UNIVERSITY OF CALIFORNIA, SAN DIEGO

EFFICIENT MONTE CARLO METHODS FOR LIGHT TRANSPORT IN SCATTERING MEDIA

A dissertation submitted in partial satisfaction of the
requirements for the degree
Doctor of Philosophy

in

Computer Science

by

Wojciech Jarosz

Committee in charge:

Henrik Wann Jensen, Co-Chair
Matthias Zwicker, Co-Chair
Samuel R. Buss
Per H. Christensen
David J. Kriegman
Falko Kuester

2008

Copyright
Wojciech Jarosz, 2008
All rights reserved.

The dissertation of Wojciech Jarosz is approved, and
it is acceptable in quality and form for publication
on microfilm and electronically:

Co-Chair

Co-Chair

University of California, San Diego

2008

To Dorota and Krzysztof

TABLE OF CONTENTS

Signature Page iii

Dedication iv

Table of Contents v

List of Figures ix

List of Tables xii

List of Algorithms xiii

Acknowledgements xiv

Vita and Publications xvi

Abstract of the Dissertation xvii

1 Introduction 1
  1.1 Summary of Original Contributions 5
  1.2 Organization of the Dissertation 6

2 Fundamentals of Light Transport 8
  2.1 Assumptions About the Nature of Light 8
  2.2 Radiometry 9
    2.2.1 Radiometric Quantities 9
    2.2.2 Radiometric Relationships 11
    2.2.3 Incident and Exitant Radiance Functions 12
  2.3 Interaction of Light with Surfaces 14
    2.3.1 The BRDF 14
    2.3.2 The Rendering Equation 16
  2.4 Methods for Solving the Rendering Equation 19
    2.4.1 Finite Element Methods 19
    2.4.2 Monte Carlo Ray Tracing Methods 20
    2.4.3 Hybrid Methods 21

3 Irradiance Caching and Derived Methods 23
  3.1 Algorithm Overview 23
  3.2 Computing Irradiance 25
  3.3 Interpolating Irradiance 27
    3.3.1 The “Split-Sphere” Model 27
    3.3.2 Derivation of the “Split-Sphere” Model 30
  3.4 Irradiance Gradients 31
  3.5 Radiance Caching 33
    3.5.1 Radiance Computation 33
    3.5.2 Radiance Interpolation 34
    3.5.3 Translational Radiance Gradients 34

3.6 Derivation of Irradiance Gradients 36
  3.6.1 The Rotational Gradient 38
  3.6.2 The Translational Gradient 39
  3.6.3 Equivalence of the Gradient Formulations 43
3.7 Other Extensions 45
  3.7.1 Approximate Global Illumination 45
  3.7.2 Distributed Global Illumination 48
  3.7.3 Animation 50
3.8 Limitations 53

4 Light Transport in Participating Media 55
4.1 Assumptions About Scattering Media 56
4.2 Light Interaction Events 56
  4.2.1 Extinction 57
  4.2.2 In-Scattering and Emission 59
  4.2.3 Medium Properties 60
4.3 In-Scattered Radiance and the Phase Function 61
  4.3.1 In-Scattered Radiance 61
  4.3.2 Properties of the Phase Function 61
  4.3.3 Examples of Phase Functions 62
4.4 The Volume Rendering Equation 66
  4.4.1 The Radiative Transfer Equation 66
4.5 Methods for Solving the Volume Rendering Equation 68
  4.5.1 Deterministic Methods 69
  4.5.2 Stochastic Methods 70

5 Radiance Caching in Participating Media 72
5.1 Contributions 72
5.2 Overview 73
5.3 Single Scattering 76
  5.3.1 Monte Carlo Integration 77
  5.3.2 Point Lights 78
  5.3.3 Gradient of Geometry Terms 79
  5.3.4 Gradient of Phase Function 80
  5.3.5 Reduced Radiance and Transmittance Gradient 81
  5.3.6 Isotropic and Anisotropic Scattering 82
5.4 Multiple Scattering 84
  5.4.1 Monte Carlo Integration 85
  5.4.2 Isotropic and Anisotropic Scattering 86
5.5 Algorithm 87
  5.5.1 Cache Entry Storage 87
  5.5.2 Valid Radius and Error Tolerance 88
  5.5.3 The Extrapolated Radiance Estimate 91
5.6 Results 93
5.7 Summary and Discussion 97
5.8 Conclusion 99
5.9 Acknowledgements 100

