CHAPTER II.

ON THE VIBRATIONS OF SOLID BODIES.

§26. DIFFERENT SPECIES OF VIBRATIONS.—27. *TRANSVERSE VIBRATIONS.*—30. *WHEATSTONE'S KALEIDOPHONE.*—31. *LONGITUDINAL VIBRATIONS.*—34. *TORSIONAL VIBRATIONS.*—35. THE TUNING-FORK.—40. SOUND-BOARDS.—41. VIBRATIONS OF STRINGS.—44. *HARMONICS OF STRINGS.*—49. *THE TRUMPET-MARINE.*—51. *EARLY INVESTIGATORS.*—52. *BERNOULLI'S EXPERIMENT.*—53. *MELDE'S EXPERIMENT.*—55. HARMONICS OF VARIOUS SONOROUS BODIES.—58. SYMPATHETIC VIBRATIONS.—63. *THE FIBRES OF THE EAR: DE MAIRAN'S HYPOTHESIS*—64. *CORTI'S DISCOVERIES.*

26. Different Species of Vibrations. There are three distinct species of vibrations which it is necessary to consider in order to understand the acoustical phenomena of musical instruments. These vibrations are *transverse*, *longitudinal* and *torsional*. Each of these species may occur alone, or, as is more frequently the case, in union with either or both of the others.

27. *Transverse Vibrations.* Familiar illustrations of transverse vibration are afforded by the swaying of trees in the wind; the swinging of ropes or chains, and the oscillations of the ordinary clock-pendulum. This mode of vibration is often termed *pendular vibration.*

28. A little reflection on the motion of the pendulum will make it clear that every part of the vibrating body performs its complete oscillation in the same period of time, and that, although an equal rate may be maintained by the vibrations themselves, there must be a constant change of velocity during each oscillation.

29. A rod of metal or of wood, of certain proportions as to length and thickness, with one end fixed, and the free end drawn to one side and suddenly released, will give transverse

vibrations that can be heard as well as seen. Vibrating rods do not move in one plane only, but generally describe more or less complicated figures.

30. *The Kaleidophone.* Sir Charles Wheatstone (1827) invented an ingenious instrument, which he called the Kaleidophone, for exhibiting these vibrations. It consisted of a number of metal rods fixed at one end. To the free ends of these rods silvered glass beads were attached. When a beam of light was directed on one of the beads, during its vibration, bright figures were shown which indicated exactly the character of the vibrations. Dr. Thomas Young (1800) had previously applied the same principle, in his observations of the figures described by vibrating pianoforte strings.

31. *Longitudinal Vibrations* may be excited in a glass tube by passing a wet cloth briskly along it, or in a rod of metal or wood by friction with resined fingers. Savart's investigations of the longitudinal vibrations of rods are most interesting. He showed that a glass tube could be so strongly excited by rubbing it with a wet cloth, that one part of the tube would fall to pieces, flying off in rings. Professor Tyndall (1883) repeated and described this experiment.

32. The sounds of these vibrations are always much higher than those produced by the tranverse vibrations of the same body. Lord Rayleigh has quoted the following remarks of Professor Donkin (1870) on this subject. "If the peg of the violin be turned so as to alter the pitch of the lateral vibrations very considerably, it will be found that the pitch of the longitudinal vibrations has altered very slightly. The reason of this is that in the case of the lateral vibrations, the change of velocity of wave-transmission depends chiefly on the change of tension, which is considerable. But in the case of the longitudinal vibrations, the change of velocity of wave-transmission depends upon the change of extension, which is comparatively slight."

33. The reader can scarcely fail to perceive that transverse vibrations cause the whole vibrating body to move from side side, and that longitudinal vibrations occur in the vibrating body itself, its particles only being agitated.

34. *Torsional Vibrations* appear to be of little importance in music, except in so far as they interfere in the calculations of acousticians. No instrument of music has been constructed in which they have been the main cause of sound, and yet it is probable that few musical sounds are entirely free from their influence. Chladni (1802-9) investigated their phenomena, and found that "in cylindrical or prismatic rods, the sound of these vibrations was always deeper by a *fifth* than that of the longitudinal vibrations of the same body." Torsional vibrations are always present in strings sounded by a bow.

