# CS 10:
# Problem solving via Object Oriented Programming

Hashing

# Java provides us faster Sets and Maps using hashing instead of Trees

- Sets hold unique objects, Maps hold Key/Value pairs
- Map Keys are unique, but Values may be duplicated
- As we saw last class, using a Tree is a natural fit for implementing Sets and Maps
- Performance with a Tree is *generally* better than a List
- We can do better than Tree performance by using today's topic of discussion – hashing
- Java provides the HashSet and HashMap out-of-the-box that do a lot of the hard work for us

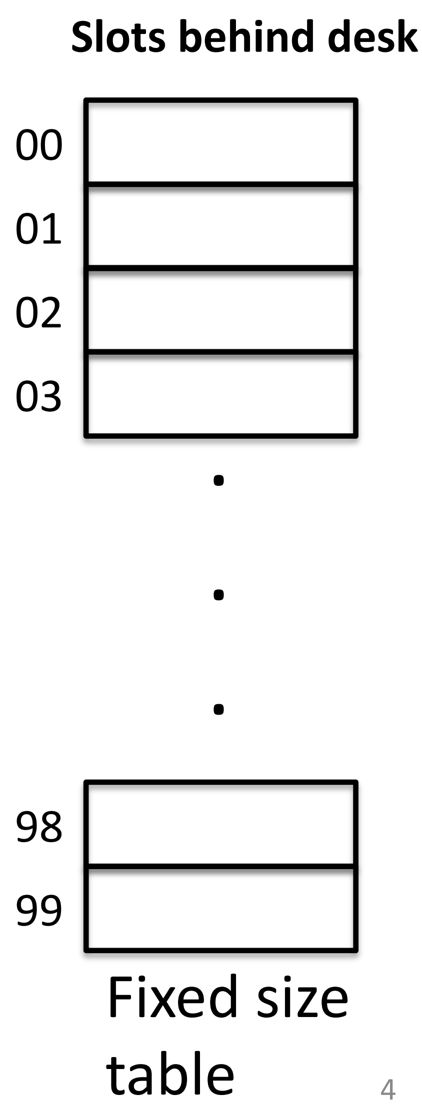
# Agenda

1. Hashing
2. Computing Hash functions
3. Implementing Maps/Sets with hashing
4. Handling collisions
    1. Chaining
    2. Open Addressing

# The old Sears catalog orders illustrate how hashing works

**Sears store implementation of hash table**

- Used to have 100 slots behind order desk, 0…99
- Shipments arrive, details of where item stored in warehouse put in slot by last two digits of customer phone number (e.g., 03)



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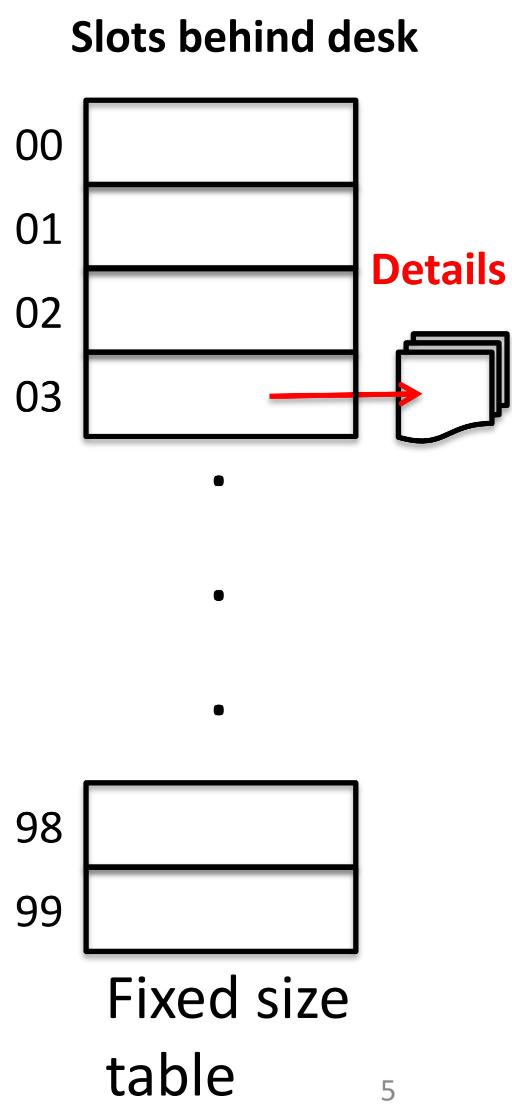
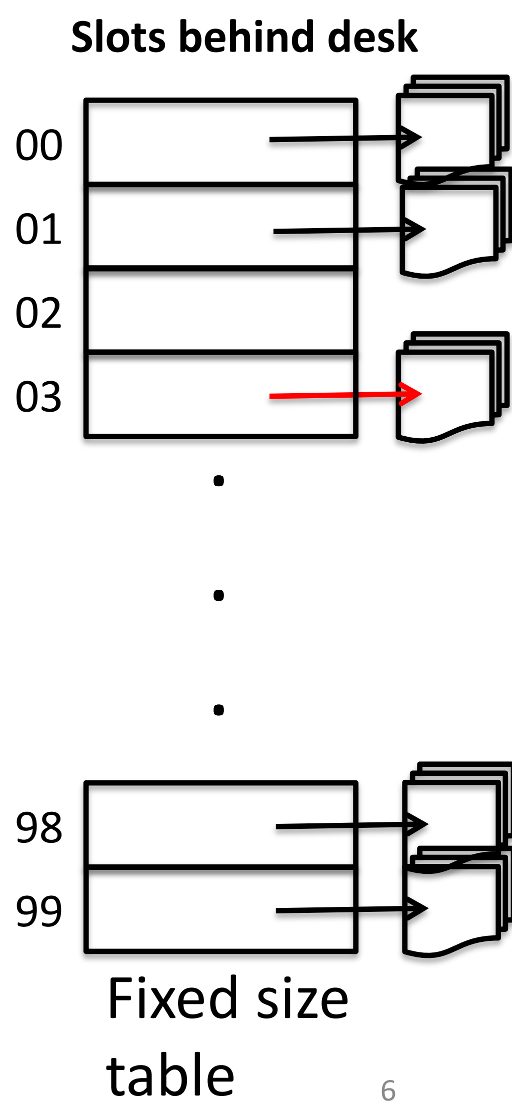


# The old Sears catalog orders illustrate how hashing works

**Sears store implementation of hash table**

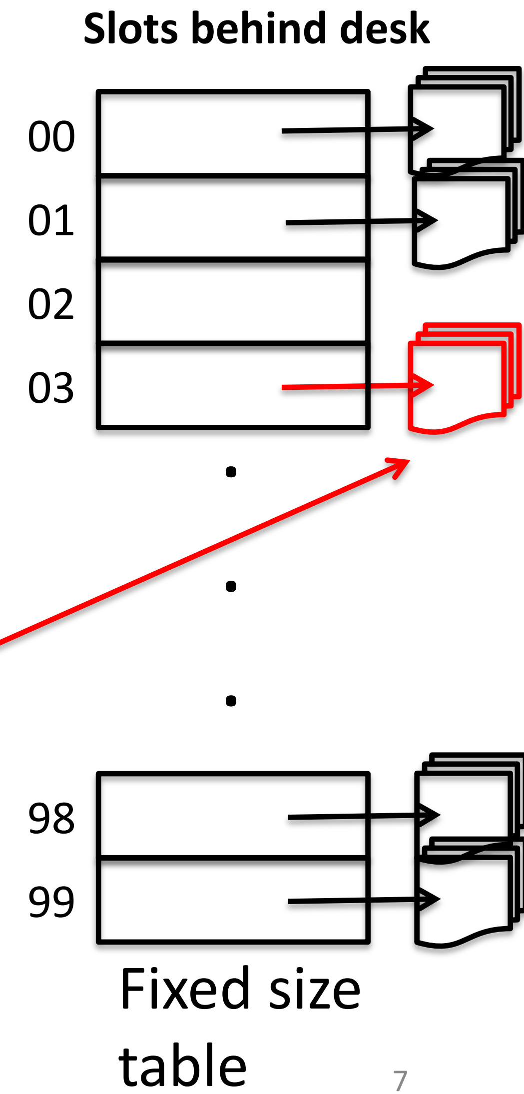
- Used to have 100 slots behind order desk, 0…99
- Shipments arrive, details of where item stored in warehouse put in slot by last two digits of customer phone number (e.g., 03)
- Customer arrives, gives last two digits of phone



# The old Sears catalog orders illustrate how hashing works

**Sears store implementation of hash table**

- Used to have 100 slots behind order desk, 0…99
- Shipments arrive, details of where item stored in warehouse put in slot by last two digits of customer phone number (e.g., 03)
- Customer arrives, gives last two digits of phone
- Clerk finds slot with that two-digit number
- Clerk searches contents of that slot only
- Could be multiple orders, but can find the order quickly because only a few orders in slot

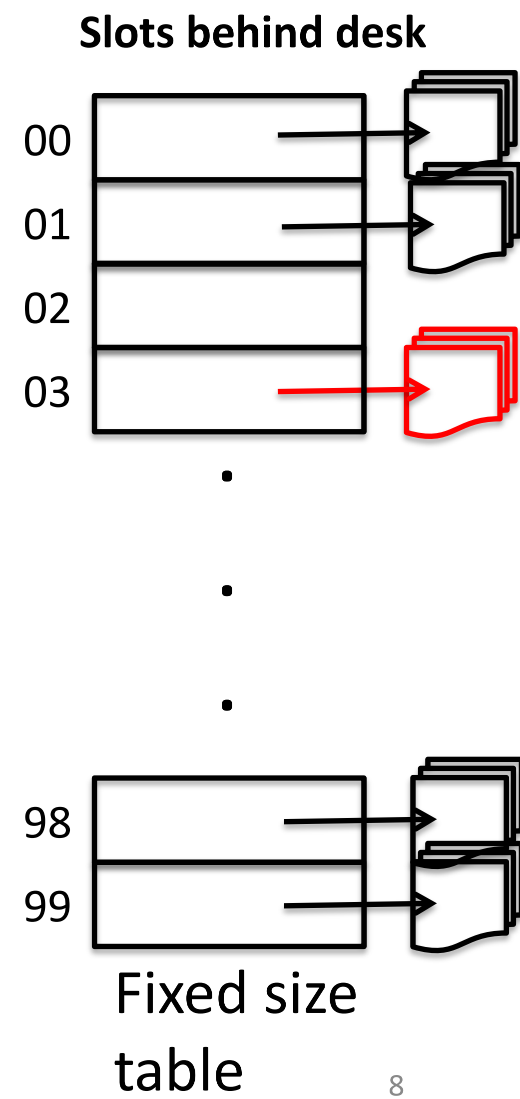



***Search only these orders, skip the rest***

# The old Sears catalog orders illustrate how hashing works

**Sears store implementation of hash table**

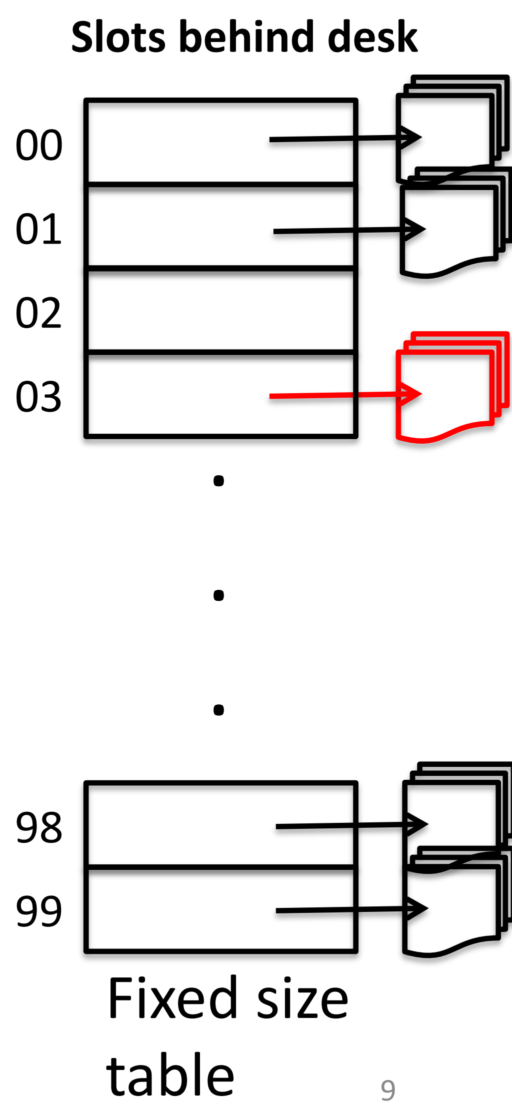
- Used to have 100 slots behind order desk, 0…99
- Shipments arrive, details of where item stored in warehouse put in slot by last two digits of customer phone number (e.g., 03)
- Customer arrives, gives last two digits of phone
- Clerk finds slot with that two-digit number
- Clerk searches contents of that slot only
- Could be multiple orders, but can find the order quickly because only a few orders in slot
- Splits set of (possibly) hundreds or thousands of orders into 100 slots of a few items each



