Extended Path Integral Formulation for Volumetric Transport

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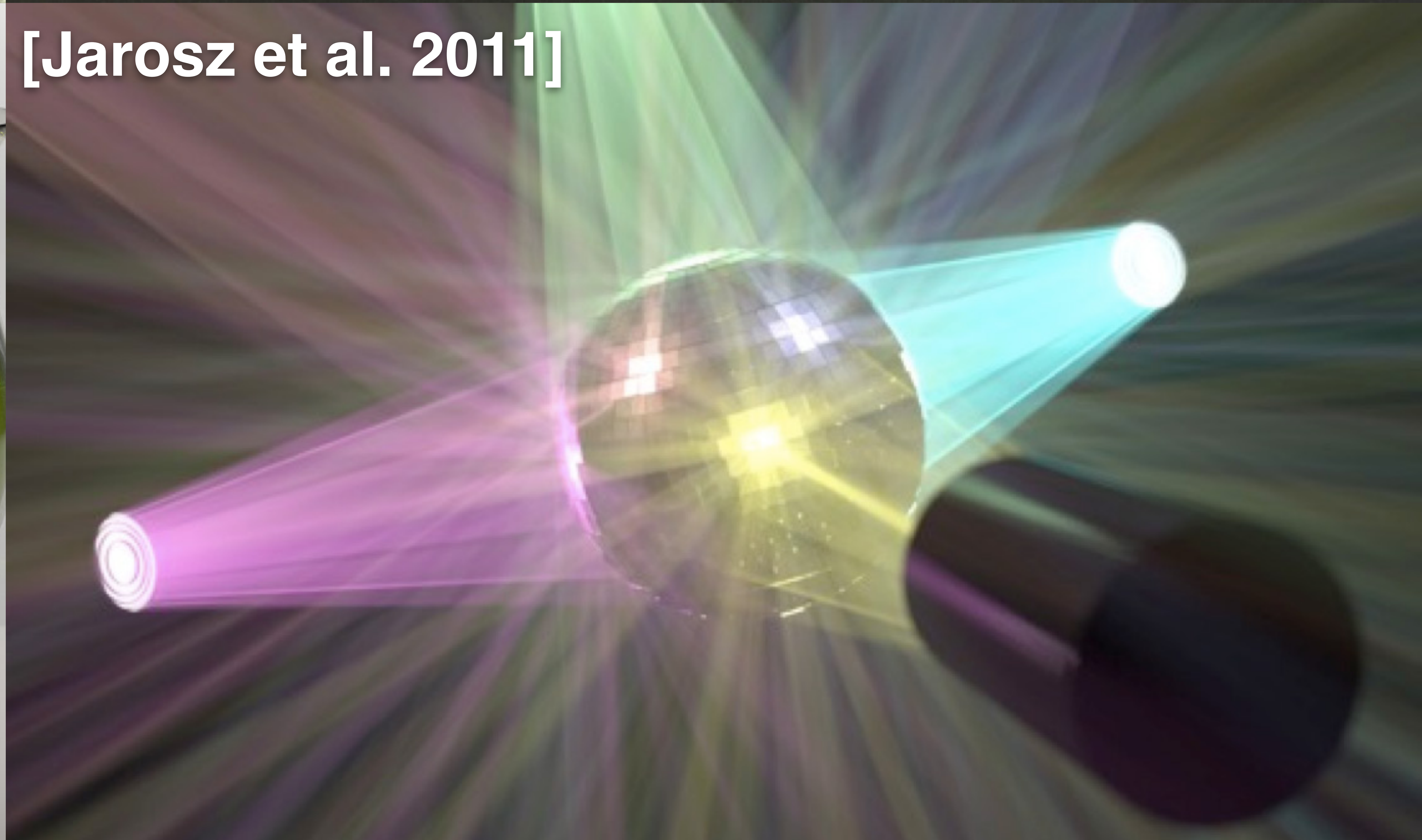
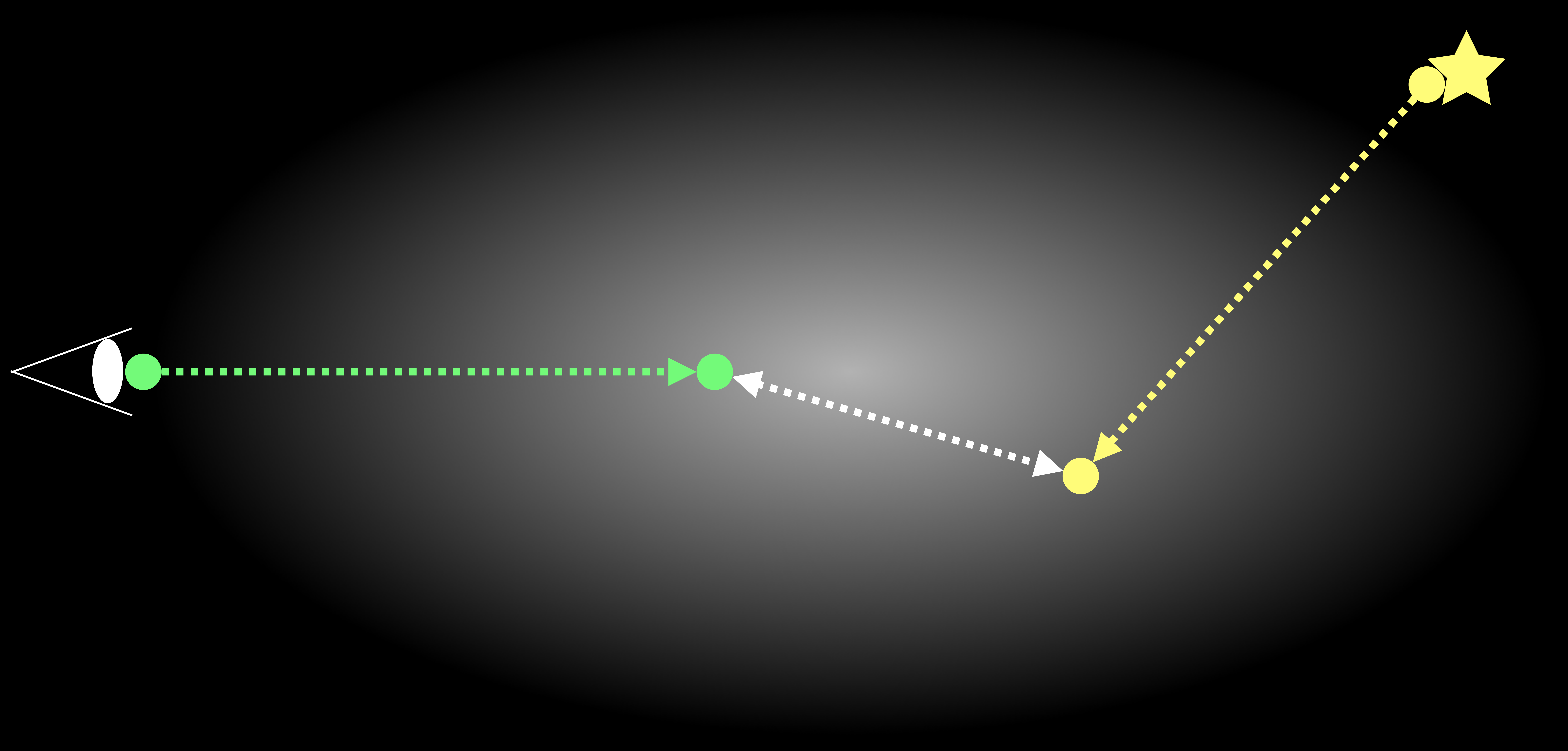
EGSR 2017


[Jensen and Christensen 1998]

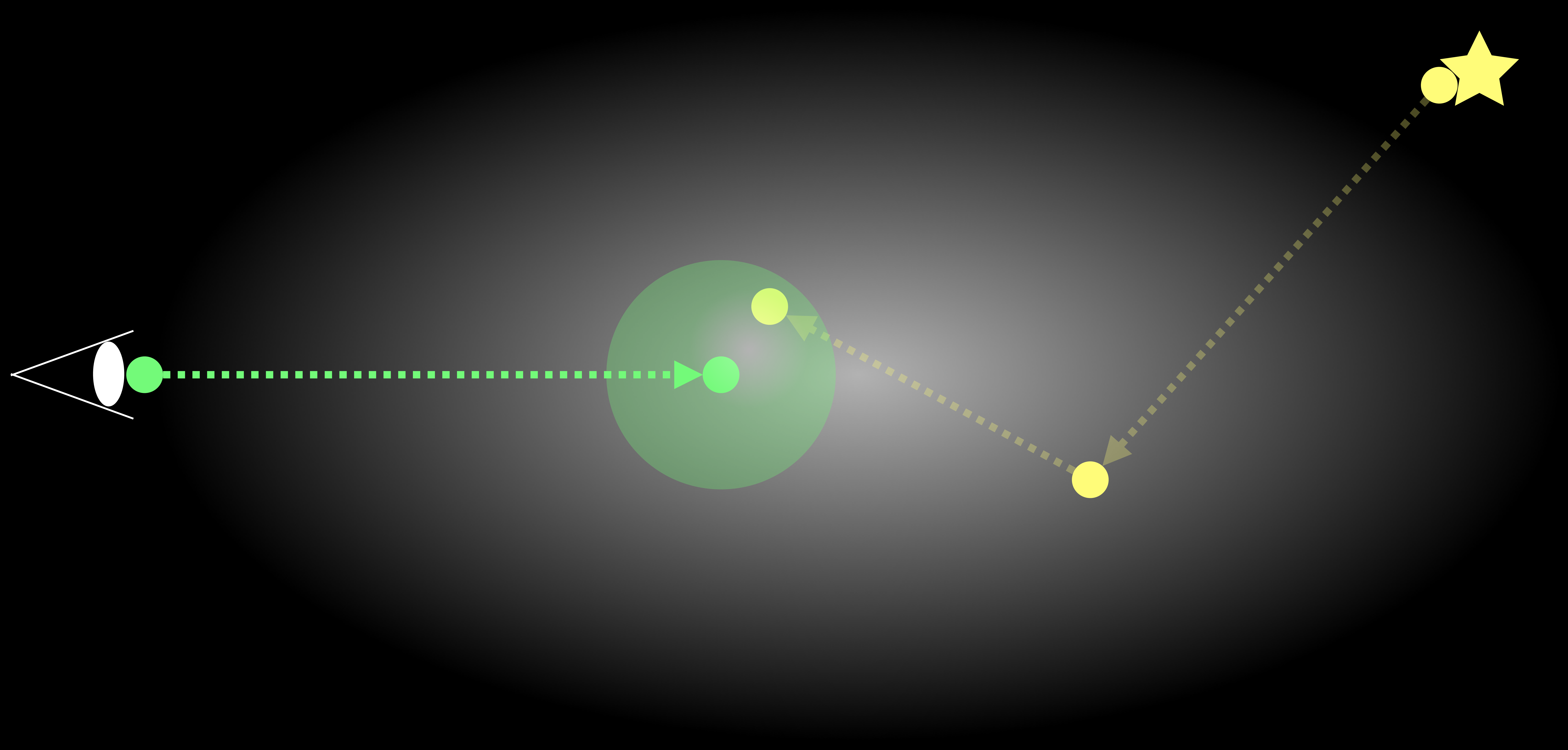

[Pauly et al. 2000]


[Křivánek et al. 2014]

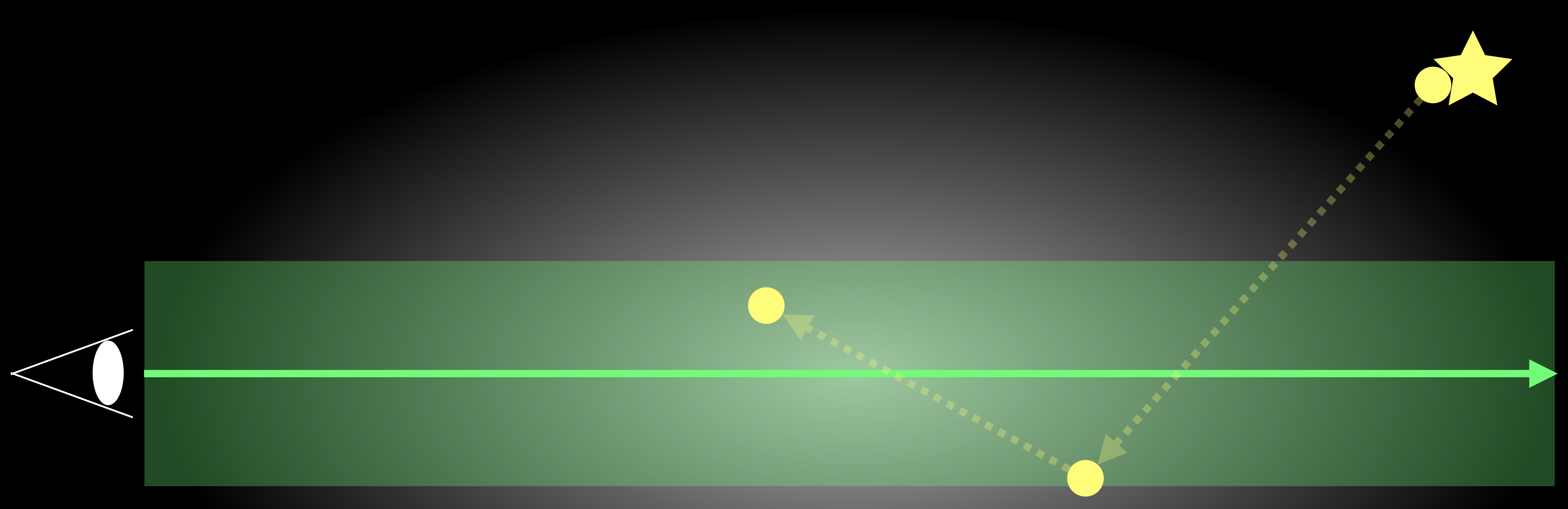

[Jarosz et al. 2011]



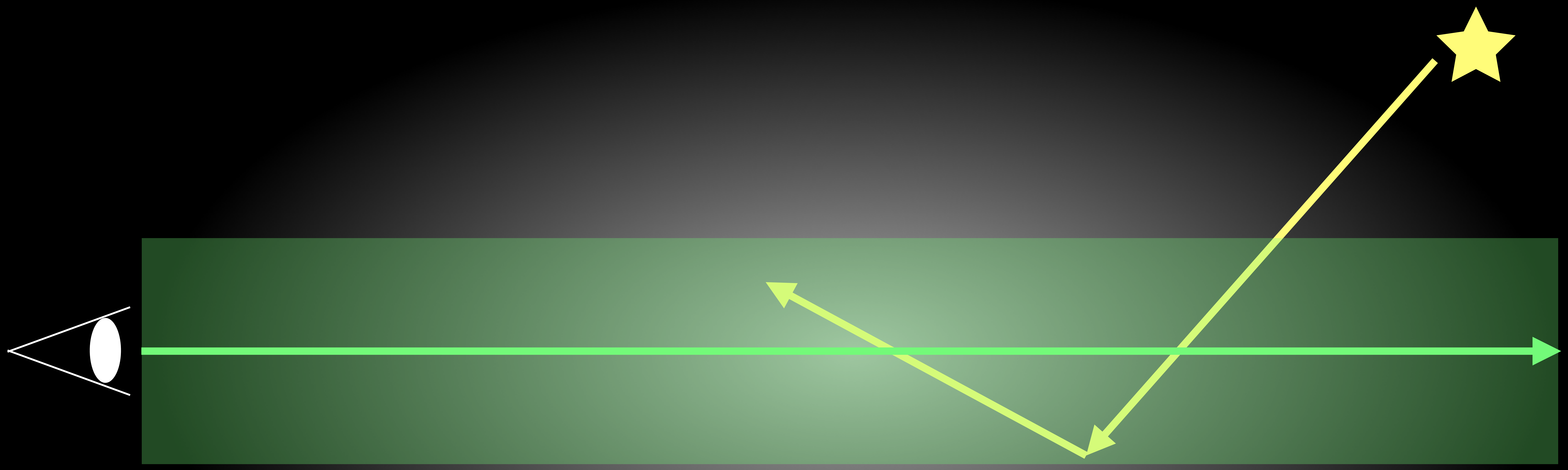
Bidirectional path tracing [Pauly et al. 2000]



Volume photon mapping [Jensen and Christensen 1998]



Beam radiance estimate [Jarosz et al. 2008]



Photon beams [Jarosz et al. 2011]

Comprehensive theory [Jarosz et al. 2011]

Comprehensive theory [Jarosz et al. 2011]

UPBP formulation

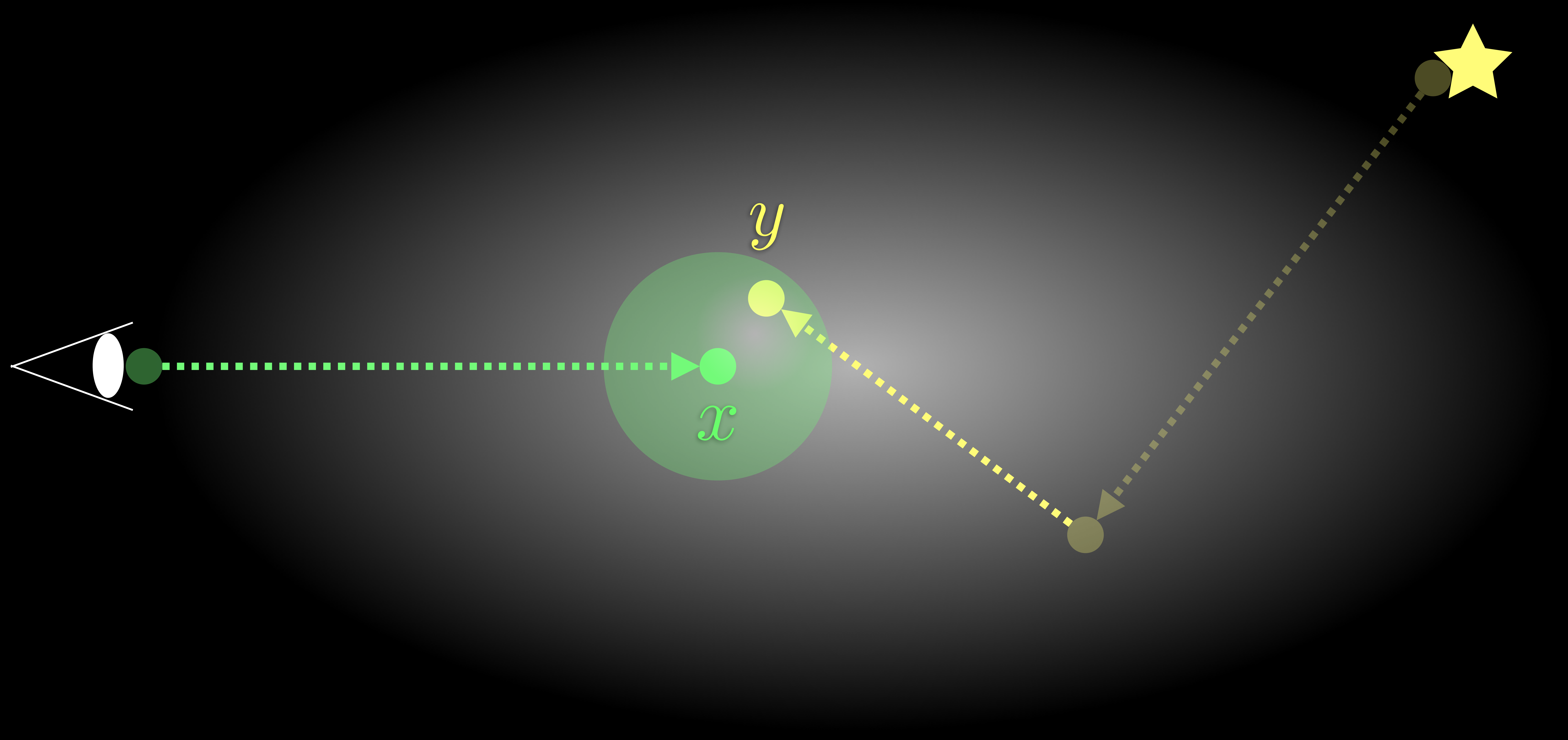
- Unified points, beams, and paths as sampling techniques for volumes

