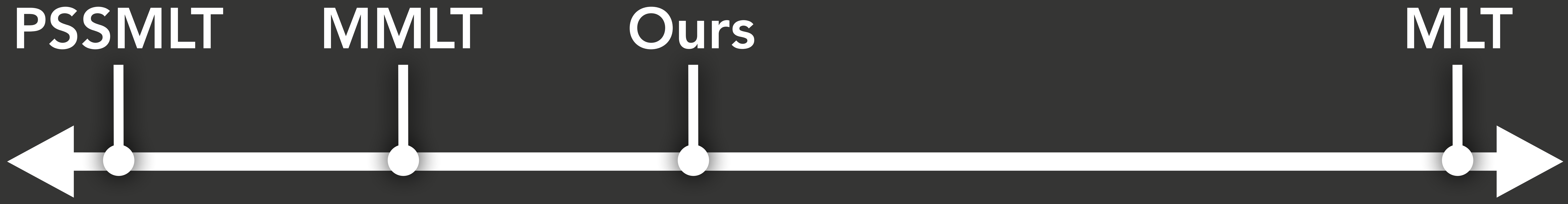
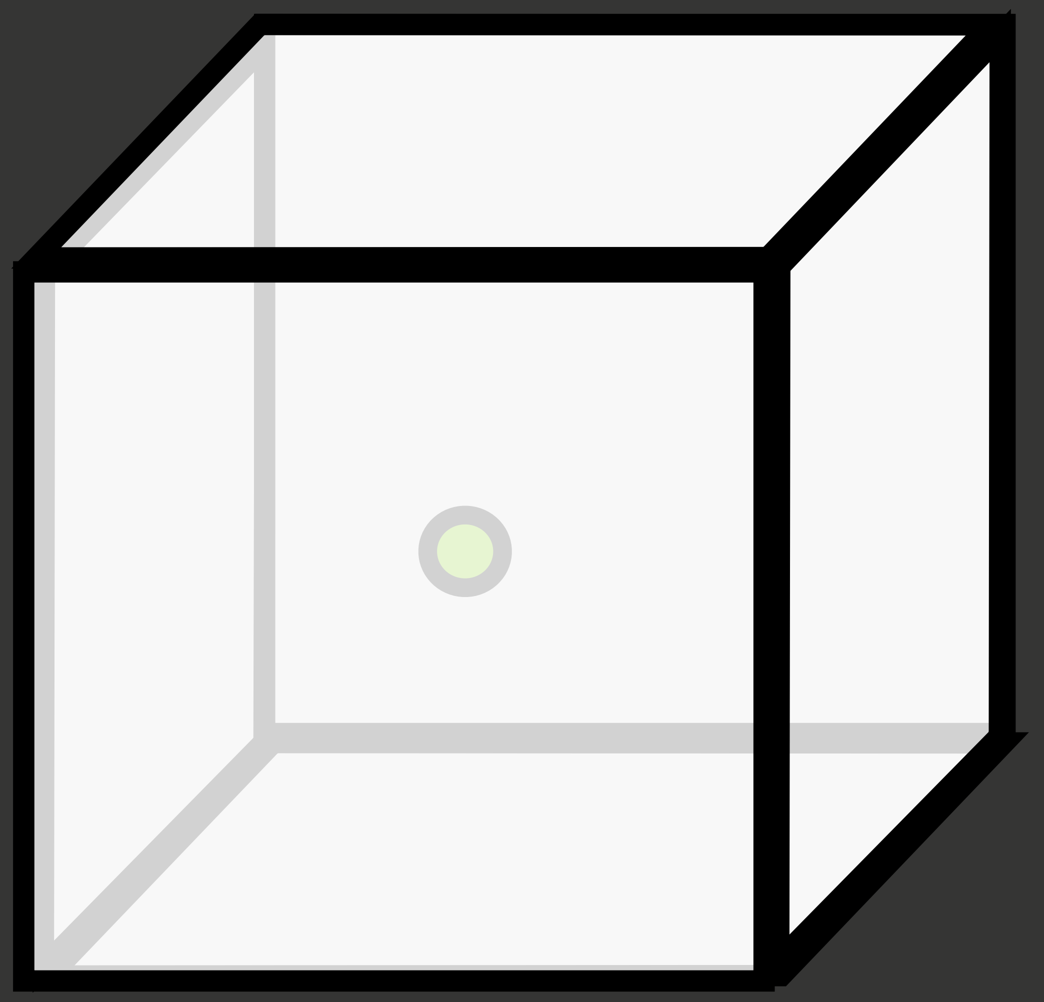
- Inversion method doesn’t cover everything
  - Rejection sampling (Woodcock tracking, …)
  - Box-Muller Transform

