Unbiased and consistent rendering using biased estimators

Zackary Misso\textsuperscript{1} Benedikt Bitterli\textsuperscript{1,2} Iliyan Georgiev\textsuperscript{3} Wojciech Jarosz\textsuperscript{1}

\textsuperscript{1}Dartmouth College, \textsuperscript{2}NVIDIA, \textsuperscript{3}Autodesk
Unbiased solutions
Unbiased solutions
Unbiased solutions

$\langle I \rangle$
Unbiased solutions

\[ \mathbb{E} \left[ \langle I \rangle \right] = I \]
Biased solutions

\[ \mathbb{E} \left[ \langle I \rangle \right] \neq I \]
Biased solutions

\[ E\left[\langle I \rangle\right] \neq I \]
Biased solutions
Motivation
Progressive photon mapping

[Hachisuka et. al. 2008]

[Knaus et. al. 2011]
Progressive photon mapping

[Knaus et. al. 2011]
Progressive photon mapping

[Knaus et. al. 2011]
Progressive photon mapping

\[ \lim_{{k \to \infty}} \{ \text{[Knaus et. al. 2011]} \} \]
Our framework
Our framework

Unbiased and consistent rendering using biased estimators
Related work

Reciprocal Estimation

[Booth 2007]

[Qin et. al. 2015]

[Zeltner et. al. 2020]
Related work

Null Collision

[Novak et. al. 2014]

[Georgiev et. al. 2019]
Applicable problems
Applicable problems

$I(k)$
Applicable problems

$I(k)$
Applicable problems

\[ \lim_{k \to \infty} I(k) = I \]
Path Tracing - max path depth
Unbiased and consistent rendering using biased estimators.

Path Tracing - max path depth

$I(1)$

$I(\infty)$
Photon mapping
Photon mapping

\[ I(k) : r = \frac{1}{k} \]
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\[ I(k) : r = \frac{1}{k} \]
Photon mapping

\[ I = \lim_{k \to \infty} I(k) \]

\[ I(k) : r = \frac{1}{k} \]
Debiasing

\[ I = \lim_{k \to \infty} I(k) \]
Debiasing

\[ I(\infty) = \lim_{k \to \infty} I(k) \]
Debiasing

\[ I(\infty) = I(k) + \left[ I(\infty) - I(k) \right] \]
Debiasing

\[ I(\infty) = I(k) + \left[ I(\infty) - I(k) \right] \]
Debiasing

\[ I(\infty) = I(k) + \left[ I(\infty) - I(k) \right] \]
Debiasing

\[ I(\infty) = I(k) + \left[ I(\infty) - I(k) \right] \]
Debiasing

$$I(\infty) = I(k) + \left[ I(\infty) - I(k) \right]$$
Debiasing

\[ I(\infty) = I(k) + \left[ I(\infty) - I(k) \right] \]
### Debiasing

<table>
<thead>
<tr>
<th>GT</th>
<th>Biased</th>
<th>Bias Correction</th>
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\[
I(\infty) = I(k) + [I(\infty) - I(k)]
\]

\[
I(\infty) = I(k) + \sum_{j=k}^{\infty} [I(j+1) - I(j)]
\]
Debiasing

\[ I(\infty) = I(k) + \sum_{j=k}^{\infty} \left[ I(j + 1) - I(j) \right] \]
Debiasing

\[ I(\infty) = I(k) + \frac{I(j + 1) - I(j)}{p(j)} \]
Debiasing

\[ \langle I(\infty) \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)} \]
Unbiased Photon-mapping

\[ \langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)} \]
Unbiased Photon-mapping

\[ \langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)} \]
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\[ \langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)} \]
Unbiased and consistent rendering using biased estimators

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}$$
Unbiased and consistent rendering using biased estimators

\[
\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}
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Unbiased and consistent rendering using biased estimators

\[ \langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)} \]
\[
\langle I \rangle = \left\langle I(k) + \frac{I(j + 1) - I(j)}{p(j)} \right\rangle
\]
Unbiased Ray-marching

\[ \langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)} \]
Unbiased and consistent rendering using biased estimators

Unbiased Ray-marching

\[ \langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)} \]

\[ I = g \left( \int f(x) dx \right) \]
Unbiased Ray-marching

\[
\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)}
\]

\[
\langle I(k) \rangle = g \left( \sum_{j=1}^{k} f(x_j) \Delta x \right)
\]
Unbiased Ray-marching

\[ \langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)} \]

\[ \langle I(k) \rangle = g \left( \sum_{j=1}^{k} f(x_j) \Delta x \right) \]

Unbiased and consistent rendering using biased estimators
Unbiased Ray-marching

\[ \langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)} \]

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Unbiased ray-marching

\[ \langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)} \]

steps \( \propto 2^{j+1} \)

\[ \langle I(k) \rangle = g \left( \sum_{j=1}^{k} f(x_j) \Delta x \right) \]

\[ \langle I(j + 1) \rangle \]

\[ \langle I(k) \rangle \]
Unbiased and consistent rendering using biased estimators

Unbiased ray-marching

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)}$$

steps = 8

$$\langle I(k) \rangle = g \left( \sum_{j=1}^{k} f(x_j) \Delta x \right)$$
Unbiased ray-marching

\[ \langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)} \]

\[ \langle I(k) \rangle = g \left( \sum_{j=1}^{k} f(x_j) \Delta x \right) \]
Results
Transmittance estimation

[Bitterli et. al. 2018]
Probability mass function

$$\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j + 1) \rangle - \langle I(j) \rangle}{p(j)}$$
Probability mass function

\[
\langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)}
\]

\[
E \left[ \langle I \rangle \right] = I
\]
Probability mass function

\[ \langle I \rangle = \langle I(k) \rangle + \frac{\langle I(j+1) \rangle - \langle I(j) \rangle}{p(j)} \]

\[ E \left[ \langle I \rangle \right] = I \quad V \left[ \langle I \rangle \right] = \infty \]
Photon mapping
Additional Contributions

• Recipe
• Taylor series
• Infinite variance
• Finite differences
Thank you!