CHAPTER VIII.

ON SCALES AND TEMPERAMENT.

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250. The Tetrachord. The music of the ancient Greeks,

"When Music, heavenly maid, was young,"

was based on the system of the *tetrachord* (τετράχορδος, *yielding four sounds*), the name given to the four strings of the lyre. In the seventh century B.C., Terpander added three strings to that instrument, which then gave two tetrachords, the middle string being common to both, thus:

1, 2, 3, 4
4, 5, 6, 7

In the sixth century B.C., Pythagoras invented the musical

canon, afterwards known as the monochord. By means of this he was enabled to investigate theoretically the intervals of the musical scale, and to determine the ratios of those intervals. Pythagoras divided his string into two parts, in the ratio of 1 : 2, and proved that the smaller of these parts gave a sound an *octave* higher than that given by the larger part. By further divisions, in the ratios of 2 : 3 and 3 : 4, he discovered the ratios of the *fifth* and the *fourth* respectively.

251. The difference between the *fourth* and the *fifth* was called, as at present, a *tone*. By means of two *fourths*, separated from each other by a *tone*, Pythagoras completed, or rather re-established the *octave*, which it appears had gone out of use, still preserving the two tetrachords, thus:

1, 2, 3, 4, TONE 5, 6, 7, 8

and so laid the foundation for our present diatonic scale. The Pythagorean scale will be described in the course of this chapter; but it will be convenient to consider first the construction of the "theoretical," "mathematical," or "just" scale as it is variously called.

252. The Theoretical Scale. The completion of the theoretical scale is generally attributed to Ptolemy, the mathematician and astronomer, who flourished in the second century A.D. The principal intervals of this scale will be readily understood from the annexed table and the explanation which follows. The logarithms of the ratios afford a convenient and sufficiently accurate means of analysing or combining intervals; they also convey an excellent idea of relative value, even to those who are not familiar with this mode of expression.

Lord Brouncker employed logarithms for musical calculation in 1653.

253. *Table of the Ratios of the most important Intervals of the Theoretical Scale.*

Intervals.	Ratios.	Logarithms.
Schisma	32768 : 32805	.00049
Minor Comma	2025 : 2048	.00490
Comma $\frac{81}{80}$	80 : 81	.00539
Pythagorean Comma	524288 : 531441	.00588
Minor Chromatic Semitone	24 : 25	.01773*
Major Chromatic Semitone	128 : 135	.02312
Diatonic Semitone	15 : 16	.02803*
Minor Tone	9 : 10	.04576*
Major Tone	8 : 9	.05115
Minor Third	5 : 6	.07918
Major Third	4 : 5	.09691
Perfect Fourth	3 : 4	.12494*
Pluperfect Fourth	32 : 45	.14806
Imperfect Fifth	45 : 64	.15297*
Perfect Fifth	2 : 3	.17609
Minor Sixth	5 : 8	.20412
Major Sixth	3 : 5	.22185*
Minor Seventh	9 : 16	.24988*
Major Seventh	8 : 15	.27300
Octave	1 : 2	.30103*

* The last digit has been corrected by the addition of 1.

254. Inversion, Synthesis and Analysis of Intervals. The inversion of an interval can be effected by doubling the smaller number of the ratio, which corresponds to raising the lower note or depressing the upper one an *octave*, for example: an inverted *minor third*, 5 : 6, becomes a *major sixth*, 6 : 10=3 : 5. An inverted *major third*, 4 : 5, becomes a *minor sixth*, 5 : 8.

255. Before entering on the explanation of such of the intervals of the theoretical scale as are likely to be unfamiliar to some of my readers, it may be necessary to remark that ratios, in their usual fractional form, must be compounded by multiplication, for instance: an *octave* is thus shown to be compounded of a *fifth* and a *fourth*.

