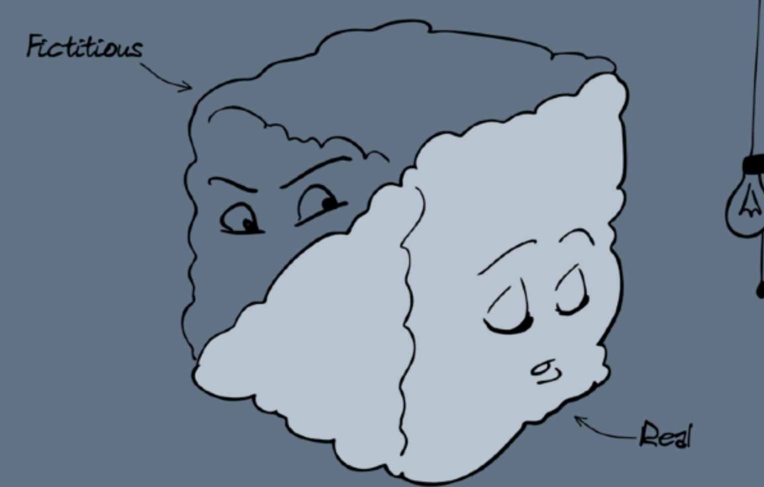
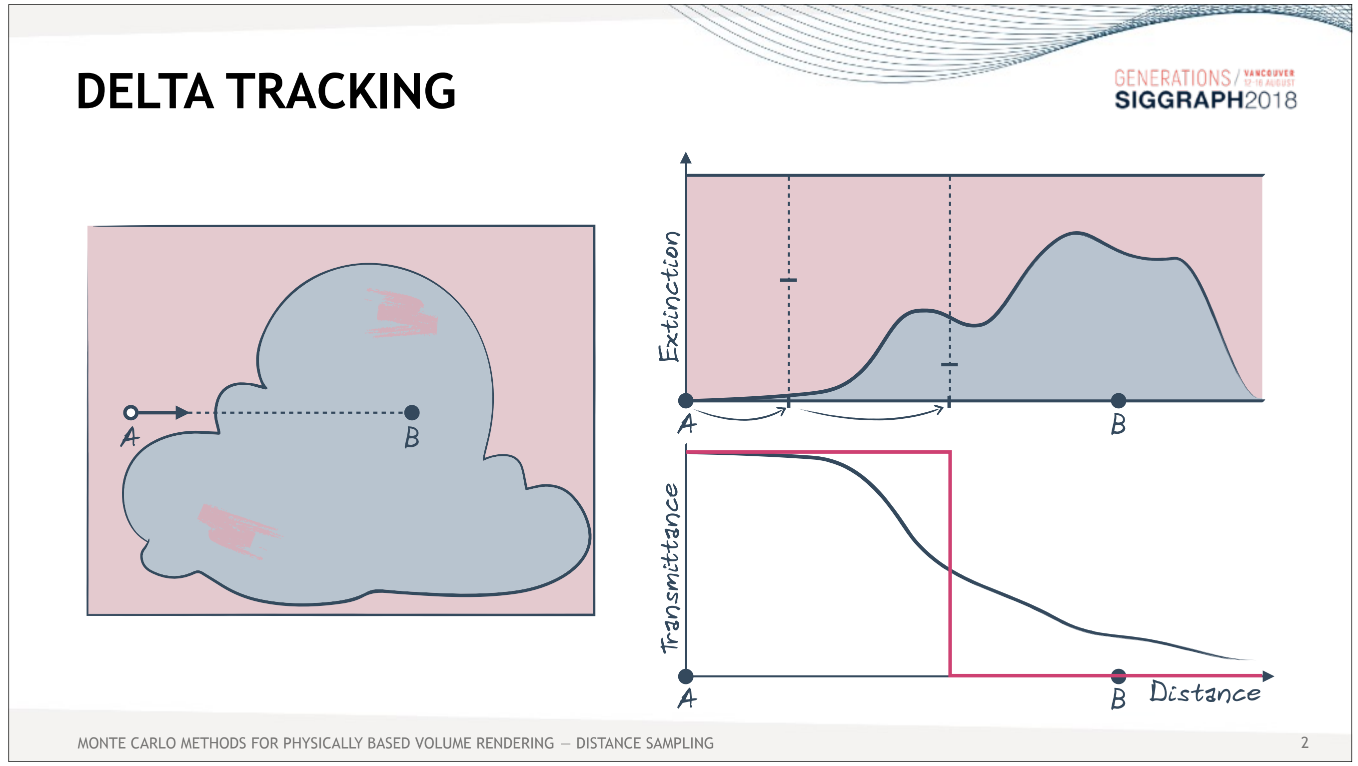
NULL-COLLISION ALGORITHMS—PART 2
TRANSMITTANCE ESTIMATION



In this second part, we will look at algorithms for transmittance estimation that are based on null collisions.



