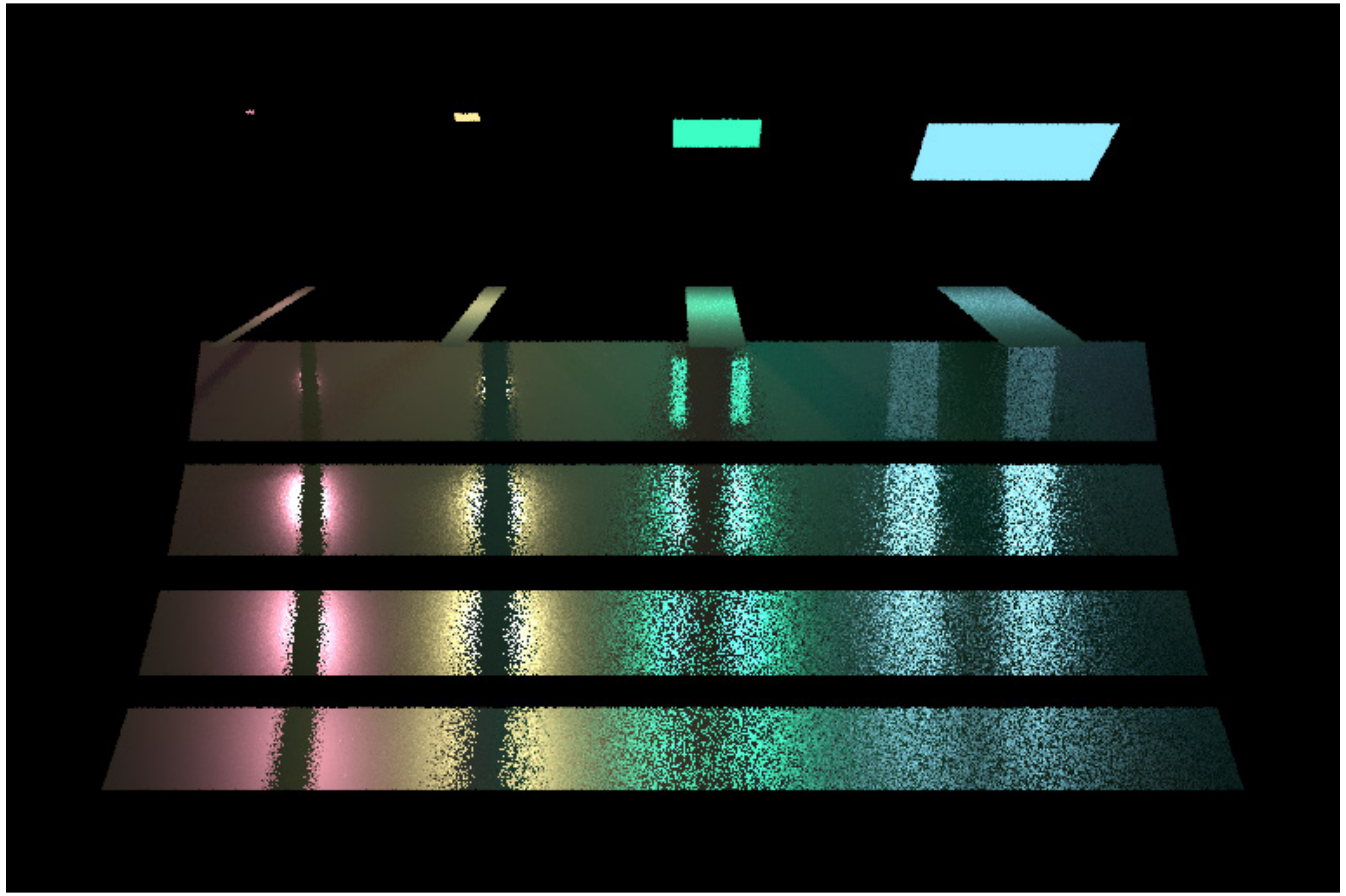
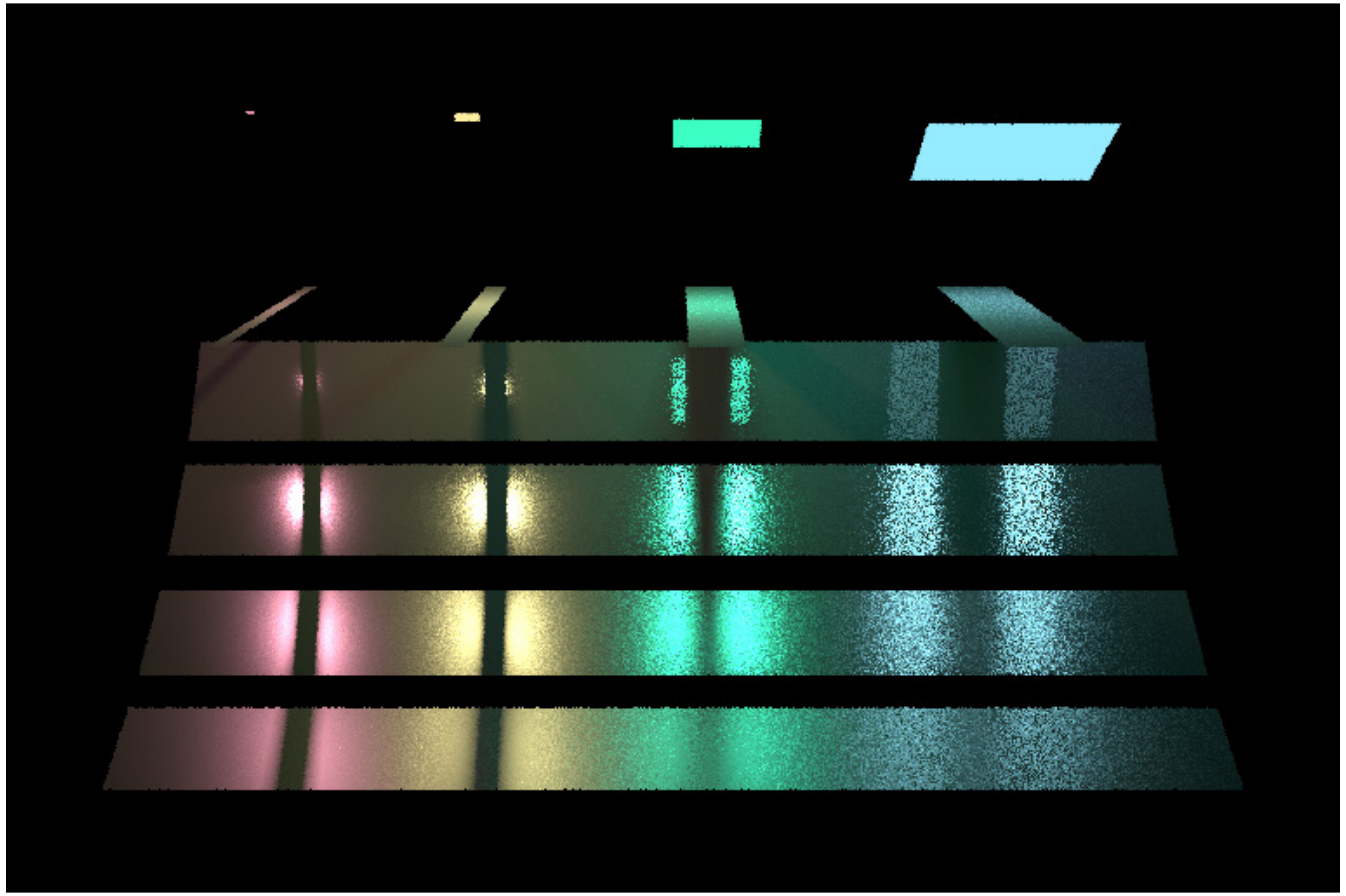
Combining Point and Line Samples for Direct Illumination

Points only



Points + Lines



Katherine Salesin

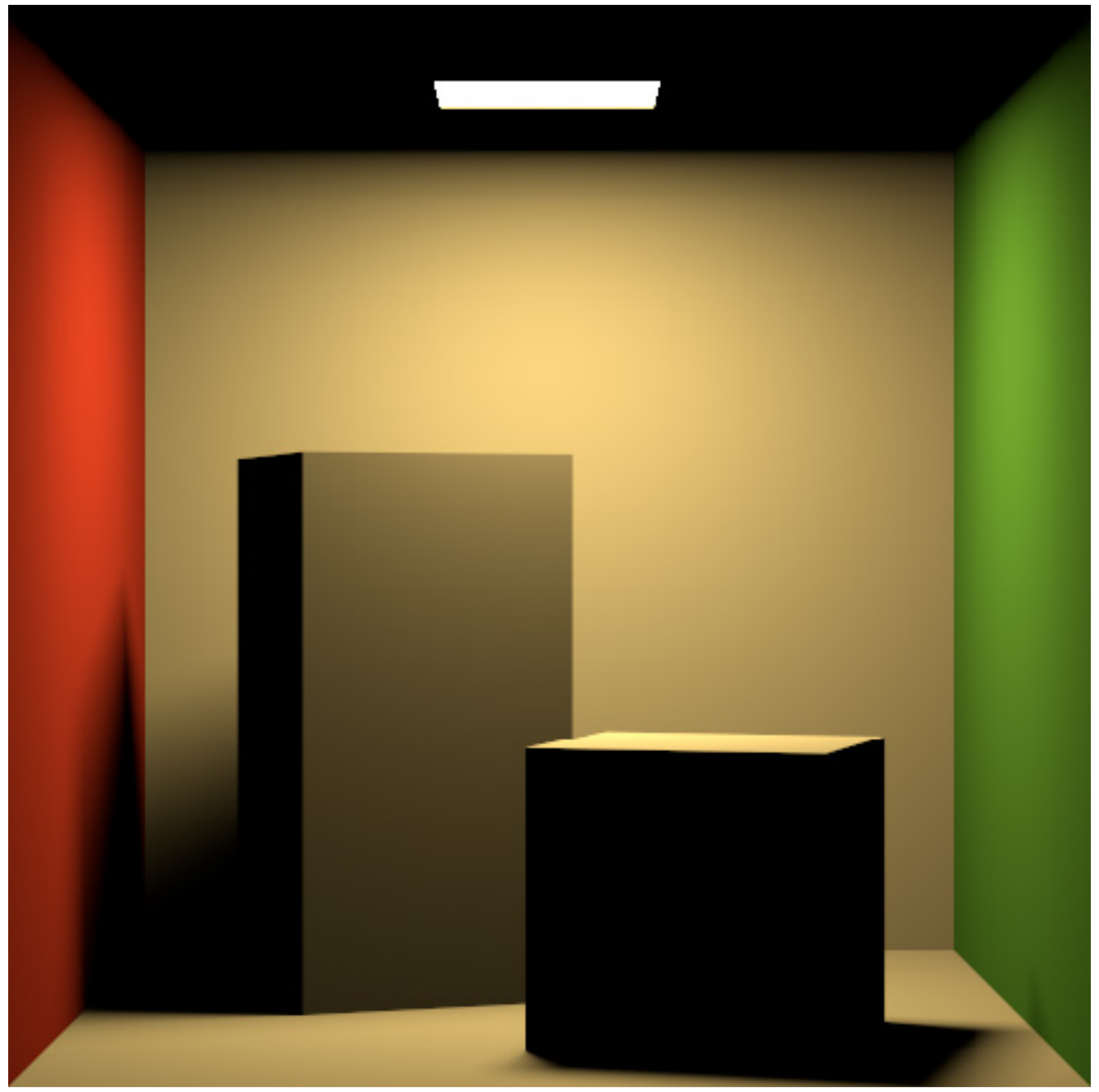
Wojciech Jarosz

DARTMOUTH
VISUAL COMPUTING LAB

Motivation

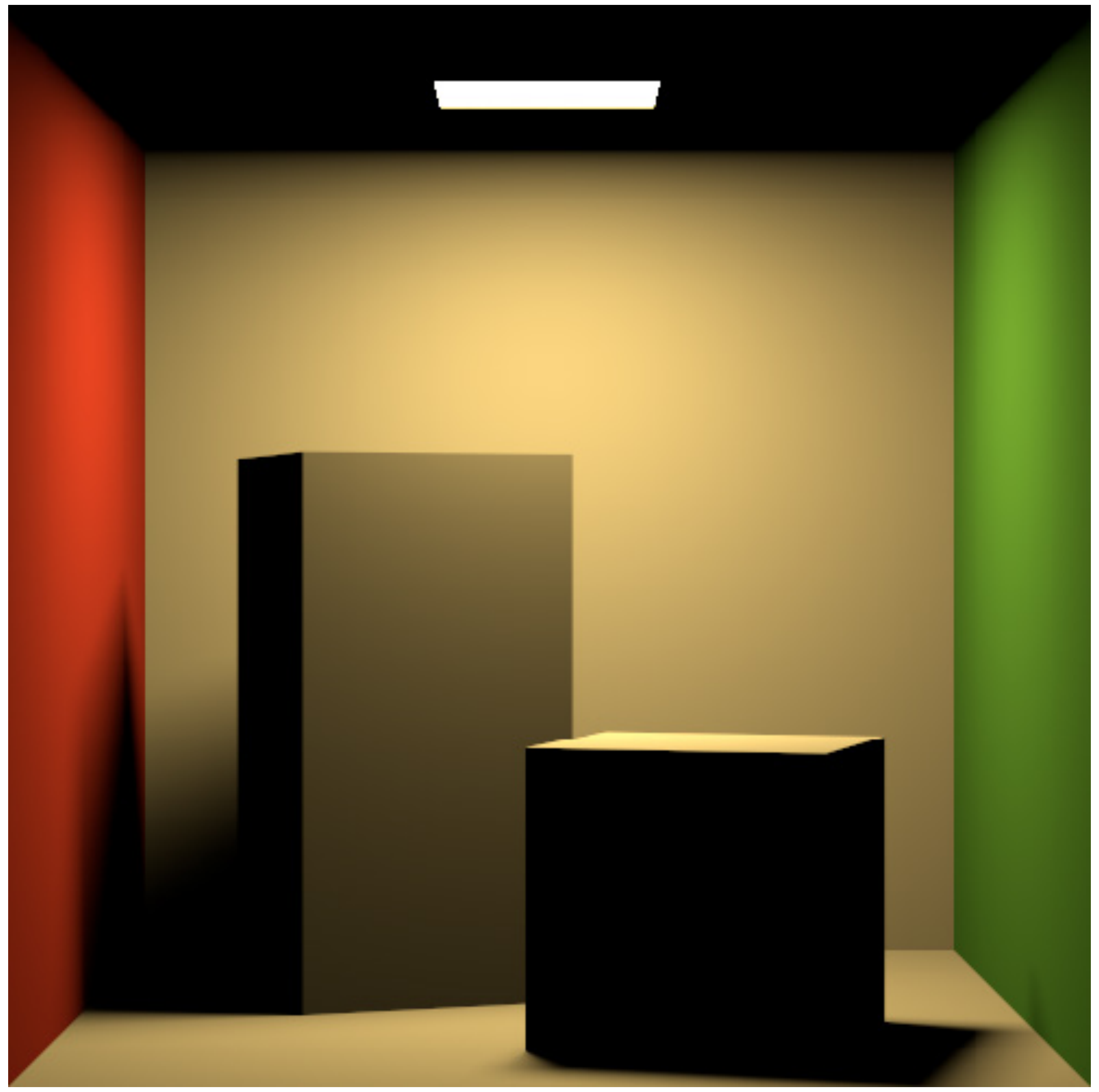
Motivation

Motivation



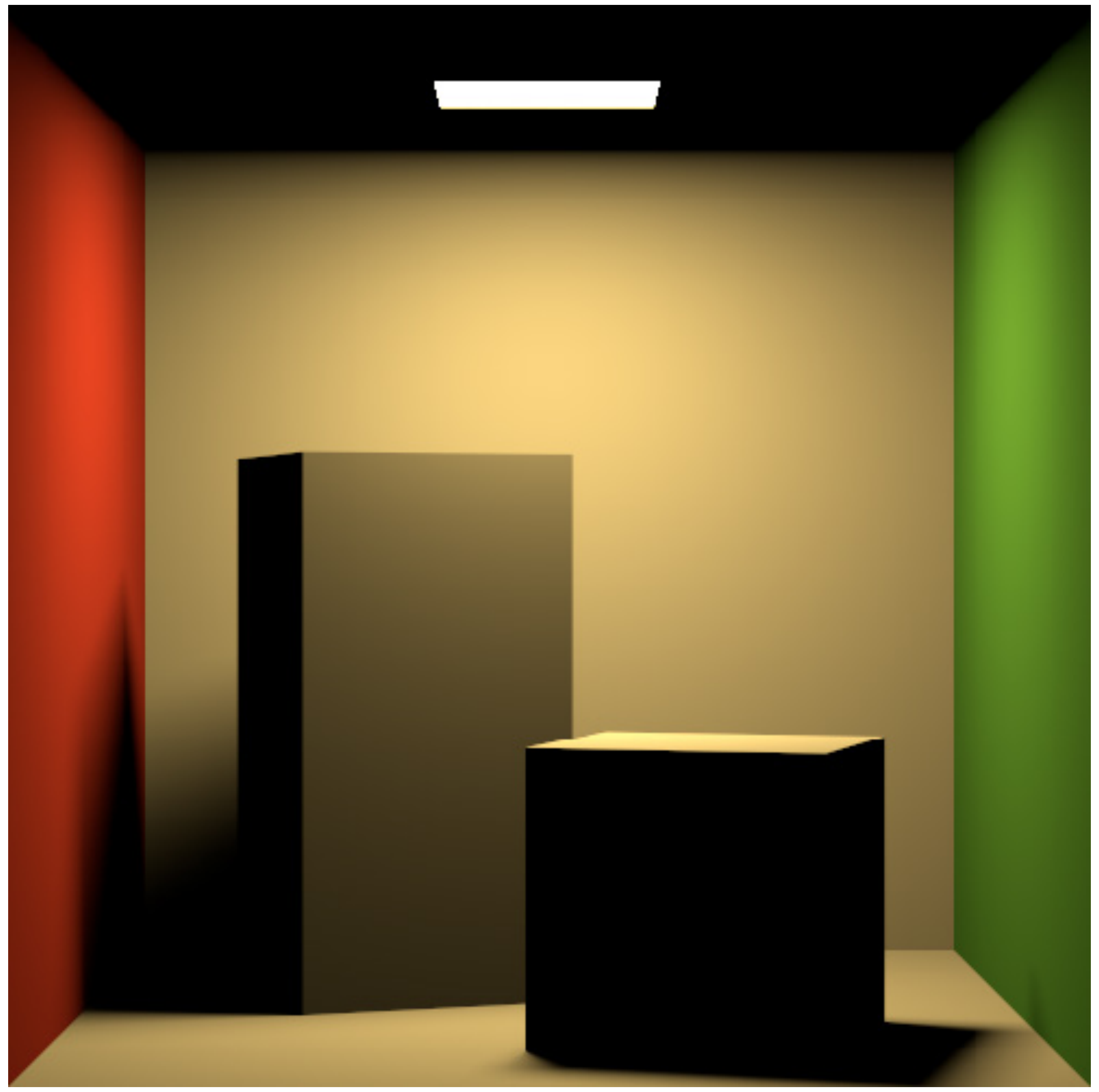
Direct lighting:

Motivation

Direct lighting: **point sampling**

Motivation


Direct lighting: **point sampling** and **line sampling**

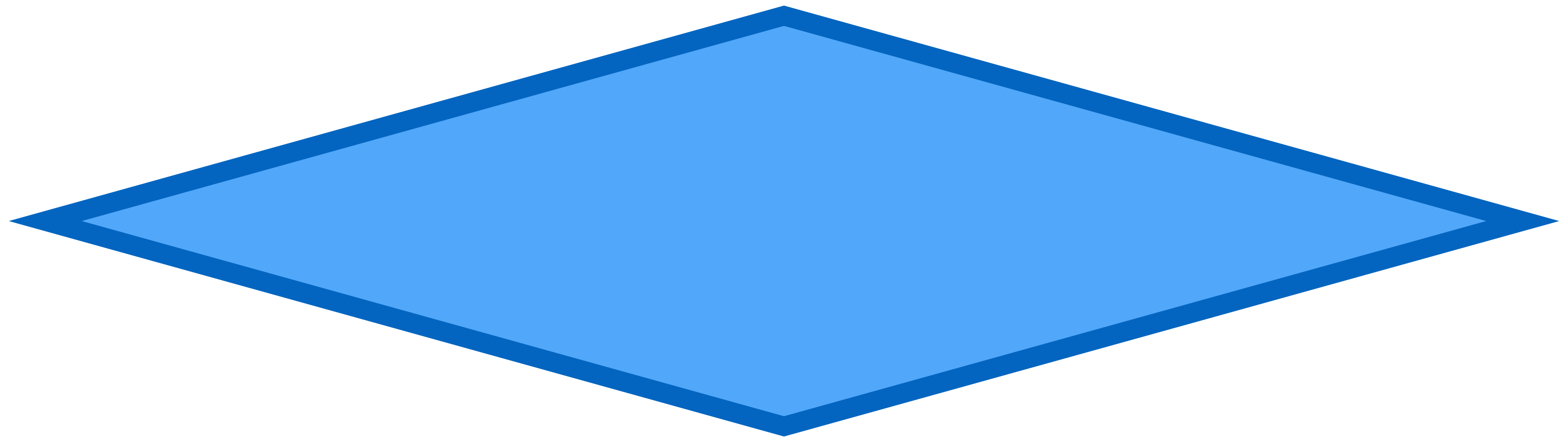
Theory:
Direct lighting + Monte Carlo sampling

Theory

Direct lighting

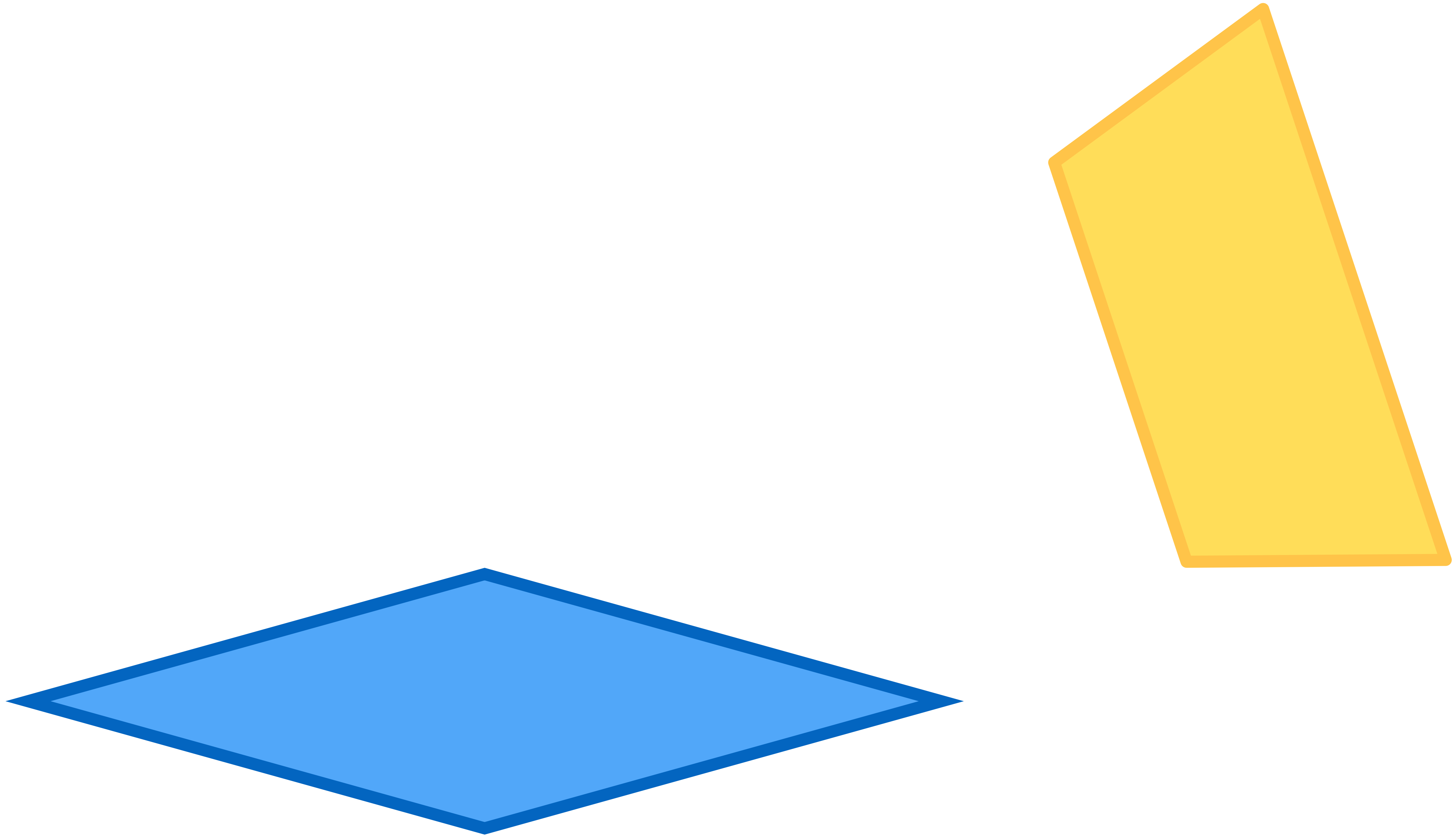
Theory

Direct lighting



Theory

Direct lighting



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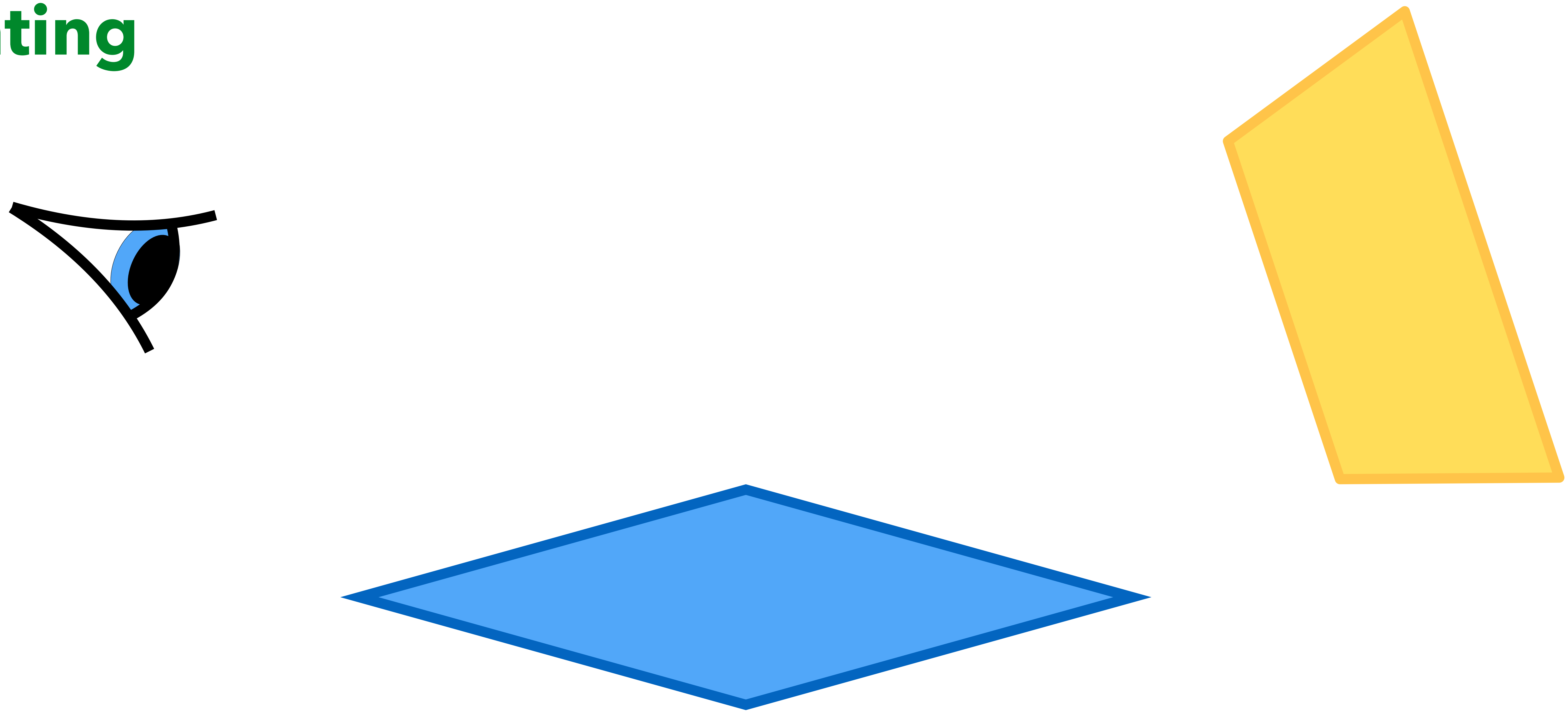

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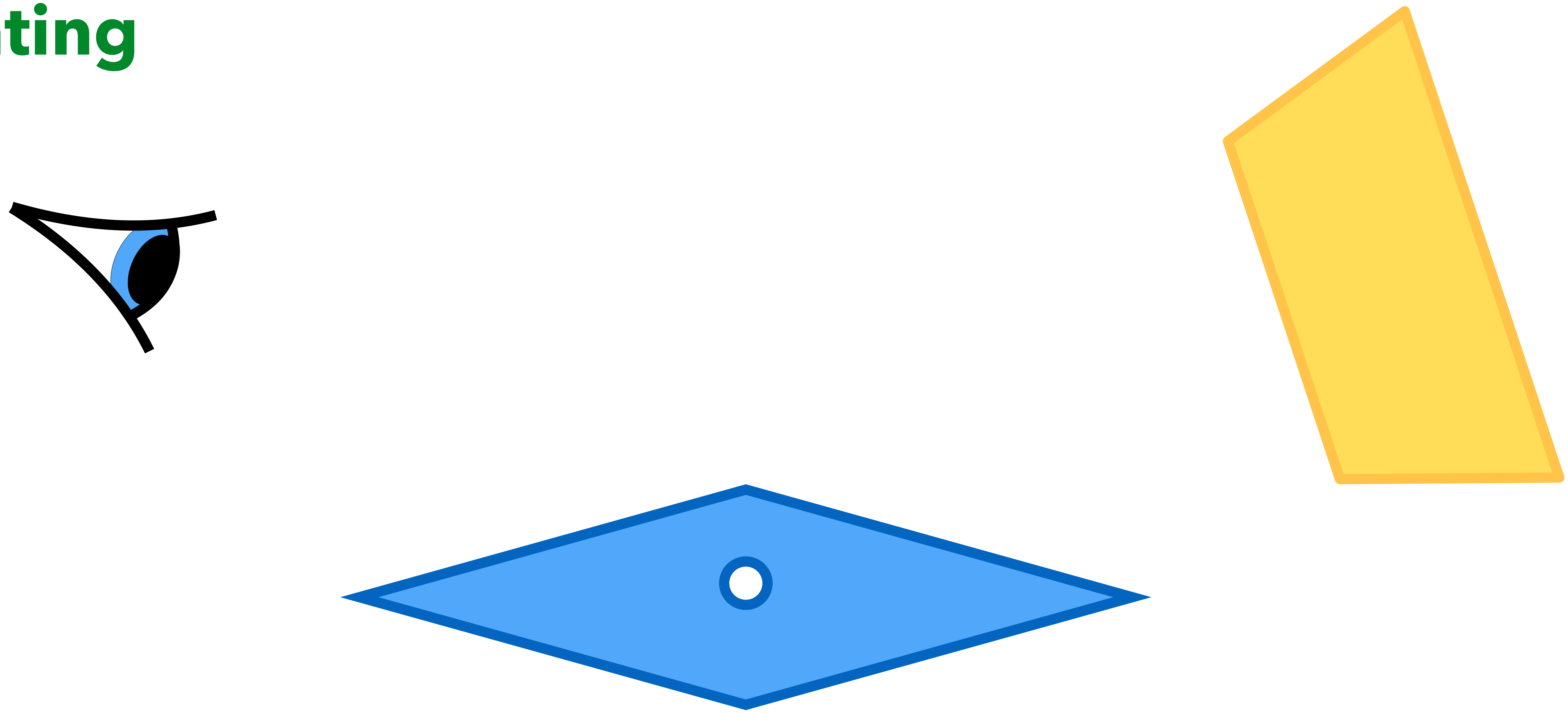

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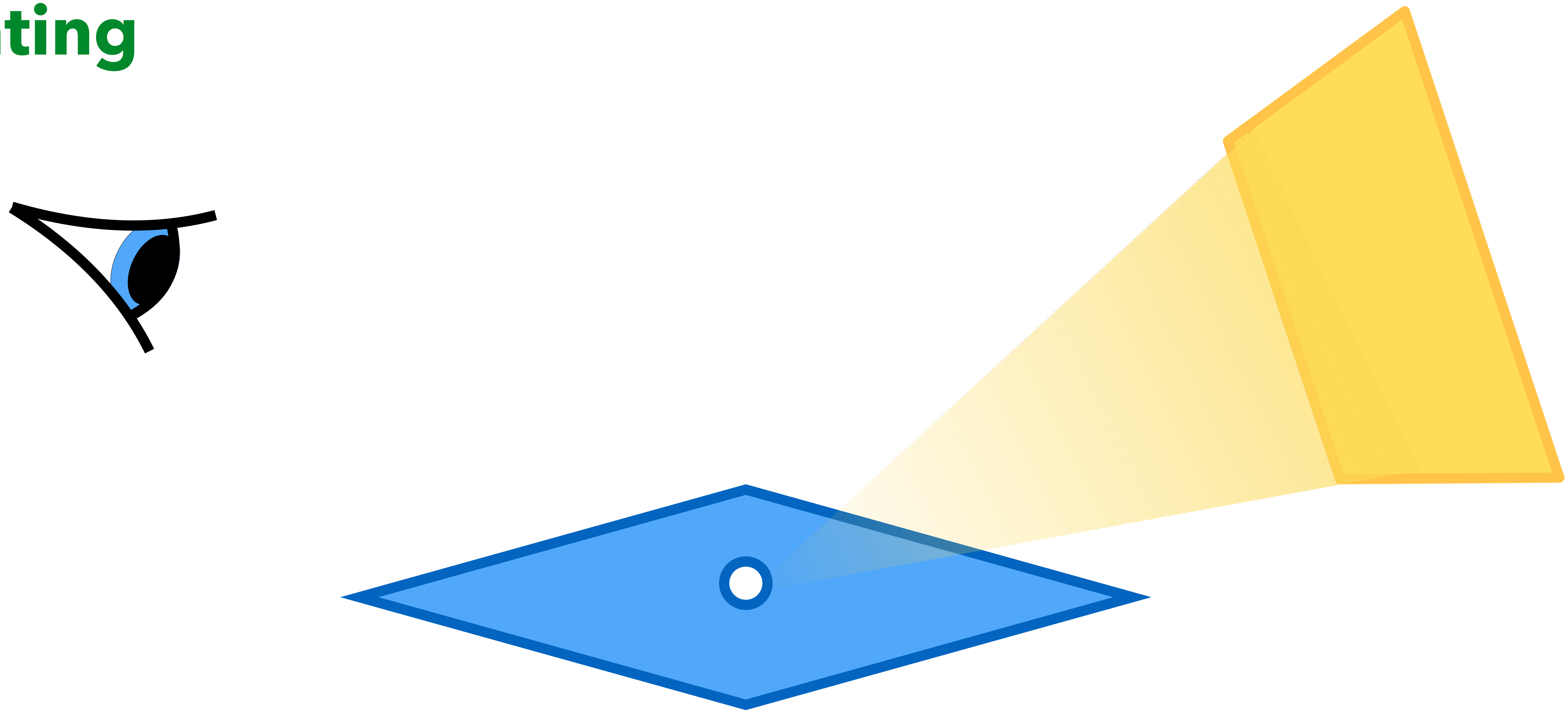

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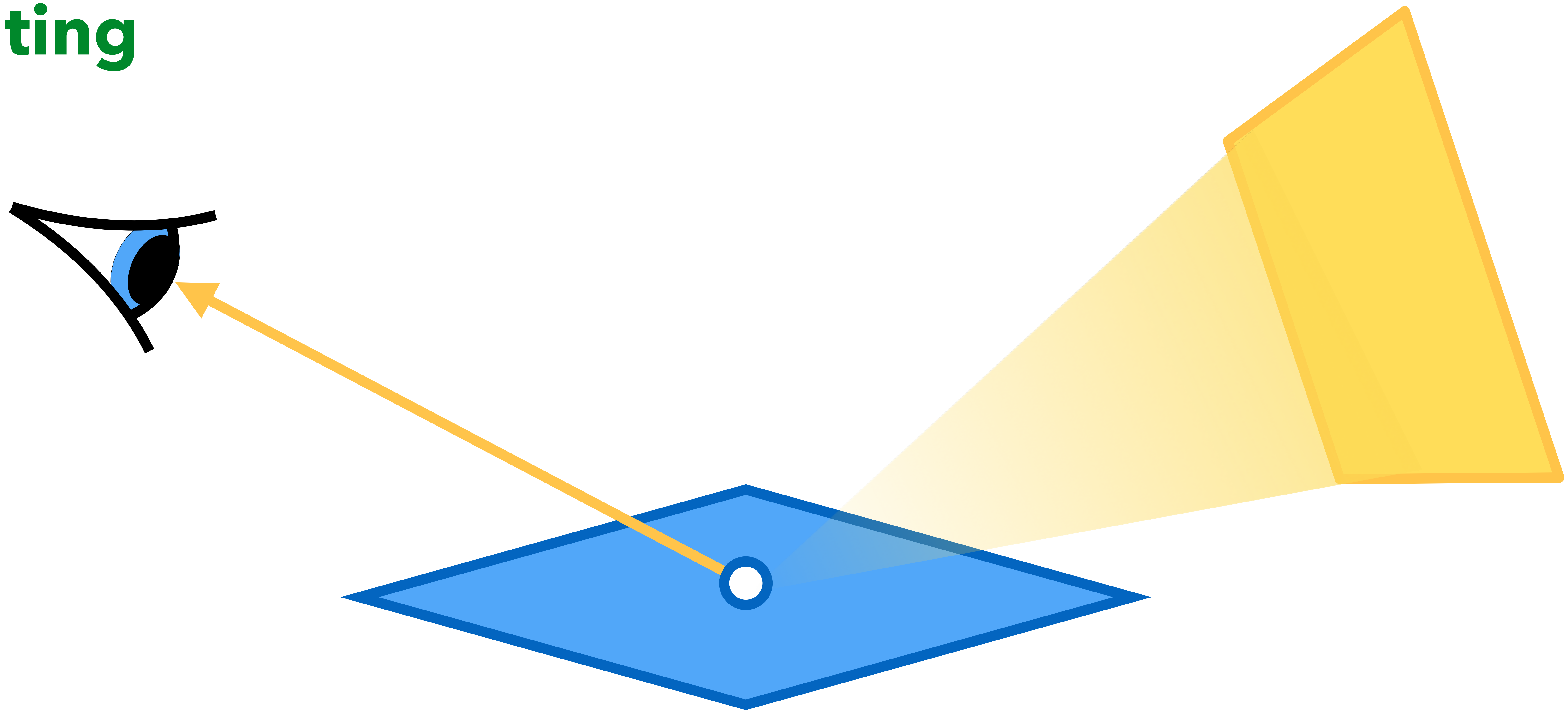

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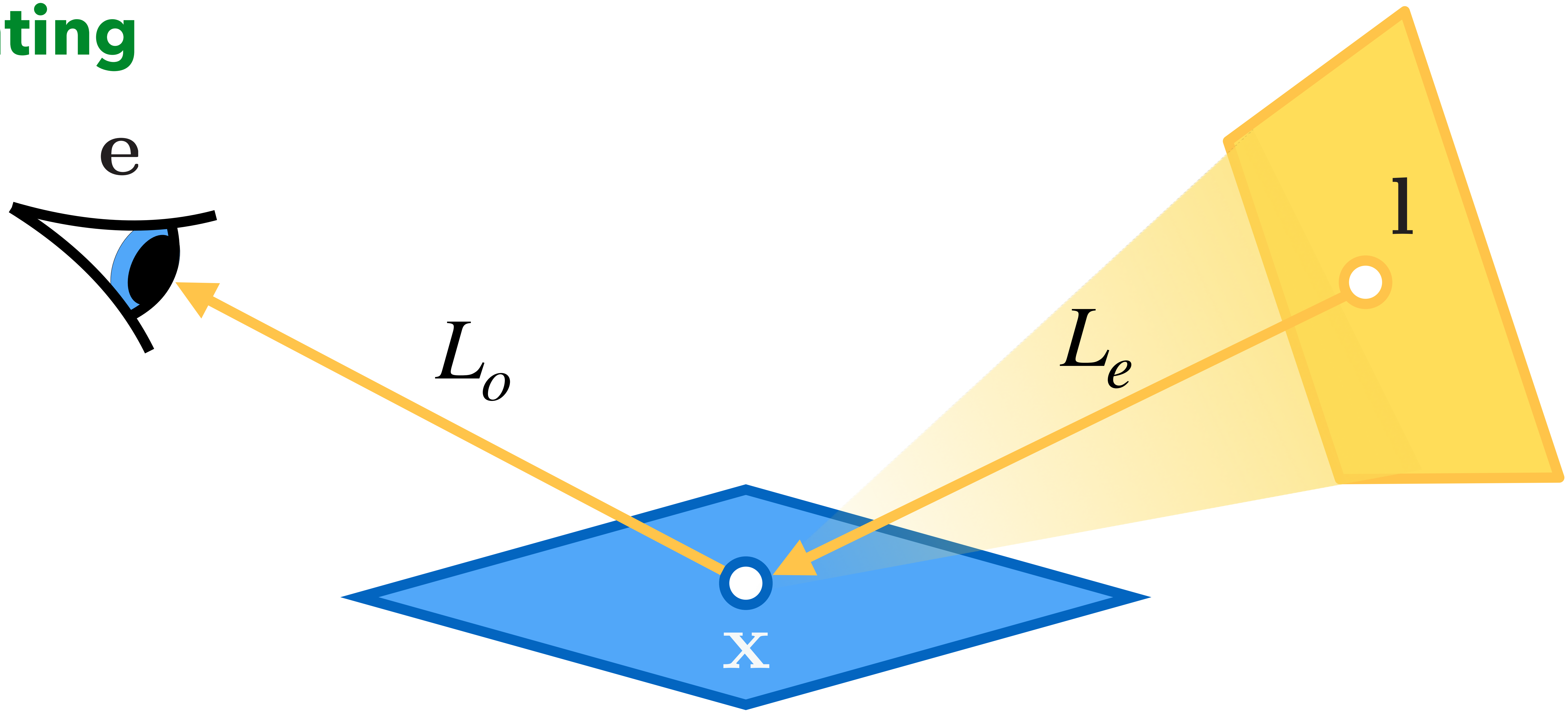
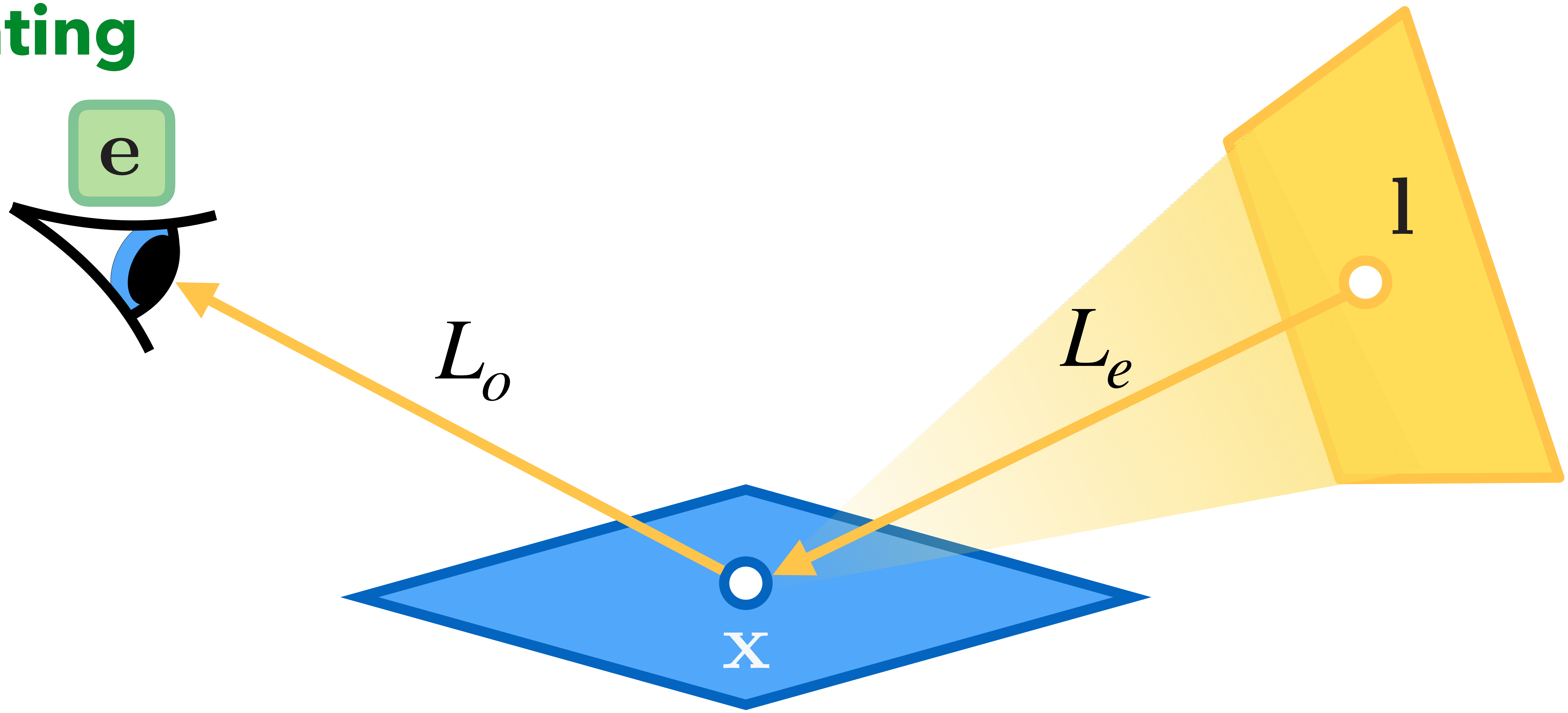


$$L_o(\mathbf{e},\mathbf{x}) = \int_{\mathcal{A}} f_r(\mathbf{e},\mathbf{x},\mathbf{l})G(\mathbf{x},\mathbf{l})V(\mathbf{x},\mathbf{l})L_e(\mathbf{x},\mathbf{l})\,\mathrm{d}A(\mathbf{l})$$

Theory

Direct lighting



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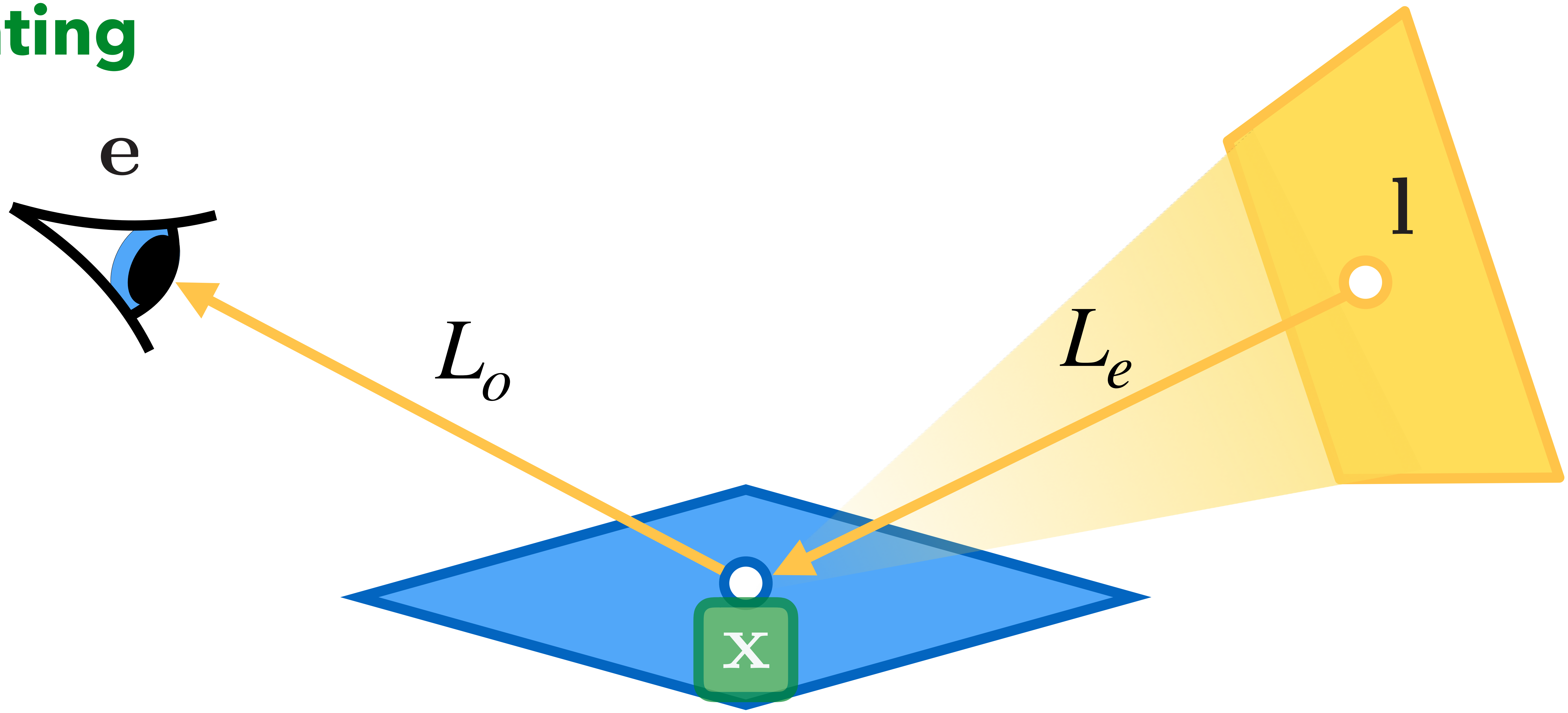

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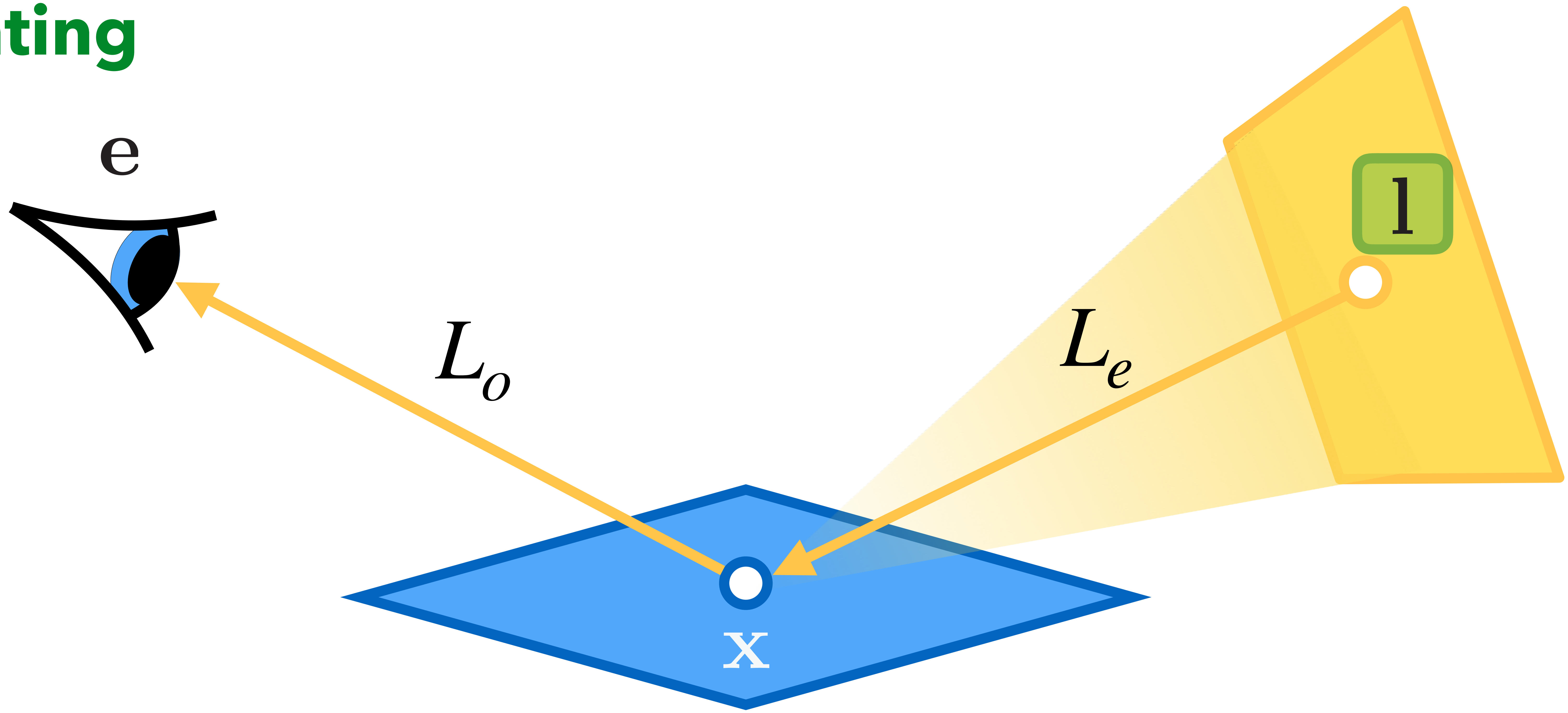

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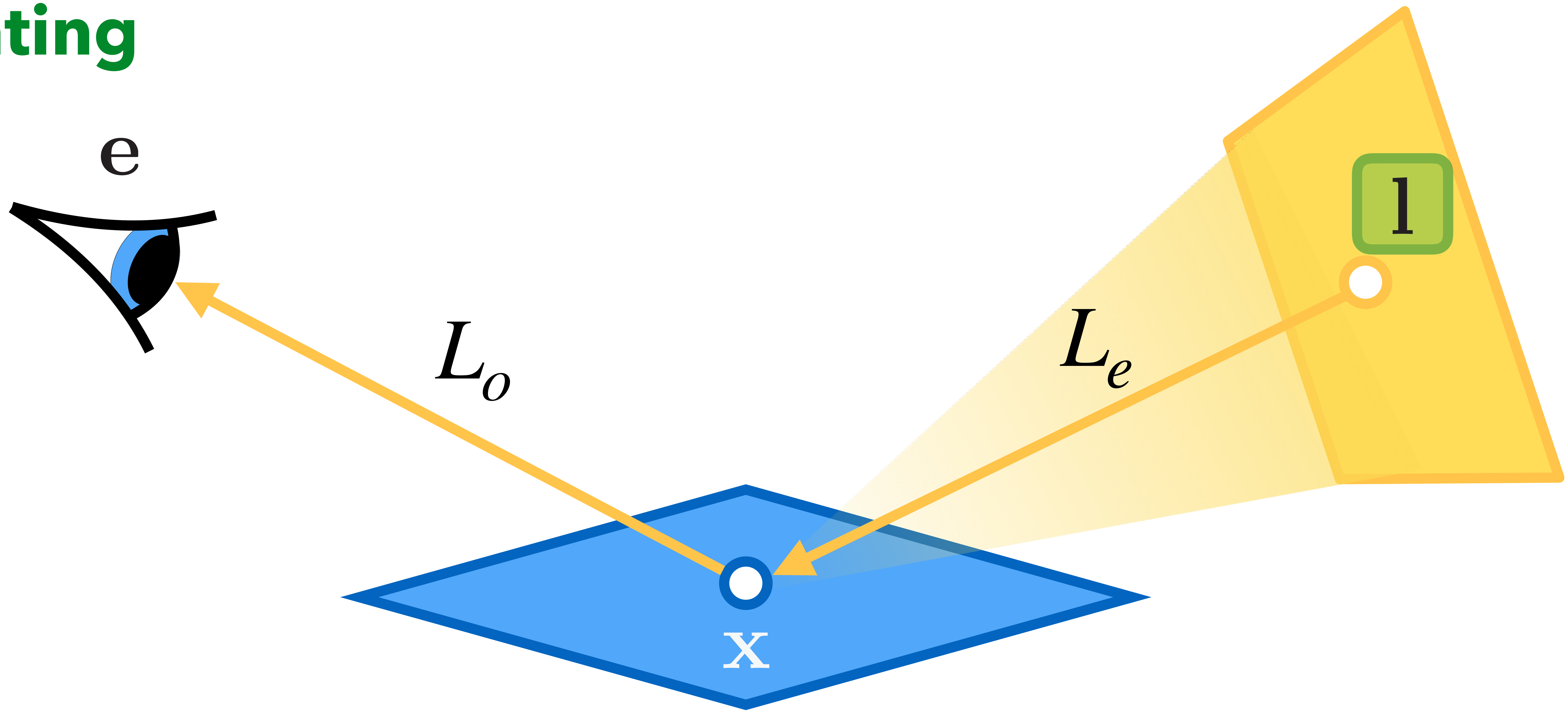

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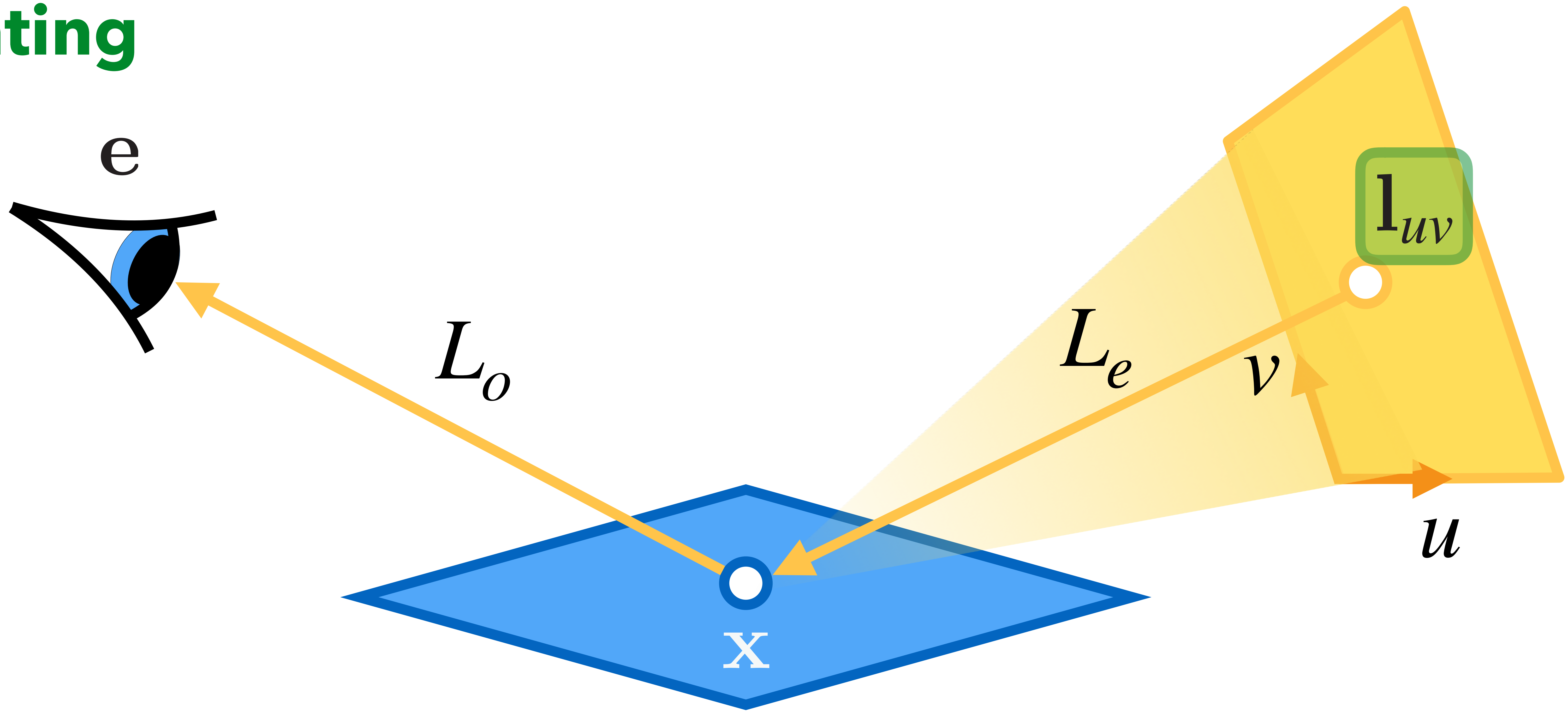

$$L_o(\mathbf{e},\mathbf{x}) = \boxed{\int_{\mathcal{U}}\int_{\mathcal{V}}} f_r(\mathbf{e},\mathbf{x},\mathbf{l}_{uv})G(\mathbf{x},\mathbf{l}_{uv})V(\mathbf{x},\mathbf{l}_{uv})L_e(\mathbf{x},\mathbf{l}_{uv})\,\boxed{\mathrm{d}v\,\mathrm{d}u}$$

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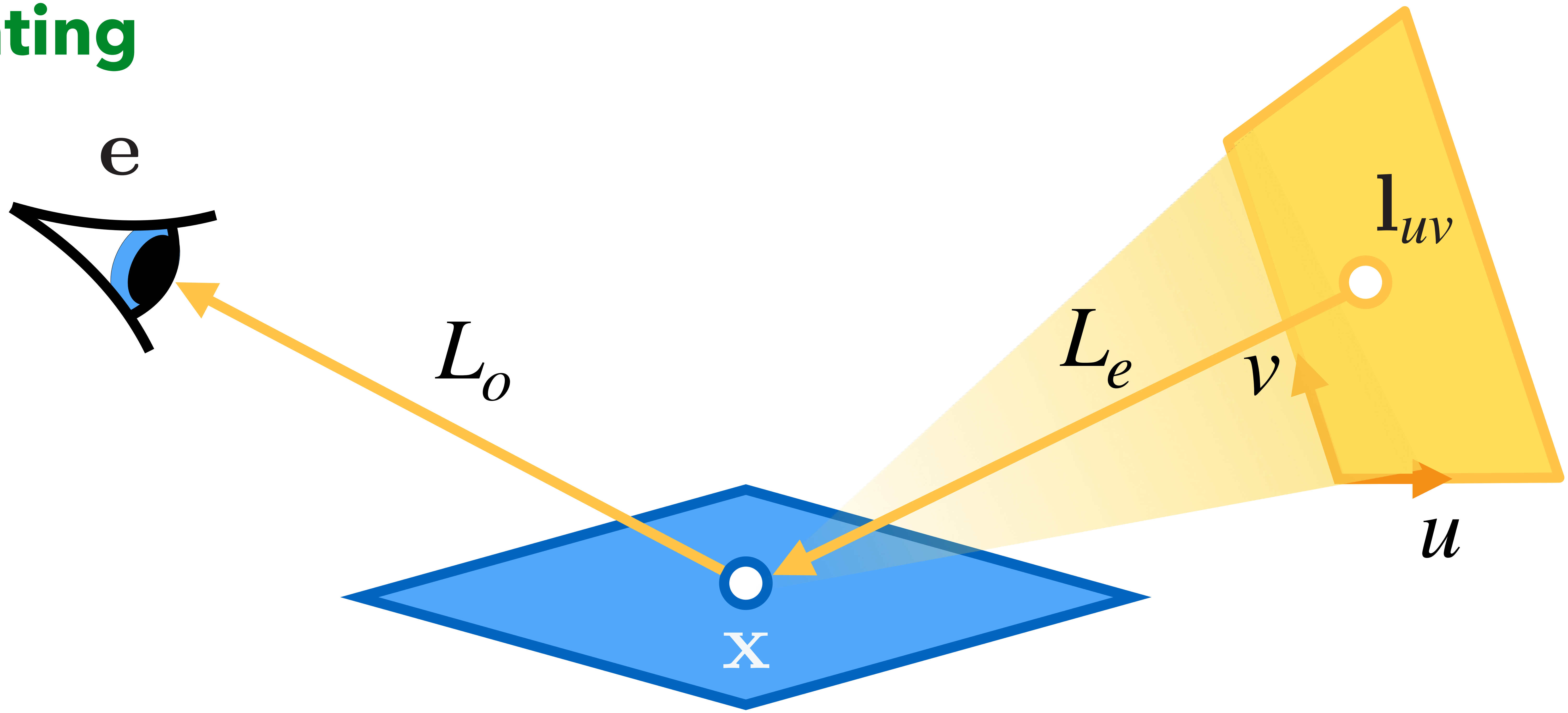

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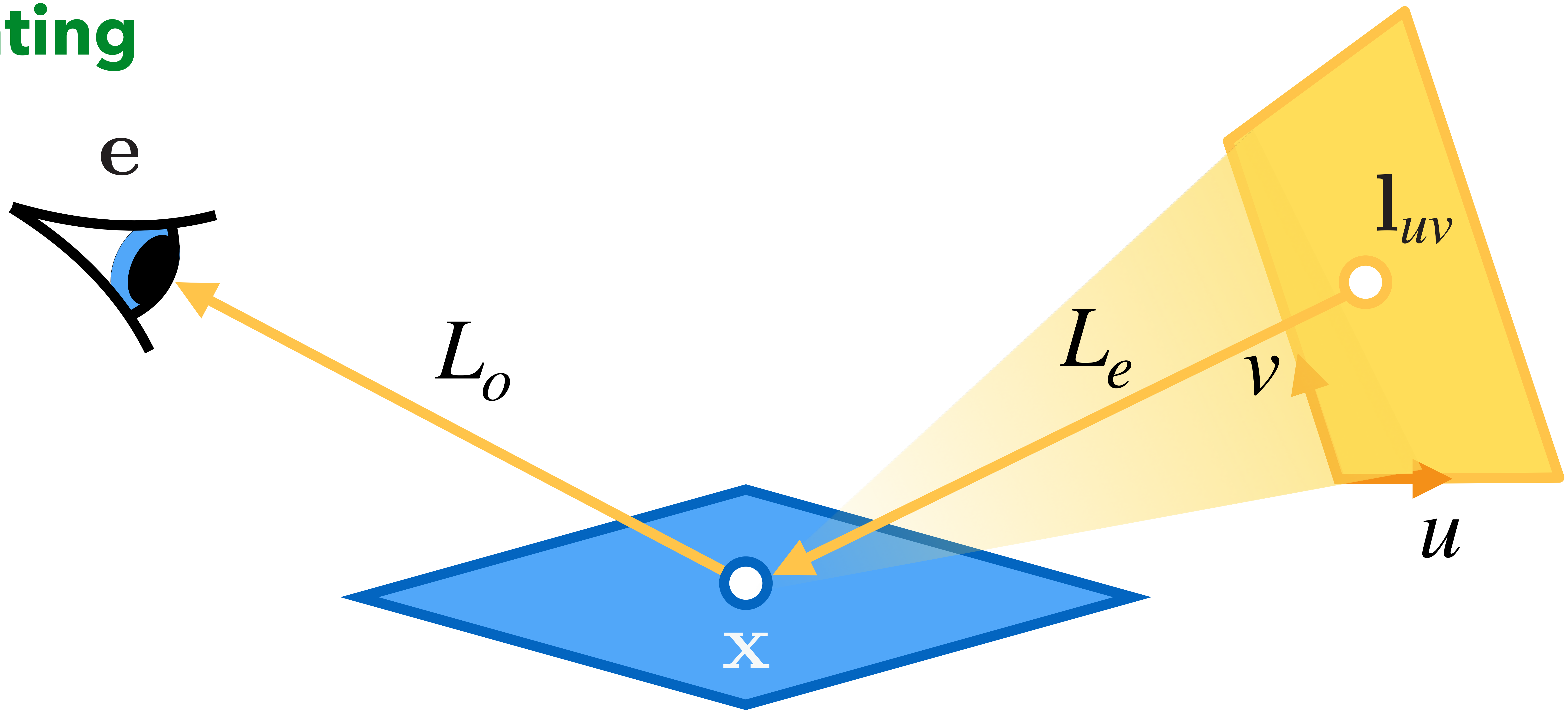

