

# Second-Order Occlusion-Aware Volumetric Radiance Caching

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Accurate simulation of light transport in participating media is expensive, due to the many scattering events. However, the band-limiting effect of scattering in media makes this kind of light transport very suitable for adaptive sampling and reconstruction techniques. In this work we present a novel algorithm that adaptively samples radiance from sparse points in the medium using up-to second-order occlusion-aware derivatives to determine when interpolation is appropriate. We derive our metric from each point's incoming light field. We use a proxy triangulation-based representation of the radiance reflected by the surrounding medium and geometry to efficiently compute the first- and second-order derivatives of the radiance at the cache points while accounting for occlusion changes. We validate the quality of our approach on a self-contained two-dimensional model for light transport in media. Then we show how our results generalize to practical three-dimensional scenarios, where we show much better results while reducing computation time up to a 30% compared to previous work.

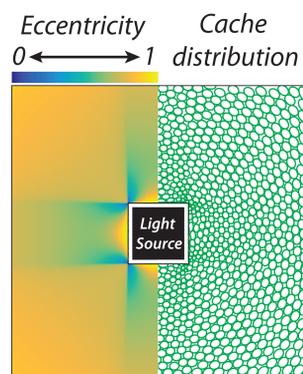
## Radiance caching in participating media

A common strategy to increase efficiency when Monte Carlo rendering participating media is to adaptively sample media radiance based on its frequency content. Existing volumetric radiance caching methods [1] estimate radiance frequency by computing its translational gradients, and adaptively placing radiance samples (caches) which are later used for radiance extrapolation. While the method significantly increases rendering efficiency, it ignores higher order derivatives and is blind to visibility changes during gradient computation. This leads to suboptimal cache distributions that fail to capture occlusions and oversample other regions of the scene.

## Second-order error bound for media radiance

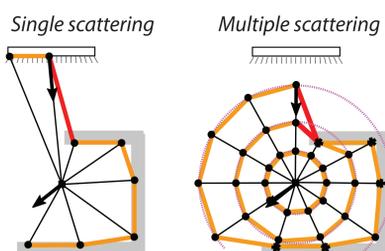
Inspired by recent work on surface irradiance Hessians [2, 3], we propose to guide cache density using radiometric Hessians as an oracle of the error of a first-order Taylor expansion. Different from surface caches, a media cache point will have a limited **3D bounding region** around its center where the point is suitable for first-order extrapolation. In particular, radiance Hessians can be used for delimiting an ellipsoid centered at the cache point, and oriented by the three principal components of radiance, so the point has longer reach along directions where radiance varies more smoothly. The three radii of the ellipsoid are given by the eigenvalues of the Hessian. In order to illustrate the benefits of our method in 2D we also provide formulae for delimiting 2D media points (ellipses), analogously oriented by the 2D Hessian principal components.

$$R_{2D}^{\lambda_i} = \sqrt[4]{\frac{4L(x)\varepsilon}{\pi|\lambda_i|}} \quad R_{3D}^{\lambda_i} = \sqrt[5]{\frac{15L(x)\varepsilon}{4\pi|\lambda_i|}}$$



## Computing occlusion-aware derivatives

Following recent work on occlusion-aware derivatives of surface irradiance [3], we propose a method for computing up-to second-order derivatives of single and multiple scattering based on surface-media form factors. By constructing subdivisions of the scene surfaces and media as seen from cached media locations, we provide closed-form approximations for occlusion-aware media radiance derivatives.



We construct occlusion-free subdivisions at surfaces locations for single scattering, and at ray marched locations within media for multiple scattering.

We estimate the derivatives of the outgoing radiance field at every media point as a summation of every element's derivatives on this piece-wise subdivision.

$$\nabla L(\mathbf{x} \rightarrow \vec{\omega}_o) \approx \sum_{r_i \in \mathcal{R}} \sum_{\ell_j \in \mathcal{L}_i} \frac{\nabla L_j(\mathbf{x} \rightarrow \vec{\omega}_o)}{\text{pdf}(r_i)}$$

$$\mathbf{H}L(\mathbf{x} \rightarrow \vec{\omega}_o) \approx \sum_{r_i \in \mathcal{R}} \sum_{\ell_j \in \mathcal{L}_i} \frac{\mathbf{H}L_j(\mathbf{x} \rightarrow \vec{\omega}_o)}{\text{pdf}(r_i)}$$

Outgoing radiance field at  $\mathbf{x}$  from every element  $j$  of these piece-wise subdivisions can be estimated by the closed form,