Ratio of Fifth.		*Ratio of Fourth.*		*Ratio of Octave.*

$$\frac{3}{2} \times \frac{4}{3} = \frac{12}{6} = \frac{2}{1}$$

To subtract one ratio from another we must proceed by division. The difference between a *fifth* and a *fourth* is thus shown to be a *major tone.*

Fifth.	*Fourth.*	*Major tone.*

$$\frac{3}{2} \div \frac{4}{3} = \frac{9}{8}$$

256. Explanation of the Intervals of the Theoretical Scale. The *Schisma* is the smallest recognised interval in music. It is the difference between 8 *fifths* plus a *major third*, and 5 *octaves*. It may be expressed in terms of the ratios, as follows:

$$\left\{\left(\frac{3}{2}\right)^8 \times \frac{5}{4}\right\} \div \left(\frac{2}{1}\right)^5 = \frac{32805}{32768}$$

The *schisma* and the *octave* are not quite commensurable: the *octave* contains very nearly 614.347 *schismas.*

257. *The Minor Comma* is the difference between a *diatonic* and a *major chromatic semitone.*

$$\frac{16}{15} \div \frac{135}{128} = \frac{2048}{2025}$$

It is also the difference between an *imperfect fifth* and a *pluperfect fourth.*

$$\frac{64}{45} \div \frac{45}{32} = \frac{2048}{2025}$$

This comma contains 10 *schismas*, as may be conveniently seen by means of the logarithms of ratio.

Logarithm of Minor Comma.		*Logarithm of Schisma.*

$$.0049 \div 10 = .00049$$

258. *The Comma with the Ratio* 80 : 81 will be mentioned in this book as the *comma* $\frac{81}{80}$. It is the difference between *three fifths*, and an *octave* plus a *major sixth*, or in terms of the ratios:

$$(\tfrac{3}{2})^3 \div (\tfrac{2}{1} \times \tfrac{5}{3}) = \tfrac{81}{80}$$

It is also the difference between a *major* and a *minor tone*.

$$\tfrac{9}{8} \div \tfrac{10}{9} = \tfrac{81}{80}$$

This *comma* contains 11 *schismas*.

Log. of Comma $\frac{81}{80}$		*Log. of Schisma.*
.00539 ÷	11 =	.00049

It is the interval generally understood when the word *comma* is used without qualification. An *octave* contains very nearly 55.8 *commas* $\frac{81}{80}$. Delezenne (1857) observes that "it is an almost universal opinion that the comma is imperceptible. This is an error that dates as far back as Rameau. . . . . . Not only is the comma appreciable to the dullest ears, but even an interval of one-tenth part of a comma is perceptible in a great number of cases."

259. *The Pythagorean Comma* is the difference between 12 *fifths* and 7 *octaves*.

$$(\tfrac{3}{2})^{12} \div (\tfrac{2}{1})^7 = \tfrac{531441}{524288}$$

It contains 12 *schismas*, and therefore consists of a *comma* $\frac{81}{80}$ and a *schisma*.

$$\tfrac{81}{80} \times \tfrac{32805}{32768} = \tfrac{2657205}{2621440} = \tfrac{531441}{524288}$$

More conveniently expressed in logarithms:

$$.00539 + .00049 = .00588$$

260. *The Minor Chromatic Semitone* is the difference between a *major* and a *minor third*.

$$\tfrac{5}{4} \div \tfrac{6}{5} = \tfrac{25}{24}$$

It is also the difference between a *minor tone* and a *diatonic semitone*.

$$\tfrac{10}{9} \div \tfrac{16}{15} = \tfrac{150}{144} = \tfrac{25}{24}$$

261. *The Major Chromatic Semitone* is a *diatonic semitone* (see §262) less a *minor comma*, see §257. It is the difference between a *major tone* and a *diatonic semitone*.

$$\tfrac{9}{8} \div \tfrac{16}{15} = \tfrac{135}{128}$$

262. *The Diatonic Semitone* is the difference between a *fourth* and a *major third*.

$$\tfrac{4}{3} \div \tfrac{5}{4} = \tfrac{16}{15}$$

It is also the difference between an *octave* and a *major seventh*.

$$\tfrac{2}{1} \div \tfrac{15}{8} = \tfrac{16}{15}$$

The remaining intervals of the theoretical scale will be understood without further explanation.

