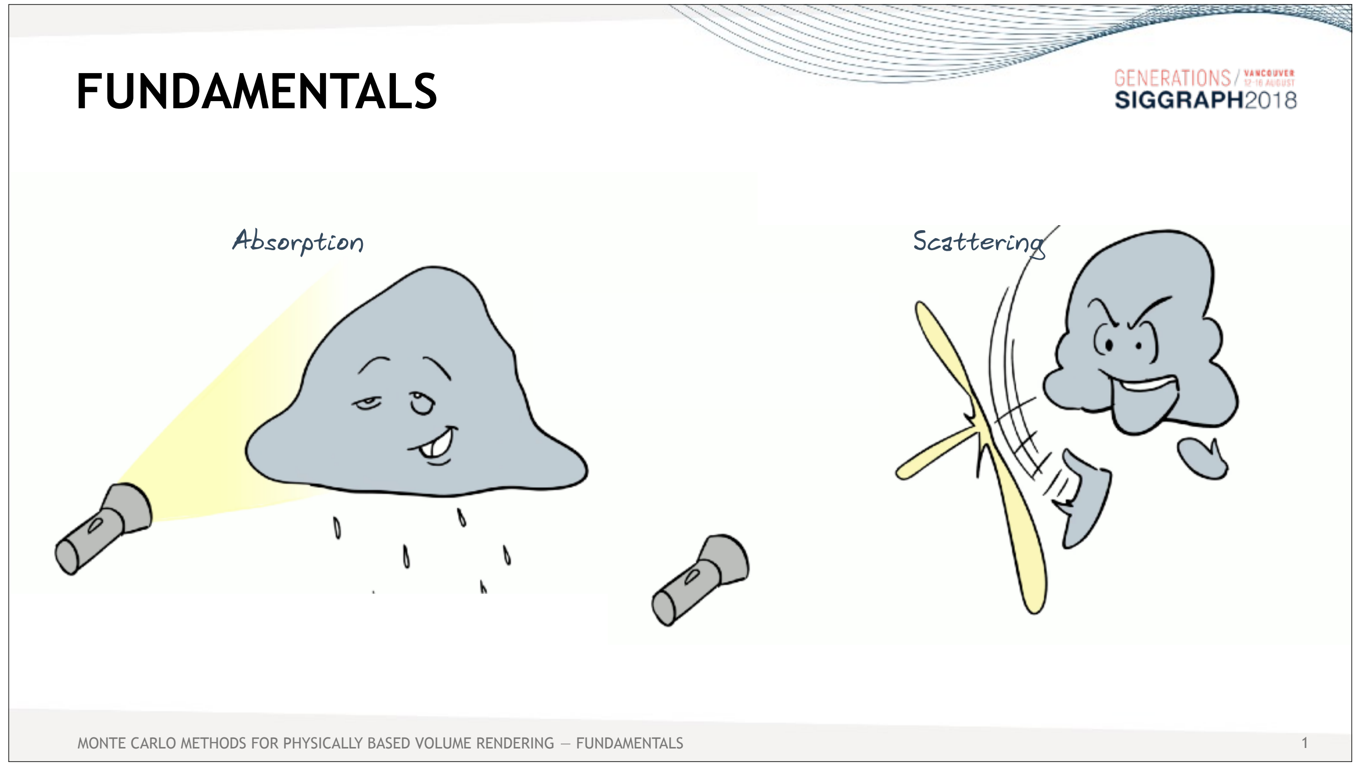
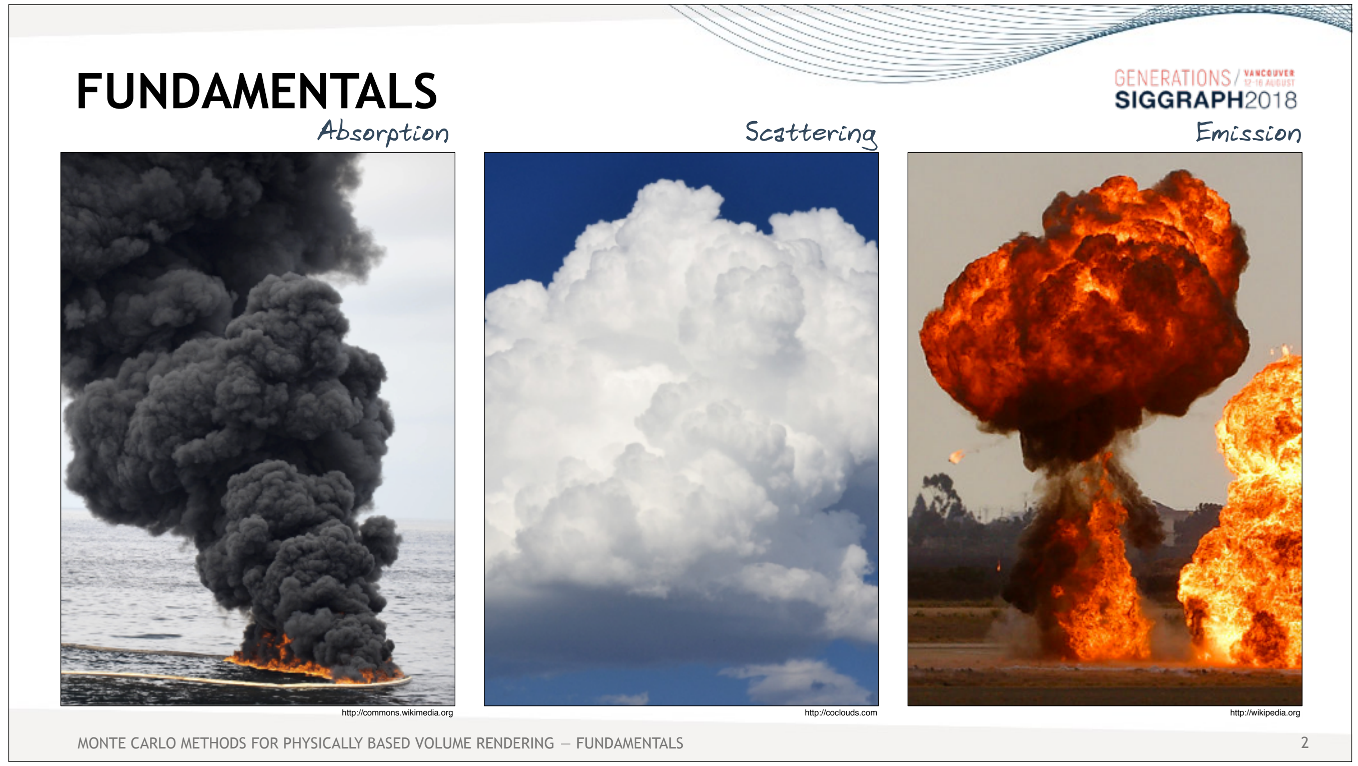