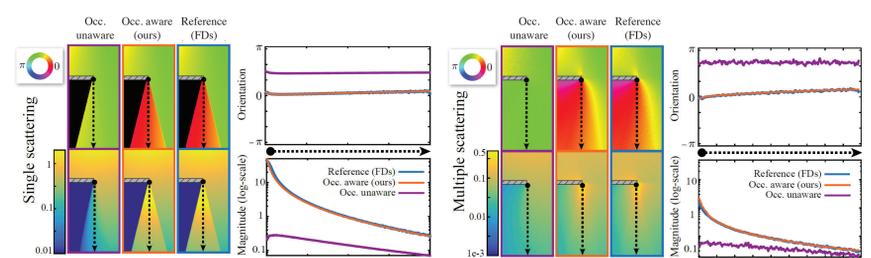
$$L_j(\mathbf{x} \rightarrow \vec{\omega}_o) = f(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_o) T_r(\mathbf{x}, \ell_j) L_o(\ell_j \rightarrow \vec{\omega}_i) F_{\ell_j}(\mathbf{x})$$

Phase function    Transmitt.    Element radiance    Surf-media form factor

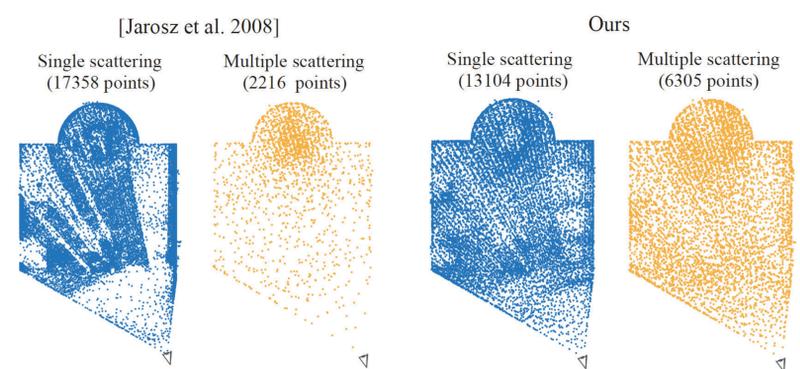
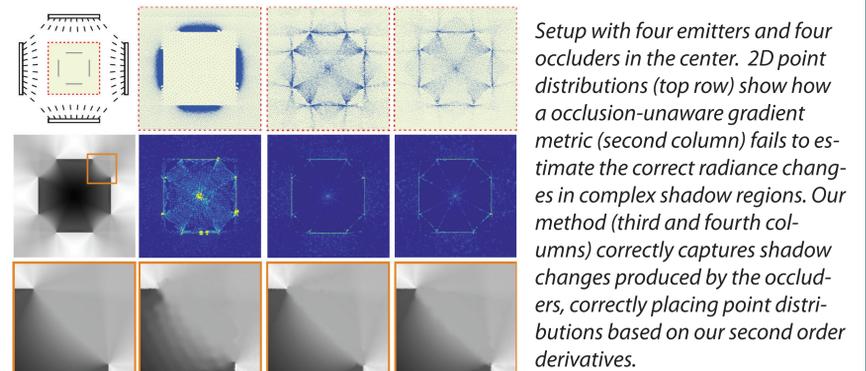
which also presents closed-form derivatives for homogeneous media and Henyey-Greenstein phase functions.

## Results

We first demonstrate the validity of our method in 2D, where radiance changes and its derivatives, and point distributions can be illustrated in 2D planes, and then show how our method equally performs in 3D scenarios.



2D single and multiple scattering gradients of an occluder and a light source on top of it. Compared against an occlusion-unaware solution [1], our method correctly captures both gradients orientation (color-coded angle), and magnitude. The graphs show the evolution of the gradient across the dotted black line for both methods (purple, orange), and the reference solution (blue).



PT1: Path tracing, 2k samples/ray, 2h. PT2: Path tracing, 500k samples/ray, 500h. Jarosz et al. 2008 [1]: Occlusion unaware, gradient-based error metric, 19k cache points, 16k spl/cache, 155 mins. Ours: Occlusion-aware, Hessian-based metric, 19k cache points, 16k samples/cache, 154 mins. Note how our method converges to the correct solution, while path tracing methods are prohibitively expensive, and occlusion-unaware radiance caching [1] fails to capture high frequency changes around the windows or in the hole in the ceiling. Point distributions on top show how our method manages to correctly capture occlusions near the windows, while occlusion unaware methods unevenly place samples near the ceiling and the walls.