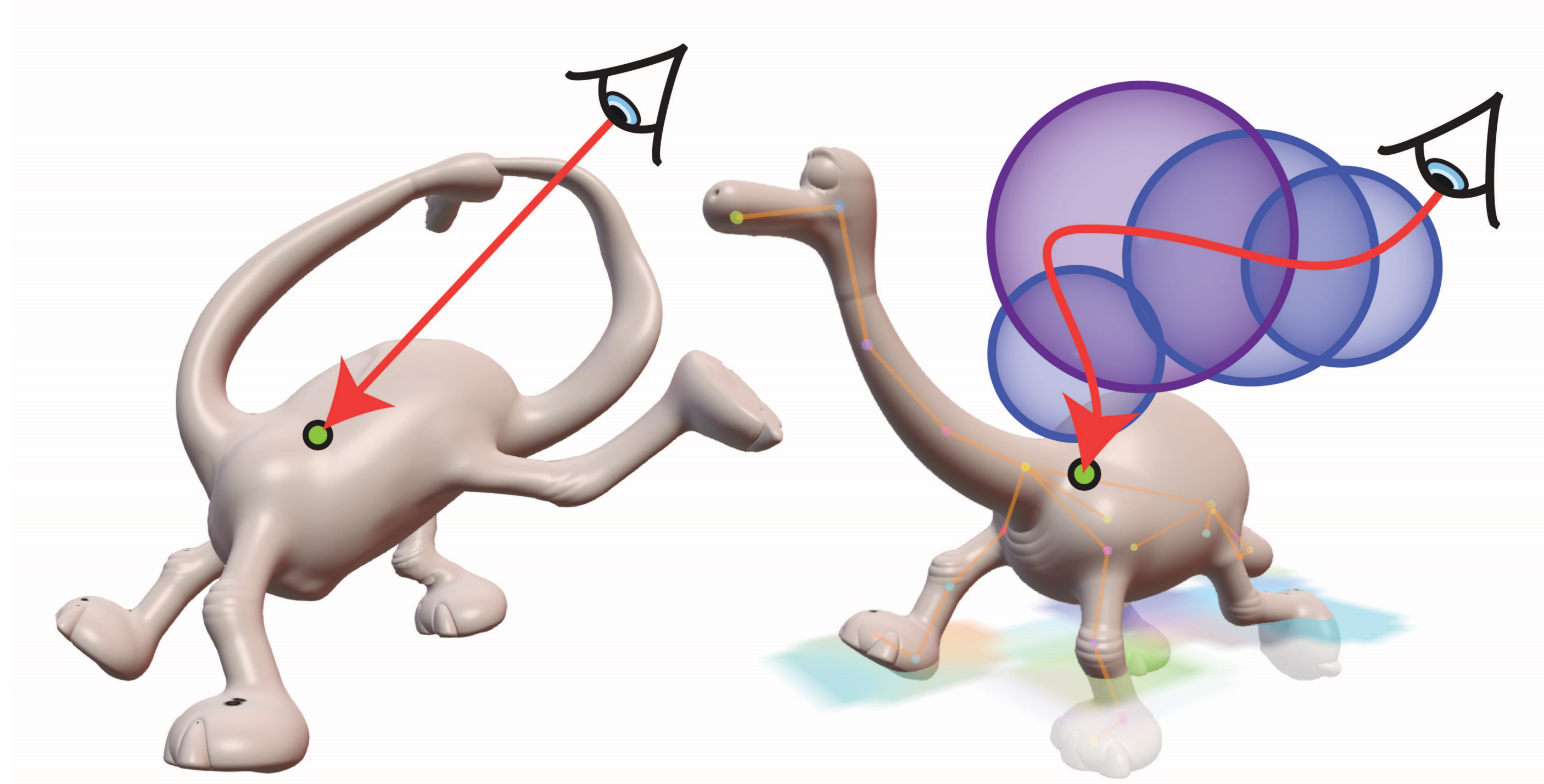


Non-linear Sphere Tracing for Rendering Deformed Signed Distance Fields



Dario Seyb¹

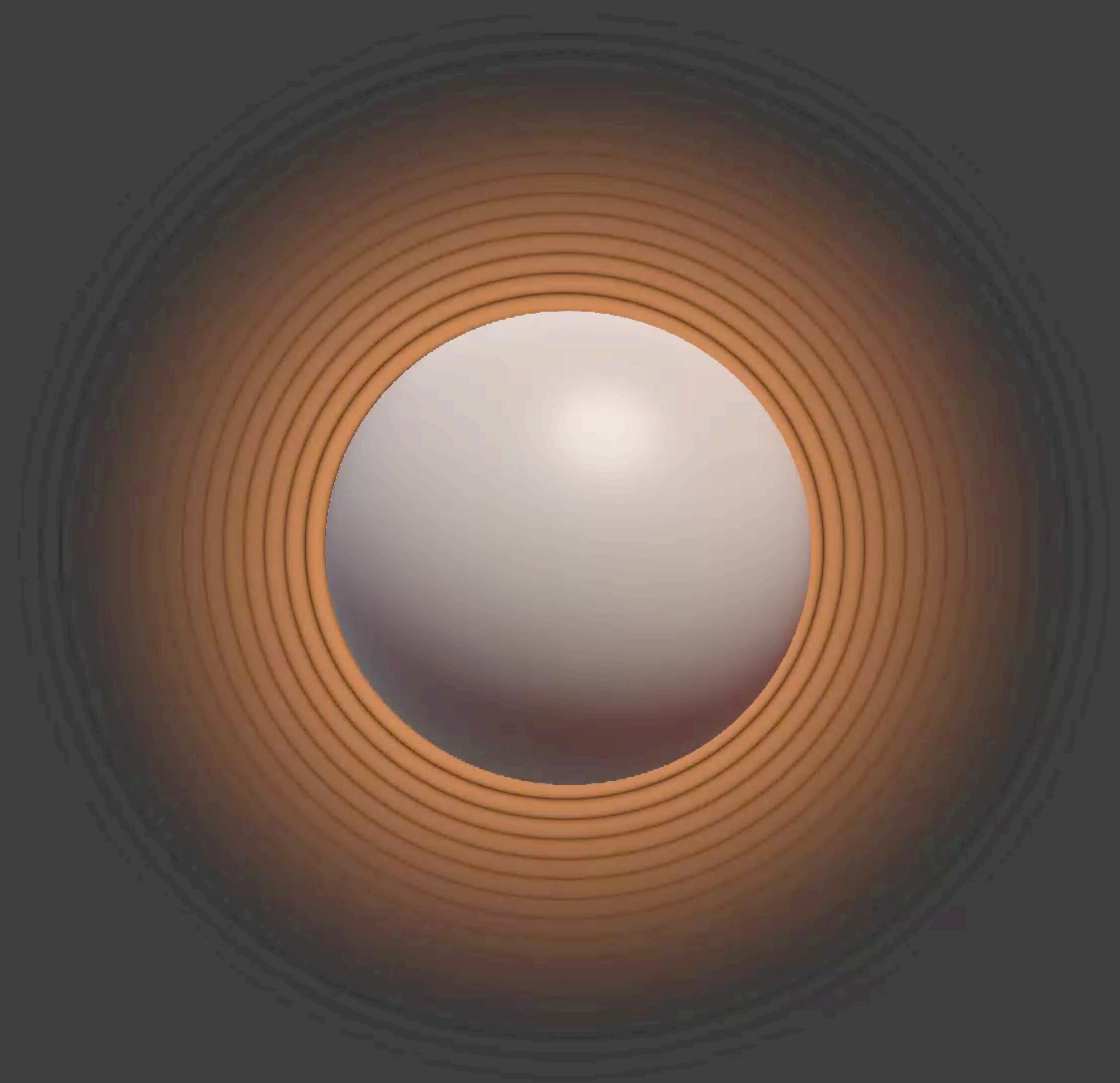
Alec Jacobson²

Derek Nowrouzezahrai³

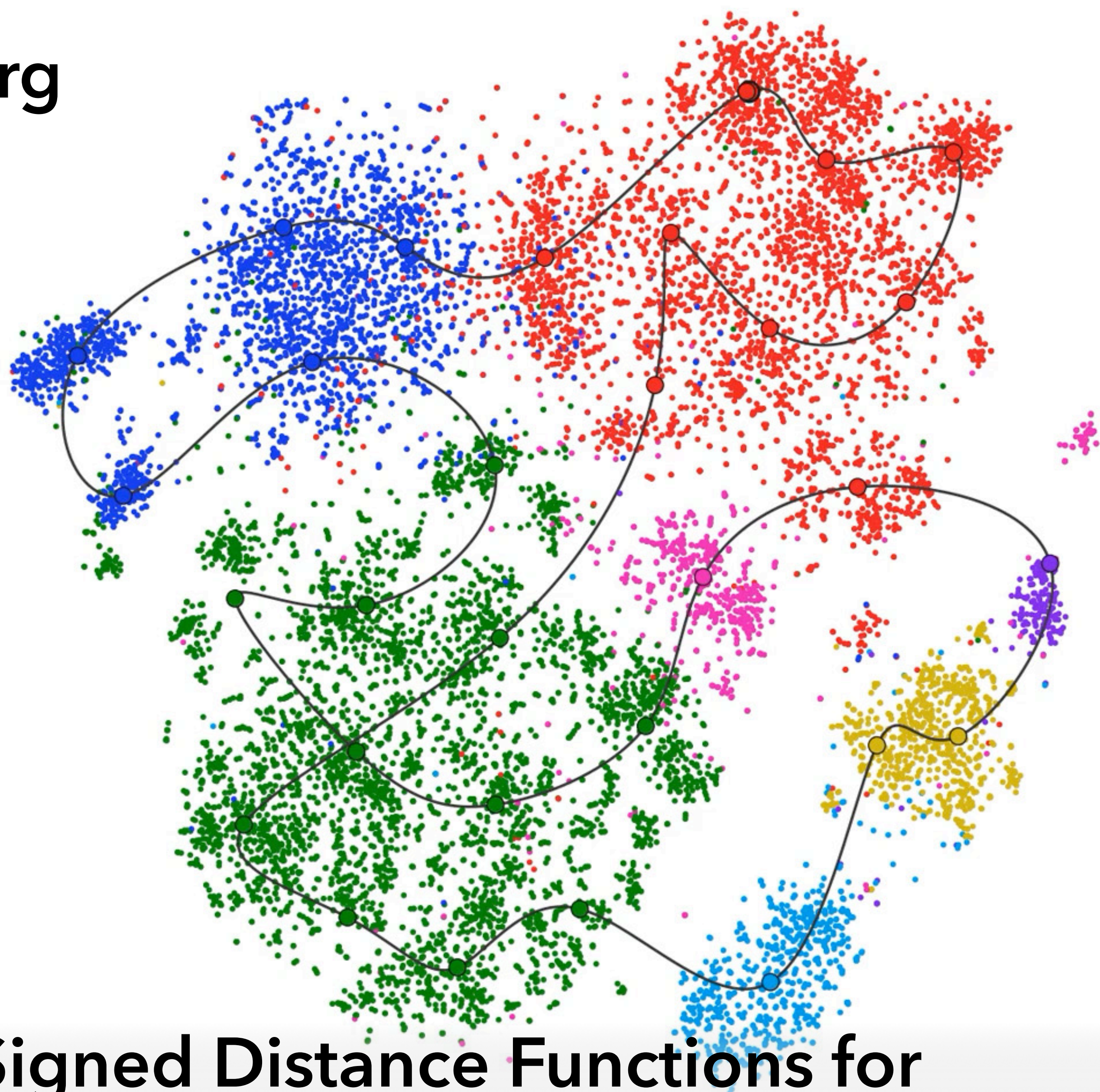
Wojciech Jarosz¹



shadertoy.com/view/lDD3DX



Visualization by Marian Kleineberg

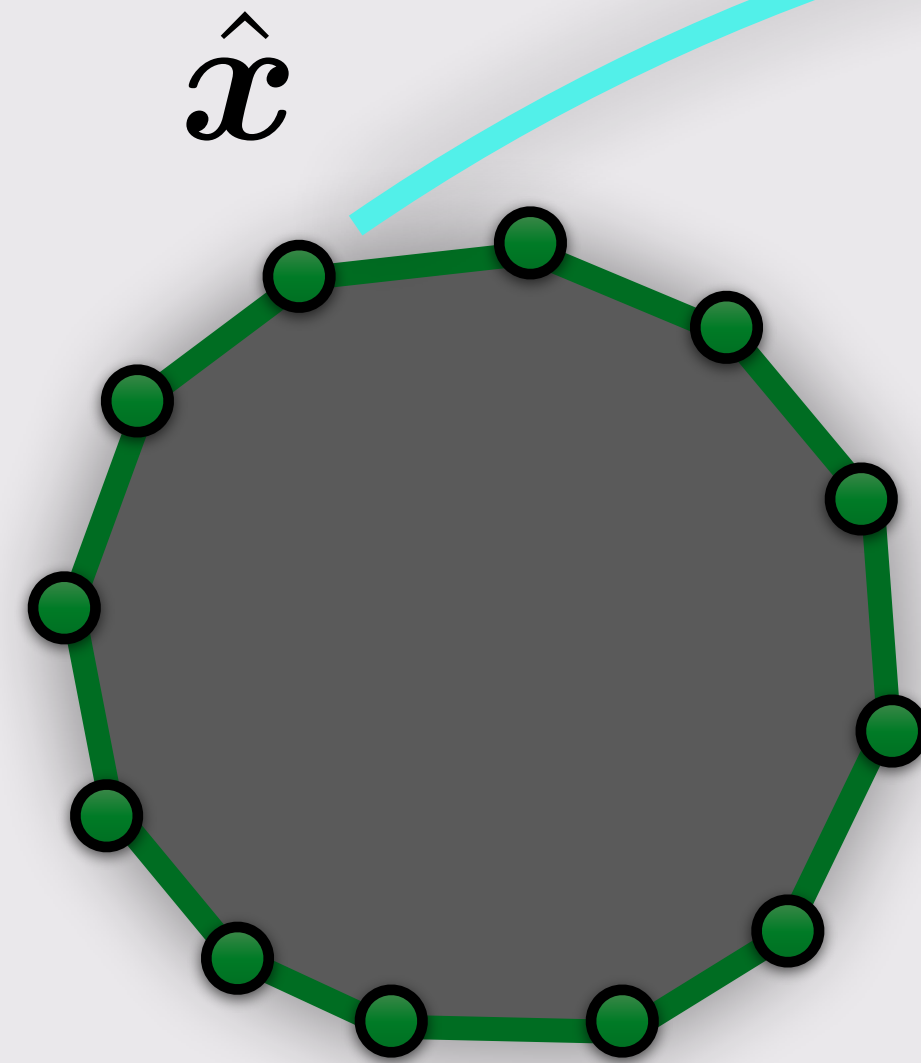


DeepSDF: Learning Continuous Signed Distance Functions for Shape Representation. Genova et. al, 2019

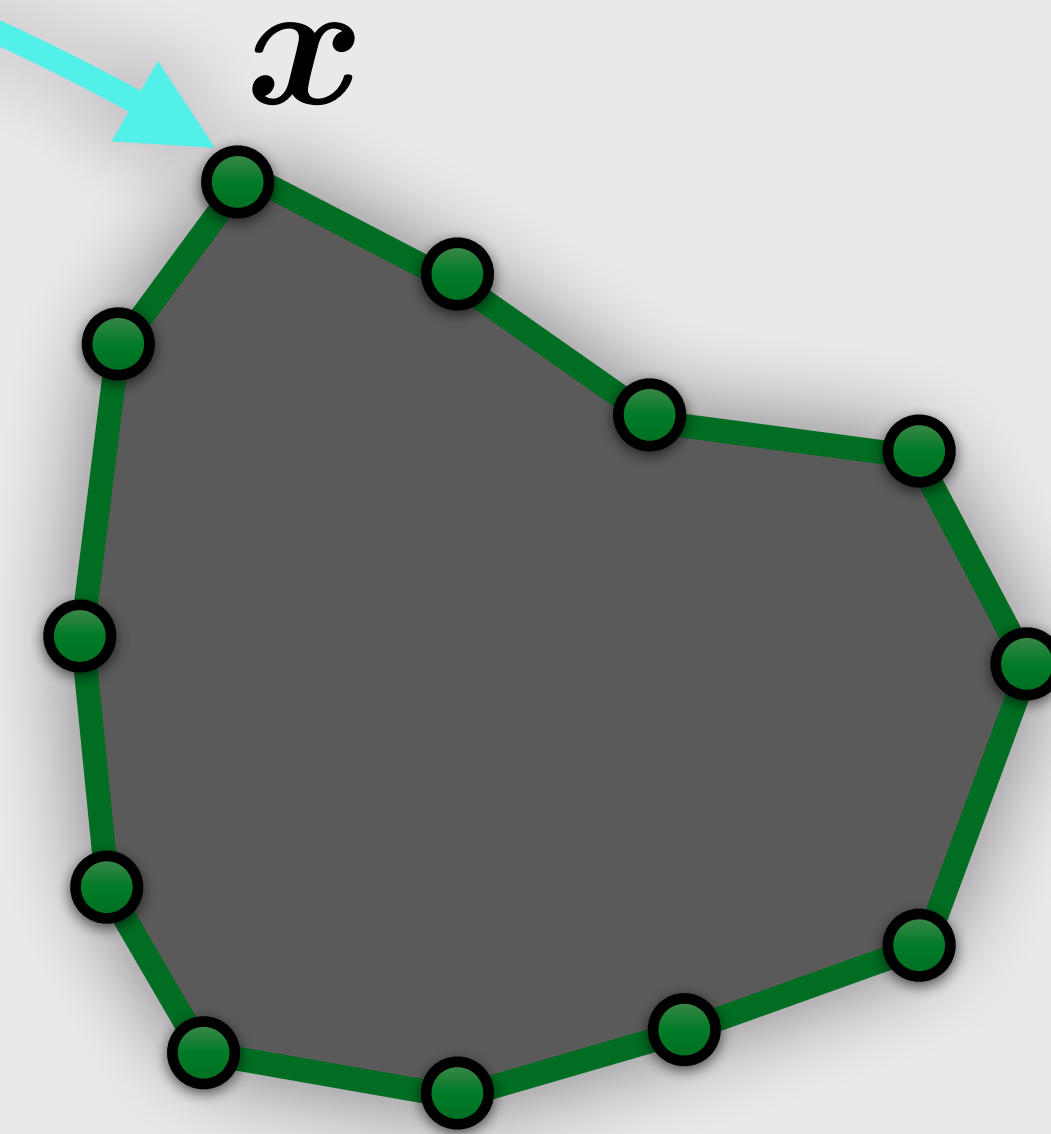


"Forward" Deformations

$$D(\hat{x})$$



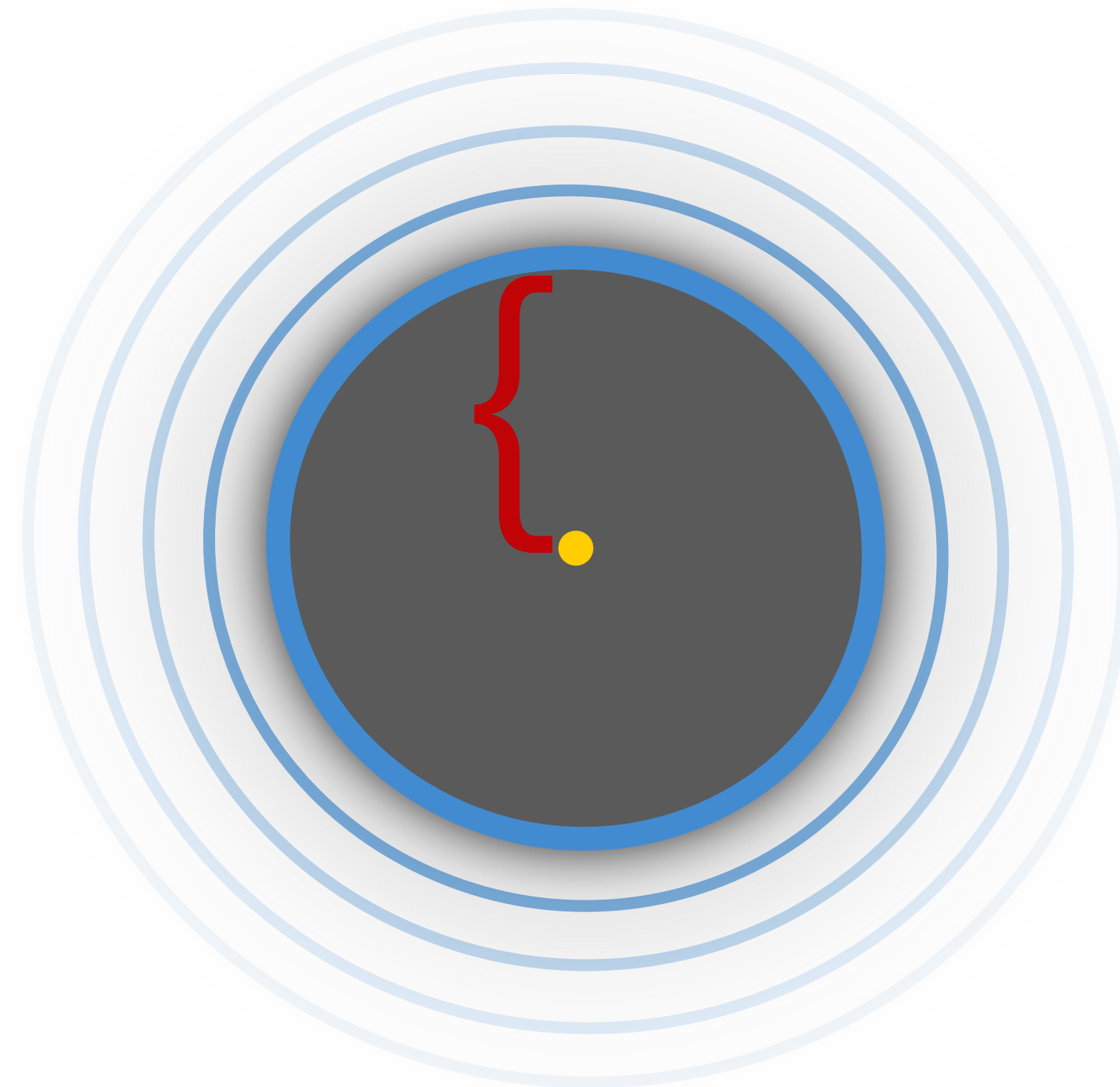
Undeformed Space



Deformed Space

Implicit Surface Representations

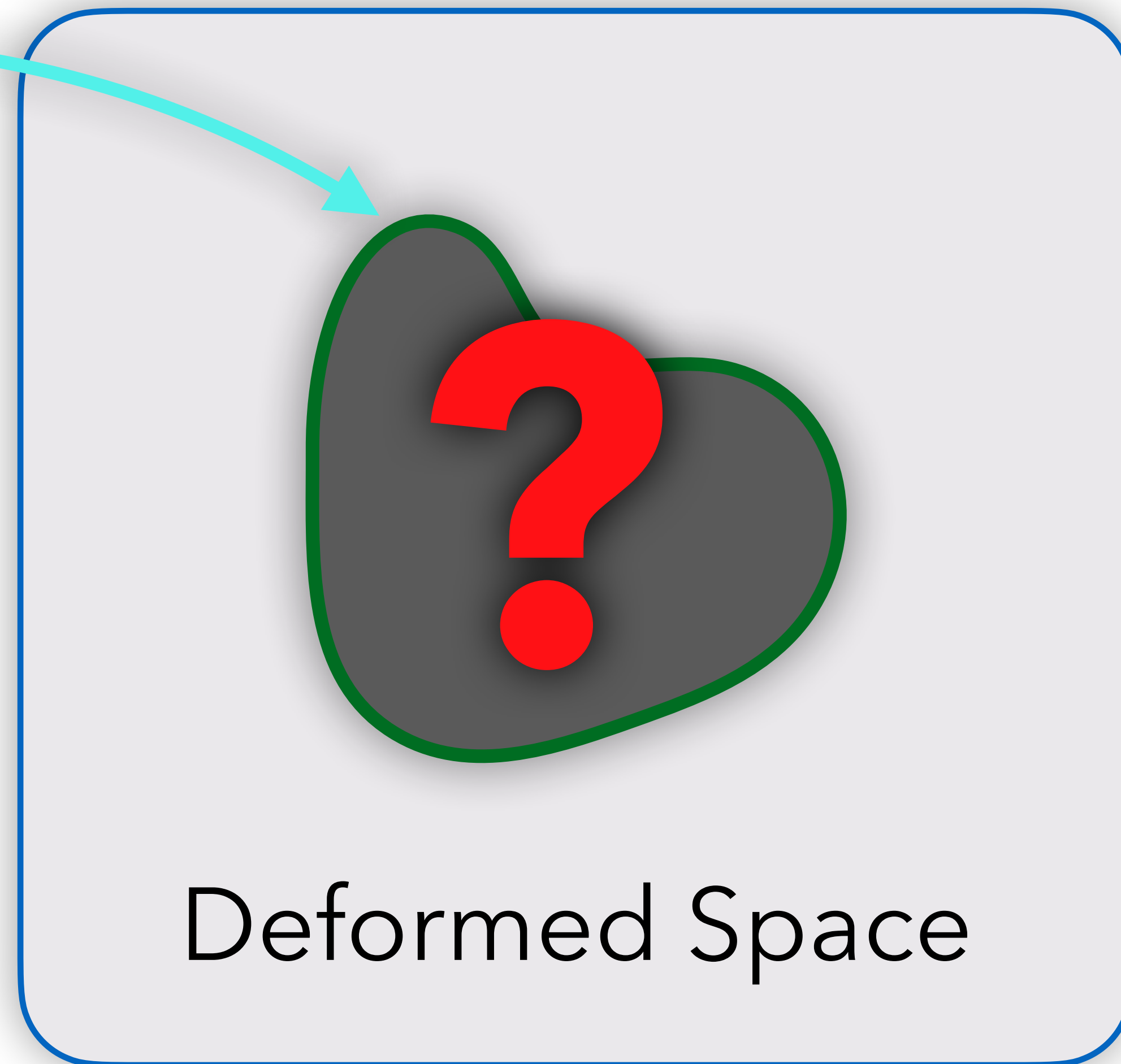
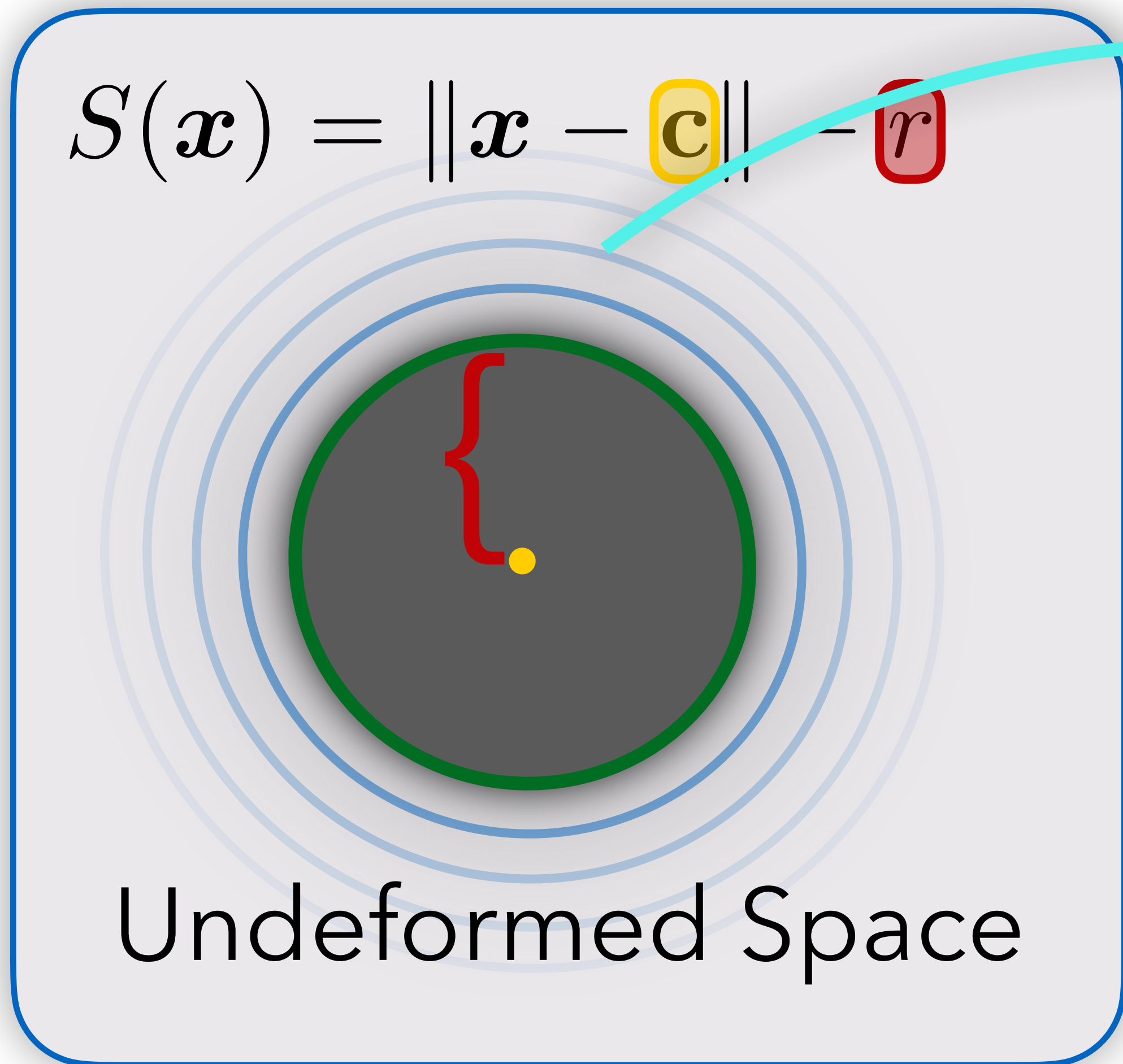
$$S(x) = \mathcal{S}(\|x - \mathbf{c}\| - r) = 0$$



