CS 109 Spring 2011 Theory of Computation: Advanced

Homework 4 Due Mon Apr 18, 5:00pm

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General Instructions: Same as in Homework 1. **Honor Principle:** Same as in Homework 1.

8. Imagine it is 1980: Savitch's Theorem is common knowledge, but no one knows yet how NSPACE(f(n)) relates to coNSPACE(f(n)). Despite this, a clever padding argument, together with the deterministic space hierarchy theorem, suffices to establish a somewhat weak nondeterministic space hierarchy theorem. Prove this. Specifically, give a proof that if α and β are real-valued constants with $0 < \alpha < \beta$, then NSPACE(n^{α}) \neq NSPACE(n^{β}). [2 points]

Hint: First, as an example, figure out how to show that $NSPACE(n^5) \neq NSPACE(n^{11})$. This should not require any padding. Now ask yourself: how would padding help reduce the gap between 5 and 11 in this example?

9. Is $\mathsf{DTIME}(2^n) = \mathsf{DTIME}(2^{0.9n})$? Why or why not (give clear reasons)? Prove that there is a constant $\alpha < 1$ such that $3\mathsf{SAT} \in \mathsf{DTIME}(2^{\alpha n})$. [You need to answer all parts of this question to receive credit.] [2 points]