Scheduling Theory **CO 454, Spring 2009**

Homework 1

Due: May 21st, 2009

1. Given precedence constraints for a machine scheduling problem as a directed graph, how would you check if the graph is acyclic (that is the precedence constraints are consistent)? What is the best time complexity (in the big-Oh notation) you can come up with?

2. Suppose γ_1 and γ_2 are regular performance measures. Which of the following are regular? If you say a measure is regular, prove it. If you say otherwise, give an example to show that it is not.
   - $w_1\gamma_1 + w_2\gamma_2$, for $w_1, w_2 \geq 0$.
   - $\gamma_1 - \gamma_2$.
   - $w_1\gamma_1 - w_2\gamma_2$, for $w_1 > w_2 \geq 0$.
   - $e^{c\gamma_1}$, for some $c > 0$. Here e is the base of the natural logarithm, $2.718\ldots$.
   - γ_1/γ_2 .

3. Given a schedule for a scheduling problem, let $N_u(t)$ denote the number of unfinished jobs at time t. Therefore, $N_u(0) = n$ and $N_u(C_{max}) = 0$. Consider the performance measure

$$\overline{N_u} := \frac{1}{C_{max}} \int_0^{C_{max}} N_u(t)dt$$

   Express this performance measure as a function of performance measures we studied in class. Is this performance measure regular? Why or why not?

4. Given an instance of a machine scheduling problem $(J \mid \mid \gamma)$ where γ is a regular performance measure, show that there exists an active schedule which is optimal.

5. In the *Traveling Salesman Problem* (TSP), we have a salesman who needs to start at his office at city 0 and visit n cities and come back to his office. The distance between city i and j for $0 \leq i \neq j \leq n$ is given as d_{ij}. The problem is to find a tour of the salesman with minimum total distance. Show that this problem is equivalent to a scheduling problem in the $(\alpha \mid \beta \mid \gamma)$ notation. (By equivalence, we mean solving one problem would imply the solution to another.)

6. Consider the following shop scheduling problem with 2 machines and 4 jobs.

Jobs	1	2	3	4
p_{1j}	8	6	4	12
p_{2j}	4	9	10	6

   Consider the two problems, $(F2 \mid \mid C_{max})$ and $(O2 \mid \mid C_{max})$, given the above data. By problem 4 above we know that there exists an optimal active schedule. How many active schedules are there for the above problem? Find optimal schedules for the above two problems and *argue* that your schedules are optimal. Try to make your argument as rigorous as you can. Arguments such as "I have tried all possible schedules" will not be considered rigorous and are not recommended. (Hint: Try to argue that C_{max} of any schedule must be at least some value depending only on the processing times and then construct the schedule)