In this part, we will look at the basic interactions between light and matter and how to describe them mathematically, deriving the so-called Volume Rendering Equation. And then we'll talk about how to solve it.



Here are a few examples that show the three main process that we will discuss:
absorption, scattering, and emission

RADIATIVE TRANSFER

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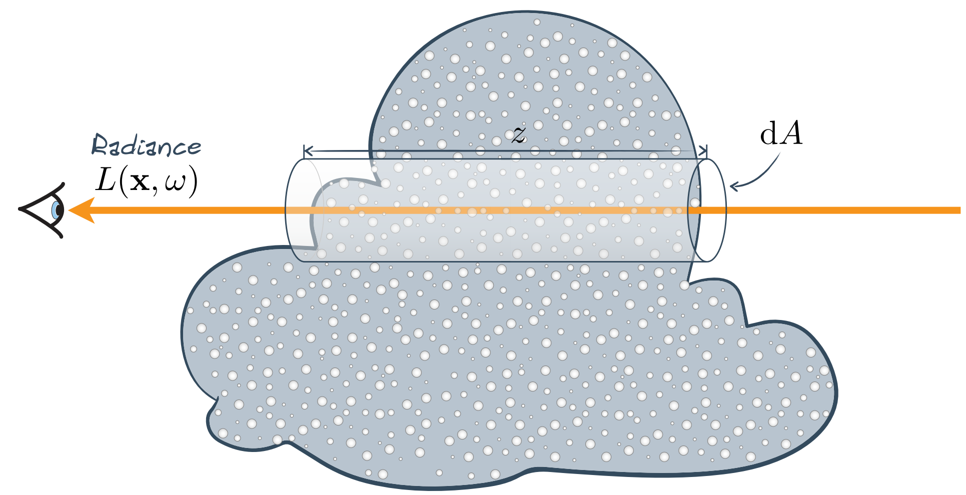

These processes impact radiance, i.e. the radiative flux traveling through a differential beam.
The flux will be increased or decreased depending on the optical properties of the medium.