35. The Tuning-fork. The vibrations of a tuning-fork, when excited in the usual manner, by percussion, are simply pendular. This important instrument has been of immense service in the practice of music as an art, and also as a means of scientific investigation. It is the only trustworthy pitch-carrier, and it has been proved to be the best instrument ever devised for pitch measurement. According to Sir John Hawkins (1776) it was invented by John Shore, a trumpeter in Queen Anne's band in 1711. At the public entry of George I. in 1714, Shore rode as Sergeant-Trumpeter in the cavalcade. He was Lutist in the Chapel Royal in 1715, and died deranged in 1753. As this is all that appears to be known of him, the date of his invention cannot be fixed within forty years. He himself described the instrument as a *pitch-fork*, a very much more appropriate name than the present one, but which fell into disuse, possibly on account of its having been previously used to designate an implement of more humble employment.

36. The tuning-fork does not appear to have become as rapidly known as might have been expected, considering its great utility. Rousseau, writing in 1764, was evidently unaware of its existence, as he laments the uncertainty of the *pitch-pipe* then in use, and "the impossibility of being certain of the same sound in two places at the same time."

37. Mr. A. J. Ellis (1880 A) gives the following valuable information concerning the tuning-fork: "It is very permanent; I have reason to believe that Scheibler's forks have not varied by one vibration in ten seconds, since his death in 1837. It varies very slightly for temperature, being (contrariwise to the organ-pipe) flattened by heat and sharpened by cold to the amount of about 1 vibration in 21,000 for each degree Fahrenheit. . . . As forks are tuned by filing, which not only heats them, but unsettles their molecular arrangements—at least in part—it is necessary to let them cool and rest for several days, sometimes for weeks, before their pitch can be depended on for scientific accuracy. They will often rise by several vibrations in ten seconds in the course of cooling. Hence copies are always apt to be too sharp, and should, if possible, be re-compared."

38. Mr. Ellis finds that the effect of rust is in all cases to flatten the fork, but that "the error is never likely to exceed 4 in 1,000." He was, however, kind enough to examine a fork which I showed him in 1885. This fork had been sent to Messrs. Rudall, Rose and Carte in 1859. It was then reputed to give 435 vibrations in a second, but it had gone down in the 26 years, through rust and "possibly bad treatment" to 424.5, a fall of 10.5 vibrations in a second. Mr. Ellis gives an account of this fork (1885).

39. Lord Rayleigh (1877) thus explains the alteration of the pitch of a tuning-fork: "To make the note higher, the equivalent inertia of the system must be reduced. This is done by filing away the ends of the prongs, either diminishing their thickness or actually shortening them. On the other hand, to lower the pitch, the substance of the prongs near the bend may be reduced, the effect of which is to diminish the force of the spring, leaving the inertia practically unchanged; or the inertia may be increased by loading the ends with wax or other material."

40. Sound-boards. The surfaces of a tuning-fork are too small to enable them to convey, unassisted, sufficient motion to the surrounding air to render the vibrations of the fork audible, excepting when the instrument is at a very short distance from the ear; the handle is therefore generally held against some large resonant body which, vibrating in accordance with it, can act as a *sound-board*. For the same reason, the sounds of strings are always too feeble to be of any musical utility unless reinforced by sound-boards of some kind, hence stringed instruments are never made without them. Sound-boards are fruitful sources of imperfection in stringed instruments, as they often carry on the vibrations, in a very objectionable manner, after those of the strings themselves have ceased.

41. Vibrations of Strings. At the first glance, it may appear as if the vibrations of strings had nothing in common with those of a wind-instrument, but it will be presently seen that a little study of the former will conduce very greatly to the understanding of the latter, and tend to prevent the confusion of ideas concerning both, which, even in this age of enlightenment, is only too prevalent.