Implicit Surface Deformation

$$D(\hat{x})$$

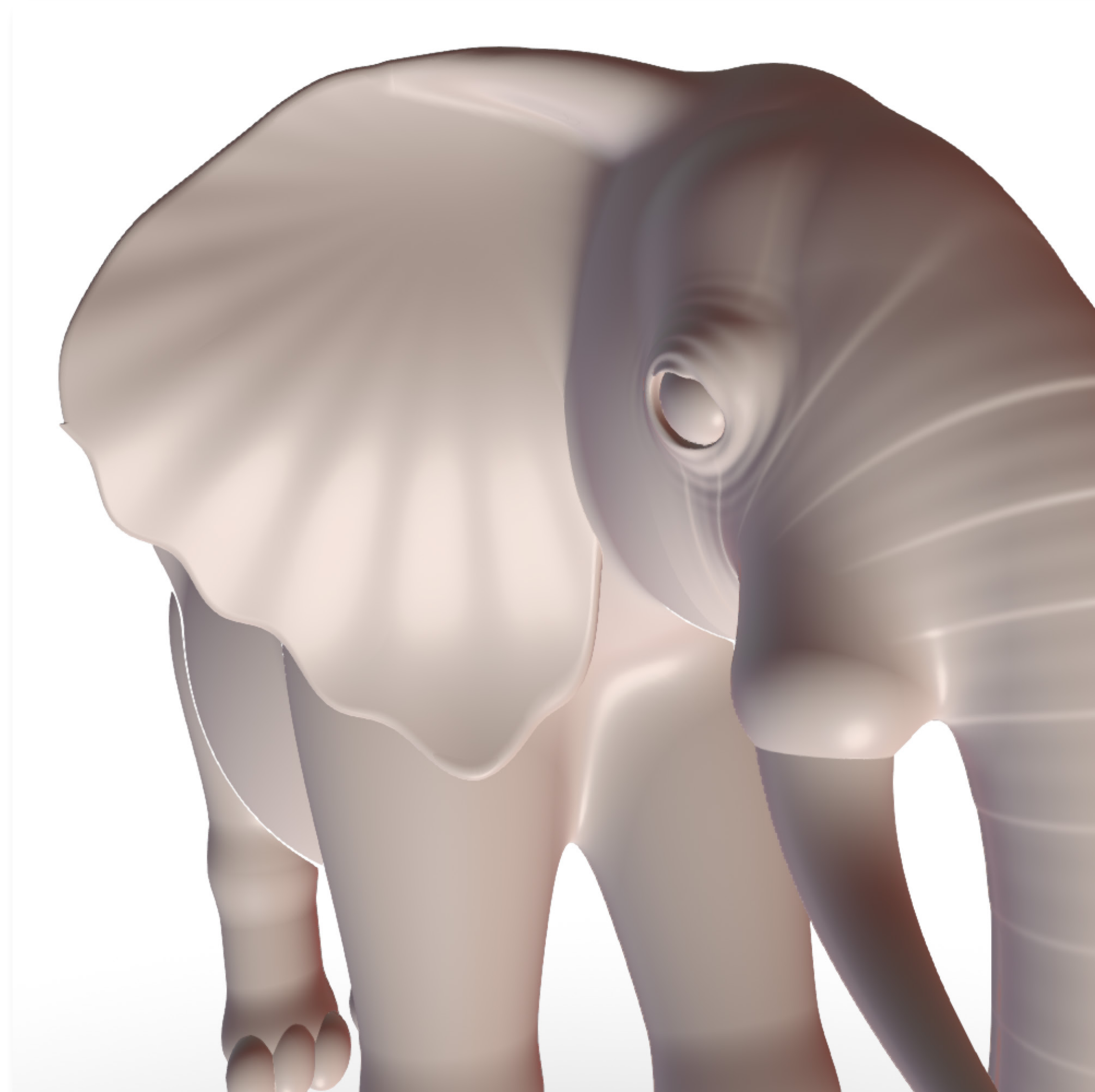
$$S(\mathbf{x}) = \|\mathbf{x} - \mathbf{c}\| - r$$



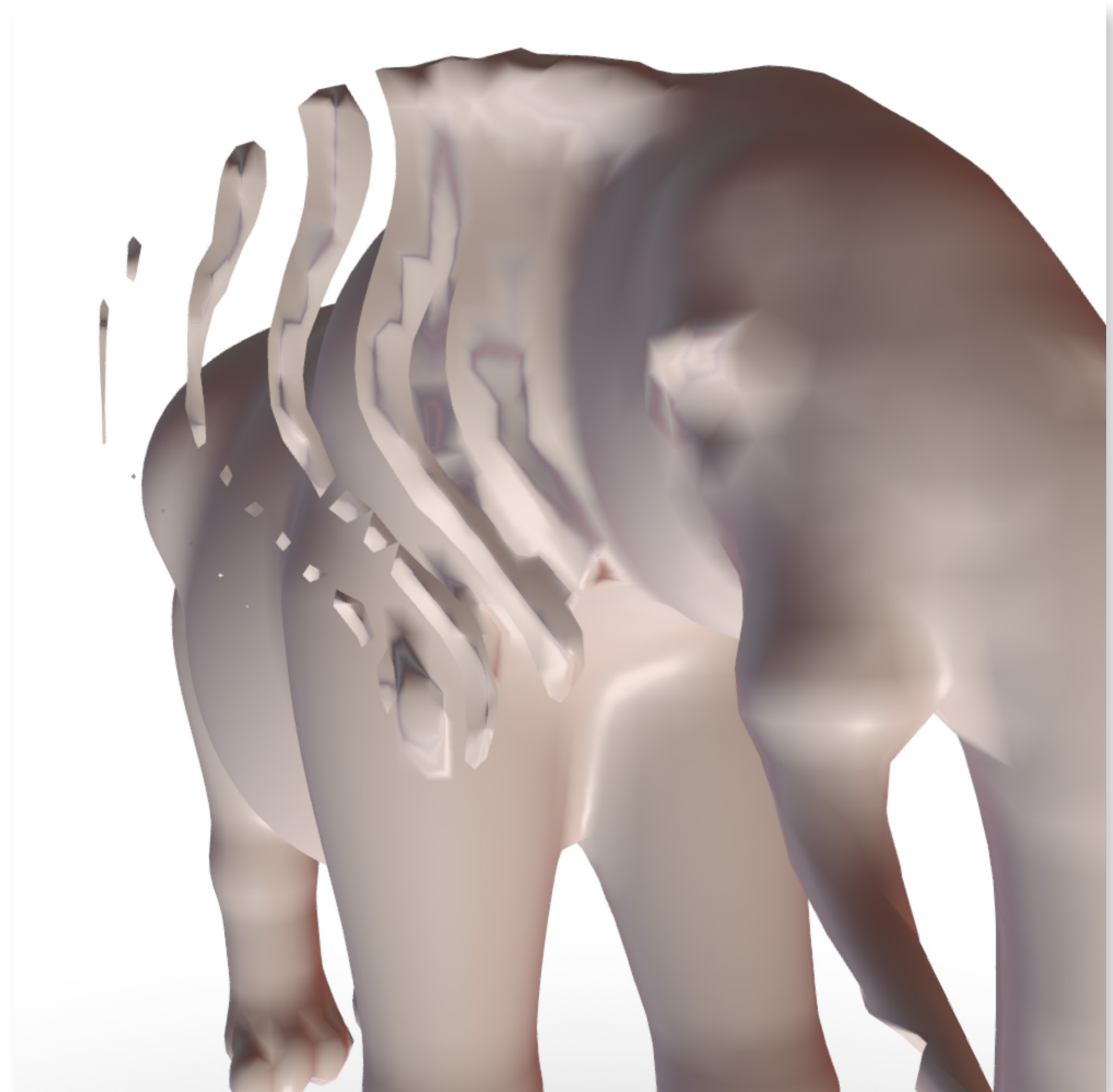
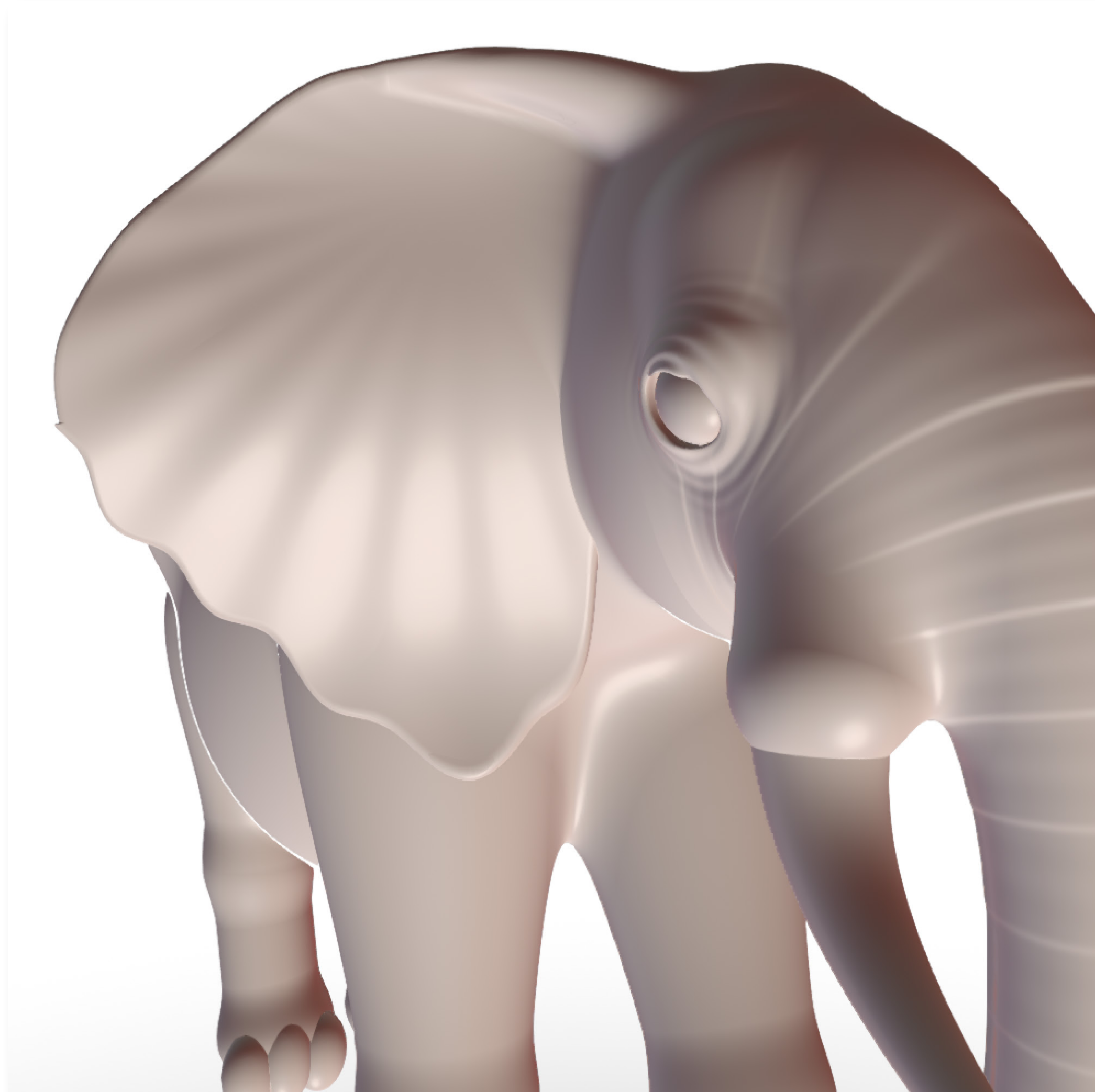
Problem Statement

Use **conventional deformation techniques** to directly render deformed **implicit surfaces**

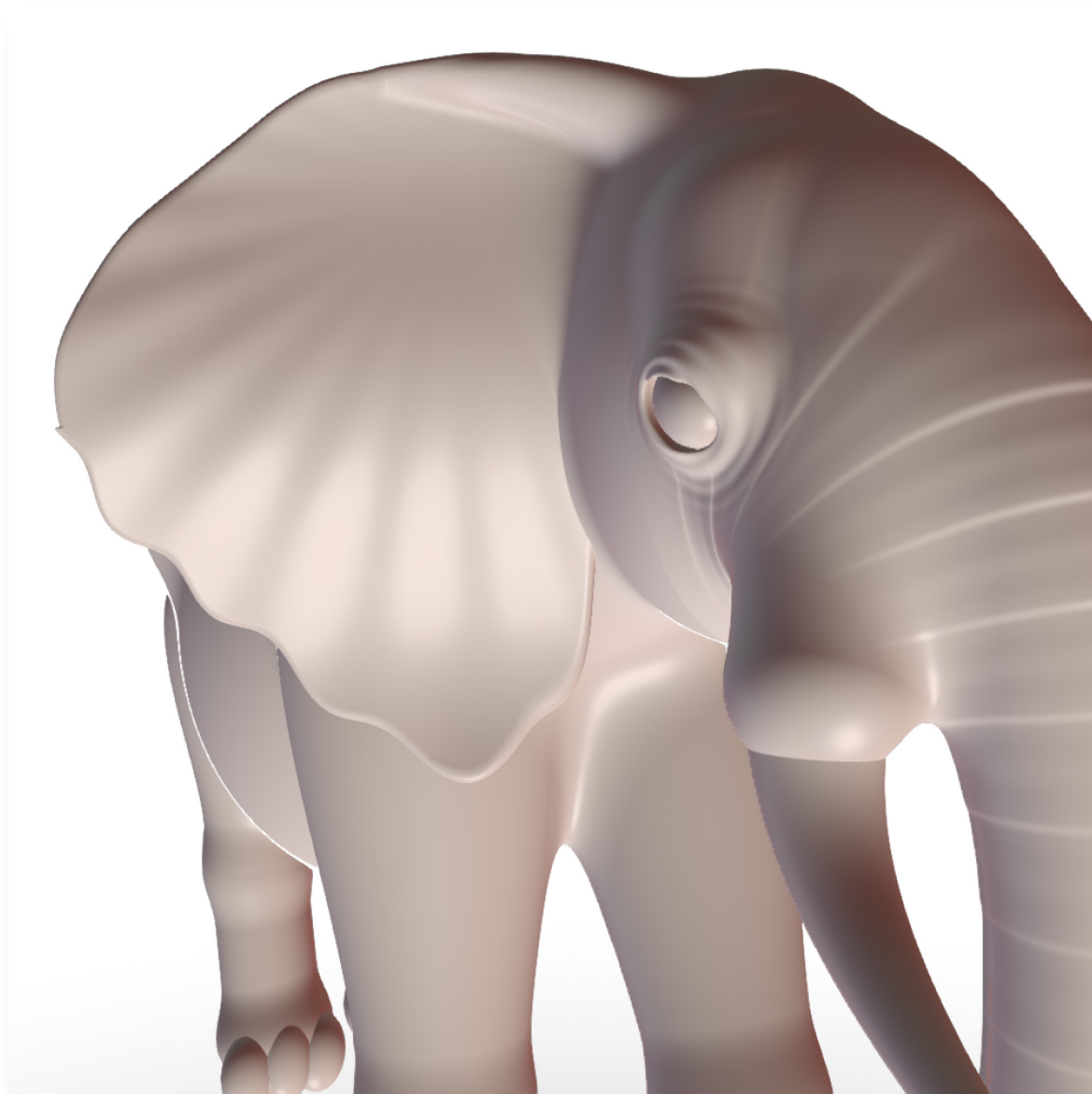
Conversion to Explicit Representation



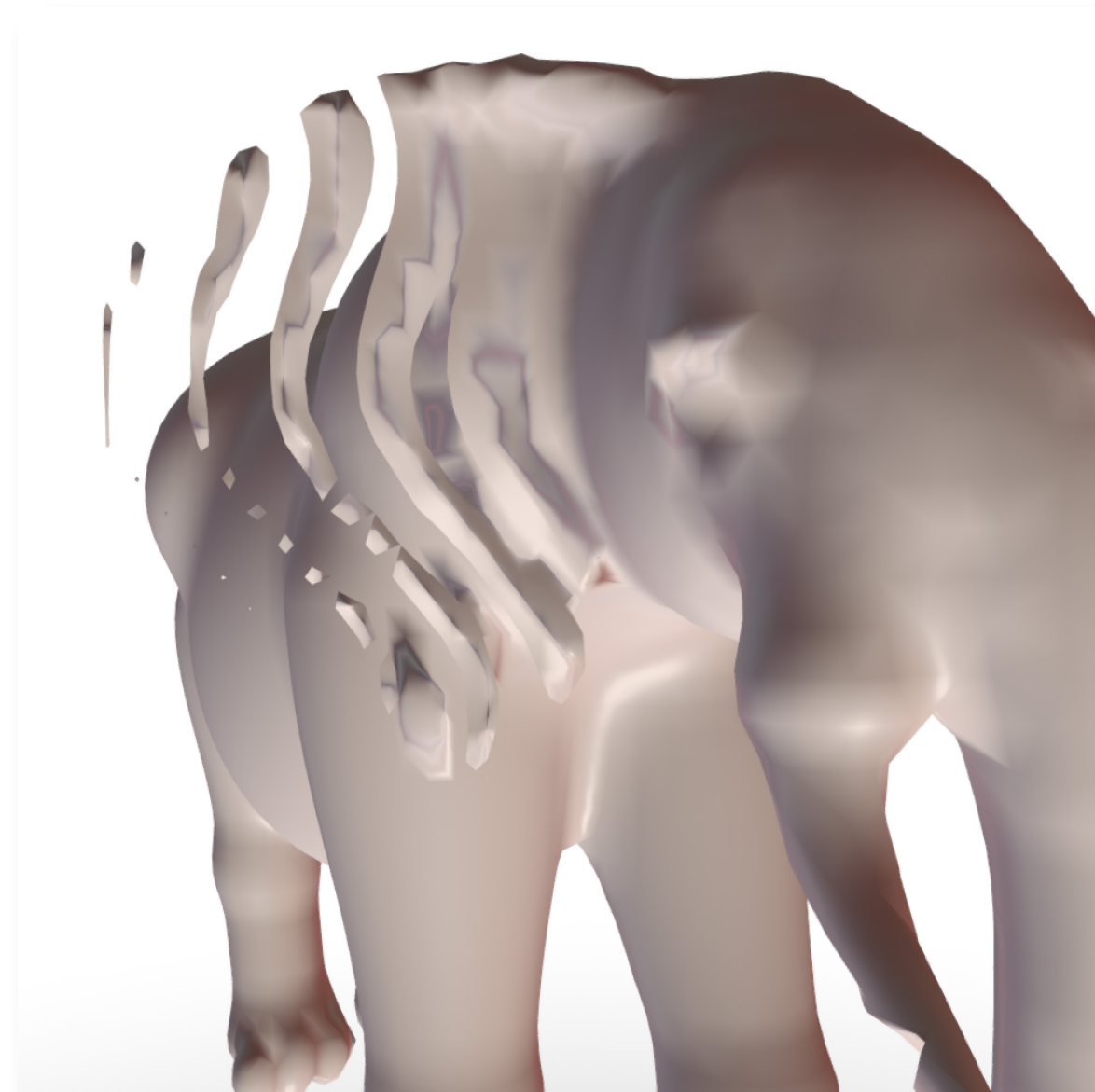
Conversion to Explicit Representation



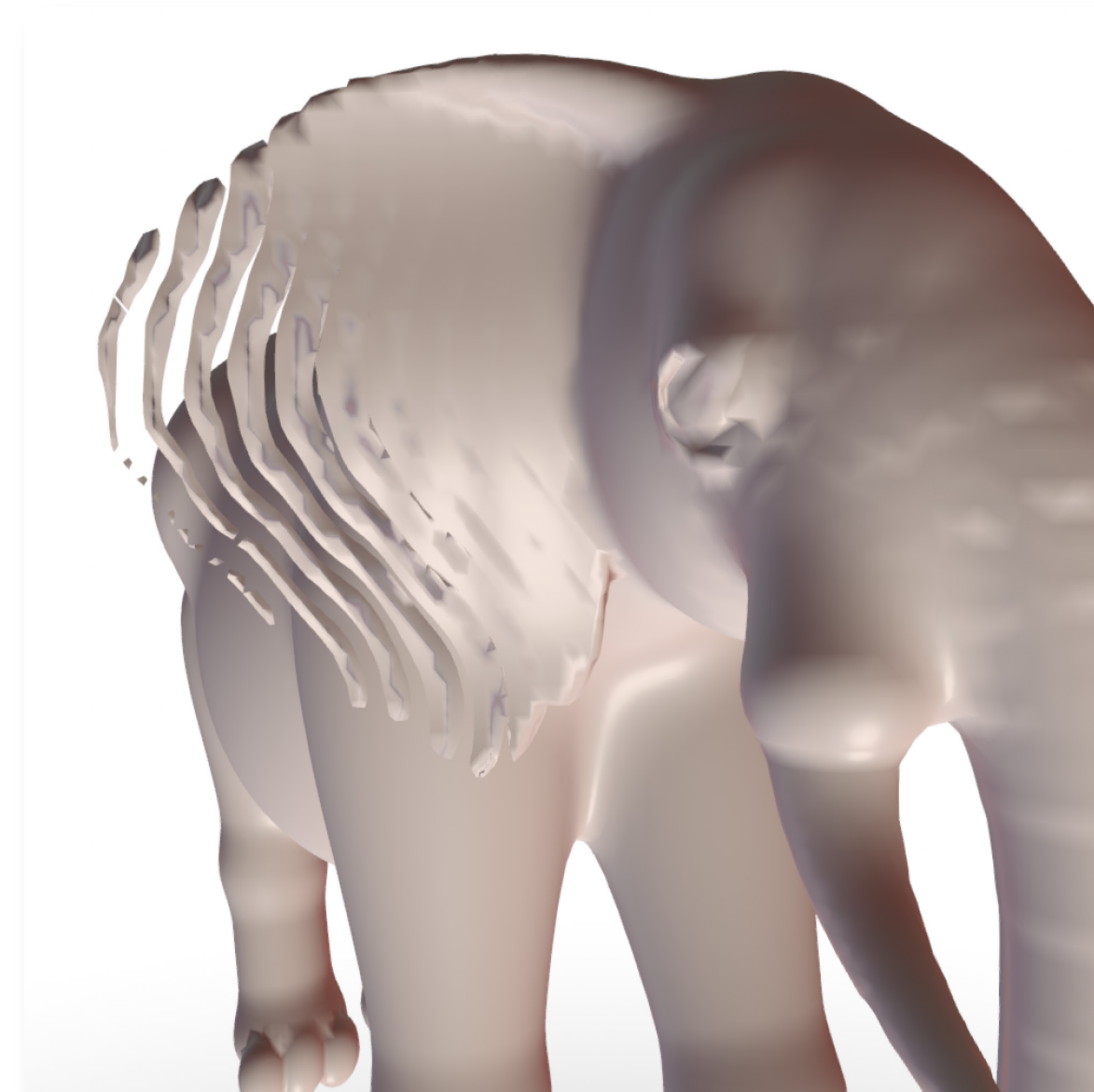
Increasing the resolution helps, but...



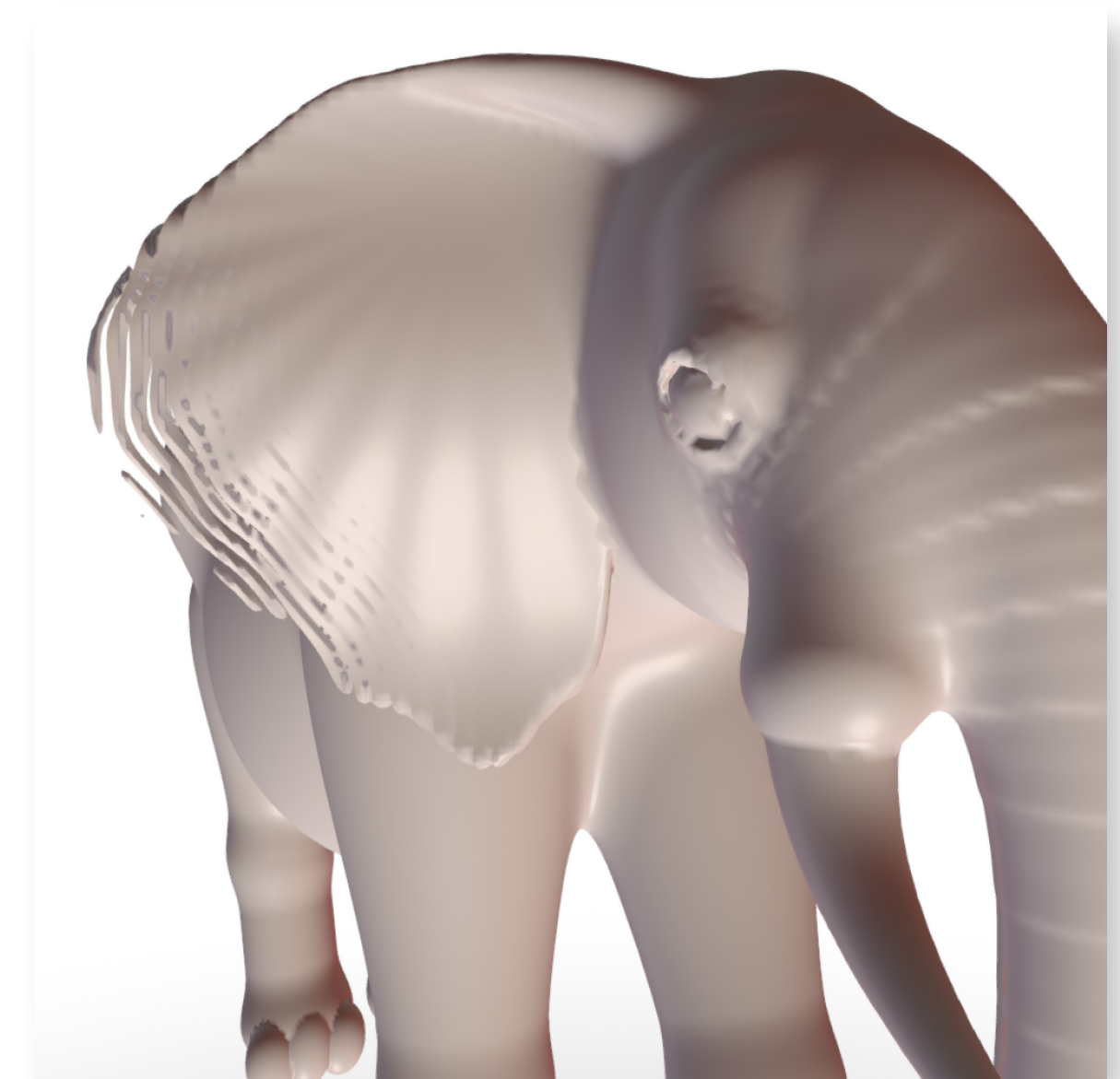
Ground Truth



128x128x128

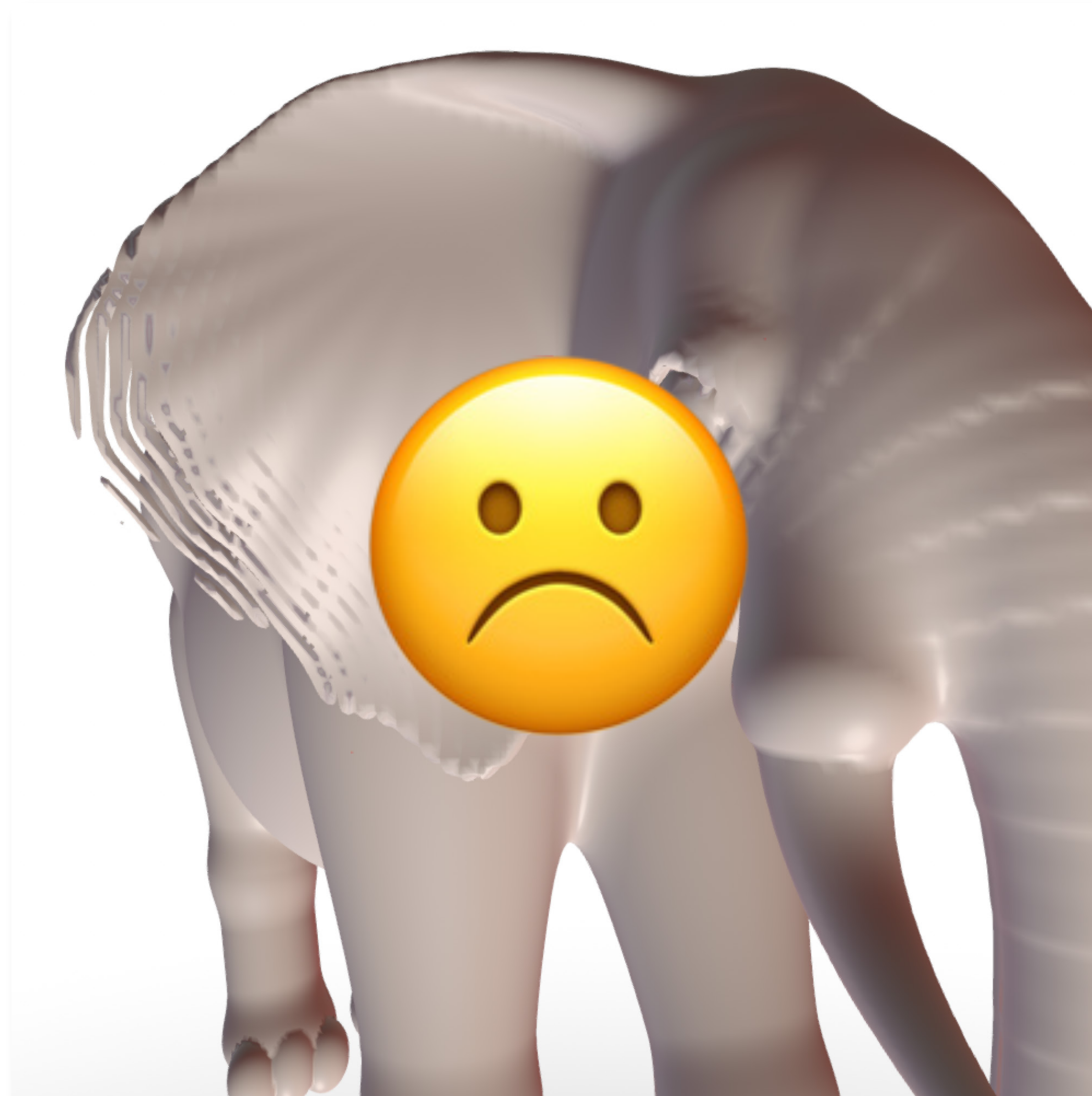


256x256x256



512x512x512

Costly and still full of artifacts!