6 Irradiance Gradients in the Presence of Participating Media and Occlusions 101
  6.1 Contributions 101
  6.2 Overview 103
  6.3 Irradiance Gradients for Surfaces 104
    6.3.1 Irradiance Gradients 104
    6.3.2 Gradient of Cell Radiance 106
  6.4 Irradiance Gradients for Media 107
    6.4.1 Visibility Gradient 109
  6.5 Implementation 110
  6.6 Results 113
  6.7 Summary and Discussion 116
  6.8 Conclusion 118
  6.9 Acknowledgements 118

7 The Photon Mapping Method 119
  7.1 Algorithm Overview 119
  7.2 Photon Tracing 120
    7.2.1 Photon Emission 120
    7.2.2 Photon Scattering 121
    7.2.3 Photon Storage 121
    7.2.4 Importance-Driven Photon Mapping 121
  7.3 Radiance Estimation 122
  7.4 Participating Media 124
    7.4.1 Photon Tracing 124
    7.4.2 Ray Marching and the Volumetric Radiance Estimate 127

8 The Beam Radiance Estimate 128
  8.1 Contributions 129
  8.2 Reformulation of Volumetric Photon Mapping 130
    8.2.1 Generalized Path Integral Formulation 131
    8.2.2 The Measurement Equation 133
    8.2.3 Volumetric Photon Tracing 134
    8.2.4 Radiance Estimation Using the Measurement Equation 135
    8.2.5 Kernel Radiance Estimation 138
  8.3 Algorithm 139
  8.4 Results 142
  8.5 Summary and Discussion 144
  8.6 Acknowledgements 144

9 Conclusions 147

A Monte Carlo Integration 149
  A.1 Probability Background 150
    A.1.1 Random Variables 150
    A.1.2 Cumulative Distributions and Density Functions 150
    A.1.3 Expected Values and Variance 151
  A.2 The Monte Carlo Estimator 152

    A.2.1 Expected Value and Convergence 153
    A.2.2 Multidimensional Integration 155
  A.3 Variance Reduction 156
    A.3.1 Importance Sampling 157
    A.3.2 Control Variates 159
    A.3.3 Uniform Sample Placement 160
    A.3.4 Adaptive Sampling 164
    A.3.5 Biased Monte Carlo 165

B Spherical Harmonics 167
  B.1 Definition 168
  B.2 Projection and Expansion 170
  B.3 Properties 172

C Density Estimation 178
  C.1 Introduction 179
  C.2 Histograms 179
  C.3 Orthogonal Series Estimation 181
  C.4 Naïve Estimator 182
  C.5 Kernel Estimator 183
  C.6 Locally Adaptive Estimators 184
    C.6.1 Balloon Estimator 185
    C.6.2 Sample-point Estimator 186

Bibliography 188

Index 200

LIST OF FIGURES

Figure 1.1: Volumetric scattering due to participating media is responsible for the appearance of a number of striking visual effects. 4

Figure 2.1: The theory of light is described by a series of increasingly complete and complex optical models. 9
Figure 2.2: Illustrations of the radiometric quantities of flux, irradiance, and radiance. . 10
Figure 2.3: Incident vs. outgoing radiance and the BRDF. 12
Figure 2.4: In a vacuum, the incident radiance at **x** from direction $\vec{\omega}$ is equal to the exitant radiance from the nearest visible surface in that direction. 13
Figure 2.5: Illustrations of BRDFs for ideal diffuse, or Lambertian, perfectly specular, and glossy surfaces. 14
Figure 2.6: Visualization of the rendering equation. 16

Figure 3.1: Irradiance caching decomposes the incident lighting into a *direct* and an *indirect* component. 24
Figure 3.2: The “split-sphere” model. 28
Figure 3.3: Comparison between irradiance caching and irradiance caching with gradients. 32
Figure 3.4: The stratified geometry used in the Ward and Heckbert [1992] gradient computation. 36
Figure 3.5: The change in cell area due to translation is decomposed into the movement of each cell wall. 40
Figure 3.6: Tabellion and Lamorlette adjusted the ray origins in the irradiance estimate in order to compensate for simplified geometry. 46
Figure 3.7: Hit point reprojection. 49

