

CS 10:


# Problem solving via Object Oriented Programming

Graph Traversals

# Main goals

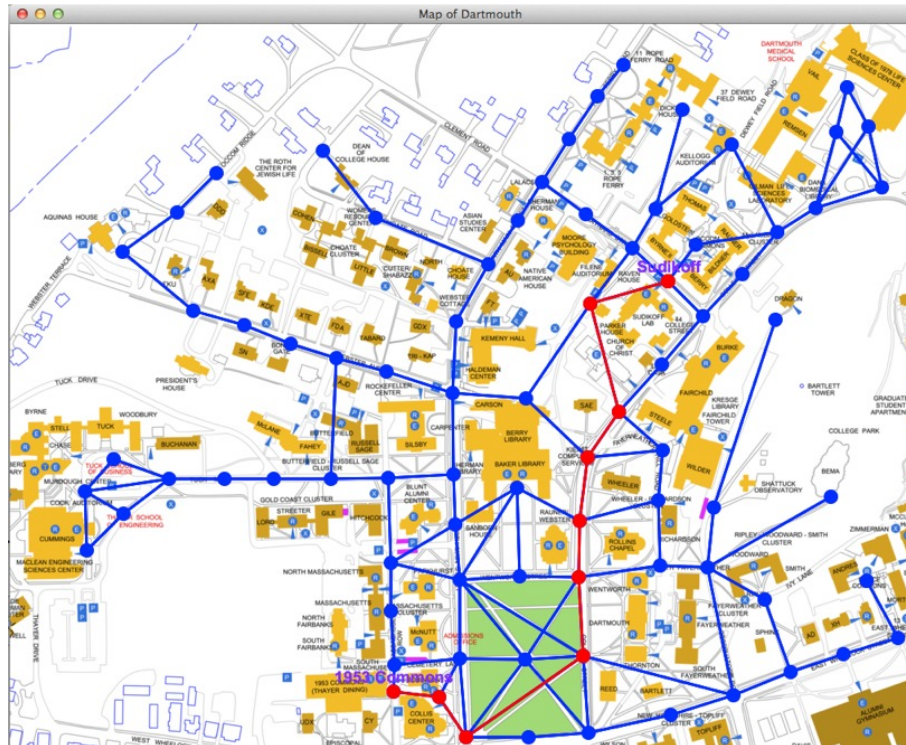
- Implement graph traversals that does not take into account cost

# Agenda

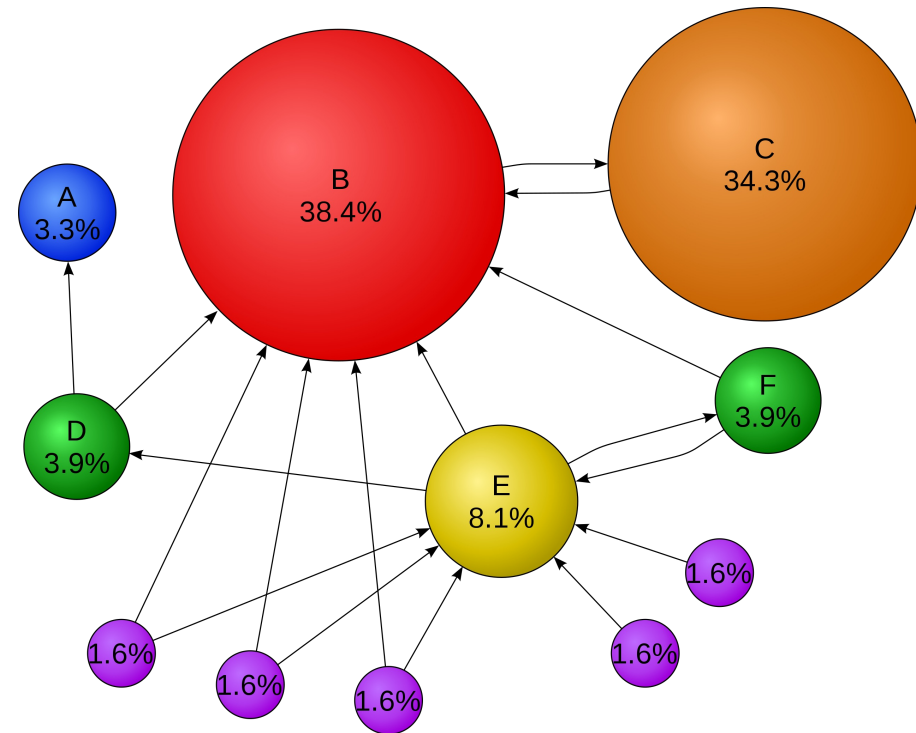
- 
1. Depth first search (DFS)
  2. Breadth first search (BFS)
  3. Examples from last class and today

# Graph traversals are useful to answer questions about relationships and reachability

## Some examples

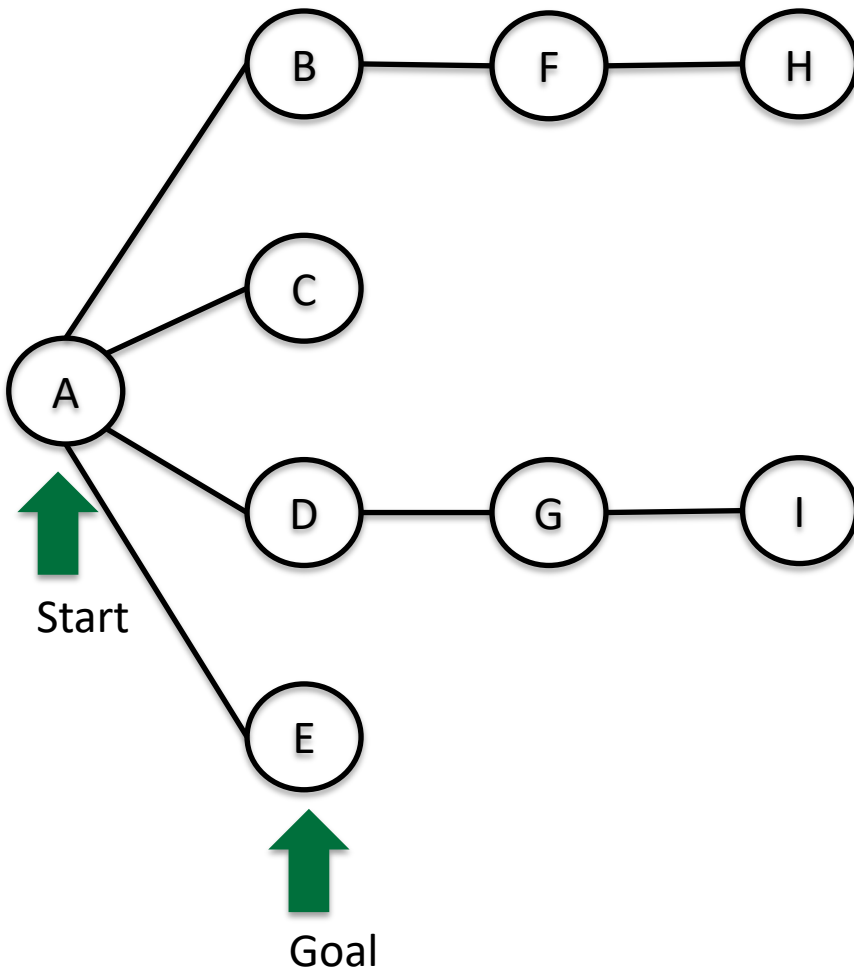


Map (CS1)



Page Rank, source: wikipedia

# Depth First Search (DFS) basic idea



- Keep going until you can't go any further, then back track
- Relies on a **Stack** (implicit or explicit) to keep track of where you've been

# Some of you did Depth First Search on Problem Set 1

## RegionFinder pseudo code

```
Loop over all the pixels
  If a pixel is unvisited and of the correct color
    Start a new region
    Keep track of pixels need to be visited, initially just one
    As long as there's some pixel that needs to be visited
      Get one to visit
      Add it to the region
      Mark it as visited
      Loop over all its neighbors
        If the neighbor is of the correct color
          Add it to the list of pixels to be visited
    If the region is big enough to be worth keeping, do so
```

# Some of you did Depth First Search on Problem Set 1

## RegionFinder pseudo code

```
Loop over all the pixels
  If a pixel is unvisited and of the correct color
    Start a new region
    Keep track of pixels need to be visited, initially just one
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      Loop over all its neighbors
        If the neighbor is of the correct color
          Add it to the list of pixels to be visited
    If the region is big enough to be worth keeping, do so
```



**If you added to end of list...**

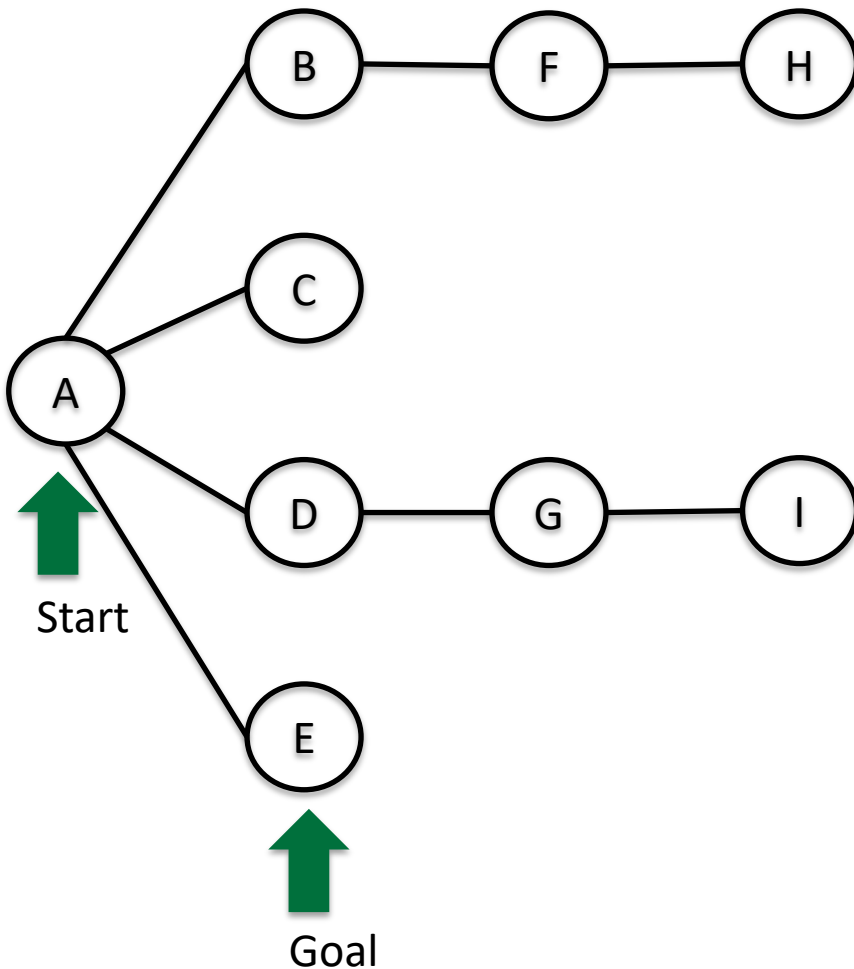
# Some of you did Depth First Search on Problem Set 1

## RegionFinder pseudo code

```
Loop over all the pixels
  If a pixel is unvisited and of the correct color
    Start a new region
    Keep track of pixels need to be visited, initially just one
    As long as there's some pixel that needs to be visited
      Get one to visit ← And if you get a pixel from end
      Add it to the region      of list, you implemented a stack
      Mark it as visited
      Loop over all its neighbors
        If the neighbor is of the correct color
          Add it to the list of pixels to be visited
    If the region is big enough to be worth keeping, do so
      ↑
      If you added to end of list...
```



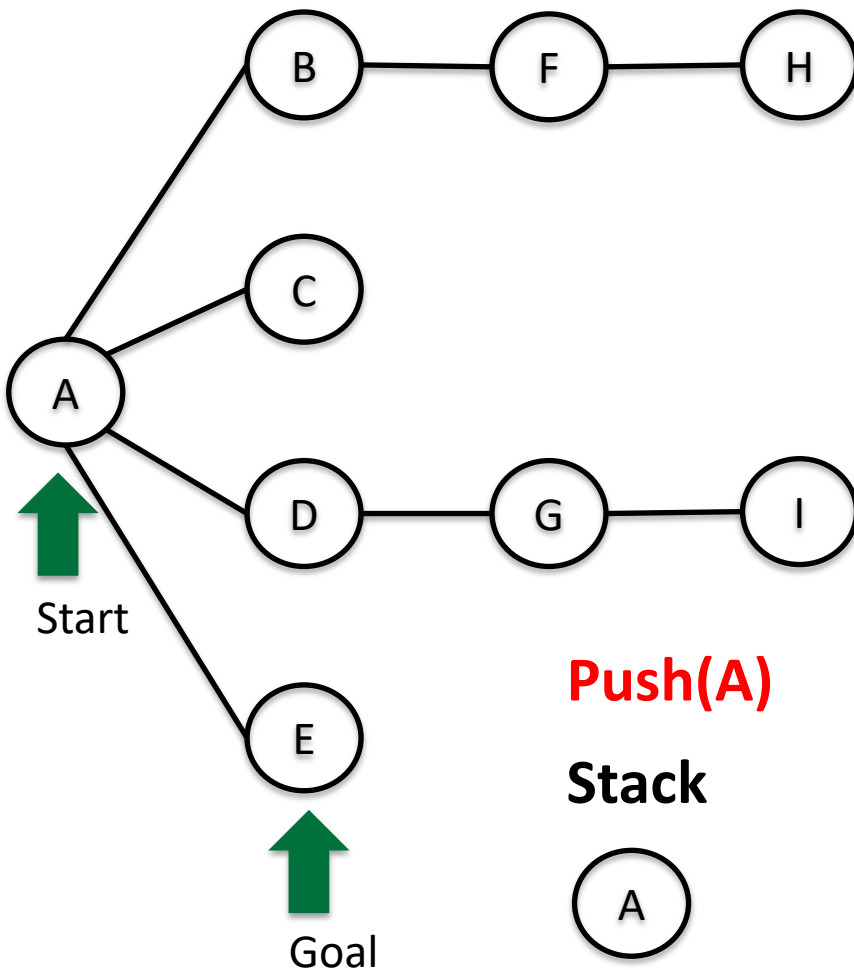
# Depth First Search (DFS) is like exploring a maze



## DFS algorithm

```
stack.push(s) //start node
repeat until find goal vertex or
stack empty:
    u = stack.pop()
    if !u.visited
        u.visited = true
        (do something while here)
        for v ∈ u.adjacent
            if !v.visited
                stack.push(v)
```

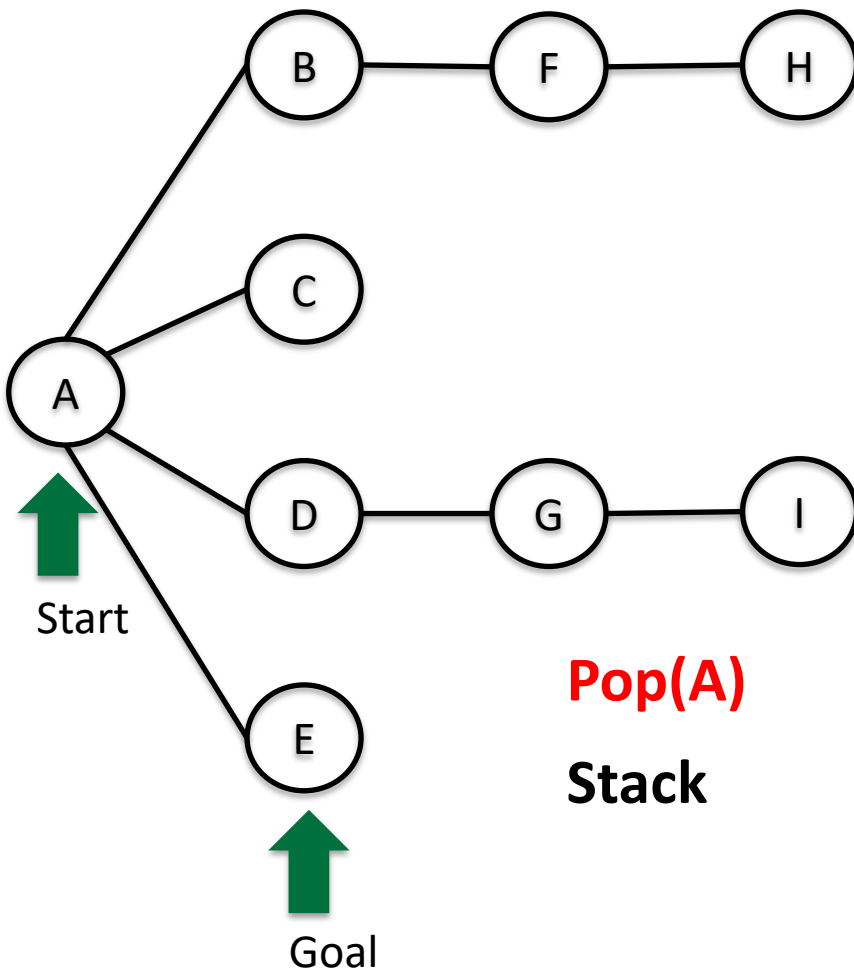
# Depth First Search (DFS) is like exploring a maze



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    u = stack.pop()
    if !u.visited
        u.visited = true
        (do something while here)
        for v ∈ u.adjacent
            if !v.visited
                stack.push(v)
```

# Depth First Search (DFS) is like exploring a maze

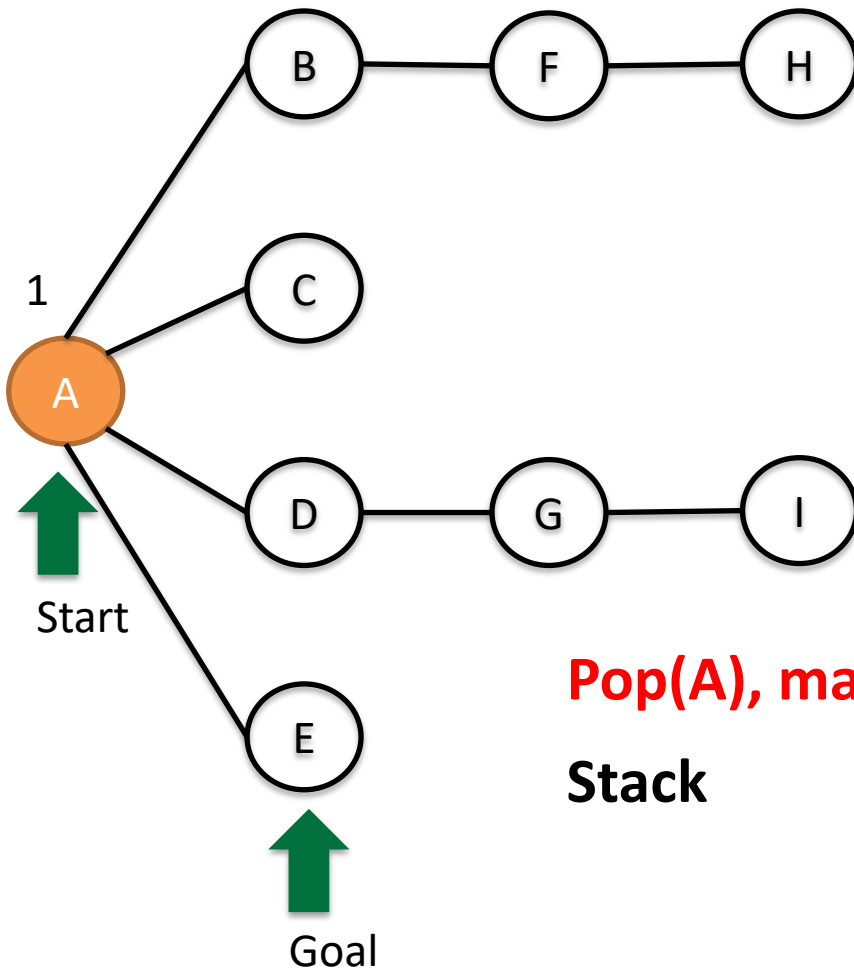


## DFS algorithm

```
stack.push(s) //start node  
repeat until find goal vertex or  
stack empty:
```

```
➔ u = stack.pop()  
  if !u.visited  
    u.visited = true  
    (do something while here)  
    for v ∈ u.adjacent  
      if !v.visited  
        stack.push(v)
```

# Depth First Search (DFS) is like exploring a maze

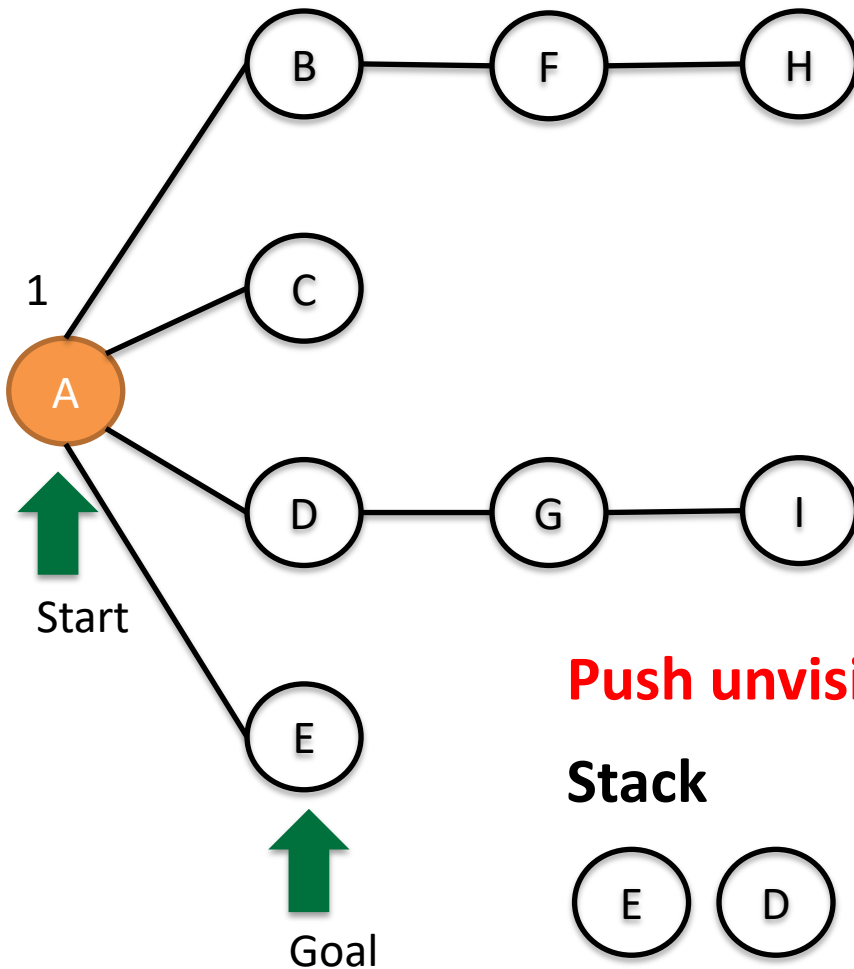


## DFS algorithm

```
stack.push(s) //start node
repeat until find goal vertex or
stack empty:
```

```
    u = stack.pop()
    if !u.visited
        Visit the node
        u.visited = true
        (do something while here)
        for v ∈ u.adjacent
            if !v.visited
                stack.push(v)
```

# Depth First Search (DFS) is like exploring a maze



## DFS algorithm

```
stack.push(s) //start node  
repeat until find goal vertex or  
stack empty:
```

```
u = stack.pop()
```

```
if !u.visited
```

```
u.visited = true
```

```
(do something while here)
```

```
for v ∈ u.adjacent
```

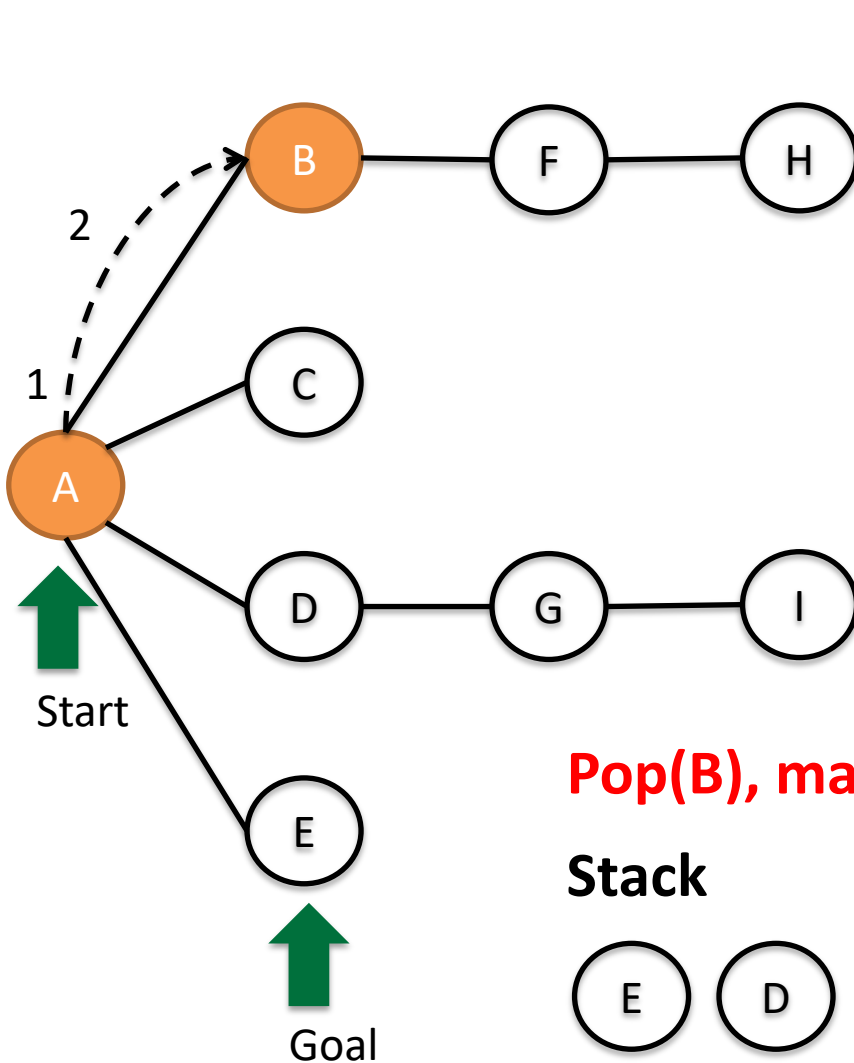
```
if !v.visited
```

```
stack.push(v)
```

**Push unvisited adjacent**

- What method would we use on our `AdjacencyMapGraph`?
- `graph.outNeighbors(u)`
- Order pushed onto stack depends on order of nodes from `outNeighbors` iterator

# Depth First Search (DFS) is like exploring a maze



## DFS algorithm