ABSORPTION

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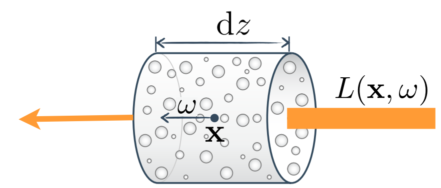

$$\frac{\mathrm{d}L}{\mathrm{d}z} = -\mu_\mathrm{a}(\mathbf{x})L(\mathbf{x},\omega)$$

μ_a - absorption coefficient

In order to formalize these changes mathematically, we will start with a differential beam segment.

I have a few particle here, but these is just for illustration.
We typically do not represent the material particles explicitly, we rather model the medium statistically quantifying the density of collisions per unit of travelled distance.

Some of these particles may be absorptive. If we look at the radiance passing through the differential segment along a given direction omega, it will be reduced, and the rate of change depends on the absorption coefficient, which quantifies the probability density of light being absorbed per unit distance.

OUT-SCATTERING

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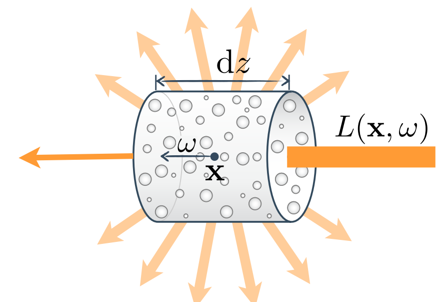

$$\frac{\mathrm{d}L}{\mathrm{d}z} = -\mu_\mathrm{s}(\mathbf{x})L(\mathbf{x},\omega)$$

μ_s – scattering coefficient

If there are scattering particles, some of the incident light will be scattered out, away from the direction of its initial travel.

Analogously to absorption, these losses are proportional to a coefficient—the scattering coefficient—which quantifies the probability density of a scattering collision per unit distance.

IN-SCATTERING

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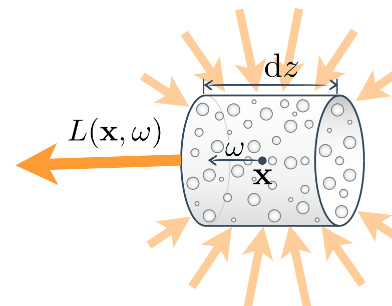

In-scattered radiance

$$L_\mathrm{s}(\mathbf{y}, \omega) = \int_{S^2} f_\mathrm{p}(\omega, \bar{\omega}) L(\mathbf{y}, \bar{\omega}) \mathrm{d}\bar{\omega}$$

$$\frac{\mathrm{d}L}{\mathrm{d}z} = \mu_\mathrm{s}(\mathbf{x}) L_\mathrm{s}(\mathbf{x}, \omega)$$

μ_s – *scattering coefficient*

Whenever light can out-scatter, there is a chance that some may also scatter into the direction omega.

The main difference to out-scattering is the sign, in-scattering is positive. The other difference is the radiance function. Here we have the so-called in-scattered radiance. We will discuss it in detail a bit later.

EMISSION

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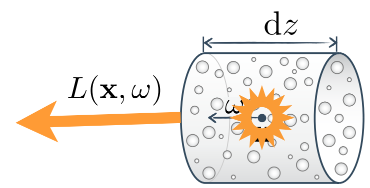

$$\frac{\mathrm{d}L}{\mathrm{d}z} = \mu_{\mathrm{a}}(\mathbf{x}) L_{\mathrm{e}}(\mathbf{x},\omega)$$

L_{e} - emitted radiance

Finally, the radiance may also increase due to emission from the volume, for instance as a result of incandescent or luminescent processes.