# The old Sears catalog orders illustrate how hashing works

**Sears store implementation of hash table**

- Used to have 100 slots behind order desk, 0…99
- Shipments arrive, details of where item stored in warehouse put in slot by last two digits of customer phone number (e.g., 03)
- Customer arrives, gives last two digits of phone
- Clerk finds slot with that two-digit number
- Clerk searches contents of that slot only
- Could be multiple orders, but can find the order quickly because only a few orders in slot
- Splits set of (possibly) hundreds or thousands of orders into 100 slots of a few items each
- Trick: find a hash function that spreads customers evenly
- Last two digits work, why not first two?

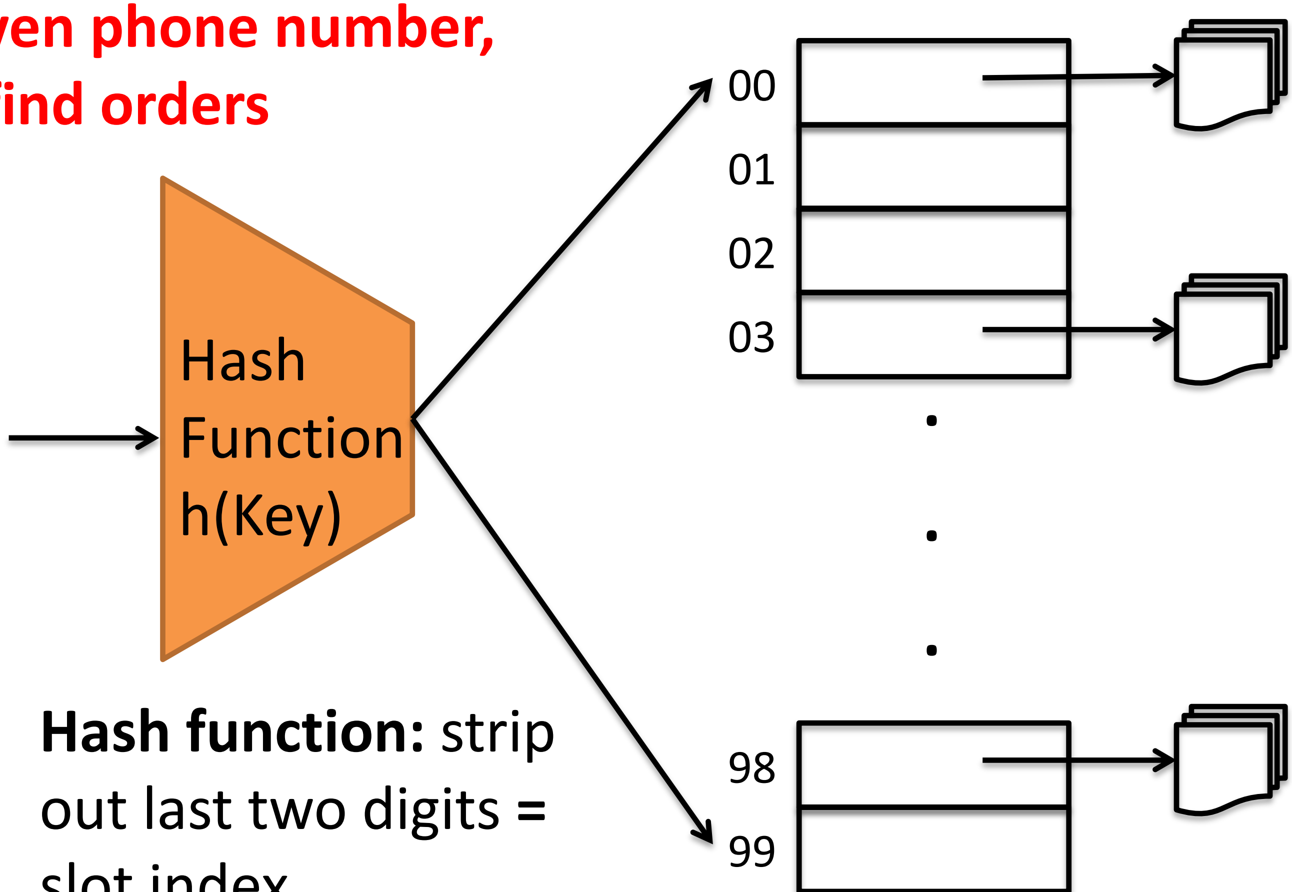


# The store is using a form of hashing based on customer’s phone number

**Hashing phone numbers to find orders**

**Goal: given phone number, quickly find orders**

**Input:** Phone number (Key)



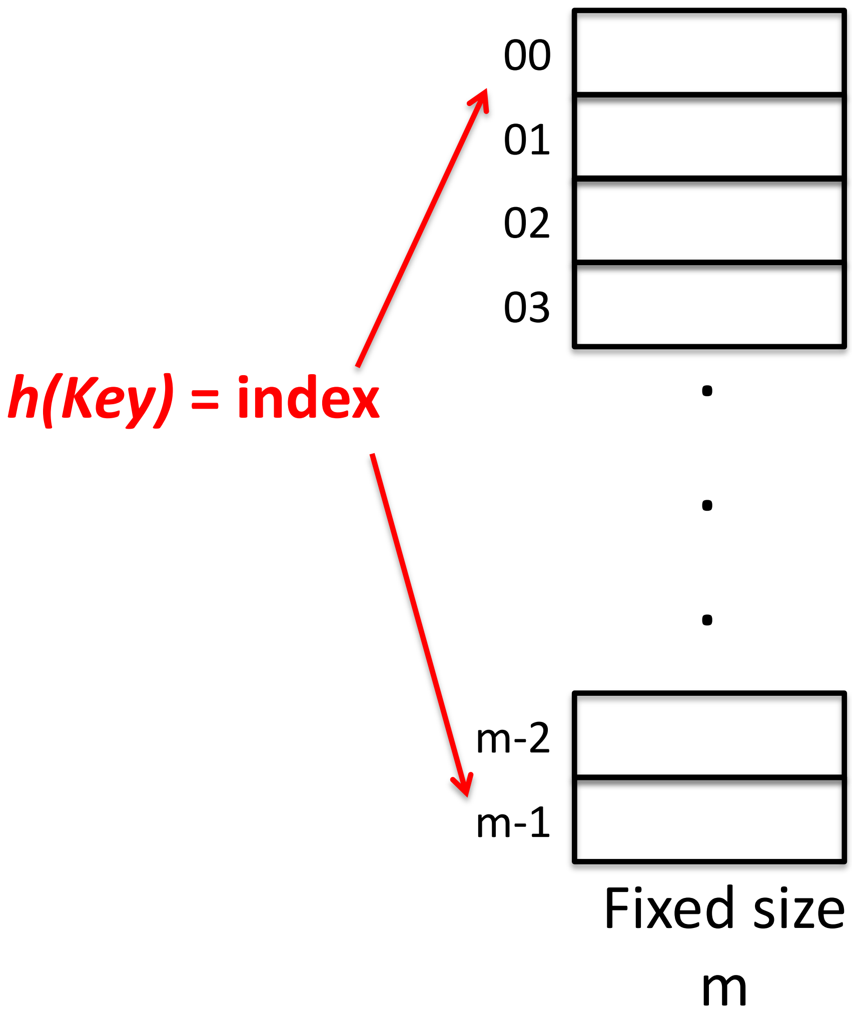

Search only small number of orders

**Hash function:** strip out last two digits = slot index

Fixed size table

Customer orders

# Hashing's big idea: map a Key to an array index, then access is fast




**Map hash table implementation**

- Begin with array of fixed size *m* (called a hash table)
- Each array index holds item we want to find (e.g., warehouse location of customer's order)
- Use hash function *h* on *Key* to give index into hash table
- ***h(Key)* = table index *i = 0..m-1***
- Get item from hash table at index given by hash function
- *Fast* to *get/set/add/remove* items
- What about a HashSet?
- Use object itself as Key
- How to hash Key or object?

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# Good hash functions map keys to indexes in table with three desirable properties

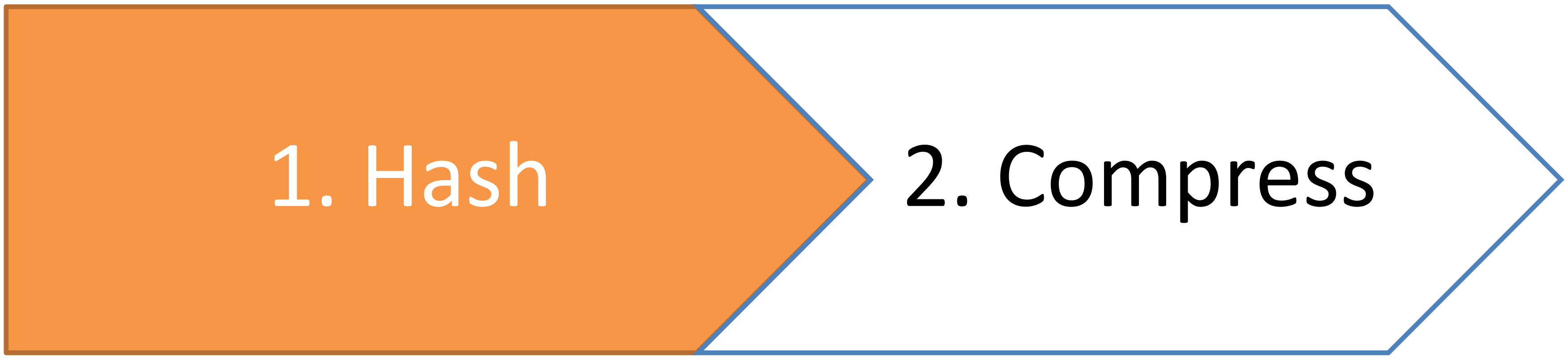
**Desirable properties of a hash function**

1. Hash can be computed quickly and consistently
2. Hash spreads the universe of keys evenly over the table (simple uniform hashing)
3. Small changes in the key (e.g., changing a character in a string or order of letters) should result in different hash value

**Cryptographic hash function also:**

- Difficult to determine key given the result of hash
- Unlikely that different keys will result in same hash
- We will not focus on crypto requirements

# Hashing is often done in two steps: hash then compress




- Get an integer representation of Key
- Integer could be in range –infinity to +infinity

Constrain integer to table index [0..m)

# First step in hashing is to get an integer representation of the key

**Goal: given key compute an index into hash table array**

**Some Java objects can be directly cast to integers**

- byte
- short
- int
- char

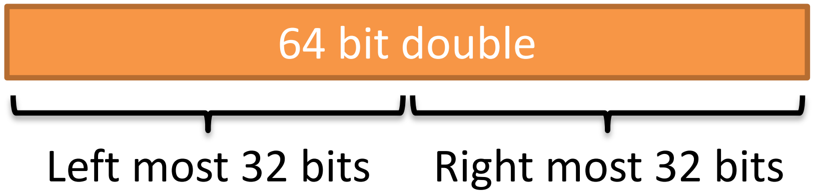
```
char a = 'a';
int b = (int)a;
```

**b = 97**

**Some items too long cast to integers**

- double (64 bits)
- long (64 bits)

