Gaussian Quadrature for Photon Beams in *Tangled*

Jared M. Johnson[†*] Dylan Lacewell[†] Andrew Selle[†] Wojciech Jarosz[‡]

[†]Walt Disney Animation Studios [*]University of Central Florida [‡]Disney Research Zürich

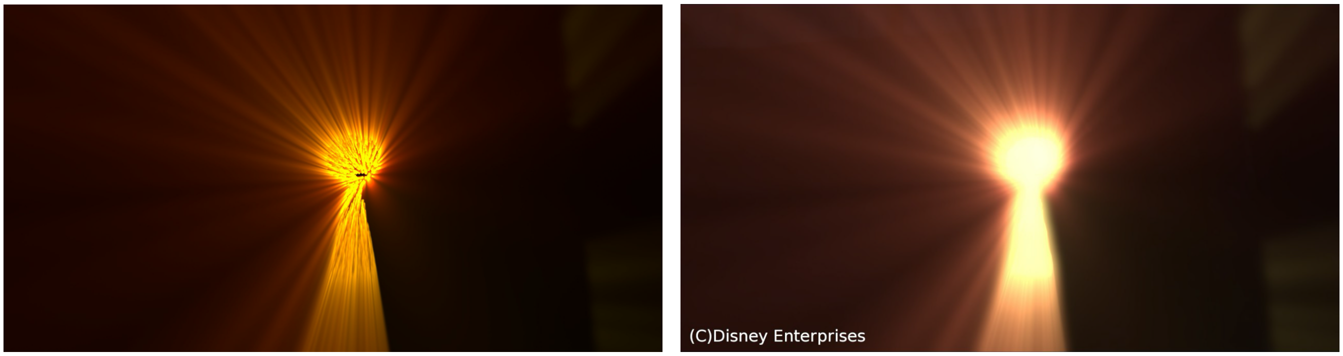

Figure 1: *Photon Beams used in the movie Tangled. (left) Photon beams only. (right) Final composited image.*

1 Introduction

We implemented the recent Photon Beams [Jarosz et al. 2011] algorithm in Photorealistic RenderMan to efficiently render artistically-directed volumetric lighting effects for the feature-length animated movie *Tangled*. Photon beams generalize volumetric photon mapping by storing the entire path of a photon instead of just the scattering location. Conceptually, each photon beam represents a truncated conical beam of light through the medium. Jarosz et. al formulated several ways to compute a radiance estimate from photon beams, promoting the so-called Beam × Beam 1D estimate which interprets each beam as a flat axial billboard.

In the original formulation, the radiance along a beam has a physically-based exponential falloff based on the scattering properties of the medium. Fortunately, the billboard representation easily handles arbitrary, artist-controlled fall-off functions along beams, making it an ideal candidate for artistic volumetric effects. Though the billboard representation gives the correct approach in the limit, it can sometimes produce artifacts. When the beams are aimed at the camera a flat billboard is a poor approximation for the conical photon beam. To avoid this problem, we use the Beam × Beam 2D estimate which treats each beams as a conical frustum with a finite cross-section. In this case, to compute the beam's contribution, we need to consider the integral along each camera ray through a beam.

While there are analytic solutions to this path integral for physically-based single-scattering [Pegoraro and Parker 2009], we need to allow for arbitrary, non-physical fall-off functions for artistic control. With the knowledge that most fall-off functions defined by our artists would be polynomial-smooth, we use Gaussian Quadrature [Stroud and Secrest 1966] to accurately and efficiently estimate the lighting contribution of these camera-containing beams. Our numerical approach allows for robust artistic control over beam appearance, while reducing the number of lighting samples compared to other numerical approaches with no loss of accuracy.

2 Gaussian Quadrature

Like the more commonly-used Riemann sum, Gaussian Quadrature is a method for approximating a definite integral. It has greater accuracy for the same number of sample points as a Riemann sum because sample locations are weighted and placed in specific locations to solve for an exact result with n-samples for polynomials of degree $2n-1$.

For a prescribed number of sample points, Gaussian Quadrature computes an integral in the canonical domain $[-1, 1]$ using a pre-computed set of positions x_i and corresponding weights w_i:

$$\int_{-1}^{1} f(x)\,\mathrm{d}x \approx \sum_{i=1}^{n} w_i f(x_i). \tag{1}$$

An integral over an arbitrary interval $[a, b]$ is computed using a simple transformation of the samples points and weights:

$$\int_{a}^{b} f(x)\,\mathrm{d}x \approx \frac{b-a}{2}\sum_{i=1}^{n} w_i f\left(\frac{b-a}{2}x_i + \frac{a+b}{2}\right). \tag{2}$$

3 Photon Beams in PRMan

We implemented photon beams in PRMan using linear `RiCurves` with two control points. When the camera is contained within a beam, `RiCurves` result in visual artifacts. We therefore replace these `RiCurves` with a screen-space quad that covers the entire screen, to ensure that every pixel receives a shading contribution. For each shading point on the quad we do a ray/cone intersection, finding the ray interval enclosed in the beam volume. We map our desired number of sample points to this ray interval, scaling sample weights using Equation 2. We then calculate the lighting at these points with our artist-defined function, and then compute the weighted summation. In practice, at most six lighting sample points were sufficient to provide a good estimate. A Riemann sum of equivalent accuracy would require more than twice as many sample points.

4 Results

The keyhole shot from the film *Tangled* (Figure 1) used photon beams in PhotoRealistic Renderman to represent light shining through a keyhole in participating media. Our artists used a one-dimensional color texture to design a color change as the beams extinguish in the media. This color change was multiplied by physically-based attenuation. As the camera passes through photon beams, our Gaussian quadrature estimate on full-screen quads blends seamlessly with the `RiCurve` estimate, accurately rendering the camera *inside* photon beams.

All of this is done within the PRMan pipeline, maintaining proper motion-blur, depth-of-field effects, and rendering correctly in stereoscopic 3D.

References

JAROSZ, W., NOWROUZEZAHRAI, D., SADEGHI, I., AND JENSEN, H. W. 2011. A comprehensive theory of volumetric radiance estimation using photon points and beams. *ACM Trans. Graph. 30* (January), 5:1–5:19.

PEGORARO, V., AND PARKER, S. G. 2009. An Analytical Solution to Single Scattering in Homogeneous Participating Media. *Computer Graphics Forum (Proceedings of the 30th Eurographics Conference) 28*, 2, 329–335.

STROUD, A. H., AND SECREST, D. 1966. *Gaussian quadrature formulas*. Prentice-Hall series in automatic computation. Prentice-Hall, Englewood Cliffs, NJ.