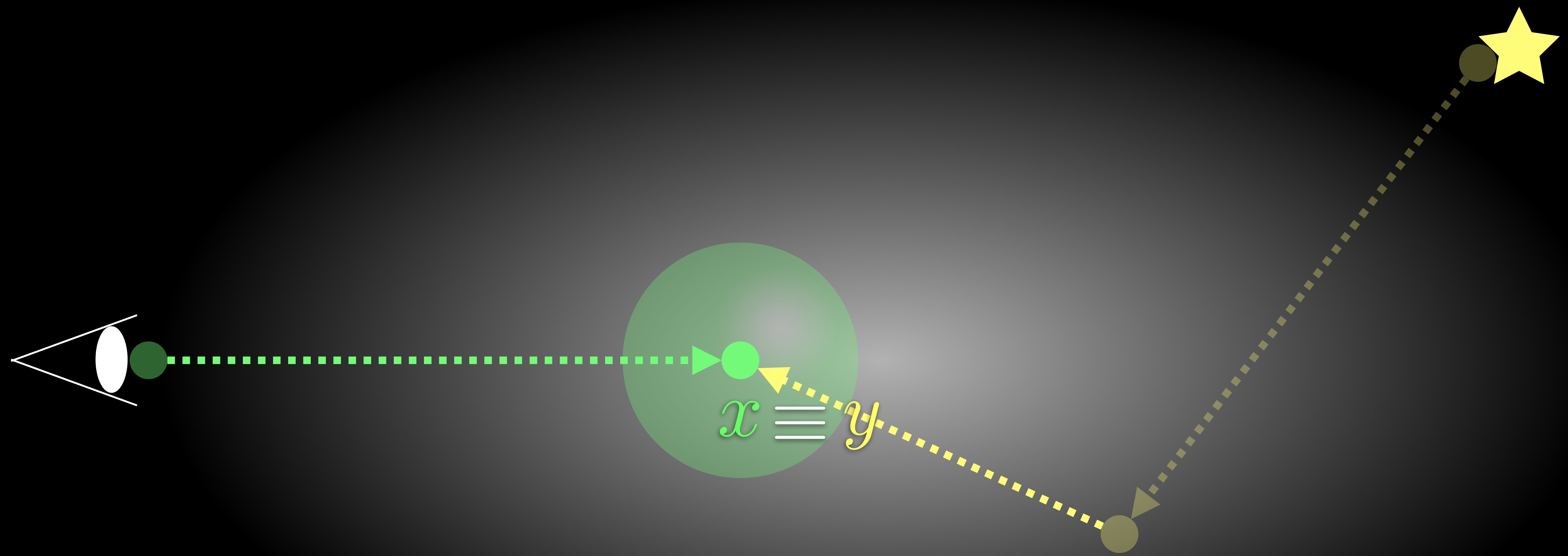
Dimensionality of paths

Path integral: **Four** vertices

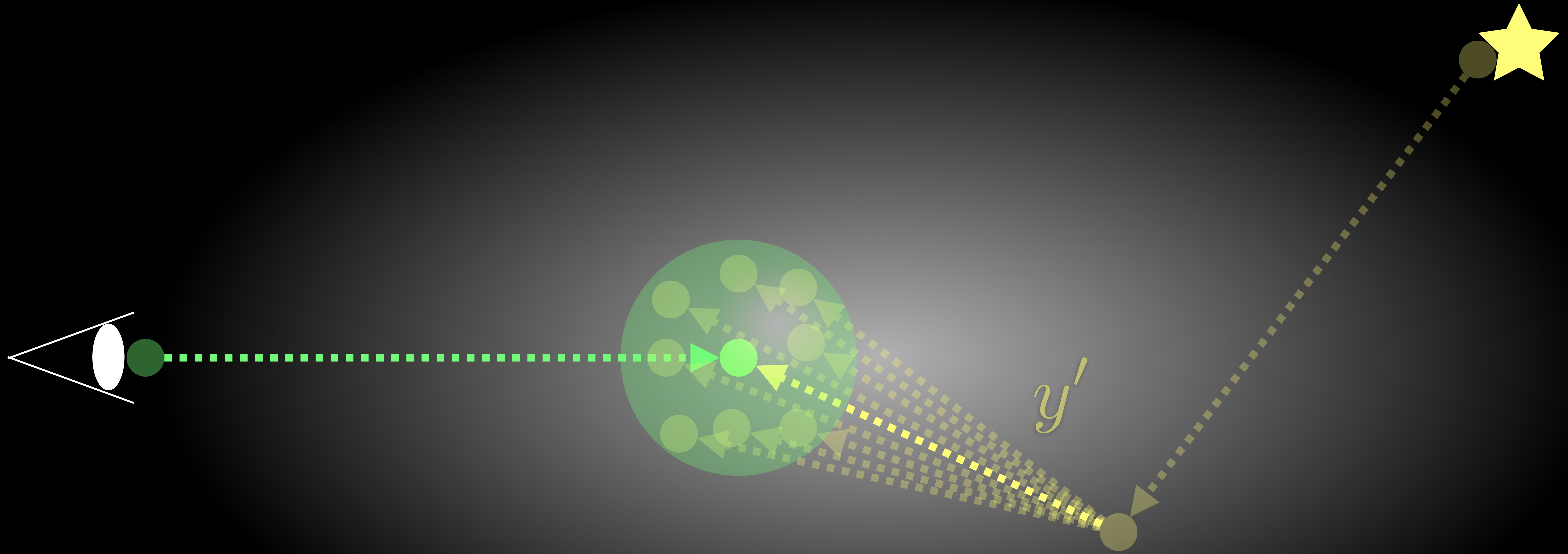
Density estimation: **Five** vertices

Same path length

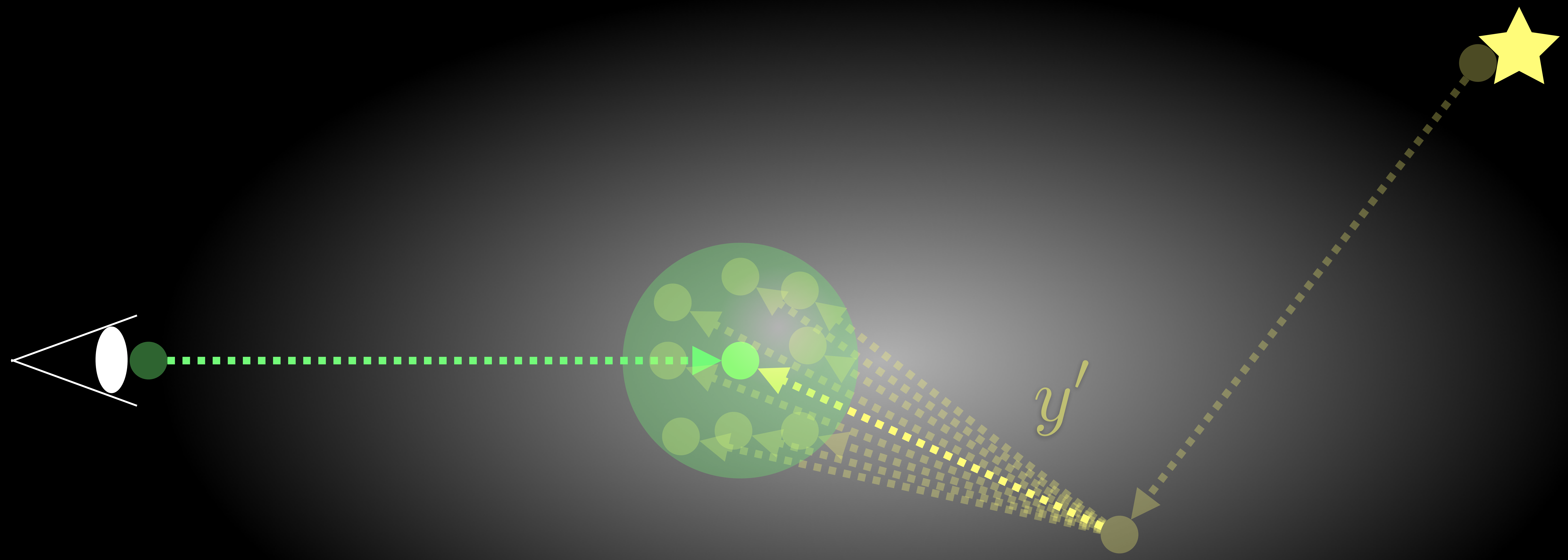

y
x



Merge vertices

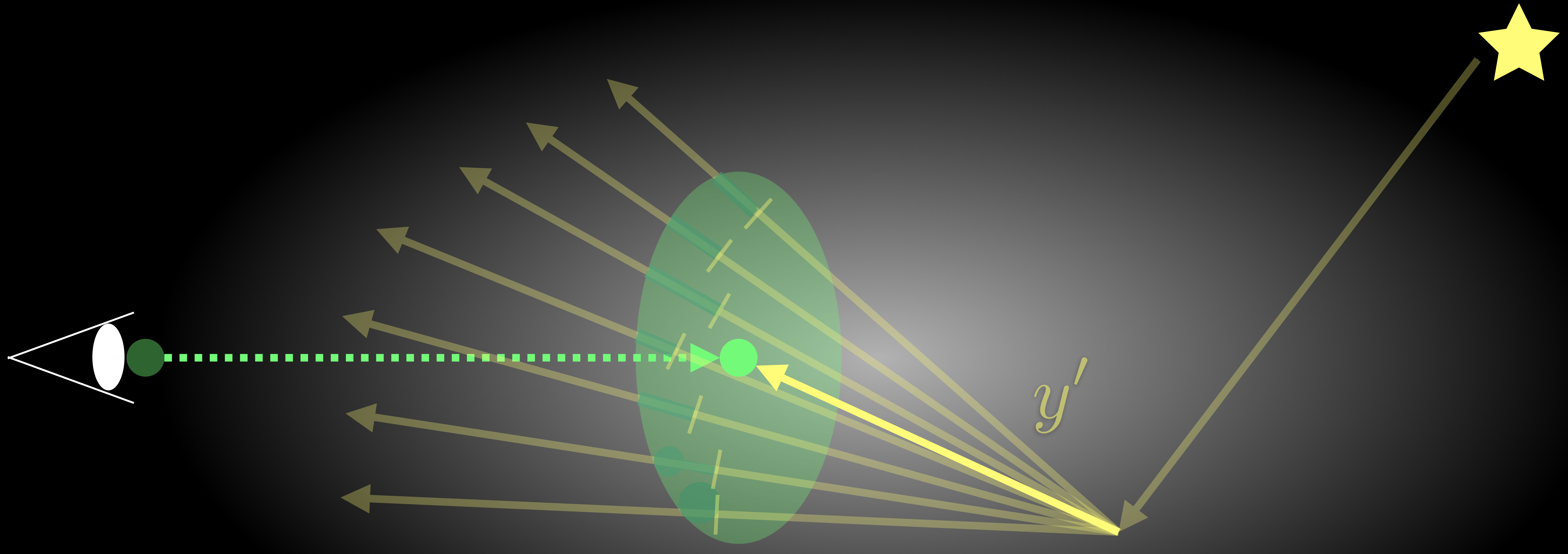


Consider all the paths which result in the same merged path



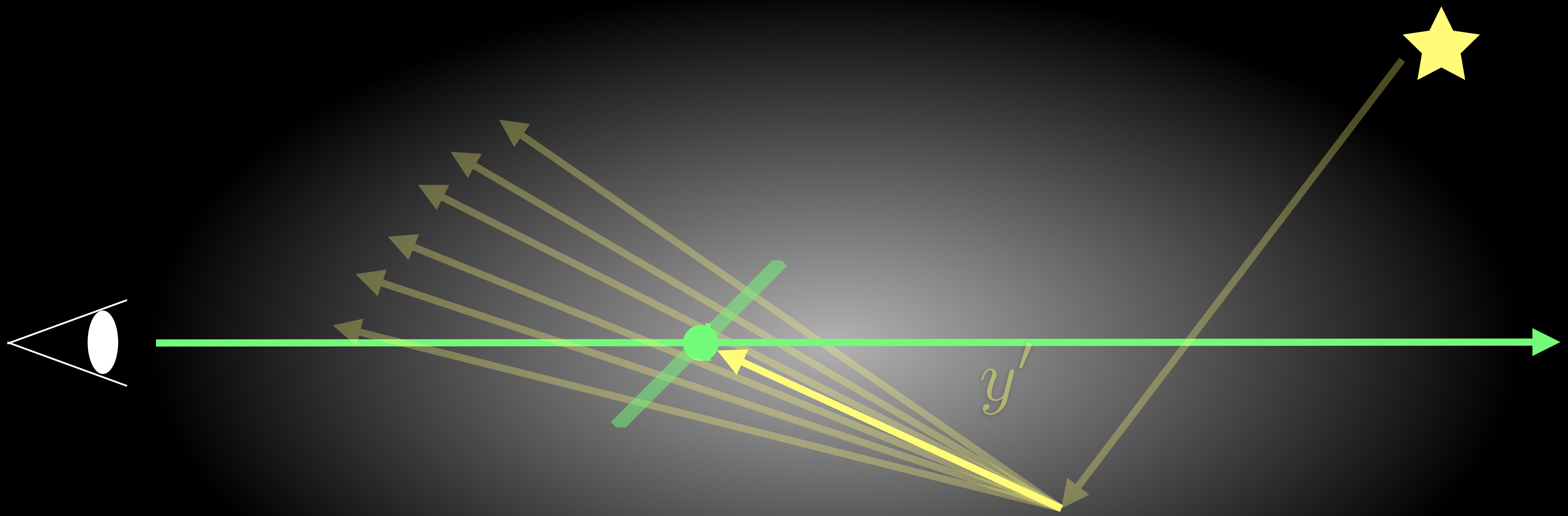
Accept according to the probability of merging

$$Prob[\,x \equiv y\,] = \int_{\bullet} p(y')\,dy'$$



Beam-Point 2D

$$Prob[\, x \equiv y \,] = \int p(y')\,dy'$$



Beam-Beam 1D

$$Prob[x \equiv y] = \int p(y')dy'$$

UPBP formulation

- Three steps to match with BDPT
  1. Merge subpaths
  2. Consider all the paths which result in the same merged path
  3. Accept the path with the probability of merging