```
stack.push(s) //start node
repeat until find goal vertex or
stack empty:
```

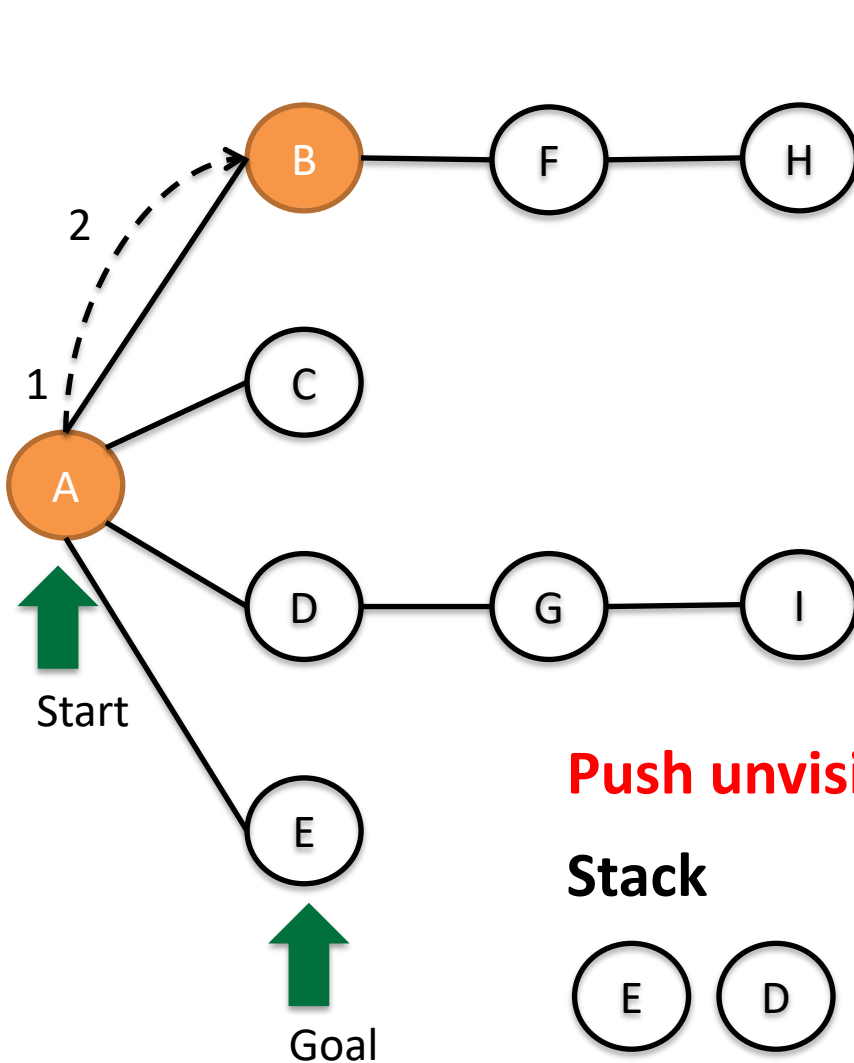
```
    → u = stack.pop()
    if !u.visited
        u.visited = true
        (do something while here)
    for v ∈ u.adjacent
        if !v.visited
            stack.push(v)
```

**Pop(B), mark visited**

**Stack**



# Depth First Search (DFS) is like exploring a maze



## DFS algorithm

```
stack.push(s) //start node
repeat until find goal vertex or
stack empty:
```

```
    u = stack.pop()
```

```
    if !u.visited
```

```
        u.visited = true
```

```
        (do something while here)
```

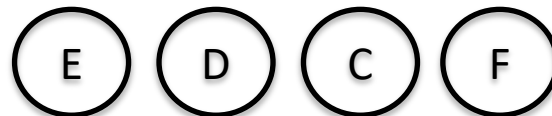
```
        for v ∈ u.adjacent
```

```
            if !v.visited
```

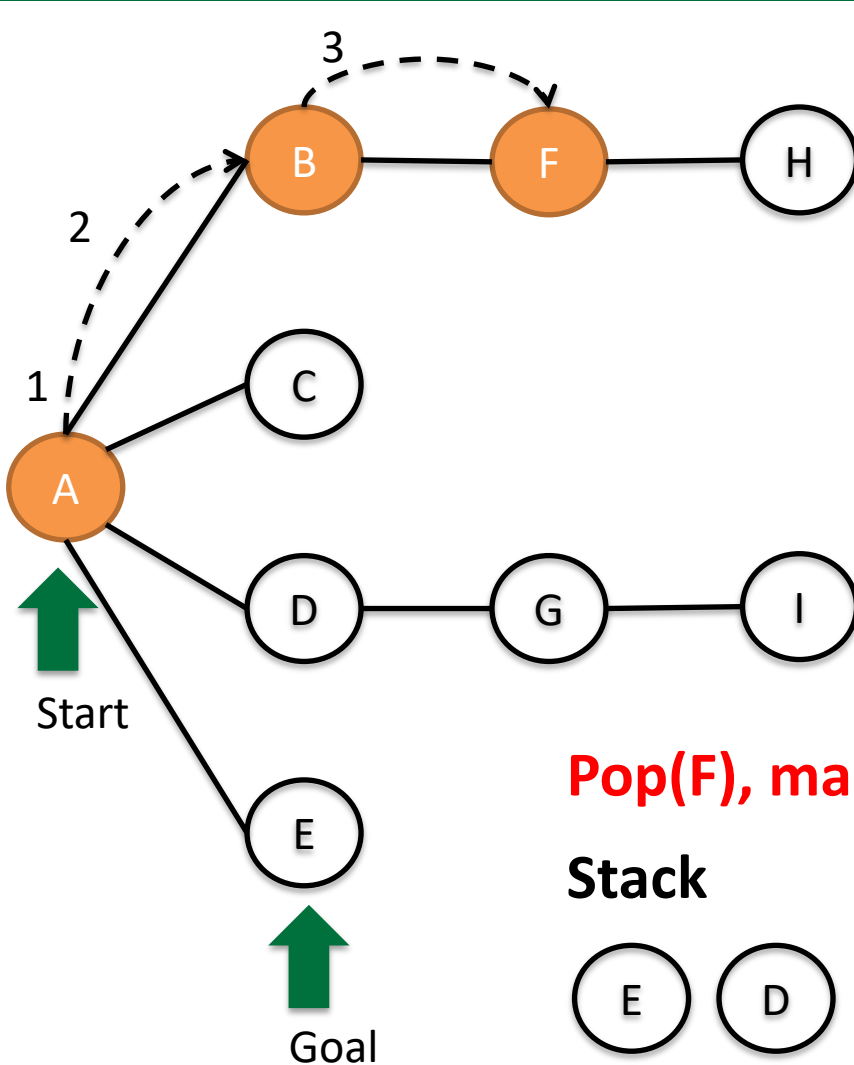
```
                stack.push(v)
```

**Push unvisited adjacent (F, but not A)**

**Stack**



# Depth First Search (DFS) is like exploring a maze



## DFS algorithm

```
stack.push(s) //start node  
repeat until find goal vertex or  
stack empty:
```

```
    → u = stack.pop()  
    if !u.visited  
        u.visited = true  
        (do something while here)  
    for v ∈ u.adjacent  
        if !v.visited  
            stack.push(v)
```

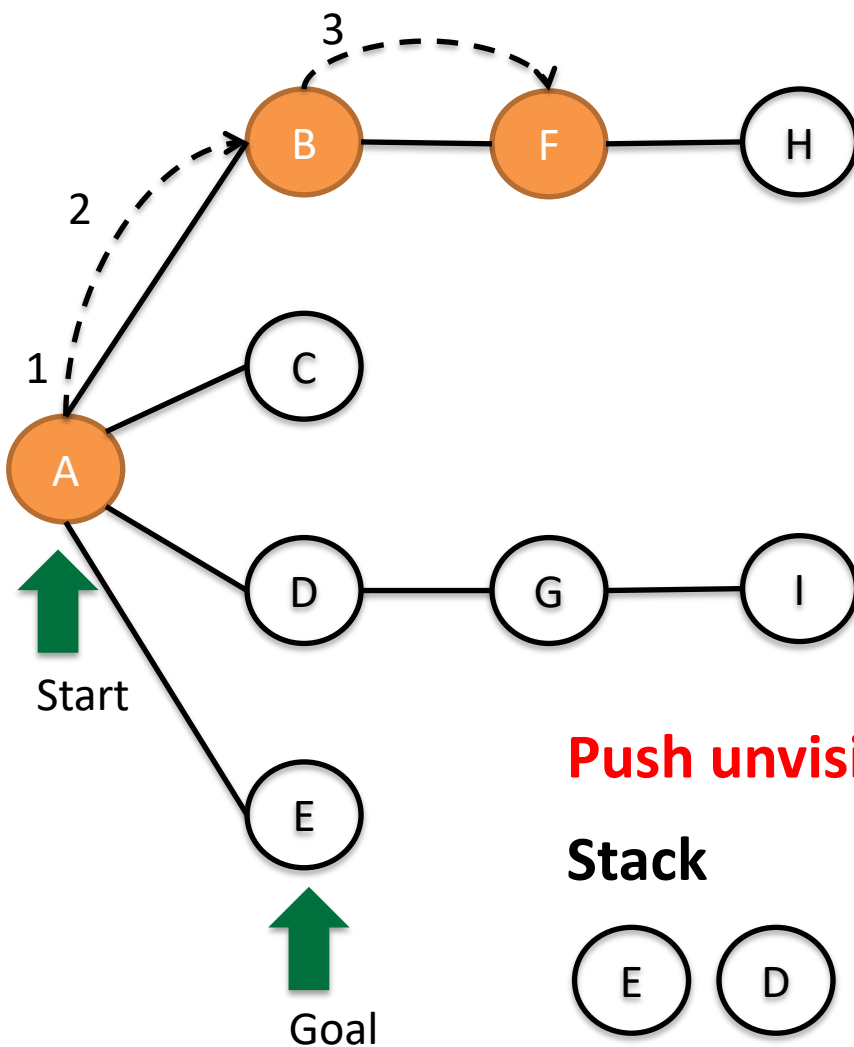
**Pop(F), mark visited**

**Stack**





# Depth First Search (DFS) is like exploring a maze



## DFS algorithm

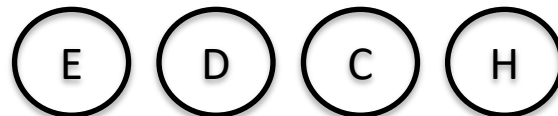
```
stack.push(s) //start node
repeat until find goal vertex or
stack empty:
```

```
    u = stack.pop()
    if !u.visited
        u.visited = true
        (do something while here)
```

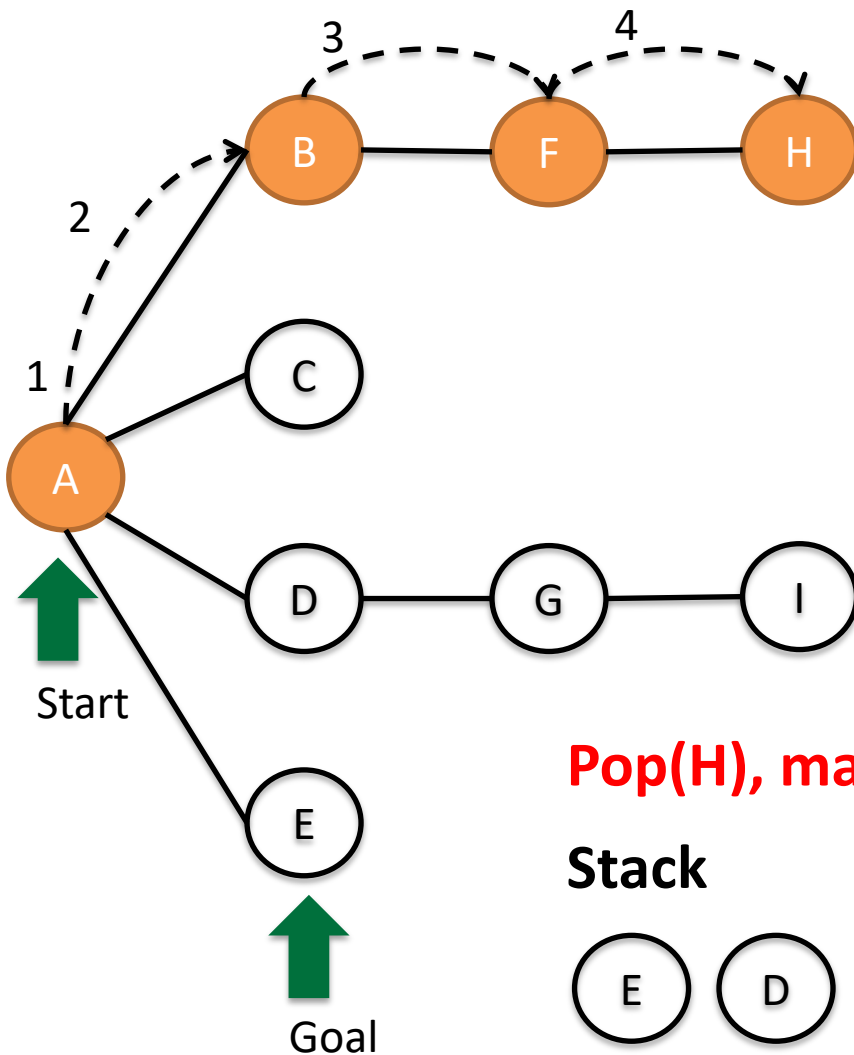
```
    for v ∈ u.adjacent
        if !v.visited
            stack.push(v)
```

**Push unvisited adjacent (H, but not B)**

**Stack**



# Depth First Search (DFS) is like exploring a maze



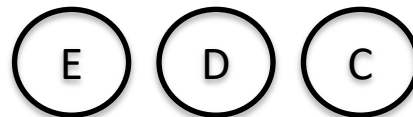
## DFS algorithm

```
stack.push(s) //start node  
repeat until find goal vertex or  
stack empty:
```

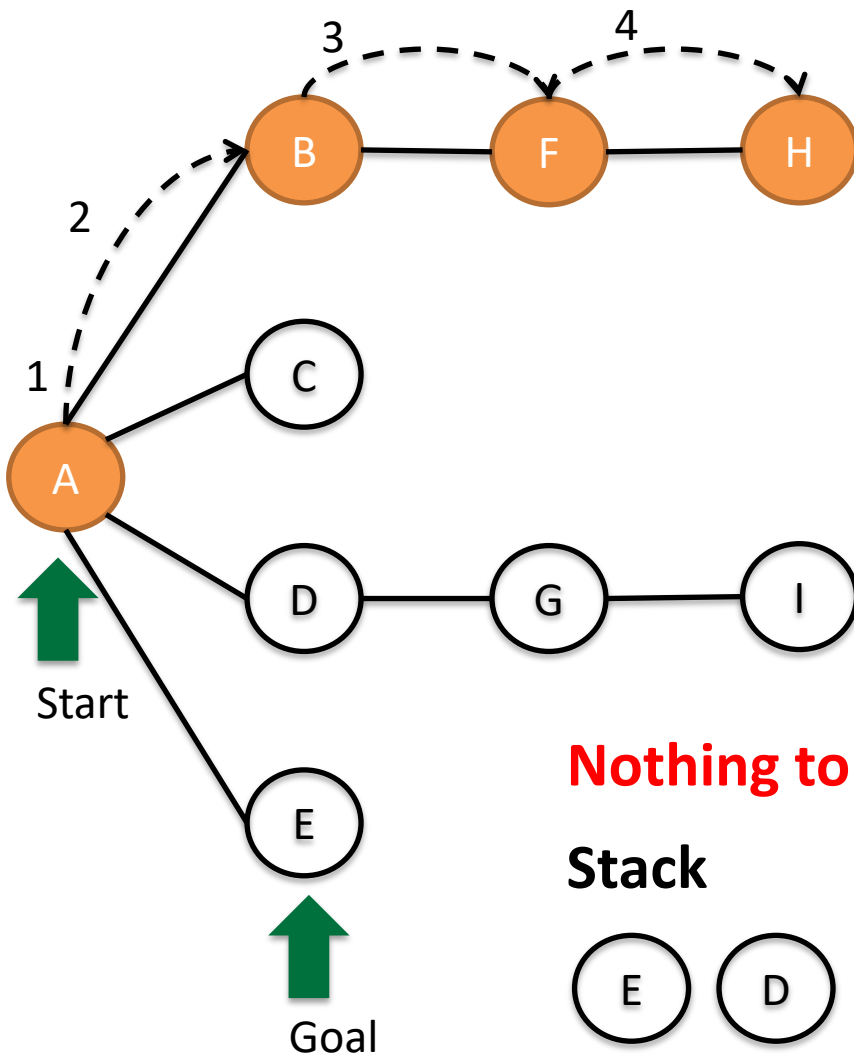
```
➔ u = stack.pop()  
  if !u.visited  
    u.visited = true  
    (do something while here)  
    for v ∈ u.adjacent  
      if !v.visited  
        stack.push(v)
```

**Pop(H), mark visited**

**Stack**



# Depth First Search (DFS) is like exploring a maze



## DFS algorithm

```
stack.push(s) //start node  
repeat until find goal vertex or  
stack empty:
```

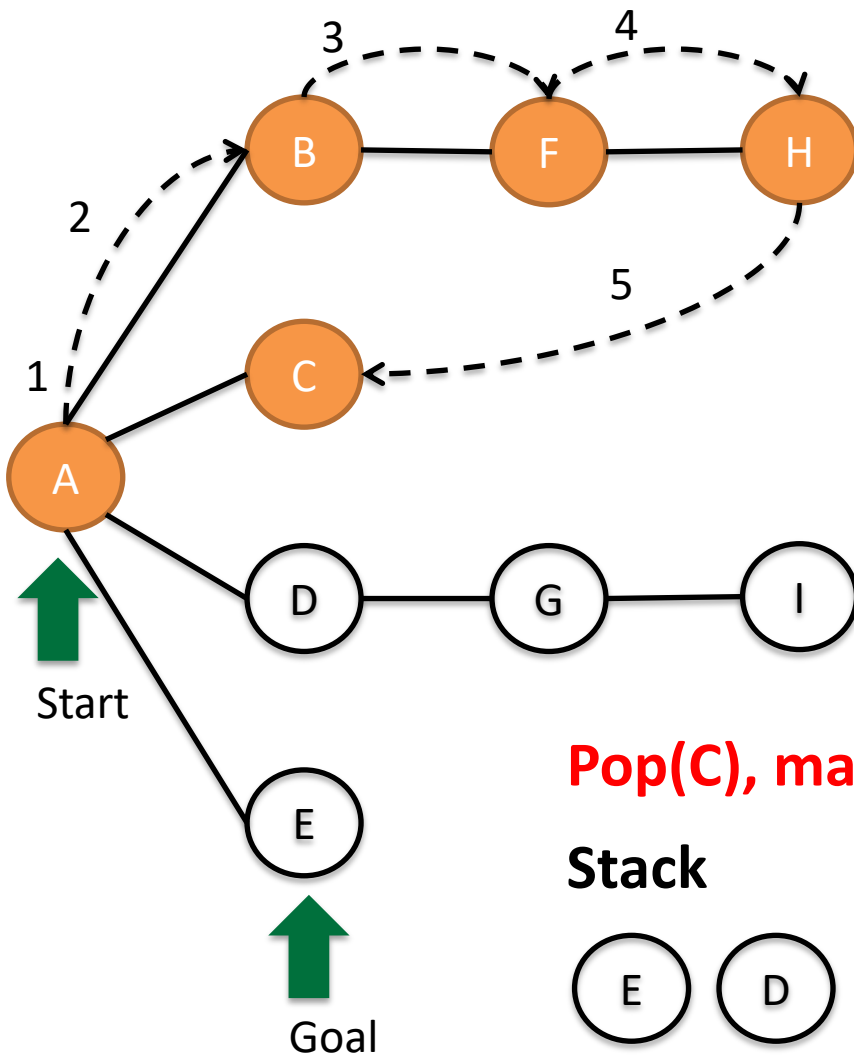
```
    → u = stack.pop()  
    if !u.visited  
        u.visited = true  
        (do something while here)  
        for v ∈ u.adjacent  
            if !v.visited  
                stack.push(v)
```

**Nothing to push, back up by popping C**

**Stack**



# Depth First Search (DFS) is like exploring a maze



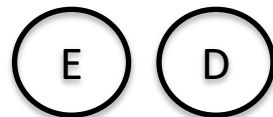
## DFS algorithm

```
stack.push(s) //start node  
repeat until find goal vertex or  
stack empty:
```

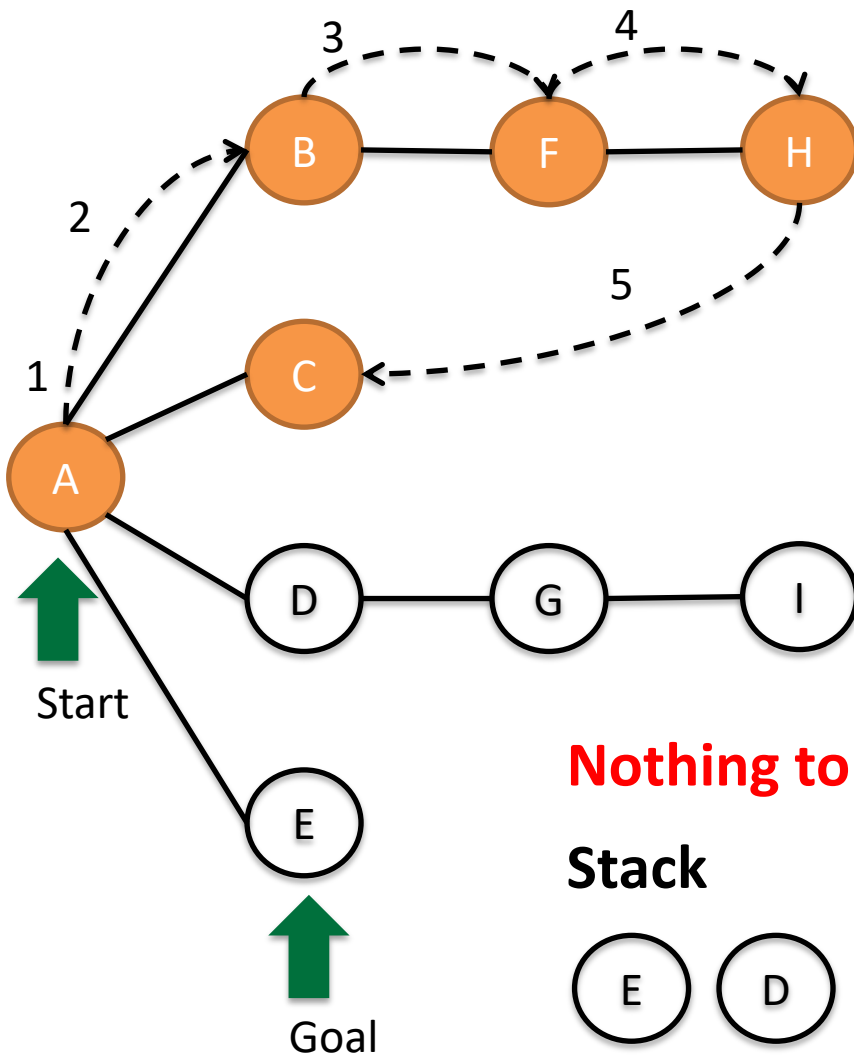
```
    → u = stack.pop()  
    if !u.visited  
        u.visited = true  
        (do something while here)  
    for v ∈ u.adjacent  
        if !v.visited  
            stack.push(v)
```

**Pop(C), mark visited**

**Stack**



# Depth First Search (DFS) is like exploring a maze



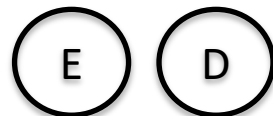
## DFS algorithm

```
stack.push(s) //start node  
repeat until find goal vertex or  
stack empty:
```

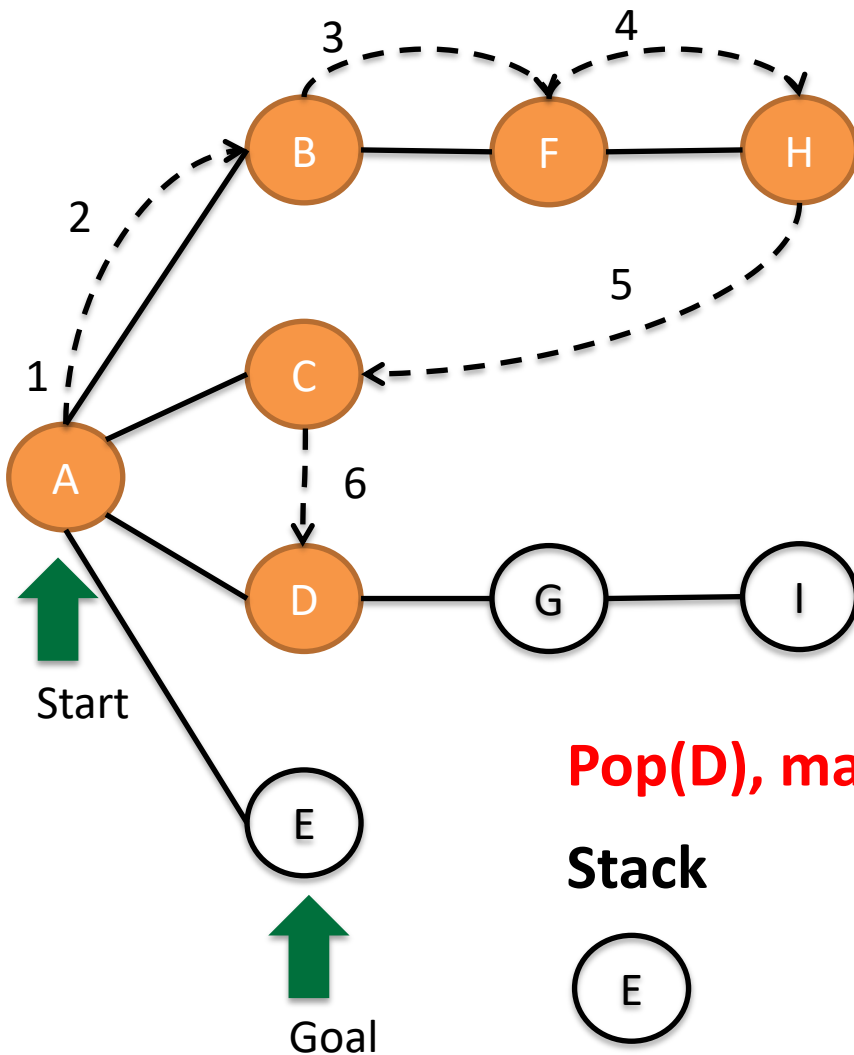
```
➔ u = stack.pop()  
  if !u.visited  
    u.visited = true  
    (do something while here)  
    for v ∈ u.adjacent  
      if !v.visited  
        stack.push(v)
```

**Nothing to push, back up by popping D**

**Stack**



# Depth First Search (DFS) is like exploring a maze



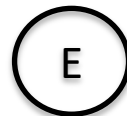
## DFS algorithm

