Logics of Emergent Computation

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Outline

- Background
- Consistency and Paraconsistency
- Kinds of Paraconsistent Logics
- Avenues of Exploration

History: George Boole

- Boolean *algebra*
  - A set
  - Two binary operators
  - One unary operator
- Basis for algebraic interpretations of logic

History: L. E. J. Brouwer and Arend Heyting

- Classical logic: “What assertions are *true*?”
- Intuitionistic logic: “What can we *construct*? Are we *justified* in doing so?”
- Classical logic: **A ^ ~A** is always false, **A v ~A** is always true
- Intuitionistic logic: **A ^ ~A** is still false, but **A v ~A** is not necessarily true!
- Classical negation: ~B means “B is false”
- Intuitionistic negation: ~B means “there exists a counterexample for B”

History: The Curry-Howard Correspondence

- *Types* correspond to *axiom schemas*
- K combinator: A -> B -> A (compare P ⊃ Q ⊃ P)
- S combinator: (A -> B -> C) -> (A -> B) -> A -> C
  (compare (P ⊃ Q ⊃ R) ⊃ (P ⊃ Q) ⊃ P ⊃ R
- Proving a proposition just means constructing a term with the type that corresponds to the proposition
- Different type theories from different logics!
  - Propositional -> function types (simply typed lambda calculus)
  - Predicate -> dependent types
  - Second-order -> parametric polymorphism

History: Per Martin-Löf and Intuitionistic Type Theory

- Dependent functions -> universal quantifier
- Dependent pairs -> existential quantifier
- BHK interpretation: what *constitutes* a proof of an assertion?
- Negation ~= "*p* implies a proposition that can't be fulfilled"
- MLTT gives rise to mechanised proof assistance

Snapshots from the Langsec Workshop: Bogk and Schöpl (2014)

- “Can we write a verified PDF parser in Coq?”
- Well ... kinda.
- A subset terminates. The entire thing doesn’t.
  - “And I’d have gotten away with it, too, if it weren’t for you crazy length fields!”
- Counterexample == nonterminating program on every PDF interpreter!
  - “The illusion that your program is manipulating its data is powerful, but it is an illusion. The data is controlling your program.”

Snapshots from the Langsec Workshop: Vanegue (2015)

- How do we ask the right questions?
- Well, it helps if you have language to ask them in
- So let's come up with a semantics for heap allocation
- "All problems in computer science can be solved by another layer of indirection."