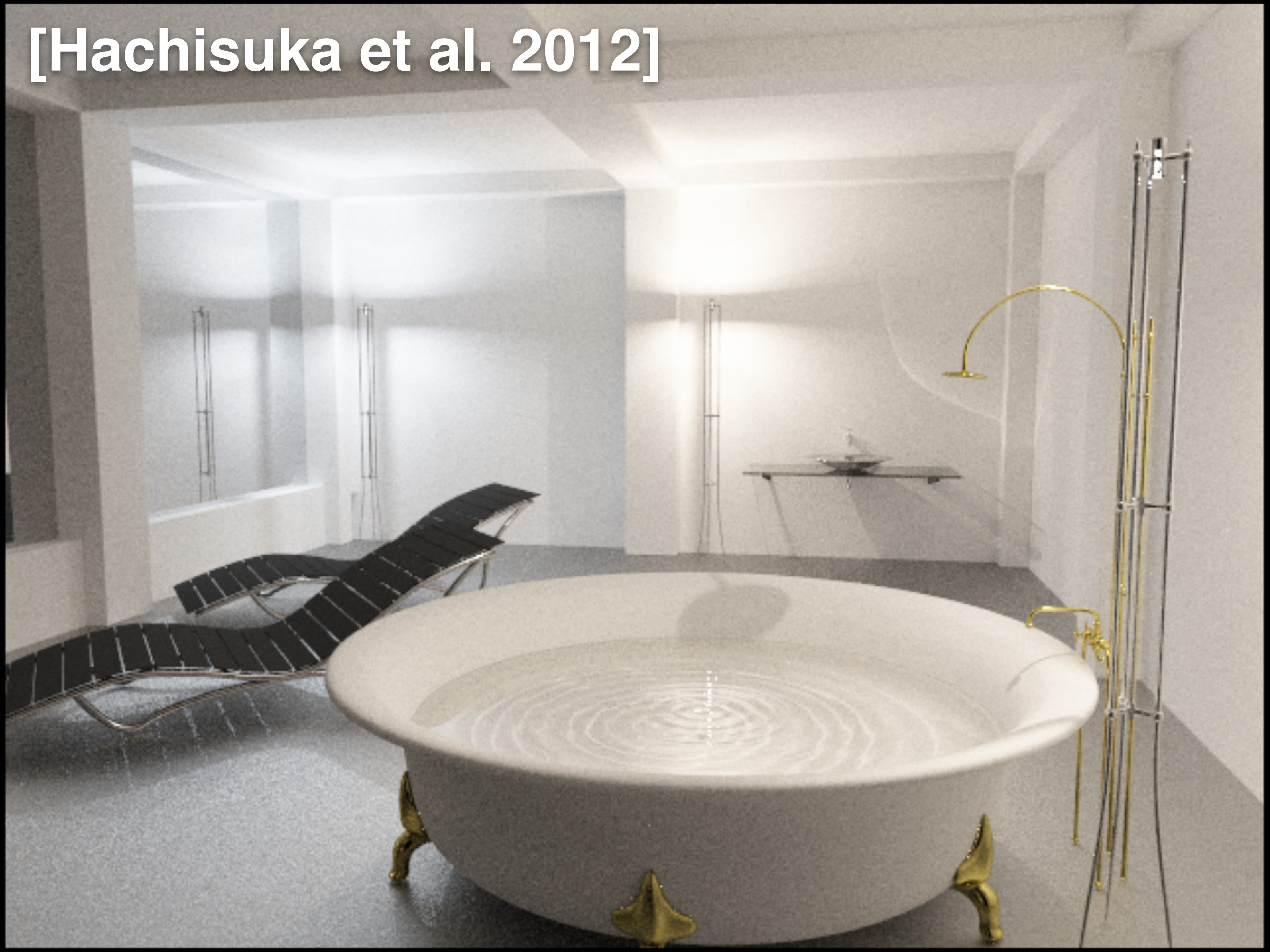
Beam-Beam 1D

Corresponds to contraction of density estimation path space

$$Prob[x \equiv y] = \int p(y')dy'$$

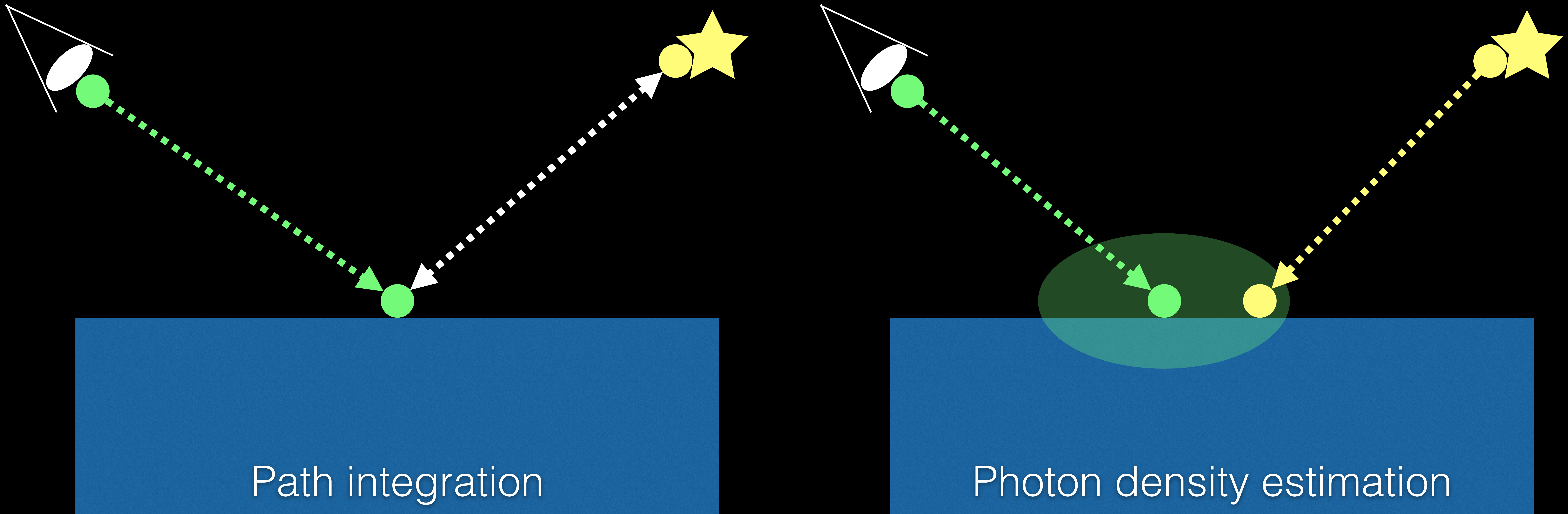
UPS/VCM formulation

- Unified path integration and photon density estimation for **surfaces**



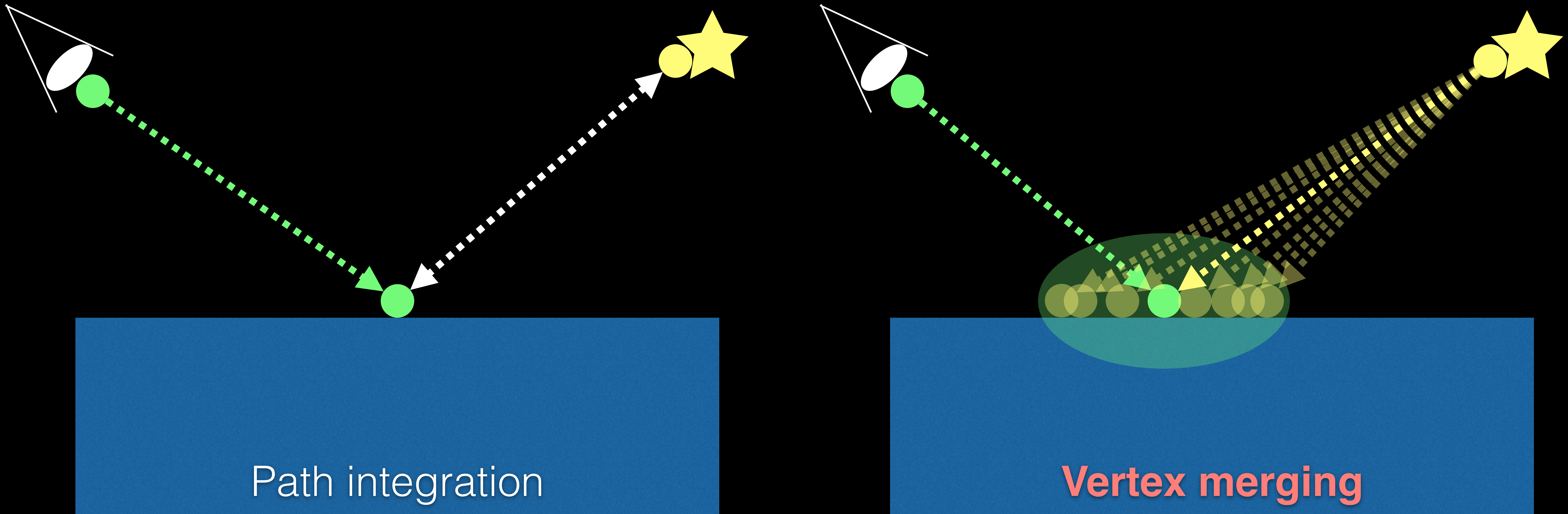
Vertex Connection and Merging

- **Contract** the space of density estimation into the original path space



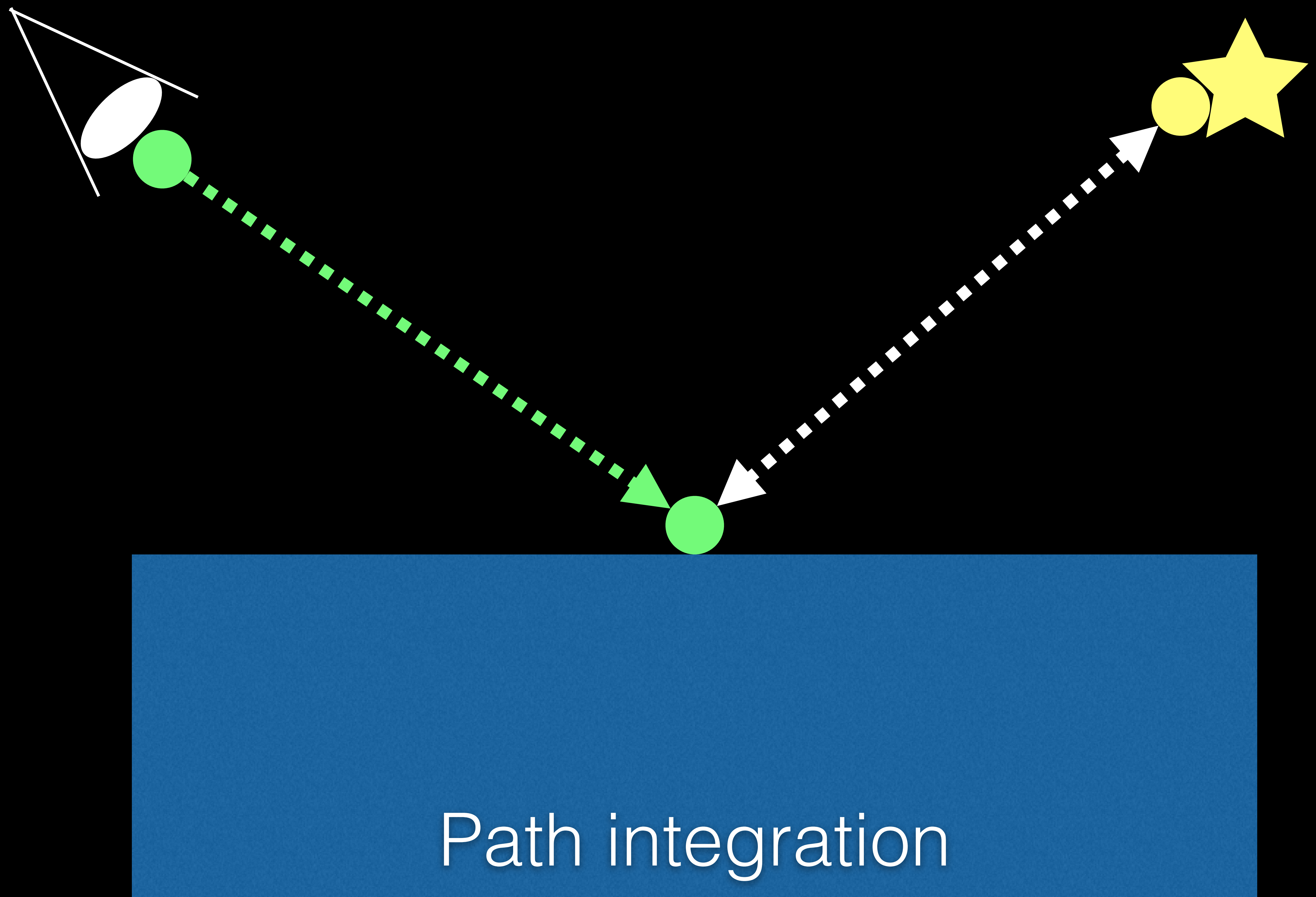
Vertex Connection and Merging

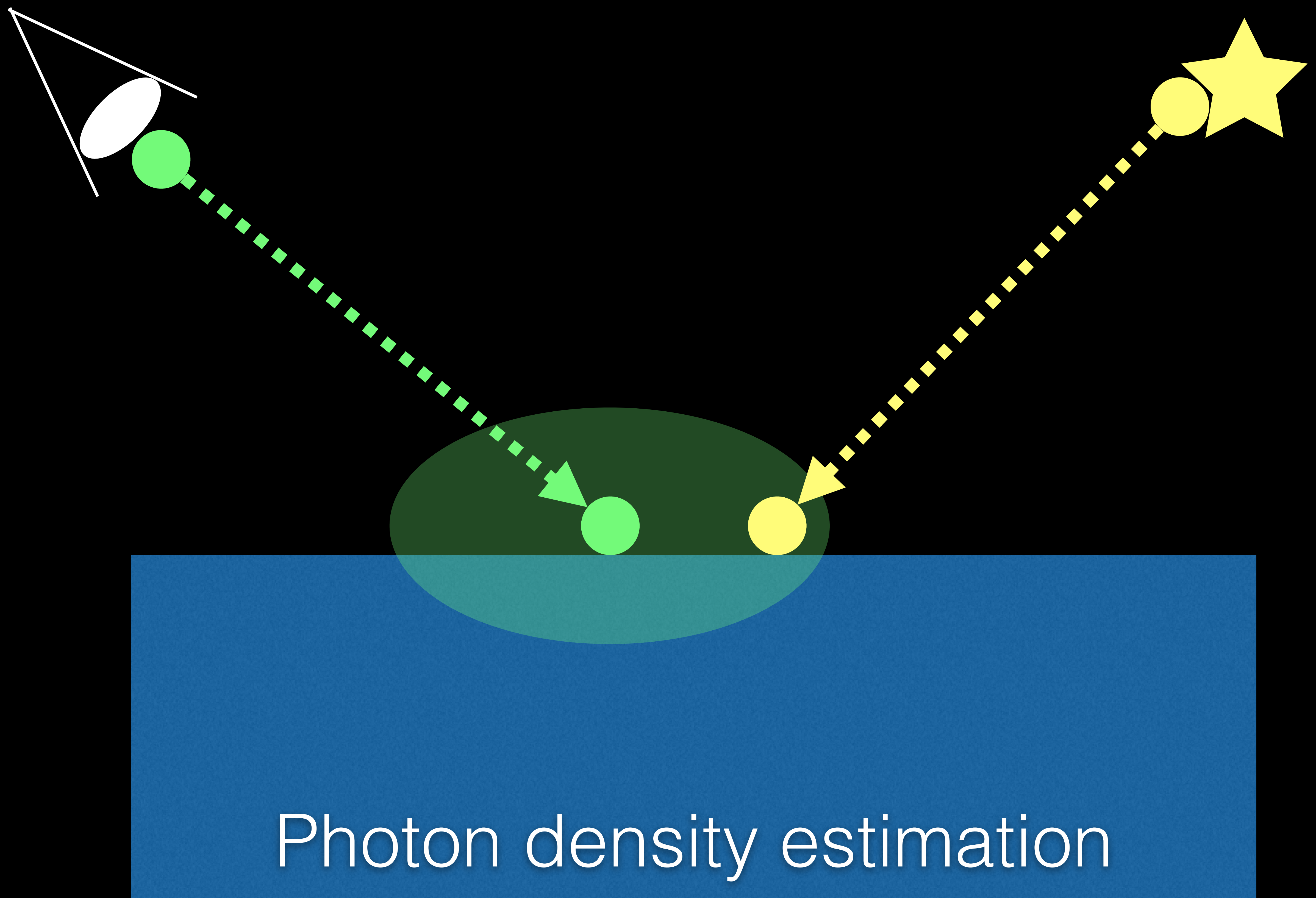
- **Contract** the space of density estimation into the original path space



Unified Path Sampling

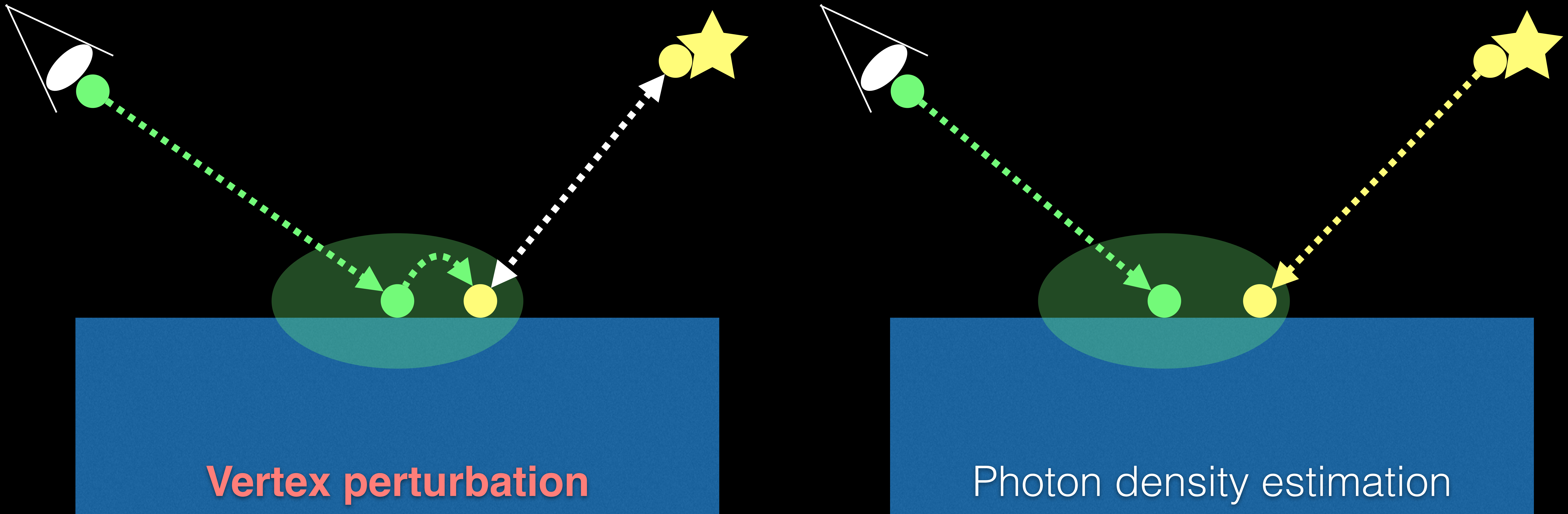
- **Extend** the original path space to include photon density estimation





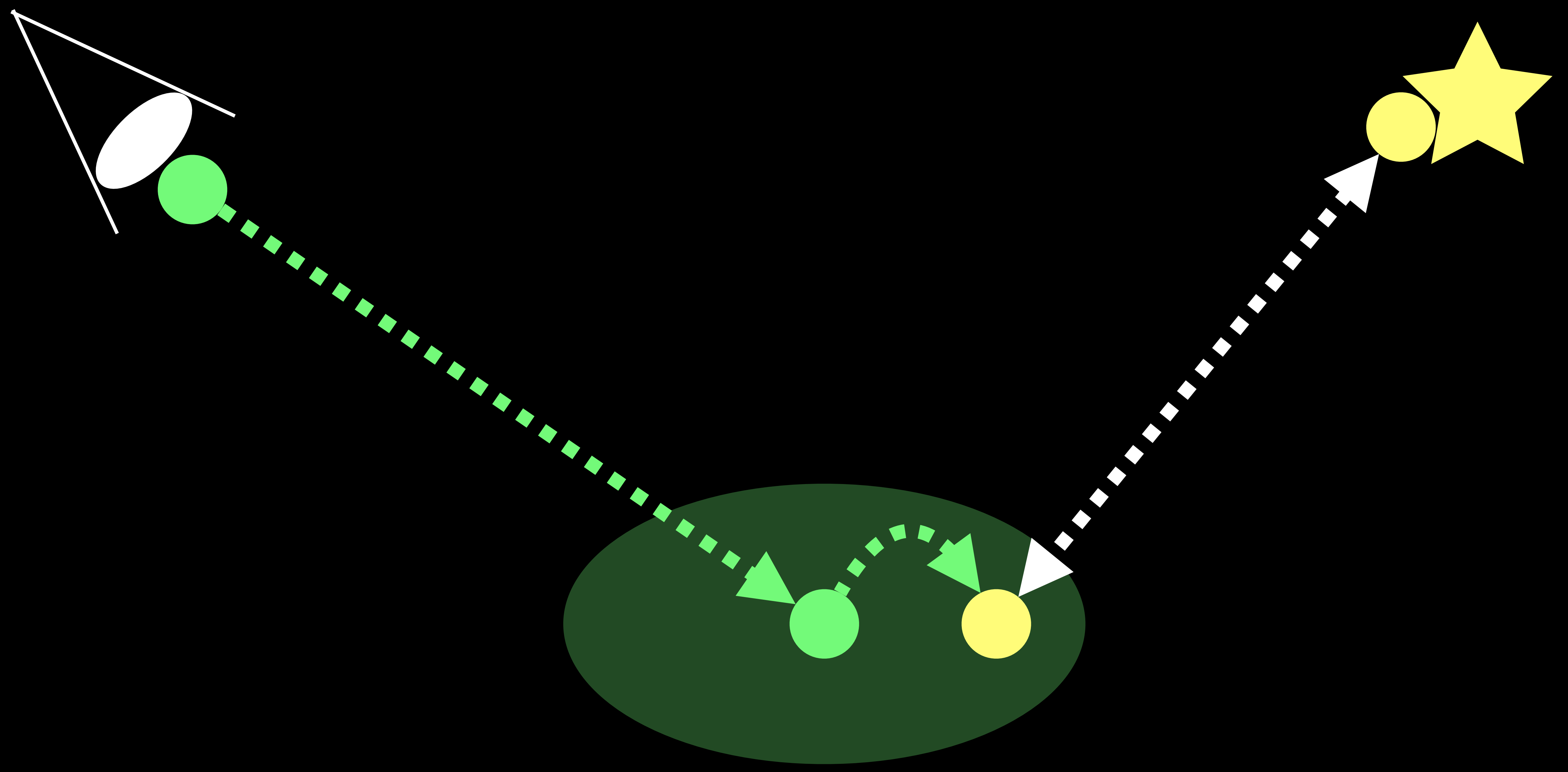
Unified Path Sampling

- **Extend** the original path space to include photon density estimation

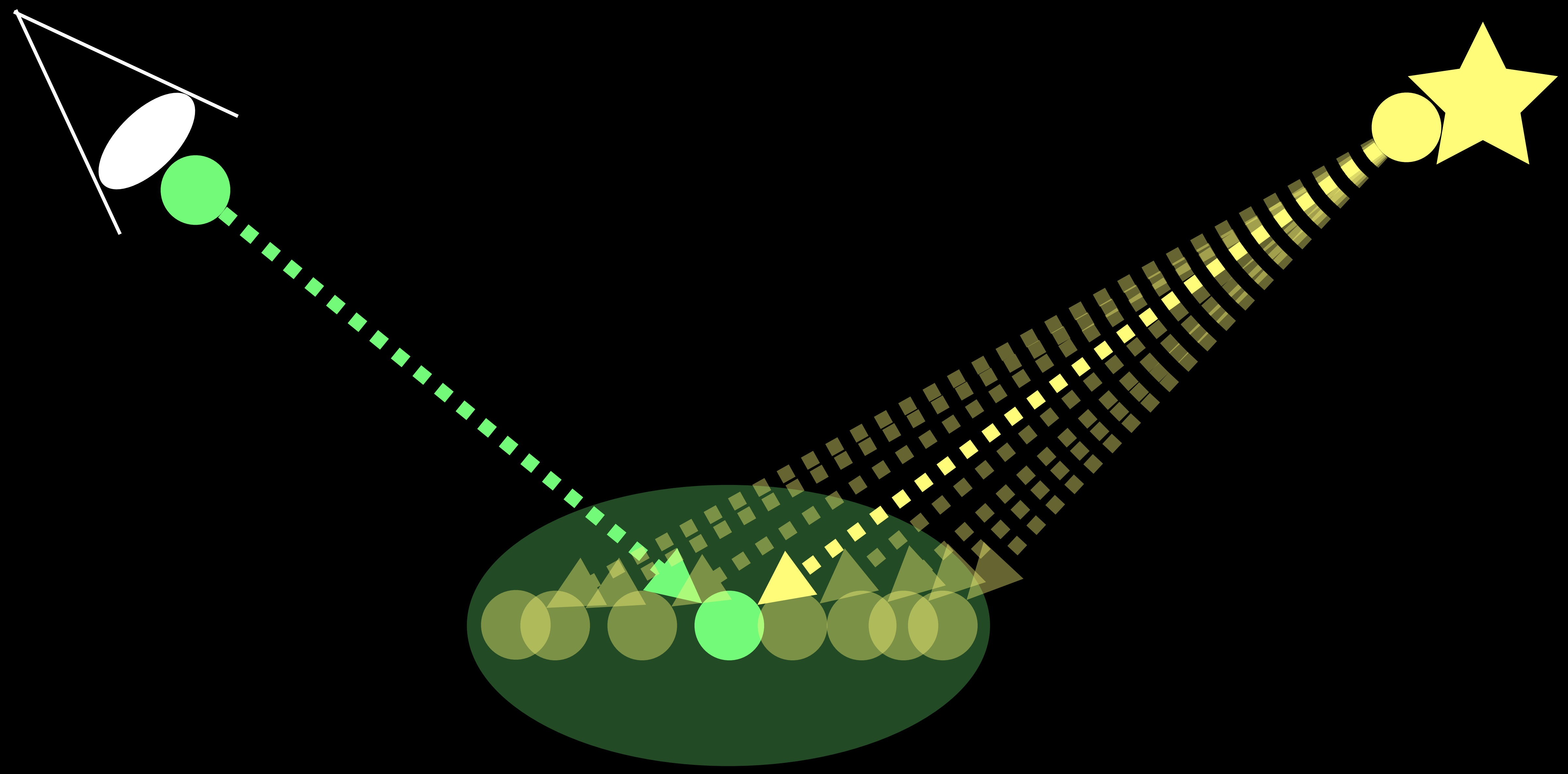


Differences

- **VCM**: precise for path integration, approximate for density estimation
- **UPS**: precise for density estimation, approximate for path integration