Delta tracking can be used as a track-length estimator to estimate transmittance in the following way. We perform the tracking and record the distance to the first real collision. If the distance is shorter than the distance of the segment of interest (here the distance to point B), the transmittance is approximated as 0. If the delta-tracking sample exceeds the distance to B, then transmittance is estimated as 1.

We can plot the transmittance approximation (pink curve), given by a single instance of delta tracking, which takes a form of a step function dropping from 1 to 0 at the distance of the real collision.

Such binary approximation is very crude. We can refine it by averaging multiple instances of the tracker.

DELTA TRACKING

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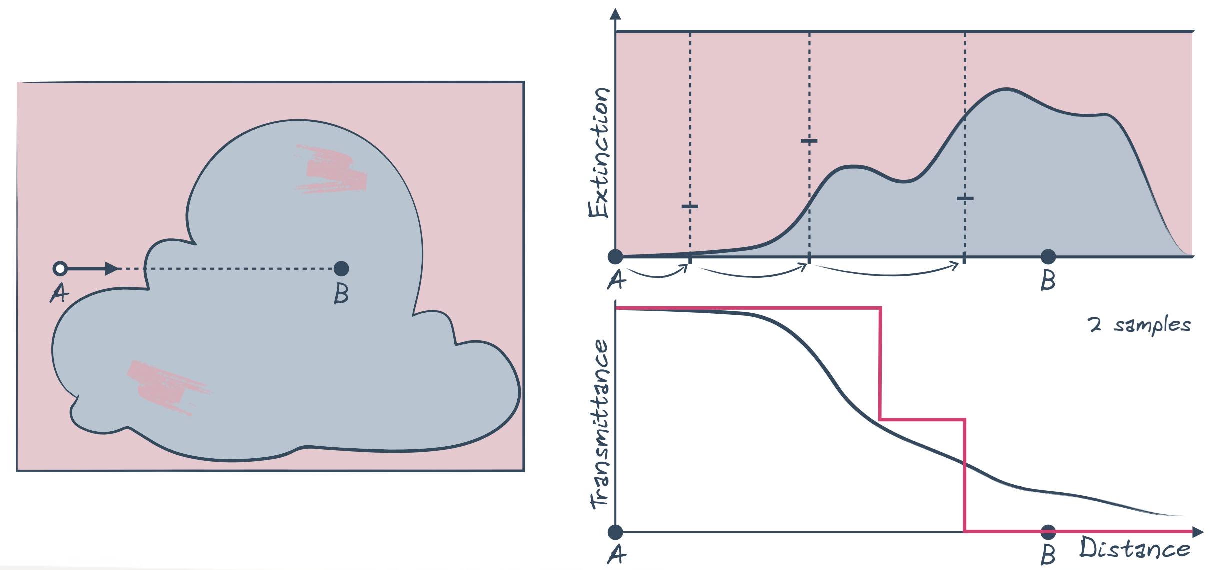
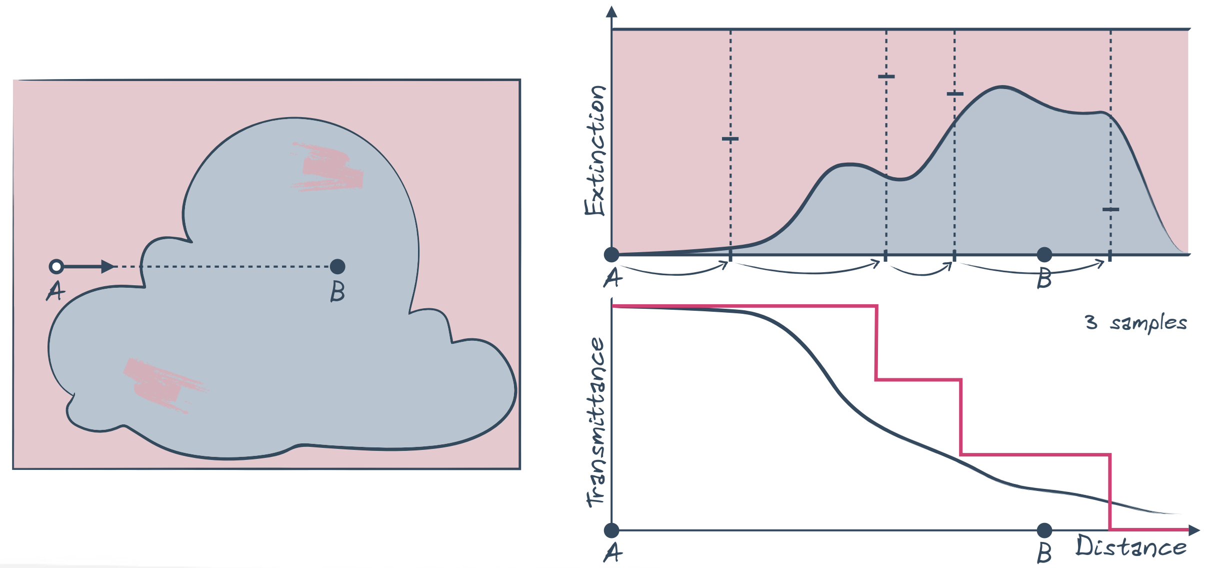
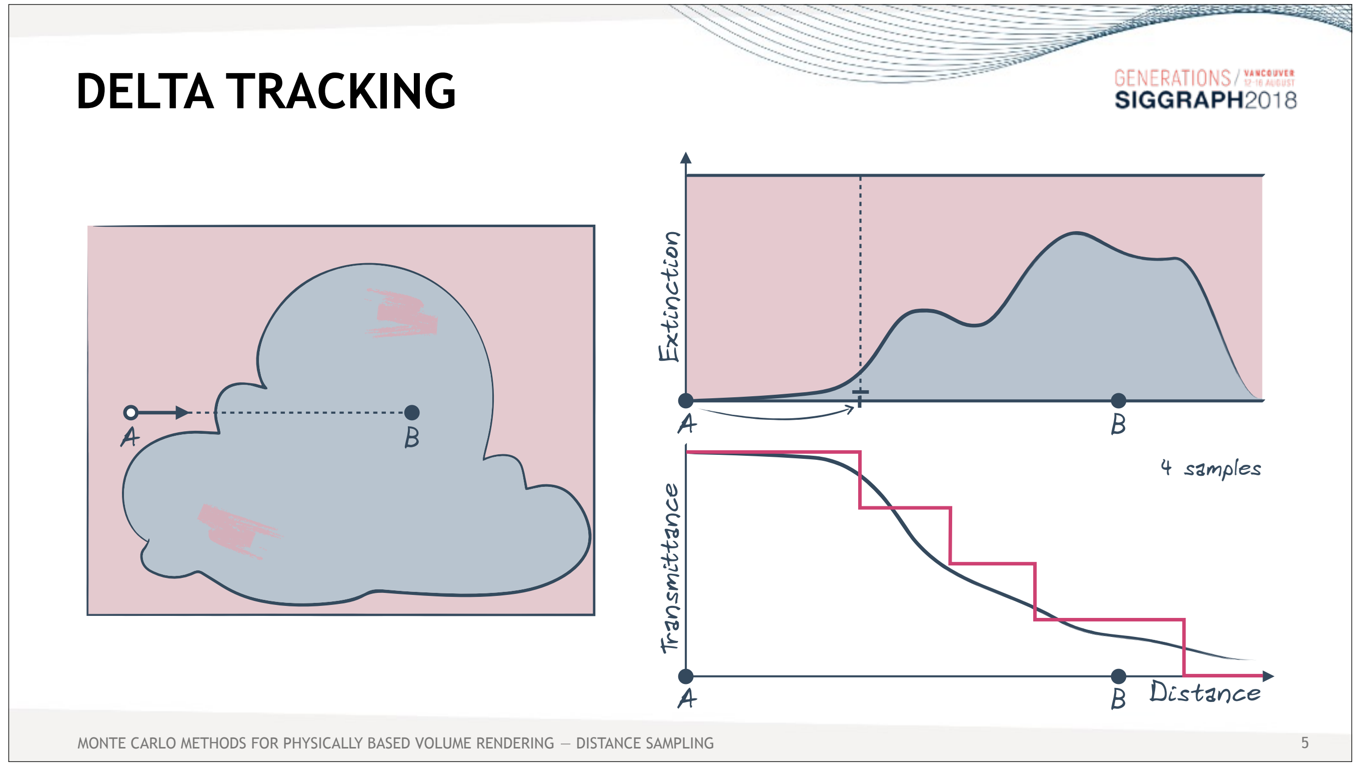