Figure 4.1: We treat participating media as a collection of microscopic scattering particles. 56
Figure 4.2: As light travels through a participating medium the radiance may change as a result of four different types of interactions: absorption, emission, outscattering and in-scattering. 58
Figure 4.3: The phase function describes the angular distribution of light scattering at any point **x** within participating media. 60
Figure 4.4: The phase function obeys Helmholtz’s reciprocity principle. 62
Figure 4.5: Polar plots visualizing the Henyey-Greenstein and Schlick phase functions as functions of θ. 64
Figure 4.6: Polar plots of phase functions arising from Rayleigh scattering and Lorenz-Mie theory. 65
Figure 4.7: The radiance reaching the eye $L(\mathbf{x} \leftarrow \vec{\omega})$ is the sum of the reduced radiance from the nearest visible surface $L(\mathbf{x}_s \rightarrow \vec{\omega})$ and the accumulated in-scattered radiance $L_i(\mathbf{x}_t \rightarrow -\vec{\omega})$ along a ray. 67

Figure 5.1: Ray marching computes lighting within participating media by dividing the ray into small discrete segments. 74
Figure 5.2: Radiance in participating media is computed by our method using a combination of ray marching and random-walk sampling. 75
Figure 5.3: Computing the single scattering radiance, L_s, and gradient, ∇L_s. 76

Figure 5.4: Computation of transmittance and transmittance gradient for single scattering. 82
Figure 5.5: Computing the multiple scattering radiance, L_m, and gradient, ∇L_m. 85
Figure 5.6: Computation of transmittance and transmittance gradient for multiple scattering. 86
Figure 5.7: Experimental validation of our error metric as compared to a numerically computed optimal radius for a 1D scene. 89
Figure 5.8: A comparison of extrapolation methods for a point-light scene in a homogeneous medium. 91
Figure 5.9: A comparison of extrapolation methods for a point-light scene in a heterogeneous medium. 92
Figure 5.10: A Cornell box filled with isotropic smoke rendered using path tracing and volumetric radiance caching. 93
Figure 5.11: A Cornell box filled with anisotropic smoke rendered using path tracing and volumetric radiance caching. 94
Figure 5.12: A visualization of the single, surface, and multiple scattering cache points used in the Cornell box scene. 95
Figure 5.13: The Sponza atrium with beams of light and multiple scattering. 96
Figure 5.14: A still frame from an animation of heterogeneous smoke. 97
Figure 5.15: Two cars in a dense fog on a road illuminated by 60 lights. 98
Figure 5.16: A equal-time, contrast-enhanced comparison between radiance caching and photon mapping in the cars scene from Figure 5.15. 99

Figure 6.1: Comparison of irradiance gradient computations. 104
Figure 6.2: Comparison of irradiance caching and extrapolation using different gradient computation techniques in a scene with an absorbing medium. 107
Figure 6.3: Comparison of irradiance caching and extrapolation using different gradient computation techniques in a scene with an emitting medium. 108
Figure 6.4: Comparison of irradiance caching and extrapolation using different gradient computation techniques in a scene with a scattering medium. 110
Figure 6.5: Visualization of the gradient magnitude along a scanline. 112
Figure 6.6: Relative error plot for computing the irradiance and irradiance gradient. . . . 113
Figure 6.7: A room illuminated by a volumetric beam of light. 114
Figure 6.8: The classic Cornell box scene with a scattering medium. 115
Figure 6.9: A disco light containing 21 volumetric beams of light illuminating a ground plane. 116

Figure 7.1: The radiance estimate and volumetric radiance estimate compute outgoing radiance using density estimation. 123
Figure 7.2: In participating media, photons are stored not only on surfaces, but also within the medium. 125
Figure 7.3: Ray marching is used to accumulate in-scattered radiance through the medium.127

Figure 8.1: Comparison of conventional and beam photon gathering. 129
Figure 8.2: Illustration of the path characteristic. 133
Figure 8.3: The cylindrical parametrization of the beam radiance estimate. 137
Figure 8.4: The relationship between the photon map kd-tree and the BBH. 138

Figure 8.5: A comparison between the convention radiance estimate and the beam radiance estimate on the Stage scene. 143
Figure 8.6: Visual comparison for the Cornell box, Cars, and Lighthouse scenes. 145

Figure A.1: The two interpretations of the basic Monte Carlo estimator. 154
Figure A.2: Comparison of three probability density functions. 158
Figure A.3: Comparison of Riemann integration and stratified Monte Carlo integration. . 162
Figure A.4: Comparison of several 2D sampling approaches. 163

Figure B.1: Plots of the real-valued spherical harmonic basis functions. 171

LIST OF TABLES

Table 2.1: Definitions of the fundamental radiometric quantities with their associated symbols and units. 11
Table 2.2: Notation used in this dissertation. 18

Table 3.1: Definitions of quantities used in the irradiance gradients derivation. 37

Table 6.1: A comparison of the capabilities of illumination gradient techniques. 102