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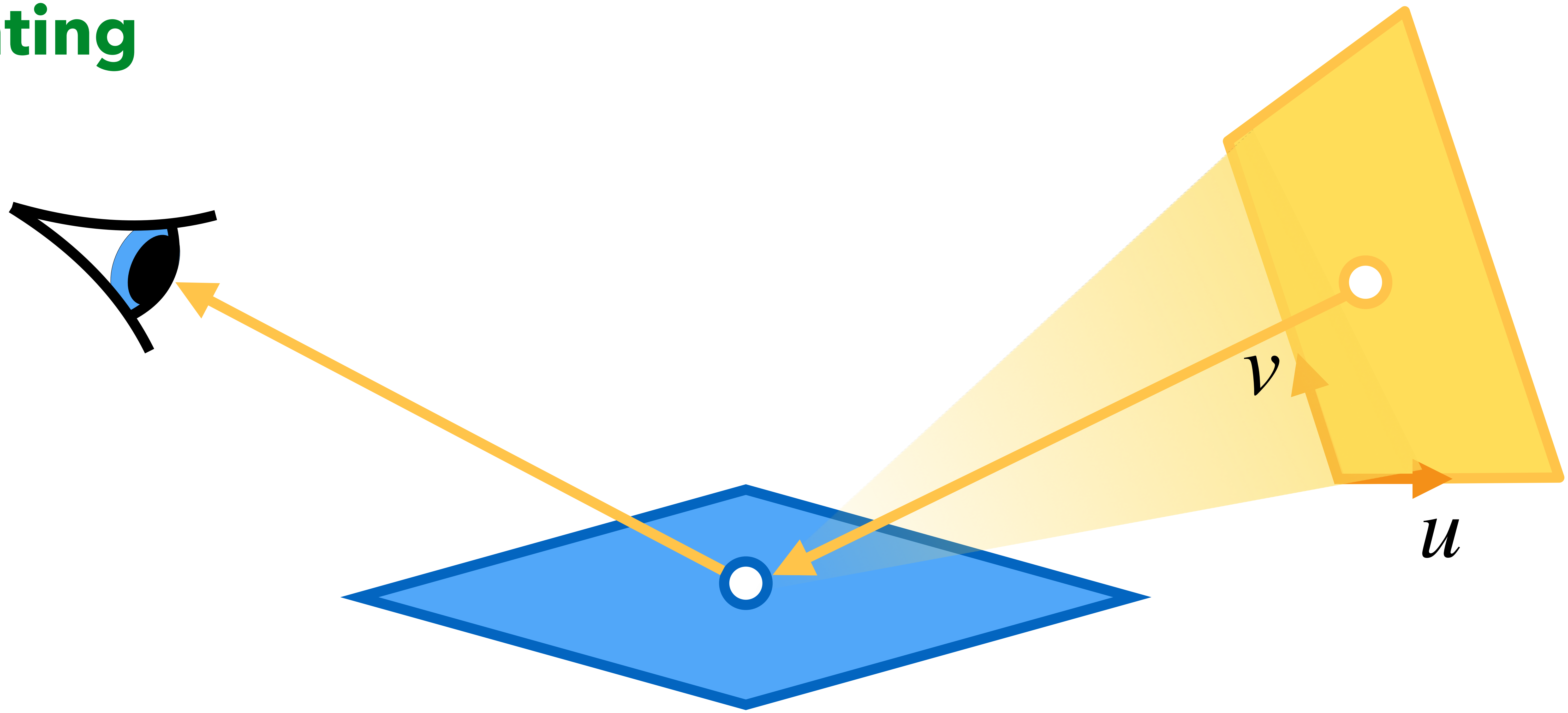

$$L_o = \int_{\mathcal{U}} \int_{\mathcal{V}} \boxed{f(u, v)}\, \mathrm{d}v\, \mathrm{d}u$$

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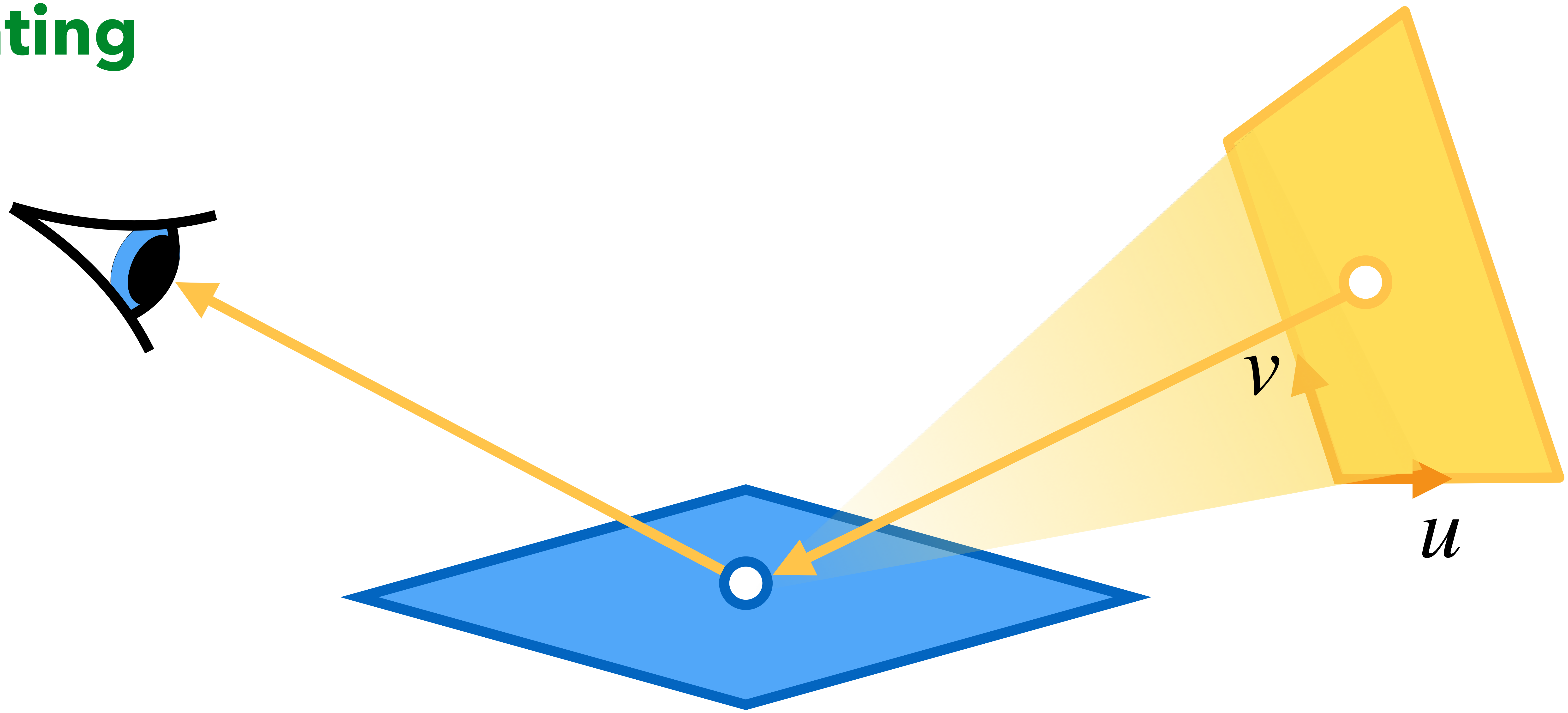

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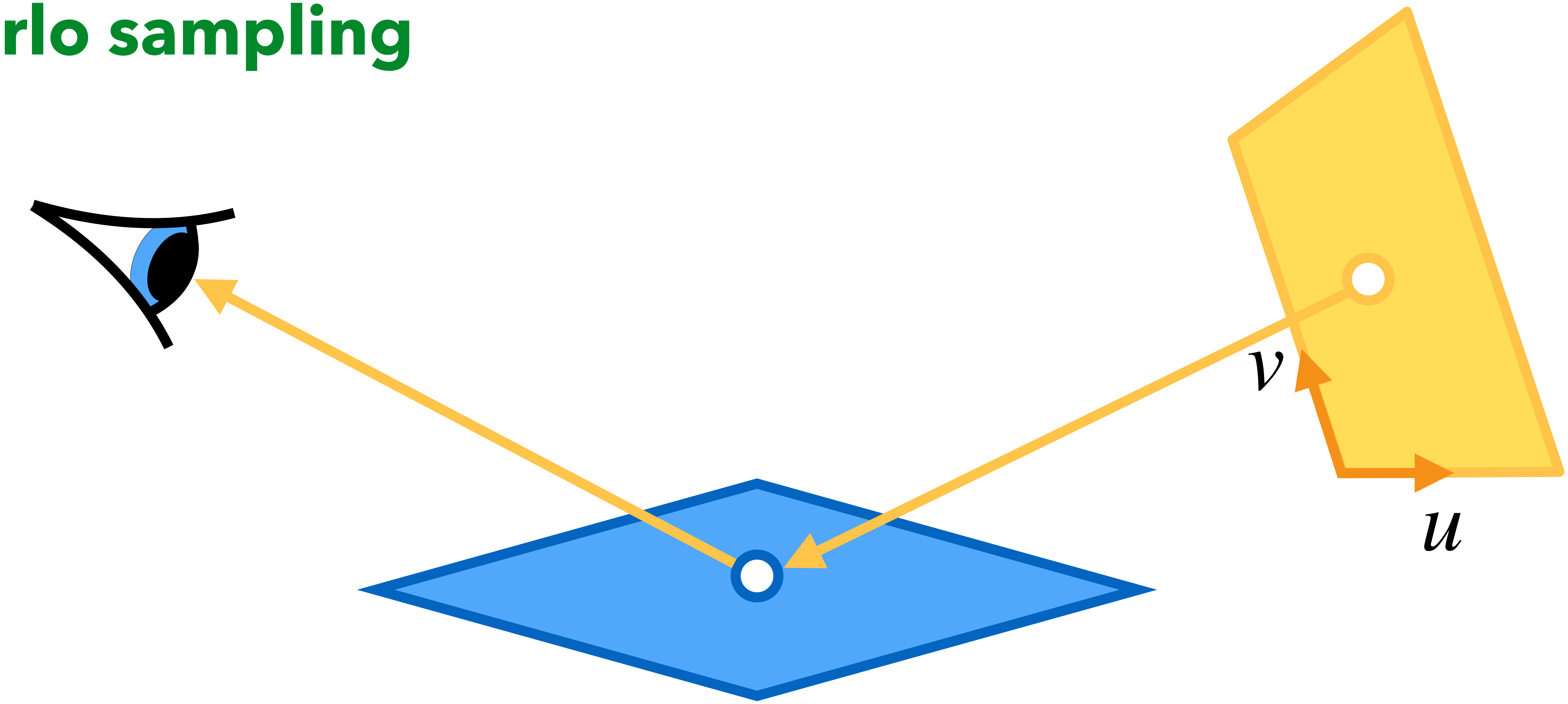
Theory

Monte Carlo sampling



$$L_o = \int_{\mathcal{U}} \int_{\mathcal{V}} f(u, v)\, \mathrm{d}v\, \mathrm{d}u \approx \langle L_o \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(u_i, v_i)}{p(u_i, v_i)}$$

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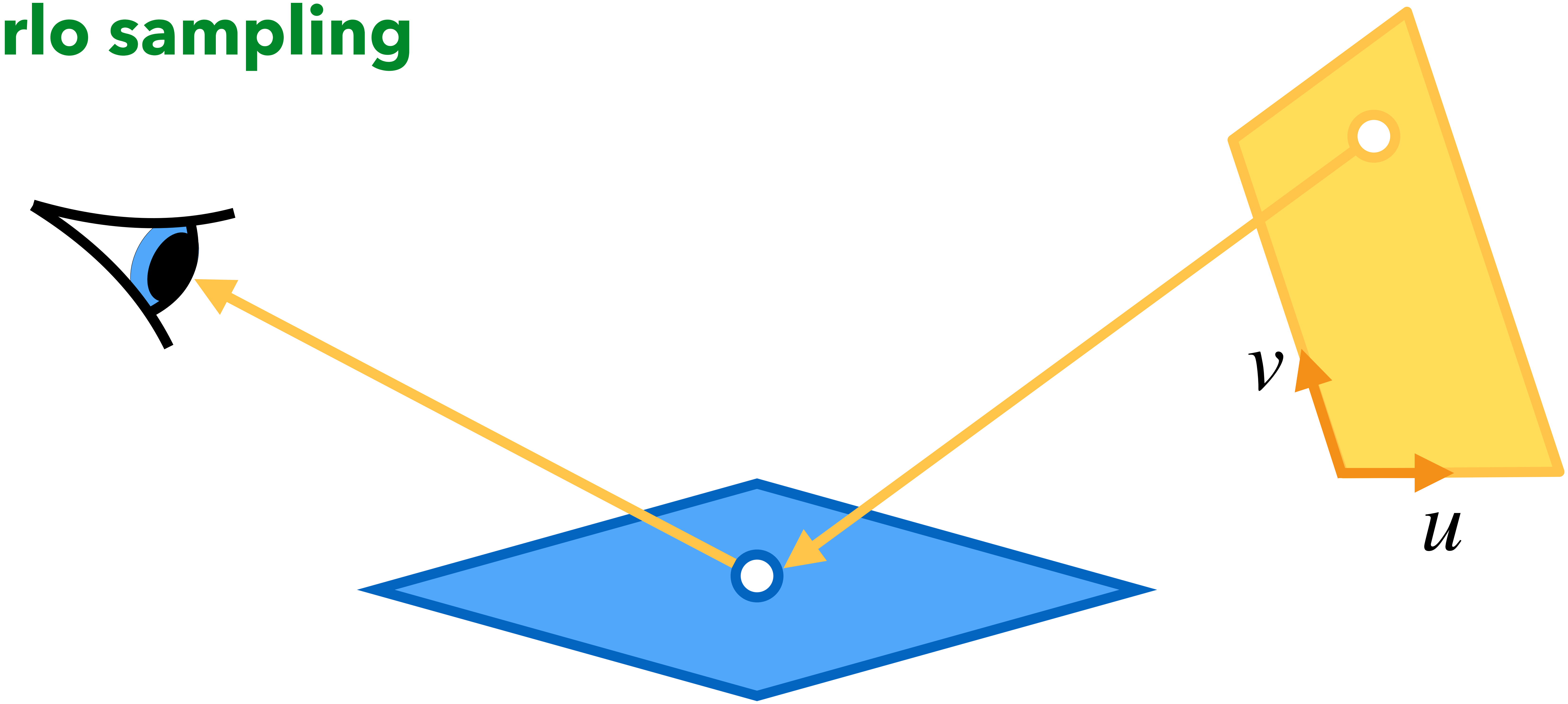

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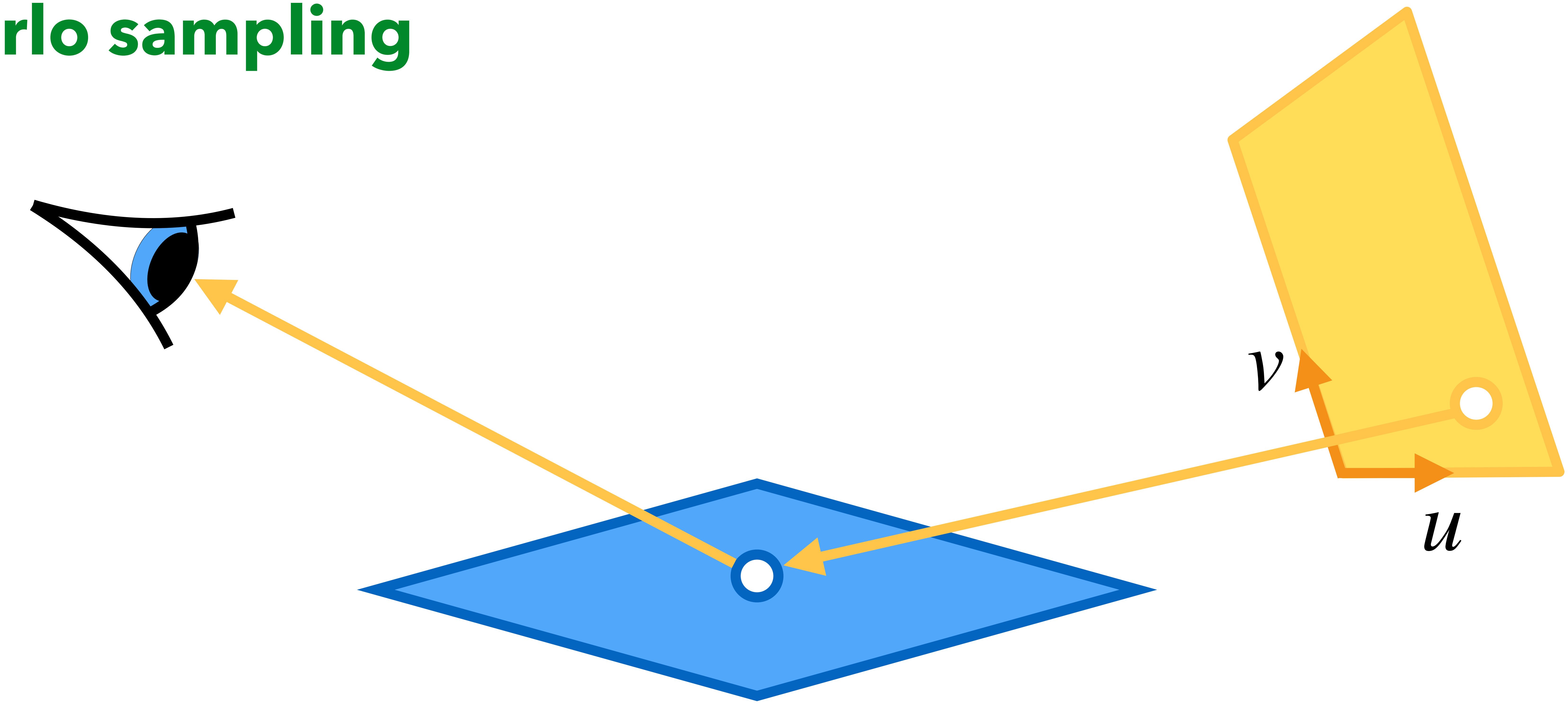

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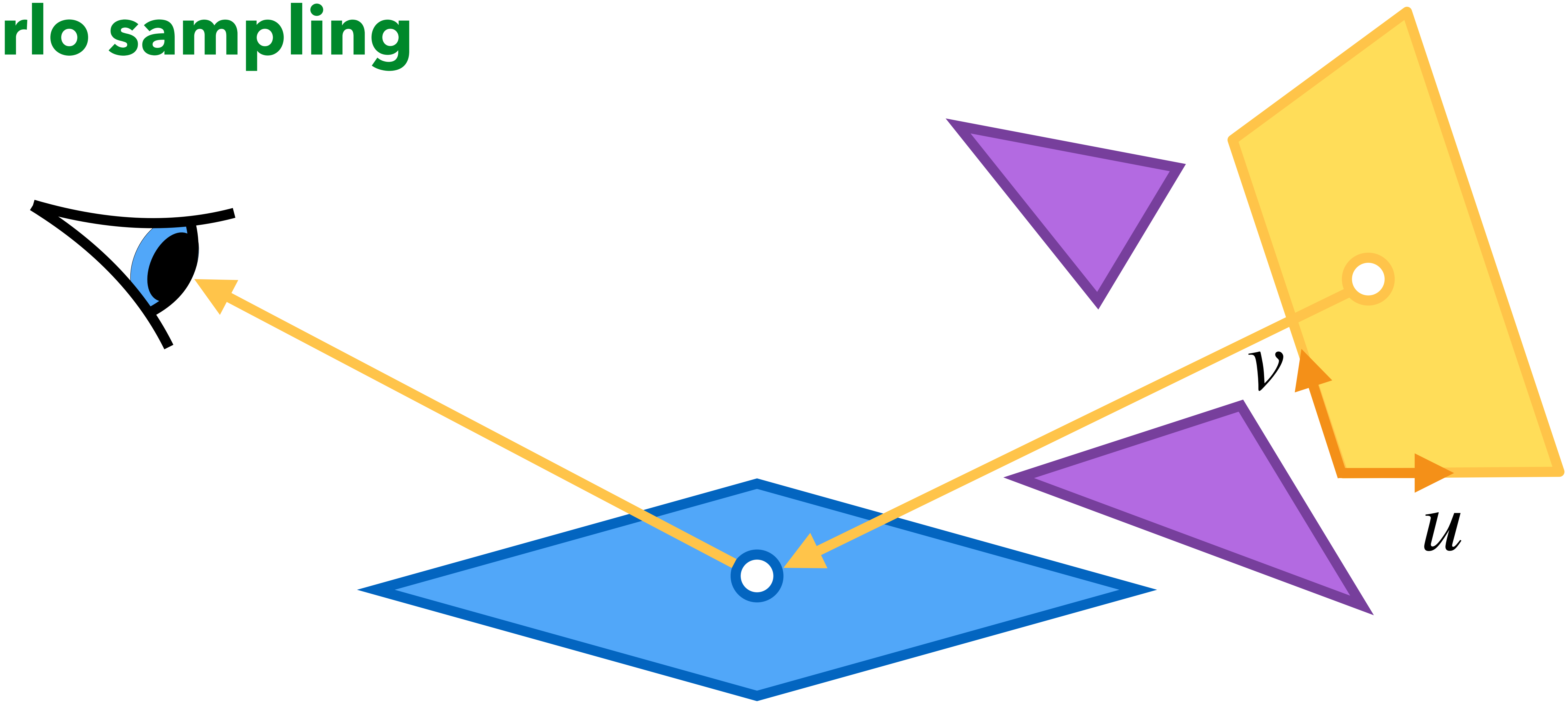

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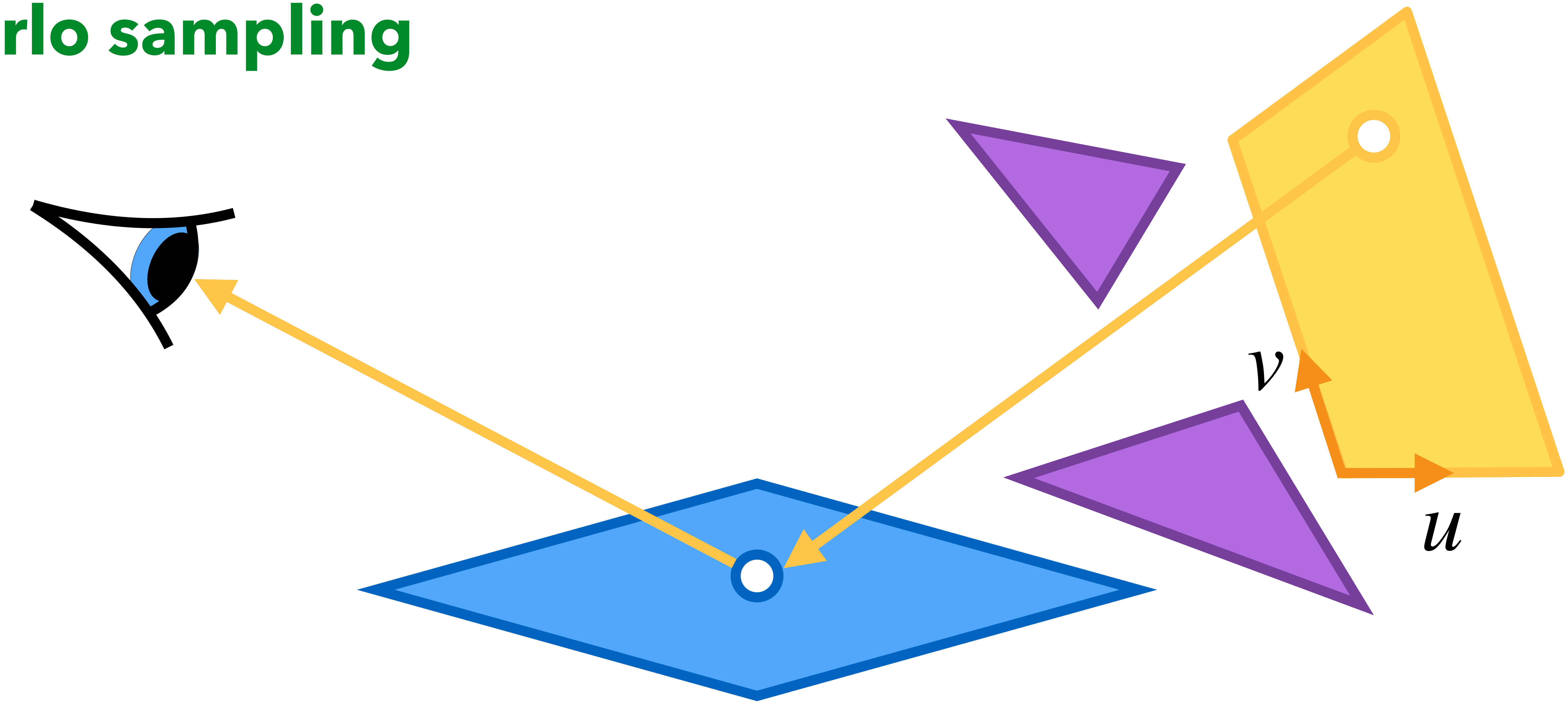

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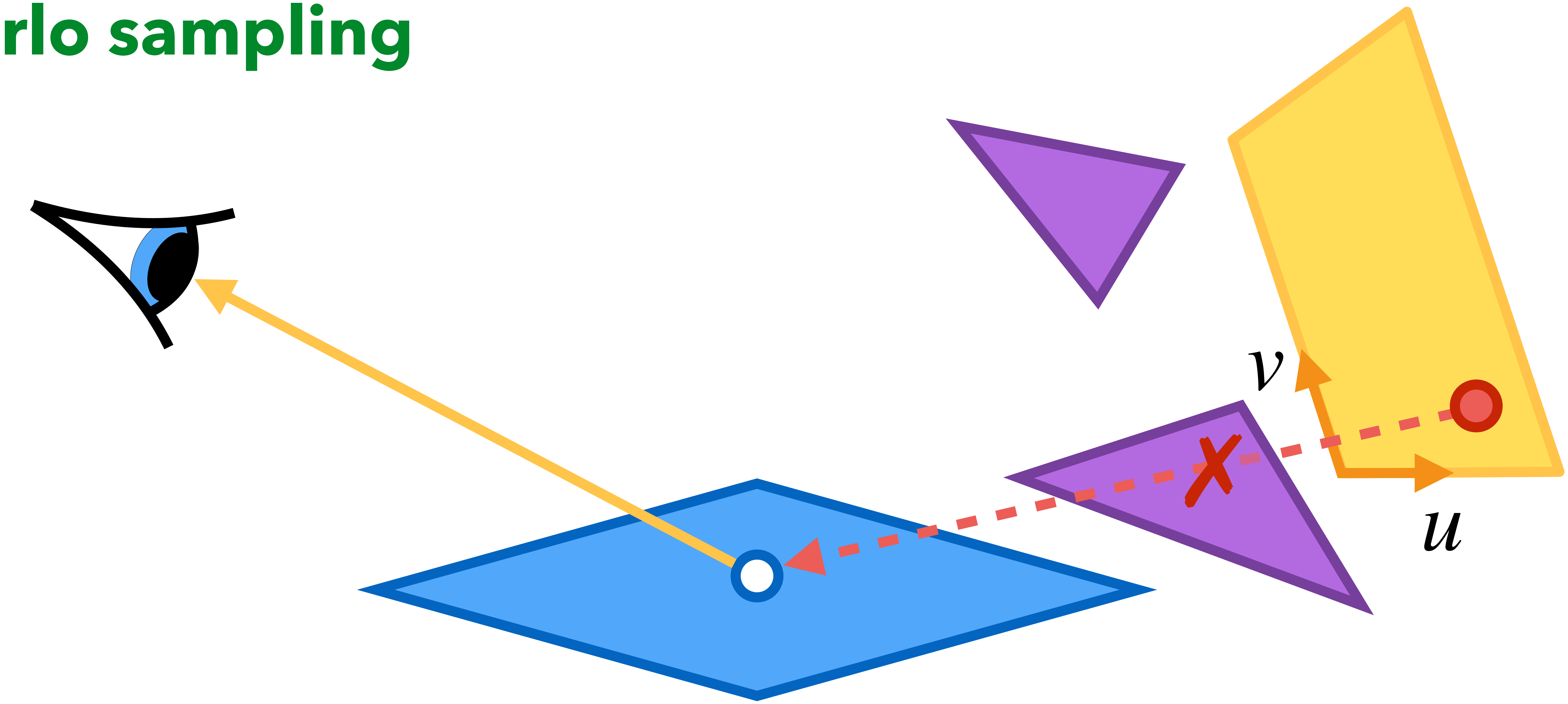

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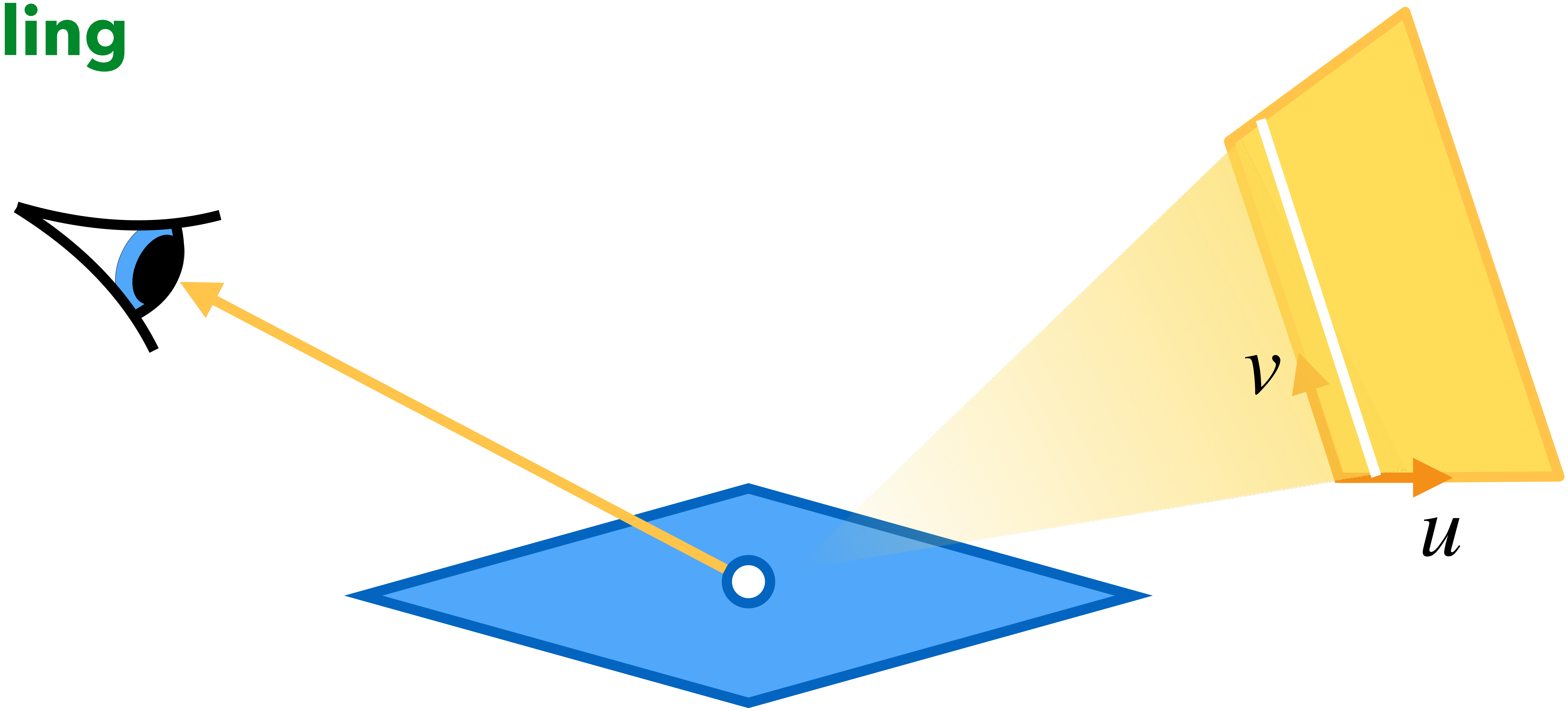
Theory

Line sampling

[BD16]



$$L_o = \int_{\mathcal{U}} \int_{\mathcal{V}} f(u, v)\, \mathrm{d}v\, \mathrm{d}u = \int_{\mathcal{U}} f_v(u)\, \mathrm{d}u$$

Theory

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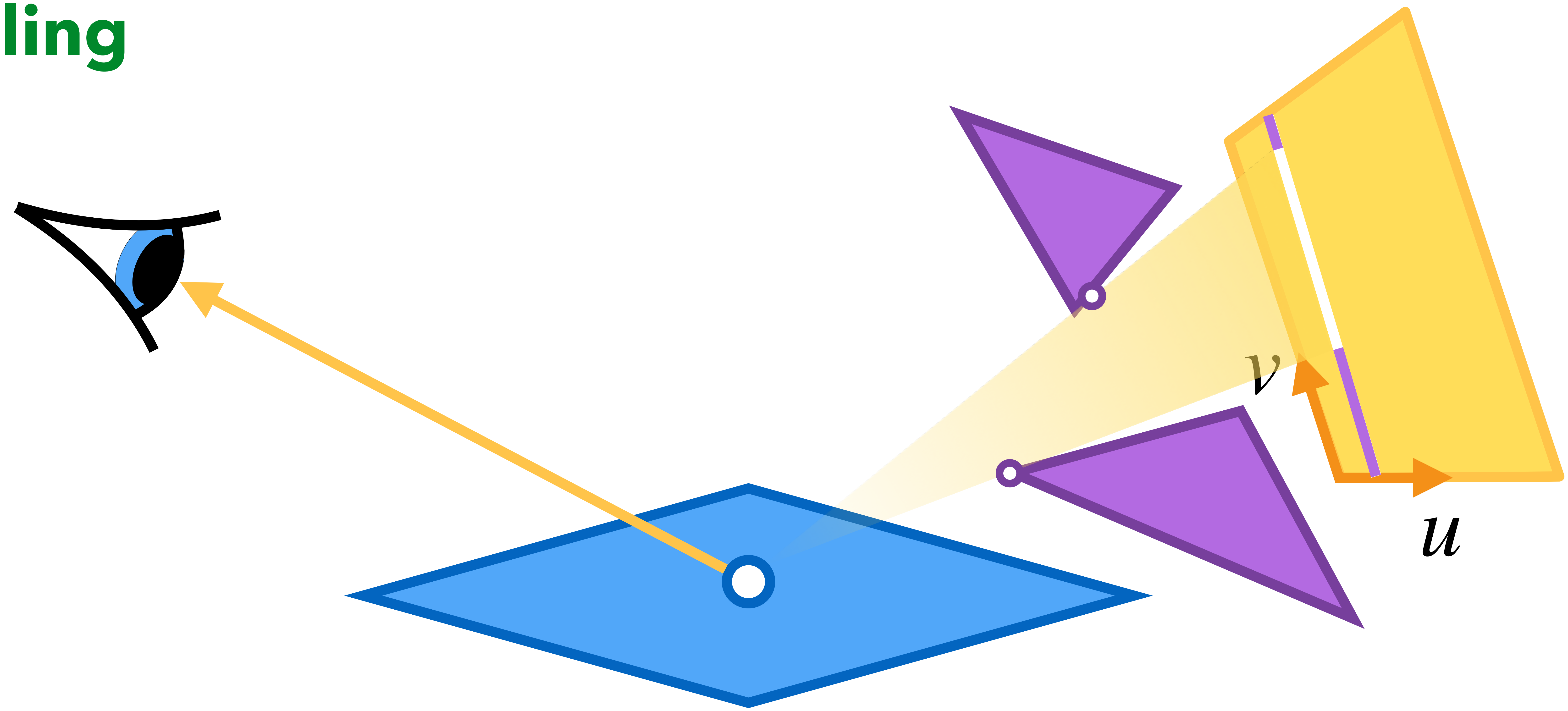

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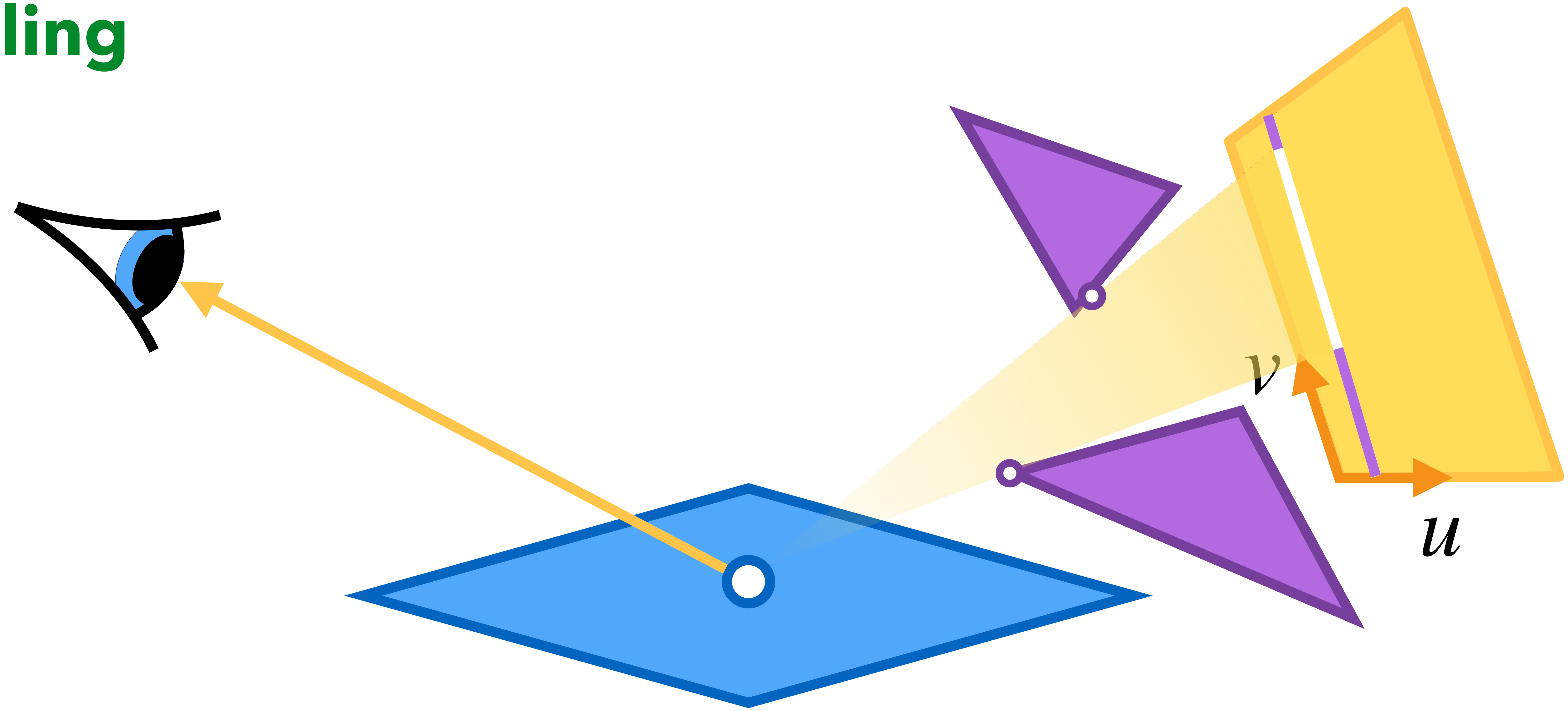

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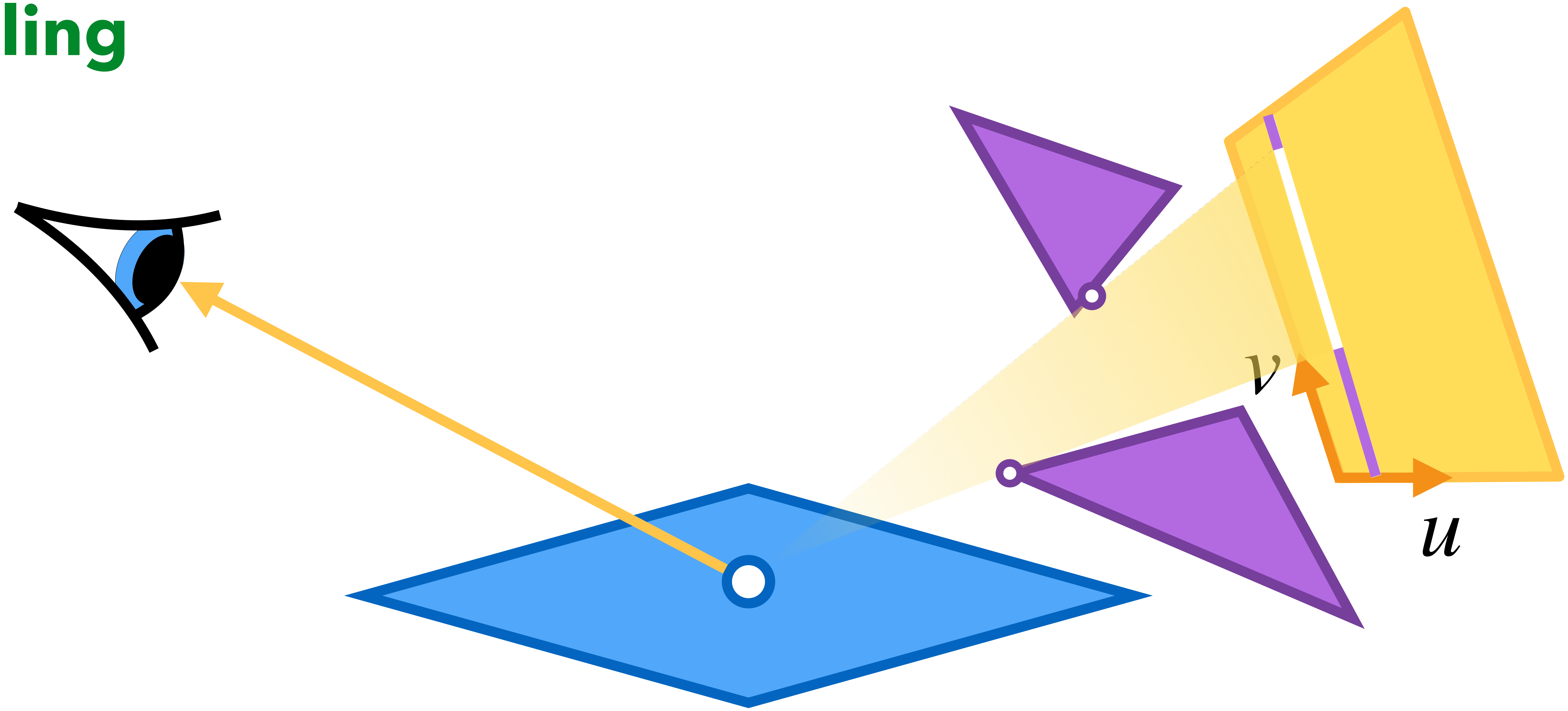

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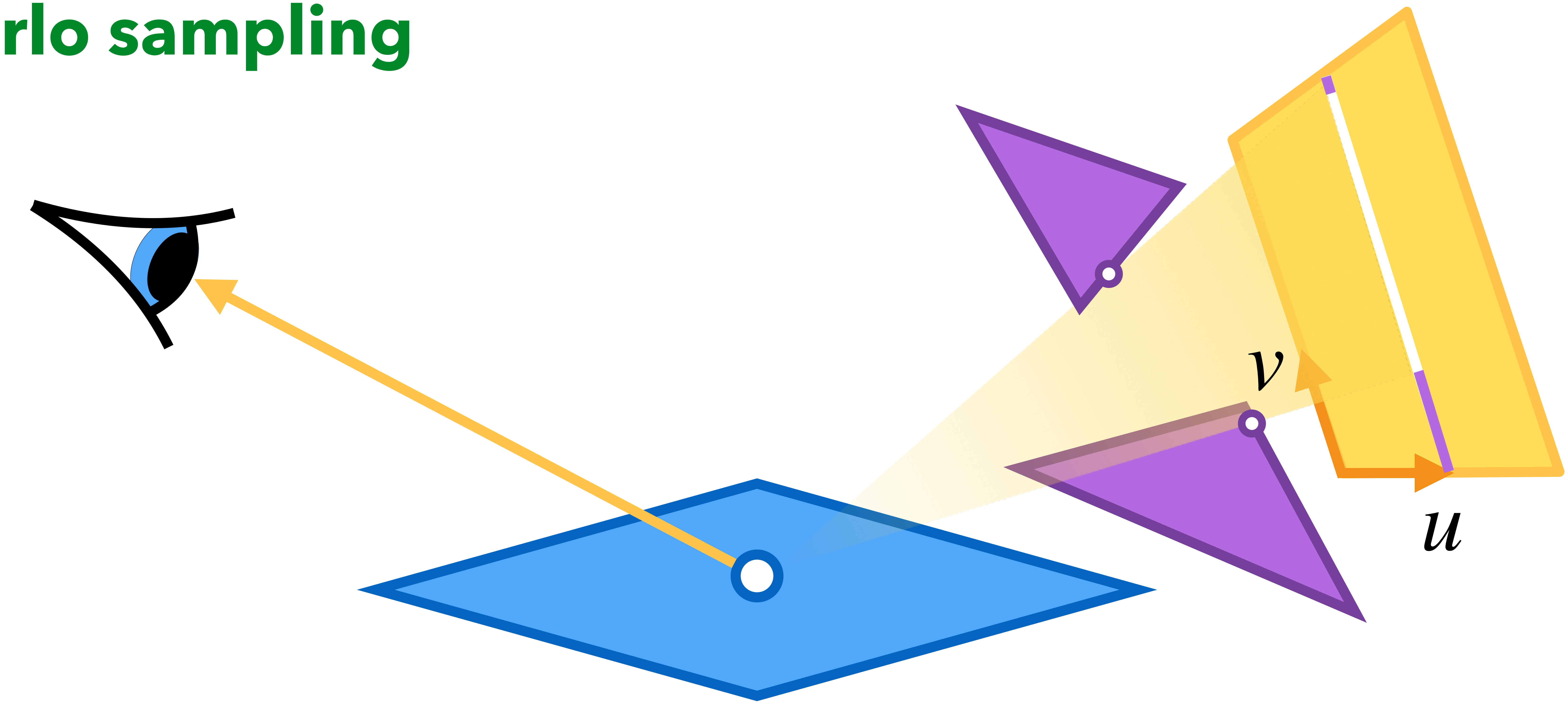

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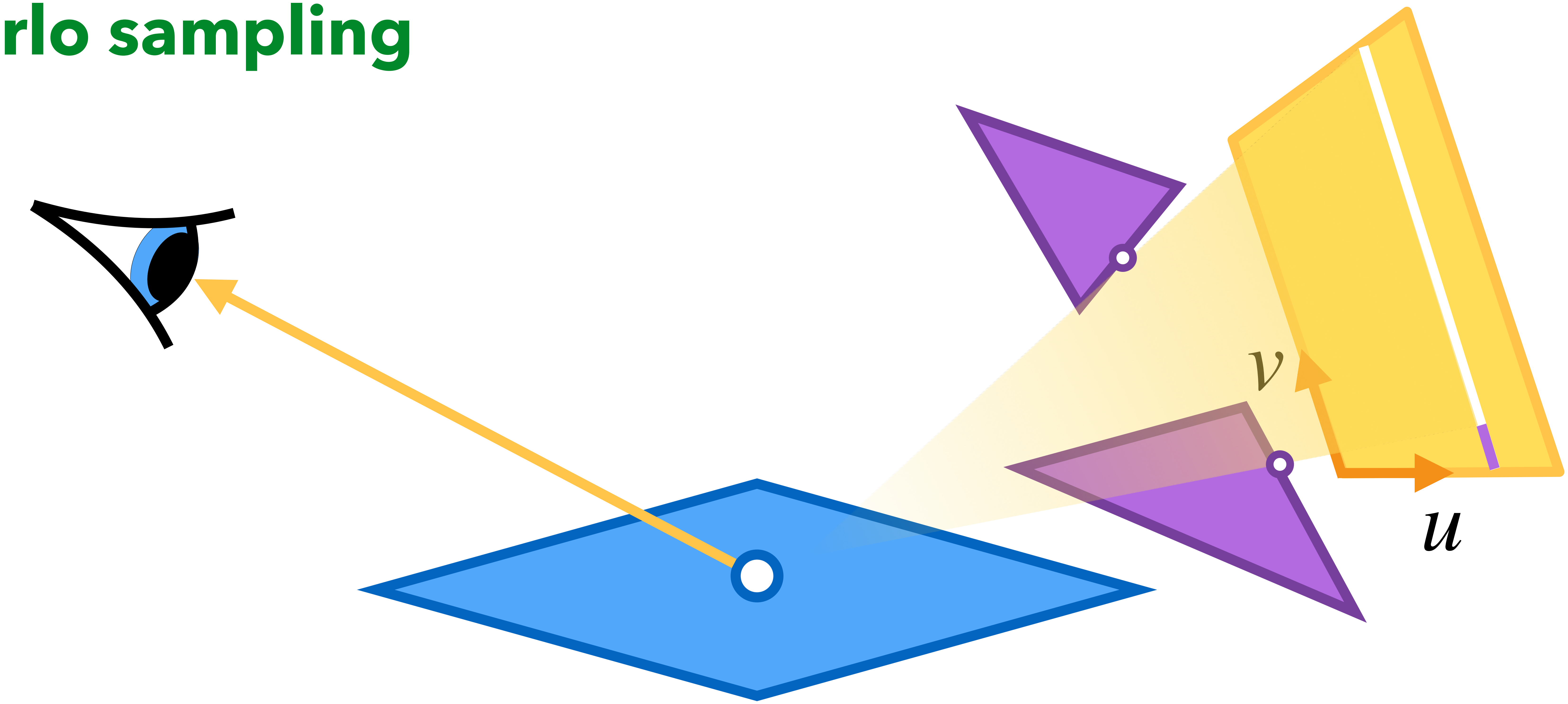

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Prior work: line sampling

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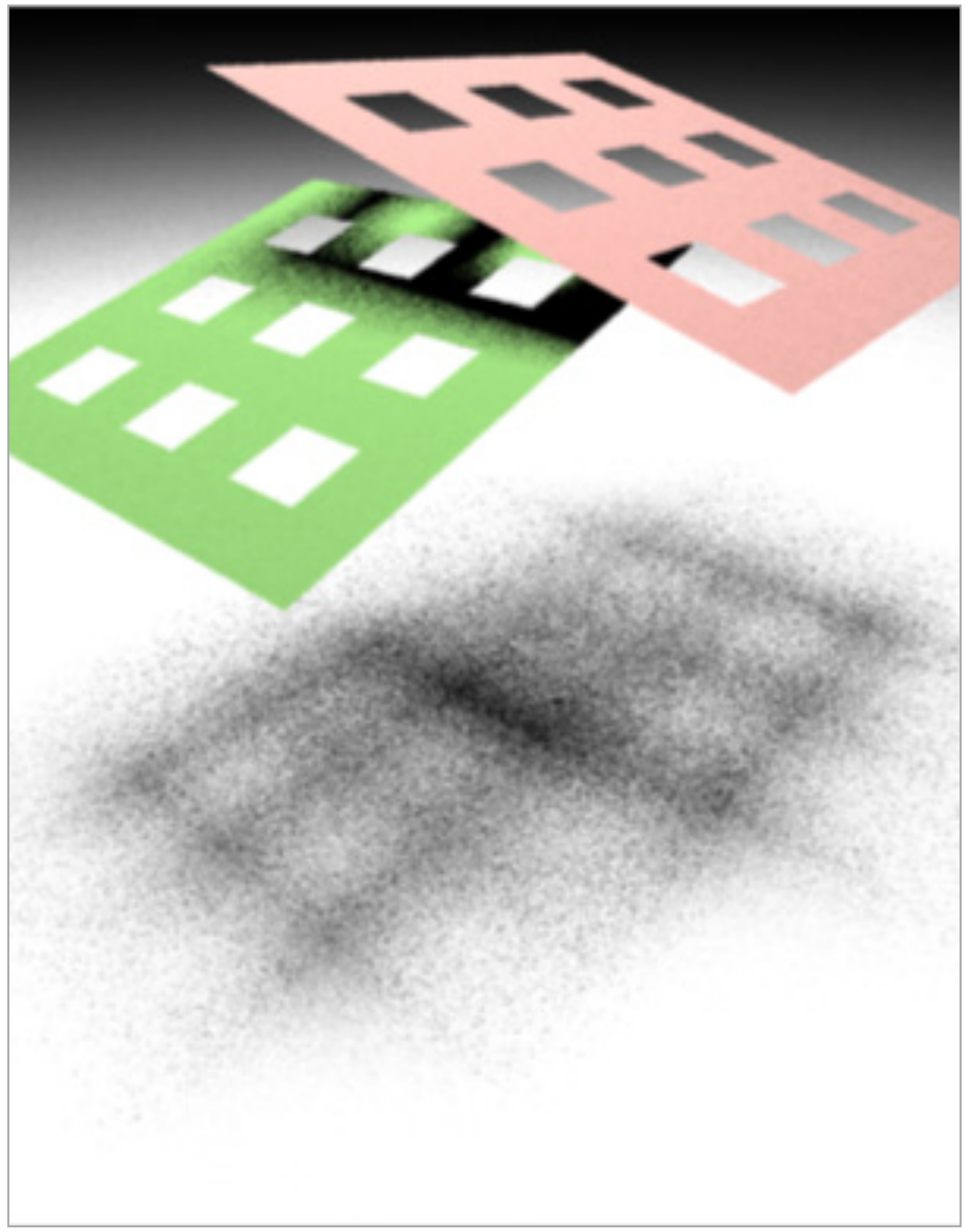
- **Direct illumination** [BD16]

[BD16]



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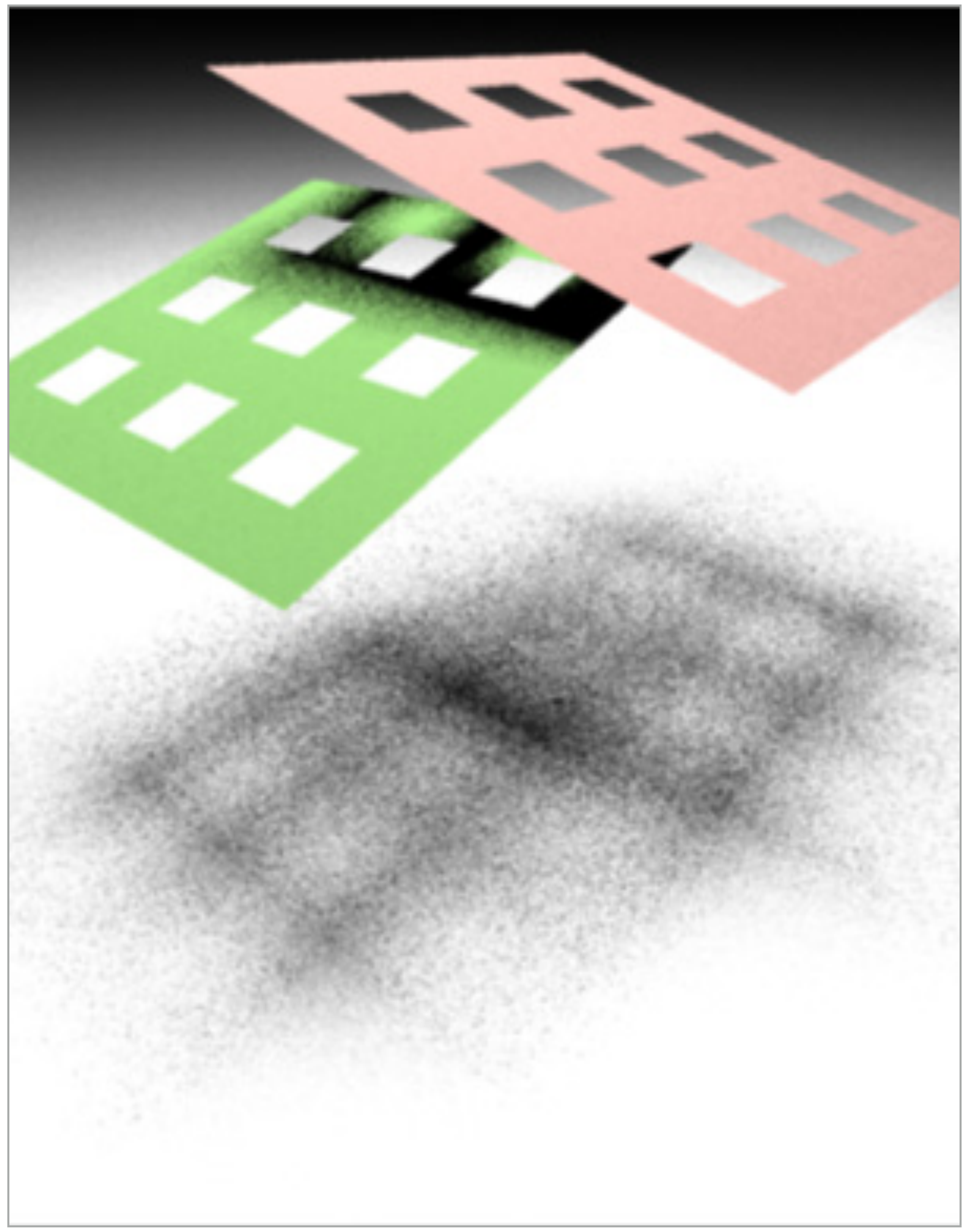

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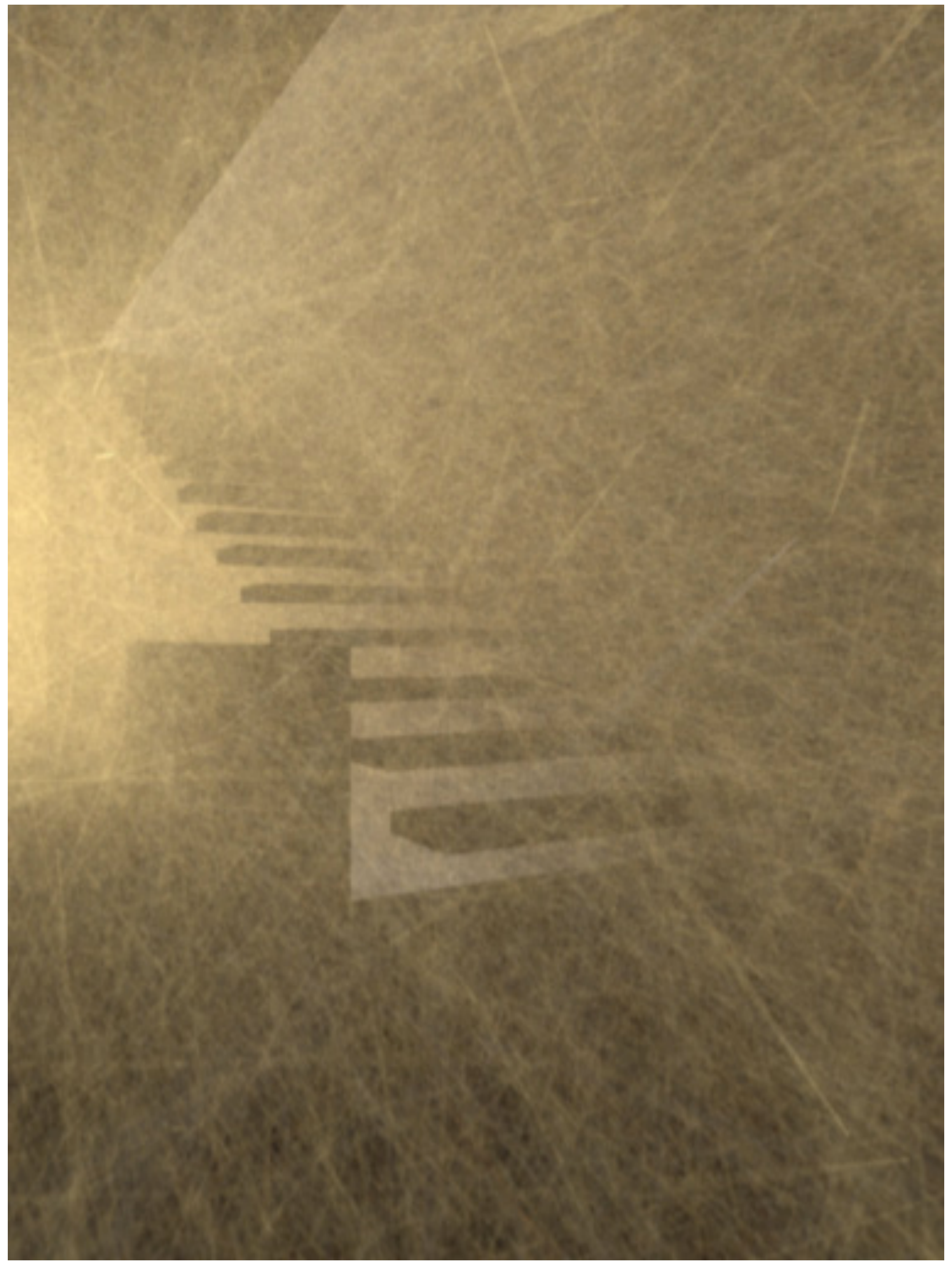

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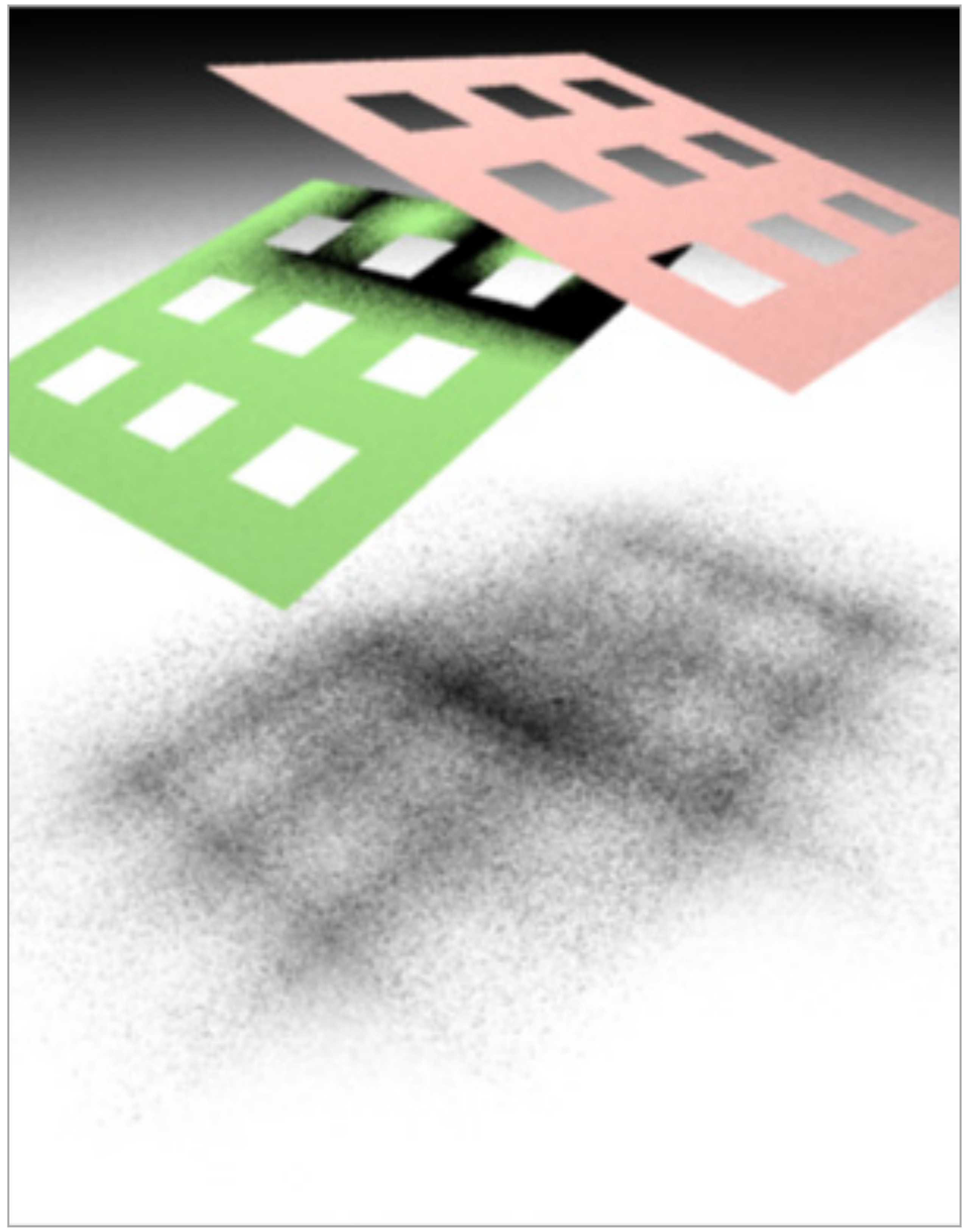

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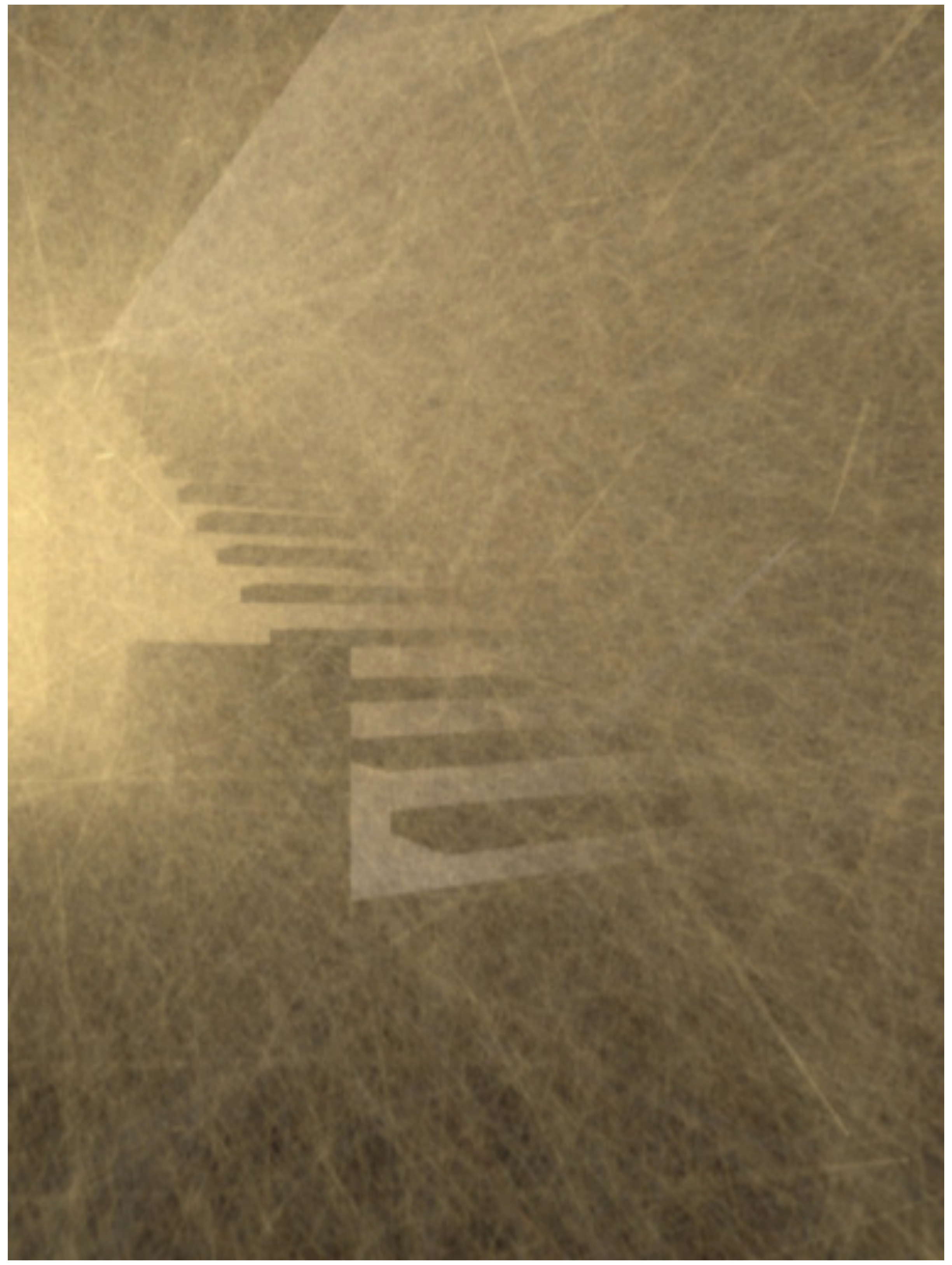

