Combining Point and Line Samples for Direct Illumination

Points only

Points + Lines

Katherine Salesin

Wojciech Jarosz

DARTMOUTH VISUAL COMPUTING LAB
Motivation

Combining point and line samples for direct illumination
Motivation

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Motivation

Direct lighting:
Motivation

Direct lighting: **point sampling**
Motivation

Direct lighting: **point sampling** and **line sampling**
Theory:

Direct lighting + Monte Carlo sampling
Combining point and line samples for direct illumination
Theory

Direct lighting
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Combining point and line samples for direct illumination
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Direct lighting

Combining point and line samples for direct illumination
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\[ L_o(e, x) = \int_A f_r(e, x, l)G(x, l)V(x, l)L_e(x, l) \, dA(l) \]
Combining point and line samples for direct illumination

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Direct lighting

\[ L_o(e, x) = \int_u \int_v f_r(e, x, l_{uv}) G(x, l_{uv}) V(x, l_{uv}) L_e(x, l_{uv}) \, dv \, du \]
Combining point and line samples for direct illumination

Theory

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Monte Carlo sampling

\[ L_0 = \int_{\mathcal{U}} \int_{\mathcal{V}} f(u, v) \, dv \, du \approx \langle L_0 \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(u_i, v_i)}{p(u_i, v_i)} \]
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Combining point and line samples for direct illumination

Line sampling

\[ L_o = \int_U \int_V f(u, v) \, dv \, du = \int_U f_v(u) \, du \]
Theory

Line sampling

$[BD16]$

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Prior work: line sampling
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- **Direct illumination** [BD16]
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- Transient light transport [MGJ*19]
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and more…
Line sampling: pros and cons
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✓ Less error per sample than points
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✓ Better convergence rate than points (if stratified)
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✗ Slow to evaluate samples
  • Expensive line sample-scene intersection
Motivation
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Make line samples play nicely with any point-based strategy
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Make line samples play nicely with any point-based strategy.

Mitigate orientation-based performance issues.
MIS Points + Lines
(Ours)
Roadmap
Roadmap

• Reframe line samples as point samples that importance sample visibility
Roadmap

- Reframe line samples as point samples that *importance sample visibility*
- Show how to *multiple importance sample* between lines and points, and lines of different orientations
Roadmap

• Reframe line samples as point samples that importance sample visibility

• Show how to multiple importance sample between lines and points, and lines of different orientations

• Propose novel MIS weighting scheme to improve convergence rate
Main idea

**Point sampling**

\[ L_o = \int_{\mathcal{U}} \int_{\mathcal{V}} f(u, v) \, dv \, du \]

**Line sampling**

\[ L_o = \int_{\mathcal{U}} f_v(u) \, du \]
Main idea

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Combining point and line samples for direct illumination
Main idea

Combining point and line samples for direct illumination

Point sampling

Line sampling

\[ L_o = \int_U \int_V f(u, v) \, dv \, du \quad \text{constant} \]

\[ L_o = \int_U f_v(u) \, du \]
Main idea

Point sampling

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\[ \langle L_0 \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(u_i, v_i)}{p(u_i, v_i)} \]

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Combining point and line samples for direct illumination
**Main idea**

\[
\langle L_0 \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(u_i, v_i)}{p(u_i, v_i)}
\]
Main idea

Point sampling

$$\langle L_o \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(u_i, v_i)}{p(u_i, v_i)}$$

Line sampling

$$\langle L_o \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(u_i, v_i)}{p(u_i) p(v_i | u_i)}$$
**Main idea**

**Point sampling**

\[
\langle L_0 \rangle = \frac{1}{N} \sum_{i=1}^{N} f(u_i, v_i)
\]

**Line sampling**

\[
\langle L_0 \rangle = \frac{1}{N} \sum_{i=1}^{N} p(u_i) \frac{f(u_i, v_i)}{f_y(u_i)}
\]