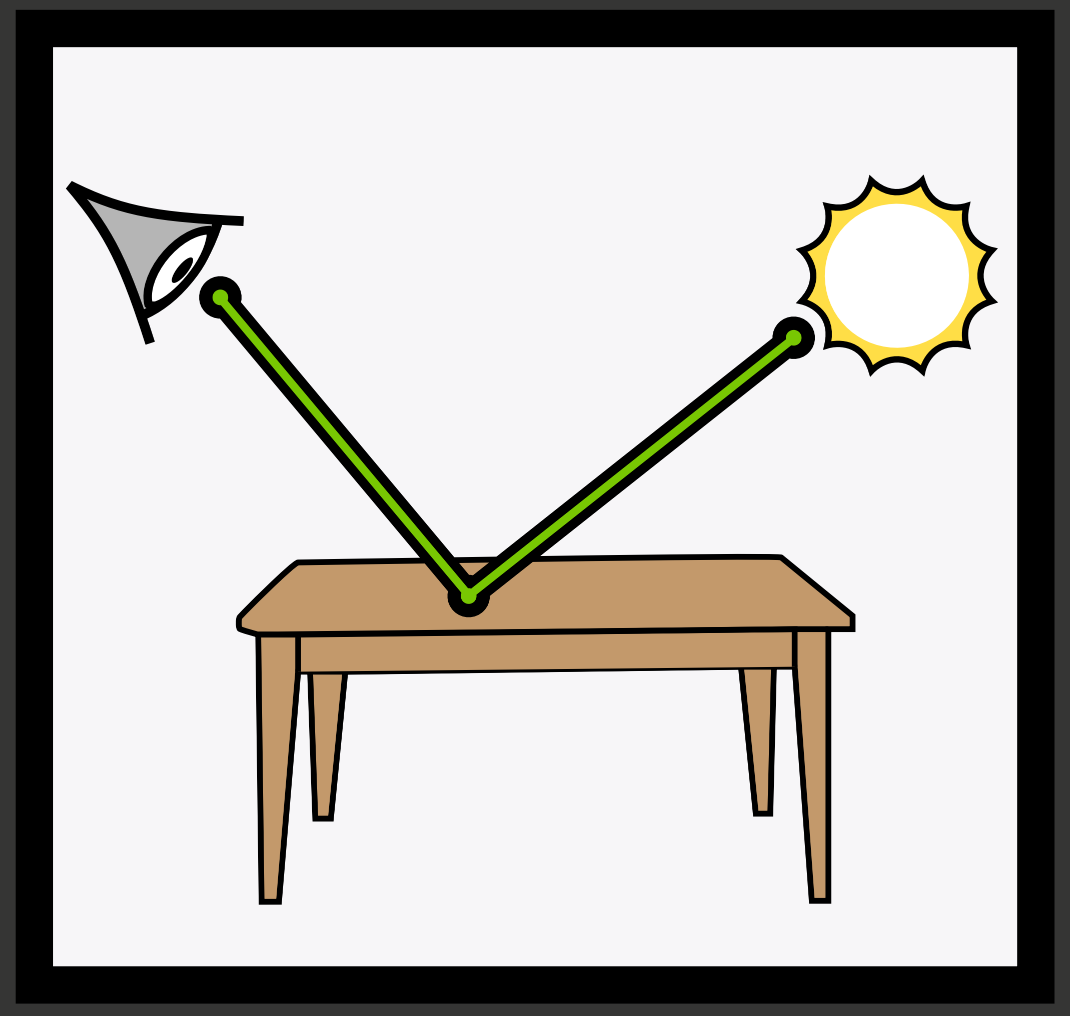
➡ Fall back to MMLT

Future Work

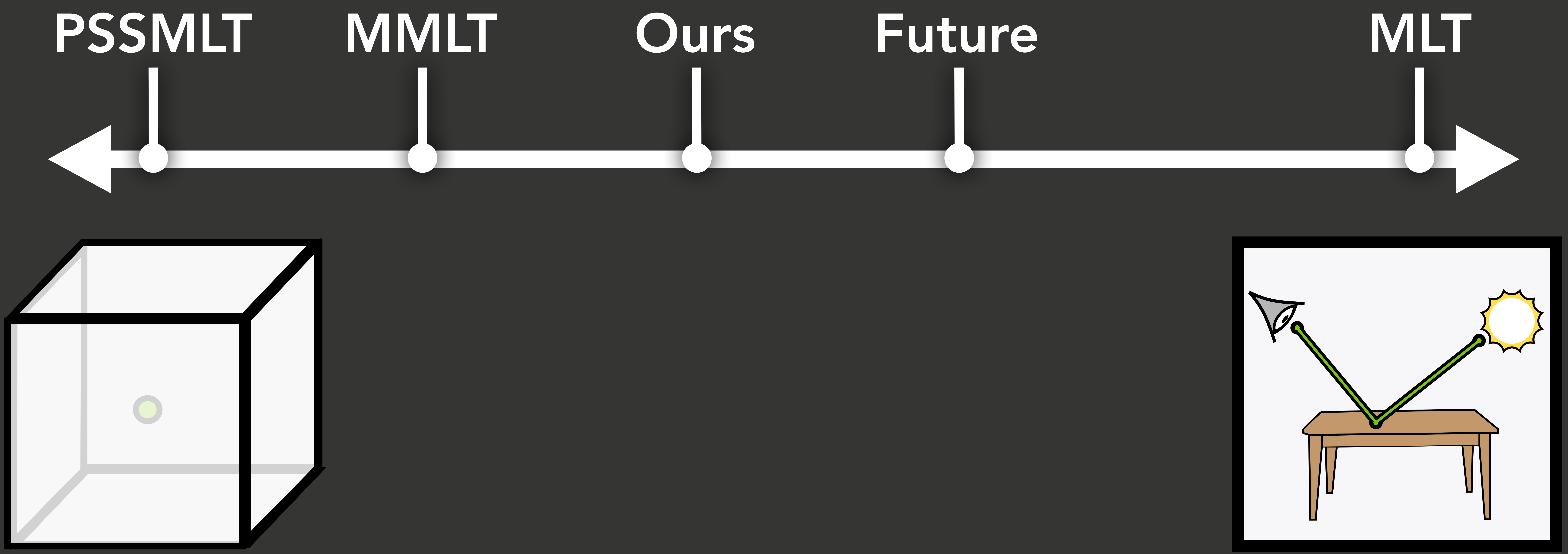
Future Work



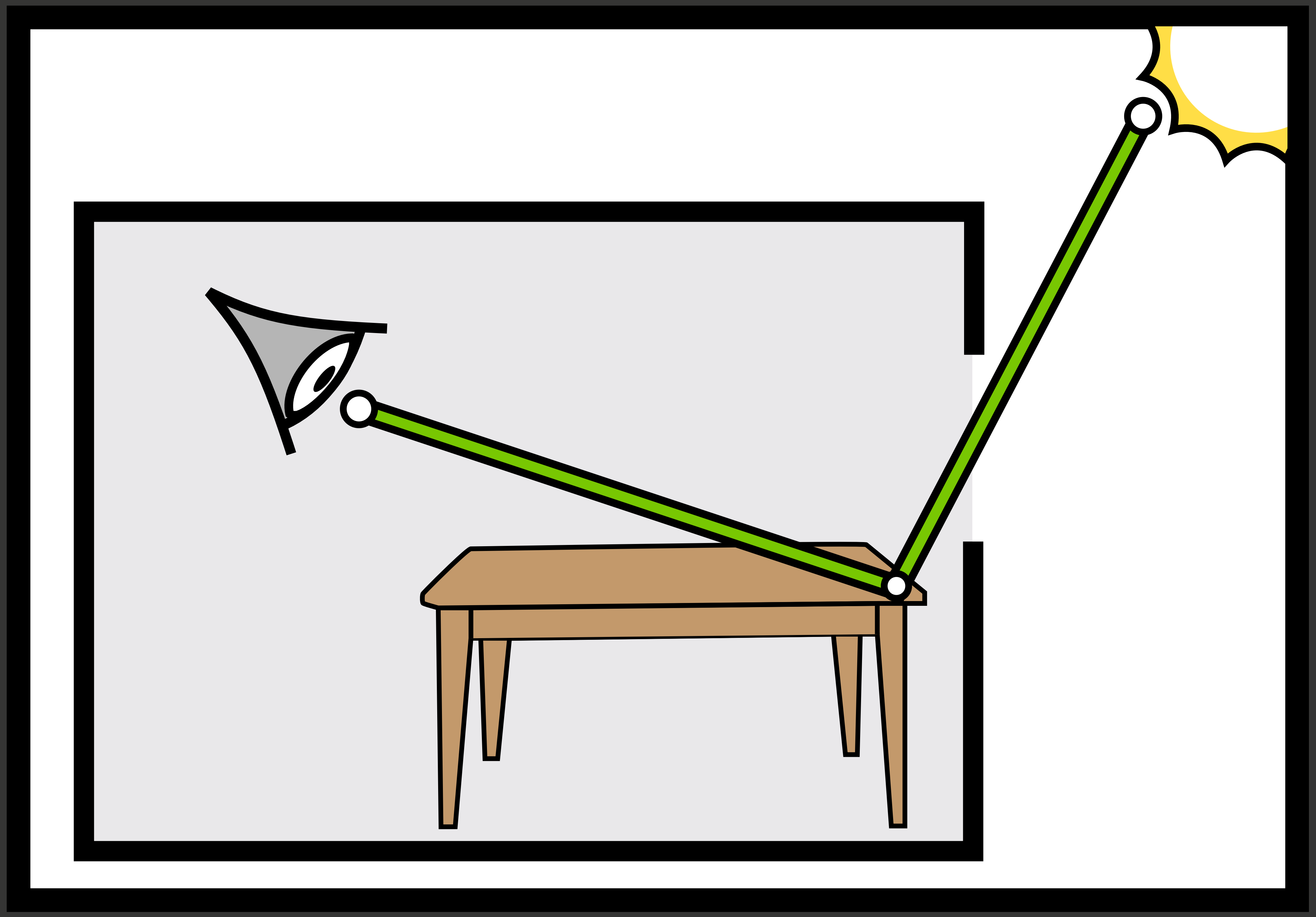




Future Work

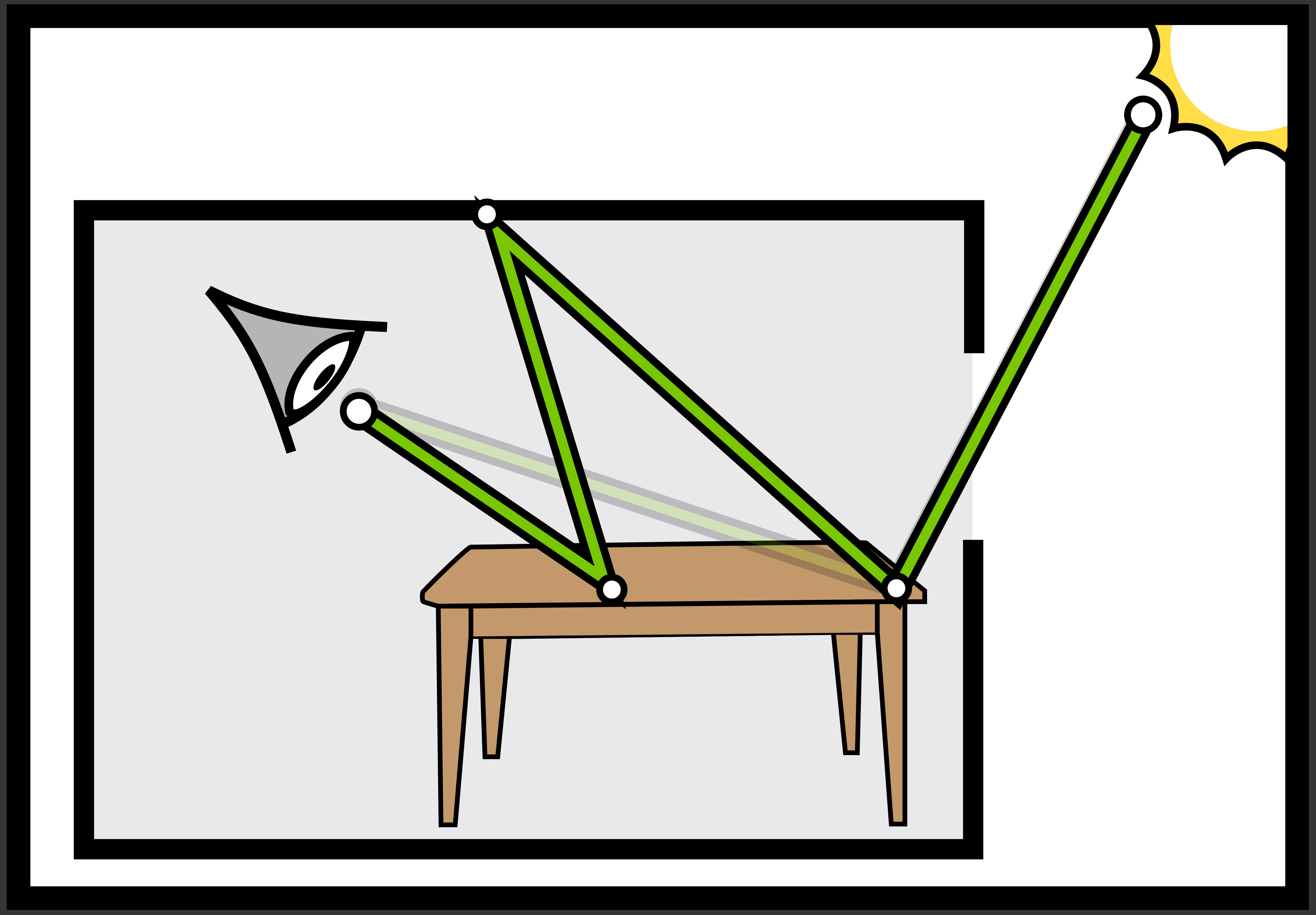


Future Work



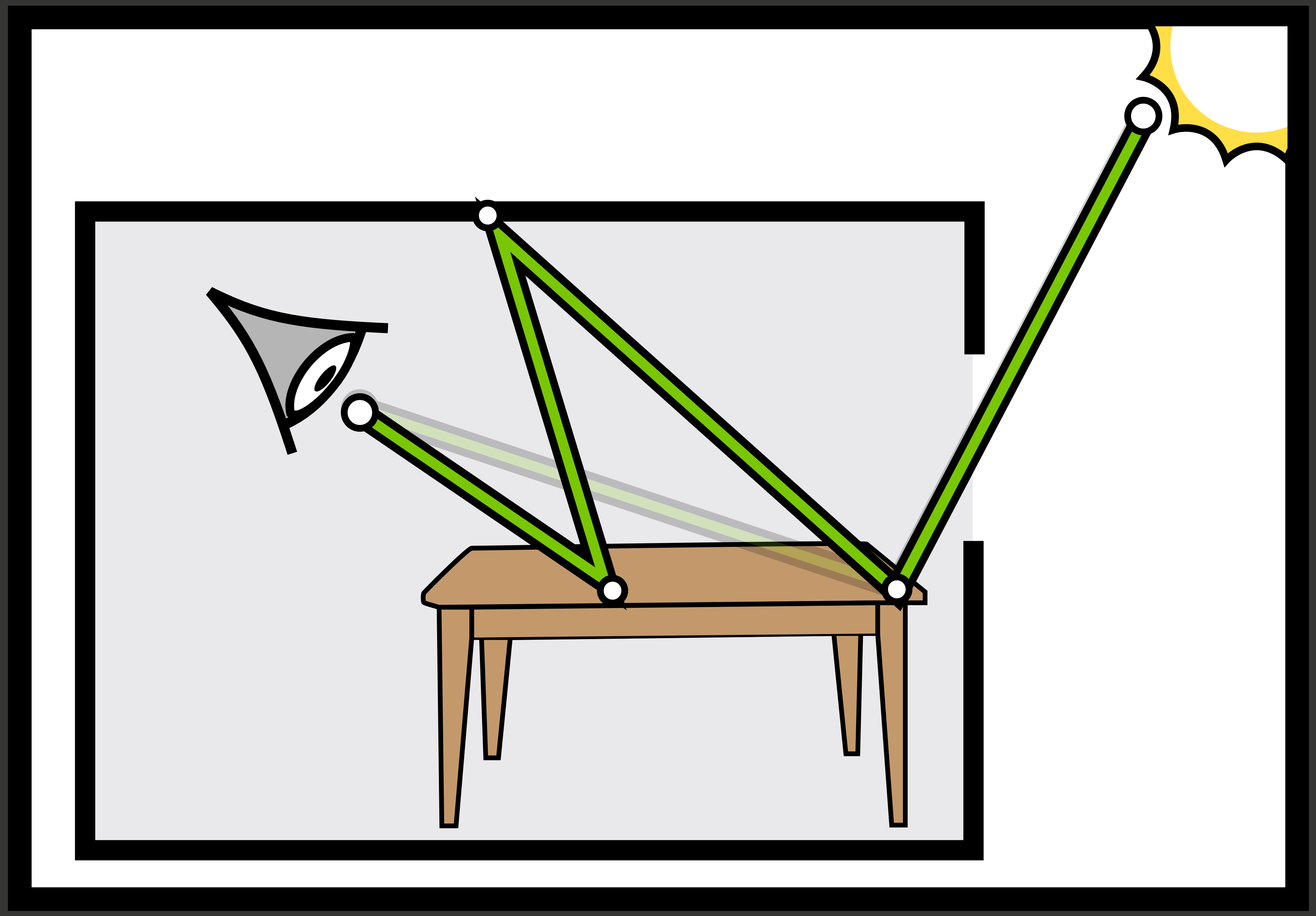
Change Path Length

Future Work

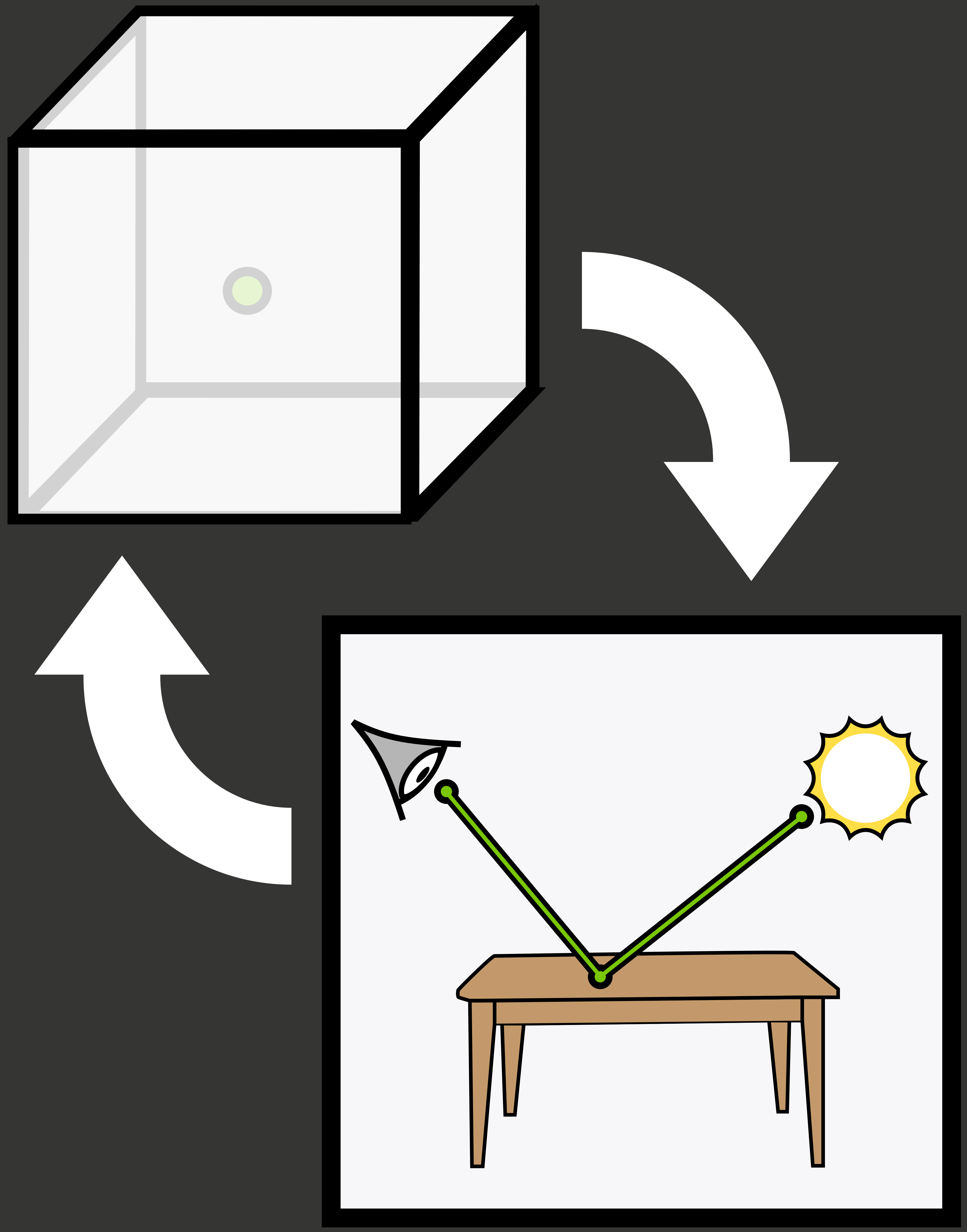


Change Path Length

Future Work

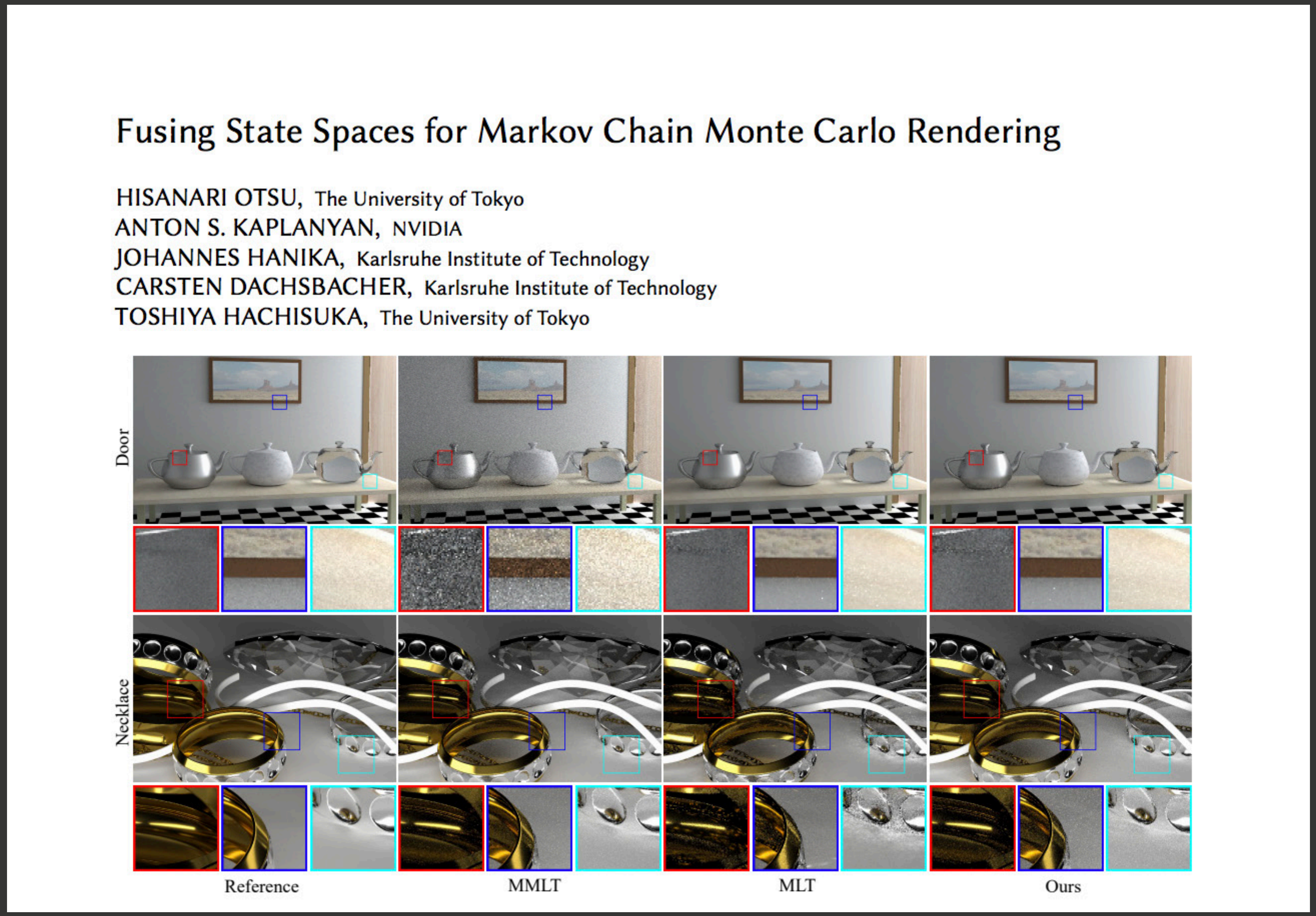


Change Path Length



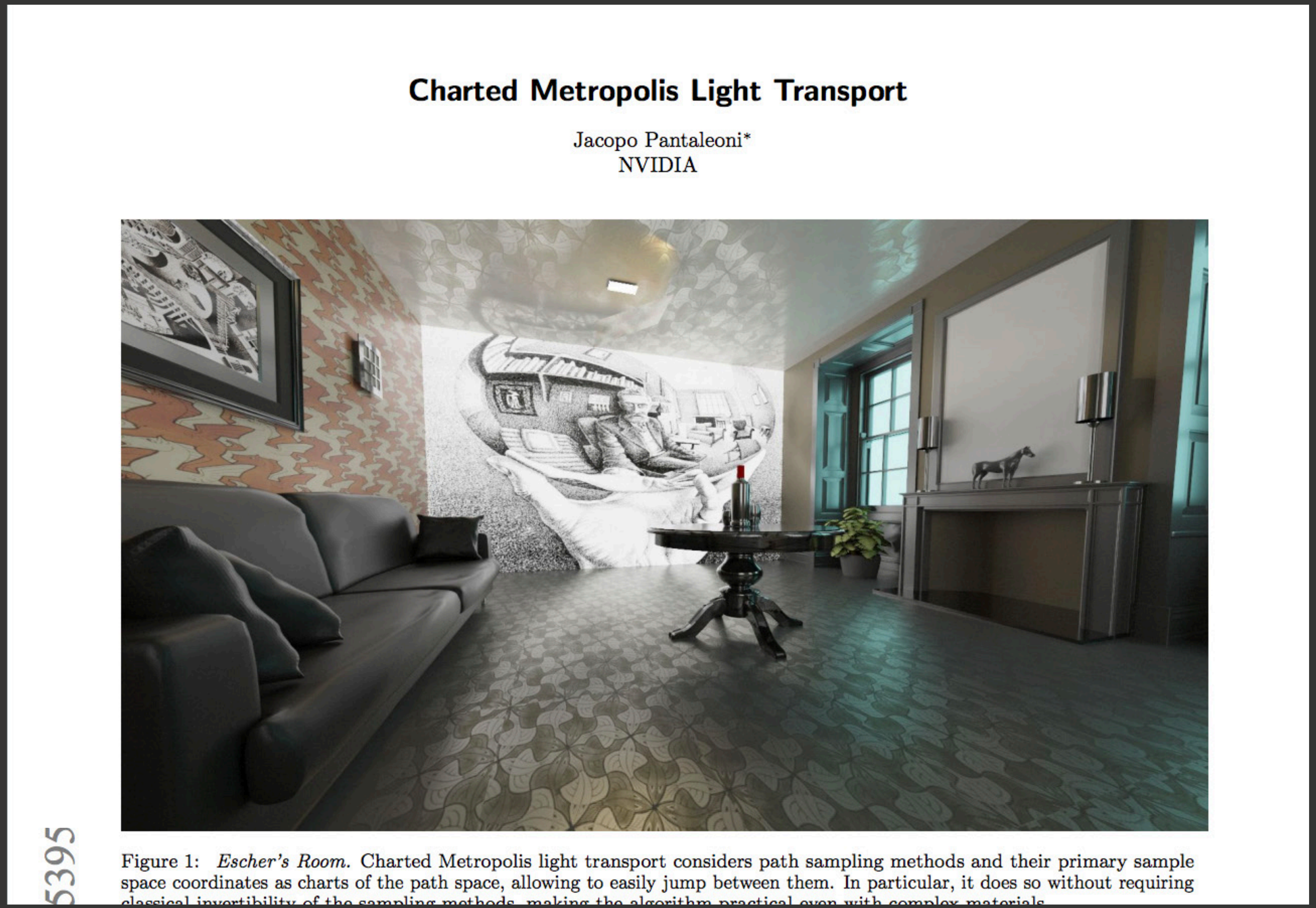
Rapidly Switch Spaces

Concurrent Work


Fusing State Spaces for Markov Chain Monte Carlo Rendering

HISANARI OTSU, The University of Tokyo
ANTON S. KAPLANYAN, NVIDIA
JOHANNES HANIKA, Karlsruhe Institute of Technology
CARSTEN DACHSBACHER, Karlsruhe Institute of Technology
TOSHIYA HACHISUKA, The University of Tokyo

[Otsu et al. '17]