Here we tracked another instance. The pink function shows the average of the two instances.

DELTA TRACKING



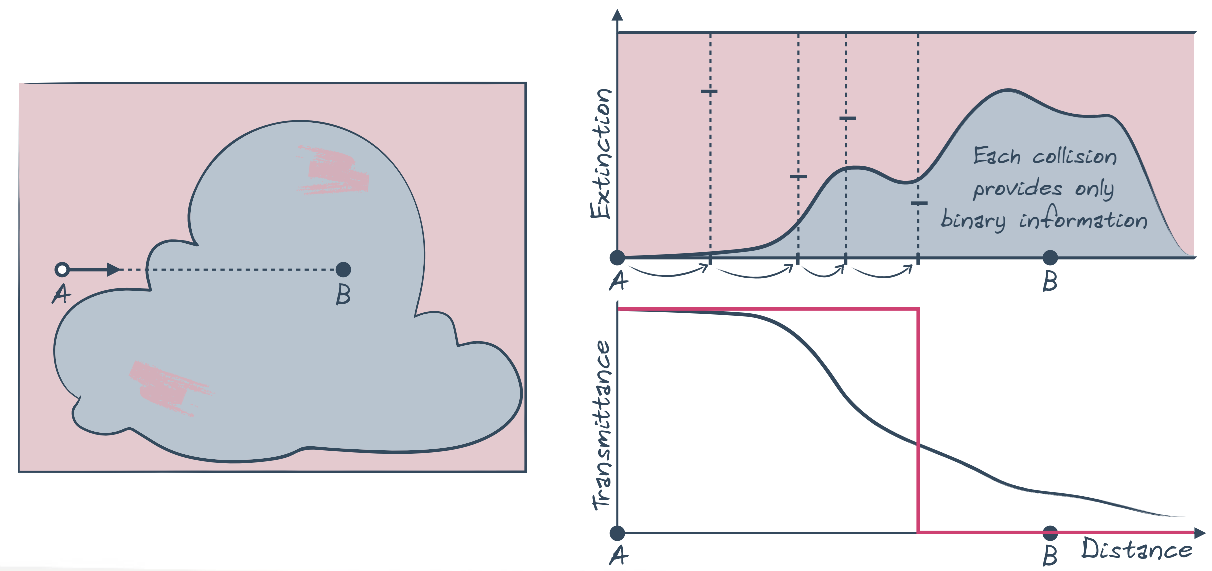
3 instances



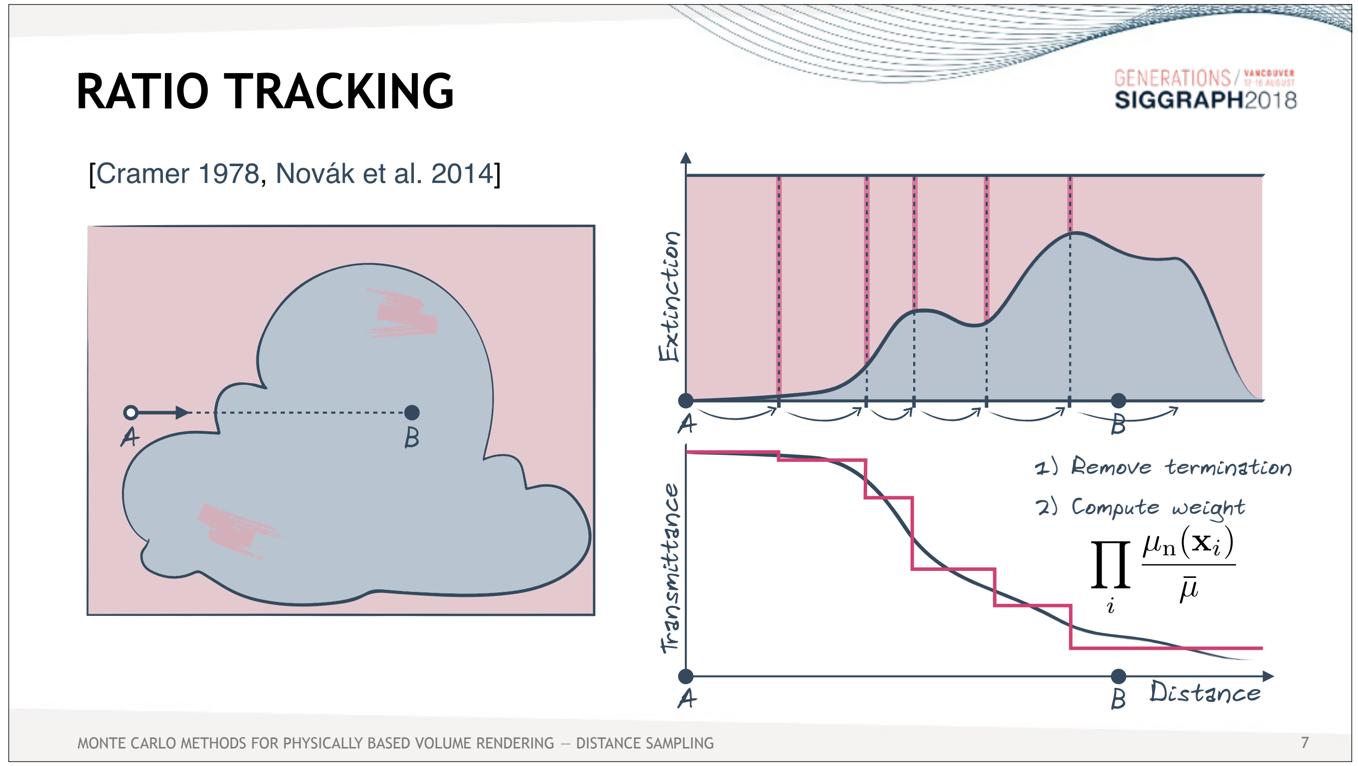
4 instances…

Refining the transmittance function estimate by invoking tracking multiple distances is however fairly expensive and computationally inefficient (in most cases).

