Homework 3
Due Mon Apr 20, 5:00pm

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General Instructions: Same as in Homework 1.
Honor Principle: Same as in Homework 1.

6. Is $\mathsf{DTIME}(2^n) = \mathsf{DTIME}(2^{0.9n})$? You can answer "yes," "no," or "I think this is an open question." Give clear reasons! Prove that there is a constant $\alpha < 1$ such that $\textsc{3sat} \in \mathsf{DTIME}(2^{\alpha n})$. [2 points]

7. Imagine it is 1980: Savitch's Theorem is common knowledge, but no one knows yet how $\mathsf{NSPACE}(f(n))$ relates to $\mathsf{coNSPACE}(f(n))$. Despite this, a clever padding argument, together with the deterministic space hierarchy theorem, suffices to establish a somewhat weak nondeterministic space hierarchy theorem. Prove this. Specifically, give a proof that if α and β are real-valued constants with $0 < \alpha < \beta$, then $\mathsf{NSPACE}(n^\alpha) \neq \mathsf{NSPACE}(n^\beta)$. [2 points]

8. Consider the language

$$\mathsf{ALL}_{\mathsf{NFA}} = \{\langle \Sigma, M \rangle : M \text{ is a nondeterministic finite automaton over } \Sigma \text{ such that } \mathcal{L}(M) = \Sigma^*\}.$$

Note that the alphabet Σ is specified as part of the encoding of the NFA, M. Prove that $\mathsf{ALL}_{\mathsf{NFA}}$ is PSPACE-complete.

Hint: While reducing from TQBF may be tempting as an approach, it may be a better idea to carefully study the proof of [Sipser, Theorem 5.13] and try to adapt that. [2 points]