- Too long to make 32 bit integers




**XOR each half**

# Complex objects such as Strings can also be hashed to a single integer

**Hashing complex objects**

- Consider String x of length n where $x = x_0x_1\ldots x_{n-2}x_{n-1}$
- Pick prime number a (book recommends 31, 37, or 41)
- Cast each character in x to an integer
- Calculate polynomial hashcode as $x_0a^{n-1} + x_1a^{n-2} + \ldots x_{n-2}a + x_{n-1}$
- Use Horner's rule to efficiently compute hash code

```
public int hashCode() {
    final int a=37;
    int sum = x[0]; //first item in array
    for (int j=1;j<n;j++) {
        sum = a*sum + x[j]; //array element j
    }
    return sum;
}
```

- Experiments show that when using a as above, 50,000 English words had fewer than 7 collisions

# Good news: Java provides a *hashCode()* method to compute hashes for us!

***hashCode()***
Java does the hashing for us for Strings and autoboxed types with *hashCode()* method

```
Character a = ‘a’;
a.hashCode() returns 97

String b = “Hello”;
b.hashCode() returns 69609650
```

# Bad news: We need to override *hashCode()* and *equals()* for our own Objects

- By default Java uses memory address of objects as a *hashCode*
- But we typically want to hash based on properties of object, not whatever memory location an object happened to be assigned
- This way two objects with same instance variables will hash to the same table location (those objects are considered equal)
- Java says that two *equal* objects must return same *hashCode()*

```
public class BlobHash extends Blob{

    @Override
    public boolean equals(Object otherBlob) {
        Blob b = (Blob)otherBlob; //cast as Blob
        if (x == b.x && y == b.y && r == b.r)
            return true;
        return false;
    }

    @Override
    public int hashCode() {
        final int a=37;
        int sum = a * a * (int)x;
        sum += a * (int)y;
        sum += (int)r;
        return sum;
    }
}
```

**Here we consider two Blobs *equal* if they have the same *x, y* and *r* values**
**<u>*equals()* IS THE RIGHT WAY TO COMPARE OBJECT EQUALITY</u> (not ==)**

**Override *hashCode()* to provide the same hash if two Blobs are *equal***

**If don't override *hashCode()* then even though two objects are considered equal, Java will look in the wrong slot**

# Java *hashCode()* example

**hashCode()**

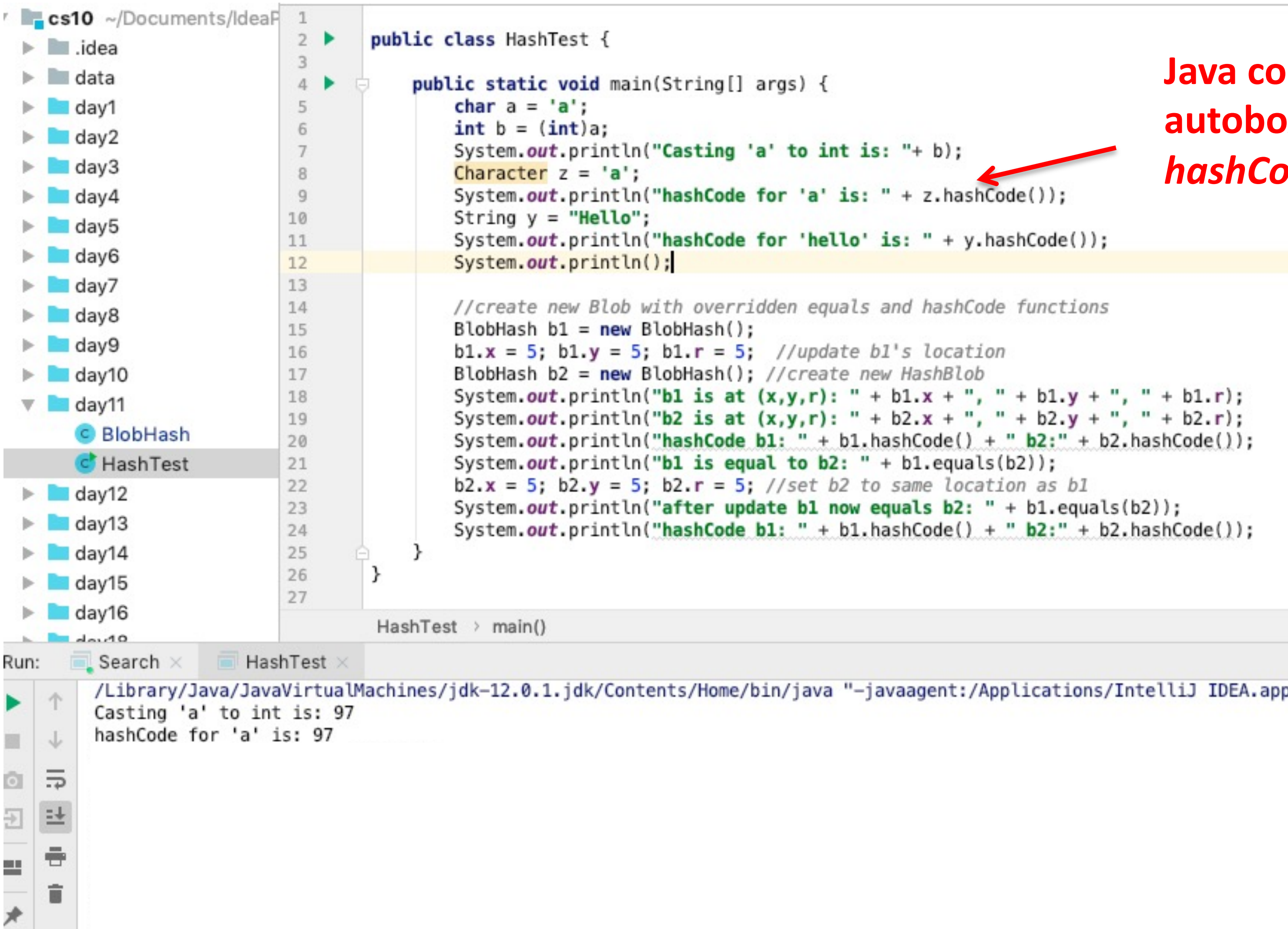
```java
public class HashTest {

    public static void main(String[] args) {
        char a = 'a';
        int b = (int)a;
        System.out.println("Casting 'a' to int is: "+ b);
        Character z = 'a';
        System.out.println("hashCode for 'a' is: " + z.hashCode());
        String y = "Hello";
        System.out.println("hashCode for 'hello' is: " + y.hashCode());
        System.out.println();

        //create new Blob with overridden equals and hashCode functions
        BlobHash b1 = new BlobHash();
        b1.x = 5; b1.y = 5; b1.r = 5;  //update b1's location
        BlobHash b2 = new BlobHash(); //create new HashBlob
        System.out.println("b1 is at (x,y,r): " + b1.x + ", " + b1.y + ", " + b1.r);
        System.out.println("b2 is at (x,y,r): " + b2.x + ", " + b2.y + ", " + b2.r);
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
        System.out.println("b1 is equal to b2: " + b1.equals(b2));
        b2.x = 5; b2.y = 5; b2.r = 5; //set b2 to same location as b1
        System.out.println("after update b1 now equals b2: " + b1.equals(b2));
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
    }
}
```

**Some types can be directly cast to an integer**

```
/Library/Java/JavaVirtualMachines/jdk-12.0.1.jdk/Contents/Home/bin/java "-javaagent:/Applications/IntelliJ IDEA.app/
Casting 'a' to int is: 97
```

# Java *hashCode()* example

**hashCode()**




**Java computes hash for autoboxed types with *hashCode()***

# Java *hashCode()* example

**hashCode()**

cs10 ~/Documents/IdeaP
- .idea
- data
- day1
- day2
- day3
- day4
- day5
- day6
- day7
- day8
- day9
- day10
- day11
  - BlobHash
  - HashTest
- day12
- day13
- day14
- day15
- day16

```java
public class HashTest {

    public static void main(String[] args) {
        char a = 'a';
        int b = (int)a;
        System.out.println("Casting 'a' to int is: "+ b);
        Character z = 'a';
        System.out.println("hashCode for 'a' is: " + z.hashCode());
        String y = "Hello";
        System.out.println("hashCode for 'hello' is: " + y.hashCode());
        System.out.println();

        //create new Blob with overridden equals and hashCode functions
        BlobHash b1 = new BlobHash();
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        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
        System.out.println("b1 is equal to b2: " + b1.equals(b2));
        b2.x = 5; b2.y = 5; b2.r = 5; //set b2 to same location as b1
        System.out.println("after update b1 now equals b2: " + b1.equals(b2));
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
    }
}
```

HashTest › main()

Run: Search × HashTest ×

```
/Library/Java/JavaVirtualMachines/jdk-12.0.1.jdk/Contents/Home/bin/java "-javaagent:/Applications/IntelliJ IDEA.app/
Casting 'a' to int is: 97
hashCode for 'a' is: 97
hashCode for 'hello' is: 69609650
```

***hashCode()*** **also works for more complex built-in types**

# Java *hashCode()* example

## hashCode()

**For our own objects, we can provide our own *hashCode()* otherwise we get the memory location by default**

```
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  .idea
  data
  day1
  day2
  day3
  day4
  day5
  day6
  day7
  day8
  day9
  day10
  day11
    BlobHash
    HashTest
  day12
  day13
  day14
  day15
  day16
```

```java
public class HashTest {

    public static void main(String[] args) {
        char a = 'a';
        int b = (int)a;
        System.out.println("Casting 'a' to int is: "+ b);
        Character z = 'a';
        System.out.println("hashCode for 'a' is: " + z.hashCode());
        String y = "Hello";
        System.out.println("hashCode for 'hello' is: " + y.hashCode());
        System.out.println();

        //create new Blob with overridden equals and hashCode functions
        BlobHash b1 = new BlobHash();
        b1.x = 5; b1.y = 5; b1.r = 5;  //update b1's location
        BlobHash b2 = new BlobHash(); //create new HashBlob
        System.out.println("b1 is at (x,y,r): " + b1.x + ", " + b1.y + ", " + b1.r);
        System.out.println("b2 is at (x,y,r): " + b2.x + ", " + b2.y + ", " + b2.r);
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
        System.out.println("b1 is equal to b2: " + b1.equals(b2));
        b2.x = 5; b2.y = 5; b2.r = 5; //set b2 to same location as b1
        System.out.println("after update b1 now equals b2: " + b1.equals(b2));
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
    }
}
```

HashTest › main()

Run: Search × HashTest ×

```
/Library/Java/JavaVirtualMachines/jdk-12.0.1.jdk/Contents/Home/bin/java "-javaagent:/Applications/IntelliJ IDEA.app/
Casting 'a' to int is: 97
hashCode for 'a' is: 97
hashCode for 'hello' is: 69609650

b1 is at (x,y,r): 5.0, 5.0, 5.0
b2 is at (x,y,r): 0.0, 0.0, 5.0
hashCode b1: 7035 b2:5
```

# Java *hashCode()* example

## hashCode()

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        Character z = 'a';
        System.out.println("hashCode for 'a' is: " + z.hashCode());
        String y = "Hello";
        System.out.println("hashCode for 'hello' is: " + y.hashCode());
        System.out.println();