Table 8.1: Rendering parameters and timings for all example scenes. 141
Table 8.2: Medium scattering properties and photon tracing statistics for the four example scenes. 143

LIST OF ALGORITHMS

Algorithm 3.1: ComputeIndirectIrradiance($\mathbf{x}$) 25
Algorithm 6.1: ComputeGradient($\mathbf{x}, \vec{\mathbf{n}}$) 111
Algorithm 7.1: PhotonMapping() 119
Algorithm 7.2: PhotonTracing() 120
Algorithm 7.3: VolumetricPhotonTracing($\mathbf{x}_p$, $\vec{\omega}_p$, Φ_p) 126
Algorithm 8.1: Beam photon mapping. 140
Algorithm 8.2: ConstructBBH(p) 141

ACKNOWLEDGEMENTS

I owe many people an immense debt of gratitude for the help and support they have provided me over the course of my life and academic career.

I would like to thank my friends and family for supporting me along the way. First and foremost I would like to thank my parents Dorota and Krzysztof, from whom I inherited the interest in a topic that straddles the artistic and analytic sides of my brain, and without whom I would not be here today. This dissertation is dedicated to them.

It has been a great privilege to work with both of my advisors, Henrik Wann Jensen and Matthias Zwicker. Their complementary research approaches and advising styles have been immensely influential in the development of my work. The combined guidance I received from them has been invaluable in the culmination of this dissertation. I also owe a debt of gratitude to the other members of my dissertation committee, Sam Buss, Per Christensen, David Kriegman, and Falko Kuester, for agreeing to take their valuable time to read and evaluate this dissertation. I would furthermore like to thank my undergraduate research advisors John C. Hart and Michael Garland for introducing me to computer graphics research and giving me hands-on experience at an early stage in my academic career. I would also like to thank my high school programming and calculus teacher James Hindelang.

This dissertation would not have been possible if not for the stimulating environment created by the other members and visitors of the UCSD graphics lab, in particular: Neil Alldrin, Will Chang, Piotr Dollar, Diego Gutierrez, Toshiya Hachisuka, Neel Joshi, Arash Keshmirian, Alex Kozlowski, Wan-Yen Lo, Krystle de Mesa, Iman Mostafavi, Vincent Rabaud, Steve Rotenberg, and Iman Sadeghi. I thank each of them for their fruitful suggestions, countless conversations, and for agreeing to read my papers. I'd also like to thank everyone with whom I had the privilege of collaborating: Craig Donner, Tomas Akenine-Möller, Petrik Clarberg, Kevin Dale, Frédo Durand, Greg Humphreys, Oleg Kozhushnyan, Wojciech Matusik, Sylvain Paris, and Richard Weistroffer.

I would like to thank Industrial Light & Magic for providing me with three exciting and stimulating summer internships and for making the movies that sparked my interest in computer graphics in the first place. I would particularly like to thank Florian Kainz with whom I worked for three summers and whose knowledge and experience made them so rewarding. I would also like to thank my friend of countless years, Eric Froemling, who first inspired me to take up 3D modeling and animation, which led to where I am now.

Finally, I would particularly like to thank Kelly, my fiance, for sharing this chapter of my life with me and embarking with me on the next one.

Portions of this dissertation are based on papers which I have co-authored with others. My contributions to each of these papers is listed below.

- Chapter 5 is a reproduction of the material published in the article:

  > Wojciech Jarosz, Craig Donner, Matthias Zwicker, and Henrik Wann Jensen. "Radiance Caching for Participating Media." In *ACM Transactions on Graphics*, 27(1):1–11, 2008.

  I was the primary investigator and author of this paper.

- Chapter 6 is based on the work that appears in the paper:

  > Wojciech Jarosz, Matthias Zwicker, and Henrik Wann Jensen. "Irradiance Gradients in the Presence of Participating Media." In *Computer Graphics Forum (Proceedings of EGSR 2008)*, 27(4), 2008.

  I was the primary investigator and author of this paper.

- Chapter 8 is based on material published in the following paper and extended technical report:

  > Wojciech Jarosz, Matthias Zwicker, and Henrik Wann Jensen. "The Beam Radiance Estimate for Volumetric Photon Mapping." In *Computer Graphics Forum (Proceedings of Eurographics 2008)*, 27(2):557–566, 2008.
  >
  > Wojciech Jarosz, Matthias Zwicker, and Henrik Wann Jensen. "The Beam Radiance Estimate for Volumetric Photon Mapping." Technical Report CS2008-0914, University of California, San Diego, 2008.

  I was the primary investigator and author of both papers.

