selection of advanced methods:
- Dwivedi sampling / zero variance random walks
- spectral tracking
- acceleration data structures
  - for regular tracking
  - inside these: null collision based
- emissive media
Dwivedi sampling

- regular path tracing random walk
- tends to get lost inside a **volume bounded by a shape**
Dwivedi sampling

- random walk biased to exit **bounded volume** as quickly as possible
  - assumes constant illumination from the outside [Kd14]

- assume **homogeneous slab** with **isotropic** phase function
- approximate closed-form solution of transport using this simplified setting
  - known as **zero-variance theory** (term may be a bit bold)
Dwivedi sampling

- random walk biased towards light source
- aims to exit towards light [MHD16]
Dwivedi sampling

- random walk biased towards light source
- aims to exit towards light [MHD16]

achieved by **biasing the PDF** to sample direction and distance
estimator remains unbiased!
spectral tracking

- another problem with skin: chromatic media
  - collision coefficients $\mu$ depend on wavelength $\lambda$
  - for instance free flight distance much longer for long wavelengths:

makes path invalid for different wavelength?

- can we still exploit coherence?
spectral tracking via MIS

- hero wavelength sampling [WND $\times 14$]
- sample perfectly for one single wavelength $\lambda_0$
- evaluate path for a stratified set of wavelengths $\lambda_i$ at the same time
- optimally weighted combination via MIS (balance heuristic)
  - limited to regular tracking because it requires explicit evaluation of PDF

\[
\frac{f(\bar{x}, \lambda_i)}{\sum_j p(\bar{x}, \lambda_j)}
\]
spectral tracking via MIS

image comparison 64spp

- skin material with 1 wavelength
spectral tracking via MIS

image comparison 64spp

- skin material with 4 wavelengths (SSE)
spectral tracking via MIS

image comparison 64spp

- skin material with 8 wavelengths (AVX)
spectral tracking without PDF [KHLN17]

- sample by common majorant $\bar{\mu}$
- how do decide for null collision, scattering, or absorption?
- probability according to $\mu_n(\lambda), \mu_s(\lambda), \mu_a(\lambda)$
  - pick by maximum over $\lambda_i$
  - pick by average weighted by spectral path throughput history
spectral tracking without PDF \[\text{[KHLN17]}\]

- sample by common majorant $\bar{\mu}$
- how do decide for null collision, scattering, or absorption?
- probability according to $\mu_n(\lambda), \mu_s(\lambda), \mu_a(\lambda)$
  - pick by maximum over $\lambda_i$
  - pick by average weighted by spectral path throughput history
- results in different noise patterns:
speed!

low variance estimators are important

but also, in volumes most of the run time is memory fetching
acceleration data structures

- grid, super voxels [SKTM11], kd-tree [YIC\*11], adaptive blocks

- adaptivity driven by
  - pixel footprint / camera tessellation
  - heterogeneity / variation

- two-level modelling (super voxel, kd nodes) store majorants $\mu$ in coarse blocks
acceleration data structures

- grid, super voxels [SKTM11], kd-tree [YIC11], adaptive blocks

- adaptivity driven by
  - pixel footprint / camera tessellation
  - heterogeneity / variation
- two-level modelling (super voxel, kd nodes) store majorants $\bar{\mu}$ in coarse blocks
  - perform **regular tracking** on coarse blocks [SKTM11]
acceleration data structures

- grid, super voxels [SKTM11], kd-tree [YIC*11], adaptive blocks

- adaptivity driven by
  - pixel footprint / camera tessellation
  - heterogeneity / variation

- two-level modelling (super voxel, kd nodes) store majorants $\bar{\mu}$ in coarse blocks
  - perform regular tracking on coarse blocks [SKTM11]
  - access $\mu_s(\lambda), \mu_a(\lambda)$ on fine levels to sample collision type
acceleration data structures

regular tracking

- needs to step through every voxel, bad for fine tessellations
- well chosen tessellation is a big advantage!
acceleration data structures

null collision-based tracking

- is independent of tessellation and is efficient in thin media (few events)
- high number of events in dense media, regardless of tessellation!

- accessing the memory within the same voxel is still expensive
- alleviated by decomposition tracking [KHLN17]
  - separate $\mu$ into sum of coarse and fine, to sample distance pick shortest (and early out!)
  - also profits full regular tracking
emissive media

thin/dense media make a difference

> no event inside the medium means we cannot pick up emission:
emissive media

thin/dense media make a difference

- following the idea of beams, collect emission along a ray

- particularly well suited for regular tracking, touching all voxels anyways
emissive media

thin/dense media make a difference

- direct application of MIS with NEE [VH13] introduces noise:

<table>
<thead>
<tr>
<th>Point + NEE</th>
<th>Line + NEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Image 1]</td>
<td>![Image 2]</td>
</tr>
<tr>
<td>![Image 3]</td>
<td>![Image 4]</td>
</tr>
</tbody>
</table>

- reason: NEE cannot create paths with end point outside the medium
  - forward scattering PDF is poor, however, and now it picks up line emission!
emissive media

thin/dense media make a difference

- need to teach next event estimation about line emission [SHZD17]:

<table>
<thead>
<tr>
<th>Point + NEE</th>
<th>Line + NEE</th>
<th>Line + FNEE</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
</tbody>
</table>
end of content

up next:

summary and open research problems
summary

free flight distance sampling
  - woodcock/delta tracking

transmittance estimation
  - track-length
  - residual ratio
  - free flight versions

path sampling
  - path space formulation
  - summary of advanced methods

acceleration structures
  - for regular tracking
  - for null collisions (bottom-level)
open research problems

null collision algorithms and MIS

- missing link to integrate into powerful framework
  - for instance combine with equi-angular sampling
- can we estimate the PDF?
  - expectation and division do not commute!

\[ X = \frac{f(\bar{x})}{p(\bar{x})} \]
open research problems

leverage recent advances in machine learning

- special purpose denoising
  - including a volume prior?
- path guiding for volumes?
  - importance sampling for multiple vertices?
open research problems

joint handling of surfaces and geometry

- still often surface transport is handled separately
- makes inclusion of all interreflections hard
- custom-cut algorithms increase maintenance cost
open research problems

generalisation to correlated scatterers

- core assumption of exponential path length: uncorrelated particles!
- particle repulsion such as in cell growth is very correlated
- really, no collision can be found inside the current particle (min distance)
- some existing work

[d'Eon 2018, Bitterli et al. 2018]
thank you!

any questions?

acknowledgements:

- Peter Kutz
  - for tracing down many of early delta tracking papers
- Jaroslav Křivánek & reviewers
  - for feedback on the paper draft
- Maurizio Nitti
  - for help w/ fast forward and illustrations