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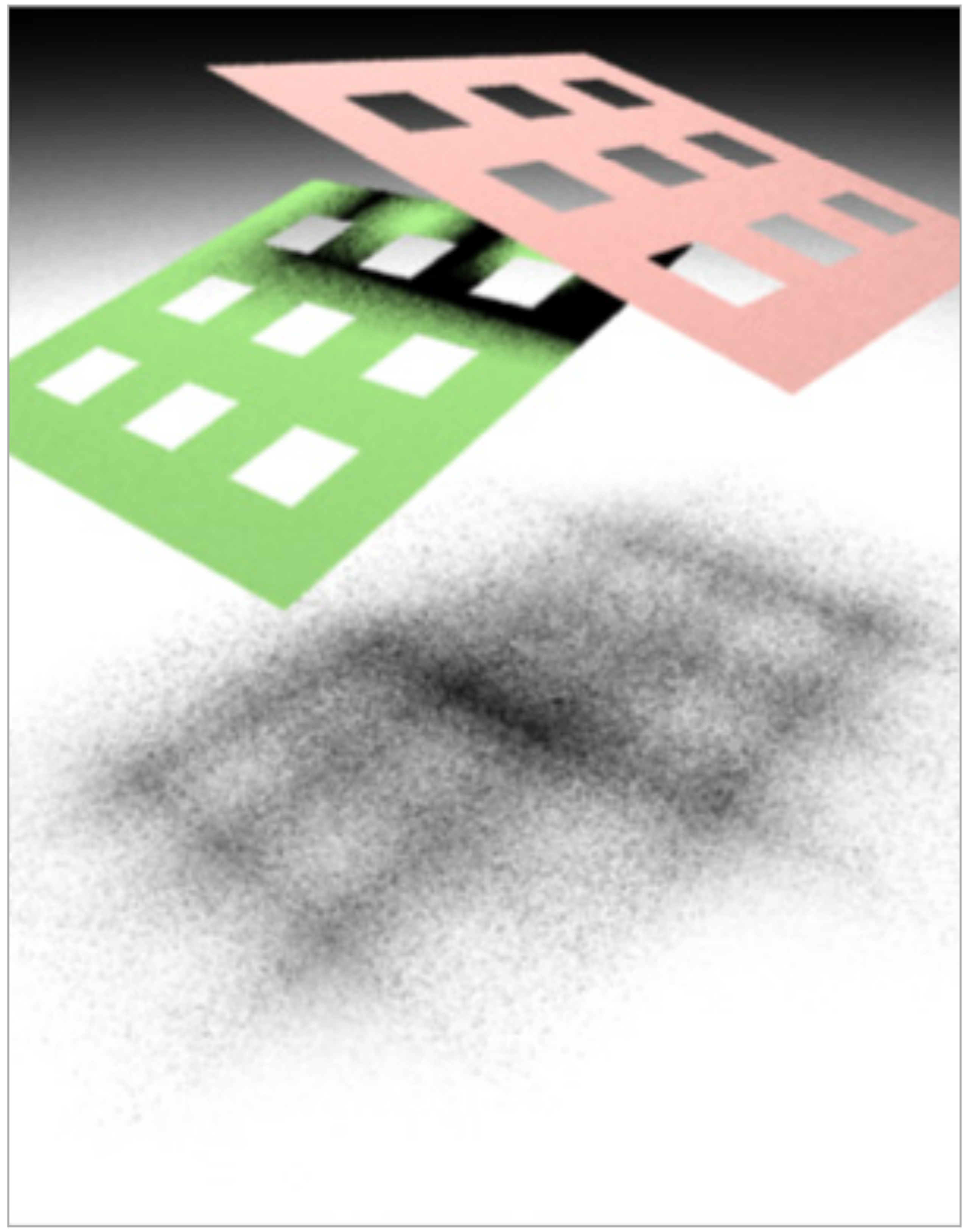

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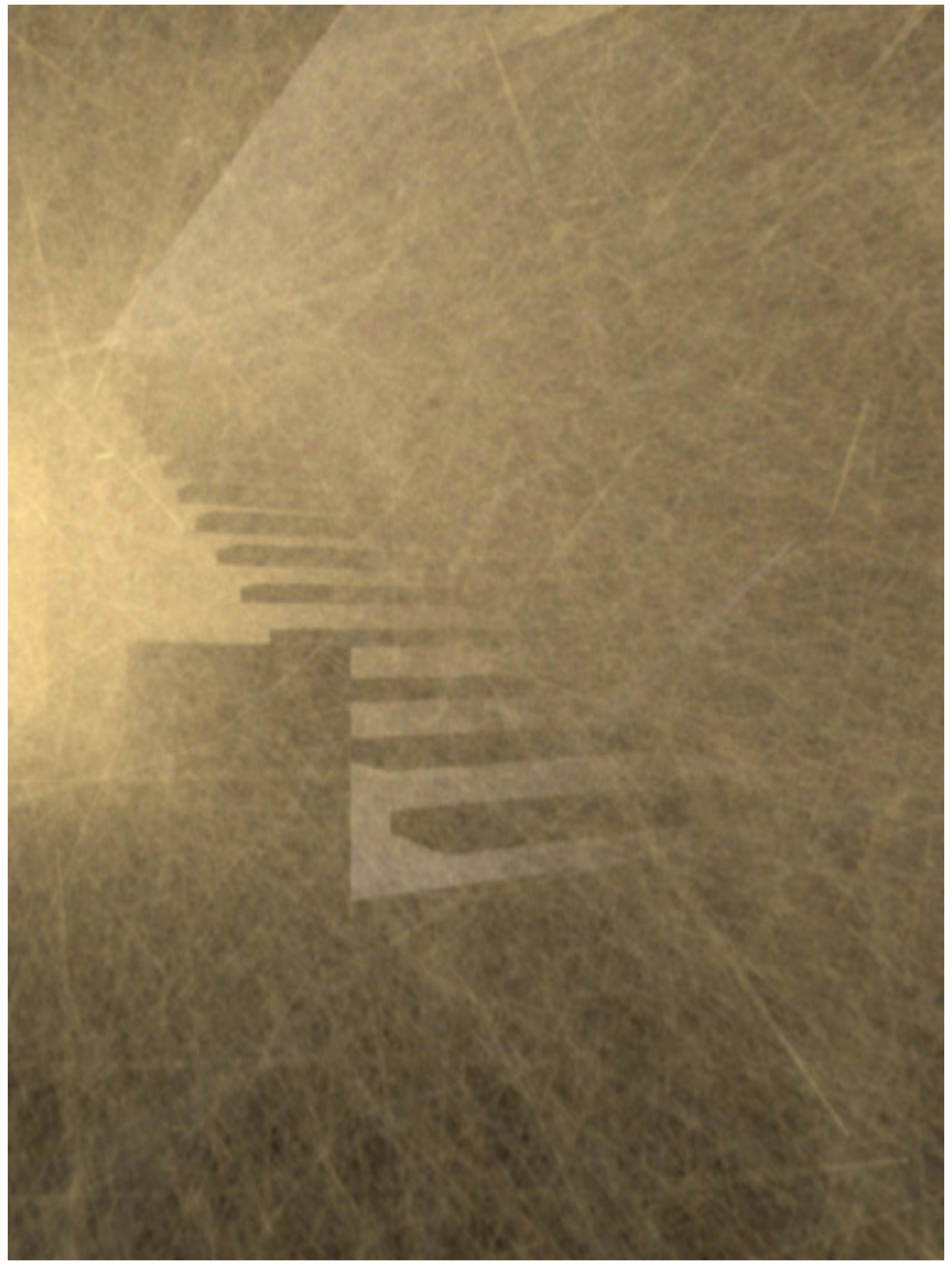

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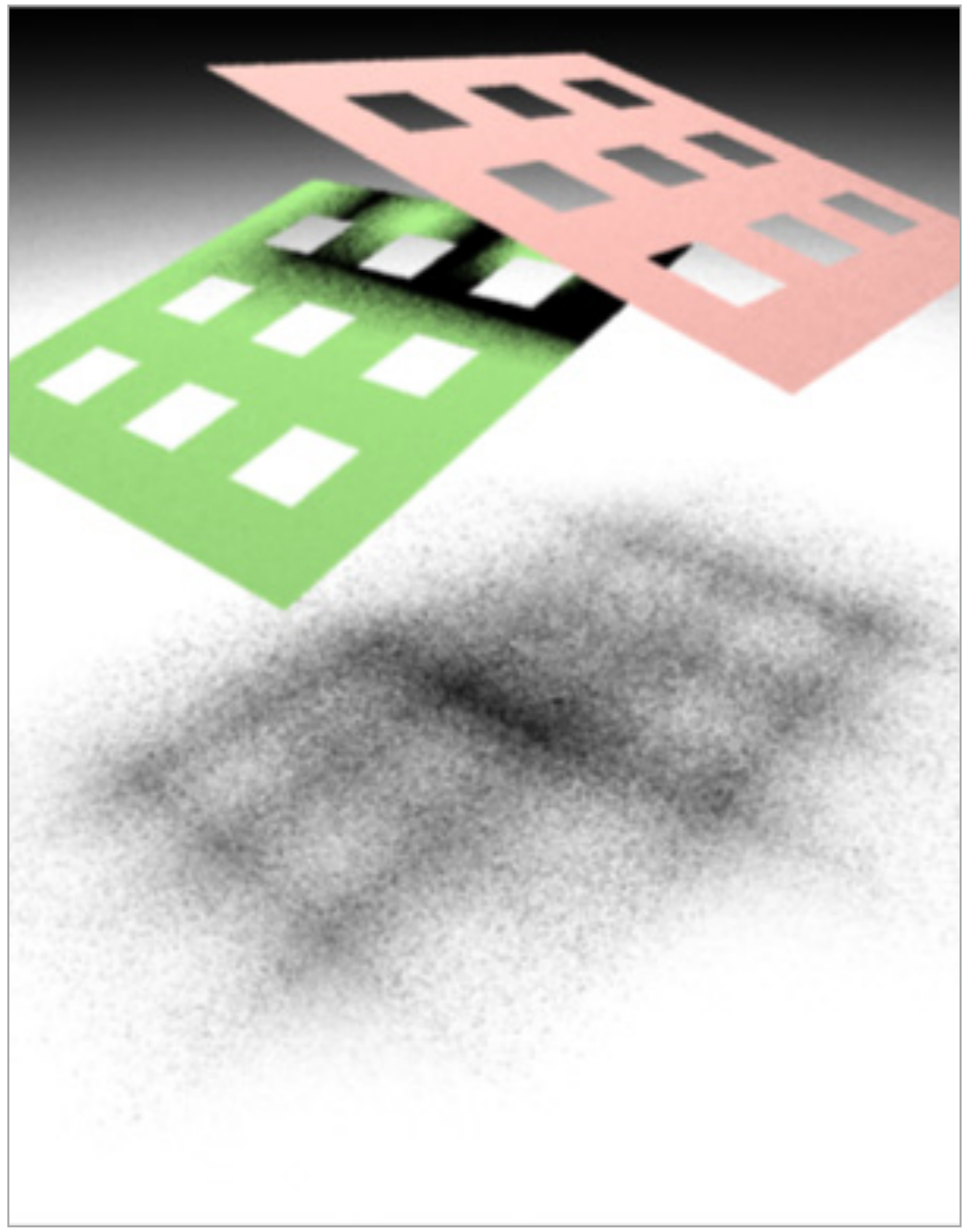

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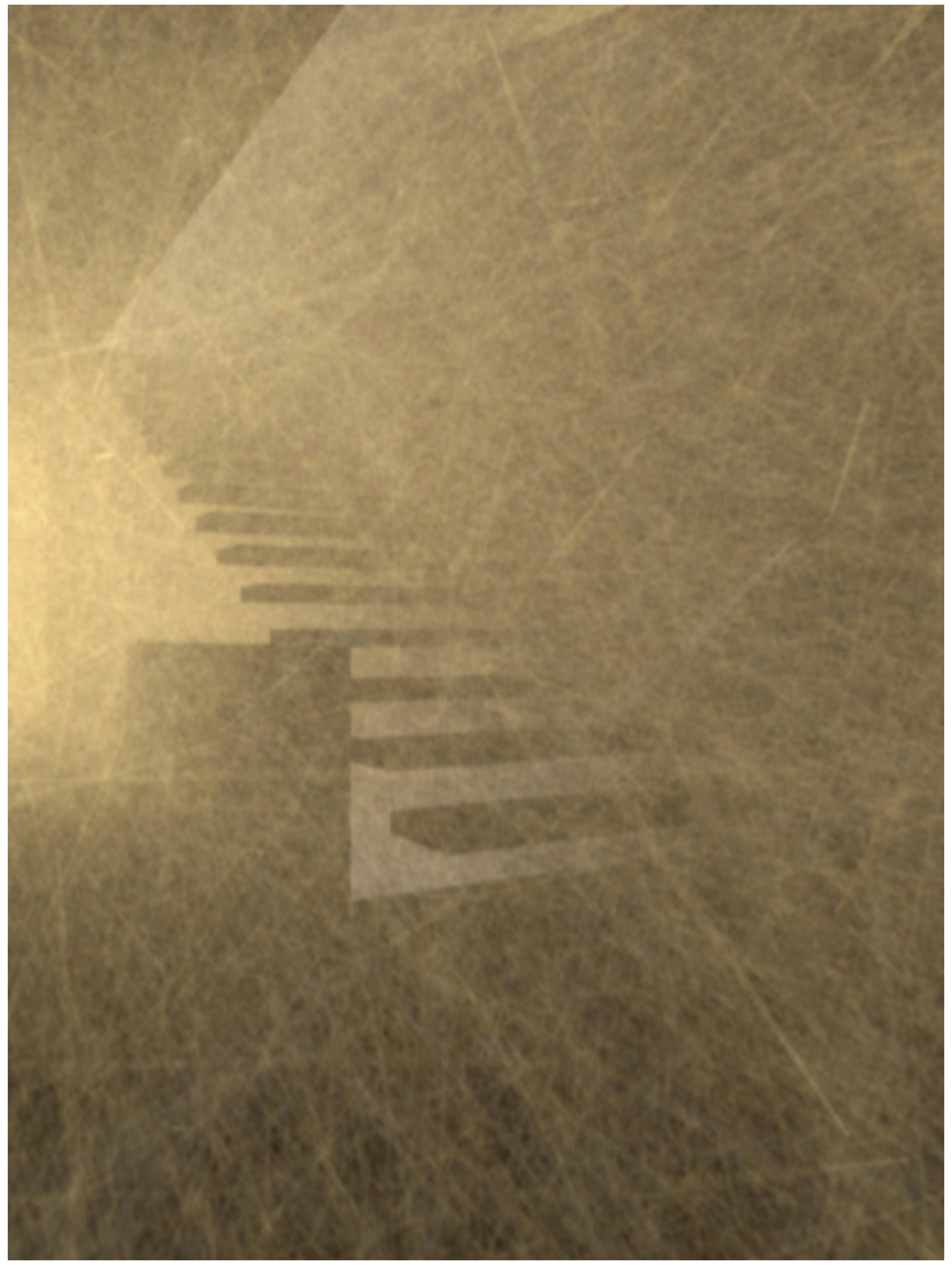

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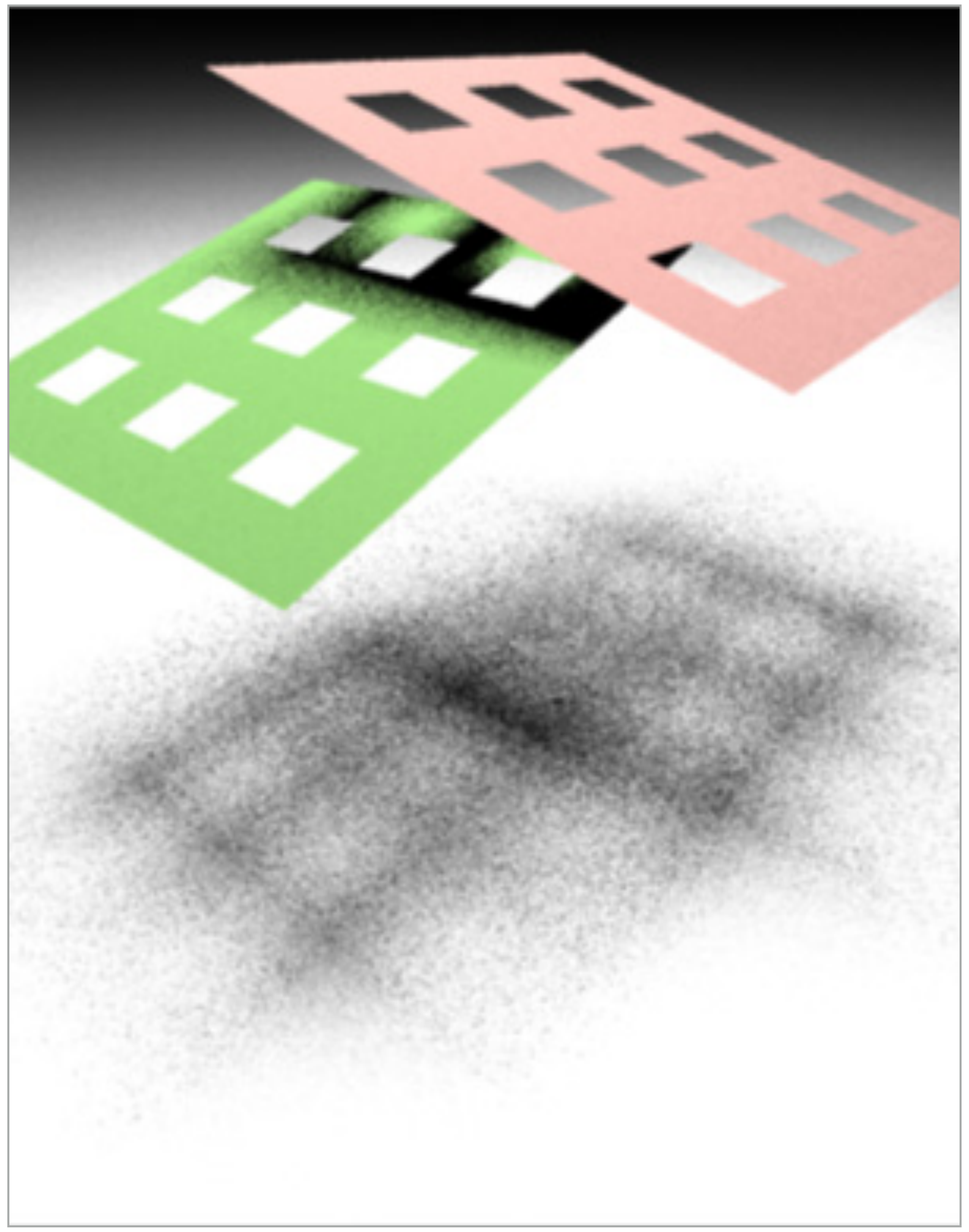

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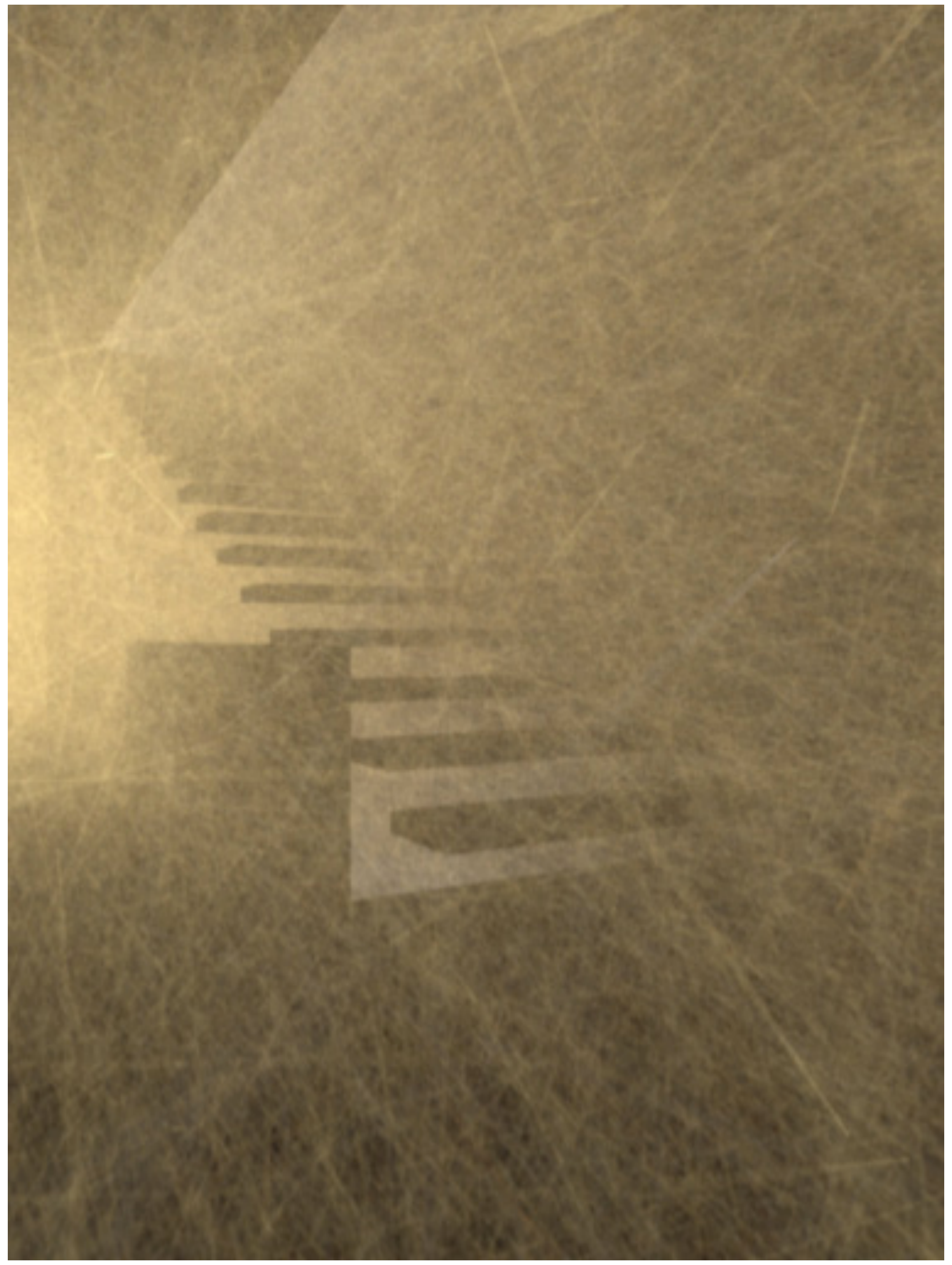

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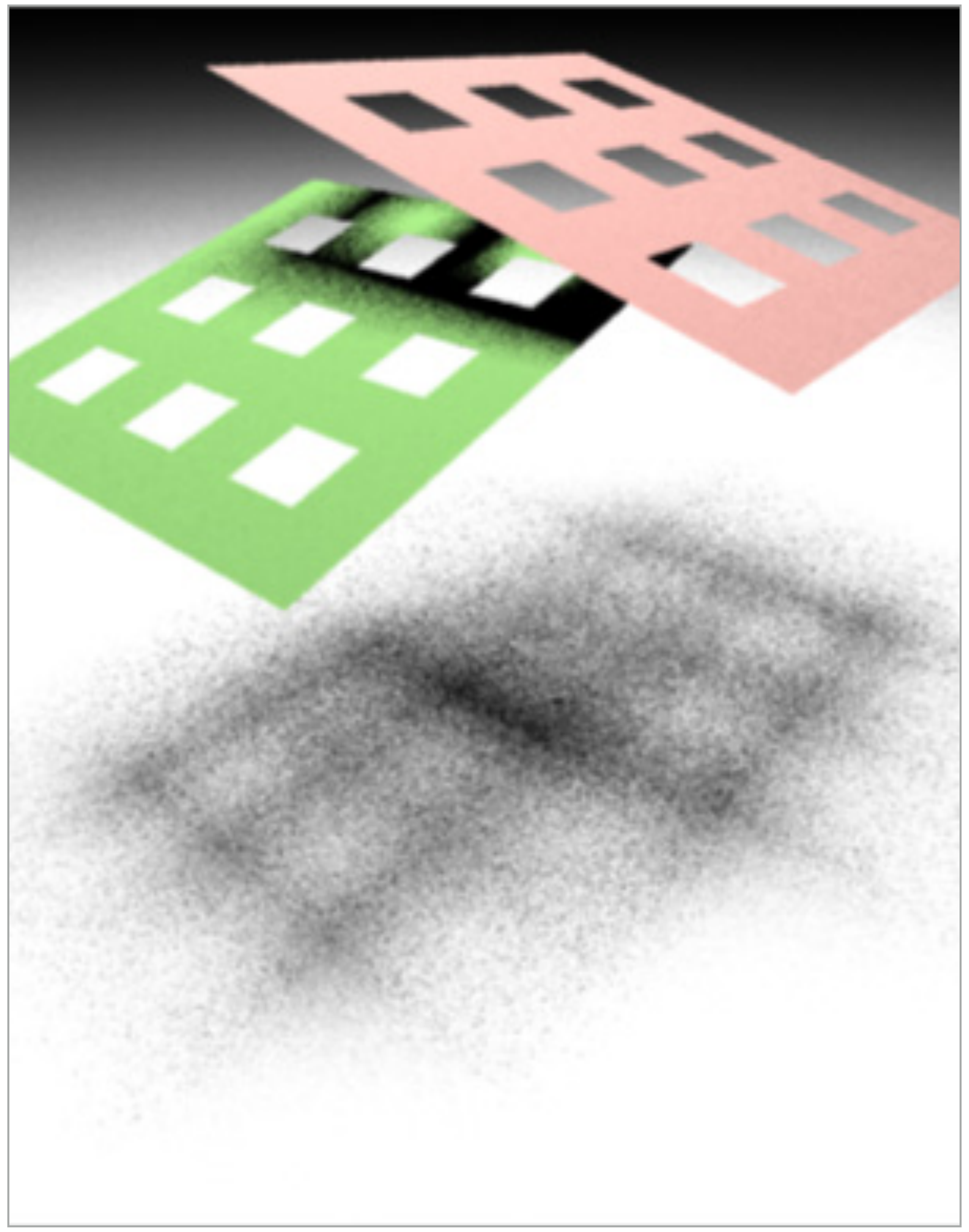

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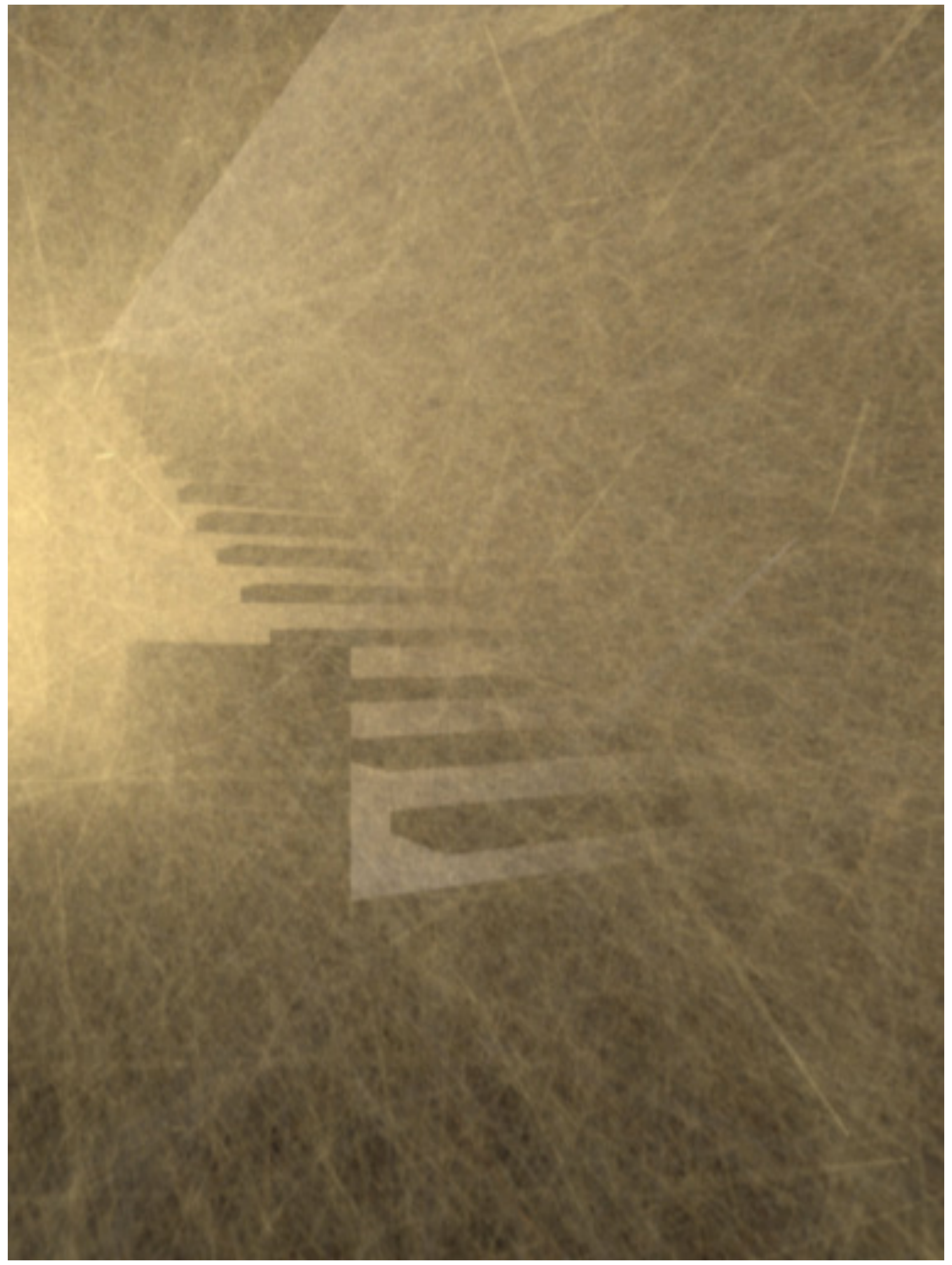
and more…

[BJ17]



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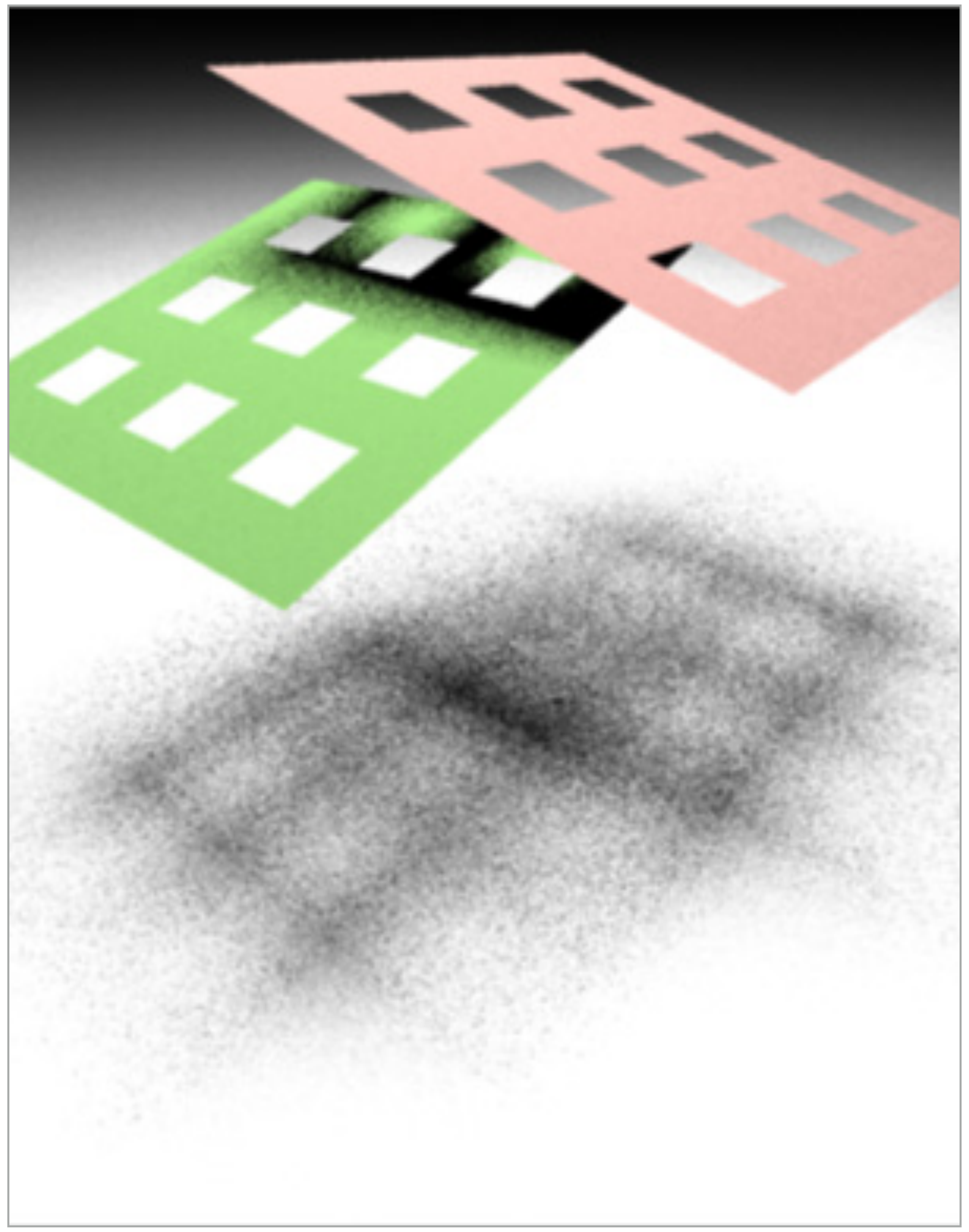

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✗ Slow to evaluate samples

- Expensive line sample-scene intersection

Motivation

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Make line samples play nicely with **any point-based strategy**

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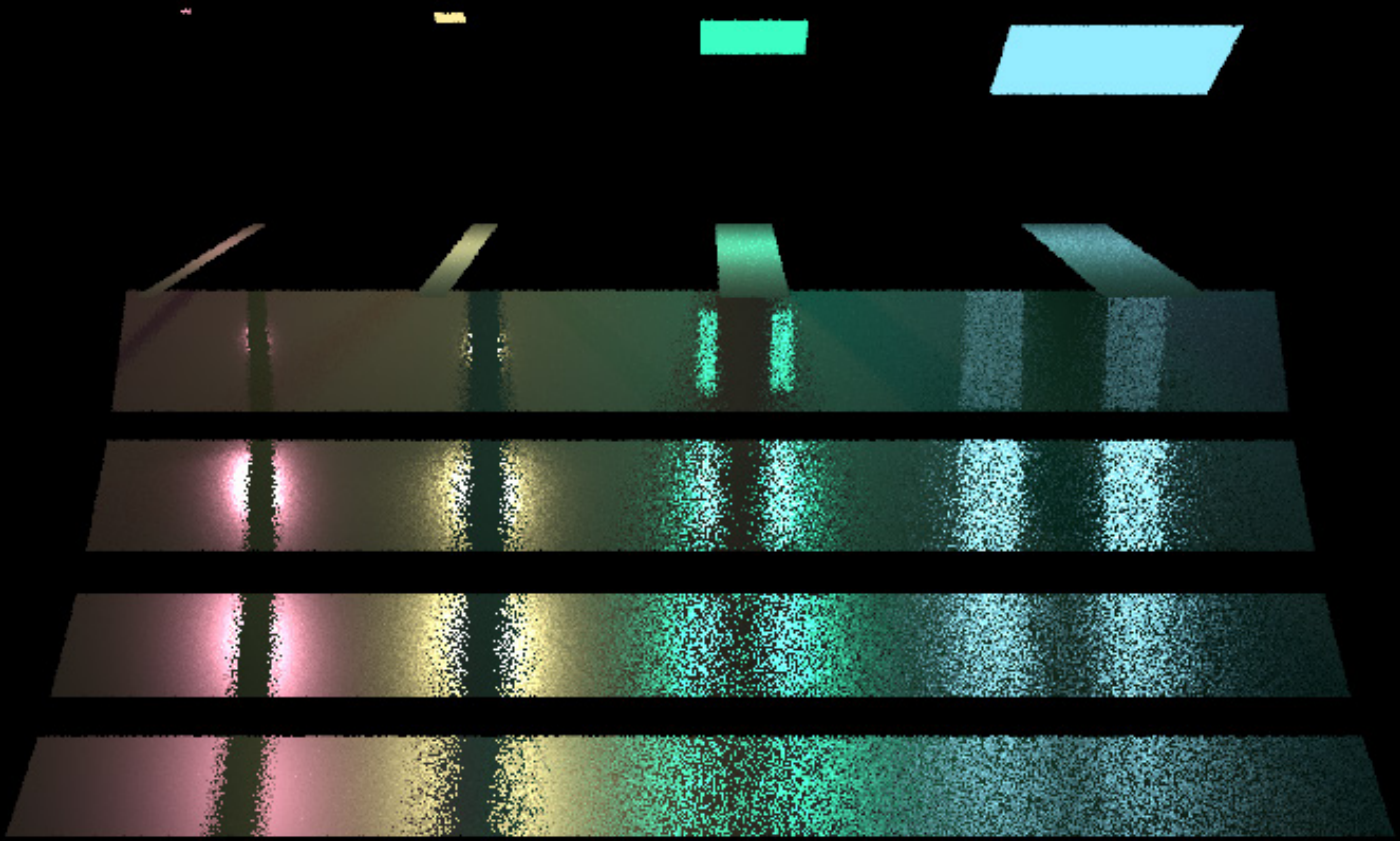
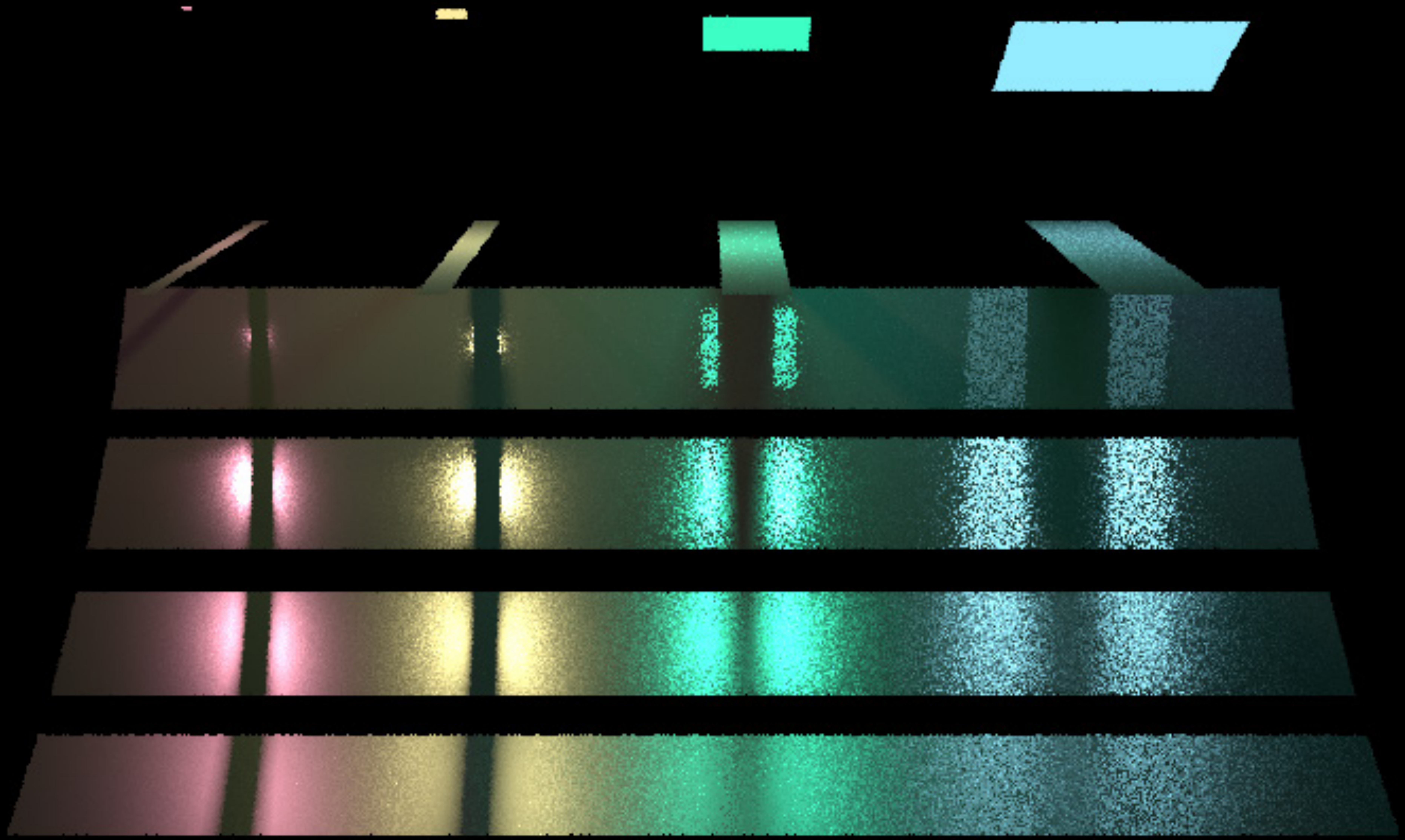
Mitigate orientation-based **performance issues**

MIS Points + Points



MIS Points + Lines
(Ours)



Roadmap

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- Reframe line samples as point samples that **importance sample visibility**

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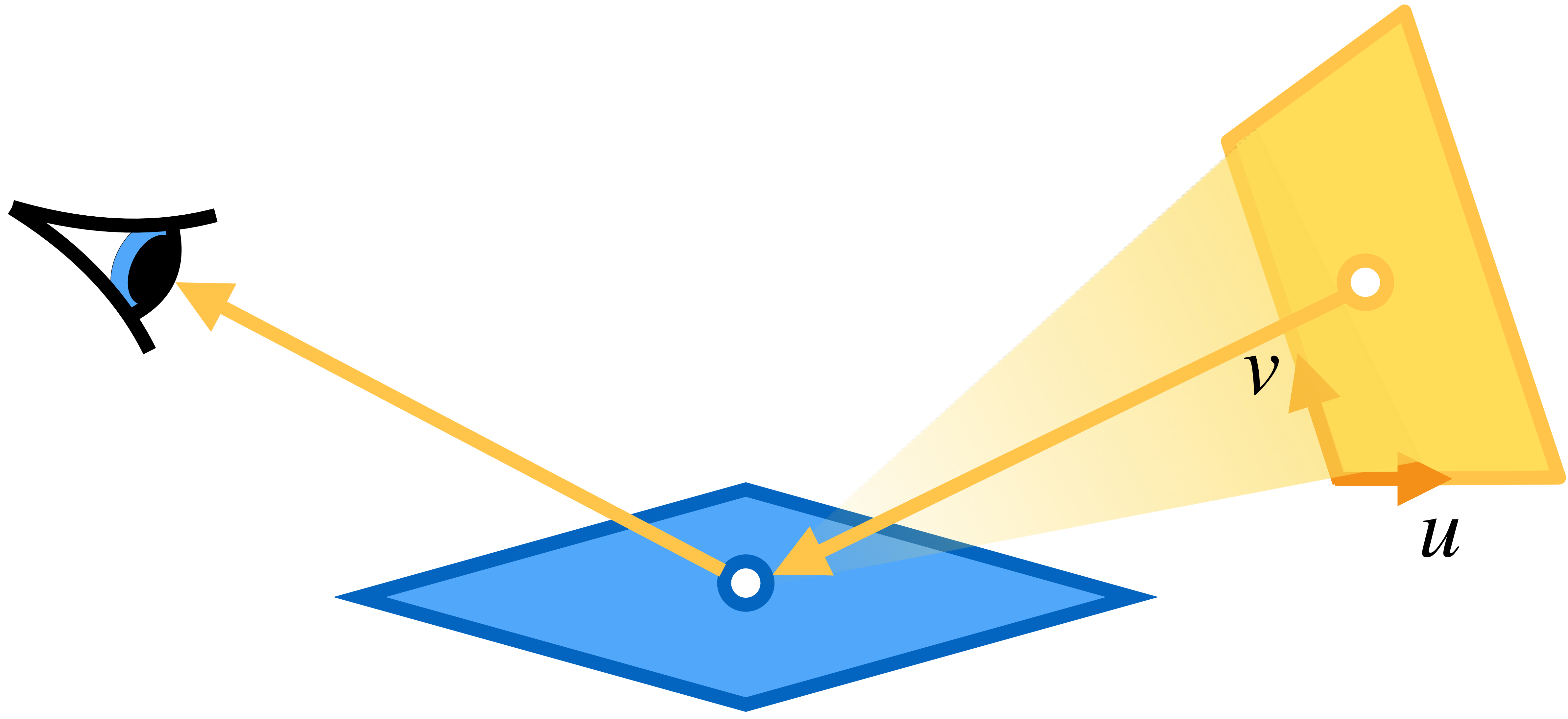
- Show how to **multiple importance sample** between lines and points, and lines of different orientations
- Propose novel MIS weighting scheme to **improve convergence rate**

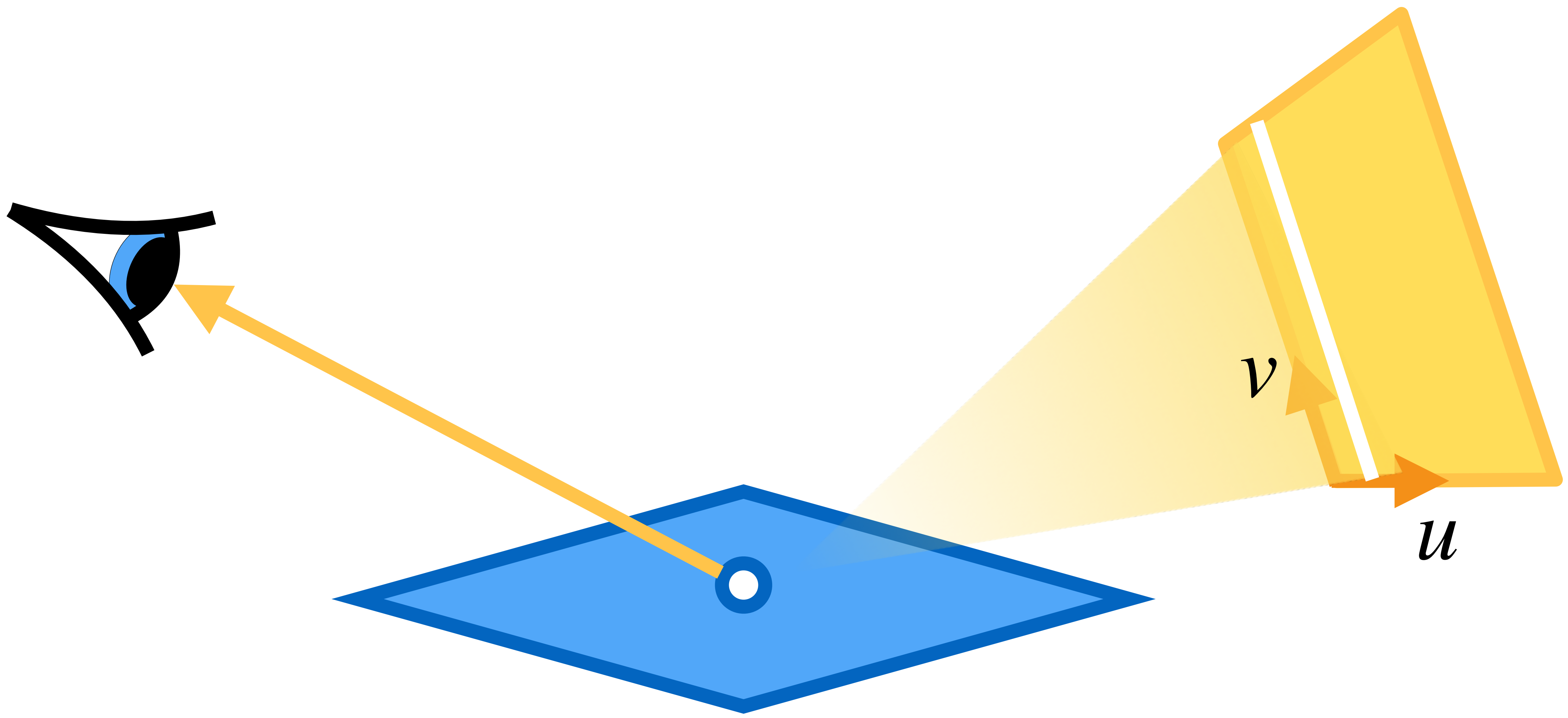
Main idea

Point sampling



$$L_o = \int_{\mathcal{U}} \int_{\mathcal{V}} f(u, v)\, \mathrm{d}v\, \mathrm{d}u$$

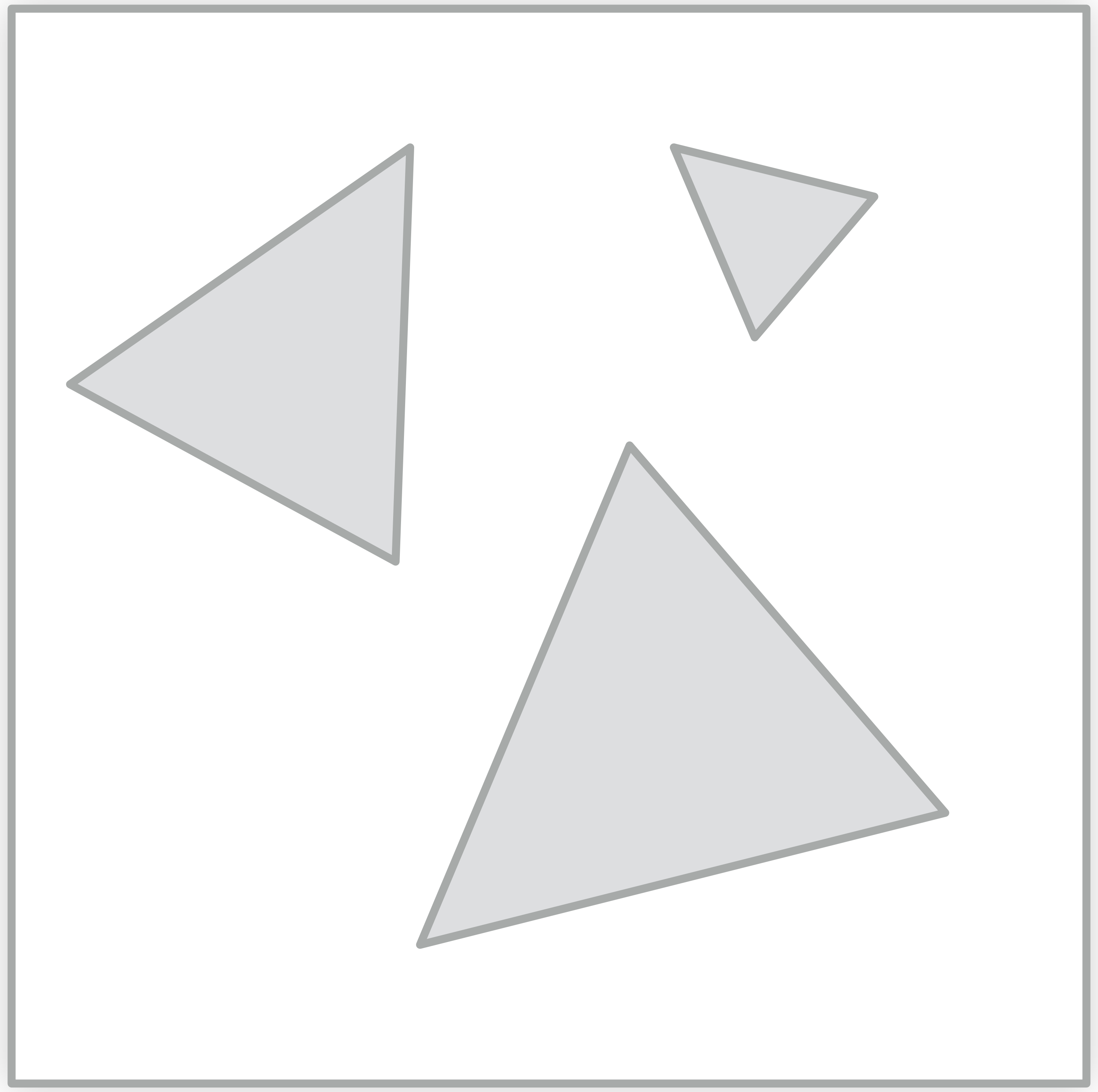
Line sampling



$$L_o = \int_{\mathcal{U}} f_v(u)\, \mathrm{d}u$$

Main idea

Point sampling



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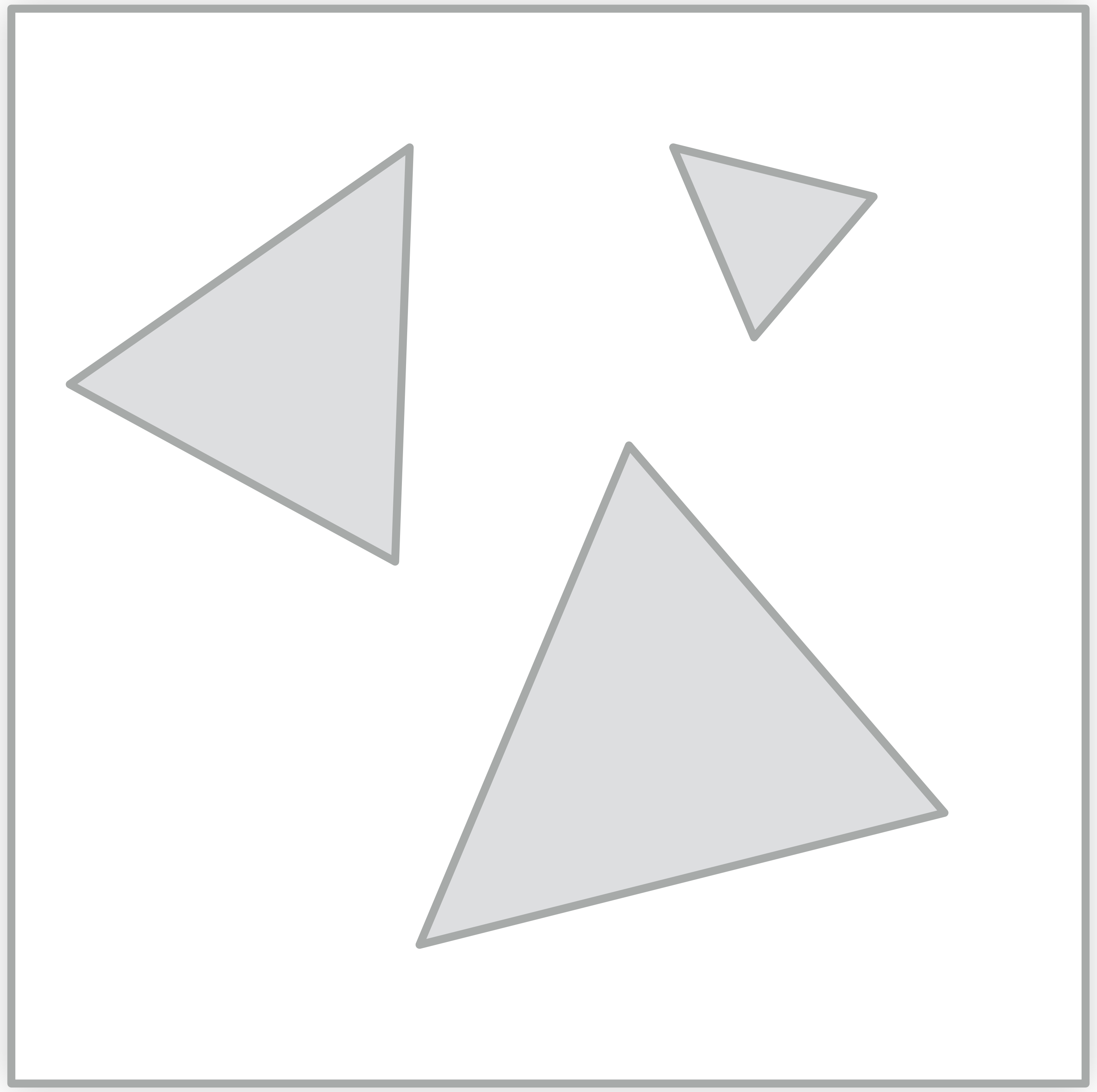

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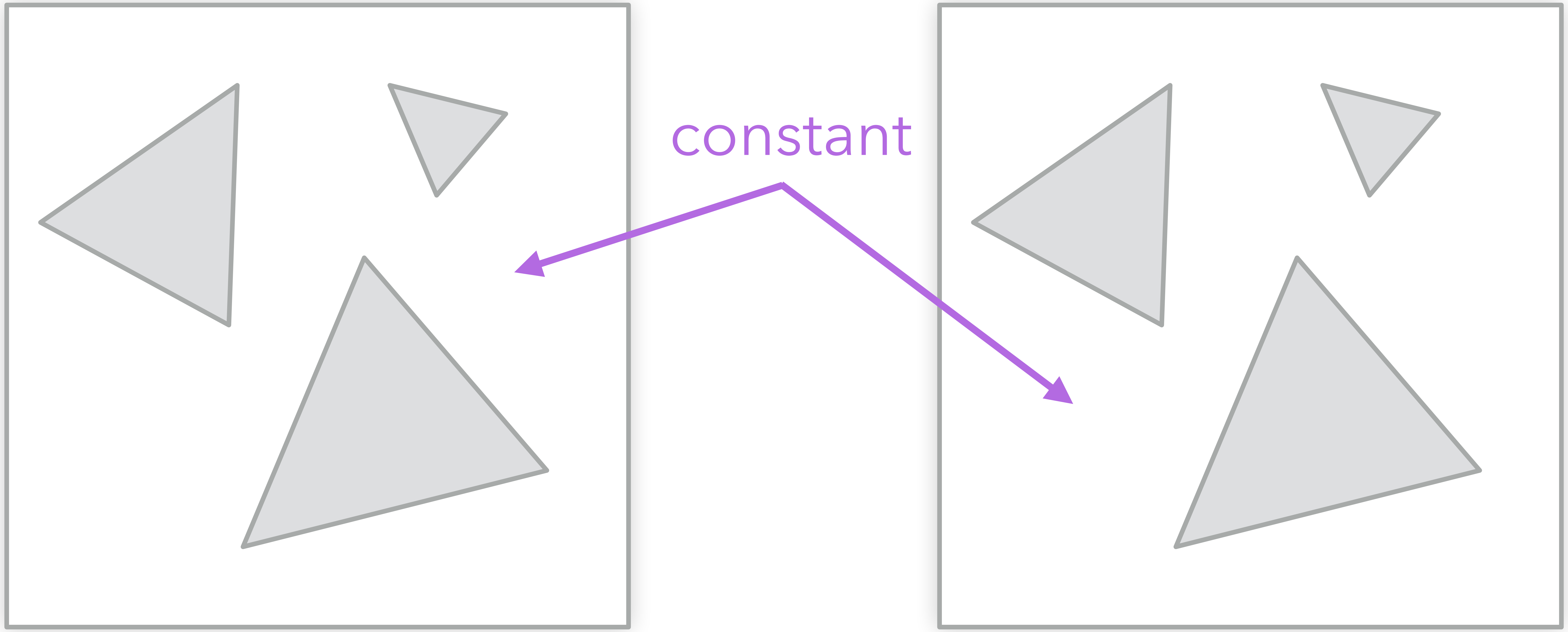
Point sampling **Line sampling**



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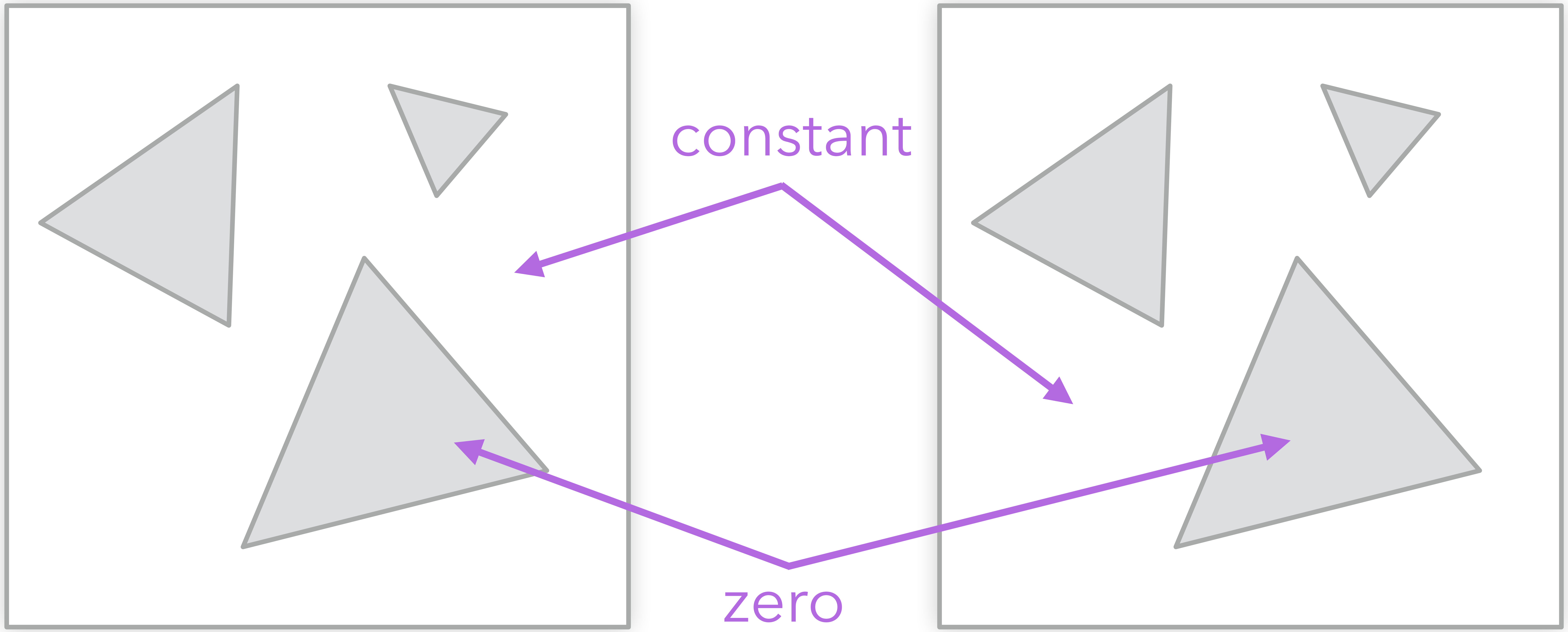
Main idea

