

General Instructions: Same as in Homework 1.

Honor Principle: Same as in Homework 1.

5. We proved in class that $\overline{\text{STCON}} \in \text{NL}$, thereby concluding $\text{NL} = \text{coNL}$. We remarked that our proof in fact shows that $\text{NSPACE}(f(n)) = \text{coNSPACE}(f(n))$ for any function $f : \mathbb{N} \rightarrow \mathbb{N}$ with $f(n) \geq \log_2 n$. Prove this remark rigorously.

At some point, you will have to take care of the technical difficulty that computing the *value* of $f(n)$ might require more than $O(f(n))$ space — after all, you have no idea what kind of crazy function $f(n)$ is. Consult the proof of Savitch's theorem in Sipser's book for a hint on how to handle this. [2 points]

6. Prove that $\{ \langle G \rangle : G \text{ is a strongly connected directed graph} \}$ is NL-complete. [2 points]

7. Consider the language

$$\text{ALL}_{\text{NFA}} = \{ \langle \Sigma, M \rangle : M \text{ is a nondeterministic finite automaton over } \Sigma \text{ such that } \mathcal{L}(M) = \Sigma^* \}.$$

Note that the alphabet Σ is specified as part of the encoding of the NFA, M . Prove that ALL_{NFA} is PSPACE-complete.

Hint: While reducing from TQBF may be tempting as an approach, it may be a better idea to carefully study the proof of [Sipser, Theorem 5.13] and try to adapt that. [2 points]