```
stack.push(s) //start node  
repeat until find goal vertex or  
stack empty:
```

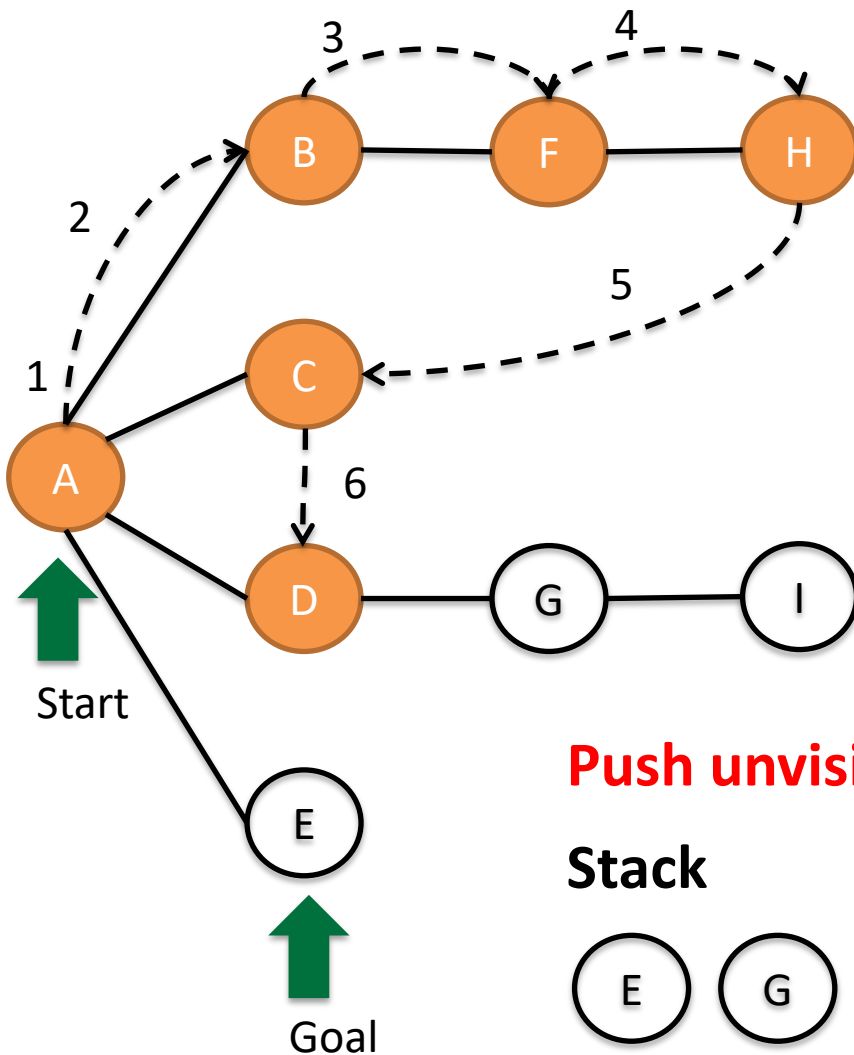
```
➔ u = stack.pop()  
  if !u.visited  
    u.visited = true  
    (do something while here)  
  for v ∈ u.adjacent  
    if !v.visited  
      stack.push(v)
```

**Pop(D), mark visited**

**Stack**



# Depth First Search (DFS) is like exploring a maze



---> Order nodes visited

## DFS algorithm

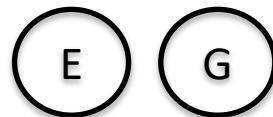
```
stack.push(s) //start node  
repeat until find goal vertex or  
stack empty:
```

```
    u = stack.pop()  
    if !u.visited  
        u.visited = true  
        (do something while here)
```

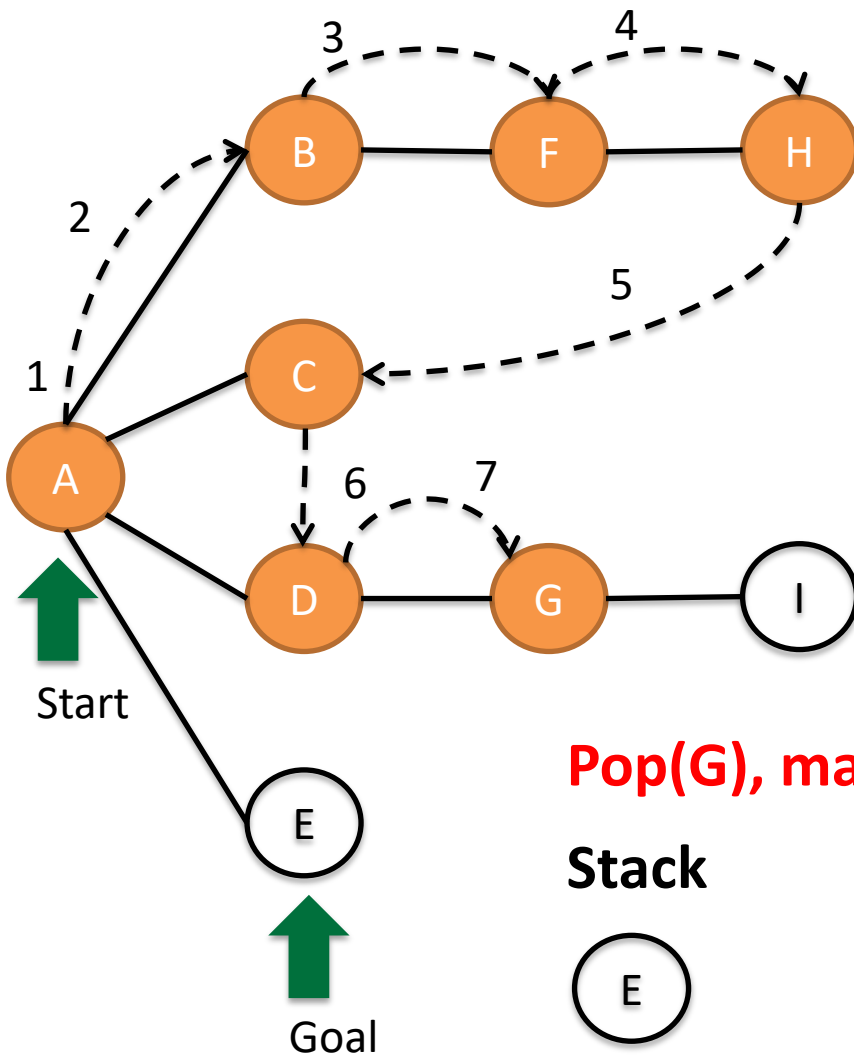
```
    for v ∈ u.adjacent  
        if !v.visited  
            stack.push(v)
```

**Push unvisited adjacent (G, but not A)**

**Stack**



# Depth First Search (DFS) is like exploring a maze



--> Order nodes visited

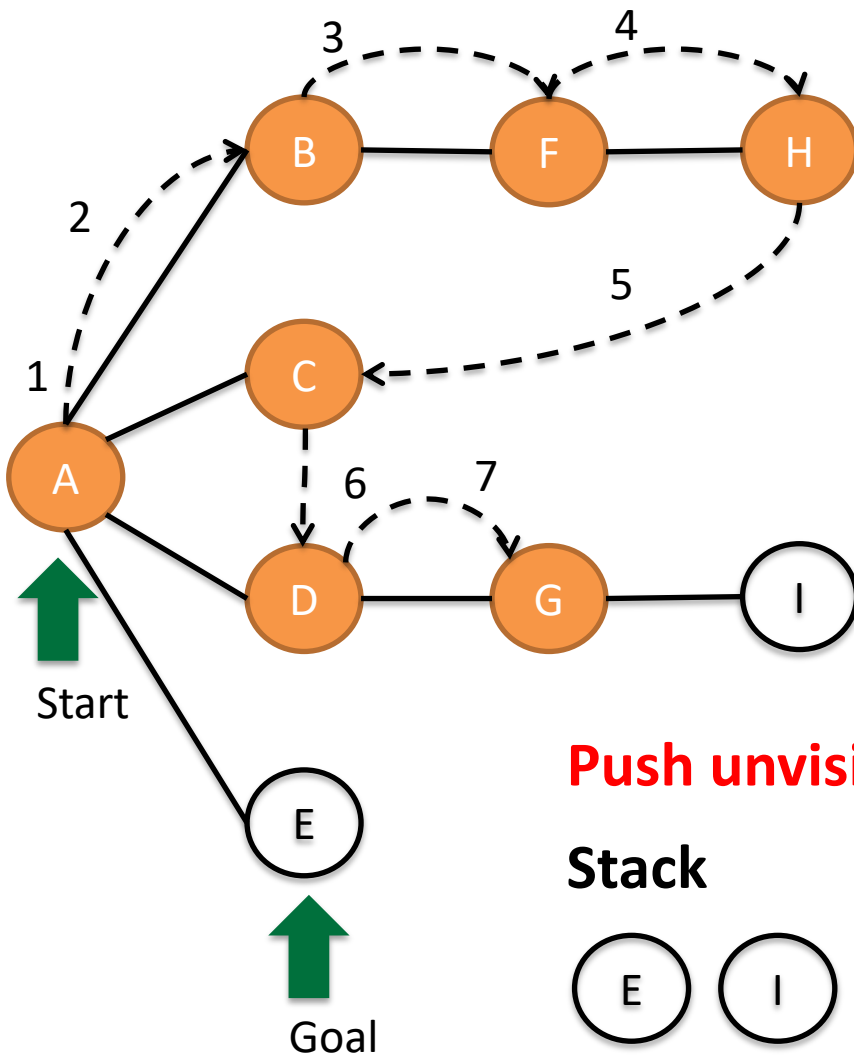
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stack.push(s) //start node
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stack empty:
```

```
    → u = stack.pop()
    if !u.visited
        u.visited = true
        (do something while here)
        for v ∈ u.adjacent
            if !v.visited
                stack.push(v)
```



# Depth First Search (DFS) is like exploring a maze



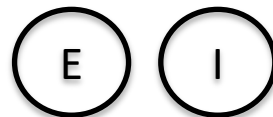
## DFS algorithm

```
stack.push(s) //start node
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```

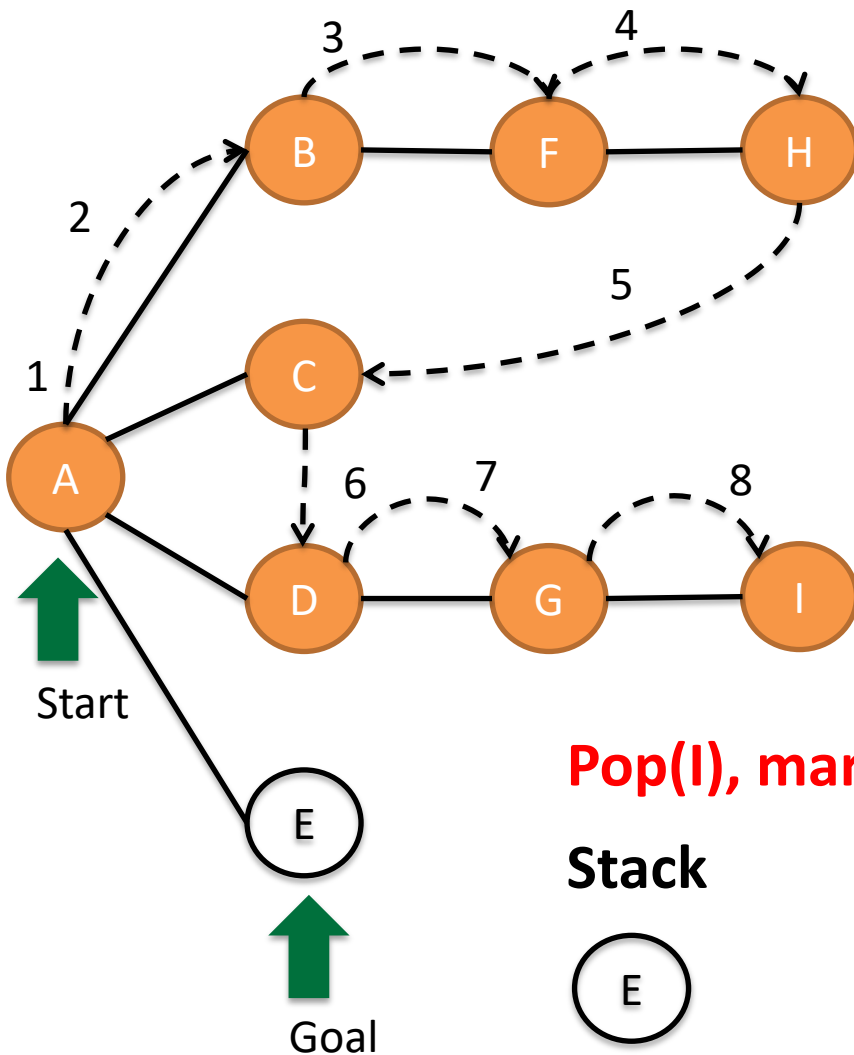
```
    u = stack.pop()
    if !u.visited
        u.visited = true
        (do something while here)
        for v ∈ u.adjacent
            if !v.visited
                stack.push(v)
```

**Push unvisited adjacent (I, but not D)**

**Stack**



# Depth First Search (DFS) is like exploring a maze



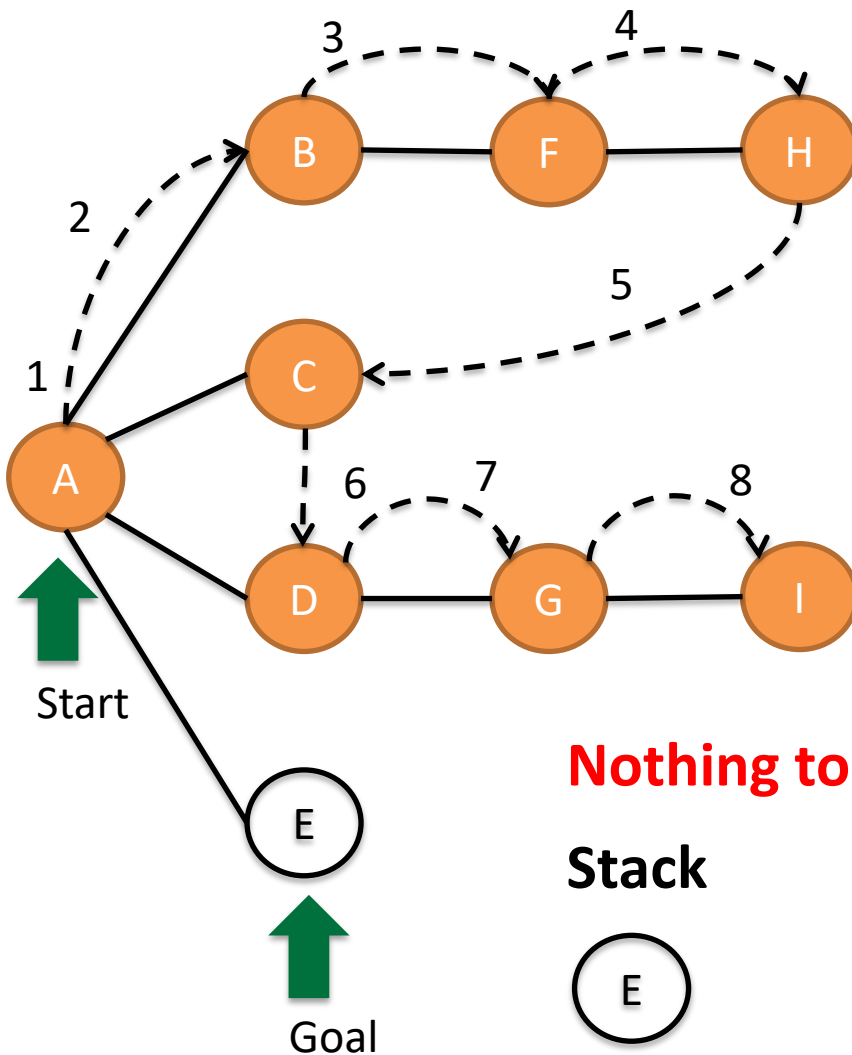
--> Order nodes visited

## DFS algorithm

```
stack.push(s) //start node  
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stack empty:
```

```
    → u = stack.pop()  
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        for v ∈ u.adjacent  
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                stack.push(v)
```

# Depth First Search (DFS) is like exploring a maze



## DFS algorithm

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repeat until find goal vertex or  
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```

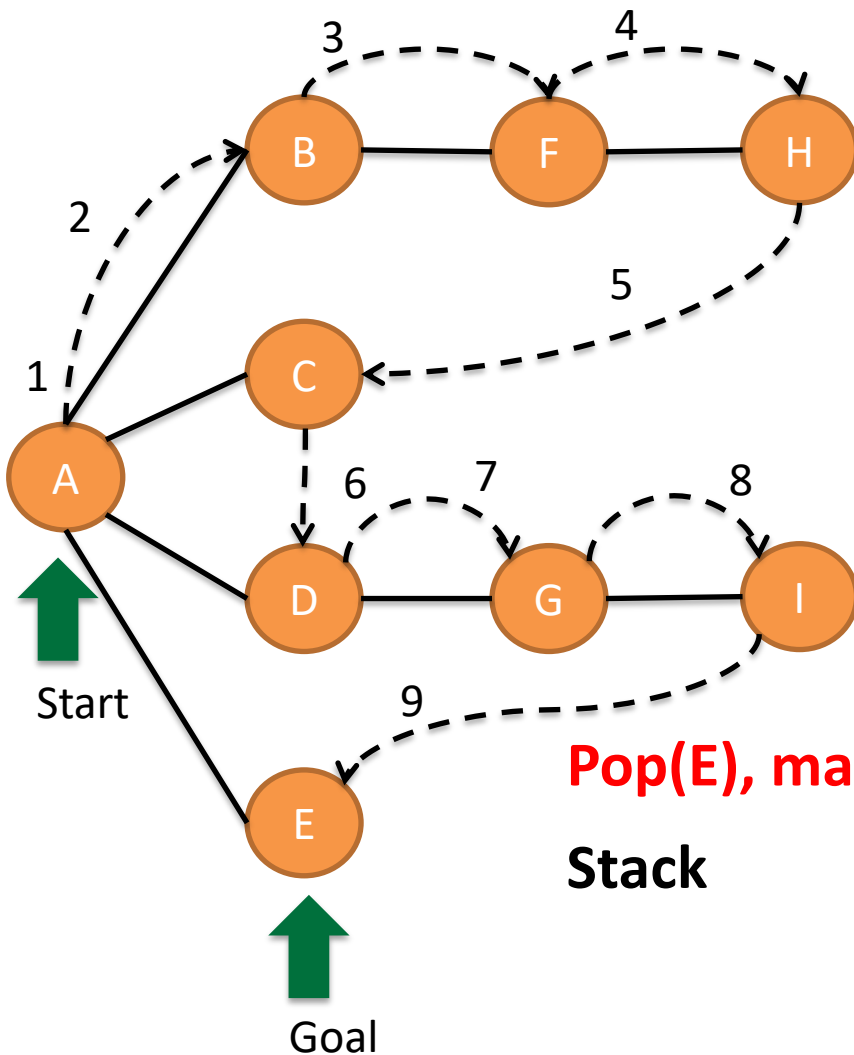
```
    → u = stack.pop()  
    if !u.visited  
        u.visited = true  
        (do something while here)  
        for v ∈ u.adjacent  
            if !v.visited  
                stack.push(v)
```

**Nothing to push, back up by popping E**

**Stack**



# Depth First Search (DFS) is like exploring a maze



--> Order nodes visited

## DFS algorithm

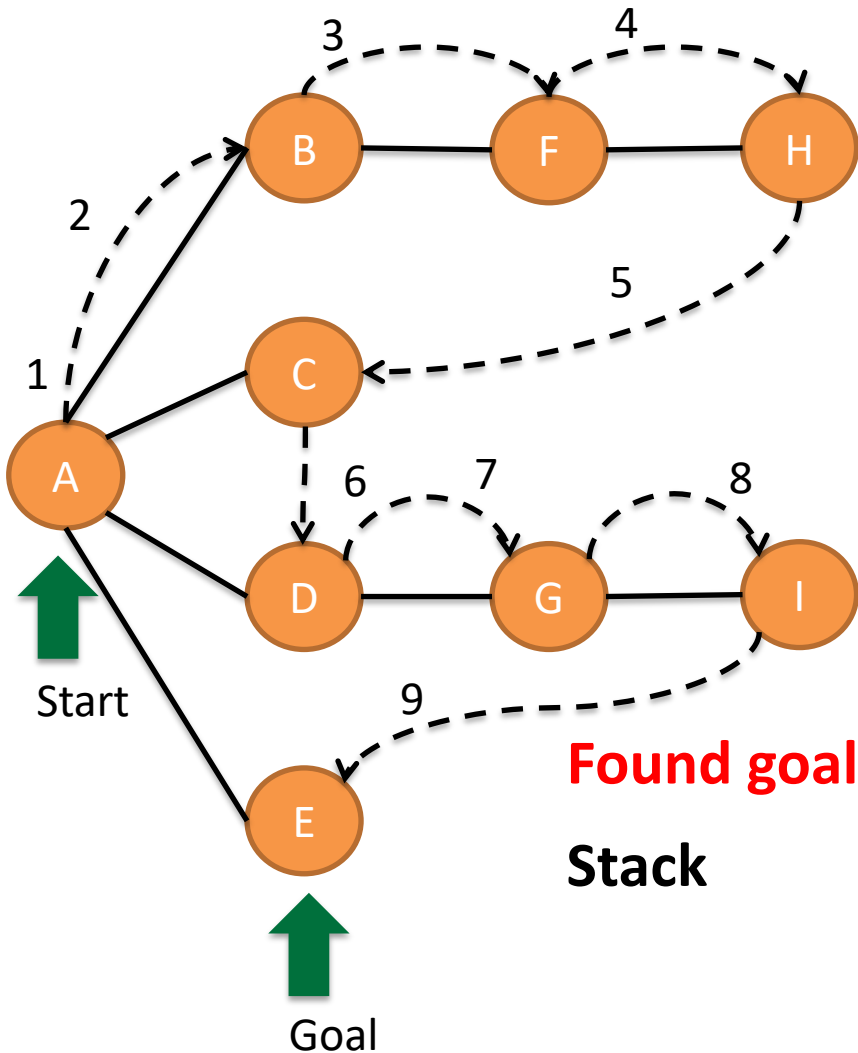
```
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repeat until find goal vertex or  
stack empty:
```

```
    → u = stack.pop()  
    if !u.visited  
        u.visited = true  
        (do something while here)  
        for v ∈ u.adjacent  
            if !v.visited  
                stack.push(v)
```

**Pop(E), mark visited**

**Stack**

# Depth First Search (DFS) is like exploring a maze



--> Order nodes visited

## DFS algorithm

```
stack.push(s) //start node
repeat until find goal vertex or
stack empty:
```

```
    u = stack.pop()
```

```
    if !u.visited
```

```
        u.visited = true
```

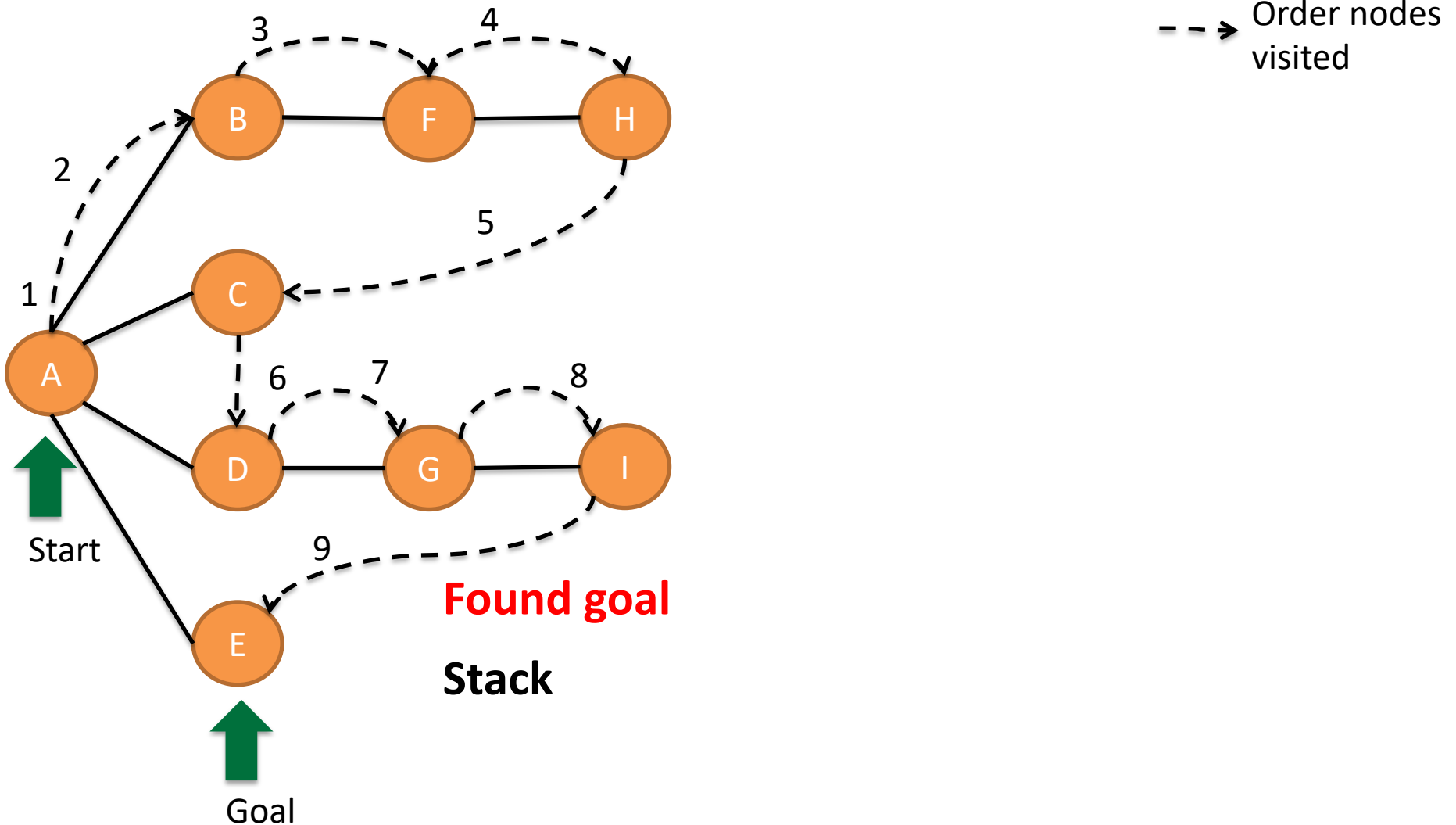
```
        (do something while here)
```