Point sampling | **Line sampling**



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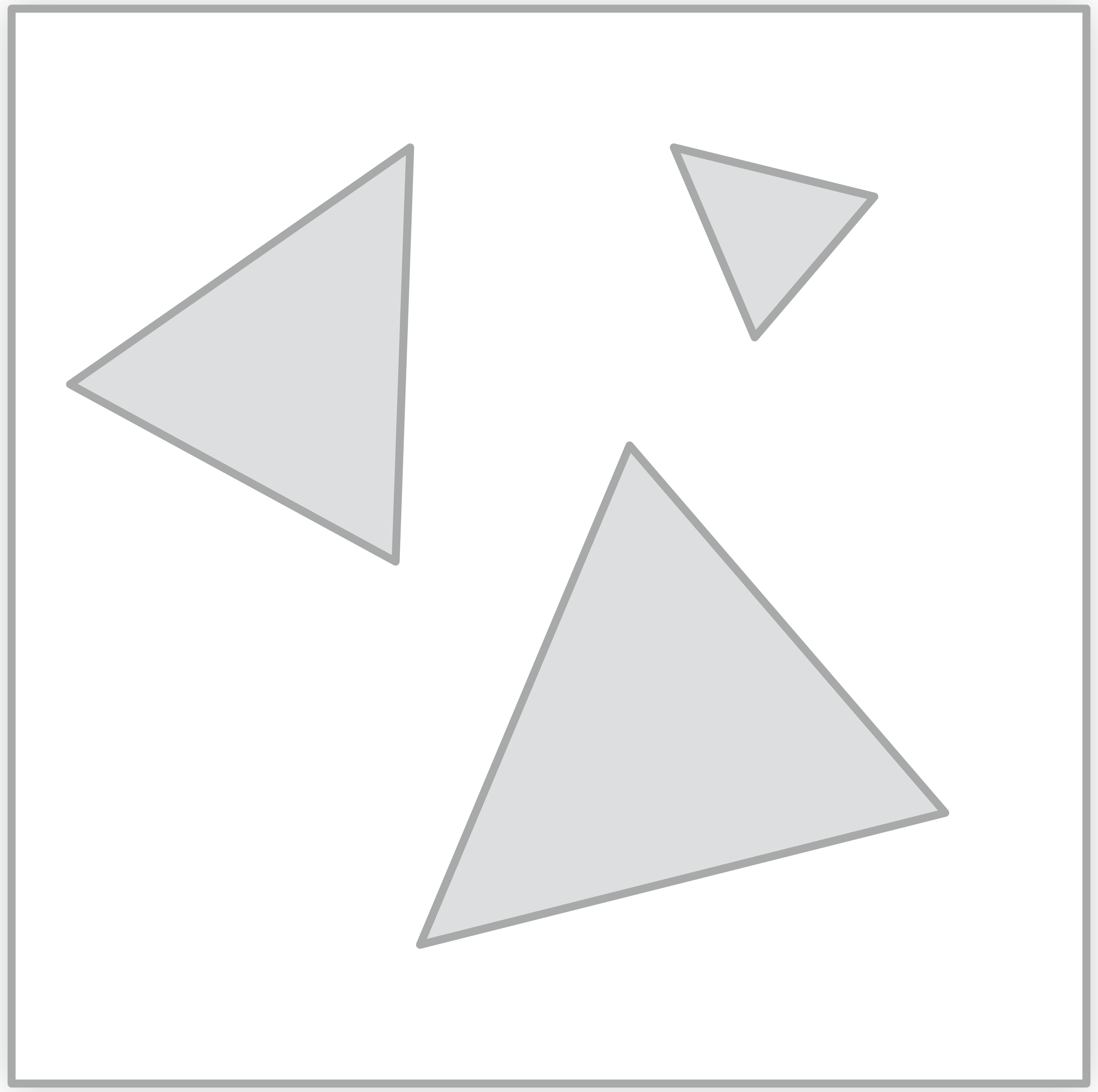
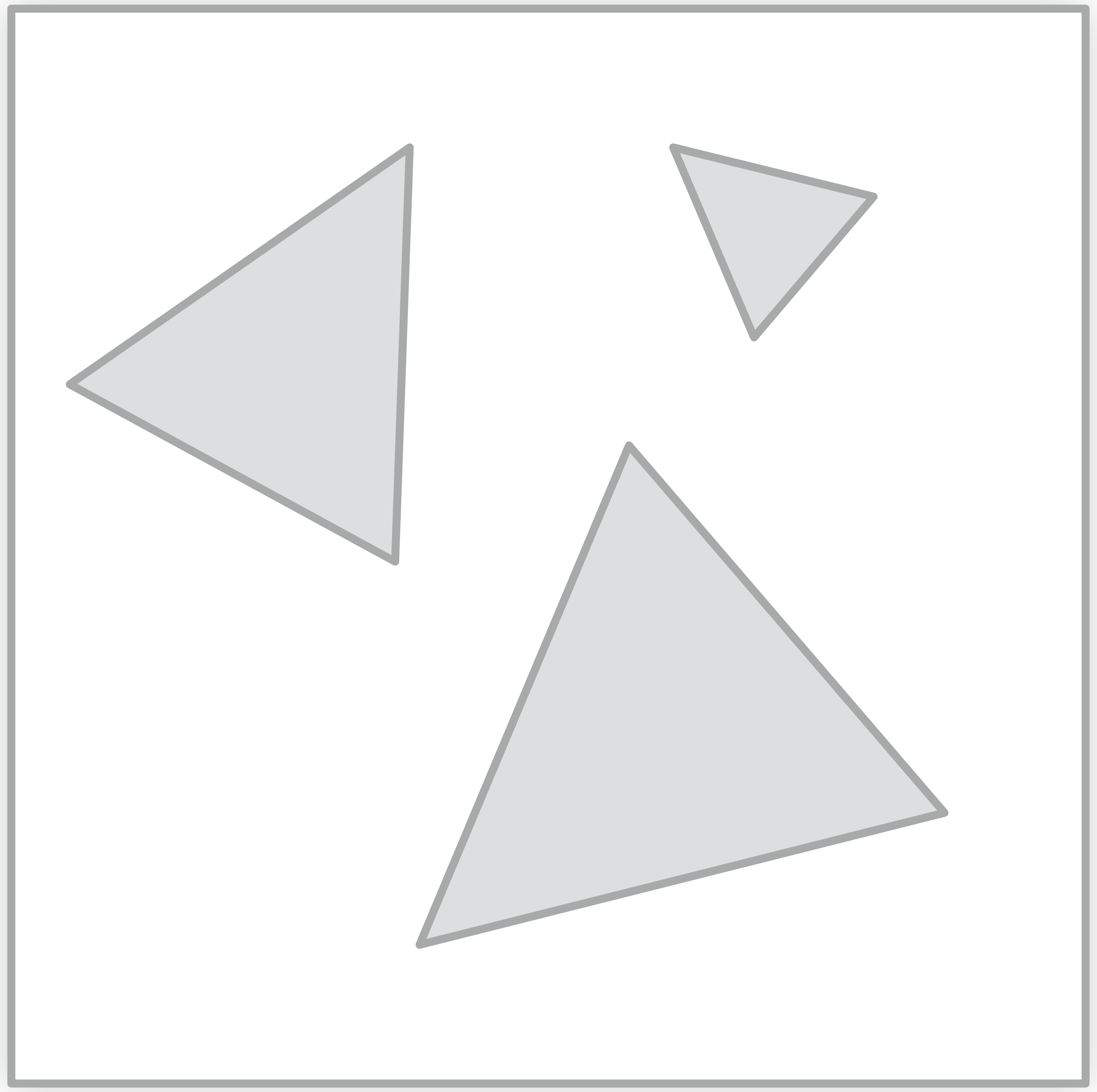


$$\langle L_o \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(u_i, v_i)}{p(u_i, v_i)}$$

Line sampling



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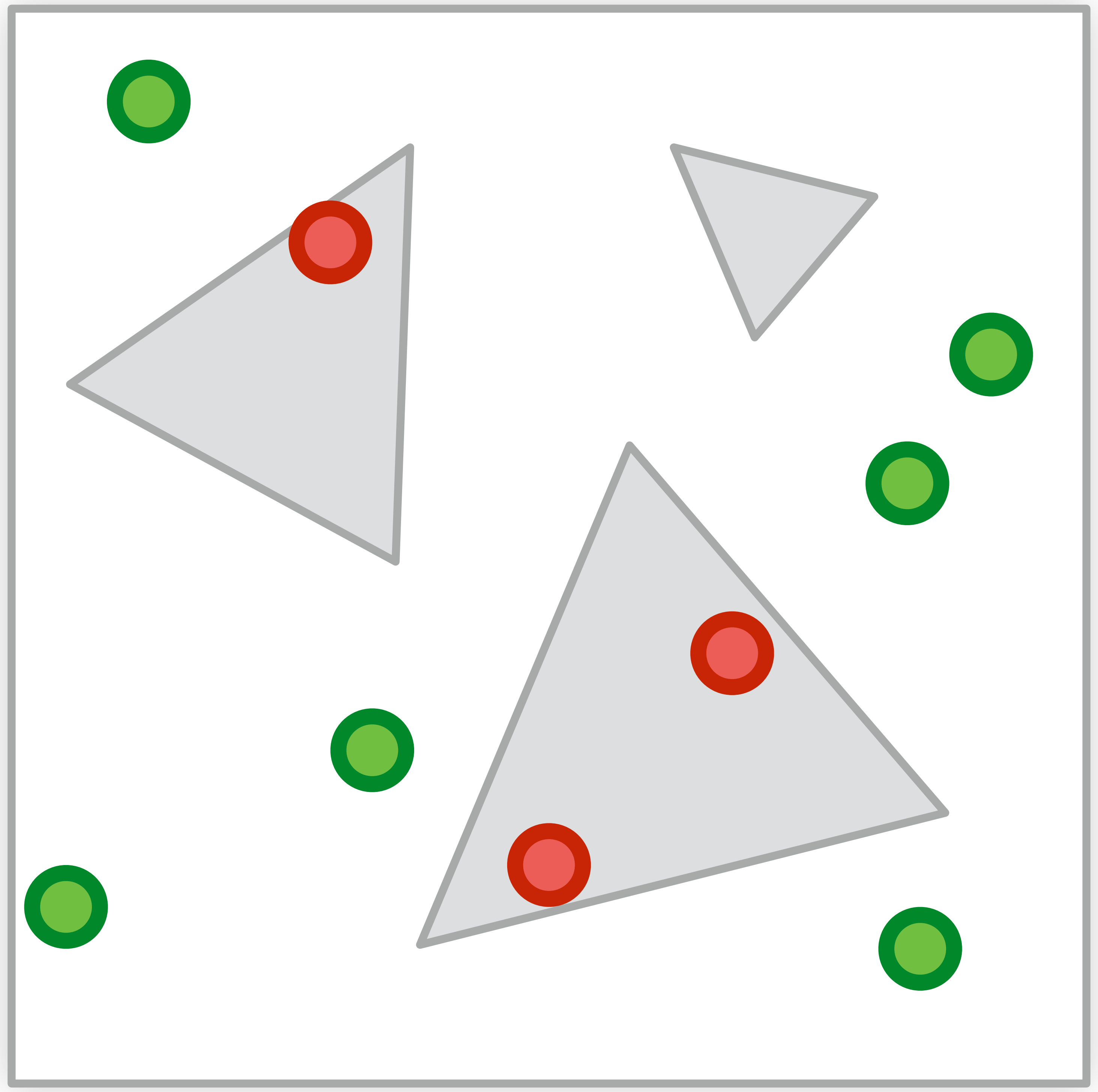

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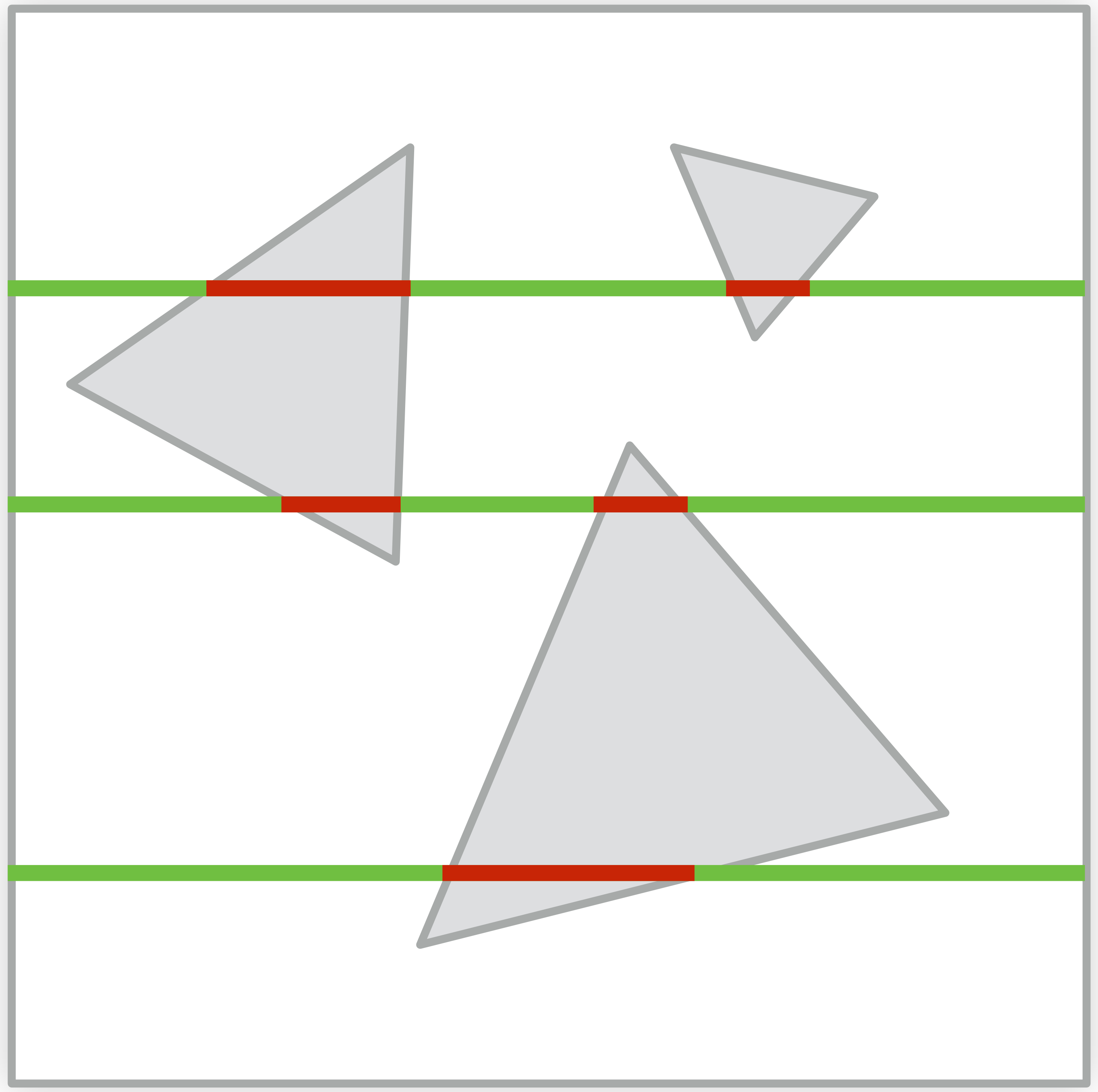

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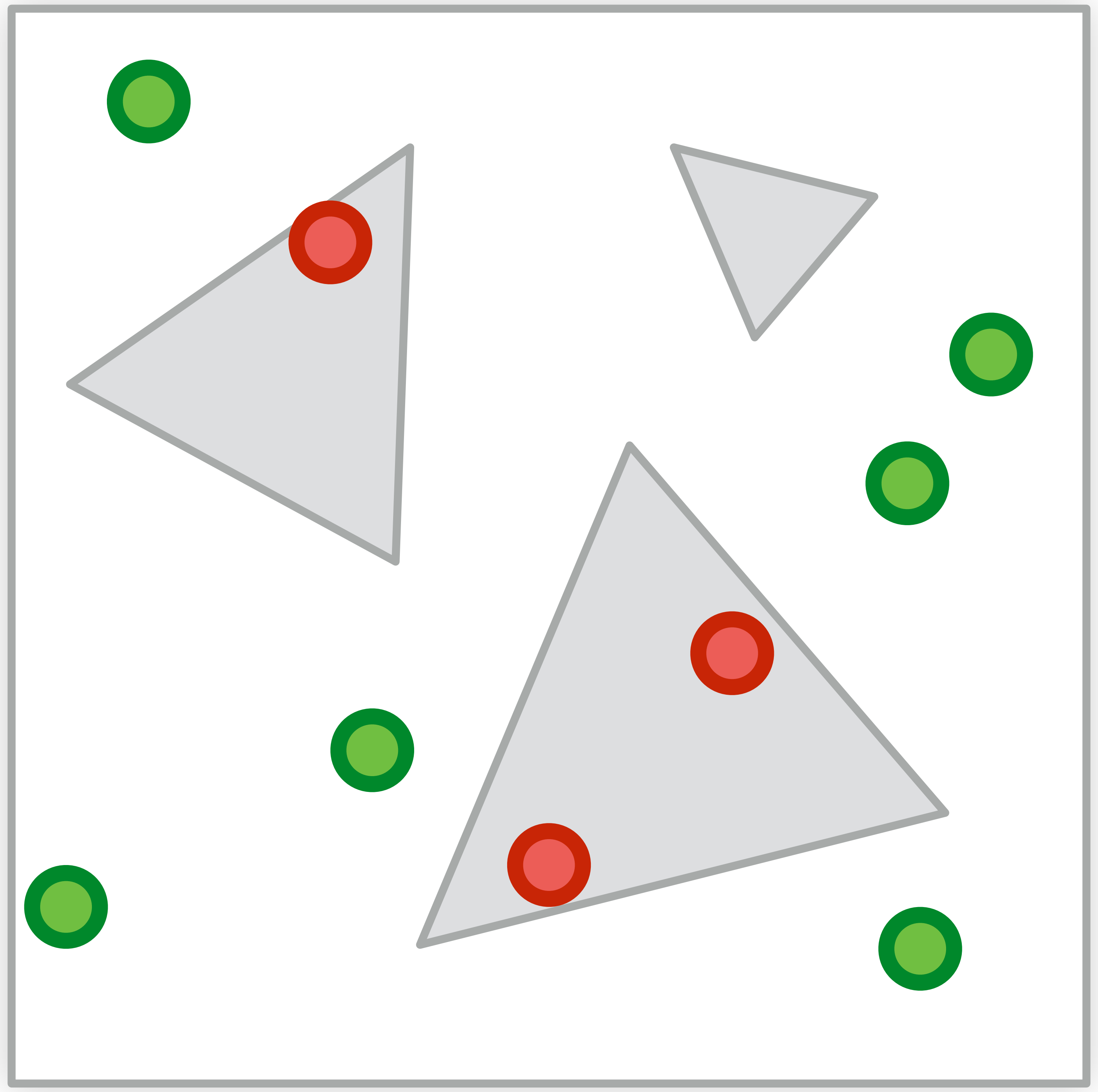

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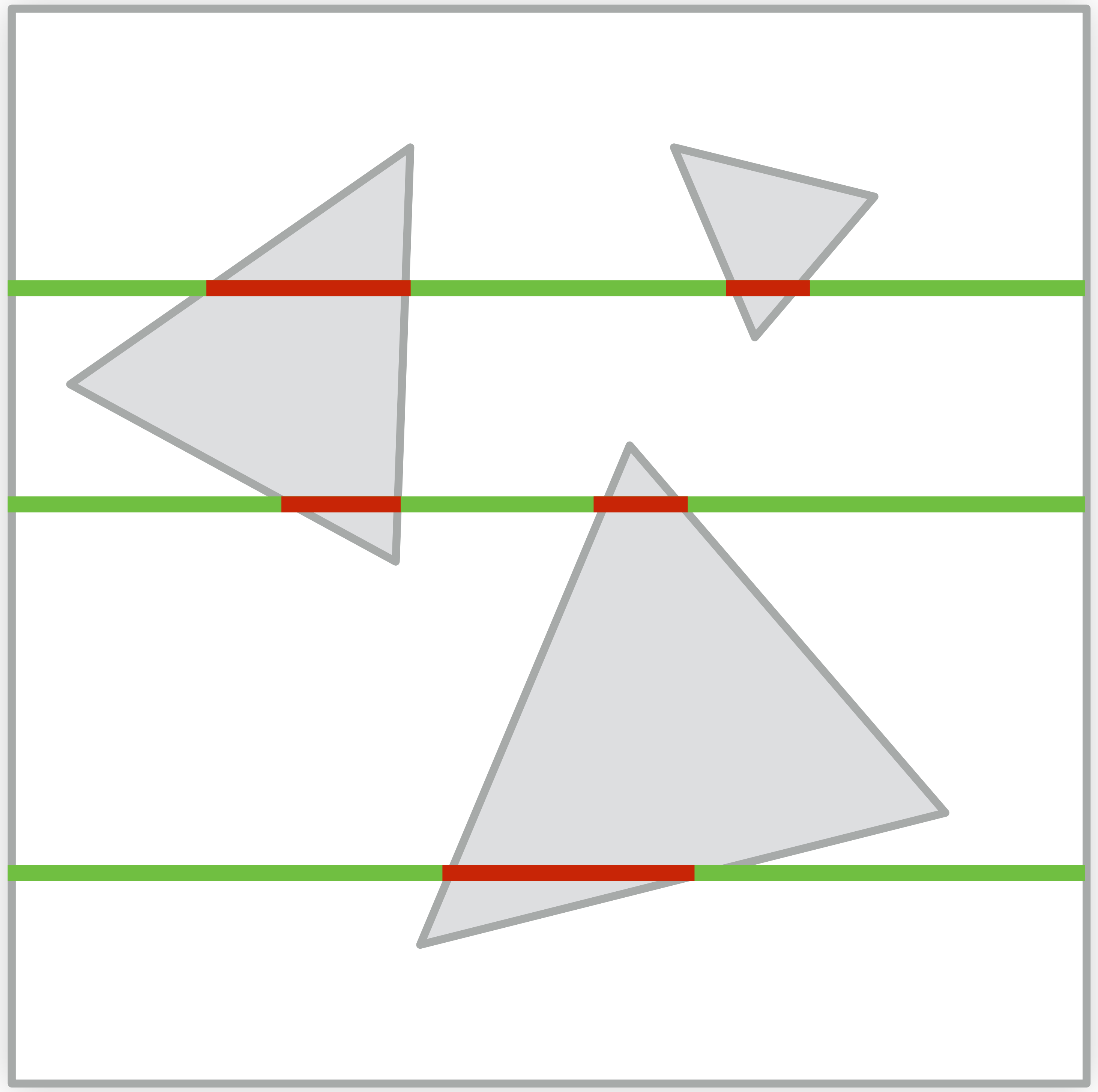

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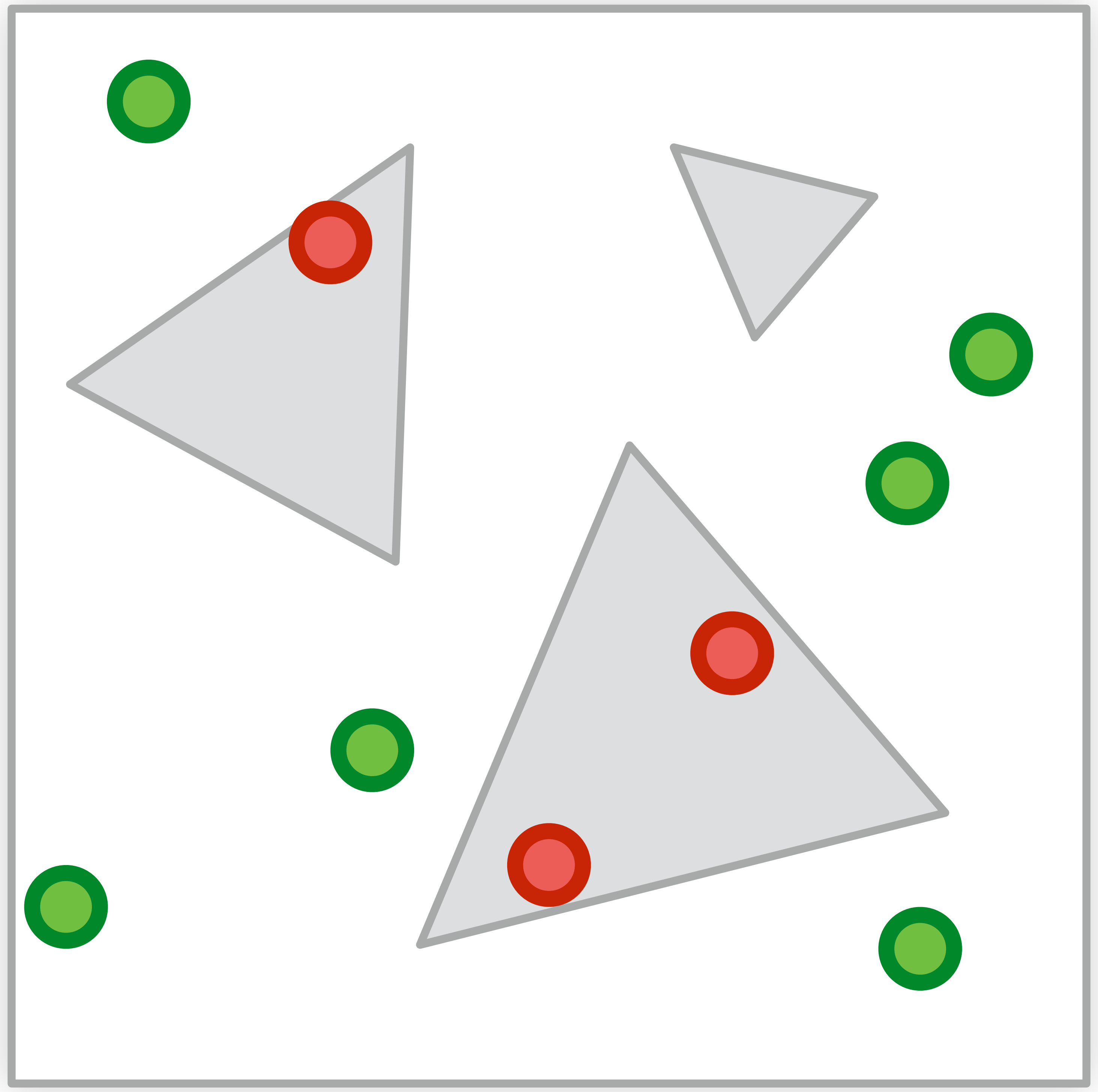

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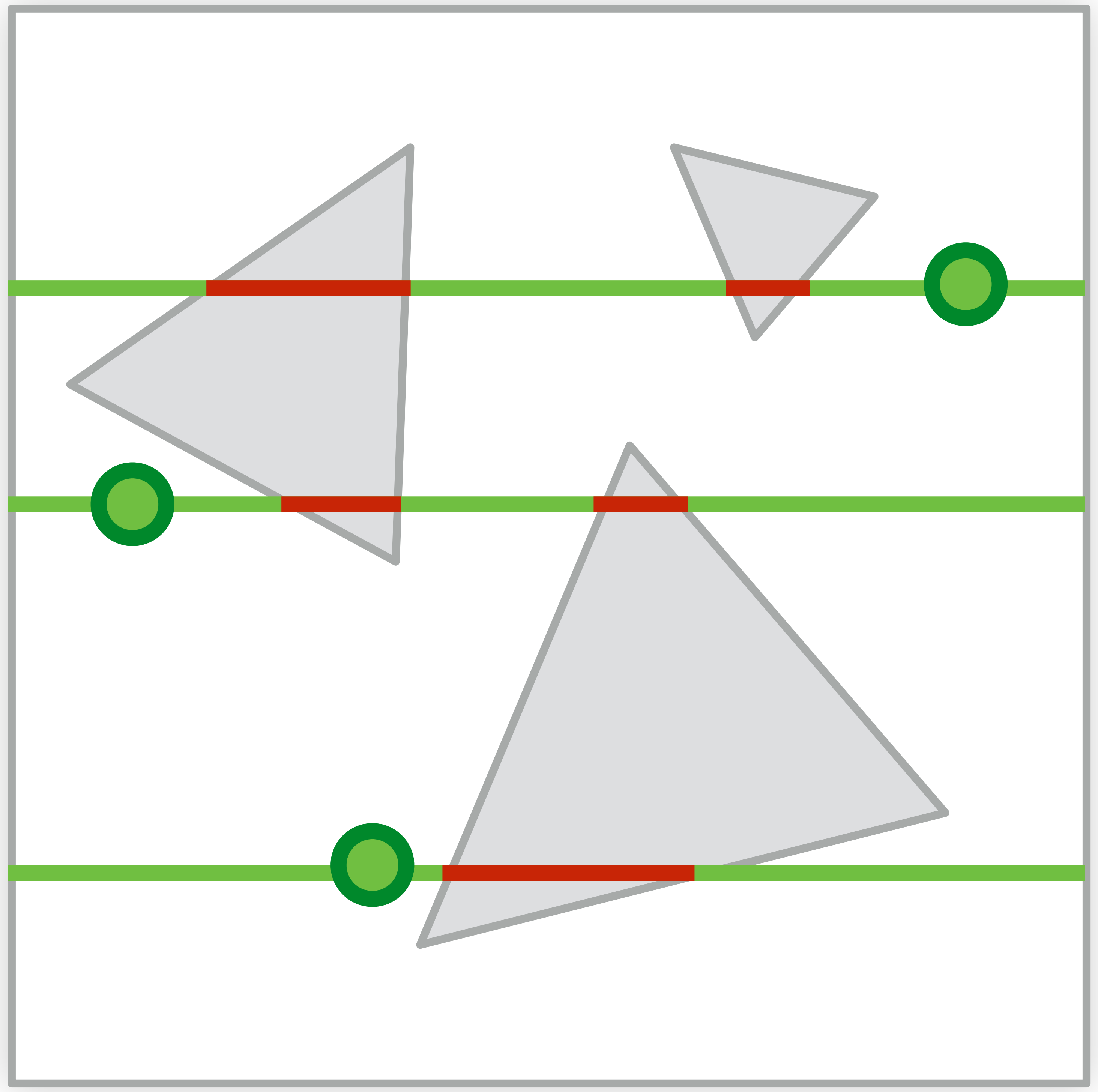

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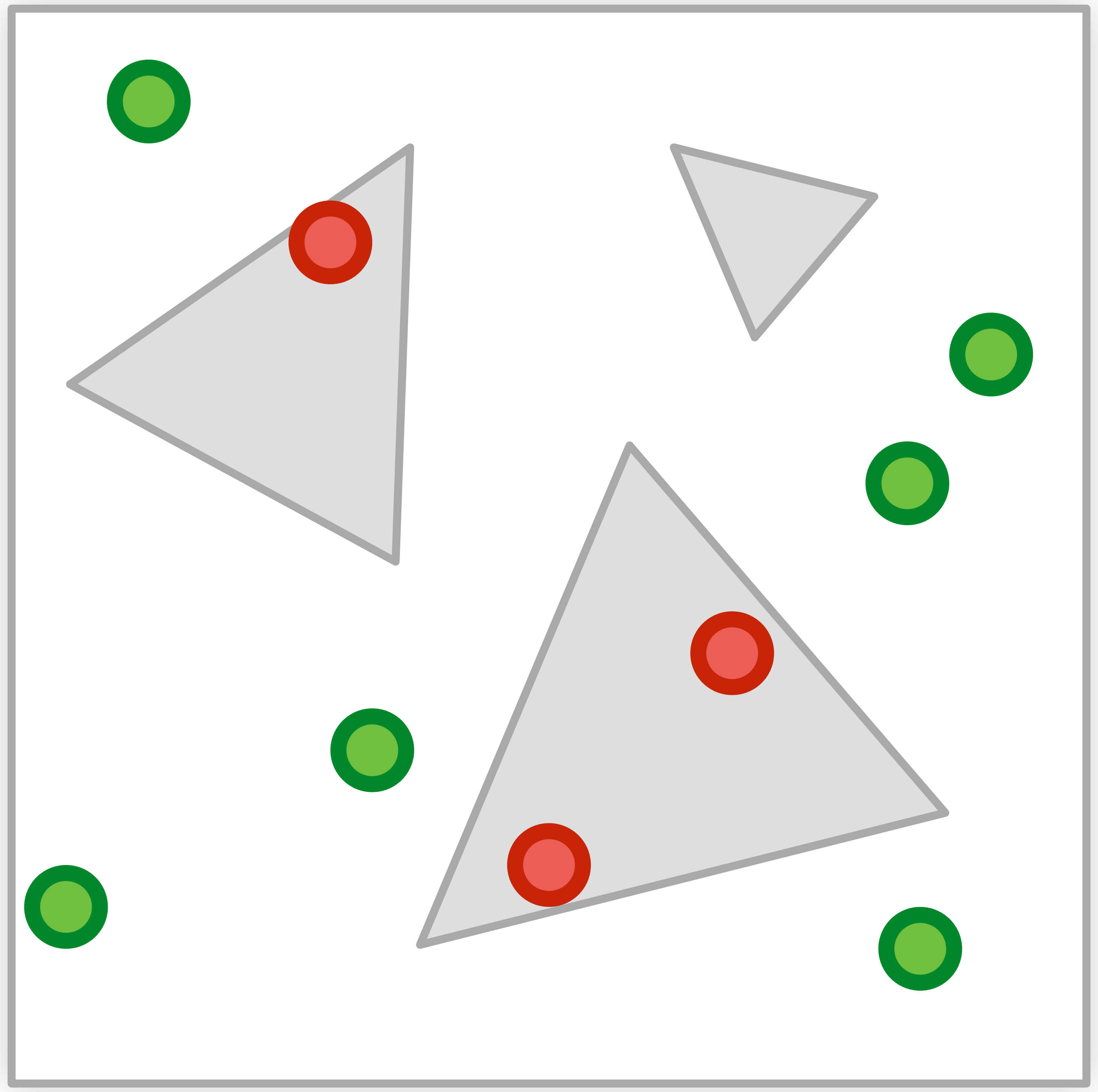

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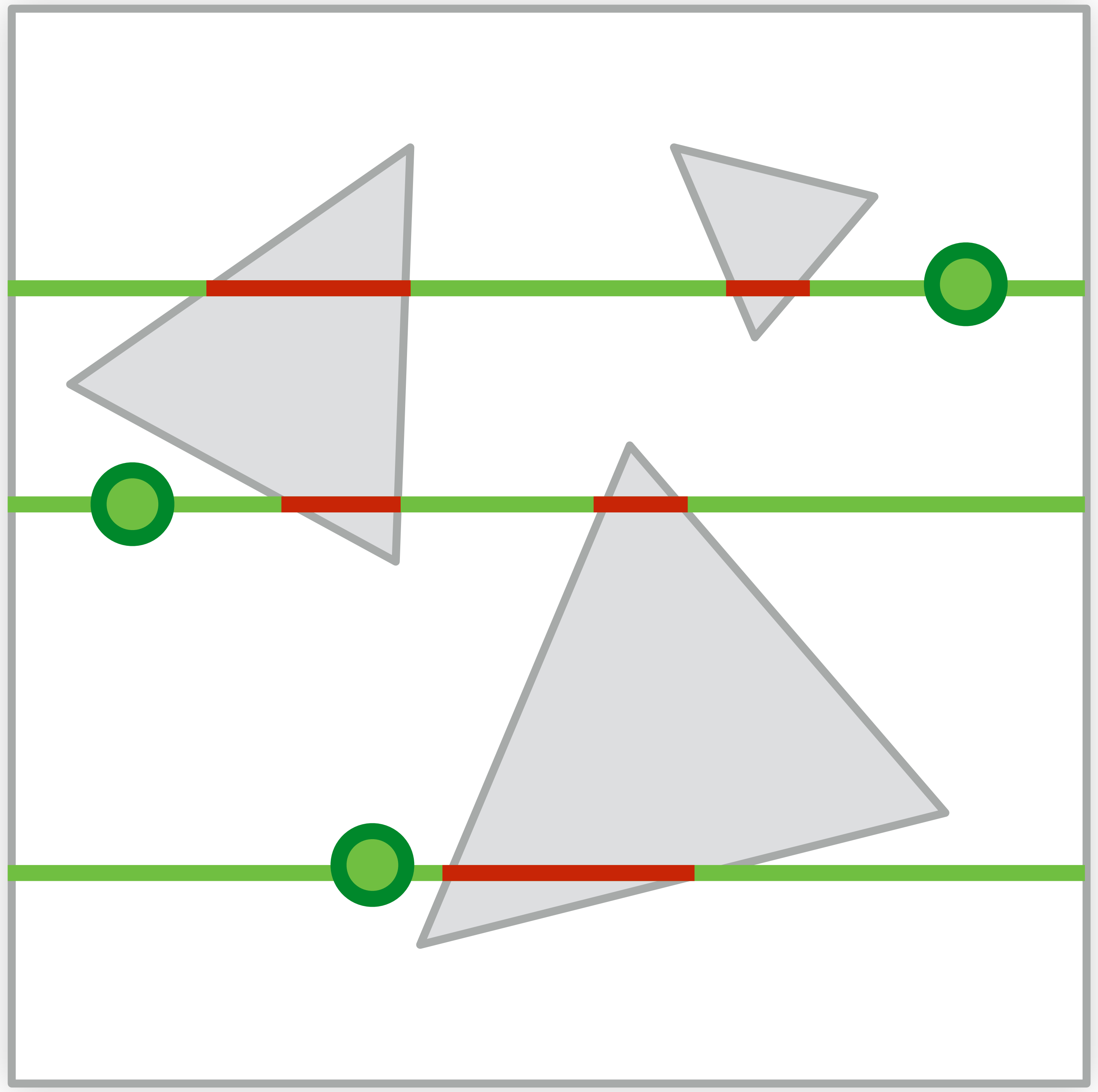

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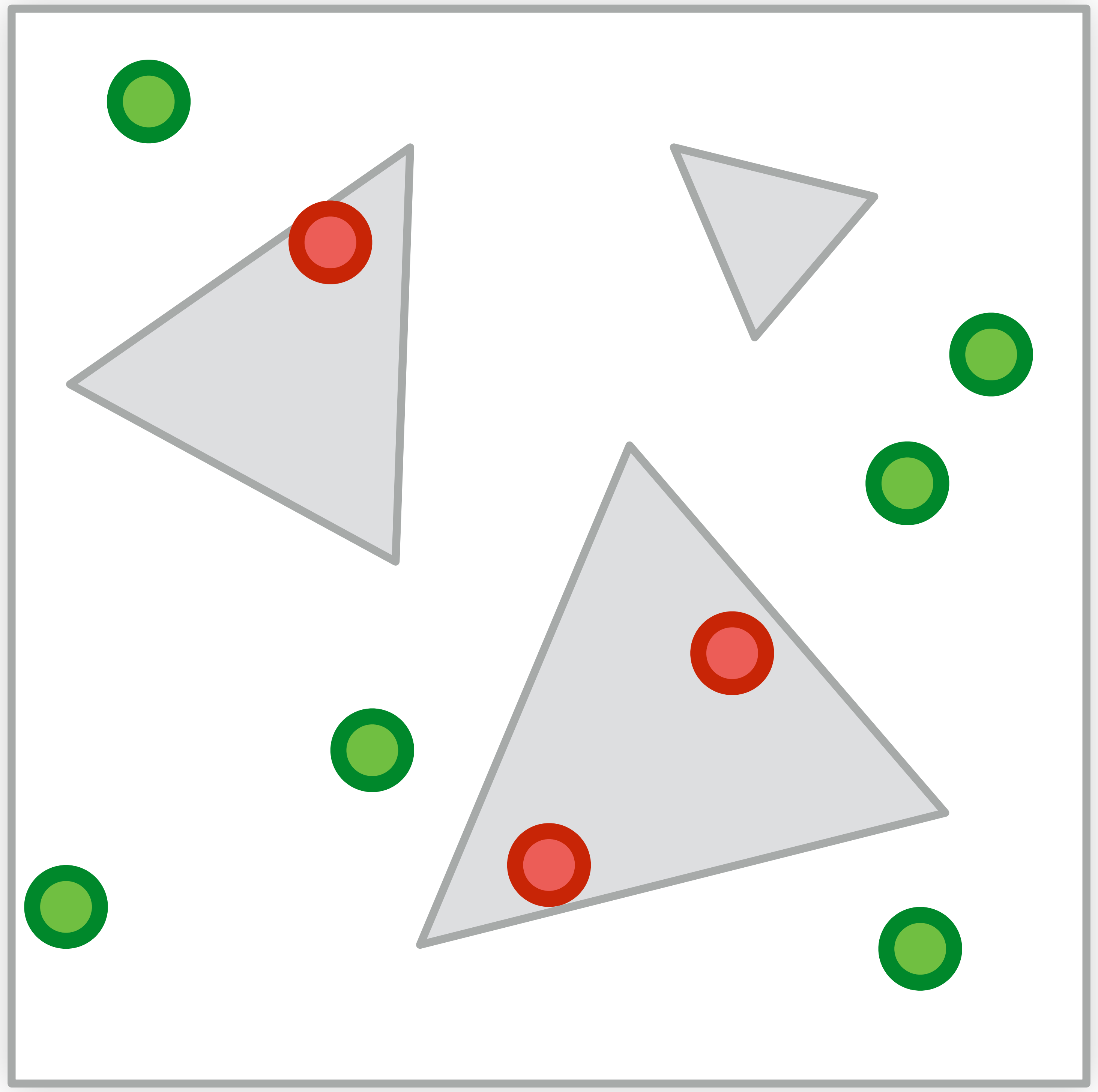

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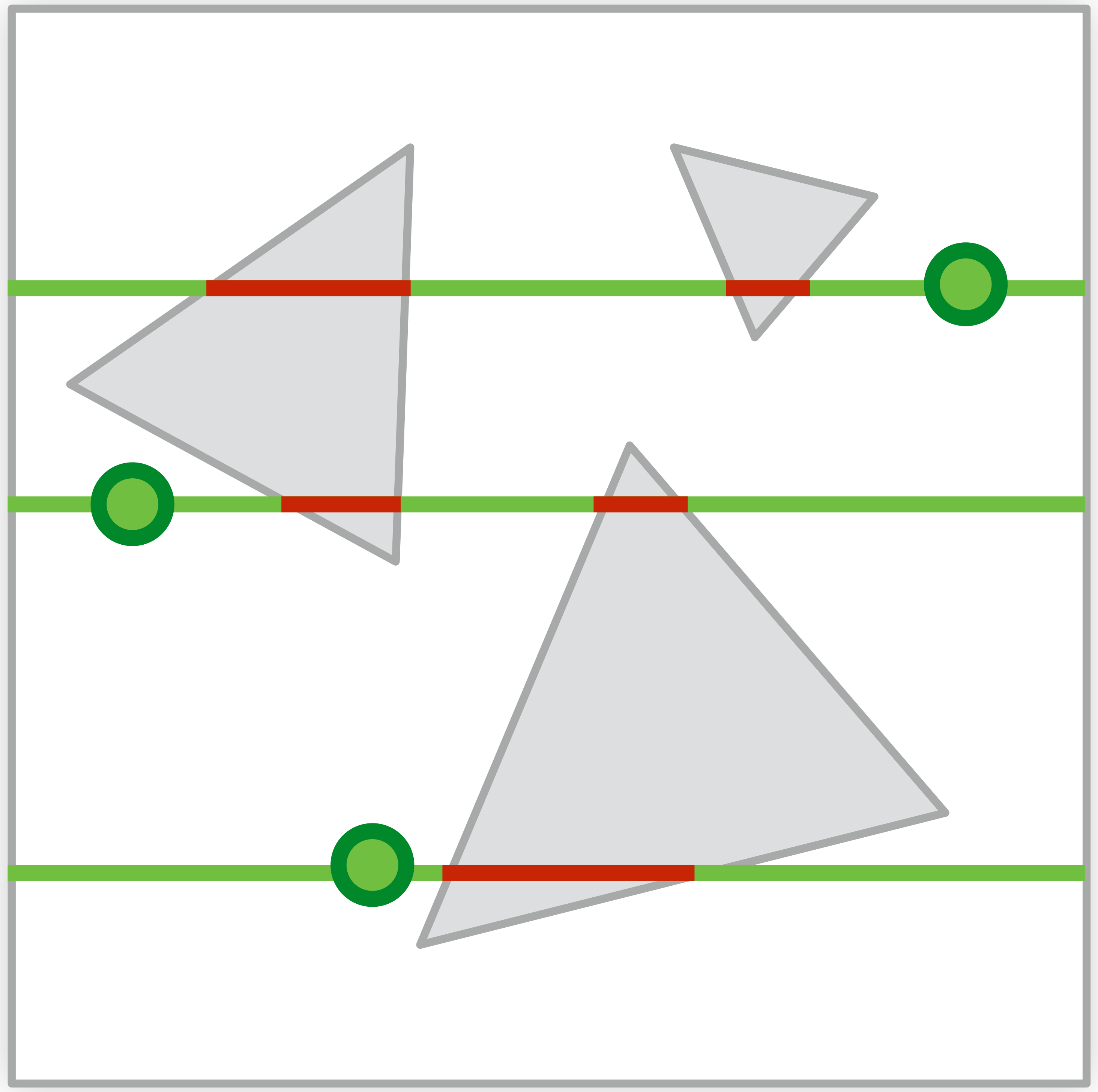

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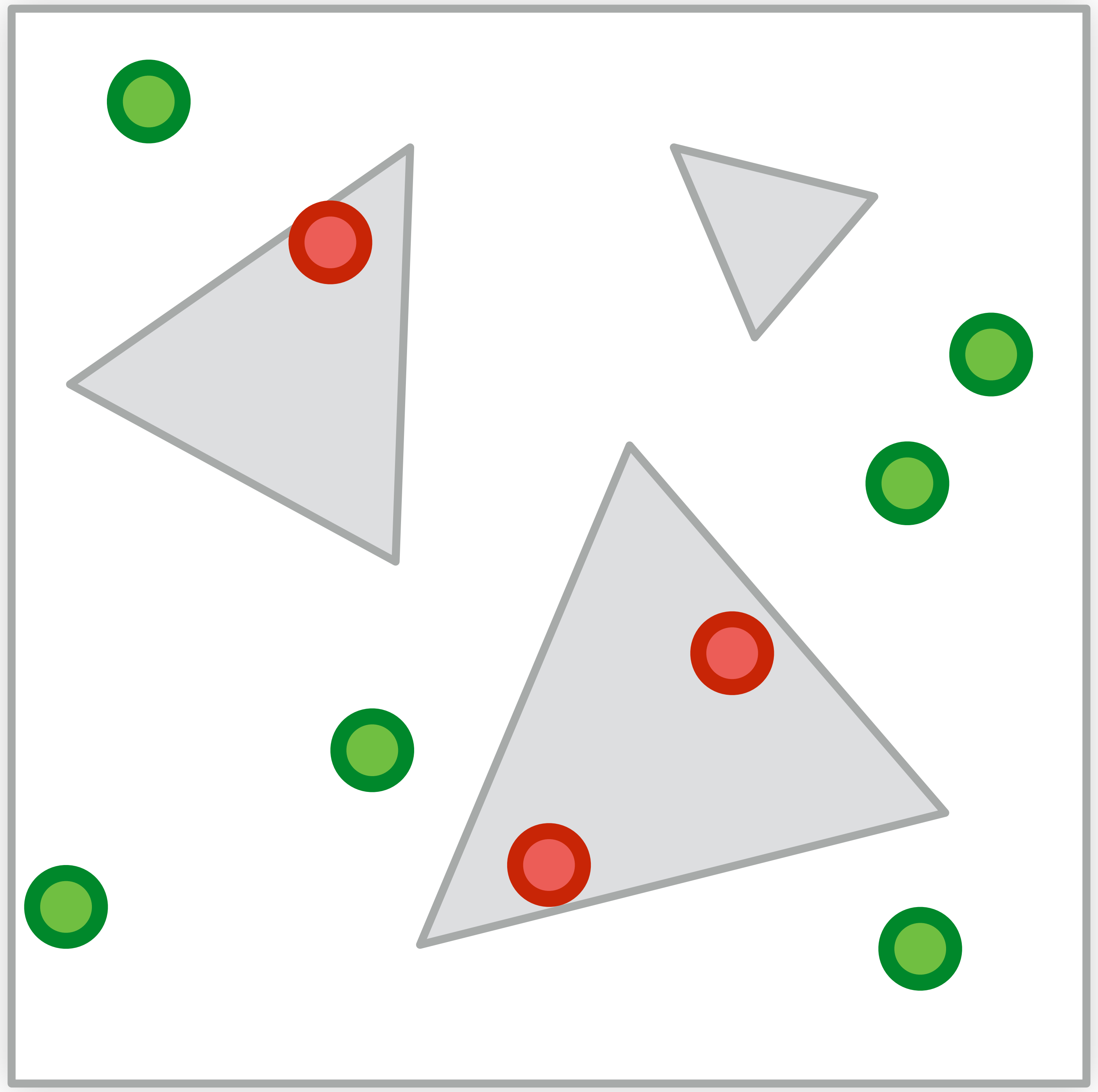

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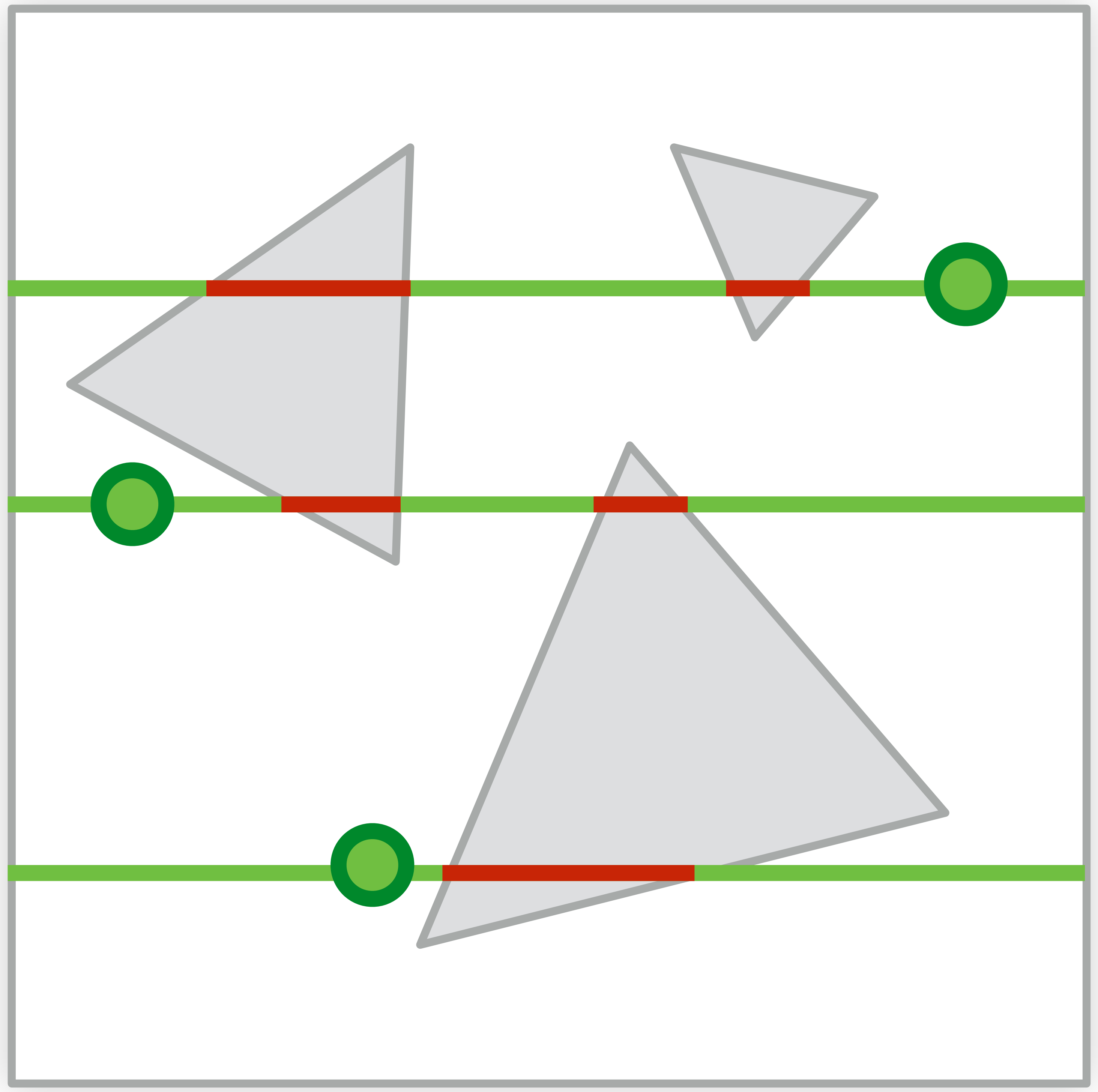

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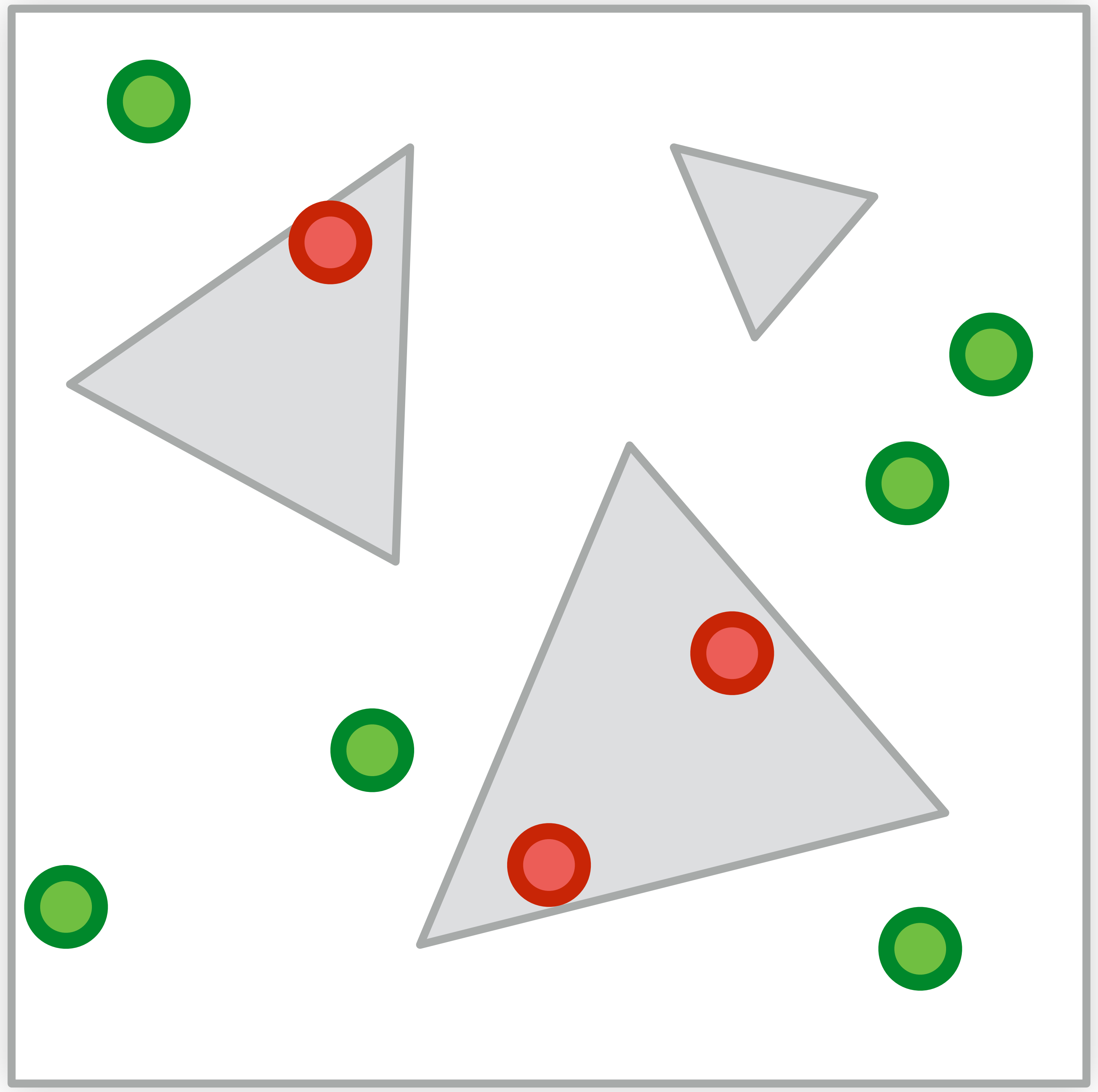

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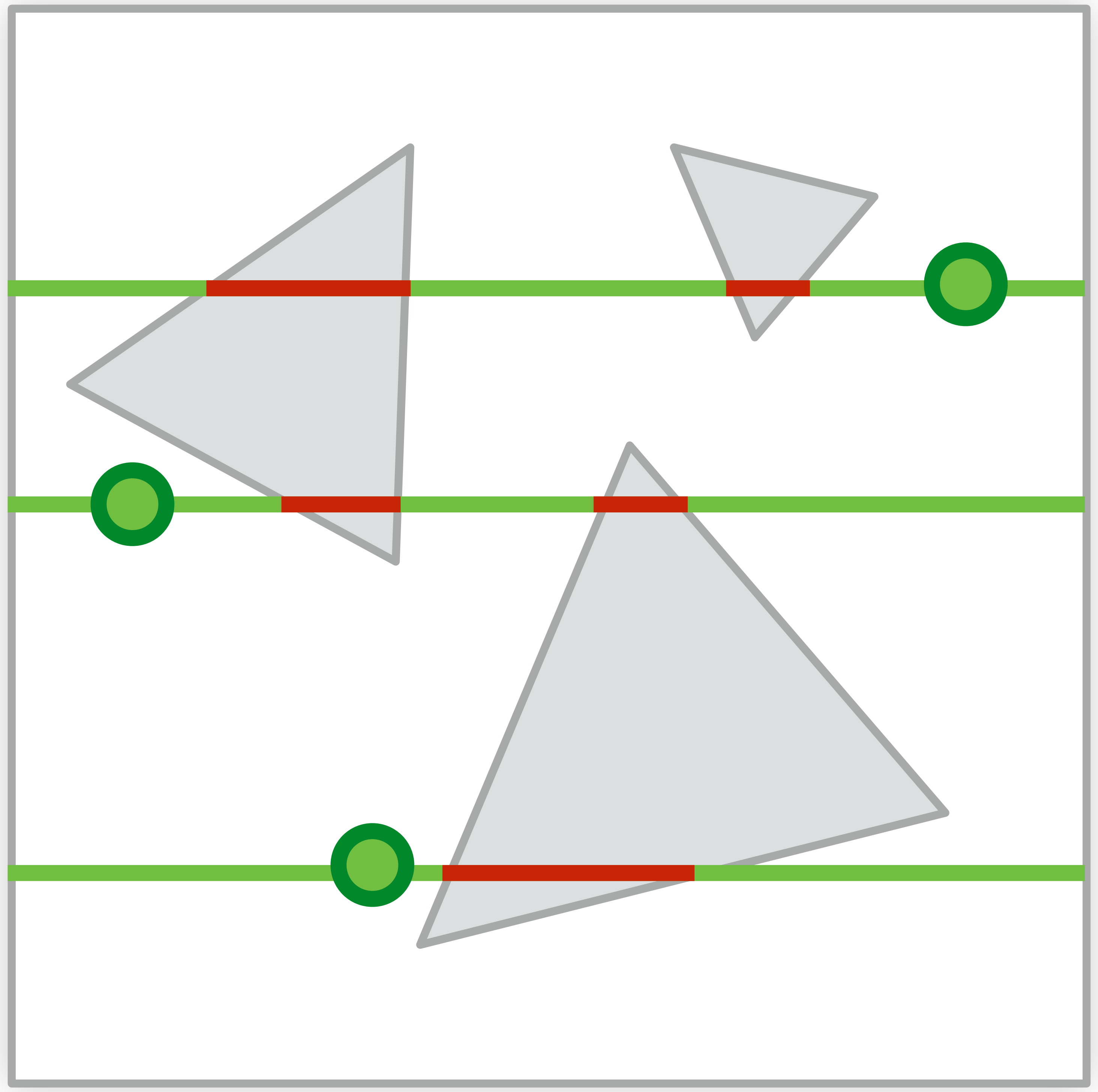

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Main idea

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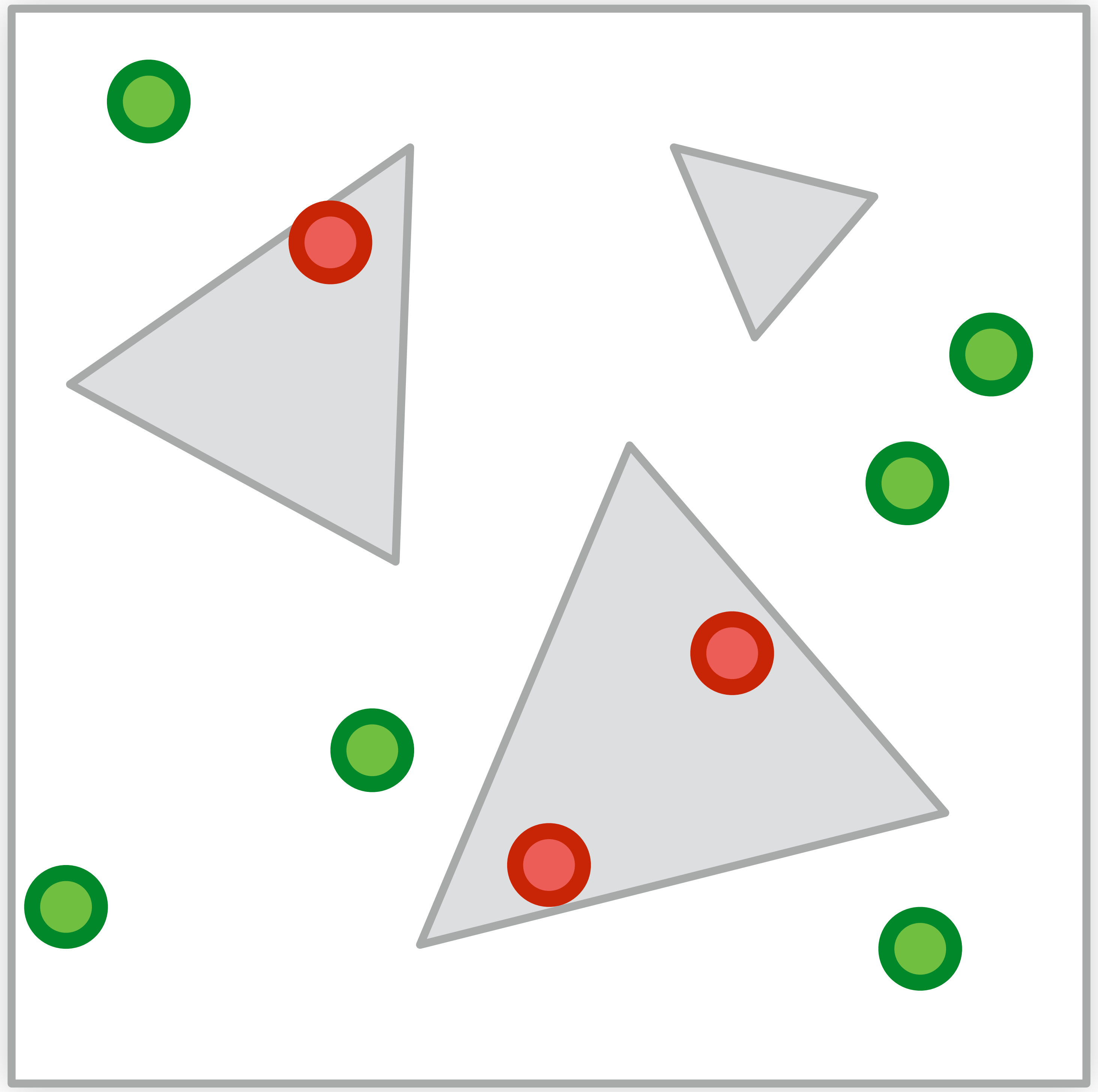

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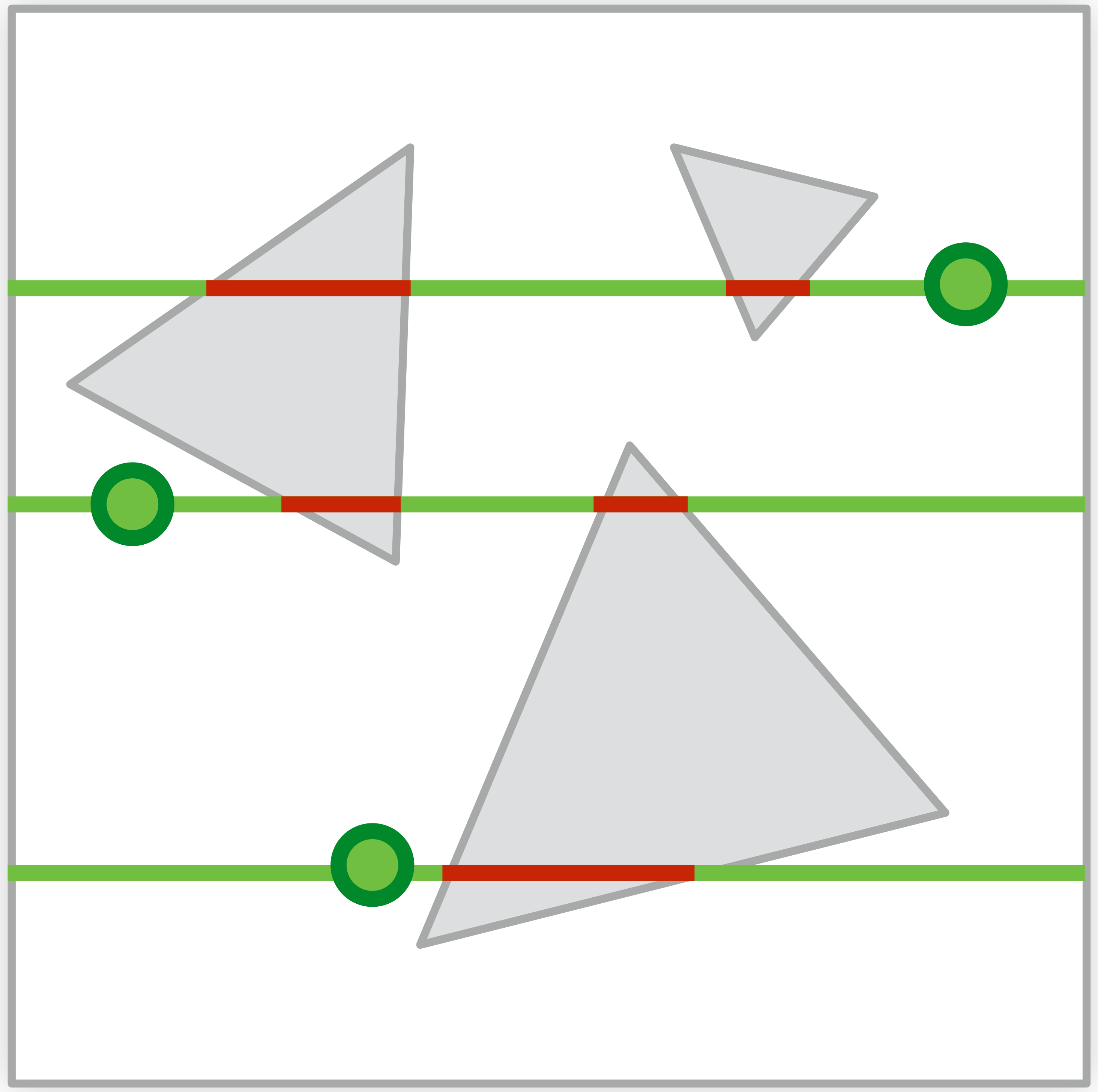

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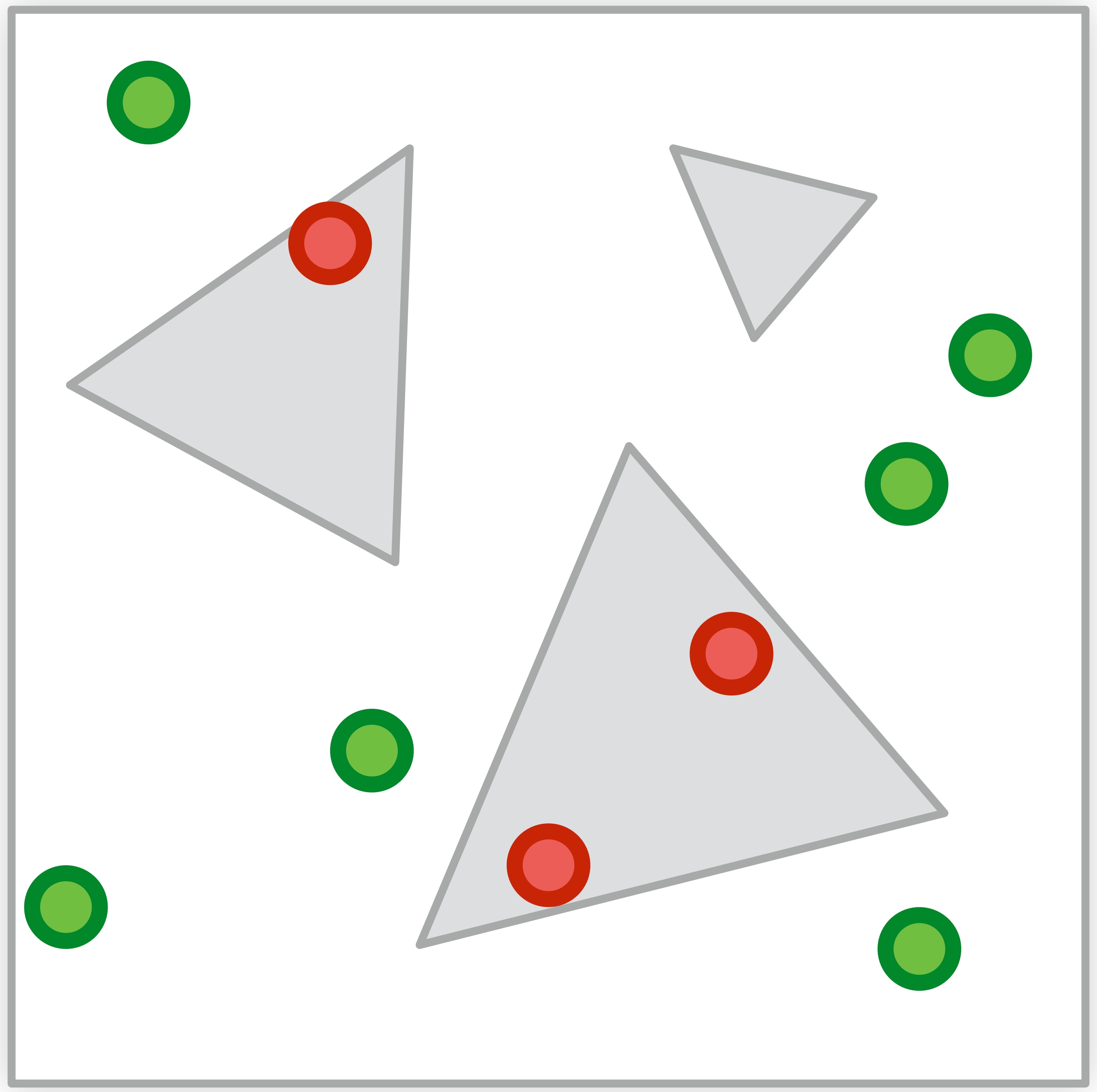

