### Combining Point and Line Samples for Direct Illumination

### Points only



### **Katherine Salesin**

### Points + Lines



Wojciech Jarosz

Image: Second state
Image: Second state

Imag













### Direct lighting:







### Direct lighting: point sampling







### Direct lighting: point sampling and line sampling





# Theory: **Direct lighting + Monte Carlo sampling**



















































### $L_o(\mathbf{e}, \mathbf{x}) = \int f_r(\mathbf{e}, \mathbf{x}, \mathbf{l}) G(\mathbf{x}, \mathbf{l}) V(\mathbf{x}, \mathbf{l}) L_e(\mathbf{x}, \mathbf{l}) \, dA(\mathbf{l})$

































 $L_o = \int_{\mathcal{U}} \int_{\mathcal{V}} f(u, v) \, \mathrm{d}v \, \mathrm{d}u$ 





























Combining point and line samples for direct illumination 



# $L_o = \int_{\mathcal{U}} \int_{\mathcal{V}} f(u, v) \, \mathrm{d}v \, \mathrm{d}u = \int_{\mathcal{U}} f_v(u) \, \mathrm{d}u$



Combining point and line samples for direct illumination



 $f_v(u) \,\mathrm{d} u$ JU

 $L_o =$ 




























• **Direct illumination** [BD16]

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### [BD16]







- **Direct illumination** [BD16]
- Transient light transport [MGJ\*19]









- **Direct illumination** [BD16]
- Transient light transport [MGJ\*19]
- Transmittance [BJ17]







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- Transient light transport [MGJ\*19]
- Transmittance [BJ17]
- Motion blur [GDA10]

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- **Direct illumination** [BD16]
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- Depth of field [TPD\*12]







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- **Direct illumination** [BD16]
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- Motion blur [GDA10]
- Depth of field [TPD\*12]
- Environment lights [NBMJ14]
- Hair [BGA12]

and more...

▼/ € | • Combining point and line samples for direct illumination









✓ Less error per sample than points



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- **X** Hard to analytically integrate one dimension
  - [BD16] derived solution only for diffuse and Phong materials
- X Slow to evaluate samples
  - Expensive line sample-scene intersection



## Motivation



## Motivation

## Make line samples play nicely with any point-based strategy

## Motivation

Make line samples play nicely with any point-based strategy Mitigate orientation-based performance issues



### MIS Points + Lines (Ours)





 Reframe line samples as point samples that importance sample visibility



- Reframe line samples as point samples that importance sample visibility

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## Show how to multiple importance sample between lines and points, and lines of different orientations



- Reframe line samples as point samples that importance sample visibility
- Propose novel MIS weighting scheme to improve convergence rate

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# Show how to multiple importance sample between lines and points, and lines of different orientations







### **Point sampling**



Combining point and line samples for direct illumination













### **Point sampling**



V C -Combining point and line samples for direct illumination





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V C -Combining point and line samples for direct illumination





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V C -Combining point and line samples for direct illumination





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V C |-Combining point and line samples for direct illumination





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▼ € |• Combining point and line samples for direct illumination




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▼ € . Combining point and line samples for direct illumination





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▼/ € |· Combining point and line samples for direct illumination





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V C |-Combining point and line samples for direct illumination





## **Point sampling**



V C -Combining point and line samples for direct illumination





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the full direct lighting integral

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# But a perfect conditional pdf would be hard to find for



- the full direct lighting integral

# But a perfect conditional pdf would be hard to find for

Instead, we use simpler conditional pdfs that work well in practice



- the full direct lighting integral
- Effectively importance sampling visibility

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But a perfect conditional pdf would be hard to find for

Instead, we use simpler conditional pdfs that work well in practice



## **Conditional point pdfs**

## We propose two options:

Combining point and line samples for direct illumination 



## **Conditional point pdfs**

## We propose two options:

Surface-area-based sampling (uniform over surface area)



V C |-Combining point and line samples for direct illumination



## **Conditional point pdfs**

## We propose two options:

Surface-area-based sampling (uniform over surface area)



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## **Solid-angle**-based sampling (uniform over solid angle)







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## • We have reframed line sampling as point sampling that importance samples visibility

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- We have reframed line sampling as point sampling that importance samples visibility
- We can now use line sampling with any BRDF

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- We have reframed line sampling as point sampling that importance samples visibility
- We can now use line sampling with any BRDF
- But line samples are still bad at importance sampling some terms - can we do better?