```
        for v ∈ u.adjacent
```

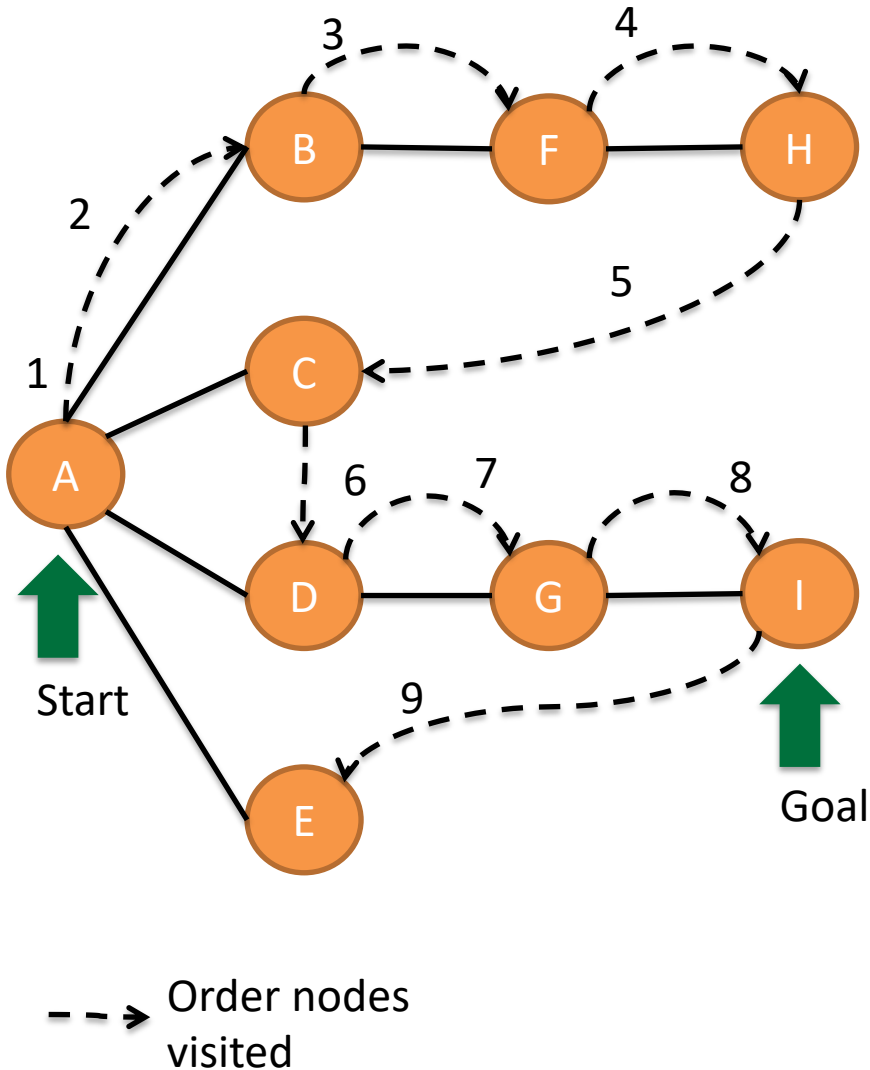
```
            if !v.visited
```

```
                stack.push(v)
```

# Node discovery tells us something about the graph



# To find path from *start* to *v*, keep track of previous node as nodes are “discovered”



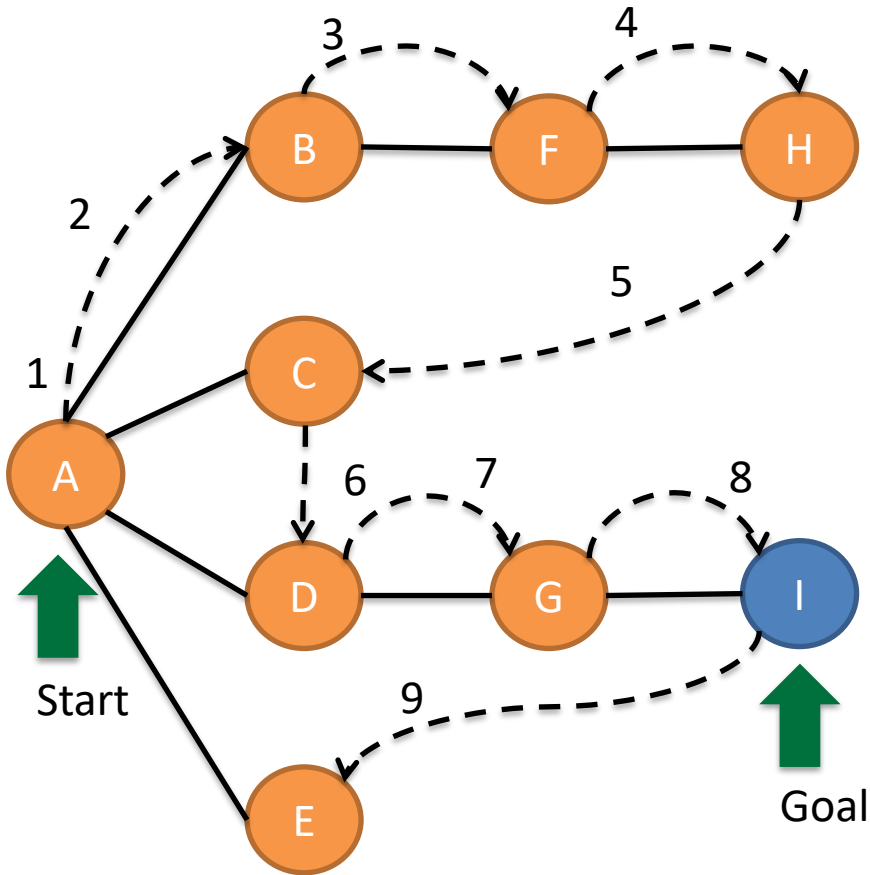
## Path from start to *v*

- Each time node discovered, keep pointer to previous
- Could keep Map with node as Key and previous as Value
- Start at goal node (*v*)
- Track backward until find starting node
- Will find a path if it exists, but not necessarily the shortest path (wait for BFS)

## Path A to I

Key	Value
A	Null
B	A
C	A
D	A
E	A
F	B
G	D
H	F
I	G

# To find path from *start* to *v*, keep track of previous node as nodes are “discovered”



## Path from *start* to *v*

- Each time node discovered, keep pointer to previous
- Could keep Map with node as Key and previous as Value
- Start at goal node (*v*)
- Track backward until find starting node
- Will find a path if it exists, but not necessarily the shortest path (wait for BFS)

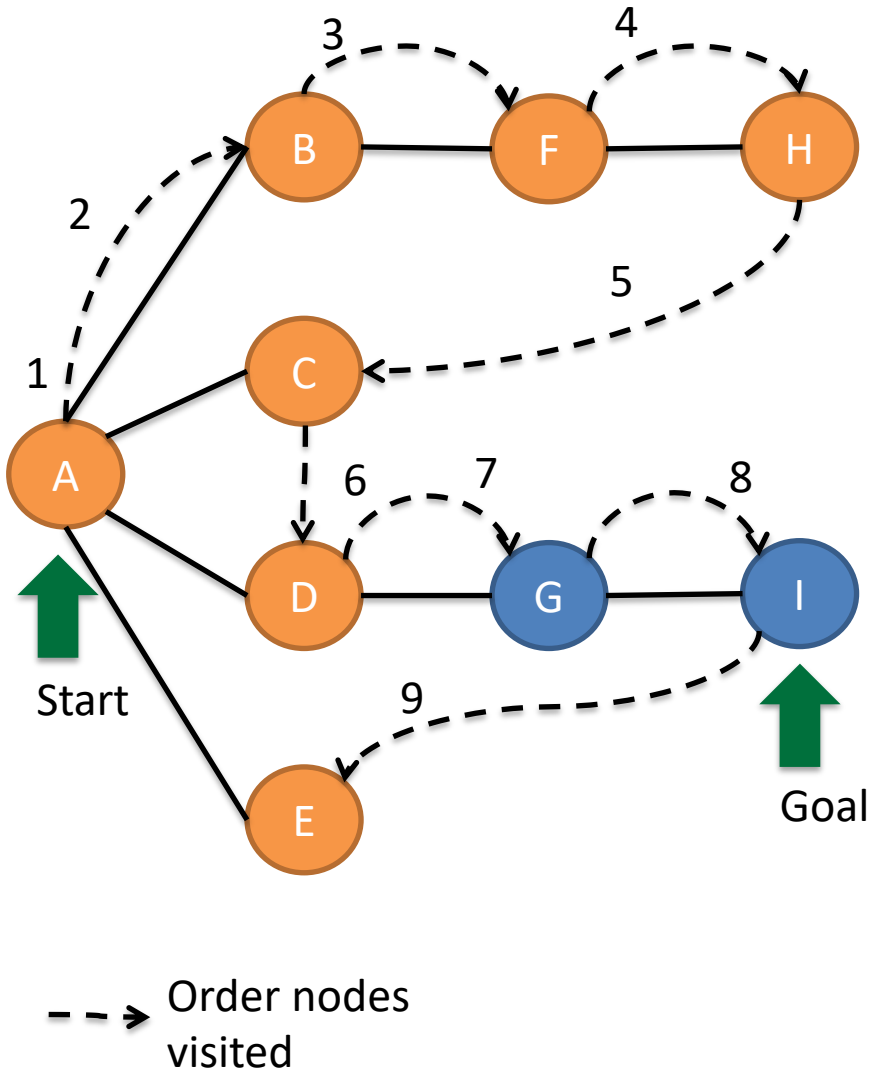
## Path A to I

Key	Value
A	Null
B	A
C	A
D	A
E	A
F	B
G	D
H	F
I	G

## Path G,I



# To find path from *start* to *v*, keep track of previous node as nodes are “discovered”



## Path from *start* to *v*

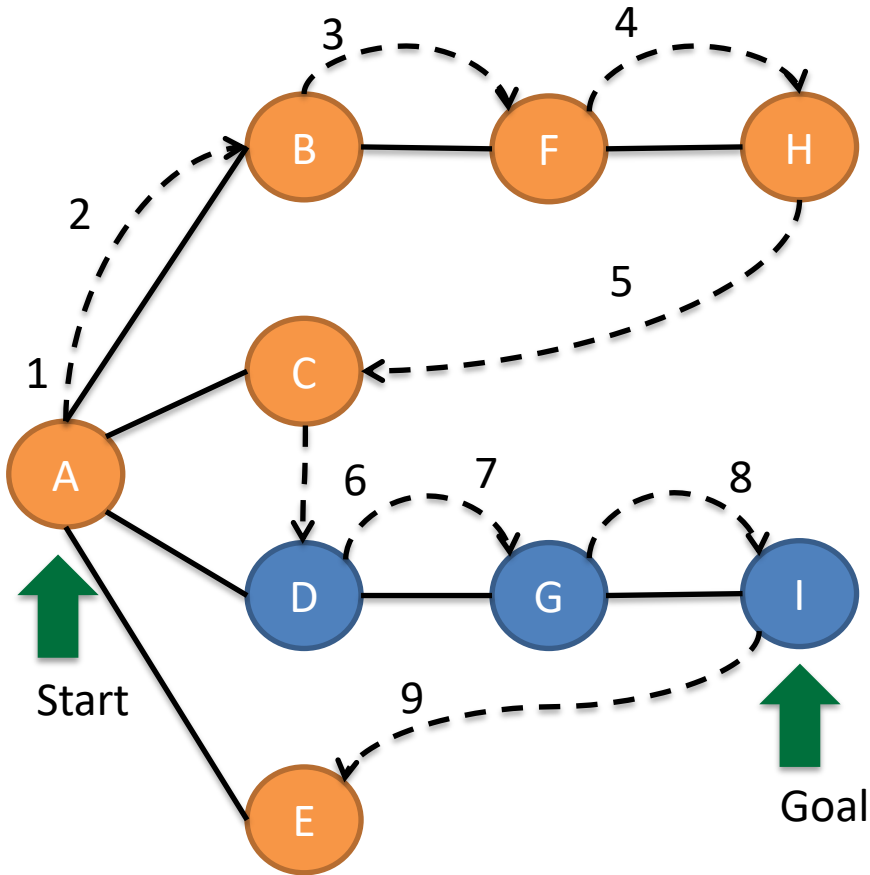
- Each time node discovered, keep pointer to previous
- Could keep Map with node as Key and previous as Value
- Start at goal node (*v*)
- Track backward until find starting node
- Will find a path if it exists, but not necessarily the shortest path (wait for BFS)

## Path A to I

Key	Value
A	Null
B	A
C	A
D	A
E	A
F	B
<b>G</b>	<b>D</b>
H	F
I	G

**Path  
D,G,I**

# To find path from *start* to *v*, keep track of previous node as nodes are “discovered”



---> Order nodes visited

## Path from *start* to *v*

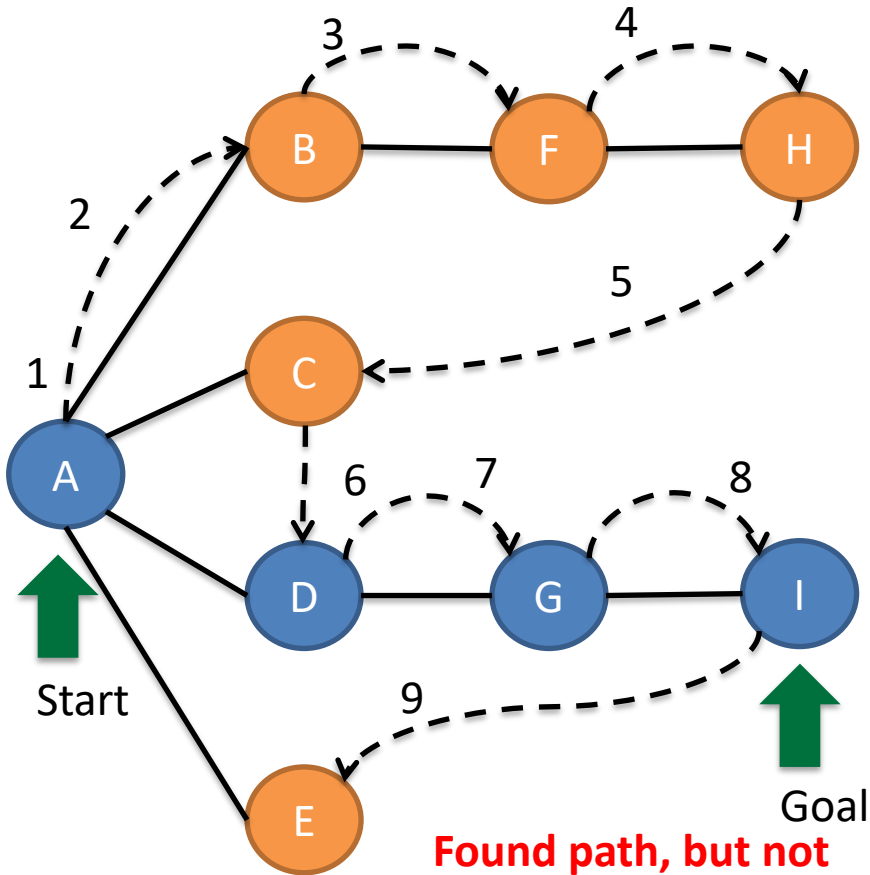
- Each time node discovered, keep pointer to previous
- Could keep Map with node as Key and previous as Value
- Start at goal node (*v*)
- Track backward until find starting node
- Will find a path if it exists, but not necessarily the shortest path (wait for BFS)

## Path A to I

Key	Value
A	Null
B	A
C	A
<b>D</b>	<b>A</b>
E	A
F	B
G	D
H	F
I	G

**Path  
A,D,G,I**

# To find path from *start* to *v*, keep track of previous node as nodes are “discovered”



**Found path, but not necessarily shortest path**

**After DFS from *start* can find a path from *start* to any other reachable node**

## Path from *start* to *v*

- Each time node discovered, keep pointer to previous
- Could keep Map with node as Key and previous as Value
- Start at goal node (*v*)
- Track backward until find starting node
- Will find a path if it exists, but not necessarily the shortest path (wait for BFS)

## Path A to I

Key	Value
A	Null
B	A
C	A
<b>D</b>	<b>A</b>
E	A
F	B
G	D
H	F
I	G

**Path A,D,G,I**

**Could we start from node other than A? <sup>35</sup> No**

---> Order nodes visited

# GraphTraversal.java: DFS code

```
17 public class GraphTraversal<V,E> {
18
19     public Map<V,V> backTrack; //keep track of prior vertex when v
20
21     /**
22      * Constructor. Initialize backTrack to new HashMap.
23      */
24     public GraphTraversal() {
25         backTrack = new HashMap<V,V>();
26     }
27
28     /**
29      * Depth First Search
30      * @param G -- graph to search
31      * @param start -- starting vertex
32      */
33     public void DFS(AdjacencyMapGraph<V,E> G, V start) {
34         System.out.println("\nDepth First Search from " + start);
35         backTrack = new HashMap<V,V>(); //initialize backTrack
36         backTrack.put(start, null); //load start node with null pa
37         Set<V> visited = new HashSet<V>(); //Set to track which ve
38         Stack<V> stack = new Stack<V>(); //stack to implement DFS
39
40         stack.push(start); //push start vertex
41         while (!stack.isEmpty()) { //loop until no more vertices
42             V u = stack.pop(); //get most recent vertex
43             if (!visited.contains(u)) { //if not already visited
44                 visited.add(u); //add to visited Set
45                 for (V v : G.outNeighbors(u)) { //loop over out ne
46                     if (!visited.contains(v)) { //if neighbor not
47                         stack.push(v); //push non-visited neighbor
48                         backTrack.put(v, u); //save that this vert
49                     }
50                 }
51             }
52         }
53     }
54 }
```

# DFS run time

## Run time

- Assume graph with  $n$  nodes and  $m$  edges
- Visit each node at most one time (due to visited indicator)
- Examine each edge at most one time
- Run-time complexity is  $O(n+m)$

## DFS algorithm

```
stack.push(s) //start node
repeat until find goal vertex or
stack empty:
    u = stack.pop()
    if !u.visited
        u.visited = true
        (do something while here)
        for v ∈ u.adjacent
            if !v.visited
                stack.push(v)
```

# After DFS (or BFS) *findPath()* finds a path from start to end if it exists

## GraphTraversal.java

```
88= public ArrayList<V> findPath(V start, V end) {
89     //check that DFS or BFS have already been run from start
90     if (backTrack.isEmpty() || !backTrack.containsKey(start) ||
91         (backTrack.containsKey(start) && backTrack.get(start) != null)) {
92         System.out.println("Run DFS or BFS on " + start + " before trying to find a path");
93         return new ArrayList<V>();
94     }
95     System.out.println("Path from " + start + " to " + end);
96     //make sure end vertex in backTrack
97     if (!backTrack.containsKey(end)) {
98         System.out.println("\tNo path found");
99         return new ArrayList<V>();
100    }
101    //start from end vertex and work backward to start vertex
102    ArrayList<V> path = new ArrayList<V>(); //this will hold the path from start to end vertex
103    V current = end; //start at end vertex
104    //loop from end vertex back to start vertex
105    while (current != null) {
106        path.add(0, current); //add this vertex to front of arraylist path
107        current = backTrack.get(current); //get vertex that discovered this vertex
108    }
109    System.out.println(path);
110    return path;
111 }
```

- **Run time complexity?**
- **Length of path from *start to end***

# Agenda

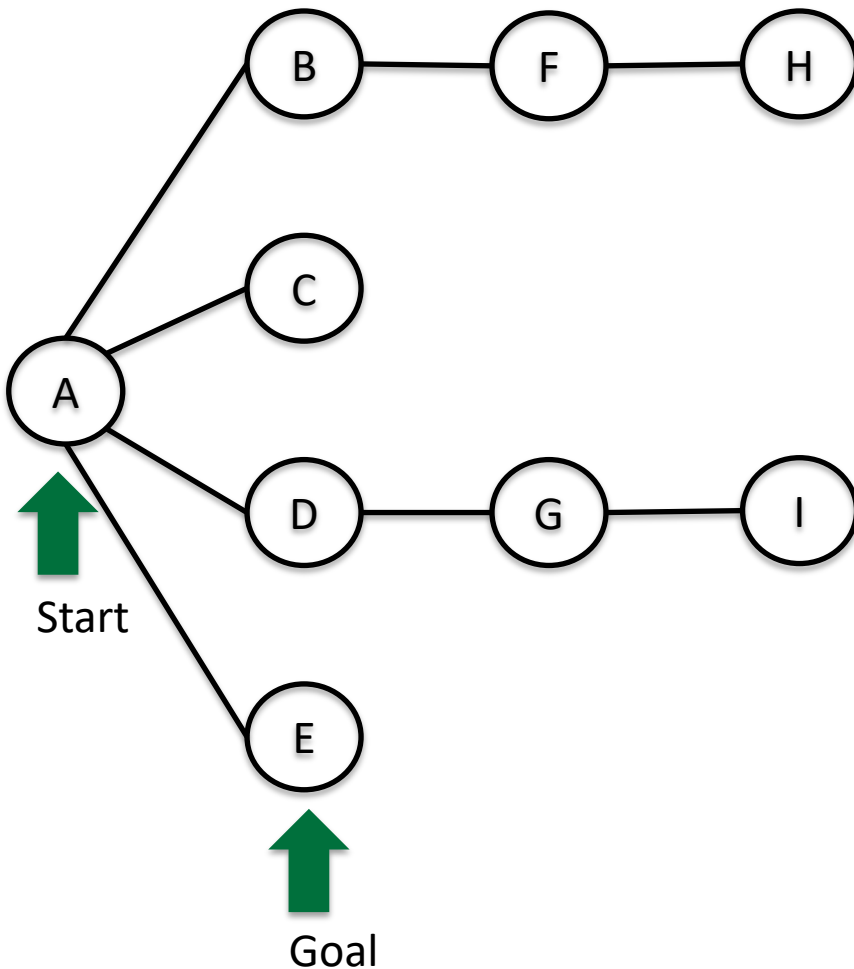
1. Depth first search (DFS)



2. Breadth first search (BFS)

3. Examples from last class and today

# Breadth First Search (BFS) can find the shortest path between nodes



## BFS basic idea

- Explore outward in “ripples”
- Look at all nodes 1 step away, then all nodes 2 steps away...
- Relies on a **Queue** (implicit or explicit) implementation
- Path from  $s$  to any other vertex is shortest



# Some of you did Breadth First Search on Problem Set 1

## RegionFinder

```
Loop over all the pixels
  If a pixel is unvisited and of the correct color
    Start a new region
    Keep track of pixels need to be visited, initially just one
    As long as there's some pixel that needs to be visited
      Get one to visit
      Add it to the region
      Mark it as visited
      Loop over all its neighbors
        If the neighbor is of the correct color
          Add it to the list of pixels to be visited
    If the region is big enough to be worth keeping, do so
```

# Some of you did Breadth First Search on Problem Set 1

## RegionFinder

Loop over all the pixels

    If a pixel is unvisited and of the correct color

        Start a new region

        Keep track of pixels need to be visited, initially just one

        As long as there's some pixel that needs to be visited

            Get one to visit

            Add it to the region

            Mark it as visited

            Loop over all its neighbors

                If the neighbor is of the correct color

**Add it to the list of pixels to be visited**

        If the region is big enough to be worth keeping, do so



**If you added to end of list...**

# Some of you did Breadth First Search on Problem Set 1

## RegionFinder

Loop over all the pixels

    If a pixel is unvisited and of the correct color

        Start a new region

        Keep track of pixels need to be visited, initially just one

        As long as there's some pixel that needs to be visited

**Get one to visit**



**And if you get a pixel from front of list, you implemented a Queue**

            Add it to the region

            Mark it as visited

            Loop over all its neighbors

                If the neighbor is of the correct color

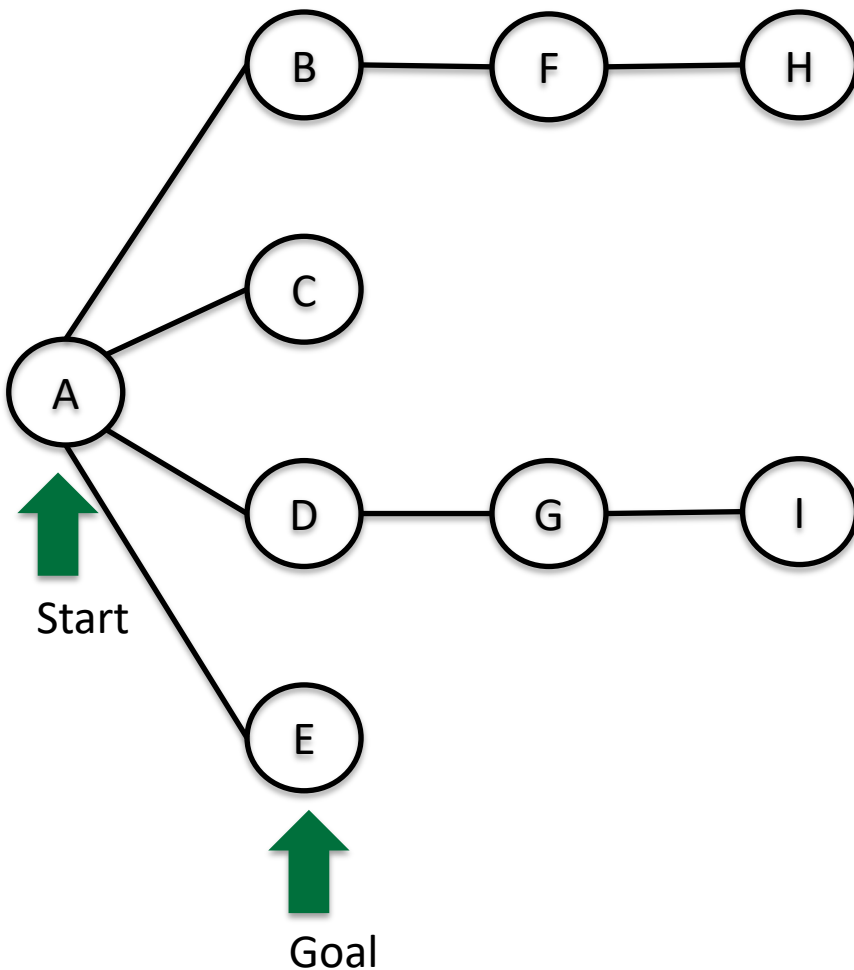
**Add it to the list of pixels to be visited**

        If the region is big enough to be worth keeping, do so



**If you added to end of list...**

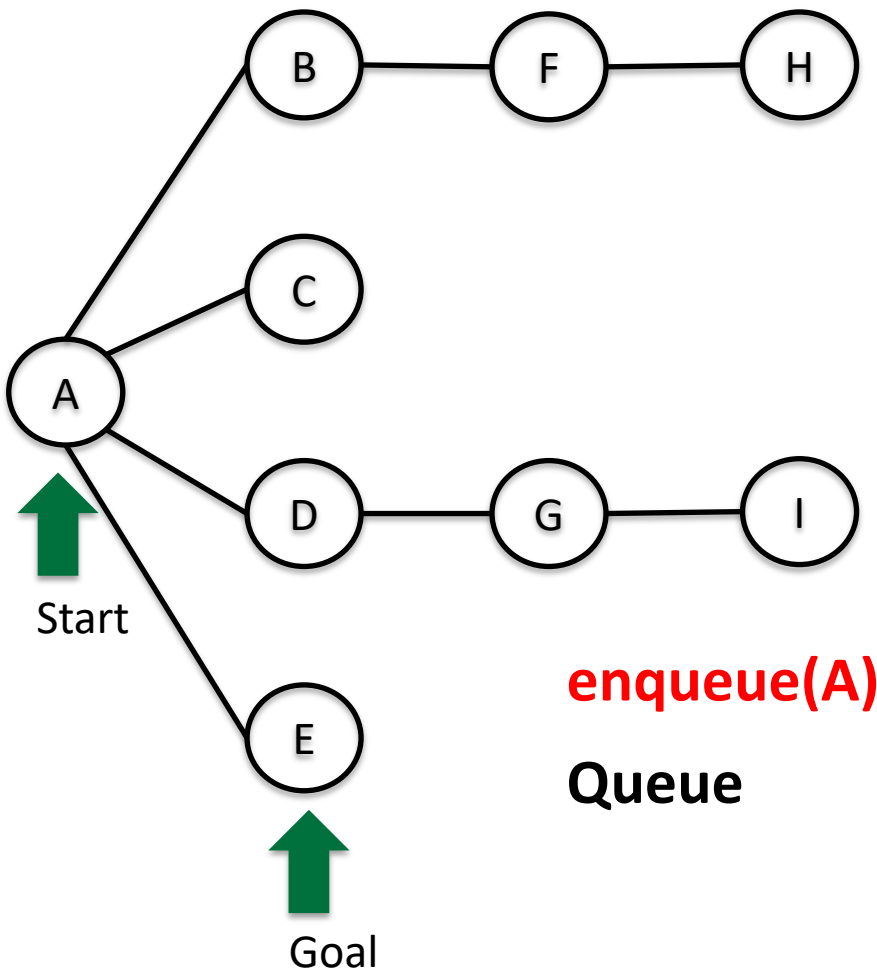
# Breadth First Search (BFS) can find the shortest path between nodes



## BFS algorithm