Conversion to Explicit Representation

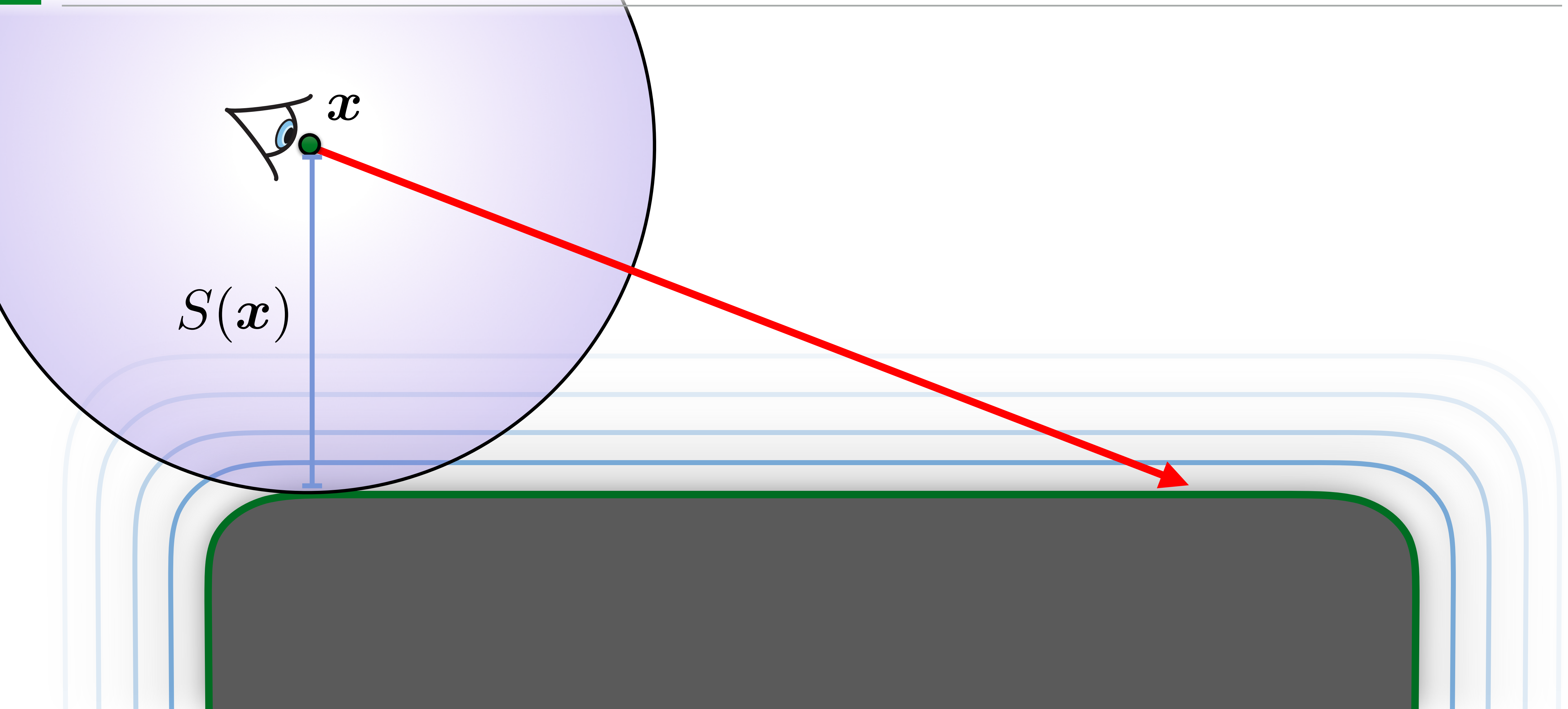
- Data Structure for Soft Objects [Wyvill et al. 1986]
- ➔ **Marching Cubes [Lorensen et al. 1987]**
- Dual Contouring [Ju et al. 2002]
- ...

Problem Statement – Solved?

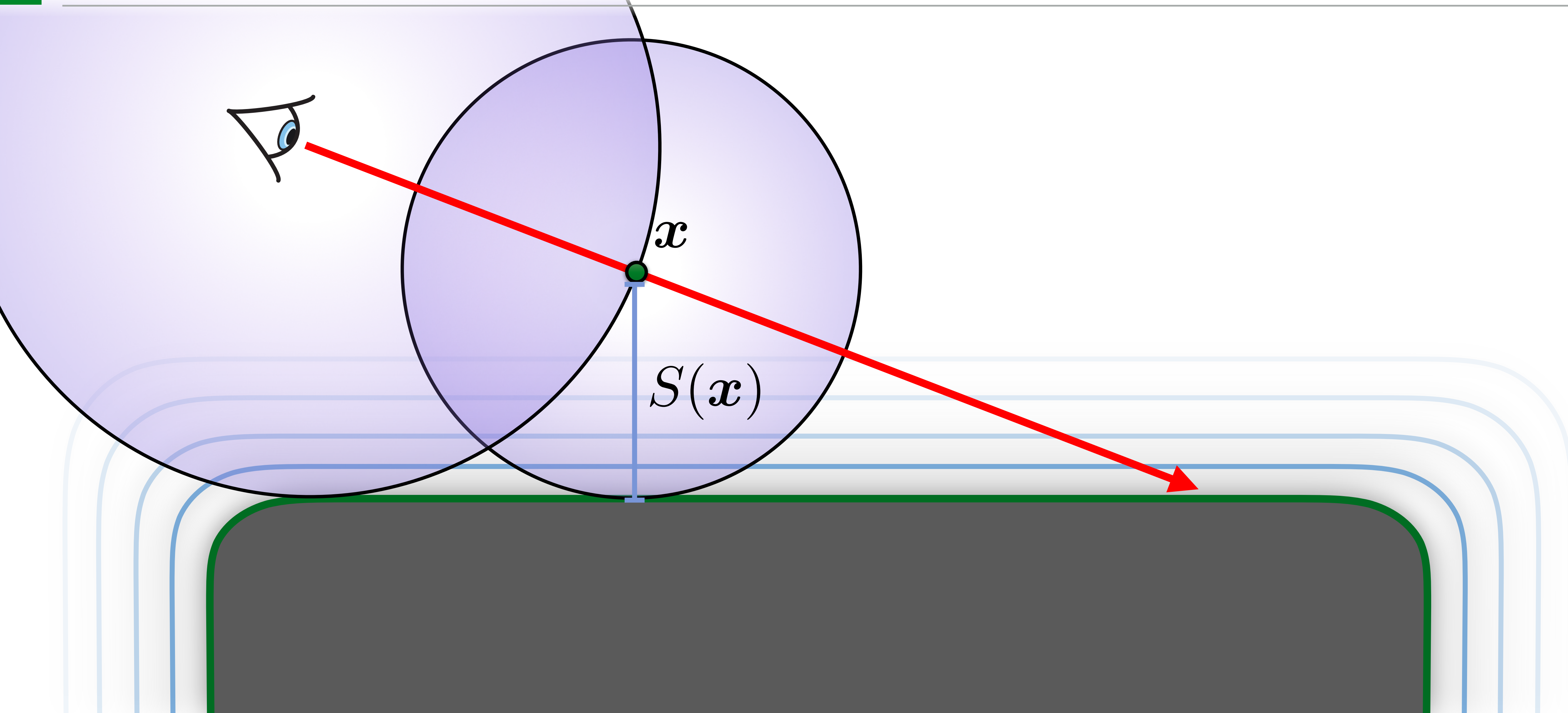
Use **conventional**  **deformation techniques** to directly render deformed **implicit**  **surfaces**

Implicit Surface Rendering

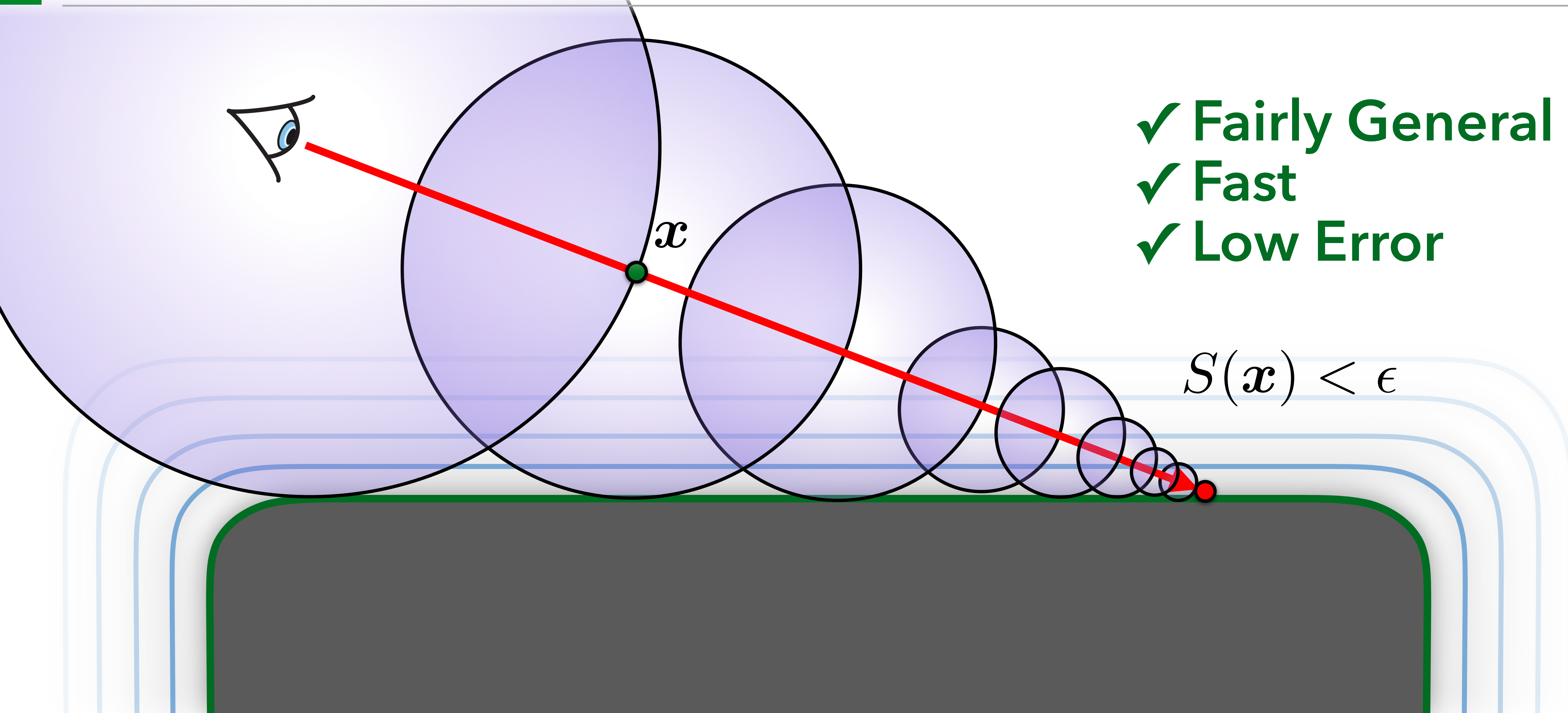
Sphere Tracing [Hart1996]



Sphere Tracing [Hart1996]



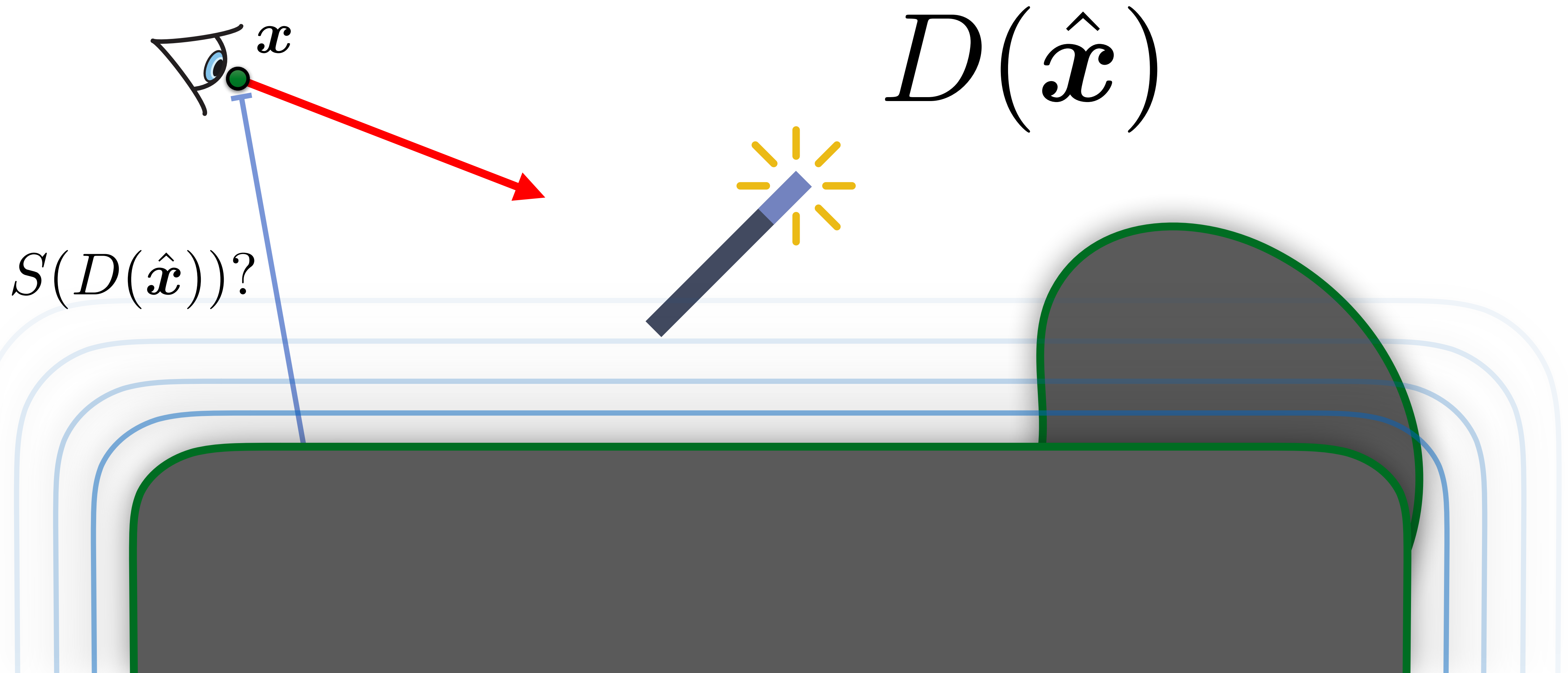
Sphere Tracing [Hart1996]



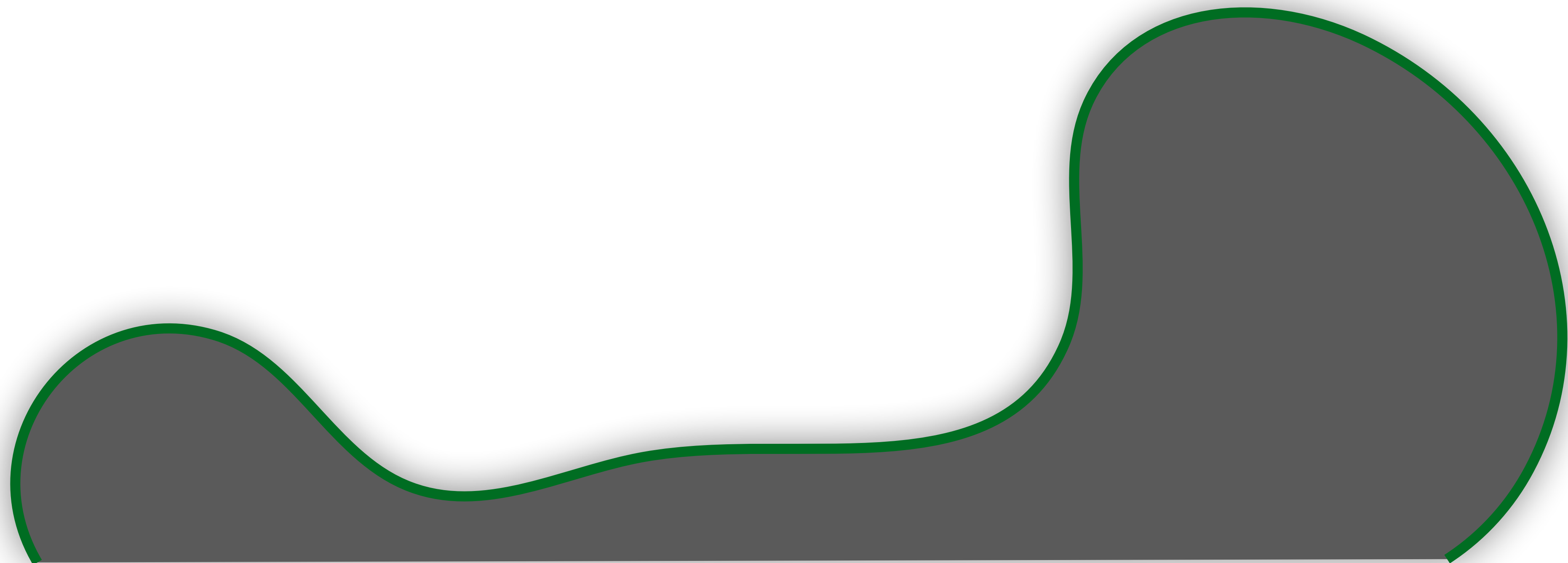
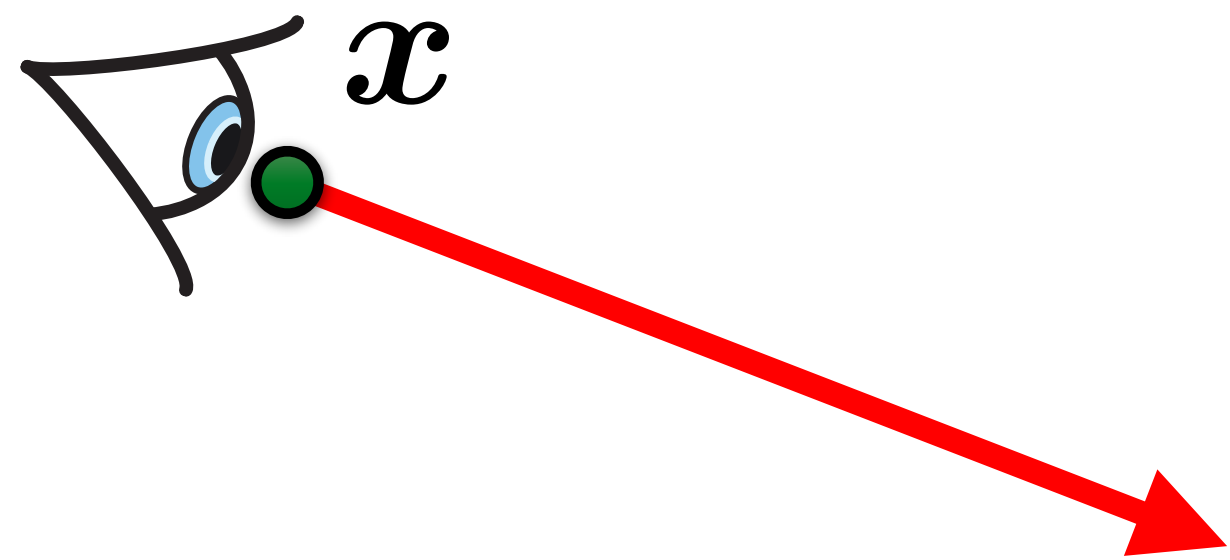
- ✓ Fairly General
- ✓ Fast
- ✓ Low Error