Charted Metropolis Light Transport

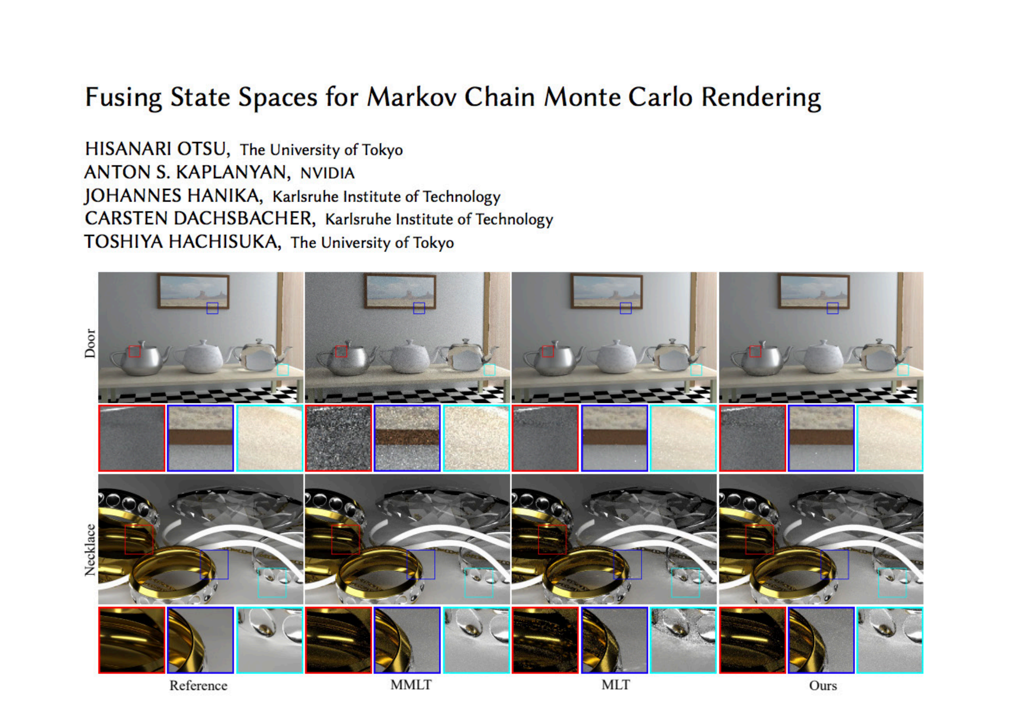
Jacopo Pantaleoni*
NVIDIA

Figure 1: *Escher's Room.* Charted Metropolis light transport considers path sampling methods and their primary sample space coordinates as charts of the path space, allowing to easily jump between them. In particular, it does so without requiring

[Pantaleoni '17]

Concurrent Work

[Otsu et al. '17]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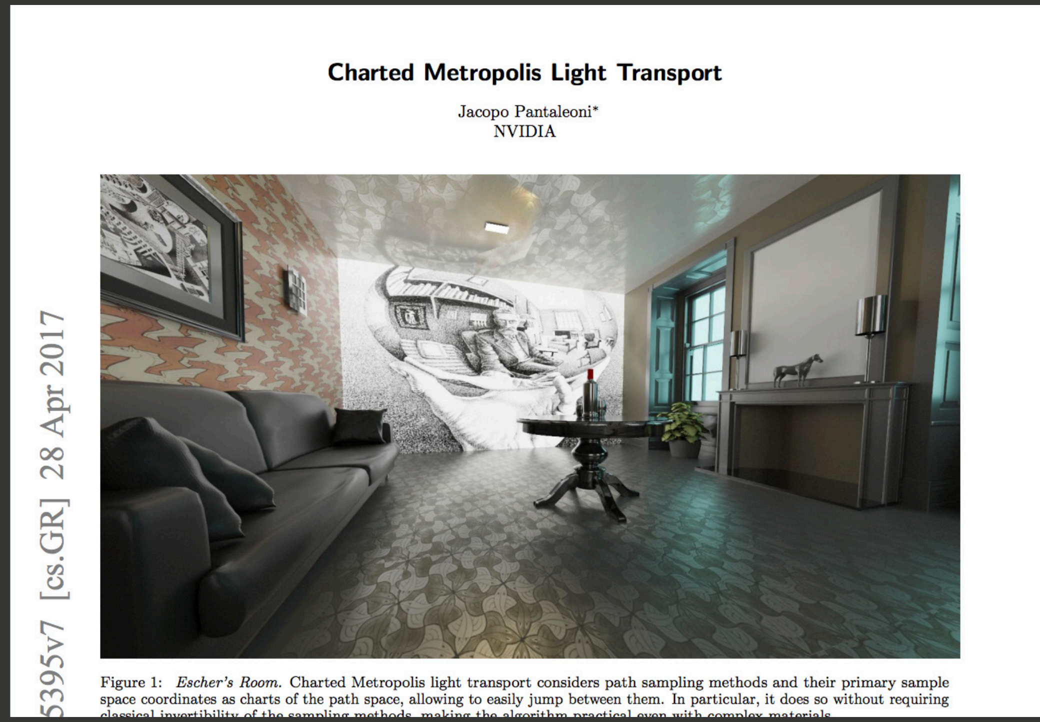
TOSHIYA HACHISUKA, The University of Tokyo

Door
Necklace
Reference MMLT MLT Ours

[Pantaleoni '17]


Charted Metropolis Light Transport

Jacopo Pantaleoni*
NVIDIA

5395v7 [cs.GR] 28 Apr 2017

Figure 1: *Escher's Room.* Charted Metropolis light transport considers path sampling methods and their primary sample space coordinates as charts of the path space, allowing to easily jump between them. In particular, it does so without requiring

MLT — 1997

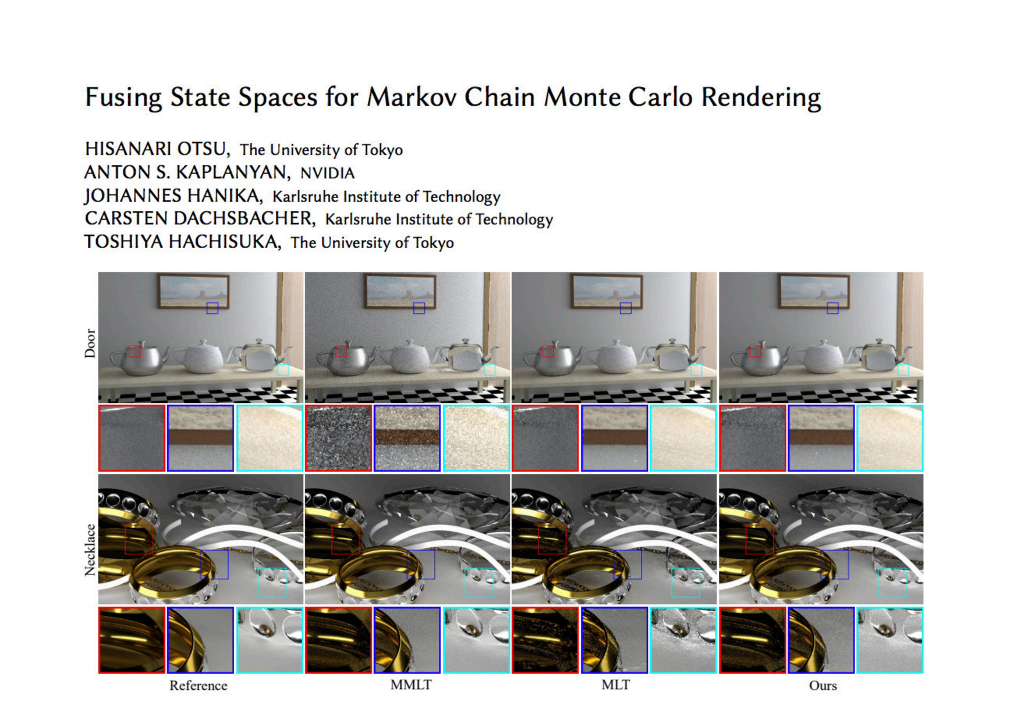
PSSMLT — 2002

MMLT — 2014

Charted MLT, Fused MLT, Ours — 2017

Concurrent Work

[Otsu et al. '17]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Focus on Path
Space Perturbations

[Pantaleoni '17]


Charted Metropolis Light Transport

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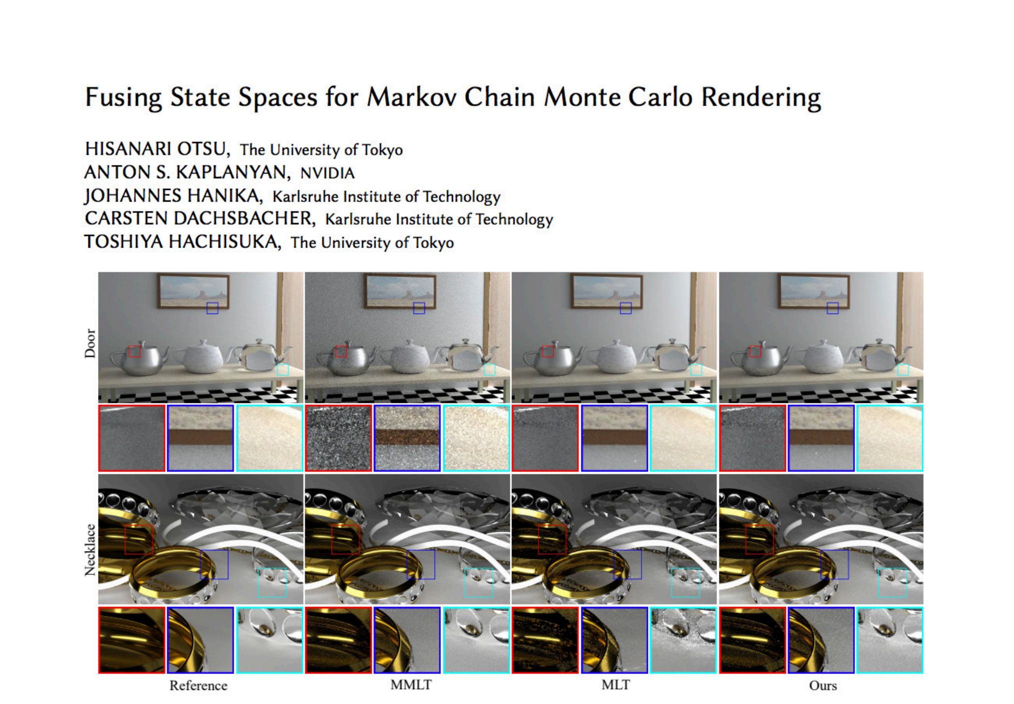
NVIDIA

5395v7 [cs.GR] 28 Apr 2017

Figure 1: *Escher's Room.* Charted Metropolis light transport considers path sampling methods and their primary sample space coordinates as charts of the path space, allowing to easily jump between them. In particular, it does so without requiring

Concurrent Work

[Otsu et al. '17]



Fusing State Spaces for Markov Chain Monte Carlo Rendering

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Focus on Path
Space Perturbations

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NVIDIA

5395v7 [cs.GR] 28 Apr 2017

Figure 1: *Escher's Room.* Charted Metropolis light transport considers path sampling methods and their primary sample space coordinates as charts of the path space, allowing to easily jump between them. In particular, it does so without requiring

No RJMCMC

Charted MLT
Fused MLT
Ours

MLT — 1997
PSSMLT — 2002
MMLT — 2014
Charted MLT / Fused MLT / Ours — 2017

Thanks!

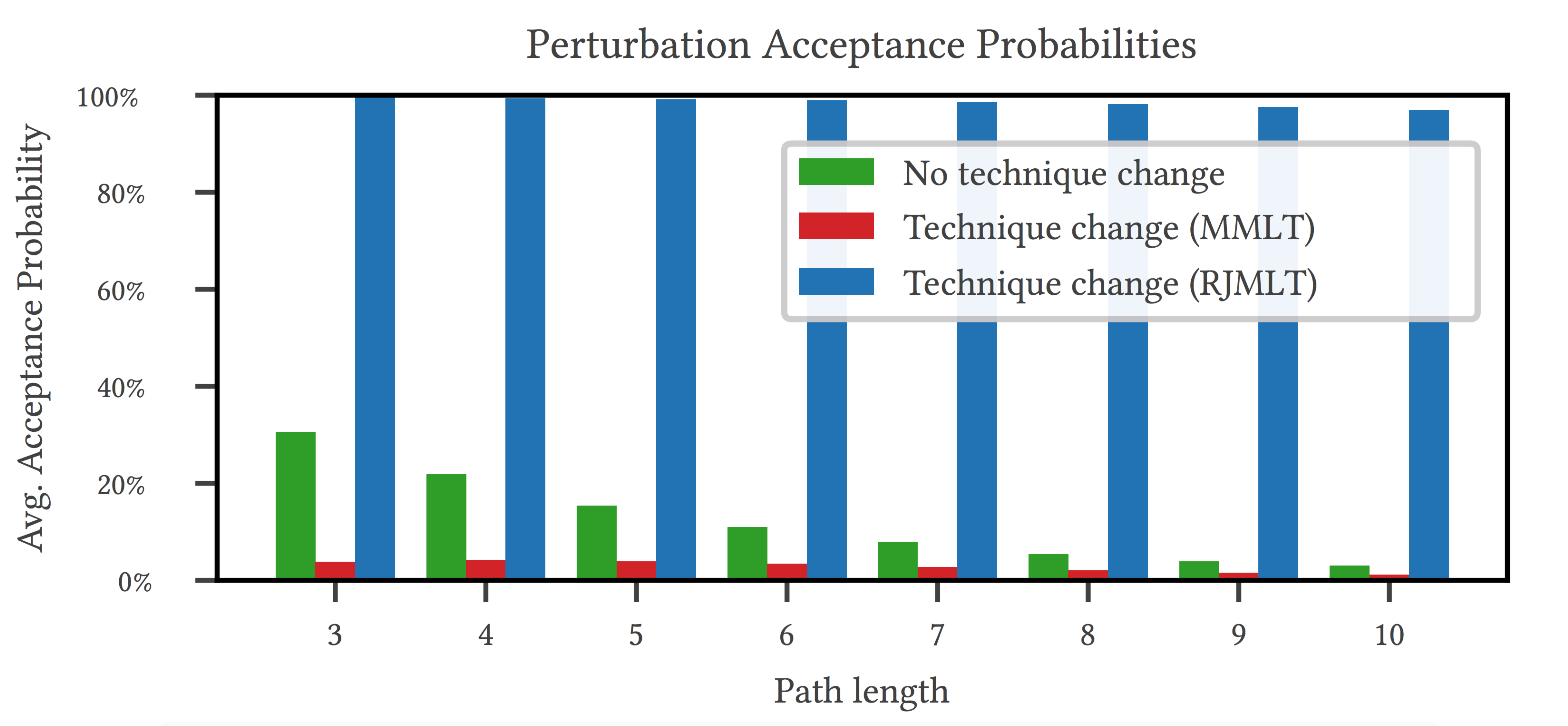
Questions?

Big thanks to
Activision
National Science Foundation

Scenes by Jay-Artist

Bonus Slides

Acceptance Probabilities



A Problem of Dimensions

- PSSMLT suffers from same problem!

The pixel contribution can also be obtained as an integral in the primary sample space:

$$\Phi_j = \int_{\mathcal{U}} F(S(\mathbf{u})) \cdot \left| \frac{dS(\mathbf{u})}{d\mathbf{u}} \right| d\mathbf{u},$$

where

$$\left| \frac{dS(\mathbf{u})}{d\mathbf{u}} \right| = \frac{1}{p_S(\mathbf{u})}$$

is the Jacobi determinant of the inverse mapping.