```
enqueue(s) //start node
s.visited = true
repeat until find goal vertex or
queue empty:
    u = dequeue()
    (do something here)
    for v ∈ u.adjacent
        if !v.visited
            v.visited = true
            enqueue(v)
```

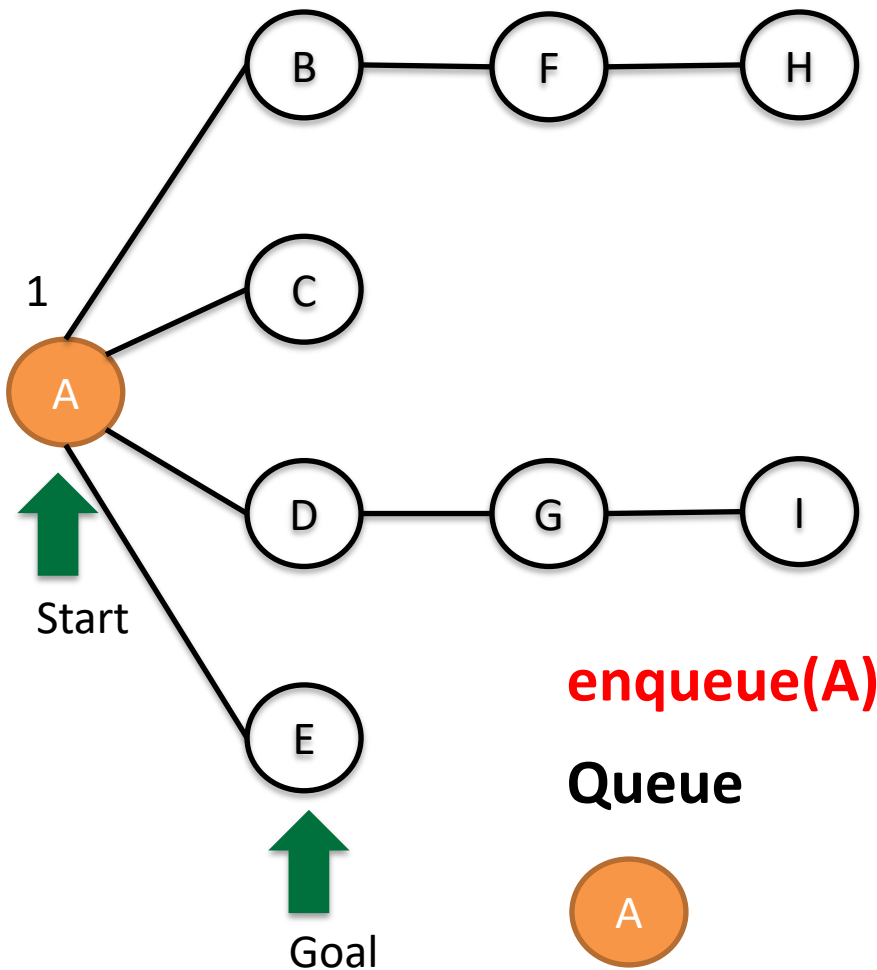
# Breadth First Search (BFS) can find the shortest path between nodes



## BFS algorithm

```
enqueue(s) //start node
s.visited = true
repeat until find goal vertex or
queue empty:
    u = dequeue()
    (do something here)
    for v ∈ u.adjacent
        if !v.visited
            v.visited = true
            enqueue(v)
```

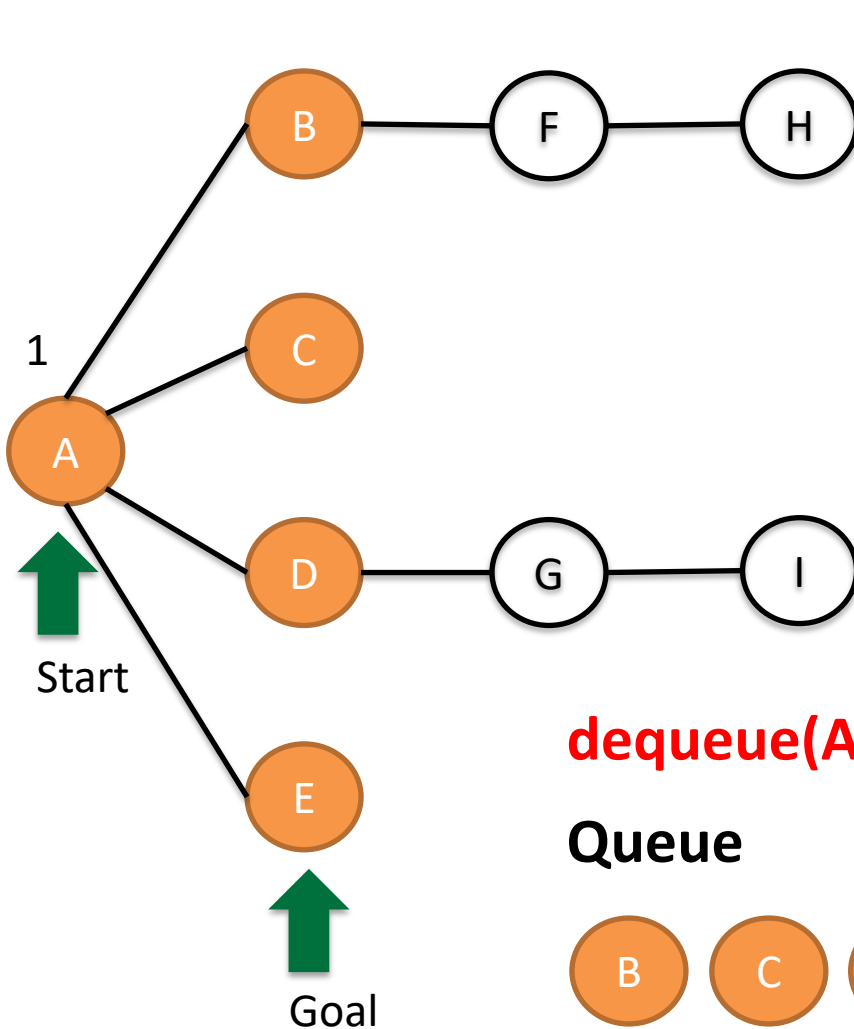
# Breadth First Search (BFS) can find the shortest path between nodes



## BFS algorithm

```
enqueue(s) //start node  
→ s.visited = true  
repeat until find goal vertex or  
queue empty:  
    u = dequeue()  
    (do something here)  
    for v ∈ u.adjacent  
        if !v.visited  
            v.visited = true  
            enqueue(v)
```

# Breadth First Search (BFS) can find the shortest path between nodes



## BFS algorithm

```
enqueue(s) //start node  
s.visited = true  
repeat until find goal vertex or  
queue empty:
```

```
    → u = dequeue()  
    (do something here)  
    for v ∈ u.adjacent  
        if !v.visited  
            v.visited = true  
            enqueue(v)
```

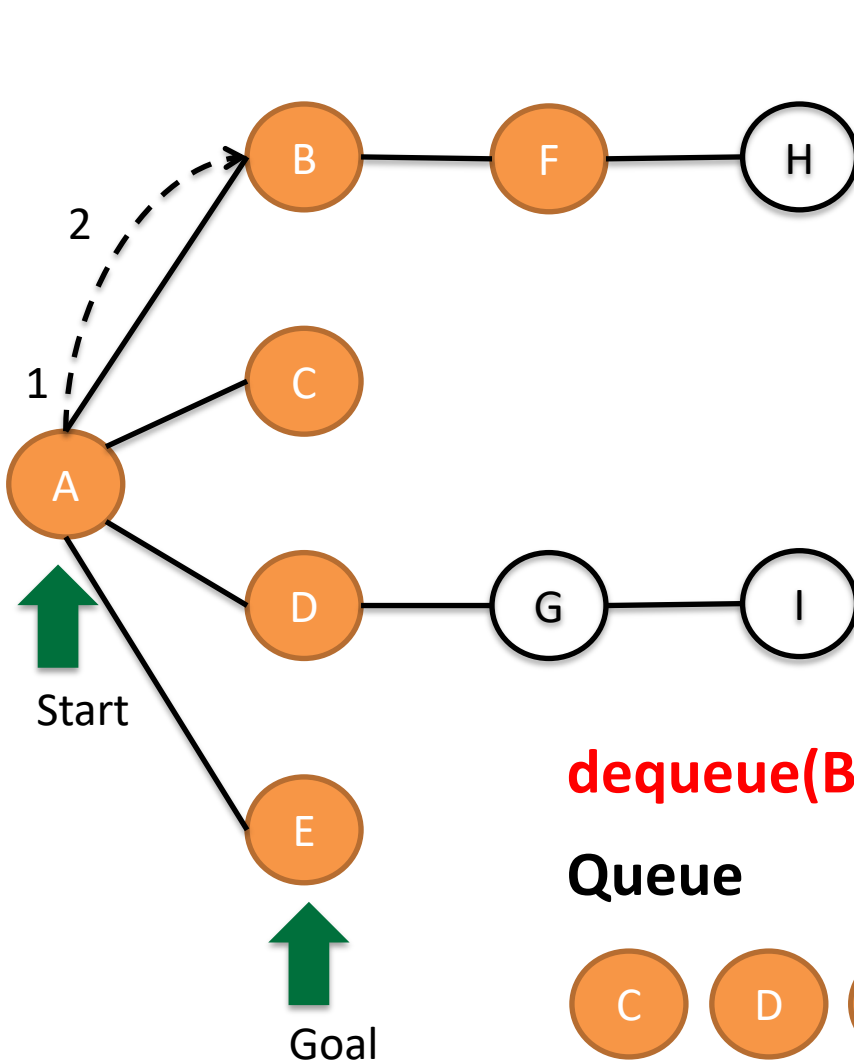
**“Visit” node u**

**dequeue(A), enqueue unvisited adjacent**

Queue



# Breadth First Search (BFS) can find the shortest path between nodes



--> Order nodes visited

## BFS algorithm

```
enqueue(s) //start node  
s.visited = true  
repeat until find goal vertex or  
queue empty:
```

```
    → u = dequeue()  
    (do something here)  
    for v ∈ u.adjacent  
        if !v.visited  
            v.visited = true  
            enqueue(v)
```

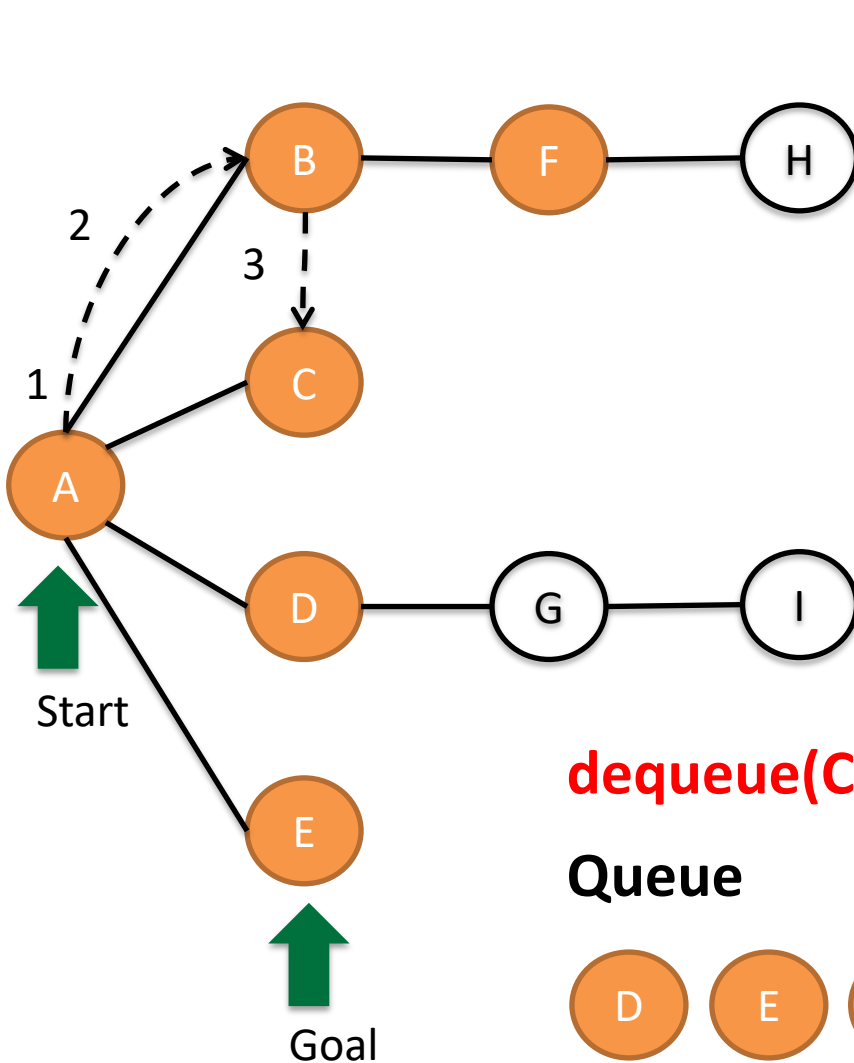
**dequeue(B), enqueue unvisited adjacent F (not A)**

Queue





# Breadth First Search (BFS) can find the shortest path between nodes



## BFS algorithm

```
enqueue(s) //start node  
s.visited = true  
repeat until find goal vertex or  
queue empty:
```

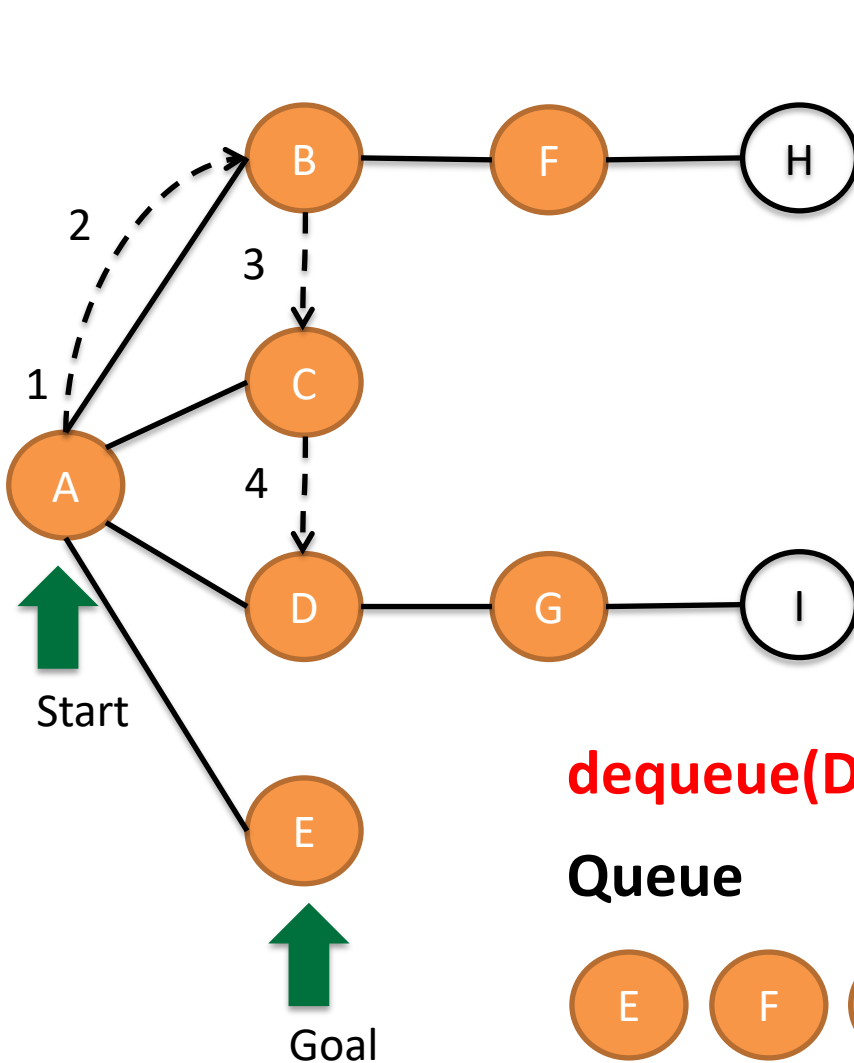
```
    u = dequeue()  
    (do something here)  
    for v ∈ u.adjacent  
        if !v.visited  
            v.visited = true  
            enqueue(v)
```

**dequeue(C), enqueue unvisited adjacent (none)**

Queue



# Breadth First Search (BFS) can find the shortest path between nodes



## BFS algorithm

```
enqueue(s) //start node
s.visited = true
repeat until find goal vertex or
queue empty:
```

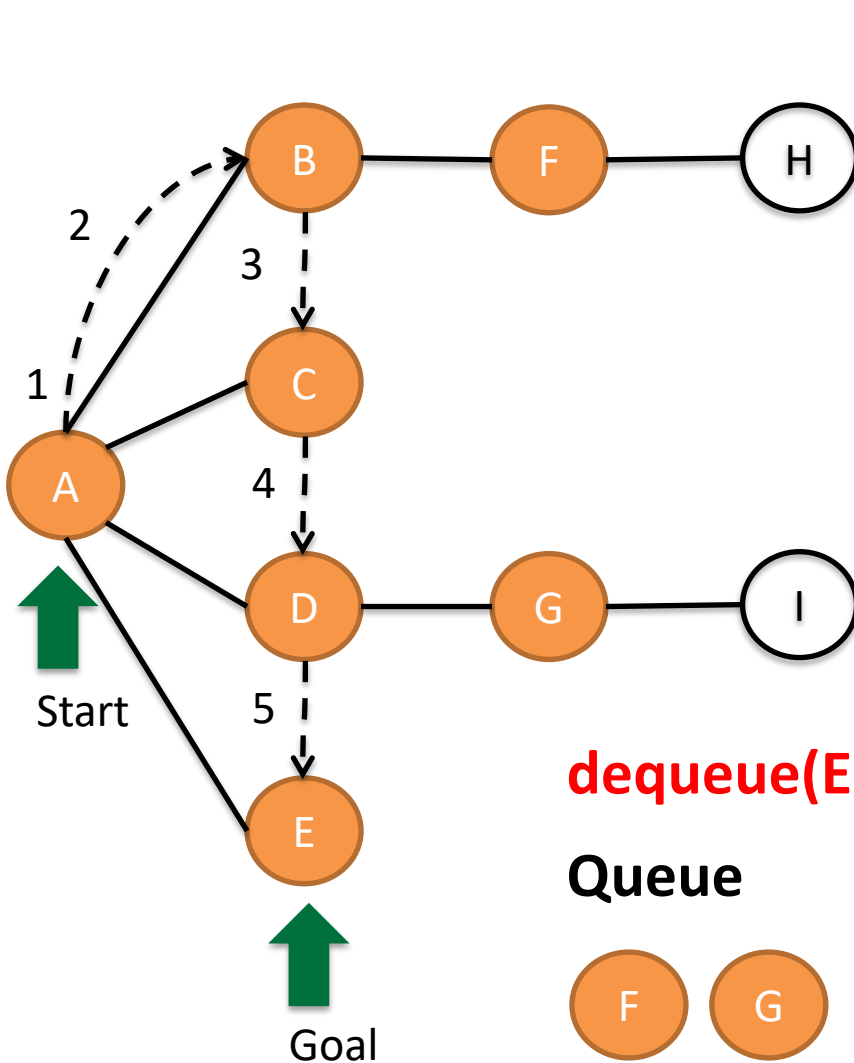
```
    u = dequeue()
    (do something here)
    for v ∈ u.adjacent
        if !v.visited
            v.visited = true
            enqueue(v)
```

**dequeue(D), enqueue unvisited adjacent G (not A)**

Queue



# Breadth First Search (BFS) can find the shortest path between nodes



## BFS algorithm

```
enqueue(s) //start node
s.visited = true
repeat until find goal vertex or
queue empty:
```

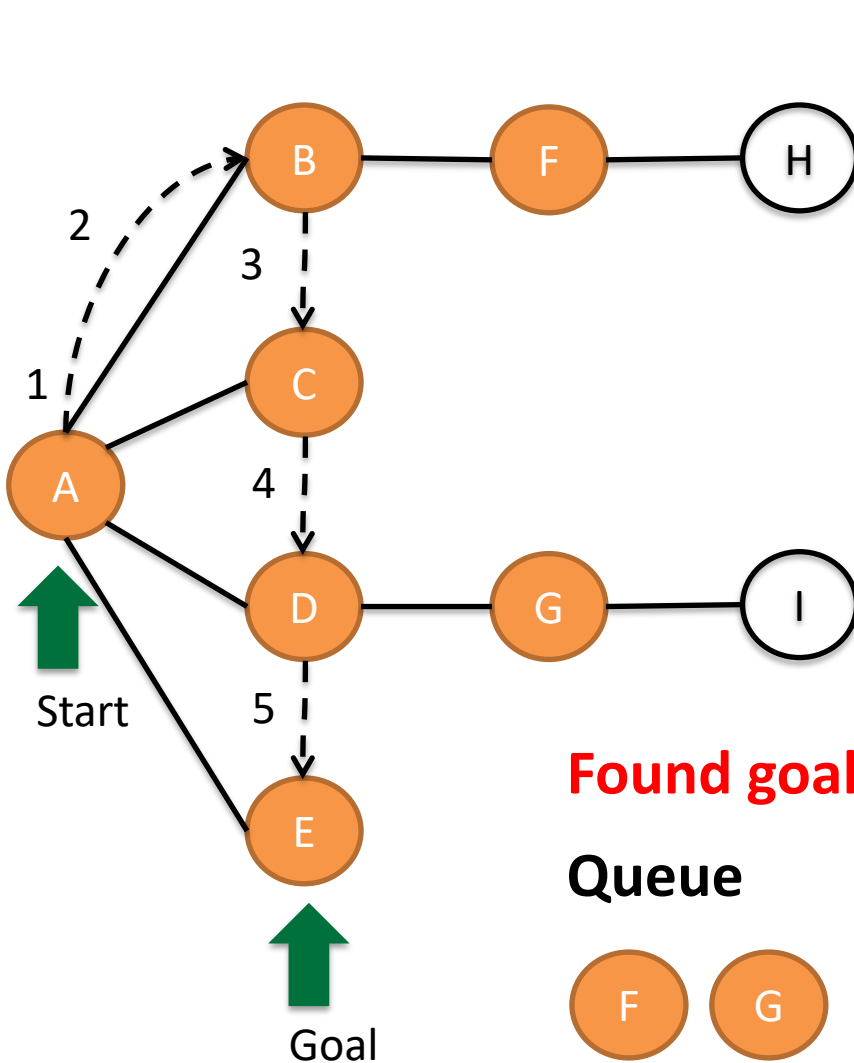
```
    u = dequeue()
    (do something here)
    for v ∈ u.adjacent
        if !v.visited
            v.visited = true
            enqueue(v)
```

**dequeue(E), enqueue unvisited adjacent (none)**

Queue



# Breadth First Search (BFS) can find the shortest path between nodes



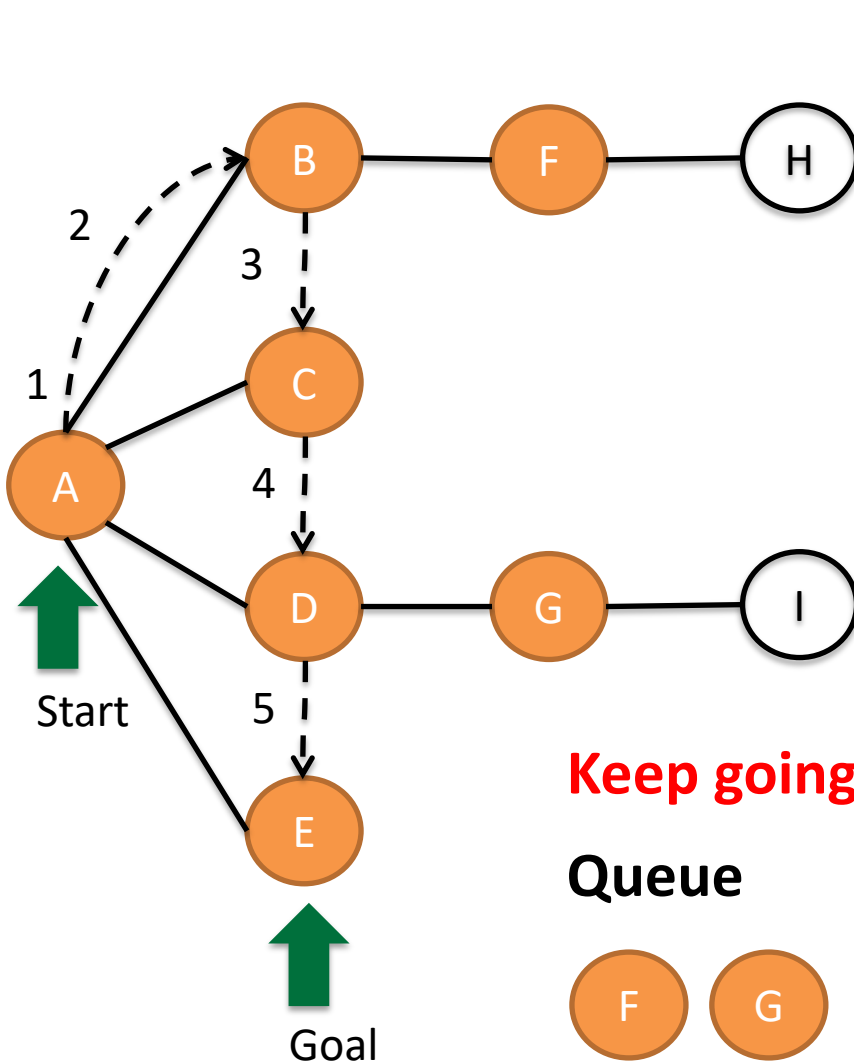
--> Order nodes visited

## BFS algorithm