42. The longitudinal and torsional vibrations of strings are of little or no practical importance; it will therefore be only necessary to consider their transverse vibrations. These are somewhat similar to those of tuning-forks, the chief differences being that the equilibrium of a vibrating tuning-fork is restored by its rigidity, and that of a string by the effect of its tension between two fixed points; also that whereas the greatest motion of a tuning-fork occurs at the free ends of the prongs, that of a string is midway between its fixed points. The two ends of a vibrating string, the points of least vibration, are properly termed *nodes;* the point of greatest motion is the *antinode*. A string during vibration is said to form a *loop*.

43. It has been known from very early times that the pitch of the sound given by a stretched string, depends on the *length*, the *thickness*, the *density* and the *tension* of the string: increase of tension raising the pitch, while increase of length, of thickness, or of density, lowers it. Dr. Robert Plot (1676) appears to have been the first English author who wrote anything of importance on the subject of string vibration. He clearly demonstrated that a string of the same length, thickness, density and tension always vibrates at the same rate, "a greater *force* only making it fly out to a greater *distance*, or fetch a greater *compass* in its *vibrations*, and thereby *move* (but not *vibrate*) faster." The laws of string vibration are as follows: *The rapidity of vibration varies inversely as the length; as the diameter, and as the square root of the density of the string: it varies directly as the square root of the weight which causes the tension.*

44. *Harmonics of Strings.* A string vibrating as a whole, between its terminal nodes, gives the lowest sound that can be obtained from it as long as the four conditions previously named are maintained. If the vibrations should be slow enough to be seen, the string would assume the form shown at *fig.* 1, in which *N. N.* are the two nodes, and *L.* the loop. The position of the string, at the alternative oscillations, is shown by the dotted line. The sound thus produced is termed a *fundamental*, or *prime tone*. A vibrating string lightly *damped*, or *shaded*, in the centre, no longer vibrates as a whole, but another node is formed where the string is touched, and two loops or *segments* are the result, as shown in *fig.* 2, where *N. N. N.* are the three nodes, and *S. S.* the two segments. Each of these segments vibrates twice as fast as the whole string would

FIG. 1.

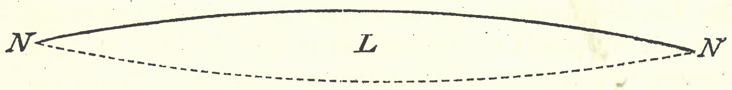
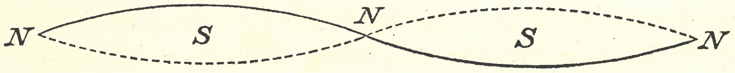
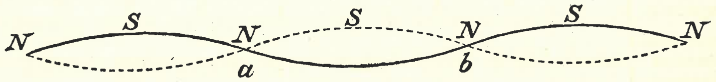


FIG. 2.



vibrate, giving the sound expressed by the well-known term, *an octave higher;* the rate of vibration being the same as if the halves of the string were completely separated from each other. A string shaded at one third of its length, appears as in *fig.* 3; it vibrates three times as fast as at first, and its

FIG. 3.

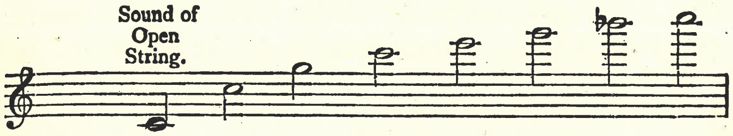


sound is a *twelfth* higher than that of the whole string. These sounds are generally called *harmonics*; those which are given by strings that are *stopped*, or actually shortened, in the ordinary manner, are, of course, fundamentals.

45. Space will not permit the discussion of the interesting subject of the motion of the sound-pulse in strings. It may suffice to state that a string shaded at any aliquot part of its length, smaller than the half, will form nodes at all the corresponding points, consequently a string shaded at the point *a*, *fig.* 3, will not only form a node there, but one at *b* also. Rousseau (1764) gives a very good explanation of this law. I translate the French: "Let a string, 6, be divided into two parts, 4 and 2; the *harmonic sound* will be according to the length of the smaller part, 2, which is the aliquot of the larger part, 4; but if a string, 5, be divided into 2 and 3, then, as the smaller part is not a measure of the greater, the *harmonic sound* will be only according to the [length of the] half, 1, of this smaller part, which half is the greatest common measure of the two parts, 3 and 2, and of the whole string, 5."