        //create new Blob with overridden equals and hashCode functions
        BlobHash b1 = new BlobHash();
        b1.x = 5; b1.y = 5; b1.r = 5;  //update b1's location
        BlobHash b2 = new BlobHash(); //create new HashBlob
        System.out.println("b1 is at (x,y,r): " + b1.x + ", " + b1.y + ", " + b1.r);
        System.out.println("b2 is at (x,y,r): " + b2.x + ", " + b2.y + ", " + b2.r);
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
        System.out.println("b1 is equal to b2: " + b1.equals(b2));
        b2.x = 5; b2.y = 5; b2.r = 5; //set b2 to same location as b1
        System.out.println("after update b1 now equals b2: " + b1.equals(b2));
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
    }
}
```

HashTest › main()

Run: Search × HashTest ×

```
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b1 is at (x,y,r): 5.0, 5.0, 5.0
b2 is at (x,y,r): 0.0, 0.0, 5.0
hashCode b1: 7035 b2:5
```

For our own objects, we can provide our own *hashCode()* otherwise we get the memory location by default

```java
@Override
public int hashCode() {
    final int a=37;
    int sum = a * a * (int)x;
    sum += a * (int)y;
    sum += (int)r;
    return sum;
}
```

*hashCode()* should compute hash:

1. Quickly and consistently
2. Spread keys evenly
3. Small changes = different hash

# Java *equals()* example

**equals()**

**Override *equals()* to test if objects are equivalent**
**Otherwise *equals()* checks if same memory location**

```java
@Override
public boolean equals(Object otherBlob) {
    Blob b = (Blob)otherBlob; //cast as Blob
    if (x == b.x && y == b.y && r == b.r)
        return true;
    return false;
}
```

```
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    HashTest
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  day13
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```

```java
public class HashTest {

    public static void main(String[] args) {
        char a = 'a';
        int b = (int)a;
        System.out.println("Casting 'a' to int is: "+ b);
        Character z = 'a';
        System.out.println("hashCode for 'a' is: " + z.ha
        String y = "Hello";
        System.out.println("hashCode for 'hello' is: " +
        System.out.println();

        //create new Blob with overridden equals and hash
        BlobHash b1 = new BlobHash();
        b1.x = 5; b1.y = 5; b1.r = 5;  //update b1's location
        BlobHash b2 = new BlobHash(); //create new HashBlob
        System.out.println("b1 is at (x,y,r): " + b1.x + ", " + b1.y + ", " + b1.r);
        System.out.println("b2 is at (x,y,r): " + b2.x + ", " + b2.y + ", " + b2.r);
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
        System.out.println("b1 is equal to b2: " + b1.equals(b2));
        b2.x = 5; b2.y = 5; b2.r = 5; //set b2 to same location as b1
        System.out.println("after update b1 now equals b2: " + b1.equals(b2));
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
    }
}
```

HashTest › main()

Run: Search × HashTest ×

```
/Library/Java/JavaVirtualMachines/jdk-12.0.1.jdk/Contents/Home/bin/java "-javaagent:/Applications/IntelliJ IDEA.app/
Casting 'a' to int is: 97
hashCode for 'a' is: 97
hashCode for 'hello' is: 69609650

b1 is at (x,y,r): 5.0, 5.0, 5.0
b2 is at (x,y,r): 0.0, 0.0, 5.0
hashCode b1: 7035 b2:5
b1 is equal to b2: false
```

# Java *equals()* example

**equals()**

**Override *equals()* to test if objects are equivalent**
**Otherwise *equals()* checks if same memory location**

```
@Override
public boolean equals(Object otherBlob) {
    Blob b = (Blob)otherBlob; //cast as Blob
    if (x == b.x && y == b.y && r == b.r)
        return true;
    return false;
}
```

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public class HashTest {

    public static void main(String[] args) {
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        Character z = 'a';
        System.out.println("hashCode for 'a' is: " + z.ha
        String y = "Hello";
        System.out.println("hashCode for 'hello' is: " +
        System.out.println();

        //create new Blob with overridden equals and hash
        BlobHash b1 = new BlobHash();
        b1.x = 5; b1.y = 5; b1.r = 5;  //update b1's location
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        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
        System.out.println("b1 is equal to b2: " + b1.equals(b2));
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        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
    }
}
```

HashTest › main()

Run: Search × HashTest ×

```
/Library/Java/JavaVirtualMachines/jdk-12.0.1.jdk/Contents/Home/bin/java "-javaagent:/Applications/IntelliJ IDEA.app/
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b1 is at (x,y,r): 5.0, 5.0, 5.0
b2 is at (x,y,r): 0.0, 0.0, 5.0
hashCode b1: 7035 b2:5
b1 is equal to b2: false
```

**This is the right way to compare if two objects are equivalent (not b1 == b2)**

# Java *equals()* example

**equals()**

**Override *equals()* to test if objects are equivalent**
**Otherwise *equals()* checks if same memory location**

- cs10 ~/Documents/IdeaP
  - .idea
  - data
  - day1
  - day2
  - day3
  - day4
  - day5
  - day6
  - day7
  - day8
  - day9
  - day10
  - day11
    - BlobHash
    - HashTest
  - day12
  - day13
  - day14
  - day15
  - day16

```java
public class HashTest {

    public static void main(String[] args) {
        char a = 'a';
        int b = (int)a;
        System.out.println("Casting 'a' to int is: "+ b);
        Character z = 'a';
        System.out.println("hashCode for 'a' is: " + z.ha
        String y = "Hello";
        System.out.println("hashCode for 'hello' is: " +
        System.out.println();

        //create new Blob with overridden equals and hash
        BlobHash b1 = new BlobHash();
        b1.x = 5; b1.y = 5; b1.r = 5;  //update b1's location
        BlobHash b2 = new BlobHash(); //create new HashBlob
        System.out.println("b1 is at (x,y,r): " + b1.x + ", " + b1.y + ", " + b1.r);
        System.out.println("b2 is at (x,y,r): " + b2.x + ", " + b2.y + ", " + b2.r);
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
        System.out.println("b1 is equal to b2: " + b1.equals(b2));
        b2.x = 5; b2.y = 5; b2.r = 5; //set b2 to same location as b1
        System.out.println("after update b1 now equals b2: " + b1.equals(b2));
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
    }
}
```

HashTest › main()

```java
@Override
public boolean equals(Object otherBlob) {
    Blob b = (Blob)otherBlob; //cast as Blob
    if (x == b.x && y == b.y && r == b.r)
        return true;
    return false;
}
```

Run: Search × HashTest ×

```
/Library/Java/JavaVirtualMachines/jdk-12.0.1.jdk/Contents/Home/bin/java "-javaagent:/Applications/
Casting 'a' to int is: 97
hashCode for 'a' is: 97
hashCode for 'hello' is: 69609650

b1 is at (x,y,r): 5.0, 5.0, 5.0
b2 is at (x,y,r): 0.0, 0.0, 5.0
hashCode b1: 7035 b2:5
b1 is equal to b2: false
after update b1 now equals b2: true
```

**This is the right way to compare if two objects are equivalent (not b1 == b2)**

**After updating *x,y,* and *r* two Blobs are now equal**

# Java *equals()* example

**equals()**

**Override *equals()* to test if objects are equivalent**
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    BlobHash
    HashTest
  day12
  day13
  day14
  day15
  day16
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b1 is equal to b2: false
after update b1 now equals b2: true
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```

**This is the right way to compare if two objects are equivalent (not b1 == b2)**

**After updating *x,y,* and *r* two Blobs are now equal**

***hashCode()* also returns the same value for equivalent objects**

# Java *equals()* example

**equals()**

**Override *equals()* to test if objects are equivalent**
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```java
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Run: Search × HashTest ×

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hashCode b1: 7035 b2:5
b1 is equal to b2: false
after update b1 now equals b2: true
hashCode b1: 7035 b2:7035
```

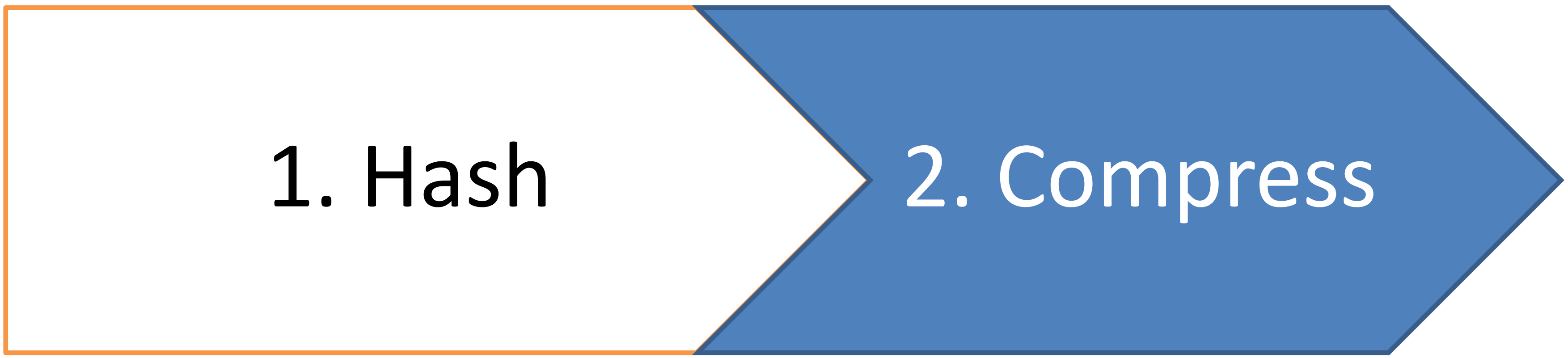
**This is the right way to compare if two objects are equivalent (not b1 == b2)**

**After updating *x,y,* and *r* two Blobs are now equal**

**HashMap and HashSet will now put equivalent objects in the same slot in the table (after compression)**

***hashCode()* also returns the same value for equivalent objects**

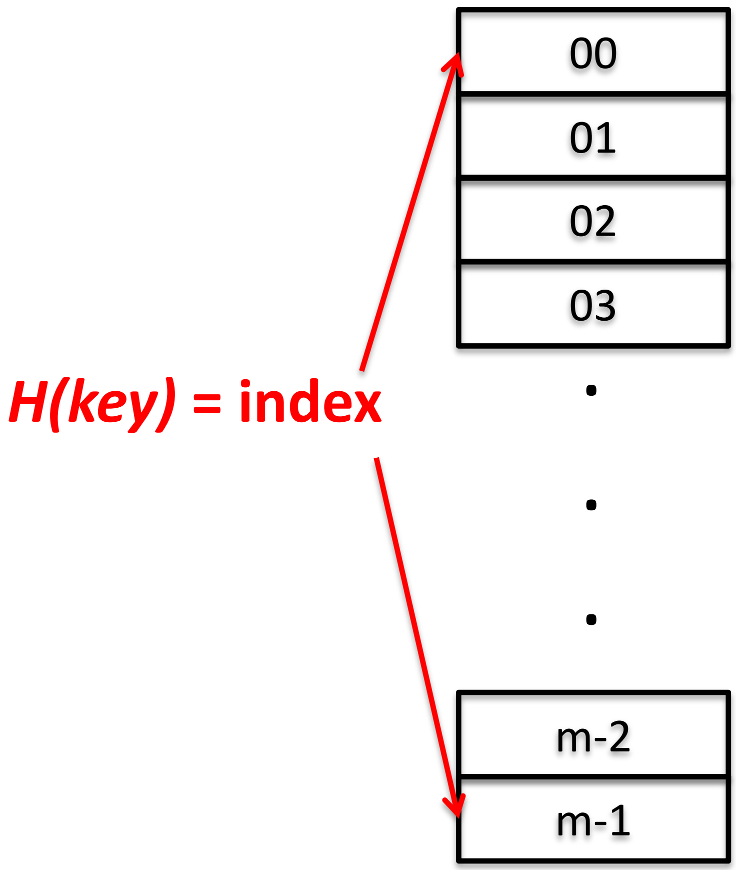
# Hashing is often done in two steps: hash then compress




- Get an integer representation of Key
- Integer could be in range –infinity to +infinity

Constrain integer to table index [0..m)

