Homework 4
Due Mon Jun 2, 5:00pm

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General Instructions: This homework is just two short exercises. You shouldn't sweat it too much!

1. Prove the formula version of Shannon's Theorem, i.e., prove that for every large enough integer n there exists a function $f_n : \{0,1\}^n \to \{0,1\}$ such that $\mathrm{L}(f_n) = \Omega(2^n / \log n)$. Here $\mathrm{L}(f_n)$ denotes the length of the shortest Boolean formula, using the operators $\{\vee, \wedge, \neg\}$, that evaluates f_n. [10 points]

2. The Karchmer-Wigderson game corresponding to a nonconstant Boolean function $f : \{0,1\}^n \to \{0,1\}$ is as follows: Alice gets an input $x \in \{0,1\}^n$ such that $f(x) = 1$ and Bob gets an input $y \in \{0,1\}^n$ such that $f(y) = 0$. The goal of the game is for Alice and Bob to agree on an index $i \in [n]$ such that $x_i \neq y_i$. Let $\mathrm{d}(f)$ denote the minimum depth of a Boolean circuit that computes f. Let $\mathrm{CC}(f)$ denote the complexity of the shortest communication protocol that solves the above Karchmer-Wigderson game. Prove that $\mathrm{d}(f) = \mathrm{CC}(f)$. [15 points]

   Note: One direction should be a straightforward generalization of what we did in class for the STCON game. The other direction is also an induction argument: use induction on the complexity of the best protocol for the game.