

# COMPUTATIONAL ASPECTS OF DIGITAL PHOTOGRAPHY

## Light & Color (continued)



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# Administrivia

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## Assignment 2 available now

- back to programming
- due next Wednesday

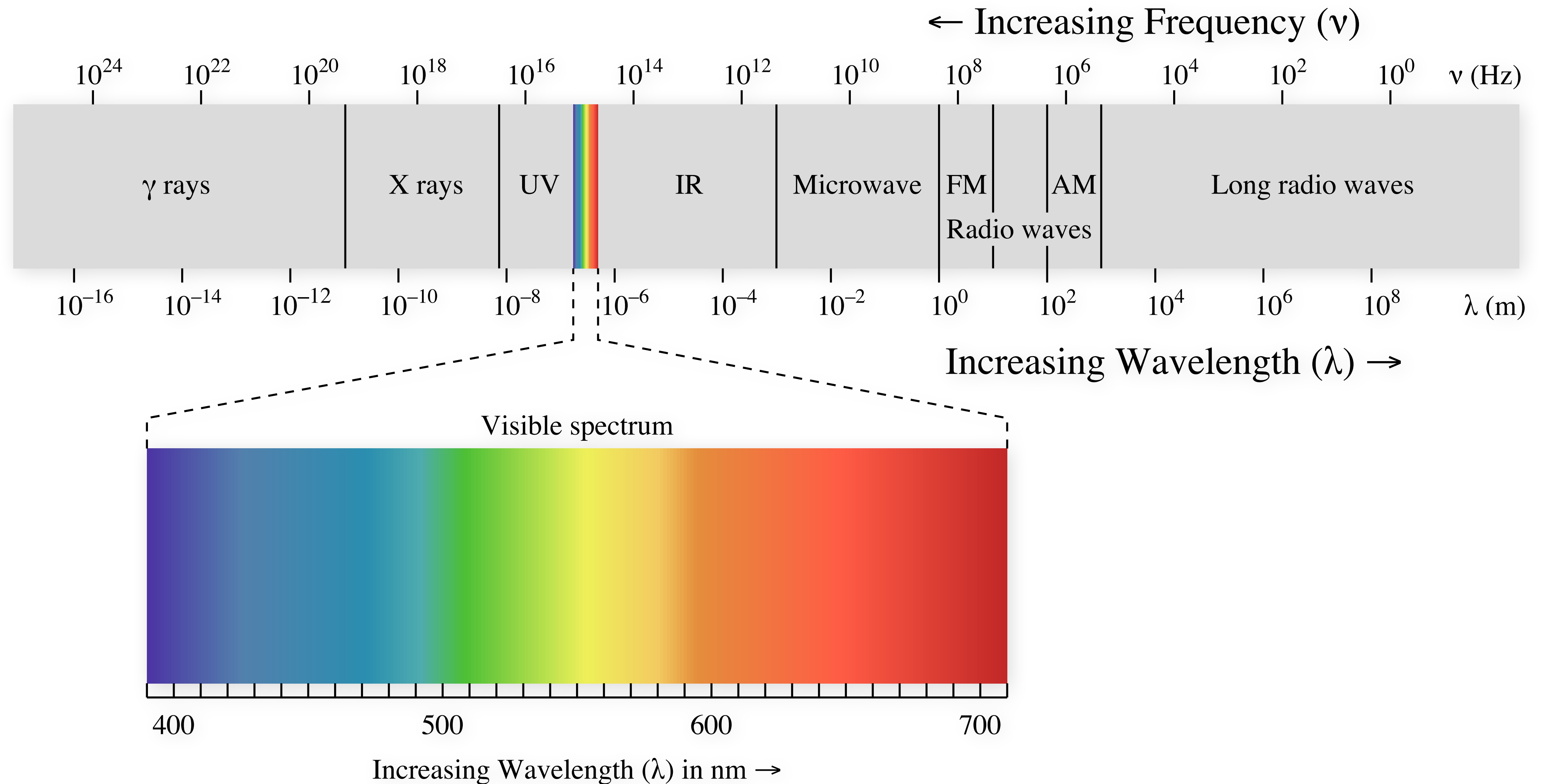
# Last time...

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## Light & Color

- Physics background
- Color perception & measurement
- Color reproduction
- Color spaces

# What is light?

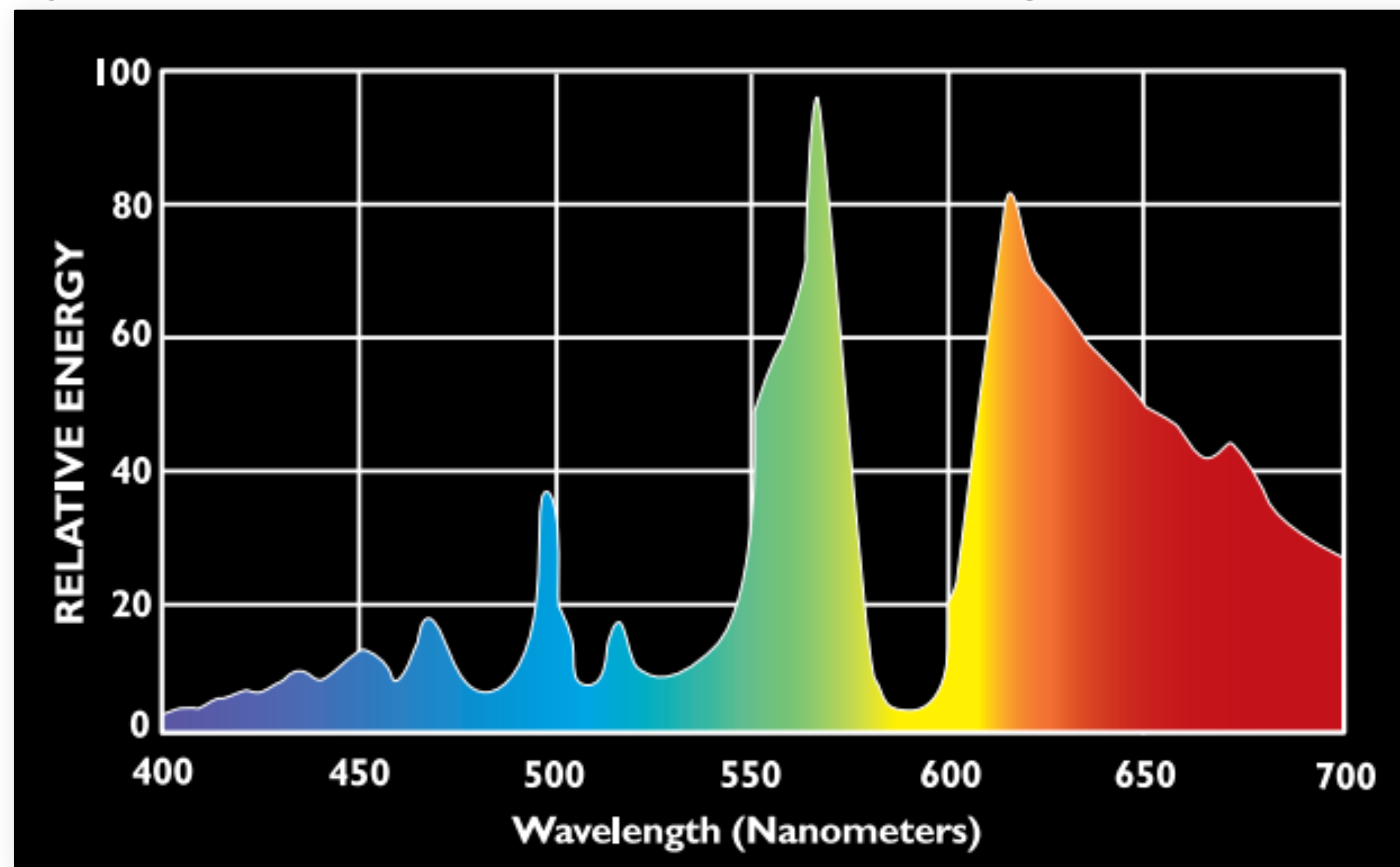




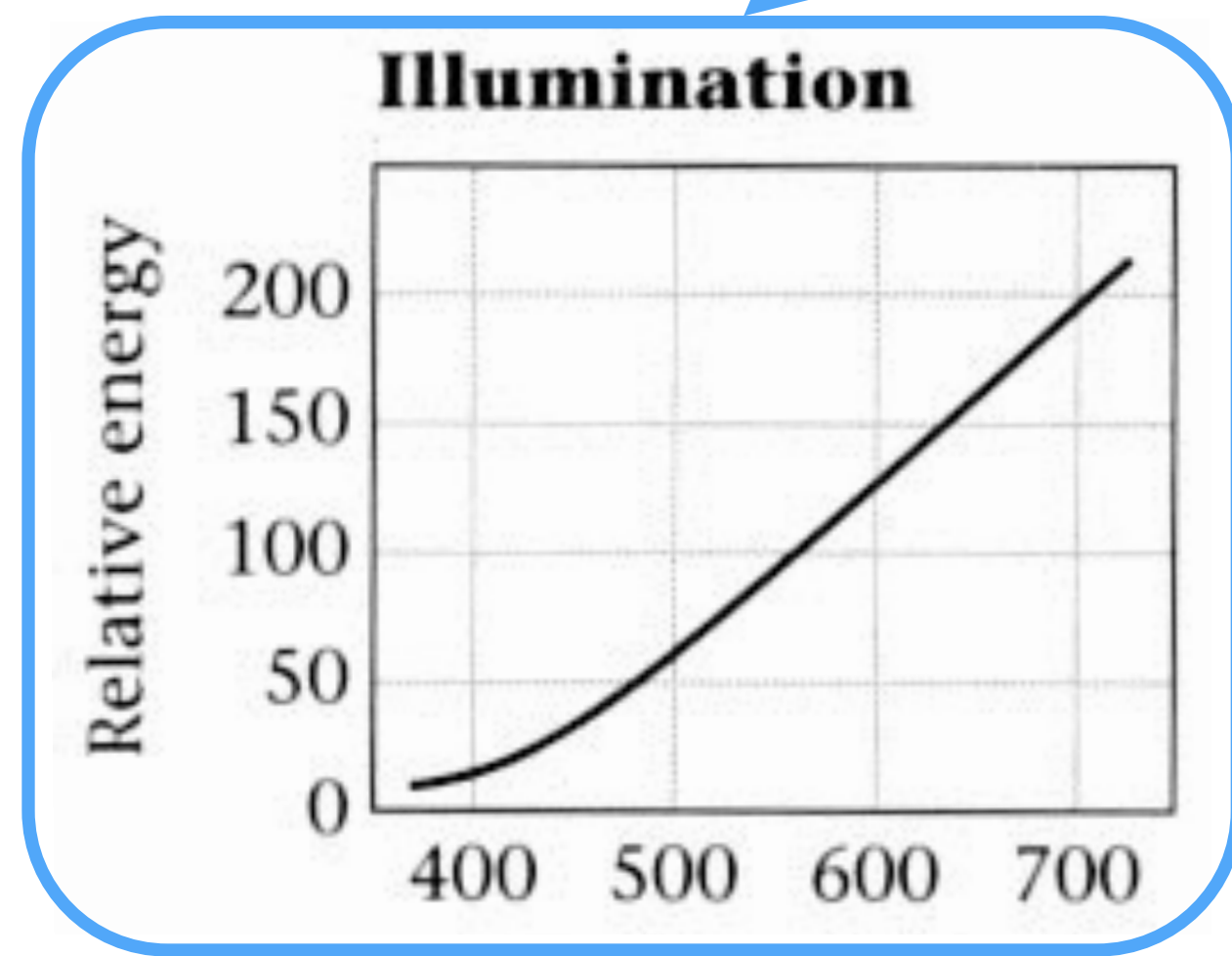
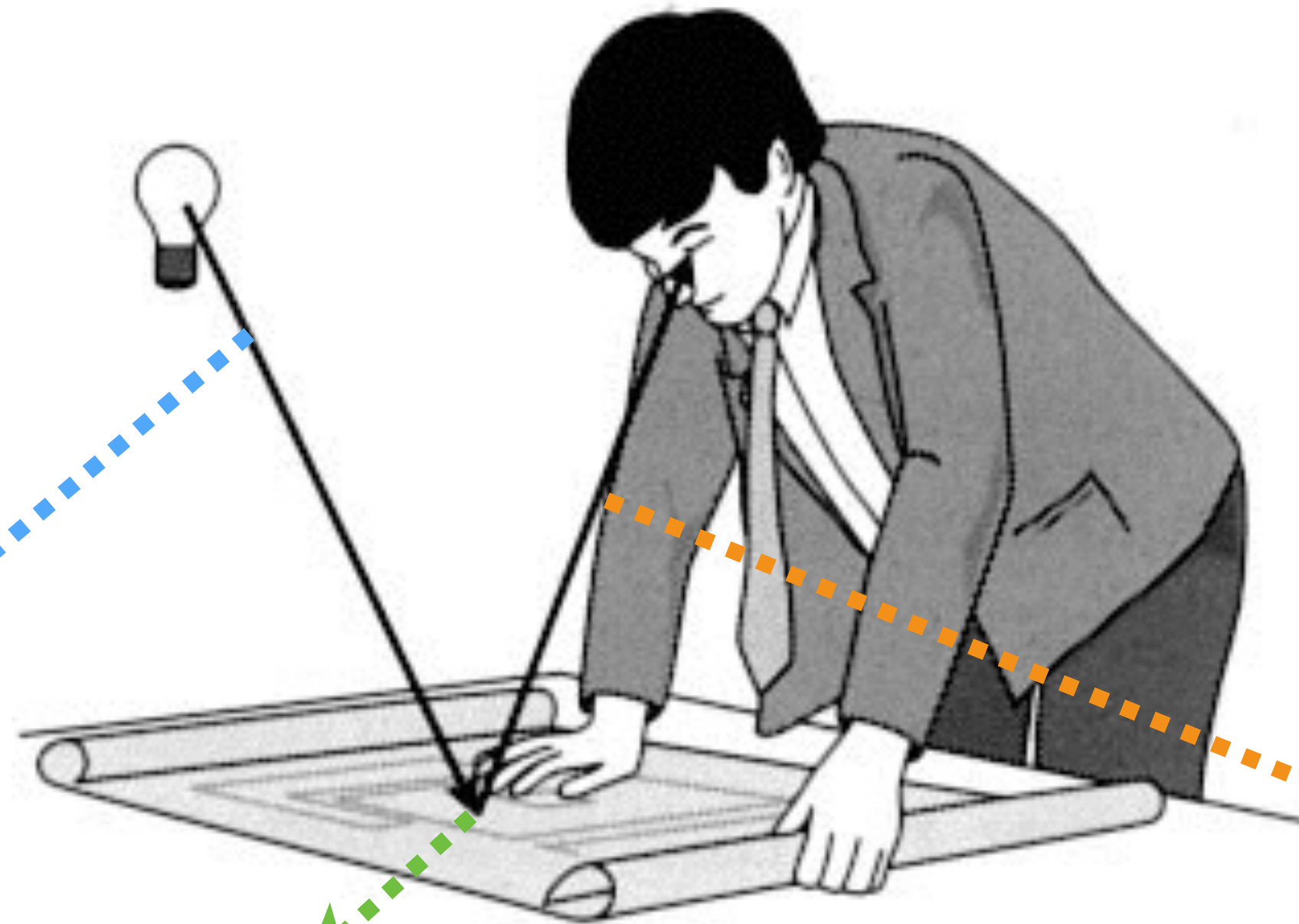
# Spectral distribution function (SPD)

Light can be a mixture of many wavelengths

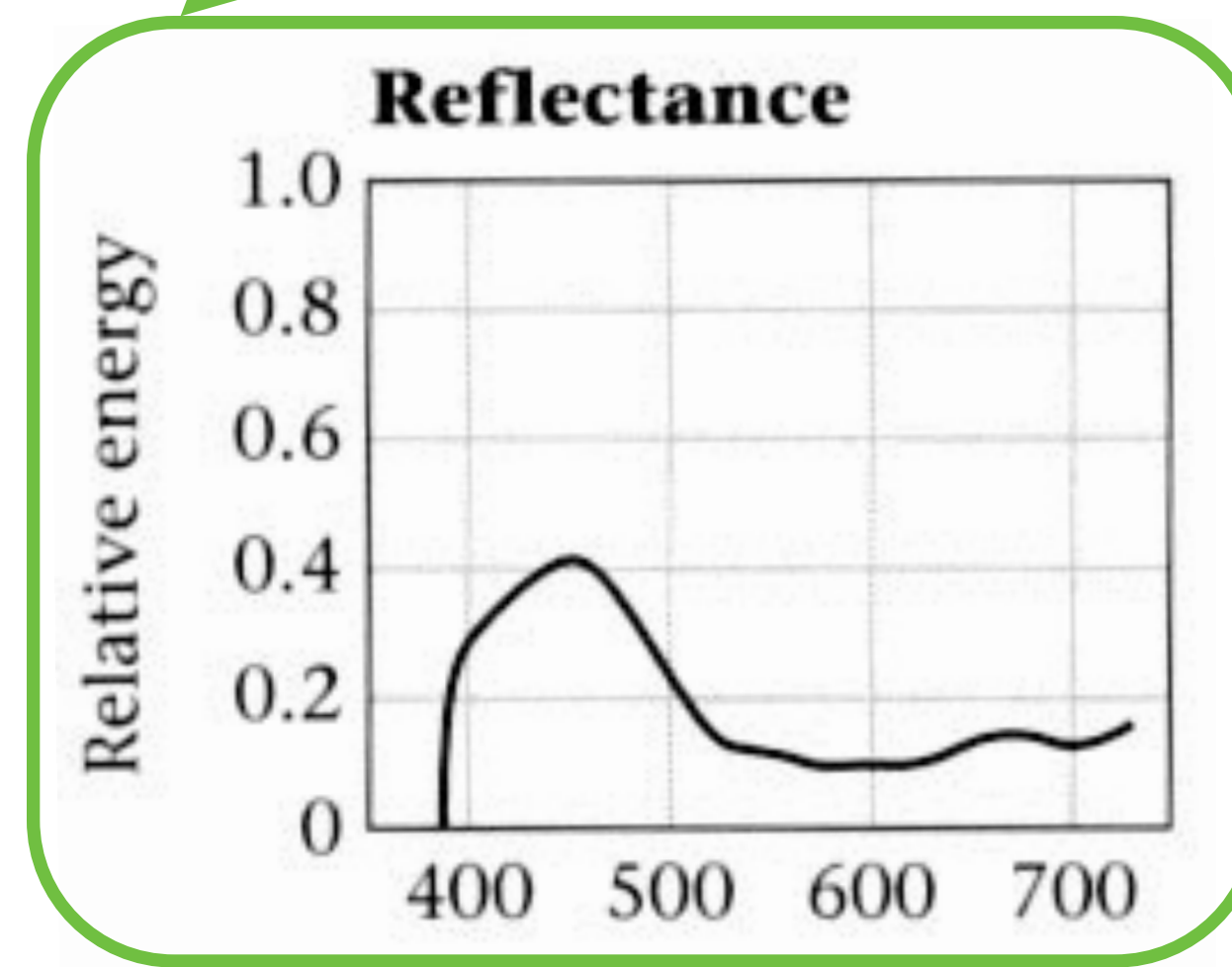
- SPD: intensity as a function of wavelength over entire spectrum