```
enqueue(s) //start node
s.visited = true
repeat until find goal vertex or
queue empty:
    u = dequeue()
    (do something here)
    for v ∈ u.adjacent
        if !v.visited
            v.visited = true
            enqueue(v)
```

**Found goal! Can stop now**

# Breadth First Search (BFS) can find the shortest path between nodes



--> Order nodes visited

## BFS algorithm

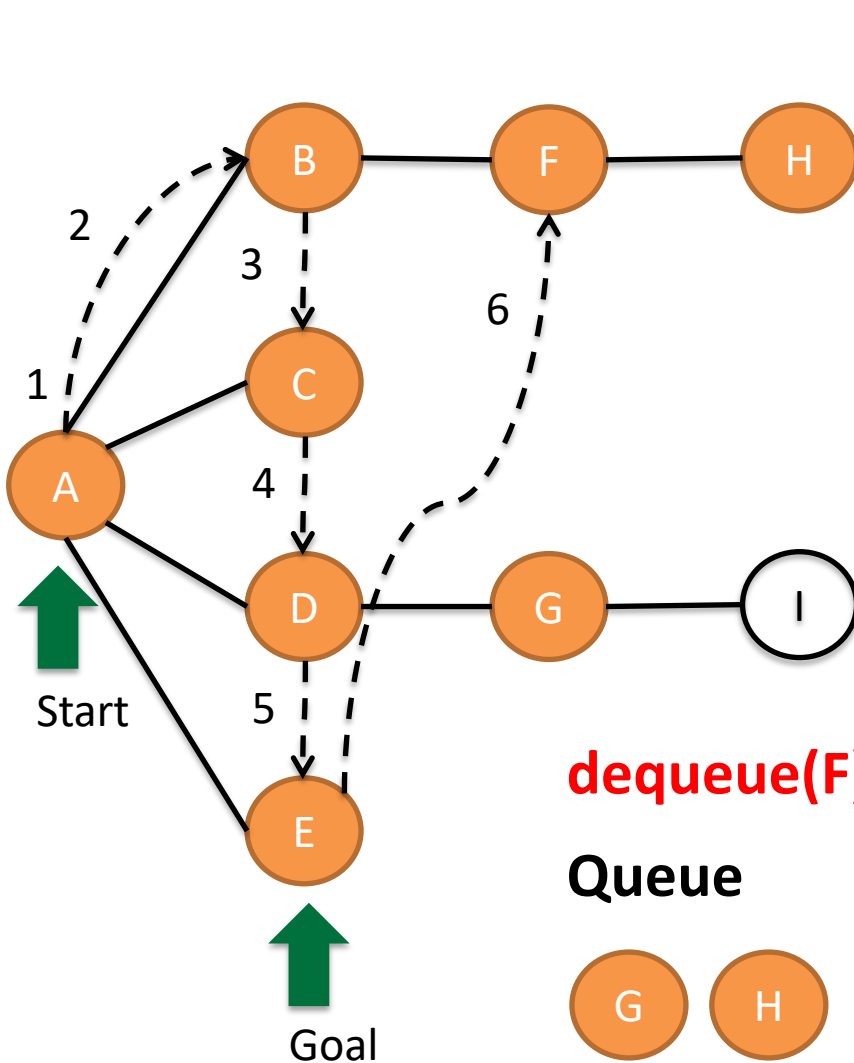
```
enqueue(s) //start node
s.visited = true
repeat until find goal vertex or
queue empty:
    u = dequeue()
    (do something here)
    for v ∈ u.adjacent
        if !v.visited
            v.visited = true
            enqueue(v)
```

**Keep going to further illustrate BFS process**

Queue



# Breadth First Search (BFS) can find the shortest path between nodes



## BFS algorithm

```
enqueue(s) //start node
s.visited = true
repeat until find goal vertex or
queue empty:
```

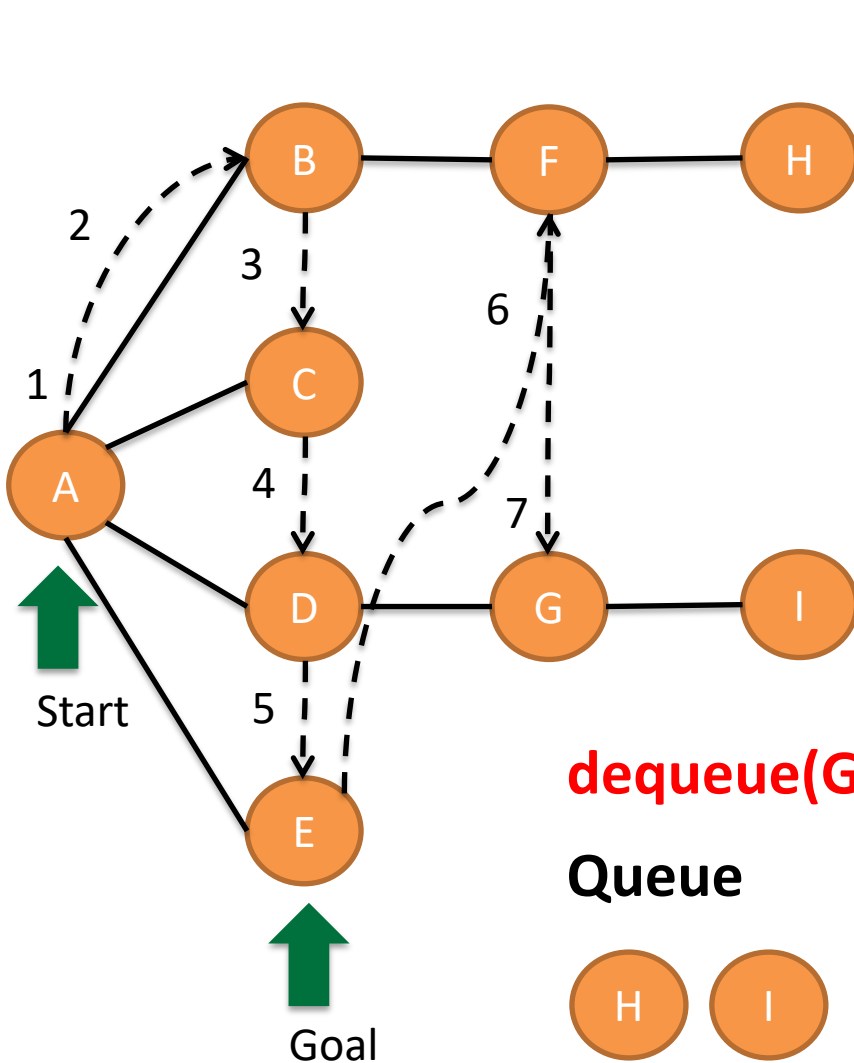
```
    u = dequeue()
    (do something here)
    for v ∈ u.adjacent
        if !v.visited
            v.visited = true
            enqueue(v)
```

**dequeue(F), enqueue unvisited adjacent H**

Queue



# Breadth First Search (BFS) can find the shortest path between nodes



## BFS algorithm

```
enqueue(s) //start node  
s.visited = true  
repeat until find goal vertex or  
queue empty:
```

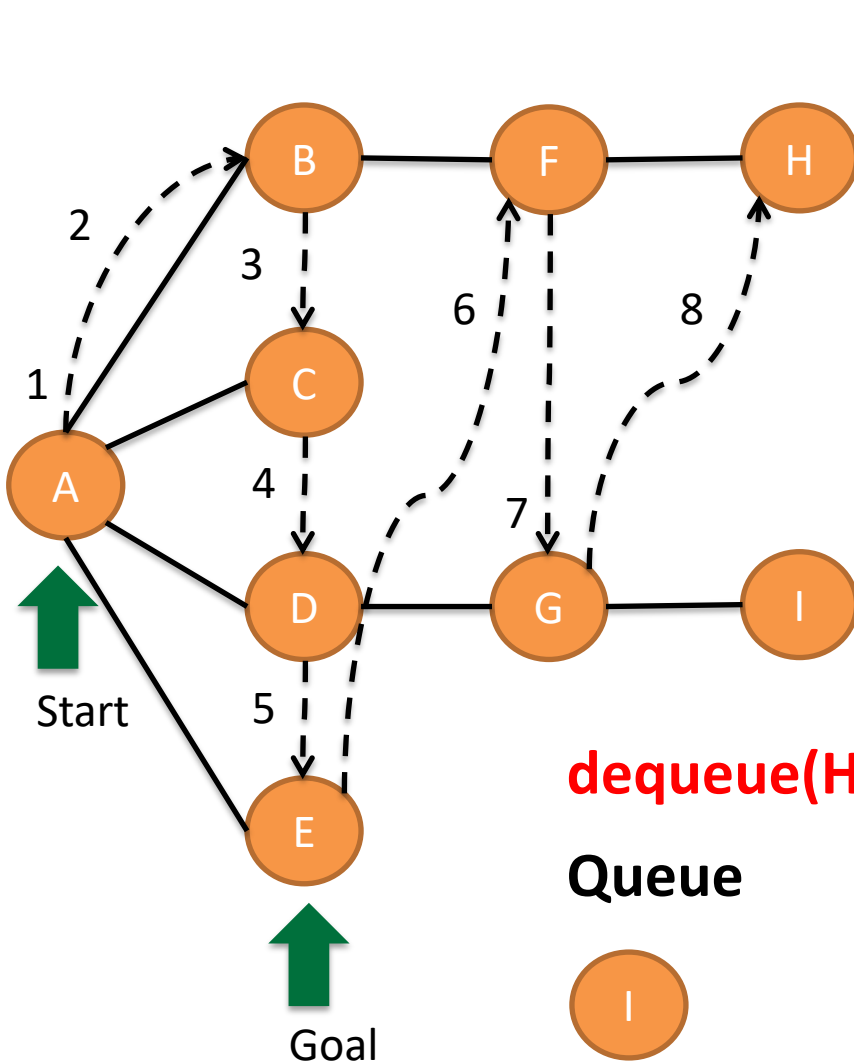
```
    → u = dequeue()  
      (do something here)  
      for v ∈ u.adjacent  
        if !v.visited  
          v.visited = true  
          enqueue(v)
```

**dequeue(G), enqueue unvisited adjacent I**

Queue



# Breadth First Search (BFS) can find the shortest path between nodes



--> Order nodes visited

## BFS algorithm

```
enqueue(s) //start node  
s.visited = true  
repeat until find goal vertex or  
queue empty:
```

```
    → u = dequeue()  
      (do something here)  
    for v ∈ u.adjacent  
      if !v.visited  
        v.visited = true  
        enqueue(v)
```

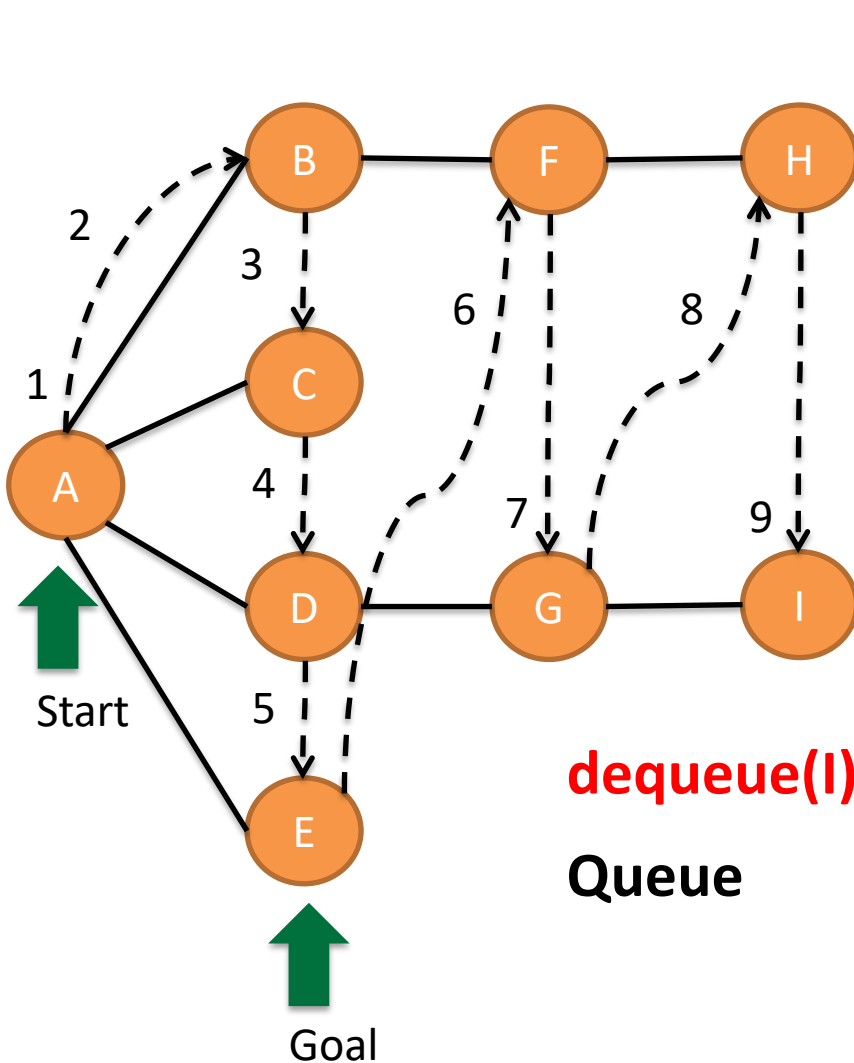
**dequeue(H), enqueue unvisited adjacent (none)**

Queue





# Breadth First Search (BFS) can find the shortest path between nodes



## BFS algorithm

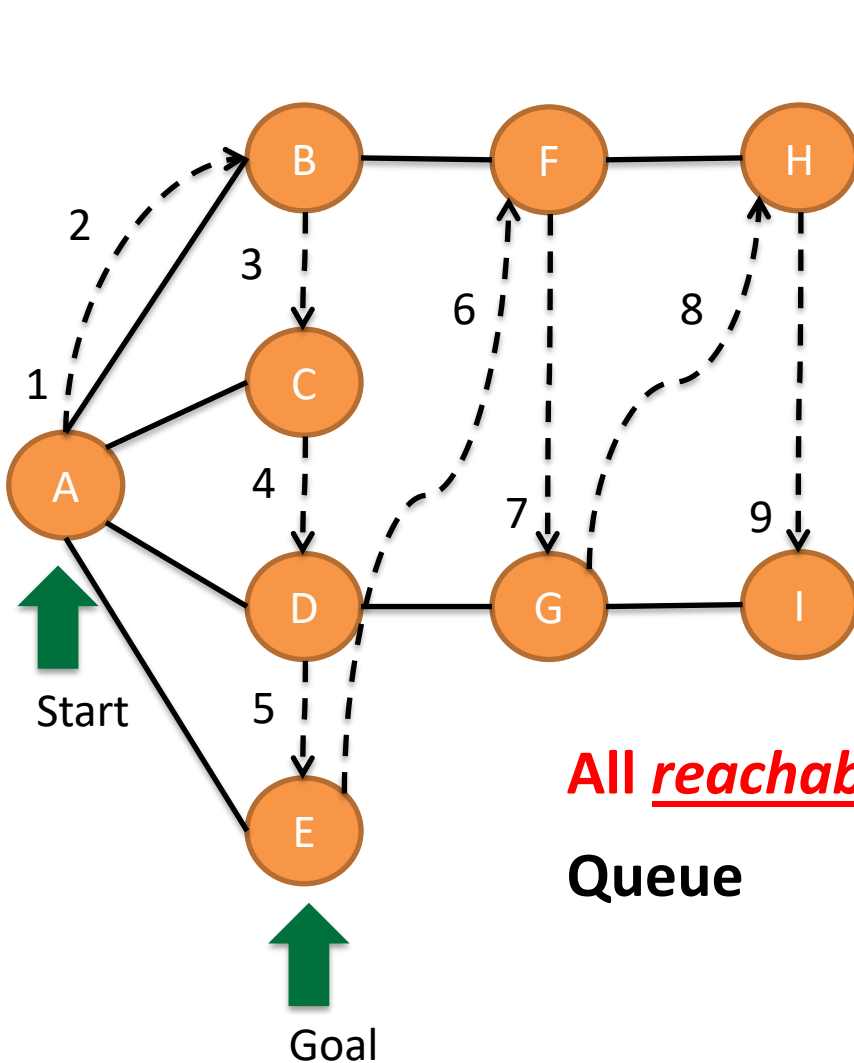
```
enqueue(s) //start node  
s.visited = true  
repeat until find goal vertex or  
queue empty:
```

```
    → u = dequeue()  
      (do something here)  
    for v ∈ u.adjacent  
      if !v.visited  
        v.visited = true  
        enqueue(v)
```

**dequeue(I), enqueue unvisited adjacent (none)**

Queue

# Breadth First Search (BFS) can find the shortest path between nodes



--> Order nodes visited

## BFS algorithm

```
enqueue(s) //start node
s.visited = true
repeat until find goal vertex or
queue empty:
    u = dequeue()
    (do something here)
    for v ∈ u.adjacent
        if !v.visited
            v.visited = true
            enqueue(v)
```

**All reachable nodes from *start* explored**

Queue

# Node discovery tells us something about the graph

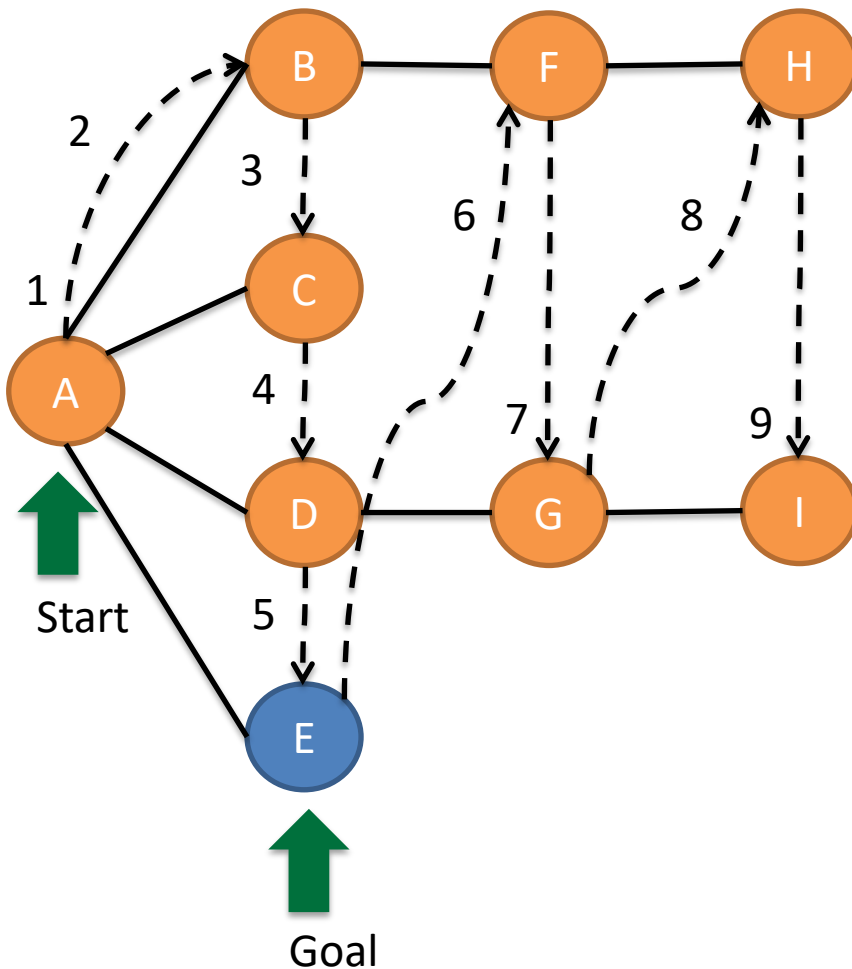
## Discovery edges

- Edges that lead to unvisited nodes called *discovery edges*
- Discovery edges form a tree on the graph (root, no cycles)
- Can traverse from start to goal on tree (if goal reachable)
- Can tell us which nodes are not reachable (not on path formed by discovery edges)
- **Path guaranteed to have smallest number of edges**

## Can track how we got to node to find shortest path

- Keep track of parent vertex
- Parent of each vertex is vertex that discovered it
- Parent is unique because we don't visit vertices twice

# To find path from *start* to *v*, keep track of previous node as nodes are “discovered”



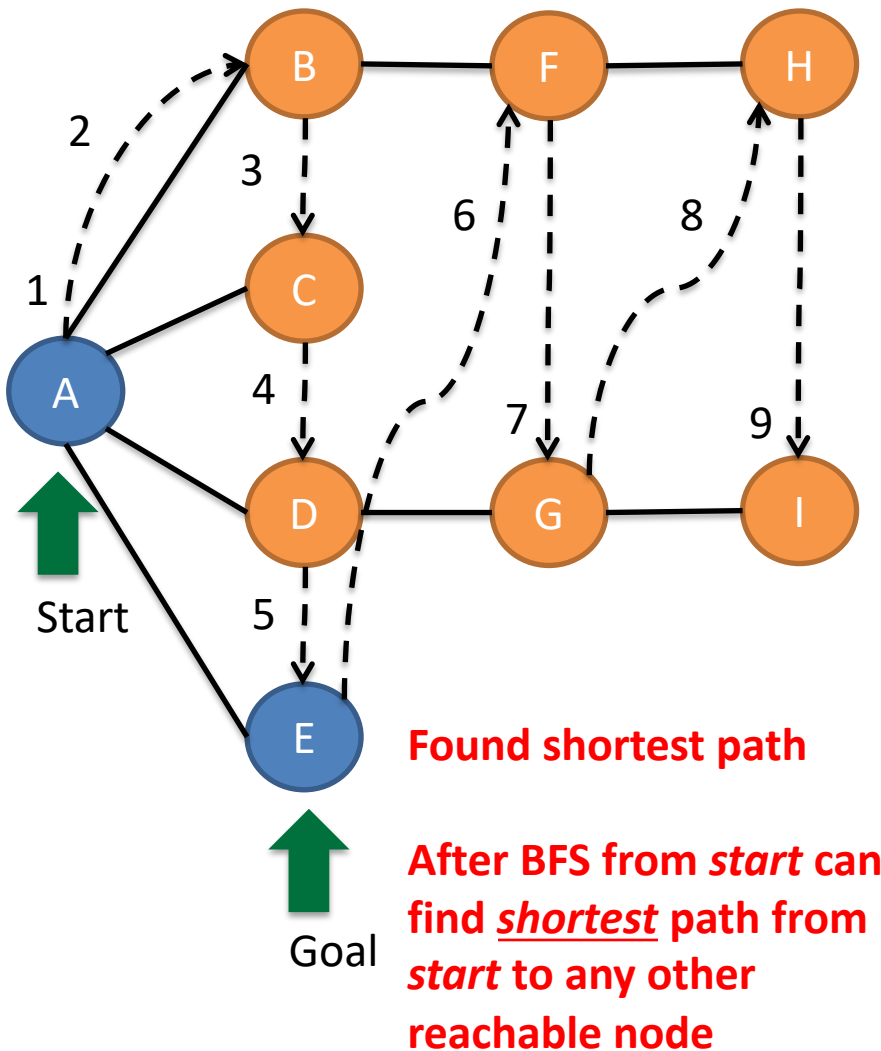
## Path from *start* to *v*

- Each time node discovered, keep pointer to previous
- Could keep Map with node as Key and previous as Value
- Start at goal node (*v*)
- Track backward until find starting node
- **Will find shortest path if it exists**

## Path A to E

Key	Value
A	Null
B	A
C	A
D	A
E	A
F	B
G	D
H	F
I	G

# To find path from *start* to *v*, keep track of previous node as nodes are “discovered”



## Path from *start* to *v*

- Each time node discovered, keep pointer to previous
- Could keep Map with node as Key and previous as Value
- Start at goal node (*v*)
- Track backward until find starting node
- **Will find shortest path if it exists**

## Path A to E

Key	Value
A	Null
B	A
C	A
D	A
<b>E</b>	<b>A</b>
F	B
G	D
H	F
I	G

Path  
A,E

# BFS run-time complexity is $O(n+m)$

## Run time

- Assume graph with  $n$  nodes and  $m$  edges
- Visit each node at most one time (due to visited indicator)
- Visit each edge at most one time
- Run-time complexity  $O(n+m)$

## BFS algorithm

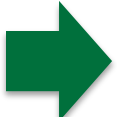
```
enqueue(s) //start node
s.visited = true
repeat until find goal vertex or
queue empty:
    u = dequeue()
    (do something here)
    for v ∈ u.adjacent
        if !v.visited
            v.visited = true
            enqueue(v)
```

# GraphTraversal.java: BFS code

## GraphTraversal.java

