

## Shifting Bottleneck Heuristic

1. Start with  $M_0 = \emptyset$ ,  $G[M_0] \equiv$  fixed arcs + disj arcs for  $M_0$

2. For each machine  $i \in M - M_0$ , check the effect of adding  $i$ 's disjunctive edges on  $C_{\max}(M_0)$  as follows.

- Calculate  $r_j$  for each  $j$  as longest path from  $s$  to  $(i, j)$

- Calculate  $d_j$  for each  $j$  as ~~longest path from~~

$$C_{\max}(M_0) - (\text{longest path from } (i, j) \text{ to } t) + P_{ij}$$

- Solve  $(1|r_j|L_{\max})$  with this data. Let  $L_{\max}$  be  $L(i)$  and schedule be  $(j_1, j_2, \dots, j_n)$

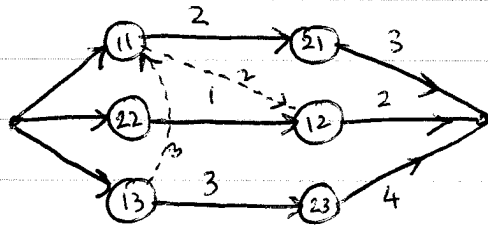
• Add machine  $i$  with largest  $L(i)$  to  $M_0$   
Add arcs  $(i, j_1), (j_1, j_2), (j_2, j_3), \dots, (j_{n-1}, j_n)$   
for machine  $i$

③ For every "old" machine,  $i' \in M_0 - i$ , ~~recalculate~~ <sup>re-evaluate</sup> their ~~orientation~~ orientation of their disjunctive arcs by solving another  $(1|r_j|L_{\max})$  problem.

4. End when  $M_0 = M$ .

# Shifting Bottleneck heuristic

- Decides which the orientation one machine at a time
- Start with  $M_0 = \emptyset$

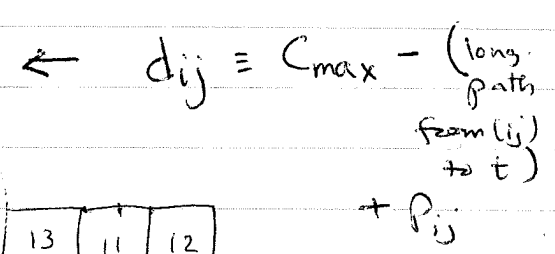


$C_{max}(M_0) \equiv$  Longest path from  $s-t$  where ~~disj.~~ edges corr. to  $M_0$  have been added

$C_{max}\{\emptyset\} = 7$

- If m/c 1 ~~some~~ disj. edges were to be added, how much would it affect  $C_{max}$ .

Think of a single m/c problem  
 $r_{11} = 0, r_{13} = 0, r_{12} = 1$   
 $d_{11} = 4, d_{13} = 3, d_{12} = 7$   
 $p_{11} = 2, p_{13} = 3, p_{12} = 2$

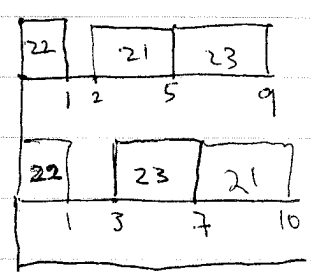


jobs: (11, 12, 13) ~~13, 11, 12~~  
 $L_{max} = 1$

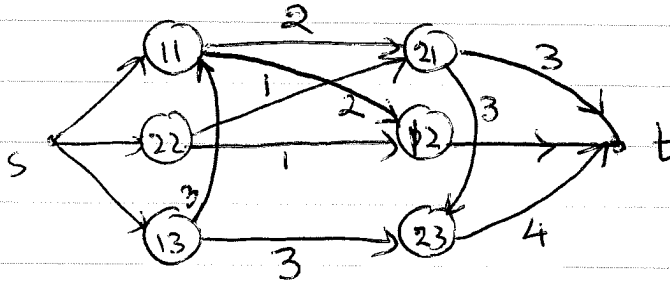
$C_{max}\{\emptyset, m1\} = 7 + 1 = 8$

- If m/c 2's disj. edges were to be added, how much would it affect  $C_{max}$

$r_{21} = 2, r_{22} = 0, r_{23} = 3$   
 $d_{21} = 7, d_{22} = 5, d_{23} = 7$   
 $p_{21} = 3, p_{22} = 1, p_{23} = 4$

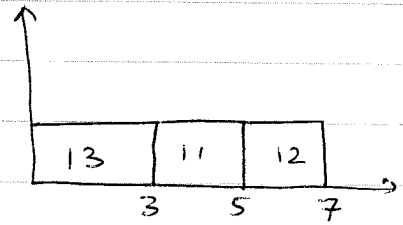


Add m/c 2 to  $M_0$



$$C_{max} \{ 2 \} = 9$$

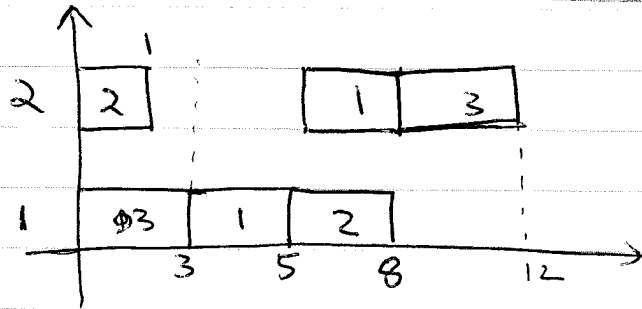
$r_{11} = 0$	$r_{12} = 1$	$r_{13} = 0$
$d_{11} = 6$	$d_{12} = 9$	$d_{13} = 5$
$p_{11} = 2$	$p_{12} = 2$	$p_{13} = 3$



13, 11, 12

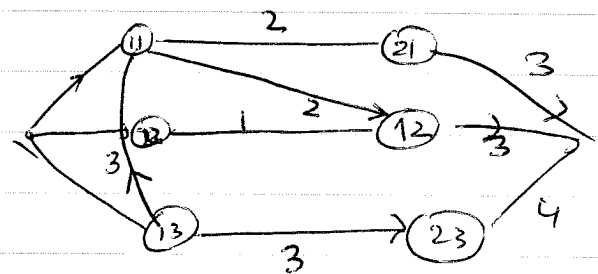
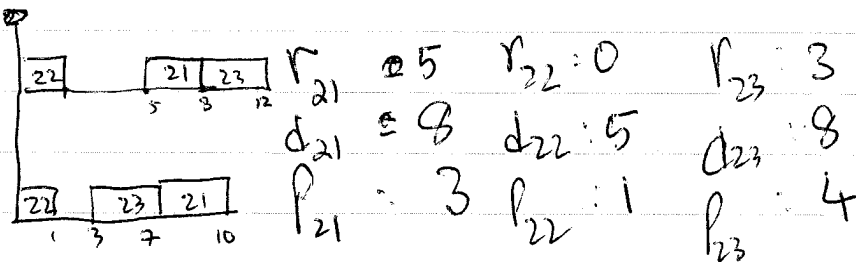
$$L_{max} = 0$$

$t_{13} = 0$
$t_{22} = 0$
$t_{11} = 3$
$t_{12} = 5$



$t_{21} = 5$
$t_{23} = 9$

Re-evaluation step :



$$C_{max} = 9$$