Combining point and line samples for direct illumination
Main idea

Point sampling

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\langle L_o \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(u_i, v_i)}{p(u_i, v_i)}
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Line sampling

\[
\langle L_o \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(u_i, v_i)}{p(u_i)} \cdot \frac{f(u_i, v_i)}{f_v(u_i)}
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Combining point and line samples for direct illumination
Main idea

• But a perfect conditional pdf would be hard to find for the full direct lighting integral
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• Instead, we use simpler conditional pdfs that work well in practice
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• But a perfect conditional pdf would be hard to find for the full direct lighting integral
  • Instead, we use simpler conditional pdfs that work well in practice
• Effectively **importance sampling visibility**
Main idea

Conditional point pdfs

We propose two options:
Main idea

Conditional point pdfs

We propose two options:

**Surface-area**-based sampling
(uniform over surface area)
Main idea

Conditional point pdfs

We propose two options:

**Surface-area**-based sampling (uniform over surface area)

**Solid-angle**-based sampling (uniform over solid angle)

from [UFK13]
Combining point and line samples for direct illumination
Summary

• We have reframed line sampling as point sampling that **importance samples visibility**
Summary

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• We can now use line sampling with **any BRDF**
Summary

• We have reframed line sampling as point sampling that **importance samples visibility**

• We can now use line sampling with **any BRDF**

• But line samples are still bad at importance sampling some terms – can we do better?
Theory

Combining point and line samples for direct illumination
Theory

• We can use **multiple importance sampling** (MIS) to combine the strengths of different strategies
• We can use **multiple importance sampling** (MIS) to combine the strengths of different strategies

• MIS uses a set of **weights** to favor each strategy where it is strongest (i.e. where a strategy’s pdf is largest relative to other strategies’ pdfs)
Combining point and line samples for direct illumination
Summary

- We can now MIS lines with lines of other orientations
MIS between lines:
Equal time comparison
## Relative MSE

<table>
<thead>
<tr>
<th>Lines (average) [BD16]</th>
<th>Full image</th>
<th>Green</th>
<th>Purple</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.6 x 10^{-1}</td>
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Equal time comparison
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<td><strong>MIS lines</strong></td>
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Summary

• We can now MIS lines with lines of other orientations
Summary

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• We can now MIS lines with points that importance sample other distributions (like BRDFs)
MIS between points and lines:

Equal time comparisons
<table>
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<tr>
<td>BSDF</td>
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**Relative MSE**

Full image

| BSDF     | $5.5 \times 10^1$ |
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<th>Purple</th>
</tr>
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<tbody>
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<td>4 points : 0 lines</td>
<td>$4.0 \times 10^{-1}$</td>
<td>$5.7 \times 10^{-2}$</td>
<td>$4.4 \times 10^{-1}$</td>
</tr>
<tr>
<td>3 points : 1 line</td>
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</tr>
<tr>
<td>2 points : 2 lines</td>
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<td>$5.2 \times 10^{-1}$</td>
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</tr>
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<td>1 point : 3 lines</td>
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<td>$1.3 \times 10^{-1}$</td>
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Equal time comparison
Equal time comparison

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Summary

• We can now MIS lines with lines of other orientations
• We can now MIS lines with points that importance sample other distributions (like BRDFs)
Summary

• We can now MIS lines with lines of other orientations
• We can now MIS lines with points that importance sample other distributions (like BRDFs)
• But MIS inherits the worst convergence rate of its strategies – can we do better?
Discontinuity-smoothing MIS

Combining point and line samples for direct illumination
Discontinuity-smoothing MIS

• We know convergence rate improves when discontinuities in effective integrand are smoothed
Discontinuity-smoothing MIS

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• MIS estimator:
Discontinuity-smoothing MIS

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\[
\langle L_o \rangle^{\text{mis}} = \frac{1}{S} \sum_{s=1}^{S} \left( \frac{1}{N_s} \sum_{i=1}^{N_s} w_s(u_i, v_i) \frac{f(u_i, v_i)}{p_s(u_i, v_i)} \right) \quad \text{for S strategies}
\]
Discontinuity-smoothing MIS

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effective integrand
Discontinuity-smoothing MIS

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for \( S \) strategies

effective integrand

[SSC*19]
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for S strategies.