263. The Vibration Numbers of the Theoretical Scale. Bearing in mind the principles already explained, it will be easy to see that the vibration number of any note, and the ratio of the required interval, being given, the theoretical vibration number of any other note may be found by ordinary "rule of three," as follows:

Ratio of Fifth.		*Vib. numb. of c'.*		*Vib. numb. of g'.*
2 : 3	: :	271.2	:	406.8

or by multiplying any vibration number by the decimal fraction which represents the desired interval.

Vib. numb. of c'.		*Ratio of Fifth.*		*Vib. numb. of g'.*
271.2	×	1.5	=	406.8

264. *Table of the Intervals and the Vibration Numbers of the Theoretical Scale.*

Notes.	Vibration Numbers.	Intervals from Note to Note.	Intervals from *c'*.
e''	678.0	-	Major Tenth.
		Minor Tone.	
d''	610.2	-	Major Ninth.
		Major Tone.	
c''	542.4	-	Octave.
		Diatonic Semitone.	
b'	508.5	-	Major Seventh.
		Major Tone.	
a'	452.0	-	Major Sixth.
		Minor Tone.	
g'	406.8	-	Perfect Fifth.
		Major Tone.	
f'	361.6	-	Perfect Fourth.
		Diatonic Semitone.	
e'	339.0	-	Major Third.
		Minor Tone.	
d'	305.1	-	Major Tone.
		Major Tone.	
c'	271.2	-	(Key-note.)
		Diatonic Semitone.	
b	254.25	-	Major Semitone.
		Major Tone.	
a	226.0	-	Minor Third.
		Minor Tone.	
g	203.4	-	Perfect Fourth.

The logarithms of ratio will be found in §276.

This table includes the three *perfect fifths* of the strings of the violin, see §258. In order to make *d'* and *d''* agree with all the other notes of the scale, their vibrations must change to 301.3 and 602.6, respectively, otherwise several of the intervals will be false to the extent of a *comma* $\frac{81}{80}$: for example, *g* to *d'* and *d'* to *a'*.

265. It will be evident that the intervals of this scale are incongruous by their very nature, otherwise the schisma and the commas could not exist. Mr. Ellis (1885) thus succinctly describes the causes of these discrepancies: "No recurrence of notes formed by taking intervals of *fifths*, *major thirds* and *octaves*, is possible, because no powers of the numbers $\frac{3}{2}$, $\frac{5}{2}$, 2, or of any combination of them, however often repeated, can produce a power of any single one of them." It is therefore indisputable that the notes of the theoretical scale cannot agree, even for a single octave, in any key.

A true chromatic scale is altogether incompatible with "just" intonation, but any sound may be taken as the key-note of a diatonic scale, and the notes will, of course, occupy correct positions if the calculations be properly made.

The annexed series of notes is intended to illustrate the further complications of the "just" scale. I have not thought it necessary to give the ratios or the vibration numbers, but each of these notes would have a different sound, the interchangeable notes of our modern scale, such as B♯ and C♮, being altogether foreign to natural intonation. This series contains but a small selection from the appalling multitude of notes which belong to the system. The difficulties that would ensue from taking even this comparatively small number, as alternative key-notes, may be left to the reader's imagination.

C♮
C♭
B♯
B♮
B♭
A♯
A♮
A♭
G♯
G♮
G♭
F♯
F♮
F♭
E♯
E♮
E♭
D♯
D♮
D♭
C♯
C♮

266. Temperament. Pythagoras appears to have been not only the first to discover the natural proportions of the scale, but also the first to see the necessity for tempering, or modifying, certain of its degrees, so that by sacrificing somewhat of the perfection of the less sensitive consonances, the *thirds* and the *sixths*, the *perfect fifth* and the *octave* might be preserved as nearly as possible in their natural integrity. Thus, twenty-five centuries ago, arose the great question of temperament, which has never since ceased to agitate the souls of musicians.