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# Theory

▼/ <u>€</u> |• Combining point and line samples for direct illumination



# Theory

## • We can use multiple importance sampling (MIS) to combine the strengths of different strategies

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# Theory

- combine the strengths of different strategies
- it is strongest (i.e. where a strategy's pdf is largest relative to other strategies' pdfs)

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# • We can use multiple importance sampling (MIS) to

MIS uses a set of weights to favor each strategy where



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Combining point and line samples for direct illumination 

## • We can now MIS lines with lines of **other orientations**



# **MIS between lines:** Equal time comparison













	Relative MSE						
				Fullimag	ge	Green	Р
Lin	es (av [BD1		je)	2.6 x 10 <sup>.</sup>	-1	1.2 x 10-2	1.
Solid	-angl [UFK <sup>*</sup>		oints	<b>2.0</b> x 10	)-1	<b>1.4 x 10</b> -3	1.9





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Combining point and line samples for direct illumination

## • We can now MIS lines with lines of **other orientations**



- We can now MIS lines with points that importance sample other distributions (like BRDFs)

## • We can now MIS lines with lines of **other orientations**



# MIS between points and lines: Equal time comparisons













### **Relative MSE**

### Full image

### BSDF

### **5.5 x 10<sup>1</sup>**

























































### **Relative MSE**

Full image

### BSDF

### **5.5 x 10<sup>1</sup>**
























































### **Relative MSE**

Full image

BSDF

5.5 x 10<sup>1</sup>

### Solid-angle points [UFK13]

**4.5 x 10**-1











	<b>Relative MSE</b> Full image	*
BSDF	5.5 x 10 <sup>1</sup>	
Solid-angle points [UFK13]	4.5 x 10 <sup>-1</sup>	
MIS BSDF + SA points	<b>2.0 x 10-1</b>	





	<b>Relative MSE</b> Full image	
BSDF	5.5 x 10 <sup>1</sup>	
Solid-angle points [UFK13]	4.5 x 10 <sup>-1</sup>	
MIS BSDF + SA points	<b>2.0 x 10</b> -1	
Solid-angle lines [Ours]	4.2 x 10 <sup>-1</sup>	





	<b>Relative MSE</b> Full image	
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Solid-angle points [UFK13]	4.5 x 10 <sup>-1</sup>	
MIS BSDF + SA points	2.0 x 10 <sup>-1</sup>	
Solid-angle lines [Ours]	4.2 x 10 <sup>-1</sup>	
MIS BSDF + SA lines	6.8 x 10-2	































### **Relative MSE**

	Full image	Green	Purple
4 points : 0 lines	<b>4.0 x 1</b> 0-1	<b>5.7 x 10</b> -2	4.4 x 10
3 points : 1 line	1.0 x 10 <sup>0</sup>	<b>1.1</b> x 10-1	1.1 x 10
2 points : 2 lines	1.0 x 10 <sup>1</sup>	5.2 x 10-1	<b>9.8</b> x 1
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			*



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2 points : 2 lines	1.0 x 10 <sup>1</sup>	5.2 x 10-1	<b>9.8</b> x 1
1 point : 3 lines	2.8 x 10-1	5.5 x 10 <sup>-1</sup>	1.3 x 1
<b>0 points : 4 lines</b>	5.3 x 10-1	1.9 x 10 <sup>1</sup>	1.1 x 1(



### Summary

- We can now MIS lines with points that importance sample other distributions (like BRDFs)

### • We can now MIS lines with lines of **other orientations**



### Summary

- We can now MIS lines with points that importance sample other distributions (like BRDFs)
- But MIS inherits the worst convergence rate of its strategies - can we do better?