Augmented Path Space

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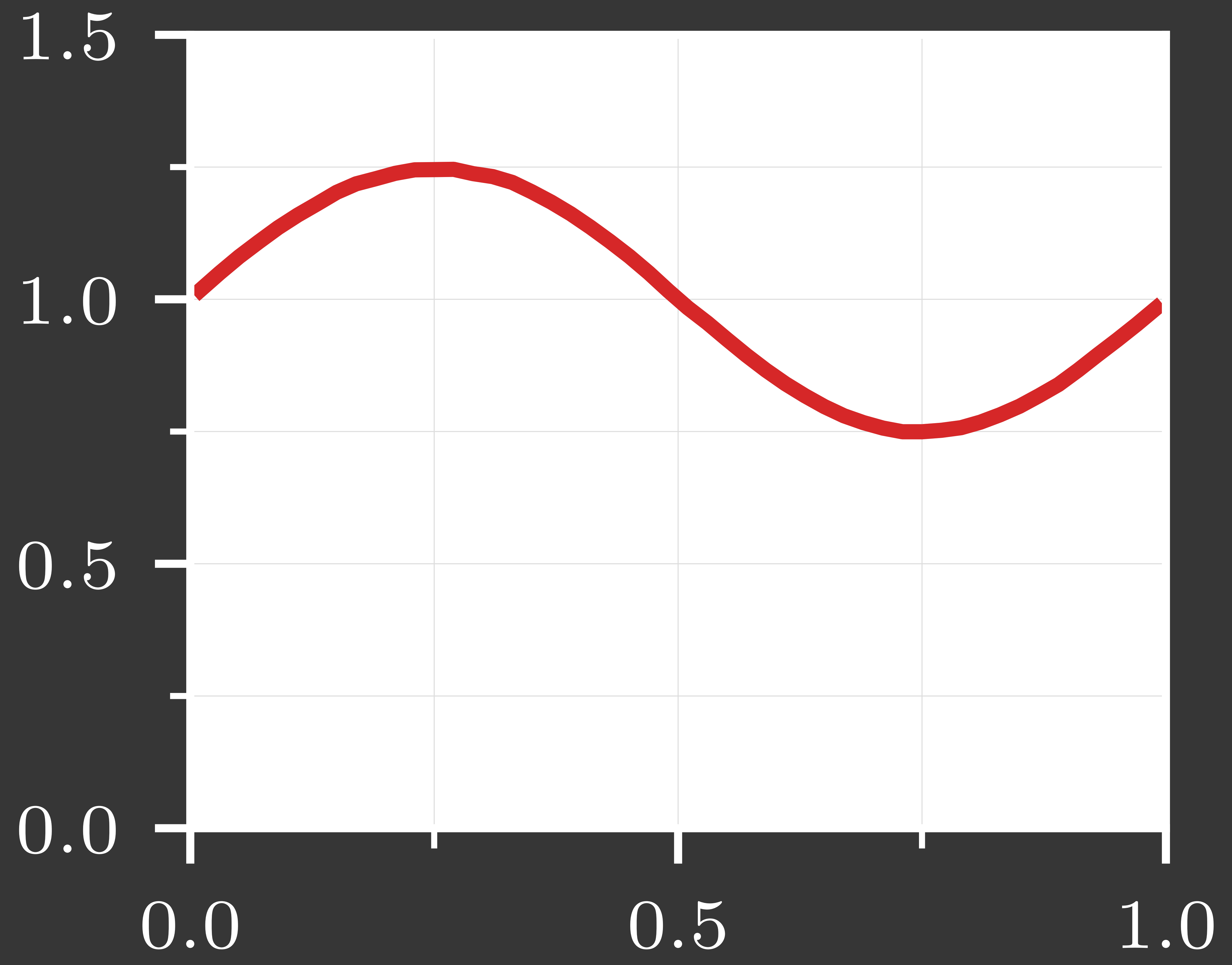
$$\bar{\mathbf{u}} = S^{-1}(\bar{\mathbf{x}}, \bar{\gamma})$$

1D Experiments



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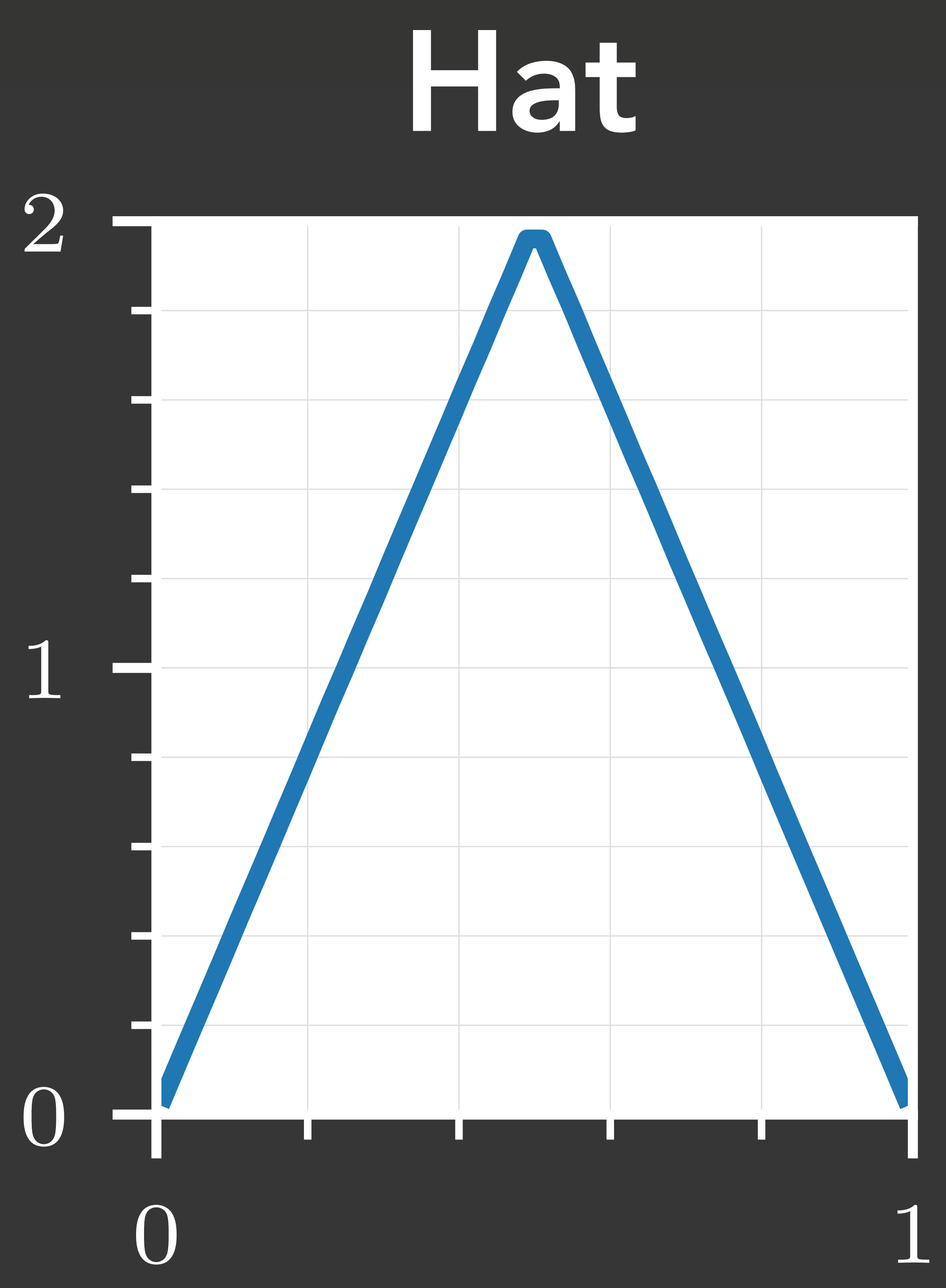
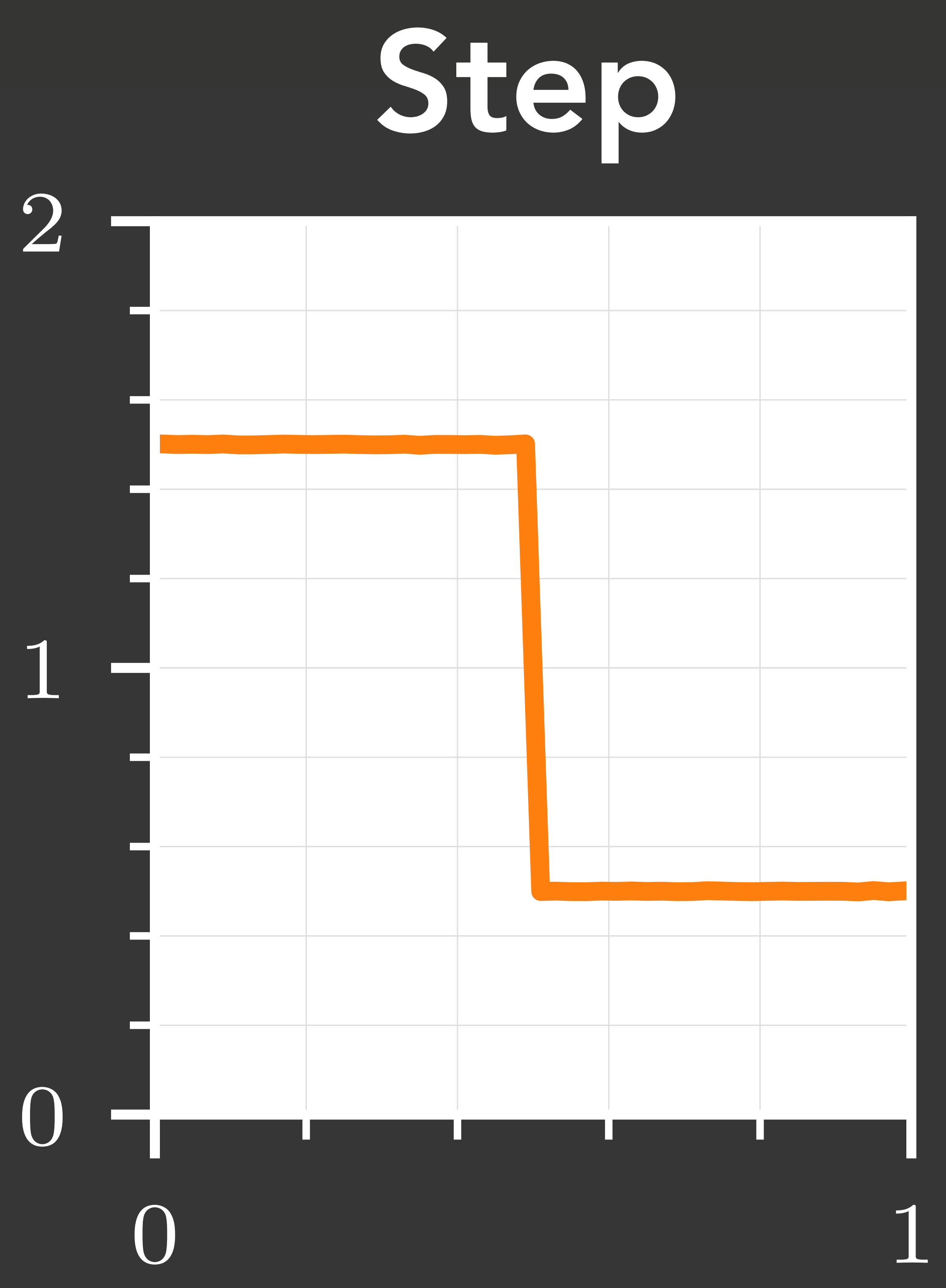
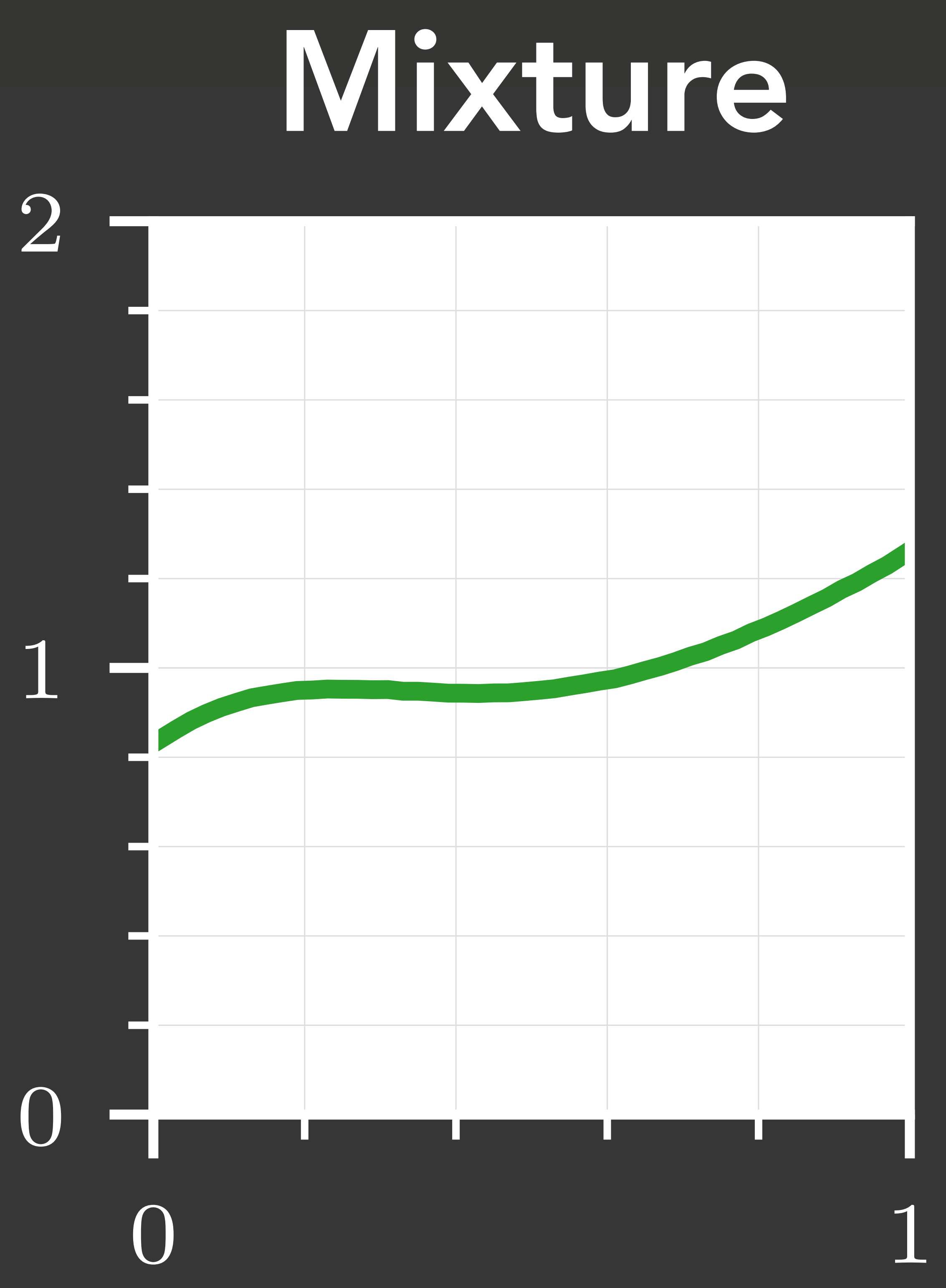
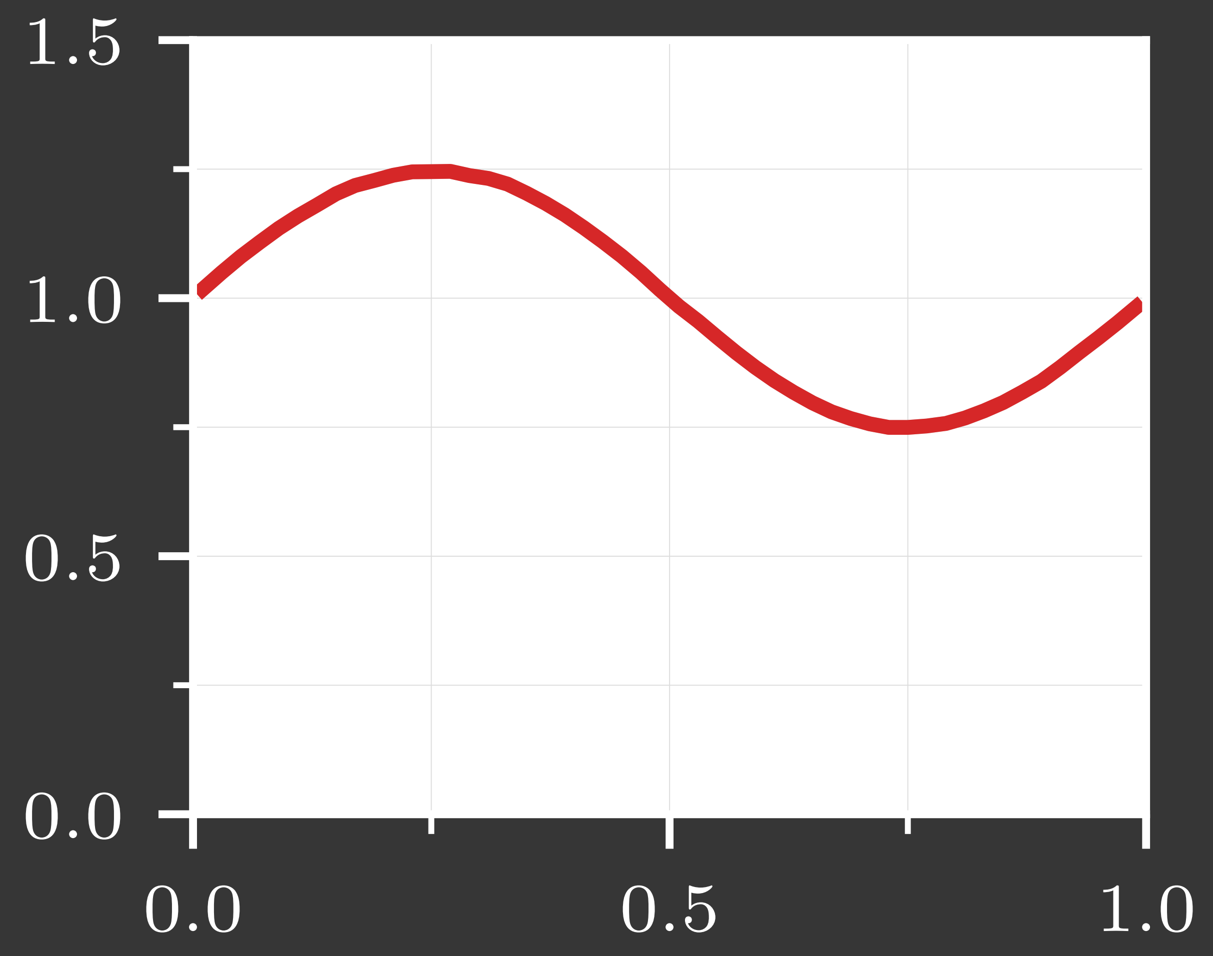