Here we made the choice to model the emission as a product of emitted radiance and absorption coefficient. This has two motivations, although other definitions are justifiable as well and may indeed be better in specific situations. The first reason is the notational convenience in later part of the talk. The second reason is based on a physically motivated argument. For emission to exist, there should be some material that has the ability to absorb energy and release it in the form of photons—e.g. through the process of incandescence.

RADIATIVE TRANSFER EQUATION

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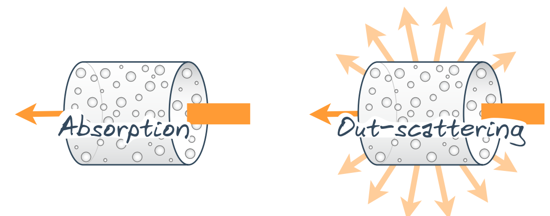
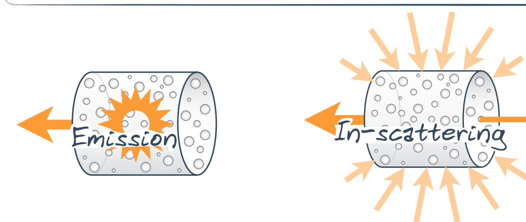


$$\frac{\mathrm{d}L(\mathbf{x},\omega)}{\mathrm{d}z} = -\mu_\mathrm{a}(\mathbf{x})L(\mathbf{x},\omega) \; -\mu_\mathrm{s}(\mathbf{x})L(\mathbf{x},\omega) \quad \textit{Losses}$$

$$+\mu_\mathrm{a}(\mathbf{x})L_\mathrm{e}(\mathbf{x},\omega) + \mu_\mathrm{s}(\mathbf{x})L_\mathrm{s}(\mathbf{x},\omega) \quad \textit{Gains}$$



[Chandrasekhar 1960]

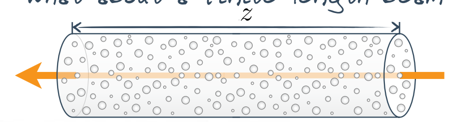
We can combine all the losses and the gains into a single differential equation—the so-called *radiative transfer equation* (RTE).

RADIATIVE TRANSFER

Extinction coefficient $\mu_{\mathrm{t}}(\mathbf{x}) = \mu_{\mathrm{a}}(\mathbf{x}) + \mu_{\mathrm{s}}(\mathbf{x})$

$$\frac{\mathrm{d}L(\mathbf{x},\omega)}{\mathrm{d}z} = \underbrace{-\mu_{\mathrm{t}}(\mathbf{x})L(\mathbf{x},\omega)}_{\text{Losses}} \underbrace{+\mu_{\mathrm{a}}(\mathbf{x})L_{\mathrm{e}}(\mathbf{x},\omega) + \mu_{\mathrm{s}}(\mathbf{x})L_{\mathrm{s}}(\mathbf{x},\omega)}_{\text{Gains}}$$

What about a finite-length beam?



[Chandrasekhar 1960]

Since both absorption and out-scattering affect the same radiance function L, we can merge them into a single term. Here, mu_t is the sum of the absorption and scattering coefficient, and it is referred to as the extinction coefficient. It describes the probability density of any collision per unit distance.

The RTE is a differential equation operating on a differential beam segment. In rendering, however, we are typically working with beams with finite length. As such, we integrate both sides of the equation spatially, along direction omega, to obtain the integral form of RTE, which is more suitable for our needs.

RTE — INTEGRAL FORM

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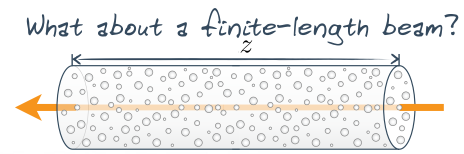
$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})\Big[\mu_\mathrm{a}(\mathbf{y})L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y})L_\mathrm{s}(\mathbf{y},\omega)\Big]\mathrm{d}y$$



The integral form tells us how to compute radiance traveling through point x in direction omega: we need to integrate “back” along the ray and at each point y, we need to evaluate the gains and weight them down by the transmittance function that takes into account the losses on the way towards the point x.