46. There can, of course, be no theoretical limit to the divisions of a string. The subjoined table will explain them as far as it is necessary to follow them at present. The open string is assumed to give *c'*. The vibration numbers are obtained by multiplying the number of the vibrations of the open string, by the number of its segments.

Harmonics of a String.



Numbers of Segments.		2	3	4	5	6	7	8
Theoretical Vibration Numbers.	271,2	542.4	813.6	1084.8	1356	1627.2	1898.4	2169.6

47. In order that these sounds may be given correctly, the string must be true. If false, that is, not of equal thickness and density throughout its length, the harmonics given by it will be incorrect and uncertain. Even the fundamental note of a very false string is bad.

48. The seventh sound of the above scale, though commonly called *b♭*, is not a note recognised in music. It lies between *a* and the real *b♭*.

49. *The Trumpet-Marine.* The harmonics of strings appear to have been known and used for many years before they were understood. An obsolete instrument, called the *trumpet-marine*, or, according to Dr. Thomas Young, *trumpet-marigny*, had no other notes. This instrument may be described as an exceedingly long and narrow viol, generally of triangular section. It was seldom furnished with more than one string: this rested on a *bridge*, one foot of which was supported by the sound-board, while the other was slightly raised, so that it jarred against the sound-board when the string was in vibration. According to Filippo Bonanni (1722) it was played by means of a bow at the narrow, or upper, end. Its tone is said to have somewhat resembled that of a trumpet.

50. During the seventeenth century, the sounds of this curious instrument seem to have considerably exercised the minds of philosophers. The shading of the string was no doubt also practised on other instruments played with bows, but the trumpet-marine appears to have been the prime motive of the researches which were prosecuted with so much ardour.

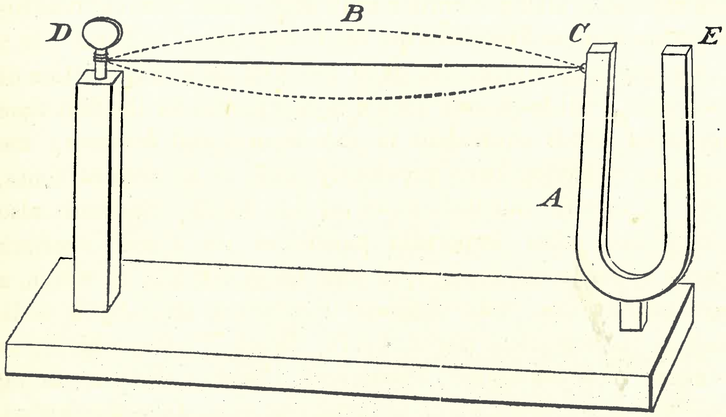
51. *Early Investigators.* Des Cartes (1618); Mersenne (1636), who gives an amusing account of his bewilderment, and wonders why the string does not always make the interval of an octave at each "*saut*"*;* Kircher (1650); Marsh, Pigot and Noble, as quoted by Dr. Plot (1676); Dr. Wallis (1677); the Hon. Francis Roberts (1692); Philippe de la Hire (1694), and many others, recorded their experiences, but not much real progress appears to have been made in the knowledge of the phenomena of string-vibration until 1697, when Sauveur read his *Traité de Musique Spéculative* at the *Collège Royal* in Paris. In a subsequent paper (1701) he gave the first clear explanation of that which had been until then a mystery, and for the first time appeared words equivalent to *node*, *segment* and *harmonic;* the latter term having been previously used, as in ancient times, only to signify musical tones of all kinds. Sauveur also contributed other important papers to the Paris *Académie Royale* (1700, 1702, 1707, 1711 and 1713), the last of which, a posthumous one, was followed two years later by a well-known mathematical paper, by Dr. Brook Taylor, which was printed in *Philosophical Transactions.* During the succeeding hundred years, most valuable results accrued from the investigations of D'Alembert, Euler, Dr. Robert Smith, Daniel Bernoulli, La Grange, Riccati, Dr. Thomas Young and others.