# May have to compress hash value to table index [0..m)




Fixed size *m* hash table

**Compressing**

- *hashCode()* value may be larger than the table (or negative!)
- Need to constrain value to one of the table slots [0..*m*)
- "Division method" is simple:
  *h(key) = key.hashCode() % m*
- Works well if *m* is prime
- Book gives a more advanced version called Multiply-Add-And-Divide (MAD)
- Java takes care of this for us ☺
- Eventually will encounter collisions where multiple keys map to the same slot ☹

# Agenda

1. Hashing
2. Computing Hash functions
3. Implementing Maps/Sets with hashing
4. Handling collisions
   1. Chaining
   2. Open Addressing

# Map methods can be easily implemented with hashing

***put(key, value)***

- Hash key to get table index
  - Get i=key.hashCode()
  - Compress i to 0..m-1
- Store key/value

***get(key)***

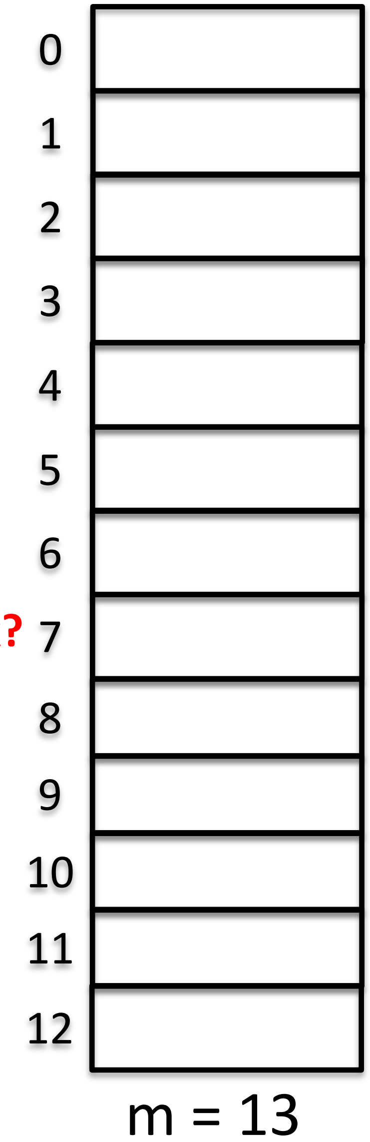
- Hash key to get table index
  - Get i=key.hashCode()
  - Compress i to 0..m-1
- Return stored value

***remove(key)***

- Hash key to get table index
  - Get i=key.hashCode()
  - Compress i to 0..m-1
- Remove stored key/value

**Open questions:**

- **What if multiple items hash to the same index?**
- **What if table fills up?**



# Agenda

1. Hashing
2. Computing Hash functions
3. Implementing Maps/Sets with hashing
4. Handling collisions
    1. Chaining
    2. Open Addressing

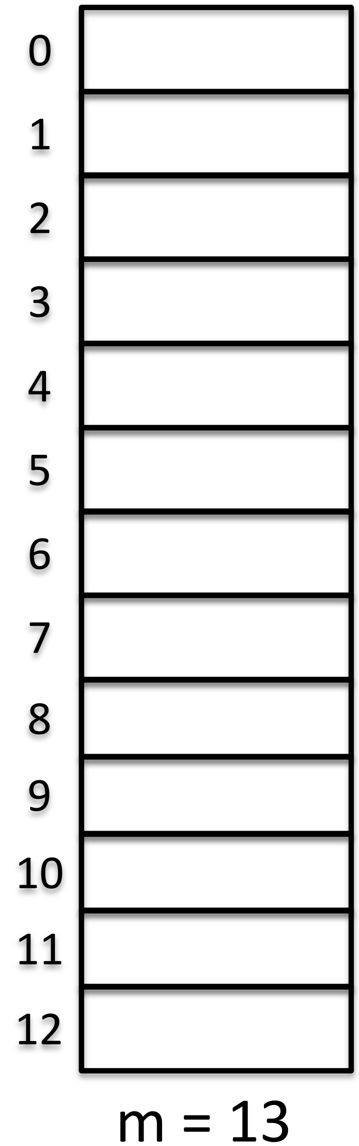
# Collisions happen when multiple keys map to the same table index

**Integer keys**

Given table size m = 13
put(key,value)

- Hash & constrain key
- Store value at index

**index = key.*hashCode()* % *m***



# Collisions happen when multiple keys map to the same table index

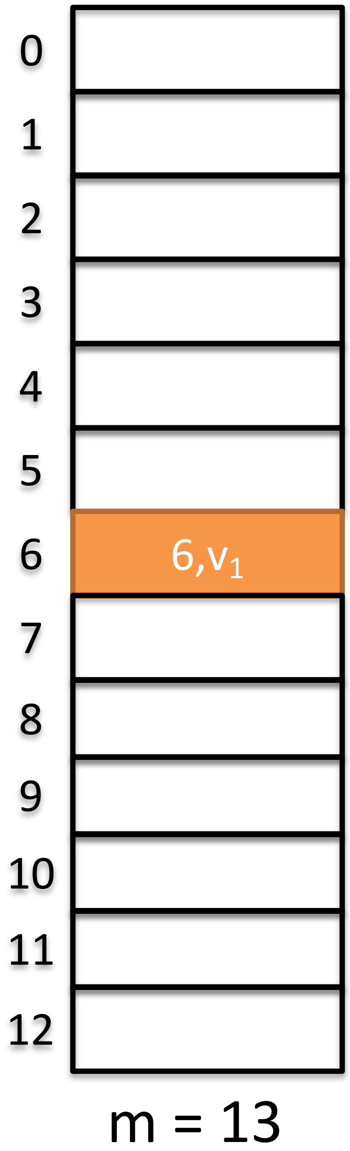
**Integer keys**

Given table size m = 13
put(key,value)

- Hash & constrain key
- Store value at index

index = key.*hashCode() % m*
Example

- **put(6,$v_1$) = 6 % 13 = 6**



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**Integer keys**

Given table size m = 13

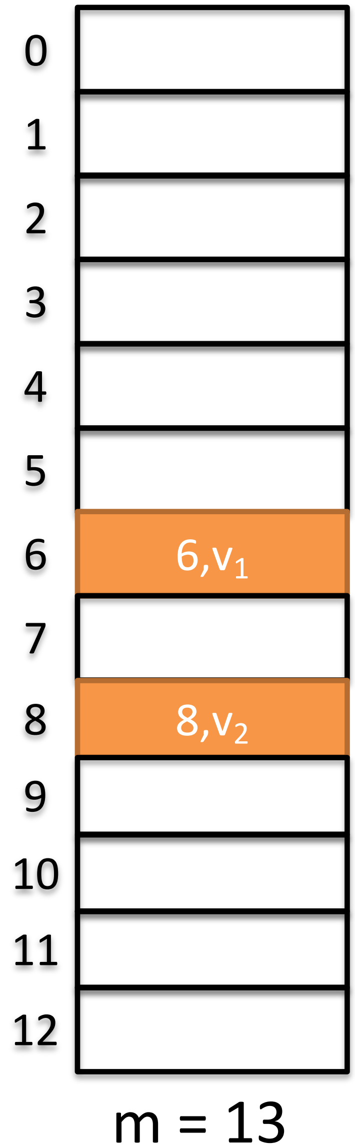
put(key,value)

- Hash & constrain key
- Store value at index

index = key.*hashCode() % m*

Example

- put(6,$v_1$) = 6 % 13 = 6
- **put(8,$v_2$) = 8 % 13 = 8**



# Collisions happen when multiple keys map to the same table index

**Integer keys**

Given table size m = 13

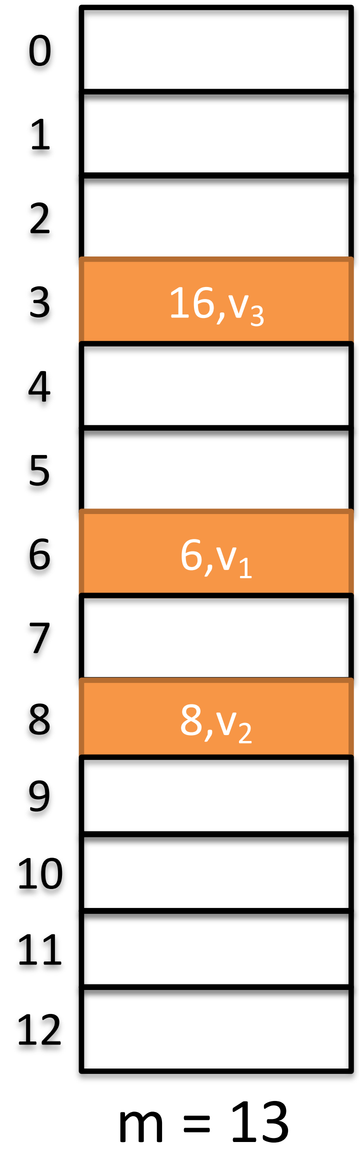
put(key,value)

- Hash & constrain key
- Store value at index

index = key.*hashCode() % m*

Example

- put(6,$v_1$) = 6 % 13 = 6
- put(8,$v_2$) = 8 % 13 = 8
- **put(16,$v_3$) = 16 % 13 = 3**



# Collisions happen when multiple keys map to the same table index

**Integer keys**

Given table size m = 13

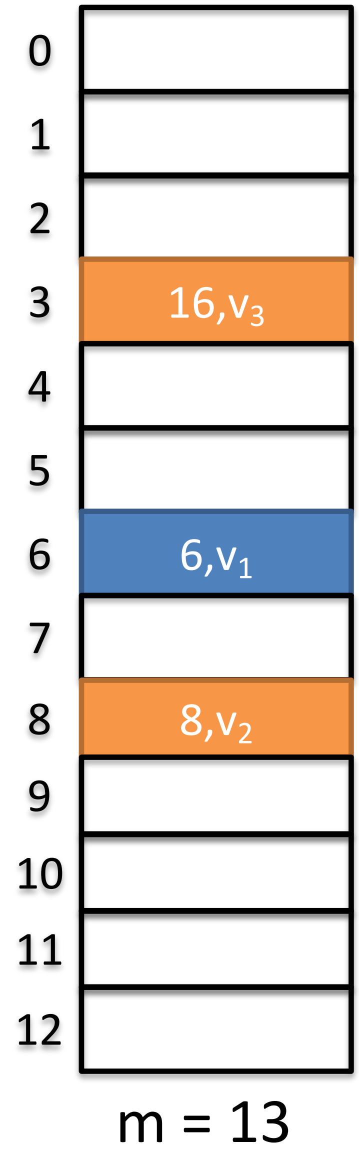
put(key,value)

- Hash & constrain key
- Store value at index

index = key.*hashCode() % m*

Example

- put(6,$v_1$) = 6 % 13 = 6
- put(8,$v_2$) = 8 % 13 = 8
- put(16,$v_3$) = 16 % 13 = 3
- **put(19,$v_4$) = 19 % 13 = 6**




Collision!