$$S(x) < \epsilon$$

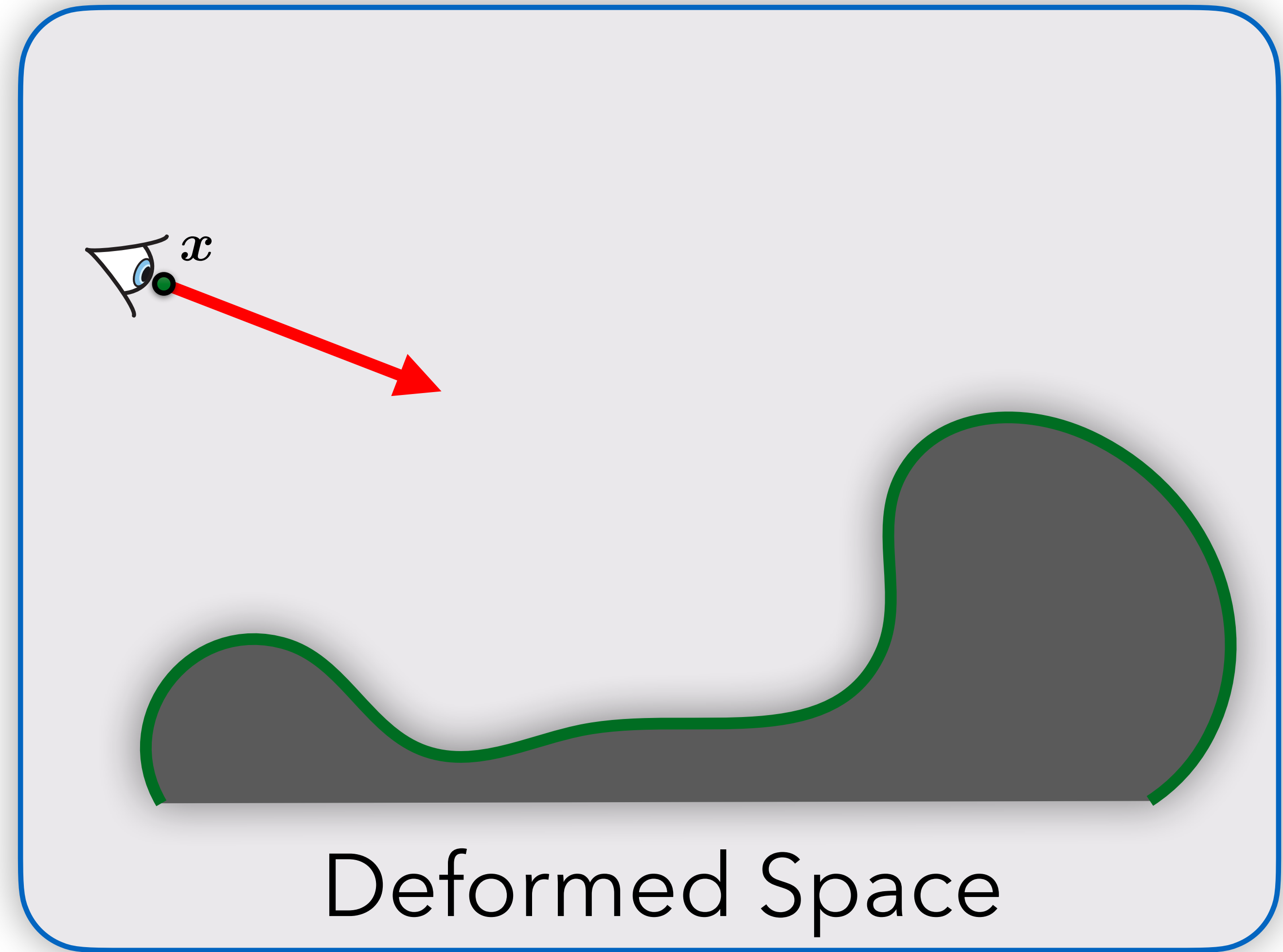
Sphere Tracing - Deformation



Sphere Tracing - Deformation

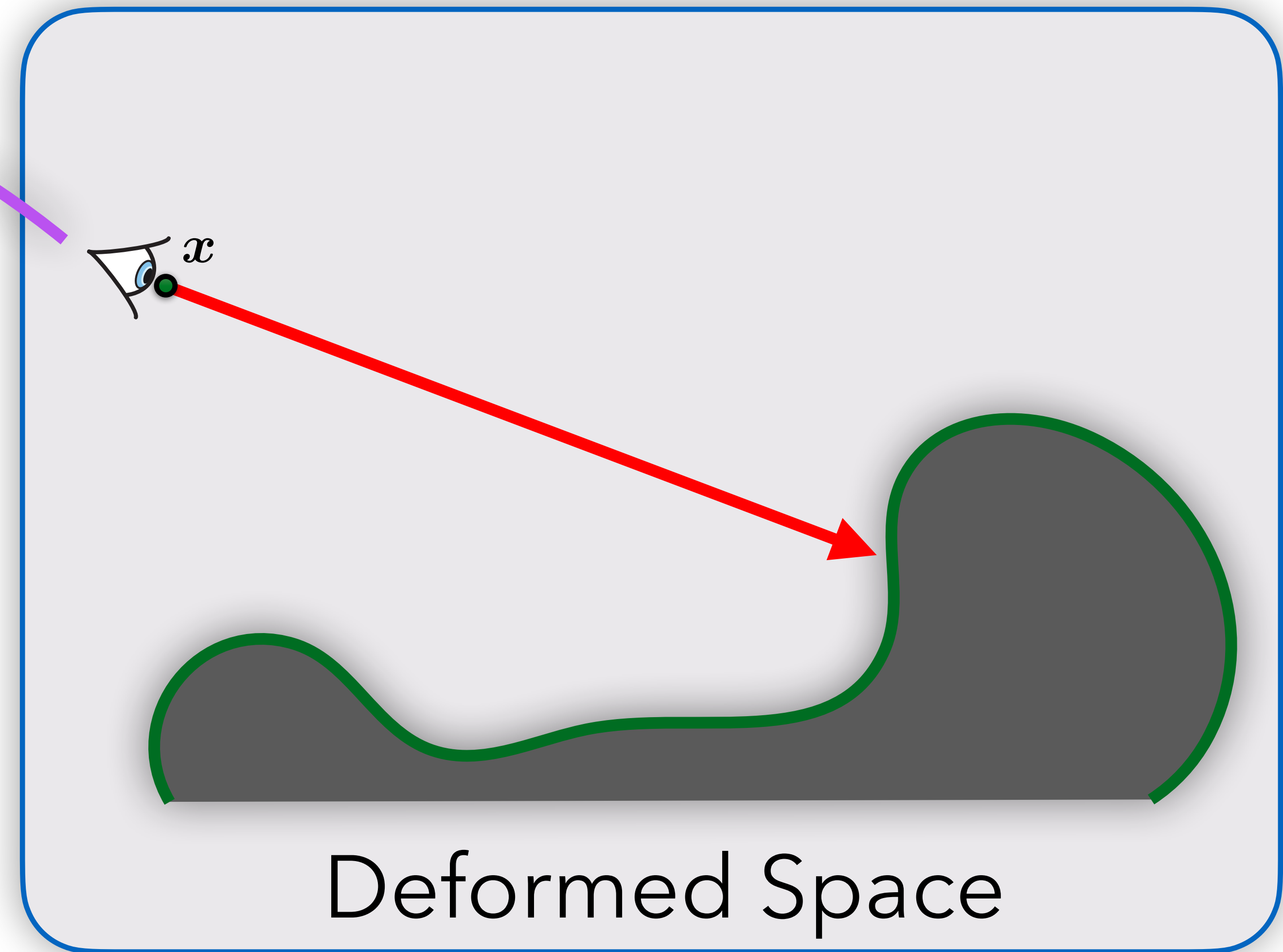
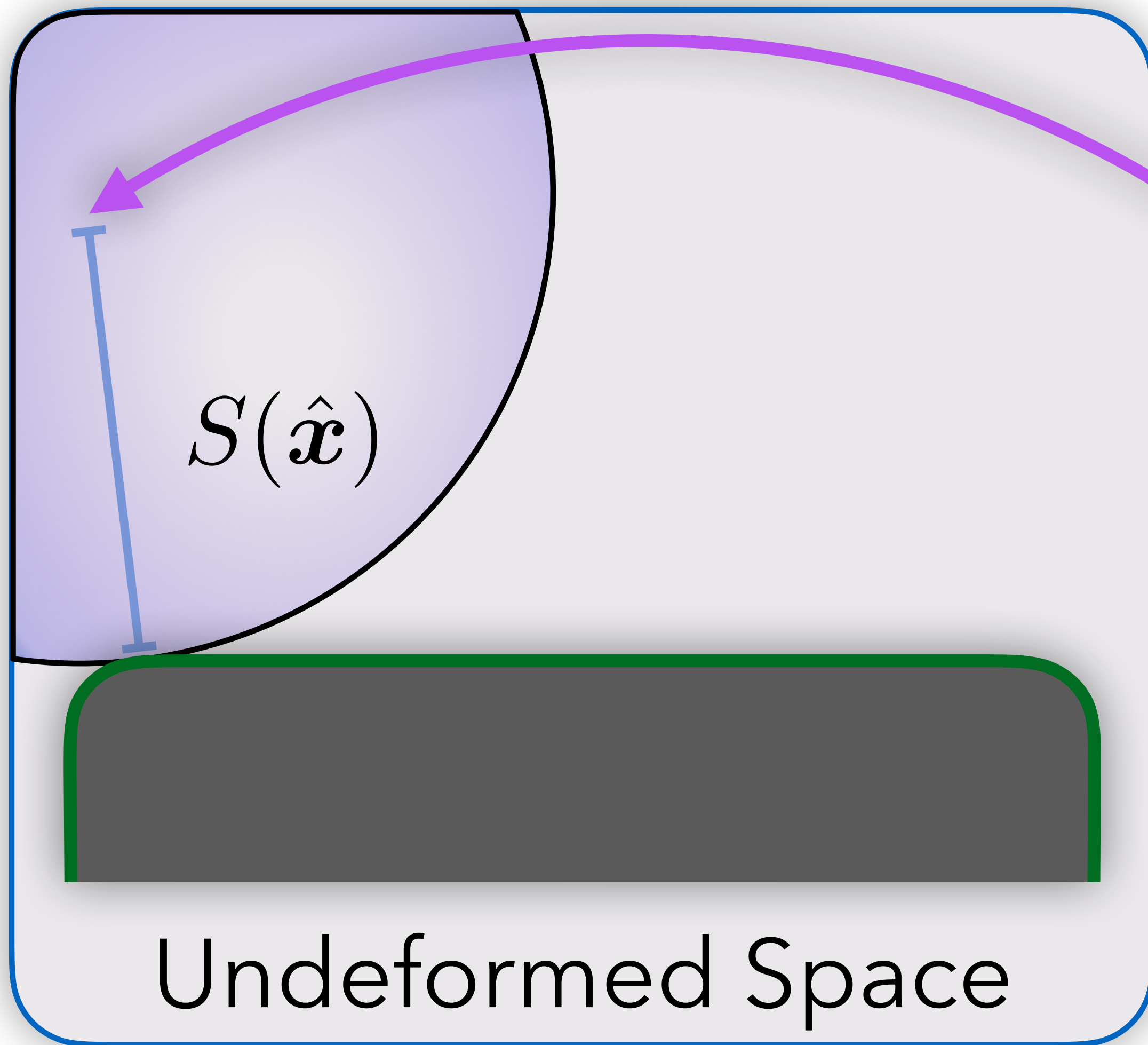


Sampling in Undeformed Space



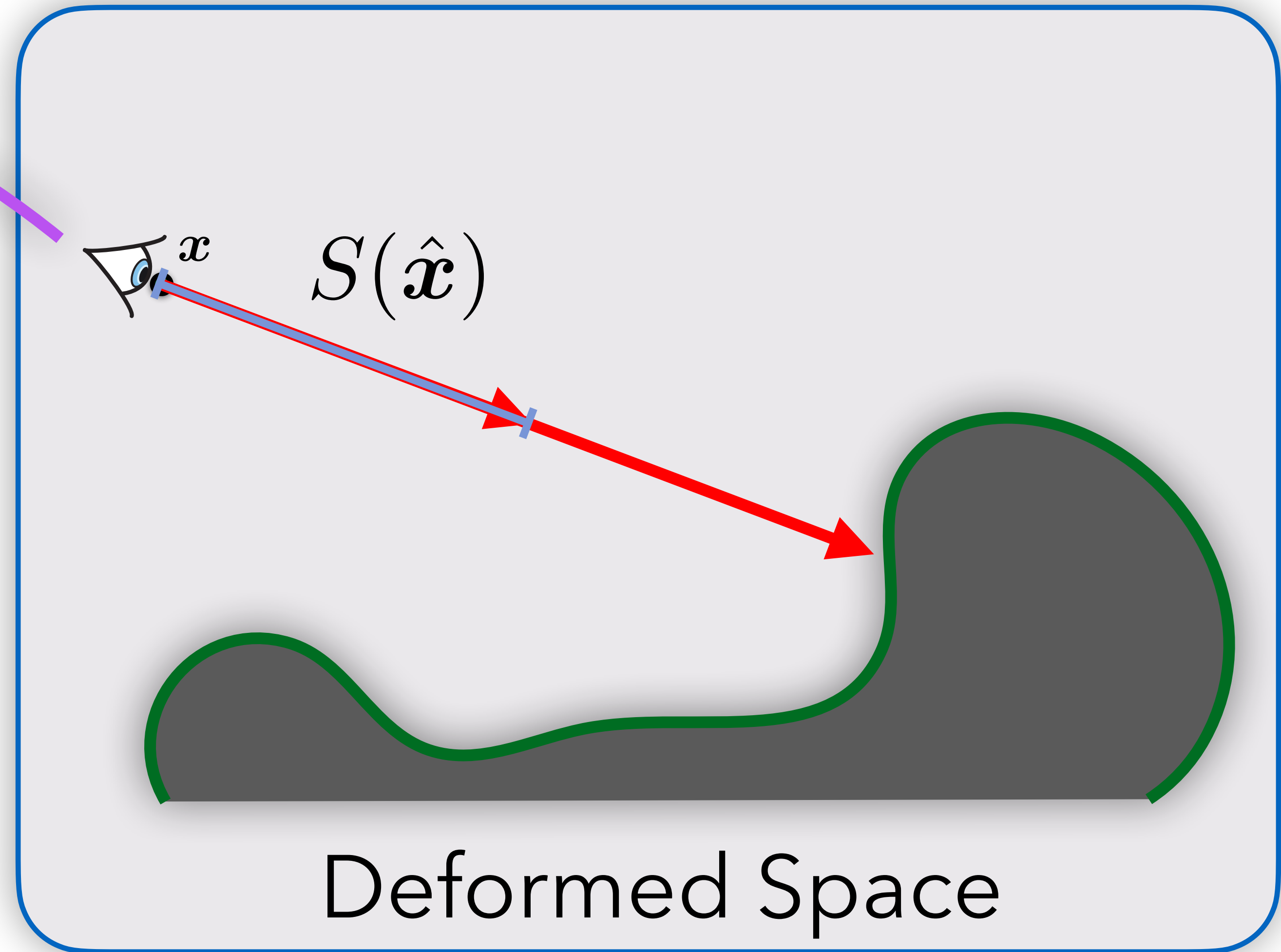
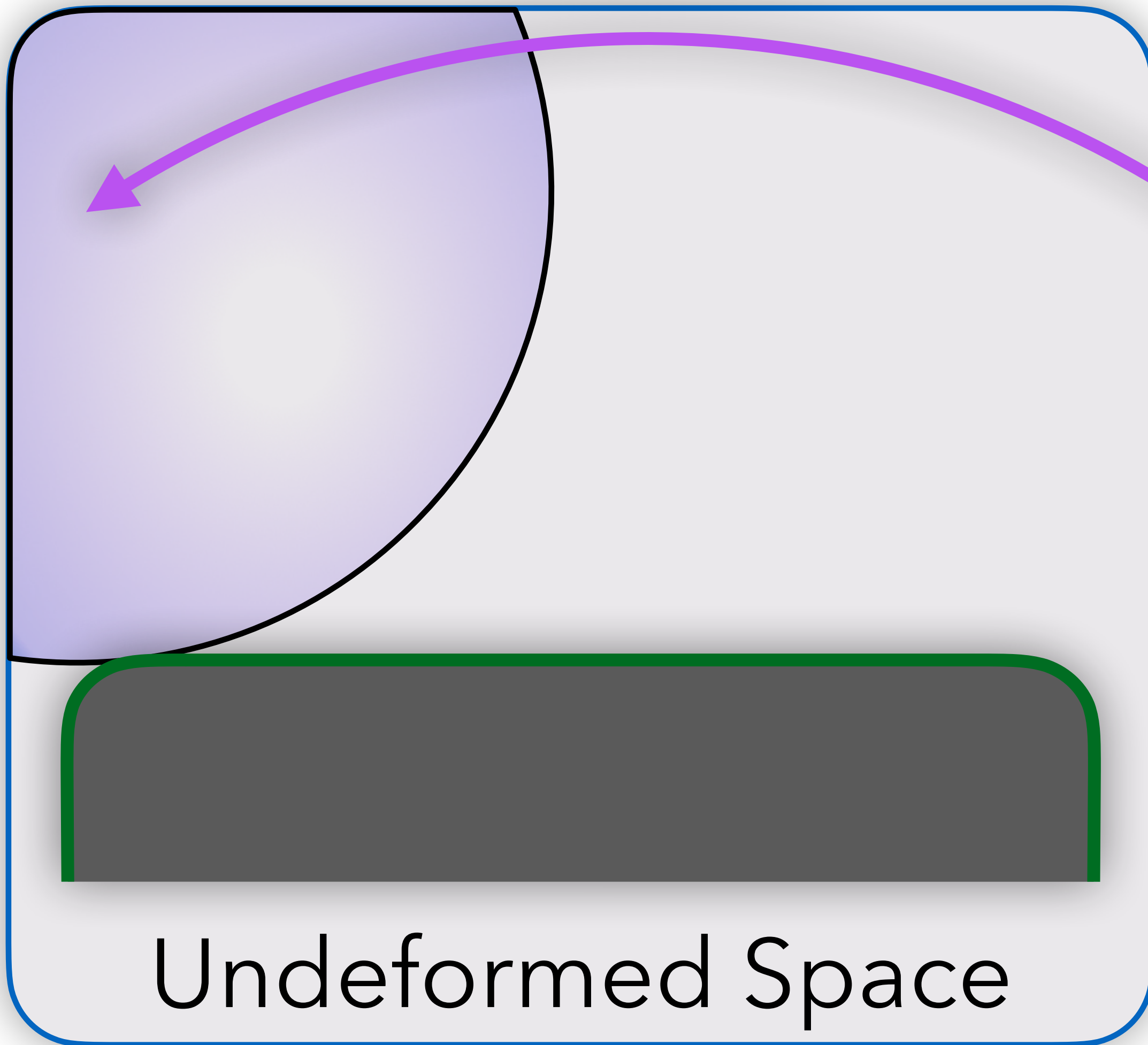
Sampling in Undeformed Space

$$D^{-1}(x)$$



Sampling in Undeformed Space

$$D^{-1}(x)$$

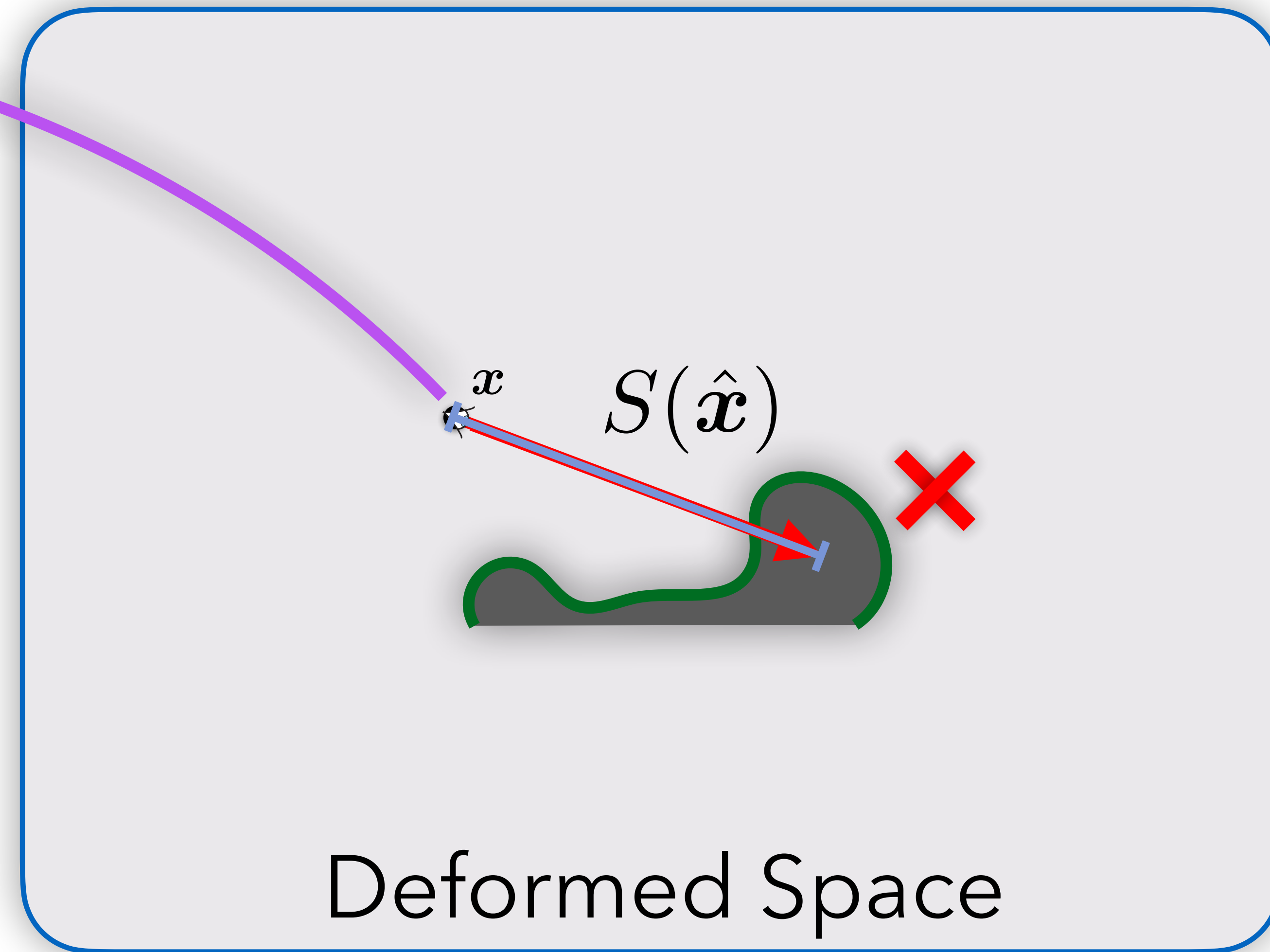
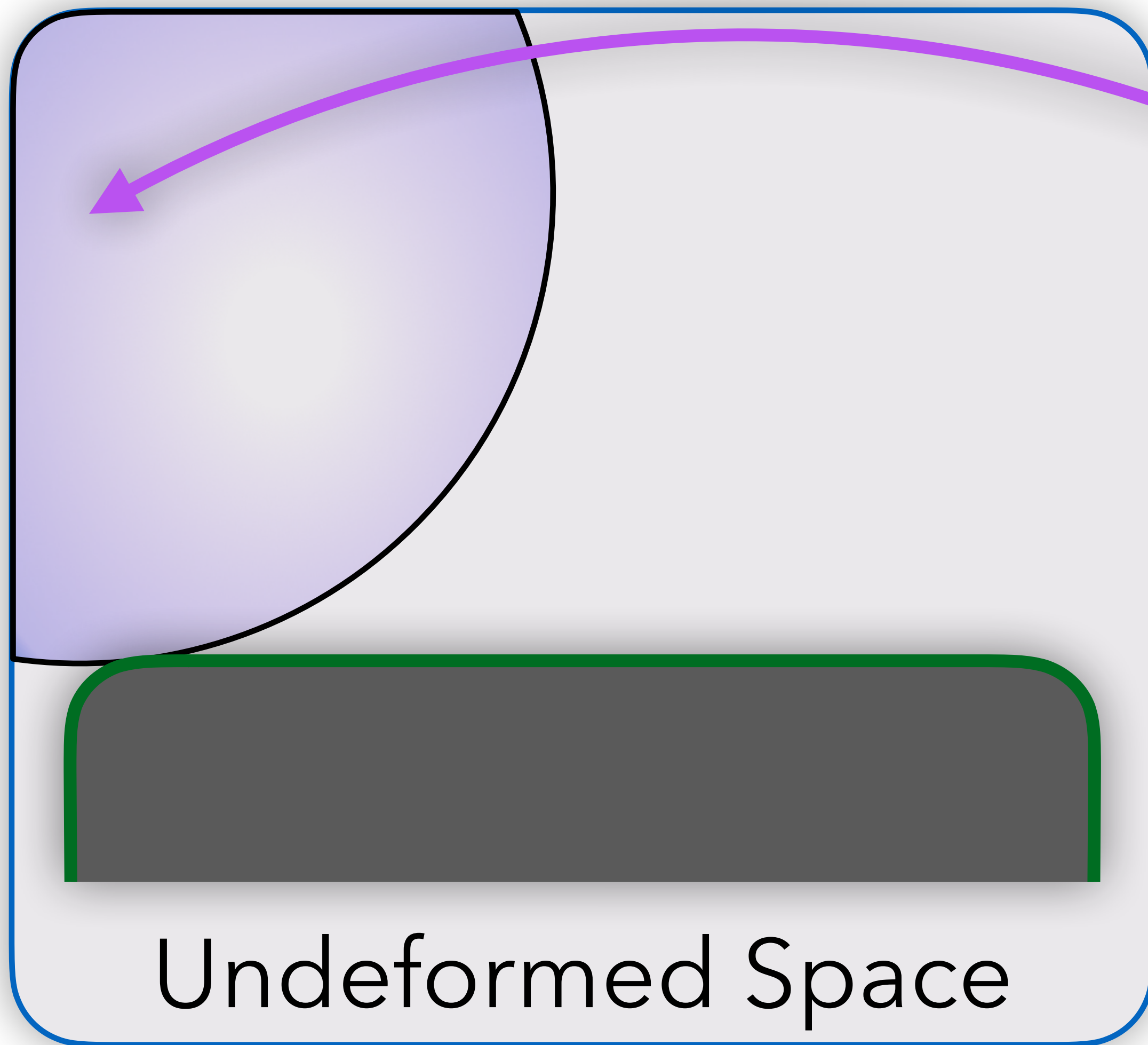


$$S(\hat{x})$$

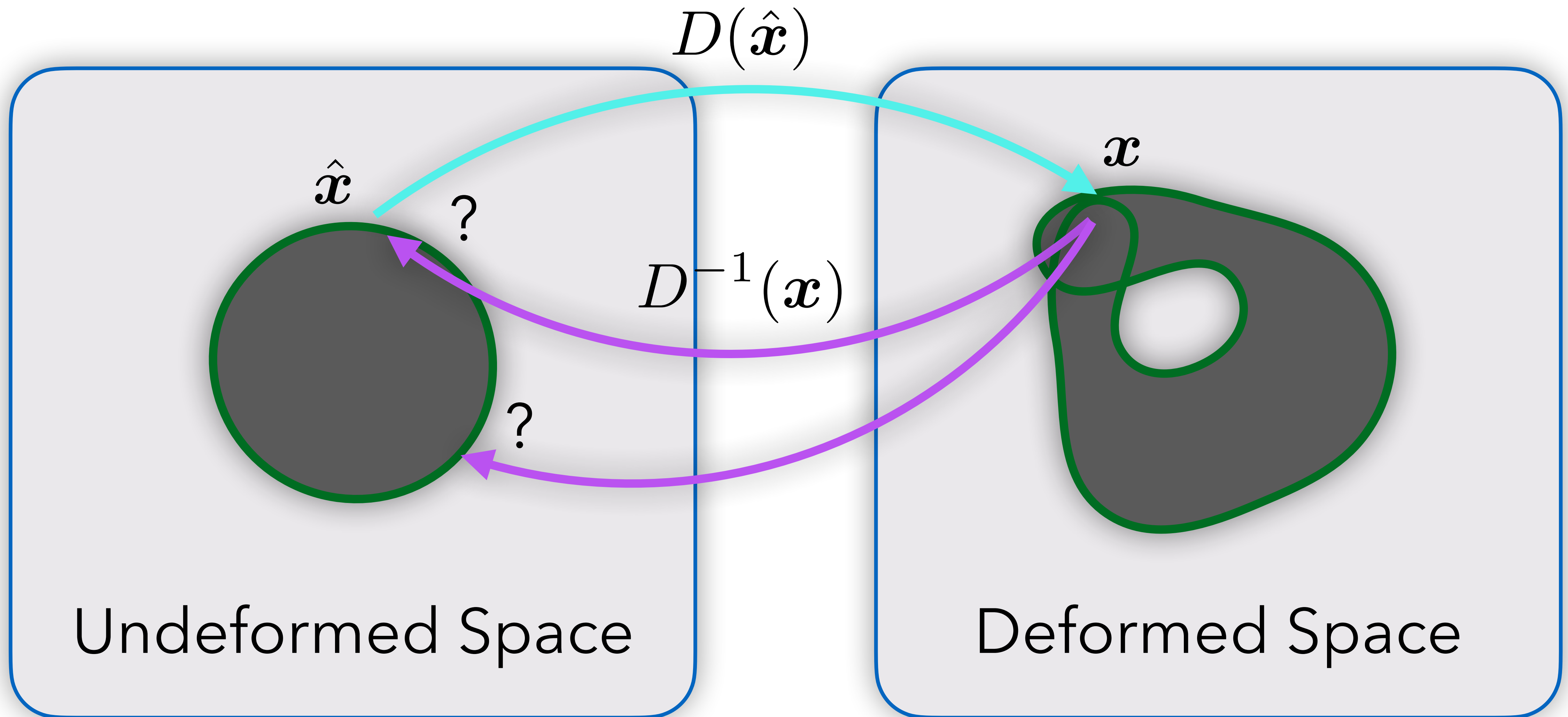
$$x$$

Issue #1: Remapping the Distance

$$D^{-1}(x)$$



Issue #2: Non Invertible Deformations



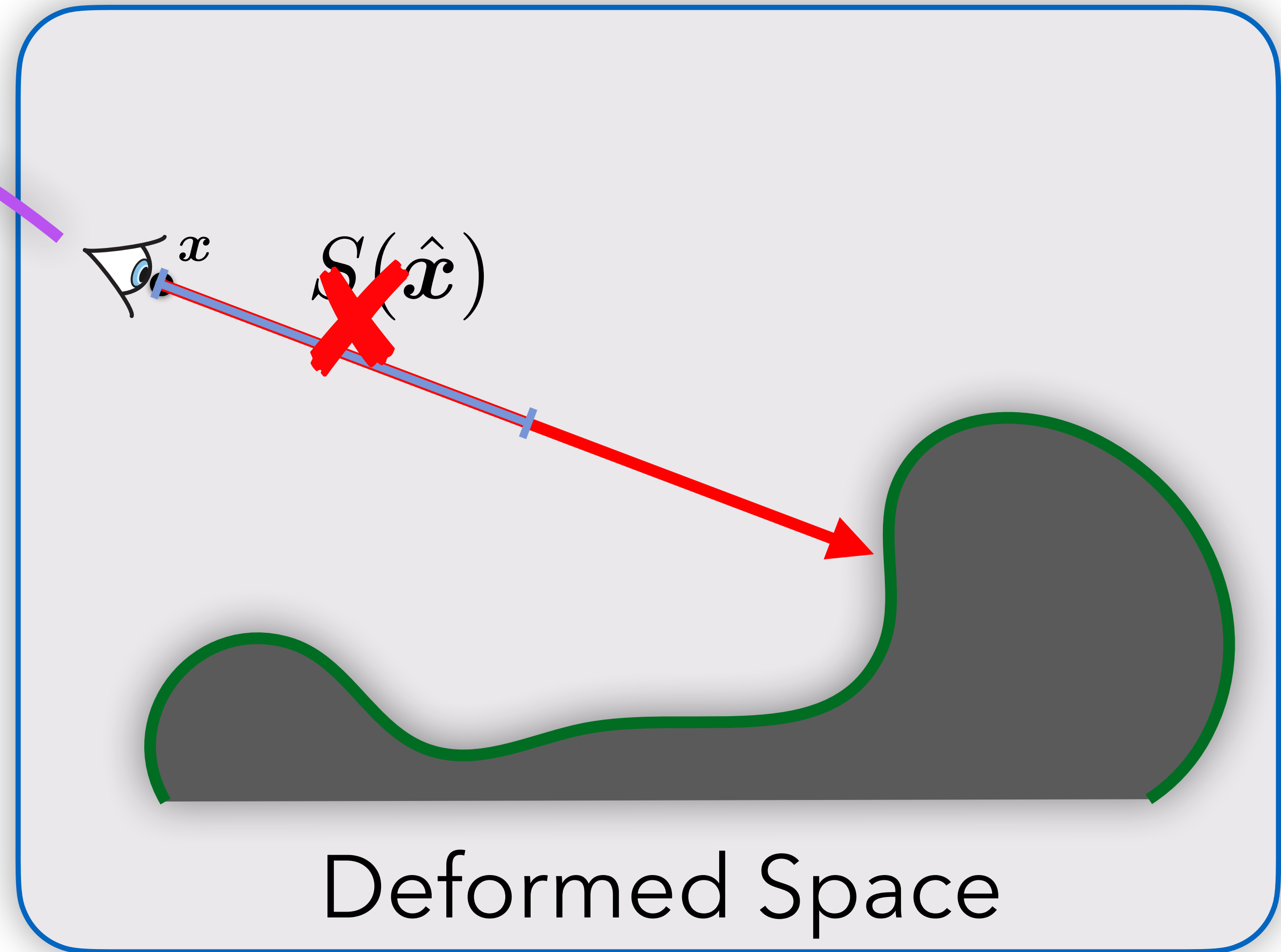
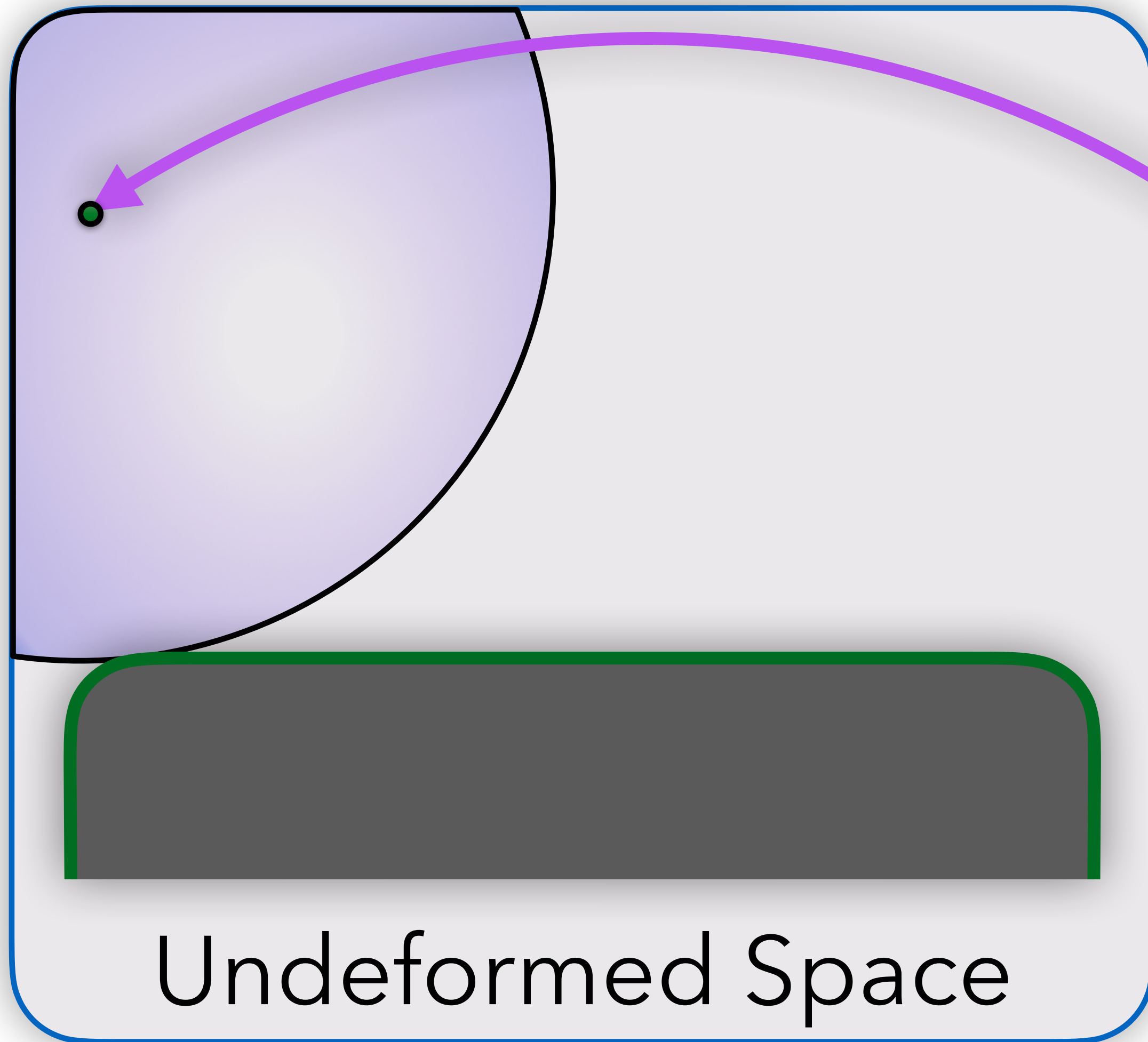
Problem Statement – Solved?