```
60 public void BFS(AdjacencyMapGraph<V,E> G, V start) {
61     System.out.println("\nBreadth First Search from " + start);
62     backTrack = new HashMap<V,V>(); //initialize backTrack
63     backTrack.put(start, null); //load start vertex with null parent
64     Set<V> visited = new HashSet<V>(); //Set to track which vertices have already been visited
65     Queue<V> queue = new LinkedList<V>(); //queue to implement BFS
66
67     queue.add(start); //enqueue start vertex
68     visited.add(start); //add start to visited Set
69     while (!queue.isEmpty()) { //loop until no more vertices
70         V u = queue.remove(); //dequeue
71         for (V v : G.outNeighbors(u)) { //loop over out neighbors
72             if (!visited.contains(v)) { //if neighbor not visited, then neighbor is discovered
73                 visited.add(v); //add neighbor to visited Set
74                 queue.add(v); //enqueue neighbor
75                 backTrack.put(v, u); //save that this vertex was discovered from prior vertex
76             }
77         }
78     }
79 }
```

# Agenda

1. Depth first search (DFS)
2. Breadth first search (BFS)
-  3. Examples from last class and today

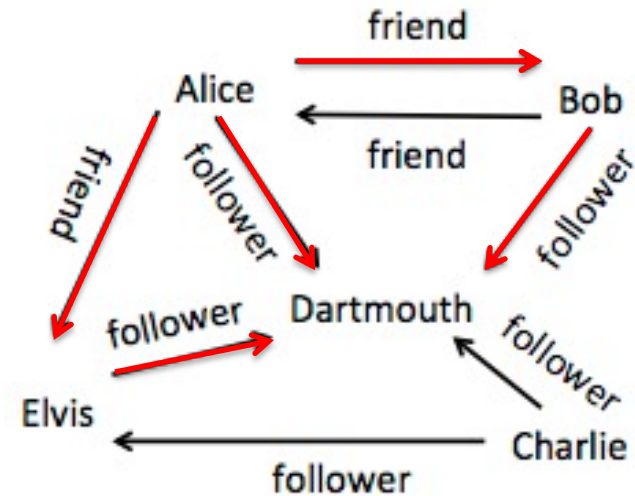


# GraphTraversal.java: DFS and BFS on graph we looked at last class

```

113 public static void main(String[] args) {
114     //set up graph from class introducing Graphs
115     GraphTraversal<String,String> GT = new GraphTraversal<String,String>();
116     AdjacencyMapGraph<String,String> g = new AdjacencyMapGraph<String,String>();
117     g.insertVertex("Alice");
118     g.insertVertex("Bob");
119     g.insertVertex("Charlie");
120     g.insertVertex("Dartmouth");
121     g.insertVertex("Elvis");
122     g.insertDirected("Alice", "Dartmouth", "follower");
123     g.insertDirected("Bob", "Dartmouth", "follower");
124     g.insertDirected("Charlie", "Dartmouth", "follower");
125     g.insertDirected("Elvis", "Dartmouth", "follower");
126     g.insertUndirected("Alice", "Bob", "friend"); // symmetric, undirected edge
127     g.insertDirected("Alice", "Elvis", "friend"); // not symmetric, directed edge!
128     g.insertDirected("Charlie", "Elvis", "follower");
129
130     //run DFS from Alice
131     GT.DFS(g, "Alice");
132     //find path from start to end
133     GT.findPath("Bob", "Dartmouth"); //DFS wasn't run from Bob, should reject this
134     GT.findPath("Alice", "Dartmouth");
135     GT.findPath("Alice", "Charlie");
136     GT.findPath("Alice", "Alice");
137
138     //run BFS
139     GT.BFS(g, "Alice");
140
141     //find path from start to end
142     GT.findPath("Alice", "Dartmouth");
143     GT.findPath("Alice", "Charlie");
144

```



Key	Value
Alice	Null
Bob	Alice
Dartmouth	Elvis
Elvis	Alice

```

Problems  Javadoc  Declaration  Console  Debug  Expressions  Error Log  Call Hierarchy
<terminated> GraphTraversal [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 4:31:06 PM)
Depth First Search from Alice
Run DFS or BFS on Bob before trying to find a path
Path from Alice to Dartmouth
[Alice, Elvis, Dartmouth]
Path from Alice to Charlie
No path found
Path from Alice to Alice
[Alice]

Breadth First Search from Alice
Path from Alice to Dartmouth
[Alice, Dartmouth]
Path from Alice to Charlie
No path found

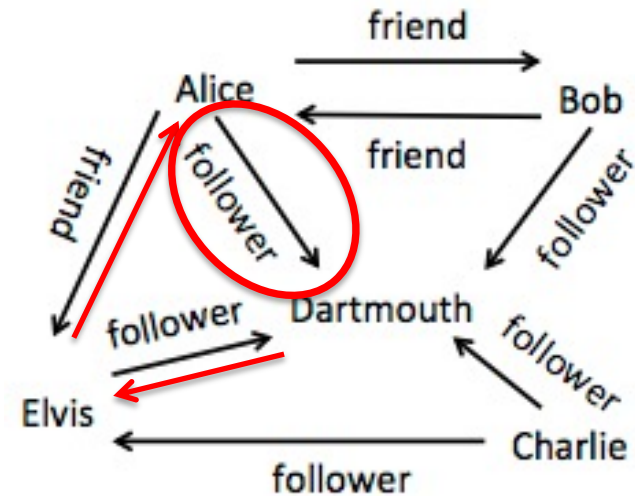
```

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119     g.insertVertex("Charlie");
120     g.insertVertex("Dartmouth");
121     g.insertVertex("Elvis");
122     g.insertDirected("Alice", "Dartmouth", "follower");
123     g.insertDirected("Bob", "Dartmouth", "follower");
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125     g.insertDirected("Elvis", "Dartmouth", "follower");
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134     GT.findPath("Alice", "Dartmouth");
135     GT.findPath("Alice", "Charlie");
136     GT.findPath("Alice", "Alice");
137
138     //run BFS
139     GT.BFS(g, "Alice");
140
141     //find path from start to end
142     GT.findPath("Alice", "Dartmouth");
143     GT.findPath("Alice", "Charlie");
144

```



Key	Value
Alice	Null
Bob	Alice
Dartmouth	Elvis
Elvis	Alice

```

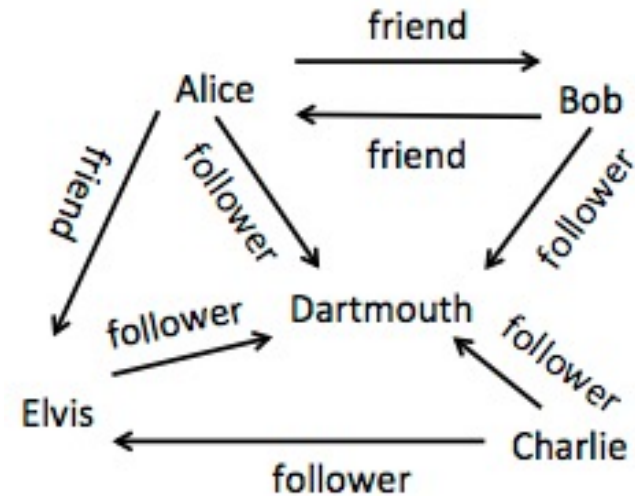
Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy
<terminated> GraphTraversal [Java Application] /Library/Java/JavaVirtualMachines/dk1.8.0_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 4:31:06 PM)
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[Alice, Elvis, Dartmouth]
Path from Alice to Charlie
No path found
Path from Alice to Alice
[Alice]

Breadth First Search from Alice
Path from Alice to Dartmouth
[Alice, Dartmouth]
Path from Alice to Charlie
No path found

```

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119     g.insertVertex("Charlie");
120     g.insertVertex("Dartmouth");
121     g.insertVertex("Elvis");
122     g.insertDirected("Alice", "Dartmouth", "follower");
123     g.insertDirected("Bob", "Dartmouth", "follower");
124     g.insertDirected("Charlie", "Dartmouth", "follower");
125     g.insertDirected("Elvis", "Dartmouth", "follower");
126     g.insertUndirected("Alice", "Bob", "friend"); // symmetric, undirected edge
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130     //run DFS from Alice
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136     GT.findPath("Alice", "Alice");
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138     //run BFS
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144 }
```



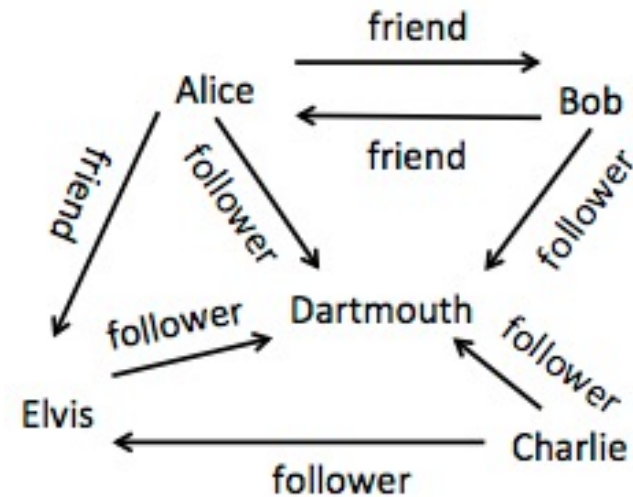
Key	Value
Alice	Null
Bob	Alice
Dartmouth	Elvis
Elvis	Alice

```
Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy
<terminated> GraphTraversal [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 4:31:06 PM)
Depth First Search from Alice
Run DFS or BFS on Bob before trying to find a path
Path from Alice to Dartmouth
[Alice, Elvis, Dartmouth]
Path from Alice to Charlie
No path found
Path from Alice to Alice
[Alice]

Breadth First Search from Alice
Path from Alice to Dartmouth
[Alice, Dartmouth]
Path from Alice to Charlie
No path found
```

# GraphTraversal.java: DFS and BFS on graph we looked at last class

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113 public static void main(String[] args) {
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115     GraphTraversal<String,String> GT = new GraphTraversal<String,String>();
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143     GT.findPath("Alice", "Charlie");
144 }
```



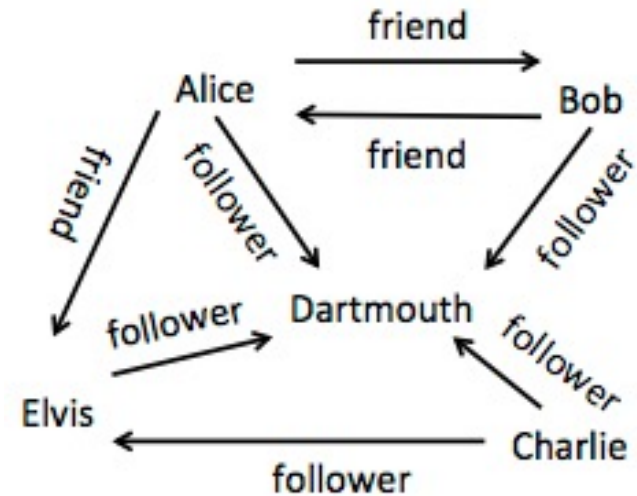
Key	Value
Alice	Null
Bob	Alice
Dartmouth	Elvis
Elvis	Alice

```
Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy
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Breadth First Search from Alice
Path from Alice to Dartmouth
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Path from Alice to Charlie
No path found
```

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144 }
```



Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy  
c-terminated- GraphTraversal [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0\_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 4:31:06 PM)

Depth First Search from Alice  
Run DFS or BFS on Bob before trying to find a path

Path from Alice to Dartmouth

[Alice, Elvis, Dartmouth]

Path from Alice to Charlie

No path found

Path from Alice to Alice

[Alice]

Breadth First Search from Alice

Path from Alice to Dartmouth

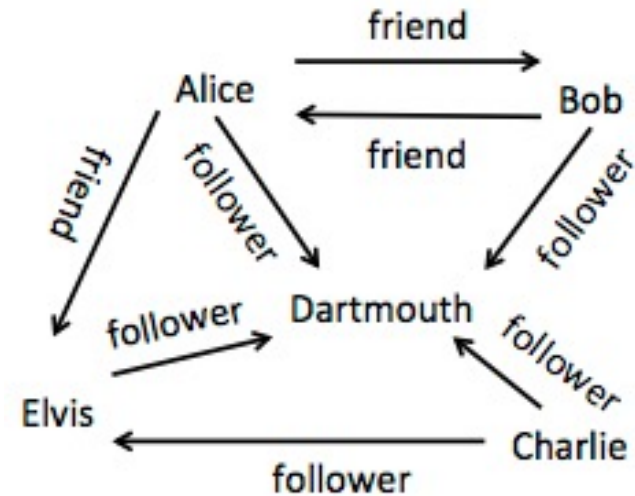
[Alice, Dartmouth]

Path from Alice to Charlie

No path found

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143     GT.findPath("Alice", "Charlie");
144 }
```



Key	Value
Alice	Null
Bob	Alice
Dartmouth	Alice
Elvis	Alice

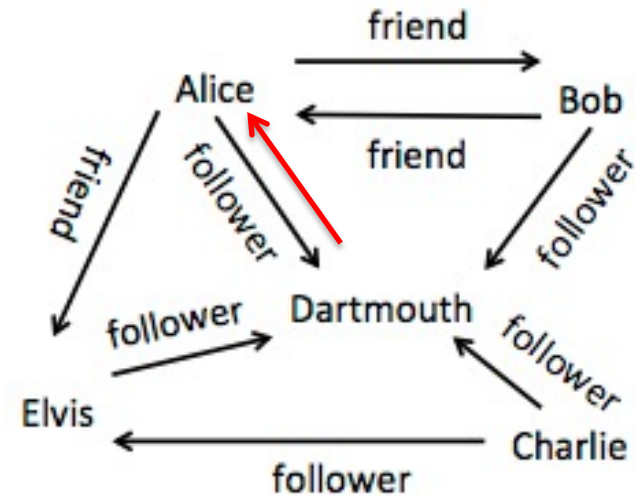
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c-terminated- GraphTraversal [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0\_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 4:31:06 PM)

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No path found
Path from Alice to Alice
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Breadth First Search from Alice
Path from Alice to Dartmouth
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```



Key	Value
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Bob	Alice
Dartmouth	Alice
Elvis	Alice

```
Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy
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```

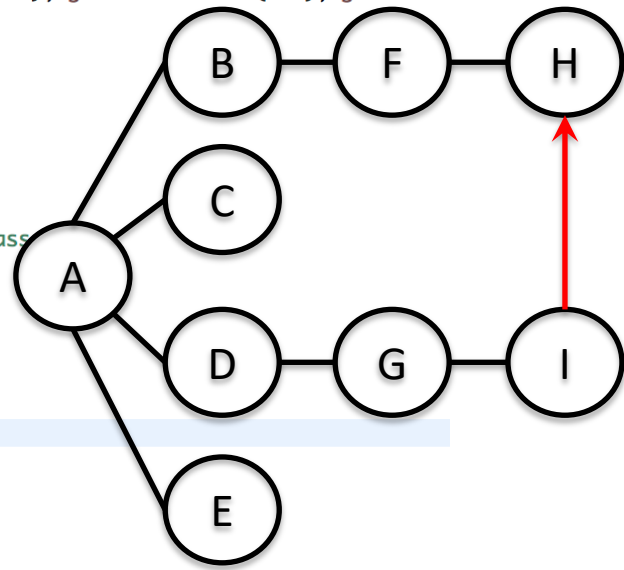
# DFS on today's graph

## GraphTraversal.java

```

145 //set up graph from Graph Traversal class
146 AdjacencyMapGraph<String,String> g2 = new AdjacencyMapGraph<String,String>();
147 g2.insertVertex("A"); g2.insertVertex("B"); g2.insertVertex("C"); g2.insertVertex("D");
148 g2.insertVertex("E"); g2.insertVertex("F"); g2.insertVertex("G"); g2.insertVertex("H"); g2.insert
149 g2.insertUndirected("A", "B", "");
150 g2.insertUndirected("B", "F", "");
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154 g2.insertUndirected("D", "G", "");
155 g2.insertUndirected("G", "I", "");
156 g2.insertUndirected("A", "E", "");
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162
163 //run BFS from A and find path to H
164 GT.BFS(g2,"A");
165 GT.findPath("A", "B");
166
167 }
168
169 }
170

```



### DFS

Key	Value
A	Null
B	F
C	A
D	A
E	A
F	H
G	D
H	I
I	G



Depth First Search from A  
 Path from A to B  
 [A, D, G, I, H, F, B]

Breadth First Search from A  
 Path from A to B  
 [A, B]



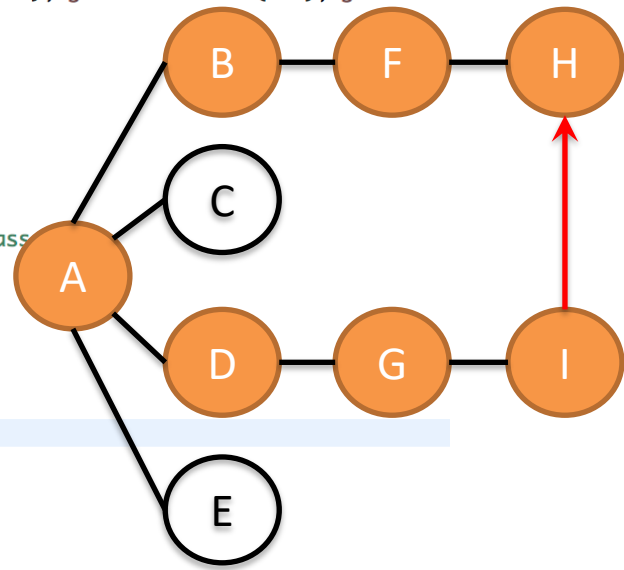
# DFS on today's graph

## GraphTraversal.java

```

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167 }
168
169 }
170

```



### DFS

Key	Value
A	Null
B	F
C	A
D	A
E	A
F	H
G	D
H	I
I	G



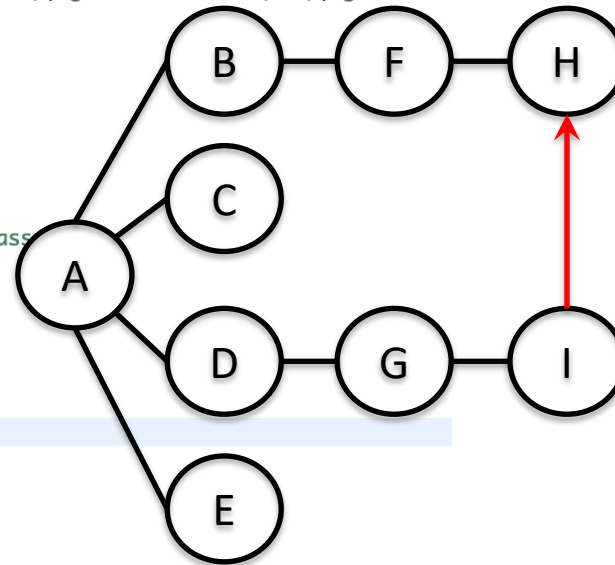
Depth First Search from A  
 Path from A to B  
 [A, D, G, I, H, F, B]

Breadth First Search from A  
 Path from A to B  
 [A, B]

# BFS on today's graph

## GraphTraversal.java

```
145 //set up graph from Graph Traversal class
146 AdjacencyMapGraph<String,String> g2 = new AdjacencyMapGraph<String,String>();
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153 g2.insertUndirected("A", "D", "");
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166
167 }
168
169 }
170
```



**BFS**

Key	Value
A	Null
<b>B</b>	<b>A</b>
C	A
D	A
E	A
F	B
G	D
H	F
I	G

Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy  
<terminated> GraphTraversal [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0\_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 5:14:16 PM)

Depth First Search from A  
Path from A to B  
[A, D, G, I, H, F, B]

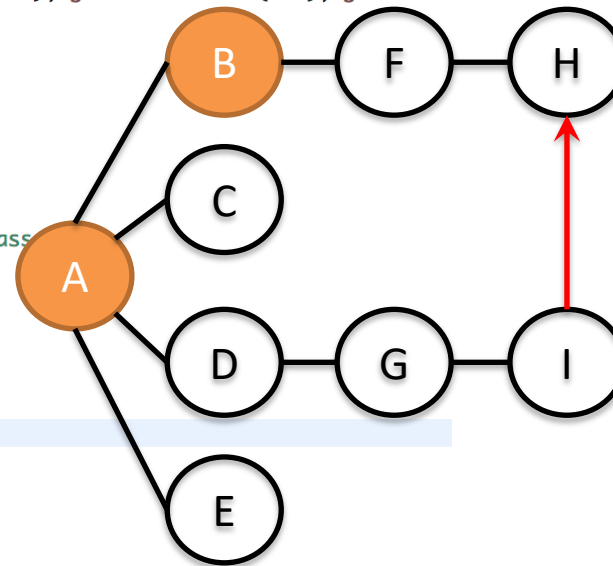
Breadth First Search from A  
Path from A to B  
[A, B]

# BFS on today's graph

## GraphTraversal.java

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165 GT.findPath("A", "B");
166
167 }
168
169 }
170
```

- Run BFS
- Find path A to B



### BFS

Key	Value
A	Null
<b>B</b>	<b>A</b>
C	A
D	A
E	A
F	B
G	D
H	F
I	G

Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy  
<terminated> GraphTraversal [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0\_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 5:14:16 PM)

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Path from A to B  
[A, D, G, I, H, F, B]

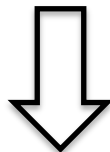
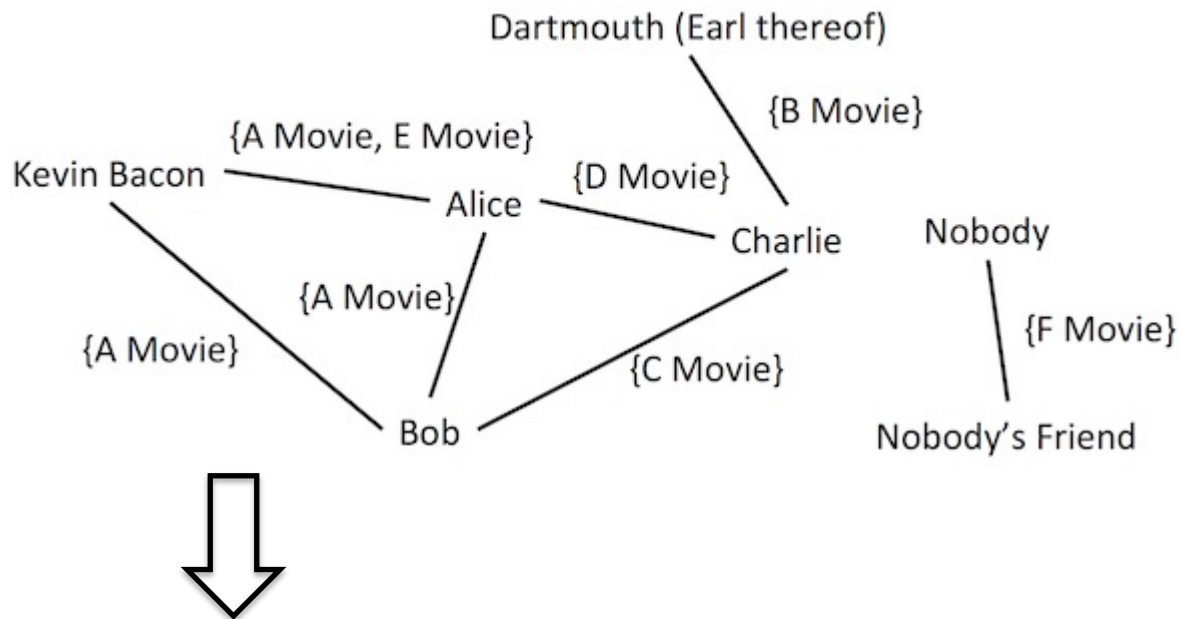
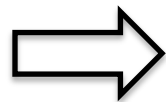
Breadth First Search from A  
Path from A to B  
[A, B]

# PS-4

actors.txt  
...  
18|Loretta Devine  
19|Whitney Houston

movies.txt  
...  
1|Toy Story (1995)  
2|Jumanji (1995)  
...

movie-actors.txt  
...  
1|2  
1|3  
...



```
Run: BaconGame x
Commands:
c <#>: list top (positive number) or bottom (negative) <#> centers of the universe, sorted by average separation
d <low> <high>: list actors sorted by degree, with degree between low and high (inclusive)
i: list actors with infinite separation from the current center
p <name>: find path from <name> to current center of the universe
s <low> <high>: list actors sorted by non-infinite separation from the current center, with separation between low and high (inclusive)
u <name>: make <name> the center of the universe
q: quit game

Kevin Bacon is now the center of the acting universe, connected to [redacted] /9235 actors with average separation [redacted]

Kevin Bacon game >
```

# Summary

- To find paths from a start to another node
  - DFS uses stack to find paths from a start node to another node
  - BFS uses queue
- Both have a complexity of  $O(n+m)$  when keeping track of visited nodes
- Backtrack map to keep track of paths

# Next

- Graph traversals that use cost

# Additional Resources

# GRAPH TRAVERSAL NOTES



# Graph traversals are useful to answer questions about relationships

## Some Graph traversals uses

- Uses are often around *reachability*
- Computing *path* from vertex  $u$  to vertex  $v$
- Given start vertex  $s$  of Graph  $G$ , compute a path with the minimum number of edges between  $s$  and all other vertices (or report no such path exists)
- Testing whether  $G$  is fully connected (e.g., all vertices reachable)
- Identifying *cycles* in  $G$  (or reporting no cycle exists)
- Today's examples have ~~no~~ few cycles (next class)

# DFS Node discovery tells us something about the graph

## Discovery edges

- Edges that lead to unvisited nodes called *discovery edges*
- Discovery edges form a tree on the graph (root, no cycles)
- Can traverse from start to goal on tree (if goal reachable)
- Can tell us which nodes are not reachable (not on path formed by discovery edges)
- **With DFS, path not guaranteed to be shortest path!**

## Back, cross, and forward edges

- Edges that lead to previously discovered nodes
- Back edges lead to ancestor nodes, forward edges to descendants, cross edges to non-ancestor or descendant
- These edges indicate presence of a cycle in the Graph
- Today's focus on graphs without cycles

# BFS Node discovery tells us something about the graph

## Discovery edges

- Edges that lead to unvisited nodes called *discovery edges*
- Discovery edges form a tree on the graph (root, no cycles)
- Can traverse from start to goal on tree (if goal reachable)
- Can tell us which nodes are not reachable (not on path formed by discovery edges)
- **Path guaranteed to have smallest number of edges**

## Can track how we got to node to find shortest path

- Keep track of parent vertex
- Parent of each vertex is vertex that discovered it
- Parent is unique because we don't visit vertices twice