DELTA TRACKING



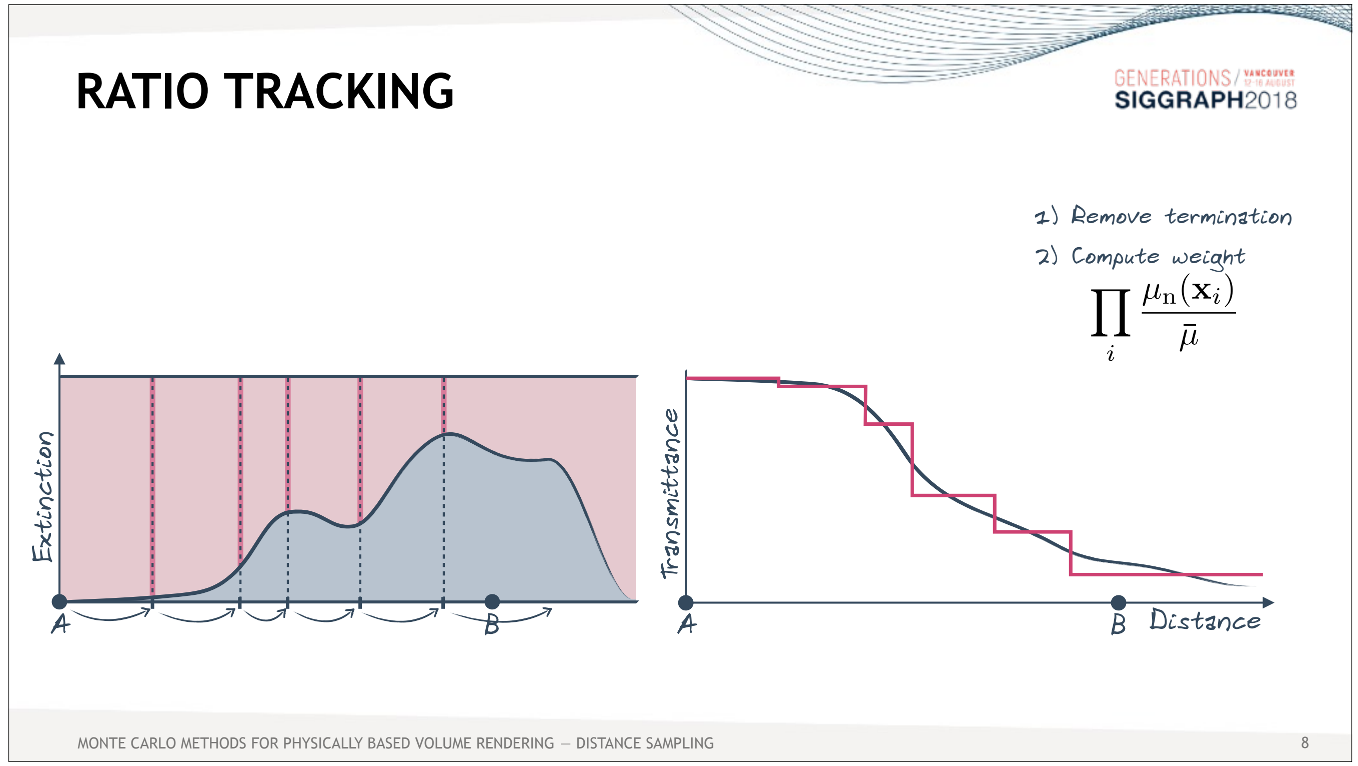
During a single tracking, we extract a fair amount of information about the medium (the collision coefficients at each individual tentative collision), but we reduce the information to only a binary value by probabilistically classifying the collision as either real or null. It seems somewhat inefficient to just flip a coin and reduce all the information to a binary outcome.