RTE — INTEGRAL FORM

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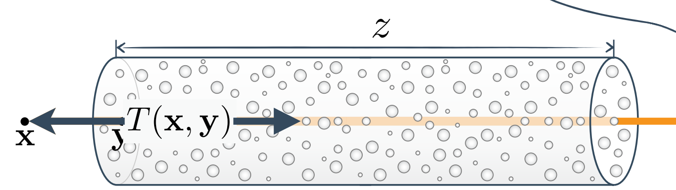
$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})\Big[\mu_\mathrm{a}(\mathbf{y})L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y})L_\mathrm{s}(\mathbf{y},\omega)\Big]\mathrm{d}y$$

Transmittance $T(\mathbf{x},\mathbf{y}) = e^{-\int_0^y \mu_\mathrm{t}(\mathbf{s})\mathrm{d}s}$ is the fraction of light that makes it from y to x



Optical thickness

$$\tau(\mathbf{x},\mathbf{y}) = \int_0^y \mu_\mathrm{t}(\mathbf{s})\mathrm{d}s$$

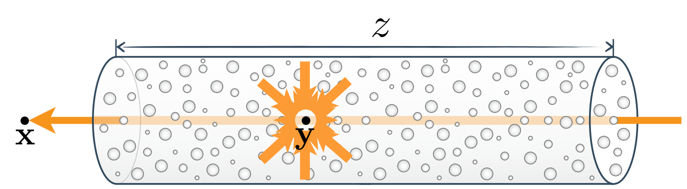
The transmittance function quantifies the fraction of light that makes it from one point to another. It’s derivation is know as the Beer-Lambert law, which tells us that light is reduced exponentially as a function of negative optical thickness.

The optical thickness is the integral of the extinction coefficient along the ray and it represents all the material that light could potentially interact with in-between x and y.

RTE — INTEGRAL FORM

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y}) \left[\mu_\mathrm{a}(\mathbf{y}) L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y}) L_\mathrm{s}(\mathbf{y},\omega) \right] \mathrm{d}y$$

Emission In-scattering



Let's now have a closer look at the gains.

The first term here represents volumetric emission. The function L_e is typically defined by the artist, e.g. by measuring real objects, or using a fluid simulation.

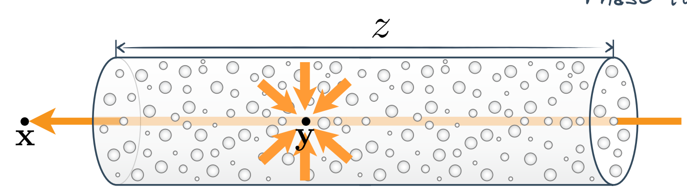
The second term is a little more complicated. It accounts for the effects of in-scattering.

RTE — INTEGRAL FORM

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})\left[\mu_a(\mathbf{y})L_e(\mathbf{y},\omega) + \mu_s(\mathbf{y})L_s(\mathbf{y},\omega)\right]\mathrm{d}y$$

$$L_s(\mathbf{y},\omega) = \int_{S^2} f_p(\omega,\bar{\omega})L(\mathbf{y},\bar{\omega})\mathrm{d}\bar{\omega}$$