GraphTraversal.java: DFS

# **ANNOTATED SLIDES**

# GraphTraversal.java: DFS code

```
17 public class GraphTraversal<V,E> {
18
19     public Map<V,V> backTrack; //keep track of prior vertex when v
20
21     /**
22      * Constructor. Initialize backTrack to new HashMap.
23      */
24     public GraphTraversal() {
25         backTrack = new HashMap<V,V>();
26     }
27
28     /**
29      * Depth First Search
30      * @param G -- graph to search
31      * @param start -- starting vertex
32      */
33     public void DFS(AdjacencyMapGraph<V,E> G, V start) {
34         System.out.println("\nDepth First Search from " + start);
35         backTrack = new HashMap<V,V>(); //initialize backTrack
36         backTrack.put(start, null); //load start node with null pa
37         Set<V> visited = new HashSet<V>(); //Set to track which ve
38         Stack<V> stack = new Stack<V>(); //stack to implement DFS
39
40         stack.push(start); //push start vertex
41         while (!stack.isEmpty()) { //loop until no more vertices
42             V u = stack.pop(); //get most recent vertex
43             if (!visited.contains(u)) { //if not already visited
44                 visited.add(u); //add to visited Set
45                 for (V v : G.outNeighbors(u)) { //loop over out ne
46                     if (!visited.contains(v)) { //if neighbor not
47                         stack.push(v); //push non-visited neighbor
48                         backTrack.put(v, u); //save that this vert
49                     }
50                 }
51             }
52         }
53     }
54 }
```

- When running DFS (or BFS), keep track of prior vertex when a vertex is discovered
- Map Key is current vertex, Value is prior vertex

DFS – given Graph *G* and *start* vertex

- Use *Set* to track visited vertices
- Use *Stack* to track vertices to visit
- Follow pseudo code from previous slides
- Add vertex to *backTrack* when discovered
- Only discovered vertices are added, non-reachable vertices not added to *backTrack*

After DFS can get from *start* to any reachable node in Graph using *backTrack*

GraphTraversal.java: findPath

# **ANNOTATED SLIDES**

# After DFS (or BFS) *findPath()* finds a path from start to end if it exists

## GraphTraversal.java

```
88= public ArrayList<V> findPath(V start, V end) {
89     //check that DFS or BFS have already been run from start
90     if (backTrack.isEmpty() || !backTrack.containsKey(start) ||
91         (backTrack.containsKey(start) && backTrack.get(start) != null)) {
92         System.out.println("Run DFS or BFS on " + start + " before trying to find a path");
93         return new ArrayList<V>();
94     }
95     System.out.println("Path from " + start + " to " + end);
96     //make sure end vertex in backTrack
97     if (!backTrack.containsKey(end)) {
98         System.out.println("\tNo path found");
99         return new ArrayList<V>();
100    }
101    //start from end vertex and work backward to start vertex
102    ArrayList<V> path = new ArrayList<V>(); //this will hold the path from start to end vertex
103    V current = end; //start at end vertex
104    //loop from end vertex back to start vertex
105    while (current != null) {
106        path.add(0, current); //add this vertex to front of arraylist path
107        current = backTrack.get(current); //get vertex that discovered this vertex
108    }
109    System.out.println(path);
110    return path;
111 }
112
```

Make sure DFS or BFS has been previously run from *start* vertex

Make sure *end* vertex reachable

- Run time complexity?
- Length of path from *start* to *end*

- Loop backward from *end* to *start* (parent null)
- Add new vertices to front of *path*
- Return *path* when done

GraphTraversal.java: BFS

# **ANNOTATED SLIDES**



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- Loop backward from *end* to *start* (parent null)
- Add new vertices to front of *path*
- Return *path* when done

GraphTraversal.java: From example of last class

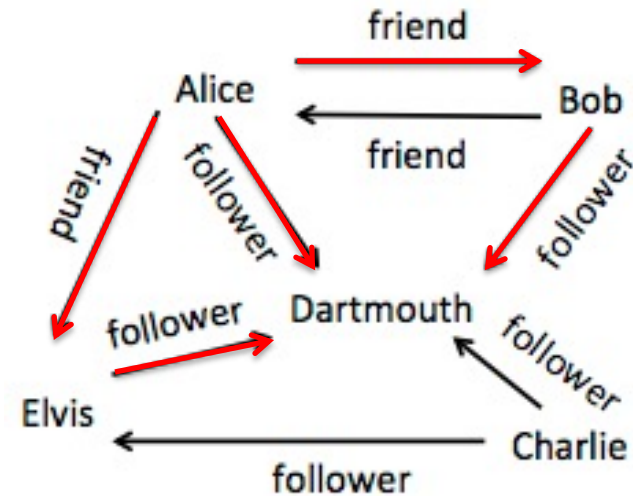
# **ANNOTATED SLIDES**

# GraphTraversal.java: DFS and BFS on graph we looked at last class

```
113 public static void main(String[] args) {
114     //set up graph from class introducing Graphs
115     GraphTraversal<String,String> GT = new GraphTraversal<String,String>();
116     AdjacencyMapGraph<String,String> g = new AdjacencyMapGraph<String,String>();
117     g.insertVertex("Alice");
118     g.insertVertex("Bob");
119     g.insertVertex("Charlie");
120     g.insertVertex("Dartmouth");
121     g.insertVertex("Elvis");
122     g.insertDirected("Alice", "Dartmouth", "follower");
123     g.insertDirected("Bob", "Dartmouth", "follower");
124     g.insertDirected("Charlie", "Dartmouth", "follower");
125     g.insertDirected("Elvis", "Dartmouth", "follower");
126     g.insertUndirected("Alice", "Bob", "friend"); // symmetric, undirected edge
127     g.insertDirected("Alice", "Elvis", "friend"); // not symmetric, directed edge!
128     g.insertDirected("Charlie", "Elvis", "follower");
129
130     //run DFS from Alice
131     GT.DFS(g, "Alice");
132     //find path from start to end
133     GT.findPath("Bob", "Dartmouth"); //DFS wasn't run from Bob, should reject this
134     GT.findPath("Alice", "Dartmouth");
135     GT.findPath("Alice", "Charlie");
136     GT.findPath("Alice", "Alice");
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138     //run BFS
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140
141     //find path from start to end
142     GT.findPath("Alice", "Dartmouth");
143     GT.findPath("Alice", "Charlie");
144 }
```

Create graph

- Run DFS with start=Alice
- Find paths from Alice



Key	Value
Alice	Null
Bob	Alice
Dartmouth	Elvis
Elvis	Alice

```
Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy
<terminated> GraphTraversal [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 4:31:06 PM)
Depth First Search from Alice
Run DFS or BFS on Bob before trying to find a path
Path from Alice to Dartmouth
[Alice, Elvis, Dartmouth]
Path from Alice to Charlie
No path found
Path from Alice to Alice
[Alice]

Breadth First Search from Alice
Path from Alice to Dartmouth
[Alice, Dartmouth]
Path from Alice to Charlie
No path found
```

findPath("Bob", "Dartmouth")

DFS not run from Bob

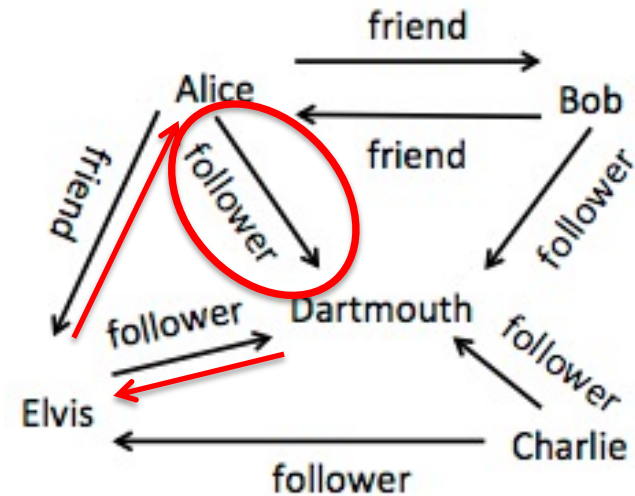
DFS was run from Alice

So do not do this search (check findPath code)

# GraphTraversal.java: DFS and BFS on graph we looked at last class

```
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114     //set up graph from class introducing Graphs
115     GraphTraversal<String,String> GT = new GraphTraversal<String,String>();
116     AdjacencyMapGraph<String,String> g = new AdjacencyMapGraph<String,String>();
117     g.insertVertex("Alice");
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124     g.insertDirected("Charlie", "Dartmouth", "follower");
125     g.insertDirected("Elvis", "Dartmouth", "follower");
126     g.insertUndirected("Alice", "Bob", "friend"); // symmetric, undirected edge
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129
130     //run DFS from Alice
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135     GT.findPath("Alice", "Charlie");
136     GT.findPath("Alice", "Alice");
137
138     //run BFS
139     GT.BFS(g, "Alice");
140
141     //find path from start to end
142     GT.findPath("Alice", "Dartmouth");
143     GT.findPath("Alice", "Charlie");
144 }
```

Create graph



- Run DFS with start=Alice
- Find paths from Alice

Key	Value
Alice	Null
Bob	Alice
Dartmouth	Elvis
Elvis	Alice

```
Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy
<terminated> GraphTraversal (Java Application) /Library/Java/JavaVirtualMachines/jdk1.8.0_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 4:31:06 PM)
Depth First Search from Alice
Run DFS or BFS on Bob before trying to find a path
Path from Alice to Dartmouth
[Alice, Elvis, Dartmouth]
Path from Alice to Charlie
No path found
Path from Alice to Alice
[Alice]

Breadth First Search from Alice
Path from Alice to Dartmouth
[Alice, Dartmouth]
Path from Alice to Charlie
No path found
```

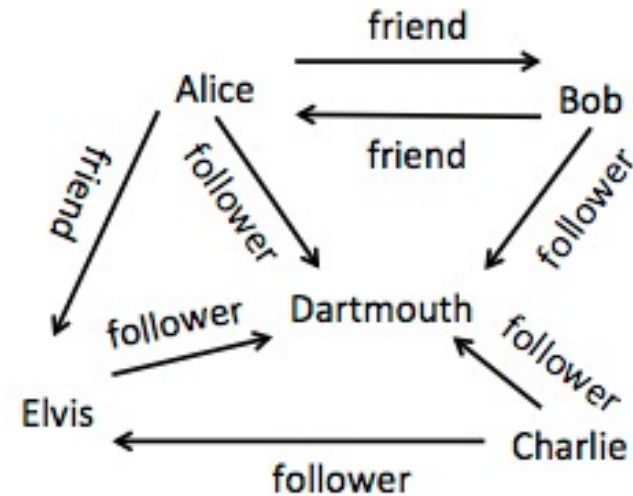
**findPath("Alice", "Dartmouth") finds path  
Alice->Elvis->Dartmouth  
Path yes, but not shortest path  
Shortest is Alice->Dartmouth**

# GraphTraversal.java: DFS and BFS on graph we looked at last class

```
113 public static void main(String[] args) {
114     //set up graph from class introducing Graphs
115     GraphTraversal<String,String> GT = new GraphTraversal<String,String>();
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144 }
```

Create graph

- Run DFS with start=Alice
- Find paths from Alice



Key	Value
Alice	Null
Bob	Alice
Dartmouth	Elvis
Elvis	Alice

Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy

```
Run DFS or BFS on Bob before trying to find a path
Path from Alice to Dartmouth
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Breadth First Search from Alice
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Path from Alice to Charlie
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```

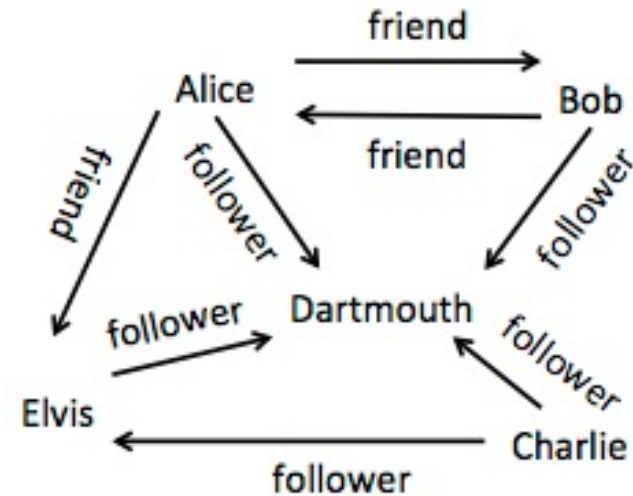
Alice can't reach Charlie in this graph  
Charlie is not in *backTrack*

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144 }
```

Create graph

- Run DFS with start=Alice
- Find paths from Alice



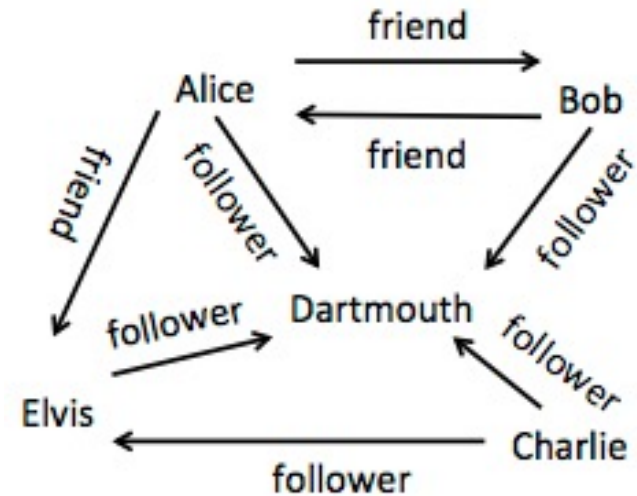
Key	Value
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```
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Breadth First Search from Alice
Path from Alice to Dartmouth
[Alice, Dartmouth]
Path from Alice to Charlie
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```

Alice can reach herself

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- Run BFS start=Alice

Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy  
<terminated> GraphTraversal [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0\_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 4:31:06 PM)

Depth First Search from Alice

Run DFS or BFS on Bob before trying to find a path

Path from Alice to Dartmouth

[Alice, Elvis, Dartmouth]

Path from Alice to Charlie

No path found

Path from Alice to Alice

[Alice]

Breadth First Search from Alice

Path from Alice to Dartmouth

[Alice, Dartmouth]

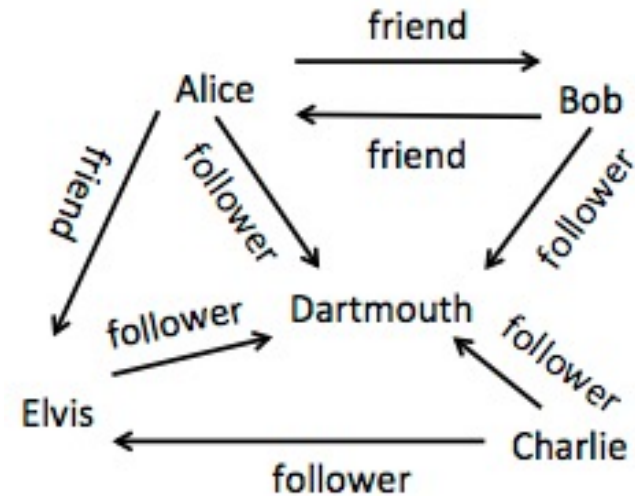
Path from Alice to Charlie

No path found

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```

- Run BFS start=Alice



Key	Value
Alice	Null
Bob	Alice
Dartmouth	Alice
Elvis	Alice

```
Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy
<terminated> GraphTraversal [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 4:31:06 PM)
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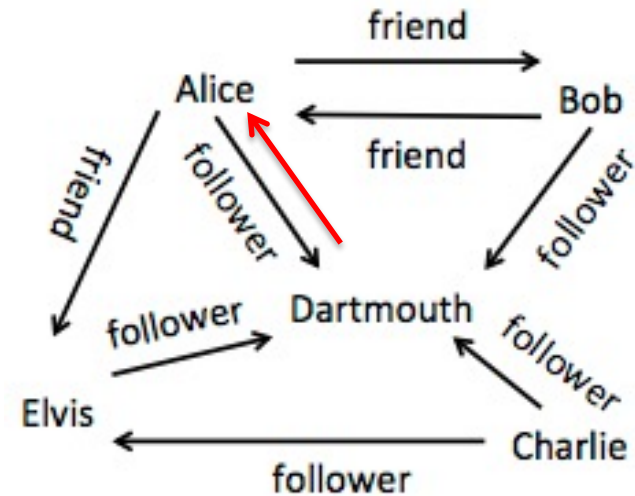


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144

```



- Run BFS start=Alice
- Find paths from Alice

Key	Value
Alice	Null
Bob	Alice
Dartmouth	Alice
Elvis	Alice

```

Problems  Javadoc  Declaration  Console  Debug  Expressions  Error Log  Call Hierarchy
<terminated> GraphTraversal (Java Application) /Library/Java/JavaVirtualMachines/jdk1.8.0_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 4:31:06 PM)
Depth First Search from Alice

```

```

Run DFS or BFS on Bob before trying to find a path
Path from Alice to Dartmouth
[Alice, Elvis, Dartmouth]
Path from Alice to Charlie
No path found
Path from Alice to Alice
[Alice]

Breadth First Search from Alice
Path from Alice to Dartmouth
[Alice, Dartmouth]
Path from Alice to Charlie
No path found

```

**BFS**  
**findPath("Alice", "Dartmouth") finds shortest path**  
**Alice->Dartmouth (DFS went through Elvis before Dartmouth)**

GraphTraversal.java: From today's graph

# **ANNOTATED SLIDES**

# DFS on today's graph

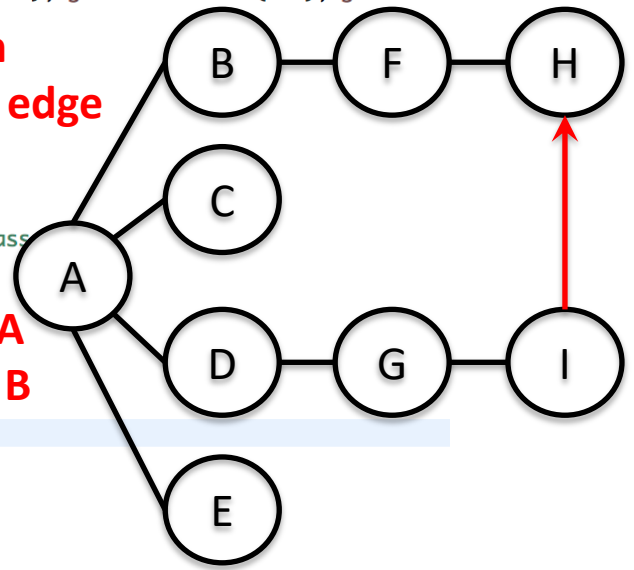
## GraphTraversal.java

```

145 //set up graph from Graph Traversal class
146 AdjacencyMapGraph<String,String> g2 = new AdjacencyMapGraph<String,String>();
147 g2.insertVertex("A"); g2.insertVertex("B"); g2.insertVertex("C"); g2.insertVertex("D");
148 g2.insertVertex("E"); g2.insertVertex("F"); g2.insertVertex("G"); g2.insertVertex("H"); g2.insert
149 g2.insertUndirected("A", "B", "");
150 g2.insertUndirected("B", "F", "");
151 g2.insertUndirected("F", "H", "");
152 g2.insertUndirected("A", "C", "");
153 g2.insertUndirected("A", "D", "");
154 g2.insertUndirected("D", "G", "");
155 g2.insertUndirected("G", "I", "");
156 g2.insertUndirected("A", "E", "");
157 g2.insertDirected("I", "H", ""); //directed edge not from class
158
159 //run DFS from A and find path to H
160 GT.DFS(g2,"A");
161 GT.findPath("A", "B");
162
163 //run BFS from A and find path to H
164 GT.BFS(g2,"A");
165 GT.findPath("A", "B");
166
167 }
168
169 }
170

```

- Create graph
- Added extra edge
- Run DFS from A
- Find path A to B



### DFS

Key	Value
A	Null
B	F
C	A
D	A
E	A
F	H
G	D
H	I
I	G



Depth First Search from A  
 Path from A to B  
 [A, D, G, I, H, F, B]

Breadth First Search from A  
 Path from A to B  
 [A, B]

# DFS on today's graph

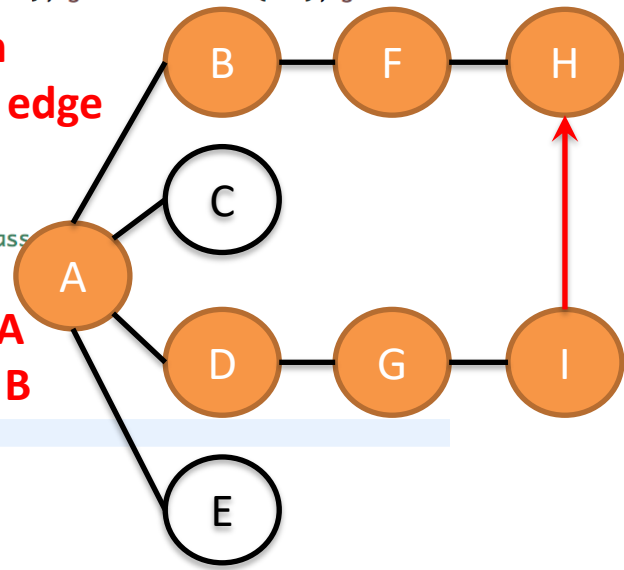
## GraphTraversal.java

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149 g2.insertUndirected("A", "B", "");
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151 g2.insertUndirected("F", "H", "");
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153 g2.insertUndirected("A", "D", "");
154 g2.insertUndirected("D", "G", "");
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164 GT.BFS(g2,"A");
165 GT.findPath("A", "B");
166
167 }
168
169 }
170

```

- Create graph
- Added extra edge
- Run DFS from A
- Find path A to B



### DFS

Key	Value
A	Null
B	F
C	A
D	A
E	A
F	H
G	D
H	I
I	G

Depth First Search from A  
 Path from A to B  
 [A, D, G, I, H, F, B]

Breadth First Search from A  
 Path from A to B  
 [A, B]

**DFS findPath("A", "B") finds path, but not shortest path!**

**A->D->G->I->H->F->B**

**Why?**

**DFS explores as in a maze, as far as it can go before backing up**

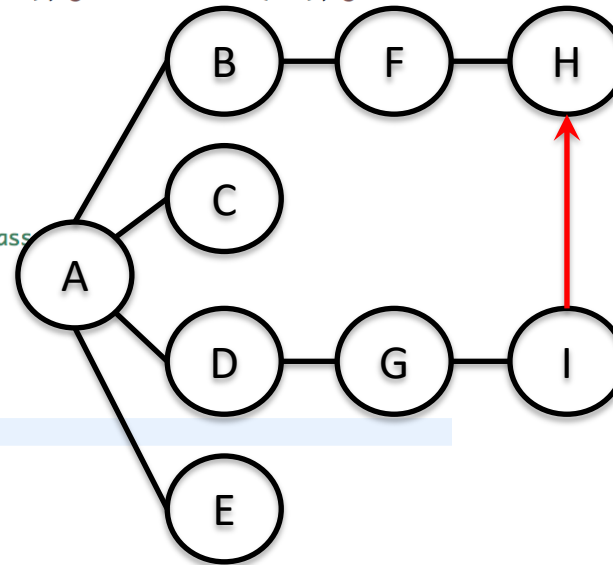
**Here DFS popped D from Stack before it popped B and explored until B found**

# BFS on today's graph

## GraphTraversal.java

```
145 //set up graph from Graph Traversal class
146 AdjacencyMapGraph<String,String> g2 = new AdjacencyMapGraph<String,String>();
147 g2.insertVertex("A"); g2.insertVertex("B"); g2.insertVertex("C"); g2.insertVertex("D");
148 g2.insertVertex("E"); g2.insertVertex("F"); g2.insertVertex("G"); g2.insertVertex("H"); g2.insert
149 g2.insertUndirected("A", "B", "");
150 g2.insertUndirected("B", "F", "");
151 g2.insertUndirected("F", "H", "");
152 g2.insertUndirected("A", "C", "");
153 g2.insertUndirected("A", "D", "");
154 g2.insertUndirected("D", "G", "");
155 g2.insertUndirected("G", "I", "");
156 g2.insertUndirected("A", "E", "");
157 g2.insertDirected("I", "H", ""); //directed edge not from class
158
159 //run DFS from A and find path to H
160 GT.DFS(g2,"A");
161 GT.findPath("A", "B");
162
163 //run BFS from A and find path to H
164 GT.BFS(g2,"A");
165 GT.findPath("A", "B");
166
167 }
168
169 }
170
```

- Run BFS
- Find path A to B



## BFS

Key	Value
A	Null
<b>B</b>	<b>A</b>
C	A
D	A
E	A
F	B
G	D
H	F
I	G

Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy  
<terminated> GraphTraversal [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0\_112.jdk/Contents/Home/bin/java (Feb 12, 2018, 5:14:16 PM)

Depth First Search from A  
Path from A to B  
[A, D, G, I, H, F, B]

Breadth First Search from A  
Path from A to B  
[A, B]

# BFS on today's graph

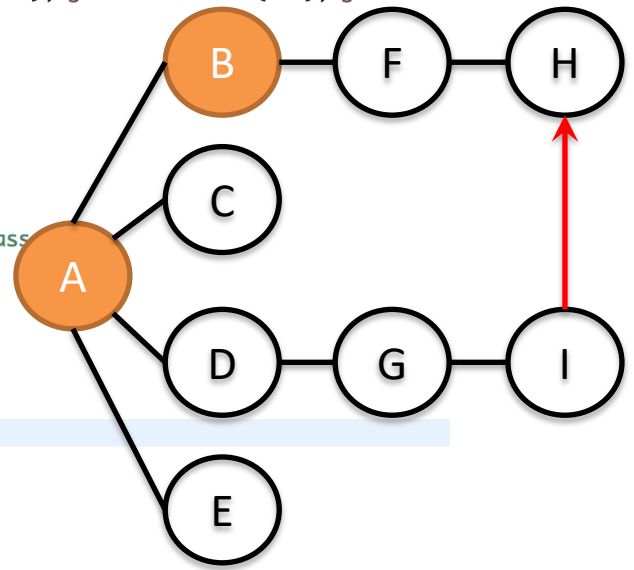
## GraphTraversal.java

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148 g2.insertVertex("E"); g2.insertVertex("F"); g2.insertVertex("G"); g2.insertVertex("H"); g2.insert
149 g2.insertUndirected("A", "B", "");
150 g2.insertUndirected("B", "F", "");
151 g2.insertUndirected("F", "H", "");
152 g2.insertUndirected("A", "C", "");
153 g2.insertUndirected("A", "D", "");
154 g2.insertUndirected("D", "G", "");
155 g2.insertUndirected("G", "I", "");
156 g2.insertUndirected("A", "E", "");
157 g2.insertDirected("I", "H", ""); //directed edge not from class
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165 GT.findPath("A", "B");
166
167 }
168
169 }
170

```

- Run BFS
- Find path A to B



### BFS

Key	Value
A	Null
<b>B</b>	<b>A</b>
C	A
D	A
E	A
F	B
G	D
H	F
I	G



Depth First Search from A  
 Path from A to B  
 [A, D, G, I, H, F, B]

Breadth First Search from A  
 Path from A to B  
 [A, B]

**BFS findPath("A", "B") finds shortest path**  
**A->B**  
**Why?**  
**BFS explores outward in ripples**