UPS

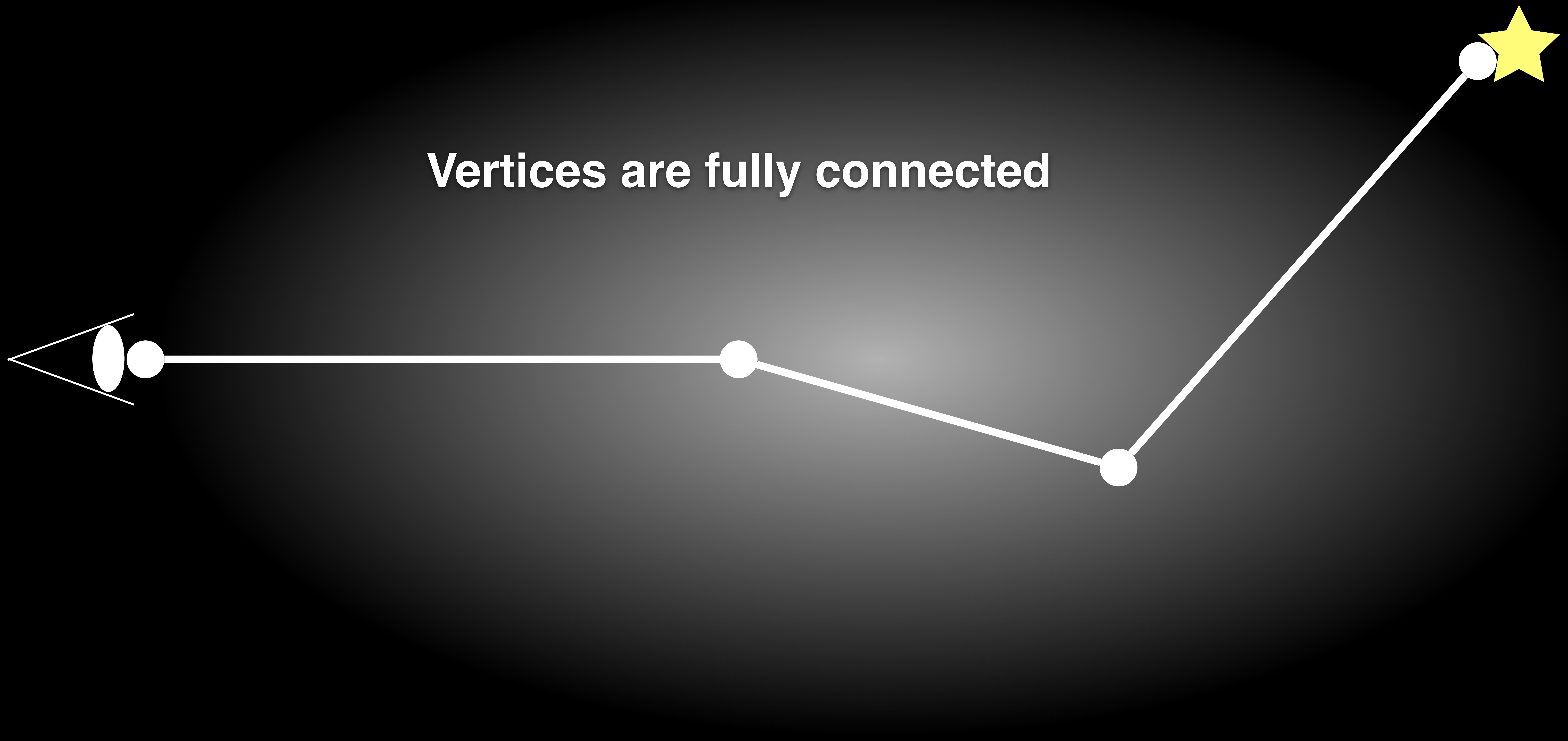


VCM

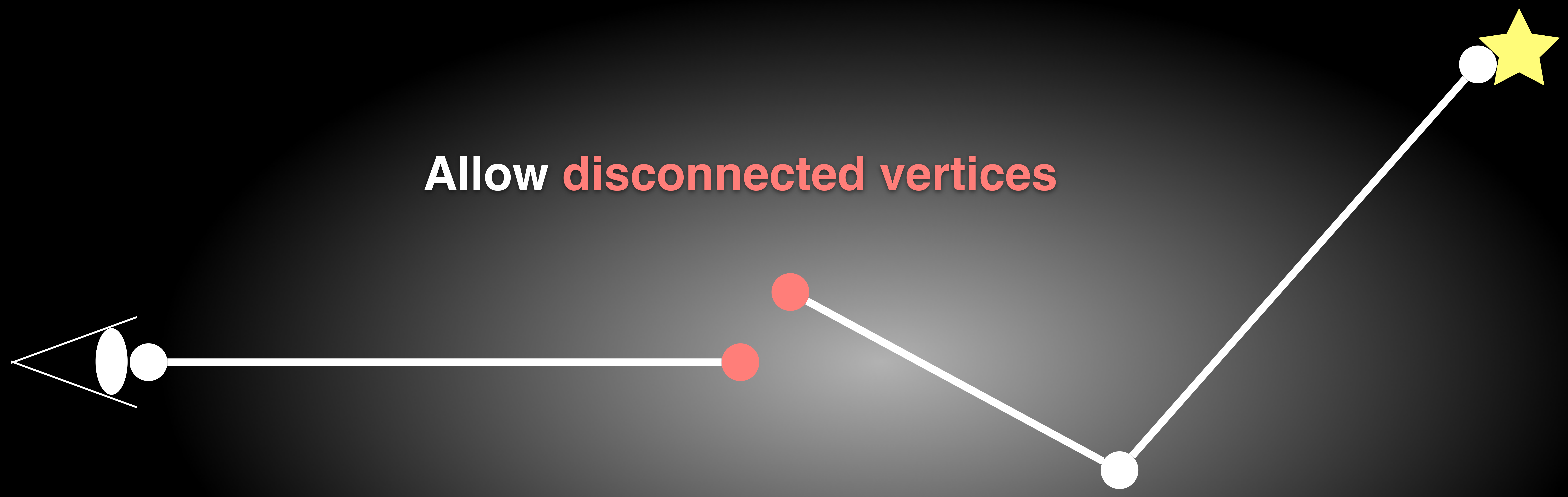
	Surfaces	Volumes
Contraction	VCM	UPBP
Extension	UPS	**Ours (UVPS)**