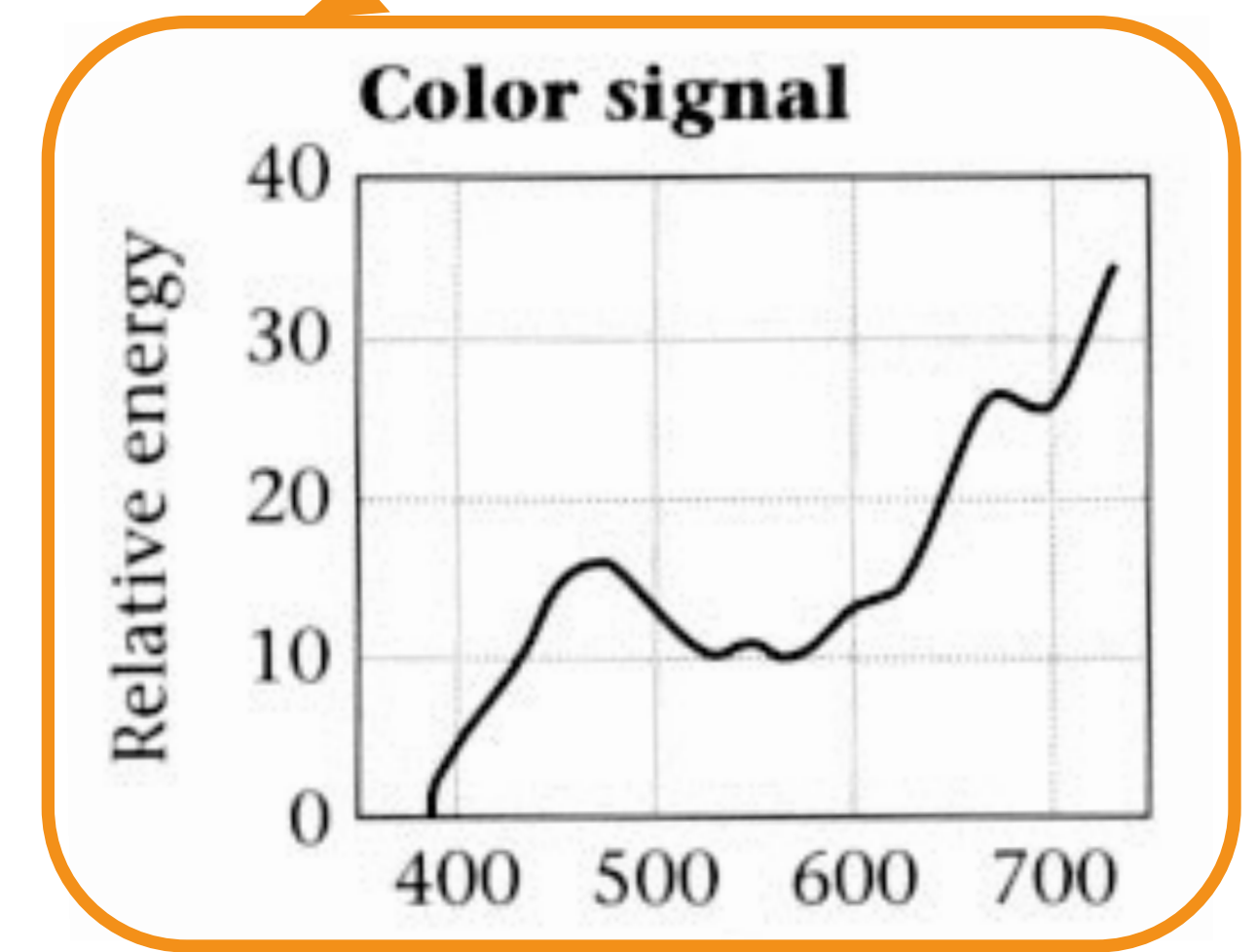
# Light-matter interaction



×

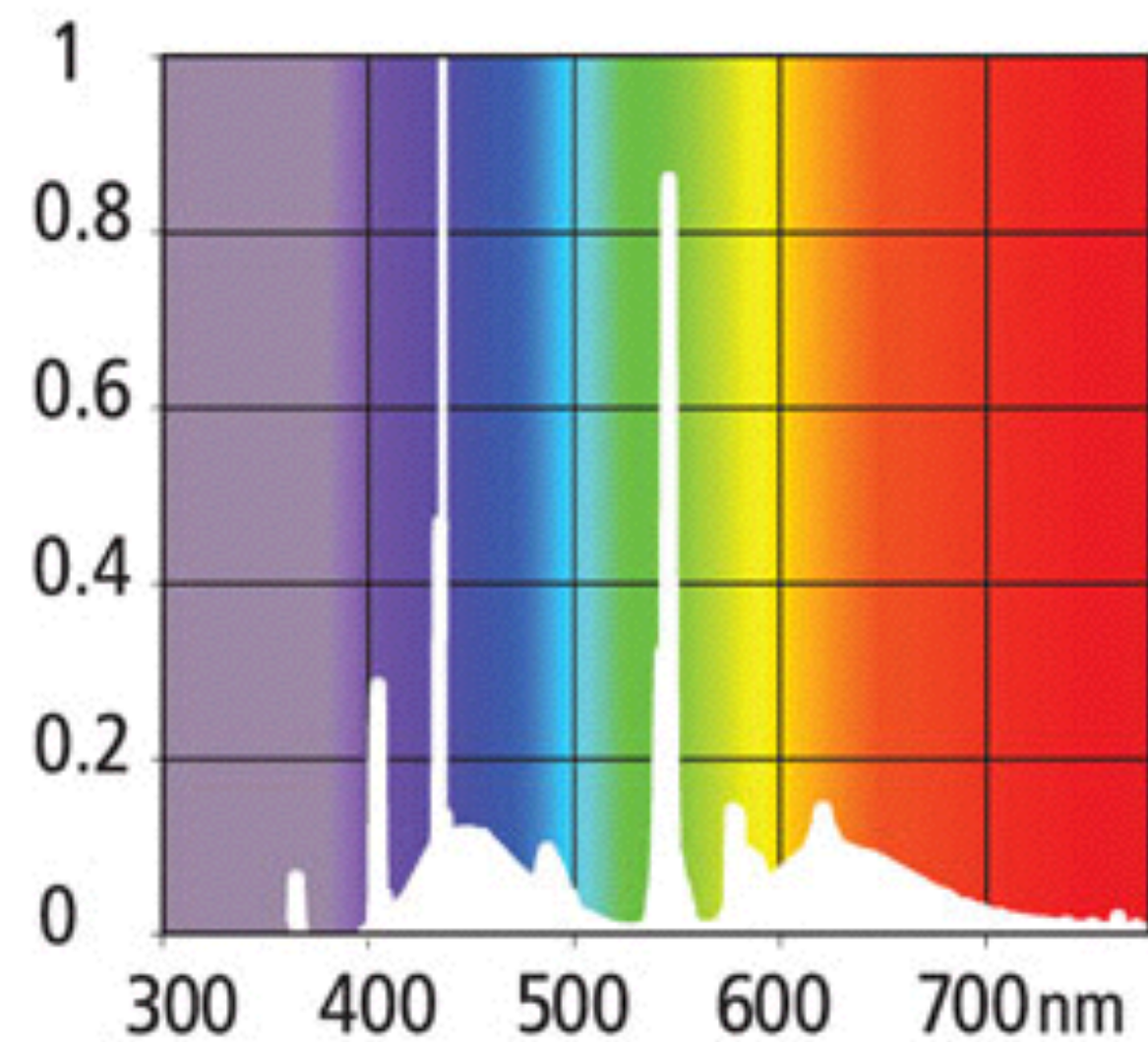


=

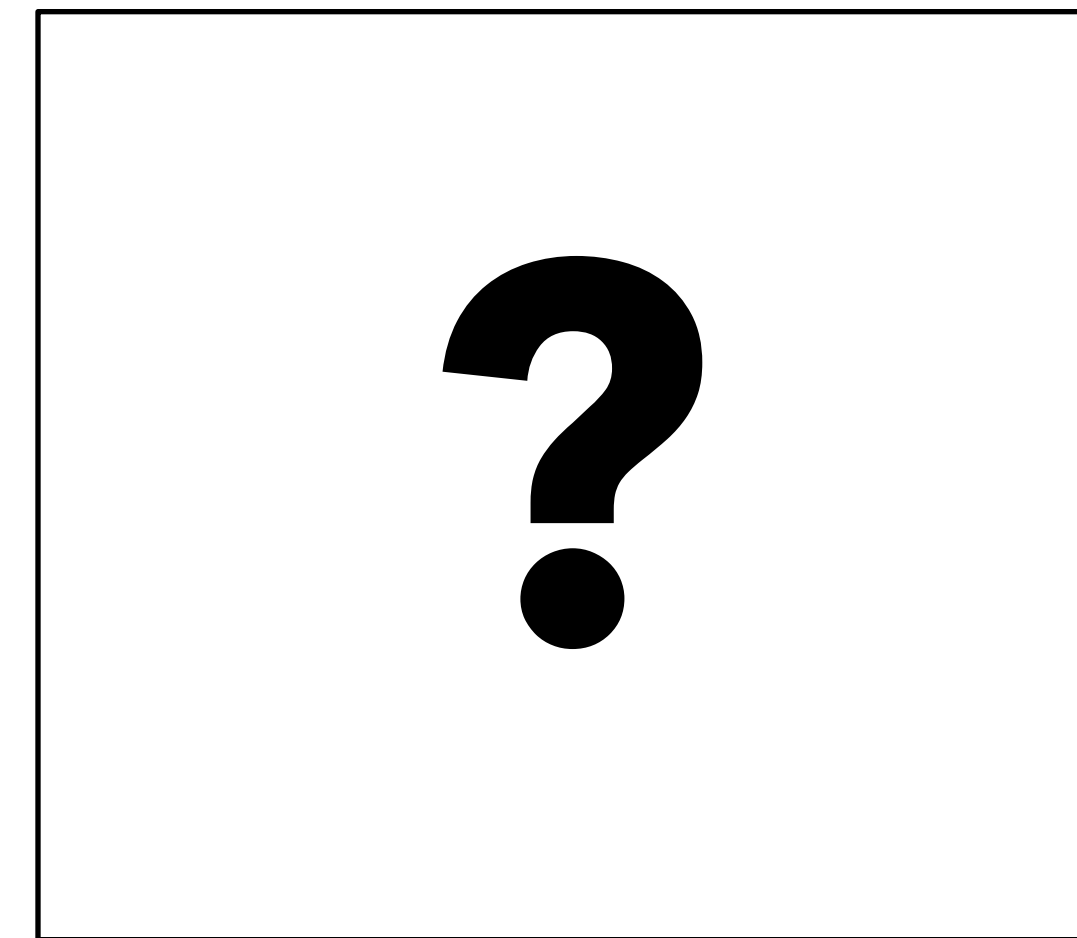




# Physical light to perceptual color



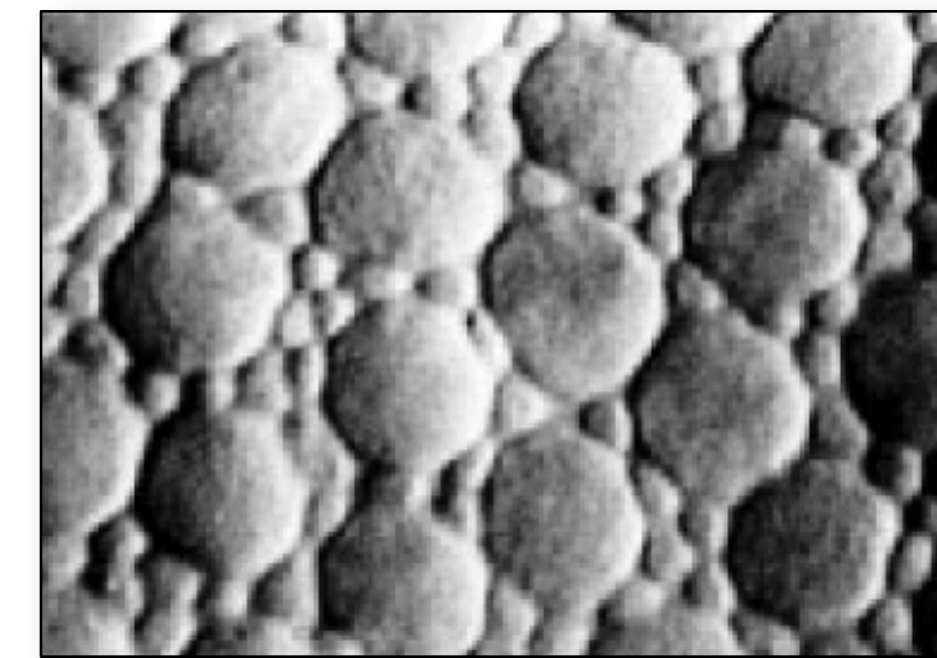
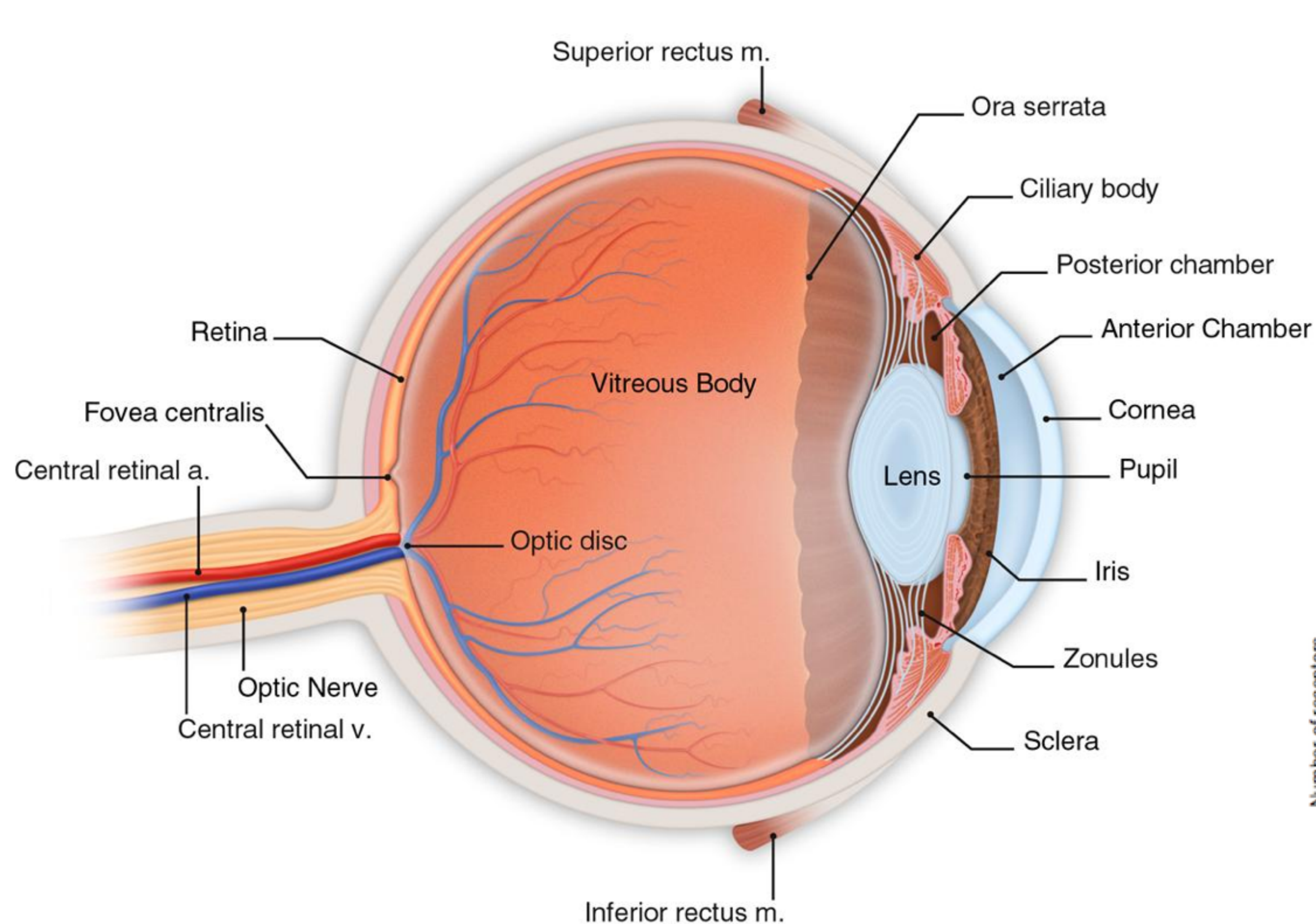
Physical