267. The Pythagorean Temperament. Pythagoras considerably simplified the scale by increasing the *major third*, *sixth* and *seventh* from the key-note by a *comma* $\frac{81}{80}$ each, thus changing all the *minor tones* into *major tones*, and correspondingly reducing the *semitones*.

Table showing the variations of the Intervals of the Pythagorean Scale from those of the Theoretical Scale.

	Pythagorean Intervals and Ratios from Note to Note.	Difference in Commas 80 : 81	Theoretical Intervals and Ratios from Note to Note.	Pythagorean Intervals and Ratios from Key-note.	Difference in Commas 80 : 81	Theoretical Intervals and Ratios from Key-note.
C		-	- -	Octave 1 : 2	- =	Octave 1 : 2
	Semitone 243 : 256	1 <	Diatonic Semitone 15 : 16			
B		-	- -	Major Seventh 128 : 243	- 1 >	Major Seventh 8 : 15
	Major Tone 8 : 9	=	Major Tone 8 : 9			
A		-	-	Major Sixth 48 : 81	- 1 >	Major Sixth 3 : 5
	Major Tone 8 : 9	1 >	Minor Tone 9 : 10			
G		-	- -	Perfect Fifth 2 : 3	- =	Perfect Fifth 2 : 3
	Major Tone 8 : 9	=	Major Tone 8 : 9			
F		-	- -	Perfect Fourth 3 : 4	- =	Perfect Fourth 3 : 4
	Semitone 243 : 256	1 <	Diatonic Semitone 15 : 16			
E		-	- -	Major Third 64 : 81	- 1 >	Major Third 4 : 5
	Major Tone 8 : 9	1 >	Minor Tone 9 : 10			
D		-	- -	Major Tone 8 : 9	=	Major Tone 8 : 9
	Major Tone 8 : 9	=	Major Tone 8 : 9			
C		-	-	Key-note		Key-note

The logarithms of ratio will be found in §276.

268. The system of Pythagoras errs on account of a too rigid adherence to accuracy in the *fifth*, which necessitates so wide a deviation from it in the *thirds* and *sixths* as to render the scale almost unfit for harmony, however agreeable it may be for melody. It is, nevertheless, much used by vocalists and by stringed instrument-players, who, however, generally contrive to modify its discordant notes when these form integral parts of the accompanying harmony. Let no one imagine that violinists ever use the theoretical scale. Dr. Pole (1875, 1877) has made some interesting observations on this topic, and given important quotations from Hauptmann's letters in proof of the truth of his statements.

269. Meantone Temperament. The disadvantages of tuning instruments with fixed sounds according to the Pythagorean temperament, and the impossibility of tuning them according to the theoretical scale, have long been recognised, and numerous have been the attempts to remove the discrepancies of the latter. The only one of these which has attained notoriety is the *meantone* temperament. To whomsoever may be attributed the first idea of this temperament, there appears to be no doubt that it was "perfected" by Salinas, who described it in an important and well-known work entitled *De Musica libri septem. Salamanca*, 1577. The main principle of the system was the division of each of the *major thirds* of the scale into two equal parts, each part being a *mean* between the *major* and *minor tones;* hence the name, *meantone.*

As there are three *major*, and but two *minor tones* to the *octave* in the theoretical scale, this mode of division caused too much to be given to the *semitones*, the difference, half a *comma*, between a *major* and a *mean tone* having to be apportioned between the two *semitones*, each of which was thus made a quarter of a *comma* too wide. The *thirds* were generally correct, but the *fifths* were a quarter of a *comma* too small: in short, the authors of this system distributed the incongruities of the theoretical scale amongst the intervals least tolerant of any deviation from accuracy. According to Mr. Ellis, this system was not complete without twenty-seven notes to the octave. When it was applied to instruments with fixed tones, and with only twelve divisions to the octave, it had the defect common to all unequal temperaments: certain keys were made tolerably good at the expense of the rest. The group of keys into which the worst imperfections were thrown, was termed "the wolf." Happily this wolf has been exterminated by equal temperament, and meantone temperament may be pronounced obsolete, but enquiry into its defects and their causes is none the less necessary, if only to assist us in intelligent appreciation of the blessings of the truly philosophical temperament which has now thoroughly supplanted its older rival.