Unified Volumetric Path Sampling

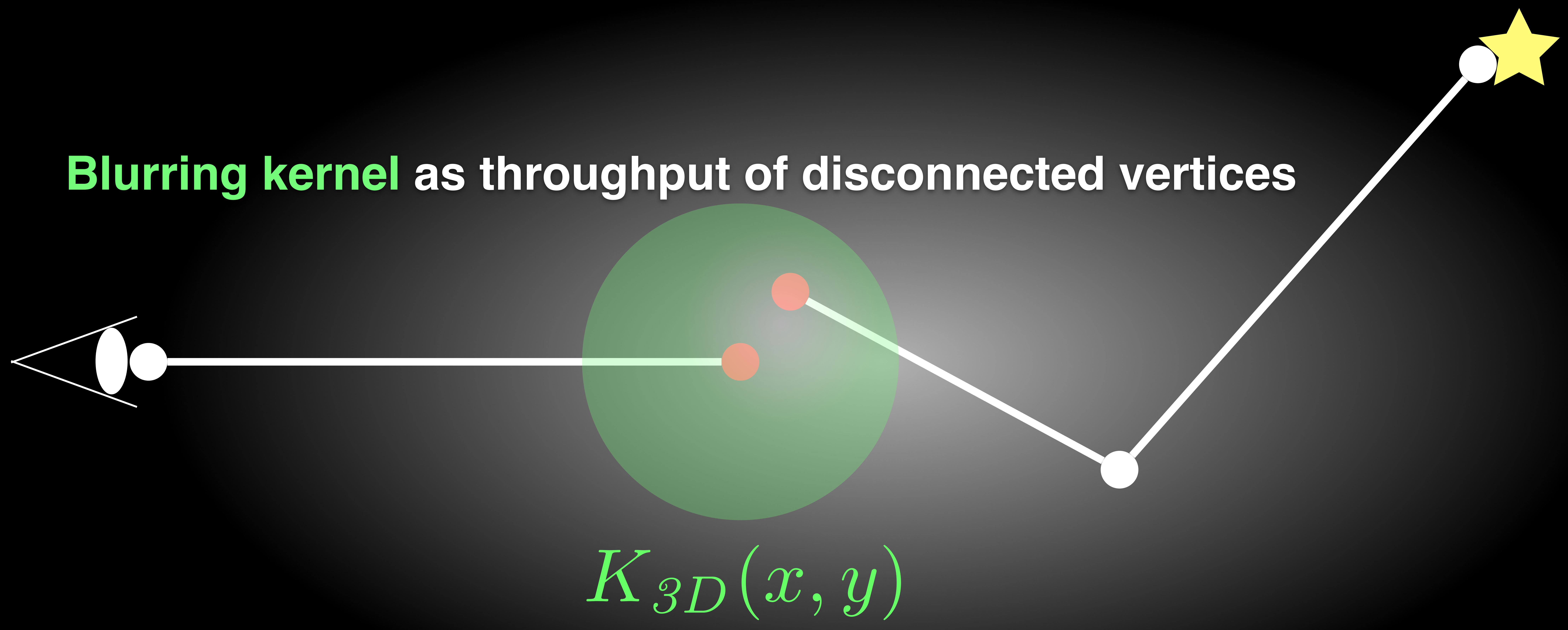
Path integral formulation



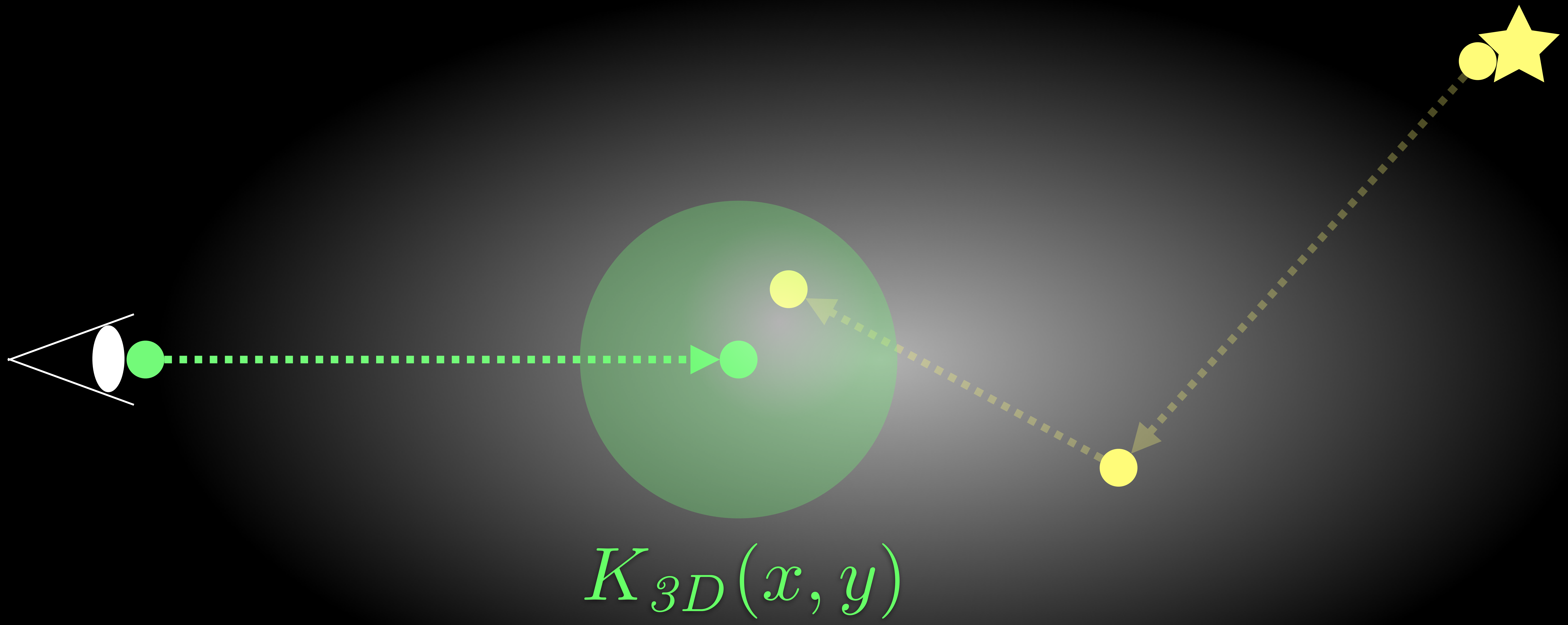
Extended path integral formulation



Extended path integral formulation

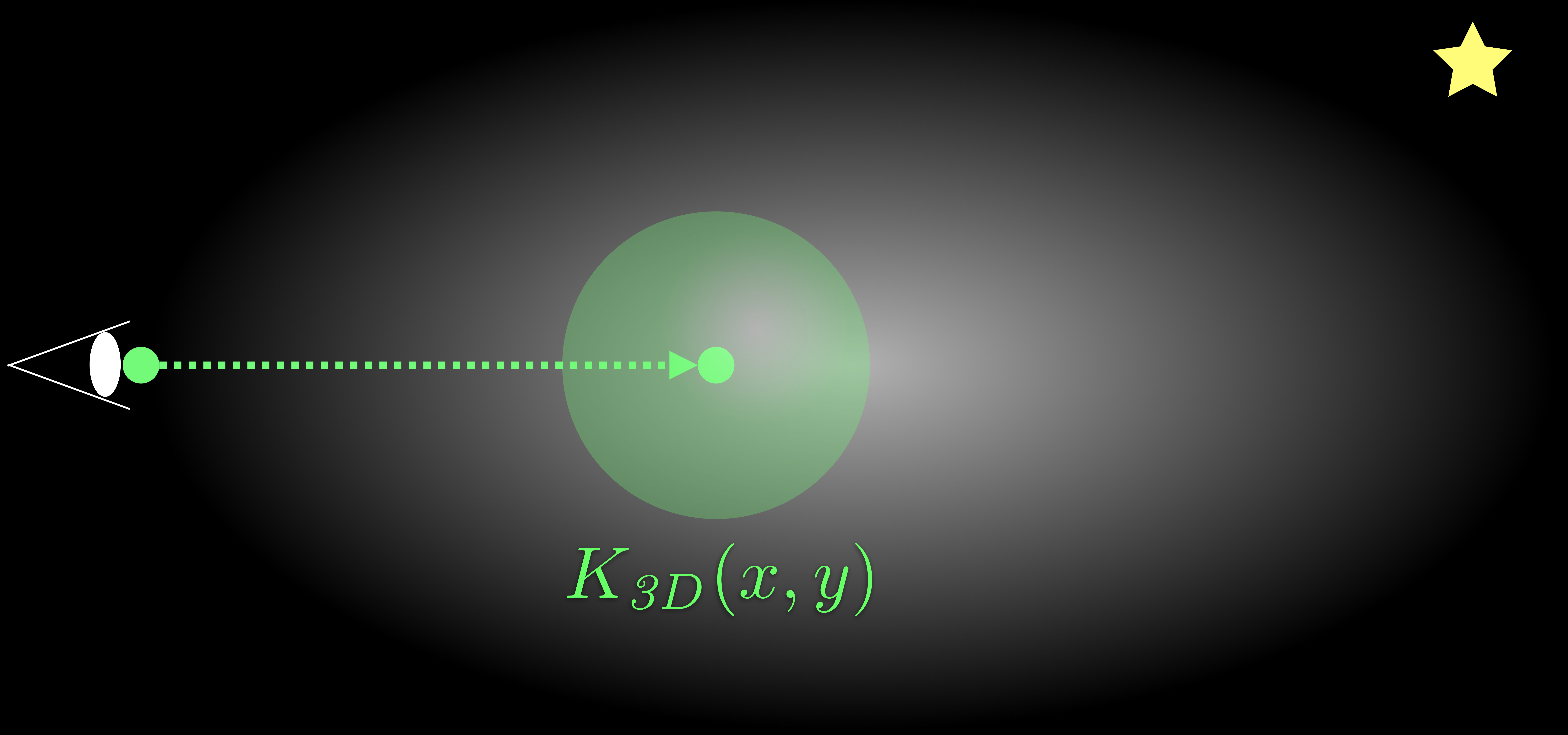


Point-Point 3D

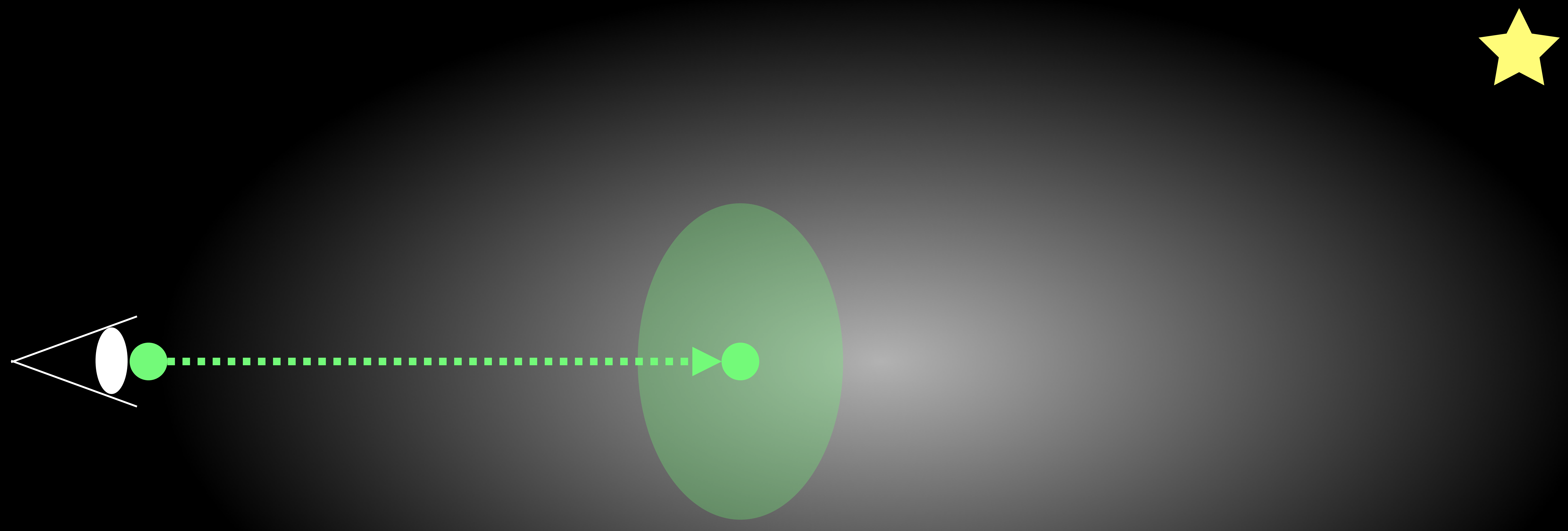


Precisely models photon density estimation

3D blur to 2D blur



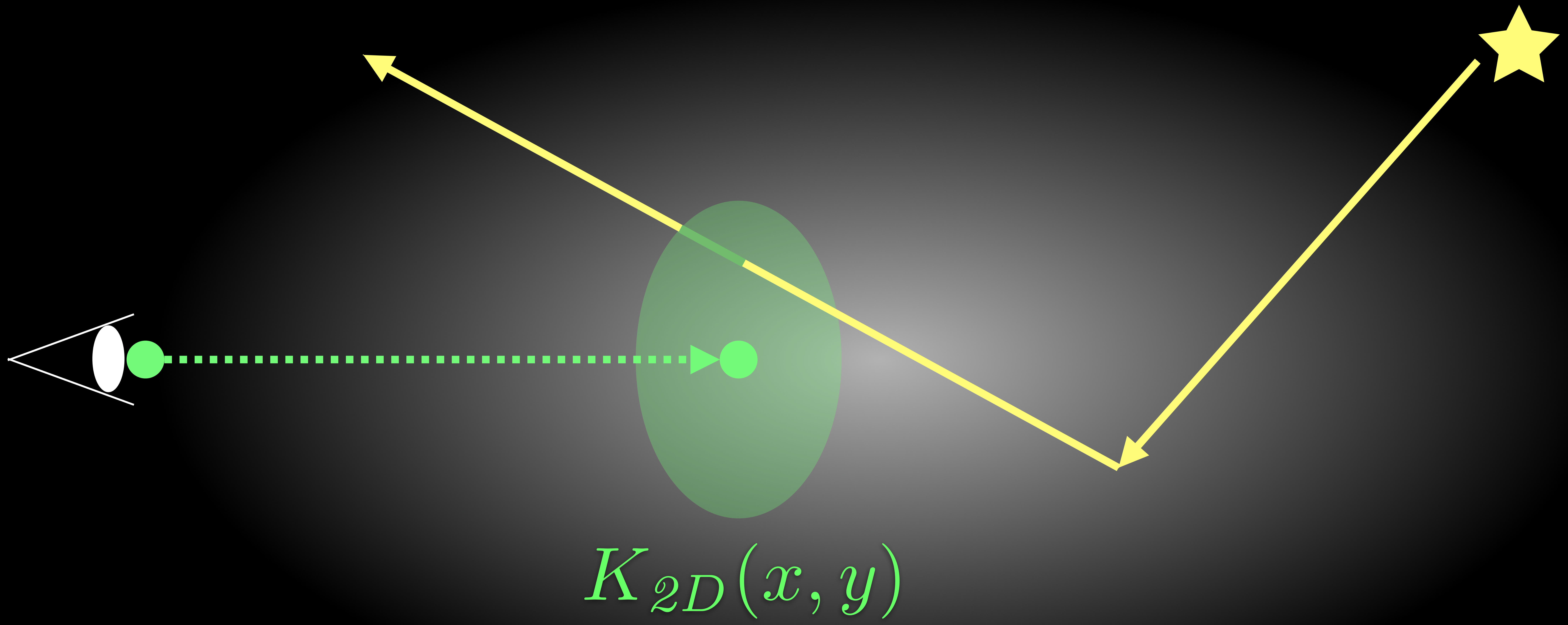
3D blur to 2D blur



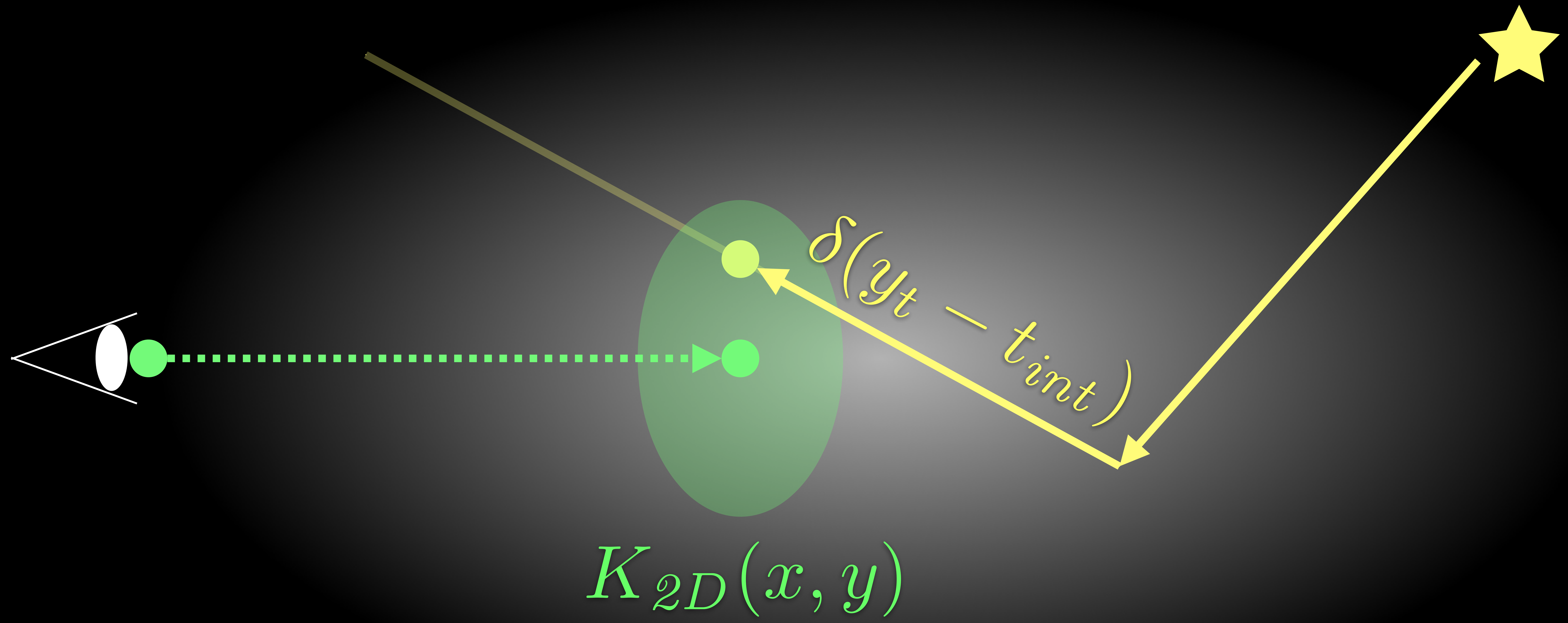
$$K_{2D}(x, y) = K_{3D}(x, y)\delta(x_t - t_K)$$

Flatten a sphere into a disc

Beam-Point 2D

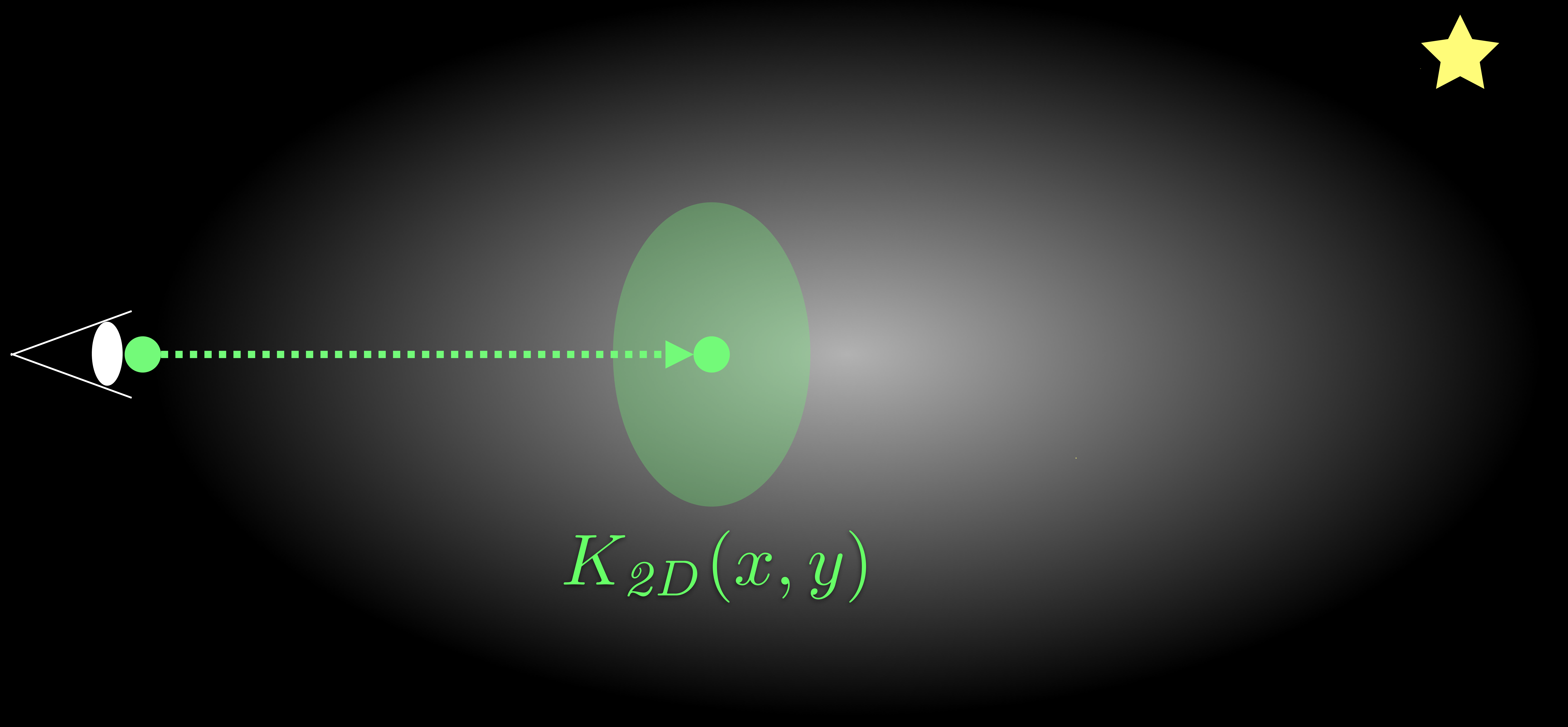


Beam-Point 2D

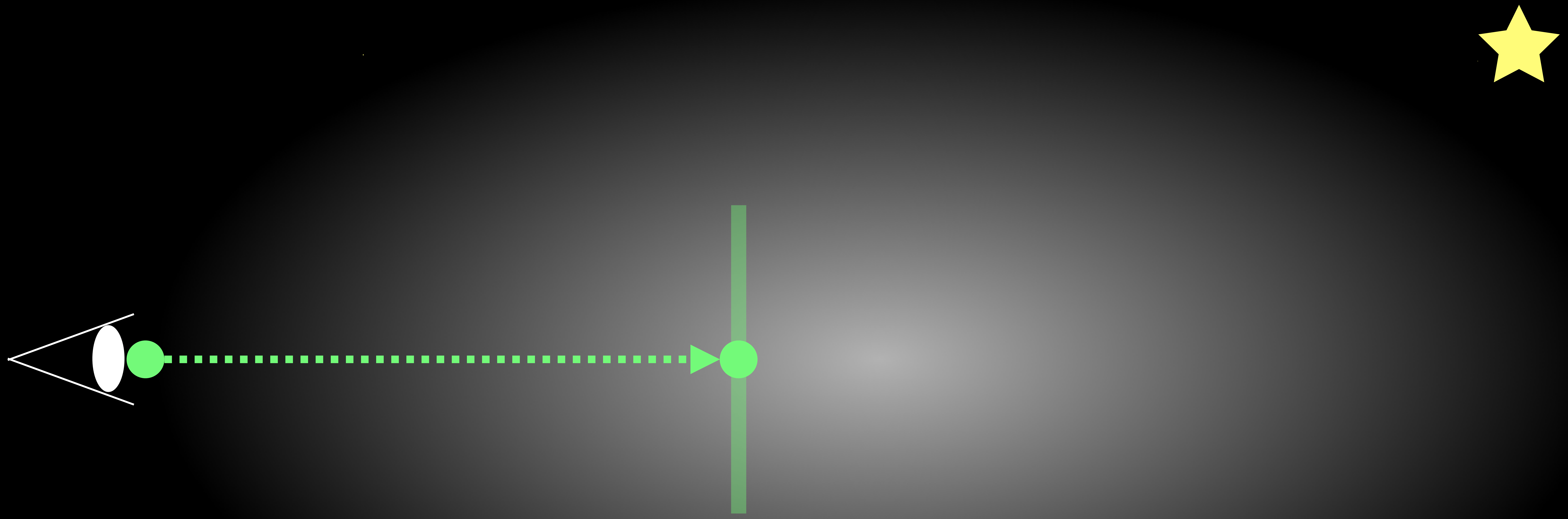


Beam-point 2D = deterministic sampling of one distance

2D blur to 1D blur



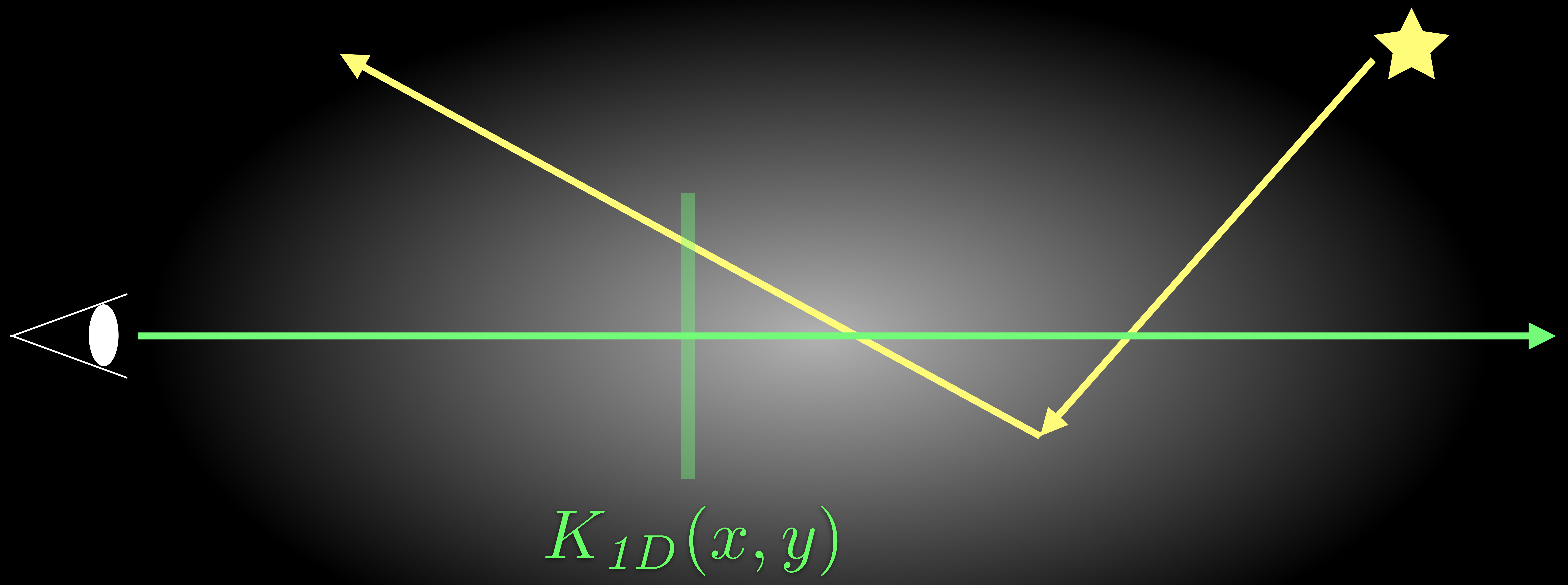
2D blur to 1D blur



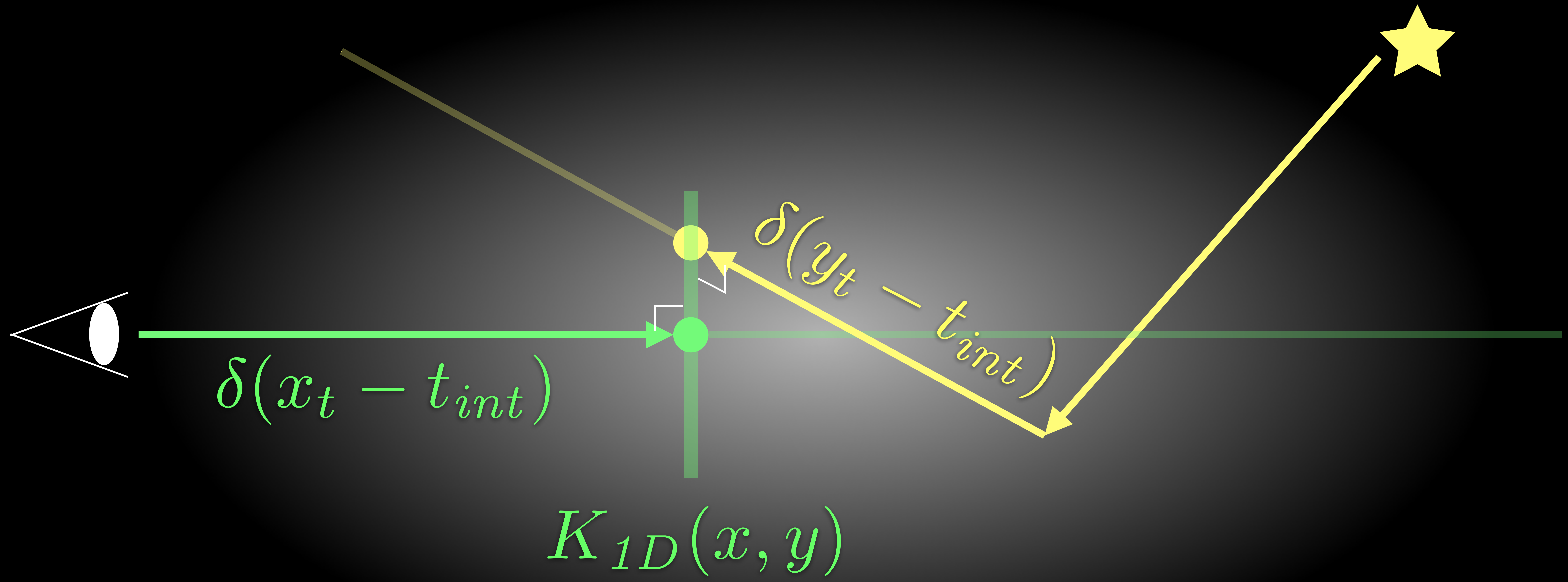
$$K_{1D}(x, y) = K_{2D}(x, y)\delta(x_t - t'_K)$$