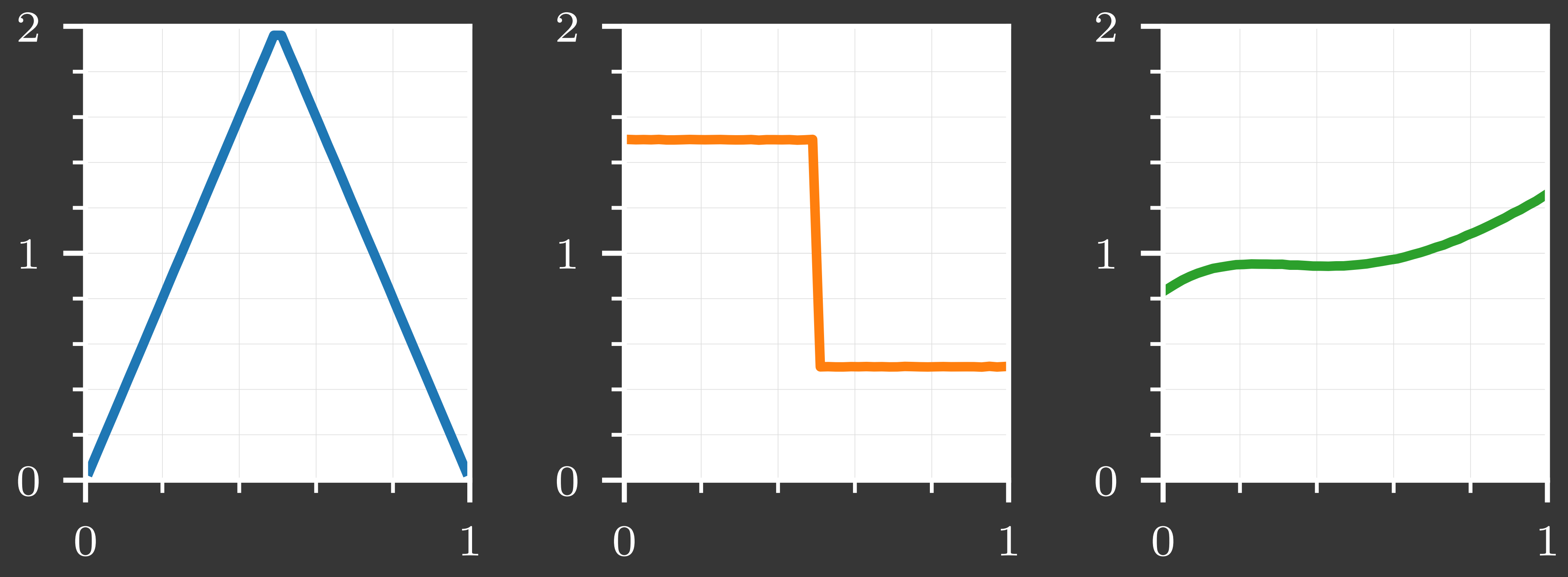




1D Experiments

MMLT





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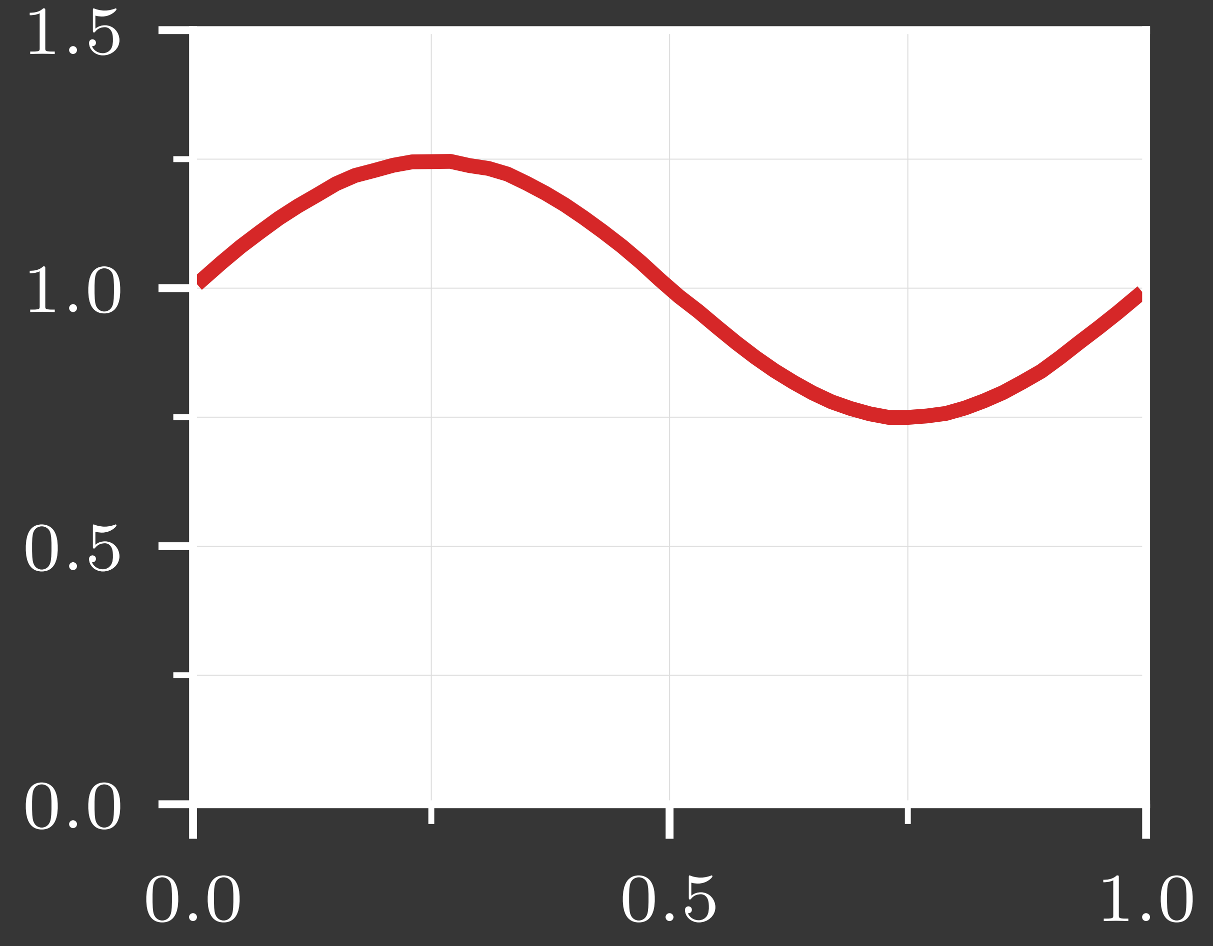
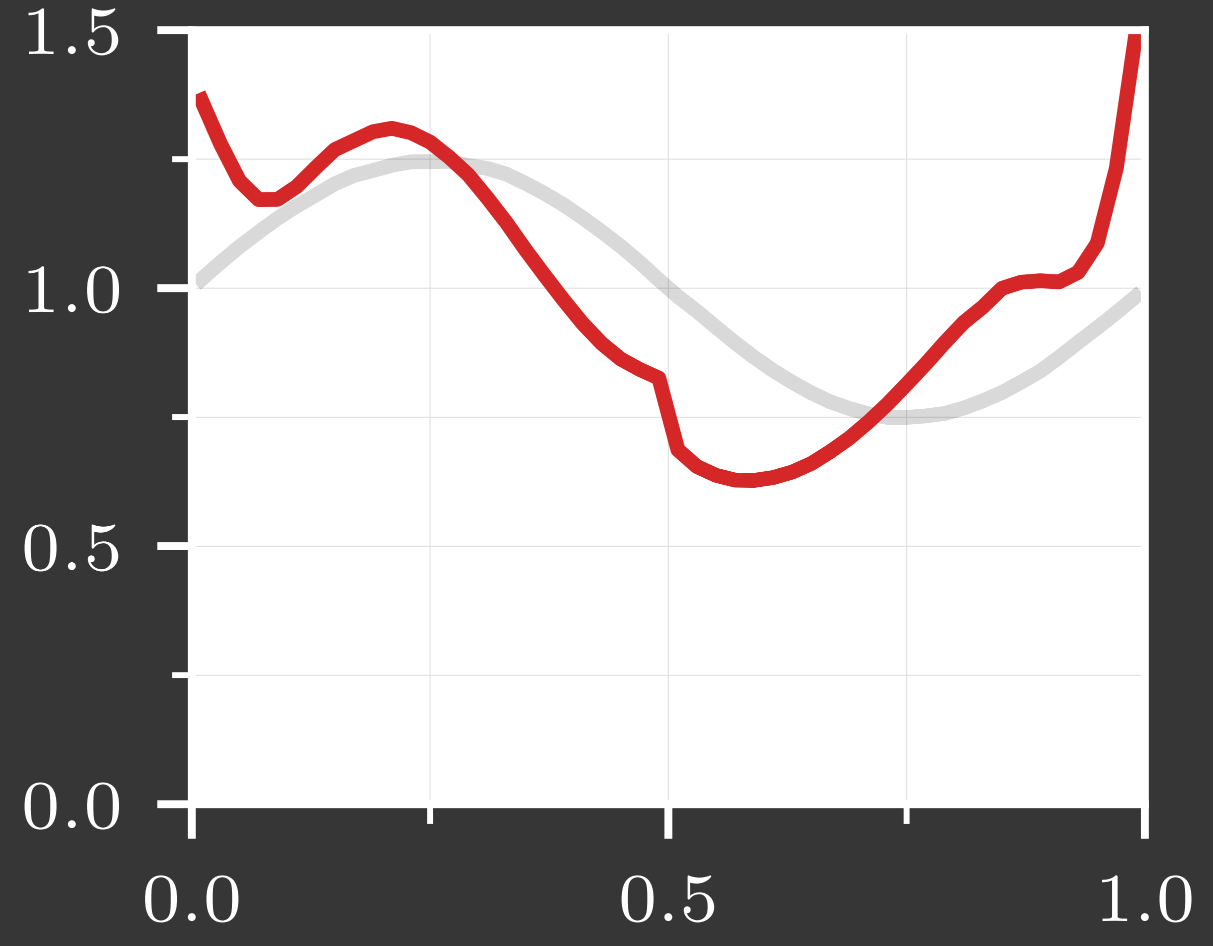
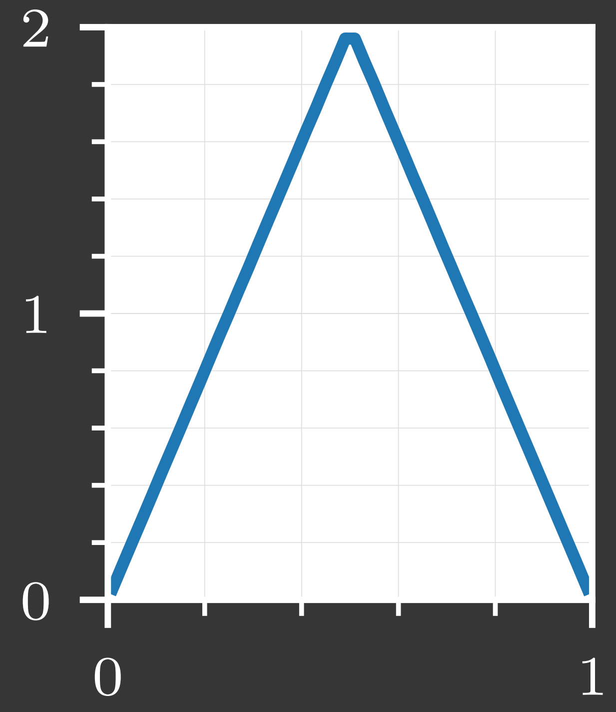
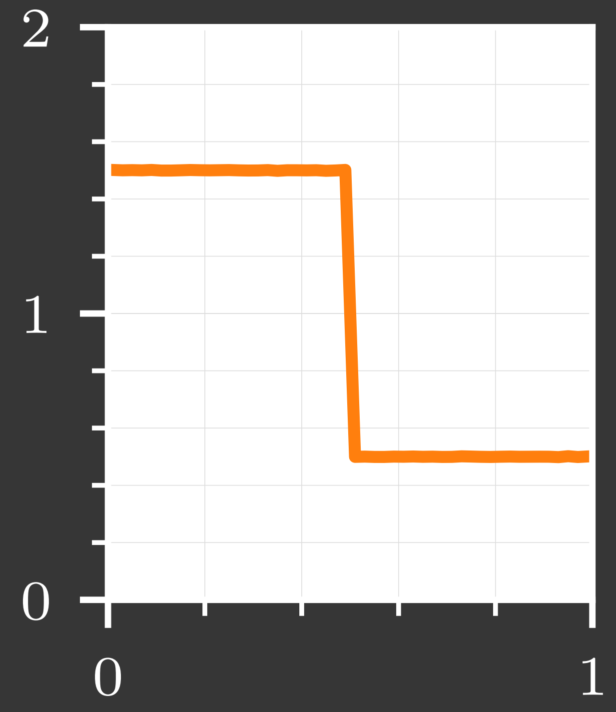
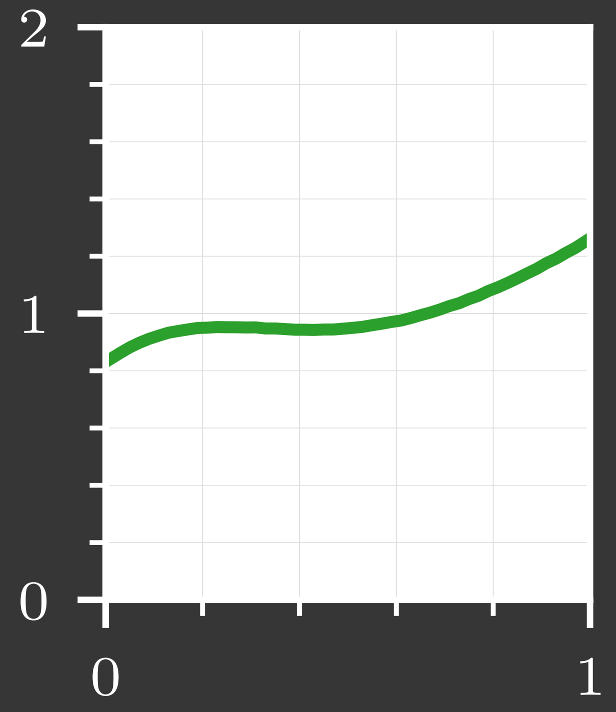