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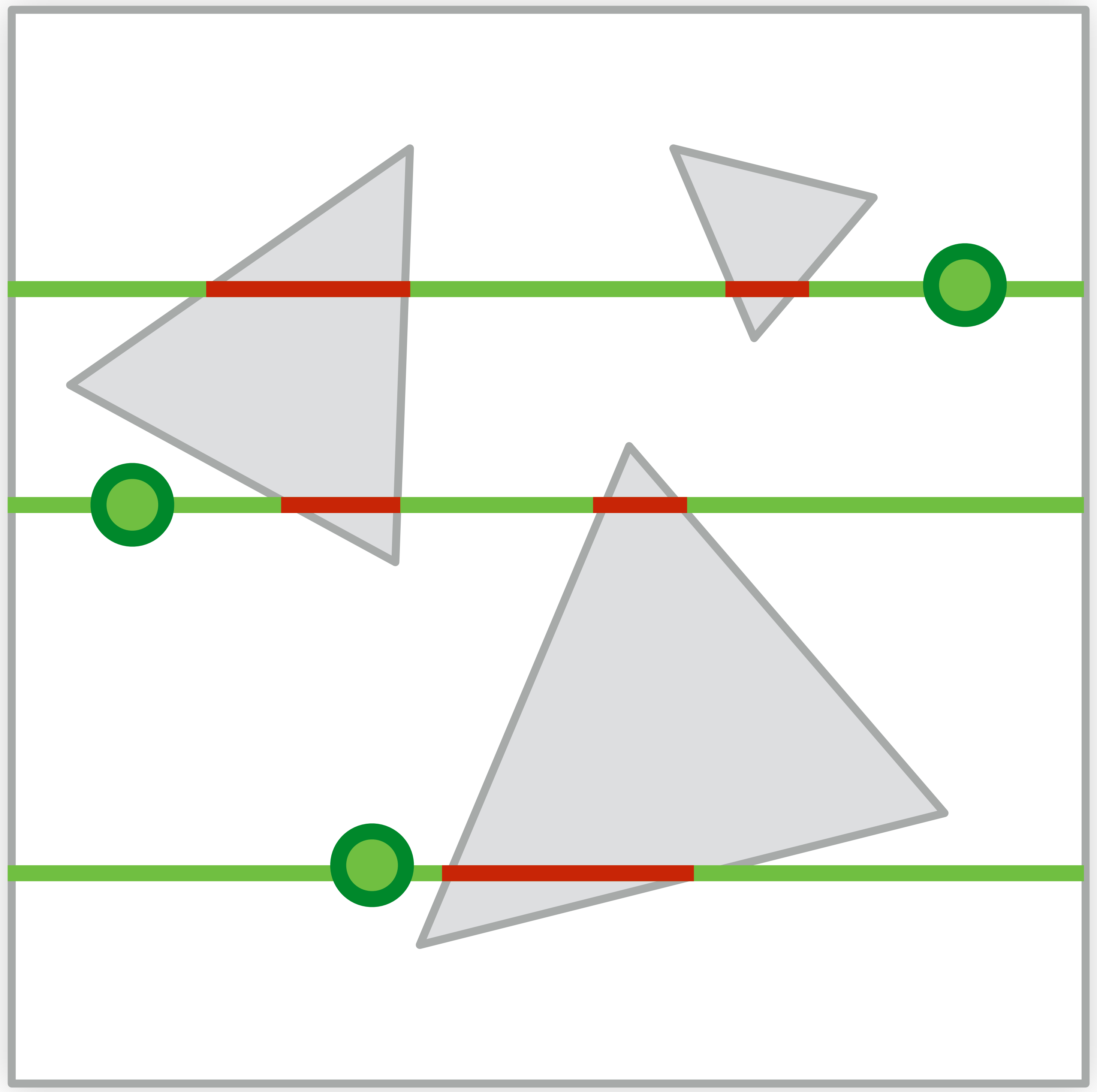

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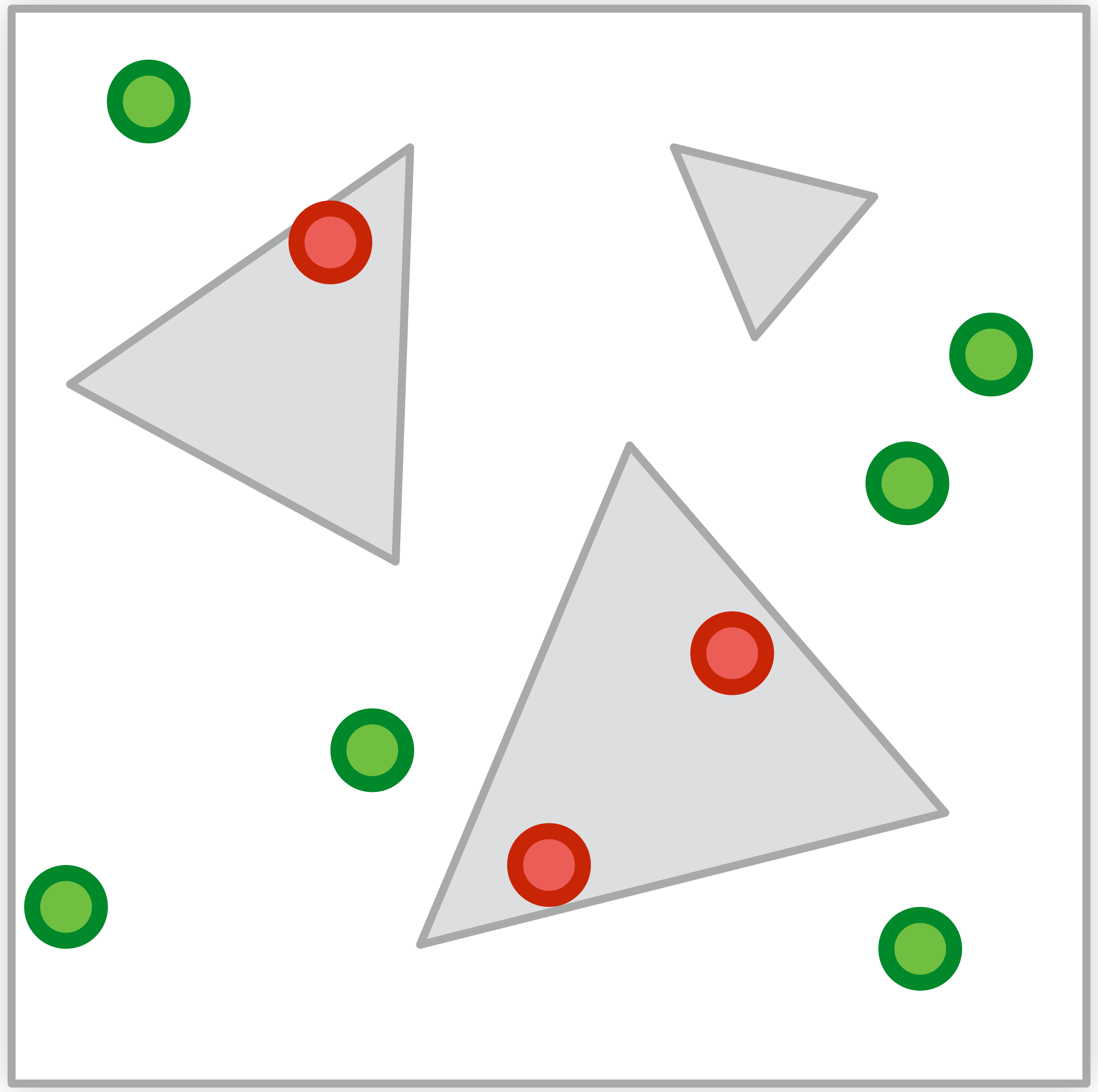

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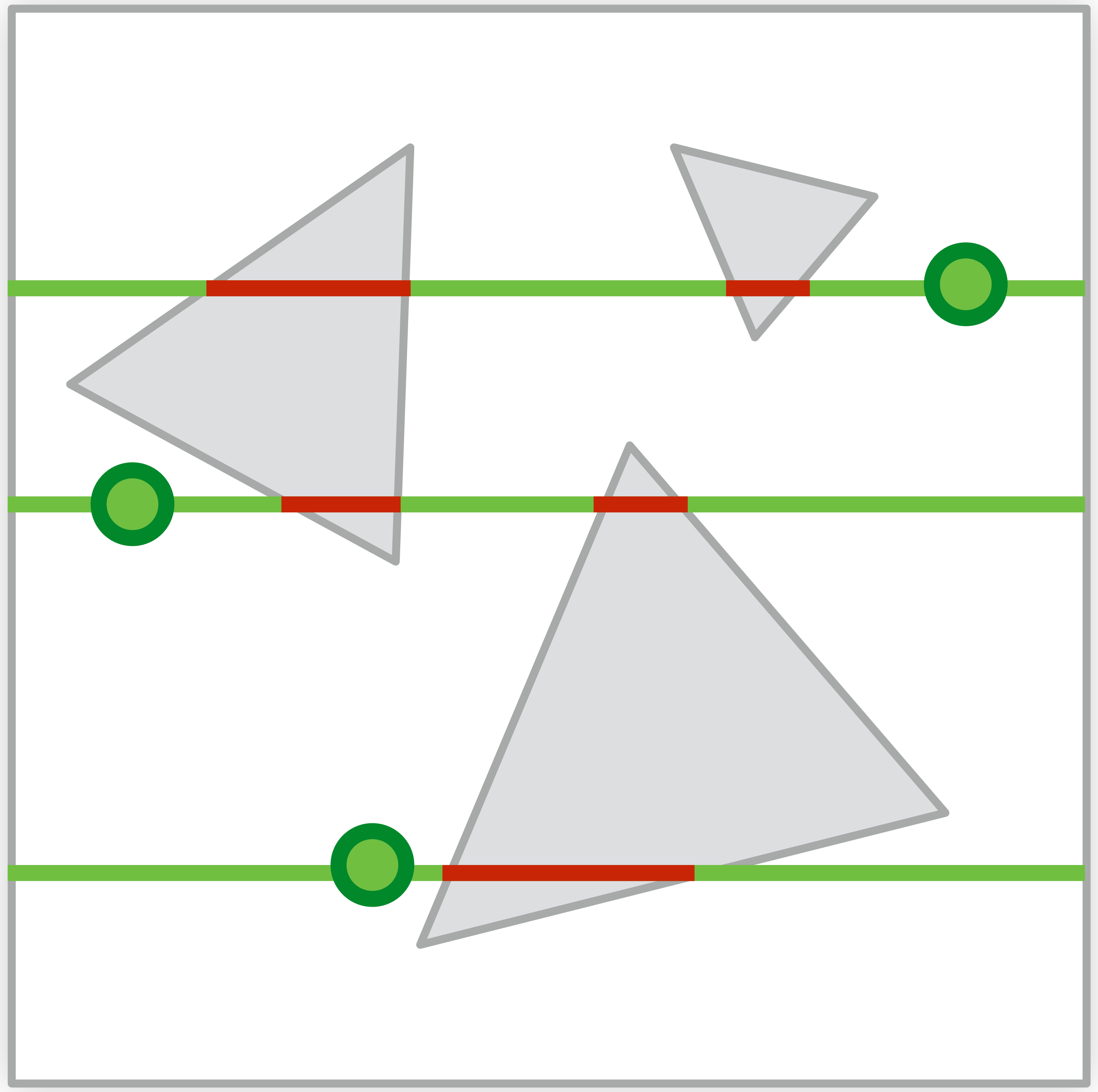

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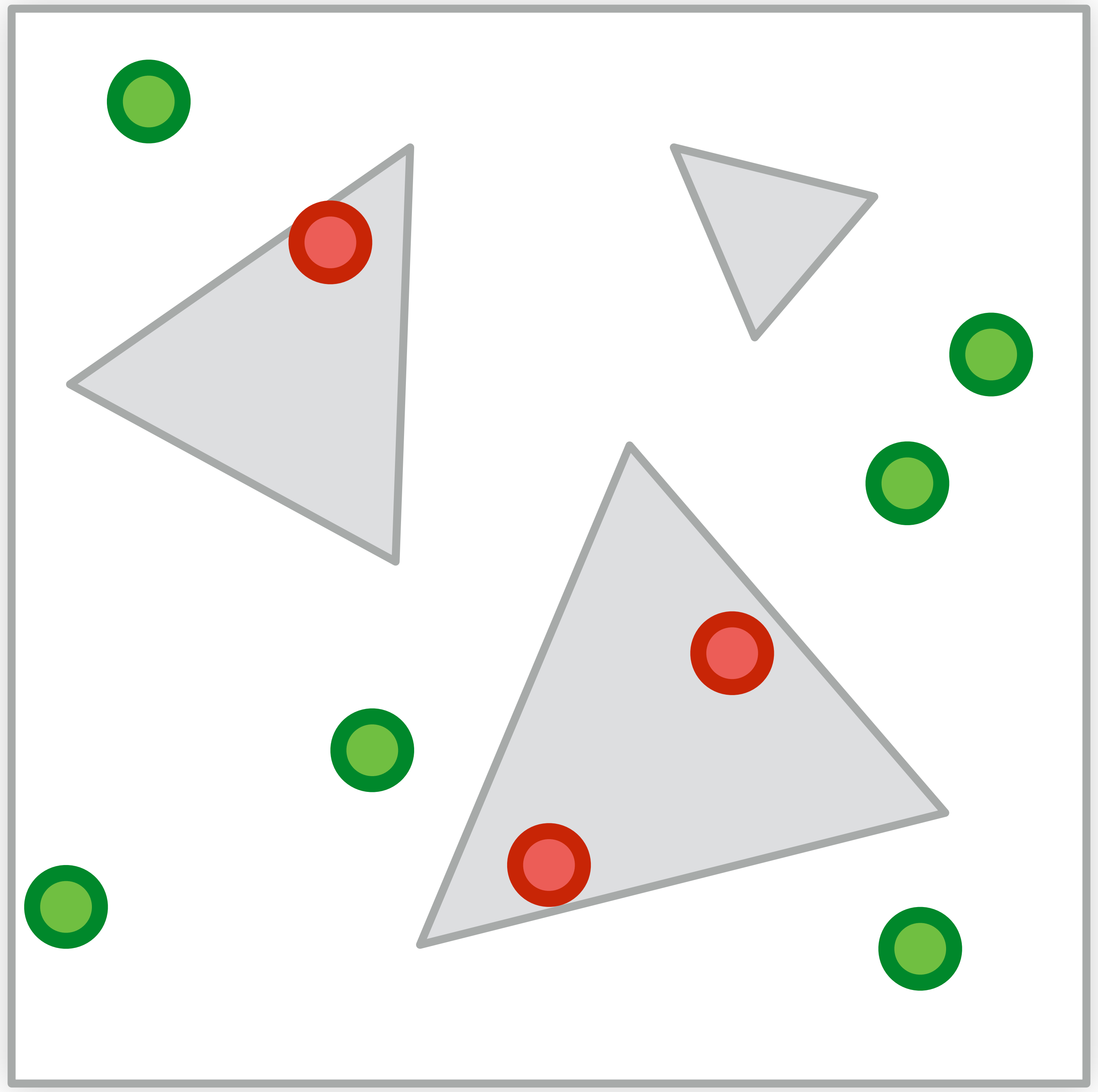

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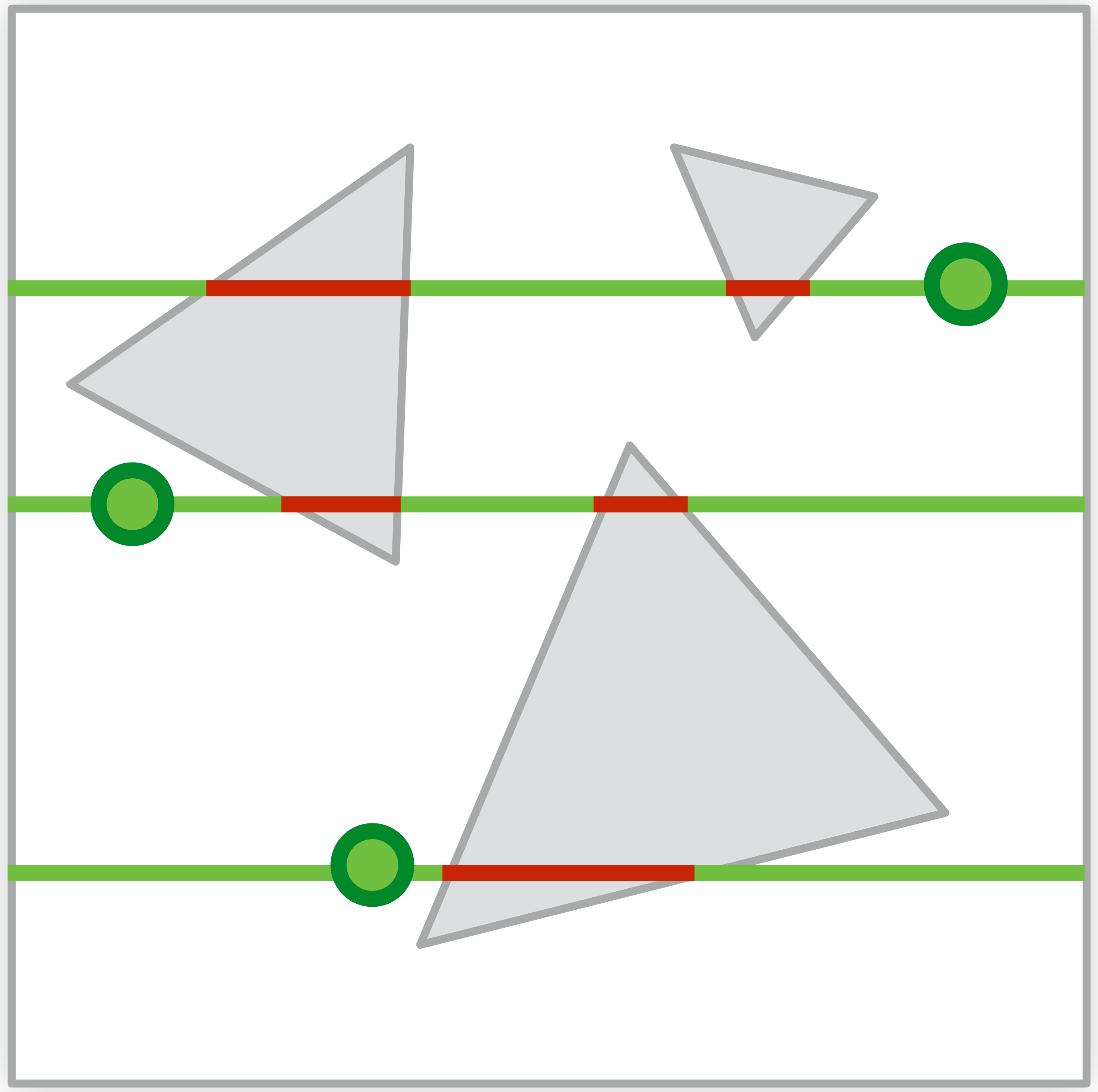

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Main idea

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  - Instead, we use simpler conditional pdfs that work well in practice
- Effectively **importance sampling visibility**

Main idea

Conditional point pdfs

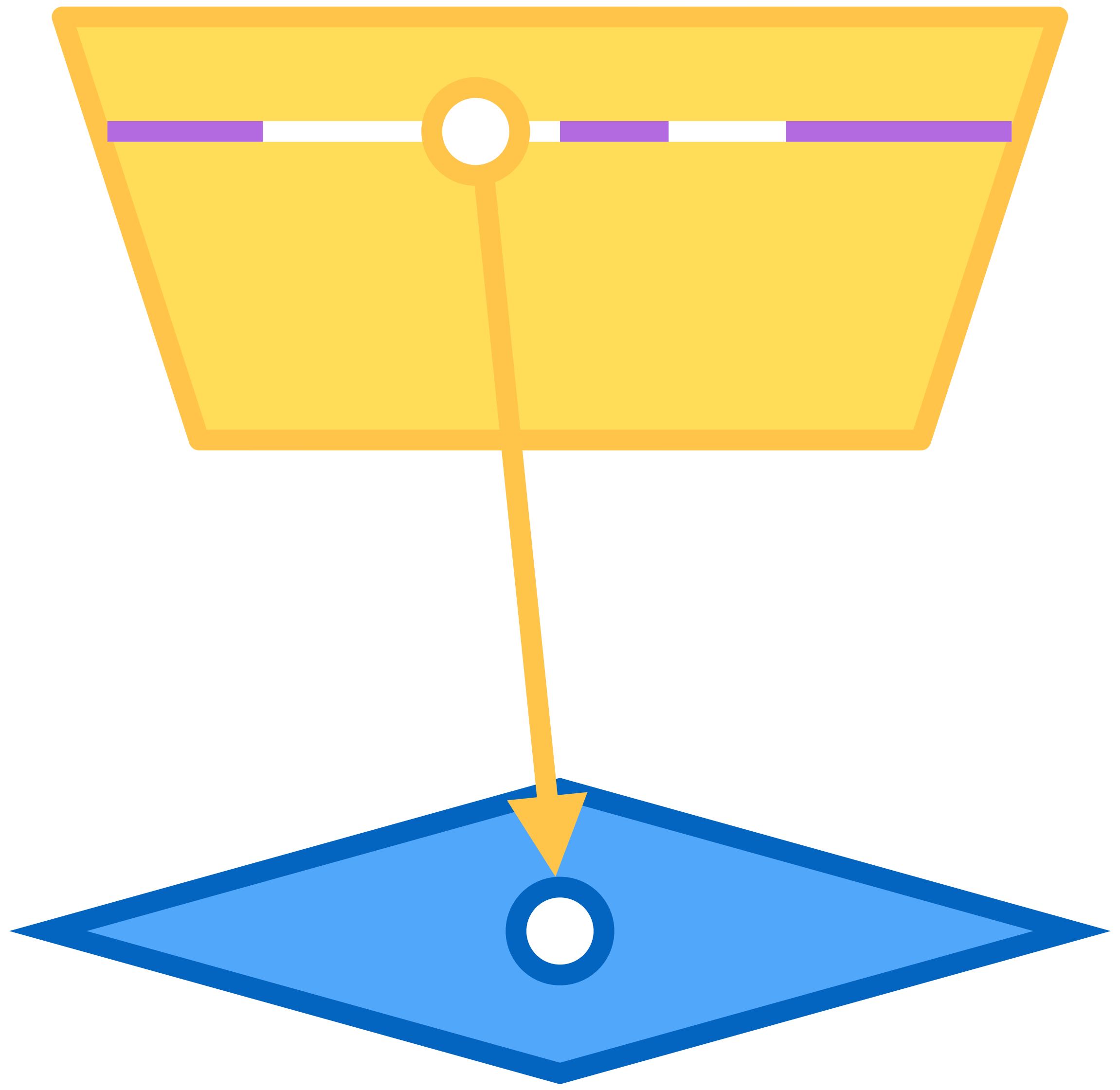
We propose two options:

Main idea

Conditional point pdfs

We propose two options:

Surface-area-based sampling
(uniform over surface area)



Main idea

Conditional point pdfs

We propose two options:

Surface-area-based sampling
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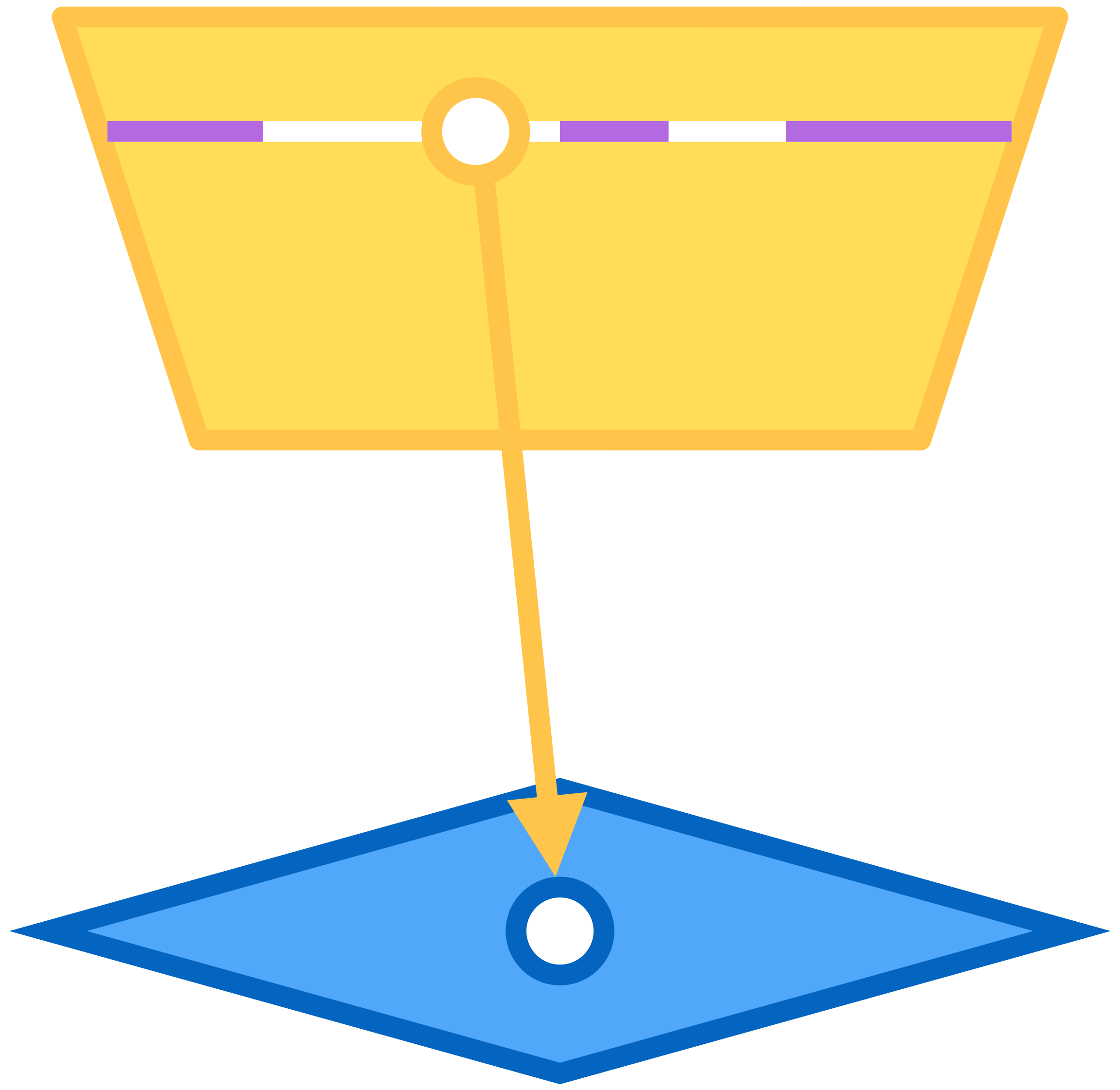
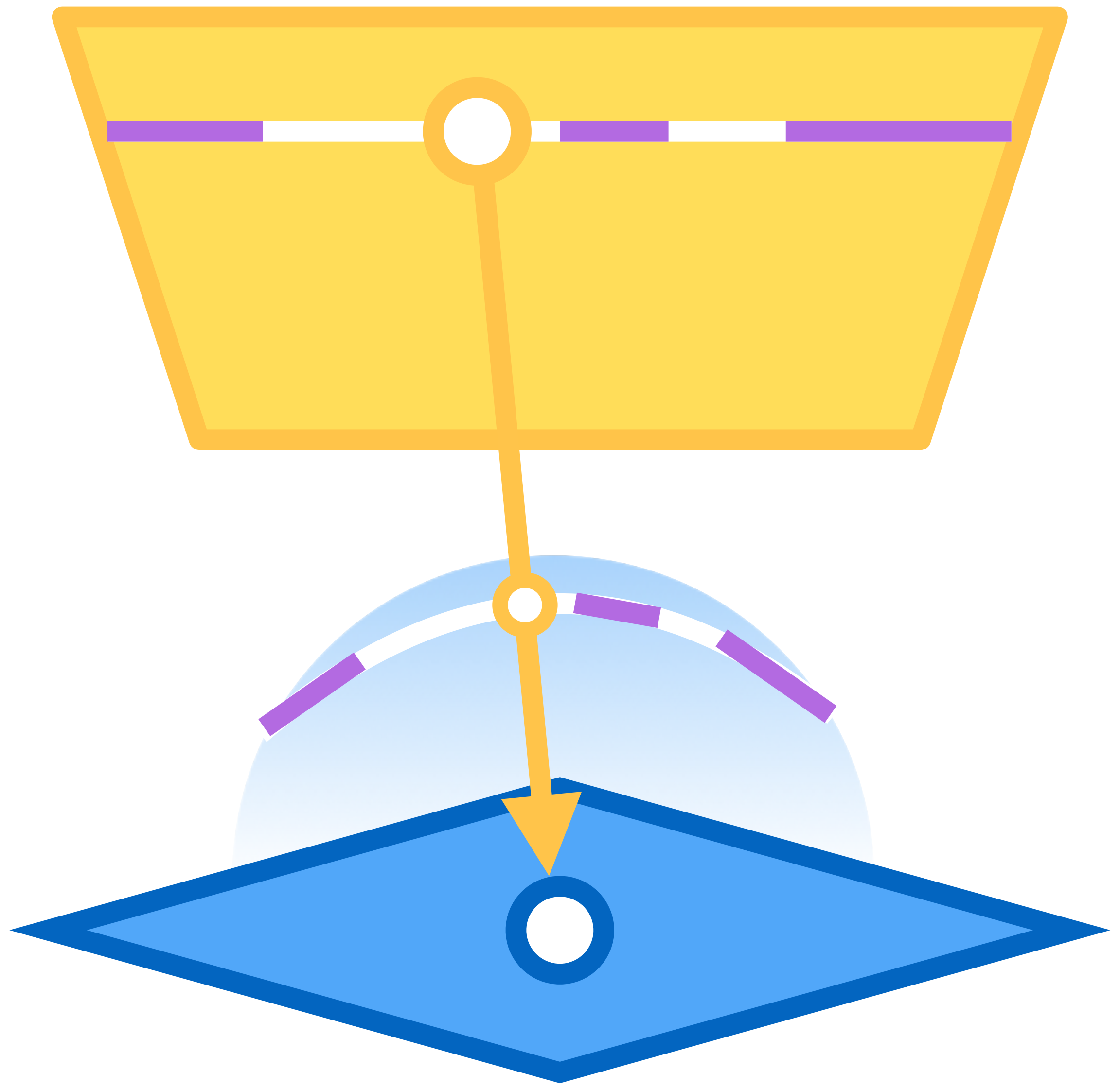


Solid-angle-based sampling
(uniform over solid angle)



from [UFK13]

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- We have reframed line sampling as point sampling that **importance samples visibility**

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- We can now use line sampling with **any BRDF**
- But line samples are still bad at importance sampling some terms – can we do better?

Theory

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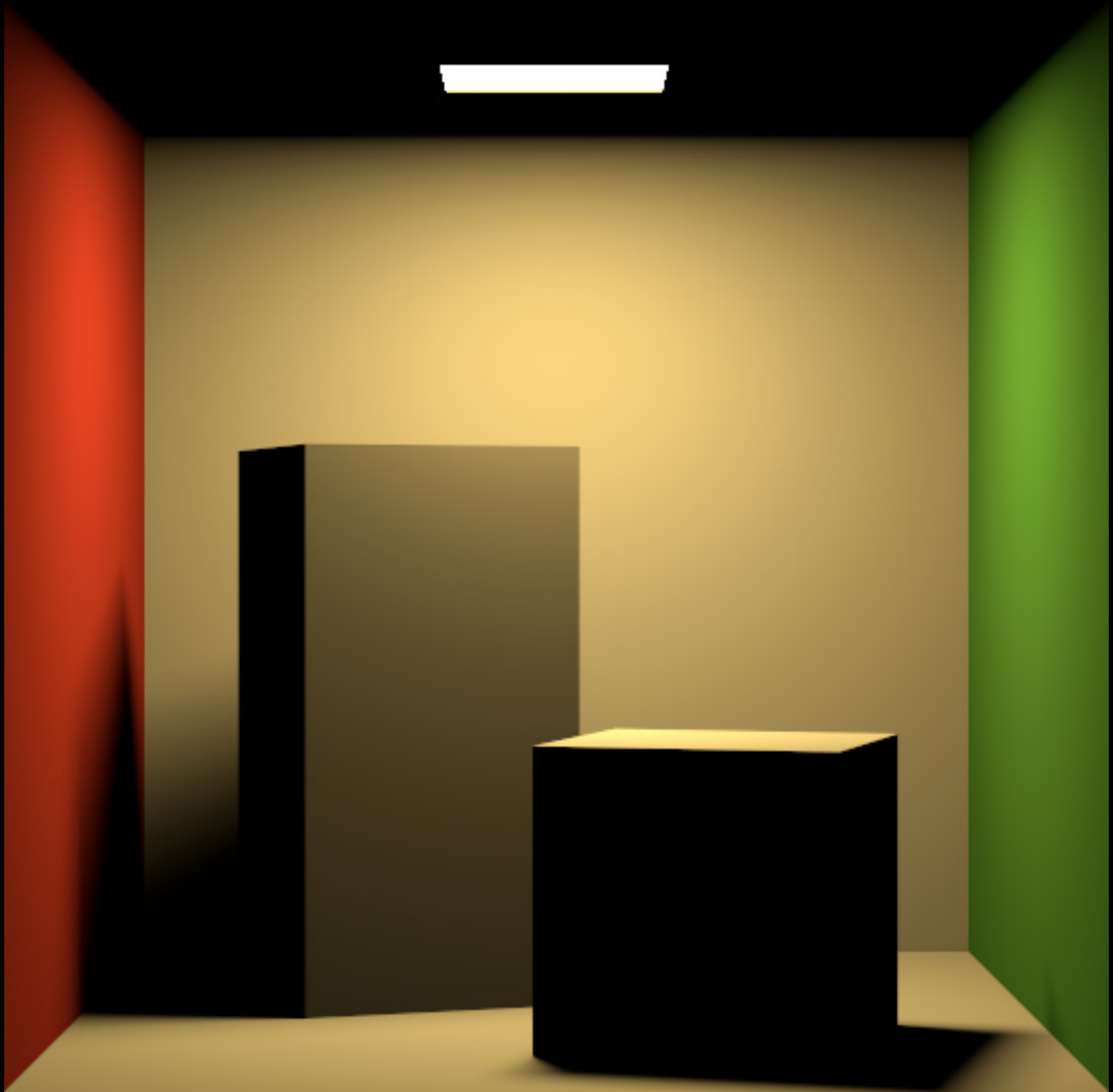
- MIS uses a set of **weights** to favor each strategy where it is strongest (i.e. where a strategy's pdf is largest relative to other strategies' pdfs)

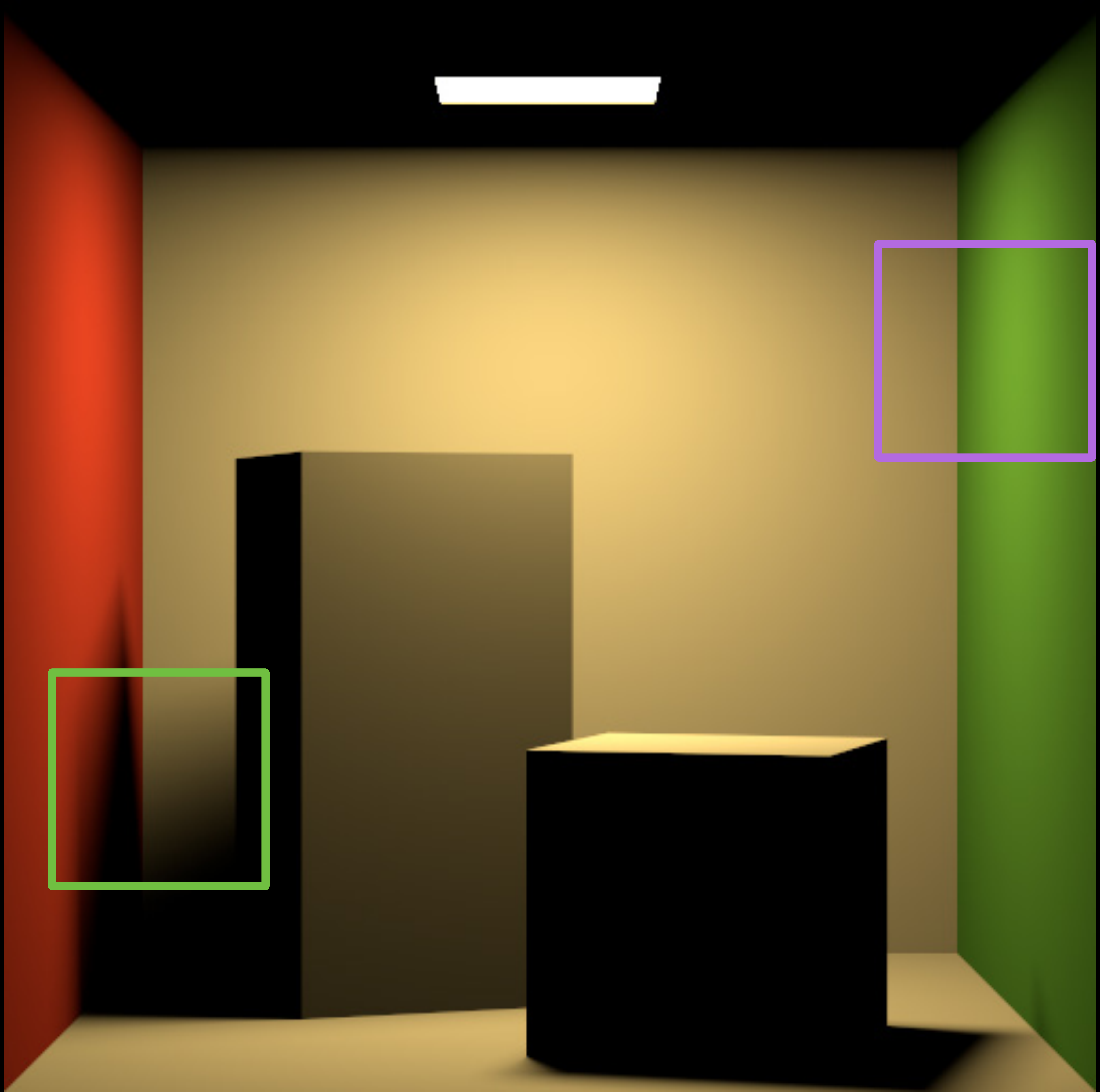
Summary

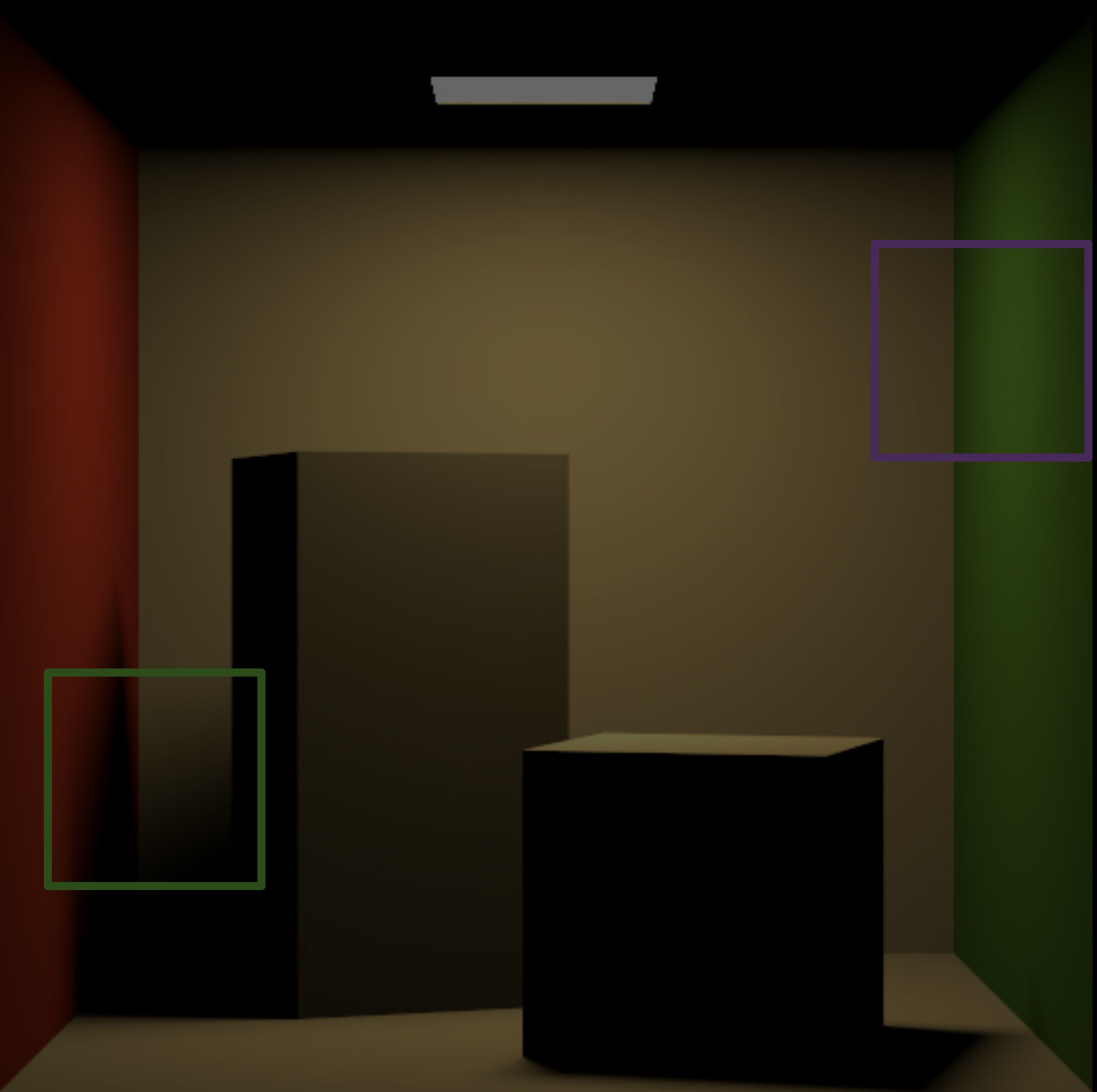
Summary

- We can now MIS lines with lines of **other orientations**

MIS between lines:
Equal time comparison



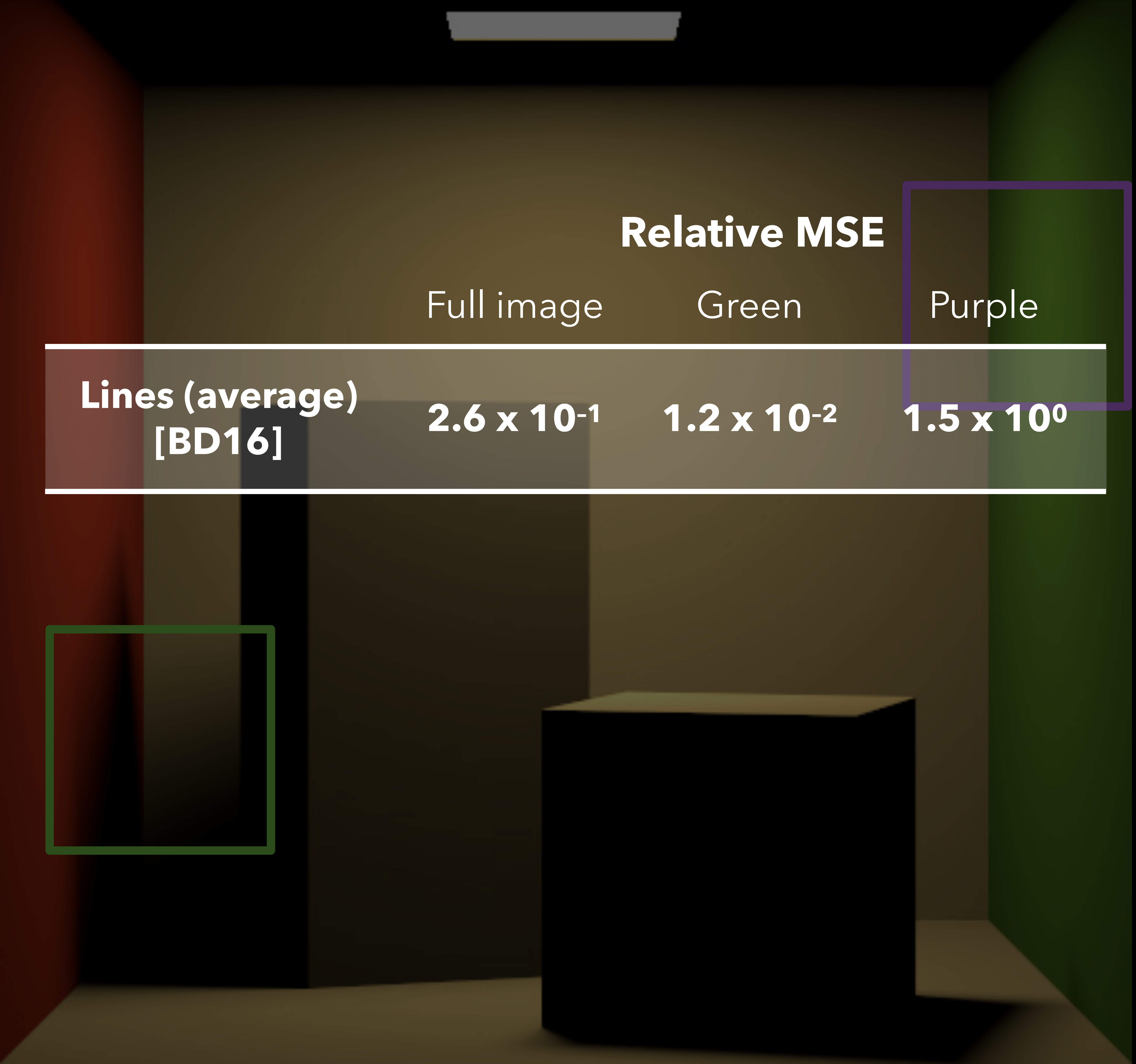


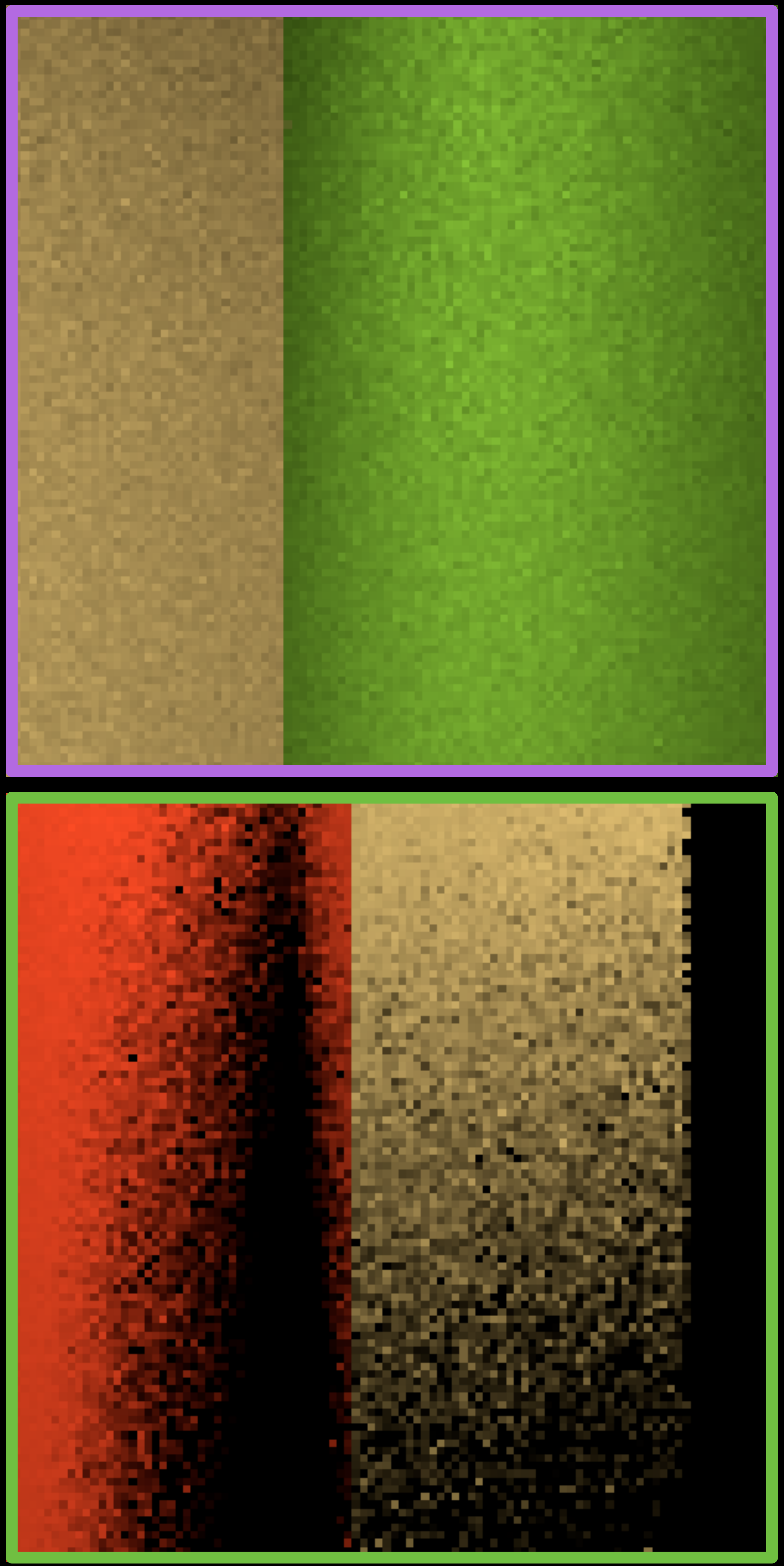


Equal time comparison

Relative MSE

	Full image	Green	Purple
Lines (average) [BD16]	**2.6×10^{-1}**	**1.2×10^{-2}**	**1.5×10^{0}**





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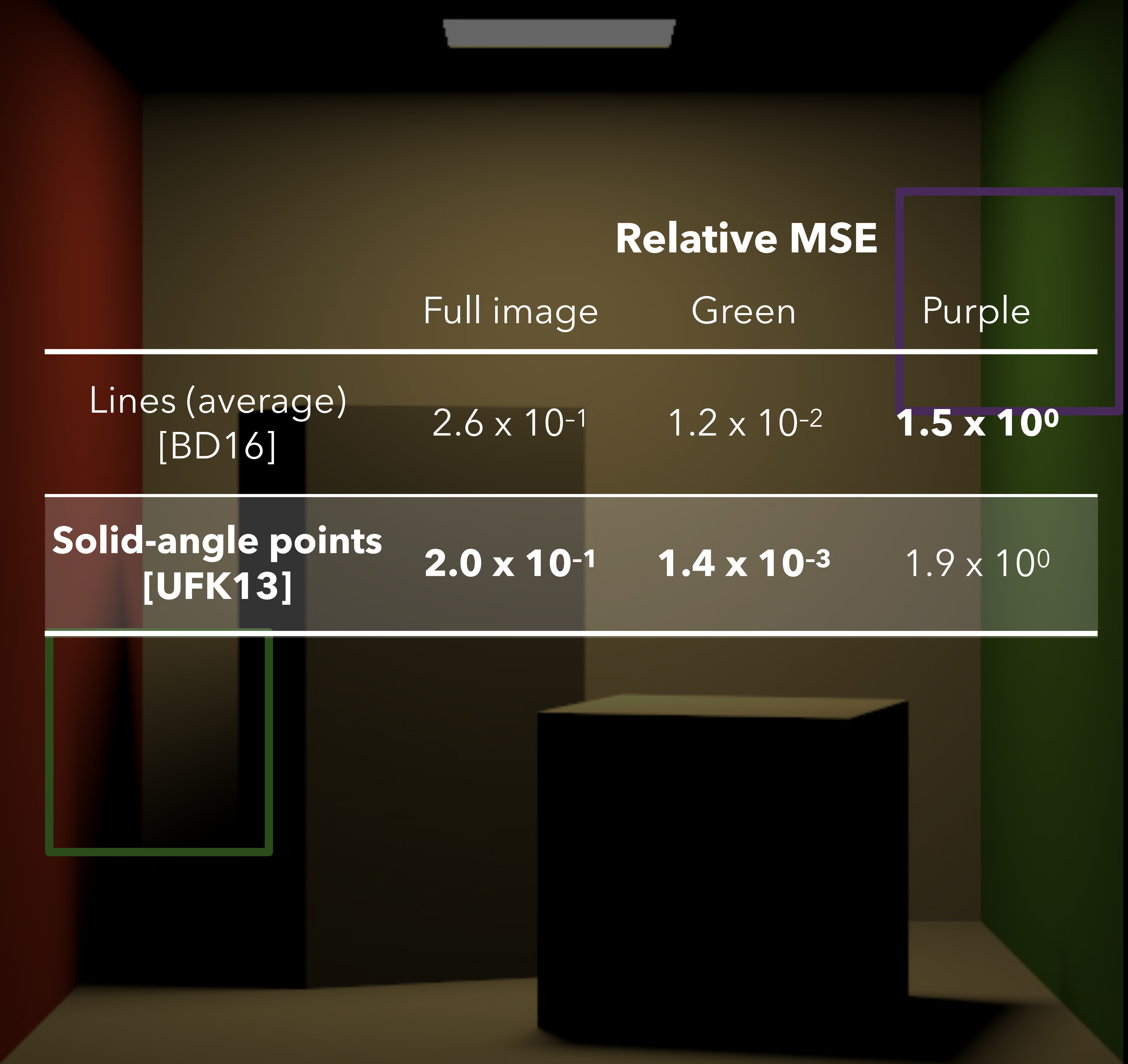

	Relative MSE		
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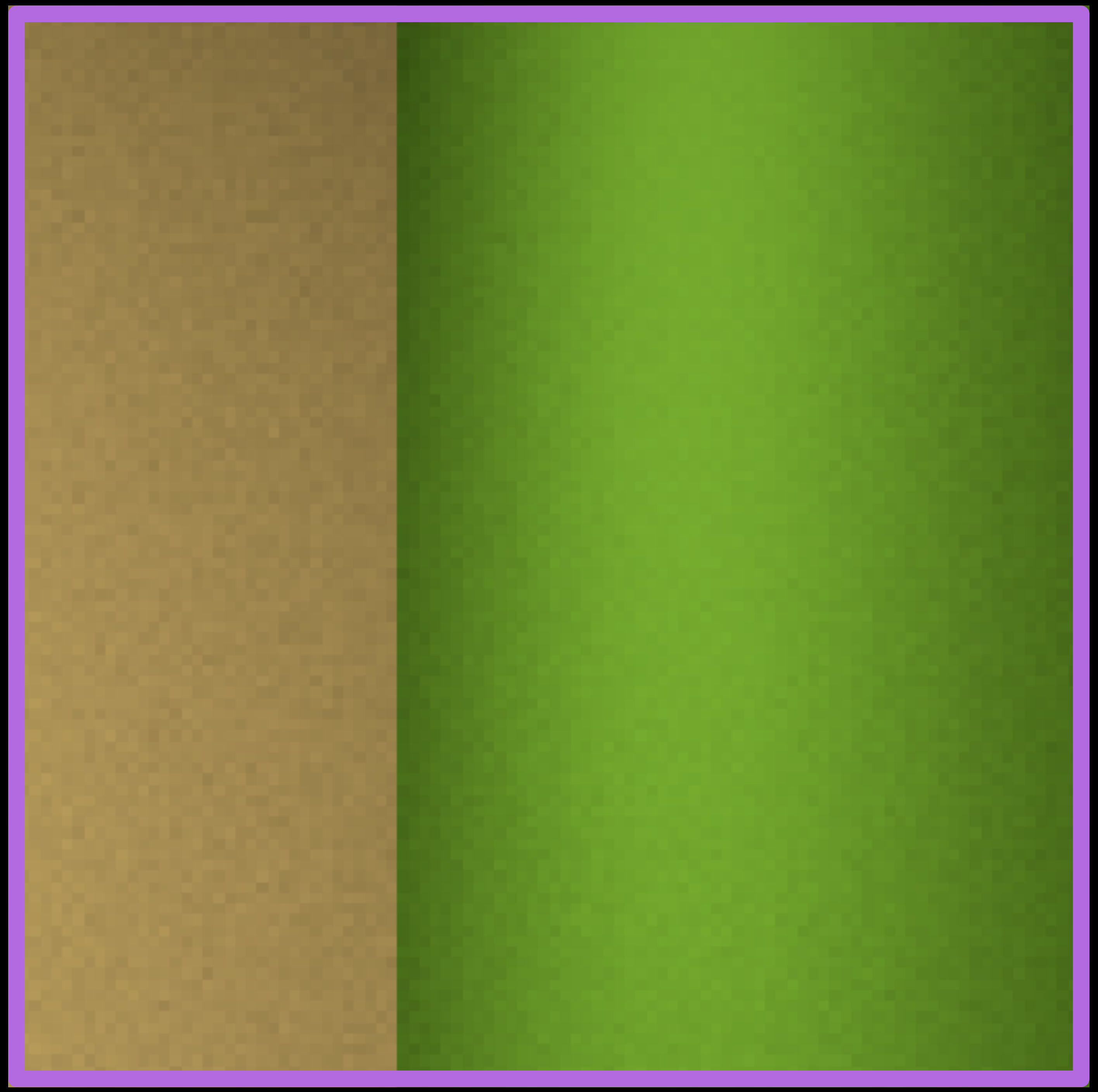
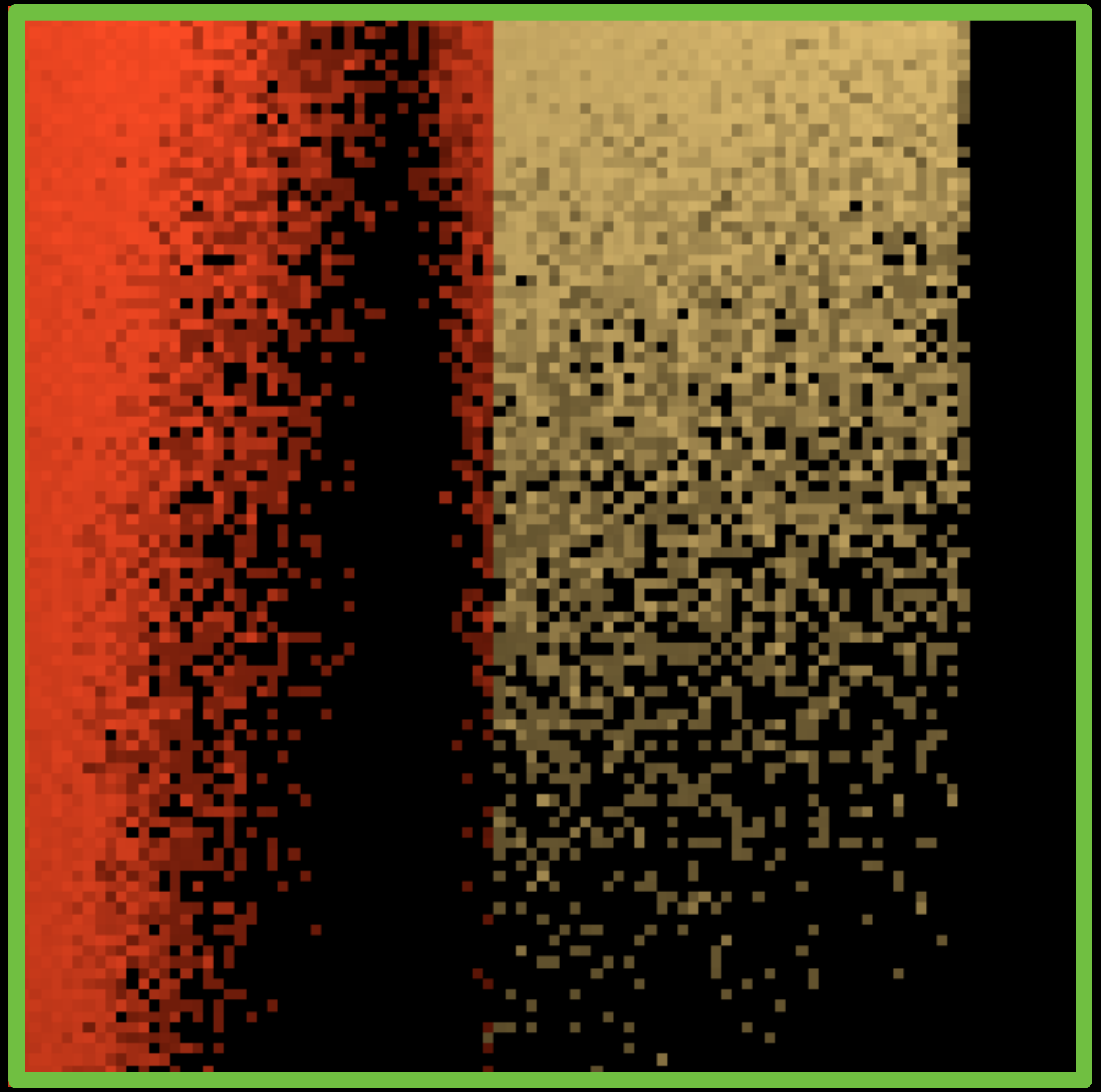
Solid-angle points [UFK13]	$\mathbf{2.0 \times 10^{-1}}$	$\mathbf{1.4 \times 10^{-3}}$	1.9×10^{0}





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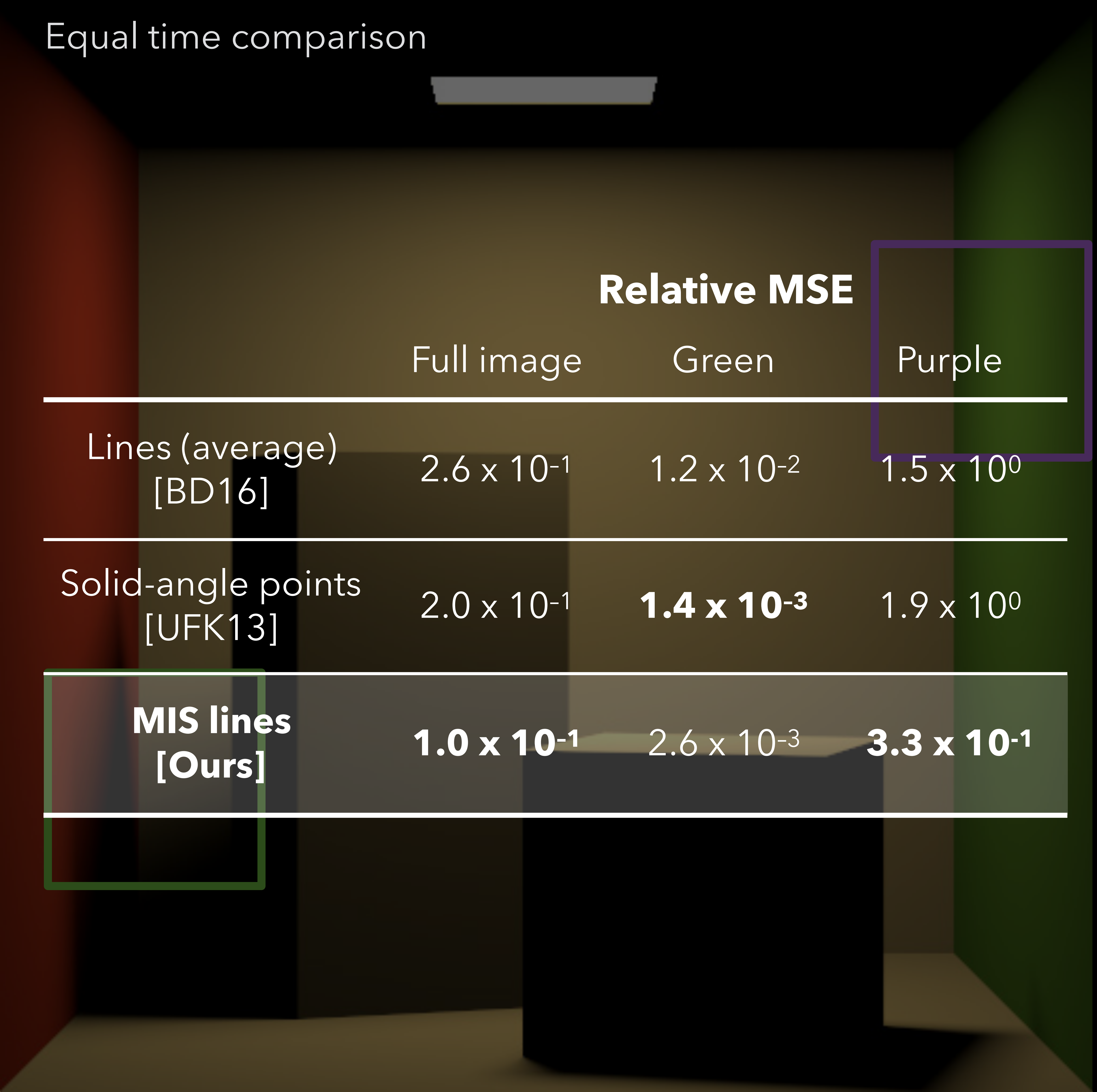