What We Talk About When We Talk About Consistency

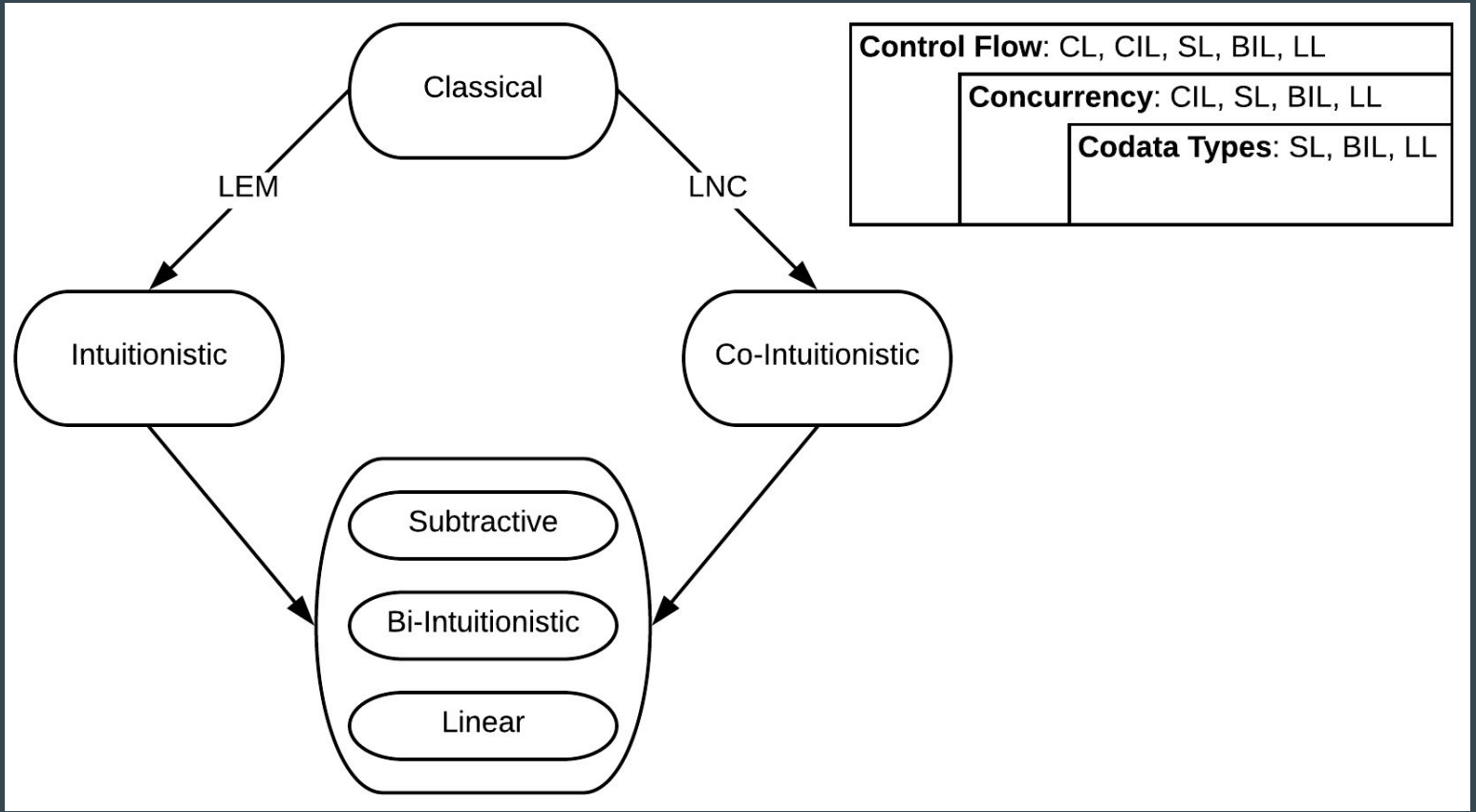
A ^ ~A -> B

(traditional notation)

A, ~A ⊢ B

(turnstile notation)

What We Talk About When We Talk About Paraconsistency



Bi-intuitionistic Logic

- Int *and* co-Int living under the same roof
- Both implication and co-implication
- Both classical negation and paraconsistent negation

Linear Logic

- *Substructural* logics: remove one or more structural rules
- Forbids contraction and weakening
- Instead of implication/coimplication, new operators!
  - Multiplicative conjunction, ⊗ (“times”) - simultaneous resource use, consumer-directed
  - Additive disjunction, ⊕ (“plus”) - alternative resource use, producer-directed
  - Additive conjunction, & (“with”) - alternative resource use, consumer-directed
  - Multiplicative disjunction, ⅋ (“par”) - simultaneous resource use, producer-directed
  - ! (“of course!”)
  - ? (“why not?”)
- Undecidable :(
  - But *affine logic* (which adds global weakening) is ok!
  - Ohai Rust

Subtractive Logic

- Subtraction as dual of implication
- A C-H correspondence has been demonstrated (Tristan Crolard)
  - I don't entirely understand it yet
  - But, it's an extension of the lambda-calculus, like the other correspondences
- What kinds of computation do these new calculi allow us to model more effectively?

Now What?

- Get our bearings. Exploits as dual of intended computation -> dualised logics for reasoning about interaction
  - Not all interaction is exploitative. But introduce interaction, and you introduce that possibility.
- Which maps best fit the territory?
- Category-theoretic interpretations
- Mechanise all the logics