270. *Table showing the Variations of the Intervals of the Meantone Scale from those of the Theoretical Scale.*

Meantone Notes.	Fractions of Comma 80 : 81.		Theoretical Intervals from Note to Note.	Fractions of Comma 80 : 81.		Theoretical Intervals from Key-note.
C					=	Octave.
	$\frac{1}{4}$	>	Diatonic Semitone.			
B				$\frac{1}{4}$	<	Major 7th.
	$\frac{1}{2}$	<	Major Tone.			
A				$\frac{1}{4}$	>	Major 6th.
	$\frac{1}{2}$	>	Minor Tone.			
G				$\frac{1}{4}$	<	Perfect 5th.
	$\frac{1}{2}$	<	Major Tone.			
F				$\frac{1}{4}$	>	Perfect 4th.
	$\frac{1}{4}$	>	Diatonic Semitone.			
E					=	Major 3rd.
	$\frac{1}{2}$	>	Minor Tone.			
D				$\frac{1}{2}$	<	Major Tone.
	$\frac{1}{2}$	<	Major Tone.			
C						Key-note.

The logarithms will be found in § 276. The ratios are too complicated to be conveniently expressed otherwise.

271. Equal Temperament. The principle of equal temperament is the division of the *octave* into twelve *equal semitones;* the vibration numbers of the consecutive notes of the chromatic scale are therefore proportional, and all intervals of the same name have the same ratio. By the adoption of this principle the incongruities inseparable from all other systems are entirely removed, and it fortunately happens that in this system the intervals that suffer most are those which are the best able to bear some deviation from theoretical accuracy. As in all other systems, the *octave* is left intact, its nature being so sensitive that the preservation of its absolute integrity is indispensable. The *perfect fifth* and *fourth* can bear a slight amount of tempering, and the imperfect consonances, the *sixths* and the *thirds*, a still greater amount. The discords are, of course, less sensitive than any of the concords. Such errors as inherently belong to this scale are fairly set forth in the tables §§273, 274 and 276.

A *semitone* in equal temperament contains very nearly 51.15 *schismas*, or 4.65 *commas* $\frac{81}{80}$.

272. *The Vibrations of the Notes of the Equally tempered Scale.* Of several methods of calculating the vibration numbers of a scale in equal temperament, the following is perhaps as simple as any. The ratio of the *semitone* in equal temperament is the twelfth root of 2. If we take this as $\sqrt[12]{2} = 1.0594631$, which is very slightly in excess of the true figures, it will be sufficiently near for the working out of the vibration numbers to four decimal places.

The multiplication of the vibration number of any note by the number given above, will give the vibrations of the note one *semitone* higher. We may, therefore, begin on any note of which we have the vibration number, and, discarding all decimals beyond seven places, work upwards to the highest note of the octave required. The division of the vibration number of that note by 2 should, of course, give the vibrations of the note one octave lower. Working upwards from that note to the starting point we have an excellent test of the correctness of the result. For example: on multiplying 452, the vibration number of *a'*, by

1.0594631, we get 478.8773, the number for *a'*♯: continuing upwards to *c''* 537.5216, we divide that number by 2, which gives *c'* 268.7608, and, proceeding as before we should arrive at *a'* 452.0000. If we begin with the lowest note, we should, of course, find the vibrations of the highest note exactly double those of the former.

273.

Notes.		Equal Temperament Vibrations.		Theoretical Vibrations.
c''	-	- 537.5216	-	- 542.4
b'	-	- 507.3528	-	- 508.5
a'♯	-	- 478.8773		
a'	-	- 452.0000	-	- 452.0
g'♯	-	- 426.6312		
g'	-	- 402.6862	-	- 406.8
f'♯	-	- 380.0852*		
f'	-	- 358.7526	-	- 361.6
e'	-	- 338.6174	-	- 339.0
d'♯	-	- 319.6123*		
d'	-	- 301.6738	-	- 305.1
c'♯	-	- 284.7422*		
c	-	- 268.7608	-	- 271.2

* The last digit has been corrected by the addition of 1.