Naive









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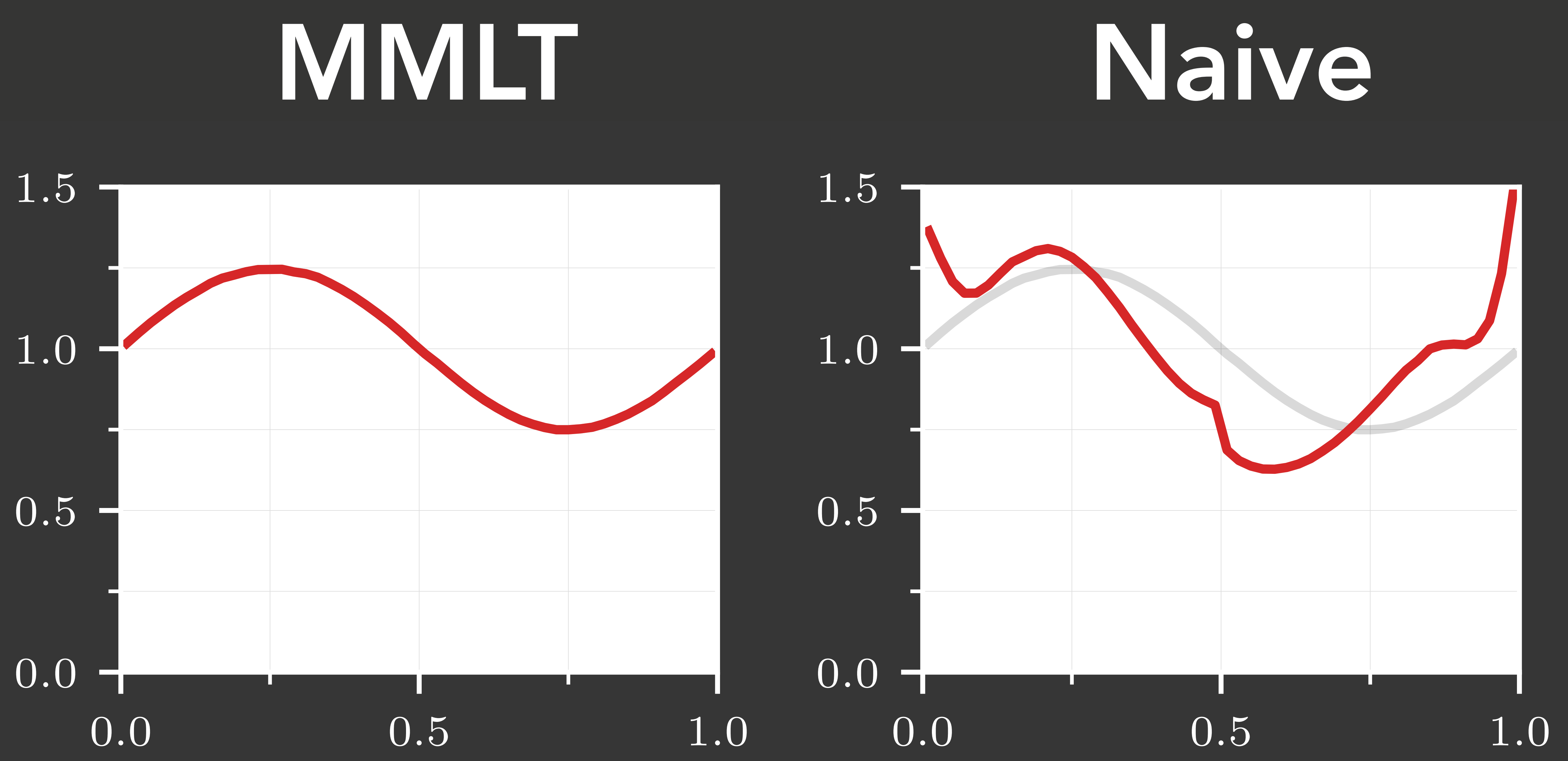
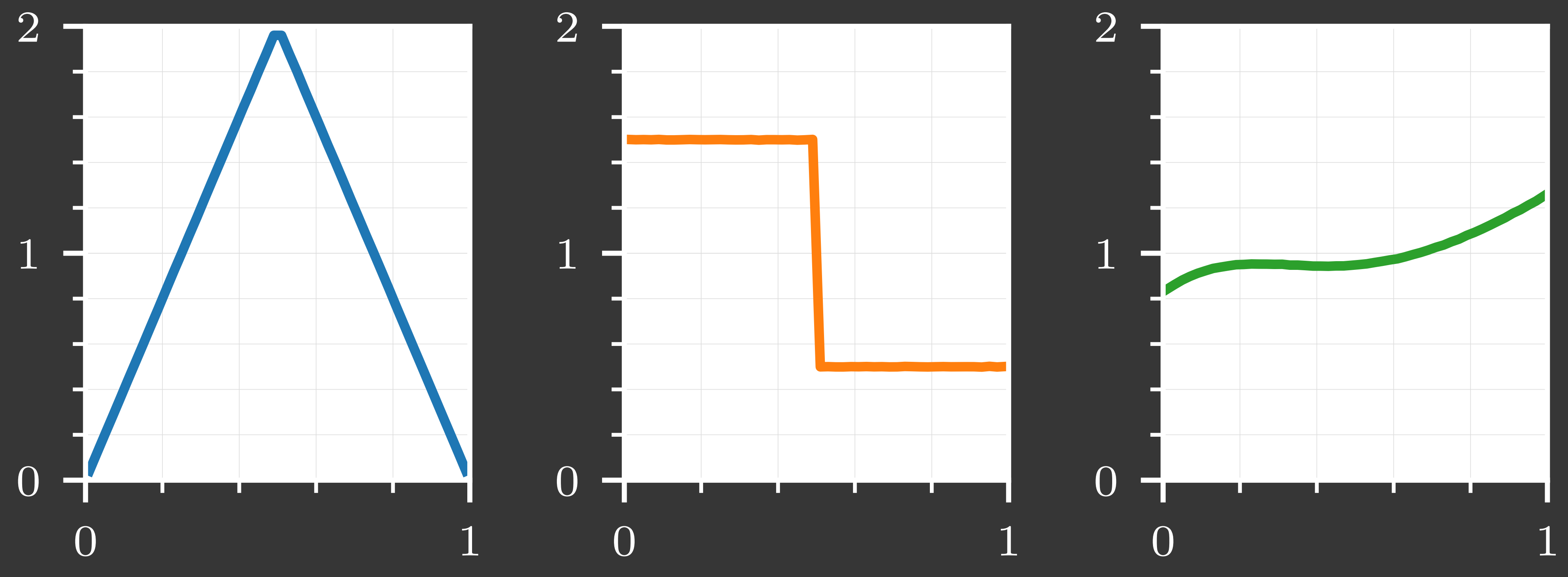


$$\Pr = \frac{f(\bar{\mathbf{v}})T(\bar{\mathbf{v}}, \bar{\mathbf{u}})}{f(\bar{\mathbf{u}})T(\bar{\mathbf{u}}, \bar{\mathbf{v}})} ||\partial h(\bar{\mathbf{u}})||$$



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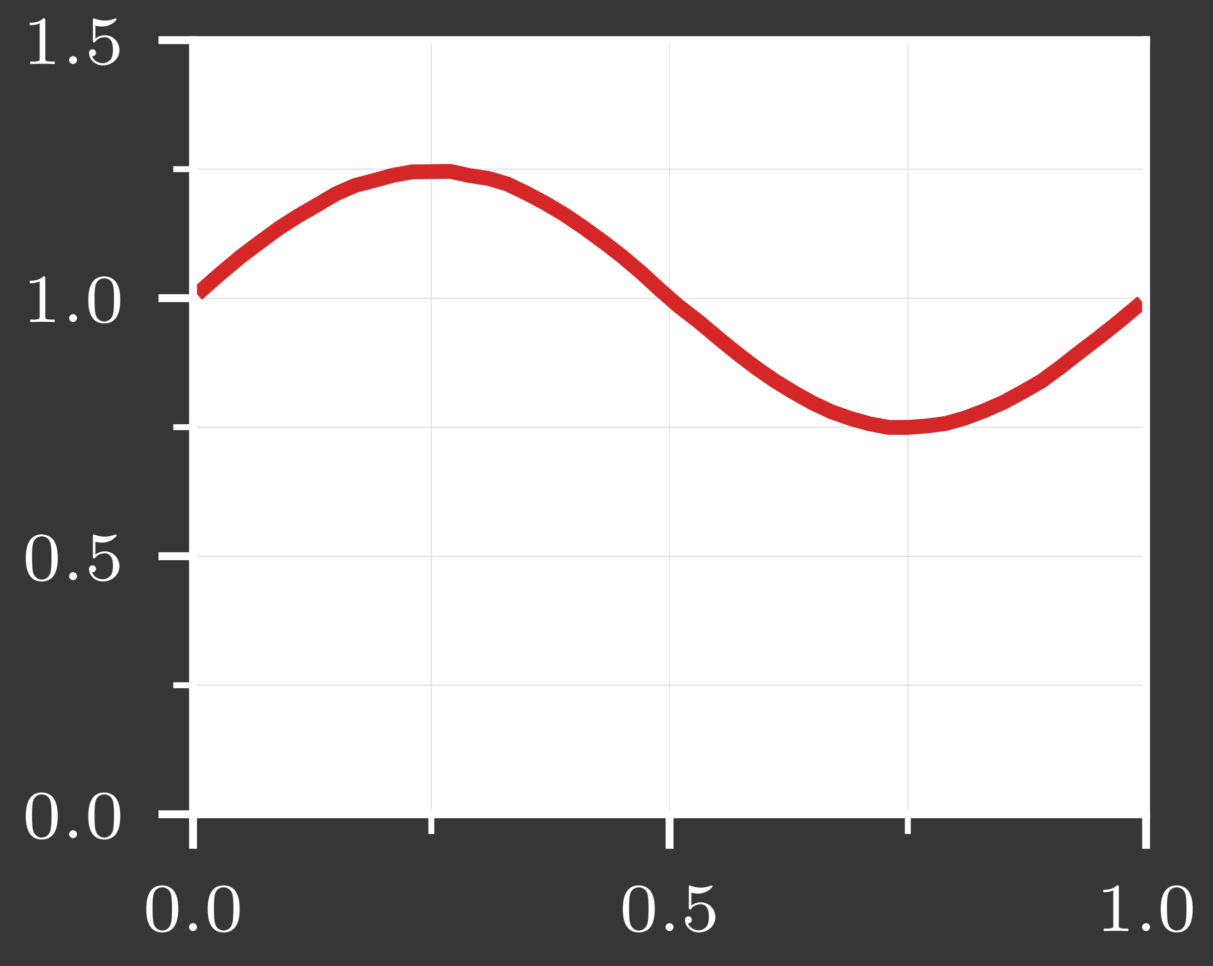

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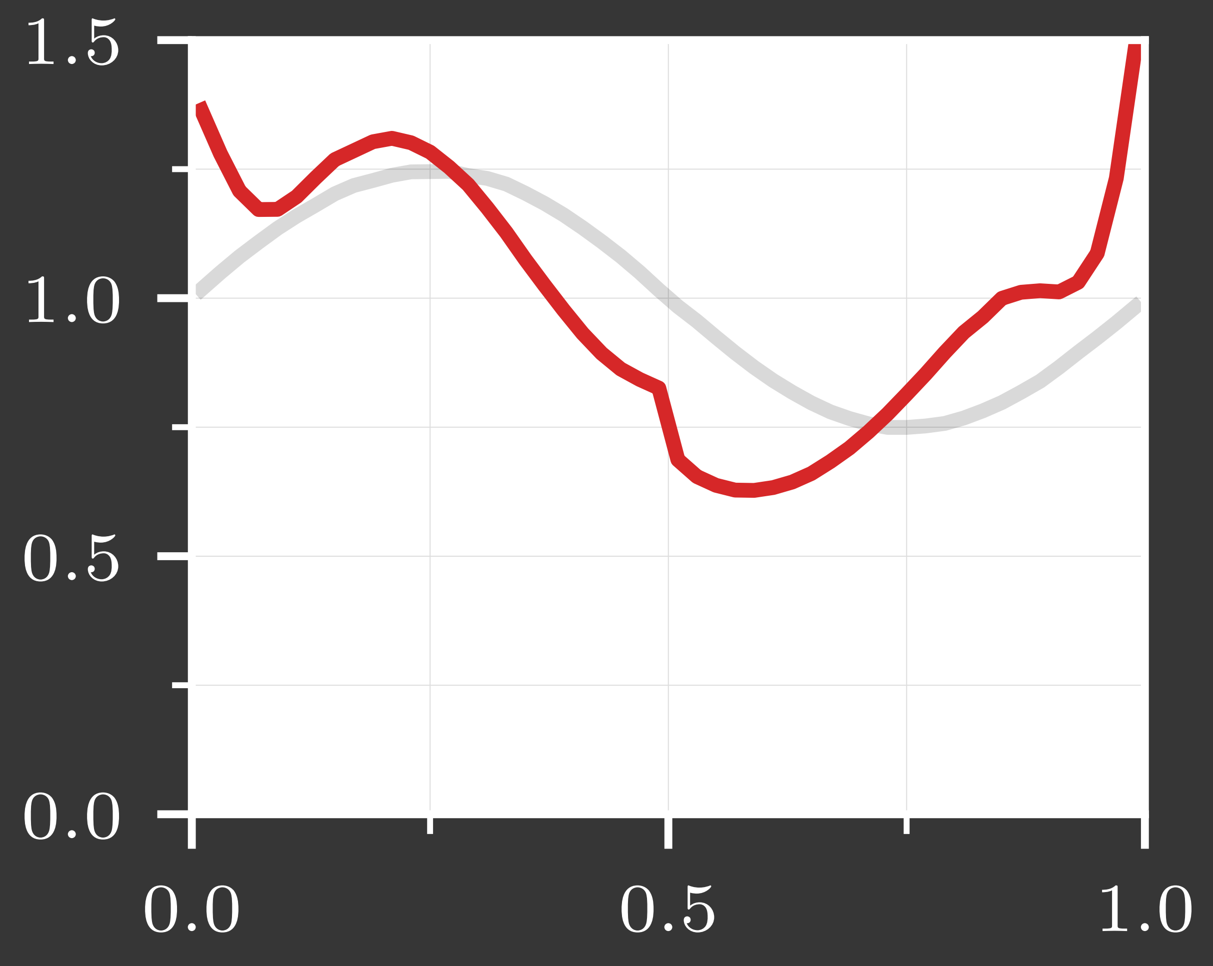

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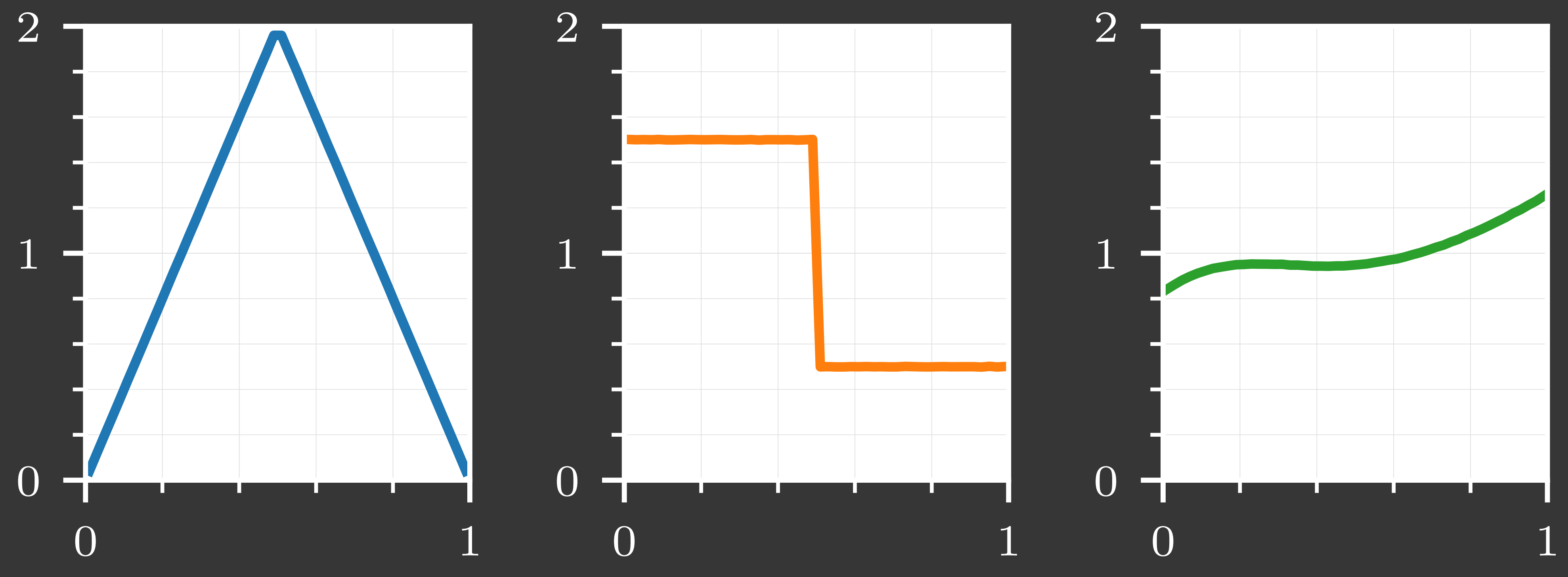

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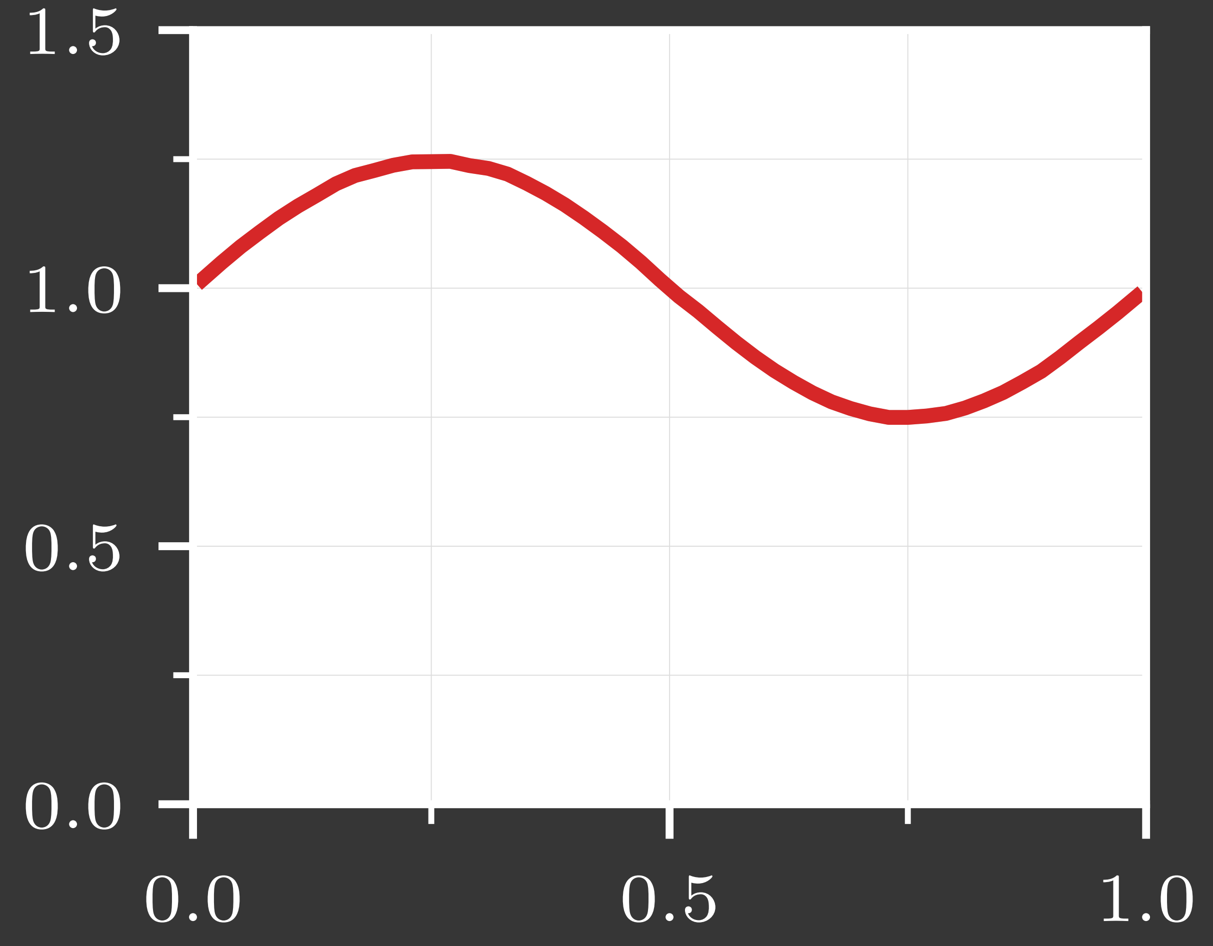

