DELTA TRACKING

Each collision provides only binary information.

Distance sampling.
RATIO TRACKING

[Cramer 1978, Novák et al. 2014]
RATIO TRACKING

1) Remove termination
2) Compute weight
\[ \prod_i \frac{\mu_n(x_i)}{\bar{\mu}} \]
RATIO TRACKING

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\[ \prod_{i} \frac{\mu_n(x_i)}{\bar{\mu}} \]
RATIO TRACKING

1) Remove termination
2) Compute weight
\[ \prod_i \frac{\mu_n(x_i)}{\bar{\mu}} \]

Extra steps => higher cost than delta tracking
RATIO TRACKING

Probabilistic TERMINATION replaced by WEIGHTING

 prá 
  - Rational score instead of binary
  - Requires more steps than a delta-tracking estimator (must reach $B$)
  - Reduces the need for tight majorants
    - Loose majorants produce (more null collisions and therefore) finer estimates
RESIDUAL RATIO TRACKING

Compute part of the transmittance analytically

- [Novák et al. 2014]
RESIDUAL RATIO TRACKING

Residual transmittance estimated via ratio tracking

Control transmittance computed analytically

Distance

Residual component

Control component
RESIDUAL RATIO TRACKING

\[ \langle T(t) \rangle = T_{\text{control}}(t) \langle T_{\text{residual}}(t) \rangle \]
HOMOGENEOUS and RESIDUAL HETEROGENEOUS components

- Reduces noise by handling part of the transmittance analytically
- Requires a space-partitioning data structure (e.g. octree) to be practical
- Can handle negative residual extinctions
NEXT-FLIGHT ESTIMATORS

Score a weight at every tentative collision

- Cramer [1978] combines next-flight estimation with delta and ratio tracking

NEXT-FLIGHT DELTA TRACKING

\[
\langle T(t) \rangle = T_{\mu}(0, t) + \sum_{j=1}^{n} \frac{\mu_n(t_j)}{\mu(t_j)} T_{\mu}(t_j, t)
\]

Transmittance along the remaining segment through real + fictitious matter

Fraction of fictitious matter
COMPARISON

(a) Canonical scene

(b) Equal-cost renders

(c) Variance of $\langle T \rangle$

(d) Cost $\langle T \rangle$

(e) Variance $\times$ cost
SUMMARY

DELTA TRACKING estimator

- Relatively cheap but binary, inefficient w/ loose majorants

RATIO TRACKING estimator

- More expensive, but also more accurate especially w/ loose majorants

RESIDUAL TRACKING estimators

- Reduces variance by employing analytic computation for part of the transmittance function

NEXT-FLIGHT estimators

- Further improve performance by scoring a weight at each step
- Not fully explored yet in the context of rendering…