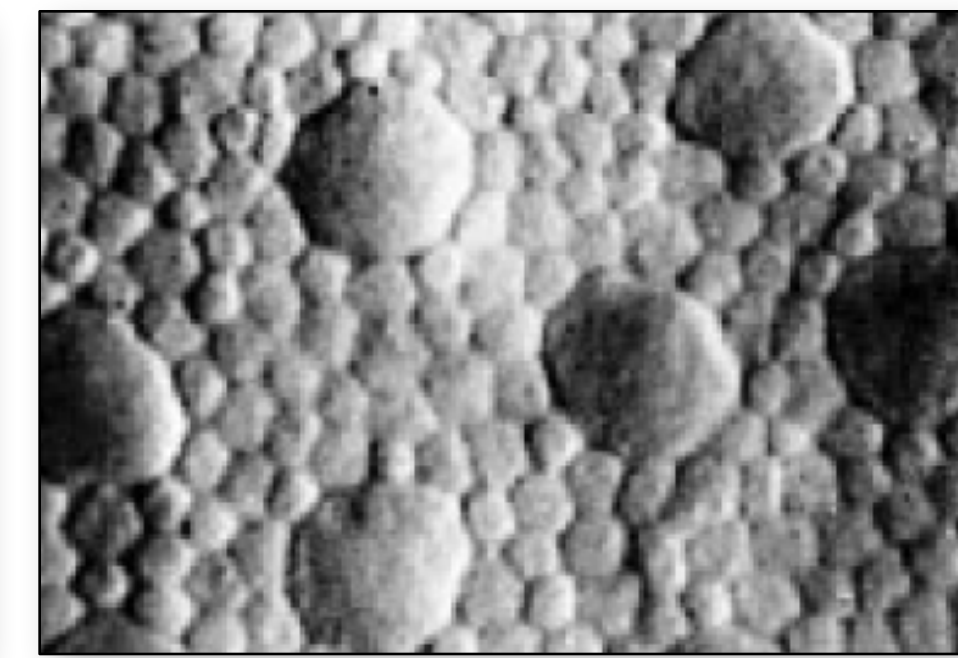
Perceptual



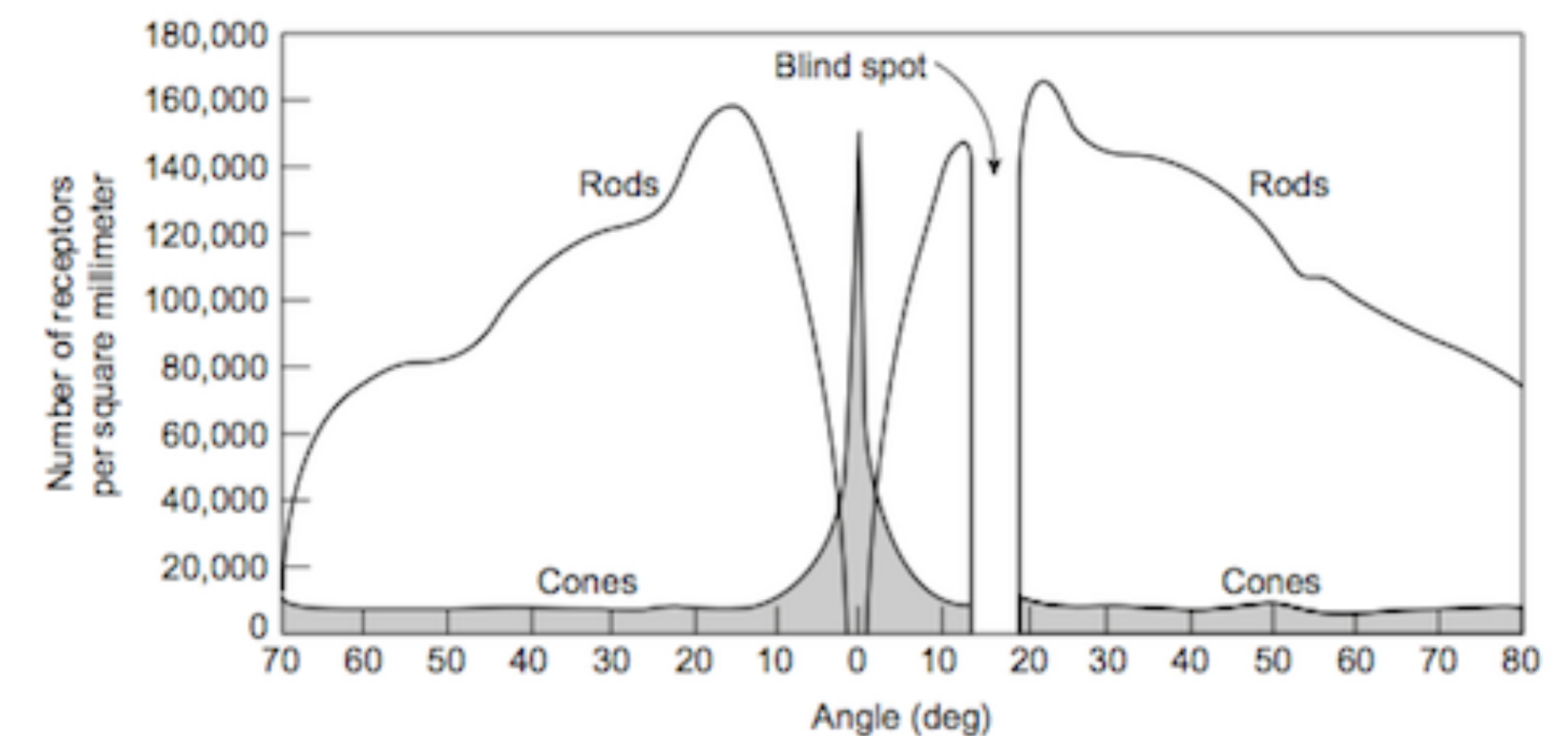
# The eye as a measurement device



near fovea

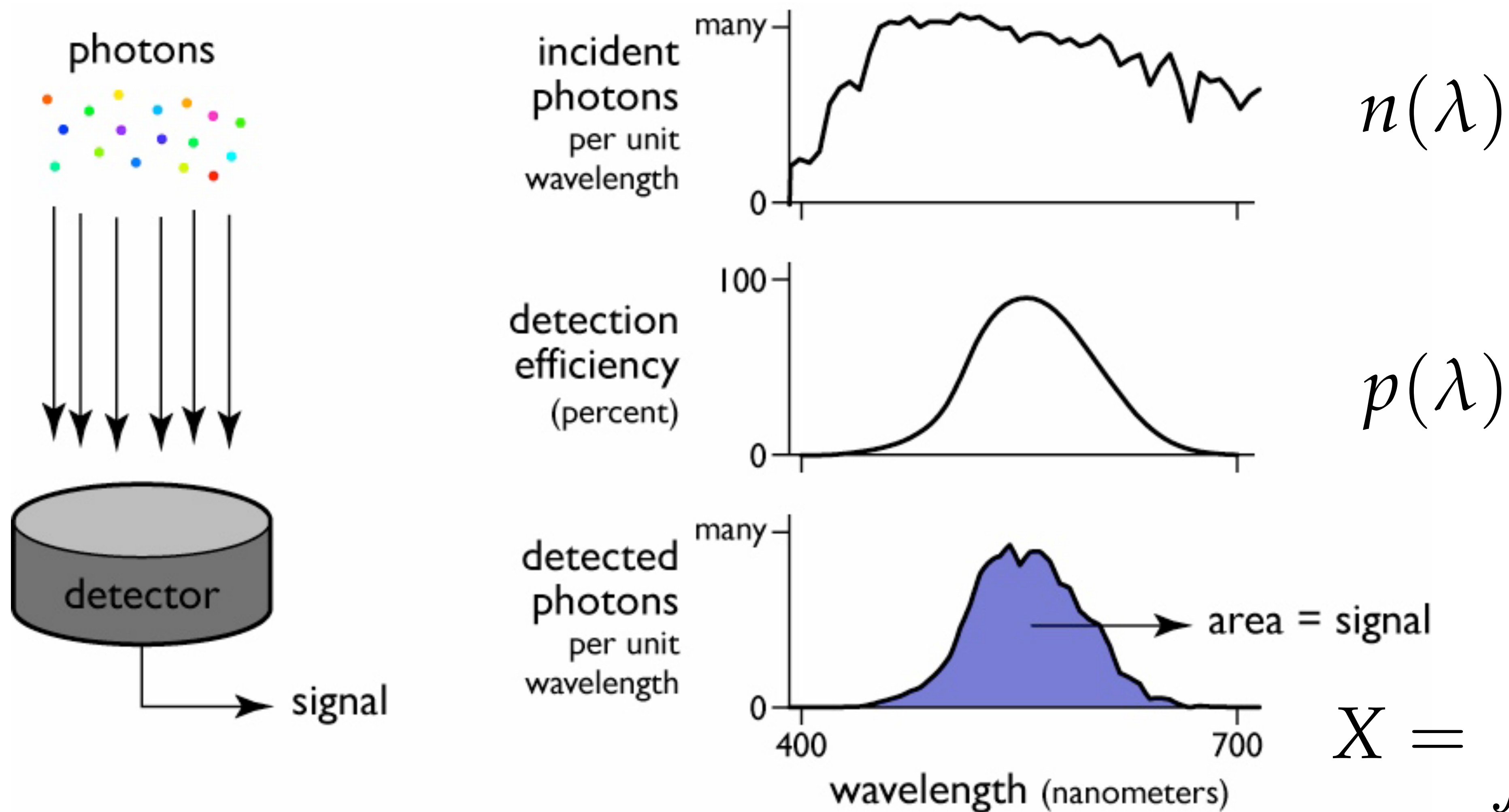


away from fovea





# A simple light detector model



$$X = \int n(\lambda) p(\lambda) d\lambda$$

# Light detection math

$$X = \int s(\lambda) r(\lambda) d\lambda$$

measured signal

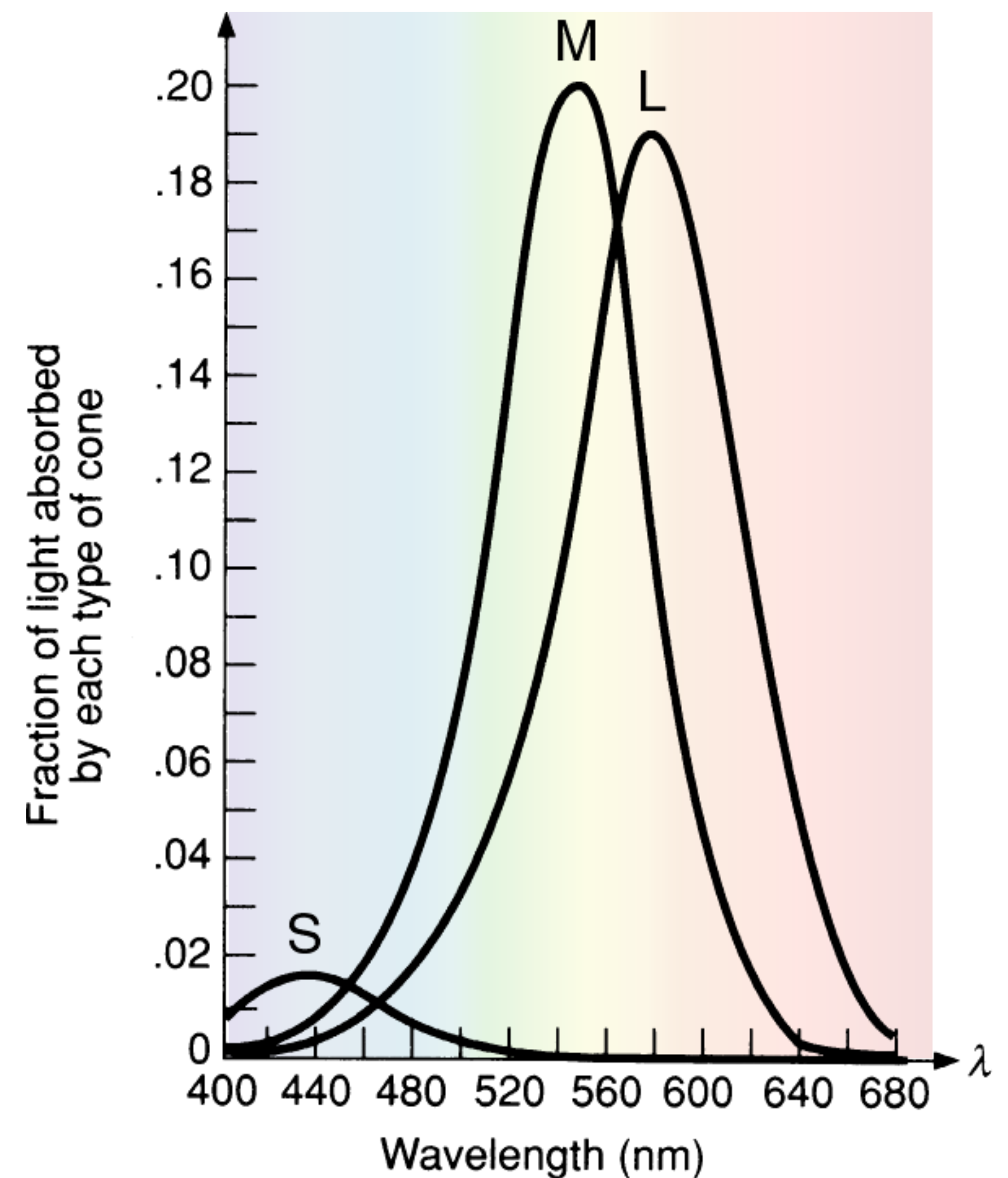
input spectrum

detector's sensitivity

$$S = \int r_S(\lambda) s(\lambda) d\lambda$$

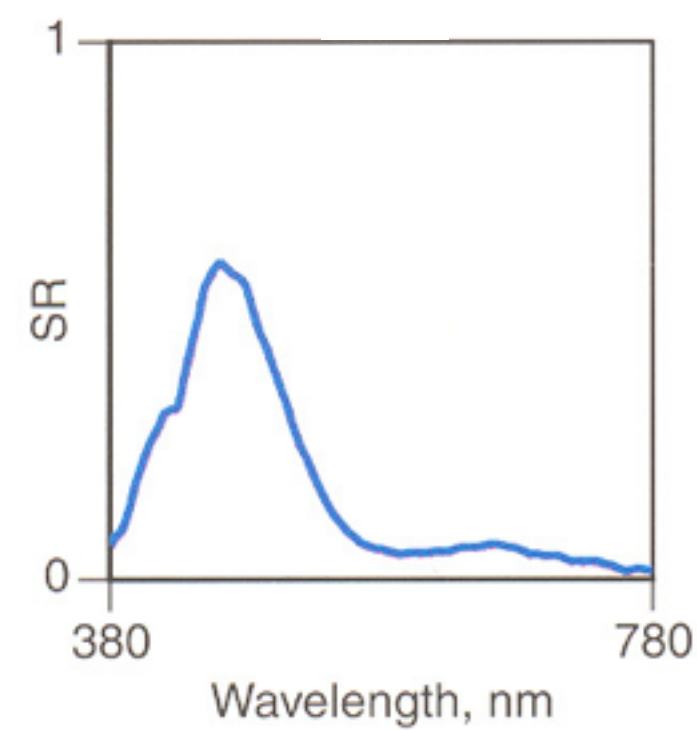
$$M = \int r_M(\lambda) s(\lambda) d\lambda$$

$$L = \int r_L(\lambda) s(\lambda) d\lambda$$



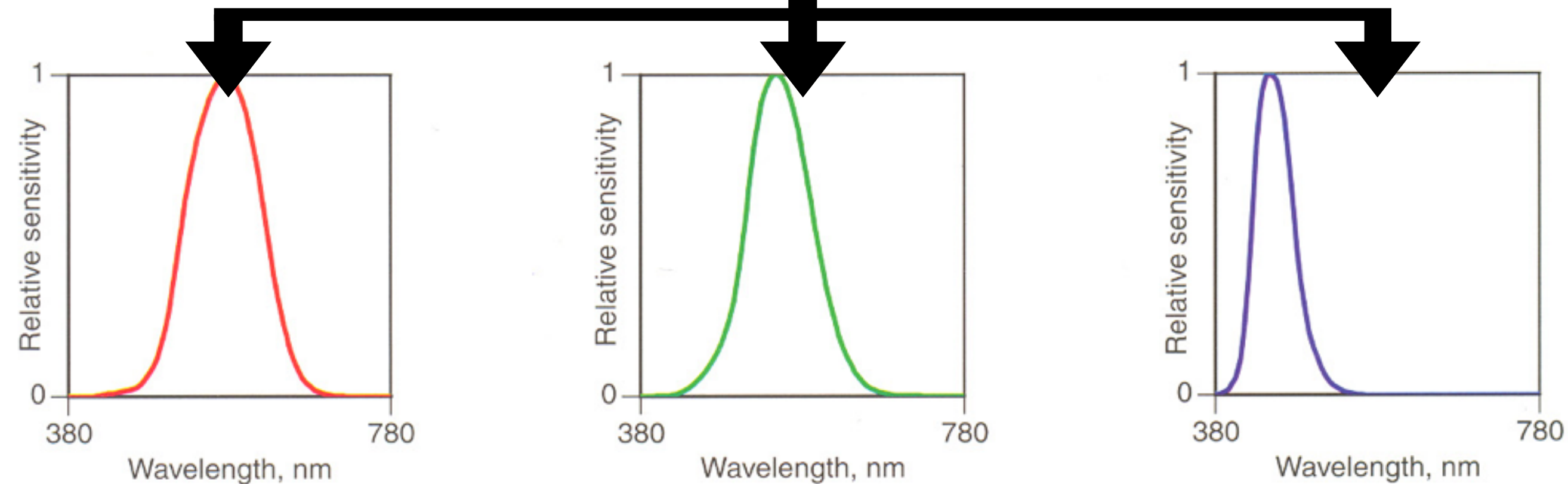


Stimulus  
(arbitrary spectrum)

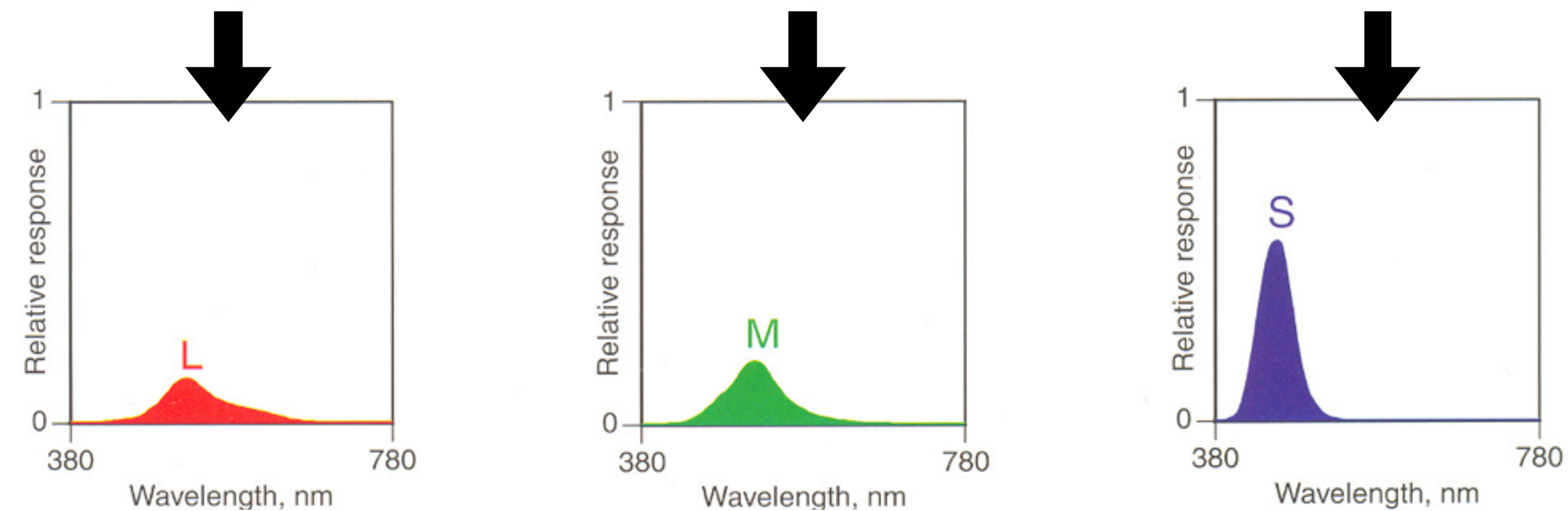


Start with infinite  
number of values  
(one per wavelength)

Response curves



Multiply



Integrate

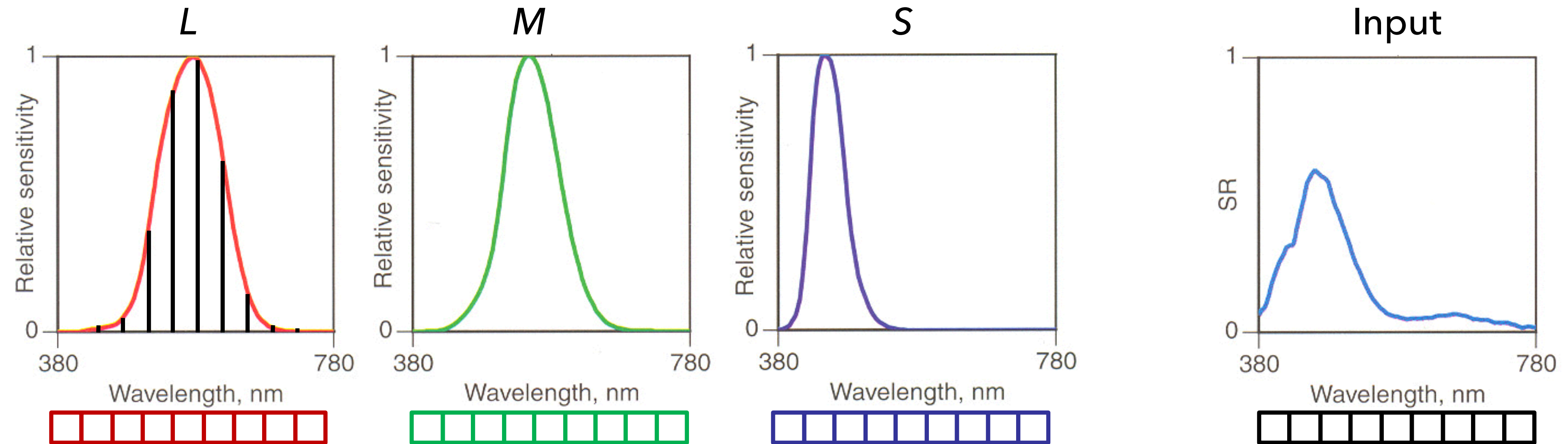
1 number

1 number

1 number

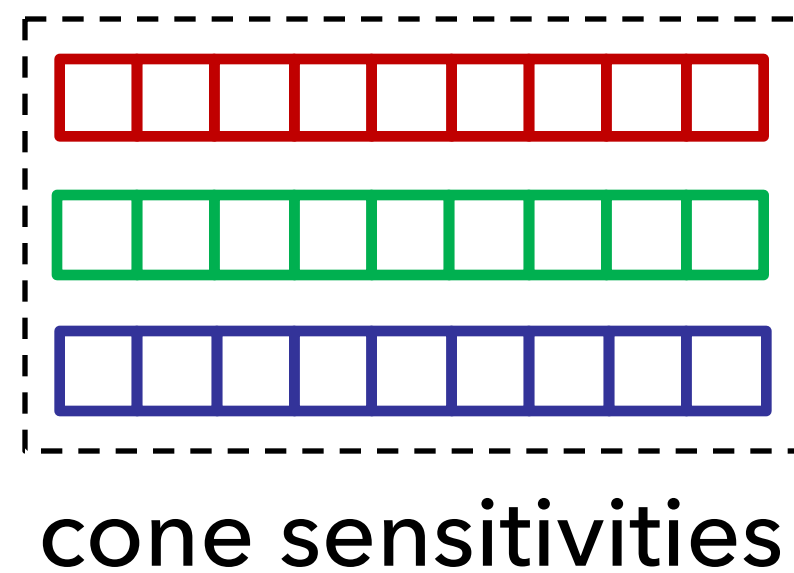
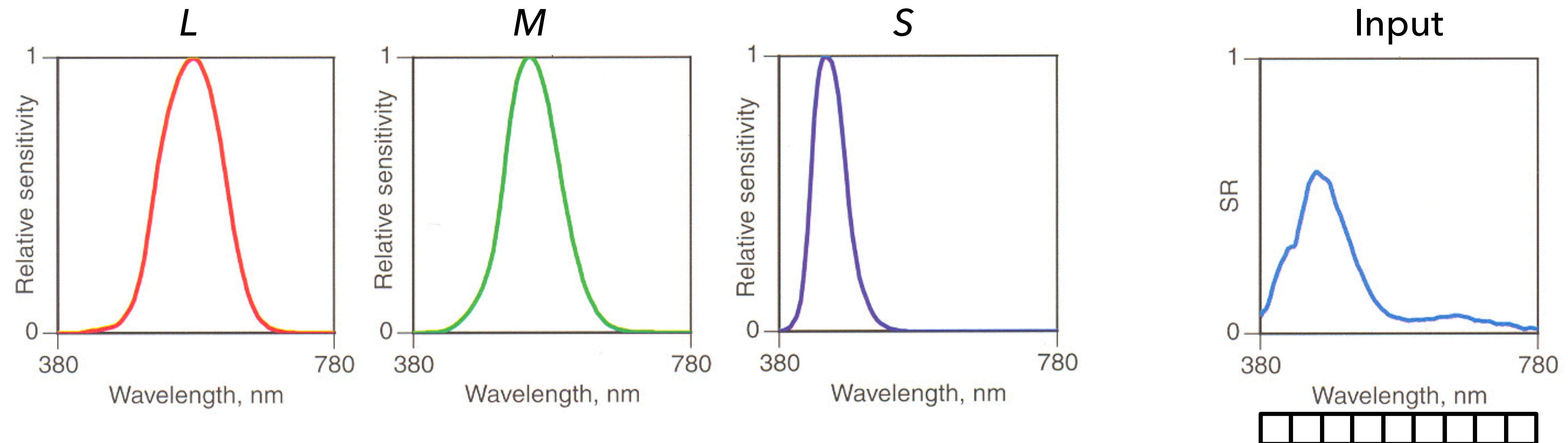
End up with 3 values  
(one per cone type)