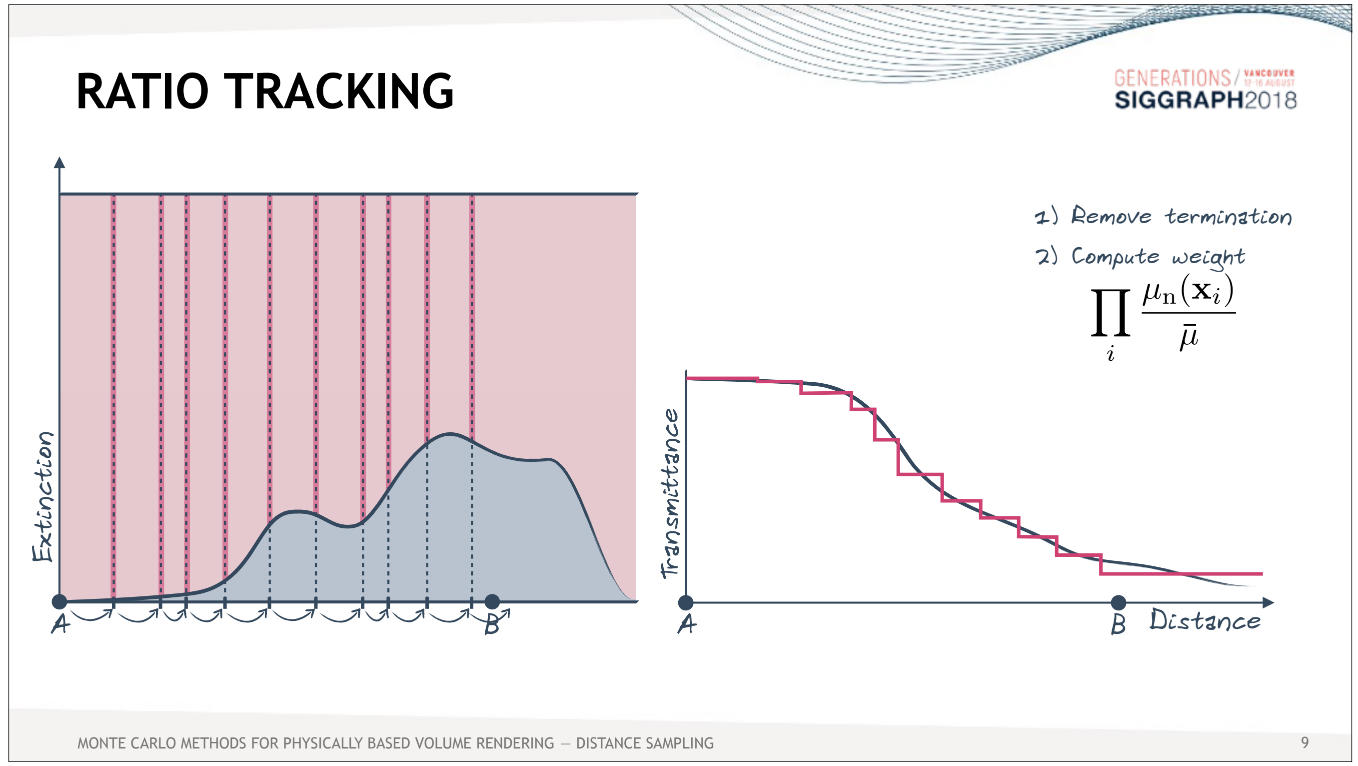
Ratio tracking addresses the inefficiency of delta tracking. The idea of ratio tracking is to remove the random termination, and replace it by its expectation, which serves as a weight (this is also known as Rao-Blackwelization).

Instead of scoring a binary answer, the tracker scores a rational weight: the product of ratios (of the null-collision coefficient to the majorant) at all points visited before reaching distance B.

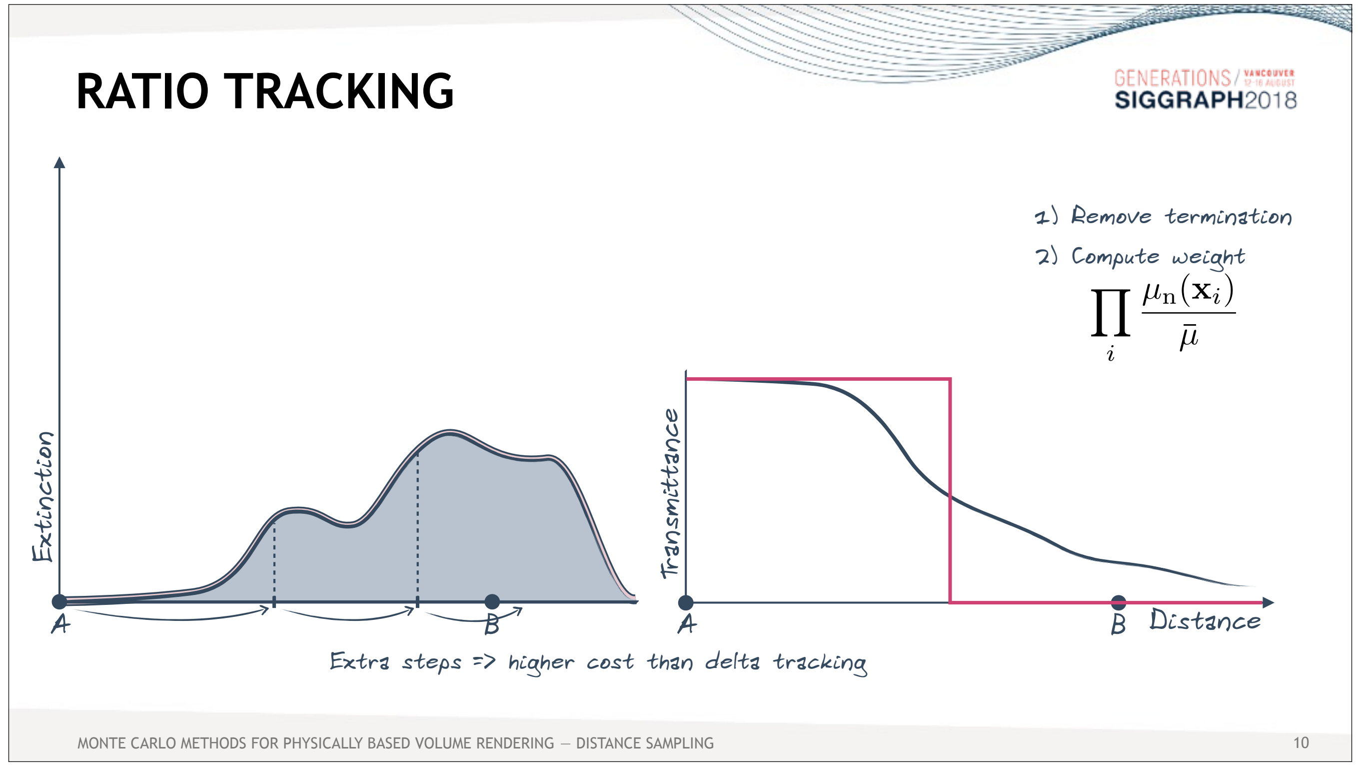
The resulting approximation of transmittance is piecewise constant (instead of binary).