Flatten a disc into a line

Beam-Beam 1D

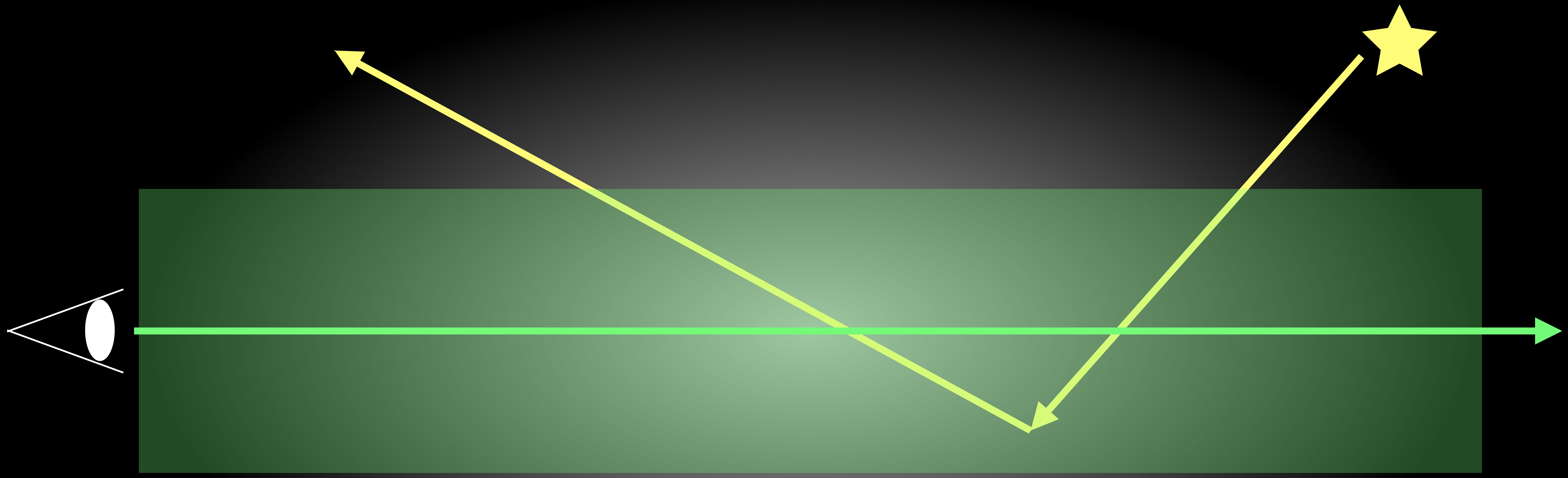


Beam-Beam 1D

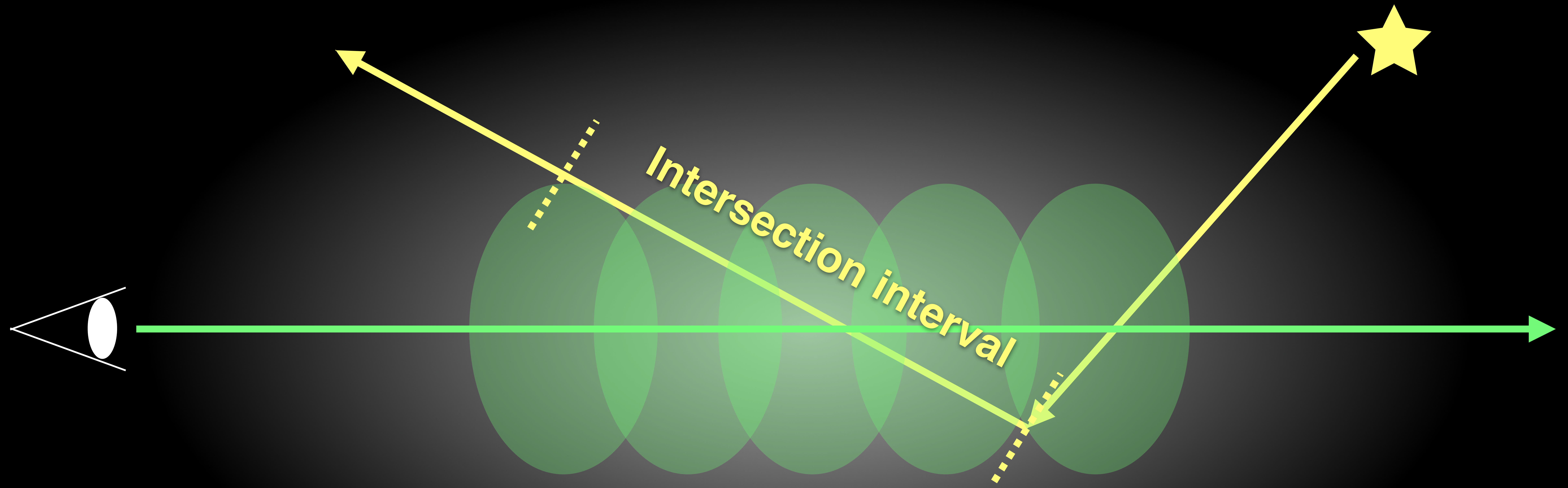


Beam-beam 1D = deterministic sampling of two distances

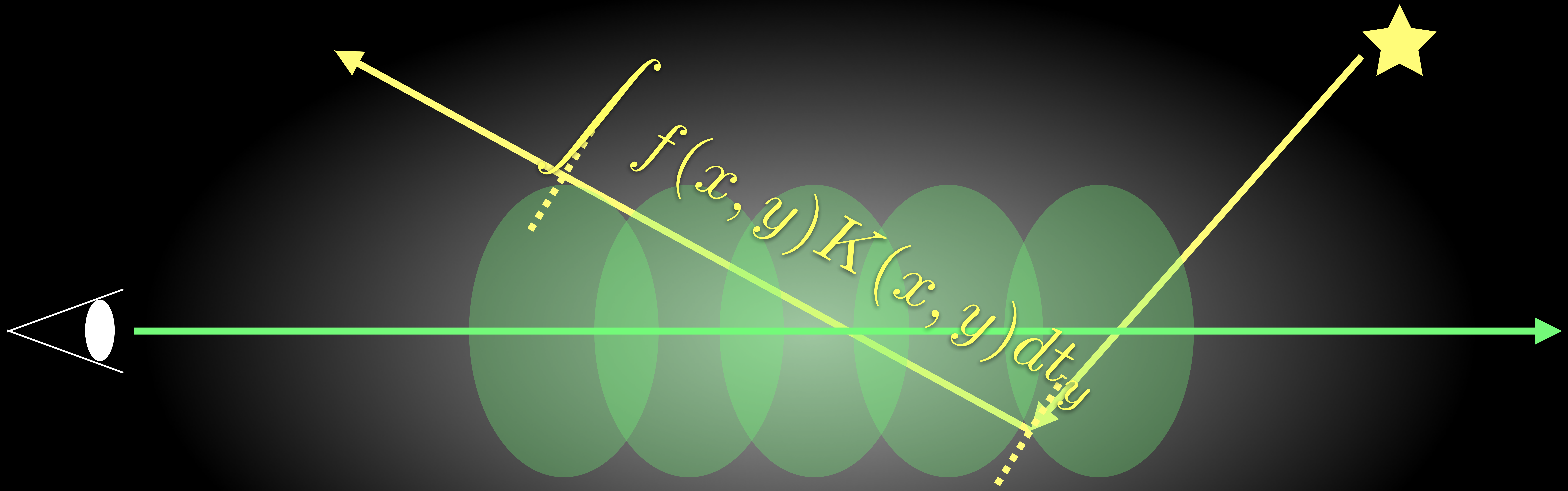
Beam-Beam 2D



Beam-Beam 2D

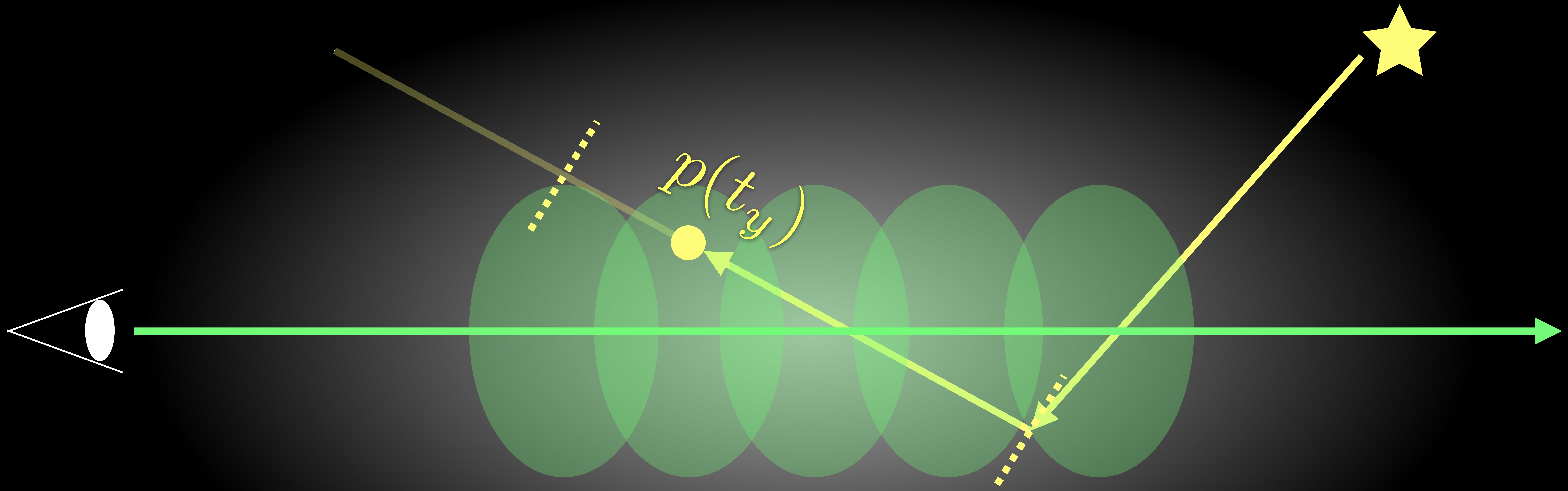


Beam-Beam 2D [Jarosz et al. 2011]