52. *An Experiment of Daniel Bernoulli* (1762) has a direct bearing on the production of sound in wind-instruments, as will be seen hereafter. Near one end of a horizontal string, twenty-four feet long, with such a tension that it oscillated once in a quarter of a second, he placed a vertical toothed wheel which grazed the string so that on the wheel being turned, each tooth gave a slight impulse to the string, and escaped. He found that if these impulses were timed to $\frac{1}{2}$ sec., 1 sec., $1\frac{1}{2}$ sec., or 2 sec., the vibrations of the string were regular and well sustained. When the strokes of the teeth occurred four times in a second,

the string divided into two segments; a further increase of speed compelled it to divide into three or more.

53. *Dr. Franz Melde*, of Marburg, devised a very ingenious and instructive modification of this experiment (1864), which I have ventured to simplify. Let a large tuning-fork, *A*, *Fig.* 4, be screwed into a slab of wood. Let a silk thread, *B*, be attached to one prong of the fork at *C*, and let the other end of the thread be wound round a moveable peg, *D*, set at a convenient distance. Let the fork be sounded by a bow at the prong *E*, and when the thread shall have received sufficient tension to enable it to oscillate once during each complete vibration of the fork, it will form a loop as shown by the dotted lines. If the tension of the thread be relaxed so as to cause it to vibrate at half its previous rate, it will divide into two segments. A further slackening will cause it to divide into three. Of course this process might be continued indefinitely, and the same effect would be produced if, instead of the tension of the thread being reduced, the fork could be made to vibrate twice or three times as rapidly as at first.

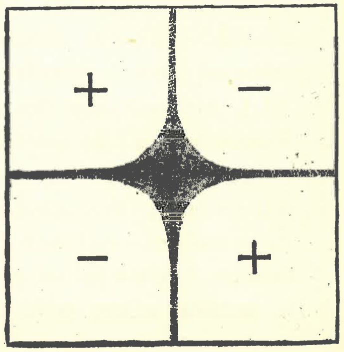
FIG. 4.



54. This experiment demonstrates that transverse vibrations may be induced by longitudinal impulses, and that the vibrations of a string may be excited at a node. The application of the principles of these two experiments to wind-instruments will be found in chapters III. and IV.

55. Harmonics of various sonorous bodies. Nodes and segments, with their corresponding harmonics, are formed not only in strings, but in all kinds of sonorous bodies. The investigation of the harmonics of rods, plates and bells, has long been a very attractive subject to philosophers, and few experiments have more pleasing results than the well-known *sand-figures* of Chladni (1802, 1809).

FIG. 5.



56. Professor Tyndall (1883, p. 138) has given a lucid account of these beautiful figures, from which I borrow his explanation of the production of one figure, which is sufficient for the purposes of this book. “A square plate of glass is held by a suitable clamp at its centre. The plate might be held with the finger and thumb, if they could only reach far enough. Scattering fine sand over the plate, the middle point of one of its edges is damped by touching it with the finger nail, and a bow is drawn across the edge of the plate, near one of its corners. The sand is tossed away from certain parts of the surface, and collects along two *nodal lines* which divide the large square into four smaller ones, as in *fig.* 5. This division of the plate corresponds to its deepest tone. The signs + and — employed in this figure denote that the two squares on which they occur are always moving in opposite directions. When the squares marked + are above the average level of the plate, those marked — are below it; and when those marked — are above the average level, those marked + are below it. The nodal lines

mark the boundaries of these opposing motions. They are the places of transition from the one motion to the other, and are therefore unaffected by either.”

57. The utilisation of the different sections of a vibrating plate for the purpose of exciting the vibrations of the air partially enclosed in a tube, is described in chapters III. and IV.

