Homework 6
Due Mon May 4, 5:00pm

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General Instructions: Same as in Homework 1.

Honor Principle: For this homework, you should work entirely on your own and not discuss with anyone.

15. Give a full formal proof that $\mathsf{ZPP} = \mathsf{RP} \cap \mathsf{coRP}$. [2 points]

16. For constants $0 < \alpha < \beta < 1$, define the class $\mathsf{BPP}_{\alpha,\beta}$ to be the class of all languages $L \subseteq \Sigma^*$ such that there exists a PTM M that runs in polynomial time and behaves as follows on an input $x \in \Sigma^*$:

$$x \notin L \quad \Rightarrow \quad \Pr_R[M(x,r) = 1] \le \alpha\,,$$
$$x \in L \quad \Rightarrow \quad \Pr_R[M(x,r) = 1] \ge \beta\,.$$

Note that our definition of BPP in class coincides with $\mathsf{BPP}_{\frac{1}{3},\frac{2}{3}}$ in this notation.

Using Chernoff bounds, give a full formal proof that for all α and β as above, $\mathsf{BPP}_{\alpha,\beta} = \mathsf{BPP}$. [2 points]

Recall that the Chernoff bound we saw in class had the following general form. Let $\{X_1, \ldots, X_n\}$ be independent indicator random variables with $\mathbb{E}[X_i] = p_i$. Suppose $X = \sum_{i=1}^n X_i$ and let p be such that $np = p_1 + \cdots + p_n$. Then, for any $\delta > 0$:

$$\Pr[X \ge (1+\delta)np] \;\le\; \left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right)^{np}.$$

We also have a similar inequality bounding deviations of X *below* its mean. For $0 < \delta < 1$:

$$\Pr[X \le (1-\delta)np] \;\le\; \left(\frac{e^{-\delta}}{(1-\delta)^{1-\delta}}\right)^{np}.$$