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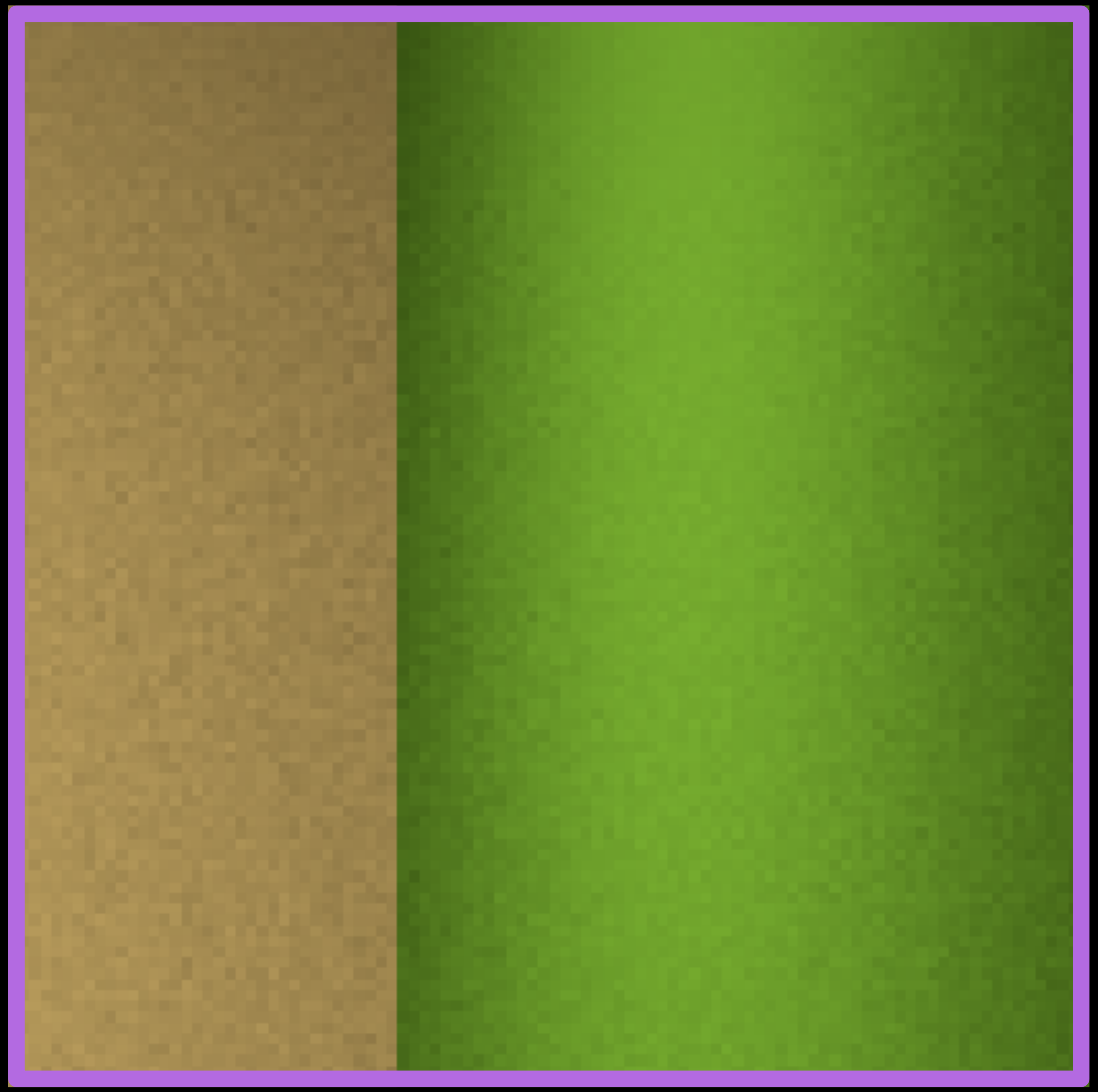
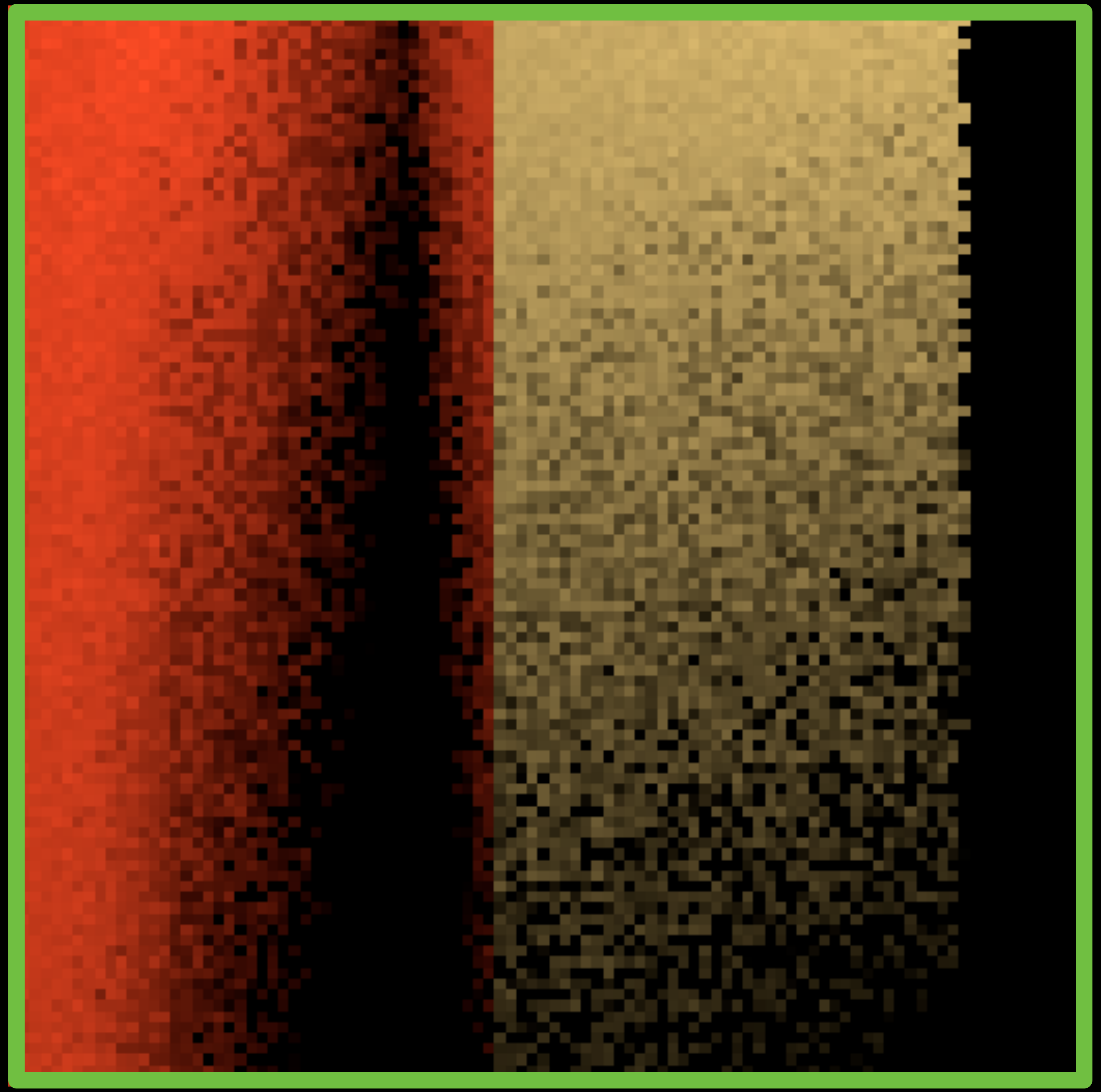
MIS lines [Ours]	$\mathbf{1.0 \times 10^{-1}}$	2.6×10^{-3}	$\mathbf{3.3 \times 10^{-1}}$





Summary

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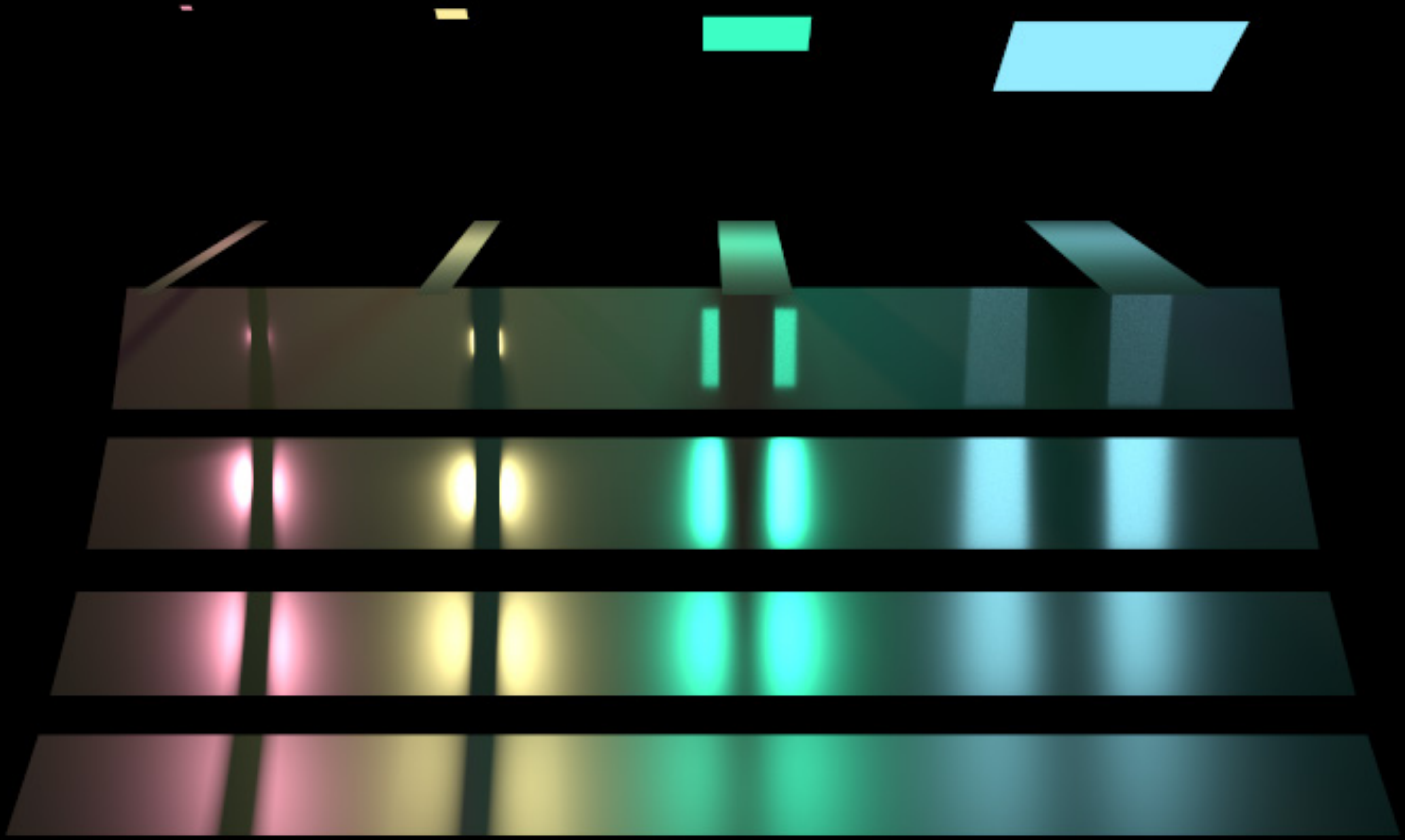
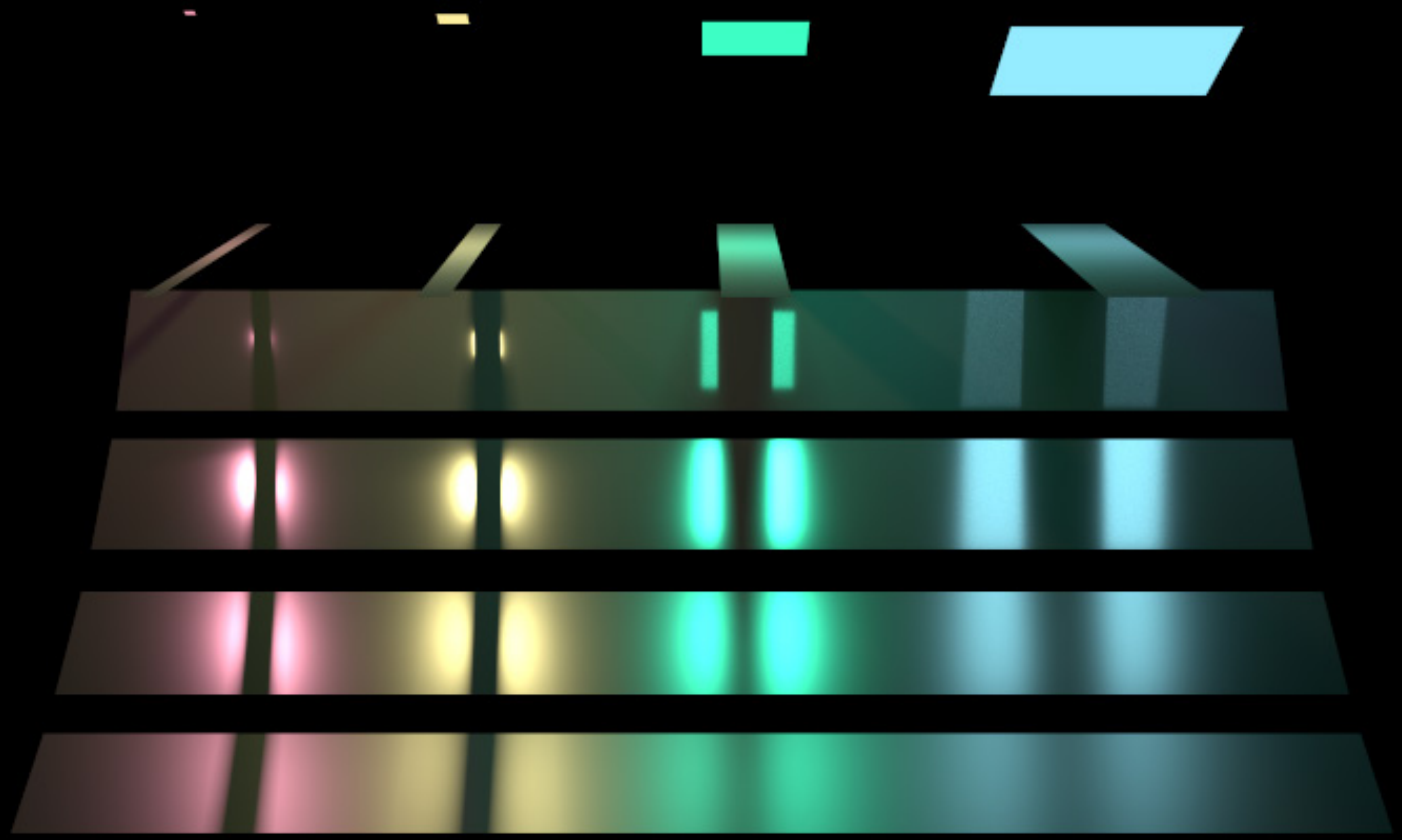
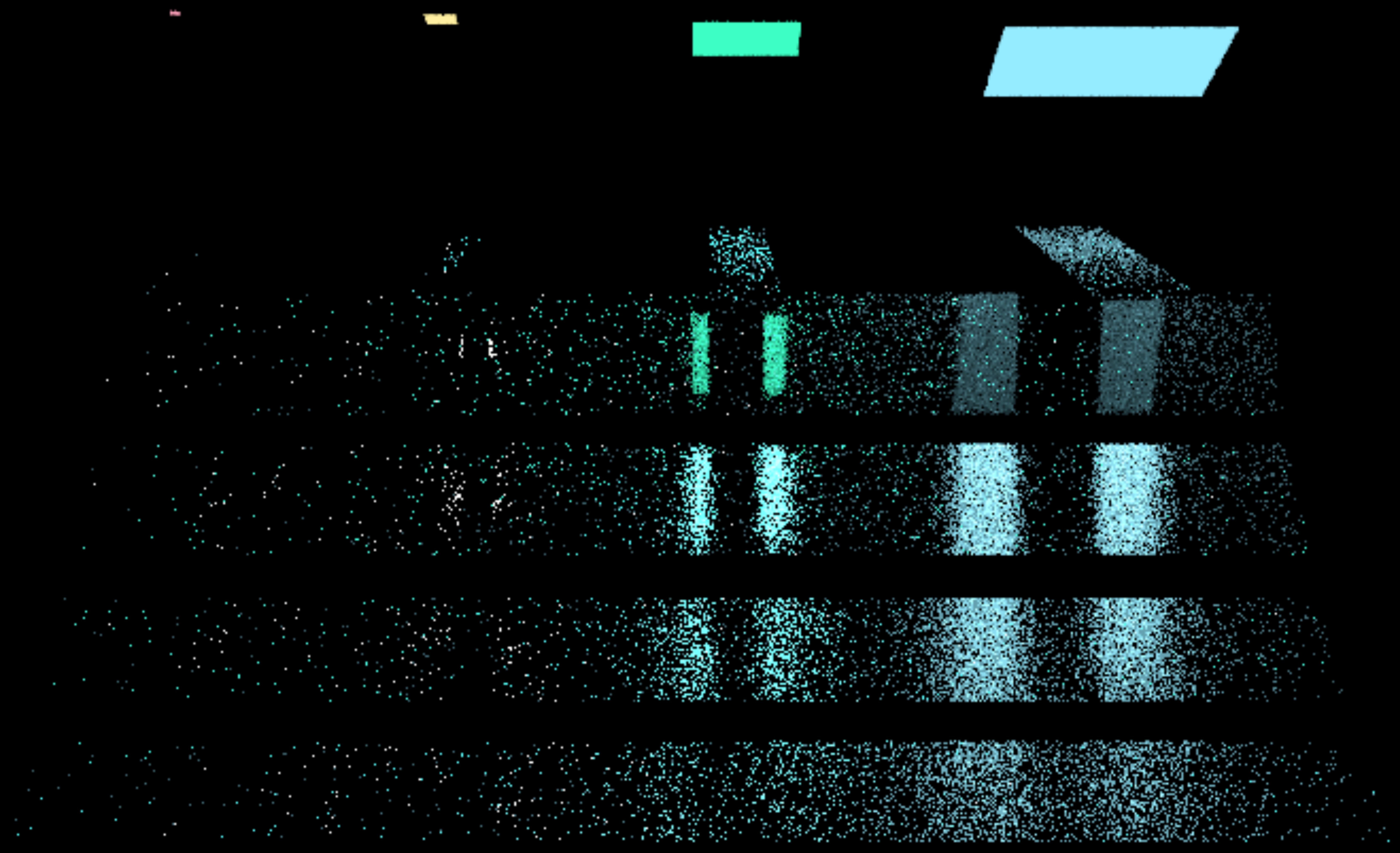
- We can now MIS lines with points that importance sample **other distributions** (like BRDFs)

MIS between points and lines:
Equal time comparisons





	Relative MSE Full image
BSDF	5.5×10^{1}



Equal time comparison

	Relative MSE Full image
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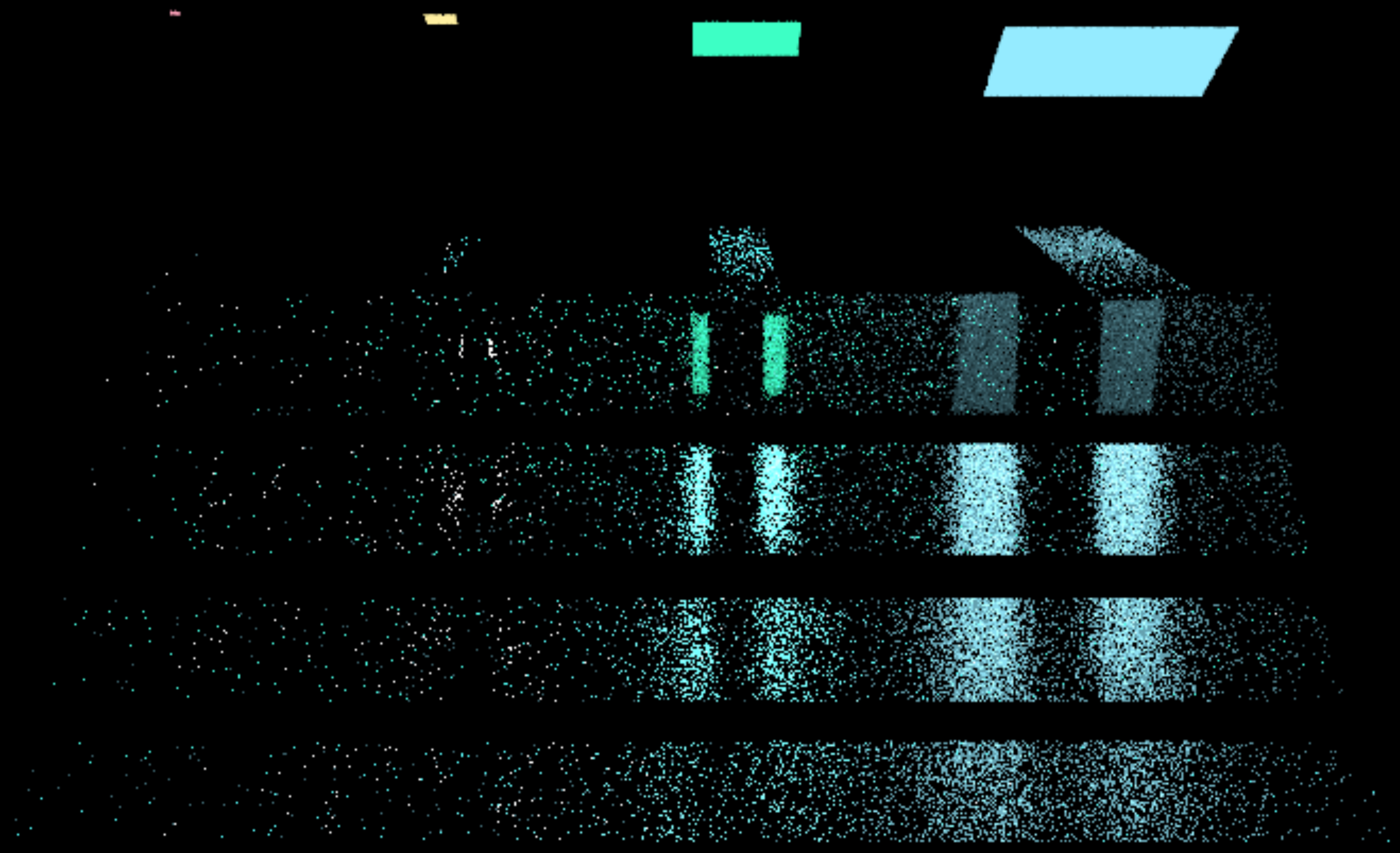
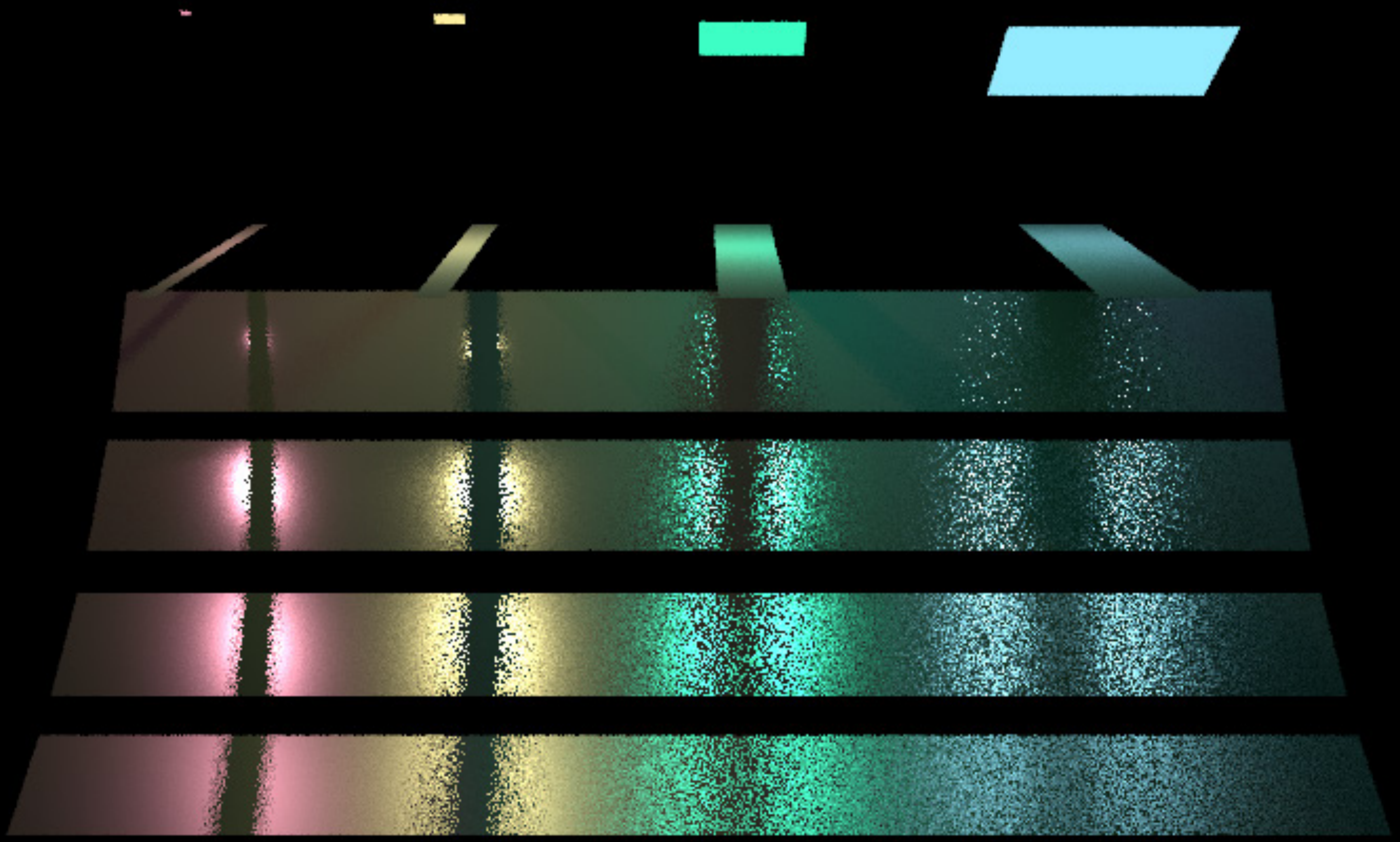


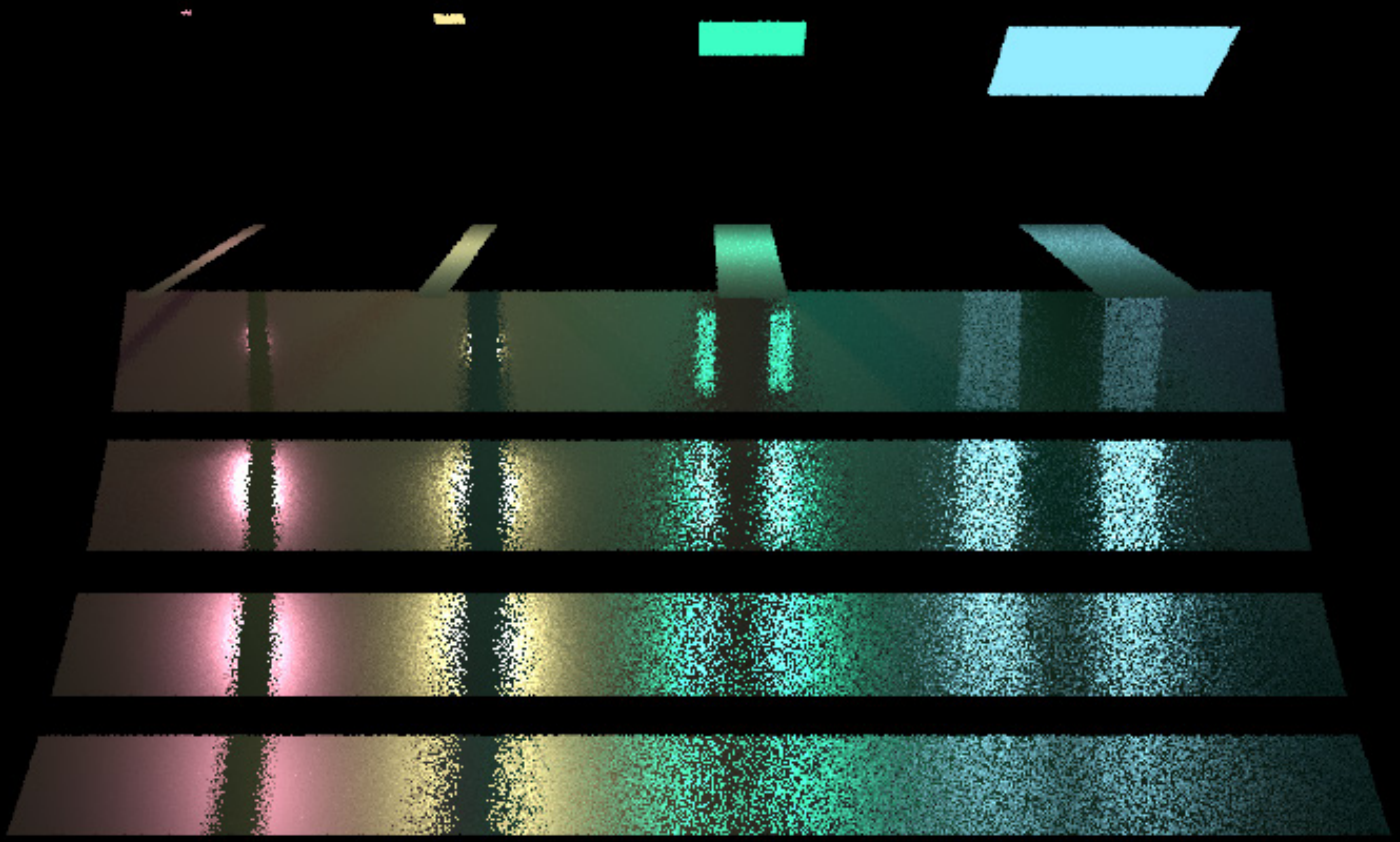
Equal time comparison

	Relative MSE
	Full image
BSDF	5.5×10^{1}
Solid-angle points [UFK13]	$\mathbf{4.5 \times 10^{-1}}$



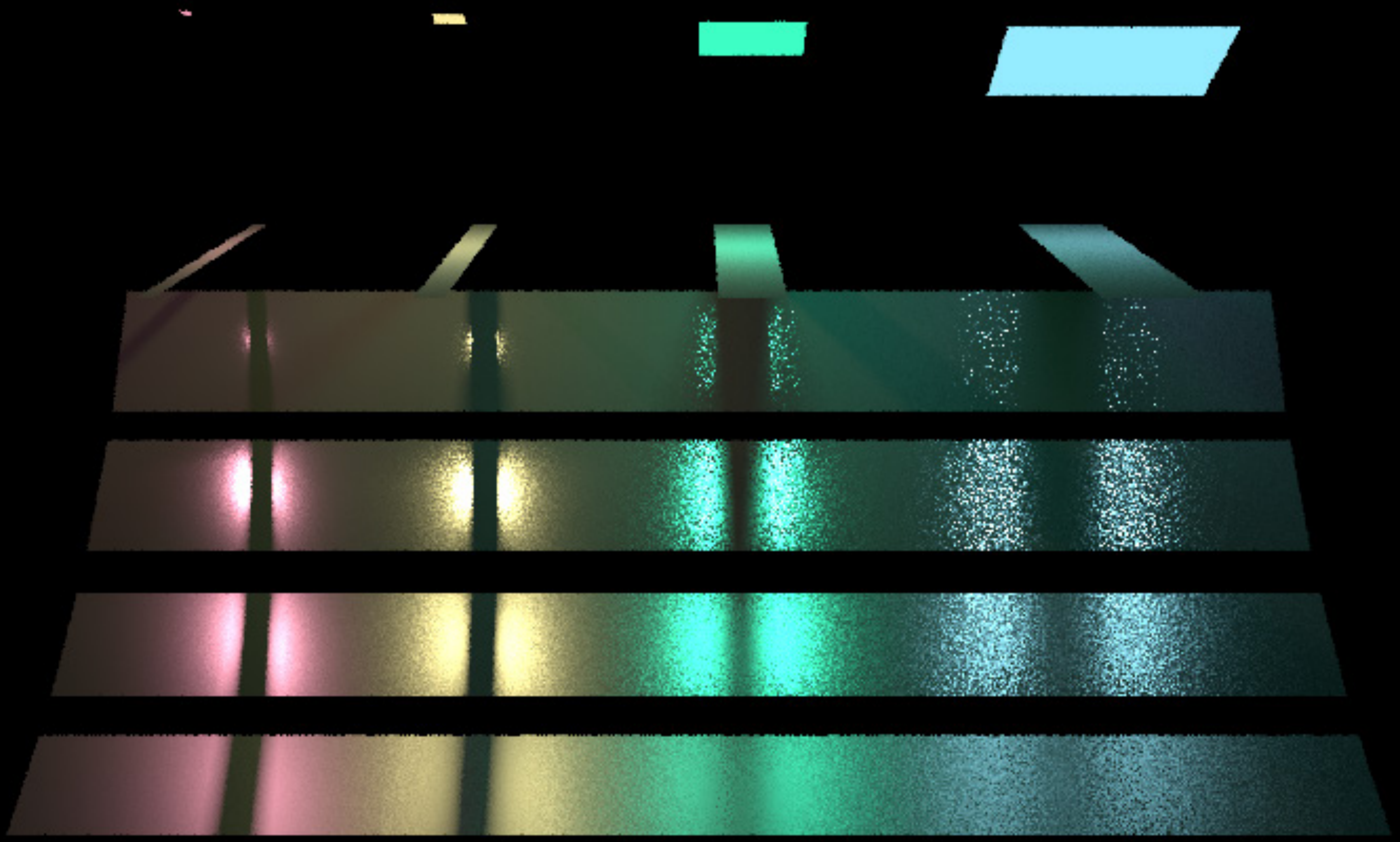
Equal time comparison

	Relative MSE Full image
BSDF	5.5×10^{1}
Solid-angle points [UFK13]	4.5×10^{-1}
MIS BSDF + SA points	$\mathbf{2.0 \times 10^{-1}}$



Equal time comparison

	Relative MSE Full image
BSDF	5.5×10^{1}
Solid-angle points [UFK13]	4.5×10^{-1}
MIS BSDF + SA points	**2.0×10^{-1}**
Solid-angle lines [Ours]	4.2×10^{-1}



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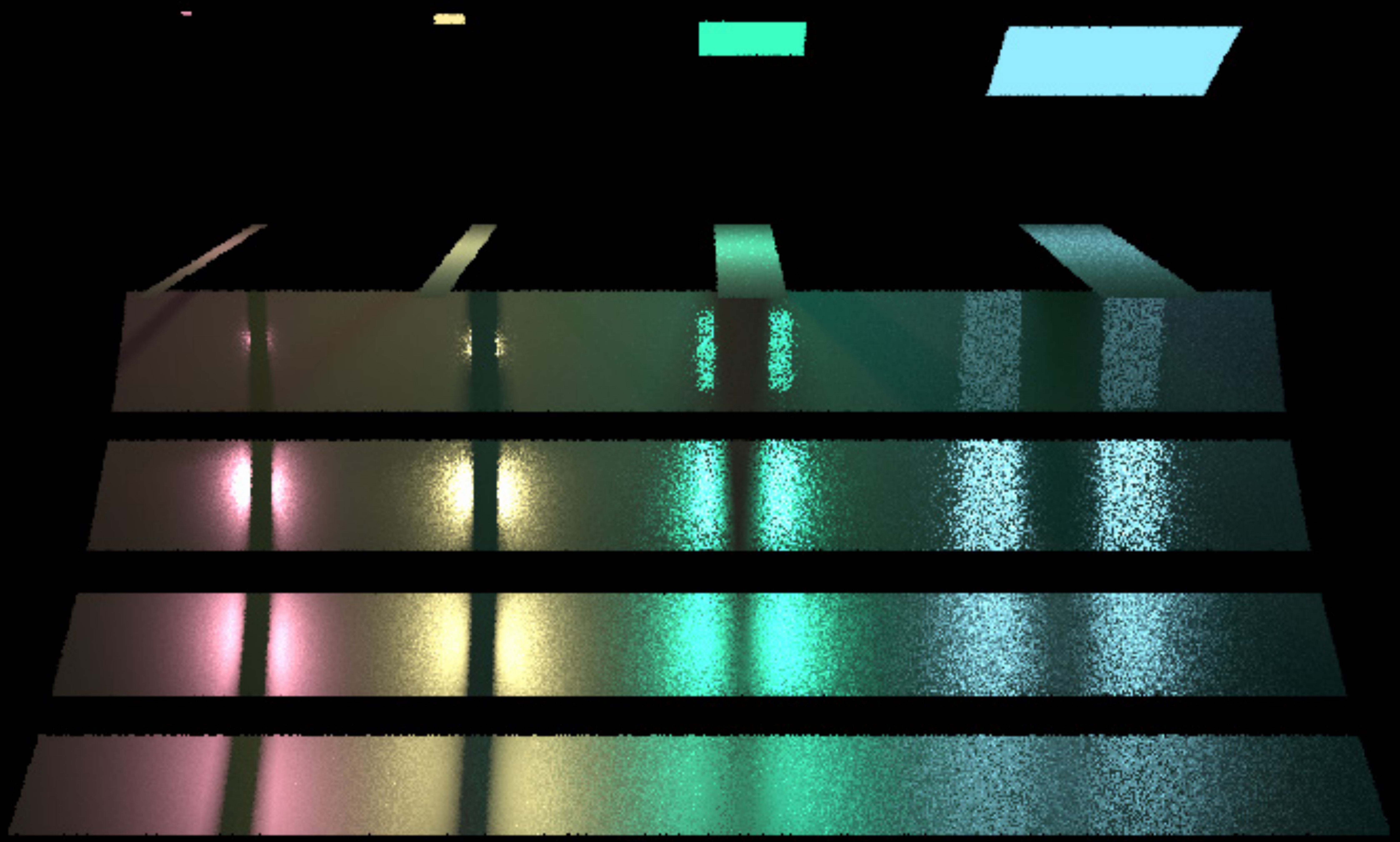
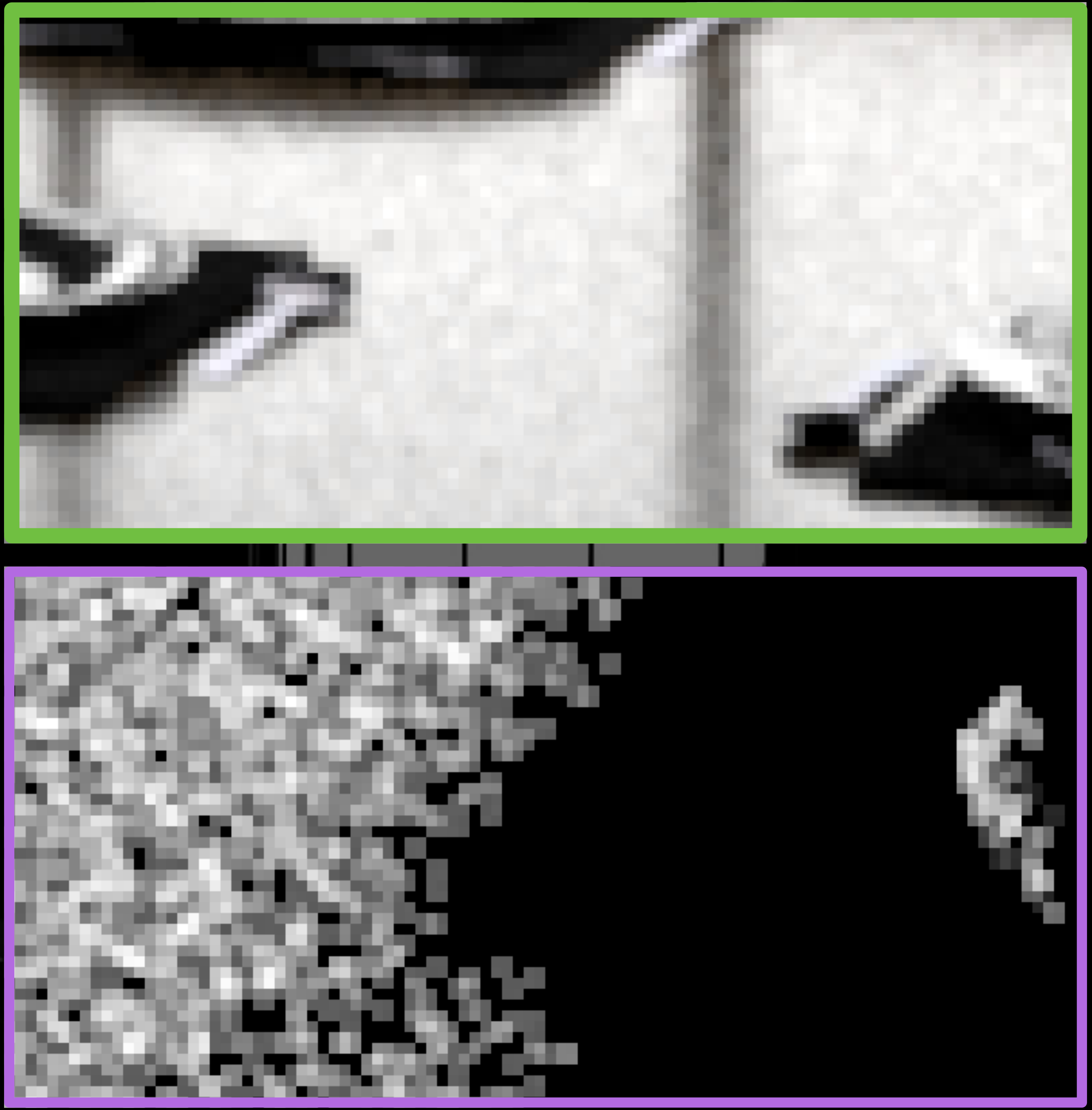
MIS BSDF + SA points	2.0×10^{-1}
Solid-angle lines [Ours]	4.2×10^{-1}
MIS BSDF + SA lines	**6.8×10^{-2}**



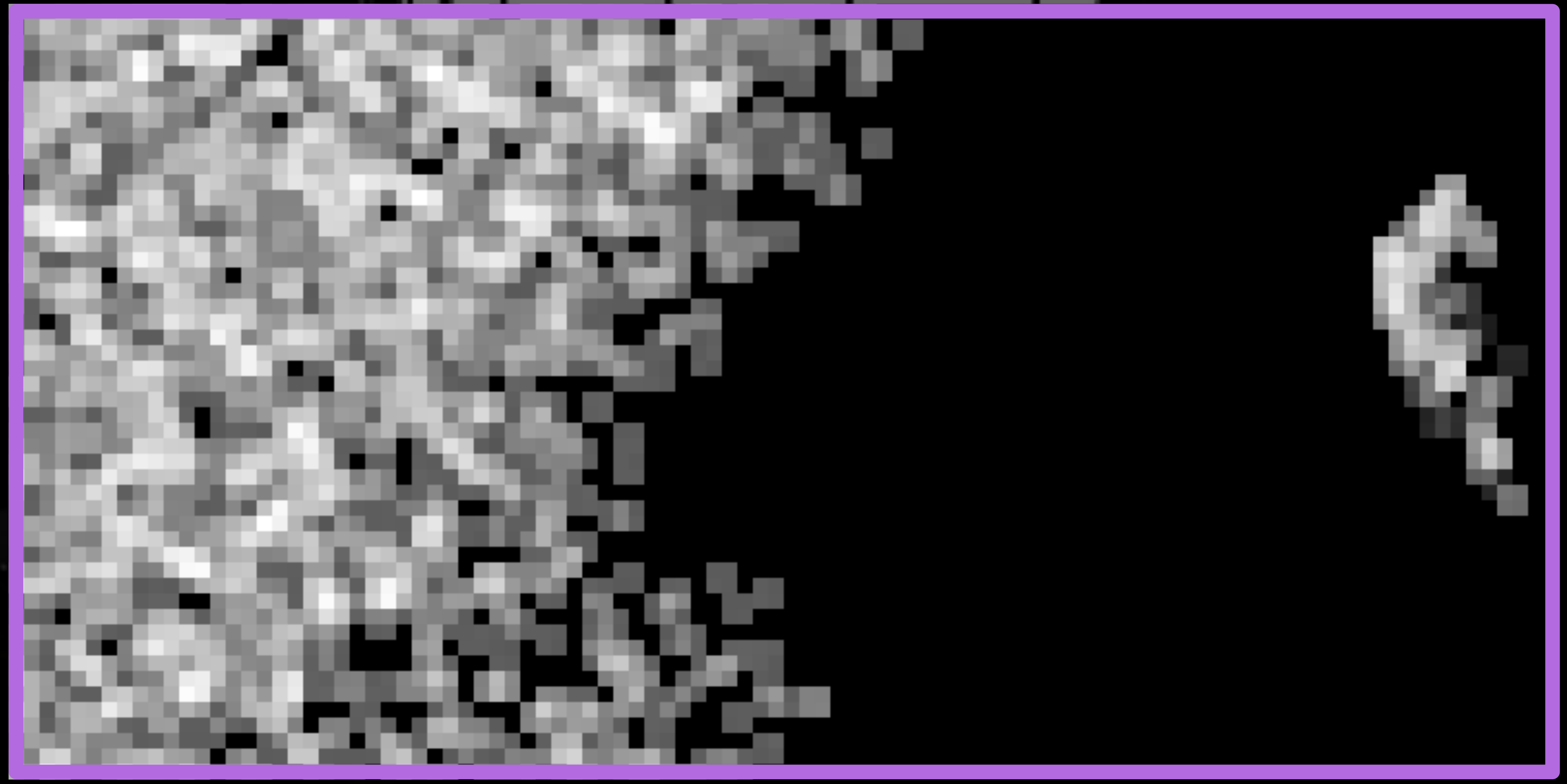
Relative MSE

	Full image	Green	Purple
4 points : 0 lines	**4.0×10^{-1}**	**5.7×10^{-2}**	**4.4×10^{-1}**



Equal time comparison

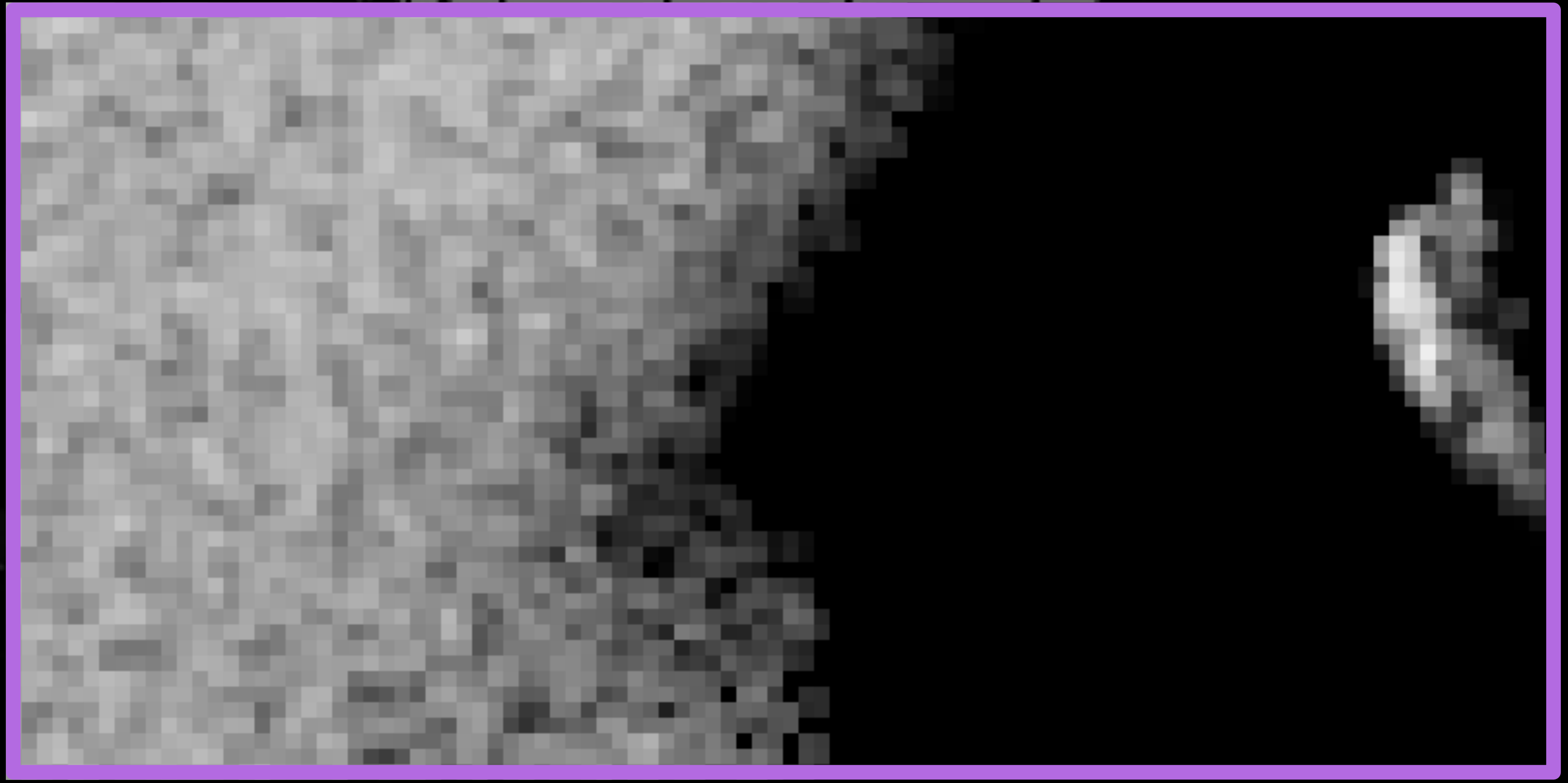
	Relative MSE		
	Full image	Green	Purple
4 points : 0 lines	**4.0×10^{-1}**	**5.7×10^{-2}**	**4.4×10^{-1}**



Equal time comparison

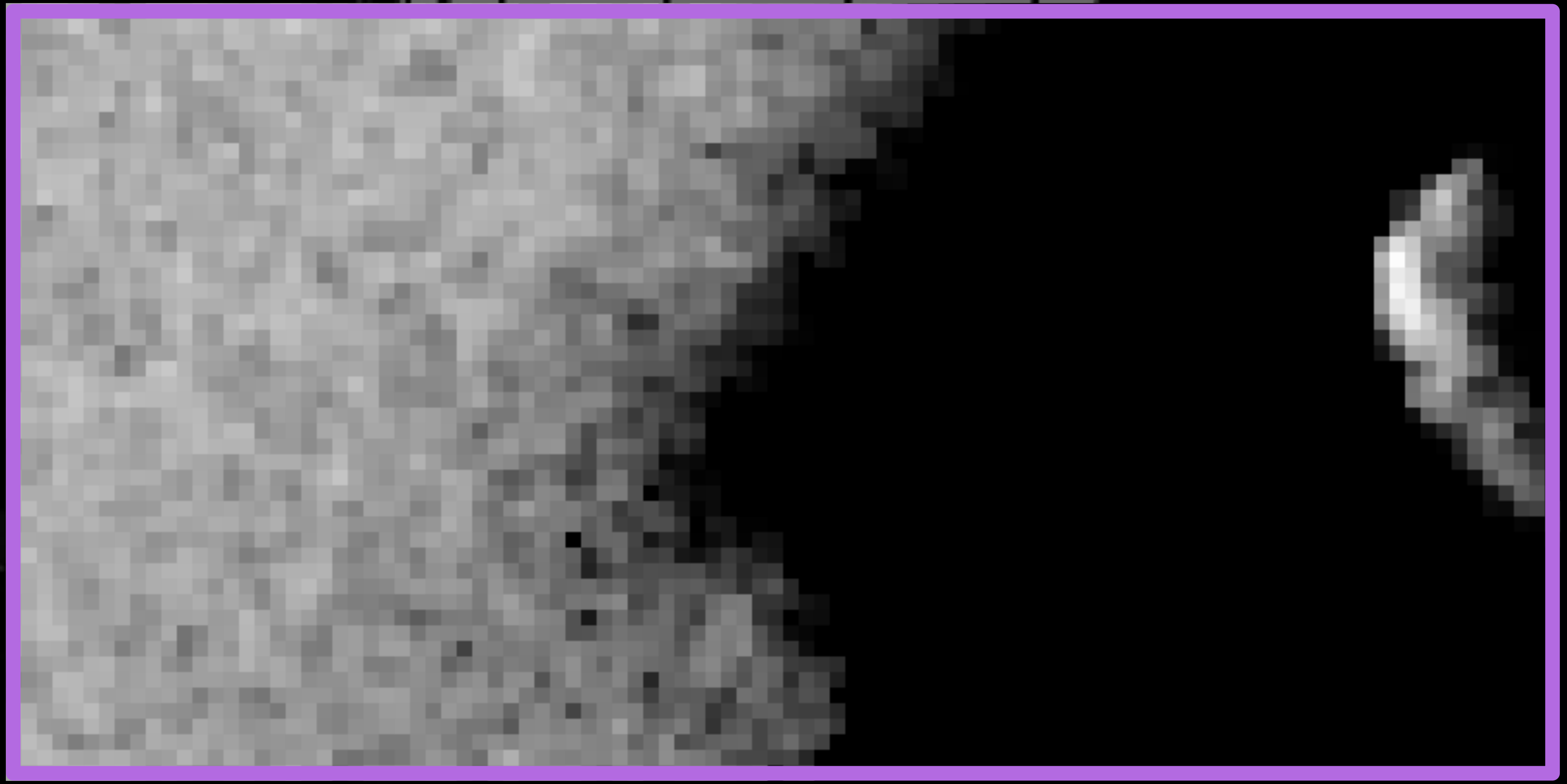
Relative MSE

	Full image	Green	Purple
4 points : 0 lines	**4.0×10^{-1}**	**5.7×10^{-2}**	4.4×10^{-1}
3 points : 1 line	1.0×10^{0}	1.1×10^{-1}	**1.1×10^{-1}**



Equal time comparison

	Relative MSE		
	Full image	Green	Purple
4 points : 0 lines	**4.0×10^{-1}**	**5.7×10^{-2}**	4.4×10^{-1}
3 points : 1 line	1.0×10^{0}	1.1×10^{-1}	1.1×10^{-1}
2 points : 2 lines	1.0×10^{1}	5.2×10^{-1}	**9.8×10^{-2}**



Equal time comparison

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4 points : 0 lines	4.0×10^{-1}	**5.7×10^{-2}**	4.4×10^{-1}
3 points : 1 line	1.0×10^{0}	1.1×10^{-1}	1.1×10^{-1}
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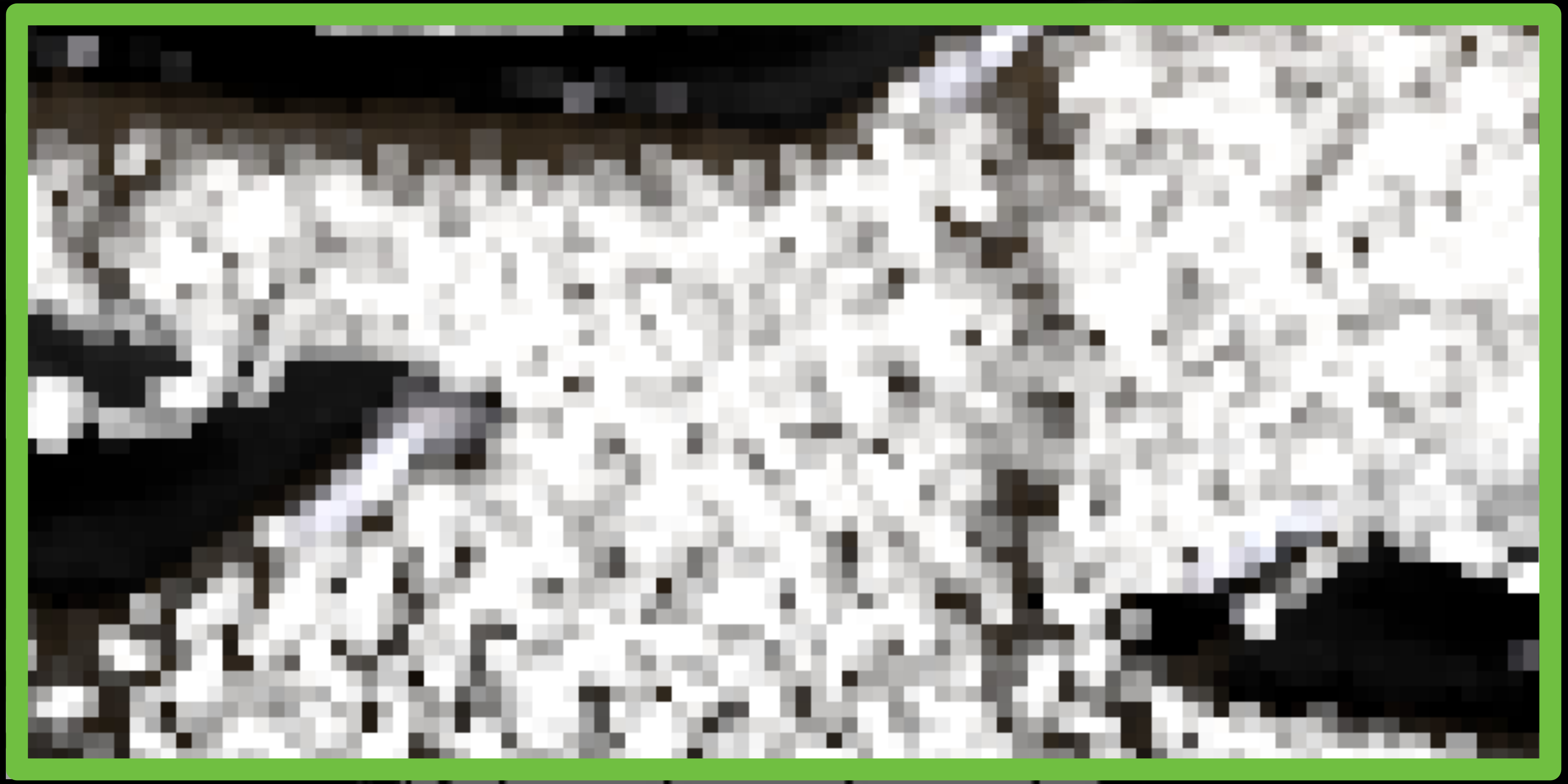
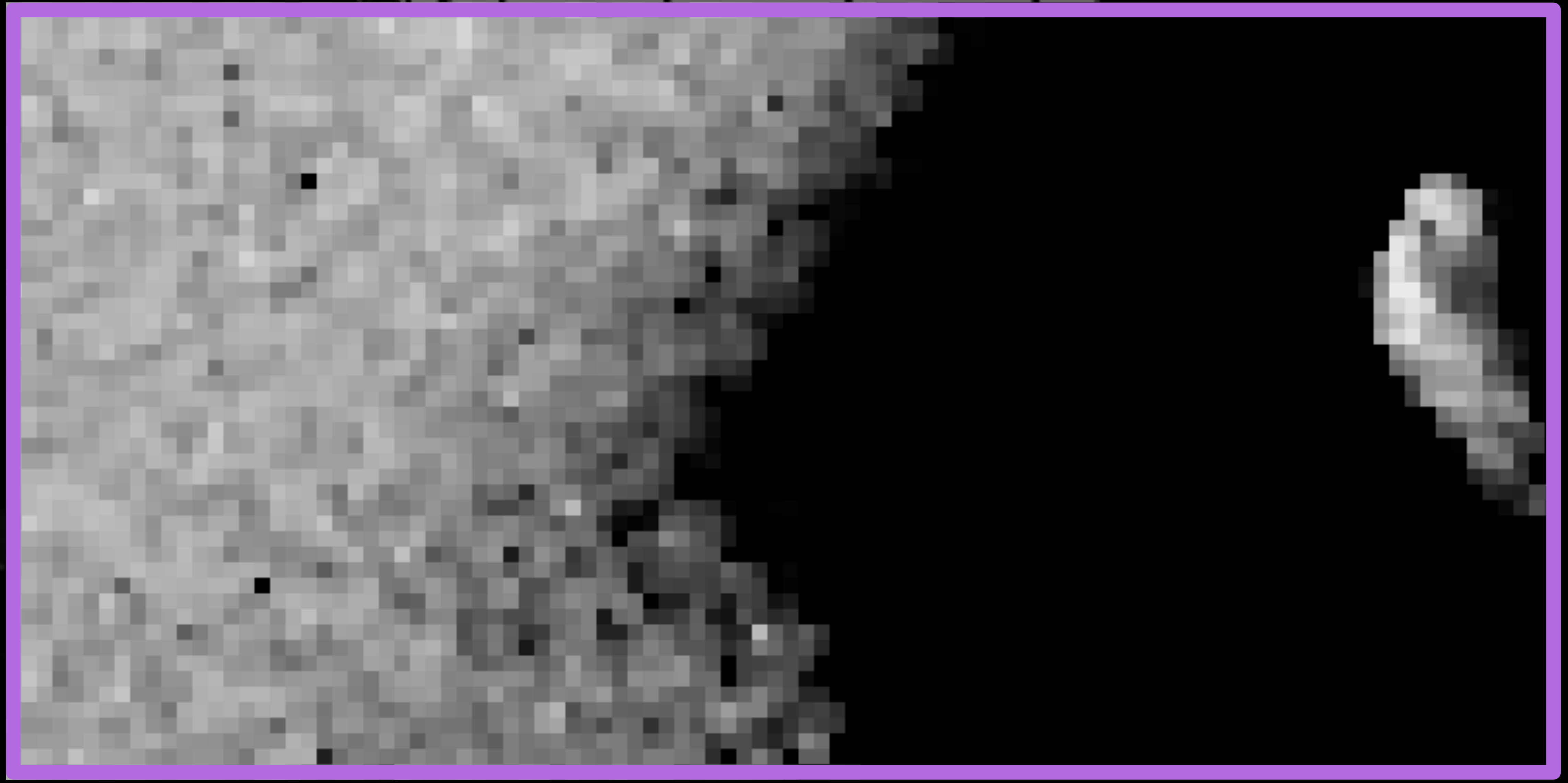
1 point : 3 lines	**2.8×10^{-1}**	5.5×10^{-1}	1.3×10^{-1}





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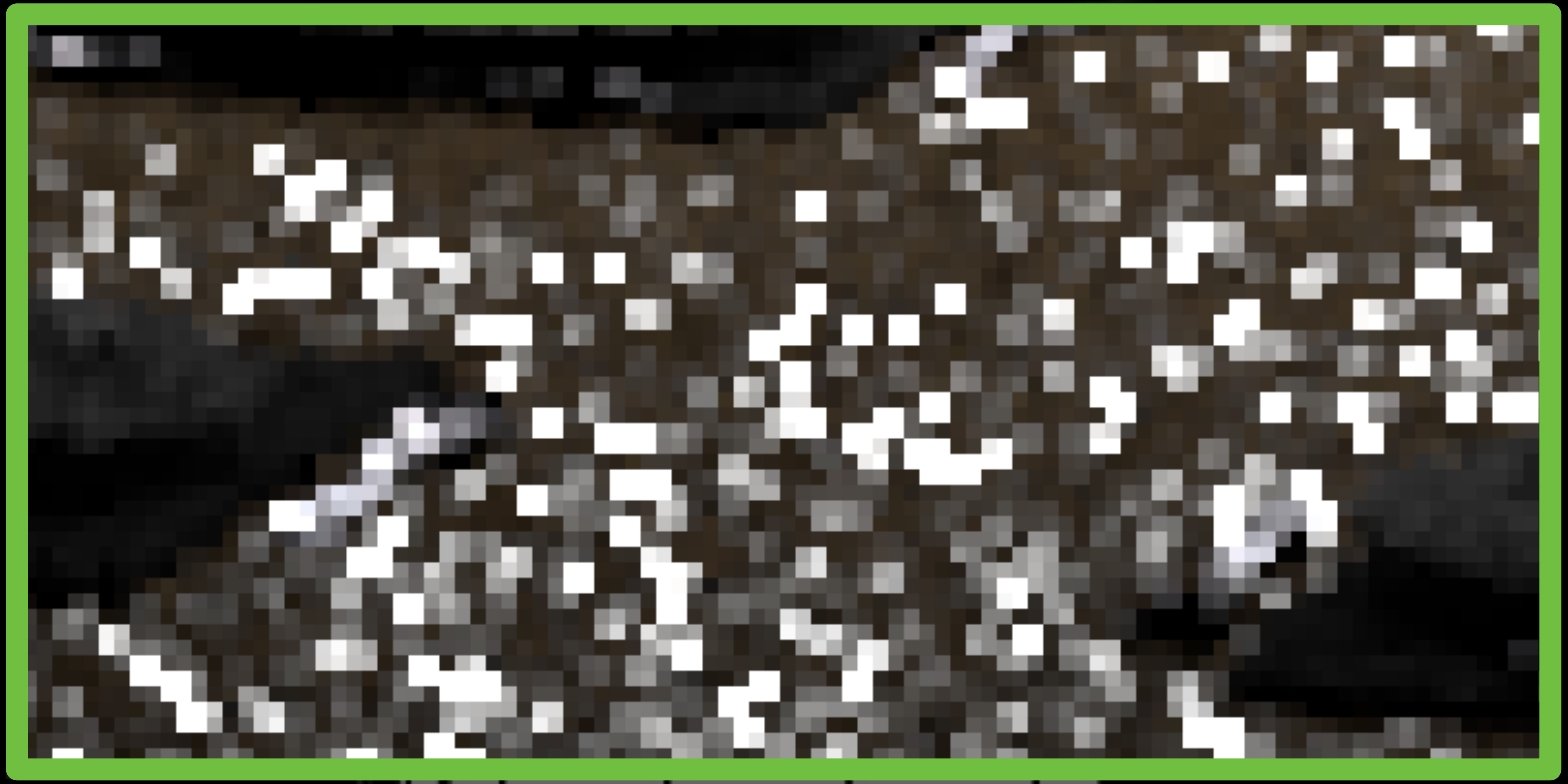
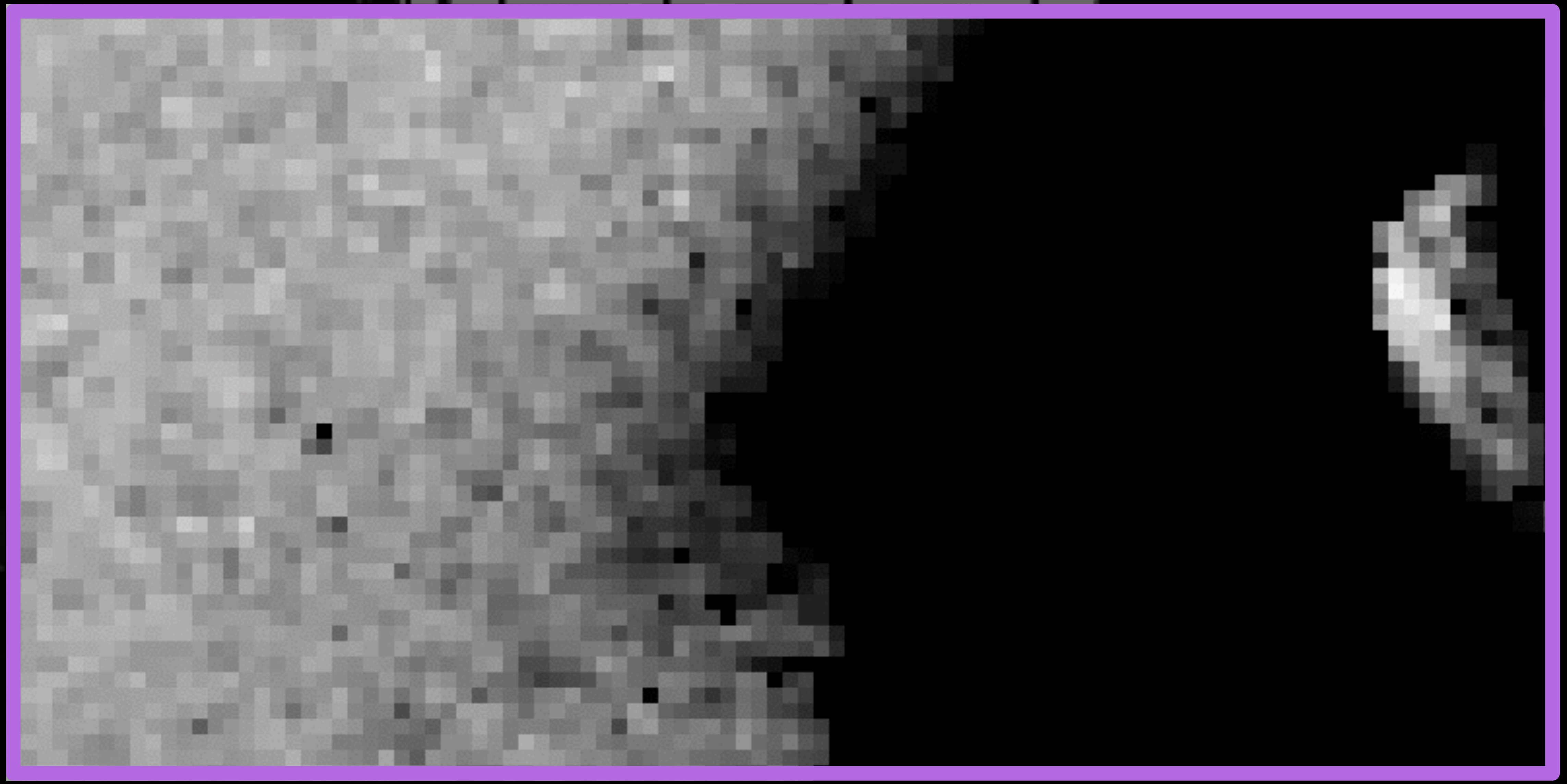
1 point : 3 lines	$\mathbf{2.8 \times 10^{-1}}$	5.5×10^{-1}	1.3×10^{-1}
0 points : 4 lines	5.3×10^{-1}	1.9×10^{1}	1.1×10^{-1}





Summary

- We can now MIS lines with lines of **other orientations**
- We can now MIS lines with points that importance sample **other distributions** (like BRDFs)

Summary

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- We can now MIS lines with points that importance sample **other distributions** (like BRDFs)
- But MIS inherits the worst convergence rate of its strategies – can we do better?

