Homework 5
Due Wed Apr 28, 5:00pm

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General Instructions: Please write concisely, but rigorously. Non-rigorous solutions won't be graded. For each problem, only "nearly flawless" solutions earn 2 points. Solutions that contain the key insights but are flawed in execution earn only 1 point. Solutions that are correct but needlessly long (usually this means over 1.5 pages long) will earn only 1 point. The purpose of this strict grading scheme is to dissuade you from writing up half-baked ideas in the hope of getting "some" credit. You are budding researchers: your writing should reflect that. *[These are the same instructions as for Homework 1, but I am repeating them to emphasize their importance.]*

Honor Principle: Same as in Homework 1.

11. Consider the language

$$\mathsf{ALL}_{\mathsf{NFA}} = \{\langle \Sigma, M \rangle : M \text{ is a nondeterministic finite automaton over } \Sigma \text{ such that } \mathcal{L}(M) = \Sigma^*\}\,.$$

Note that the alphabet Σ is specified as part of the encoding of the NFA, M. Prove that $\mathsf{ALL}_{\mathsf{NFA}}$ is PSPACE-complete.

Hint: While reducing from TQBF may be tempting as an approach, it may be a better idea to carefully study the proof of [Sipser, Theorem 5.13] and try to adapt that. [2 points]

12. For a string $x \in \{0,1\}^*$, let $N_1(x)$ denote the number of 1s in x. The *majority* function $\text{MAJ}_n : \{0,1\}^n \to \{0,1\}$ is defined as follows:

$$\text{MAJ}_n(x) = \begin{cases} 1\,, & \text{if } N_1(x) \geq n/2\,, \\ 0\,, & \text{otherwise.} \end{cases}$$

Show that MAJ_n can be computed using $O(n)$-sized circuits. [This is essentially Sipser's Problem 9.26 — if you use the approach suggested in the book, you need to first solve (in sufficient detail) any subproblems that come up, such as Sipser's Problem 9.24.] [2 points]