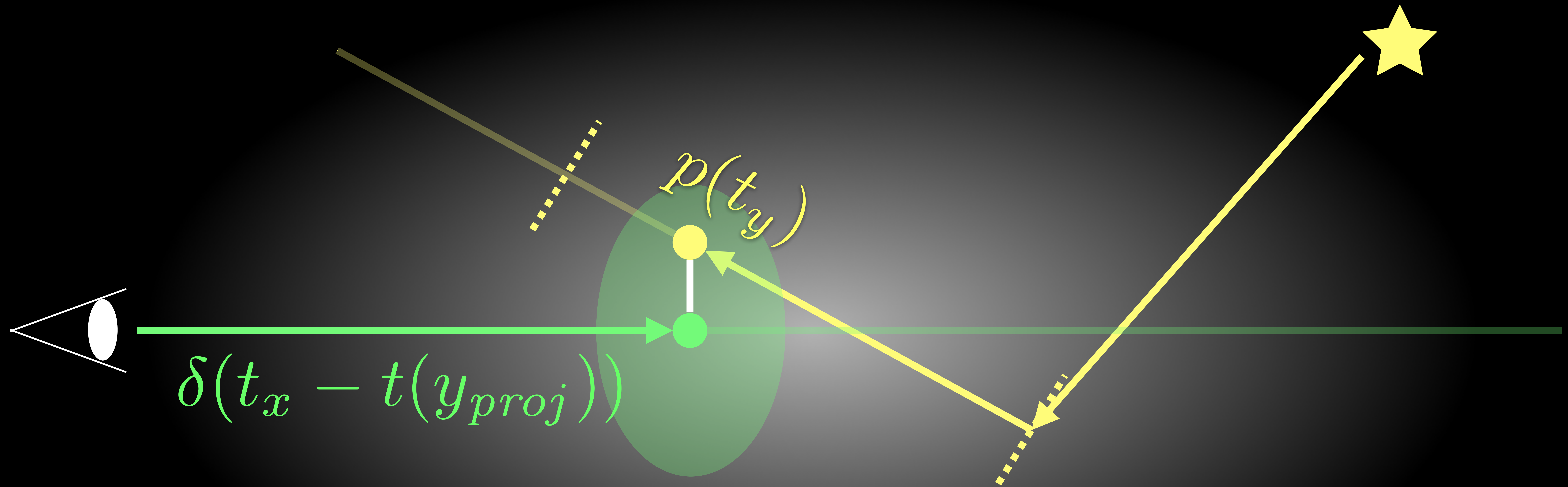
Integral over the intersection interval

Beam-Beam 2D

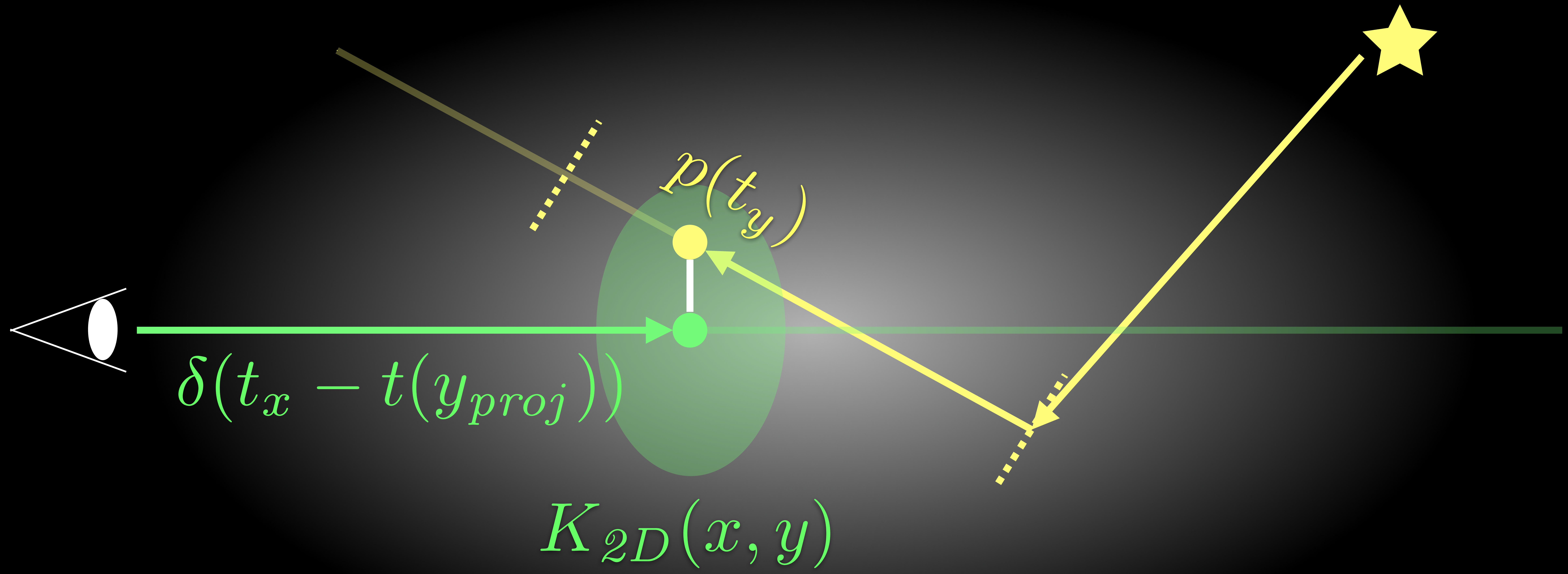


Stochastic sampling within the interval

Beam-Beam 2D

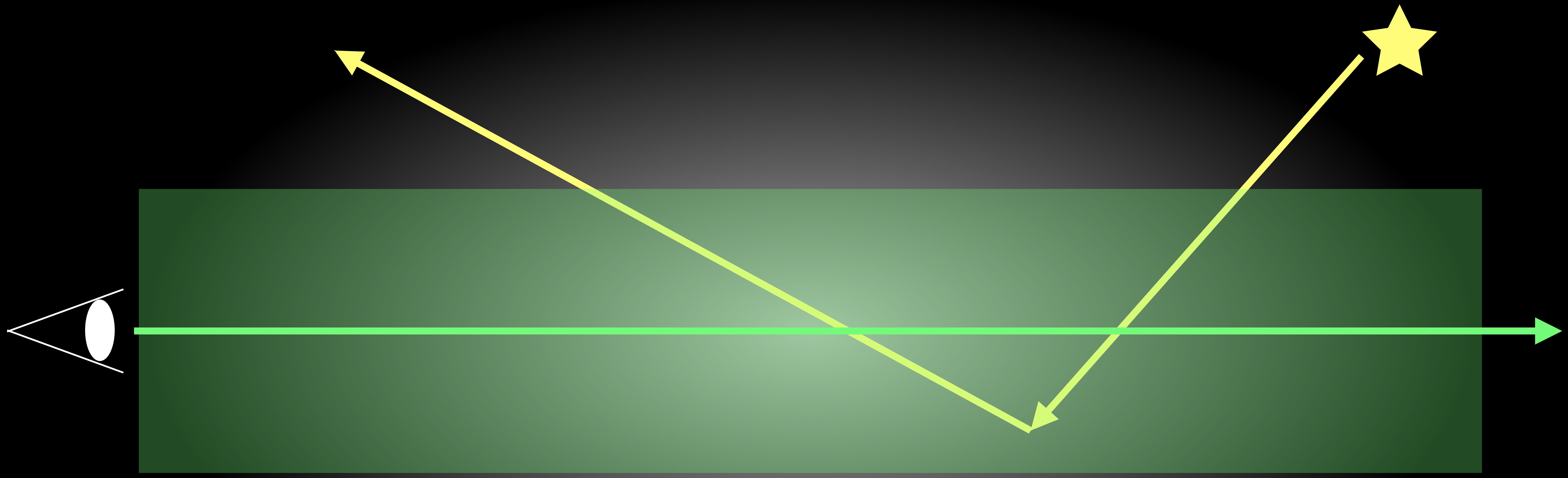


Beam-Beam 2D

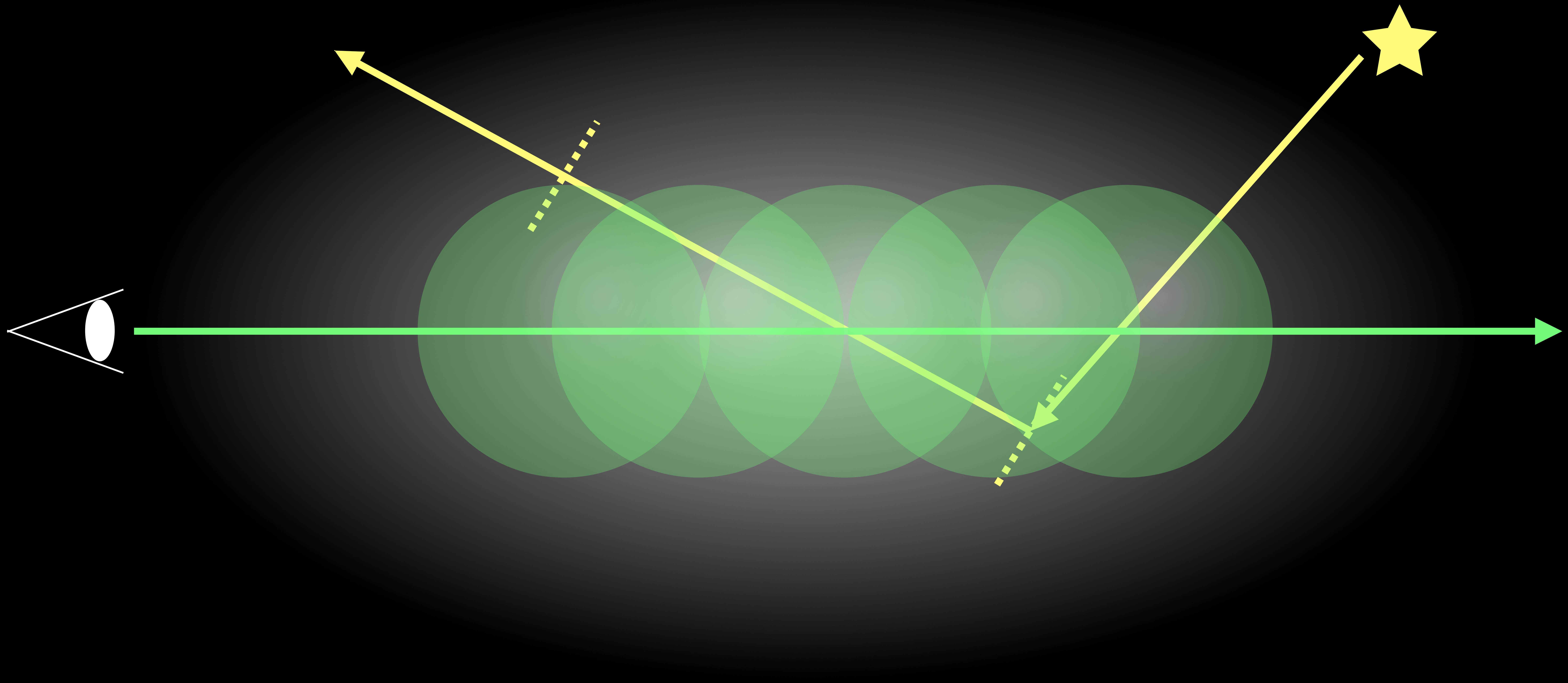


Same 2D kernel as beam-point 2D

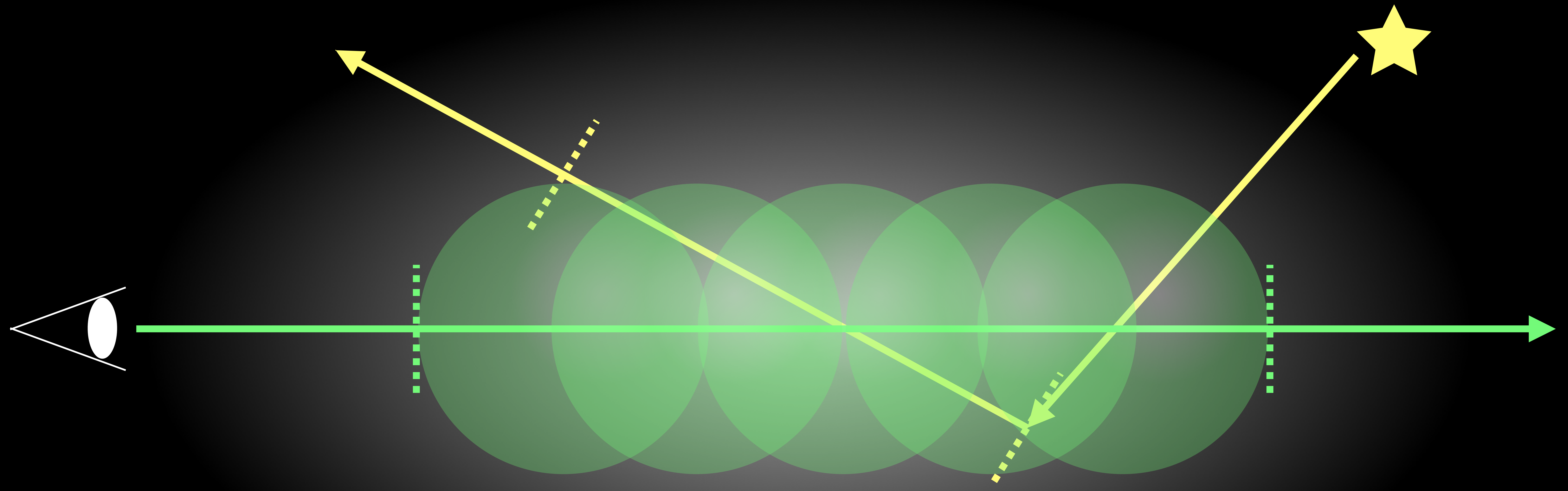
Beam-Beam 3D



Beam-Beam 3D



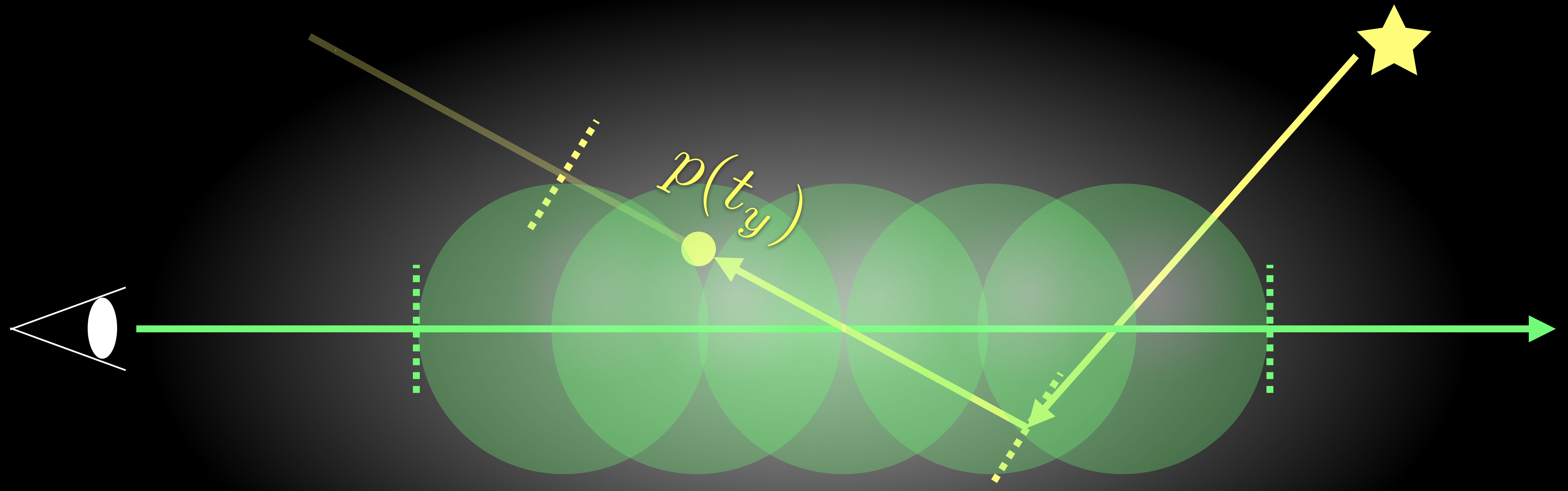
Beam-Beam 3D [Jarosz et al. 2011]



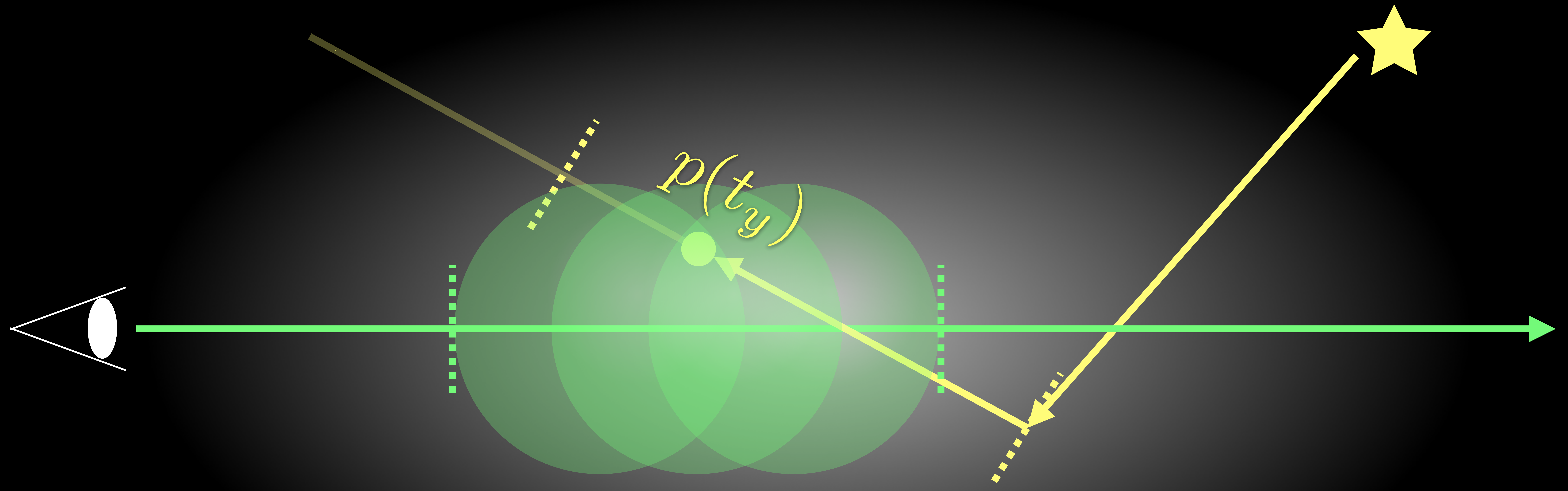
$$\iint f(x,y)K(x,y)\,dt_y\,dt_x$$

Double integral over the intersection intervals (usually intractable)

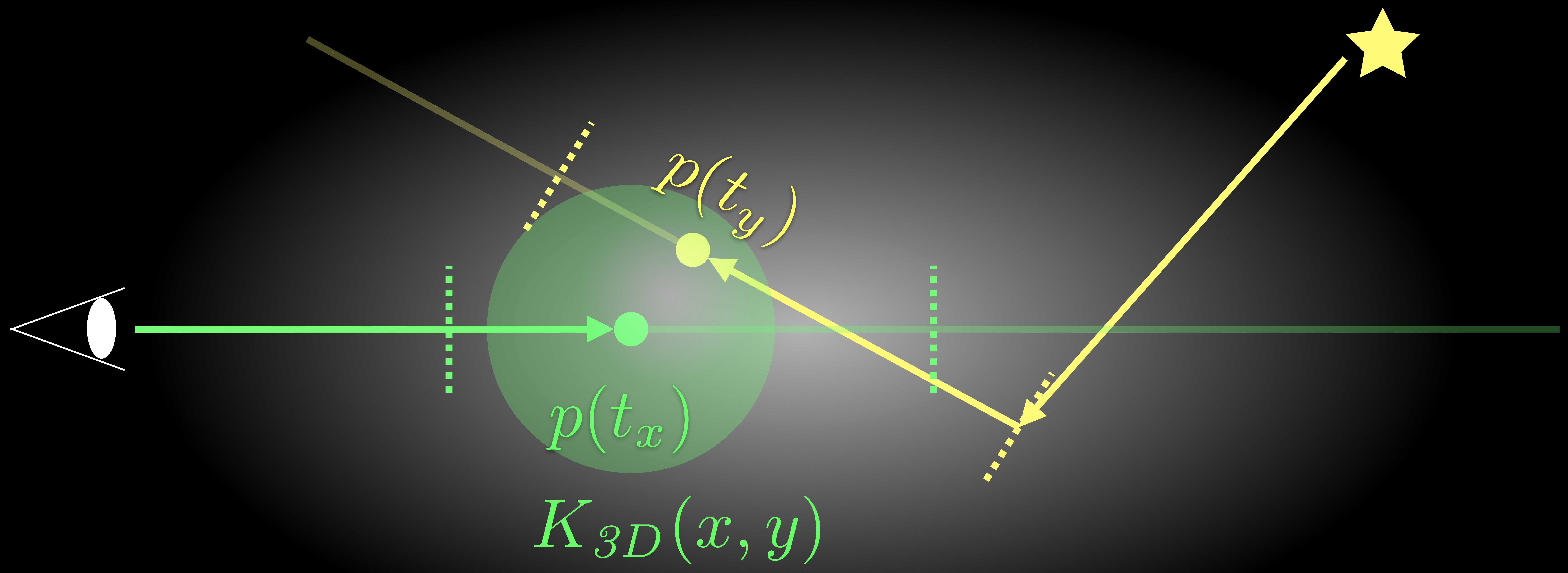
Beam-Beam 3D



Beam-Beam 3D

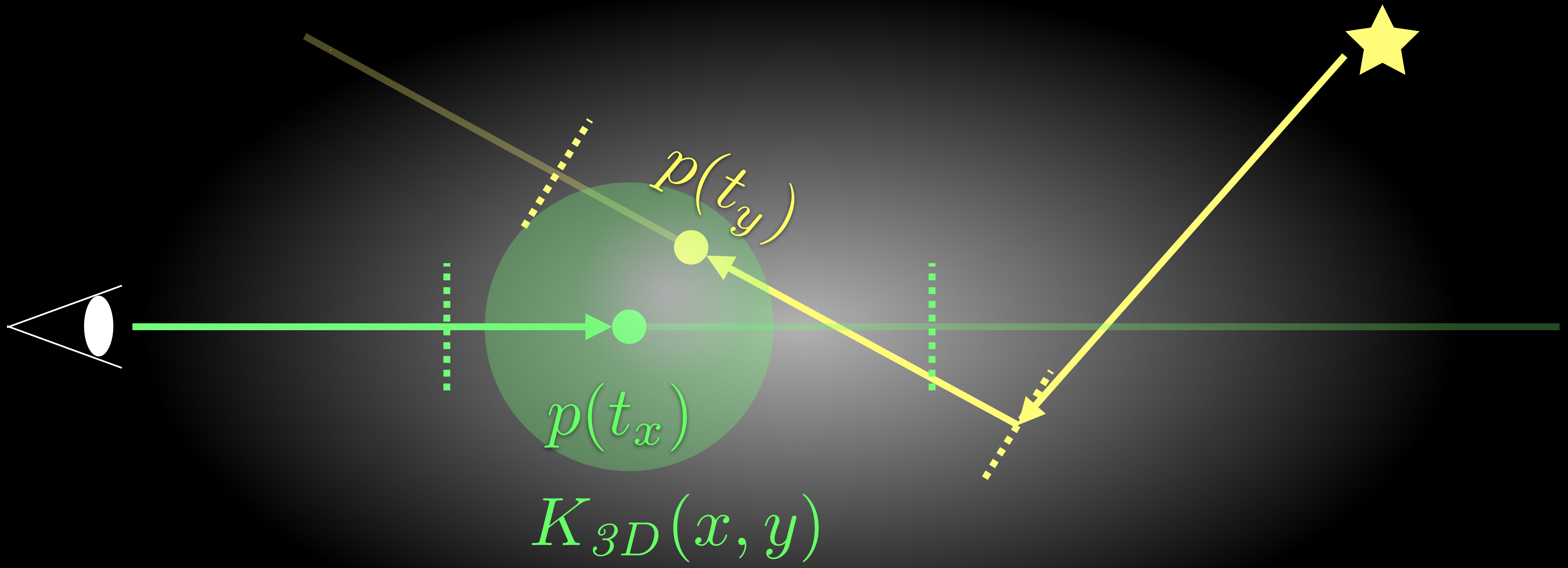


Beam-Beam 3D



Same 3D kernel as point-point 3D

Beam-Beam 3D

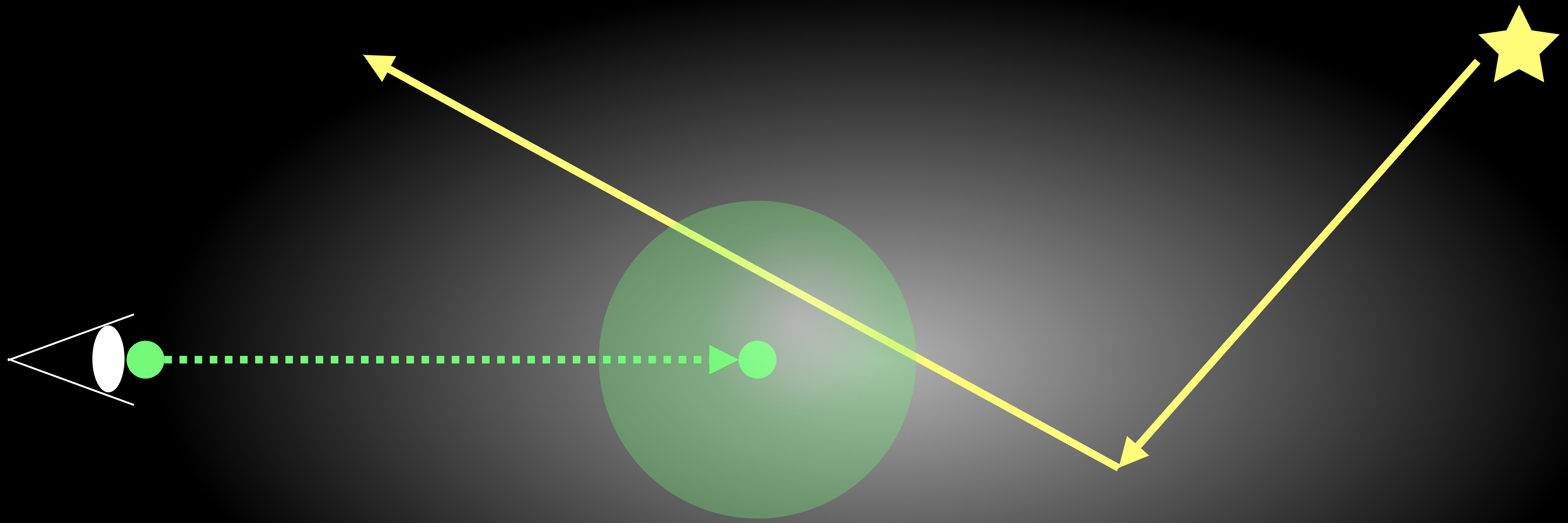


Simple Monte Carlo path sampling (no longer intractable)