Discontinuity-smoothing MIS

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$$\langle L_o \rangle^{\mathrm{mis}} = \frac{1}{S} \sum_{s=1}^{S} \left(\frac{1}{N_s} \sum_{i=1}^{N_s} w_s(u_i, v_i) \frac{f(u_i, v_i)}{p_s(u_i, v_i)} \right) \quad \text{for S strategies}$$

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[SSC*19]

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[Ours]

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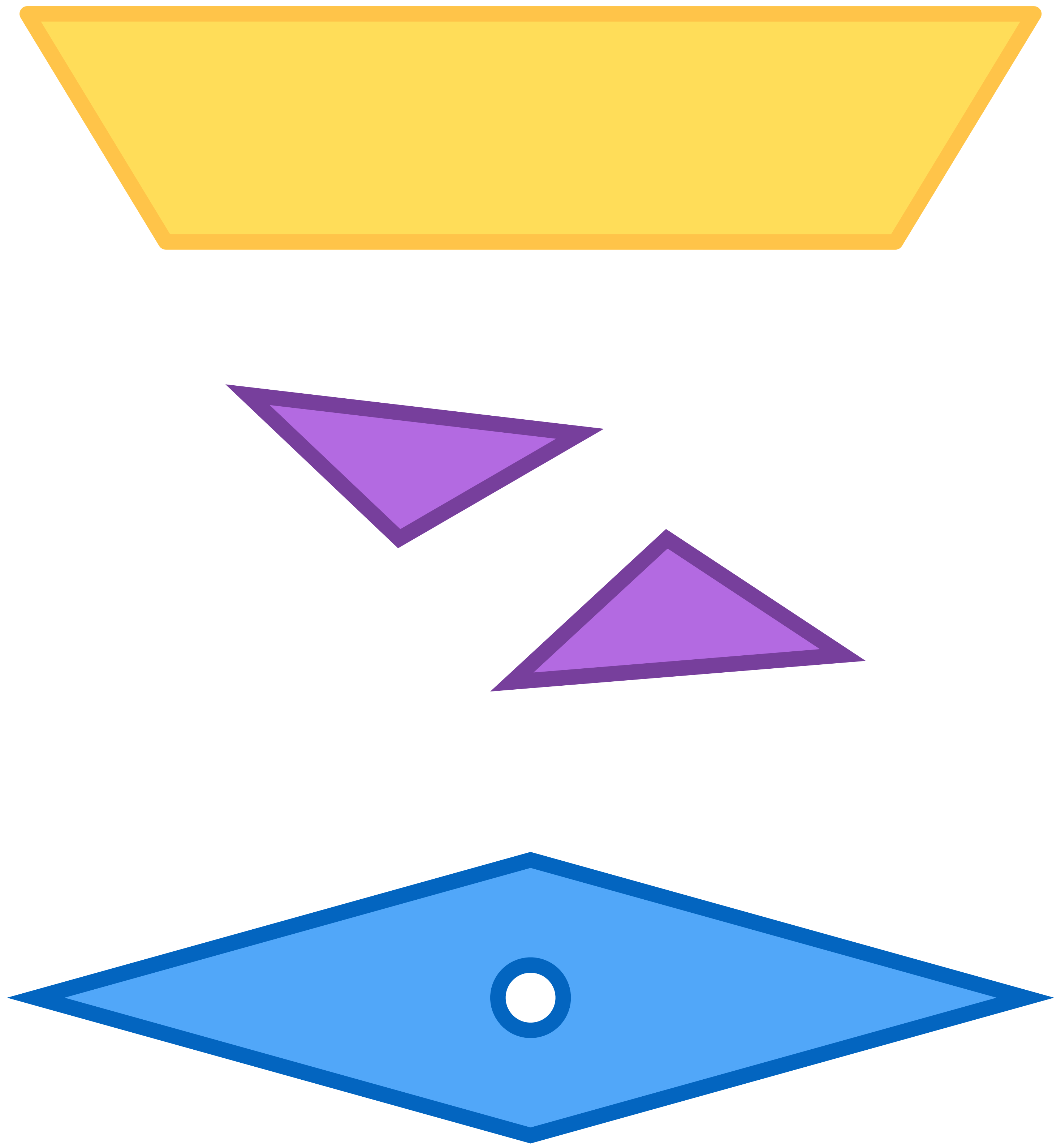
Discontinuity-smoothing MIS

Let us MIS:

1. **BRDF point samples**
2. **Vertical line samples**
3. **Horizontal line samples**

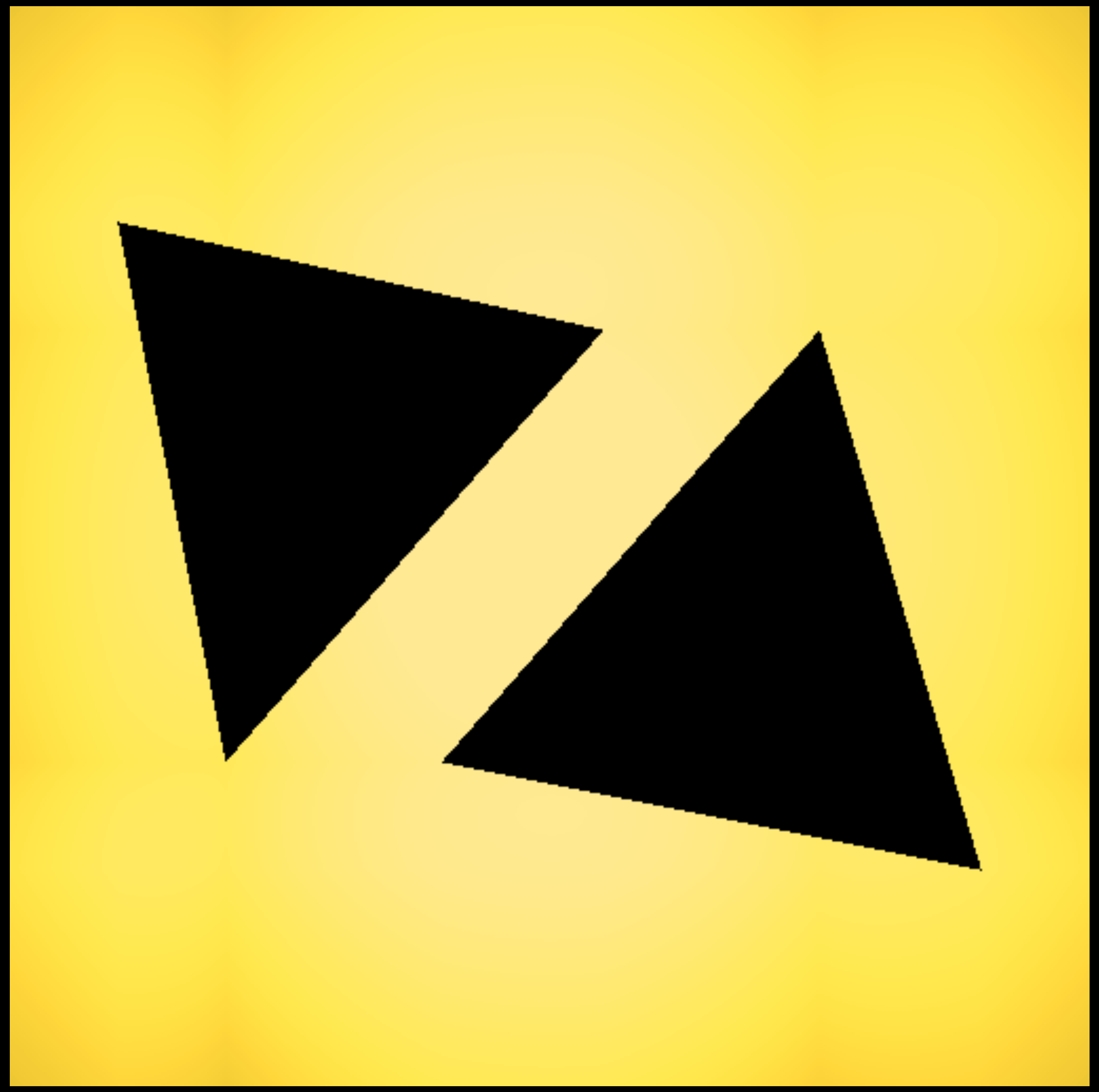


Discontinuity-smoothing MIS

Effective integrand for the

BRDF strategy

without smoothing



light

Discontinuity-smoothing MIS

Effective integrand for the **BRDF strategy**

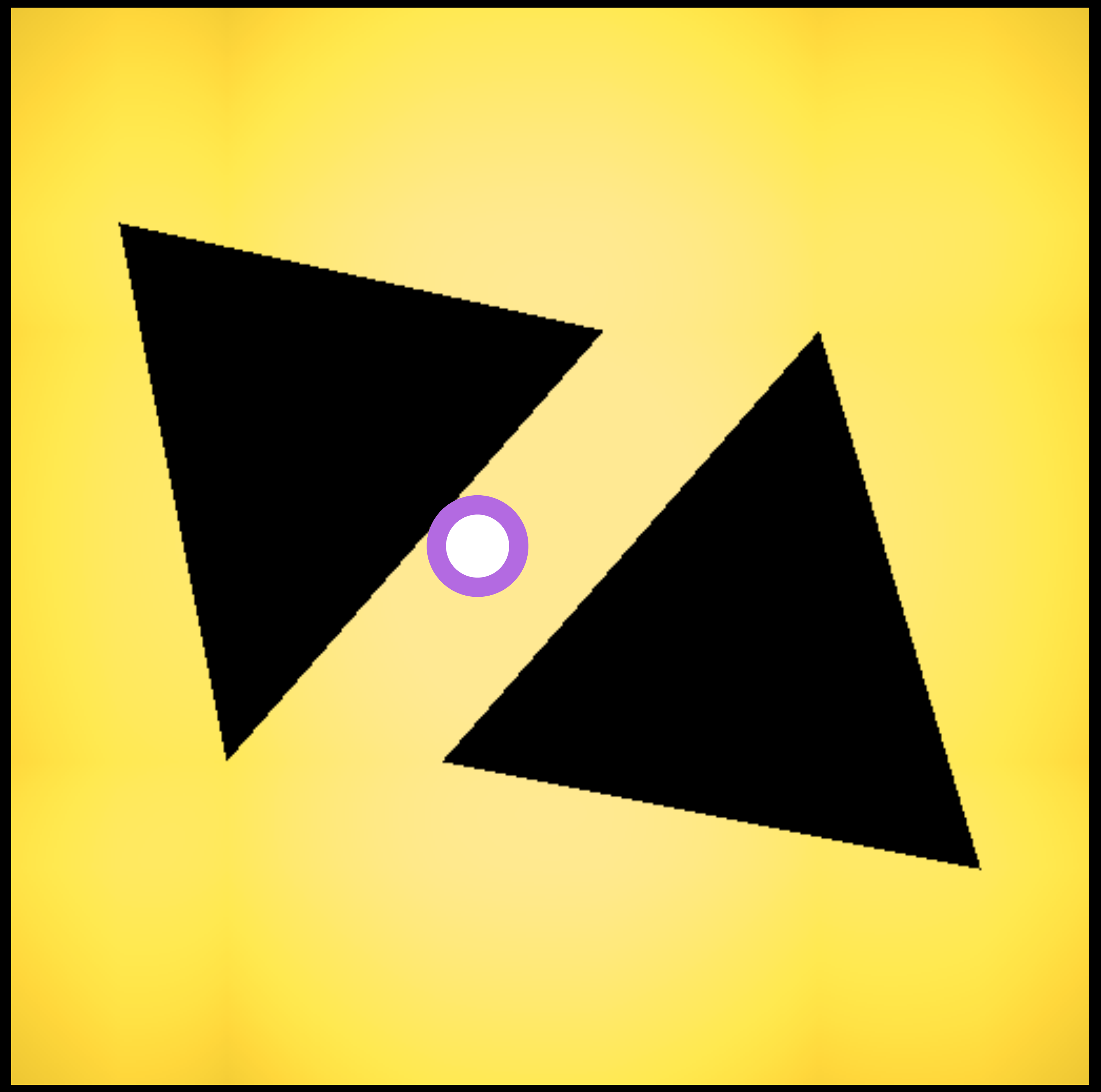
without smoothing

Discontinuity-smoothing MIS

Effective integrand for the

BRDF strategy

without smoothing



Discontinuity-smoothing MIS

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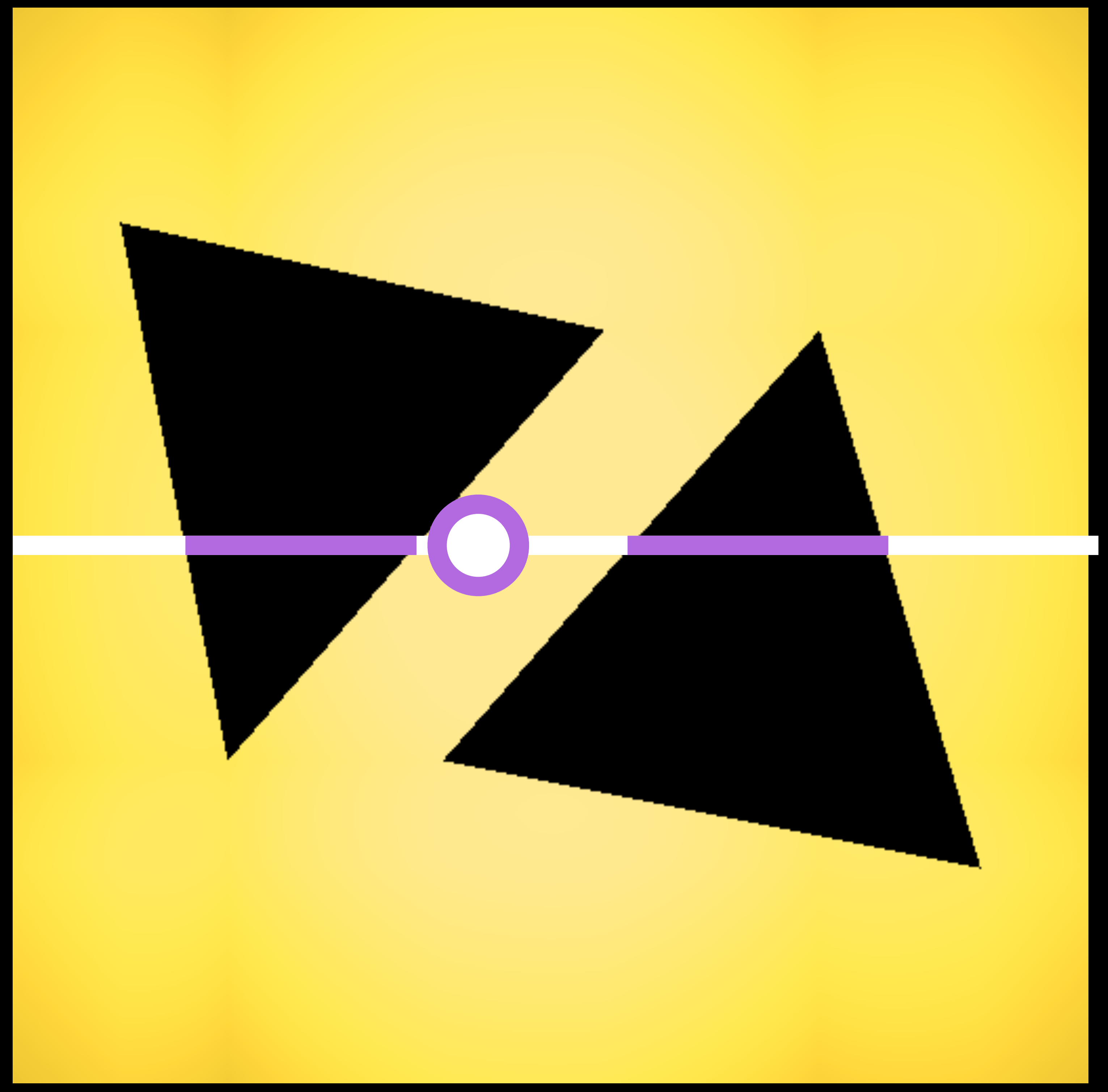

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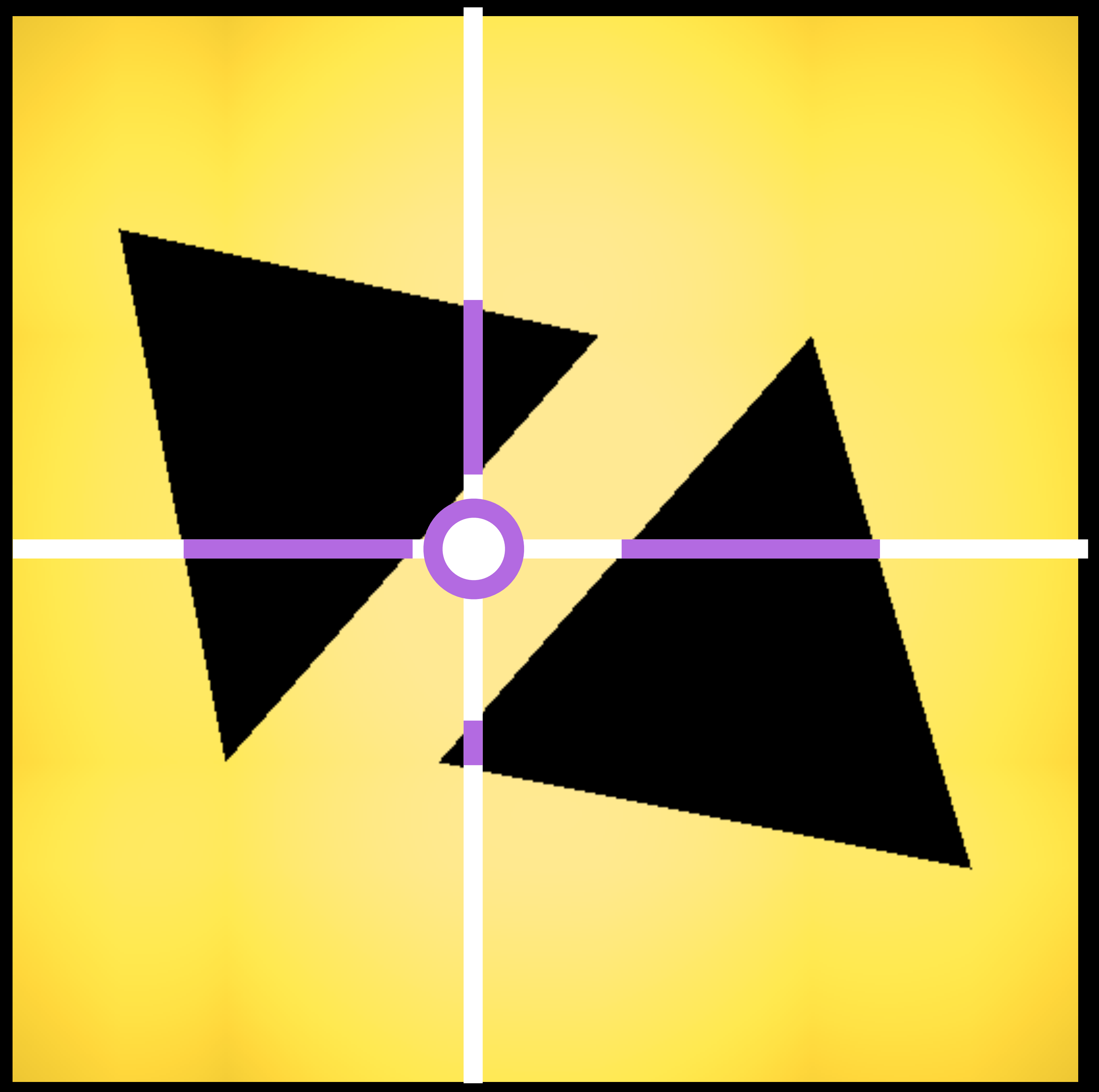

Discontinuity-smoothing MIS

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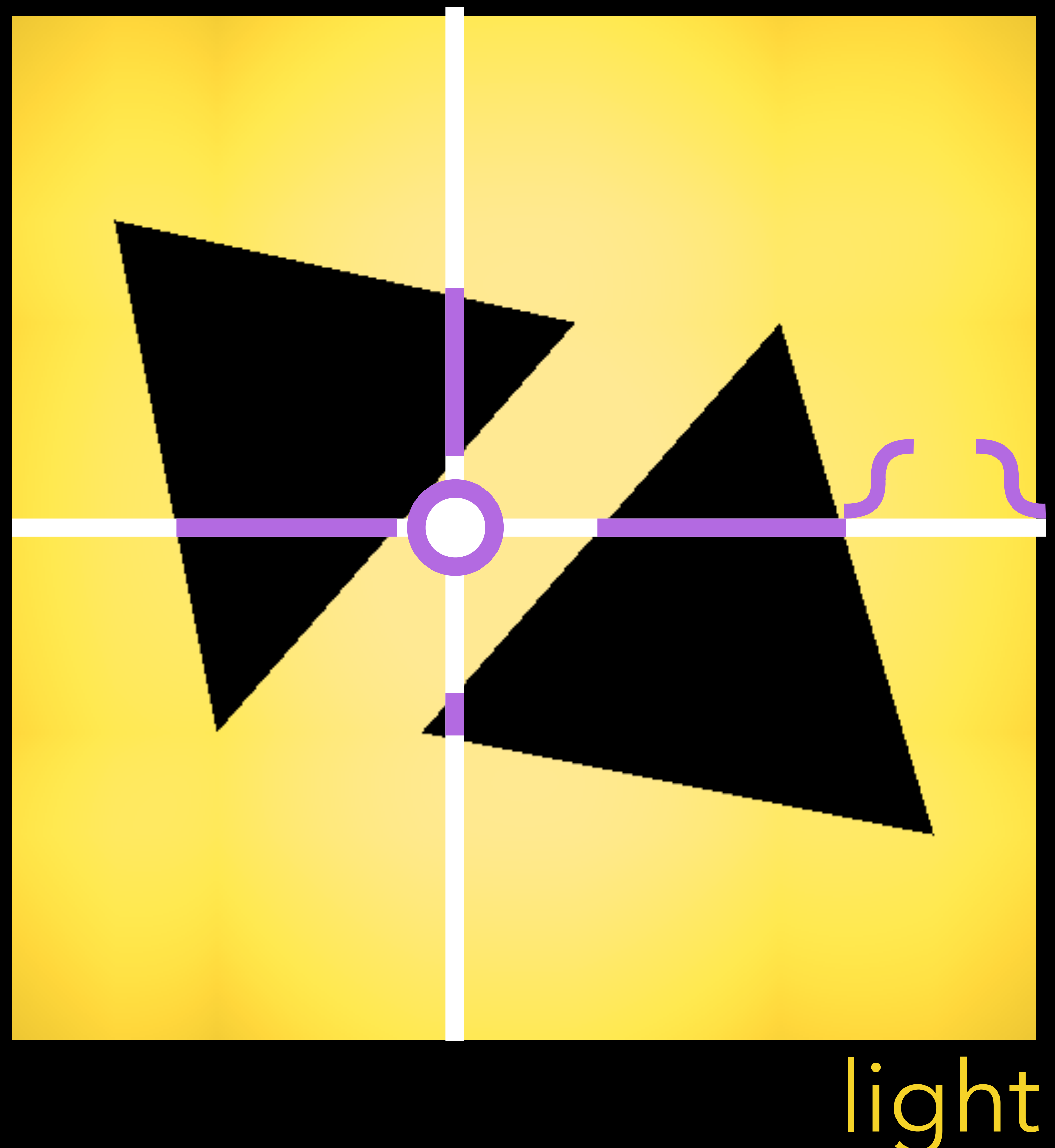
without smoothing



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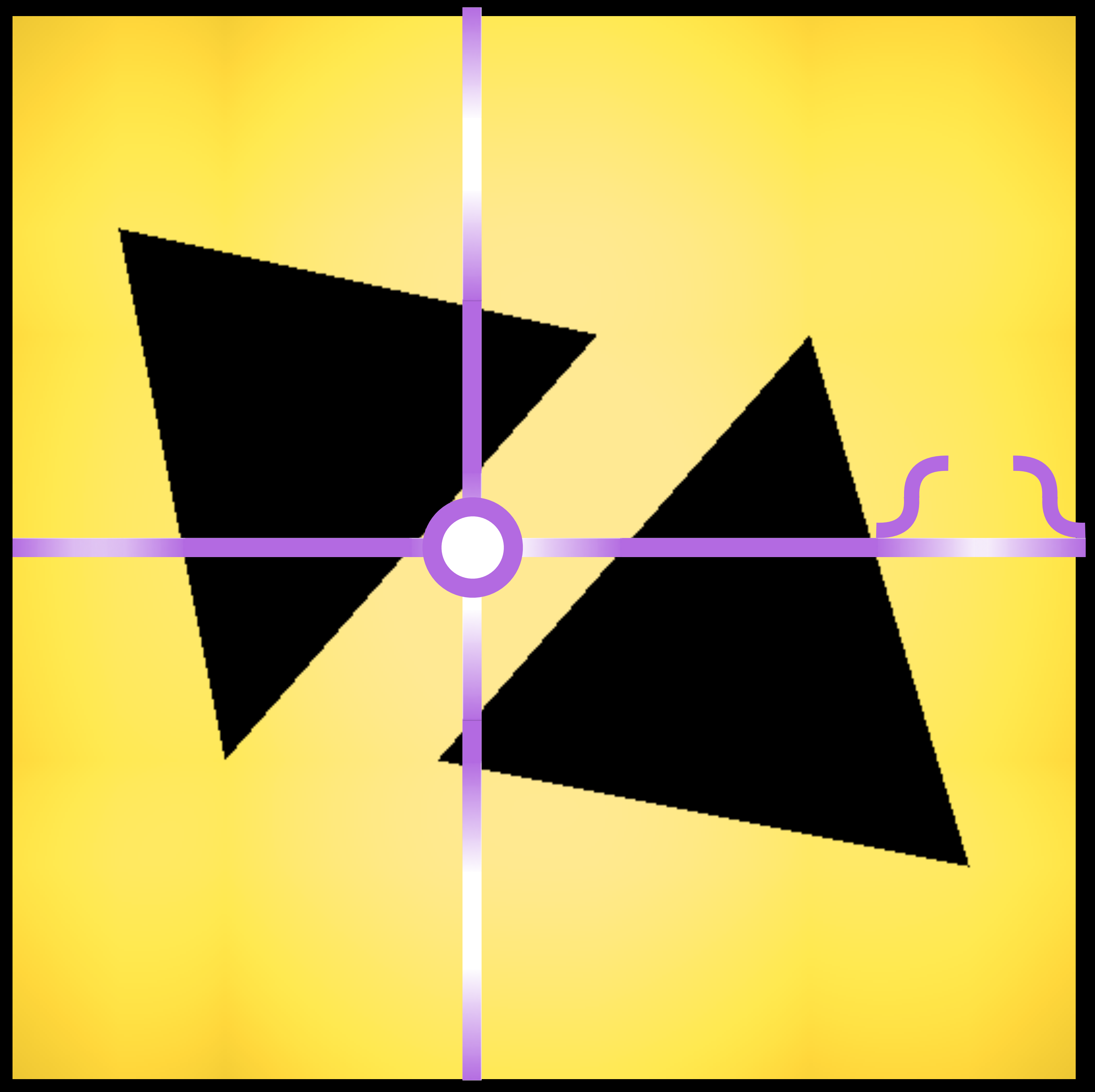


Discontinuity-smoothing MIS

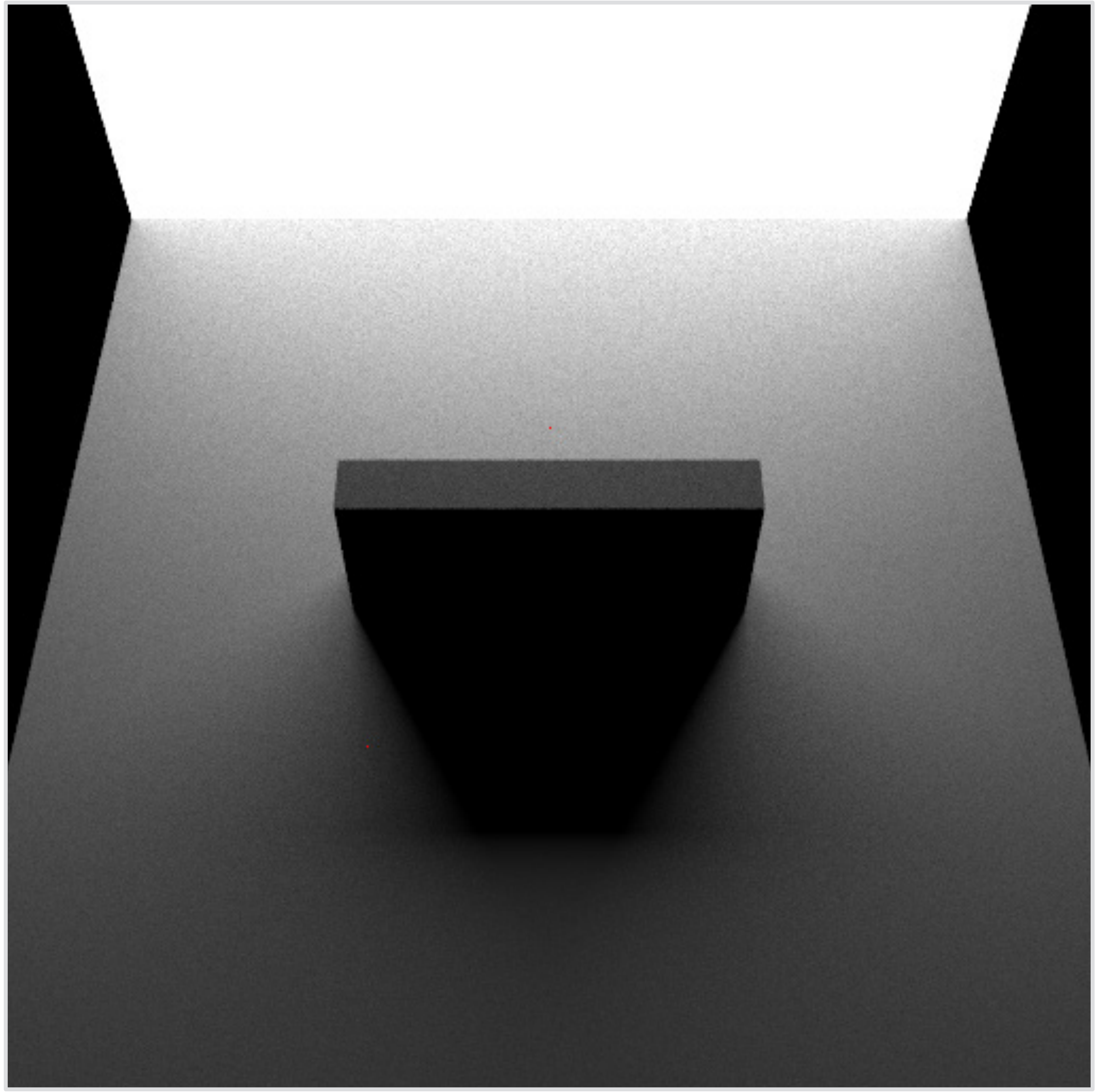
Effective integrand for the

BRDF strategy

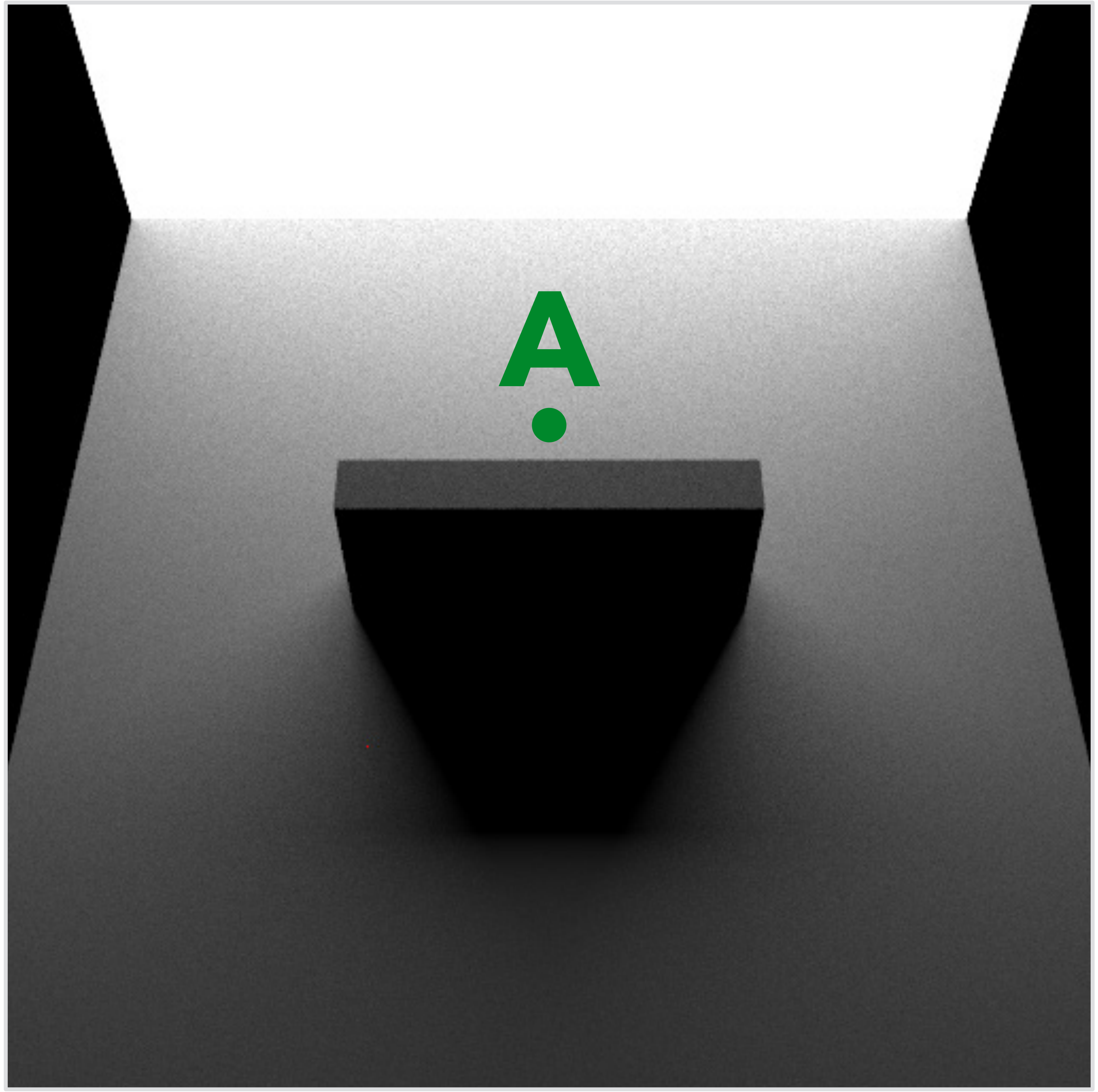
with smoothing

Smoothing MIS:
Convergence tests

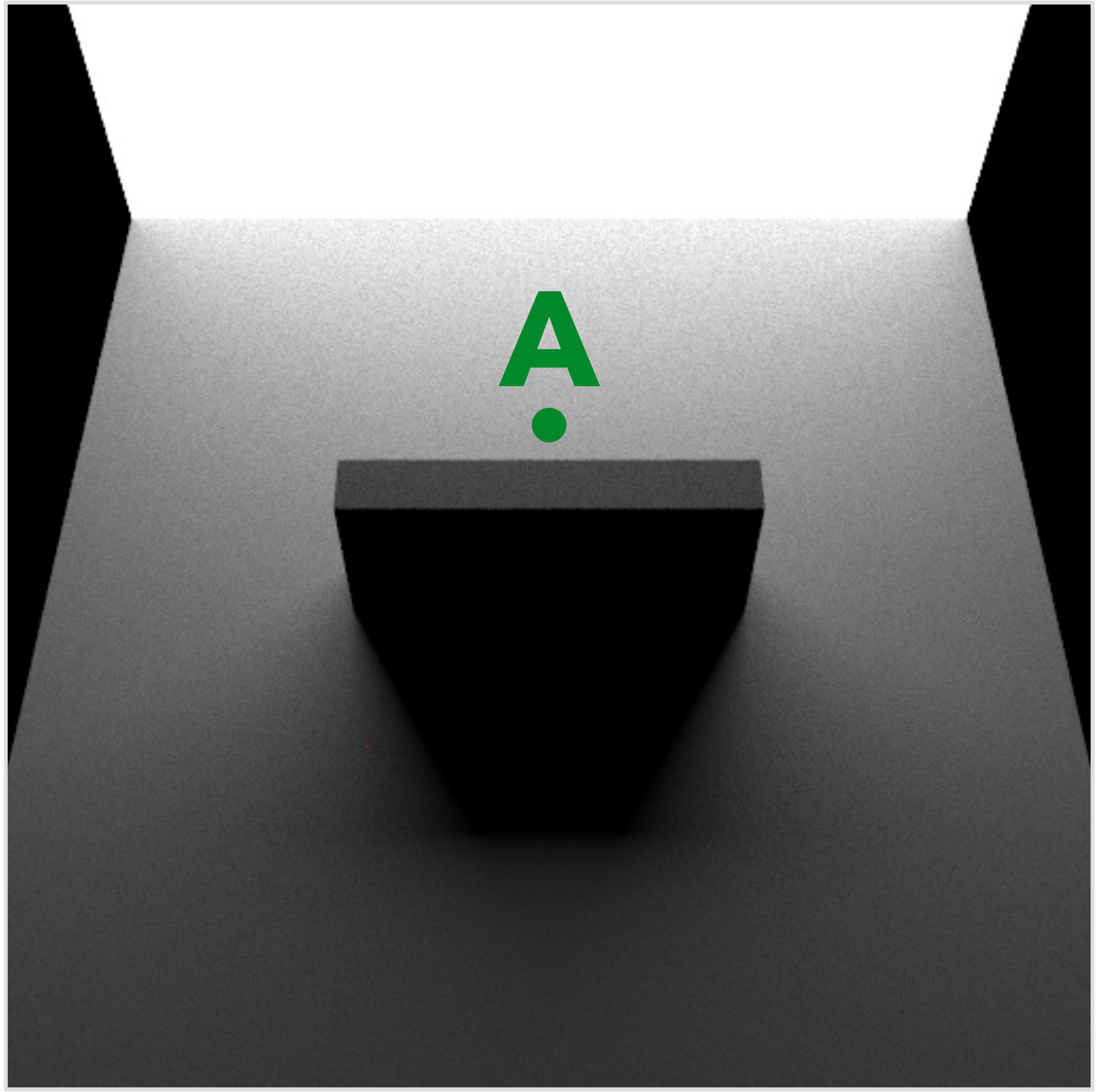
Convergence tests

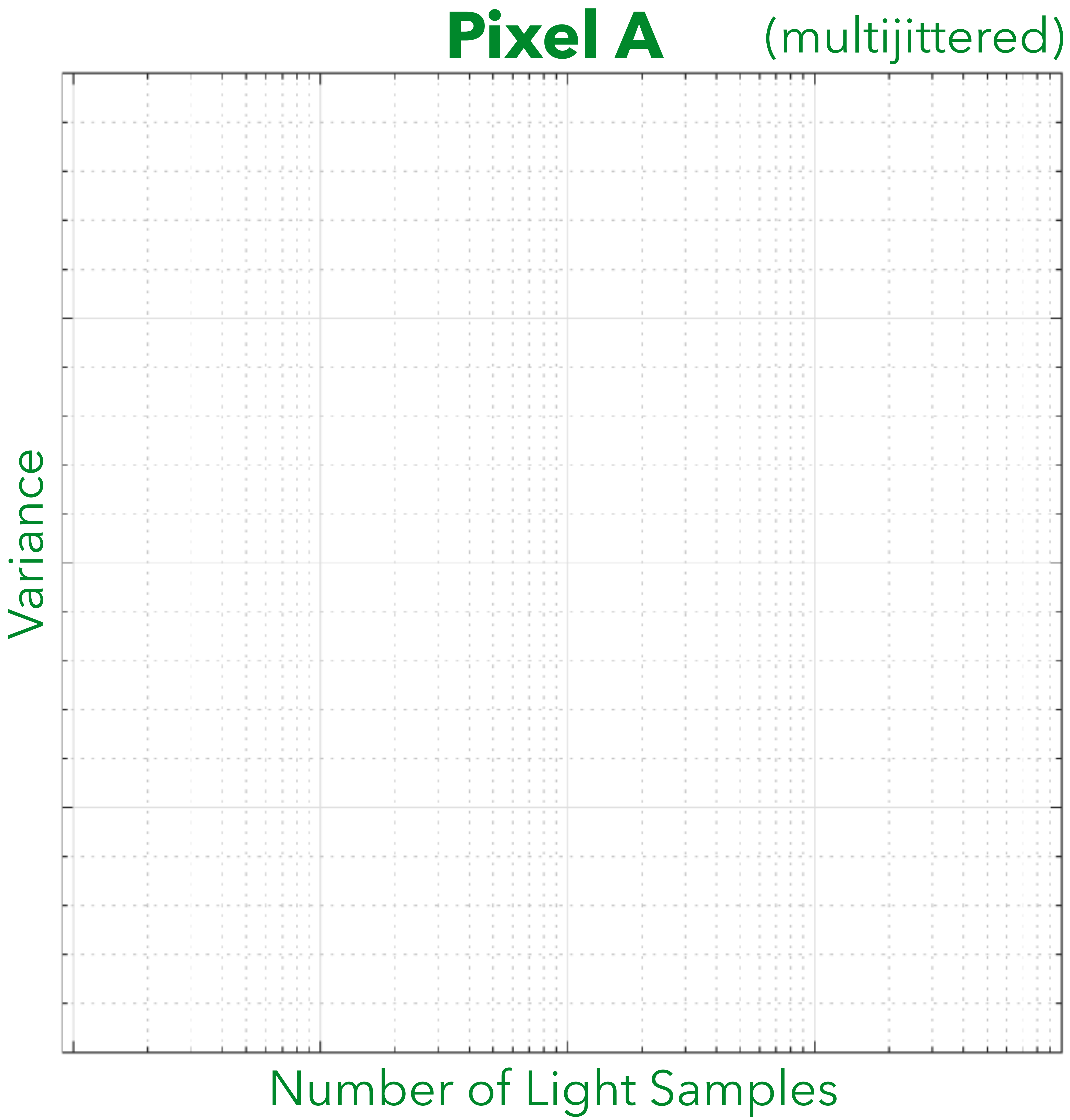


Convergence tests

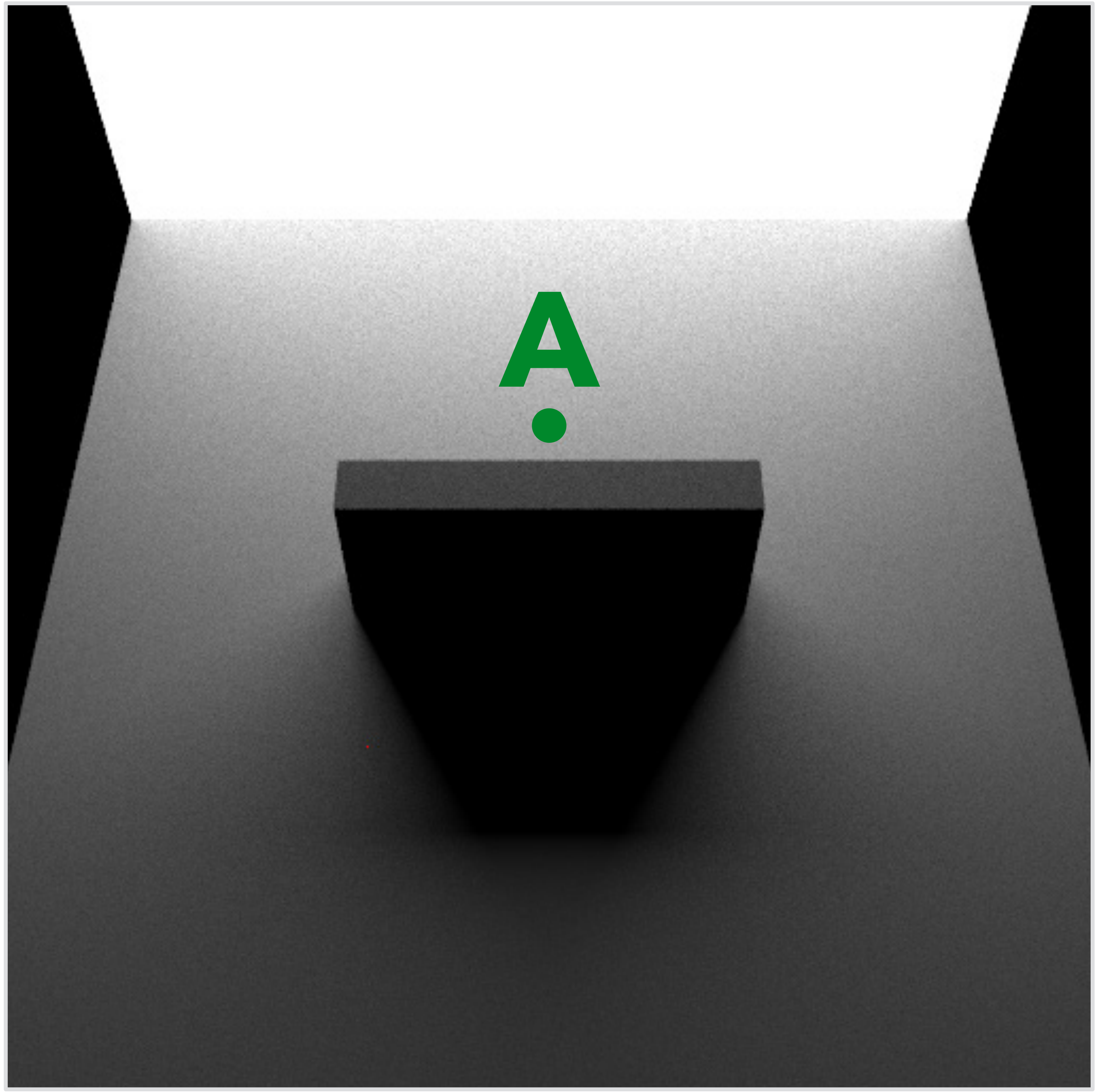


Convergence tests

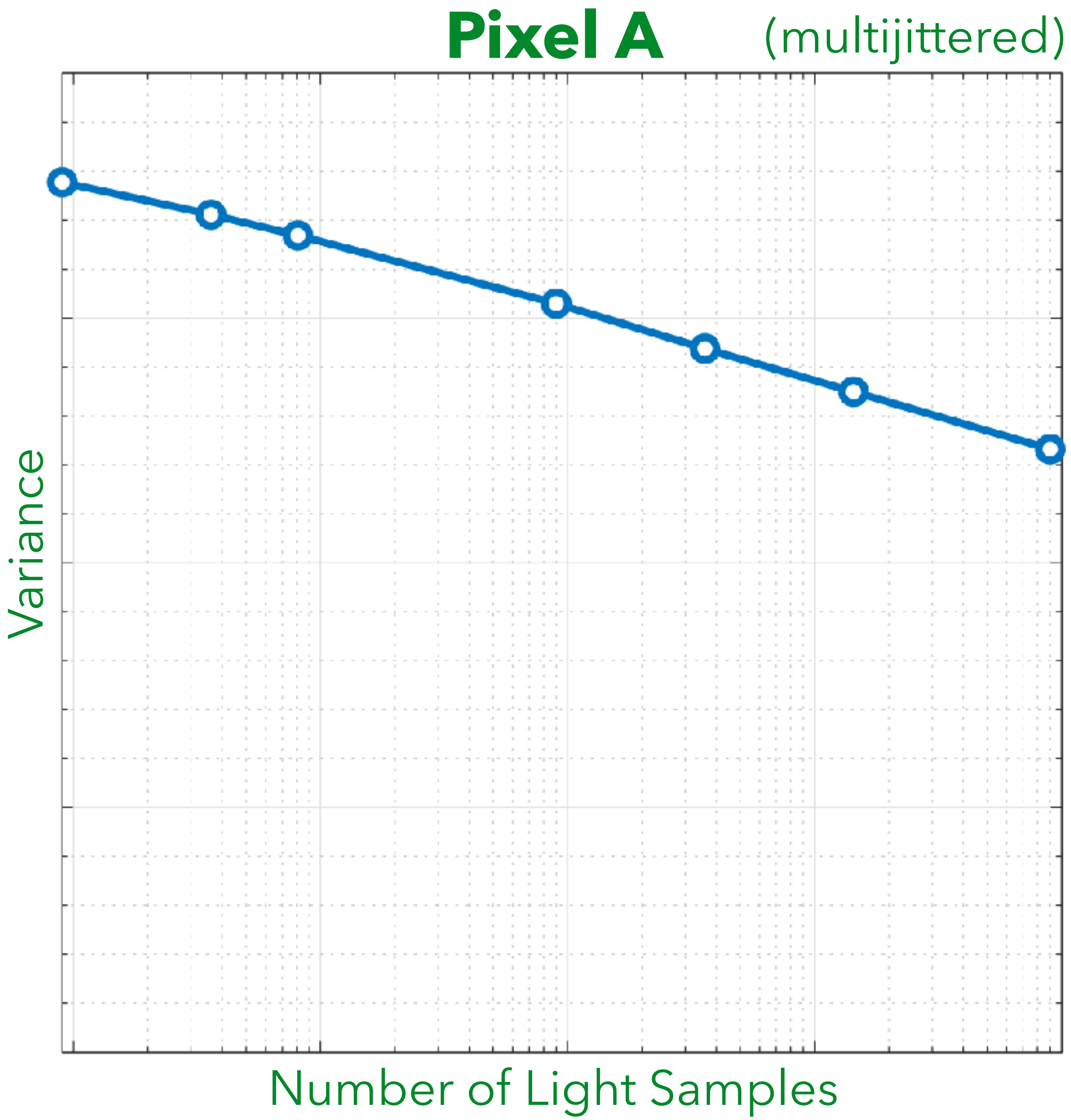




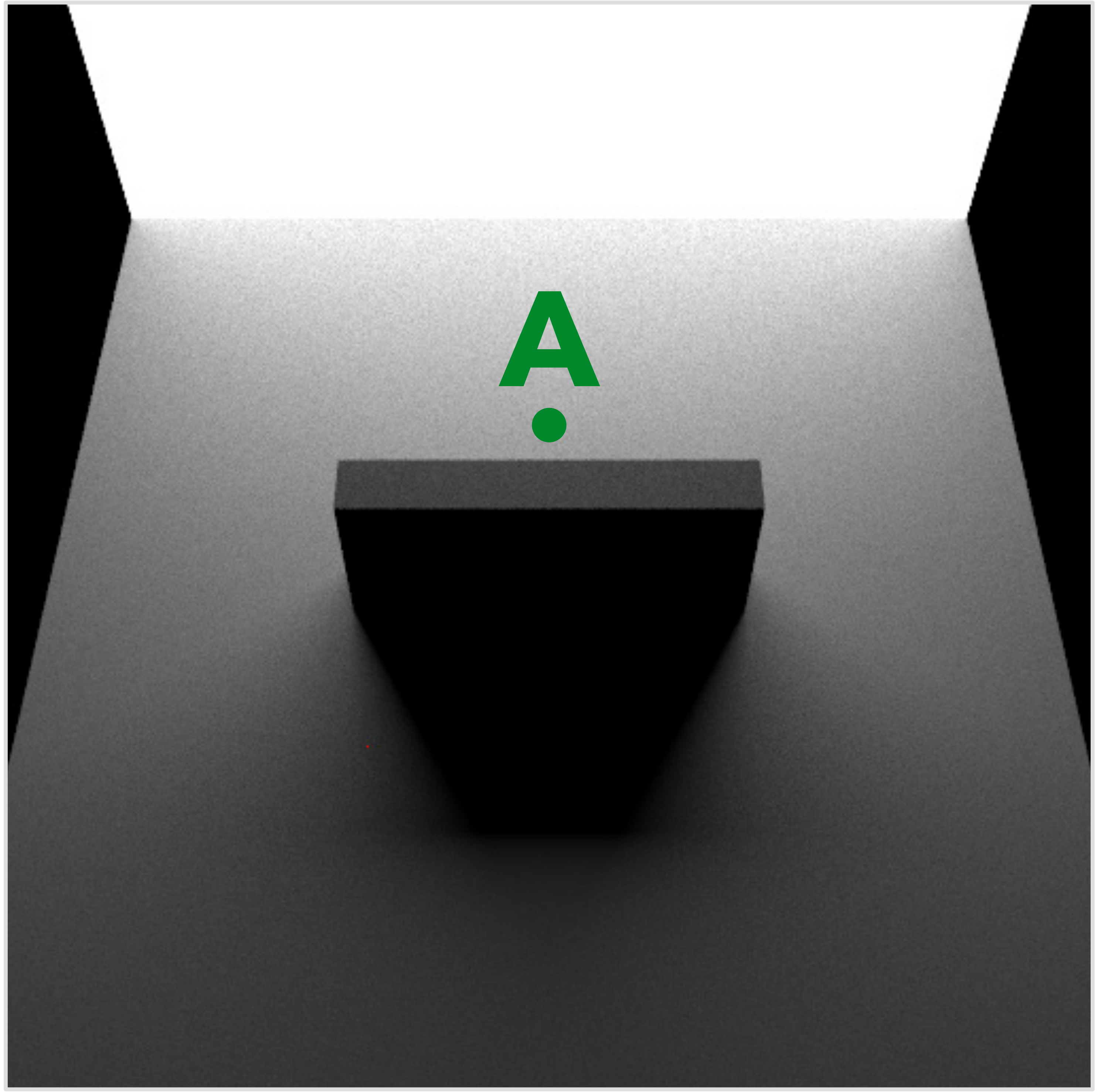
Convergence tests



BSDF ($N^{-1.38}$)

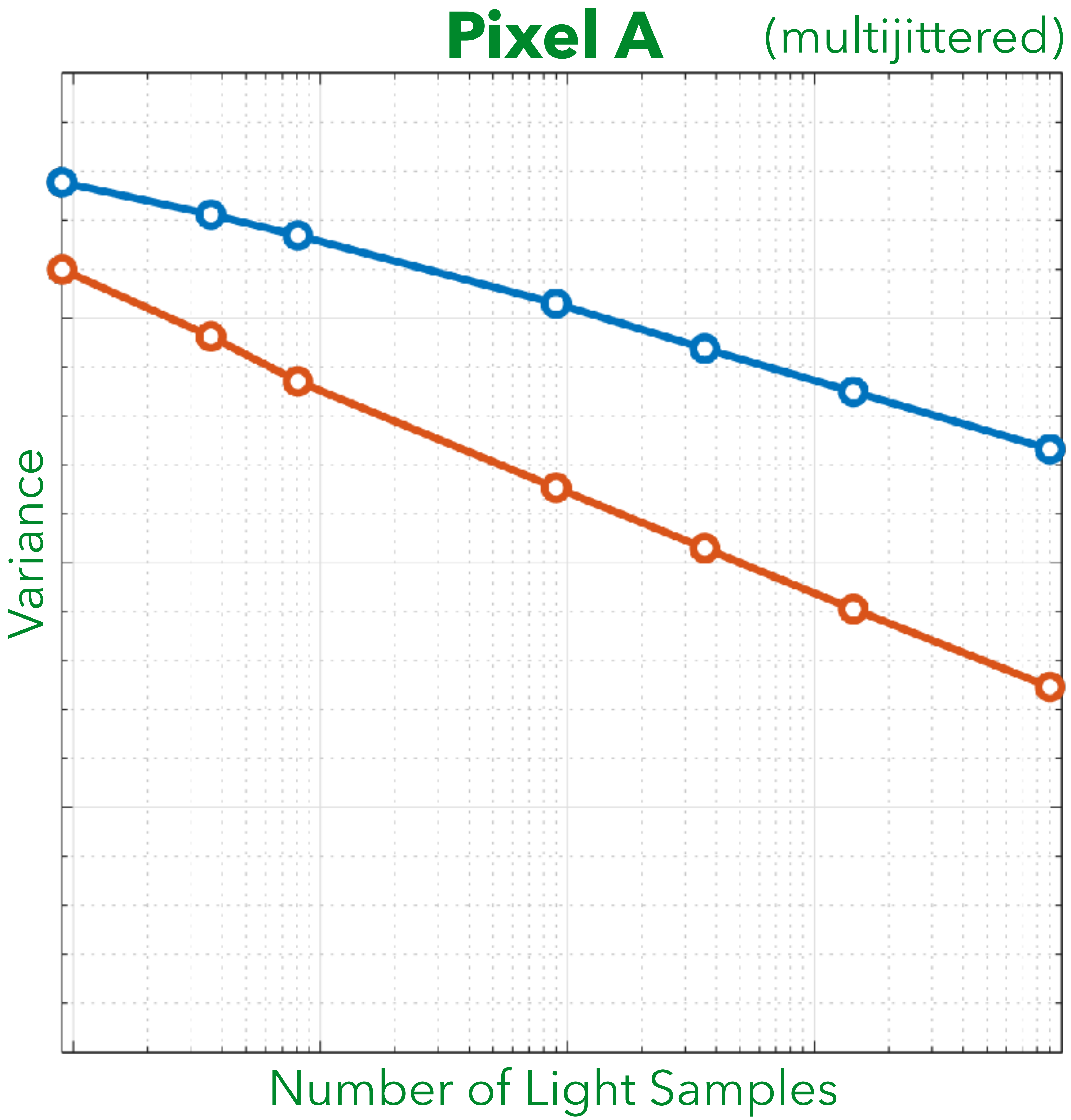


Convergence tests

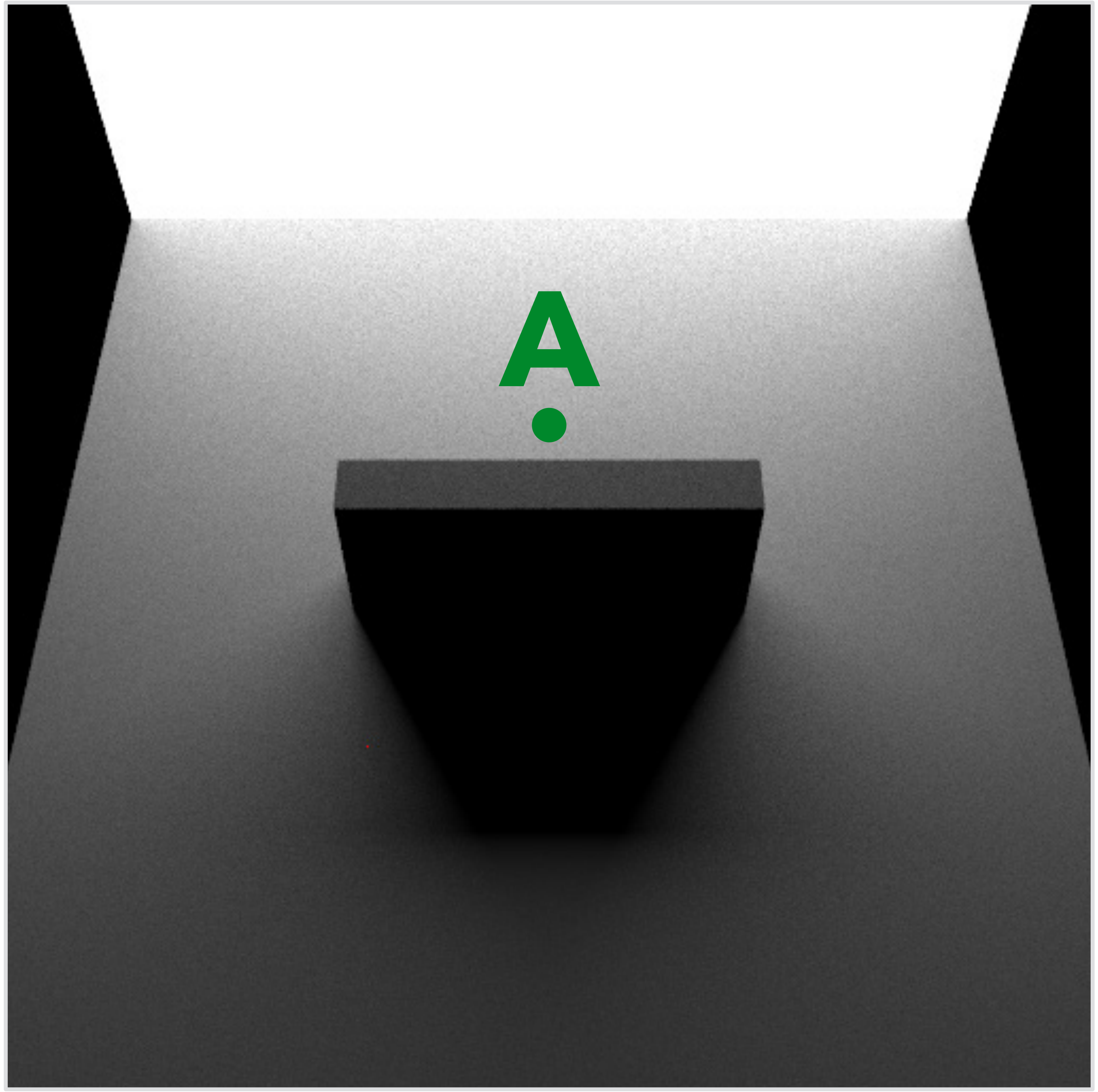


BSDF ($N^{-1.38}$)

Horiz. lines ($N^{-2.13}$)



Convergence tests

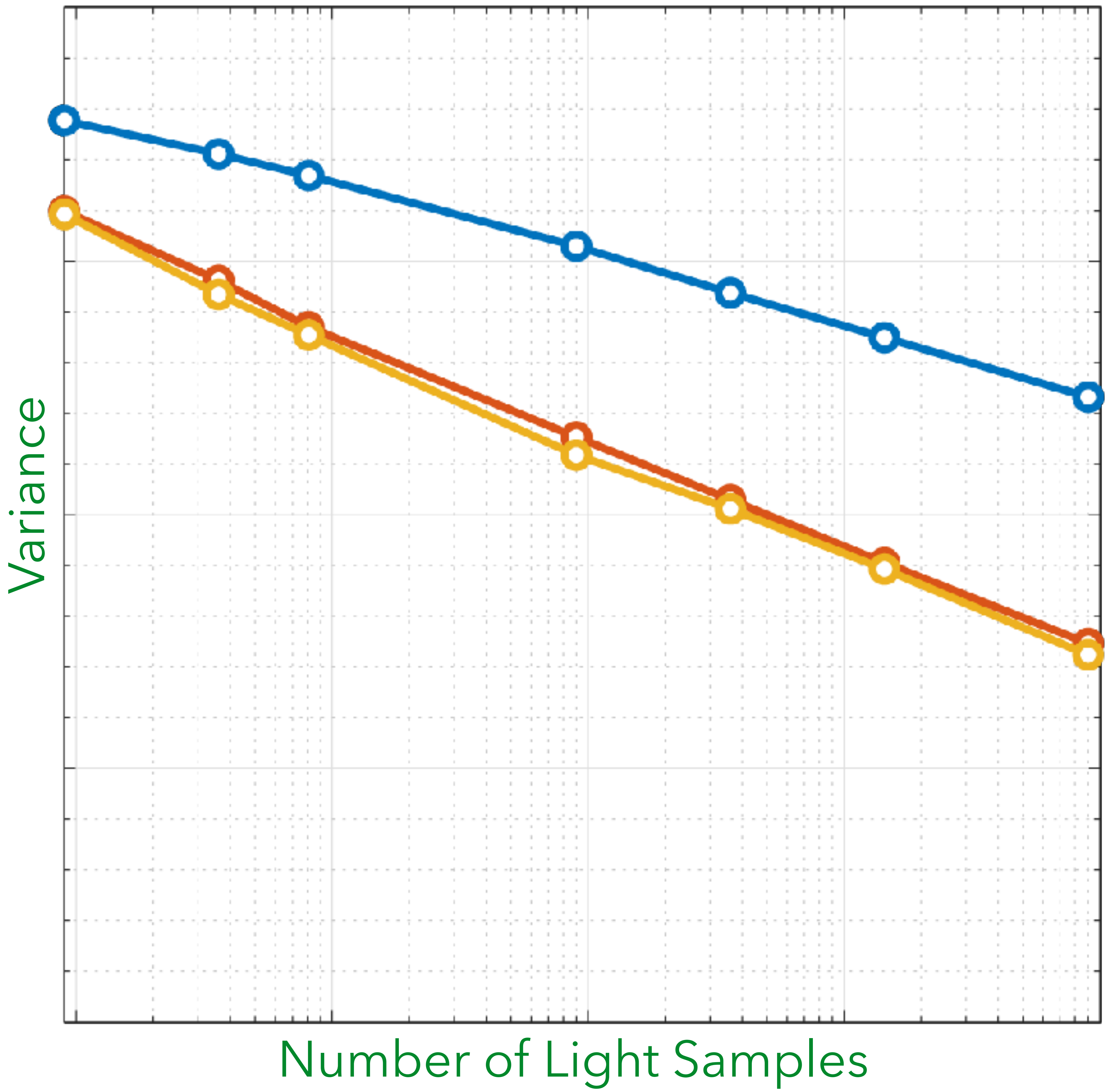


BSDF ($N^{-1.38}$)

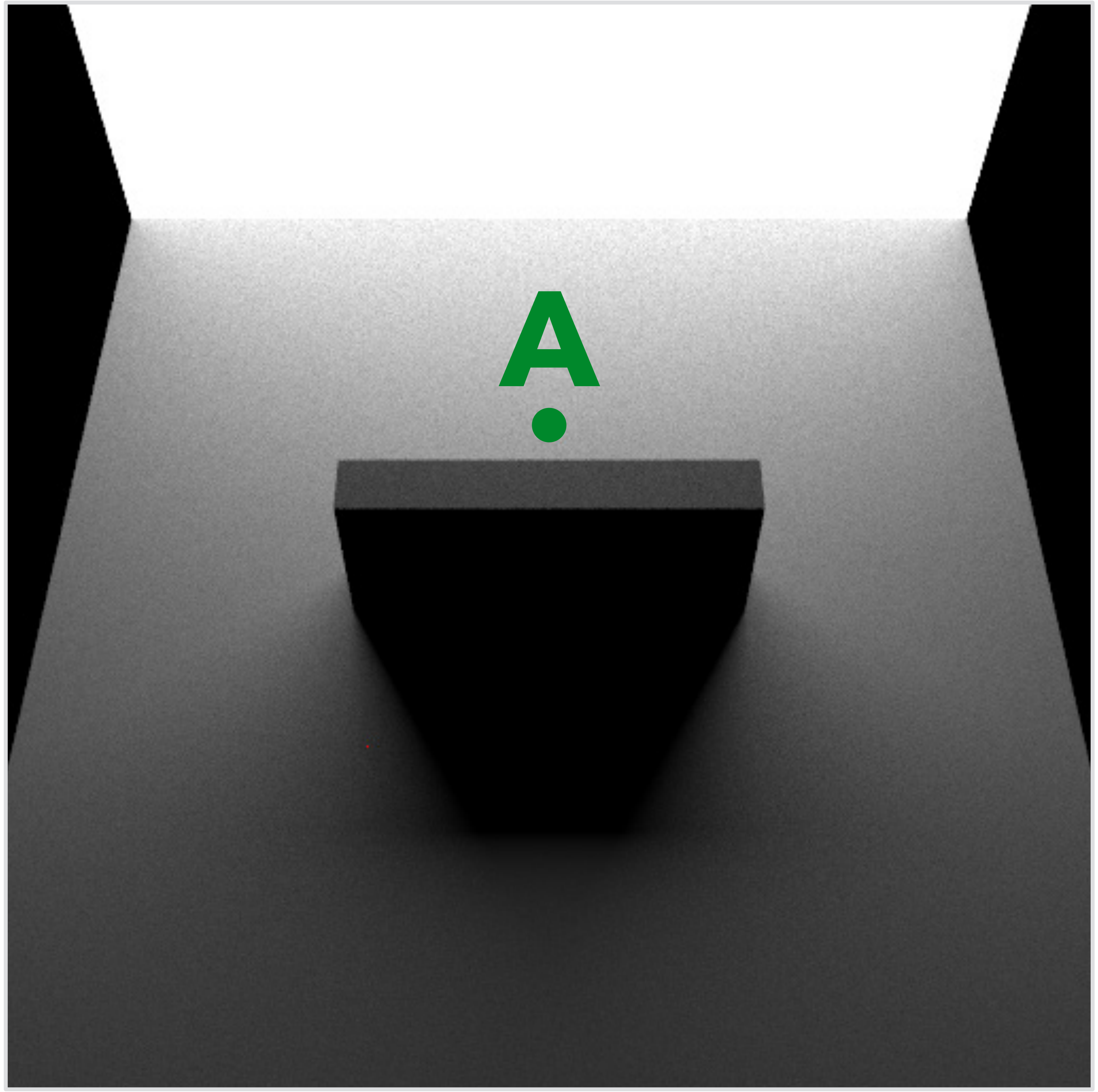
Horiz. lines ($N^{-2.13}$)

