Homework 1
Due Wed Apr 6, 5:00pm

Prof. Amit Chakrabarti
Computer Science Department
Dartmouth College

General Instructions: Please write concisely, but rigorously. Non-rigorous solutions won't be graded. For each problem, only "nearly flawless" solutions earn 2 points. Solutions that contain the key insights but are flawed in execution earn only 1 point. Solutions that are correct but needlessly long (usually this means over 1.5 pages long) will earn only 1 point.

The purpose of this strict grading scheme is to dissuade you from writing up half-baked ideas in the hope of getting "some" credit. You are budding researchers: your writing should reflect that.

Honor Principle: You are allowed to discuss the problems and exchange solution ideas, *but not complete solutions*, with your classmates. But when you write up any solutions for submission, you must work alone. You may refer to any textbook you like, including online ones. However, you may not refer to published or online solutions to the specific problems on the homework, if you intend to turn it in for credit. *If in doubt, ask the professor for clarification!*

1. For a complexity class $\mathscr{C}$, define two new complexity classes "$\exists\mathscr{C}$" and "$\forall\mathscr{C}$" as follows.

$$\begin{aligned}\exists\mathscr{C} &= \{\{x \in \Sigma^* : \exists y \in \Sigma^* \text{ with } |y| = \text{poly}(|x|) \text{ such that } \langle x, y\rangle \in L_0\} : L_0 \in \mathscr{C}\} \\ \forall\mathscr{C} &= \{\{x \in \Sigma^* : \forall y \in \Sigma^* \text{ with } |y| = \text{poly}(|x|) \text{ we have } \langle x, y\rangle \in L_0\} : L_0 \in \mathscr{C}\}\end{aligned}$$

   The notation $|y| = \text{poly}(|x|)$ means that there is some fixed polynomial p such that $|y| = O(p(|x|))$.

   Prove, rigorously, that $\exists\mathsf{P} = \mathsf{NP}$ and $\forall\mathsf{P} = \mathsf{coNP}$. Use only the basic definitions, where NP is defined using NDTMs and coNP is defined as $\{L \subseteq \Sigma^* : \overline{L} \in \mathsf{NP}\}$ (as in Sipser).

   This problem is simply asking you to write out a rigorous version of ideas we have already discussed in class, when we talked about the verifier-prover view of NP. In your proofs, make sure you precisely defined appropriate languages L_0 used in the definitions above. [2 points]

2. Define the following two complexity classes

$$\begin{aligned}\mathsf{EXPTIME} &= \bigcup_{i=1}^{\infty} \mathsf{DTIME}(2^{n^i}) \\ \mathsf{NEXPTIME} &= \bigcup_{i=1}^{\infty} \mathsf{NTIME}(2^{n^i}).\end{aligned}$$

   Prove that $\mathsf{P} = \mathsf{NP}$ implies $\mathsf{EXPTIME} = \mathsf{NEXPTIME}$. The key trick is to "pad" an input with a lot of extra symbols. [2 points]