Use ~~conventional~~ deformation
techniques to directly render
deformed implicit surfaces ✓

Non-linear Sphere Tracing

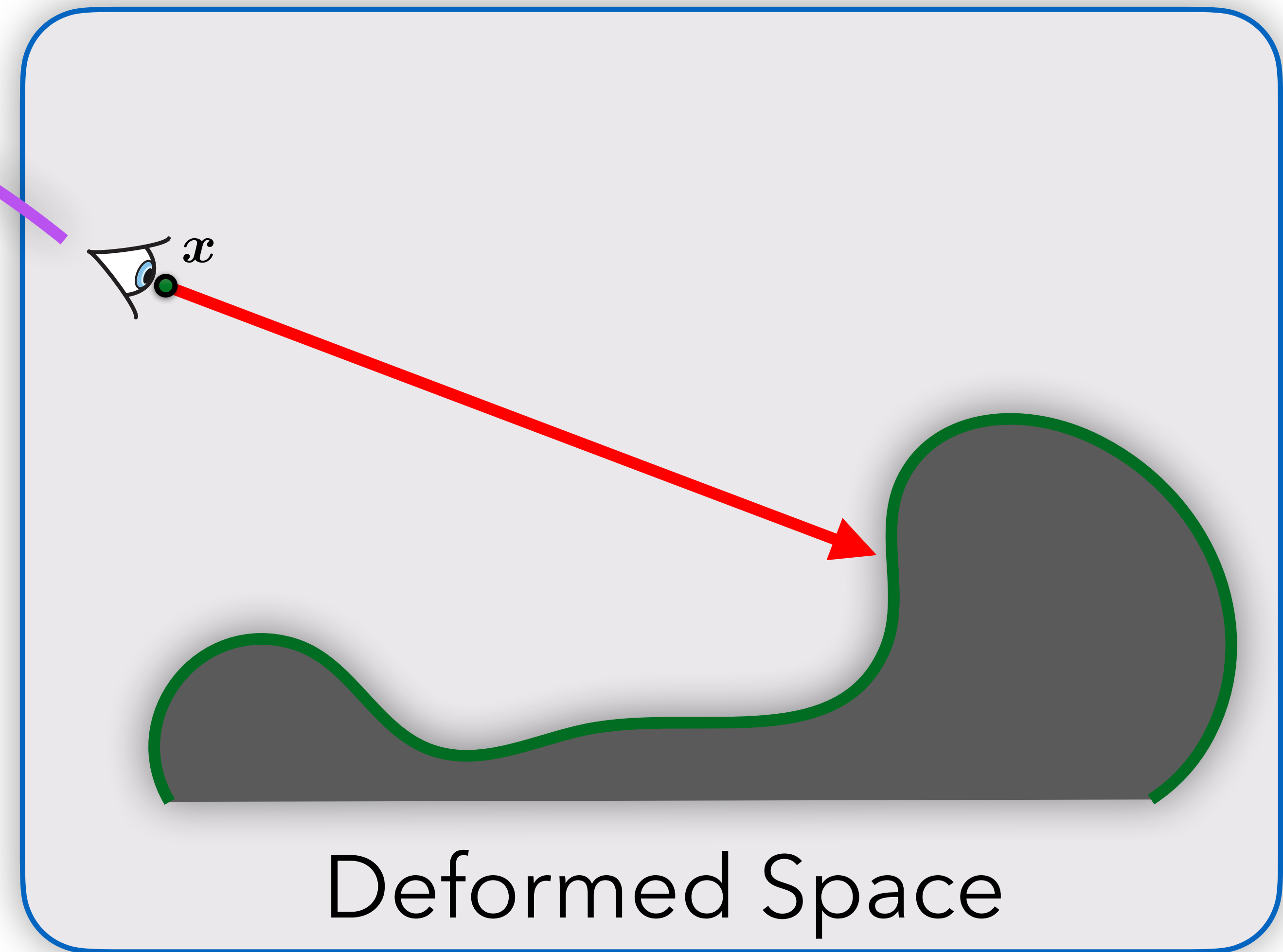
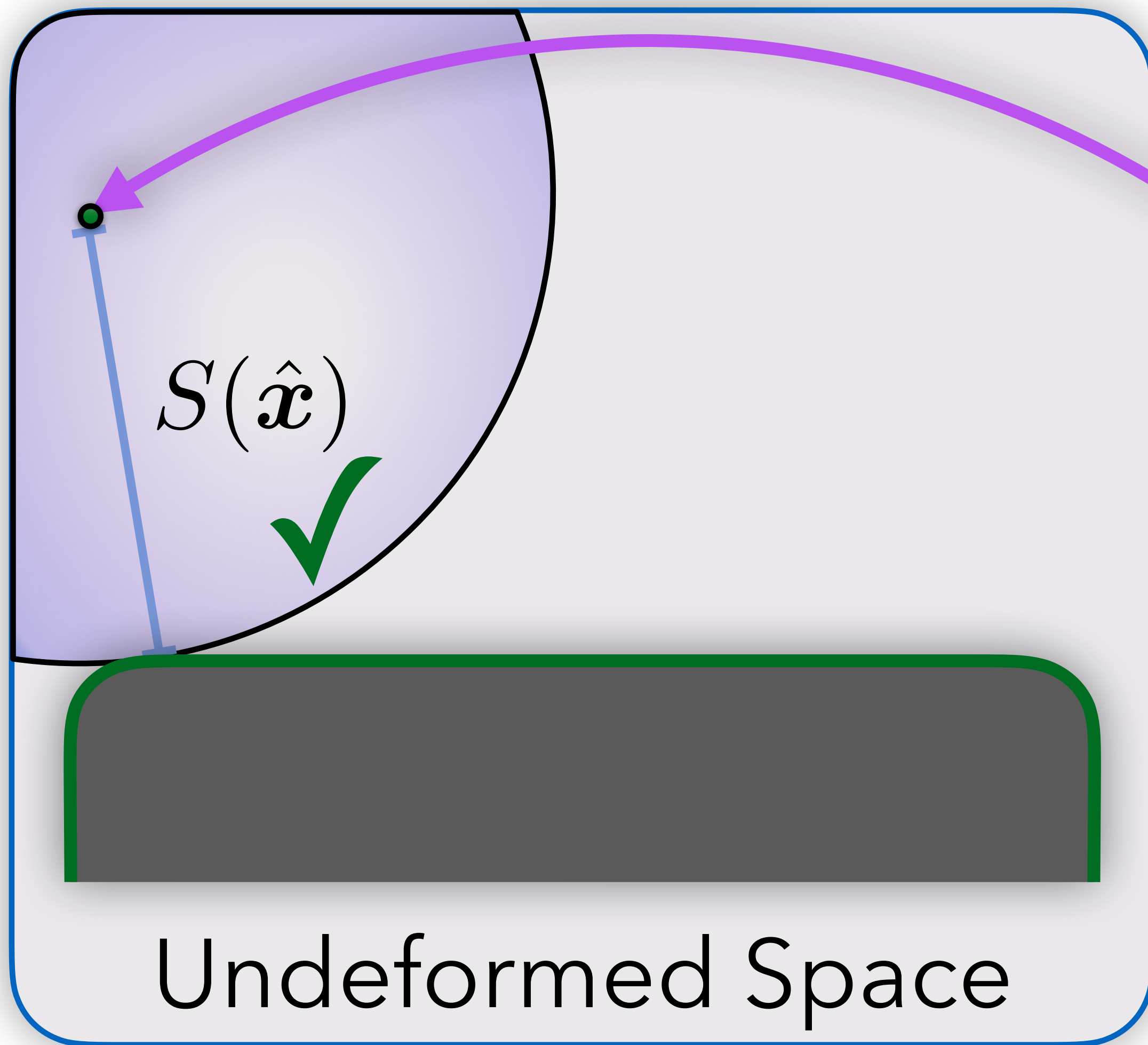
The distance isn't valid here...

$$D^{-1}(x)$$



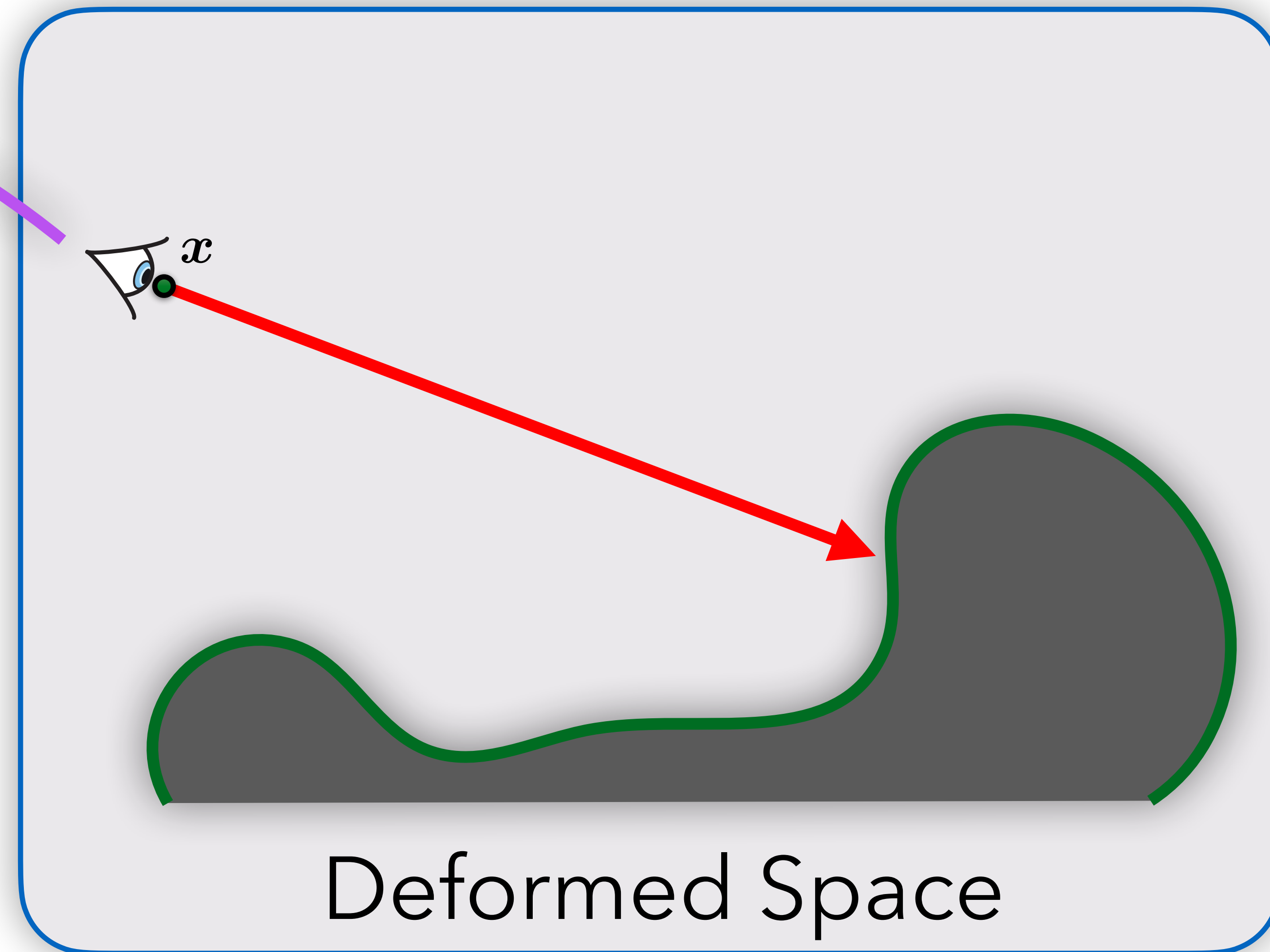
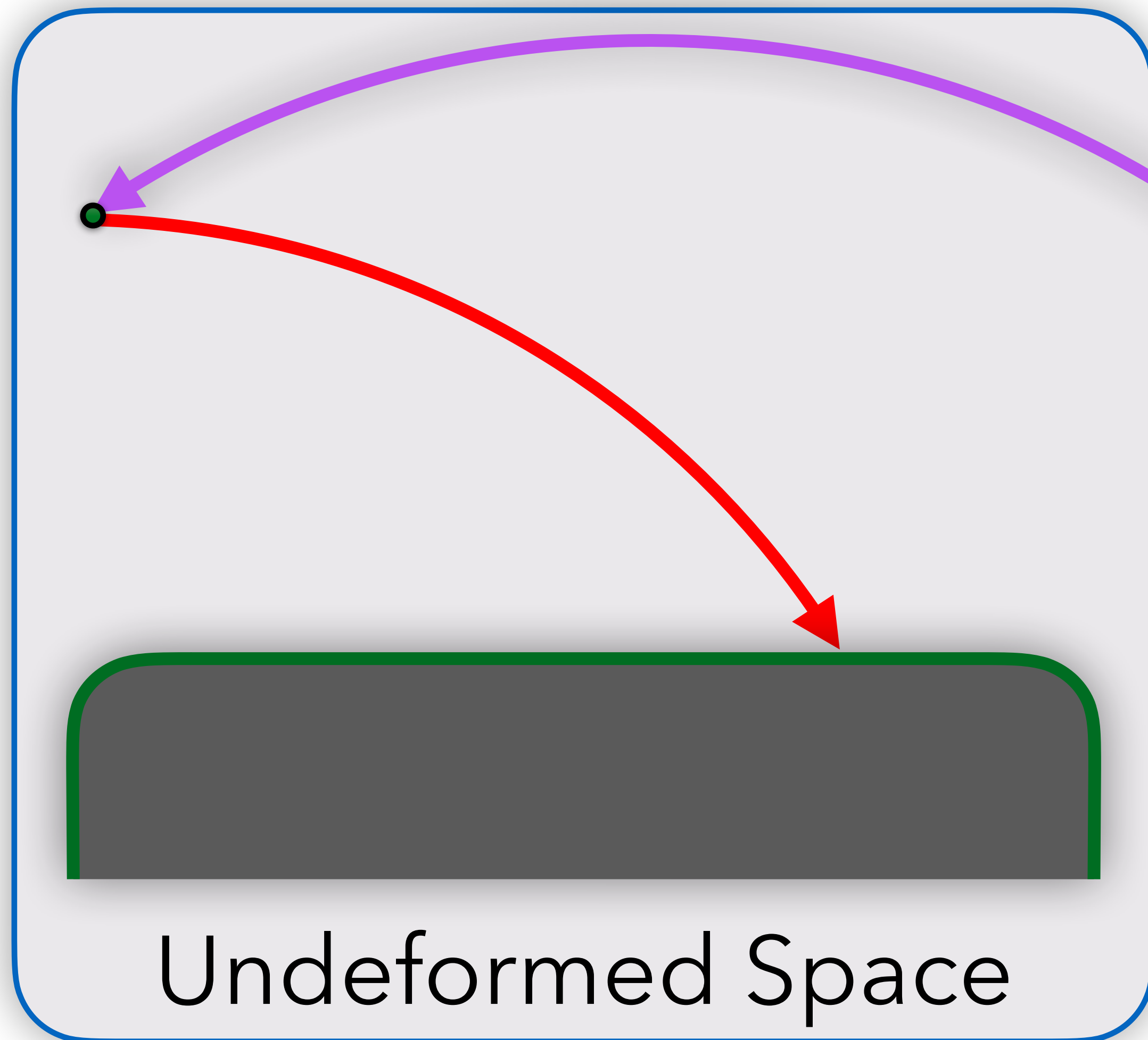
But it is here!

$$D^{-1}(x)$$



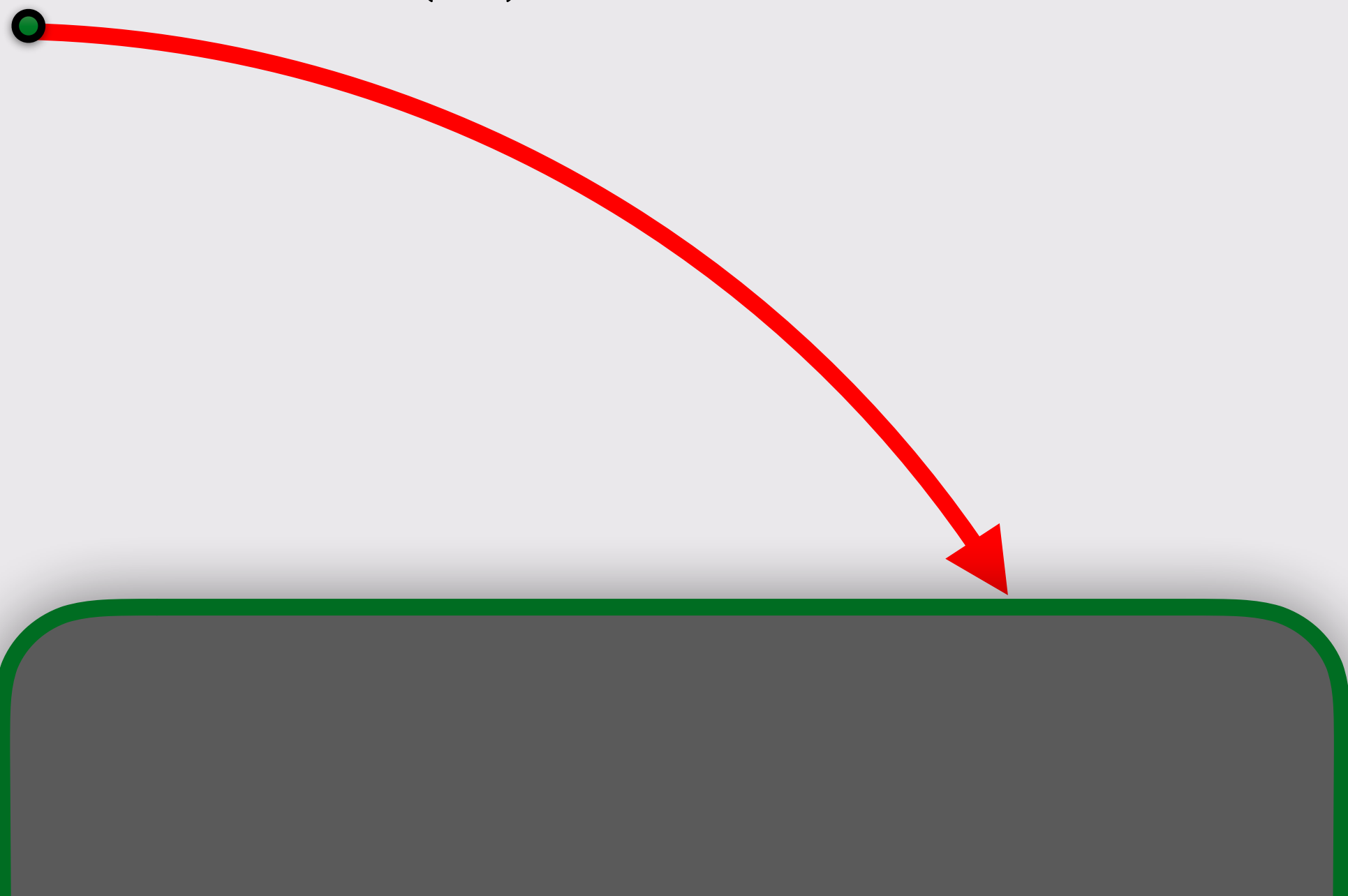
Idea: Trace in Undeformed Space

$$D^{-1}(x)$$



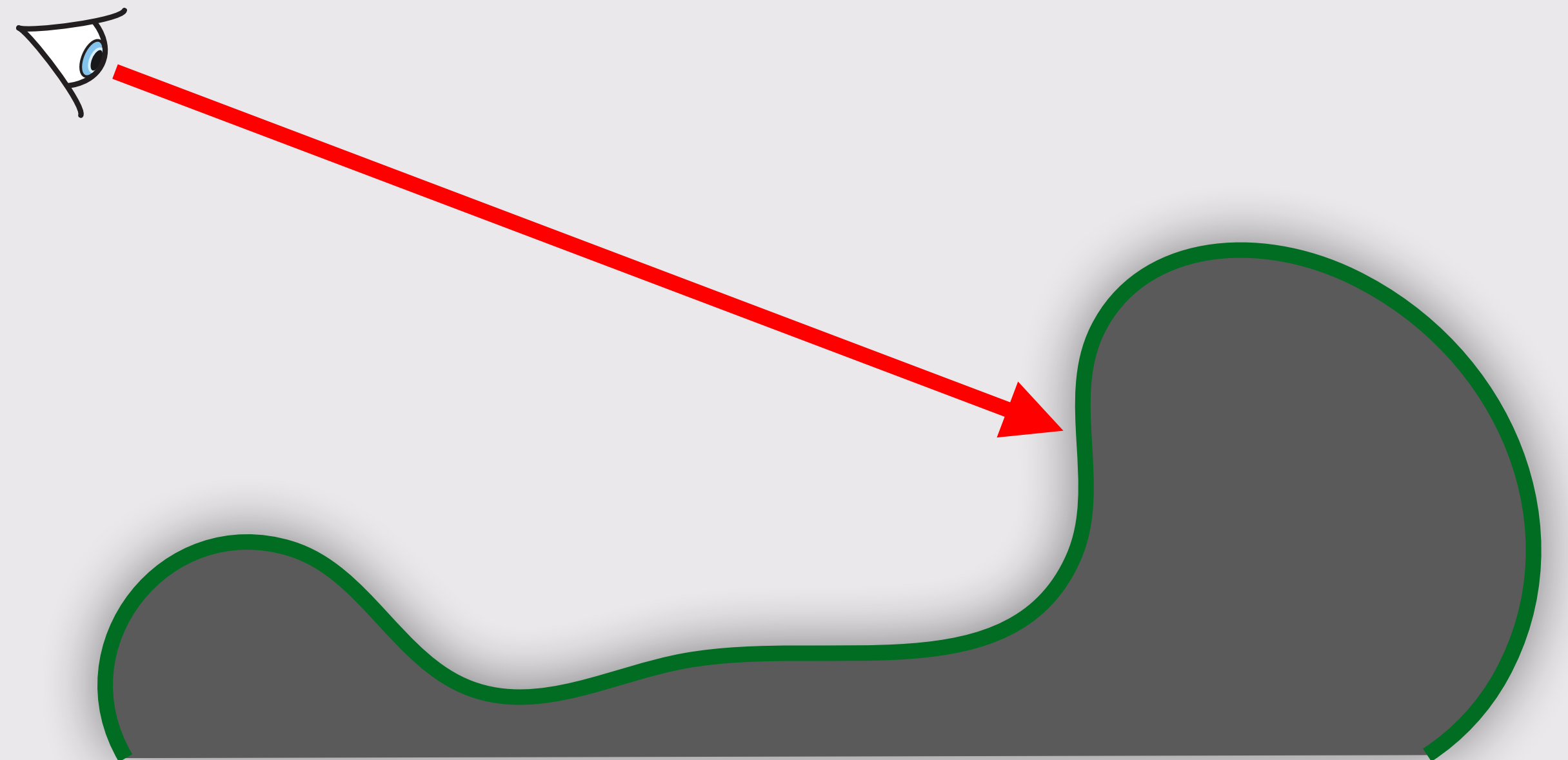
What is the *deformed* ray equation?

$$\hat{x}(s) = ?$$



Undeformed Space

$$x(s) = p + s\omega$$



Deformed Space

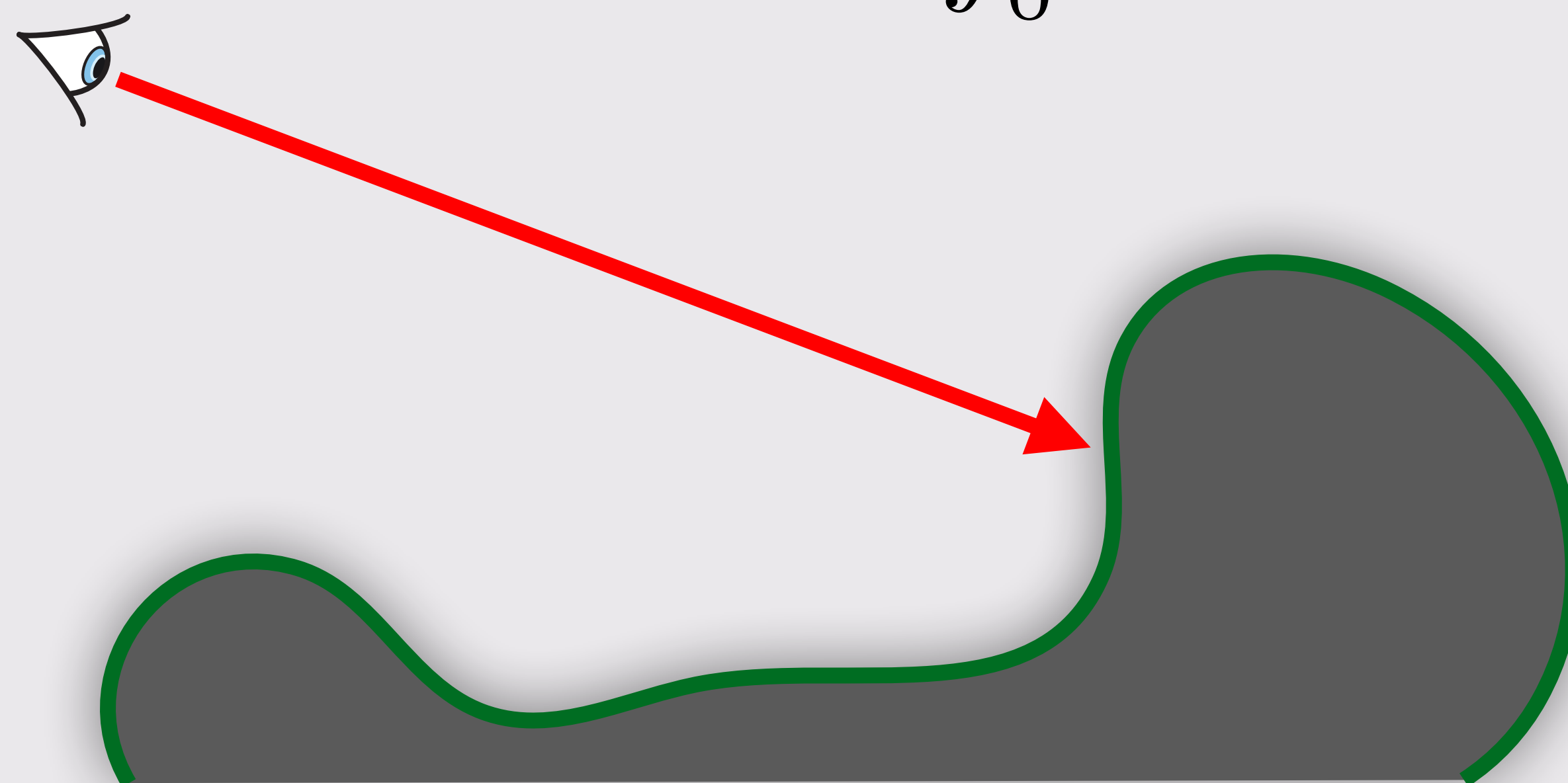
Line integrals to the rescue!