Let us know consider the impact of the amount of the fictitious material on the resulting transmittance approximation.



In cases when a significant amount of null collisions is added, the distance samples tend to be short (and many) and the local collision weights are relatively high. The piecewise-constant approximation contains many steps and estimates the transmittance fairly accurately.



In the opposite extreme when no fictitious material is added, the ratio at the first collision will be zero, and the transmittance approximation becomes again a binary function (as in the case of delta tracking).

One caveat is that the ratio tracking would keep going further until reaching the desired distance B performing (unnecessary) steps that do not further refine the transmittance estimate—it is already zero.

In practice, it is best to start with ratio tracking and once the product of weights drops below a certain threshold (e.g. 0.1), then switch to delta tracking by invoking the probabilistic termination of the walk.

RATIO TRACKING

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Probabilistic **TERMINATION** replaced by **WEIGHTING**

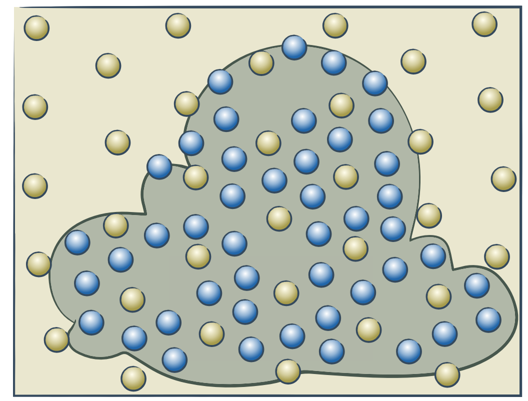
- Rational score instead of binary
- Requires more steps than a delta-tracking estimator (must reach B)
- Reduces the need for tight majorants
  - Loose majorants produce (more null collisions and therefore) finer estimates

The main benefit of ratio tracking is the robustness against loose majorants. The cost of the tracker increases in such cases, but the estimation error is significantly reduced by obtaining a fine, piecewise-constant estimate of the transmittance function.

RESIDUAL RATIO TRACKING

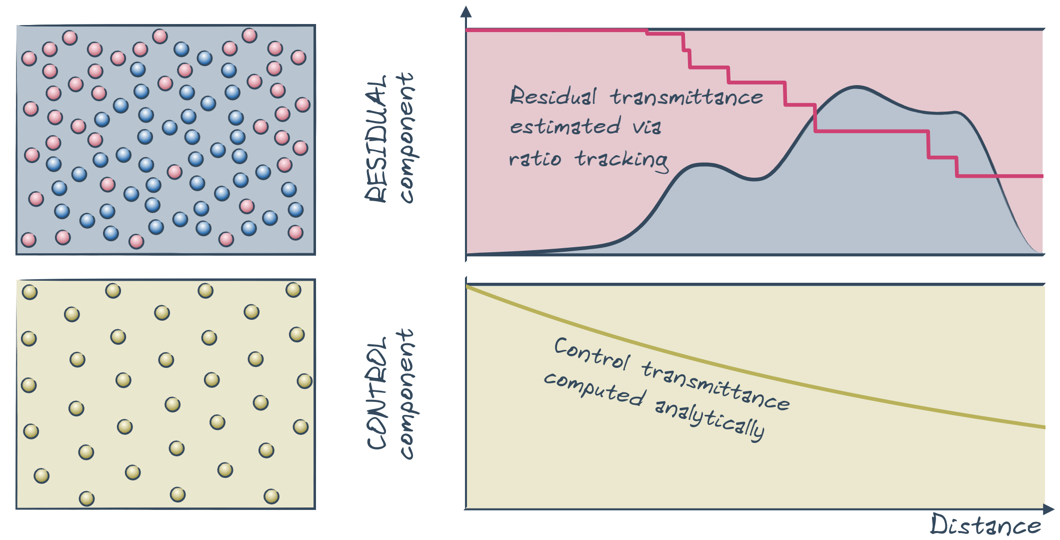
Compute part of the transmittance analytically