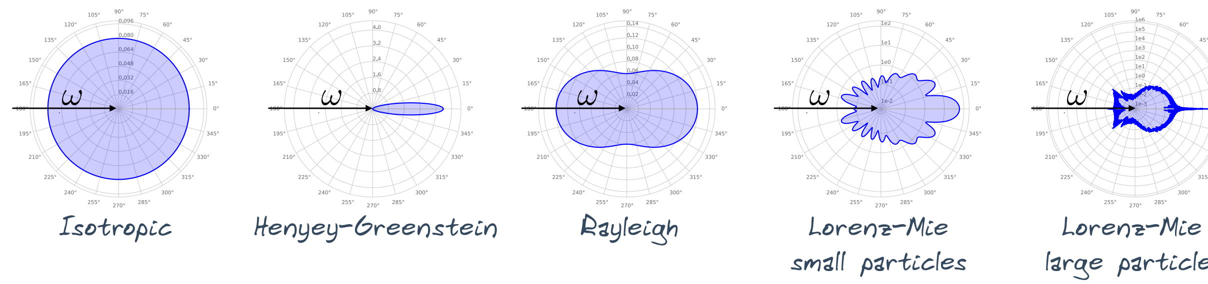
Phase function



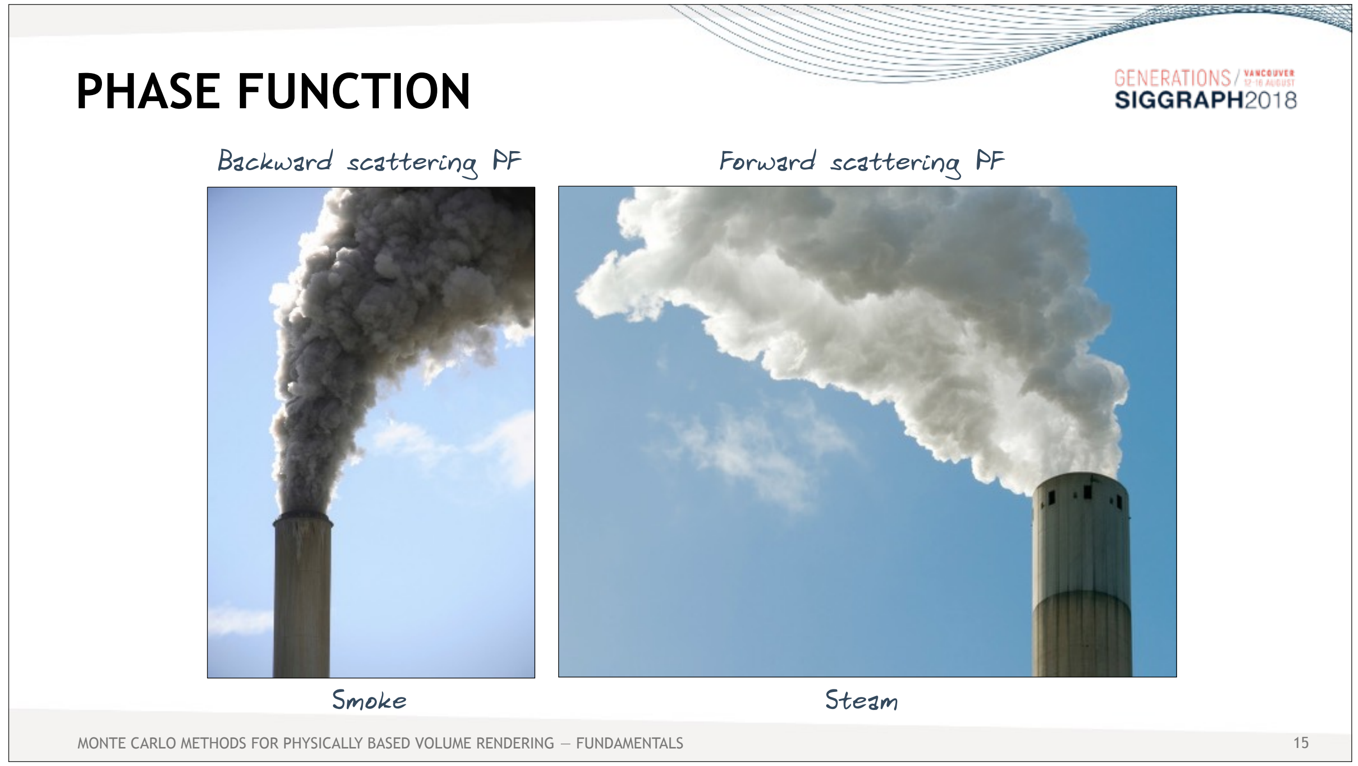
The in-scattered radiance function L_s is computed by integrating incident light over the sphere of all directions modulated by the phase function. The phase function captures the directional distribution of scattered light, and it is the volumetric analog of the BRDF.

PHASE FUNCTION

$$f_{\mathrm{p}}(\omega, \bar{\omega})$$



Here are a few different models used for expressing the phase function.



These images compare the visual appearance of predominantly backward- and forward-scattering media—smoke and steam.