6 and 19 mapped to the same index

h(6)=h(19)

# Agenda

1. Hashing

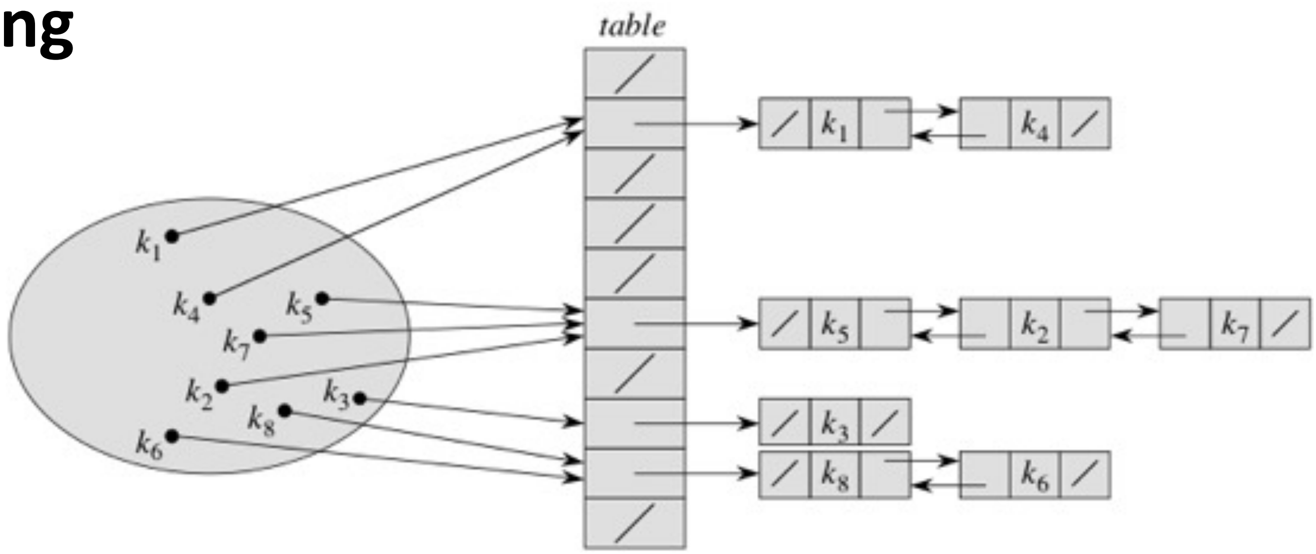
2. Computing Hash functions

3. Implementing Maps/Sets with hashing

1. Handling collisions
   1. Chaining
   2. Open Addressing

# Chaining handles collisions by creating a linked list for each table entry

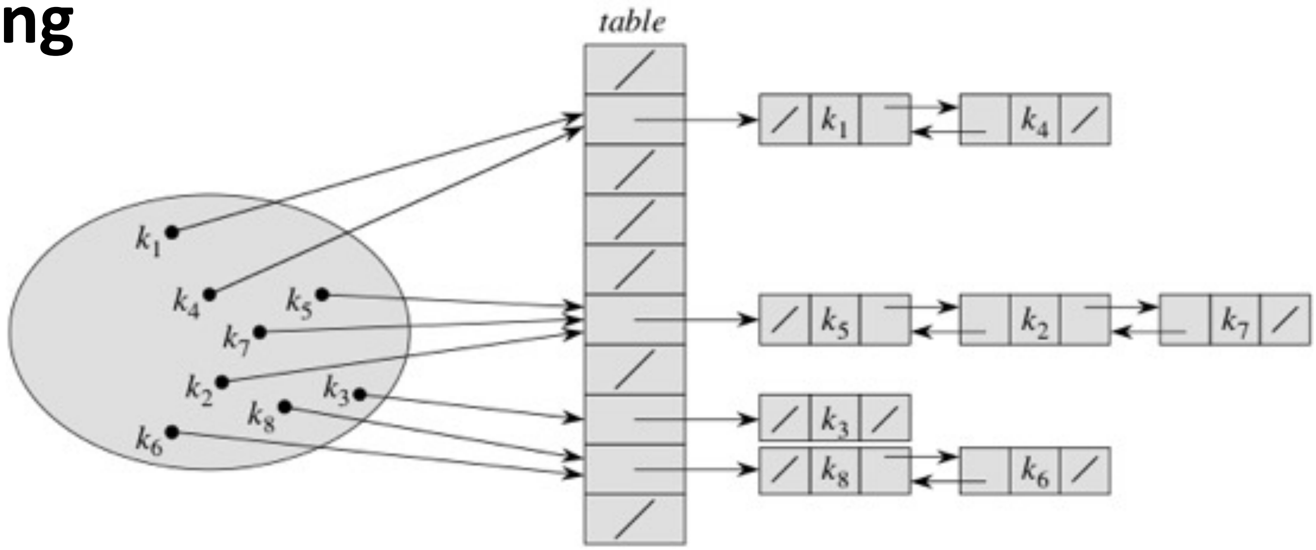
**Chaining**




- Create a table pointing to linked list of items that hash to the same index (similar to last class word positions)
- Slot *i* holds all keys *k* for which *h(k) = i*
- Splice in new elements at head
- NOTE: Values associated with Keys are not shown, here just showing Keys

# Load factor measures number of items in the list that must be searched on average

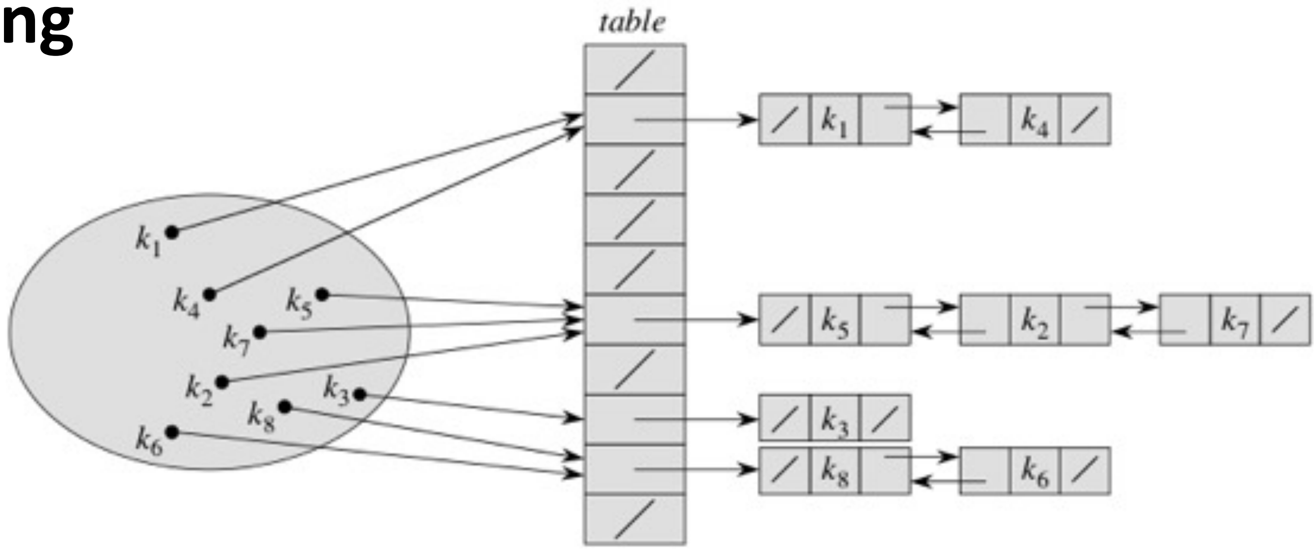
**Chaining**




- Assume table with *m* slots and *n* keys are stored in it
- On average, we expect *n/m* elements per collision list
- This is called the ***load factor*** *(λ=n/m)*
- <u>*Expected*</u> search time is *Θ(1+λ)*, assuming ***simple uniform hashing*** (each possible key equally likely to hash into a particular slot), worst case *Θ(n)* if bad hash function

# If the load factor gets too high, then we should increase the table size

**Chaining**




- If *n* (# elements) becomes larger than *m* (table size), then collisions are inevitable and search time goes up
- Java increases <u>*table size*</u> by 2X and *rehashes* into new table when $\lambda > 0.75$ to combat this problem
- Problem: memory fragmentation with link lists spread out all over, might not be good for embedded systems

# Agenda

1. Hashing

2. Computing Hash functions

3. Implementing Maps/Sets with hashing

1. Handling collisions
   1. Chaining
   2. Open Addressing

# Open addressing is different solution, everything is stored in the table itself

**Open addressing using linear probing**

- Insert item at hashed index (no linked list)
- For key *k* compute *h(k)=i,* insert at index *i*
- If collision, a simple solution is called ***linear probing***
  - Try inserting at *i+1*
  - If slot *i+1* full, try *i+2*… until find empty slot
  - Wrap around to slot 0 if hit end of table at *m-1*
  - If $\lambda < 1$ will find empty slot
  - If $\lambda \approx 1$, increase table size (*m*2*) and rehash
- Search analogous to insertion, compute key and probe until find item or empty slot (key not in table)

# Linear probing is one way of handling collisions under open addressing

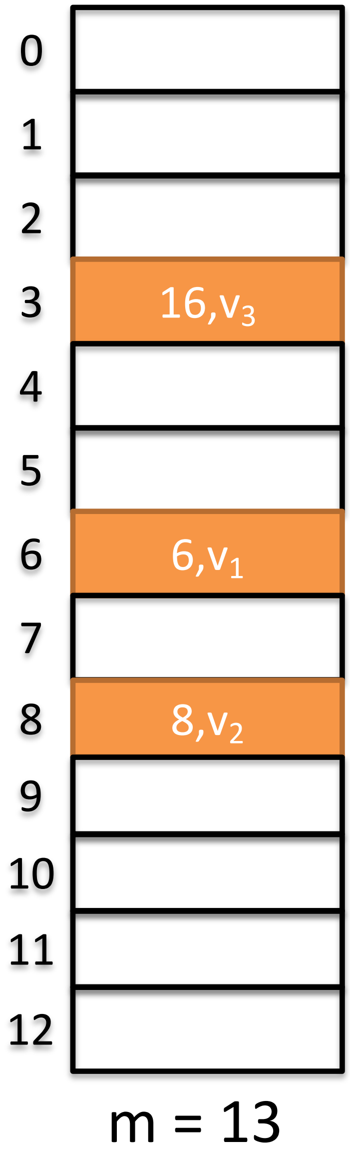
**Integer keys**

Given table size m = 13

index = key.*hashCode() % m*

Example

- put(6,$v_1$) = 6 % 13 = 6
- put(8,$v_2$) = 8 % 13 = 8
- put(16,$v_3$) = 16 % 13 = 3



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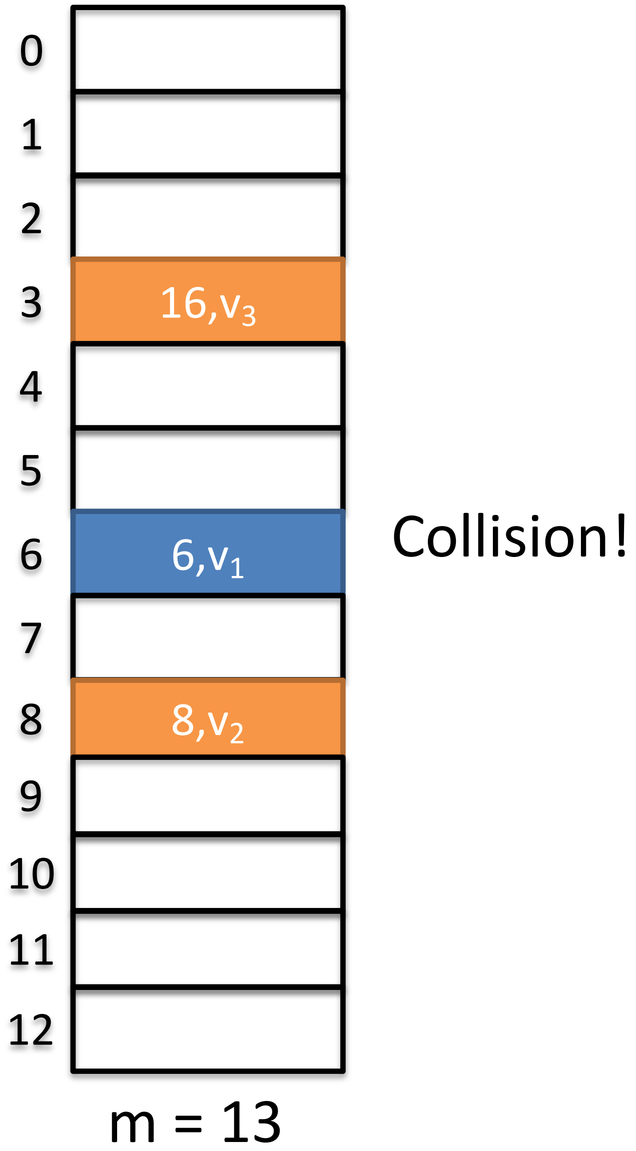