The logarithms will be found in §276.

274. *Table showing, in Schismas, the Differences between the Diatonic Intervals of the Equally Tempered and the Theoretical Scales.*

Equal Temperament.		Schismas.		Theoretical.
Semitone	-	6	<	Diatonic Semitone.
Tone	-	9	>	Minor Tone.
		2	<	Major Tone.
Minor Third	-	8	<	Corresponding Interval.
Major Third	-	7	>	" "
Perfect Fourth	-	1	>	" "
Pluperfect Fourth	-	5	>	" "
Imperfect Fifth	-	5	<	" "
Perfect Fifth	-	1	<	" "
Minor Sixth	-	7	<	" "
Major Sixth	-	8	>	" "
Minor Seventh	-	2	>	" "
Major Seventh	-	6	>	" "

275. In this system, the greatest deviation from perfect accuracy amounts to no more than three fourths of a *Pythagorean Comma:* the greatest error in a perfect consonance is one *schisma*; or less than $\frac{1}{51}$st part of an *equal temperament semitone*. Twelve *fifths*, thus reduced, lead to a note corresponding to the terminal note of the seventh *octave*, the natural error consisting of the *Pythagorean comma* being eliminated, and the *octave* and the *fifth* being rendered commensurable. The most serious deviations are in the *thirds* and the *sixths*, but even these errors are much less than those of the theoretical and the Pythagorean scales; moreover, the modification of these intervals is not an unmixed disadvantage, and it is often attended with excellent effects, as the peculiar characteristics of major and minor keys are thereby somewhat intensified, and thus we gain an addition to one of the greatest charms of music, variety of expression.

276. Comparative Table of the Logarithms of the Intervals of the Diatonic Scale, according to the various systems.

Notes.	Theoretical.	Pythagorean.	Meantone.		Equal.
C	.30103*	.30103*	.30103	0*	.30103*
B	.27300	.27839	.27165	2	.27594
A	.22185*	.22724	.22319	7	.22577
G	.17609	.17609	.17474	2	.17560
F	.12494*	.12494*	.12628	7	.12543*
E	.09691	.10230	.09691	0	.10034
D	.05115	.05115	.04845	5	.05017
C	.00000	.00000	.00000	0	.00000

* One has been added to the last digit.

Logarithms of Diatonic Semitones to seven places.

Theoretical.	Pythagorean.	Meantone.	Equal.
.0280287	.0226337	.0293774	.0250858

The logarithms of the meantone scale are given to six places, as a nearer approach to accuracy is thereby attained, but for comparison with the other scales it will be better to discard the last digits and to make the usual corrections.

277. Assuming the reader's familiarity with the differences between the major and minor modes, I have not considered it advisable to complicate matters by introducing calculations for the latter.

278. Rise and Progress of Equal Temperament. The name of the inventor of equal temperament, and the date of its introduction, are points involved in considerable obscurity: the invention has been ascribed, by some writers, to Aristoxenus of Tarentum, but there is nothing in the extant works of that philosopher (4th Century B.C.) to lead to the supposition that he ever contemplated the division of the octave into twelve equal parts, although he says that "the fourth is evidently composed of two tones and a half," and also that the tone "may be divided in three different ways, because the half, the third part, and the quarter of a tone can be sung melodiously." (Διαιρείσθω δὲ εἰς

τρεῖς διαιρέσεις. Μελῳδείσθω γὰρ αὐτοῦ τό, τε ἥμισυ, καὶ τὸ τρίτον μέρος, καὶ τέταρτον.) Euclid, who takes only a mathematical view of the subject (4th Century B.C.), absolutely denies the possibility of such a division and avers that "a tone cannot be divided into two or more equal parts" (Ὁ τόνος οὐ διαιρεθήσεται εἰς δύο ἴσους, οὔτε εἰς πλείους. *Theor. XVI.*)

279. The first account of this system, that I have been able to find, is that given by Zarlino (1588). In the earlier volumes of that author (1558, 1571) no mention is made of equal temperament, but the meantone system is highly extolled as being the only "reasonable" one. We may therefore conclude that if Zarlino was acquainted with equal temperament when writing those works, he was not impressed with its advantages. In his third volume (1588, *Lib. quarto*, *Cap. XXVII-XXXII*) he not only advocates the division of the octave into twelve equal semitones, but gives the most precise directions for dividing the scales of lutes and other instruments according to that system.

