Homework 2
Due Jan 23, 2012 (before class)

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Please think carefully about how you are going to organise your answers *before* you begin writing. Make sure your answers are complete, clean, concise and rigorous.

1. Construct NFAs for the languages generated by each of the following regular expressions:

   1.1. $ba \cup (a \cup bb)a^*b$ [7 points]

   1.2. $(0 \cup 1)((0 \cup 1)(0 \cup 1))^* \cup ((0 \cup 1)(0 \cup 1)(0 \cup 1))^*$
   More readable version: $X(XX)^* \cup (XXX)^*$, where $X := 0 \cup 1$ [7 points]

2. Give regular expressions for the following languages.

   2.1. $\{w \in \{0,1\}^* : w$ has three consecutive 0's or three consecutive 1's or both$\}$. [7 points]

   2.2. $\{w \in \{0,1\}^* : w$ has three consecutive 0's and three consecutive 1's$\}$. [7 points]

   2.3. The set of strings in $\{0,1\}^*$ with an equal number of 0's and 1's such that no prefix has two more 0's than 1's nor two more 1's than 0's. [10 points]

   2.4. Let us define a valid floating point number as $u.v$, where u and v are (finite) strings of decimal digits (0..9) satisfying the following constraints: (the symbol "." between u and v is the decimal point.)

   i. Neither u nor v may be ε.
   ii. u can be just 0. If u is not 0, u has no leading 0's.
   iii. v can be just 0. If v is not 0, v has no trailing 0's.

   (Thus, for example, 0.0, 231.0 and 5.608 are valid, but 0.00, 05.68, .65, 12. and 4.5100 are not valid.)
   Give a regular expression for the set of valid floating point numbers described above. You might want to introduce some notation first to keep your expression small and readable. [10 points]

3. Let L be a nonempty language and M an NFA that recognizes L. Prove that M can be converted into an NFA M' which recognizes the same language L and has exactly one accept state. Your proof *must* describe M' both informally, using plain English, *and* formally, using mathematical notation. [10 points]

4. Let L be the language over the alphabet $\{a,b\}$ given by the regular expression $(ab \cup aab \cup aba)^*$.

   4.1. Design an NFA for L that has no ε-transtions and has only 4 states. [6 points]

   4.2. Convert the above NFA into a DFA for L by mechanically using the *subset construction* we studied in class. [10 points]

   4.3. Remove all states that are unreachable from the start state of the resulting DFA, to get a 7-state DFA for L. [3 points]

4.4. If you carefully observe this DFA, you will notice two states that can be replaced by a single state. Do this and draw the resulting DFA. Your final DFA should have exactly 6 states. [7 points]

5. For a language L over alphabet Σ, define $\textsc{Half}(L) = \{x \in \Sigma^* : \exists\, y \in \Sigma^* \,(|x| = |y| \text{ and } xy \in L)\}$. Prove that if L is regular, then so is $\textsc{Half}(L)$. Your proof *must* be formal; proofs not written in a formal mathematical style get very little credit even if they express the right intuition. [16 points]

   Hint: Since L is regular, you know that there exists some DFA M that recognizes L, but you know absolutely nothing else about M. How do you make use of M? Here are two different approaches you can try. *Approach 1:* Build an NFA for $\textsc{Half}(L)$. Suppose x is the input string. Nondeterministically guess which state M will end up in after reading x and nondeterministically guess a y to append to x as in the definition of $\textsc{Half}(L)$. *Approach 2:* Build a DFA for $\textsc{Half}(L)$. As you read x, work forwards and backwards simultaneously inside M and try to meet in the middle.

Challenge Problems

Please read about challenge problems on the course website (click "administrative details"). Remember that challenge problems carry no regular credit, but are intended to provide a higher level of challenge for those who want to think further about the theory of computing.

CP1: For the language L from Problem 4 above, prove that it is impossible to design a DFA with 5 or fewer states.

CP2: For a language L over alphabet Σ, define $\textsc{Log}(L) = \{x \in \Sigma^* : \exists\, y \in \Sigma^* \,(|y| = 2^{|x|} \text{ and } xy \in L)\}$. Prove that if L is regular, then so is $\textsc{Log}(L)$.