# Linear algebra interpretation

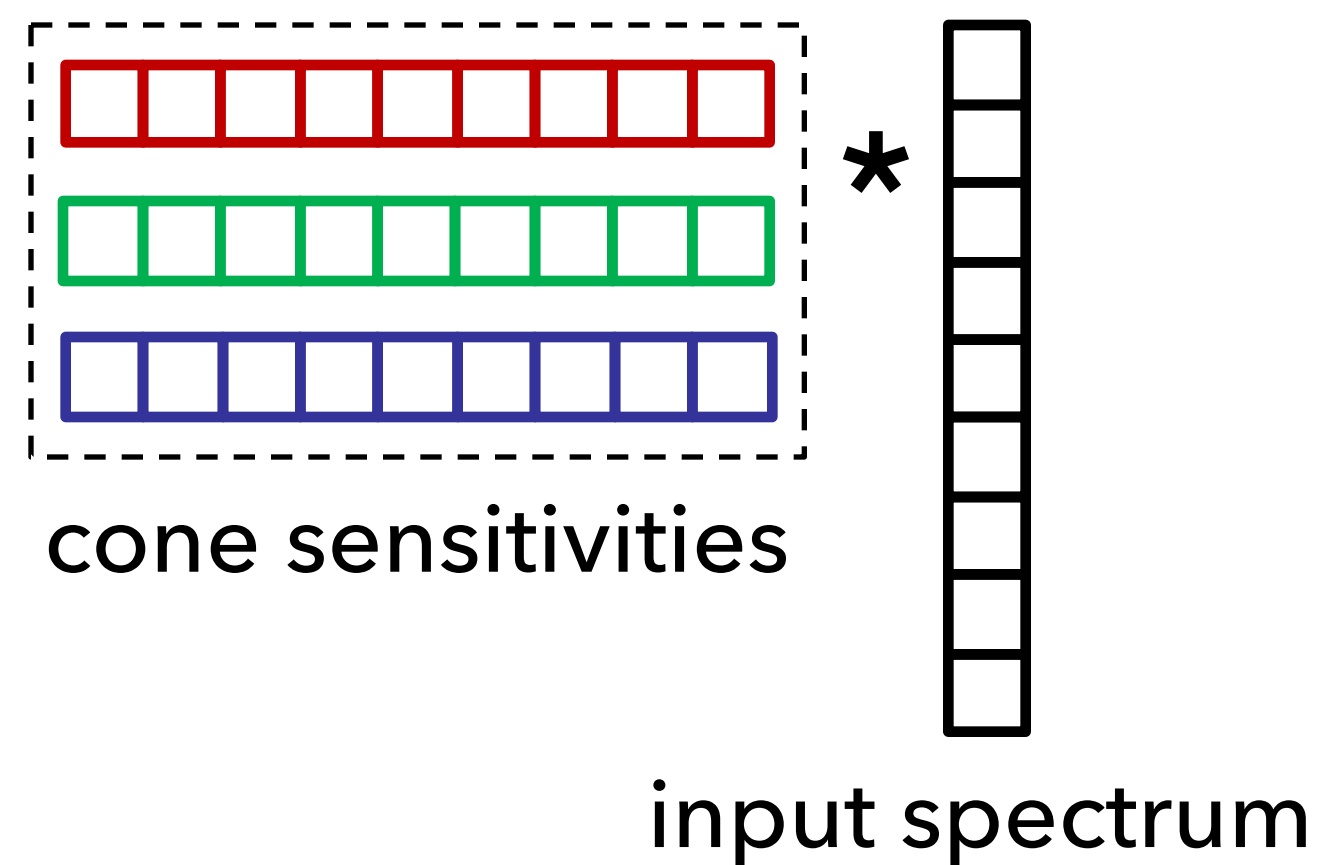
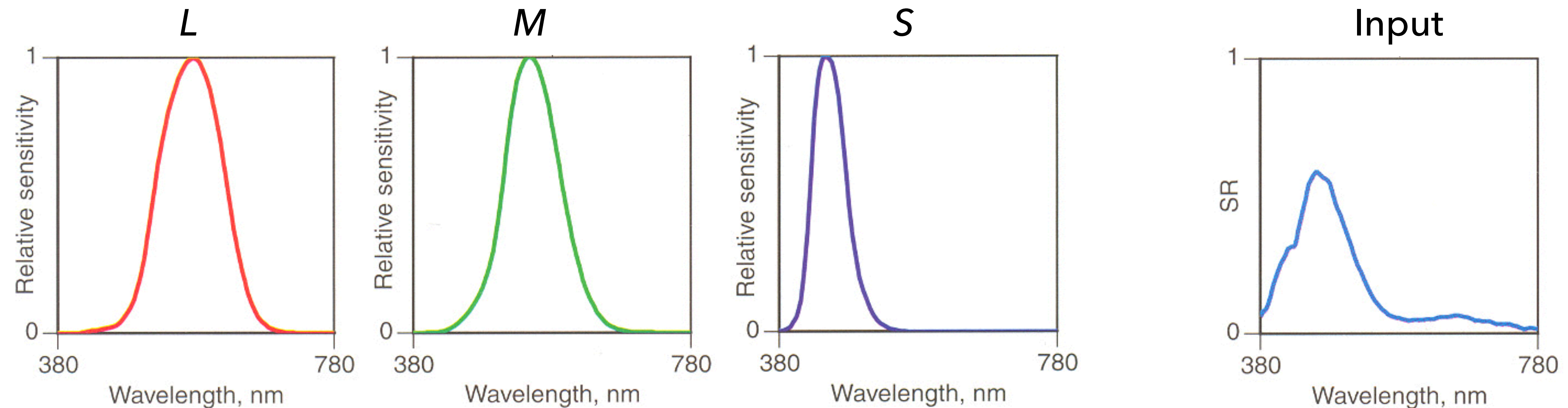




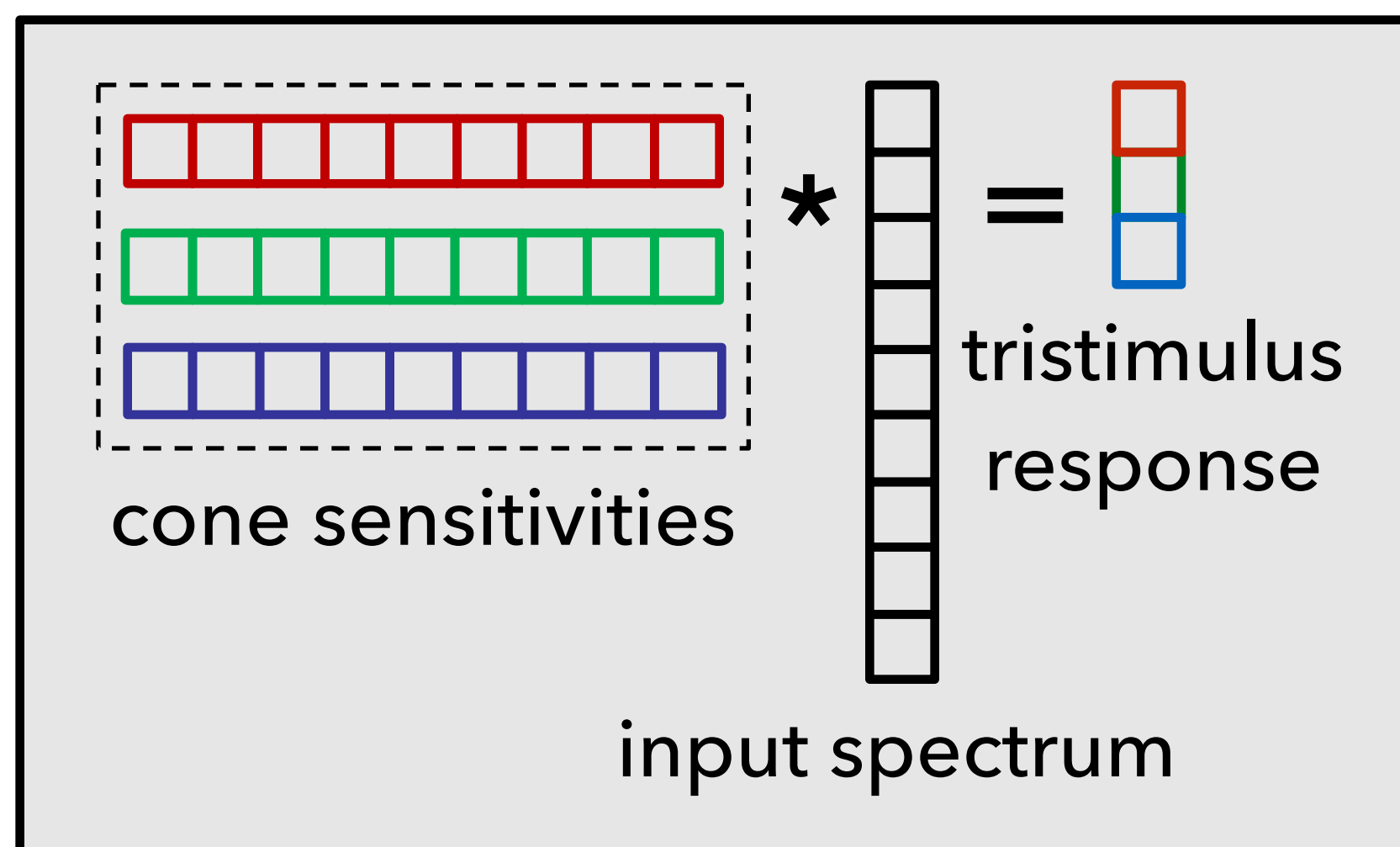
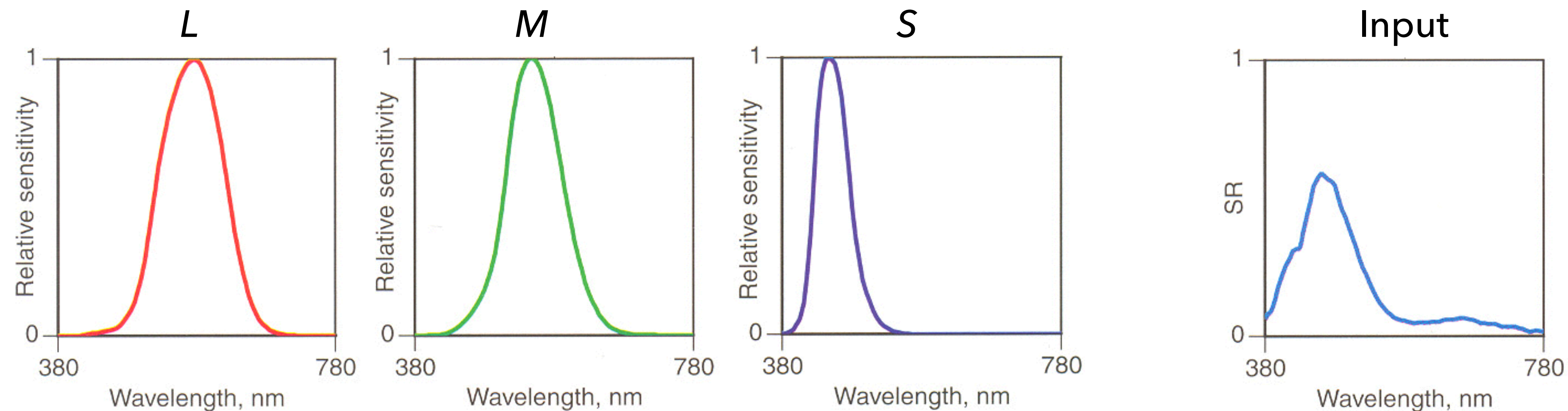
# Linear algebra interpretation



# Linear algebra interpretation



# Linear algebra interpretation



Tristimulus response is a matrix-vector multiplication



# Cone responses to a spectrum $s$

Integral notation:

$$S = \int r_S(\lambda) s(\lambda) \mathrm{d}\lambda = r_S \cdot s$$

$$M = \int r_M(\lambda) s(\lambda) \mathrm{d}\lambda = r_M \cdot s$$

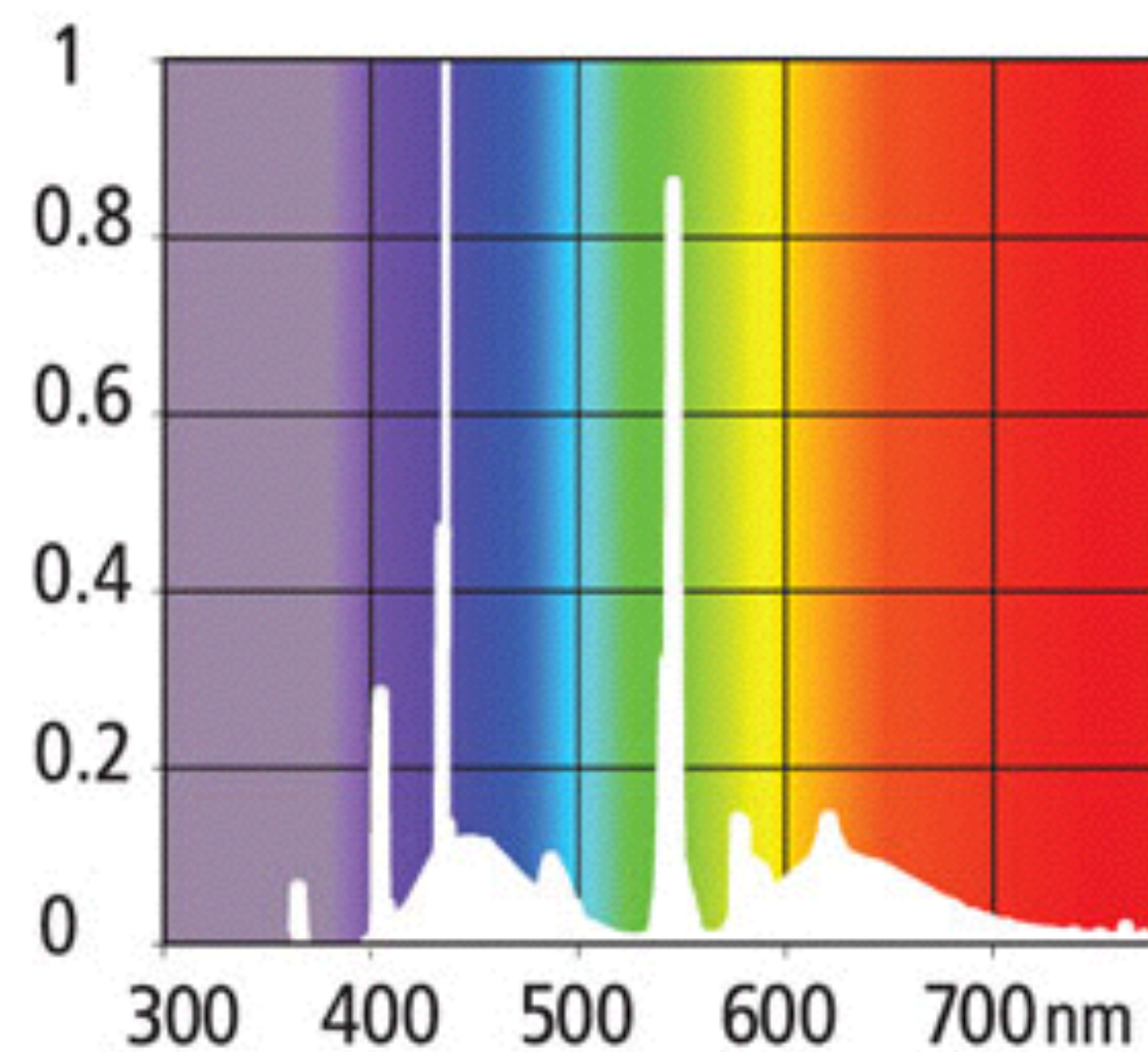
$$L = \int r_L(\lambda) s(\lambda) \mathrm{d}\lambda = r_L \cdot s$$

Matrix notation:

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

$r_S$ ,  $r_M$  and  $r_L$  are  $N$ -dimensional vectors, where  $N = \infty$

# Physical light to perceptual color



Physical



$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ S \\ | \end{bmatrix}$$

Perceptual

# Basic fact of colorimetry

Take a spectrum (which is an infinity of numbers)

Eye produces three numbers (a projection to 3D)

This throws away a lot of information!

- many spectra can produce same S, M, L tristimulus values!
- *metamers*
- affected by illuminant





# Warning: tricky thing with color

Cone responses overlap & are not orthogonal!

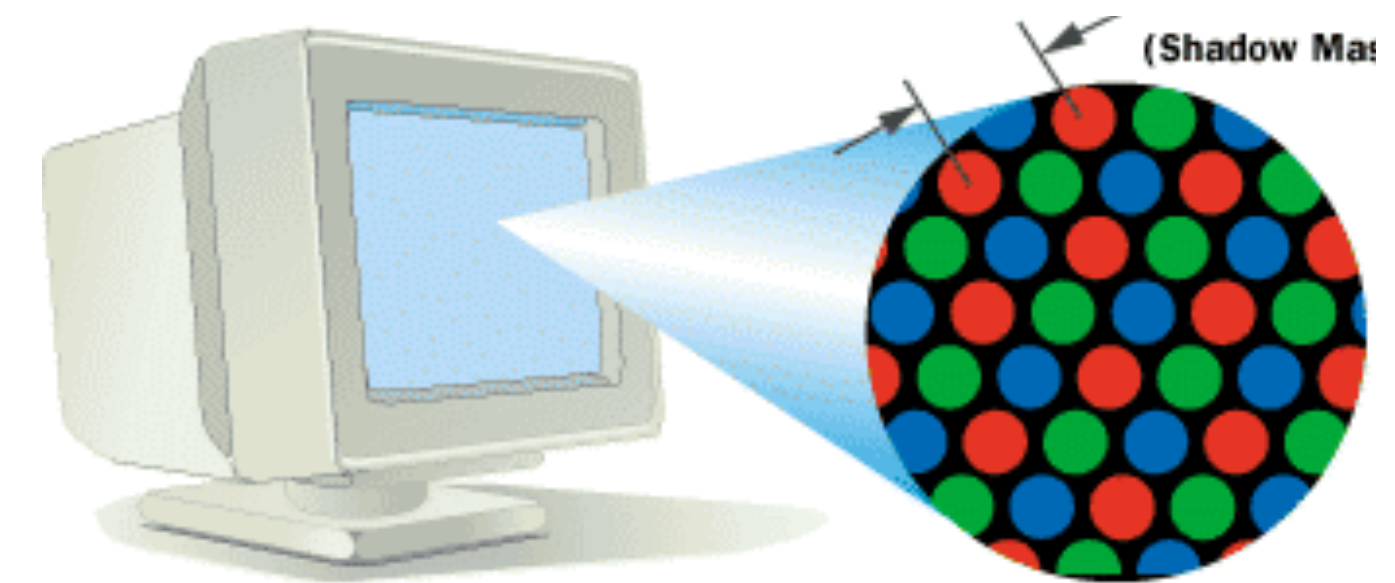
Basis functions for analysis

- eyes, cameras

are different than for synthesis

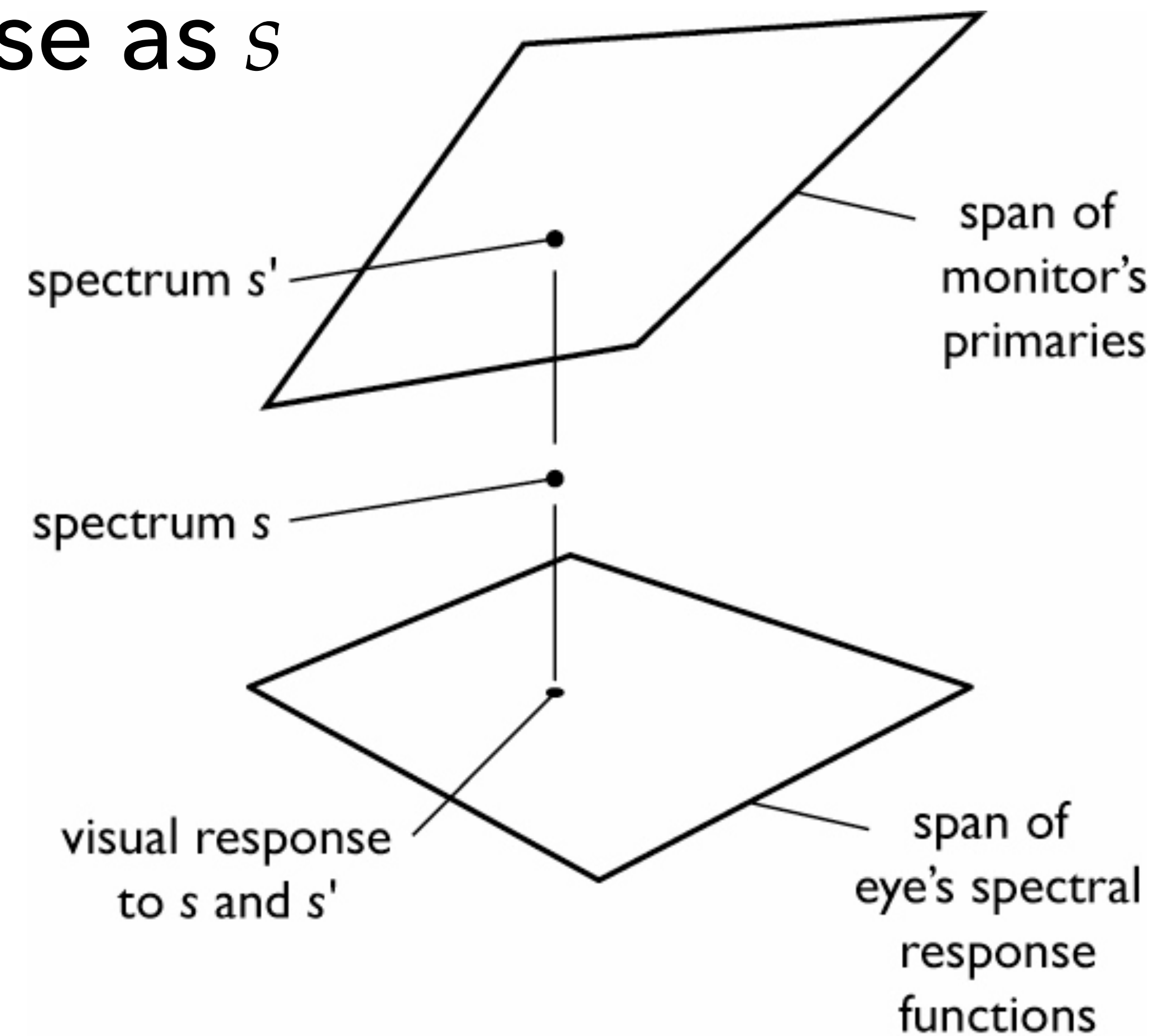
- lights, monitors

The RGB in your camera is different than the RGB in your monitor!