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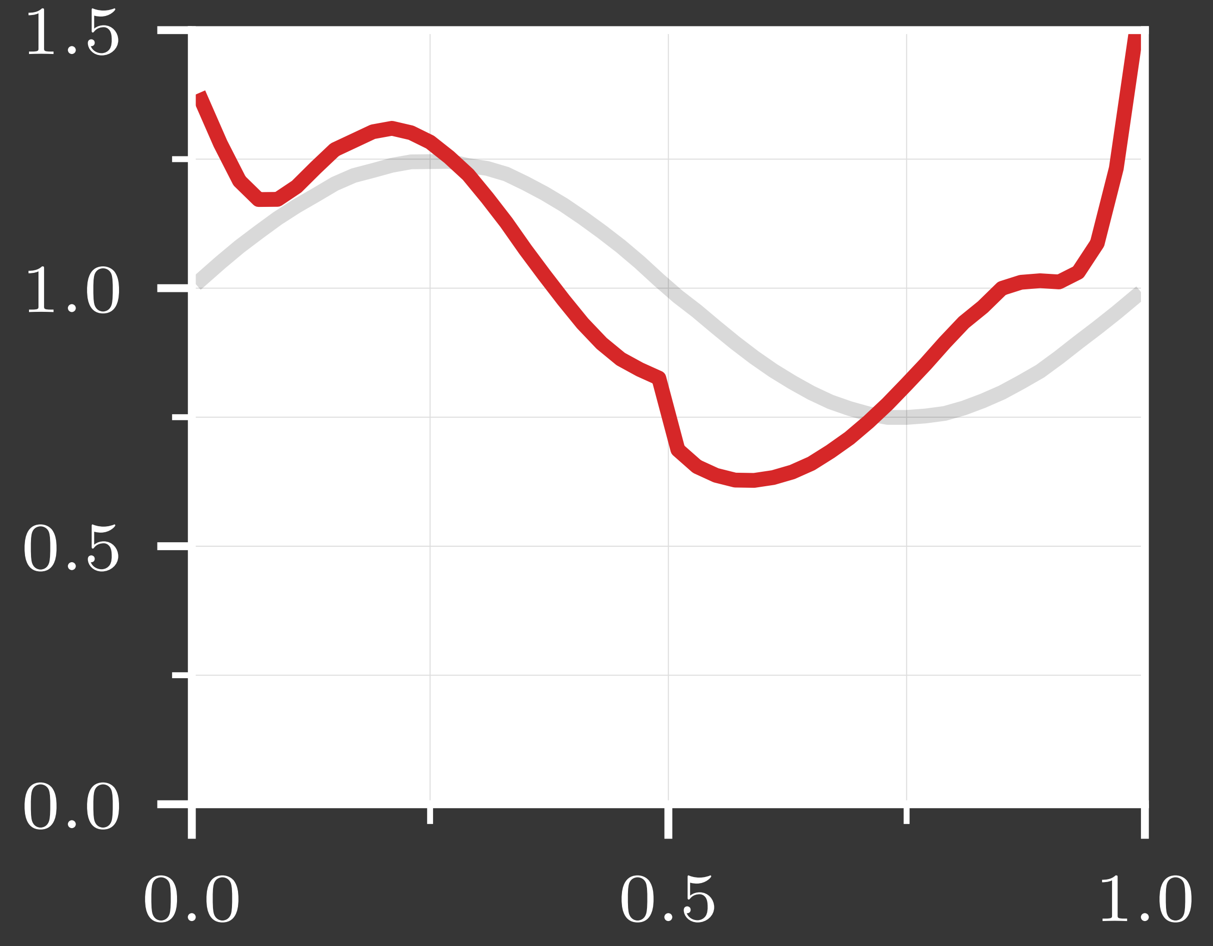

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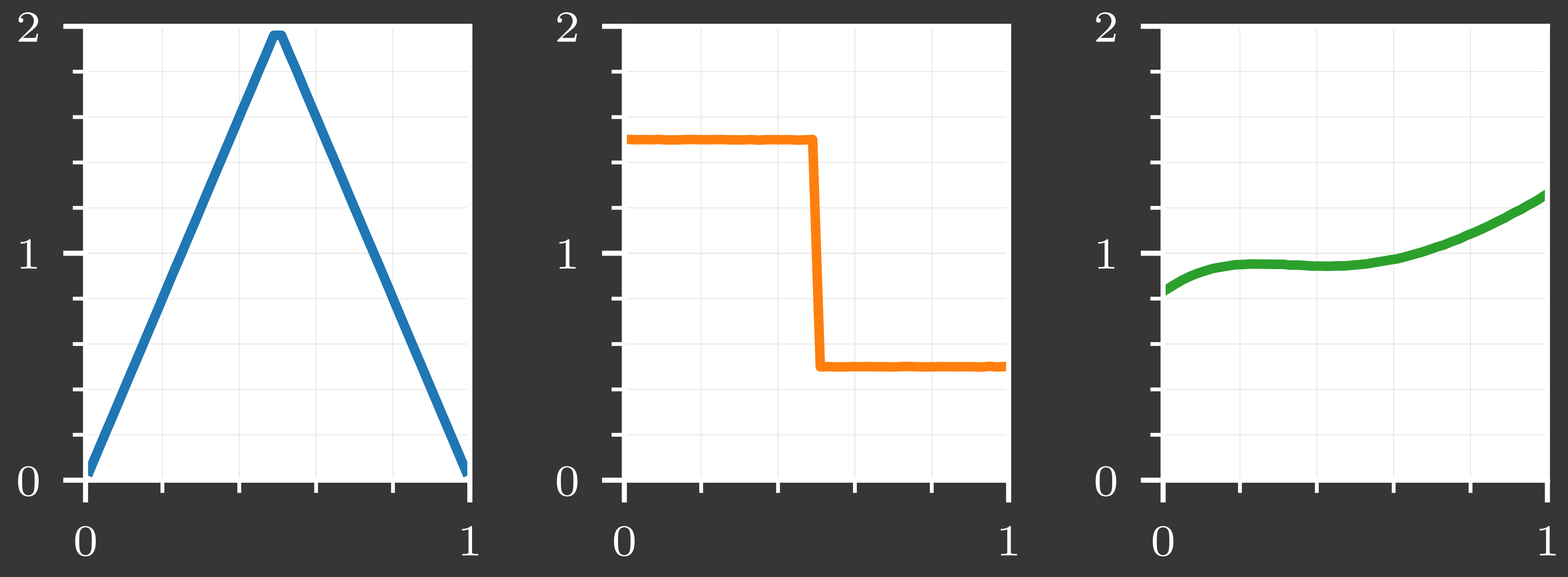

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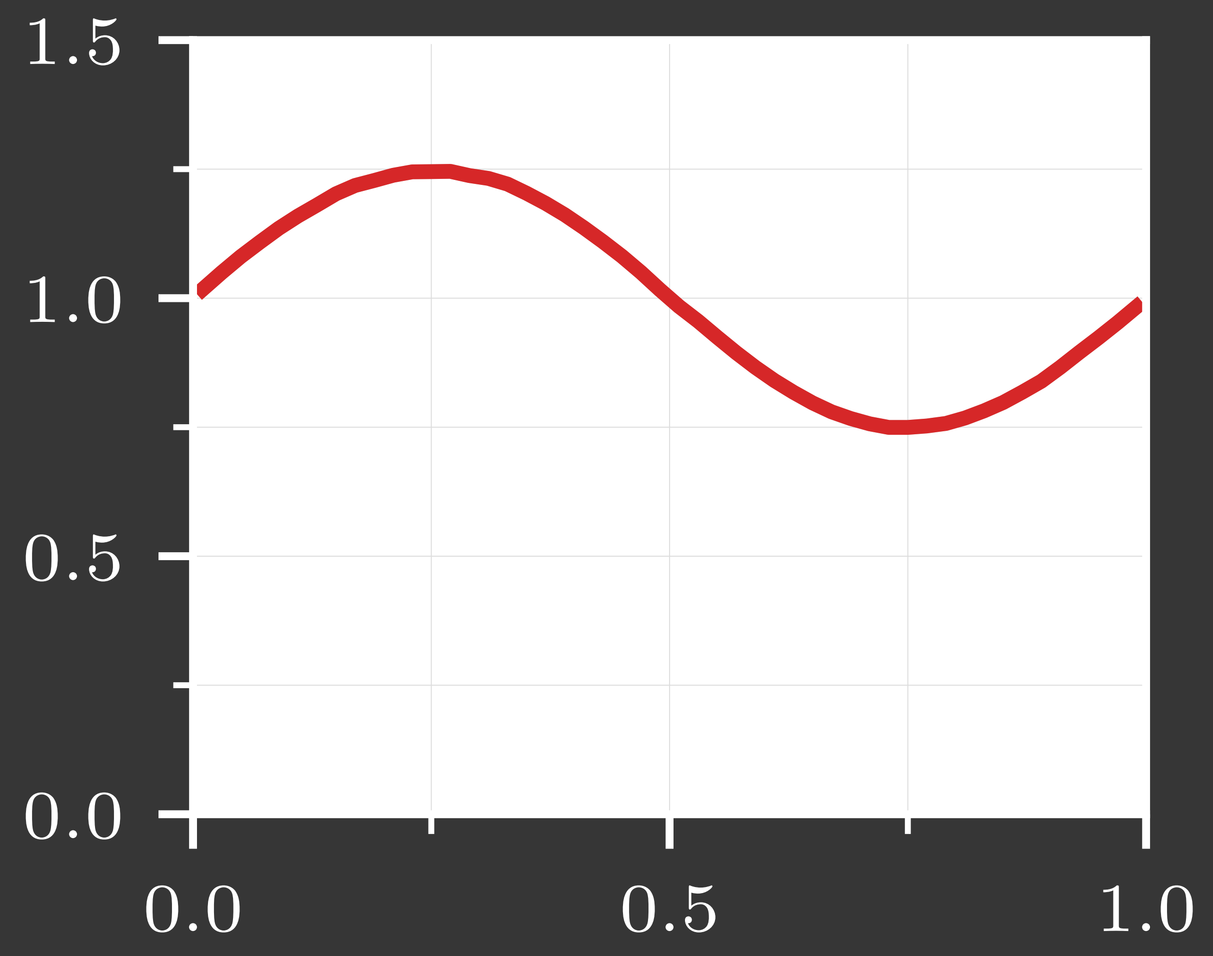

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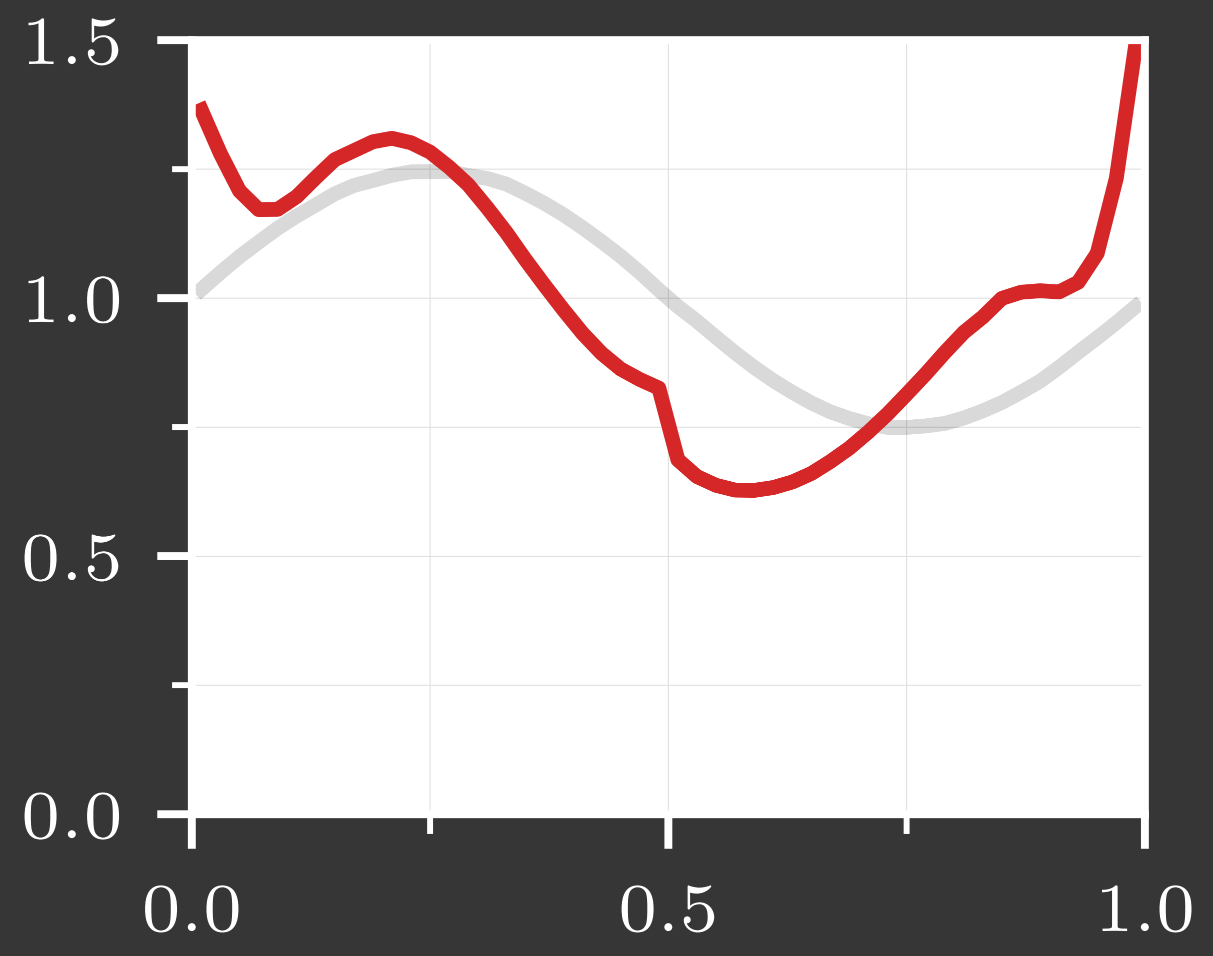
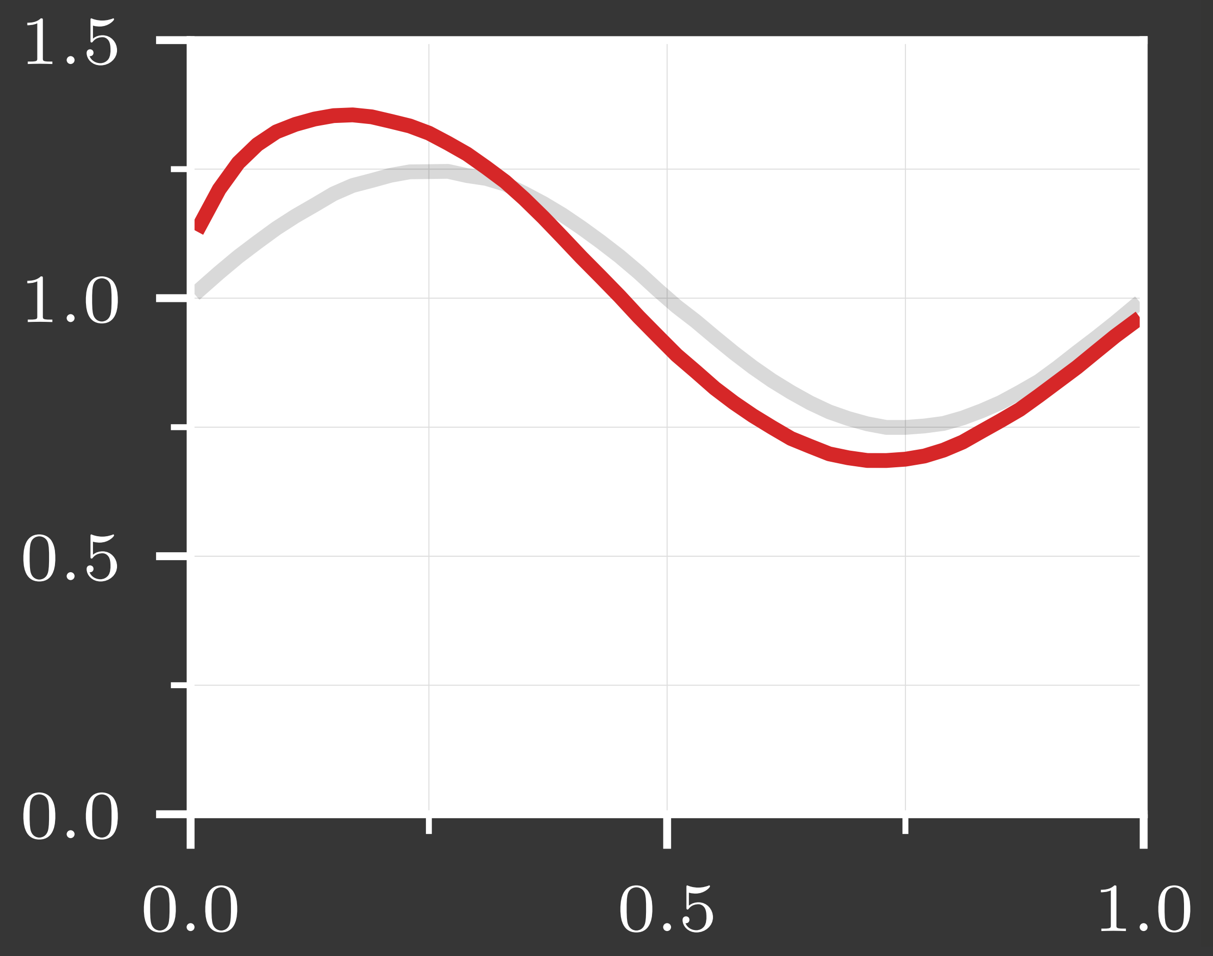
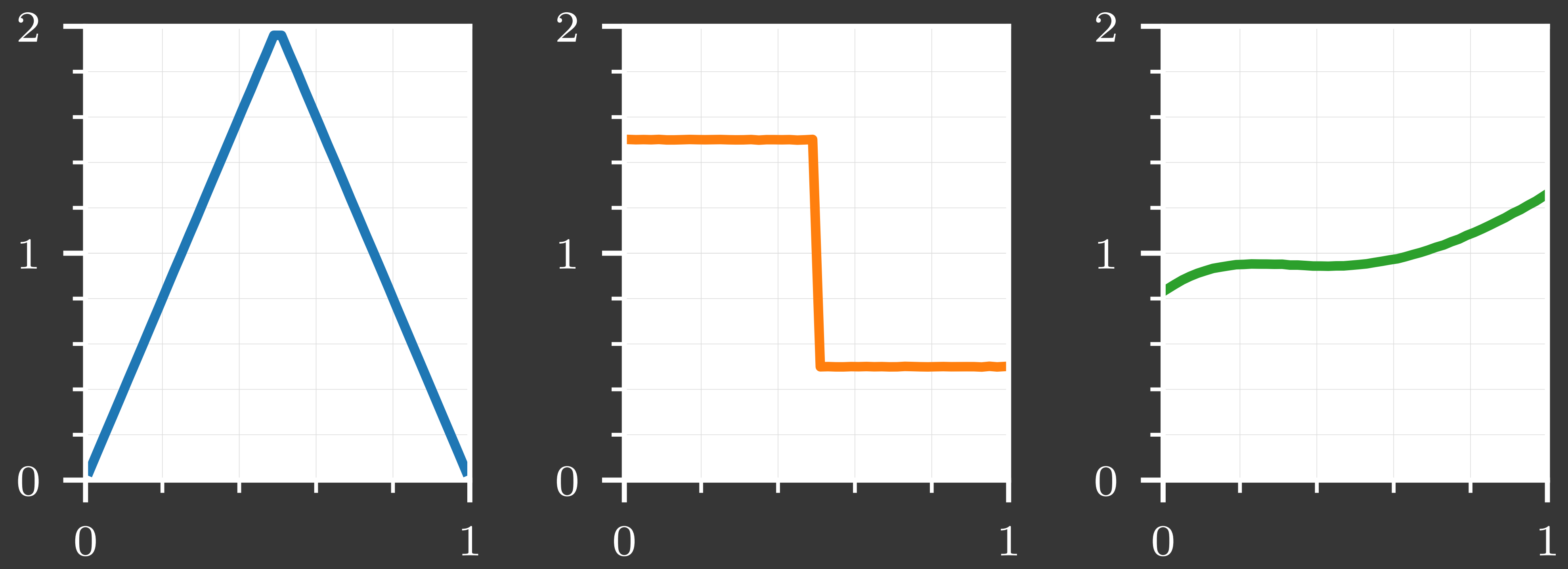


