Extended Path Integral Formulation for Volumetric Transport

T. Hachisuka  I. Georgiev  W. Jarosz  J. Křivánek  D. Nowrouzezahrai

The University of Tokyo  Solid Angle  Dartmouth College  Charles University in Prague  McGill University
Bidirectional path tracing [Pauly et al. 2000]
Volume photon mapping [Jensen and Christensen 1998]
Beam radiance estimate [Jarosz et al. 2008]
Photon beams [Jarosz et al. 2011]
Comprehensive theory [Jarosz et al. 2011]
Comprehensive theory [Jarosz et al. 2011]
UPBP formulation

- Unified points, beams, and paths as sampling techniques for volumes

[Křivánek et al. 2014]
Dimensionality of paths

Path integral: **Four** vertices

Density estimation: **Five** vertices

Same path length
Merge vertices
Consider all the paths which result in the same merged path.
Accept according to the probability of merging

$$Prob[ x \equiv y ] = \int p(y') dy'$$
\[ Prob[x \equiv y] = \int p(y') dy' \]
$\text{Beam-Beam 1D}$

$$\text{Prob}[x \equiv y] = \int p(y') dy'$$
UPBP formulation

• Three steps to match with BDPT
  1. Merge subpaths
  2. Consider all the paths which result in the same merged path
  3. Accept the path with the probability of merging

Corresponds to contraction of density estimation path space

\[ \text{Prob} [ x \equiv y ] = \int p(y') dy' \]
• Unified path integration and photon density estimation for surfaces

[Hachisuka et al. 2012]  [Georgiev et al. 2012]
Vertex Connection and Merging

- **Contract** the space of density estimation into the original path space
Vertex Connection and Merging

- **Contract** the space of density estimation into the original path space
Unified Path Sampling

- **Extend** the original path space to include photon density estimation
Unified Path Sampling

- Extend the original path space to include photon density estimation
Differences

- **VCM**: precise for path integration, approximate for density estimation
- **UPS**: precise for density estimation, approximate for path integration
<table>
<thead>
<tr>
<th></th>
<th>Surfaces</th>
<th>Volumes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Contraction</strong></td>
<td>VCM</td>
<td>UPBP</td>
</tr>
<tr>
<td><strong>Extension</strong></td>
<td>UPS</td>
<td><strong>Ours (UVPS)</strong></td>
</tr>
</tbody>
</table>
Unified Volumetric Path Sampling
Path integral formulation

Vertices are fully connected
Extended path integral formulation

Allow disconnected vertices
Extended path integral formulation

Blurring kernel as throughput of disconnected vertices

$$K_{3D}(x, y)$$
Point-Point 3D

$K_{3D}(x, y)$

Precisely models photon density estimation
3D blur to 2D blur

\[ K_{3D}(x, y) \]
3D blur to 2D blur

\[ K_{2D}(x, y) = K_{3D}(x, y) \delta(x_t - t_K) \]

Flatten a sphere into a disc
Beam-Point 2D

\[ K_{2D}(x, y) \]
Beam-Point 2D

Beam-point 2D = deterministic sampling of one distance

\[ K_{2D}(x, y) \]

\[ \delta(y_t - t_{int}) \]
2D blur to 1D blur

\[ K_{2D}(x, y) \]
2D blur to 1D blur

\[ K_{1D}(x, y) = K_{2D}(x, y) \delta(x_t - t'_K) \]

Flatten a disc into a line
Beam-Beam 1D

\[ K_{1D}(x, y) \]
Beam-beam 1D = deterministic sampling of two distances
Beam-Beam 2D
Beam-Beam 2D [Jarosz et al. 2011]

\[ \int_{\text{intersection interval}} f(x, y) K(x, y) \, dt \]
Beam-Beam 2D

Stochastic sampling within the interval
Beam-Beam 2D

\[ \delta(t_x - t(y_{proj})) \]
Beam-Beam 2D

\[ K_{1D}(x, y) = K_{2D}(x, y) \]

Same 2D kernel as beam-point 2D
Beam-Beam 3D
Beam-Beam 3D [Jarosz et al. 2011]

Double integral over the intersection intervals (usually intractable)

\[ \int \int f(x, y)K(x, y)dt_y dt_x \]
Beam-Beam 3D

\[ p(t_y) \]
Beam-Beam 3D

Same 3D kernel as point-point 3D
Beam-Beam 3D

$K_{3D}(x, y) = K_{2D}(x, y)$

Simple Monte Carlo path sampling (no longer intractable)
Beam-Beam 3D

Courtesy of Adrien Gruson
Beam-Point 3D

\[ K_{3D}(x, y) \]

Same 3D kernel as point-point 3D
Bidirectional path tracing
Bidirectional path tracing

\[ p(y) = \delta(x - y) \]

Duplicate a vertex
Bidirectional path tracing

\[ p(y) = \delta(x - y) \]

\[ K_{3D}(x, y) = \delta(x - y) \]

Delta kernel leads to the original path integral formulation
Biased bidirectional path tracing

\[ p(y) \neq \delta(x - y) \]

\[ K_{3D}(x, y) \neq \delta(x - y) \]

Take disconnected vertices via blurring kernel
Virtual perturbation

\[ p(y) \approx \delta(x - y) \]

\[ K_{3D}(x, y) \neq \delta(x - y) \]

Approximate the implementation of biased BDPT by regular BDPT
Conclusion

• Extension of the path space for volumetric light transport
  • Better explains density estimation compared to merging
  • Formulate beam as Monte Carlo distance sampling
  • Enables a practical beam-beam 3D estimator

Fills a theoretical gap in the unified formulation for volumes