# Color reproduction (the right way)

We want to compute the combination of R, G, B that will project to the same visual response as  $s$





# Color reproduction as linear algebra

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What color do we see when we look at the display?

- Feed  $C$  to display

$C$

# Color reproduction as linear algebra

---

What color do we see when we look at the display?

- Feed  $C$  to display
- Display produces  $s_a$

$$M_{RGB} C$$



# Color reproduction as linear algebra

What color do we see when we look at the display?

- Feed  $C$  to display
- Display produces  $s_a$
- Eye looks at  $s_a$  and produces  $E$

$$E = M_{SML} M_{RGB} C$$

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\ r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\ r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

# Color reproduction as linear algebra

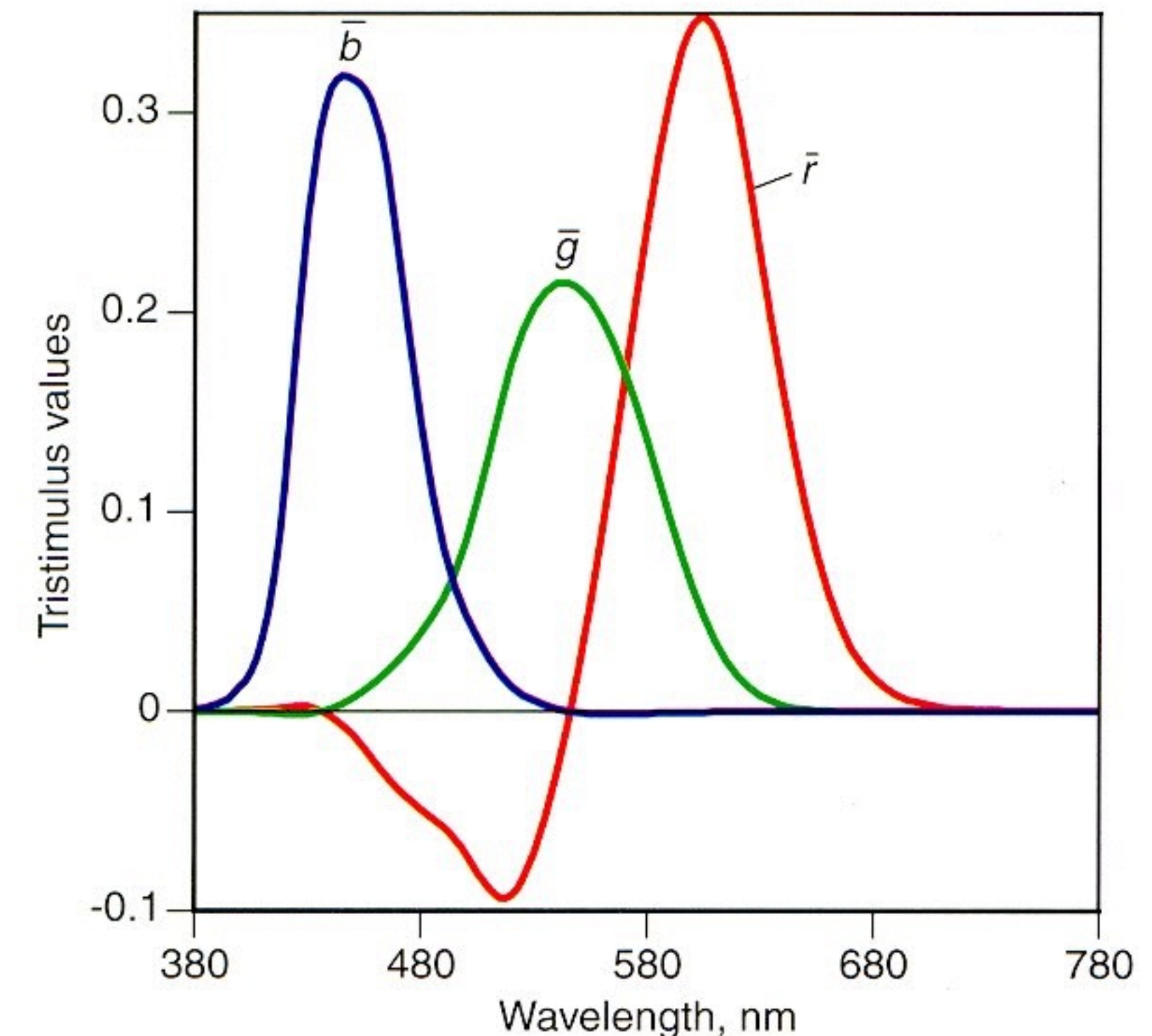
Goal of reproduction: visual response to  $s$  and  $s_a$  is the same:

$$M_{SML} s = M_{SML} s_a$$

Substitute in expression for  $s_a$ ,

$$M_{SML} s = M_{SML} M_{RGB} C$$

$$C = \underbrace{(M_{SML} M_{RGB})^{-1} M_{SML}}_{\text{color matching matrix for RGB}} s$$



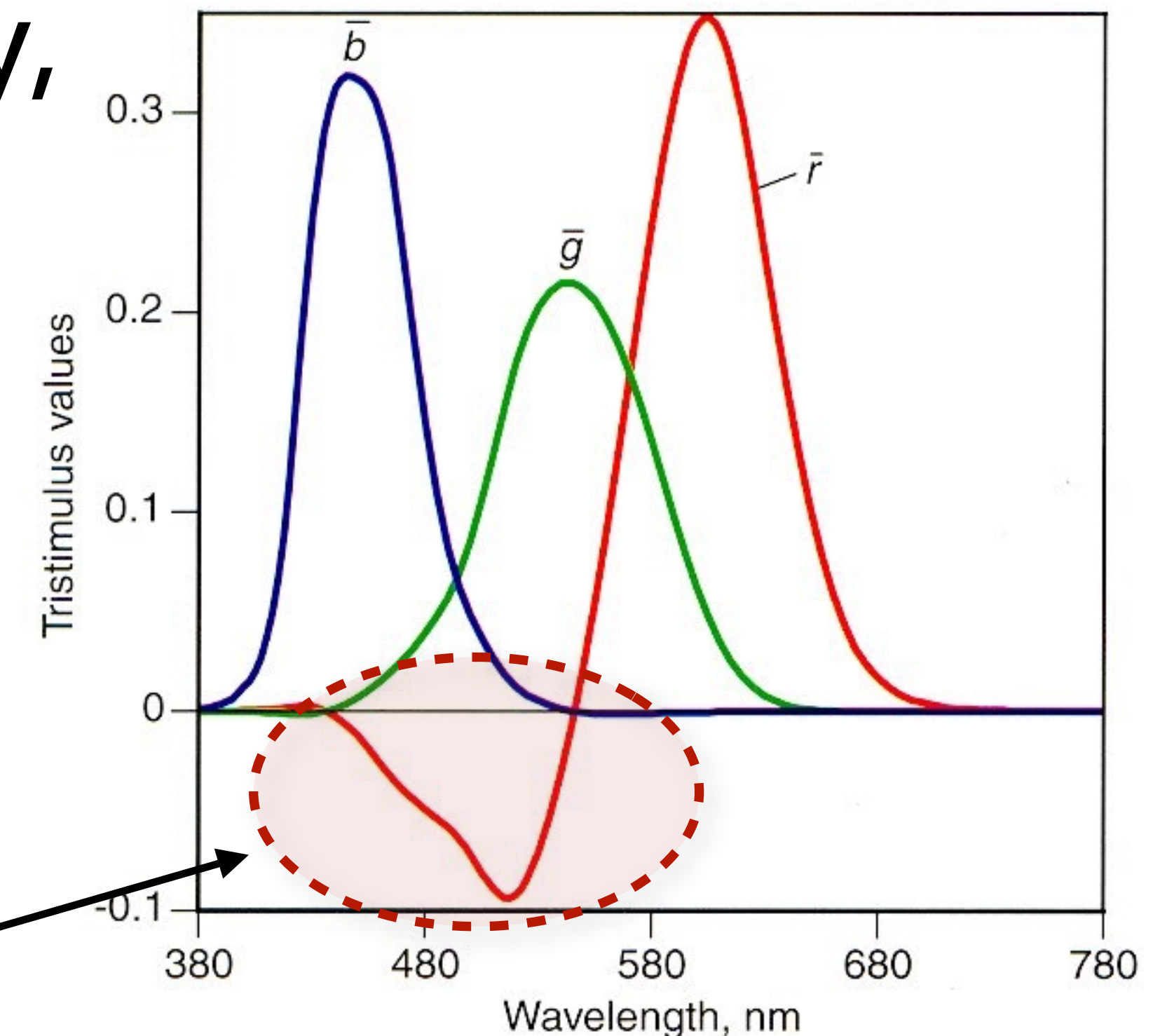
These curves are the color-matching functions for the 1931 standard observer, The average results of 17 color-normal observers having matched each wavelength of the equal-energy spectrum with primaries of 435.8 nm, 546.1 nm, and 700 nm.



# Meaning of these curves/rows

Monochromatic wavelength  $\lambda$  can be reproduced with:

+  $b(\lambda)$  amount of the 435.8nm primary,  
+  $g(\lambda)$  amount of the 546.1 primary,  
+  $r(\lambda)$  amount of the 700 nm primary



**Negative light required?**

These curves are the color-matching functions for the 1931 standard observer, The average results of 17 color-normal observers having matched each wavelength of the equal-energy spectrum with primaries of 435.8 nm, 546.1 nm, and 700 nm.



# CIE XYZ color space

Linear algebra to the rescue!

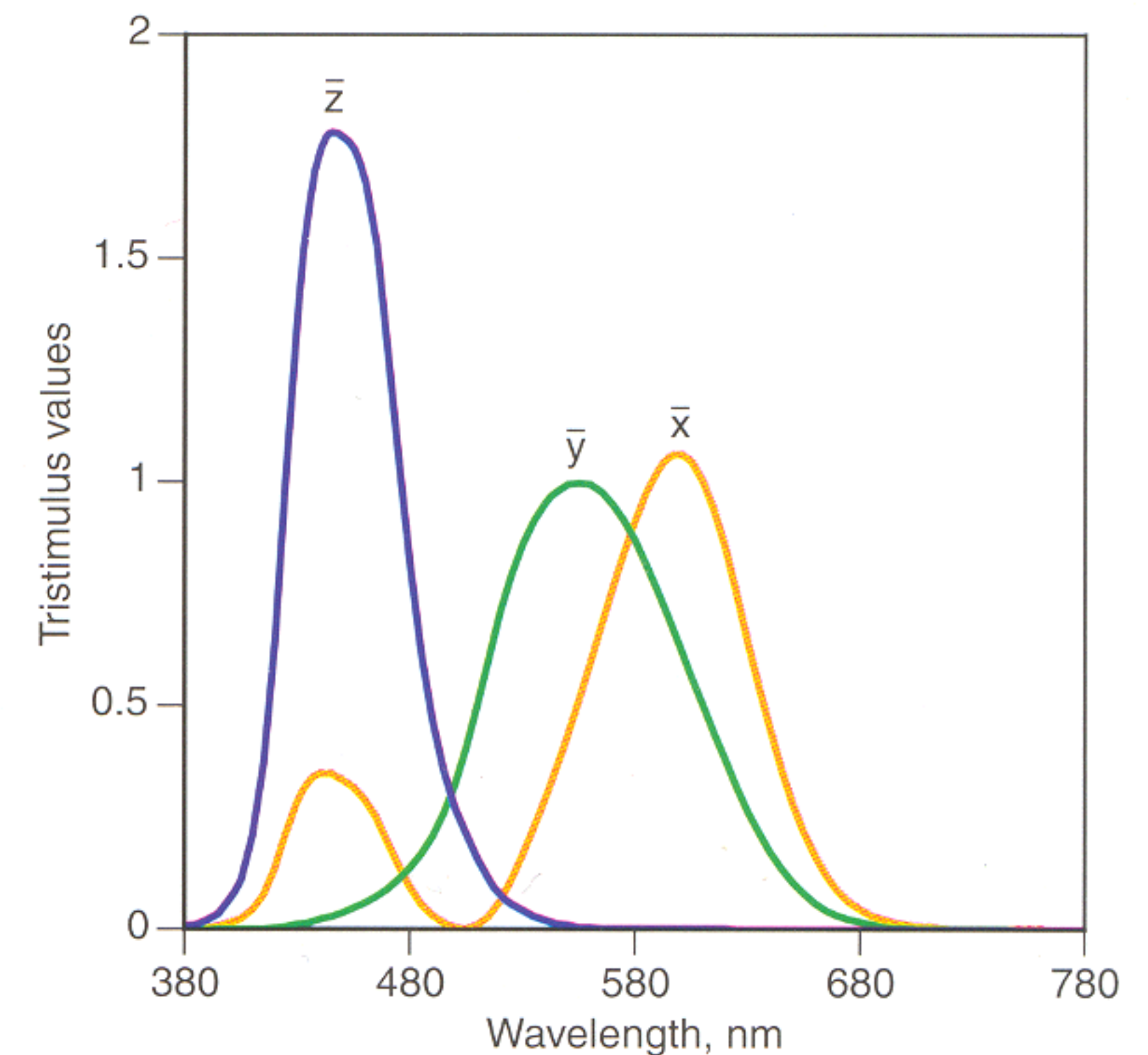
Purely positive basis functions

**Linear transformation** of CIE RGB

Non-physical primaries

CIE XYZ ←----- CIE RGB

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.17697 & 0.81240 & 0.01063 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$





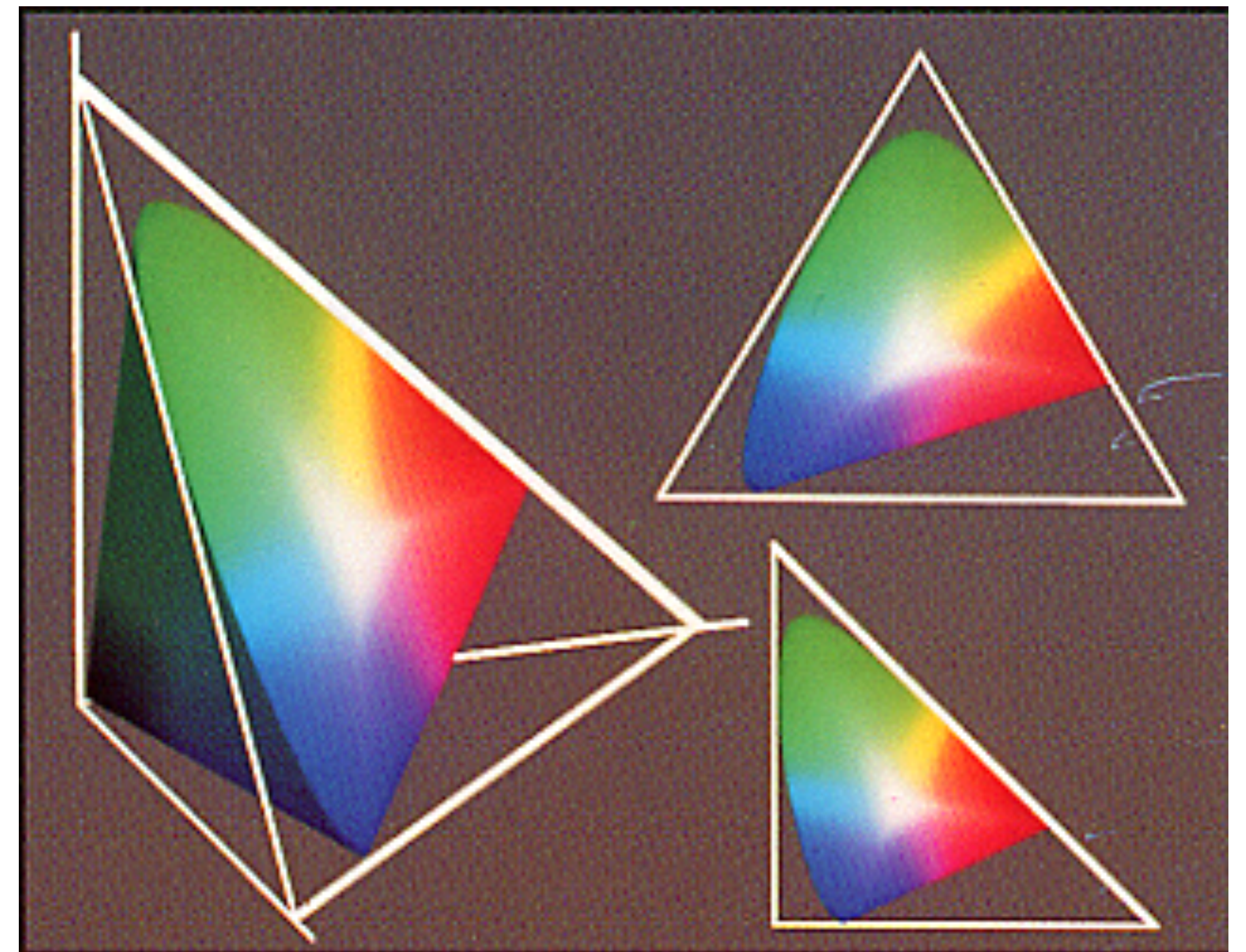
# CIE XYZ color cone


3D spaces can be hard to visualize

Chrominance is our notion of color, as opposed to brightness/luminance

Recall that our eyes correct for multiplicative scale factors

- discount light intensity





# Chromaticity Diagram



# The CIE xyY Color Space

*Chromaticity* (x,y) can be derived by normalizing the XYZ color components:

$$x = \frac{X}{X + Y + Z} \qquad y = \frac{Y}{X + Y + Z}$$

- (x,y) characterize color
- Y characterizes *brightness*

Combining xy with Y allows us to represent any color

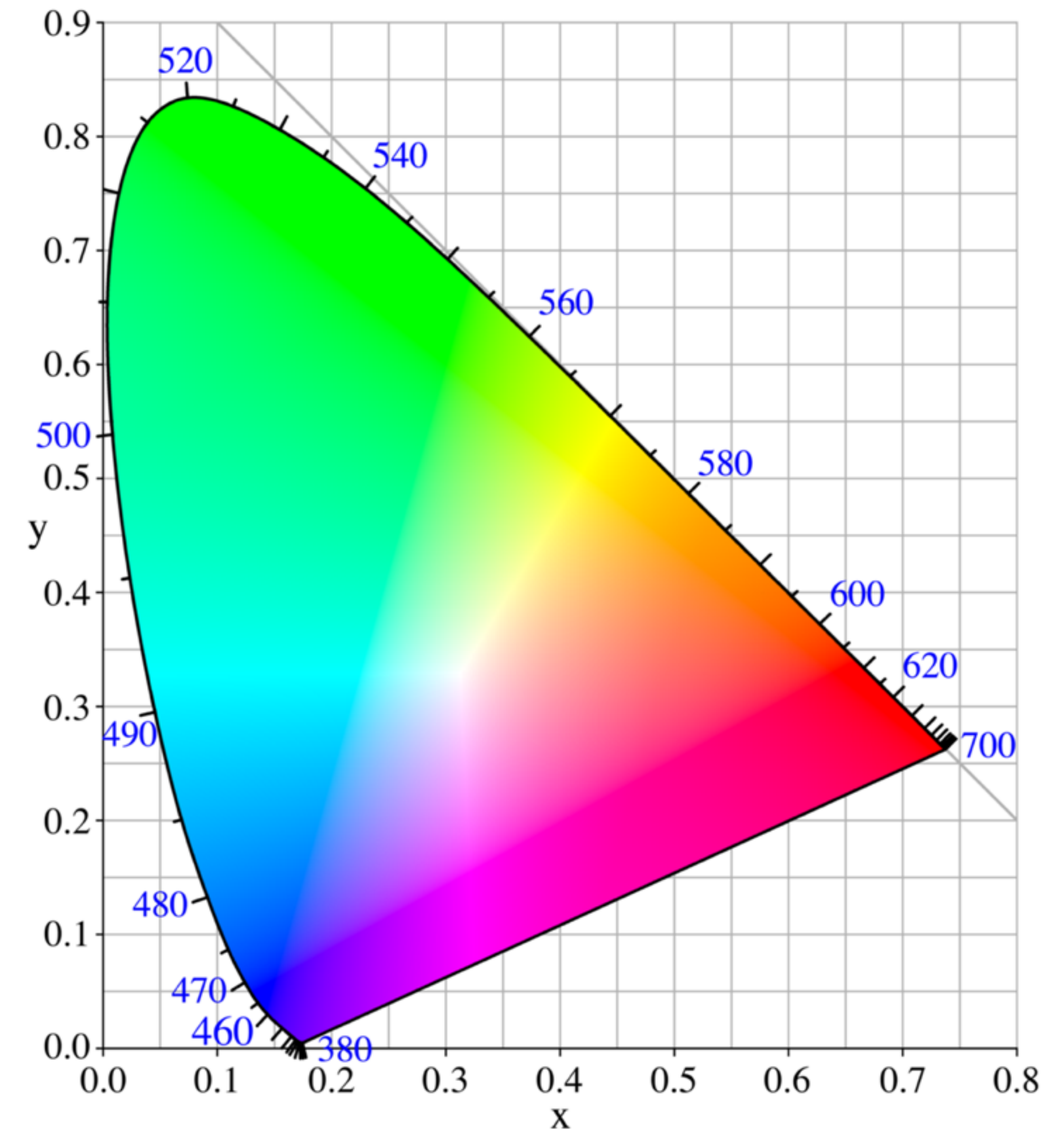
Plotting on xy plane allows us to see all colors of a single brightness