280. Mersenne's conversion was more rapid, for after having (1633) totally misinterpreted Aristoxenus, who, he tells us, "said that all tones and semitones were equal, according to the custom still practised on lutes and viols, which is nevertheless repugnant to the laws of harmony and common sense," only three years later he appears to have read his Aristoxenus better, and, while denying the possibility of his own previous conclusions with regard to that author, he alters his opinions with regard to equal temperament, and says (1636, *Traitez des Consonnances, etc., Livre 3me*), that it is the most commonly used and the most convenient of all systems, and that all practical musicians admit its advantages. In *Livre II*, *Prop. XI*, of the same part, he gives the ratios of this scale. Alluding to the excellence of the system and the defects of all others, he exclaims: "*La plus grande prudence dont l'homme puisse user, consiste à tirer le bien du mal.*"

281. Modern musical historians seem to have ascribed too recent a date to the general adoption of equal temperament. A quaint little anonymous book, kindly lent to me by my friend Mr. Henry Carte, entitled *The Modern Musick-Master* (1730), gives "Rules for tuning the Harpsichord or Spinnett." One of these rules is that "all the sharp [major] *thirds* must be as sharp as the ear will permit, and all *fifths* as flat as the ear will permit," a rule that can be applied to no other system but equal temperament.

The opinions of the great Bach on this subject are well-known. Amongst other distinguished musicians who have written in favour of the system may be mentioned Rameau (1739, 1749), who having, like others before and after him, begun by abusing it, finished by being its warm supporter. "It is well to begin with a little aversion."

282. Dr. Robert Smith, a celebrated mathematician, and the inventor of a harpsichord with stops to change the pitches of the notes, having spoken of this system as giving rise to effects "extremely coarse and disagreeable" (1759, p. 166), nevertheless admits the advantages of having all the keys with corresponding intervals, for he says (p. 215): "Till instruments are made with a changeable scale, it is proper to tune the defective scale, in present use, by making every fifth and third beat equally quick, the former flat and the latter sharp." In his preface Dr. Smith observes that instruments "would be better in tune, if all the consonances were made as equally harmonious as possible, though none of them were perfect."

283. The following opinions of Chladni are well worth recording here. I translate the French (1809) as closely as possible.

"Some persons are disposed to think that equal temperament exists only for instruments with fixed sounds; but . . . every good singer, every good performer, tempers his intervals unconsciously."

"Equal temperament is the most conformable to nature, because, on account of the equal division of the discrepancies between all the intervals except the octave, the inaccuracy of each interval is too small to offend the ear."

"As there is always but one single truth and an infinity of errors, so there is but one equal temperament and an endless number of unequal temperaments."

284. Spohr (1832) was certainly entitled to speak on this subject with authority. These are his words, literally translated from the original German, but carefully omitted in the English translation by Rudolphus: "By correct intonation, that of equal temperament is of course understood, as there is no other fit for modern music. The young violinist, therefore requires to be instructed in this one only."

285. We have seen that some of the objectors to equal temperament, became wiser as they grew older; some important authors have, however, persistently held to their opinions against it. I have selected the following specimens from the most vigorous denunciations.

286. Kollmann's chief objections to the system (1796) were, firstly, that it was difficult to obtain, an objection that is no longer tenable: secondly, that, if obtained, it would make the intervals of all the scales alike, a rather curious disparagement, which would apply equally well to the theoretical scale.