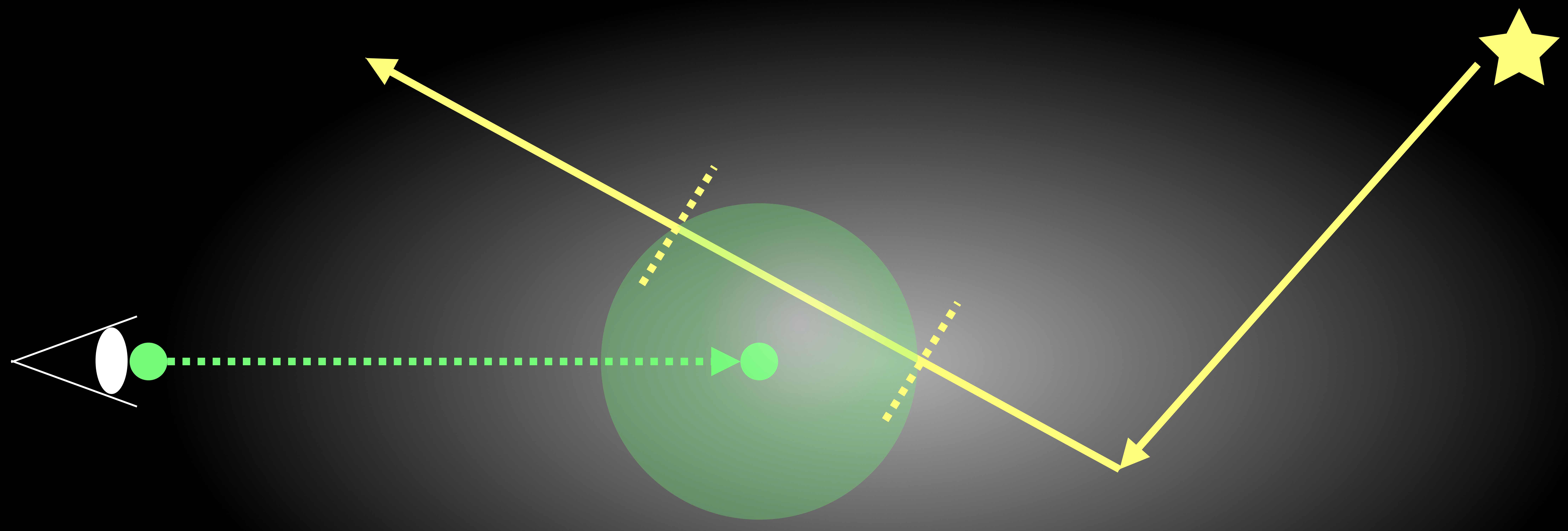
Beam-Beam 3D

Courtesy of Adrien Gruson

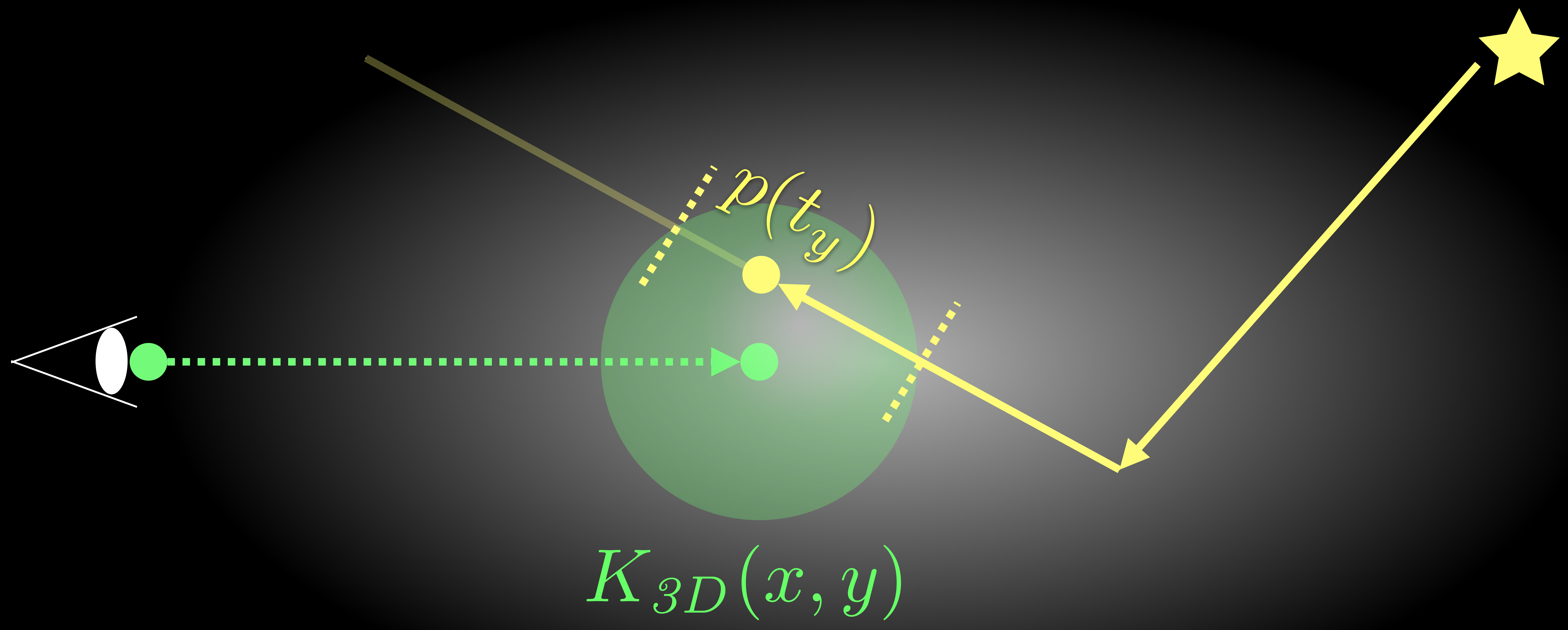
Beam-Point 3D



Beam-Point 3D

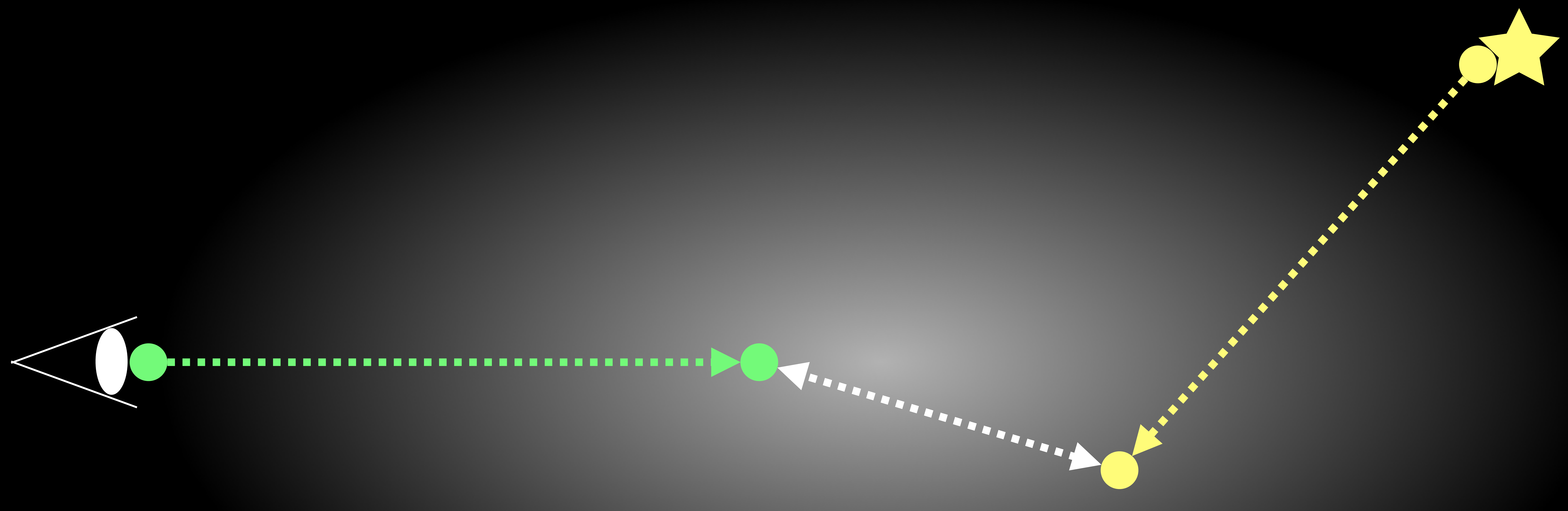


Beam-Point 3D



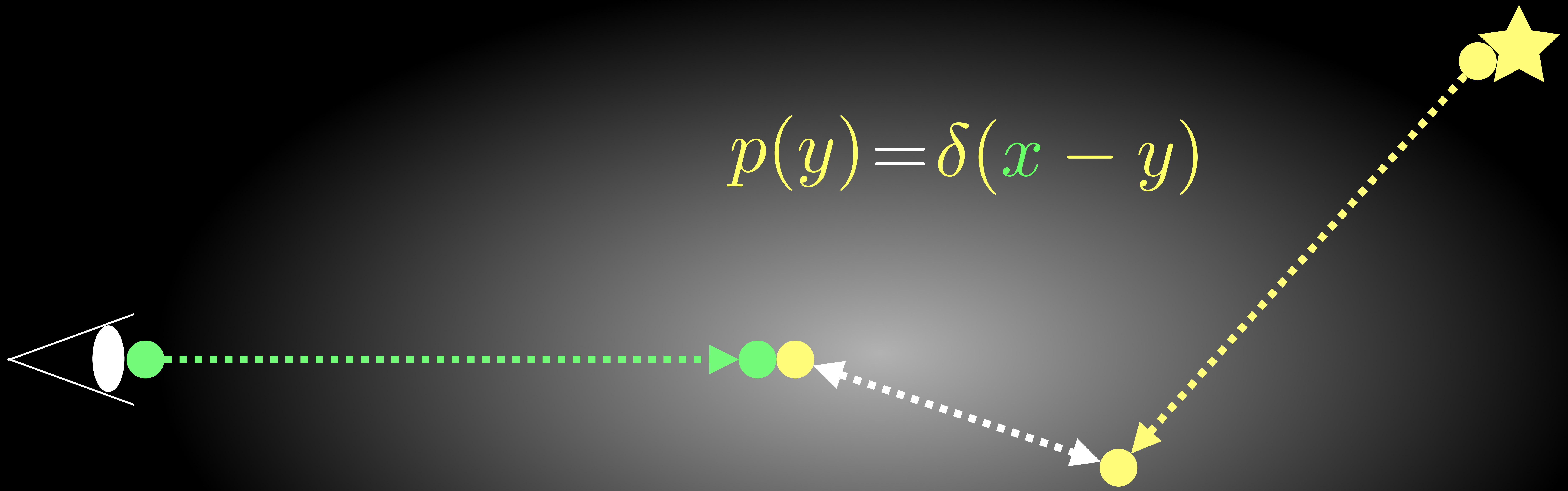
Same 3D kernel as point-point 3D

Bidirectional path tracing



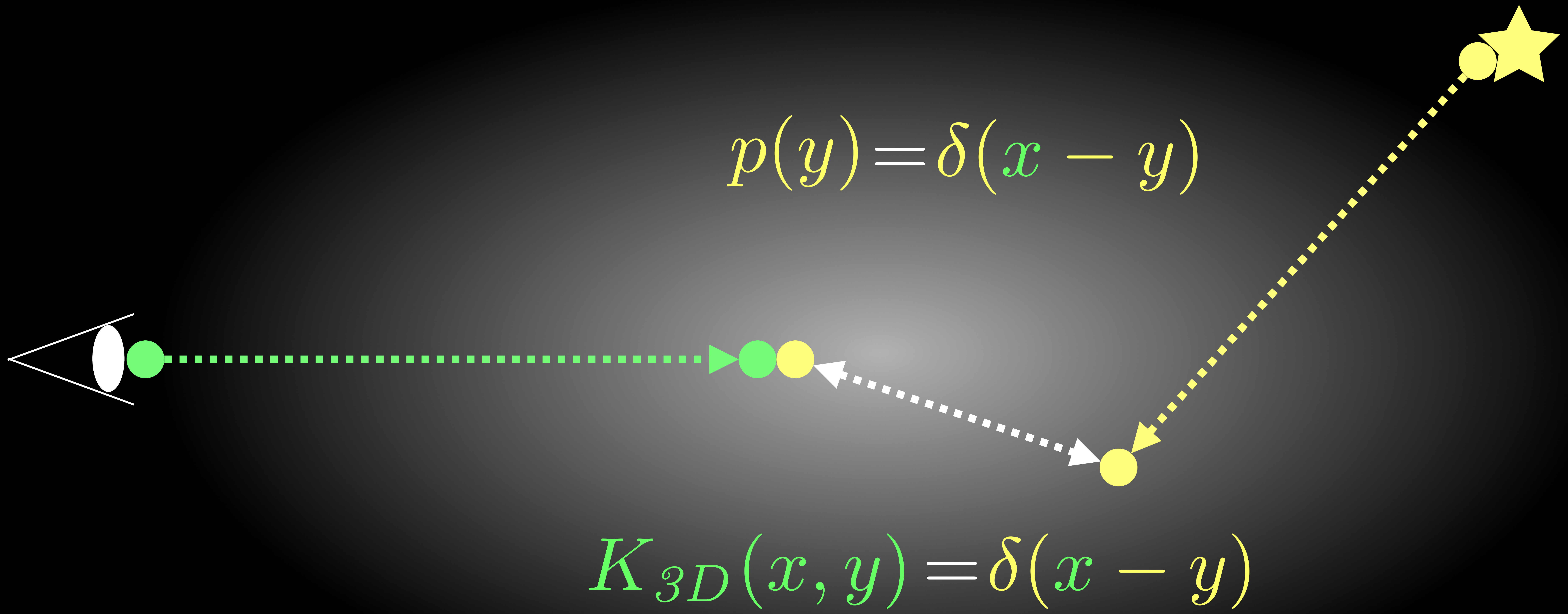
Bidirectional path tracing

$$p(y) = \delta(x - y)$$



Duplicate a vertex

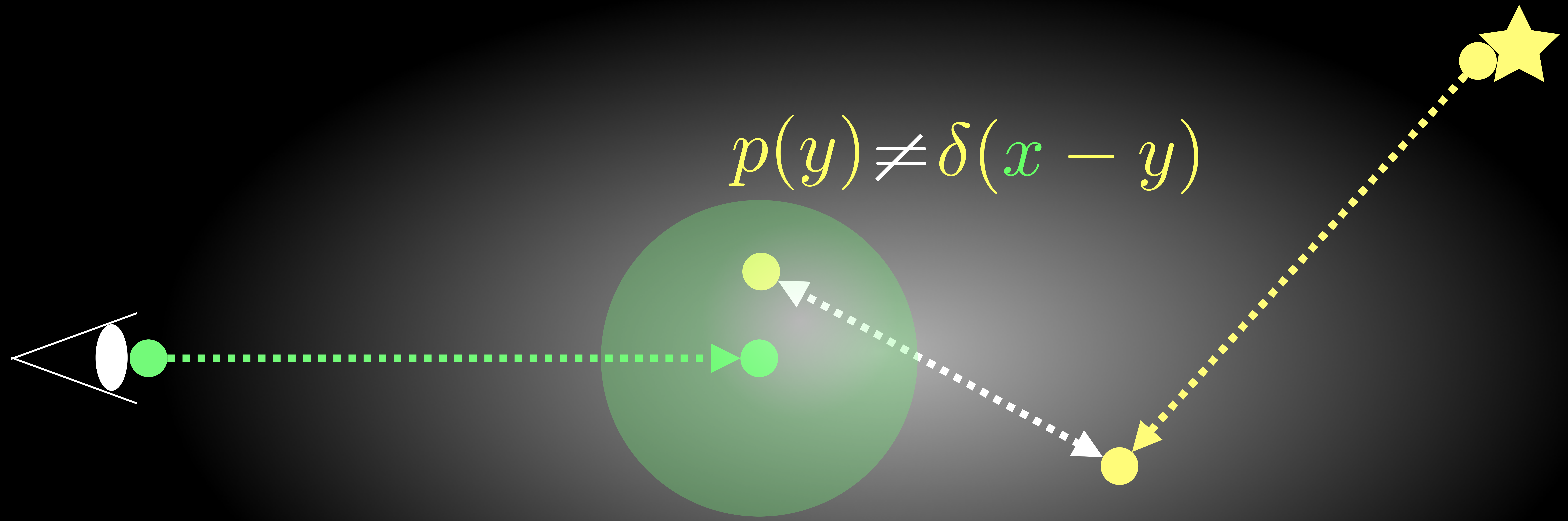
Bidirectional path tracing



Delta kernel leads to the original path integral formulation

Biased bidirectional path tracing

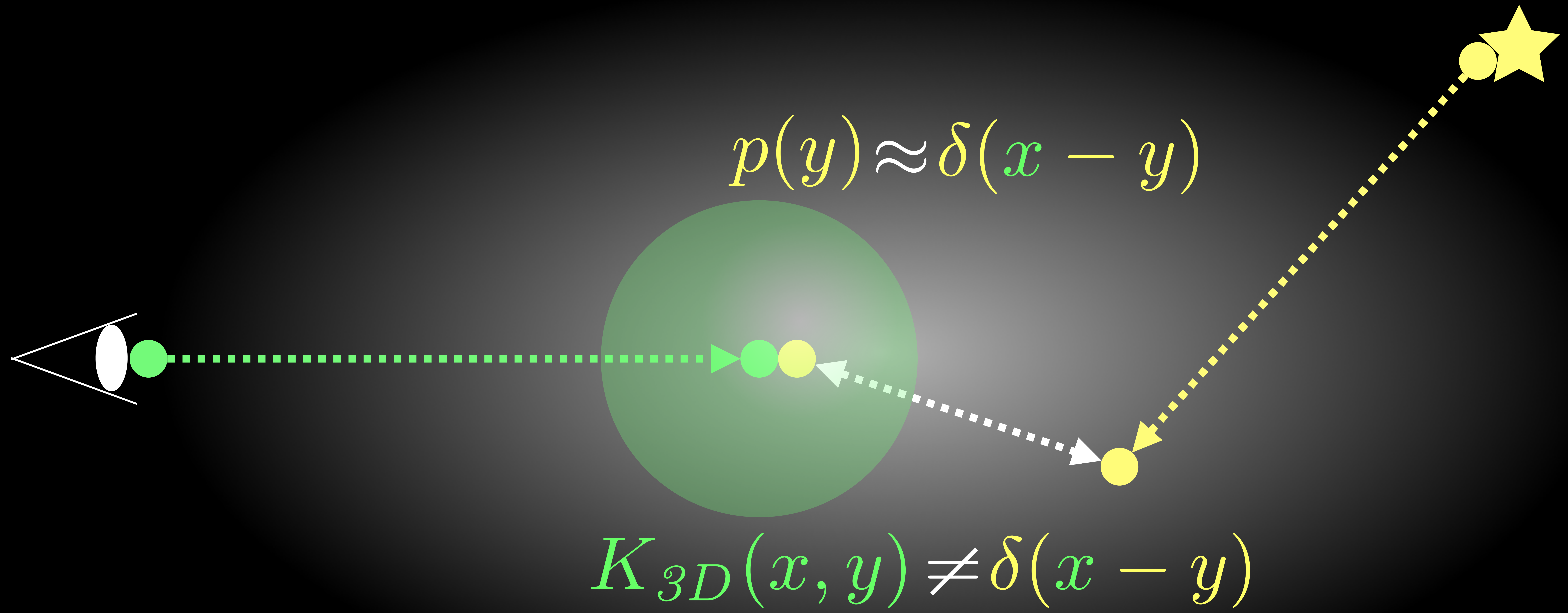
$$p(y) \neq \delta(x - y)$$



$$K_{3D}(x, y) \neq \delta(x - y)$$

Take disconnected vertices via blurring kernel

Virtual perturbation



Approximate the implementation of biased BDPT by regular BDPT

Conclusion

- Extension of the path space for volumetric light transport
  - Better explains density estimation compared to merging
  - Formulate beam as Monte Carlo distance sampling
  - Enables a practical beam-beam 3D estimator

Fills a theoretical gap in the unified formulation for volumes