$$\hat{x}(s) = ?$$



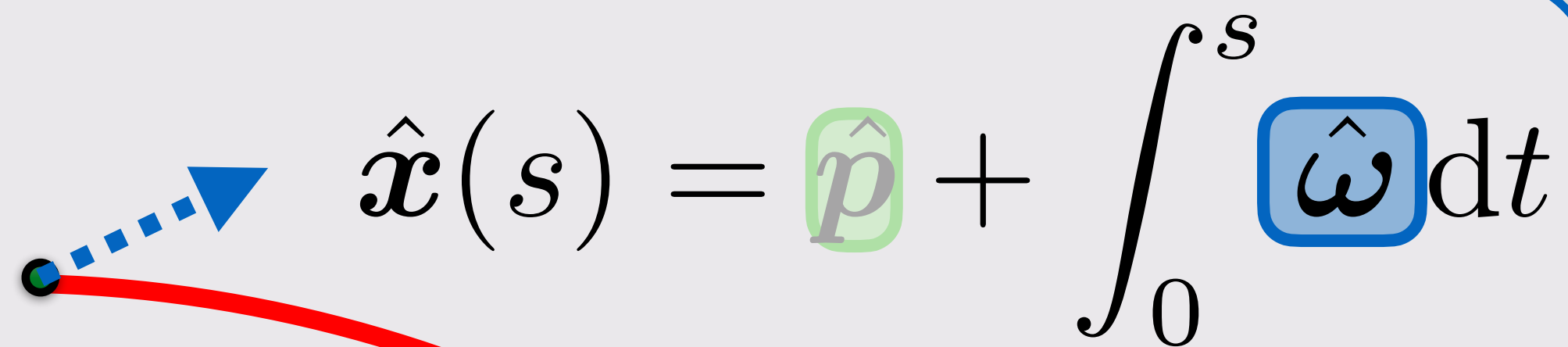
Undeformed Space

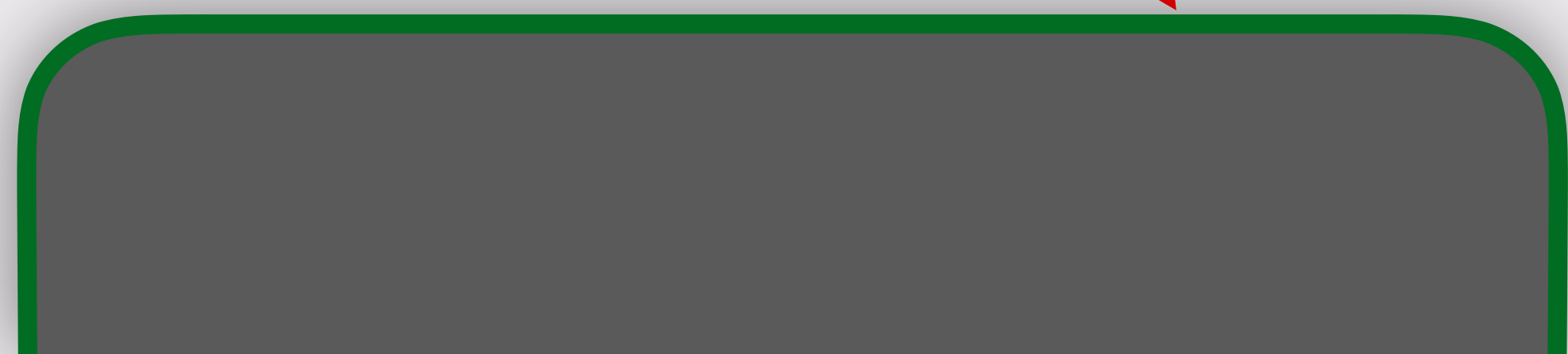
$$x(s) = p + \int_0^s \omega dt$$



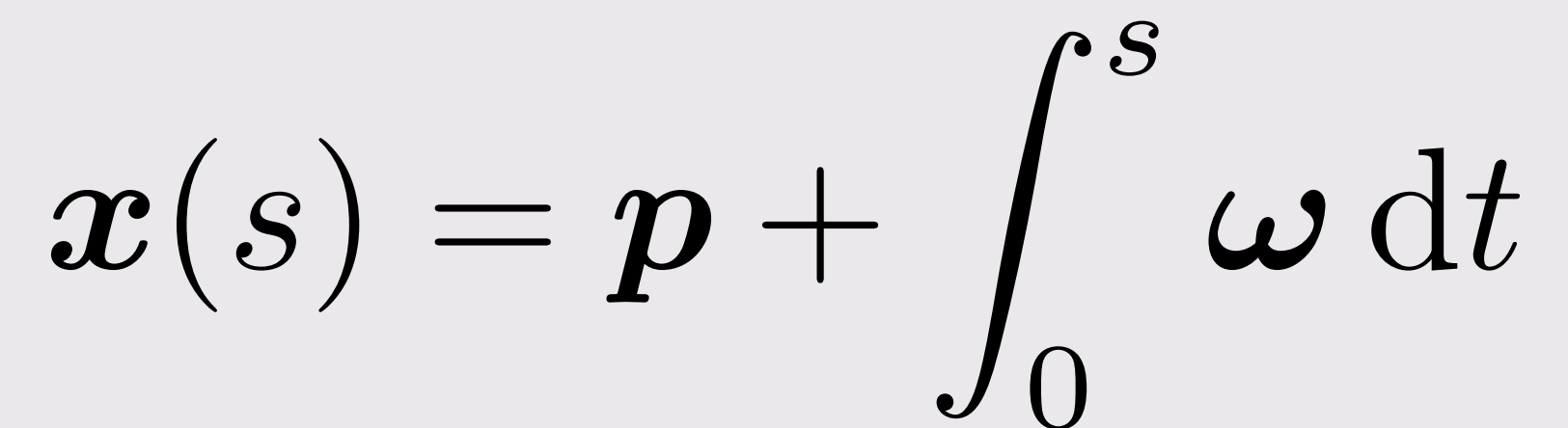
Deformed Space

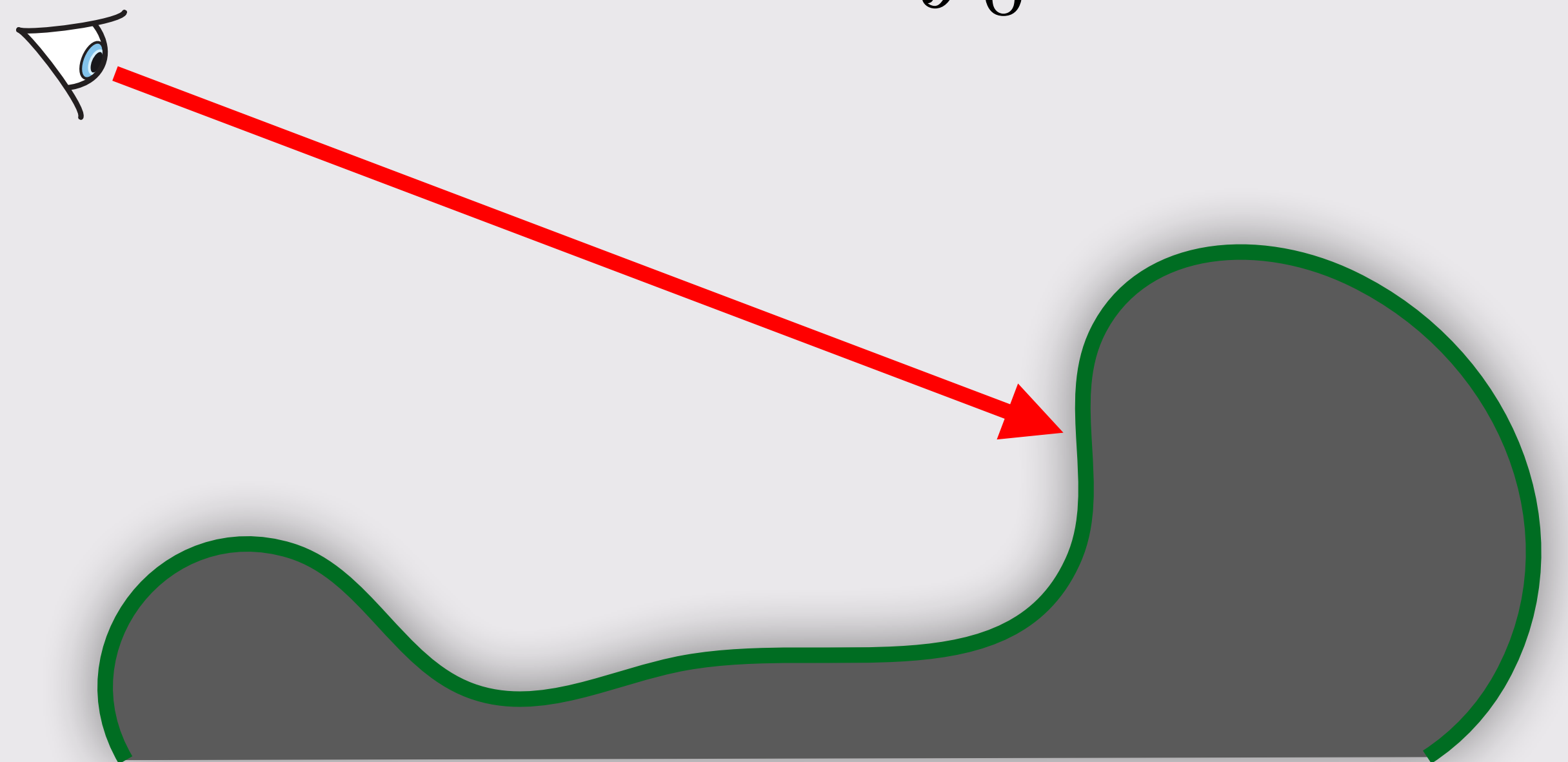
Line integrals to the rescue!

$$\hat{x}(s) = \hat{p} + \int_0^s \hat{\omega} dt$$




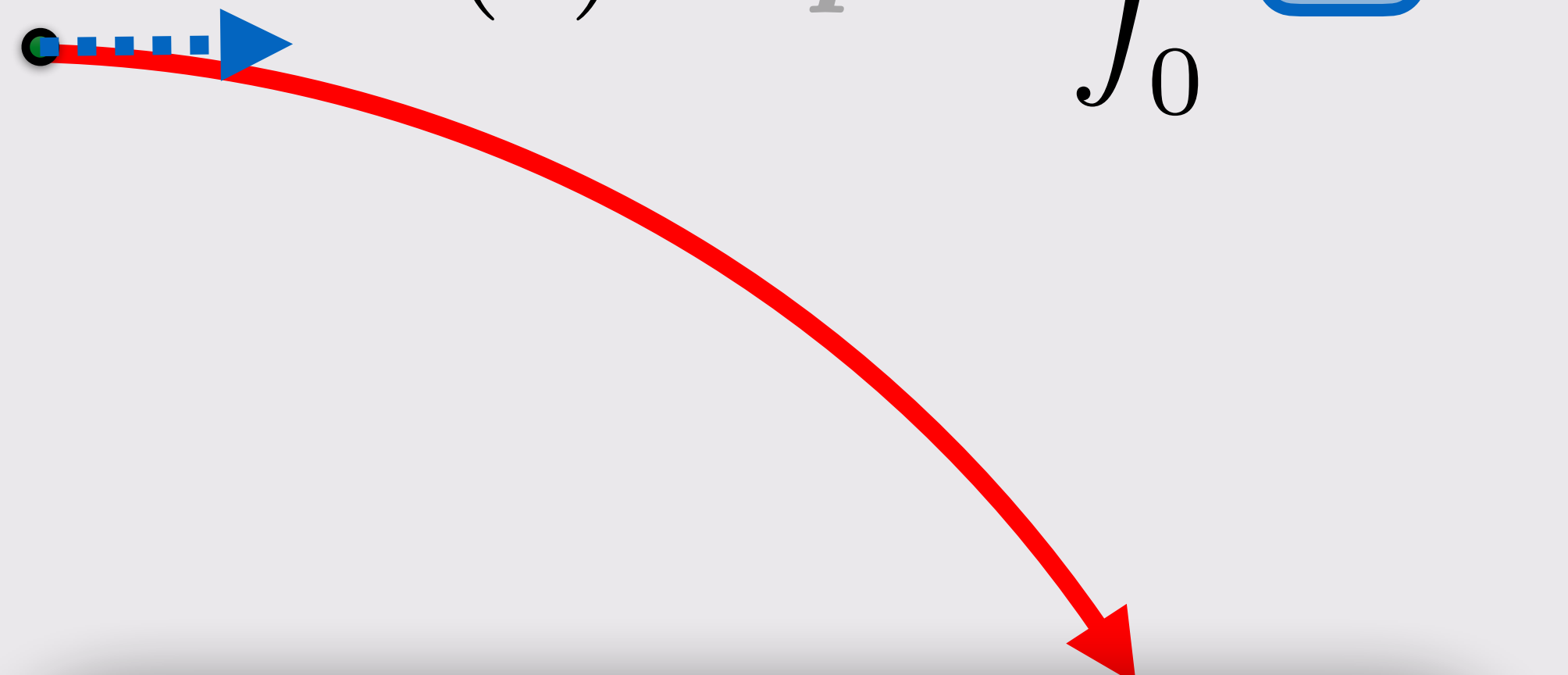
Undeformed Space

$$x(s) = p + \int_0^s \omega dt$$





Deformed Space

Line integrals to the rescue!

$$\hat{x}(s) = \hat{p} + \int_0^s \hat{\omega} dt$$
A diagram showing a point on the left with a blue dashed arrow pointing right. A red arrow curves downwards from this point to a grey rounded rectangle with a green border. The equation above shows a blue box around the symbol $\hat{\omega}$.

Undeformed Space

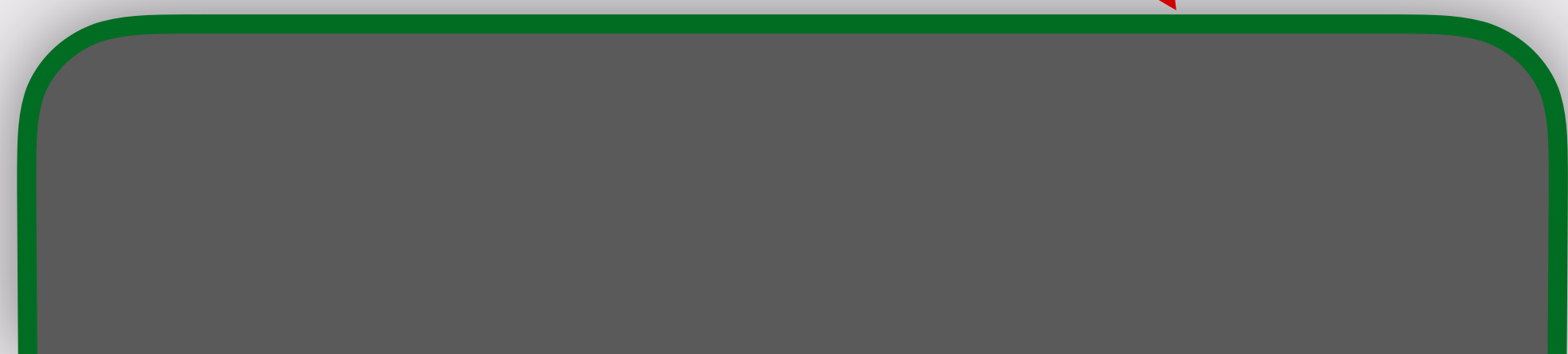
$$x(s) = p + \int_0^s \omega dt$$
A diagram showing an eye icon on the left with a red arrow pointing to a grey irregular shape with a green border. The equation above shows the symbol ω in a standard font.

Deformed Space

Finding the Tangent

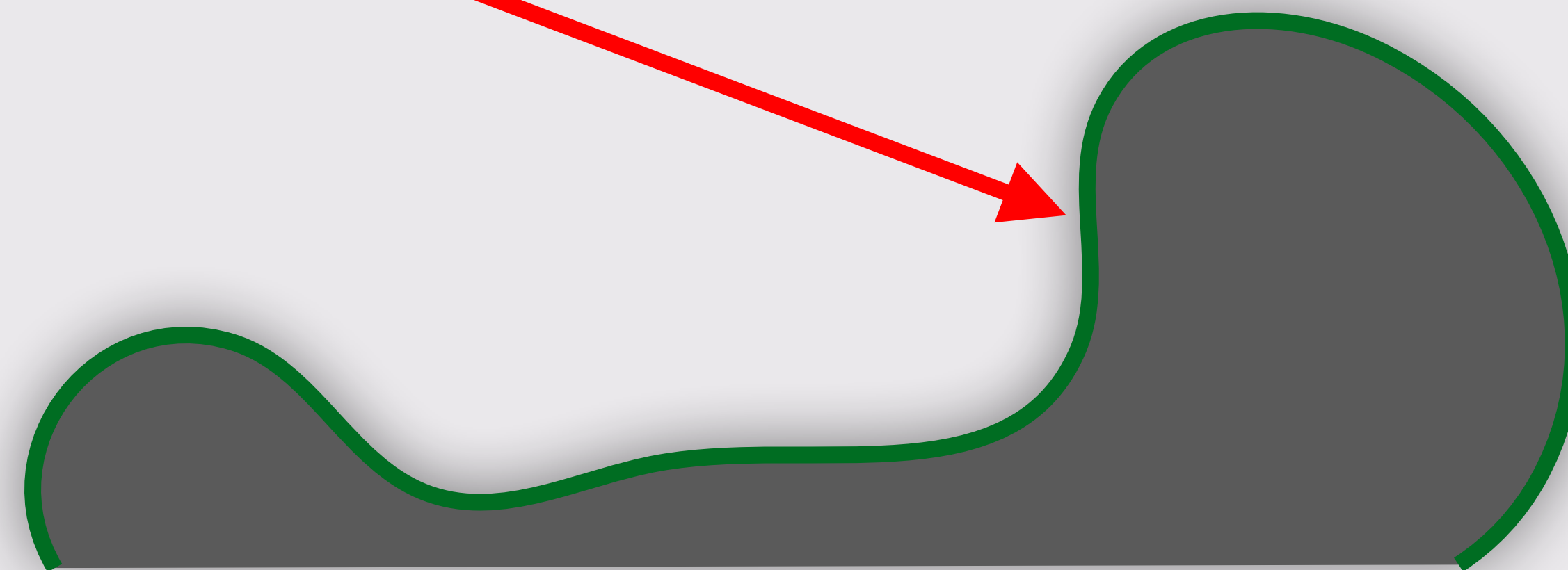
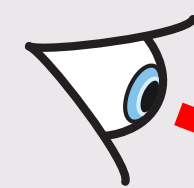
$$\hat{x}(s) = \hat{p} + \int_0^s \hat{\omega}(\hat{x}) dt$$

$\hat{\omega}(\hat{x})$



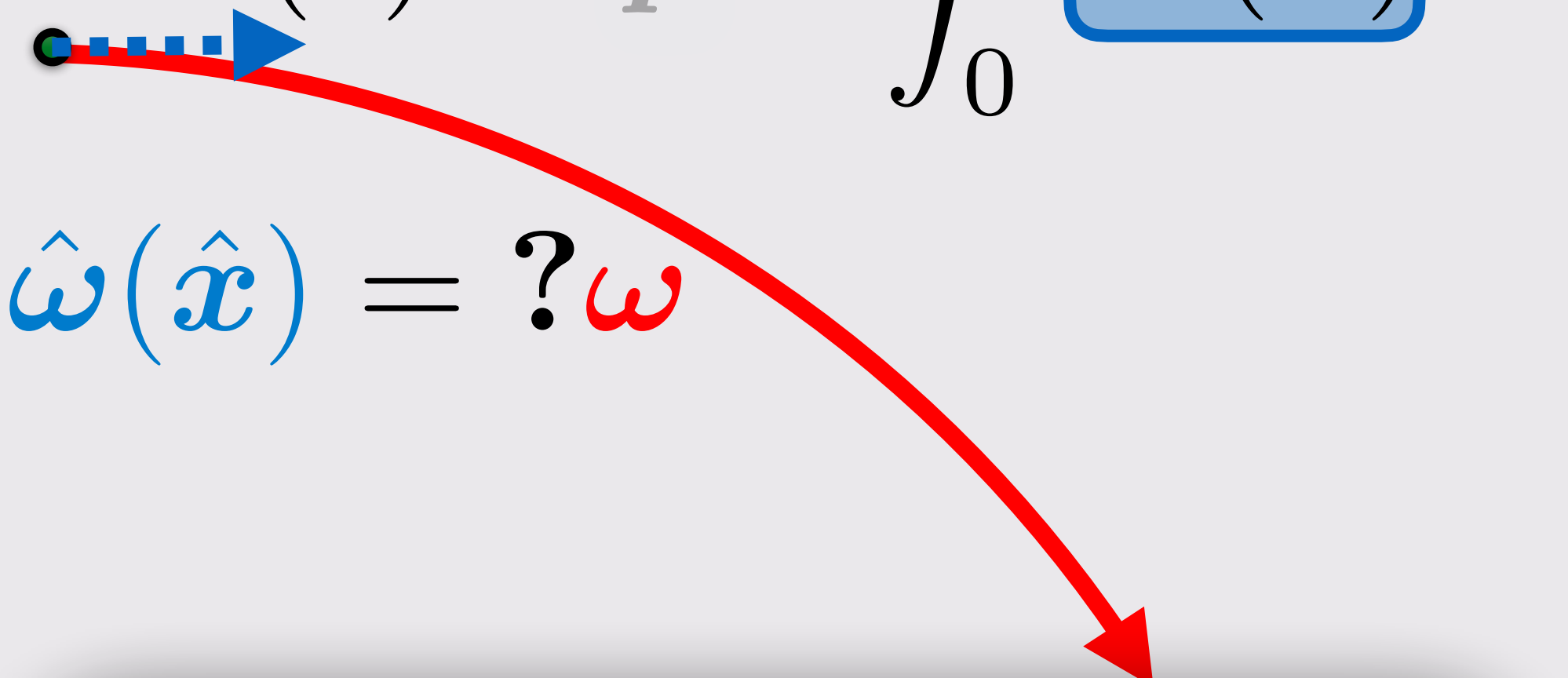
Undeformed Space

$$x(s) = p + \int_0^s \omega dt$$

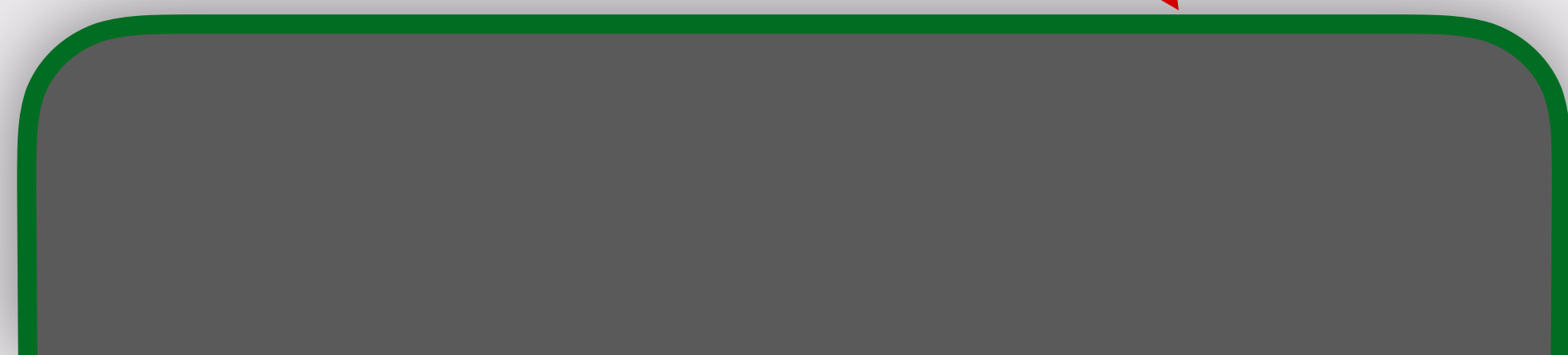


Deformed Space


Finding the Tangent

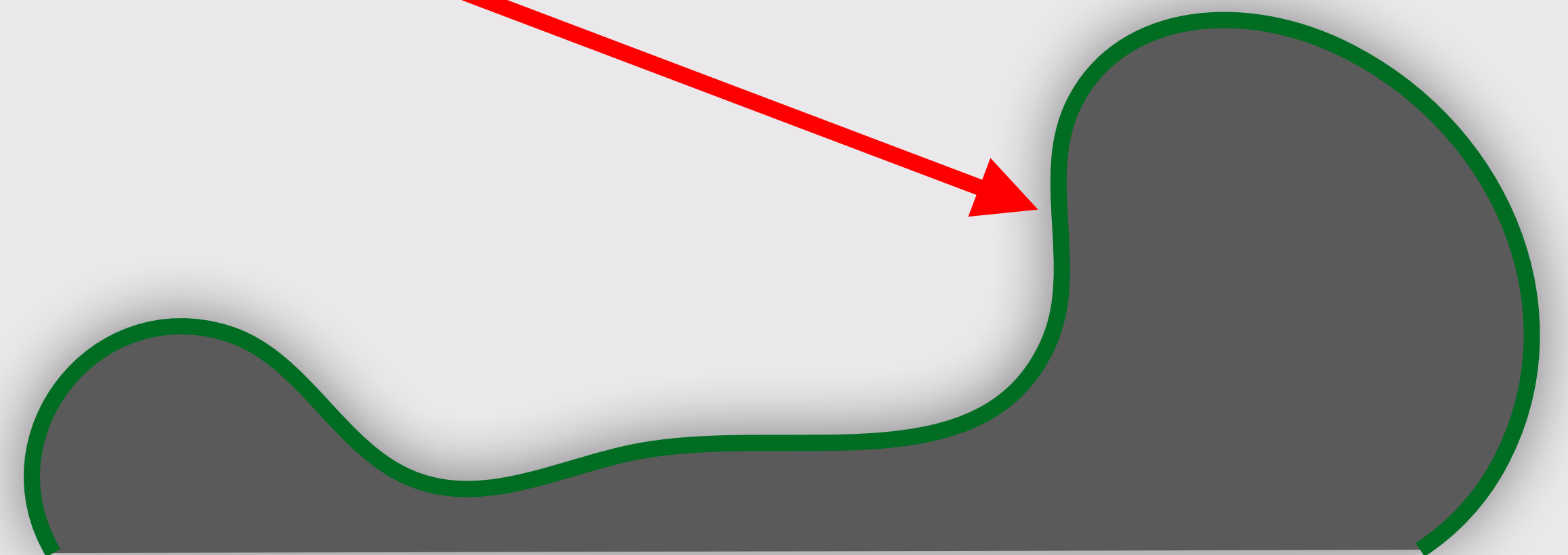
$$\hat{x}(s) = \hat{p} + \int_0^s \hat{\omega}(\hat{x}) dt$$


