Homework 3
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Please think carefully about how you are going to organise your answers *before* you begin writing. Make sure your answers are complete, clean, concise and rigorous.

1. Pick *any one* of the following two subparts and turn it in.

   [Option 1] Look at Exercise 1.21 in the textbook. Convert the two DFAs in that exercise to regular expressions, but *do not use the textbook's "GNFA method"*. Instead, use the R_{ij}^k method discussed in class, and described in the lecture notes (on the course website). You may use the shorthand X^+ to denote XX^*, where X is an arbitrary regular expression. Try to simplify the intermediate regular expressions; this will save you from a lot of pain. [6+7 points]

   [Option 2] Give a full formal proof that the subset construction, for converting an NFA into an equivalent DFA, actually works. Make every step mathematically precise, using only the formal definitions of acceptance for a DFA and an NFA, and the formal definition of the ε-closure function $E(\cdot)$. You may use informal wording like "following arrows" to add *explanation* to your proof, but not in lieu of a formal proof. [13 points]

2. For a language L over alphabet Σ, define $\textsc{Max}(L) = \{x \in L : x \text{ is not a proper prefix of any string in } L\}$. Recall that y is said to be a proper prefix of x if y is a prefix of x and $y \neq x$. Prove that if L is regular, then so is $\textsc{Max}(L)$. [8 points]

3. For a language L over alphabet Σ, define $\textsc{Cycle}(L) = \{xy : x, y \in \Sigma^* \text{ and } yx \in L\}$. Prove that if L is regular, then so is $\textsc{Cycle}(L)$. [12 points]

4. For each of the following languages, say whether or not the language is regular and prove your answer. To prove that a language is regular, specify a finite automaton or a regular expression for that language. To prove that a language is not regular, use the pumping lemma or closure properties of regular languages.

   Proofs *must* be precisely written. *Make sure you fully understand the definitions of the sets before answering.*

   4.1. $\{0^m 1^n 0^{m+n} : m, n \geq 0\}$. [7 points]

   4.2. $\{0^m 1^n : m \text{ divides } n\}$. [7 points]

   4.3. $\{xwx^R : x, w \in \{0,1\}^*, |x| > 0 \text{ and } |w| > 0\}$. [7 points]

   4.4. $\{0^{2^n} : n \geq 0\}$. [7 points]

   4.5. Problem 1.35 from the textbook. [7 points]

   4.6. $\{0^m 1^n : m, n \geq 0 \text{ and } m \neq n\}$. [7 points]

5. Are the following statements *always* true? If true, give a brief justification and if false, give a concrete counterexample. Below, A and B denote languages over some alphabet Σ.

   5.1. If $A \cup B$ is regular, then at least one of A and B is regular. [5 points]

   5.2. If $A \cap B$ is regular, then at least one of A and B is regular. [5 points]

   5.3. If $\overline{A}$ (defined as $\Sigma^* - A$) is regular, then A is regular. [5 points]

   5.4. A union of arbitrarily many regular languages is regular, even if it is an infinite union. [5 points]

   5.5. An intersection of arbitrarily many regular languages is regular, even if it is an infinite intersection. [5 points]

Challenge Problems

CP3: Let L be any subset of 0^*. Prove that L^* is regular.

This is a delightful problem and will teach you something nice about regular languages if you solve it.