58. Sympathetic Vibrations. It has been explained in chapter I. that all sonorous bodies impart vibrations, of a similar rate to their own, to the surrounding air. These aerial vibrations are capable of communicating their motion to any sufficiently elastic bodies that can vibrate at the same rate, and these are then said to vibrate *in sympathy* with the original source of sound. If two strings, tuned in a certain relationship one to the other, be placed sufficiently near together, the sounding of one will infallibly cause the other to vibrate also. According to Dr. Robert Smith (1749), Galileo explains the cause of these vibrations “on the analogy of a heavy pendulum being set in motion by the least breath of the mouth, provided the blasts be often repeated, and keep exact time with the vibrations of the pendulum.” So the second string is made to vibrate by the reiterated synchronous impulses of the air caused by the vibration of the first: *non vi, sed sæpe cadendo.*

59. Sympathetic vibrations were known in early times. Des Cartes (1618) was well acquainted with them, as was also Mersenne (1636), who says: “The strings of the Lute and of other instruments sound without being touched when anyone sings in unison with them or with their octaves, as I have proved.” Kircher describes them minutely and classes them amongst the “miracles of Nature.” (1650, *Machinamentum* XI.) He not only describes experiments similar to those of Mersenne, but he shows that a string a *twelfth* higher than the untouched one will cause the parts of the latter to vibrate in unison with the higher sound. He mentions these facts as “*pene vulgare*,” and then goes on to describe an instrument with strings, “stretched on very sonorous wood,” which were tuned in perfect unison with the pipes of an organ. On the organ being played, the strings were seen and heard to vibrate in unison with its notes. This idea was probably taken from Mersenne.

60. About the year 1674, the Rev. Dr. Narcissus Marsh seems to have heard of these vibrations, and to have described them to two graduates of Oxford University, named Thomas Pigot and William Noble, who mentioned the discovery to Dr. Plot. The latter forthwith inserted an account of it in his *Natural History of Oxfordshire* (1676). Then made their first appearance in England, the "paper-riders," since so popular in the lecture-room, which keep their seats on the nodes of a string, but which are immediately unhorsed from the vibrating segments.

61. Dr. Wallis, in a letter to *Philosophical Transactions* (1677), gave the same account, somewhat elaborated, and appended a description of a similar experiment to that of Kircher, in which strings were made to vibrate in sympathy with the sounds of organ pipes. Not content with that, he inserted the same paper, translated into Latin, in his great work, *De Algebra Tractatus* (1693), "*non quod consimilis naturæ fit cum cæteris, sed ne pereat.*"

62. The vibrations of a string in sympathy with others that give sounds an octave, a twelfth, or other wider interval, higher than its own sound, may be explained on the analogy of the experiments of Bernoulli and Melde described above.

63. *The Fibres of the Ear.* De Mairan (1720) was the author of an idea which gives immense importance to sympathetic vibrations. I translate his words: "In the organ of hearing are found an infinity of fibres which by reason of their substance, their length and their tension, are susceptible of such diversity of vibrations that it is at least very probable that Nature makes use of this means to convey to us the variety of sensations of this kind that we experience." And he further explains his hypothesis by stating that each of these fibres may vibrate sympathetically with the sounds that are in unison with it, and thus convey its own particular rate of vibration to the auditory nerve. De Mairan does not attempt to establish any claim to the discovery of these fibres, but acknowledges his obligations to "*le docteur Winslow*," probably Jacques Bénigne Winslow, a celebrated anatomist of the time.

64. *The Marquis Corti's Organ.* Whether the fibres of that part of the ear known as "the organ of Corti" are the same as those mentioned by De Mairan, I cannot pretend to decide, but at any rate a similar use has been ascribed to them. The following extracts I translate from the French of the original paper (1851) which gives an elaborate description of that wonderful apparatus. "Its structure appears to partake, at the same time, of the physical properties of stretched membranes and those of a layer of cords stretched parallel to each other, and very close together. In effect, the cylindrical enlargements may be compared to the strings of a piano brought very near to each other, and soldered together." Corti calculates that these cords, or fibres, "number about 6,900 in mice and moles, 16,000 in cats, 20,600 in pigs and sheep, and 30,000 in mankind." As these figures have been frequently quoted incorrectly, I have been very careful to transcribe them exactly as given by the Marquis himself.

65. The theory as regards the use of these marvellous fibres is not generally received in the scientific world as conclusive, but whatever difficulties there may be in the acceptance of this proffered solution of one of the most profound mysteries of hearing, it cannot be denied that the idea is truly ingenious and absolutely reasonable, although the mind unaccustomed to the contemplation of such subjects, may shrink from the attempt to realize the exquisite wonder here presented.