[Ours]
Discontinuity-smoothing MIS

Let us MIS:

1. **BRDF point samples**
2. **Vertical line samples**
3. **Horizontal line samples**
Discontinuity-smoothing MIS

Effective integrand for the
**BRDF strategy**

**without smoothing**
Effective integrand for the **BRDF strategy** without smoothing

**Discontinuity-smoothing MIS**
Discontinuity-smoothing MIS

Effective integrand for the **BRDF strategy** without smoothing
Discontinuity-smoothing MIS

Effective integrand for the 
BRDF strategy 
without smoothing
Discontinuity-smoothing MIS

Effective integrand for the BRDF strategy without smoothing
Discontinuity-smoothing MIS

Effective integrand for the BRDF strategy without smoothing
Effective integrand for the \textbf{BRDF strategy} without smoothing

\textbf{Discontinuity-smoothing MIS}
Effective integrand for the BRDF strategy with smoothing
Smoothing MIS:
Convergence tests
Convergence tests
Convergence tests
Convergence tests

Combining point and line samples for direct illumination
Convergence tests

BSDF \(N^{-1.38}\)
Convergence tests

Combining point and line samples for direct illumination

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<td>Variance</td>
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<td>Number of Light Samples</td>
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- BSDF (N^{-1.38})
- Horiz. lines (N^{-2.13})
Convergence tests

BSDF (N^{-1.38})
Horiz. lines (N^{-2.13})
Vert. lines (N^{-2.14})

Pixel A (multijittered)

Variance

Number of Light Samples
Convergence tests

- BSDF ($N^{-1.38}$)
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- MIS BSDF + Lines ($N^{-1.40}$)
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Pixel A (multijittered)

Combining point and line samples for direct illumination
Convergence tests

Combining point and line samples for direct illumination
Convergence tests
Convergence tests

Combining point and line samples for direct illumination
Convergence tests

BSDF ($N^{-1.48}$)

Pixel B (multijittered)

Variance

Number of Light Samples
Convergence tests

BSDF ($N^{-1.48}$)
Horiz. lines ($N^{-2.57}$)

Pixel B (multijittered)

Variance
Number of Light Samples
Convergence tests

BSDF \((N^{-1.48})\)
Horiz. lines \((N^{-2.57})\)
Vert. lines \((N^{-1.61})\)

Pixel B (multijittered)

Number of Light Samples vs. Variance
Convergence tests

BSDF \( (N^{-1.48}) \)
Horiz. lines \( (N^{-2.57}) \)
Vert. lines \( (N^{-1.61}) \)
MIS BSDF + Lines \( (N^{-1.53}) \)

Pixel B (multijittered)

Variance vs. Number of Light Samples
Convergence tests

BSDF (N\(^{-1.48}\))
Horiz. lines (N\(^{-2.57}\))
Vert. lines (N\(^{-1.61}\))
MIS BSDF + Lines (N\(^{-1.53}\))
MIS BSDF + Lines with smoothing (N\(^{-1.68}\))
Wrapping things up:
What’s Next?
Future work
Future work

• Optimize the line sample-scene intersection
Future work

• Optimize the line sample-scene intersection
  • Line sample-scene intersection 1.2x - 55x slower than shadow ray
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• Improve smoothing MIS heuristic to be more robust to all scenarios

• Apply novel concepts to other line sampling (or even higher-dimensional) applications
Thank you!

Please visit

dartgo.org/pointsandlines

for the full paper, supplemental document, and interactive image viewer.

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