$$\hat{\omega}(\hat{x}) = ? \omega$$



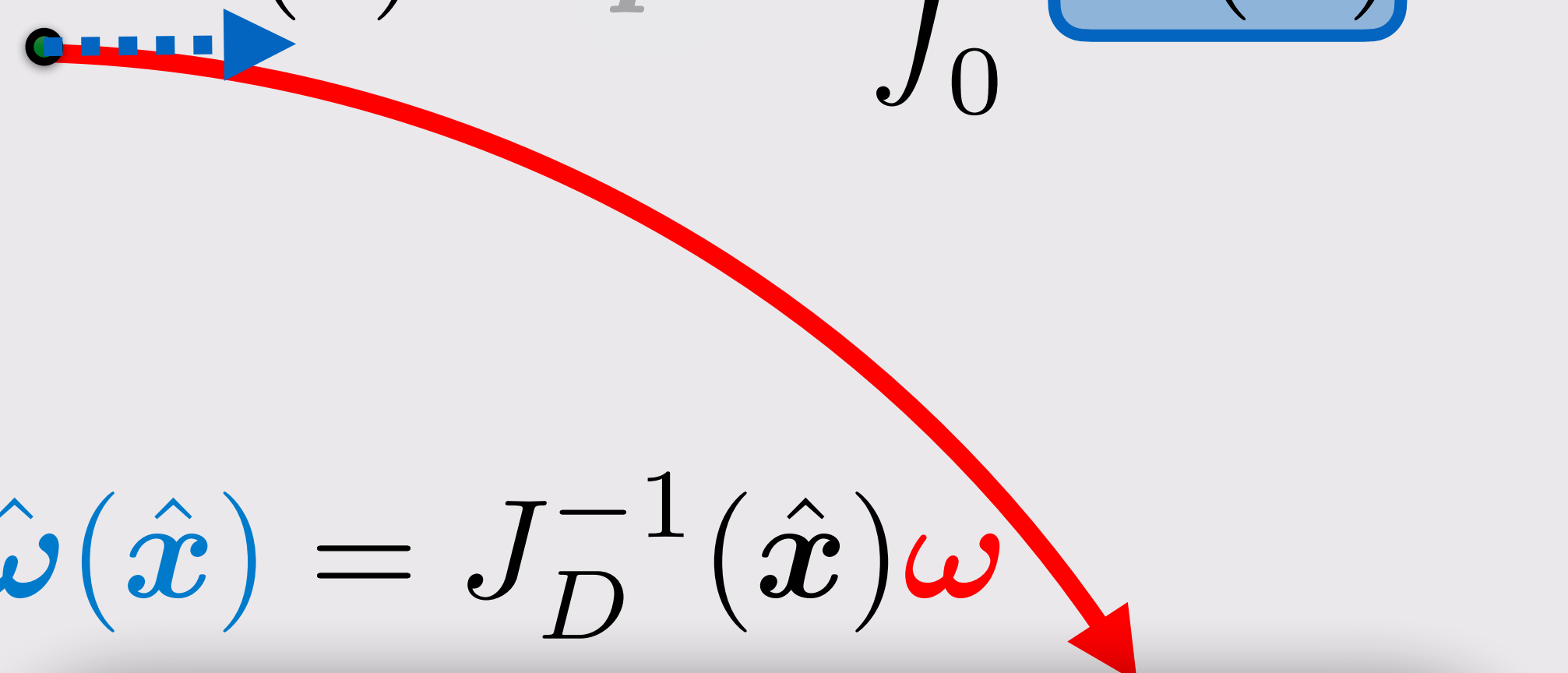
Undeformed Space

$$x(s) = p + \int_0^s \omega dt$$





Deformed Space

Finding the Tangent

$$\hat{x}(s) = \hat{p} + \int_0^s \hat{\omega}(\hat{x}) dt$$


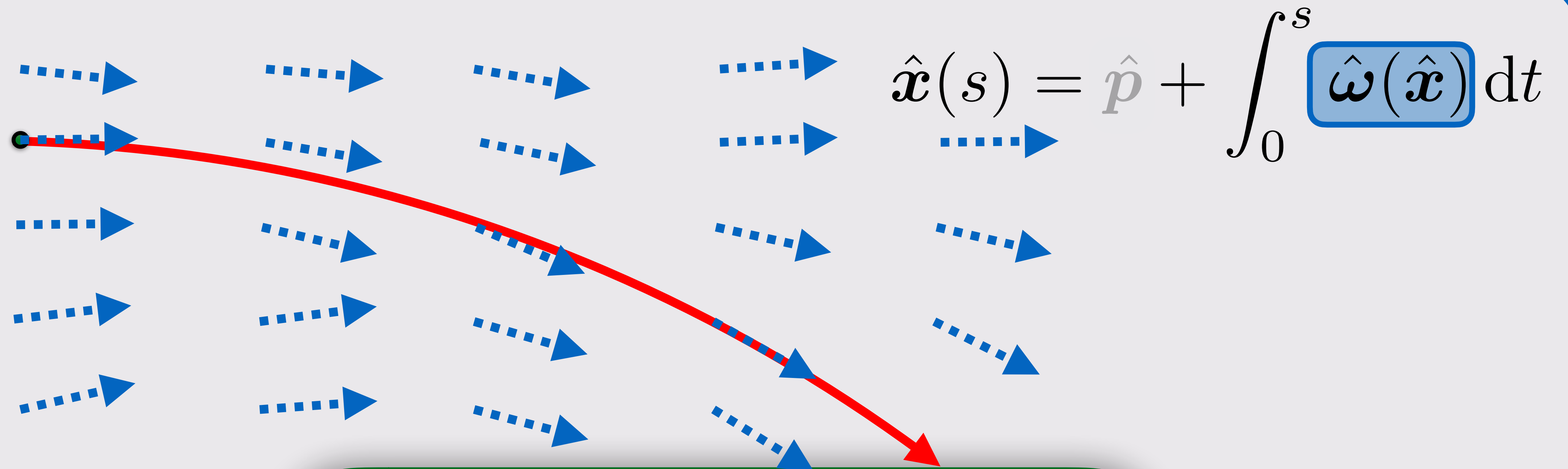
$$\hat{\omega}(\hat{x}) = J_D^{-1}(\hat{x}) \omega$$

Undeformed Space

$$x(s) = p + \int_0^s \omega dt$$


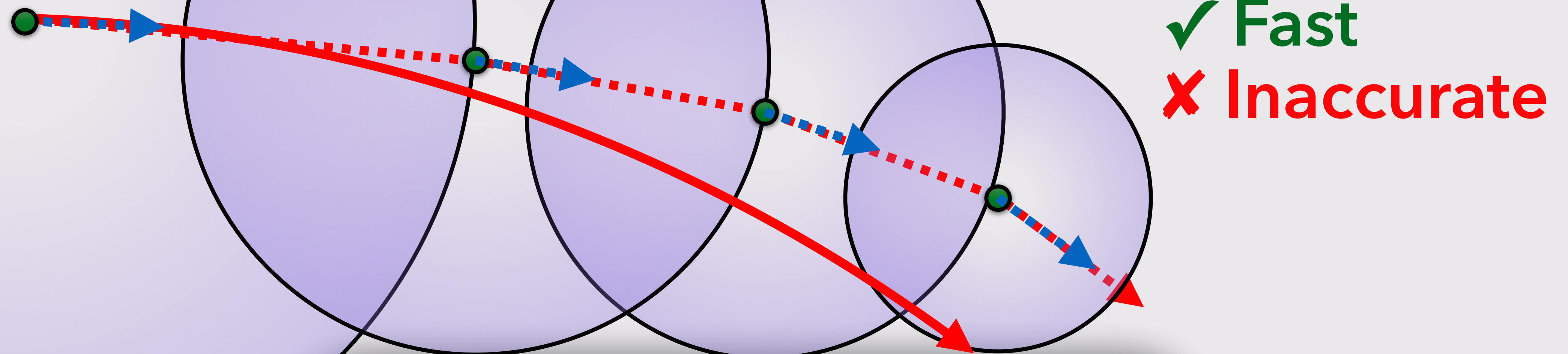
Deformed Space

Numerical Integration



Undeformed Space

Non-linear Sphere Tracing

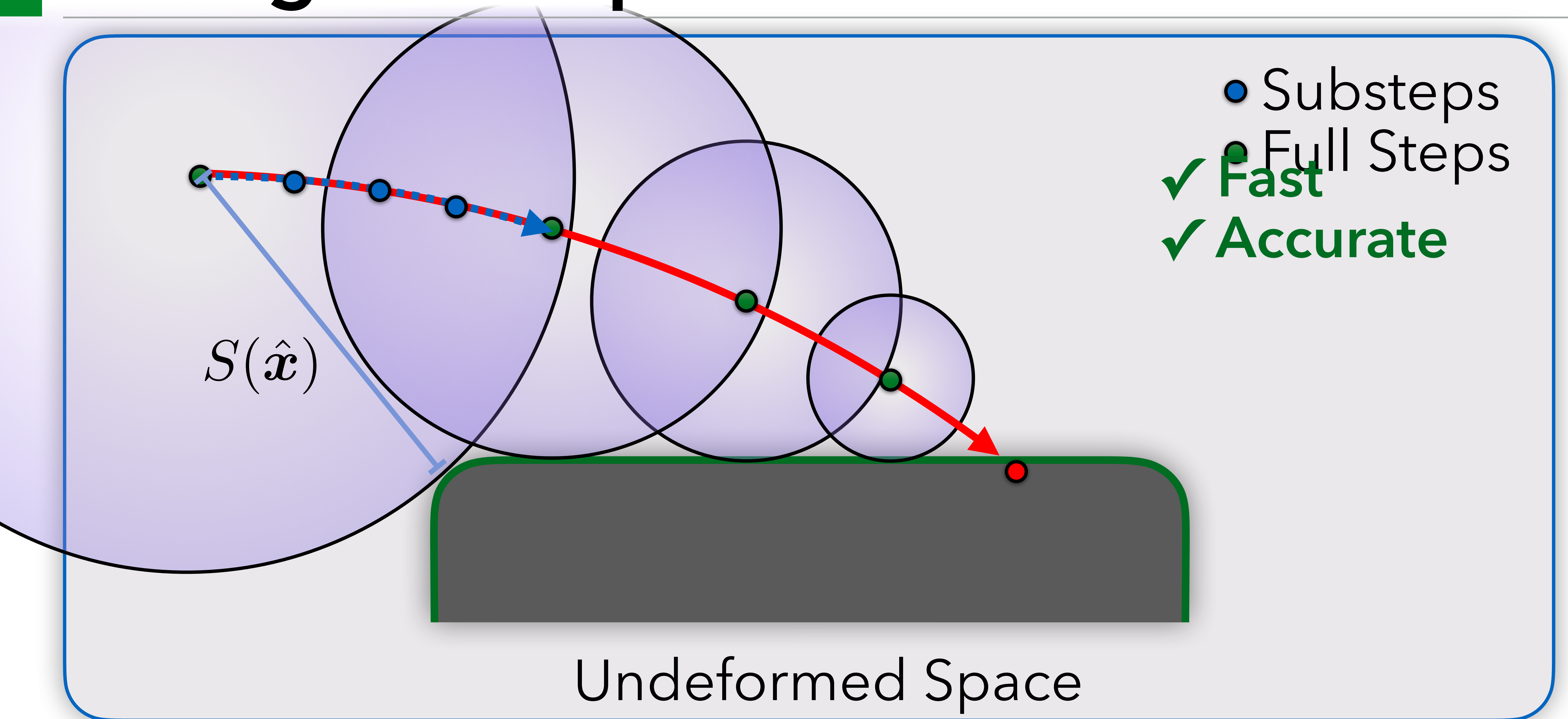


Undeformed Space


Non-linear Sphere Tracing



Taking Substeps



Integration – Summary

$$\hat{\mathbf{x}}(s) = \hat{\mathbf{p}} + \int_0^s \hat{\omega} dt$$


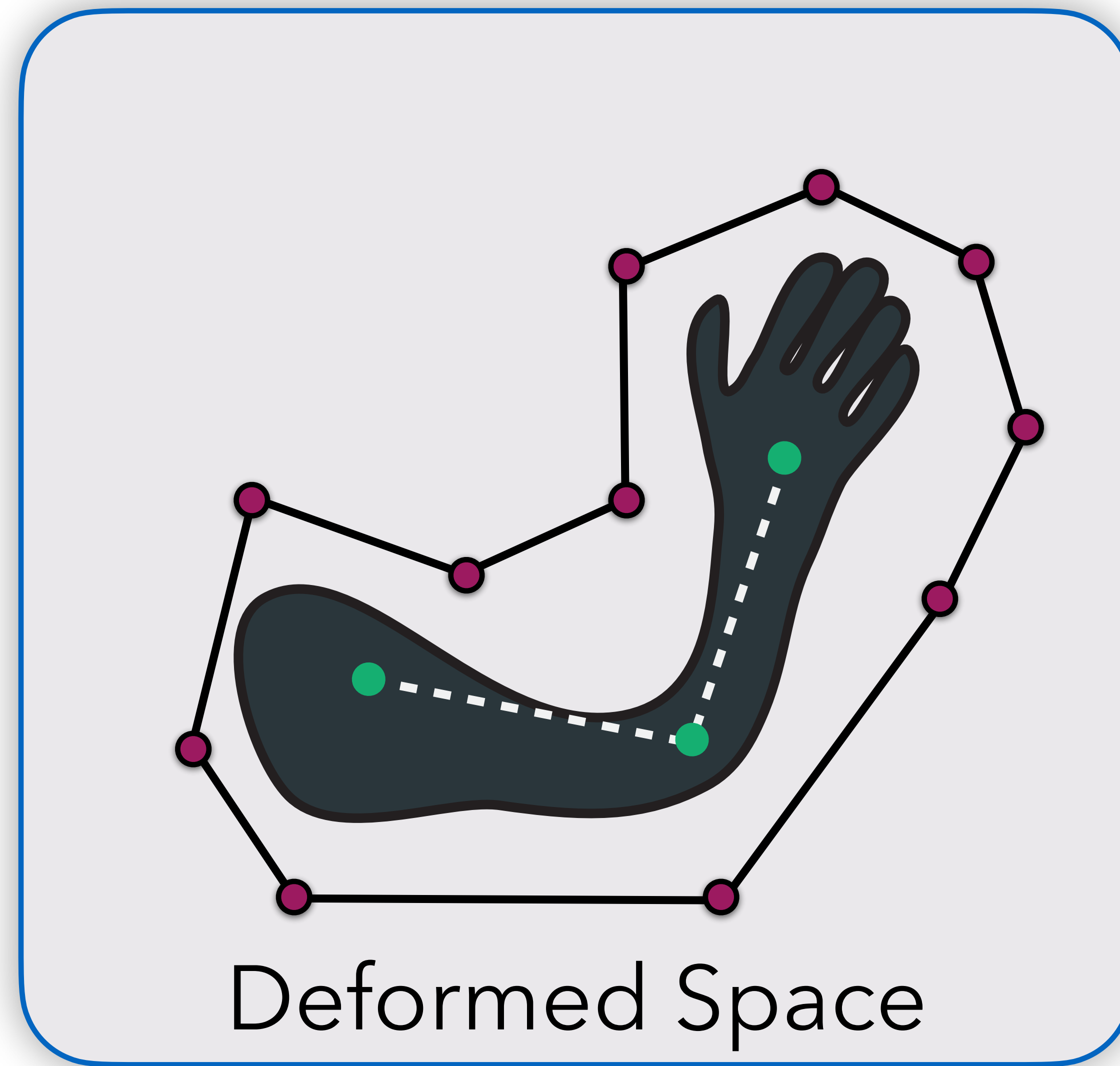
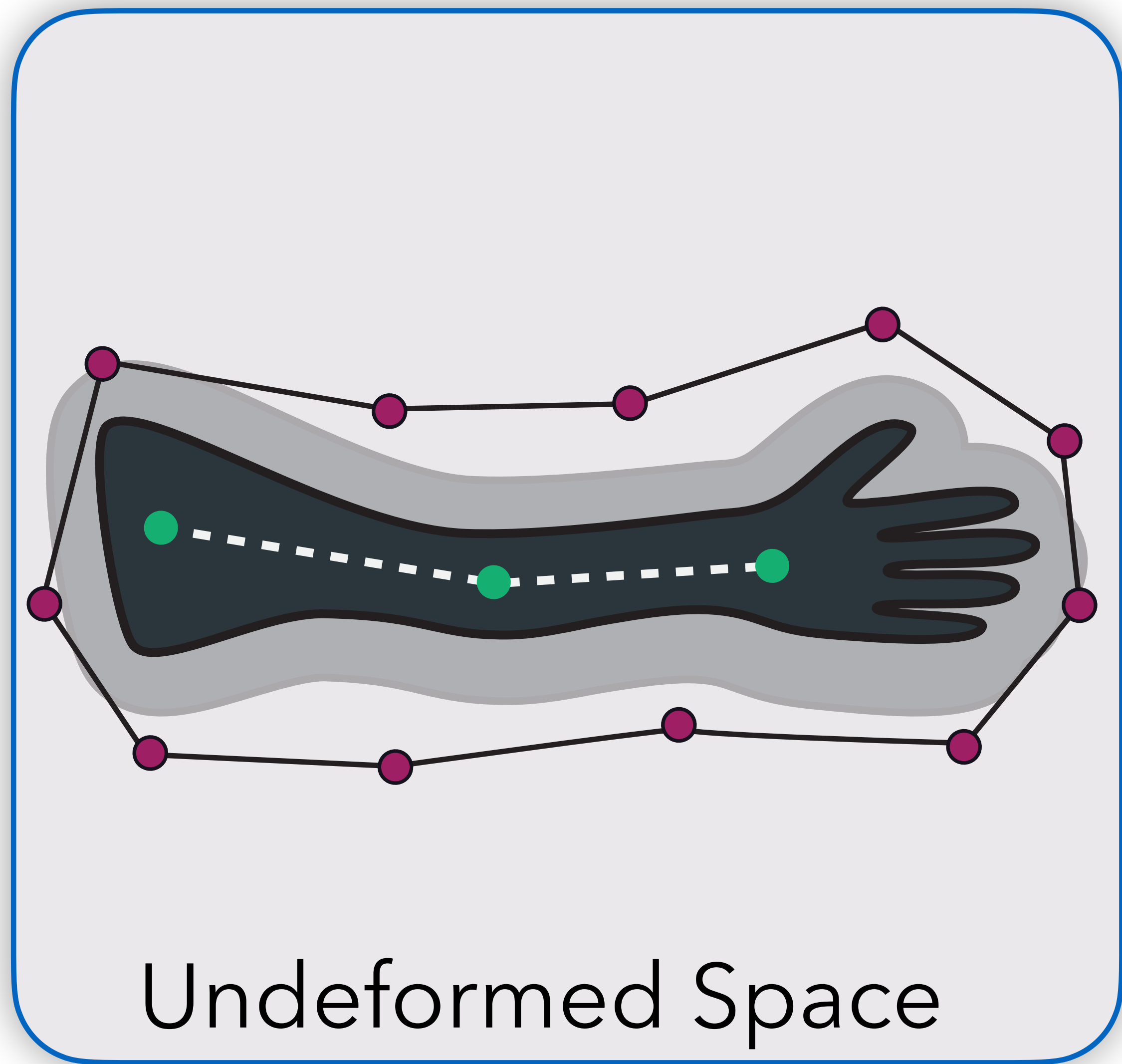
Integration – Summary

$$\hat{\mathbf{x}}(s) = \hat{\mathbf{p}} + \int_0^s \hat{\boldsymbol{\omega}} dt$$

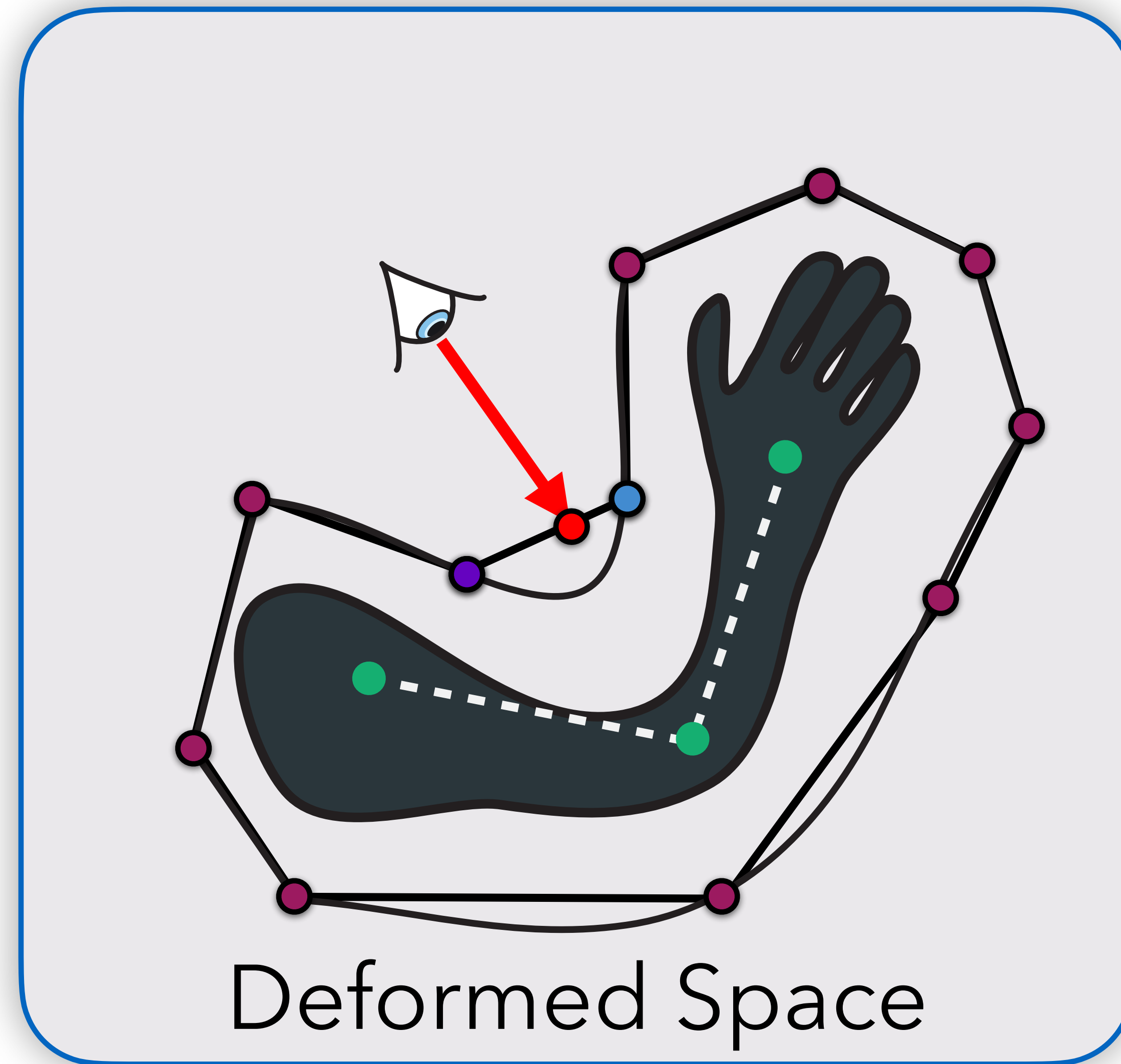
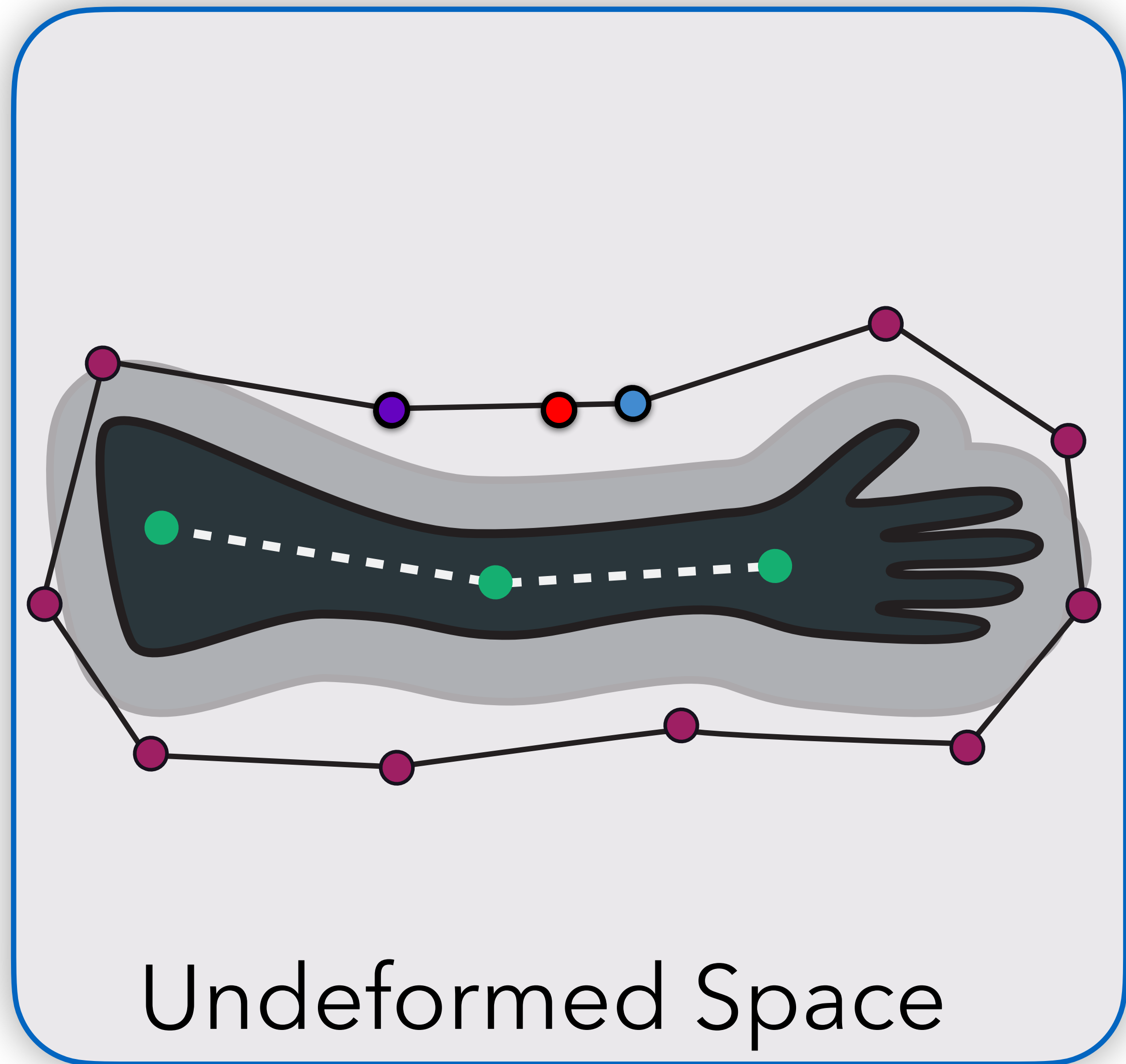
$$\hat{\mathbf{p}} = D^{-1}(\mathbf{p})$$