# CIE Chromaticity Chart

Spectral colors along curved boundary

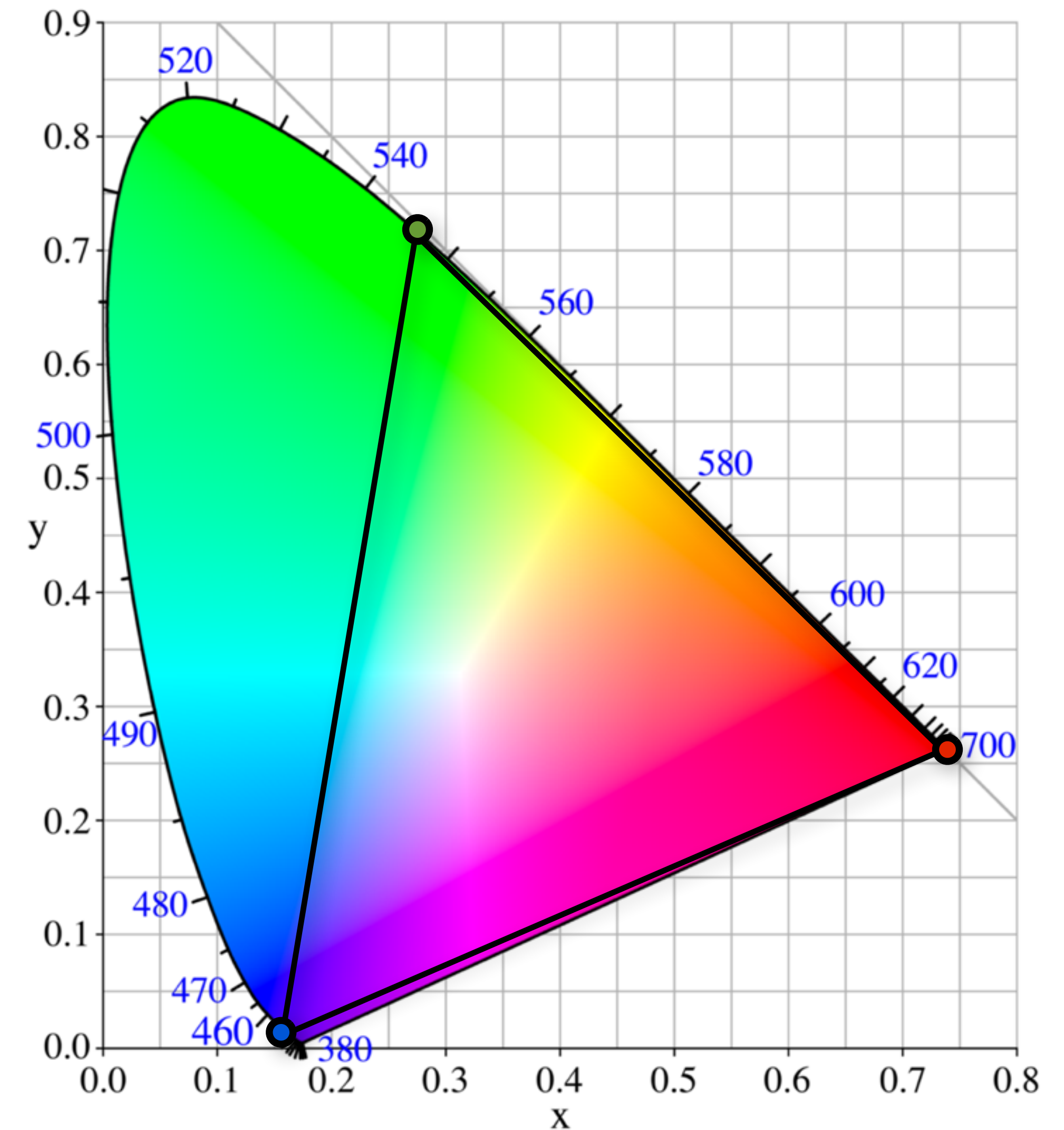
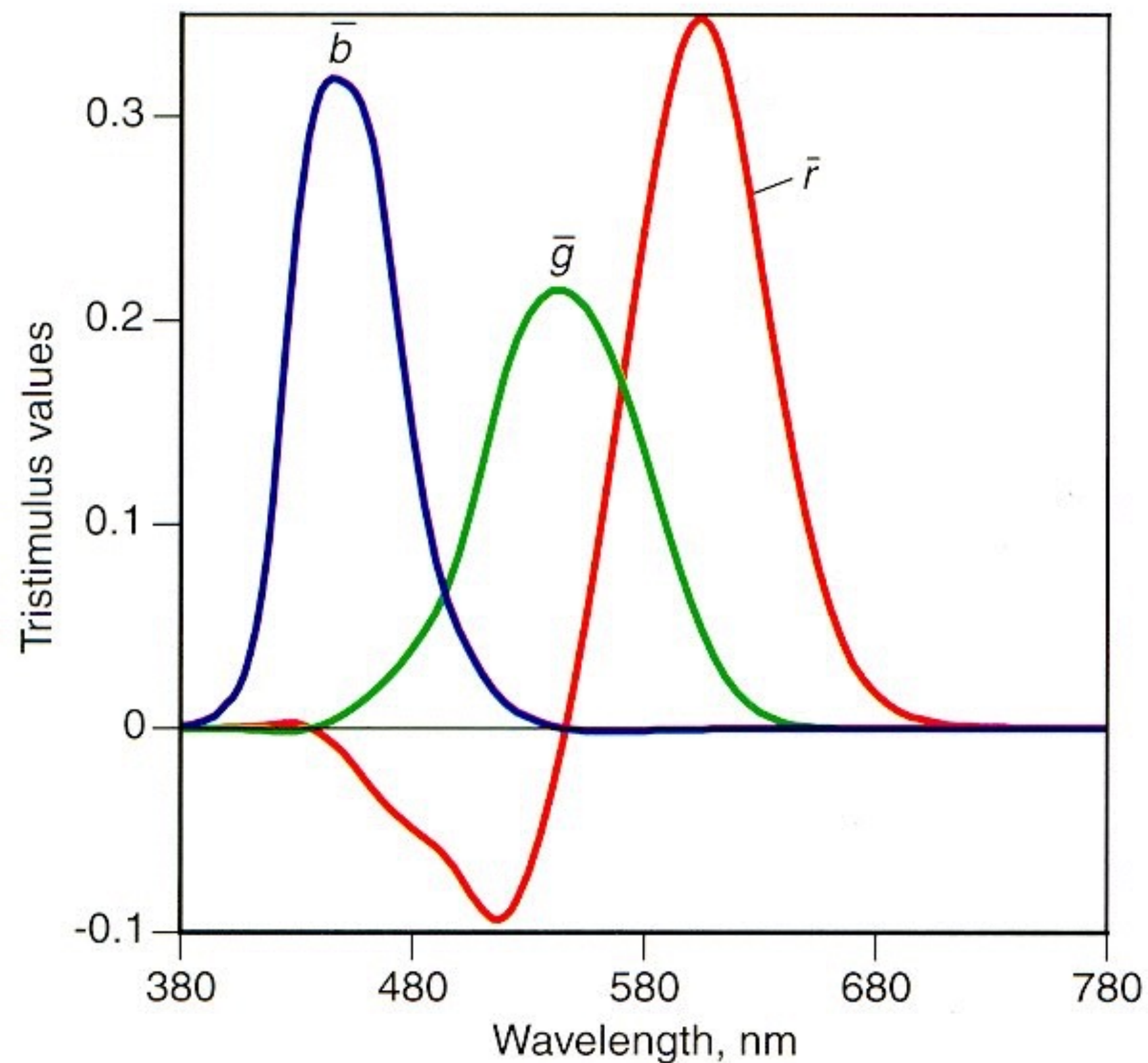
Linear combination of two colors:  
line connecting two points

Linear combination of 3 colors  
span a triangle (color gamut)



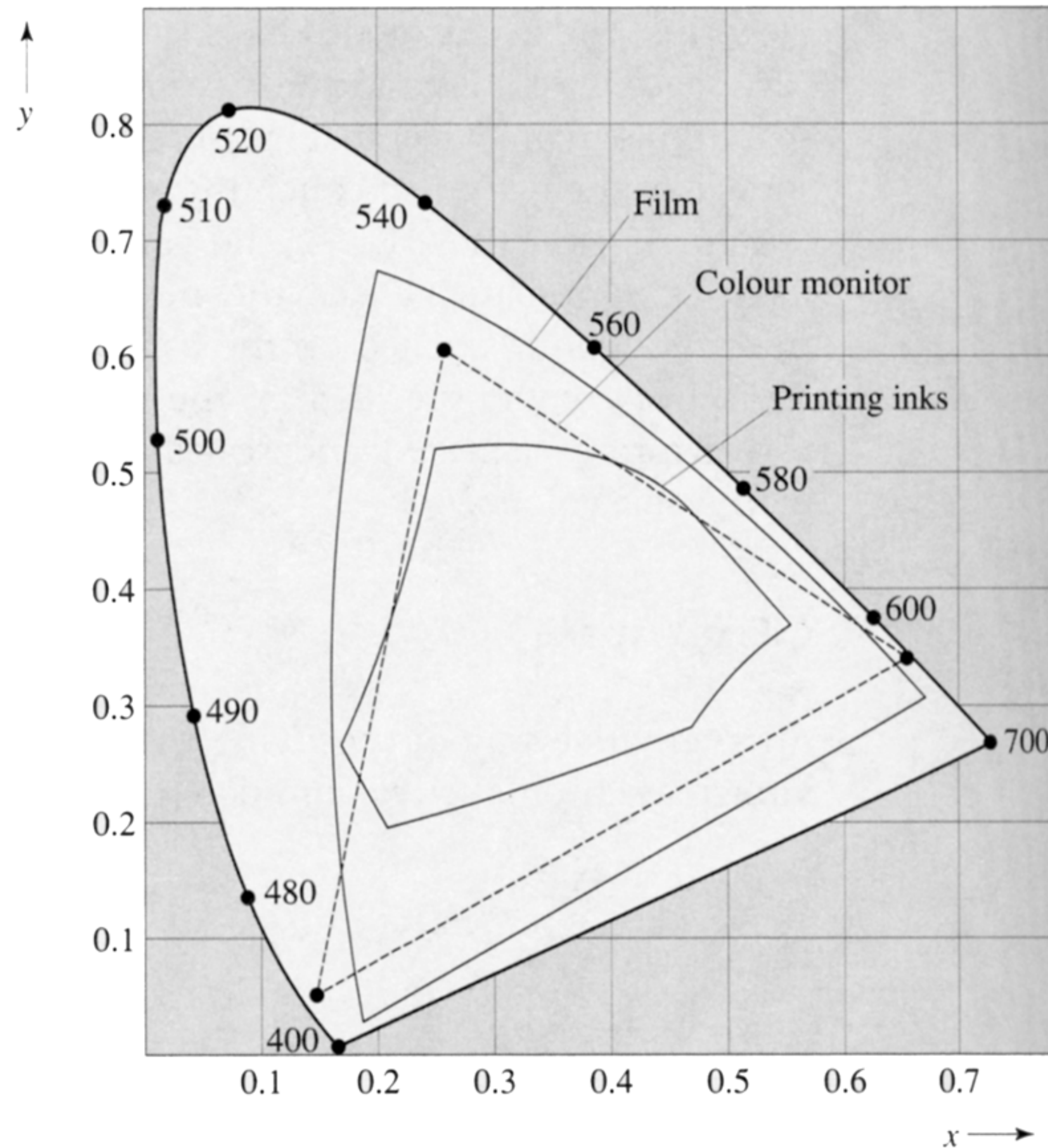
# CIE RGB Color Space

Color primaries at:  
435.8, 546.1, 700.0 nm





# Color Gamuts

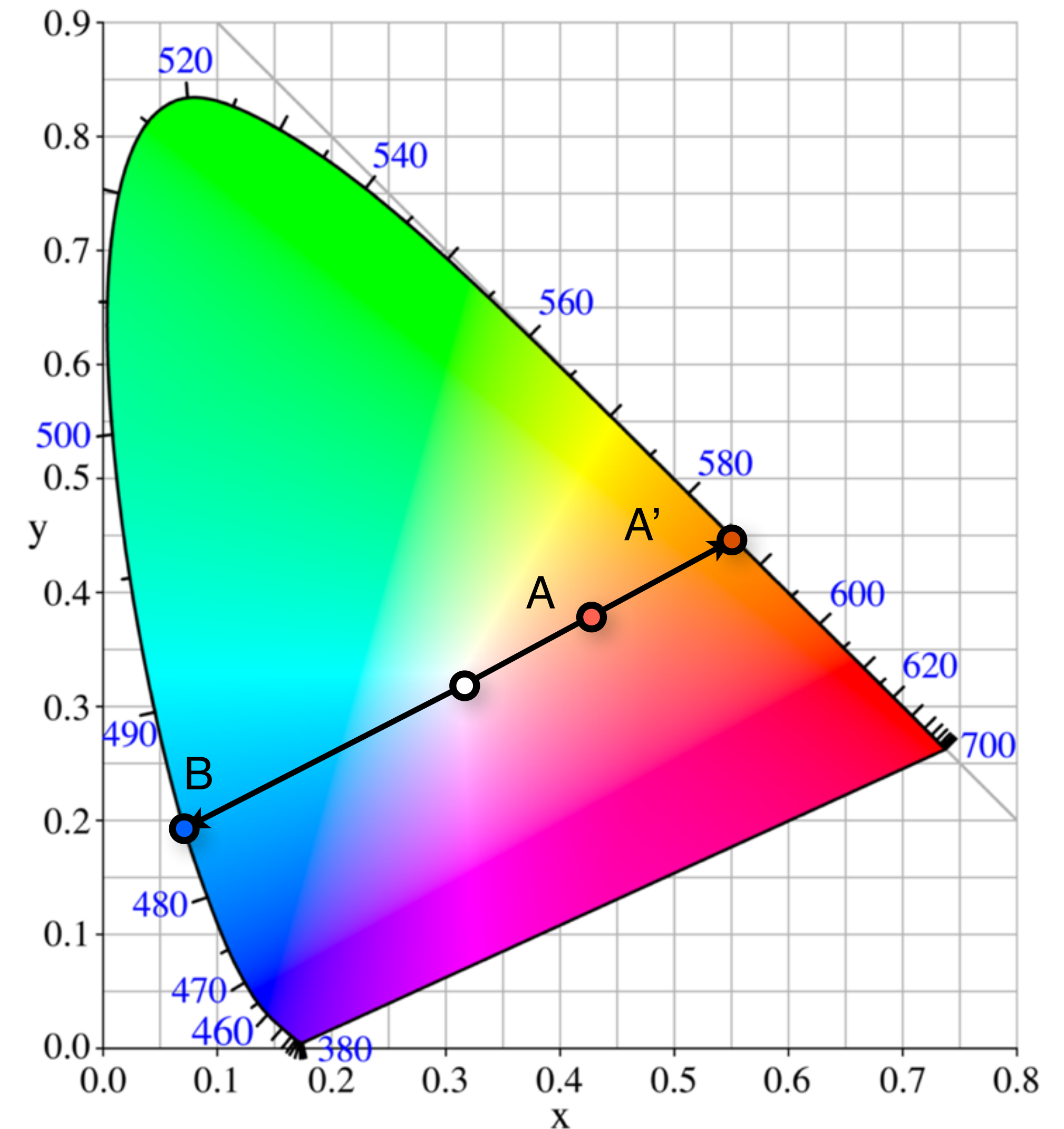


# CIE Chromaticity Chart Features

White Point

Dominant wavelength

Inverse color



# Perceptually-Uniform Color Spaces

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All these color spaces so far are perceptually non-uniform:

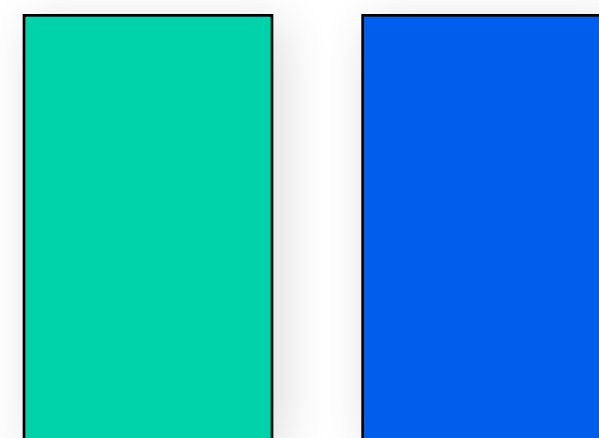
- two colors close together in space are not necessarily visually similar
- two colors far apart are not necessarily very different!