287. Biot (1816) observes that "a good violinist does not temper his intervals unless constrained to do so by an instrument with fixed sounds." As this would imply that Spohr was not a good violinist, the assertion is hardly worth contradiction.

288. General Perronet Thompson (1859) is exceedingly and amusingly emphatic in his condemnation. In his preface he says: "Among the signs of progress in the times, is a growing discontent with the thing called Temperament. Instead of being considered as the crowning exertion of musical skill, it begins to be viewed as a lazy attempt to save trouble, like nailing a telescope to one length for all eyes and distances or making the fingers of a statue of one uniform size. . . . The temptation to the old systematical teaching to play out of tune, was that performers might play 'with perfect freedom in all the keys' by playing in none."

289. Professor Blaserna (1875) evidently feels just as strongly though his language is more moderate. He observes that "music which is based on the tempered scale must be regarded as imperfect, and unworthy of our sensibilities and aspirations. That we can endure it, and even think it beautiful, only shows that our ears have become systematically vitiated."

290. There is little to be added to the *pros* and *cons* of this dispute, "the quarrel is a very pretty quarrel as it stands," but it may be well to mention a few facts which have an important bearing on the subject.

That equal temperament is practically attainable is proved emphatically by seven recorded experiences of Mr. Ellis (1885), in one of which the greatest error was $\frac{1}{100}$th of a semitone, and nine notes out of thirteen were quite correct. This case was that of a harmonium tuned by the clever acoustician, Mr. D. J. Blaikley, a year previously to the trial, and is, of course, an exceptional one, but our everyday experience shows that correct equal temperament may be easily obtained with sufficient accuracy for all useful purposes, while with it, and with no other, there is the advantage of being able to play in tune with the pianoforte and other instruments of fixed sounds. With an instrument tuned on the "just" system, if such could be had, a pianoforte accompaniment would be unbearable, and, supposing a quartet of strings were always at hand, will anyone assert that there would be any probability of its being as correctly in tune as a "well-tempered" pianoforte? Stringed instrument players, as well as their hearers, know well that theoretical perfection generally means practical imperfection, and that the very cause which gives rise to the opportunities of playing correct intervals is the reason of their being so generally incorrect.

—"*chorda sonum reddit, quem vult manus et mens,*
Poscentique gravem persaepe remittit acutum."

Even were it possible to sing or play alone in the theoretical scale, it is painful to think of the difficulties of an unfortunate musician who should be expected to conform at one time to the tempered, and at another time to the "just" scale; the two things can no more flourish side by side than can civilization and barbarism.

The utility of equal temperament may be fairly likened to that of meantime, theoretical intonation being of no more practical value for general musical purposes than is solar time for the affairs of everyday life. As the delightful Robert Herrick truly says:

"Imparity doth ever discord bring,
The mean the music makes in everything."

It is admitted that all existing music would have to be re-arranged if "just intonation" were generally adopted, and though I do not remember having seen any admission of the fact that all enharmonic changes would have to be expunged, yet it is none the less a fact, because sharpening or flattening a sound by intervals varying from one to nine schismas, would be unendurable as an intentional effect: in its unintentional form we are only too familiar with it, as it is a common practice for indifferent performers to alter their notes when they are unable at first to produce the desired sound correctly.

In the face of all these difficulties, the idea of tuning instruments according to the theoretical divisions of the scale may be pronounced Utopian, and therefore the wisest course that musical mathematicians can pursue is to educate their ears to equal intonation, as most practical musicians have long since done, and to desist from the attempt to make us dissatisfied with the inevitable. They have demonstrated, to their own entire satisfaction, that all the music we hear, except that of those very ingenious machines with from fifty-three to one hundred notes in the octave, is theoretically abominable, but even if, for the sake of peace and quietness, we are content to admit as much, I have no doubt that most of us will still continue to enjoy our music with every interval, but the octave, played "out of tune," and that instead of wasting our lives in attempts to attain the impossible we shall be content to accept the teaching of the excellent proverb: "*Si l'on ne peut pas avoir ce que l'on aime, il faut aimer ce que l'on a.*"