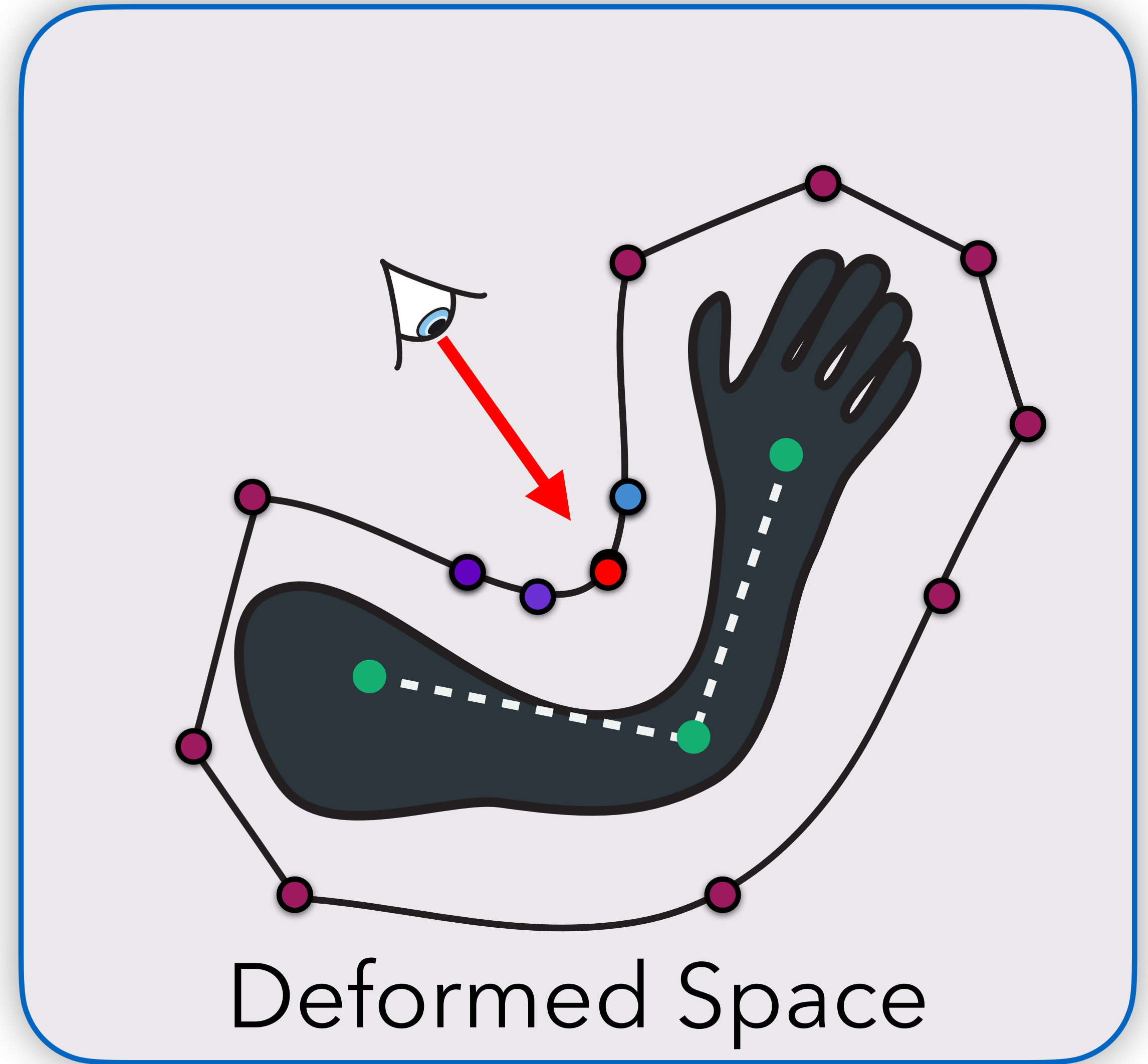
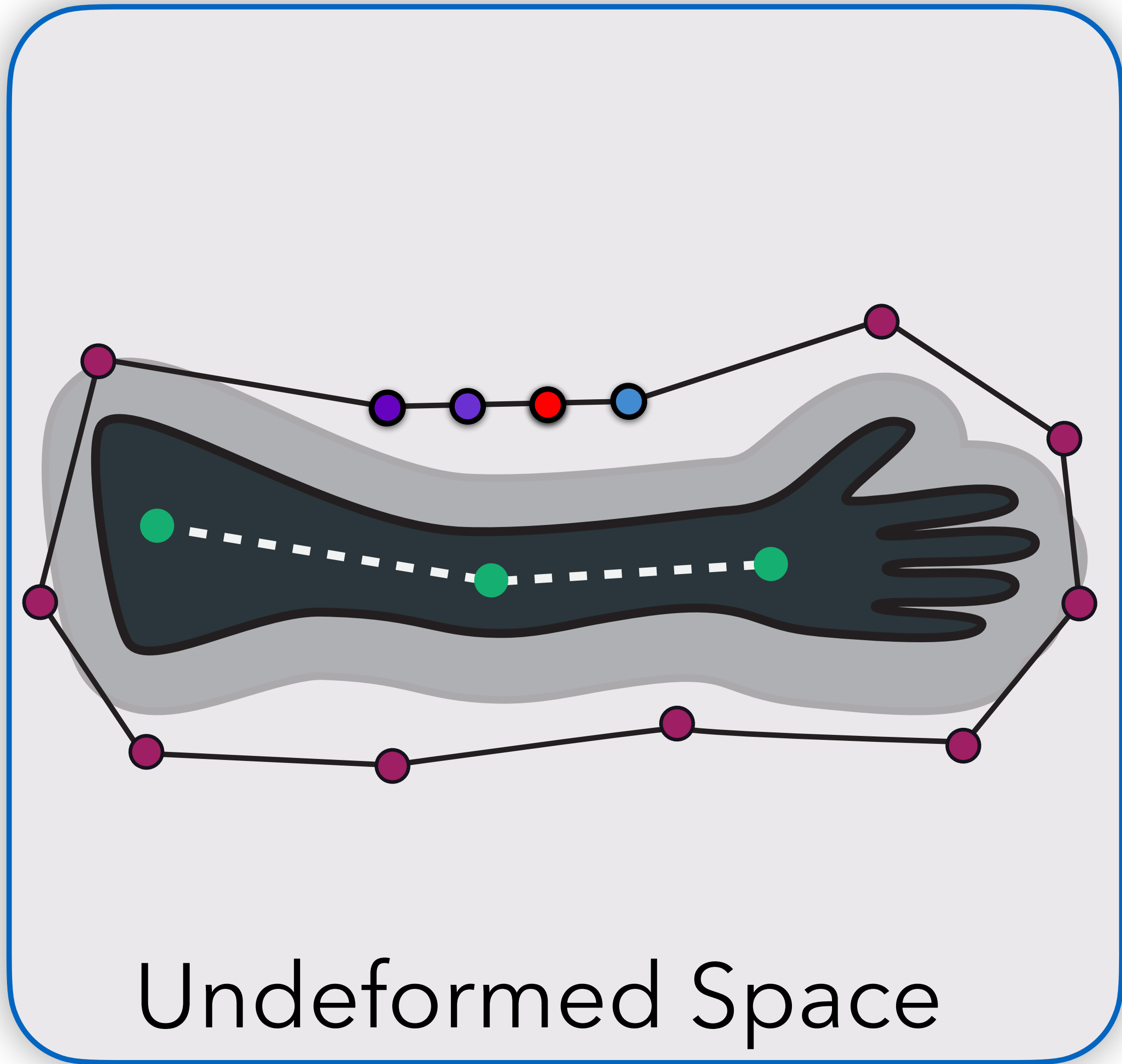
Finding the Start Point



Finding the Start Point



Subdividing the Hull



Problem Statement - Solved!

Use **conventional** deformation techniques to directly render deformed **implicit surfaces**

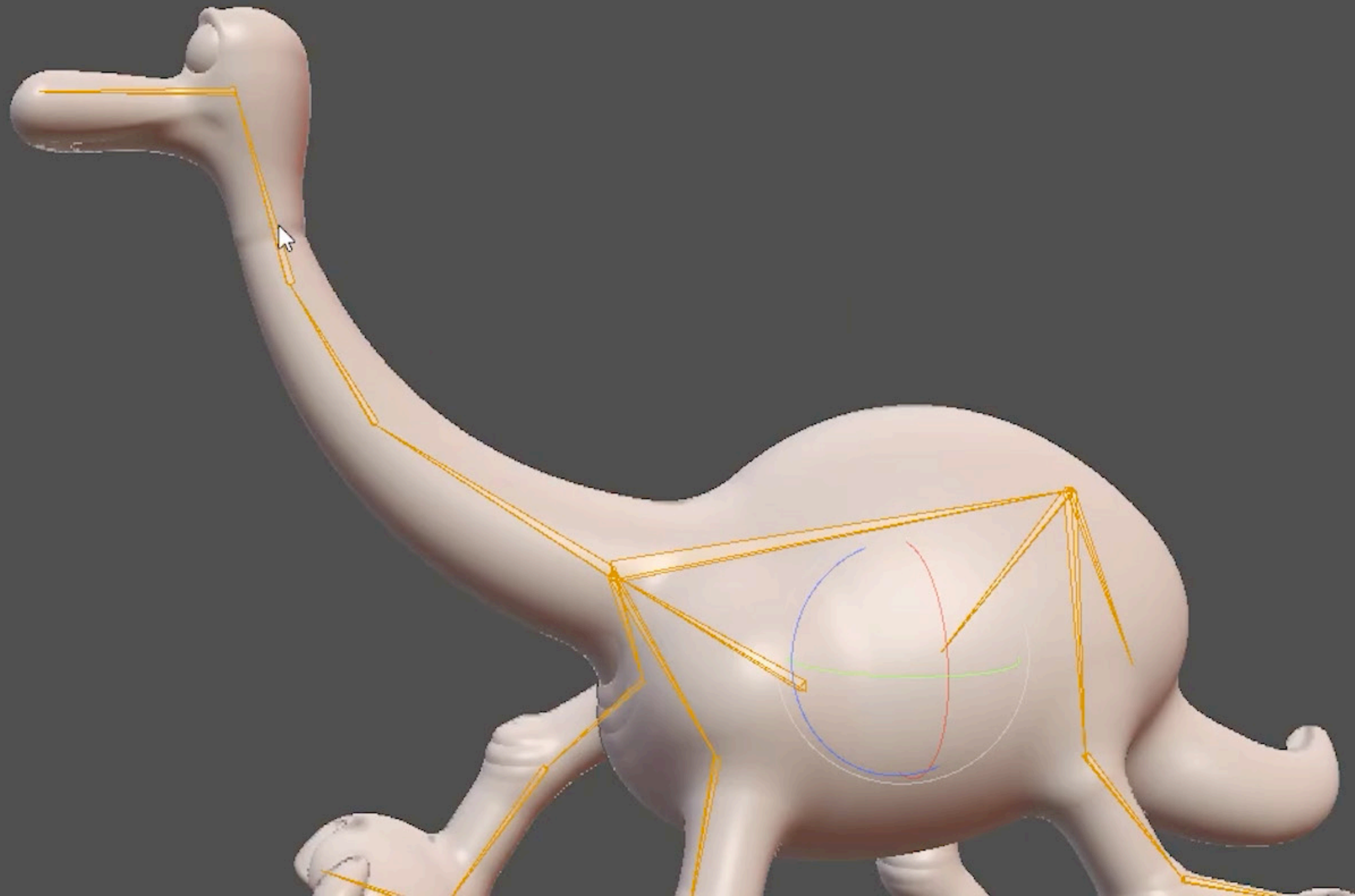
Results



Regularized Kelvinlets, Fernando de Goes and Doug L. James



Linear Blend Skinning

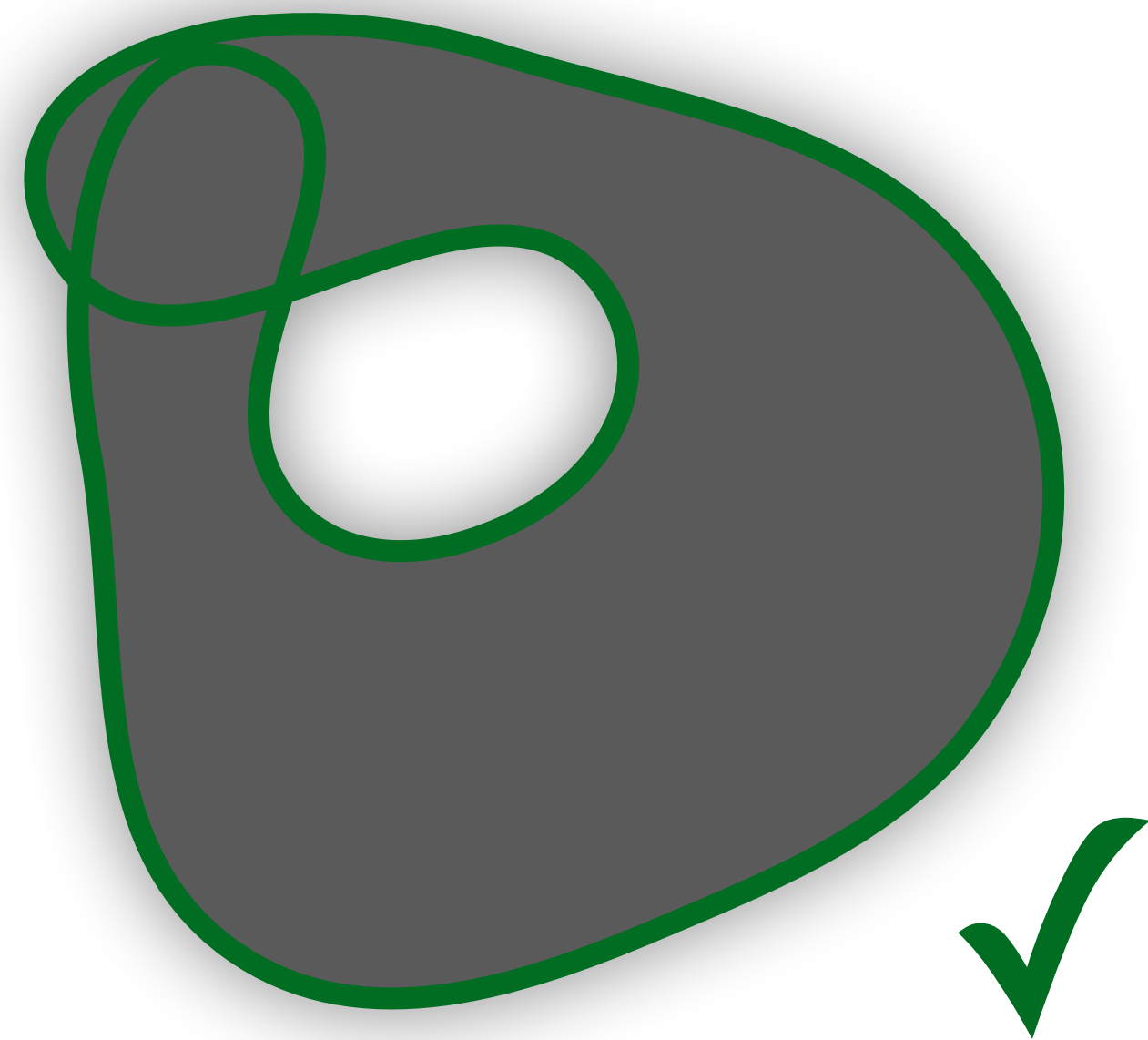


**The hull makes large
scale deformations possible.**



Limitations

- **Geometry:** Signed distance function
- **Deformation:** *Locally* foldover free



Future Work

- **Generalize** to all implicit surfaces

Future Work

- **Generalize** to all implicit surfaces
- Support **cheap layering** of deformations



Undeformed



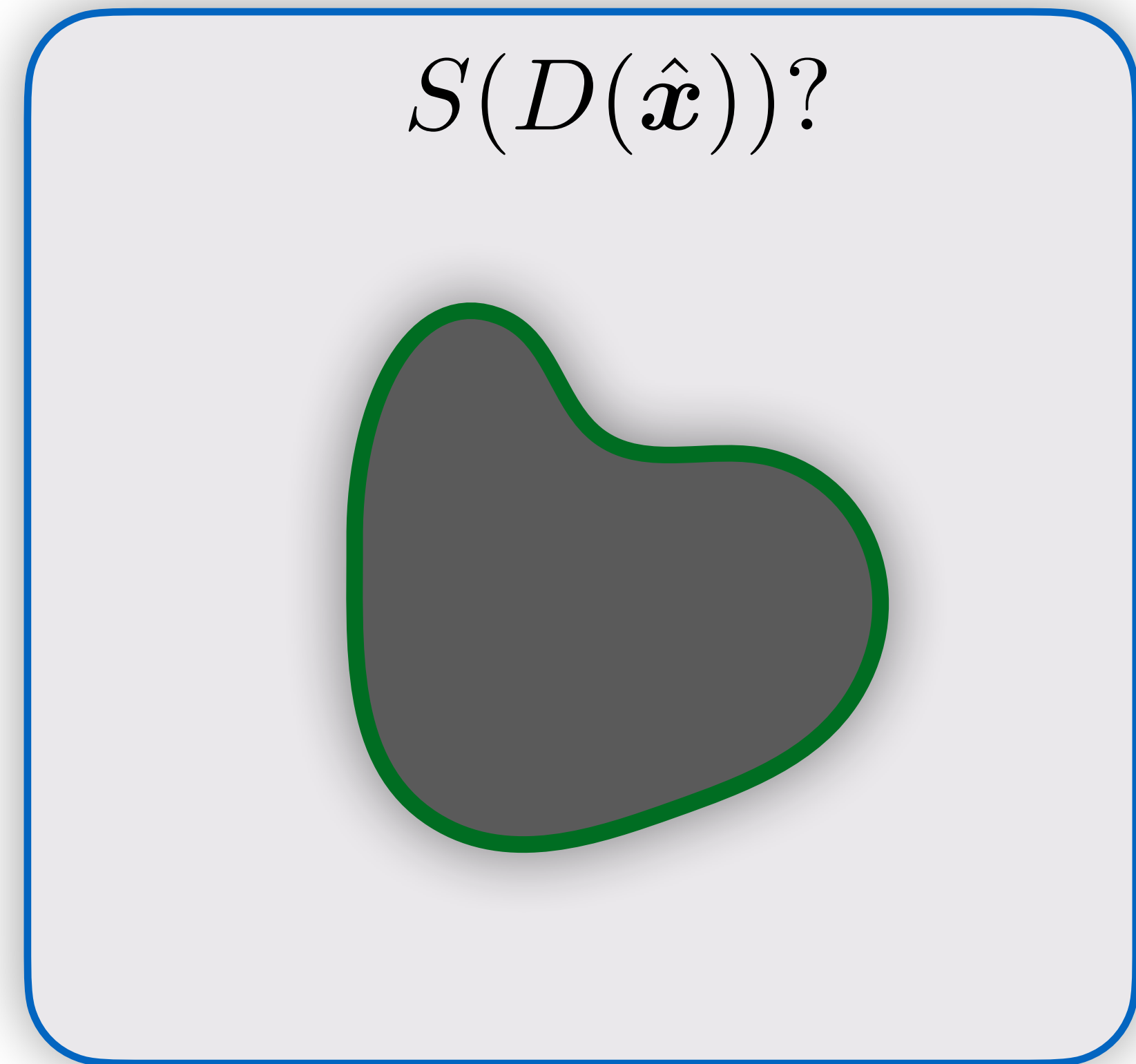
FFD



FFD + Kelvinlet

Future Work

- **Generalize** to all implicit surfaces
- Support **cheap layering** of deformations
- Evaluate **deformed SDF**



Thank you!

Please visit

dartgo.org/nlst

for the full paper and
supplemental material

Dario Seyb

dario.r.seyb.gr@dartmouth.edu

Derek Nowrouzezahrai

derek@cim.mcgill.ca

Scan Me!



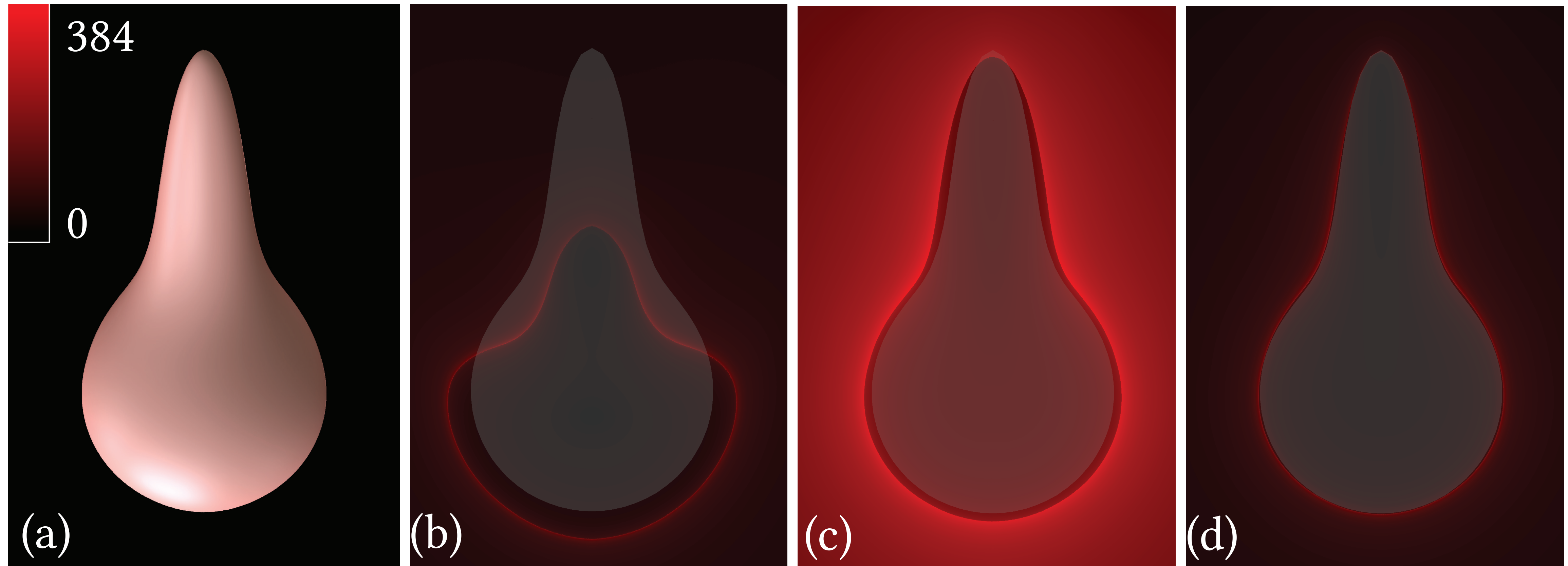
Alec Jacobson

jacobson@cs.toronto.edu

Wojciech Jarosz

wojciech.k.jarosz@dartmouth.edu

Integration – Comparison



(a)

(b)

(c)

(d)

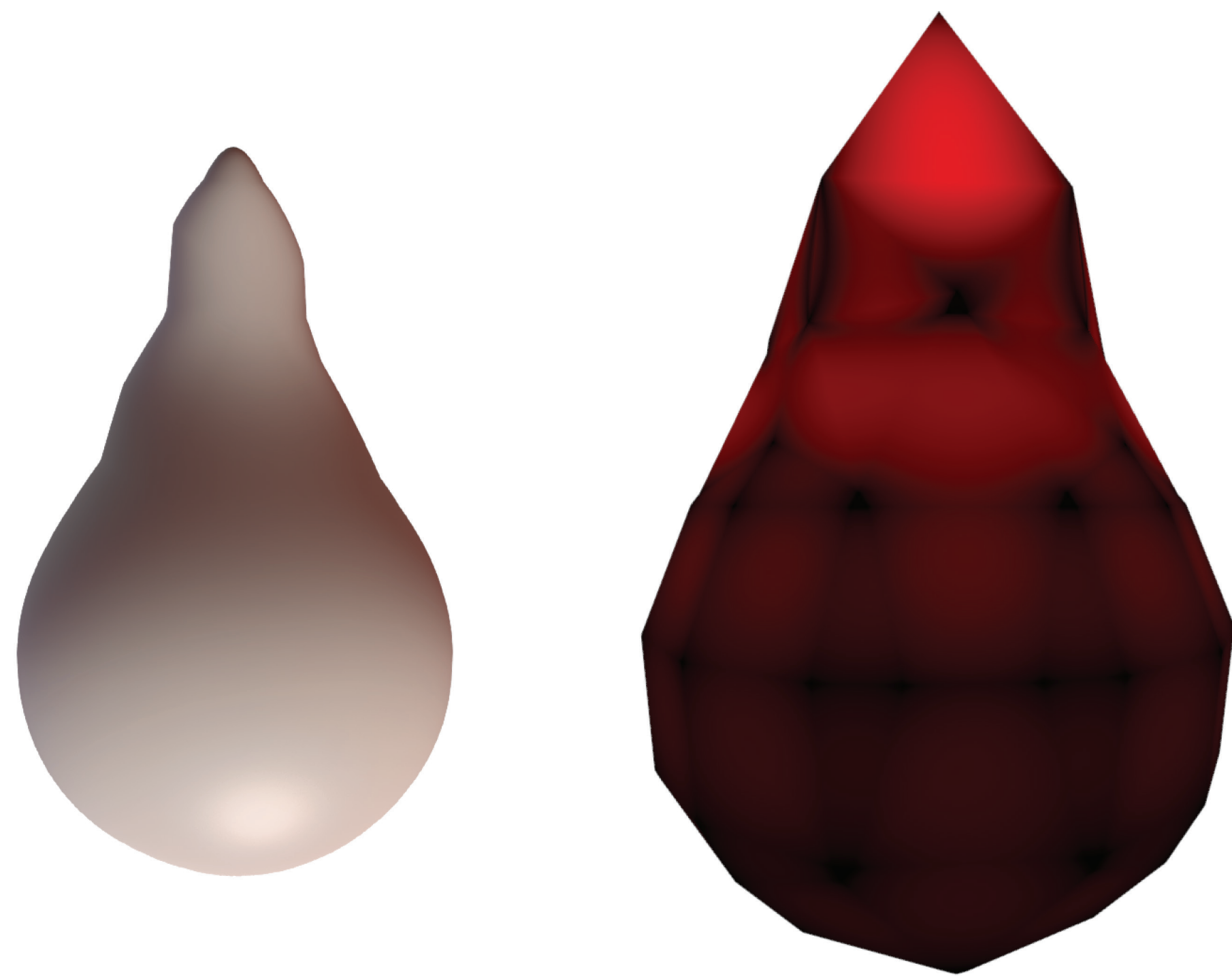
Reference

Sphere Trace

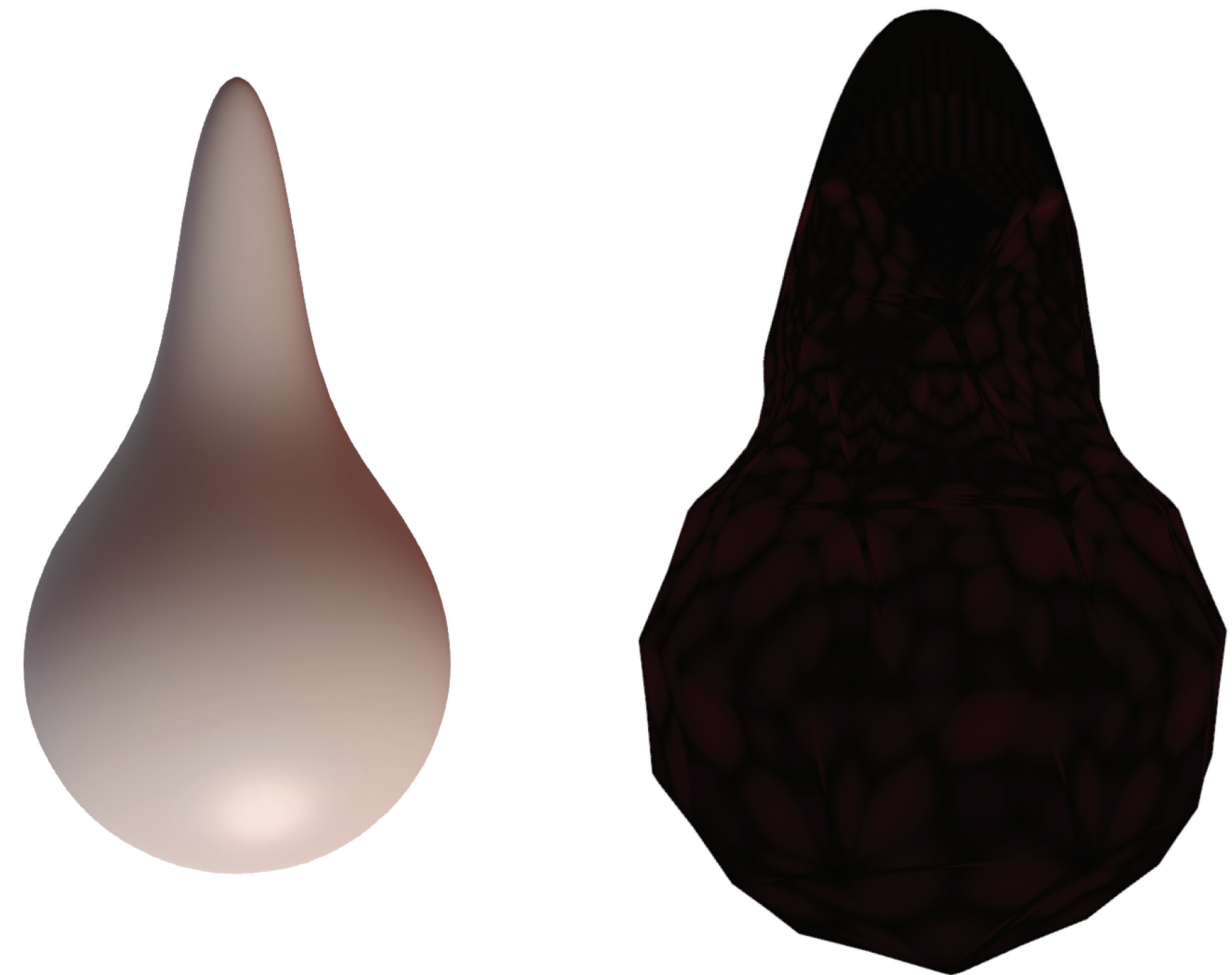
Reduced
Stepsize

Ours

Hull Subdivision – Results



No Subdivision



With Subdivision

Results - Error Control