Measuring “perceptual distance” in color spaces  
important for many industries

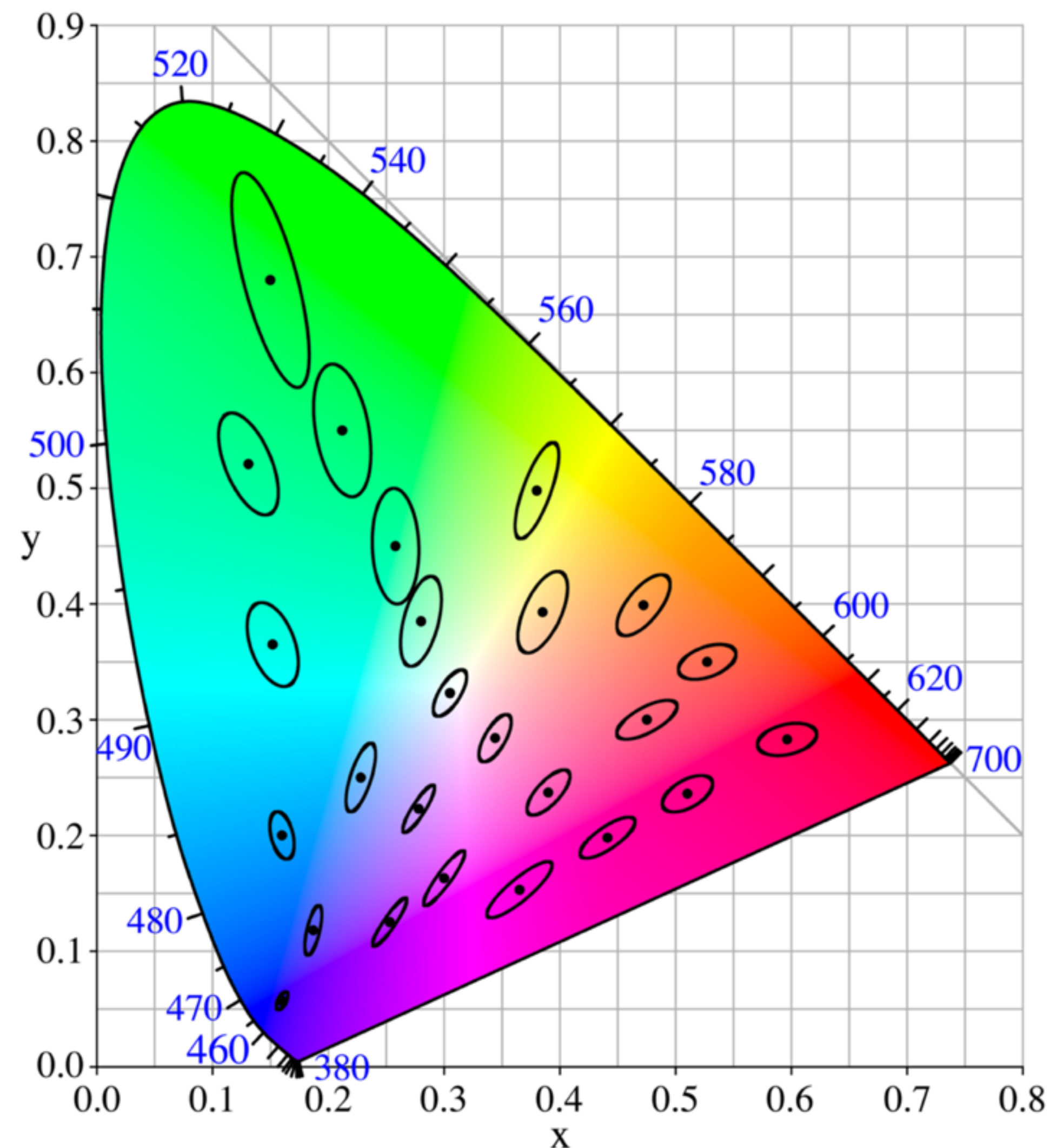
Experiments by MacAdams



# MacAdams Color Ellipses



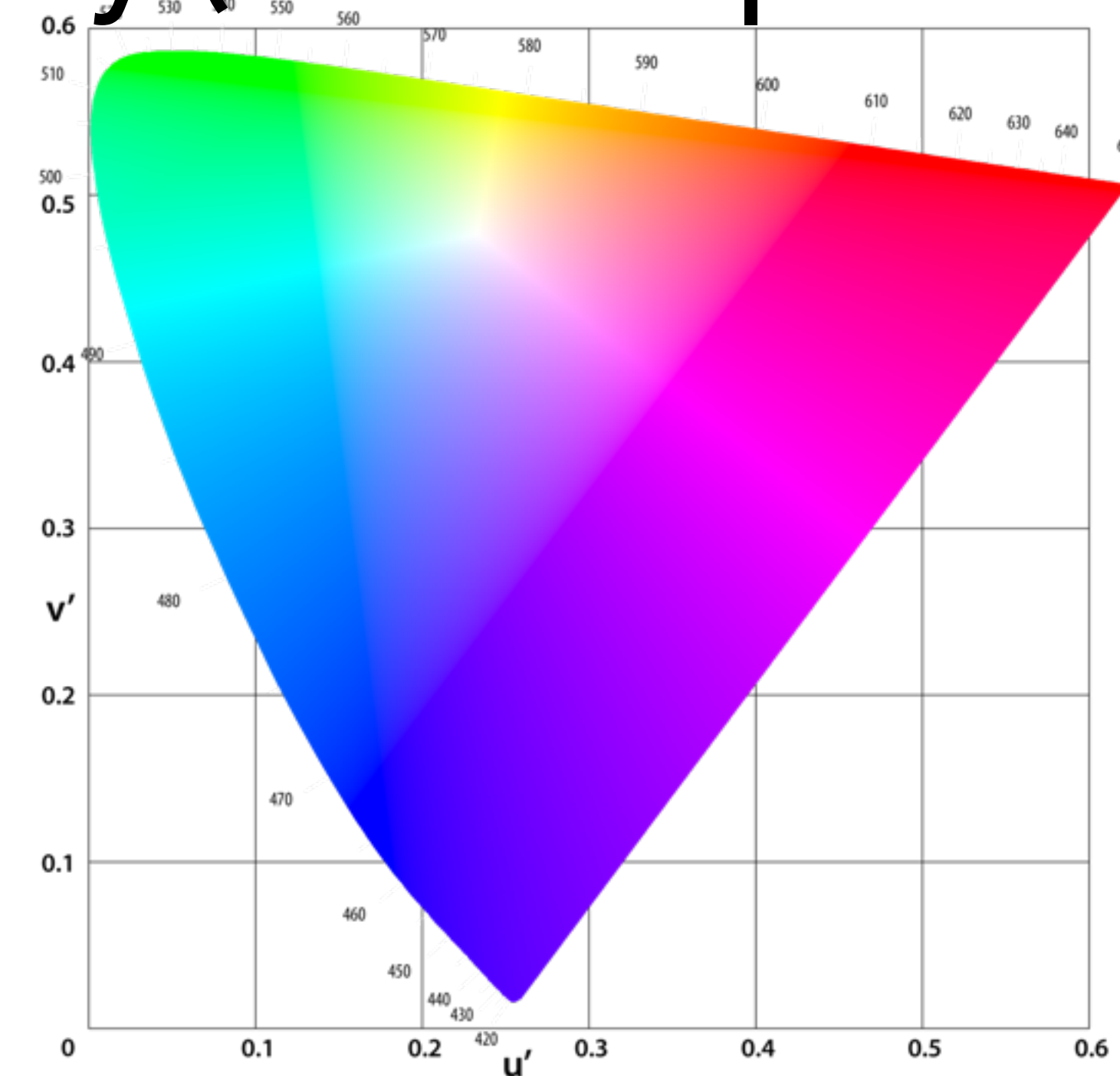
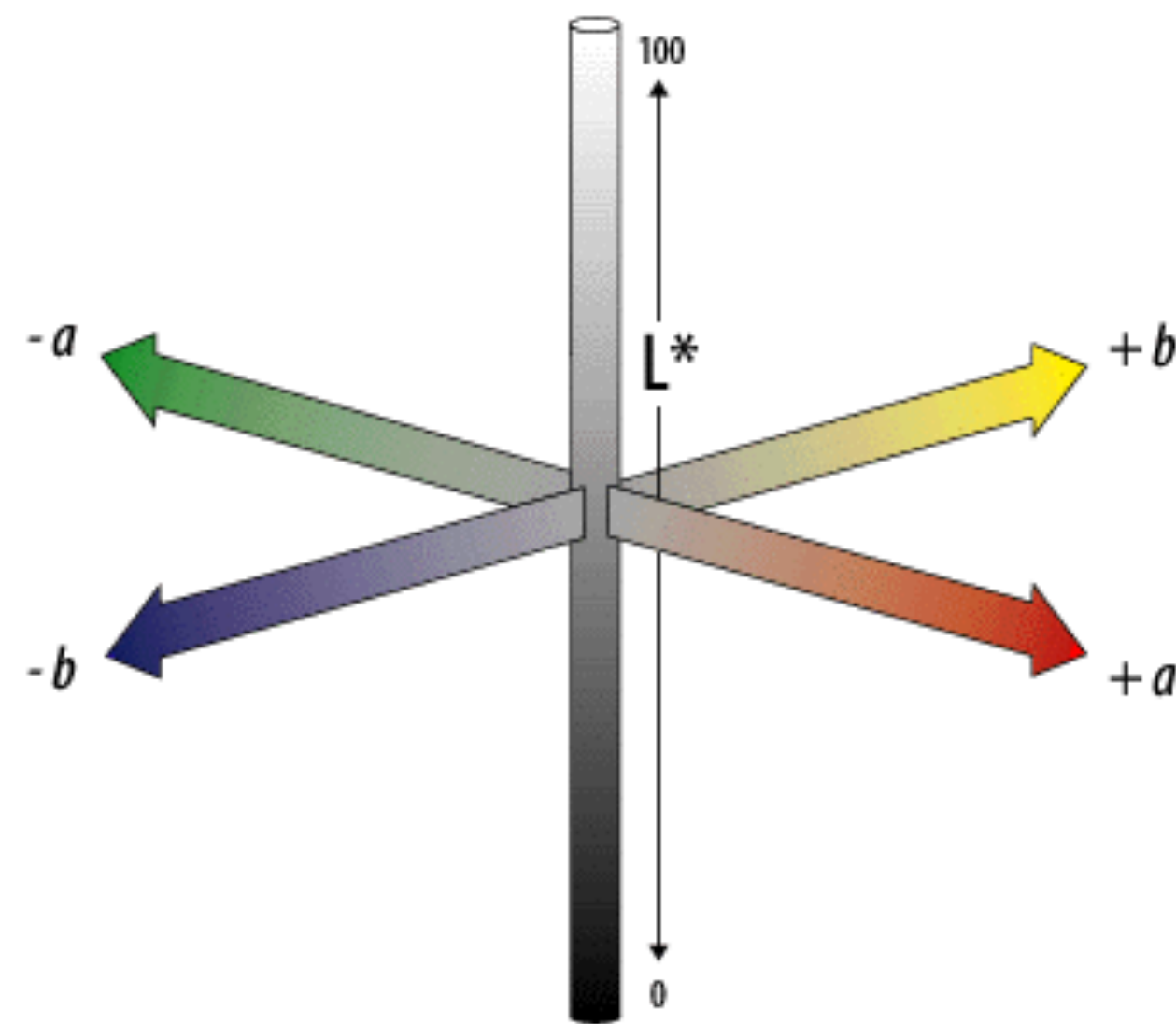
Test patches



# CIELab and CIELuv Color Spaces

Two attempts to make a perceptually-uniform color space

MacAdams ellipses become nearly (but not perfectly) circular





# Higher-level color perception

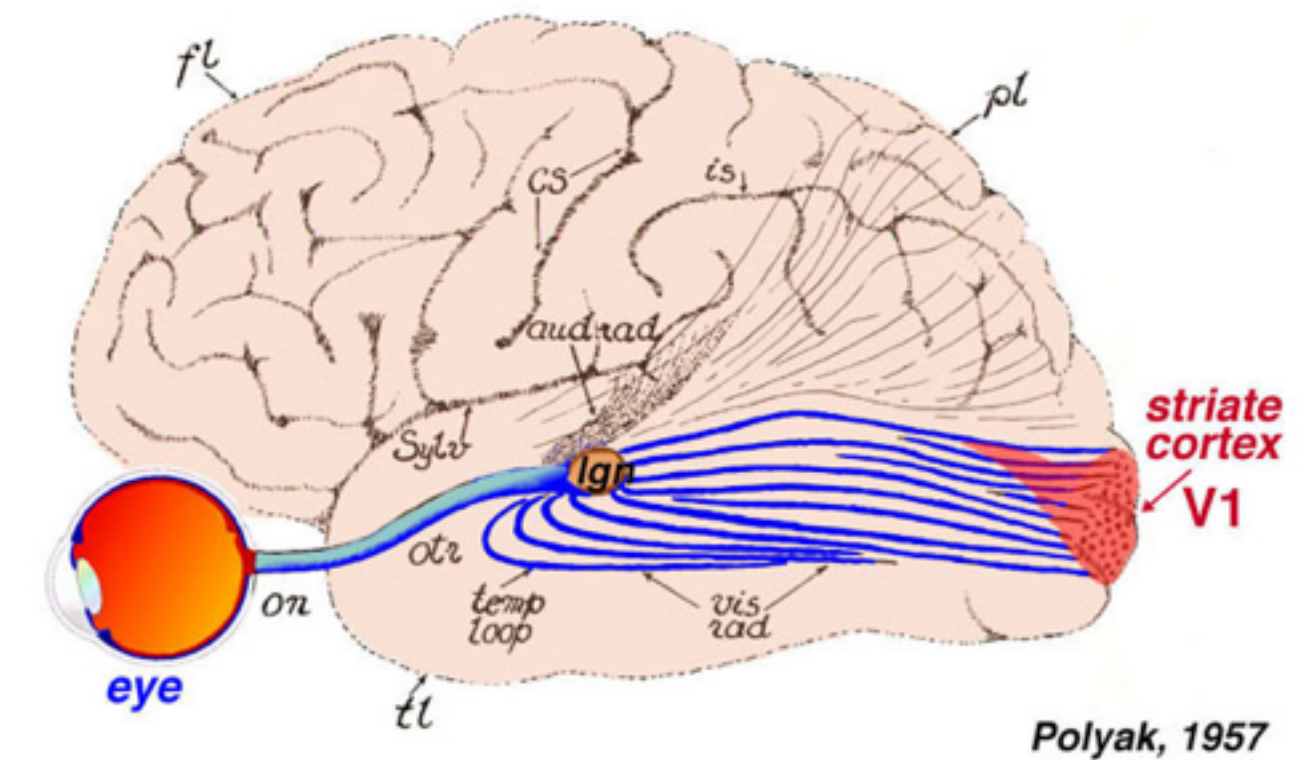


# Higher-level color perception

Color perception is much more complicated than response of SML cones...

## Visual pathway

- A lot happens after the cones
- But: cone responses are input to further processing



# Color constancy

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Also known as chromatic adaptation

Color of object is perceived as the same even under varying illumination

For example:

- A white sheet of paper under green illumination is still perceived as white, even though the reflected light is green! The human brain infers the white color from the context, which is “green-ish” too because of the green illumination.



# Color constancy

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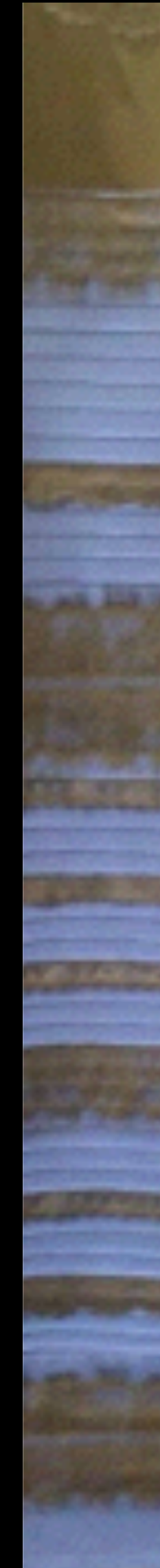




blue and black?  
or  
white and gold?



# Color constancy failure

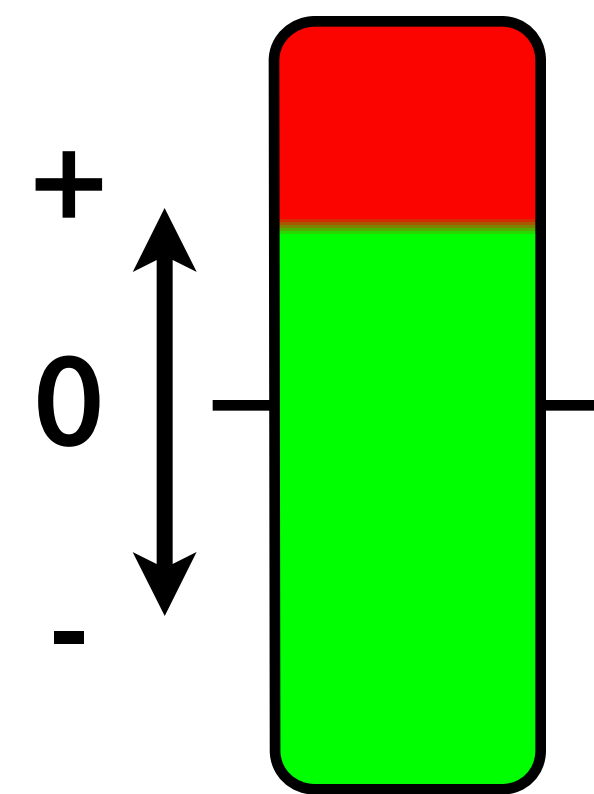




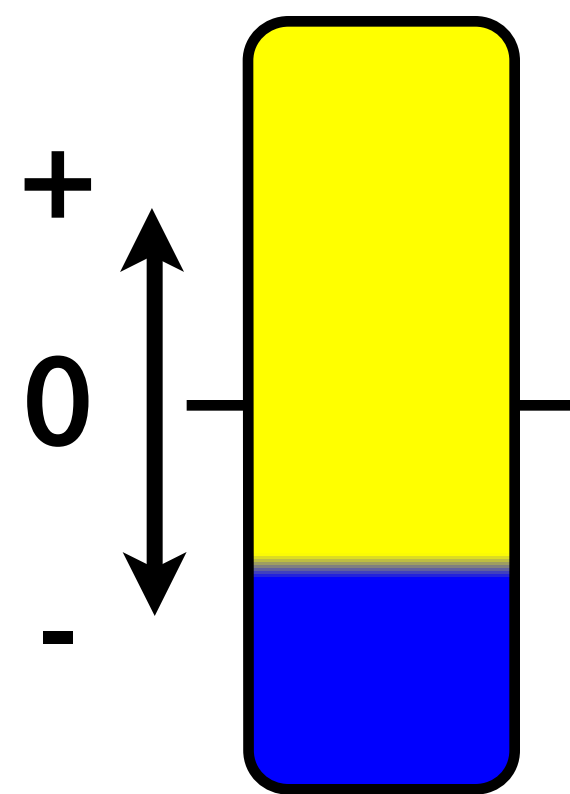
# Hering's opponent process theory (1874)

After sensing by cones, colors are encoded as red versus green, blue versus yellow, and black versus white

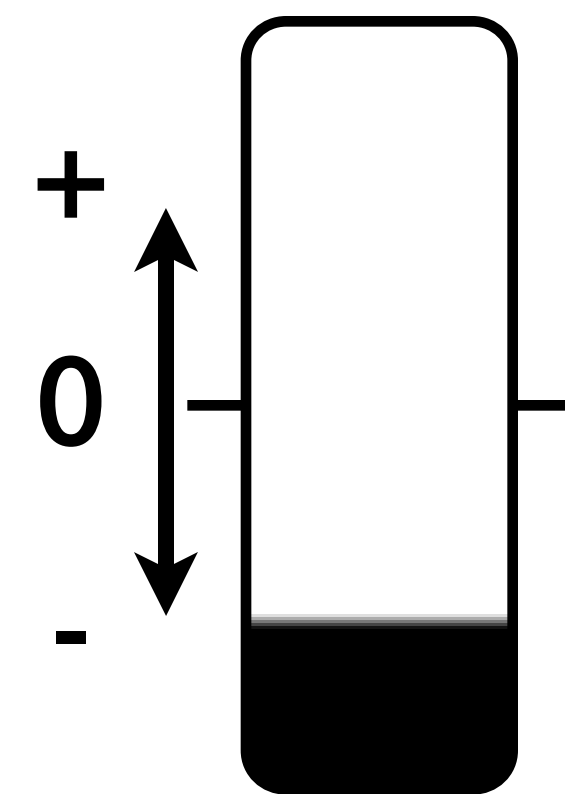
Physiological evidence found in the 1950s



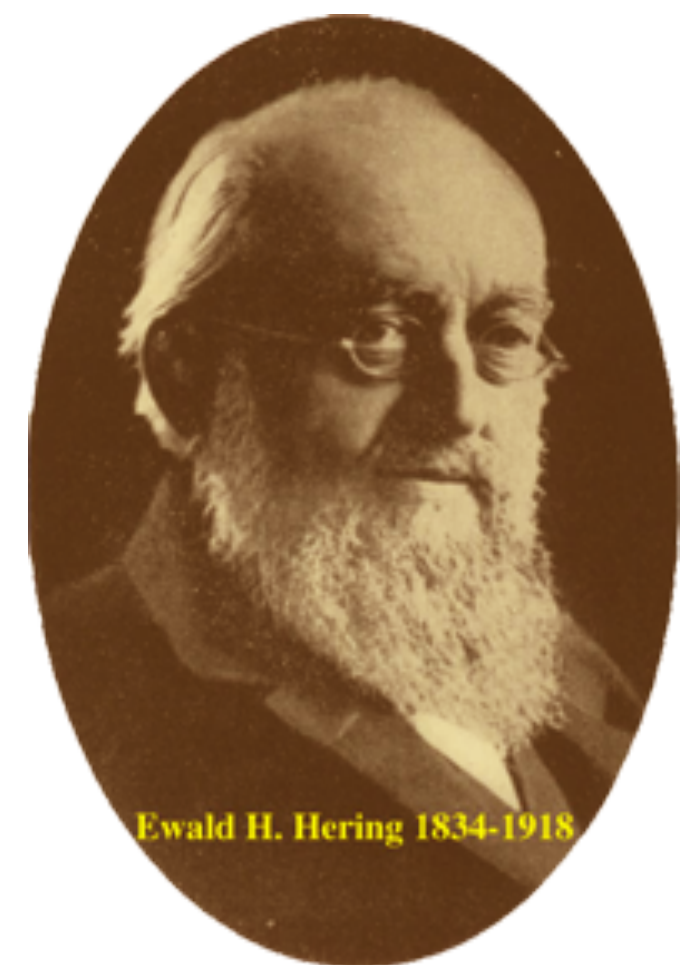
Red/Green  
Receptors



Blue/Yellow  
Receptors



Black/White  
Receptors



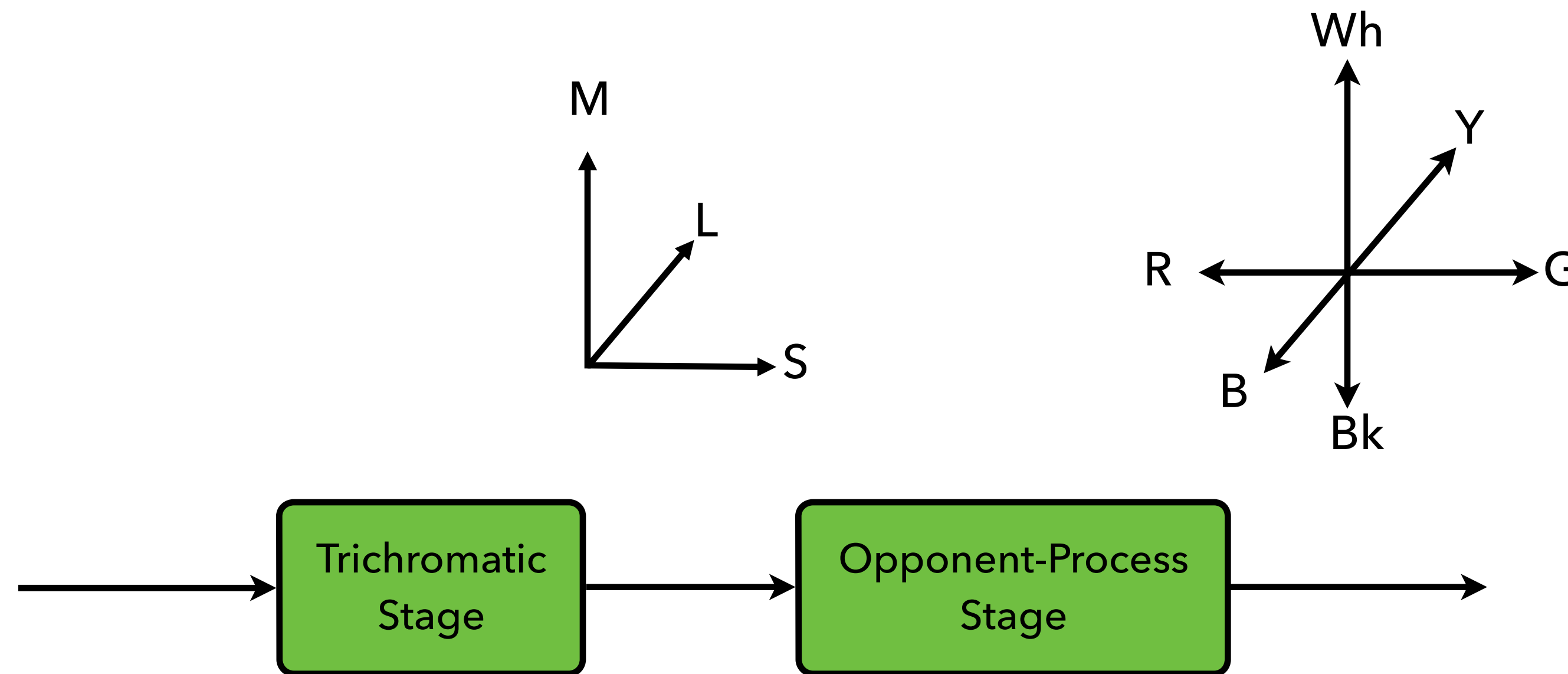


# Dual process theory

Inputs are LMS cone responses

Output has a different parameterization:

- Light-dark
- Blue-yellow
- Red-green



# Color opponents wiring

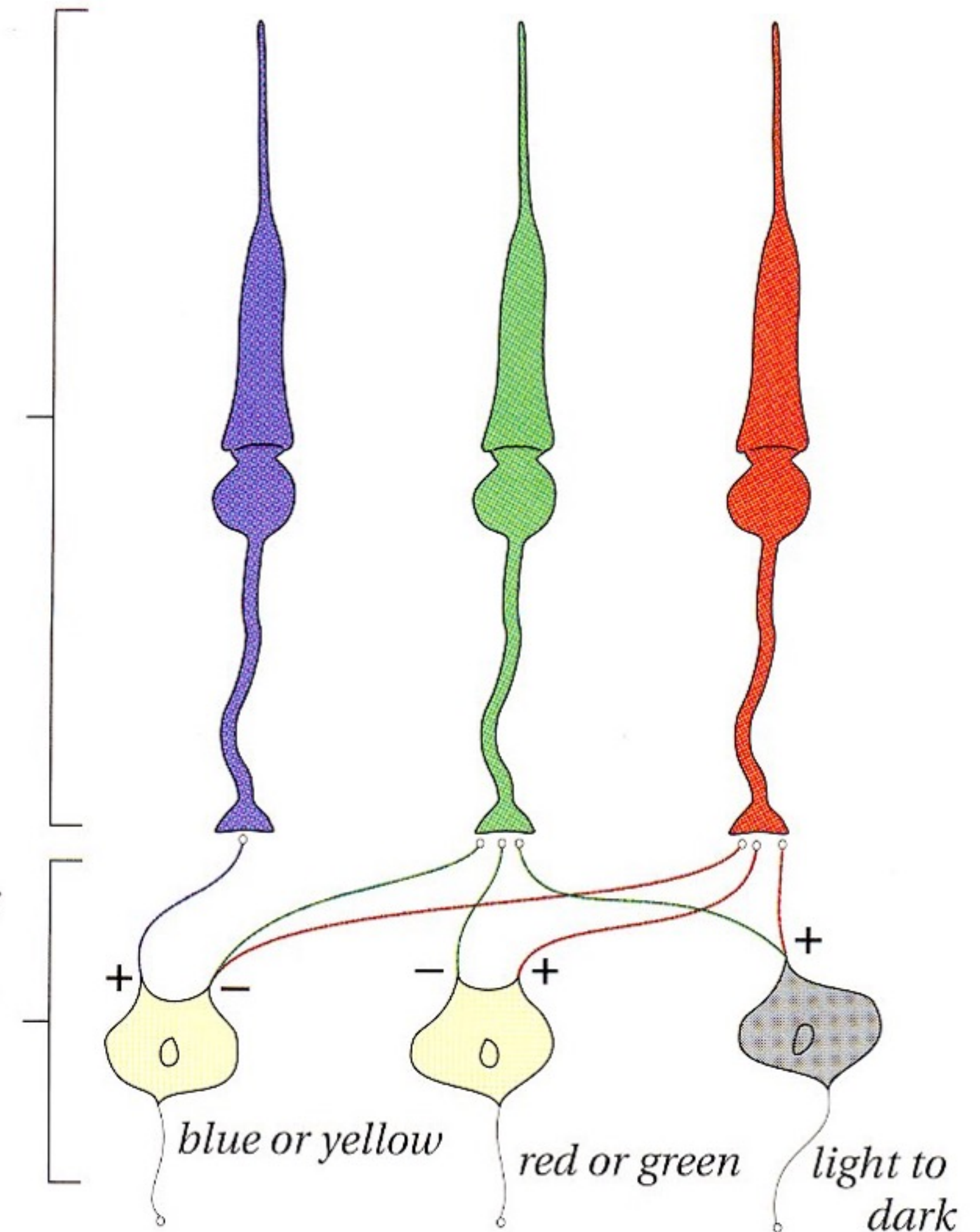
Sums for brightness

Differences for color  
opponents

At the end, it's just a 3x3  
matrix compared to LMS

*First zone (or stage):  
layer of retina with  
three independent  
types of cones*

*Second zone (or stage):  
signals from cones  
either excite or inhibit  
second layer of  
neurons, producing  
opponent signals*









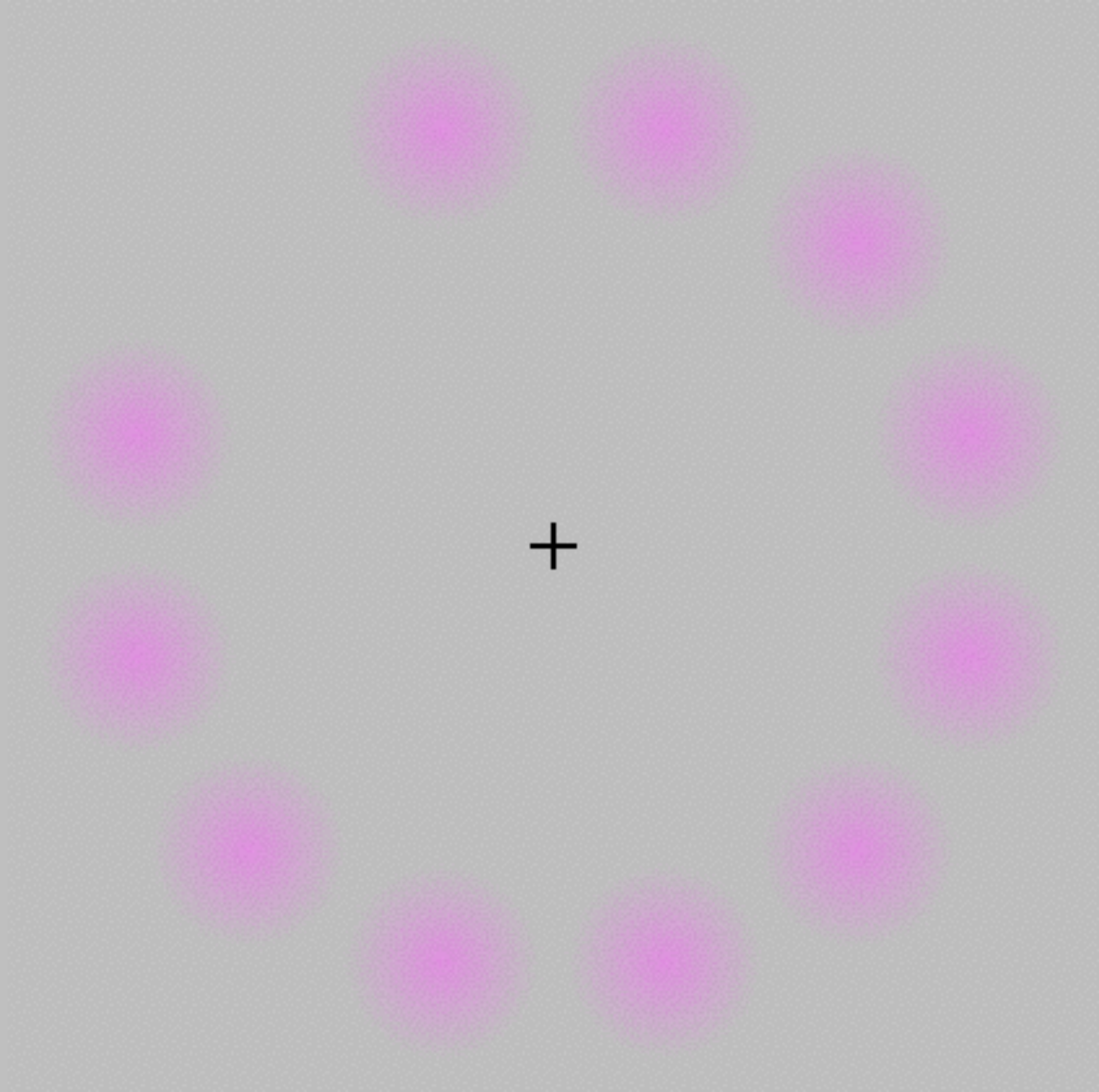
# Opponent Colors

Image



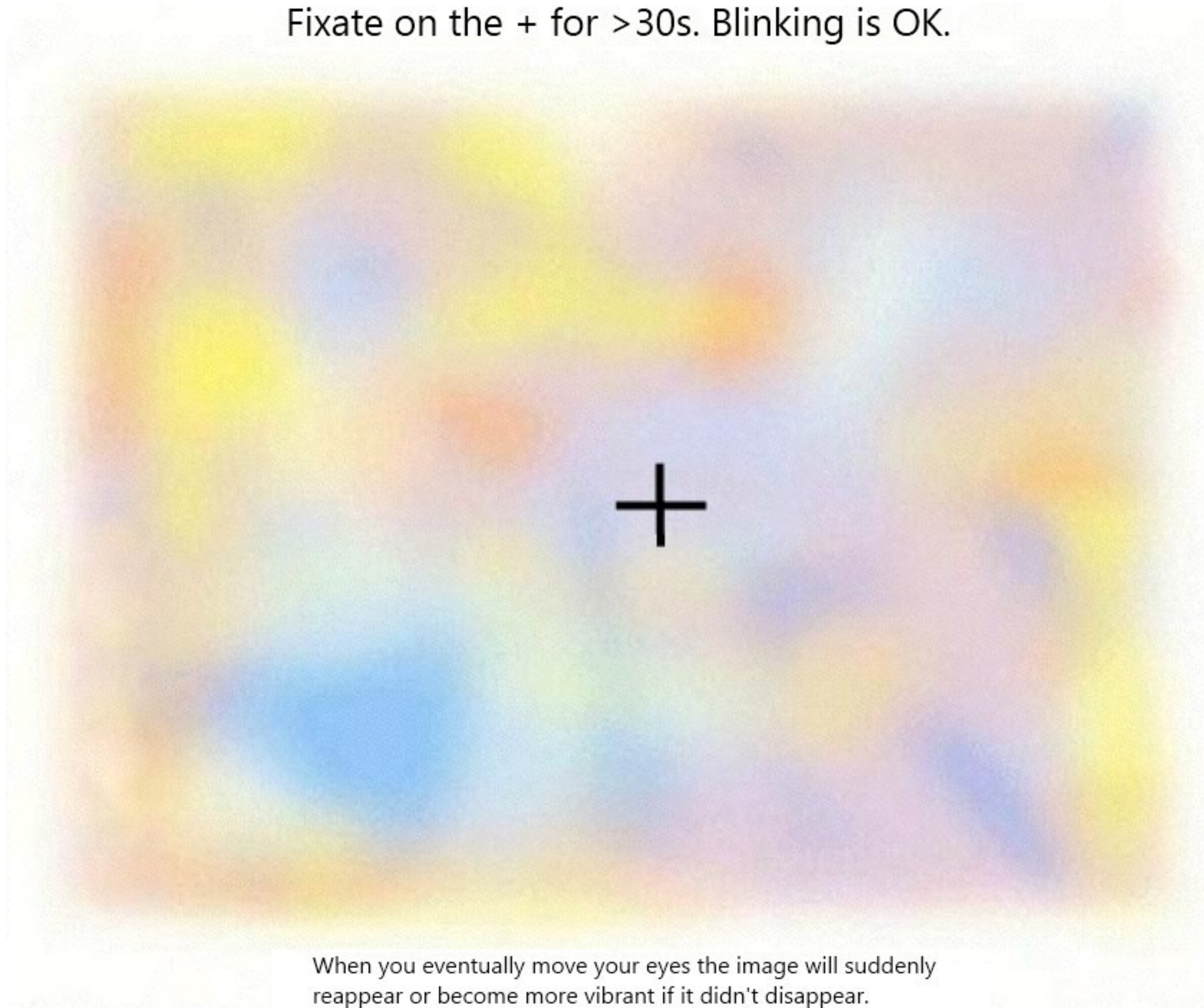
Afterimage







Fixate on the + for >30s. Blinking is OK.



When you eventually move your eyes the image will suddenly reappear or become more vibrant if it didn't disappear.



















# Opponent color spaces

Luminance, red-green, blue-yellow

CIE Lab

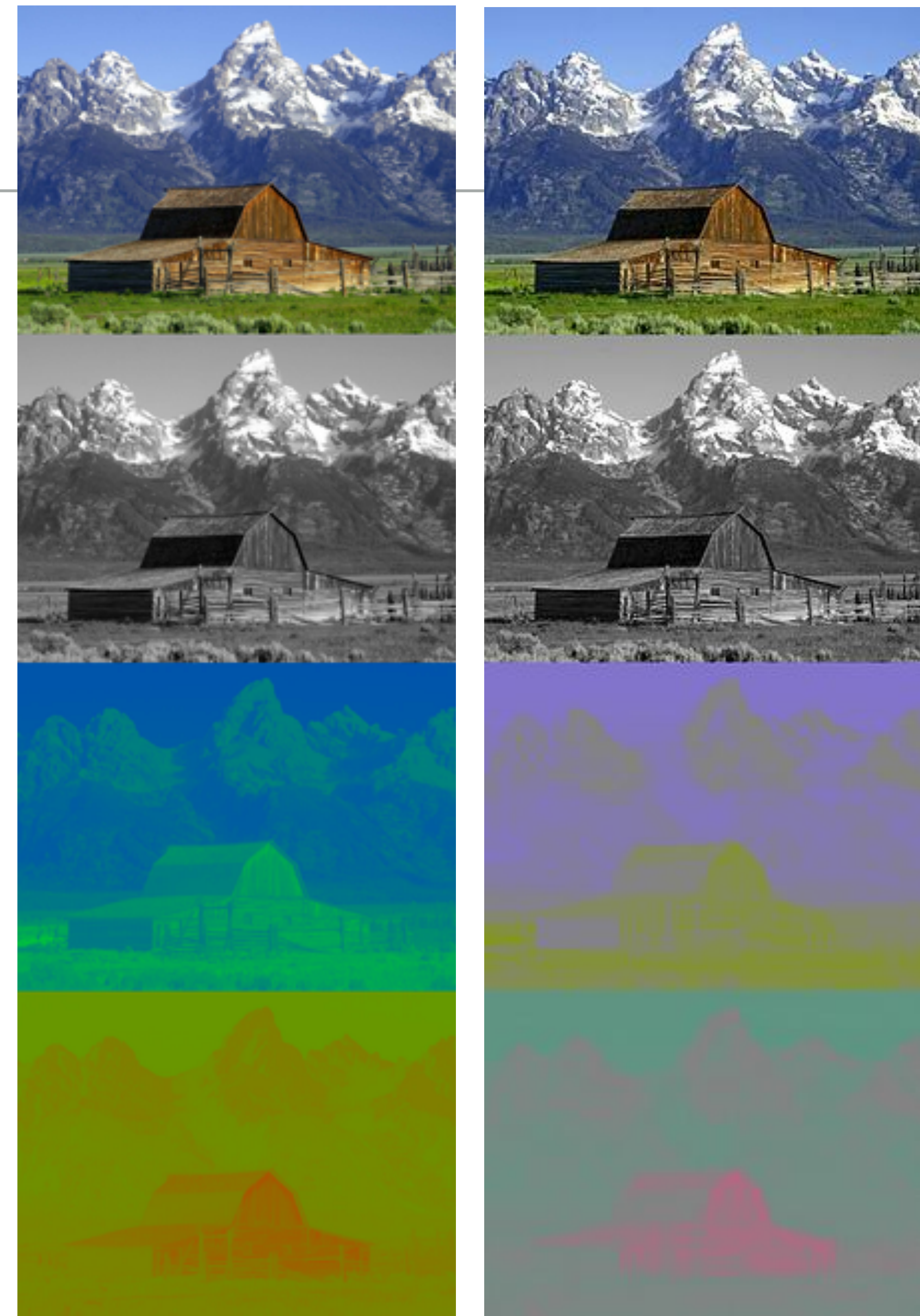
YUV

$$\begin{bmatrix} Y' \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.14713 & -0.28886 & 0.436 \\ 0.615 & -0.51499 & -0.10001 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1.13983 \\ 1 & -0.39465 & -0.58060 \\ 1 & 2.03211 & 0 \end{bmatrix} \begin{bmatrix} Y' \\ U \\ V \end{bmatrix}$$

YcrCb

- used a lot in image/video compression

$$Y' = 0 + (0.299 \cdot R'_D) + (0.587 \cdot G'_D) + (0.114 \cdot B'_D)$$
$$C_B = 128 - (0.168736 \cdot R'_D) - (0.331264 \cdot G'_D) + (0.5 \cdot B'_D)$$
$$C_R = 128 + (0.5 \cdot R'_D) - (0.418688 \cdot G'_D) - (0.081312 \cdot B'_D)$$



YUV

YCrCb

# Programming Assignment 2

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Gamma

Colorspaces

- color2gray
- luminance–chrominance separation
- luminance-only brightness/contrast
- $RGB \rightleftharpoons YUV$

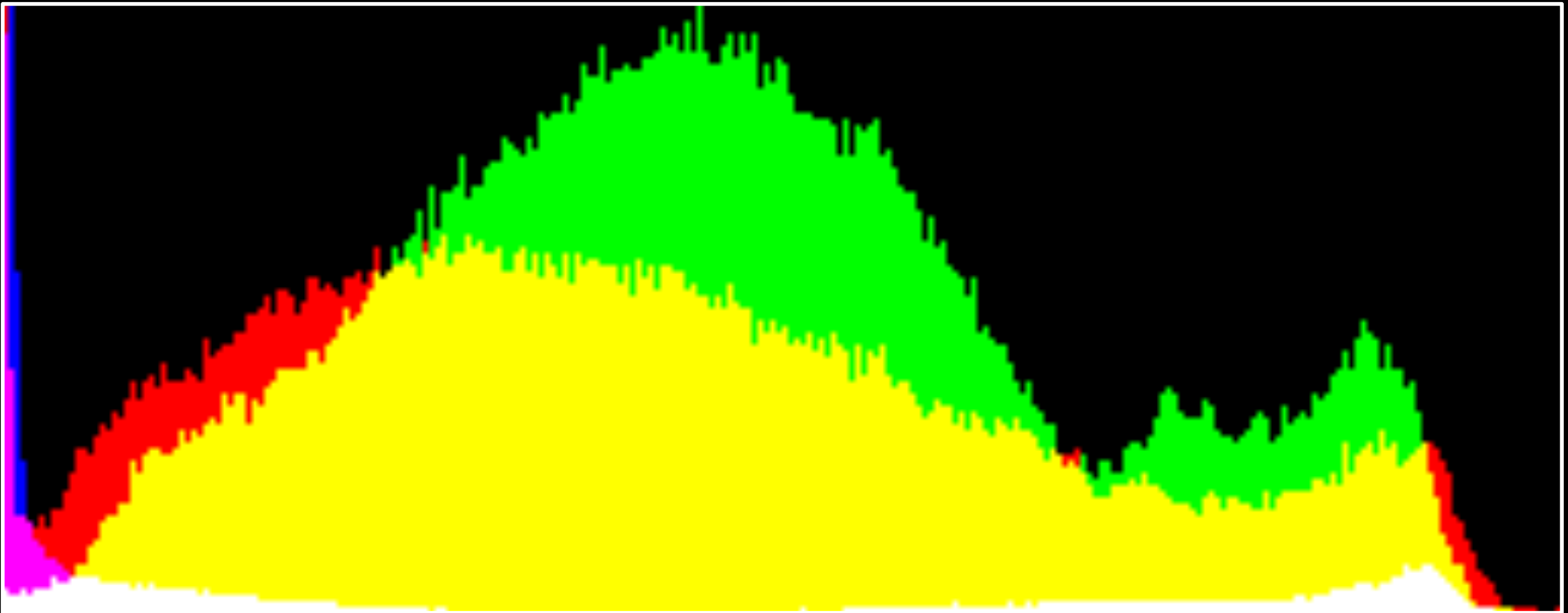
Spanish castle illusion

Grayworld whitebalance

Histograms

- autolevels
- visualize RGB histogram
- histogram equalization & histogram matching







# Slide credits

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Frédo Durand

Steve Marschner

Matthias Zwicker