PHASE FUNCTION

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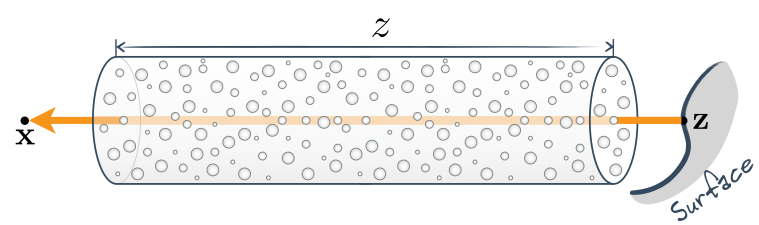
Isotropic PF

Forward scattering PF

RTE — INTEGRAL FORM

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y}) \Big[\mu_\mathrm{a}(\mathbf{y}) L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y}) L_\mathrm{s}(\mathbf{y},\omega)\Big] \mathrm{d}y$$
$$+T(\mathbf{x},\mathbf{z}) L_\mathrm{o}(\mathbf{z},\omega)$$

Background radiance



For completeness, we should also add a term that accounts for the surface at the end of the ray. We simply take the outgoing radiance from the surface and multiply it by the transmittance along the ray.

This equation here is often referred to as the *volume rendering equation* (VRE).

VOLUME RENDERING EQUATION

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$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})\Big[\mu_\mathrm{a}(\mathbf{y})L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y})L_\mathrm{s}(\mathbf{y},\omega)\Big]\mathrm{d}y$$
$$+T(\mathbf{x},\mathbf{z})L_\mathrm{o}(\mathbf{z},\omega)$$

How do we solve it?

The main question that remains now is, how do we solve it?

MONTE CARLO INTEGRATION

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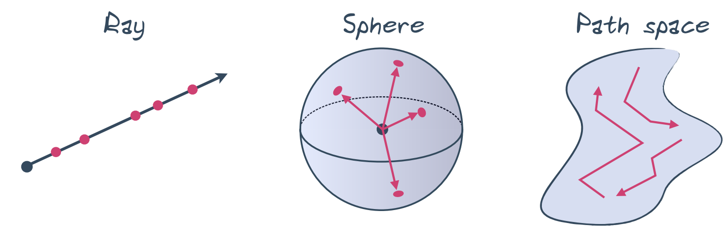
$$F = \int_{\mathcal{D}} f(x)\,\mathrm{d}x$$



$$\langle F \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

Probability density function (PDF)

In this course, we focus on the various Monte Carlo methods developed in the past few decades.

Monte Carlo integration works in the following way: in order to integrate a function f over a domain D (e.g. a ray, sphere, path space), we generate a few points in D, evaluate the integrand at these points, and estimate the integral as a weighted average where each sample is divided by the probability density of drawing it.

In the following slides, we consider taking only one sample for notation brevity.

VRE ESTIMATOR

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$$\langle L(\mathbf{x},\omega)\rangle = \frac{T(\mathbf{x},\mathbf{y})}{p(y)}\left[\mu_{\mathrm{a}}(\mathbf{y})L_{\mathrm{e}}(\mathbf{y},\omega)+\mu_{\mathrm{s}}(\mathbf{y})L_{\mathrm{s}}(\mathbf{y},\omega)\right] + \frac{T(\mathbf{x},\mathbf{z})}{P(z)}L_{\mathrm{o}}(\mathbf{z},\omega)$$

$p(y)$ - probability density of distance y

$P(z)$ - probability of exceeding distance z

Applying such a one-sample estimator to the VRE is straightforward: we evaluate the integrand at a single randomly chosen distance y divided by the probability density of sampling the distance.

If the sampled distance happens to be past the nearest surface, we evaluate the second term (the radiance from the surface) divided by the *probability* of sampling a position behind the surface.

VRE ESTIMATOR

Transmittance estimation

$$\langle L(\mathbf{x},\omega)\rangle = \frac{T(\mathbf{x},\mathbf{y})}{p(y)} \left[\mu_{\mathrm{a}}(\mathbf{y})L_{\mathrm{e}}(\mathbf{y},\omega) + \mu_{\mathrm{s}}(\mathbf{y})L_{\mathrm{s}}(\mathbf{y},\omega)\right]$$

$$+ \frac{T(\mathbf{x},\mathbf{z})}{P(z)} L_{\mathrm{o}}(\mathbf{z},\omega)$$

Distance sampling

Sampling a distance and evaluating transmittance are the key building blocks of most volume-rendering algorithms.

Next, we discuss methods for sampling distances and then we describe approaches for estimating transmittance.