# Try next index if hashed index is full, repeat if next index is also full

**Integer keys**

Given table size m = 13

index = key.*hashCode() % m*

Example

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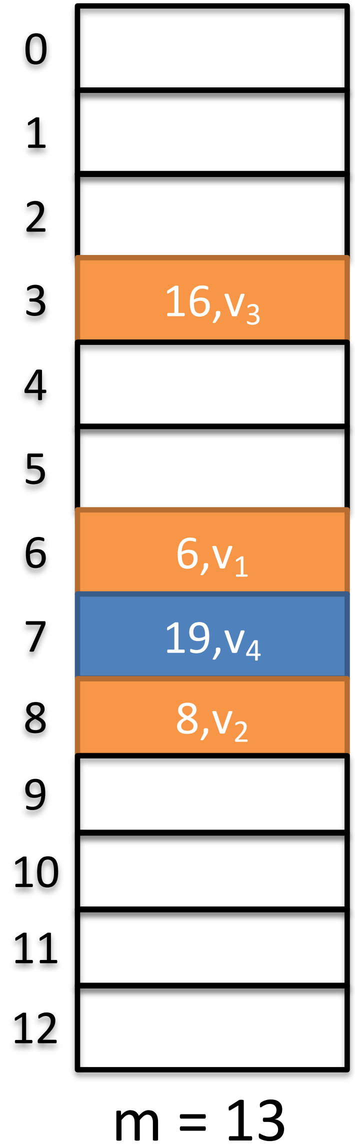


Insert at *i+1* = 7

# To find items, probe until find Key or hit an empty space

**Integer keys**

Given table size m = 13

index = key.*hashCode() % m*

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- put(6,$v_1$) = 6 % 13 = 6
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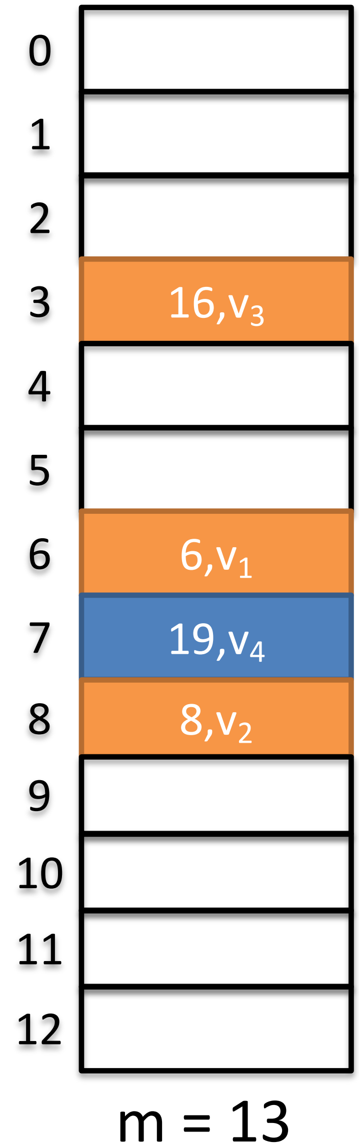
- **get(19)**




Insert at *i+1* = 7

**To find items later, hash to table index, then probe until find item or hit empty slot**

# Deleting items is tricky, need to mark deleted spot as available but not empty

**Problems deleting items under linear probing**

- Insert $k_1$, $k_2$, and $k_3$ where $h(k_1)=h(k_2)=h(k_3)$
- All three keys hash to the same slot in this example
- $k_1$ in slot *i*, $k_2$ in slot *i+1*, $k_3$ in slot *i+2*
- Remove $k_2$, creates hole at *i+1*
- Search for $k_3$
  - Hash $k_3$ to *i*, slot *i* holds $k_1 \neq k_3$, advance to slot *i+1*
  - Find hole at *i+1*, assume $k_3$ not in hash table
- Can mark deleted spaces as available for insertion, and search skips over marked spaces
- This can be a problem if many deletes create many marked slots, search approaches linear time

# Clustering of keys can build up and reduce performance

**Clustering problem**

- Long runs of occupied slots (clusters) can build up increasing search and insert time
- Clusters happen because empty slot preceded by $t$ full slots gets filled with probability $(t+1)/m$, instead of $1/m$ (e.g., $t$ keys can now fill open slot instead of just 1 key)
- Clusters can bump into each other exacerbating the problem

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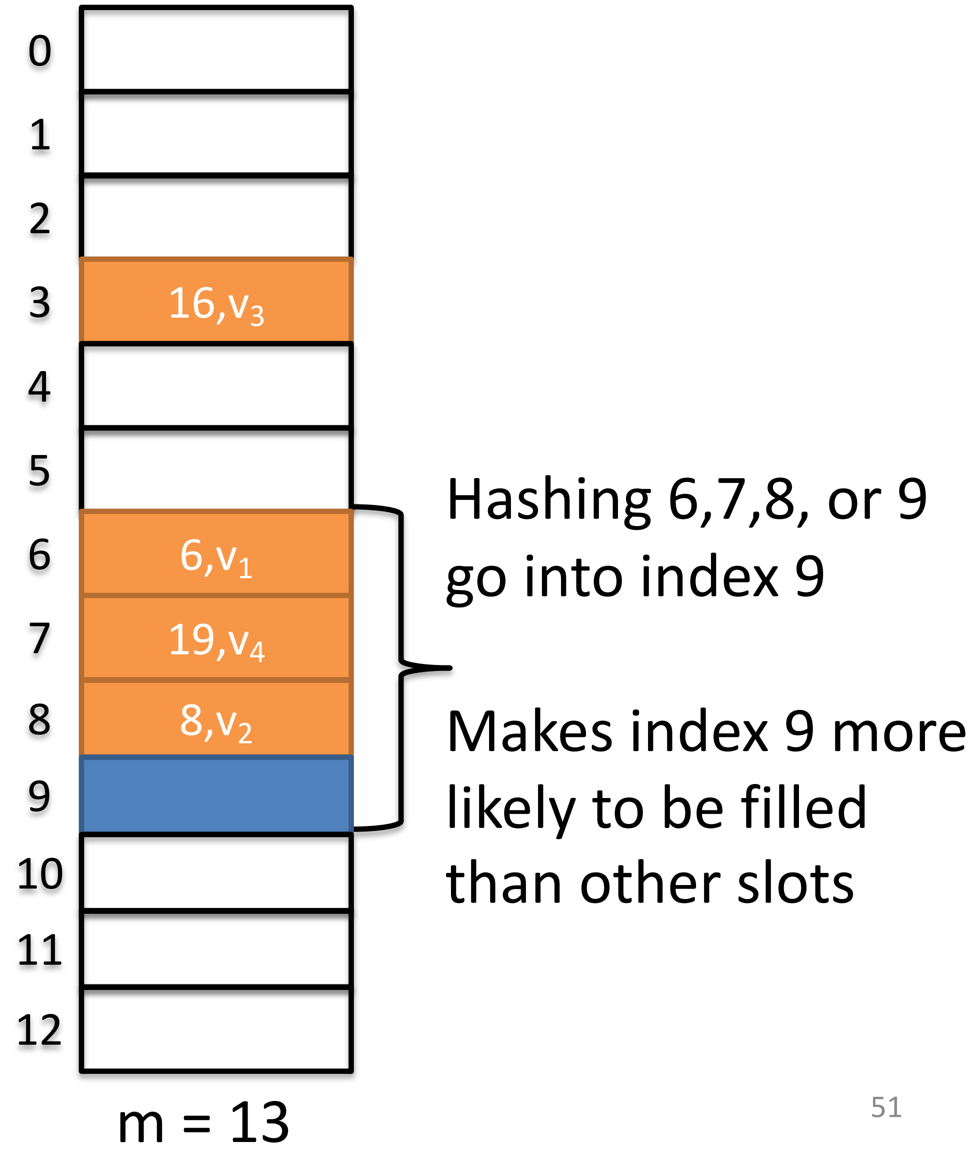


# Double hashing can help with the clustering problem

**Double hashing**

- **Big idea: instead of stepping by 1 at each collision like linear probing, step by a different amount where the step size depends on the key**
- Use two hash functions $h_1$ and $h_2$ to make a third *h'*
- $h'(k,p)=(h_1(k) + ph_2(k))$ mod $m$, where $p$ number of probes
- First probe $h_1(k)$, $p=0$, then p incremented by 1 on each collision until space is found
- Result is a step by $h_2(k)$ on each collision (then mod $m$ to stay inside table size), instead of 1
- Need to design hashes so that if $h_1(k_1)=h_1(k_2)$, then ***unlikely*** $h_2(k_1)=h_2(k_2)$

# Double hashing can help with the clustering problem

**Integer keys**

**$h_1$ same as before**
**$h_2$ new hash function**
**p = probe number (initially 0)**

Given table size m = 13

Compute

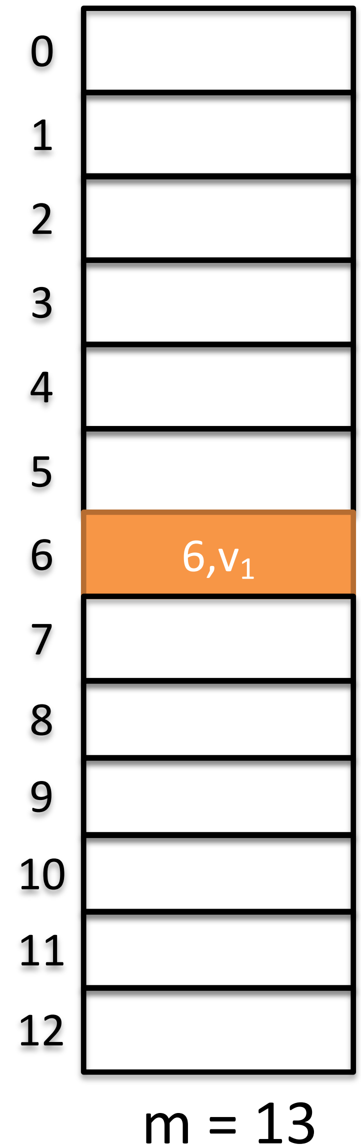
$h_1(key) = (key \% m)$

$h_2(key) = 1 + (key \% (m-1))$

$h'(k,p)=(h_1(k) + ph_2(k)) \% m$

**Example**

| Key | p | $h_1$ | $h_2$ | h' |
|---|---|---|---|---|
| 6 | 0 | 6 | 7 | (6+0*7)%13 = 6 |



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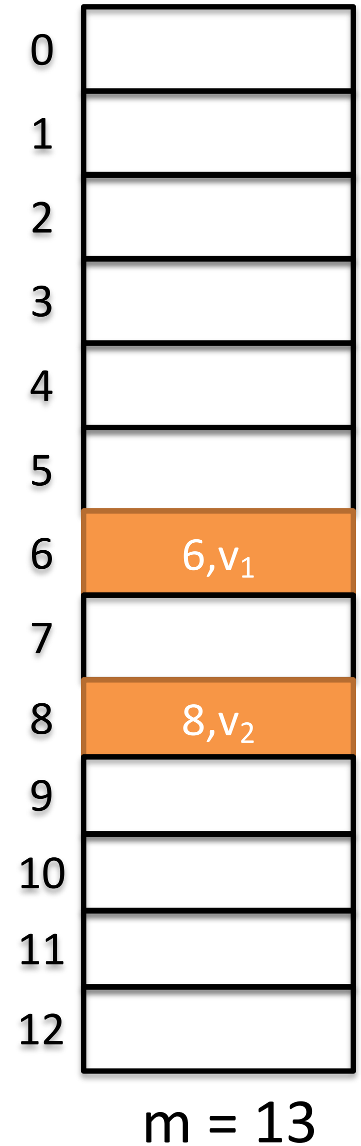
$h'(k,p)=(h_1(k) + ph_2(k)) \% m$

**Example**

| **Key** | **p** | **$h_1$** | **$h_2$** | **h'** |
|---|---|---|---|---|
| 6 | 0 | 6 | 7 | (6+0*7)%13 = 6 |
| 8 | 0 | 8 | 9 | (8+0*9)%13 = 8 |