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### We can now MIS lines with lines of other orientations



Combining point and line samples for direct illumination



We know convergence rate improves when

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# discontinuities in effective integrand are smoothed



- We know convergence rate improves when
- MIS estimator:

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# discontinuities in effective integrand are smoothed



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$$\langle L_o 
angle^{\mathrm{mis}} = rac{1}{S} \sum_{s=1}^{S} \left( rac{1}{N_s} \sum_{i=1}^{N_s} w_s(u_i) \right)$$

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# discontinuities in effective integrand are smoothed

 $(u_i, v_i) \frac{f(u_i, v_i)}{p_s(u_i, v_i)}$  for **S** strategies



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[SSC\*19]



effective integrand



- We know convergence rate improves when discontinuities in effective integrand are smoothed
- MIS estimator:

$$\langle L_o \rangle^{\text{mis}} = \frac{1}{S} \sum_{s=1}^{S} \left( \frac{1}{N_s} \sum_{i=1}^{N_s} w_s(u_i) \right)$$

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[Ours]



tive integrand



### Let us MIS:

### 1. **BRDF point samples**

### 2. Vertical line samples

### 3. Horizontal line samples

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# Effective integrand for the **BRDF strategy**







# Effective integrand for the **BRDF strategy**













# Effective integrand for the **BRDF strategy**





# Effective integrand for the **BRDF strategy**





# Effective integrand for the **BRDF strategy**





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# Effective integrand for the **BRDF strategy**





# Effective integrand for the **BRDF strategy**





### Smoothing MIS: **Convergence tests**





### Combining point and line samples for direct illumination





Combining point and line samples for direct illumination 





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BSDF (N<sup>-1.38</sup>) Horiz. lines (N<sup>-2.13</sup>)

Combining point and line samples for direct illumination



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**BSDF (N**<sup>-1.38</sup>) Horiz. lines (N<sup>-2.13</sup>) Vert. lines (N<sup>-2.14</sup>) MIS BSDF + Lines (N<sup>-1.40</sup>)



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**BSDF (N**<sup>-1.38</sup>) Horiz. lines (N<sup>-2.13</sup>) Vert. lines (N<sup>-2.14</sup>) MIS BSDF + Lines (N<sup>-1.40</sup>) MIS BSDF + Lines with smoothing (N<sup>-2.04</sup>)





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**BSDF (N**<sup>-1.48</sup>) Horiz. lines (N<sup>-2.57</sup>) Vert. lines (N-1.61)OpegaMIS BSDF + Lines (N-1.53)





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**BSDF (N**<sup>-1.48</sup>) Horiz. lines (N<sup>-2.57</sup>) Vert. lines (N-1.61)OpposeMIS BSDF + Lines (N-1.53) MIS BSDF + Lines with smoothing (N<sup>-1.68</sup>)





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### Wrapping things up: What's Next?



Optimize the line sample-scene intersection



- Optimize the line sample-scene intersection

Combining point and line samples for direct illumination 

• Line sample-scene intersection 1.2x – 55x slower than shadow ray



- Optimize the line sample-scene intersection
  - Line sample-scene intersection 1.2x 55x slower than shadow ray
- Support arbitrarily-shaped light sources and all line directions (for solid-angle lines)



- Optimize the line sample-scene intersection
  - Line sample-scene intersection 1.2x 55x slower than shadow ray
- Support arbitrarily-shaped light sources and all line directions (for solid-angle lines)
- Improve smoothing MIS heuristic to be more robust to all scenarios



- Optimize the line sample-scene intersection
  - Line sample-scene intersection 1.2x 55x slower than shadow ray
- Support arbitrarily-shaped light sources and all line directions (for solid-angle lines)
- Improve smoothing MIS heuristic to be more robust to all scenarios
- Apply novel concepts to other line sampling (or even higher-dimensional) applications



# Thank you!

Please visit

dartgo.org/pointsandlines for the full paper, supplemental document, and interactive image viewer.

Katherine Salesin katherine.a.salesin.gr@dartmouth.edu





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# Thank you!

Please visit dartgo.org/pointsandlines for the full paper, supplemental document, and interactive image viewer.

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