Homework 6
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When describing a Turing Machine, be sure to state explicitly what "extra features" (such as multiple tapes and/or nondeterminism) you are planning to use, if any. Feel free to use implementation descriptions, as opposed to formal descriptions, unless explicitly asked to do otherwise. If asked to give formal descriptions, *draw* the transition diagram rather than writing it out as a table.

As usual, please think carefully about how you are going to organize your answers *before* you begin writing. Make sure your answers are complete, clean, concise and rigorous.

1. Formally describe a single-tape deterministic TM that decides the language $\{w \in \{0,1\}^* : w = w^{\mathcal{R}}\}$. In addition to drawing the TM's transition diagram, to get credit, you must (1) explain the overall strategy **clearly** in English, and (2) describe what the important states of your machine do. (Without such explanations and comments, grading would be impossible.) [20 points]

2. Formally describe a two-tape deterministic TM that decides the language $\{w \in \{0,1\}^* : w = w^{\mathcal{R}}\}$. Provide a diagram, plus explanations and comments, exactly as above. You may assume that in a particular move one (or both) of the heads is allowed to remain stationary. Thus, the transition function for such a TM would look like

$$\delta : Q \times \Gamma^2 \longrightarrow Q \times \Gamma^2 \times \{L, R, S\}^2 .$$

   When drawing a diagram, if $\delta(q, a, b) = (r, c, d, L, S)$, for example, you would draw an arrow from q to r and label it "$(a, b) \to (c, d), (L, S)$."

   Appreciate the ease of programming with two tapes for this language. [15 points]

3. Formally describe a two-tape NDTM for the language $\{ww : w \in \{0,1\}^*\}$. Again, provide a diagram, plus explanations and comments.

   Appreciate the ease of programming resulting from nondeterminism and the availability of two tapes. [20 points]

4. Prove that decidable languages are closed under (a) union, (b) intersection, (c) complement, (d) concatenation, and (e) Kleene star. [15 points]

5. Prove that recognizable languages are closed under (a) union, (b) intersection, and (c) concatenation. [15 points]

   They are also closed under Kleene star. I'm sure you can mentally toss off a proof of this fact. Smile smugly as you figure this out (no need to turn it in).

6. Show that a k-tape TM M can be simulated by a single-tape TM M' in such a way that a computation which takes time t (i.e., t steps of one configuration yielding another) on M takes time $O(t^2)$ on M'. The big-O notation may hide a constant that depends on k. You may assume that $t \geq |x|$, where x is the input to M.

   While not strictly necessary, it might help you to use a slightly different multitape-to-single-tape encoding from the one described in class, such as one that interleaves the symbols on the k tapes. [15 points]

Challenge Problems

CP7: Prove that every context-free subset of 0^* is regular.

CP8: Make an appropriate formal definition of a *deterministic* pushdown automaton with *two* stacks. Call such an automaton a 2DPDA. Prove, by formal construction, that every decidable language can be accepted by a 2DPDA.