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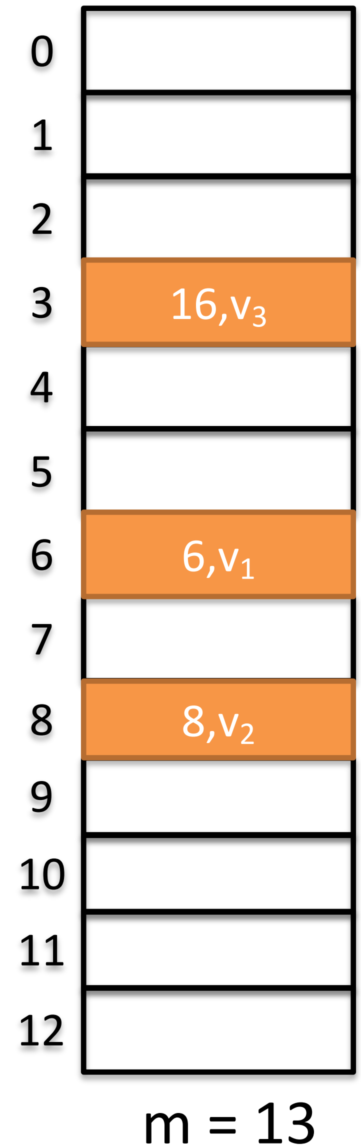
$h'(k,p)=(h_1(k) + ph_2(k)) \% m$

**Example**

| Key | p | $h_1$ | $h_2$ | h' |
|---|---|---|---|---|
| 6 | 0 | 6 | 7 | (6+0*7)%13 = 6 |
| 8 | 0 | 8 | 9 | (8+0*9)%13 = 8 |
| 16 | 0 | 3 | 5 | (3+0*5)%13 = 3 |



# Double hashing can help with the clustering problem

**Integer keys**

**$h_1$ same as before**
**$h_2$ new hash function**
**p = probe number (initially 0)**

Given table size m = 13

Compute

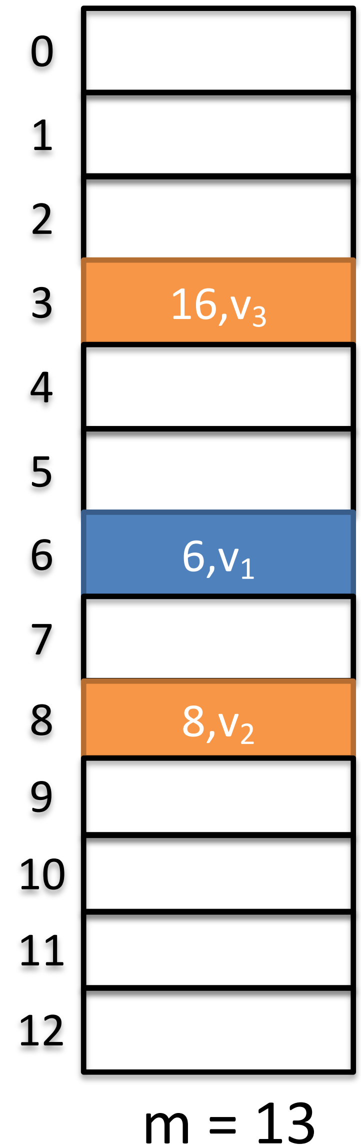
$h_1(key) = (key \% m)$

$h_2(key) = 1 + (key \% (m-1))$

$h'(k,p)=(h_1(k) + ph_2(k)) \% m$

**Example**

| Key | p | $h_1$ | $h_2$ | h' |
|---|---|---|---|---|
| 6 | 0 | 6 | 7 | (6+0*7)%13 = **6** |
| 8 | 0 | 8 | 9 | (8+0*9)%13 = 8 |
| 16 | 0 | 3 | 5 | (3+0*5)%13 = 3 |
| 19 | 0 | 6 | 8 | (6+0*8)%13 = **6** |




**Collision!**

# Double hashing can help with the clustering problem

**Integer keys**

Given table size m = 13

Compute

$h_1(key) = (key \% m)$

$h_2(key) = 1 + (key \% (m-1))$

$h'(k,p) = (h_1(k) + p h_2(k)) \% m$

**$h_1$ same as before**
**$h_2$ new hash function**
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**Example**

| Key | p | $h_1$ | $h_2$ | h' |
|---|---|---|---|---|
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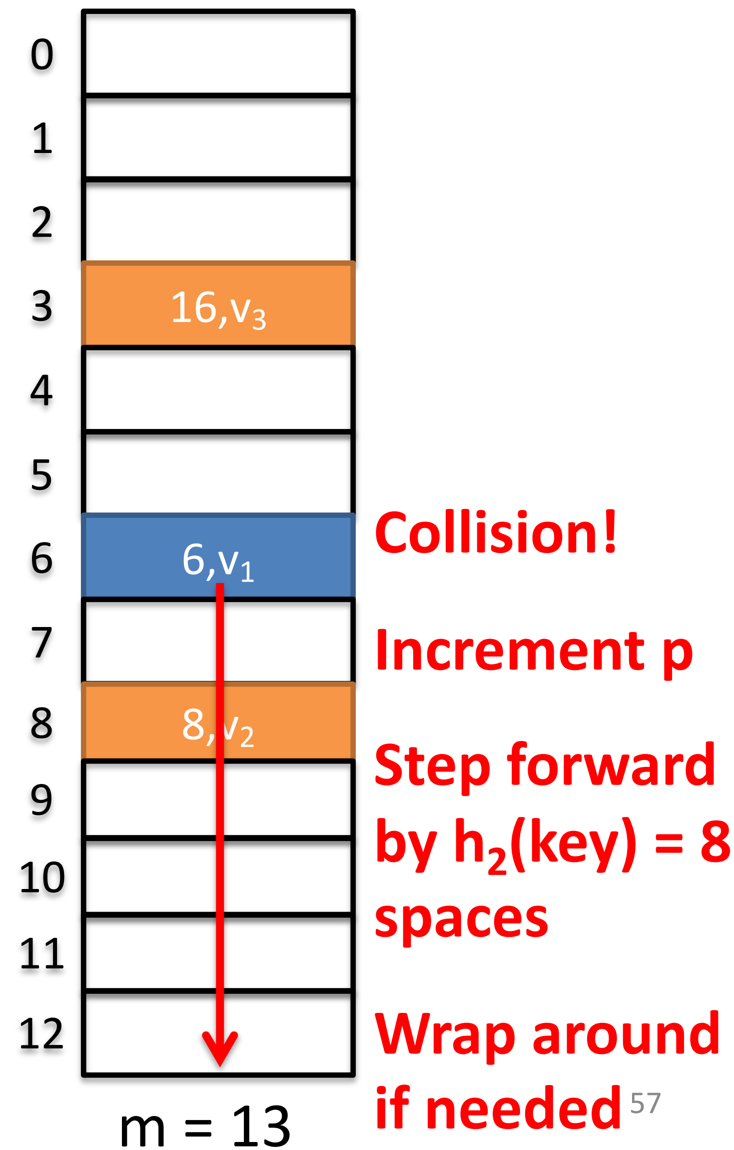
| 16 | 0 | 3 | 5 | (3+0*5)%13 = 3 |
| 19 | 0 | 6 | 8 | (6+0*8)%13 = 6 |
| 19 | 1 | 6 | 8 | (6+1*8)%13 = 1 |



# Double hashing can help with the clustering problem

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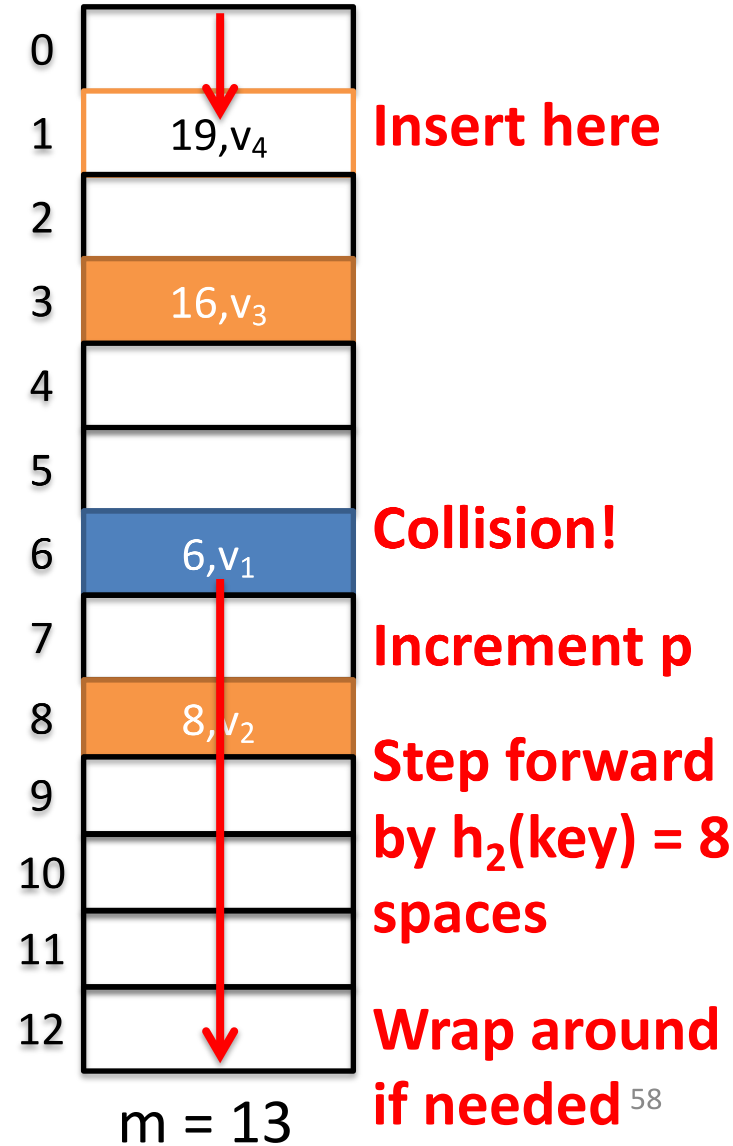
**Example**

| Key | p | $h_1$ | $h_2$ | h' |
|---|---|---|---|---|
| 6 | 0 | 6 | 7 | (6+0*7)%13 = 6 |
| 8 | 0 | 8 | 9 | (8+0*9)%13 = 8 |
| 15 | 0 | 2 | 4 | (2+0*4)%13 = 2 |
| 19 | 0 | 6 | 8 | (6+0*8)%13 = 6 |
| 19 | **1** | 6 | **8** | (6+**1*8**)%13 = **1** |



# Run time degrades as λ gets large, so keep λ small by growing hash table

**Expected insert and search time**

- Average number of probes is approximately 1/(1-λ)
- As λ ->1, expected number of probes becomes large, when λ small, number of probes approaches 1
- If table 90% full, then expect about 10 probes for unsuccessful search
- Successful search generally a little faster, about 2.5 probes (math on course web page and in book)
- Must grow table and ***rehash*** when copying to new table to keep the table sparsely populated or performance suffers

**Sparsely populated table trades memory for speed**

# Assuming load factor λ is small and hashing spreads keys, core operations are O(1)

| Operation | Expected run time | Notes |
|---|---|---|
| *hash(k)* | O(1) | • Math operations on key *k* to hash and compress, outputs 0...m-1<br>• Constant time, does not depend on number of items in Set or Map |

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| *put(k,v)* | O(1)<br>+O(1)<br>O(1) | • *Hash + find* = O(1)<br>• Plus update or add element:<br>  • Chaining: update value or add at head O(1)<br>  • Probing: store value in array O(1) |

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| *remove(k)* | O(1)<br><u>+O(1)</u><br>O(1) | • *Hash + find* = O(1)<br>• Plus remove element:<br>  • Chaining: update one pointer O(1)<br>  • Probing: mark space empty O(1) |

**Assuming a small load factor and uniform hashing, the core operations of HashSets and HashMaps are constant time!**