# Scheduling Framework and Notation

## Notation

| | |
|---|---|
| $n$ | number of jobs |
| $m$ | number of machines |
| $p_{ij}$ | processing time of job $j$ on machine $i$ |
| $C_j$ | completion time of job $j$ |
| $w_j$ | weight/importance/priority of job $j$ |
| $r_j$ | release time/first availability of job $j$ |
| $d_j$ | due date of job $j$ |
| $L_j = C_j - d_j$ | lateness of job $j$ |
| $T_j = \max(L_j, 0)$ | tardiness of job $j$ |
| $U_j = \begin{cases} 1 & \text{if } C_j > d_j \\ 0 & \text{otherwise} \end{cases}$ | lateness indicator of job $j$ |
| $(\alpha \mid \beta \mid \gamma)$ | a scheduling problem with machine environment $\alpha$, constraints and characteristics $\beta$, and objective $\gamma$ |

## Machine Environments $\alpha$

| | |
|---|---|
| 1 | one machine |
| $P$ or $Pm$ | identical parallel machines |
| $Q$ or $Qm$ | parallel machines with different speeds: machine $i$ has speed $v_i$; job $j$ takes time $p_j/v_i$ on machine $i$ |
| $R$ or $Rm$ | unrelated parallel machines: machine $i$ processes job $j$ at speed $v_{ij}$ |
| $F$ or $Fm$ | flow shop: $m$ machines in series; each job is processed on each machine in order |
| $J$ or $Jm$ | job shop: each job is processed on a job-dependent sequence of machines |
| $O$ or $Om$ | open shop: processing order of machines for each job is determined by the scheduler |

## Constraints and Characteristics $\beta$

| | |
|---|---|
| $r_j$ | non-trivial release times |
| $s_{jk}(i)$ | set-up times of a machine $i$ between jobs $j$ and $k$. |
| $pmtn$ | preemptions are allowed: a job may be removed from a machine mid process |
| $prec$ | jobs have precedence constraints |

## Objectives $\gamma$

| | |
|---|---|
| $\sum C_j$ | sum of completion times |
| $\sum w_j C_j$ | weighted sum of completion times |
| $\sum T_j$ | sum of tardiness |
| $\sum w_j T_j$ | weighted sum of tardiness |
| $C_{\max} = \max_j\{C_j\}$ | makespan/schedule length |
| $L_{\max} = \max_j\{L_j\}$ | maximum lateness |
| $\sum U_j$ | number of late jobs |
| $\sum w_j U_j$ | weighted number of late jobs |