Vert. lines ($N^{-2.14}$)

Pixel A (multijittered)



Convergence tests

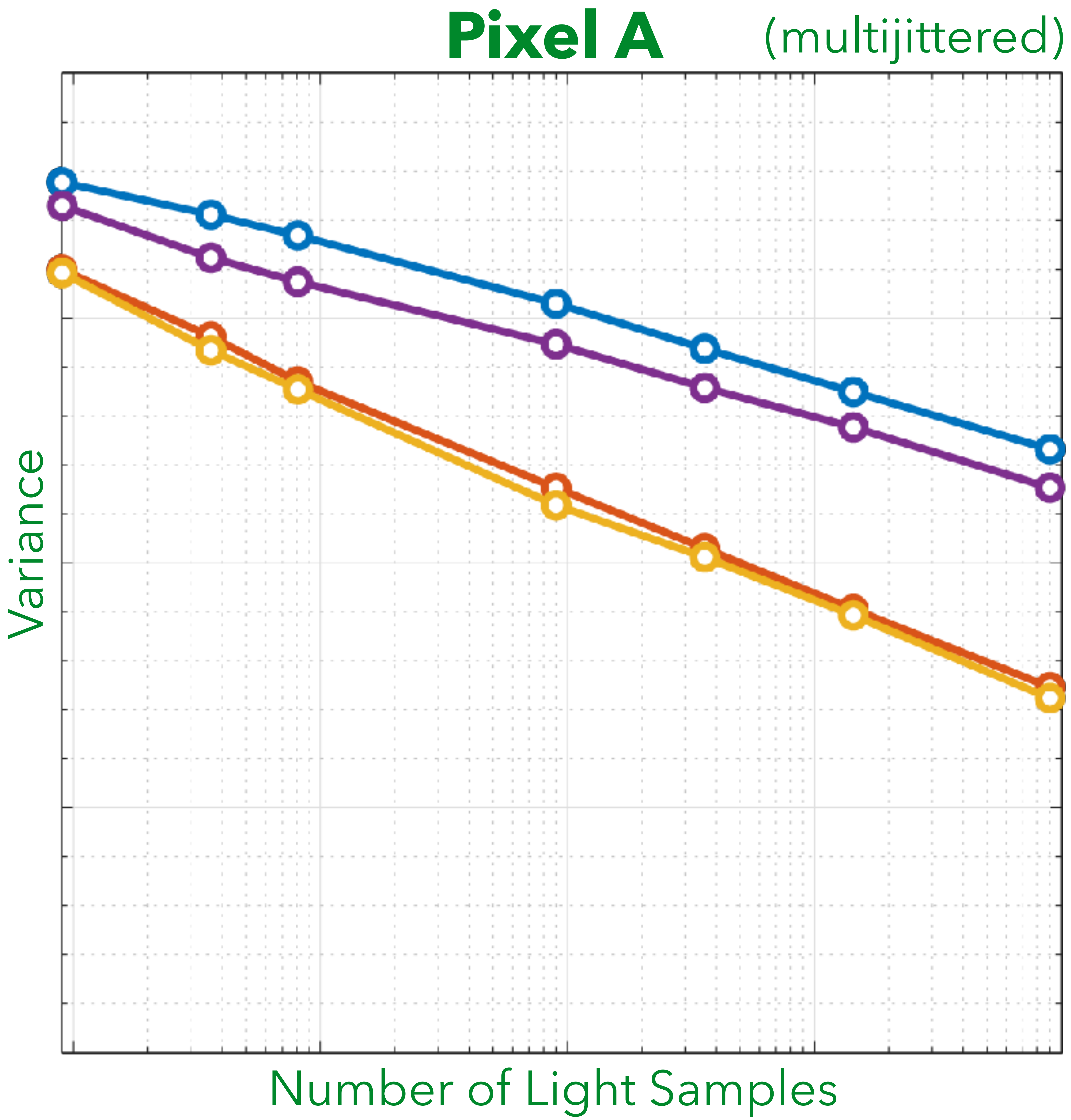


BSDF ($N^{-1.38}$)

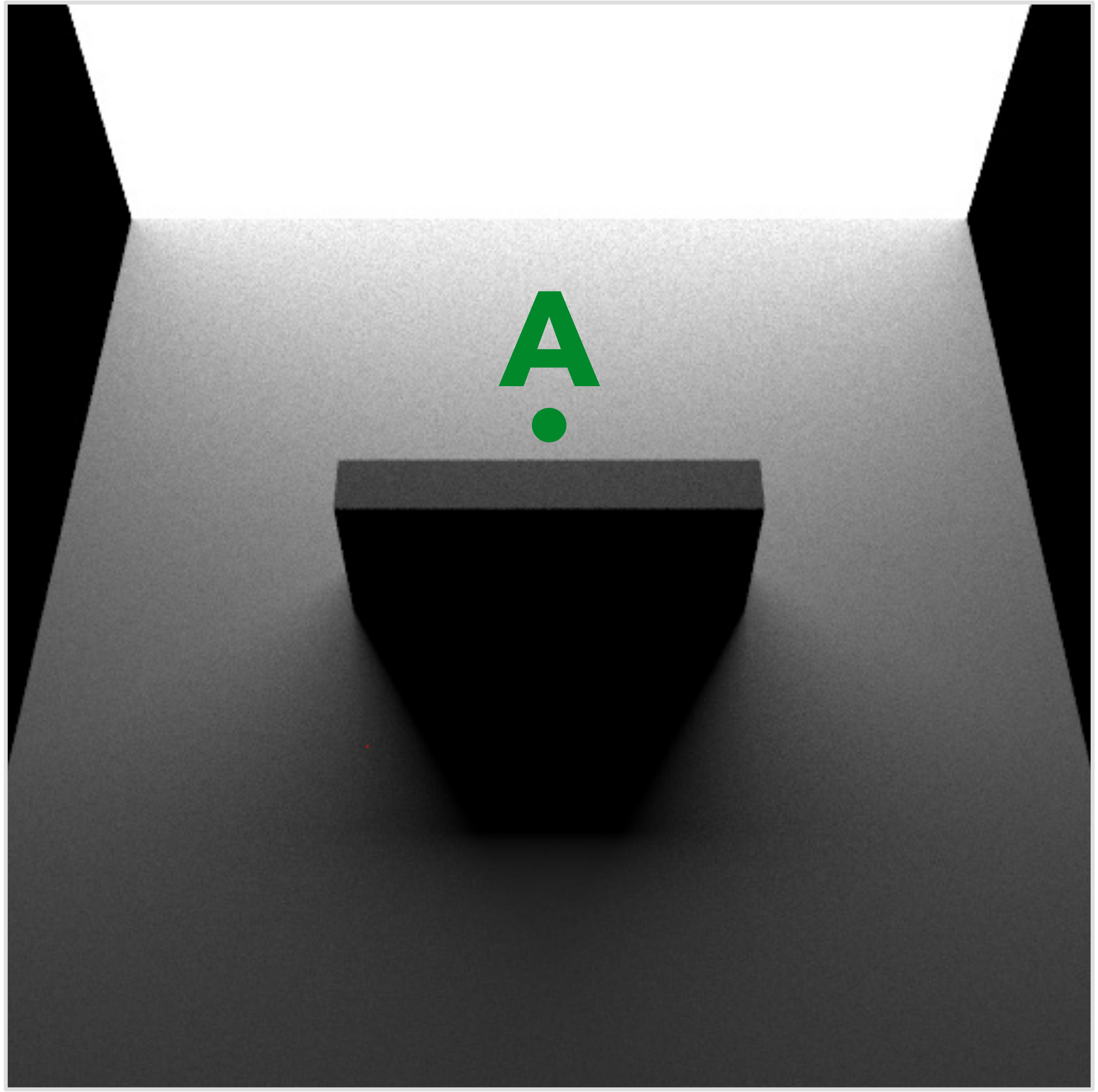
Horiz. lines ($N^{-2.13}$)

Vert. lines ($N^{-2.14}$)

MIS BSDF + Lines ($N^{-1.40}$)



Convergence tests



BSDF ($N^{-1.38}$)

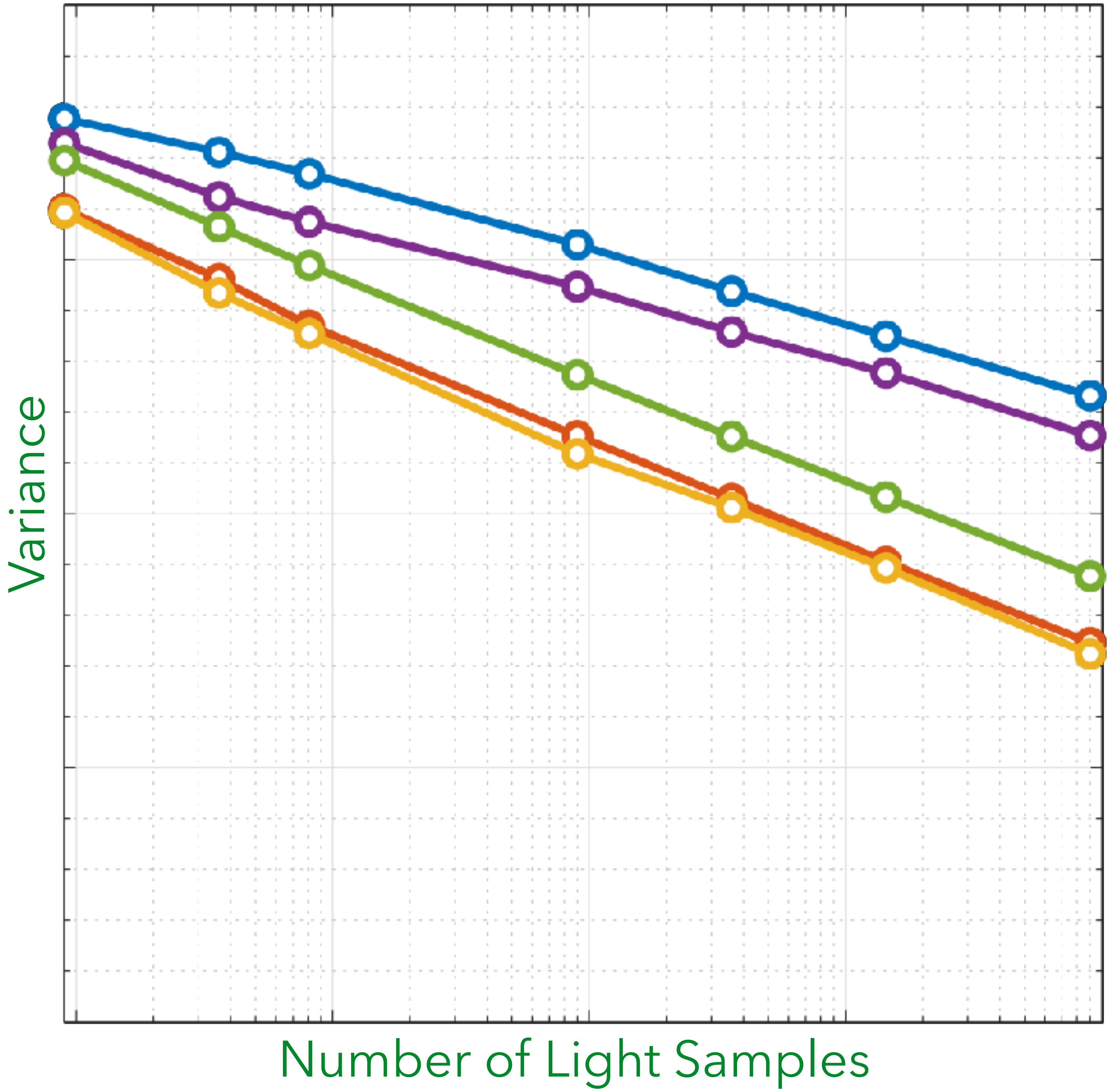
Horiz. lines ($N^{-2.13}$)

Vert. lines ($N^{-2.14}$)

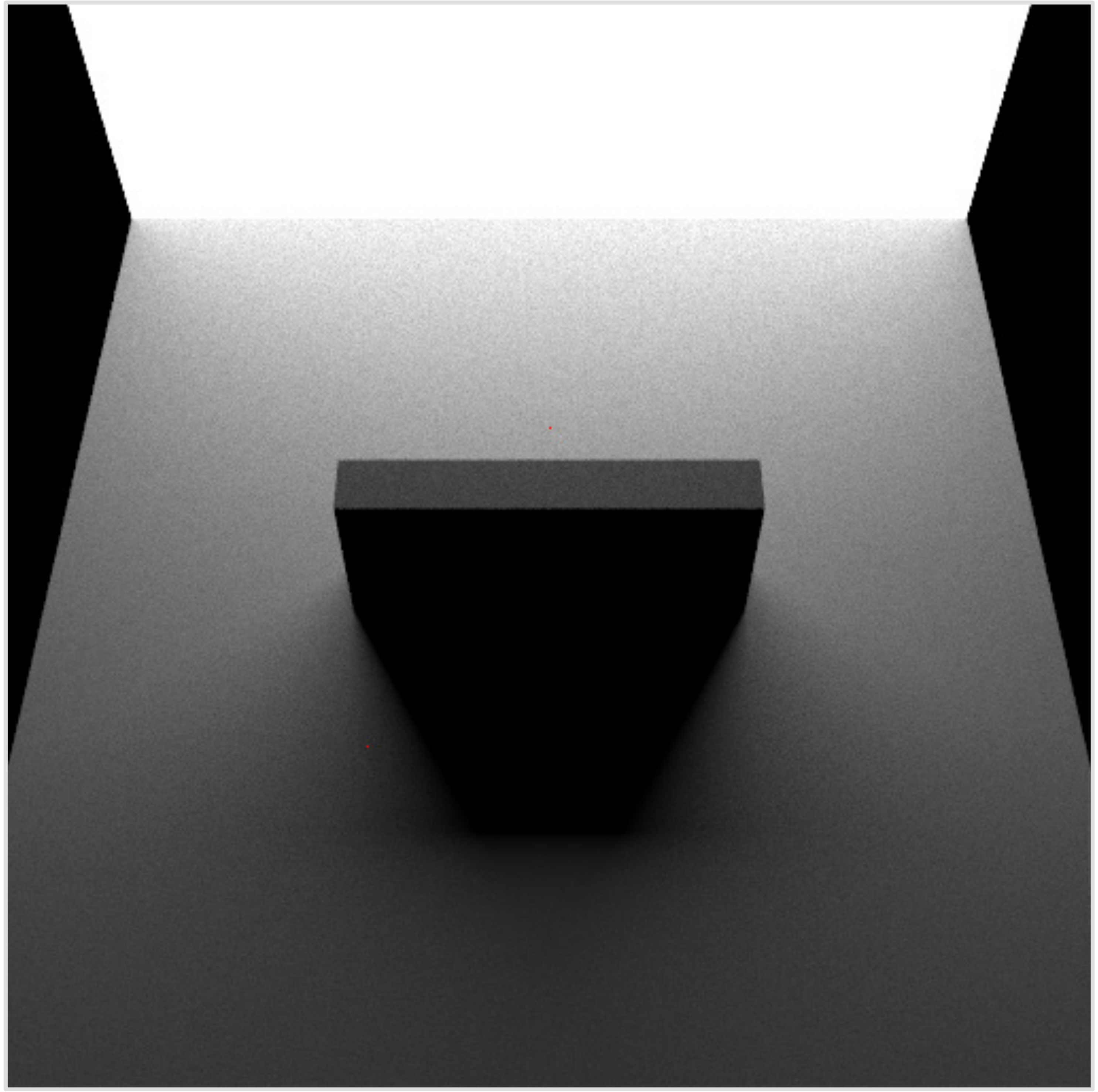
MIS BSDF + Lines ($N^{-1.40}$)

MIS BSDF + Lines
with smoothing ($N^{-2.04}$)

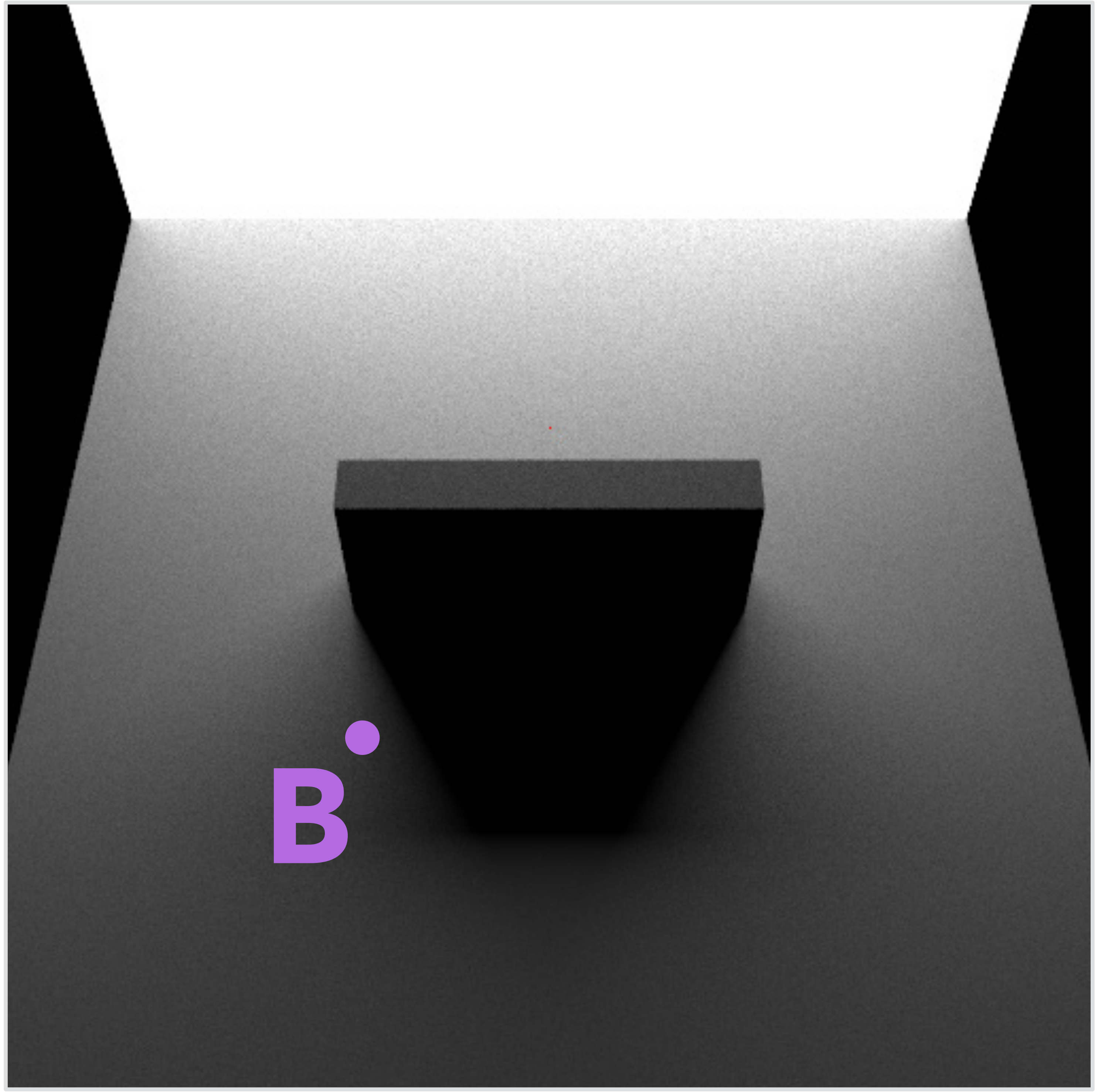
Pixel A (multijittered)



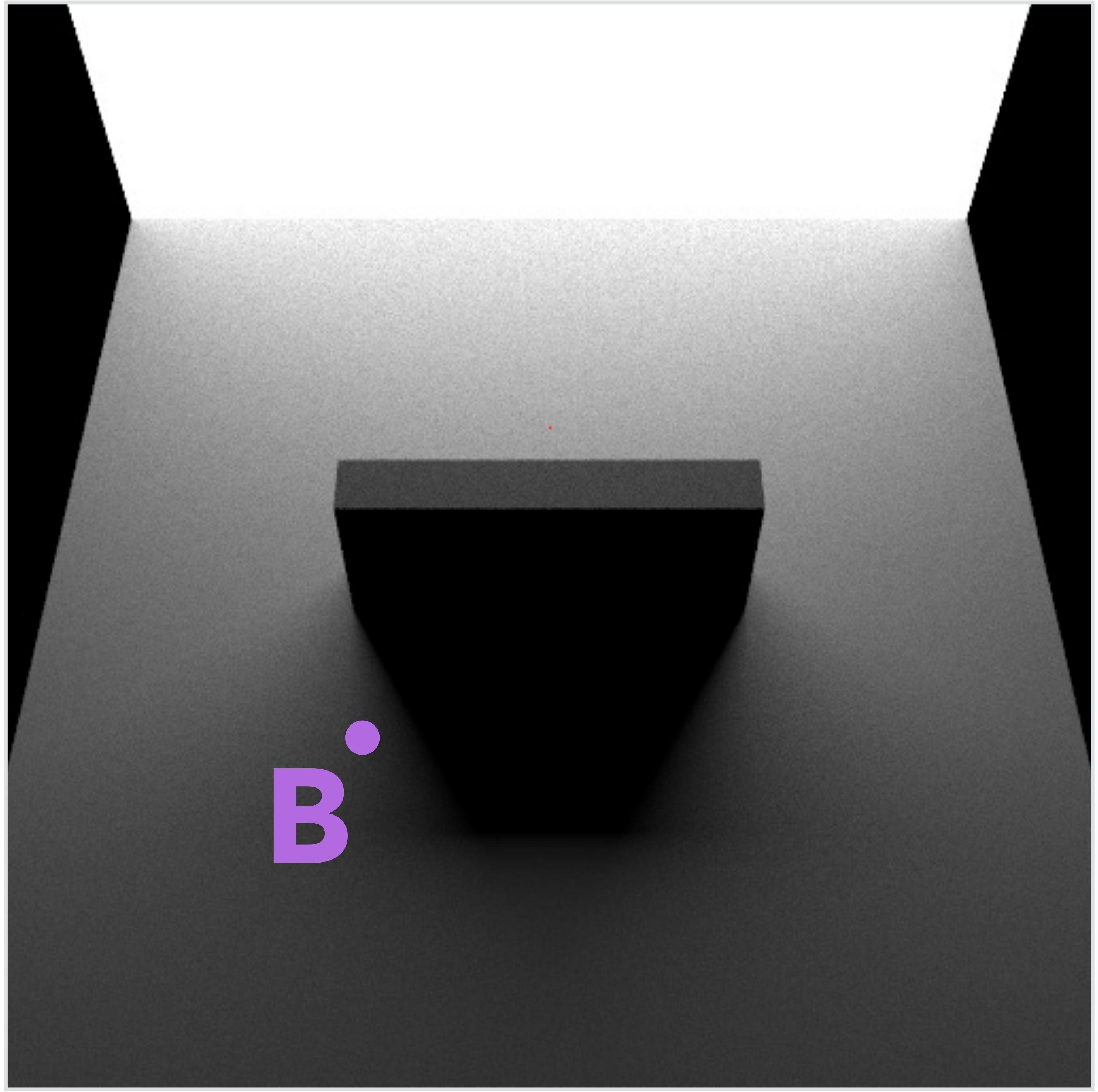
Convergence tests

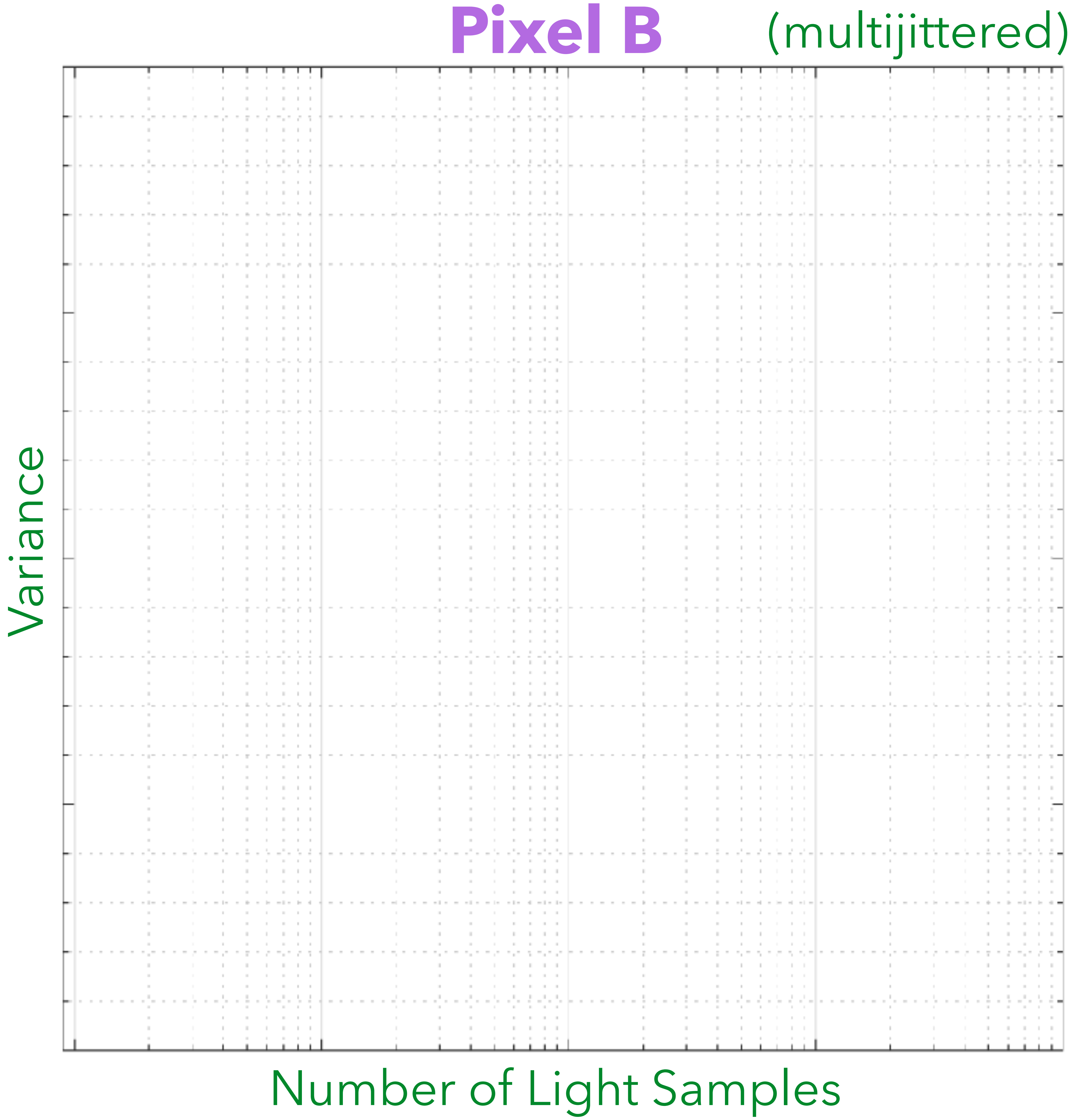


Convergence tests

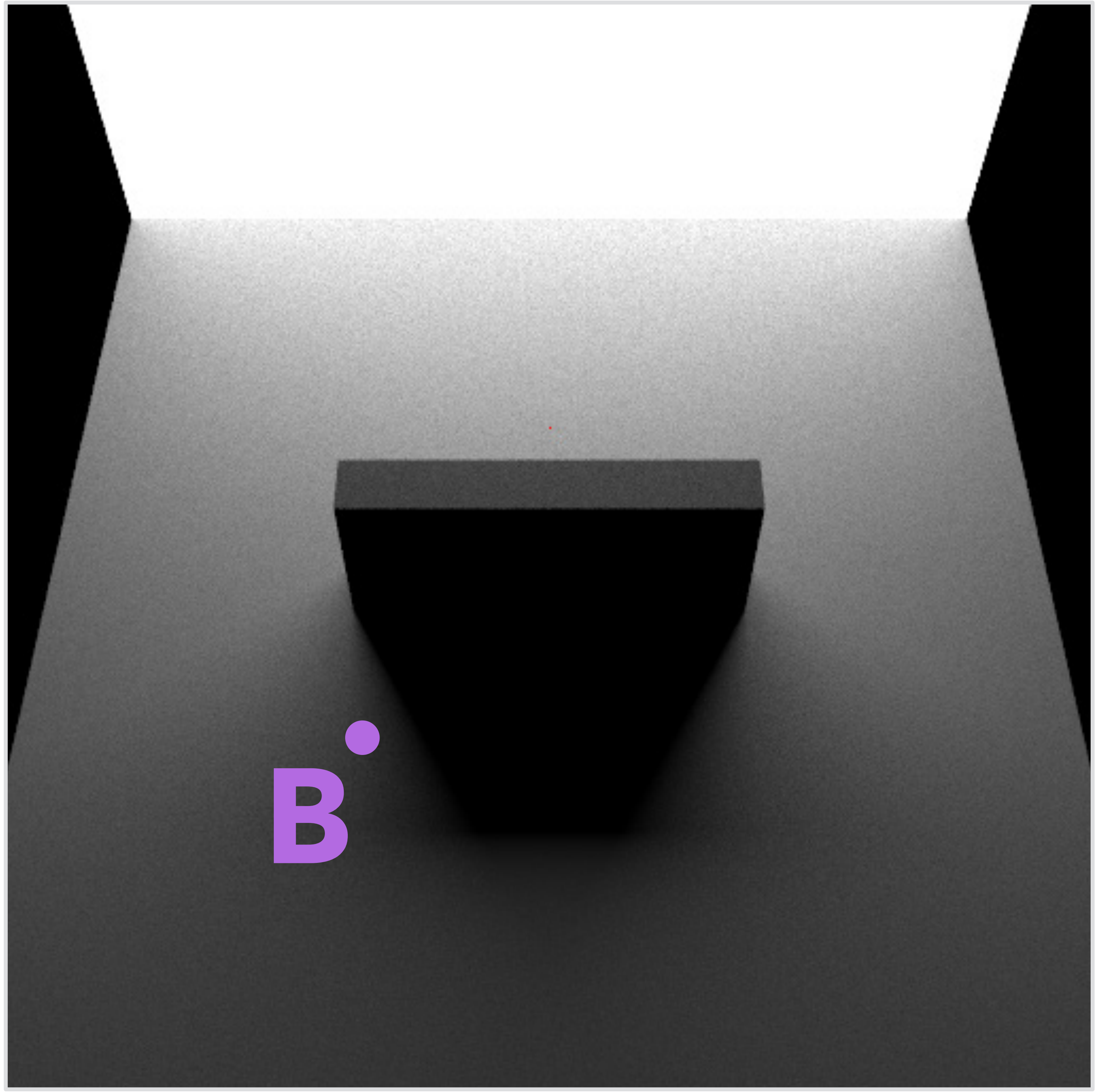


Convergence tests

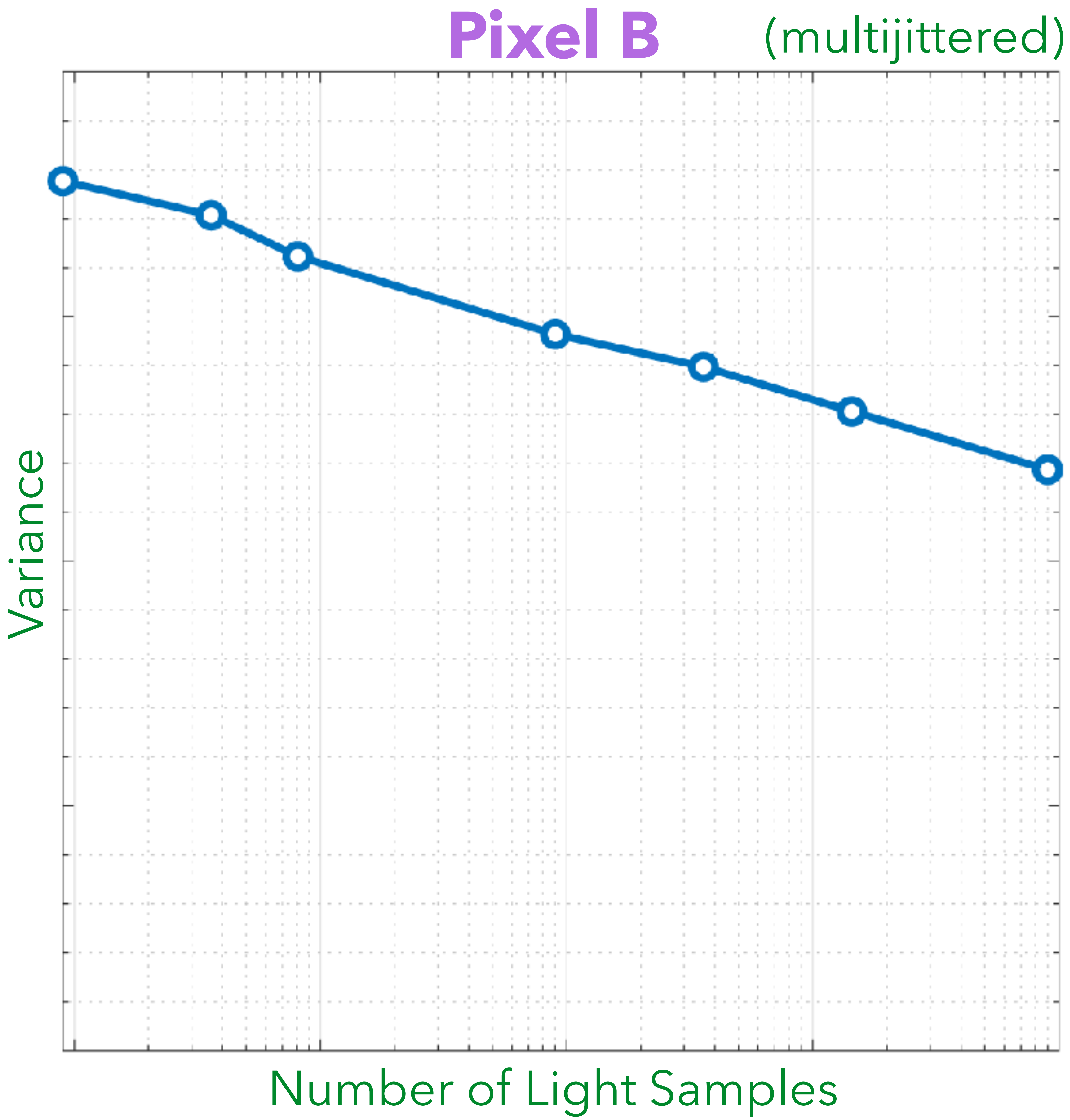




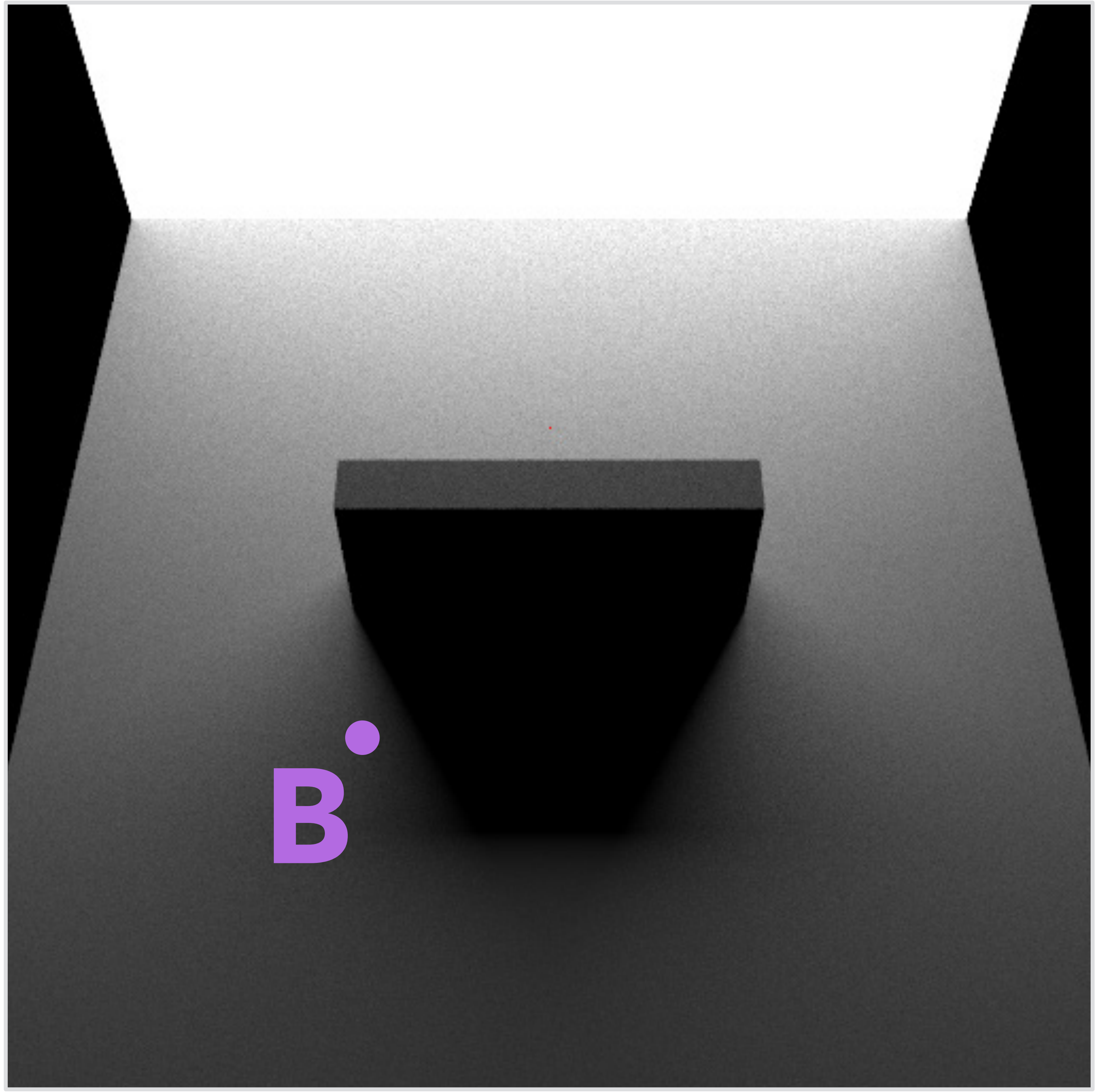
Convergence tests



BSDF ($N^{-1.48}$)

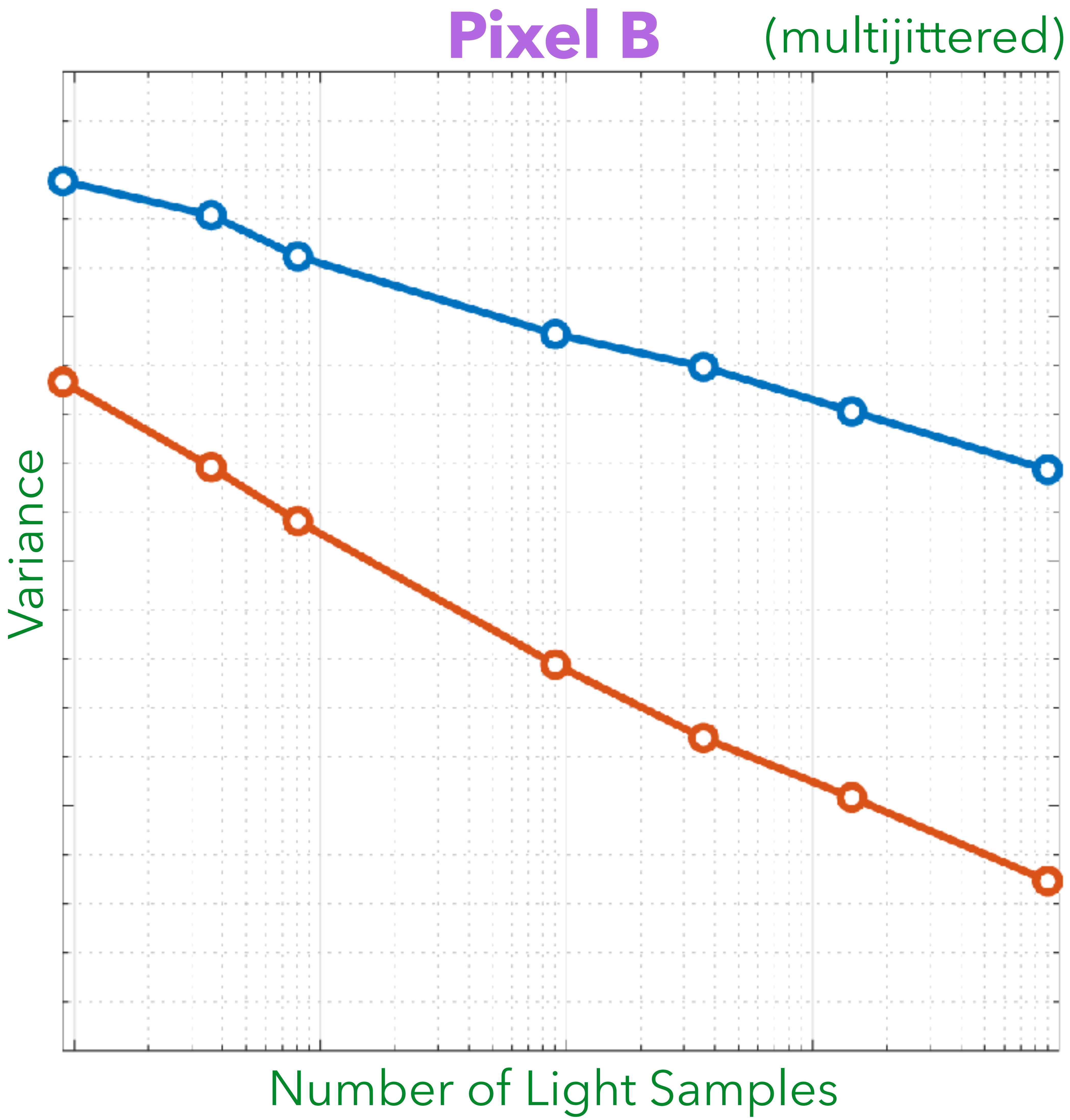


Convergence tests

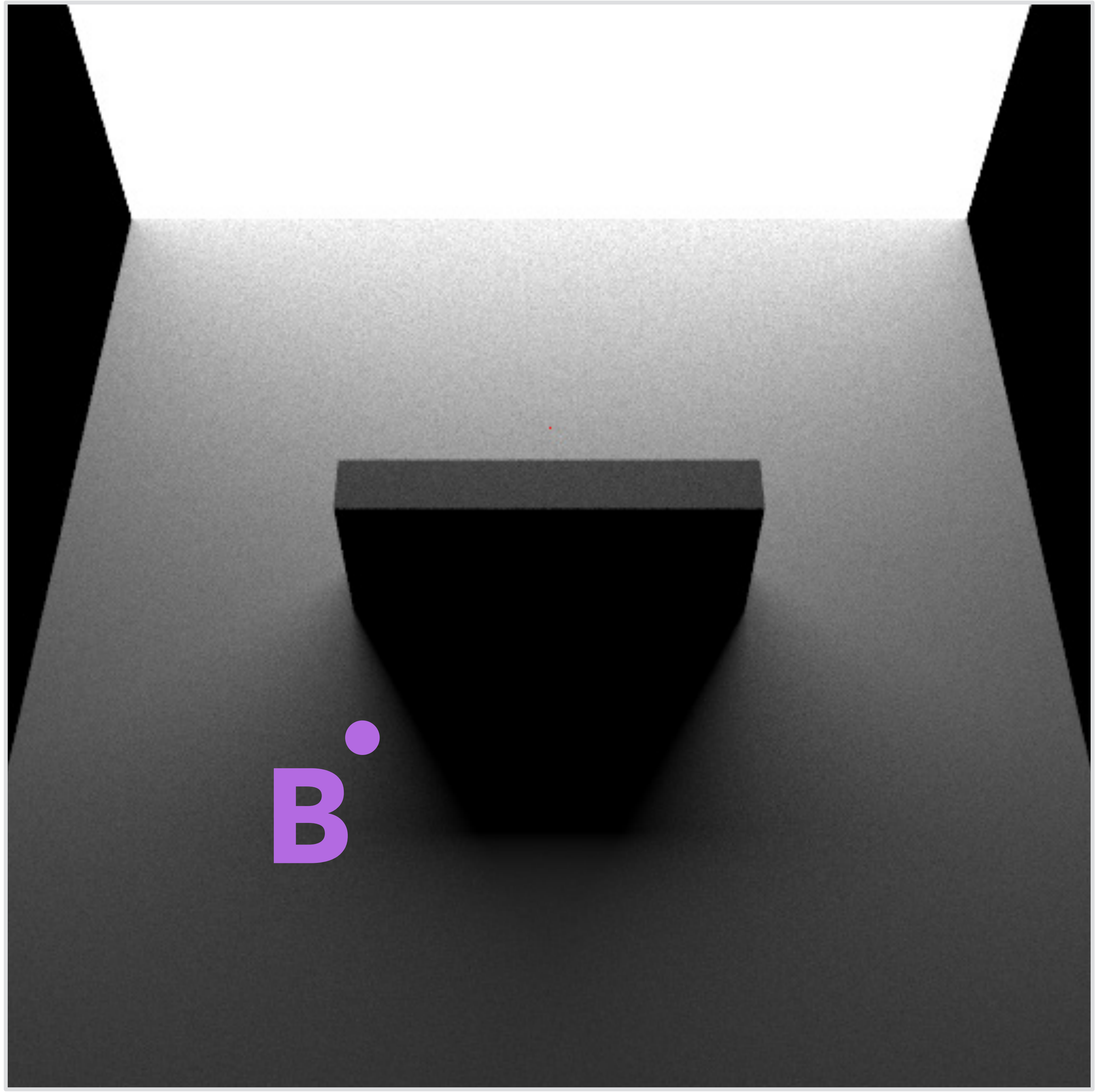


BSDF ($N^{-1.48}$)

Horiz. lines ($N^{-2.57}$)



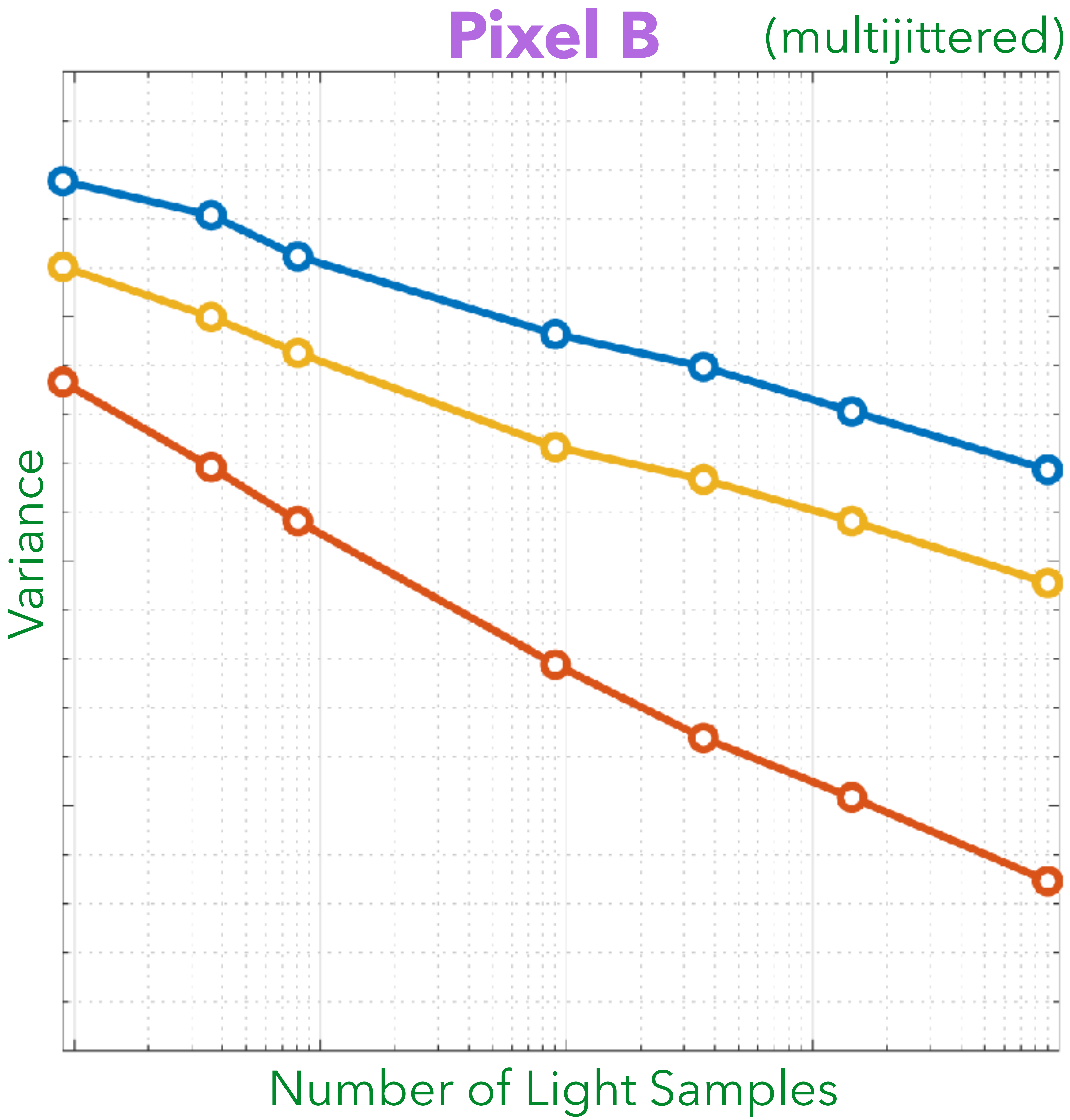
Convergence tests



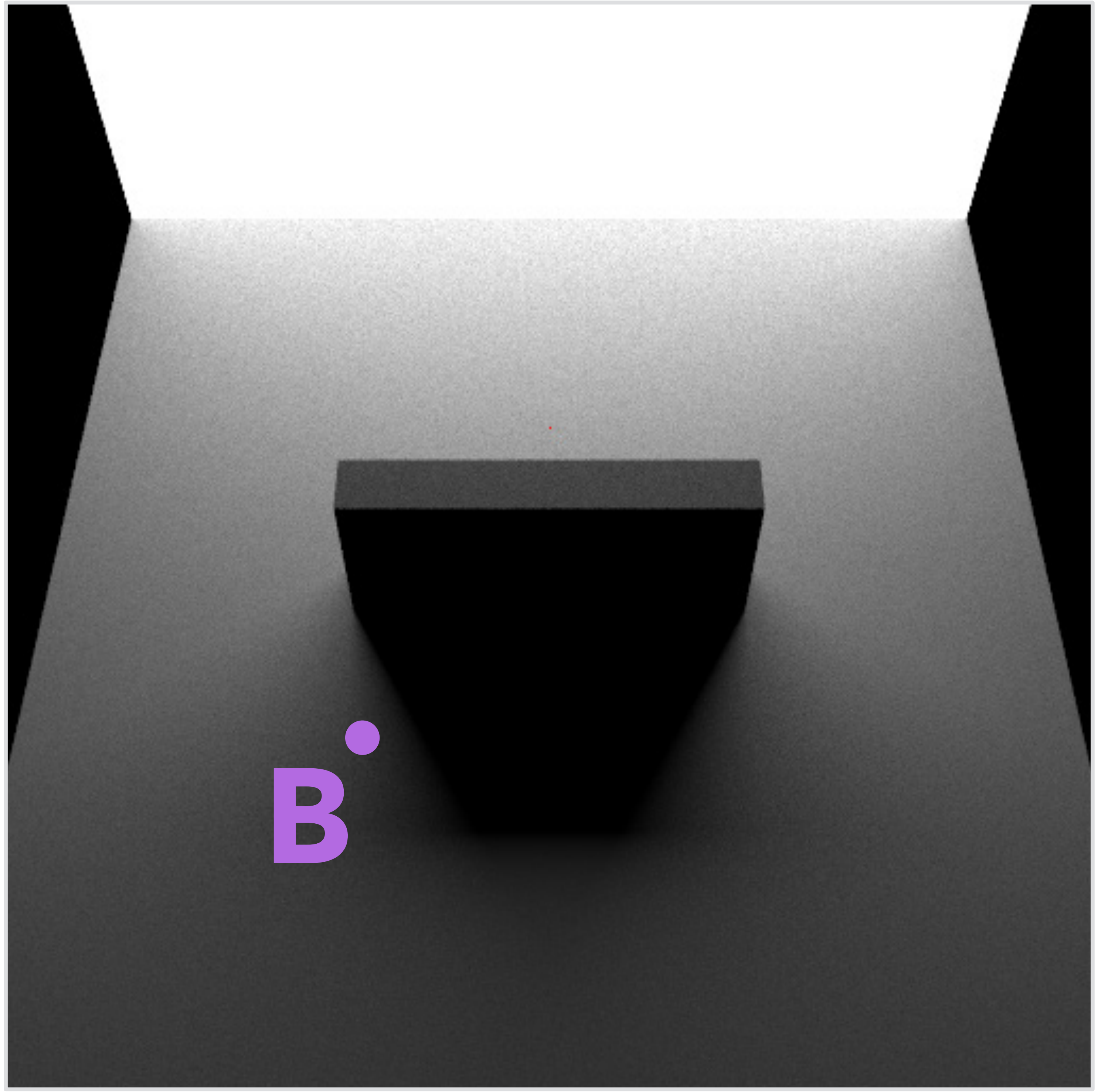
BSDF ($N^{-1.48}$)

Horiz. lines ($N^{-2.57}$)

Vert. lines ($N^{-1.61}$)



Convergence tests

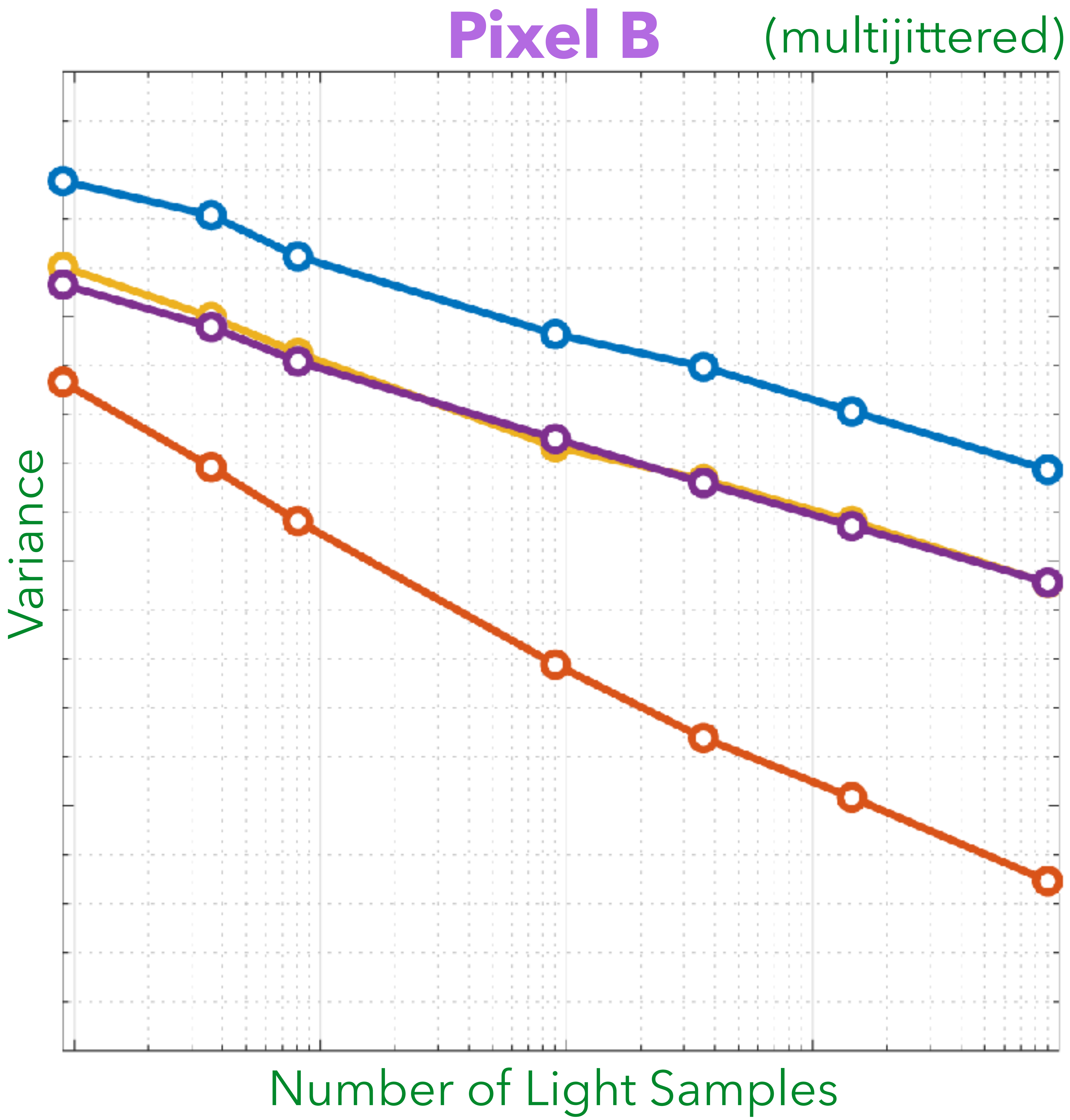


BSDF ($N^{-1.48}$)

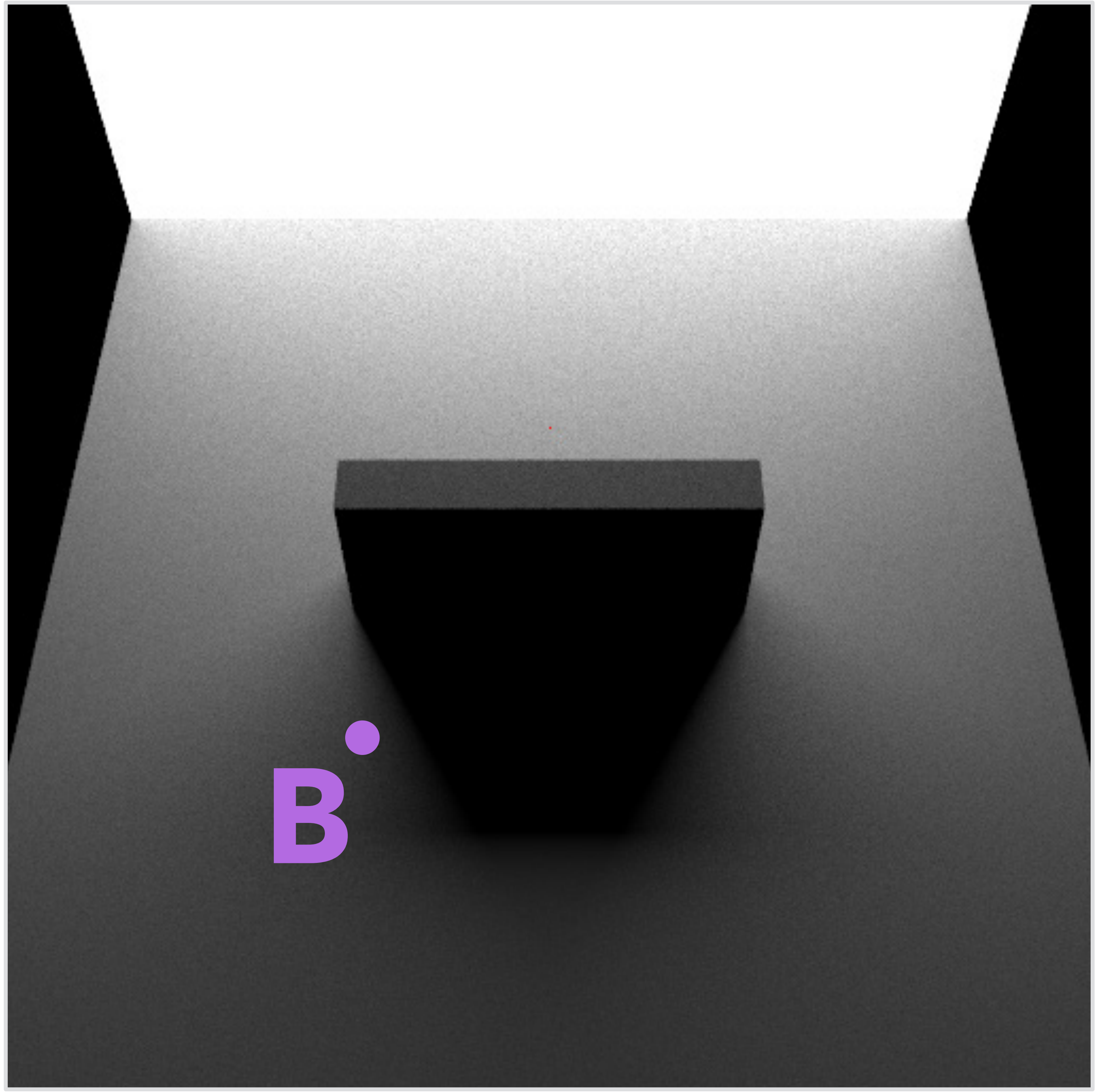
Horiz. lines ($N^{-2.57}$)

Vert. lines ($N^{-1.61}$)

MIS BSDF + Lines ($N^{-1.53}$)



Convergence tests



BSDF ($N^{-1.48}$)

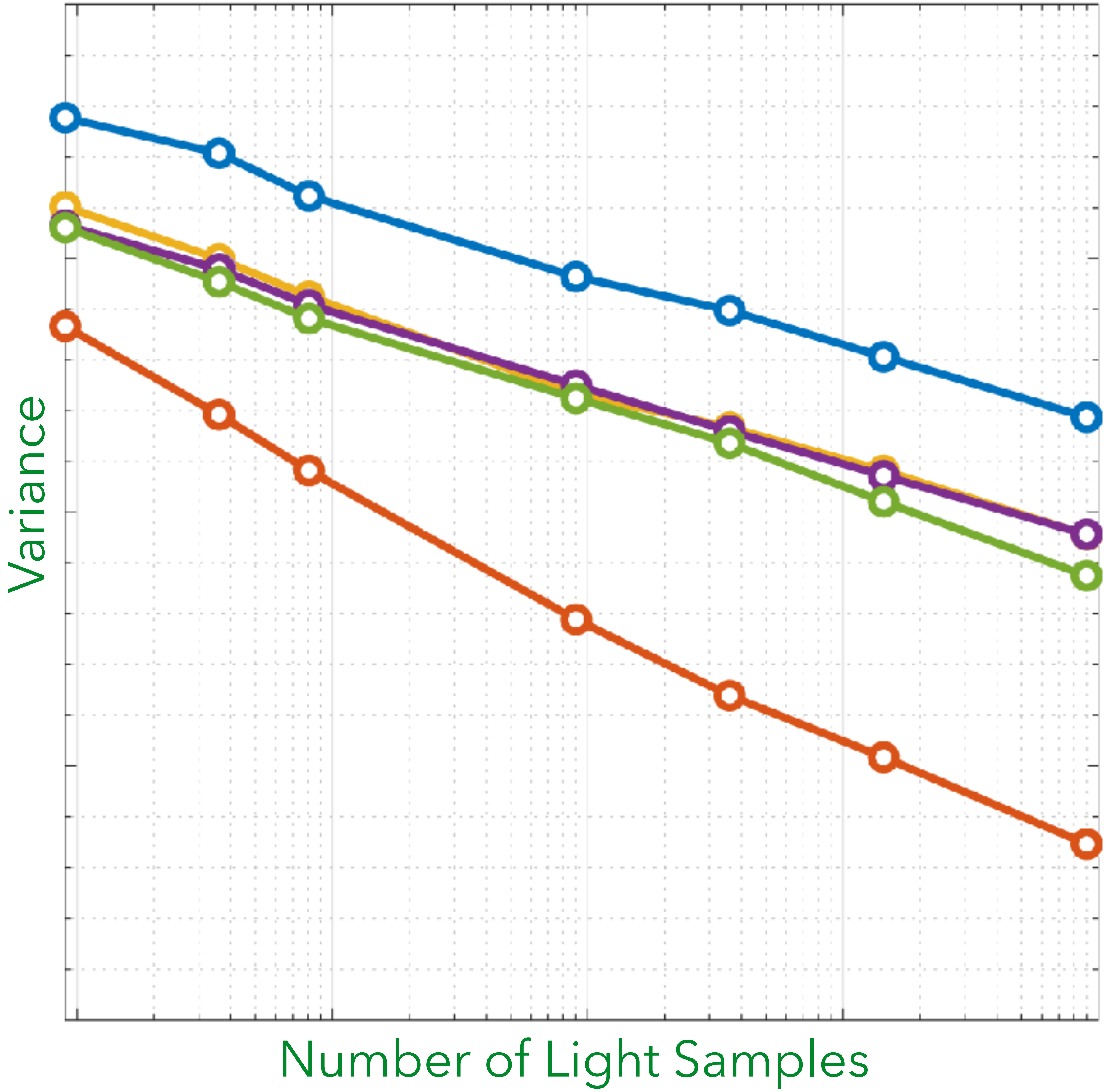
Horiz. lines ($N^{-2.57}$)

Vert. lines ($N^{-1.61}$)

MIS BSDF + Lines ($N^{-1.53}$)

MIS BSDF + Lines
with smoothing ($N^{-1.68}$)

Pixel B (multijittered)



Wrapping things up:
What's Next?

Future work

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- Optimize the line sample-scene intersection

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  - Line sample-scene intersection 1.2x – 55x slower than shadow ray

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  - Line sample-scene intersection 1.2x – 55x slower than shadow ray
- Support arbitrarily-shaped light sources and all line directions (for solid-angle lines)
- Improve smoothing MIS heuristic to be more robust to all scenarios
- Apply novel concepts to other line sampling (or even higher-dimensional) applications

Thank you!

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