VITA

2003	Bachelor of Science, *highest honors*, University of Illinois, Urbana–Champaign
2003–2008	Teaching Assistant, Department of Computer Science and Engineering, University of California, San Diego
2006	Master of Science, University of California, San Diego
2008	Doctor of Philosophy, University of California, San Diego

PUBLICATIONS

Toshiya Hachisuka, Wojciech Jarosz, Richard Peter Weistroffer, Kevin Dale, Greg Humphreys, Matthias Zwicker, and Henrik Wann Jensen. “Multidimensional Adaptive Sampling and Reconstruction for Ray Tracing.” In *ACM Transactions on Graphics (Proceedings of SIGGRAPH 2008)*, 27(3), 2008.

Sylvain Paris, Will Chang, Oleg I. Kozhushnyan, Wojciech Jarosz, Wojciech Matusik, Matthias Zwicker, and Frédo Durand. “Hair Photobooth: Geometric and Photometric Acquisition of Real Hairstyles.” In *ACM Transactions on Graphics (Proceedings of SIGGRAPH 2008)*, 27(3), 2008.

Wojciech Jarosz, Matthias Zwicker, and Henrik Wann Jensen. “Irradiance Gradients in the Presence of Participating Media.” In *Computer Graphics Forum (Proceedings of EGSR 2008)*, 27(4):1087–1096, 2008.

Wojciech Jarosz, Matthias Zwicker, and Henrik Wann Jensen. “The Beam Radiance Estimate for Volumetric Photon Mapping.” In *Computer Graphics Forum (Proceedings of Eurographics 2008)*, 27(2):557–566, 2008.

Wojciech Jarosz, Craig Donner, Matthias Zwicker, and Henrik Wann Jensen. “Radiance Caching for Participating Media.” In *ACM Transactions on Graphics*, 27(1):1–11, 2008.

Petrik Clarberg, Wojciech Jarosz, Tomas Akenine-Möller, and Henrik Wann Jensen. “Wavelet Importance Sampling: Efficiently Evaluating Products of Complex Functions.” In *ACM Transactions on Graphics (Proceedings of SIGGRAPH 2005)*, 24(3):1166–1175, 2005.

John C. Hart, Ed Bachta, Wojciech Jarosz, Terry Fleury. “Using Particles to Sample and Control More Complex Implicit Surfaces.” In *Proceedings of Shape Modeling International 2002*, May 2002, 129–136.

ABSTRACT OF THE DISSERTATION

EFFICIENT MONTE CARLO METHODS FOR LIGHT TRANSPORT IN SCATTERING MEDIA

by

Wojciech Jarosz
Doctor of Philosophy in Computer Science

University of California San Diego, 2008

Henrik Wann Jensen, Co-Chair
Matthias Zwicker, Co-Chair

In this dissertation we focus on developing accurate and efficient Monte Carlo methods for synthesizing images containing general participating media. Participating media such as clouds, smoke, and fog are ubiquitous in the world and are responsible for many important visual phenomena which are of interest to computer graphics as well as related fields. When present, the medium *participates* in lighting interactions by scattering or absorbing photons as they travel through the scene. Though these effects add atmosphere and considerable depth to rendered images they are computationally very expensive to simulate. Most practical solutions make simplifying assumptions about the medium in order to maintain efficiency. Unfortunately, accurate and efficient simulation of light transport in general scattering media is a challenging undertaking. In this dissertation, we address this problem by introducing two complementary techniques.

We first turn to the irradiance caching method for surface illumination. Irradiance caching gains efficiency by computing an accurate representation of lighting only at a sparse set of locations and reusing these values through interpolation whenever possible. We derive the mathematical concepts that form the foundation of this approach and analyze its strengths and weaknesses. Drawing inspiration from this algorithm, we then introduce a novel volumetric radiance caching method for efficiently simulating global illumination within participating media. In developing the technique we also introduce efficient methods for evaluating the gradient of the lighting within participating media. Our gradient analysis has immediate applicability for improved interpolation quality in both surface and media-based caching methods.

We also develop a novel photon mapping technique for participating media. We present a theoretical reformulation of volumetric photon mapping, which provides significant new insights. This reformulation makes it easier to qualify the error introduced by the radiance estimate but, more importantly, also allows us to develop more efficient rendering techniques. Conventional photon mapping accelerate the computation of lighting at any *point* in the scene by performing density estimation. In contrast, our reformulation accelerates the computation of accumulated lighting along the length of entire *rays*. This algorithmic improvement provides for significantly reduced render times and even the potential for real-time visualization of light transport in participating media.