Fixpoint





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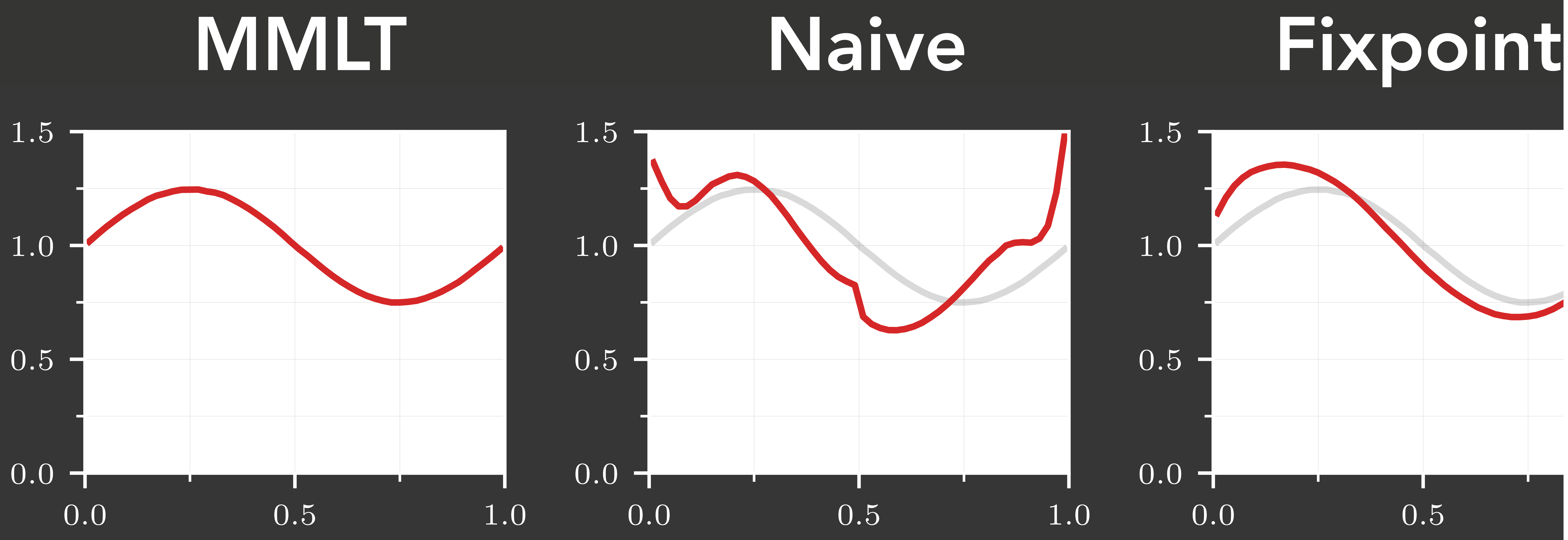

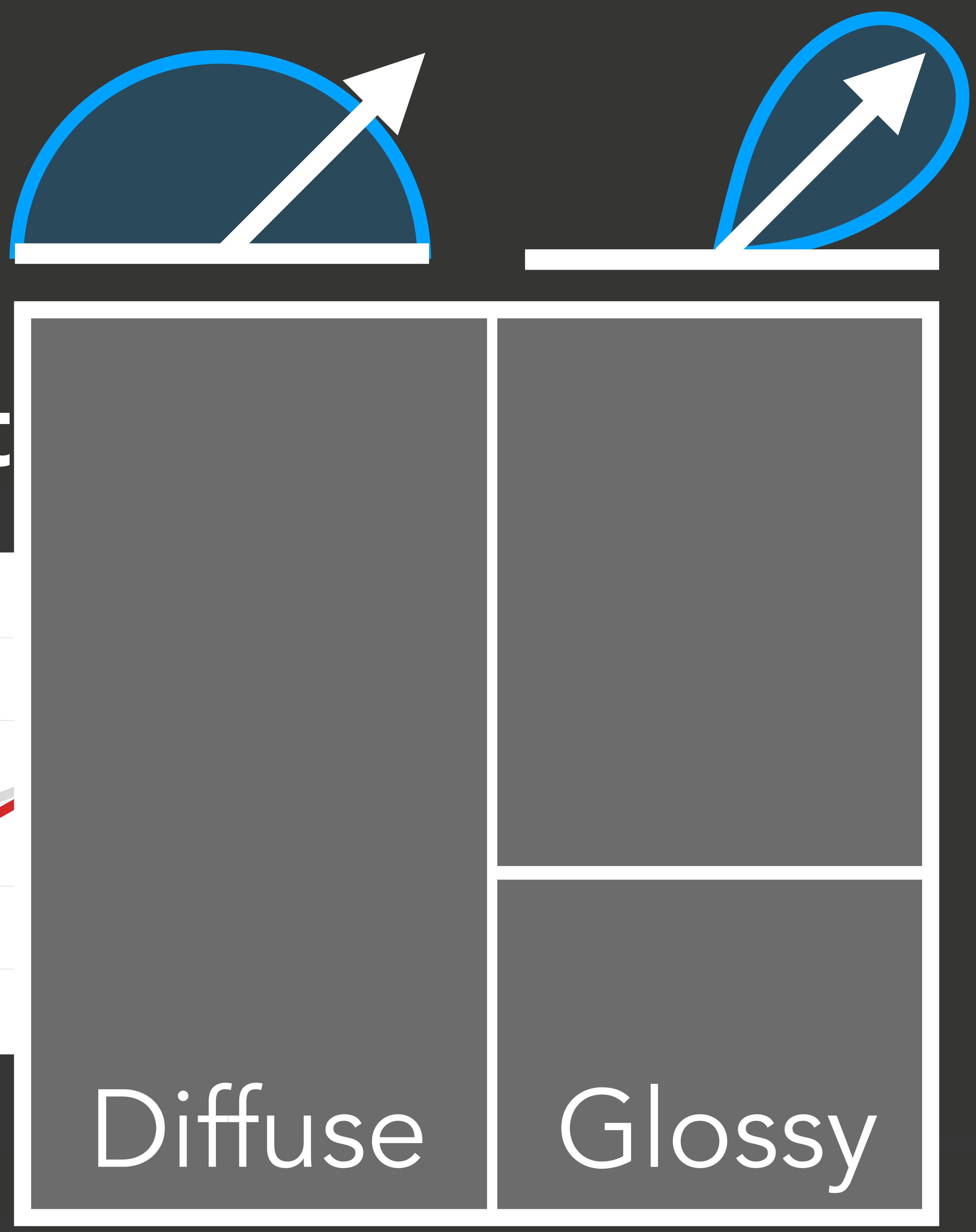

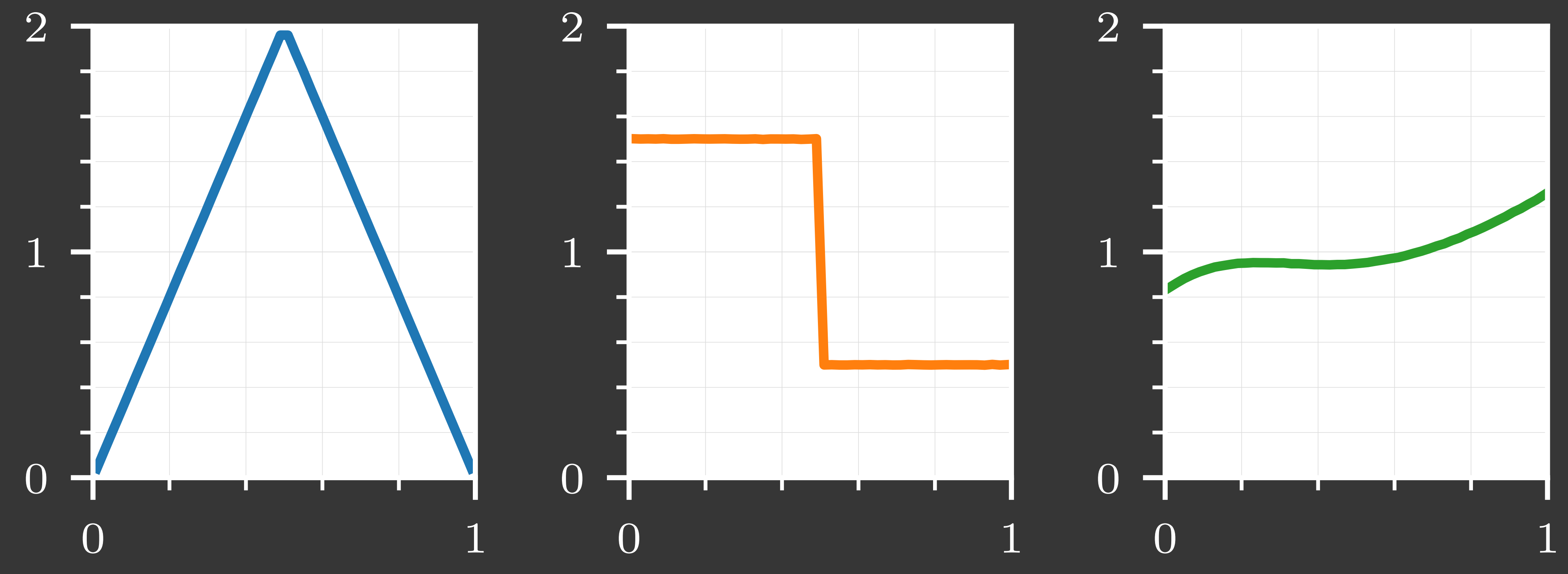

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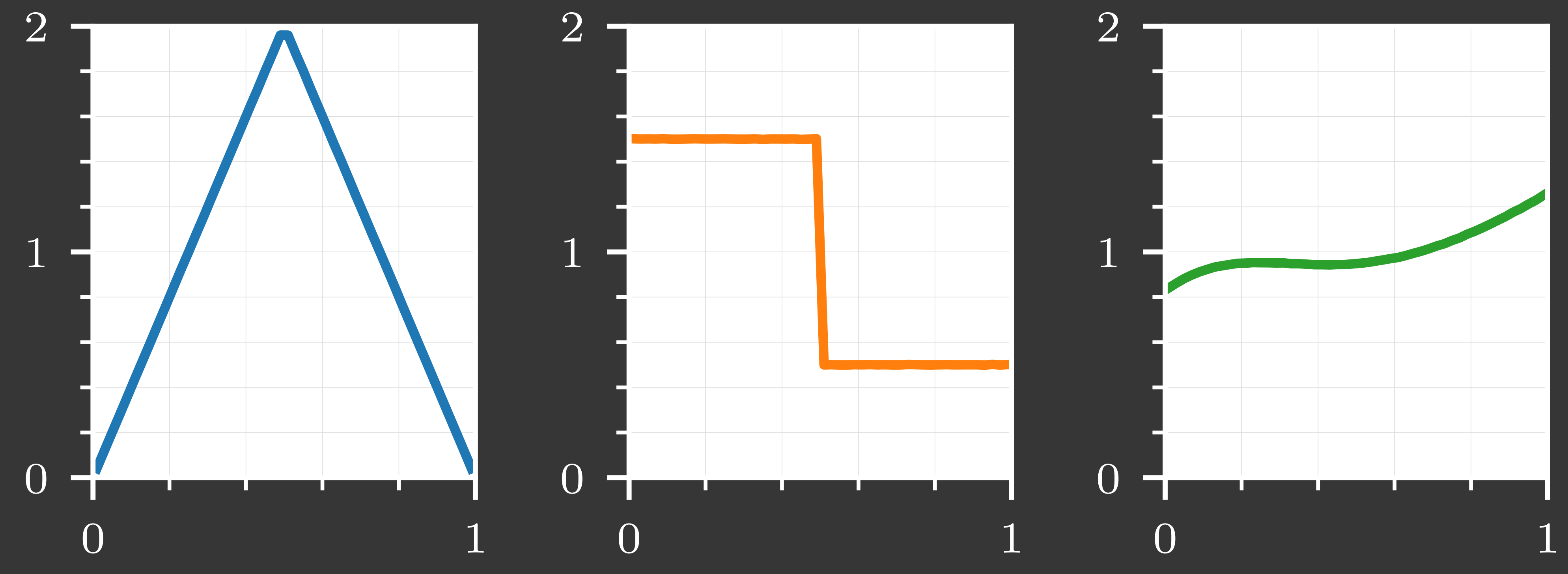

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MMLT | Naive | Fixpoint | RJMLT