- [Novák et al. 2014]



Ratio tracking can be also combined with the idea of decomposition tracking. The resulting weighted track-length estimator often further reduces estimation variance by treating part of the computation in closed form.

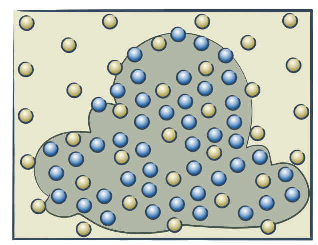
RESIDUAL RATIO TRACKING

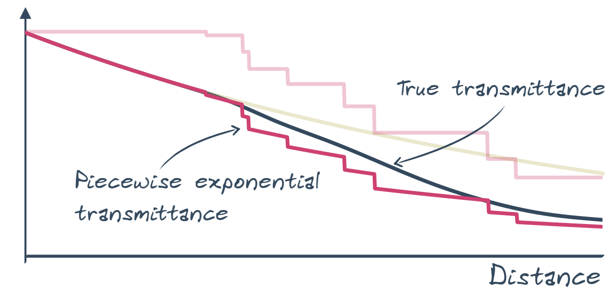


The transmittance in the homogeneous, control component (bottom) is computed analytically.

The transmittance in the heterogeneous, residual component (top) is estimated using ratio tracking.

RESIDUAL RATIO TRACKING





$$\langle T(t) \rangle = T_{\text{control}}(t) \, \langle T_{\text{residual}}(t) \rangle$$

The transmittance in the combined medium is then obtain as the product of the two transmittance functions yielding a piecewise-exponential approximation to the ground-truth function.

RESIDUAL RATIO TRACKING

HOMOGENEOUS and **RESIDUAL HETEROGENEOUS** components

- Reduces noise by handling part of the transmittance analytically
- Requires a space-partitioning data structure (e.g. octree) to be practical
- Can handle negative residual extinctions

NEXT-FLIGHT ESTIMATORS

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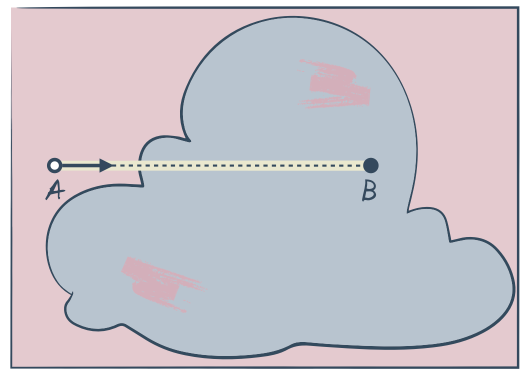
Score a weight at every tentative collision

- Cramer [1978] combines next-flight estimation with delta and ratio tracking



NEXT-FLIGHT DELTA TRACKING

$$\langle T(t)\rangle = T_{\bar{\mu}}(0,t) + \sum_{j=1}^{n} \frac{\mu_{\mathrm{n}}(t_j)}{\bar{\mu}(t_j)} T_{\bar{\mu}}(t_j,t)$$

Transmittance along the remaining segment through real + fictitious matter

Fraction of fictitious matter

There is one other approach that is based on null collisions—developed by Cramer for deep-penetration problems—that has not been yet fully evaluated in the context of rendering.

Cramer applies the idea of next-event estimation to delta-tracking and ratio-tracking based transmittance estimators. In the case of delta tracking, he proposes to score a “next-flight” contribution (analogous to a shadow ray) at each tentative collision, which amounts to the transmittance along the remaining segment towards point B, weighted by the local u_n/bar{u} ratio.

Next-flight ratio tracker works in a similar way. Please see the original publication or our EG STAR from 2018 for additional details.

SUMMARY

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DELTA TRACKING estimator

- Relatively cheap but binary, inefficient w/ loose majorants

RATIO TRACKING estimator

- More expensive, but also more accurate especially w/ loose majorants

RESIDUAL TRACKING estimators

- Reduces variance by employing analytic computation for part of the transmittance function

NEXT-FLIGHT estimators

- Further improve performance by scoring a weight at each step
- Not fully explored yet in the context of rendering…

ACKNOWLEDGEMENTS

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WALT DISNEY ANIMATION STUDIOS

PIXAR ANIMATION STUDIOS